CS584

ASSIGNMENT 4

SUPPORT VECTOR MACHINE

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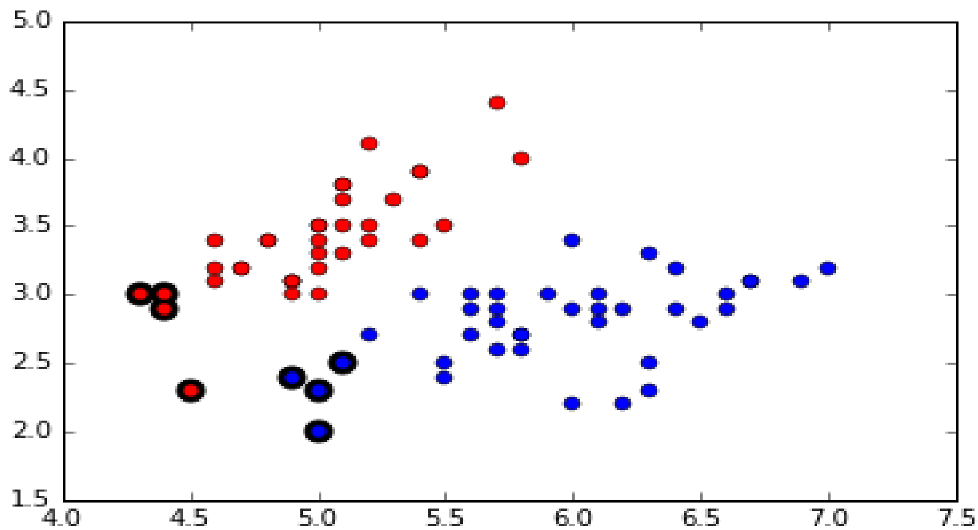
1. Generate a small 2D feature vectors of two classes such that classes are linearly separable:
   For this, I have considered iris dataset. First 2 classes i.e. setosa and versicolor are linearly separable. To make dataset 2D only 2 features from this dataset are considered.
   For non-linearly separable dataset, versicolor and virginica classes of iris dataset are considered.

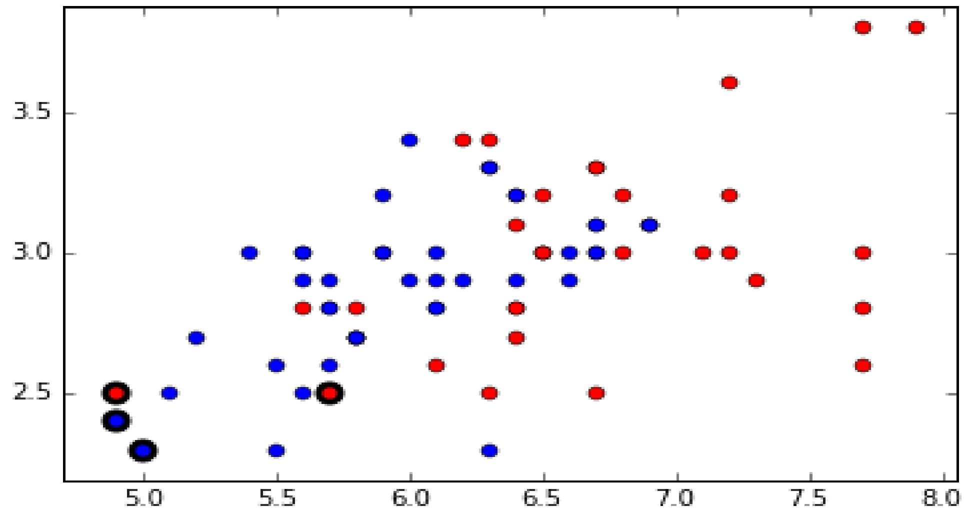
2. Implement a linear SVM with hard margins on both datasets:
   Support vector plot (linearly separable):



Accuracy for this algorithm is good.
Cvxopt package of python is used to find support vectors.
Output:

```
     pcost       dcost       gap    pres   dres
 0: -4.9681e-01 -5.9097e-01  1e+02  1e+01  1e+00
 1: -3.0112e-02 -1.5904e-03  2e+00  2e-01  2e-02
 2: -4.1519e-04 -8.6209e-04  3e-02  3e-03  3e-04
 3:  2.5357e-05 -7.7168e-04  8e-04  9e-19  1e-15
 4: -1.7980e-04 -2.6976e-04  9e-05  2e-20  4e-16
 5: -2.1924e-04 -2.5076e-04  3e-05  3e-20  2e-16
 6: -2.4057e-04 -2.4612e-04  6e-06  3e-20  3e-16
 7: -2.4518e-04 -2.4524e-04  7e-08  5e-20  3e-16
Optimal solution found.
8 support vectors out of 75 points
16 out of 25 predictions correct
Accuracy by my function 0.64
```

Support Vector plot (non-linearly separable):

