Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
C		2401871

Momentum Analysis: The movement of tennis

Summary

For the first problem, the first step is data preprocessing. The **BP neural network** is used to interpolate missing values. 8 data features of the tennis match in the dataset are extracted to **evaluate the players' performance**. The main component analysis and factor analysis models are constructed to reduce the dimensionality of the data. The **AHP-TOPSIS model** is developed to calculate the weights of three main factors respectively and then get the **performance scores of players at each moment**. The nonlinear regression equation is selected to evaluate the time point- performance score of players, and the parameter is solved by gradient descent method. The time point-performance scores of players can **contributes bettet tactics**.

As for the second question, since the definition of momentum is closely related to player performance, several influential dimensions such as scoring ability, error rate and batting quality are selected to evaluate player performance. An **evaluation system to quantify momentum** is constructed and the **integrated Stacking model** is used for training. An optimal model that can accurately predict the players' momentum in the competition is obtained. Kruskal-Wallis H is used to examine the correlation between momentum and victory in tennis matches. **The test results show that there is a significant correlation between player momentum and final victory**.

For question three, in order to know which indicators that can help judge when the flow of the match changes from favoring one player to the other, the **MNN and RBP neural network models** are mainly used.It is concluded that whether 'undefensible ball' factor can most affect the turning point, whether 'serve or not' has the least effect on the turning point. It is essential to take past matches momentum into account, so that **better suggestions can be provided for players**.

As for problem four, this study comprehensively considers Precision, Recall, F1 score and Accuracy to evaluate the model performance. The results show that **the model has excellent performance and can predict the score fluctuation reliably**.

In addition, the score fluctuation of tennis matches is also affected by many other factors, such as players' physical and technical level, mental state, strategies, environment factors. Since the above factors are difficult to quantify, **the overall ranking of players from the official website is used instead**. After adding the ranking of players, **the evalution results show that the model performance has improved**.

In the future study, physical fitness of different genders,mantal quality of players in different competitions, weather and field situation will be incorporated into the model to improve the predicting ability.

The model constructed in the study takes players' serving ability, error rate, key point scoring ability, running distance and other factors into account which **are also important in other ball games**, so **the model is applicable to predict other ball games such as badminton and volleyball**.

Finally,the **sensitivity analysis** results show that when the input matrix is changed, the predicted score change significantly.This indicates **the model has high sensitivity**.

Keywords: BP Neural Network; Factor Analysis; AHP-TOPSIS Model; MNN Neural Network; Sensitivity Analysis

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1 Introduction

1.1 Background

The Wimbledon Tennis tournament, with a rich history of about 150 years, stands as one of the most prestigious tennis events globally and the only grass-court Grand Slam. The 2023 Wimbledon Men's Singles final witnessed a clash between 20-year-old Spanish rising star Carlos Alcaraz and 36-year-old Serbian veteran Novak Djokovic.The remarkable skills and tenacity displayed by both players provided an exciting spectacle for tennis enthusiasts worldwide.

To assess the players' performance and analyze the tennis match dynamics through data, constructing a player performance evaluation model becomes crucial. This model aims to predict a player's scores and adaptability in various competitive situations.

1.2 Our Work

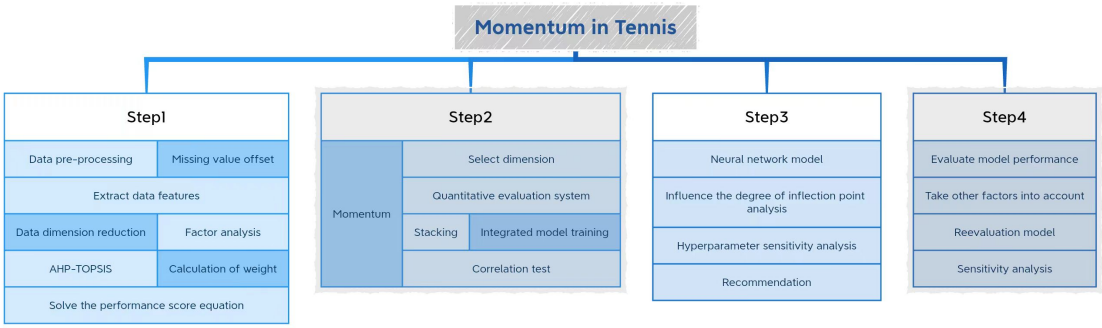


Figure 1: Our Work Function

In sports, it is difficult for teams or players to measure whether they have momentum. And even if there is momentum, it is not clear how it is generated or changed during the game.

Therefore, we should further investigate this issue. Firstly, data preprocessing was carried out using the BP neural network method for missing value interpolation; Afterwards, data features were extracted to evaluate the scoring performance of athletes; Establish a model to quantify the performance scores of athletes at each moment based on weights; Compare the performance scores of multiple athletes at different time points to facilitate a rational analysis of tactics.

Afterwards, select multiple influential dimensions to quantitatively evaluate the performance of the contestants; Establish a quantitative momentum evaluation system, use integrated models for training, and test the correlation between momentum and player victory in tennis matches.

In order to conduct more in-depth research, relevant neural network models were used to explore the correlation between various input features and the occurrence of inflection points, and sensitivity analysis of hyperparameters was conducted to more accurately predict the momentum changes in the competition.And conduct a comprehensive evaluation of the model, considering its applicability and conducting multiple evaluations.

2 Assumptions

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

***Assumption* 1:** Several important indicators, such as player stability (whether there are unforced errors, whether there are double faults), scoring ability (serve scores, return scores, net scores) and performance in key moments, comprehensively reflect player momentum, and there is a positive correlation between momentum and player score.

***Assumption* 2:** In addition to the players' momentum, the athletes' skill level, physical condition, mental state and environment factor all have an important impact on fluctuation of score. Good control and judgment of these factors will help coaches and players to change tactics and strategies in time, which will play a positive role in winning the game..

***Assumption* 3:** Considering the importance of the serve in tennis matches, the model should include the serve factor in its prediction and evaluation.

***Assumption* 4:** Momentum has an impact in a game, not just random. The coach's skepticism is based on a misunderstanding of the role of "momentum," when in fact momentum may be predictable and explicable.

***Assumption* 5:** There is available data (from at least one match) to build a predictive model.

***Assumption* 6:** Models can capture the flow of a game and identify which factors are most relevant to fluctuations in a match. Based on fluctuation model of past data, advice on strategies can be provided to better cope with different opponents.

3 Missing value interpolation based on BackPropagation neural network

Missing values for the rally_count, serve_width, serve_depth, return_depth, and speed_mph features were detected in the dataset check. To solve this problem, consider using an interpolation method based on backpropagation (BP) neural networks.

BP neural network is a kind of artificial neural network which is widely used. It uses backpropagation algorithm to adjust network parameters and realize the learning and approximation of complex patterns. BP neural networks show certain potential when dealing with missing value interpolation, especially in data situations with complex nonlinear relationships. With this interpolation method, missing values are expected to be dealt with efficiently, especially in the case of complex relationships between data. This approach is expected to improve data integrity, which in turn helps to analyze and model data sets more accurately.

First, the BP neural network model can be built by taking the non-missing values in the data set as the training set and the features corresponding to the missing values

as the target values. Through multiple iterative training, the neural network can learn the complex relationship between features and can predict the missing value relatively accurately.

The structure of BP neural network consists of input layer, hidden layer and output layer, which realizes information transmission and parameter adjustment through working signal and error signal propagation.

The format of the training set is $\{(x_1, y_1), (x_2, y_2), \ldots\}$, where x_i is the input sample, and y_i is the corresponding target value.

