3. Primal objective function of soft margins SVM :

$$L_P = \frac{1}{2}\|W\|^2 + C\sum_{i=1}^{m}\xi_i - \sum_{i=1}^{m}\alpha_i\left(y^{(i)}(W^T x_i + w_0) - 1 + \xi_i\right) - \sum_{i=1}^{m}\beta_i\xi_i$$

minimization equations : $\frac{\partial L_P}{\partial w_i} = 0$, $\frac{\partial L_P}{\partial w_0} = 0$ & $\frac{\partial L_P}{\partial \xi_i} = 0$

develop the expression of the dual L_D that has to be maximized.

$$\frac{\partial L_P}{\partial w_i} = 0 = -\sum_{i=1}^{m}\alpha_i y^{(i)} \Rightarrow \sum_{i=1}^{m}\alpha_i y^{(i)} = 0$$

$$\frac{\partial L_P}{\partial w} = 0 = \frac{1}{2}\times 2W - \sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)} = 0$$

$$\frac{\partial L_P}{\partial \xi_i} = 0 = C - \alpha_i - \beta_i$$

$$L_P = \frac{1}{2}\left(\sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)T}\right)\left(\sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)}\right) + C\sum_{i=1}^{m}\xi_i$$

$$- \sum_{i=1}^{m}\alpha_i y^{(i)}\left(\sum_{j=1}^{m}\alpha_j y^{(j)} x^{(j)T} x^{(i)}\right) -$$

$$\sum_{i=1}^{m}\alpha_i y^{(i)} w_0 + \sum_{i=1}^{m}\alpha_i - \sum_{i=1}^{m}\alpha_i\xi_i - \sum_{i=1}^{m}\beta_i\xi_i$$

$$L_D = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y^{(i)} y^{(j)} x^{(i)T} x^{(j)} + \sum_{i=1}^{m}\alpha_i$$

max L_D

s.t $\alpha_i \geq 0$, $\sum_{i=1}^{m}\alpha_i y^{(i)} = 0$

$\beta_i \geq 0$ $\quad C - \alpha_i - \beta_i = 0 \ \forall i$

$\alpha_i \geq 0$, $\beta_i = C - \alpha_i \geq 0$

$= C \geq \alpha_i$

$0 \leq \alpha_i \leq C$

Max L-D

s.t $0 \le \alpha_i \le C$, $\sum_{i=1}^{m} \alpha_i y^{(i)} = 0$