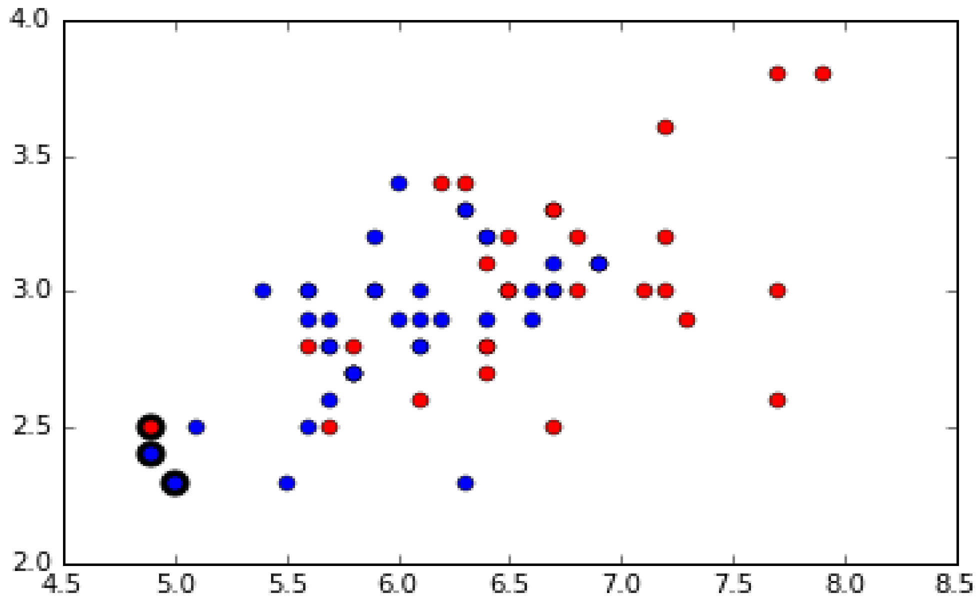


Accuracy for this is very bad. This algorithm suffers badly.

Output:

```
  pcost       dcost       gap    pres   dres
 0: -5.3831e-01 -6.6686e-01  1e+02  1e+01  1e+00
 1: -4.4484e-02 -2.0657e-03  2e+00  2e-01  2e-02
 2: -1.1008e-03 -8.5918e-04  5e-02  5e-03  6e-04
 3:  3.4474e-05 -6.8917e-04  8e-04  1e-05  1e-06
 4: -1.4955e-04 -2.5658e-04  1e-04  2e-20  4e-16
 5: -1.8615e-04 -2.2719e-04  4e-05  3e-20  3e-16
 6: -1.9981e-04 -2.2677e-04  3e-05  8e-20  3e-16
 7: -2.2124e-04 -2.2263e-04  1e-06  3e-20  3e-16
 8: -2.2212e-04 -2.2213e-04  1e-08  9e-20  3e-16
Optimal solution found.
4 support vectors out of 75 points
14 out of 25 predictions correct
Accuracy by my function 0.56
```