In the training process of a BP neural network, feedforward propagation is carried out first. For each training sample (x_i, y_i), the input sample x_i is fed into the input layer. By multiplying the weight matrix W between the input layer and the hidden layer and adding the bias $b^{(1)}$ of the hidden layer, this input, after the activation function σ, becomes the output $Z^{(1)}$ of the hidden layer. Then, the output of the hidden layer is further transmitted to the output layer. After multiplying the weight matrix $W^{(2)}$ between the hidden layer and the output layer and adding the bias $b^{(2)}$ of the output layer, the output of the neural network $Z^{(2)}$ is finally obtained.

$$Z^{(1)} = \sigma\left(W^{(1)} \cdot x_i + b^{(1)}\right)$$

$$Z^{(2)} = \sigma\left(W^{(2)} \cdot Z^{(1)} + b^{(2)}\right)$$

Next, calculate the loss function. The loss function E measures the gap between the output of the neural network and the actual target value. For each training sample, the loss function is as follows:

$$E_i = \frac{1}{2}\left|Z^{(2)} - y_i\right|^2$$

Then, the gradient of the loss function to the weight of the network is calculated by the backpropagation algorithm. By means of the chain rule, the gradient is gradually transferred from the output layer to the hidden layer and the input layer. Finally, the weights and biases of the network are updated, with gradient descent or other optimization algorithms:

$$\nabla_{W^{(2)}} E_i = \left(Z^{(2)} - y_i\right) \cdot \left(\sigma'\left(W^{(2)} \cdot Z^{(1)} + b^{(2)}\right)\right) \cdot Z^{(1)T}$$

$$\nabla_{W^{(1)}} E_i = \left(\left(W^{(2)T} \cdot \left(Z^{(2)} - y_i\right)\right) \cdot \sigma'\left(W^{(1)} \cdot X_i + b^{(1)}\right)\right) \cdot X_i^T$$

In the algorithms, $\nabla_{W^{(2)}} E_i$ is the gradient of the output layer weight, $\nabla_{W^{(1)}} E_i$ is the gradient of the hidden layer weight. σ' is the derivative of the activation function.

This process is constantly iterated. The neural network gradually improves its understanding of the relationship between input and target value by learning the samples in the training set and realizes the learning and approximation of complex patterns.

For the **speed_mph** index (Fig2), the actual batting speed distribution exhibits significant fluctuation and deviation from the mean, making the mean interpolation method unsuitable. The BP neural network interpolation, despite some inaccuracies, shows a better overall fit with the observed values, highlighting its potential in handling missing values. Similarly, for the **return_depth** indicator (Fig3), where the mean interpolation fails due to limited values, the BP neural network's fluctuation curve aligns well with the original data set, making it a suitable interpolation method. Regarding the **serve_depth** index (Fig4), the mean interpolation inaccurately predicts values, while

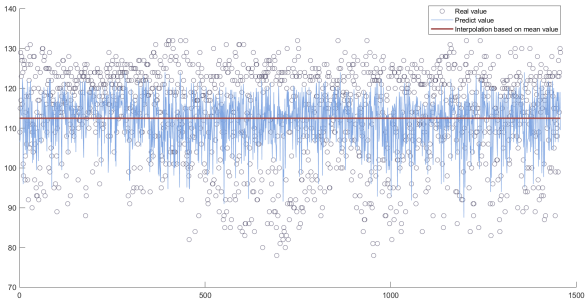


Figure 2: Comparison of **Speed_mph** Imputation Accuracy

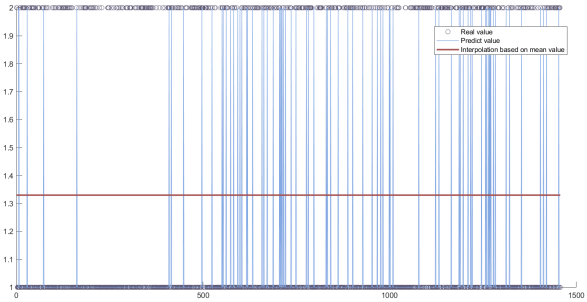


Figure 3: Comparison of **Return_depth** Imputation Accuracy

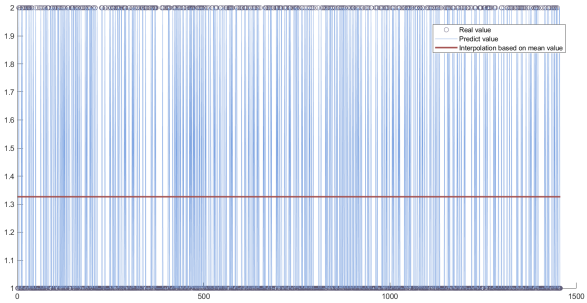


Figure 4: Comparison of **Serve_depth** Imputation Accuracy

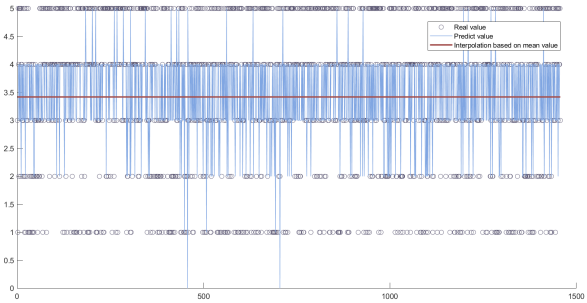


Figure 5: Comparison of **Serve_width** Imputation Accuracy

the BP neural network interpolation, although limited by data, maintains consistency with the data distribution characteristics. Lastly, for the **serve_width** indicator (Fig2), the randomness of the actual service direction and limited integer values make mean interpolation unsuitable, while the BP neural network's fluctuation curve better fits the observed values, especially in the median region.

4 How to Predict Athlete Performance Scores

4.1 Feature Extraction

In order to capture the change of athletes' performance over the course of the match, the features of data are extracted as follows:

Table 1: Data Features and Computation Methods

Feature	Calculation Method
Leading number of games	won by player minus won by opponent
Indefensible ball	shots that cannot be returned
Single serve fault rate	the ratio that serves to the total
Volley scores	points the player get by volley
distance_run	statistics, how active the player was
rally_count	statistics, how many returns the athlete
speed_mph	measured value, the speed of the player
Serve or not	statistics, the player's strategies

The leading number of games equals the number of games won by the player minus the number of games won by the opponent, which directly reflects the leading situation of the player in the competition.This feature provides an objective indicator that helps us understand the competitive advantage of the player.

Indefensible ball is the number of shots the player makes that cannot be effectively returned by an opponent during a match.This value objectively shows the performance of the player's skill level and the control of the game.

Single serve fault rate is the ratio of the number of second serves to the total number of serves.This ratio objectively reflects the stability of the player's serve quality, and a high single serve fault rate may suggest technical instability.

Volley scores are calculated by counting the number of points the player get by volley ,objectively presenting the offensive performance of the players in the game.

Distance_run and **rally_count** are two statistics that reflect how active the player was and how many returns the athlete made during the game respectively. These objective values help to assess the physical ability and endurance level of the player.

Speed_mph is a measured value,reflecting the speed of the player in the match.This objective value provides information about player's dynamic performance of the game.

Serve or not is a statistics,which objectively represents the player's strategies and help to learn more about the style of the player and tactics tendency.and further study the player's performance and advantages in the game.

4.2 Data dimensionality reduction model based on factor analysis

In order to make a more representative classification, it is necessary to reduce the dimension of features.This study contrives to use factor analysis to digitize the interdependencies within continuous attributes, and a large number of complex relationships are classified into a small number of comprehensive factors.By analyzing the variance contribution effect of each variable, the original system is represented as a new combination of factors to reproduce the internal connections. The model is built as follows:

$$
\begin{aligned}
x_1 &= a_{11}F_1 + a_{12}F_2 + \ldots + a_{1m}F_m + e_1 \\
x_2 &= a_{21}F_1 + a_{22}F_2 + \ldots + a_{2m}F_m + e_2 \\
&\vdots \\
x_p &= a_{p1}F_1 + a_{p2}F_2 + \ldots + a_{pm}F_m + e_p
\end{aligned}
$$

in which:

$X = (x_1, x_2, \ldots, x_p)^T$, where X is the observable random vector representing the continuous attributes as given in the problem. It satisfies $E(X) = 0$, and $Cov(X) = \Sigma$.

