6th Exercise in HPC

Exercise 1
Make yourself acquainted with `firstprivate` and `lastprivate`, i.e., write a little OpenMP program containing `parallel` sections making use of `firstprivate` and `lastprivate` and print the effect.

Exercise 2 (3 Points)
The Jacobi method is an iterative methods to solve a linear system

$$Ax = b.$$

The iterates are defined as

$$u_{n+1} = -D^{-1}(L + U)u_n + D^{-1}b,$$

where $A = L + D + U$ and $D = \text{diag}(A)$ is the diagonal of A, L a lower triangular matrix and U an upper triangular matrix. Use $u_0 = 0$ as a start vector. Remark: If $\|D^{-1}(L+U)\|_\infty < 1$ then this iteration converges.
Define the $n \times n$-Matrix

$$A = \begin{pmatrix} +2^0 & -2^{-2} & -2^{-4} & -2^{-8} & \dots & -2^{-2^{n-1}} \\ -2^{-2} & +2^0 & -2^{-2} & -2^{-4} & \dots & -2^{-2^{n-2}} \\ -2^{-4} & -2^{-2} & +2^0 & -2^{-2} & \dots & -2^{-2^{n-3}} \\ -2^{-8} & -2^{-4} & -2^{-2} & +2^0 & \dots & -2^{-2^{n-4}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ -2^{-2^{n-1}} & -2^{-2^{n-2}} & -2^{-2^{n-3}} & -2^{-2^{n-4}} & & +2^0 \end{pmatrix}$$

and the right hand side $b = (1, \dots, 1)^T$.
Implement a parallel Jacobi iteration using OpenMP. The iterations is stopped when the norm of the residual $r = b - Au$ is below ε. What speedups do you achieve if you change the number of threads? Choose, e.g., $n = 20000$ and $\varepsilon = 1e - 3$.