A linear regression model describes the relationship between a *dependent variable*, *y*, and one or more *independent variables*, *X*. The dependent variable is also called the *response variable*. Independent variables are also called *explanatory* or *predictor variables*. Continuous predictor variables are also called *covariates*, and categorical predictor variables are also called *factors*. The matrix *X* of observations on predictor variables is usually called the *design matrix*.

A multiple linear regression model is

$$Y_i = \beta_0+\beta_1 X_{i1}+\beta_2 X_{i2}+\cdots+\beta_p X_{ip}+\varepsilon_i, \quad i=1,\cdots,n,$$

where

- y_i is the *i*th response.
- β_k is the *k*th coefficient, where β_0 is the constant term in the model. Sometimes, design matrices might include information about the constant term. However, fitlm or stepwiselm by default includes a constant term in the model, so you must not enter a column of 1s into your design matrix *X*.
- X_{ij} is the *i*th observation on the *j*th predictor variable, *j* = 1, ..., *p*.
- ε_i is the *i*th noise term, that is, random error.

If a model includes only one predictor variable (*p* = 1), then the model is called a simple linear regression model.

In general, a linear regression model can be a model of the form

$$y_i=\beta_0+{}_{Kk=1}\beta_k f_k(X_{i1},X_{i2},\cdots,X_{ip})+\varepsilon_i, \quad i=1,\cdots,n,$$

where *f* (.) is a scalar-valued function of the independent variables, X_{ij}s. The functions, *f* (*X*), might be in any form including nonlinear functions or polynomials. The linearity, in the linear regression models, refers to the linearity of the coefficients β_k. That is, the response variable, *y*, is a linear function of the coefficients, β_k.

Some examples of linear models are:

$$y_i=\beta_0+\beta_1 X_{1i}+\beta_2 X_2+\beta_3 X_{3i}+\varepsilon_i$$
$$y_i=\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+\beta_3 X_{31i}+\beta_4 X_{22i}+\varepsilon_i$$
$$y_i=\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+\beta_3 X_{1i} X_{2i}+\beta_4 \log X_{3i}+\varepsilon_i$$

The following, however, are not linear models since they are not linear in the unknown coefficients, β_k.

$$\log y_i=\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+\varepsilon_i$$

$$y_i=\beta_0+\beta_1 X_{1i}+1\beta_2 X_{2i}+e_{\beta_3 X_{1i} X_{2i}}+\varepsilon_i$$

In a linear regression model of the form $y = \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p$, the coefficient β_k expresses the impact of a one-unit change in predictor variable, X_j, on the mean of the response E(y), provided that all other variables are held constant. The sign of the coefficient gives the direction of the effect. For example, if the linear model is E(y) = 1.8 – 2.35X_1 + X_2, then –2.35 indicates a 2.35 unit decrease in the mean response with a one-unit increase in X_1, given X_2 is held constant. If the model is E(y) = 1.1 + 1.5X_1^2 + X_2, the coefficient of X_1^2 indicates a 1.5 unit increase in the mean of Y with a one-unit increase in X_1^2 given all else held constant. However, in the case of E(y) = 1.1 + 2.1X_1 + 1.5X_1^2, it is difficult to interpret the coefficients similarly, since it is not possible to hold X_1 constant when X_1^2 changes or vice versa.