$F = (F_1, F_2, \ldots, F_m)^T$, where $m < p$, is the common factor vector. It is an unobservable random vector. It satisfies $E(F) = 0$, and $Cov(F) = I$, where I is the identity matrix. The vectors are independent.

$\epsilon = (e_1, e_2, \ldots, e_p)^T$, where ϵ is independent of F. It satisfies $E(\epsilon) = 0$, and $Cov(\epsilon)$ is a diagonal matrix with independent components.

The matrix form is indicated as:

$X = AF + \epsilon$, where $A = (a_{ij})_{(p\times m)}$ is called the factor load matrix.

4.3 The solution of factor analysis model

Perform KMO and Bartlett's on data in the game,the results are as follows:

Table 2: Data Features and Computation Methods

KMO		0.663
Bartlett's	Approximate Chi-square	94182.327764
	Degrees of Freedom	28
	Significance Level	0.000

In terms of test results, the KMO value reached 0.663, exceeding the threshold of 0.5, which indicates that there is a correlation between the eight features. At the same time, in the Bartlett's sphericity test, the Sig value is 0.000, which is less than the significance level of 0.005, so the null hypothesis is rejected, thus drawing the conclusion that there is a correlation between the variables. Therefore, there is enough reason to start factor analysis.

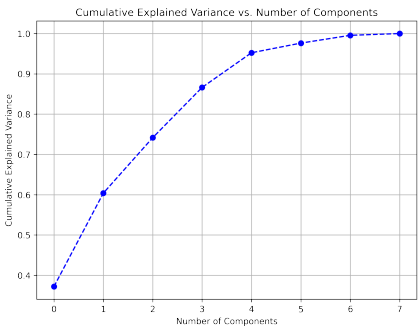


Figure 6: Cumulative Variance Explained Plot

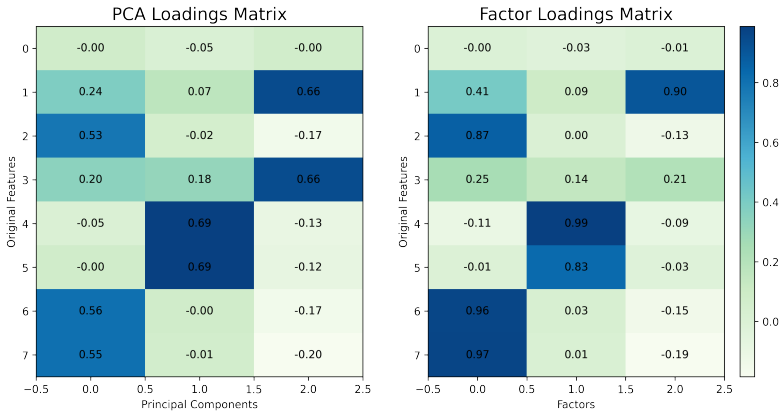


Figure 7: Component Factor Loading Matrix

By calculating the interpretability of the cumulative variance and drawing the graph, it is observed from Figure that the explanatory degree of the cumulative variance of the first three components reaches 74.990361%, and starting from the third component, a "curve of the cumulative variance gradually becomes flat, that is to say, the" inflection point "has been reached. This means that the first three components are the primary factors, while the later components are the secondary factors. Given this analysis, the three factors is kept.

By studying the component factor loading matrix, it can be observed that: among the first factor variables, single serve fault rate, service speed and whether serve or not show a large load, so these factors are defined as "service factors"; In the second factor variable, run distance and batting times show significant load, so these factors are defined as "process factors"; In the third factor variable, the volley score and the irresistible ball show a larger load, so this factor is defined as the "hit factors".

Therefore, the factor score model can be expressed as:

$$F_1 = -0.003x_1 + 0.412x_2 + 0.867x_3 + 0.250x_4 - 0.107x_5 - 0.010x_6 + 0.962x_7 + 0.969x_8$$

$$F_2 = -0.033x_1 + 0.092x_2 + 0.003x_3 + 0.142x_4 + 0.988x_5 + 0.830x_6 + 0.267x_7 + 0.131x_8$$

$$F_3 = -0.005x_1 + 0.904x_2 - 0.130x_3 + 0.210x_4 - 0.086x_5 - 0.026x_6 - 0.148x_7 - 0.186x_8$$

Table 3: Component Factor Loading Matrix

	PCA Components Matrix			Factor Components Matrix		
	PCA1	PCA2	PCA3	F1	F2	F3
Leading number of games	-0.003	-0.051	-0.003	-0.003	-0.033	-0.005
Indefensible ball	0.237	0.072	0.661	0.412	0.092	0.904
Single serve fault rate	0.533	-0.019	-0.173	0.867	0.003	-0.130
Volley scores	0.203	0.184	0.658	0.250	0.142	0.210
distance_run	-0.054	0.691	-0.132	-0.107	0.988	-0.086
rally_count	-0.001	0.693	-0.120	-0.010	0.830	-0.026
speed_mph	0.555	-0.000	-0.172	0.962	0.027	-0.148
Serve or not	0.554	-0.012	-0.198	0.969	0.013	-0.186

4.4 Player performance score evaluation model based on AHP and TOPSIS

After building the evaluation indicator, the AHP-TOPSIS model can be used to predict the score of each player at every moment.Although TOPSIS can sort according to how close a limited number of evaluation objects is to the idealized goal, it is challenging to determine the weight of each indicator when studying the "momentum" .In this respect, AHP method can overcome this difficulty relatively effectively.The influence of various factors on supply is considered comprehensively by AHP-TOPSIS model,and AHP can comprehensively give the weight of each factor.The frame diagram is as follows:

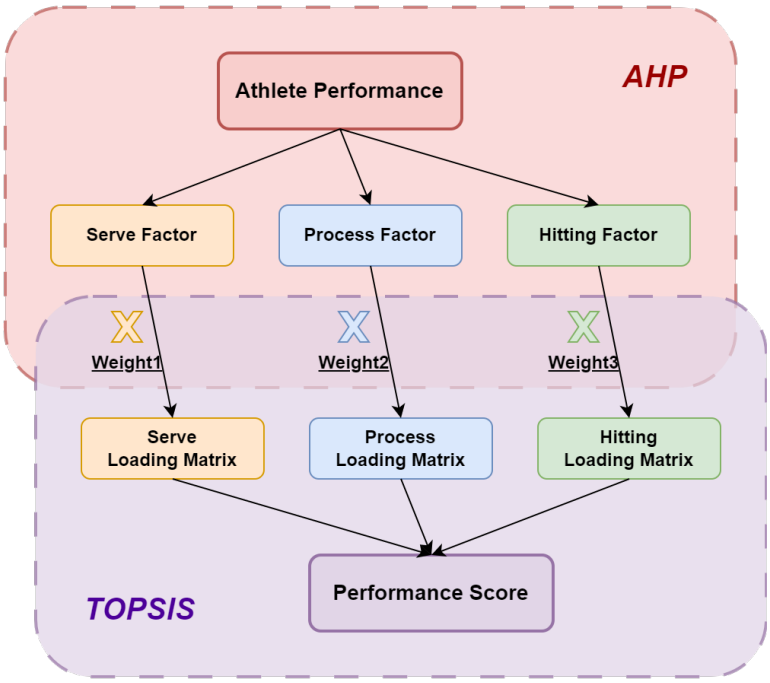


Figure 8: The Structure of AHP and TOPSIS

AHP is an analytical technique for multi-criteria decision making, which hierarchizes the problem. The following steps are involved: constructing the judgment matrix, calculating weights, and performing consistency tests. This study ultimately obtains the weights of the indicators.

