

Copula Distributions

Due
11 Jun 2023

The following soil moisture data has been made available: retrieved soil moisture over the Oklahoma region from the space-borne TRMM Microwave Imager using our LSMEM algorithm, observed Oklahoma mesonet data, averaged from 32 sites across Oklahoma, and VIC land surface model simulated soil moisture for the 32 sites. All the data are for the March-April- May (MAM) period from 1998 to 2003.

Plots of the LSMEM versus the difference (Mesonet-LSMEM) and LSMEM versus (VIC-LSMEM) are shown below. Functions that fit these plots are usually referred to as “soil moisture operators” because given a retrieved soil moisture estimation from TMI, an estimate of the mesonet or VIC soil moisture can be easily obtained for data assimilation.

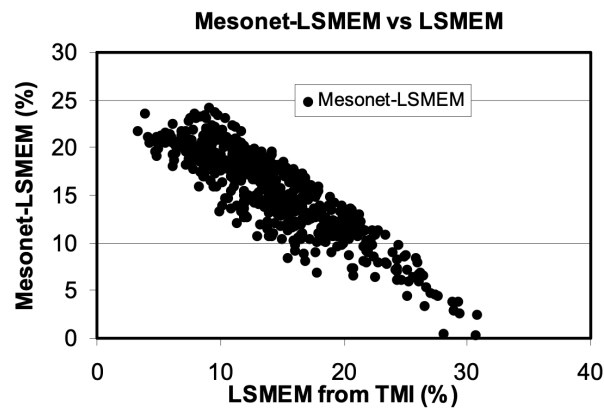


Figure 1. Comparison between soil moisture from the LSMEM retrieval and their differences from Oklahoma Mesonet field measured data. Results from 1998 to 2003 for the MAM season.

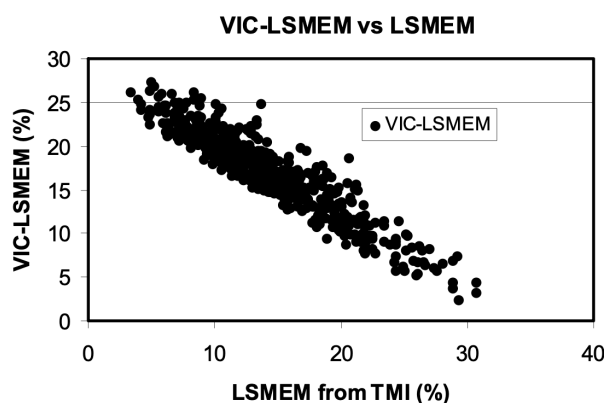


Figure 2. Same as the above figure, but y -axis is the VIC simulated soil moisture.

Task 1 – Fit the joint distribution $F(y, x)$ using copulas, where y = (VIC-LSMEM) and x = LSMEM. Assume that the marginal distributions for y and x are Gamma distributions. Fit this using three types of copulas: Clayton, Gumbel, and Frank.

Task 2 – Plot the empirical joint CDF with the joint CDFs simulated from three types of copulas: Clayton, Gumbel, and Frank. There will be a total of four plots. From comparing the empirical joint CDF with the three types of copulas, can you suggest which type fits the data the best? Could you suggest a more quantitative statistical test for this determination?

Task 3 – Using the copula that you feel best fits the data, simulate 400 pairs of (y, x) . Plot the simulated pairs (in red) and the original data (in black) on the same figure. (I would plot the original data first, then the simulated data so the red dots fall on top of the black.) Can you comment on whether you feel your choice of the particular copula and its fit is satisfactory?

Task 4 – Assuming that you obtained a retrieved TMI soil moisture value (using the LSMEM algorithm) of 10%, simulate 200 estimates of TMI soil moisture value (using the LSMEM algorithm) of 10%, simulate 200 estimates of y = (VIC-LSMEM). Plot its histogram, and estimate its mean and standard deviation.

Task 5 – Repeat parts 1 to 4 with the mesonet data and discuss your results.