Construct Judgment Matrix: Construct the judgment matrix A, where n is the number of criteria, and a_{ij} represents the importance of criterion i to criterion j.

Calculate Weight Vector: The maximum eigenvalue $\lambda_{\max}$ and corresponding eigenvector v are obtained through the eigenvalue decomposition of the judgment matrix. The eigenvalue can be expressed as $v = [v_1, v_2, \ldots, v_n]$.

Consistency Test: Calculate the Consistency Index (CI) and Consistency Ratio (CR) using the formula $CR = \frac{CI}{RI}$, where CI is the Consistency Index, and RI is the Random Index. This test ensures the consistency of the judgment matrix.

Normalize Weight: Normalize the weight vector, and the normalized vector can be represented as $W_{\text{normalized}} = \frac{W}{\sum_{i=1}^{n} W_i}$.

TOPSIS: TOPSIS is a sequencing and selection technique for multi-criteria decision making. The method involves normalizing and weighting the decision matrix, calculating the similarity of each decision object to the ideal and negative ideal solutions, and finally sorting the objects.

Standardized Decision Matrix: For a decision matrix X with m decision objects and n evaluation indicators, the standardized matrix can be calculated using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

where r_{ij} is the normalized element value, and x_{ij} are elements in the decision matrix.

Weighted Decision Matrix: The normalized matrix is weighted using the weight vectors W:

$$v_{ij} = r_{ij} \times w_j$$

where v_{ij} is the weighted value of the element, and w_j is the weight of the j-th indicator.

Ideal and Negative Ideal Solutions: Define vectors for which the ideal and negative ideal solutions are the maximum and minimum values, respectively:

$$A = (a_1, a_2, \ldots, a_n)$$

$$B = (b_1, b_2, \ldots, b_n)$$

where a_j is the maximum value of the indicator, and b_j is the minimum value of the indicator.

Similarity Measure: Calculate the Euclidean distance of each decision object to the ideal solution and the negative ideal solution:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - a_j)^2} \ldots D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - b_j)^2}$$

Similarity Measure Ratio: Calculate the similarity measure ratio for each decision object:

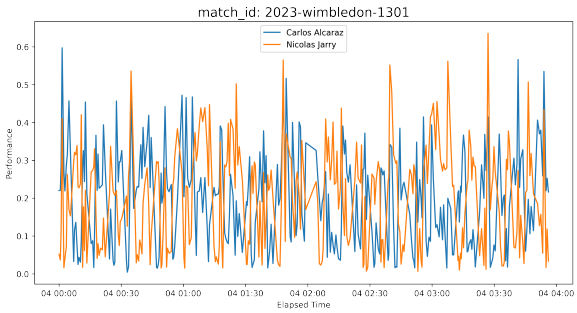
$$S_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

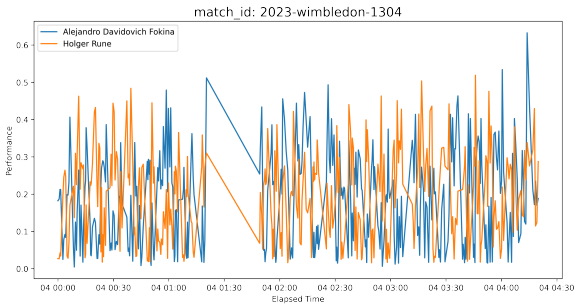
Athlete performance calculation results: The consistency ratio CR=0.0176 was calculated. CR<0.10 so the consistency of this judgment matrix is acceptable. AHP was calculated to get the weight of each criterion: **"Serve Factor"**: 0.3196, **"Process Factor"**: 0.5584, **"Hitting Factor"**: 0.1220.

4.5 Construction of point of time-athlete performance score models based on nonlinear regression

Through the above feature filtering steps, a regression analysis model is successfully constructed to reveal the relationship between time points and players' performance scores. In this model, the criterion factor weights and the weighted weights of the factor components are considered to carefully evaluate the contribution of each factor to the overall factor.

Specifically, regression analysis is constructed with athletes' performance score(y) as the dependent variable and time point(x) as the independent variable. three games for detailed analysis are selected in this study, including 2023-wimbledon-1301, 2023-wimbledon-1304 and 2023-wimbledon-1306. Here are the performance scores for each competition:





At present, most of the regression models of nonlinear regression models rely on empirical method or experimental method to select the regression model, but the empirical method will cause large error problems. Although the experimental method takes a long time, the correct model can be better selected according to the experimental results.

Therefore, in this study, the regression models suitable for this relationship are determined by experimental method as follows:

$$
\begin{aligned}
y &= p_1 \sin(p_2 x) + p_3 x^2 + p_4 \log(x) + p_5 e^{p_6 x} \\
y &= p_1 e^{p_2 x} + p_3 \tan(p_4 x) + p_5 x^{p_6} + p_7 \sin(p_8 x) \\
y &= \frac{p_1 x^2 + p_2 e^{p_3 x} + p_4 \sin(p_5 x)}{1 + p_6 \log(1 + p_7 x)} \\
y &= p_1 \sqrt{x} + p_2 \cos(p_3 x) + p_4 x^{p_5} + p_6 e^{p_7 x} \\
y &= \frac{p_1 + p_3 \sqrt{x} + p_5 x + p_7 x^{1.5} + p_9 x^2}{1 + p_2 \sqrt{x} + p_4 x + p_6 x^{1.5} + p_8 x^2 + p_{10} x^{2.5}}
\end{aligned}
$$

4.6 Parameter solution based on gradient descent method

In nonlinear regression problems, fit a model in which p are vectors containing parameters. The goal of this study is to find the optimal parameter values by minimizing the loss function so that the error between the predicted values of the model and the actual observed values is minimized. Gradient descent is an iterative optimization algorithm whose core idea is to minimize the loss function by constantly adjusting the parameters.

Algorithm 1 Gradient Descent

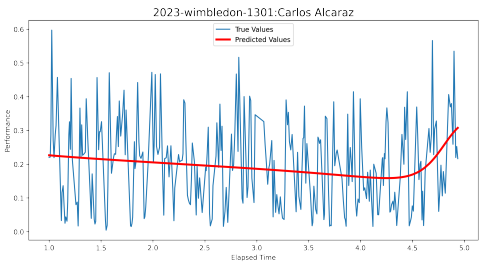
```
p_old ← random initialization
α ← learning rate
max_iterations ← maximum iterations
epsilon ← convergence criterion
iteration ← 0
while iteration < max_iterations do
    L ← compute loss(p_old)
    ∇L ← compute gradient(p_old)
    p_new ← p_old − α∇L
    if abs(L − compute loss(p_new)) < epsilon then
        break
    end if
    iteration ← iteration + 1
    p_old ← p_new
end while
Output optimal parameters: print(OptimalParameters:, p_new)
Output optimal objective value: print(OptimalObjectiveValue:, L)
```

The gradient descent method is used to find the optimal parameters, and these optimal parameters are employed for prediction. Evaluation indices such as Root Mean Squared Error (RMSE), Sum of Squared Errors (SSE), and Coefficient of Determination (DC) are selected:

Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}{n}}$$

RMSE represents the square root of the average squared differences between the actual values (y_i) and the predicted values ($\widehat{y_i}$). It measures the model's prediction accuracy, with lower RMSE values indicating better performance.



Params: [0.93598316, 6.94448383, -0.37172825, 1.72609589, 3.24535869, -2.4106305, -3.0992706, -1.64528872, 0.73489732, 0.77082411]
RMSE: 0.12337684871500512
SSE: 4.566554039653578
DC: 0.04630273293967013

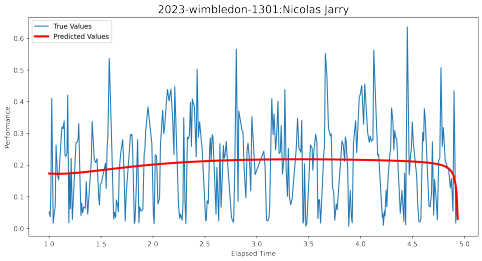
Sum of Squared Errors (SSE):

$$SSE = \sum_{i=1}^{n}(y_i - \widehat{y_i})^2$$

SSE is the sum of the squared differences between the actual values (y_i) and the predicted values ($\widehat{y_i}$). It quantifies the total error or variability in the predictions, and a smaller SSE implies a better fit of the model to the data.

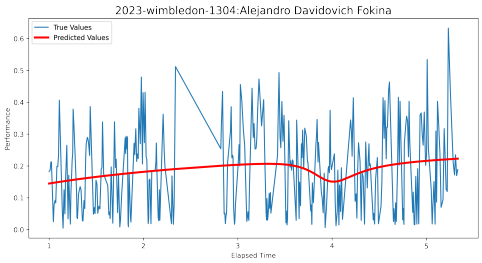
Coefficient of Determination (DC or R^2):

$$R^2 = 1 - \frac{SSE}{SST}$$

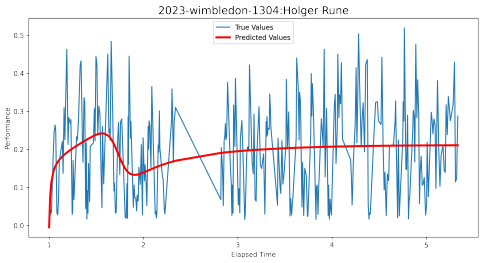


Params: [10.28055739, 8.94698495, -16.30591751, -2.98751346, 0.33575718, -6.34005924, 12.0637192, 17.09450391, -4.43231141, -6.51426063]
RMSE: 0.13144414504002405
SSE: 5.183268979590865
DC: 0.02145757246513047

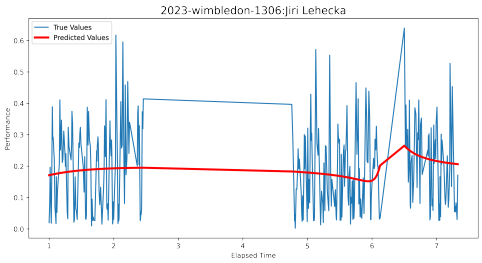
where $SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$. R^2 measures the proportion of the variance in the dependent variable (y) that is predictable from the independent variable(s) (x). Ranging from 0 to 1, higher values indicate a better fit. $R^2 = 1$ means the model perfectly predicts the dependent variable.



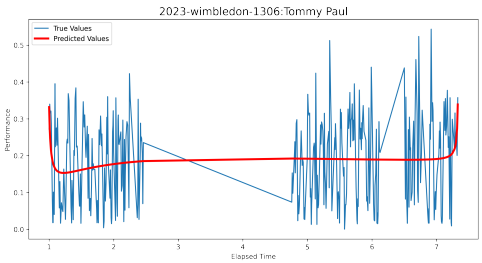
Params: [3.6791423, 64.15783426, -13.5273841, -7.03558569, 34.98790683, -40.94692422, -26.70836437, 13.73440421, 6.07291079, 0.22356109]
RMSE: 0.12610837881786316
SSE: 5.359419921119482
DC: 0.02841780549347661



Params: [352.67724667, -1082.82278201, -1473.79162816, 2359.23176415, 2203.8409484, -1347.5233311, -1413.40437132, -139.25872242, 330.6747677, 210.03275698]
RMSE: 0.11685818171177437
SSE: 4.602016271314961
DC: 0.05051313062020302



Params: [-0.27536186, 3.31635778, 0.30053074, 1.30487019, 2.10852759, -0.73362394, -1.77461626, -1.02930706, 0.36038663, 0.35056482]
RMSE: 0.13109121474764313
SSE: 5.7053889858922116
DC: 0.015594510597193856



Params: [19.70225054, -45.14051547, -29.0377477, 1.48147762, -21.30036678, 33.57198467, 42.79574647, 22.20514894, -11.79224036, -12.01008298]
RMSE: 0.11807275269186966
SSE: 4.628470076174158
DC: 0.025208297805992652

4.7 Solution analysis

By comparing the performance of players in different games, when a player has shown exceptional skills or experienced fluctuations during a specific period can be

accurately determined. This is essential for developing the targeted strategies precisely, providing coaches and athletes with flexibility to adjust their strategies based on the strengths and weaknesses of their opponents at a particular stage of the game.

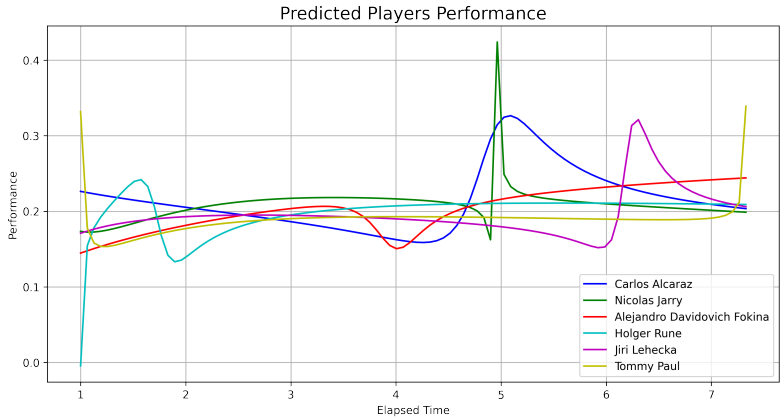


Figure 9: Athlete Performance Over Time

This comparative analysis can also provide support for the development of dynamic tactics to adapt to the changing performance level of athletes. A deeper understanding of individual athletes' performance strengths at different points in time allows teams to make smarter, more adaptable strategic decisions. For example, during periods when the opponent is weak, the team can take full advantage of the moment and adopt a more aggressive strategy; In the time period when the opponent is performing best, more conservative tactics can be chosen.

Overall, the strength of this comparative analysis is that it provides practical insight beyond the general understanding of athlete performance. This refined understanding gives the team the ability to develop more flexible, specific strategies to improve competitive advantage by taking full advantage of changes in the athlete's proficiency at different points in time. This kind of strategic analysis is an important step towards success.

5 Momentum-score correlation analysis based on the Kruskal-Wallis H-test

5.1 Modeling the momentum Impacts

In tennis competition, momentum reflects the trend of how a match is played and changes over time. Players often gain strong momentum by winning key points, their own consecutive points, the other team's consecutive mistakes, or performing stunts, so that they have great initiative and control in the game at the moment.Momentum will cause great fluctuations in the psychology of players, players with strong momentum will increase their confidence, concentrate on the spirit, quickly enter the competitive state, reach the excitement point, can play a higher level, and produce psychological pressure on the opponent, resulting in the reversal of the situation or further consolidate the leading position, thus affecting the trend of the game, and play a decisive role in the final result.

5.2 Quantization of momentum

Momentum is an abstract concept that is difficult to measure directly. Since the concept of momentum is closely related to player performance, this paper selects a number of influential dimensions such as scoring ability, error rate, batting quality, service success rate and service receiving success rate to evaluate players' performance quantitatively. An evaluation system for quantifying momentum is established.

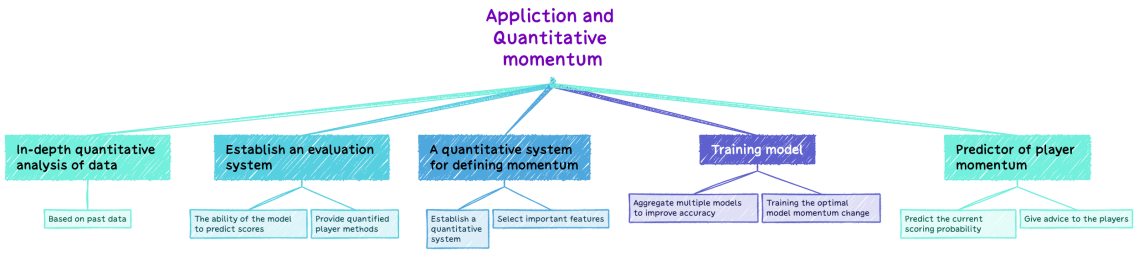


Figure 10: Application and Quantitative Momentum

This paper uses an efficient supervised learning technology, the Stacking integrated model, to train. In the first stage, the complete original training set is used to train multiple base models (level-0); in the second stage, the predictions of the base model on the original training set are combined as a new training set to train the meta-classifier (level-1 model), and the final prediction results are generated by the meta-model. The Stacking method combines multiple different base models for higher prediction accuracy. Different from the traditional classification problem, this paper needs to explore the probability that momentum affects a player's score, rather than just judging whether the score is scored or not. Here, probability can be understood as a quantitative indicator of momentum. This paper uses the Stacking method to train an optimal model that can accurately predict the momentum change of players in a match.

5.3 Correlation analysis

Kruskal-Wallis H non-parametric test has no requirements on the distribution form of data.It is applicable to testing whether there are significant differences in the medians of several independent samples when it is not normal distribution . It is an alternative to ANOVA of multiple independent samples.Kruskal-Wallis H mixed the data of multiple sets of samples from different populations and sorted them in ascending order to calculate the rank (H) of each observed value, and conducted one-way ANOVA instead of the original observed value to test whether there were significant differences in the mean values of each rank. In this paper, Kruskal-Wallis H test is used to explore the relationship between player performance and score, so as to determine the influence of momentum on player victory in tennis matches.

Null hypothesis:

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

Alternative hypothesis:

$$H_1 : \text{at least one pair: } \mu_i \neq \mu_j$$

Constructed statistic:

If n is large enough,

Levene test

Null hypothesis H_0: The variance of each group is equal, and it conforms to the homogeneity of variance.The Levene test shows significance < 0.05, and the null hypothesis H_0 is rejected. These sets of data do not have homogeneity of variance.

To sum up, the disordered variables of player scores do not follow a normal distribution nor satisfy the homogeneity of variance. Therefore, the Kruskal-Wallis H test is considered to judge the correlation.

Table 4: Levene Test

	Levin statistics	**Freedom 1**	**Freedom 2**	**Significance**
Based on mean	20.076	1	14566	0
Based on median	59.584	1	14566	0
Median with adjusted	59.584	1	14296.949	0
Average after deletion	35.982	1	14566	0

The normality test

The results of the normality test are shown below:All p-values are less than 0.05, and the null hypothesis H_0 is rejected, indicating that the scores do not follow a normal distribution.

Kruskal-Wallis H test results

The Kruskal-Wallis H test shows that the significance of an unordered variable is less than 0.001. Thus, the null hypothesis H_0 is rejected. The momentum of different players is related to the score.

Table 5: Kolmogorov-Smirnova Test

	Score or Not	**Kolmogorov-Smirnova** Statistics	Freedom	Signify
Score	1	0.258	7284	0
	2	0.148	7284	0

Table 6: Kruskal-Wallis H-test

Total N	14568
Test Statistic	2999.021
Freedom	1
Asymptotic	0

6 How to use momentum against opponents in the new game Model based on MNN and RBP

Problem 3 aims to solve how to use momentum against opponents in the new game. According to the performance and effectiveness of the model, the advanced wavelet neural network (WNN) model and RBP neural network model are used in the experimental design and result analysis, and the corresponding strategy suggestions are put forward.The following content is based on the implementation and training process of two types of neural networks, providing the steps, experimental design, suggestions, and strategies for model construction and training.

6.1 WNN and Morlet Wavelet Models

Firstly, by analyzing and studying models suitable for capturing and understanding momentum changes in tennis matches, Wavelet Neural Network and Morlet Wavelet were selected to construct the model.

Wavelet Neural Network

A small passive neural network is a type of neural network based on the topology of a BP neural network, which uses small passive basis functions as the transfer function of hidden layer nodes to propagate signals forward in the case of error back-propagation.

This neural network can be divided into three layers: input layer, hidden layer, and output layer. The schematic diagram between layers is shown below.

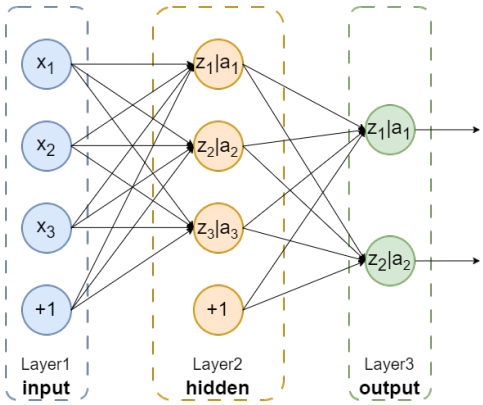


Figure 11: Neural Network Structure

When data is transmitted from the input layer to the hidden layer, the calculation formula for the input data of the m-th node in the hidden layer is as follows:

$$S_m = \sum_{k=1}^{n} \omega_{km} x_k, \quad m = 1, 2, \ldots, i$$

Where x_k is the k-th sample data in the input data, ω_{km} is the connection weight of each node in the hidden layer.

Morlet Wavelet

The introduction of activation functions in neural networks is because the vast majority of data is nonlinear, while in general neural networks, calculations are linear. Morlet Wavelet can effectively simulate nonlinear and complex patterns in time series data. Introducing activation functions is the process of introducing nonlinearity into neural networks to enhance their learning ability. This can play an important role in analyzing the competitive momentum.

The specific transformation formula for the above calculation results at the hidden layer nodes is as follows:

$$\varphi(x) = \cos(1.75x) * e^{-x^2/2}$$

$$h_m = \varphi\left(\frac{S_m - b_m}{a_m}\right)$$

Where $\varphi(x)$ is the wavelet basis function, b_m is the smoothing factor of the basis function, a_m is the scaling factor of the basis function, and h_m is the output data of the m-th node of the hidden layer.

The output data of the m-th node in the final hidden layer enters the output layer, and after calculation, it is output from the t-th node in the output layer. The calculation formula for this node is as follows:

$$y_t = \phi\left(\sum_{m=1}^{i} \omega_{mt} h_m\right), \quad t = 1, 2, \ldots, n$$

Where ω_{mt} is the connection weight of the output layer, and ϕ is the activation function.

In this question, a set of one-dimensional data (`pred`) is used to simulate the momentum changes during the competition. The target data is obtained by performing a sine transform on the `pred` and adding noise to simulate the momentum fluctuations and uncertainties in actual competitions.

The Fourier sine transform formula is as follows:

$$F[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = -2i \int_{0}^{\infty} f(t)$$

$$F_s[f(t)] = 2 \int_{0}^{\infty} f(t) \sin(\omega t) dt$$

Wavelet Neural Network Training Steps:

1. **Network Initialization.**
2. **Divide the samples:** Divide the samples into training samples and testing samples. Training samples are used to train the network, and testing samples are used to test the prediction accuracy of the network.
3. **Network Input and Error Calculation:** Input the training samples into the network, calculate the network prediction output, and compute the error between the network output and the expected output.
4. **Error Correction:** Based on error correction weights and wavelet function parameters, adjust the network prediction values to be close to the expected values.
5. **Iteration:** If the algorithm has not ended, return to step 3.

Model Training with Hyperparameter Tuning

Different hidden layer sizes (`hidden size`) and learning rates (`learning_rate`) are set and trained multiple times to produce wavelet neural network models. In this process, mean square error (MSE) is used as a loss function to optimize the predictive ability of the model.

$$\text{MSE} = \frac{1}{N} \sum_{t=1}^{N} (observed_t - predict_t)^2$$

The smaller the value of MSE, the better the accuracy of the prediction model in describing experimental data. N represents the number of measurements, $observed_t$ represents the true value of the parameter, and $predicted_t$ represents the predicted value of the parameter. As shown in above Figure, the lower the learning rate, the greater the mean square error. When the learning rate is 0.0001, there is an overall trend of decreasing mean square error before the hidden layer size reaches a plateau around 10. When the learning rates are 0.01 and 0.001, the error trends of the two are similar.

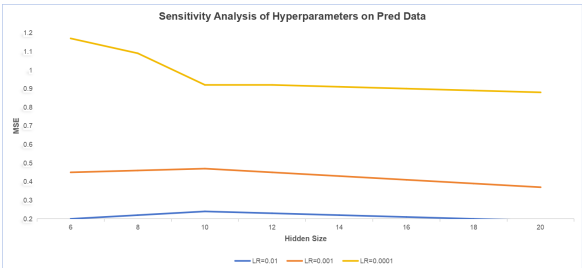


Figure 12: Sensitivity Analysis of Hyperparameters

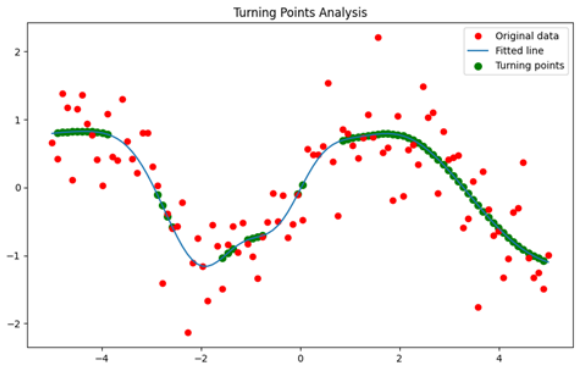


Figure 13: Turning Points Analysis

6.2 Hyperparameter sensitivity analysis

During the model training process, sensitivity analysis of hyperparameters was used to explore the effects of hidden layer size and learning rate on model performance. This helps to find the optimal model configuration, thereby more accurately predicting and understanding the momentum changes in the competition.

In deep learning techniques, adjusting hyperparameters is crucial. By adjusting hyperparameters to observe monitoring indicators (such as accuracy) during the training process, the current training status of the model can be determined, and then the model can be trained more scientifically by adjusting hyperparameters. As shown in

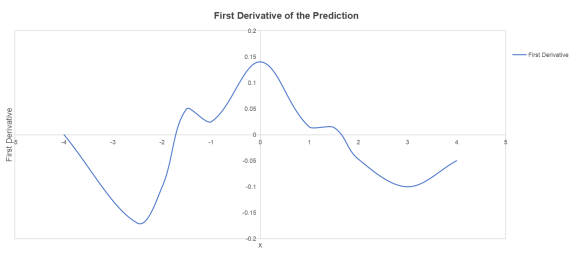


Figure 14: First Derivative of the Prediction

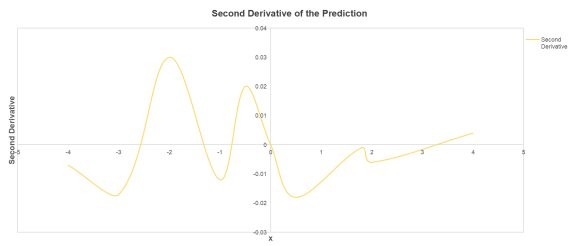


Figure 15: Second Derivative of the Prediction

the above figures, the trend of the original data and the predicted data is almost the same, indicating good prediction performance. That is, the accuracy of monitoring indicators during the training process is relatively high.

6.3 RBP (Reverse BP neural Network) model

Similar to the WNN neural network, this model adopts the RBP neural network model, which takes process factor, number of wins, service factor, number of hits, whether to serve, hitting factor, and whether to hit an untouchable ball as input features, and whether it is a turning point as output features.

- **Service factor:** In tennis, if the pitcher throws a good ball but the batter cannot hit it, the pitcher earns one point, and vice versa. Therefore, when serving, the pitcher throws a good or bad ball, which can affect the outcome of the game. This factor is called the "serving factor".
- **Process factor:** In addition to serving, the player's movement, defense, and attack also affect the outcome of the game. Therefore, these processes are referred

to as "process factors".

- **Hitting factor:** Players need to accurately determine the opponent's return path and respond promptly. If the opponent can successfully hit a good ball, they can earn one point. This factor is called the "hitting factor".

In order to explore the relationship between the input features mentioned above and the factor of whether they are inflection points, the importance of features was deconstructed.

When Ω remains constant, the larger the absolute value of c, the greater the impact of the input layer on the hidden layer. Meanwhile, the larger the absolute value of γ, the greater the impact of the hidden layer on the output layer.

$$E_k = C_k * \Omega$$

$$\Omega = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_n \end{bmatrix}^T$$

E_n is the impact of input feature k on whether it is a turning point. The formula for the contribution rate of input feature k to whether it is a turning point is as follows:

$$P_{c_k} = \frac{E_k}{\sum_{i=1}^{n} E_i} \times 100\%$$

Based on the RBP neural network model established above and the analysis of collected and processed data, the contribution rate of input feature k to whether it is a turning point is obtained as follows:

Table 7: Weights of Factor Criteria

Untouchable ball	0.235209187	**Process factor**	0.118660105
Service factor	0.198034526	**Number of wins**	0.109079016
Number of hits	0.153448093	**Whether to serve**	0.102732224
Hitting factor	0.123783743		

From the above table, it can be seen that hitting an unstoppable ball has the greatest impact on whether there is a turning point, followed by the serving factor, and whether serving has the least impact on whether there is a turning point.

6.4 Recommendation

Based on the analysis results of the neural network model, the following strategic suggestions are proposed to help athletes better utilize momentum in new competitions:

- **Correctly dealing with momentum fluctuations:** Players need to prepare strategies to cope with momentum fluctuations, make psychological adjustments and technical preparations, remain calm and focused, and not be influenced by scores and emotions.
- **Training and improvement:** By watching competition videos and participating in training, athletes discover their shortcomings and take corresponding measures to improve. Simulate the fluctuation of competition momentum during training to help athletes adapt to various situations that may arise during the competition, continuously learn and improve their skills.

- **Observing and utilizing competition data:** Participants need to collect and organize relevant competition data, including their own performance data and opponent's performance data, such as scores. By observing these data, players can discover their strengths and weaknesses in the competition, as well as the weaknesses of their opponents.
- **Develop targeted strategies:** Based on the analysis results of wavelet neural network models, if the model analysis results show that a player has high scoring ability in a certain aspect, then in the competition, they should make more use of opportunities and leverage their advantages.

7 Analysis of Model Performance, Sensitivity, and Applicability

7.1 Model Evaluation and Extension

The following indicators can test the performance of the model:

Precision:

$$Precision = \frac{TruePositives}{TruePositives + FalsePositives}$$

Recall:

$$Recall = \frac{TruePositives}{TruePositives + FalseNegatives}$$

F1 score:

$$F1score = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

Accuracy:

$$Accuracy = \frac{TruePositives + TrueNegatives}{TruePositives + TrueNegatives + FalsePositives + FalseNegatives}$$

The fluctuation prediction model including players'ranking: the score fluctuation of tennis matches is also affected by many other factors, such as players' physical and technical level, mental state, strategies, environment factors. Since the above factors are difficult to quantify, the overall ranking of 32 players from the official website(International Tennis Federation (ATP)) is used instead

Table 8: Levene Test - Table 1

Points	Value
Precision	0.897865853658537
Recall	0.920004462791476
F1 Score	0.908800352675373
Accuracy	0.886394838001098

Table 9: Levene Test - Table 2

Points	Value
Precision	0.990840517241379
Recall	0.848638670973696
F1 Score	0.914243102162565
Accuracy	0.881606039807824

Compared with the above two Table(Table9 and Table10), the performance of Precision and F1 score is improved.So the next step will consider more other factors being incorporated into the model to predict the score fluctuation of the game.

7.2 Sensitivity analysis

For the AHP-TOPSIS model constructed in this study, when the input judgment matrix is changed, the result score may change. In order to analyze the sensitivity of the score to the input judgment matrix, the following changes are made to the input:

As for the sensitivity of the AHP-TOPSIS model constructed in this study,it is assumed that changing the input matrix will affect the change of the predicted score. This assumption can be used to analyze the sensitivity of the input judgment matrix. So the study makes the following changes to the model input:

Table 10: Input Matrix Compare

	Original Input Matrix			**Change1 Input Matrix**			**Change2 Input Matrix**		
	Serve	**Process**	**Hitting**	**Serve**	**Process**	**Hitting**	**Serve**	**Process**	**Hitting**
Serve	1	0.5	3	1	0.3334	3	1	2	0.3334
Process	2	1	4	3	1	5	0.5	1	0.25
Hitting	0.3334	0.25	1	0.3334	0.2	1	3	4	1

The consistency ratio: CR1=0.0371, CR2=0.0176. Both of the CR value is less than 0.10, so the consistency of the judgment matrix can be accepted.AHP weights the data and gets the calculation results as shown in the figure below.

When the serve factor, process factor and hitting factor in the input matrix are changed many times, the predicted score changes obviously, and the tendency is significant. Especially,when the input matrix changes slightly (change 1),the predicted score changes little; when the judgment matrix changes greatly (change 2), the score changes greatly.This indicates that the model constructed in this study has high sensitivity, is suitable for different situations, and has good adaptability.

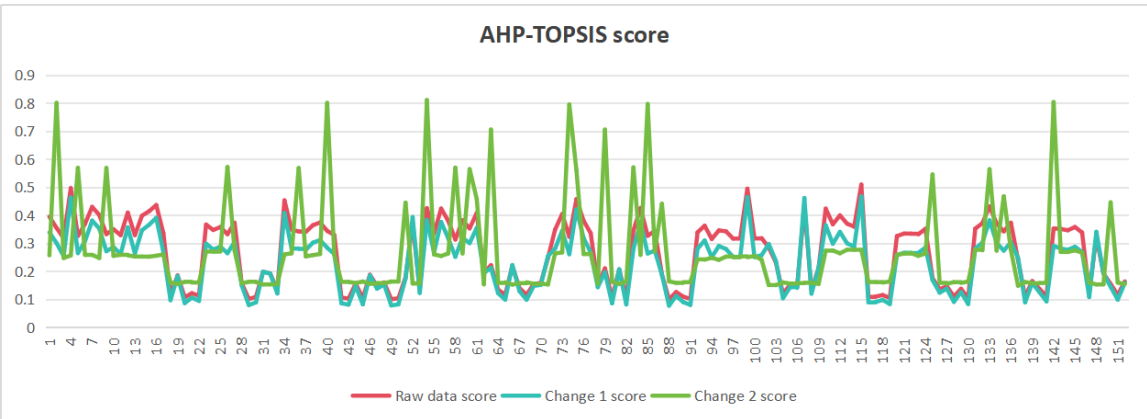


Figure 16: Time and Performance Score Chart

7.3 Model applicability

This article considers that the fluctuation of tennis scores is also influenced by various other factors, such as player physical fitness, psychological state, tactical strategies, weather and environmental factors, etc. Therefore, the indicator system constructed in this article has better applicability, as follows:

- **Cross gender or field competitions:** Consider the applicability of the model in men's and women's competitions, as well as different ground conditions on different fields. Due to differences in physical fitness, competition style, and different venue sizes and environments among athletes of different genders, these factors may affect the accuracy of the model's predictions.
- **Applicability to other sports:** It is necessary to consider the differences in different sports characteristics, such as competition rules, scoring mechanisms, etc. Explore whether the model can be applied to other sports, such as badminton, for modulation purposes. It can be easily transferred to other ball games and has a high degree of versatility.
- **Psychological state and environmental factors:** Different competition conditions may have an impact on competition rhythm and player performance. Explore other potential factors that affect the outcome of the competition, such as whether the contestant's current state is good, whether the weather is suitable, etc., and improve the input variable types of the model.
- **Strategic choices:** The player's tactical and strategic choices during the game also have an impact on score fluctuations. Different tactical choices can lead to different fluctuations in the score during the game. For example, an athlete may choose conservative or aggressive tactics to gain an advantage.

8 Conclusion

In the **data preprocessing** stage, this paper performs missing value interpolation by BP neural network and obtains **interpolation results that fit the data distribution more closely as well as more accurately** than mean value interpolation.

For **question one**, eight important data features were first extracted, and after that, using the factor analysis method, data dimensionality reduction was carried out to obtain "Serve Factor", "Process Factor", "Hitting Factor", and the factor weights were analyzed using the AHP-TOPSIS method. Finally, the weights were mapped back to the original dataset to **obtain the time-performance score evaluation model**, which realizes the time and performance of athletes, and is able to **predict the scoring probability of the athlete**, and at the same time **provides an effective reference for the tactics development**.

For **question two**, use the **Stacking ensemble model** to obtain an optimal model that can accurately predict the player's momentum changes during the competition. And the Kruskal Wallis H test results **show a significant correlation between player momentum and final victory**.

For **question three**, the wavelet neural network model and RBP model were established and trained, and a competition strategy based on previous data analysis was proposed. And whether or not an unstoppable ball is played can most affect whether there is a turning point. **In future research, more factors can be explored to influence momentum, as well as how to combine machine learning technology for more complex game analysis and strategy formulation.**

For **question four**, an evaluation of the model's performance indicates that the model has excellent performance and can **make reliable predictions about fluctuations in player scores**. And add comprehensive ranking indicators from the official website to re evaluate the model, thereby improving its performance.

9 Strengths and Weakness

9.1 Strengths

- The selected features provide objective indicators, making it easier to understand the competitive advantages of athletes in competitions. Through objective analysis of these characteristics, the performance and advantages of athletes in competitions can be better demonstrated.
- Reduce the dimensionality of features to make classification more representative. In problem one, factor analysis method is adopted to classify a large number of complex relationships within continuous attributes into a small number of comprehensive factors.
- The use of Wavelet Neural Networks to construct models is more suitable for capturing and understanding the momentum changes in tennis matches, and can effectively simulate nonlinear and complex patterns in time series data. It plays an important role in analyzing game momentum.

9.2 Weakness

- More input variables and other potential factors can be added for more accurate analysis and research.
- In the future, it is possible to consider incorporating factors such as the player's experience and level into the analysis, using more accurate methods to quantify each feature, and reusing the model's prediction results to determine whether the accuracy will be improved.

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