4. Linear SVM algorithm with soft margin :
   Support vector marked plot:

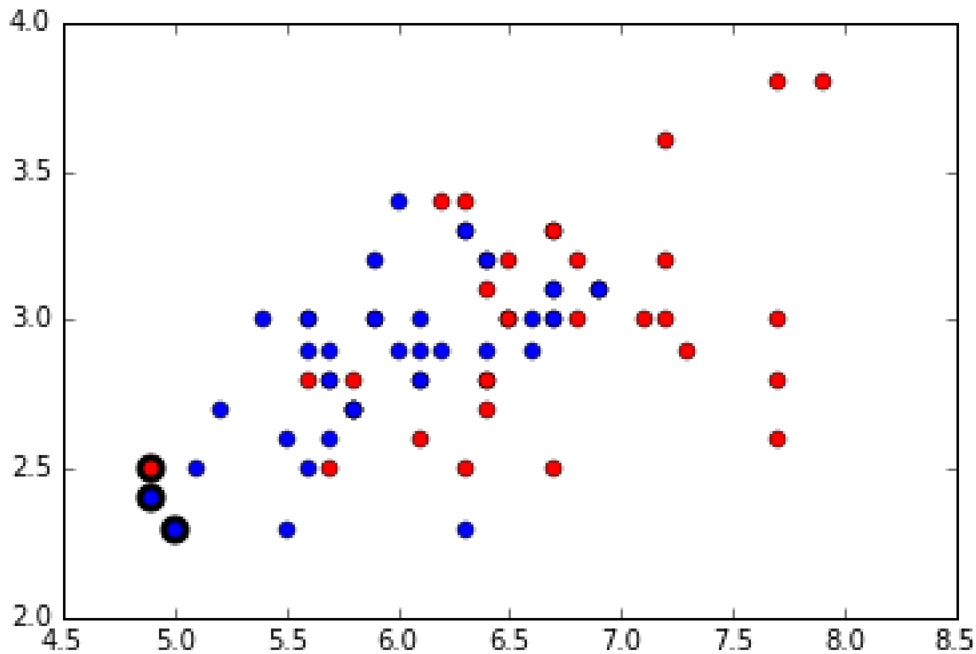


Output:

```
pcost       dcost       gap    pres   dres
 0: -2.7291e-01 -7.8823e+00  2e+02  1e+01  4e-13
 1: -4.9291e-02 -6.6868e+00  1e+01  4e-01  3e-13
 2: -5.4765e-03 -6.5301e-01  8e-01  1e-02  6e-14
 3: -2.4641e-07 -7.7935e-03  9e-03  1e-04  7e-15
 4: -1.3743e-04 -4.5742e-04  3e-04  1e-06  4e-16
 5: -1.8859e-04 -2.6029e-04  7e-05  3e-07  2e-16
 6: -2.3110e-04 -2.5058e-04  2e-05  2e-08  2e-16
 7: -2.3716e-04 -2.3880e-04  2e-06  2e-09  2e-16
 8: -2.3798e-04 -2.3801e-04  3e-08  2e-11  3e-16
Optimal solution found.
3 support vectors out of 70 points
19 out of 30 predictions correct
Accuracy by my function 0.666666666667
```

By looking at this we can say that for non-linearly separable data algorithm with soft margin performs better than with hard margin.

5. Kernel-based SVM algorithm (Polynomial and Gaussian):
   Support vector marked plot:



Output:

```
   pcost       dcost       gap    pres   dres
 0: -5.3831e-01 -6.6686e-01  1e+02  1e+01  1e+00
 1: -4.4484e-02 -2.0657e-03  2e+00  2e-01  2e-02
 2: -1.1008e-03 -8.5918e-04  5e-02  5e-03  6e-04
 3:  3.4474e-05 -6.8917e-04  8e-04  1e-05  1e-06
 4: -1.4955e-04 -2.5658e-04  1e-04  2e-20  4e-16
 5: -1.8615e-04 -2.2719e-04  4e-05  3e-20  3e-16
 6: -1.9981e-04 -2.2677e-04  3e-05  8e-20  3e-16
 7: -2.2124e-04 -2.2263e-04  1e-06  3e-20  3e-16
 8: -2.2212e-04 -2.2213e-04  1e-08  9e-20  3e-16
Optimal solution found.
3 support vectors out of 75 points
17 out of 25 predictions correct
Accuracy by my function 0.68
```

This accuracy is for Gaussian kernel. To look at the accuracy of Polynomial kernel, change the Gaussian variable to False.

For Gaussian Sigma is kept as 5 and for polynomial order is kept as 2.

Conclusion:

Hard-margin for linearly-separable data works very well. And for non-separable data soft-margin works well. Also for non-linearly separable data, kernel-based SVM works well. Comparison is done based on the accuracies obtained after implementing these algorithm.

3. Primal objective function of soft margins SVM :

$$L_p = \frac{1}{2}\|W\|^2 + C\sum_{i=1}^{m}\xi_i - \sum_{i=1}^{m}\alpha_i\left(y^{(i)}(W^T x_i + w_0) - 1 + \xi_i\right) - \sum_{i=1}^{m}\beta_i\xi_i$$

minimization equations : $\frac{\partial L_p}{\partial w_i} = 0$, $\frac{\partial L_p}{\partial w_0} = 0$ & $\frac{\partial L_p}{\partial \xi_i} = 0$

develop the expression of the dual L_D that has to be maximized.

$$\frac{\partial L_p}{\partial w_i} = 0 = -\sum_{i=1}^{m}\alpha_i y^{(i)} \Rightarrow \sum_{i=1}^{m}\alpha_i y^{(i)} = 0$$

$$\frac{\partial L_p}{\partial w} = 0 = \frac{1 \times 2W}{2} - \sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)} = 0$$

$$\frac{\partial L_p}{\partial \xi_i} = 0 = C - \alpha_i - \beta_i$$

$$L_p = \frac{1}{2}\left(\sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)T}\right)\left(\sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)}\right) + C\sum_{i=1}^{m}\xi_i$$
$$- \sum_{i=1}^{m}\alpha_i y^{(i)}\left(\sum_{j=1}^{m}\alpha_j y^{(j)} x^{(j)T} x^{(i)}\right) -$$
$$\sum_{i=1}^{m}\alpha_i y^{(i)} w_0 + \sum_{i=1}^{m}\alpha_i - \sum_{i=1}^{m}\alpha_i\xi_i - \sum_{i=1}^{m}\beta_i\xi_i$$

$$L_D = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y^{(i)} y^{(j)} x^{(i)T} x^{(j)} + \sum_{i=1}^{m}\alpha_i$$

max L_D

s.t $\alpha_i \geq 0$, $\sum_{i=1}^{m}\alpha_i y^{(i)} = 0$

$\beta_i \geq 0$ $\quad C - \alpha_i - \beta_i = 0 \;\forall i$

$\alpha_i \geq 0$, $\beta_i = C - \alpha_i \geq 0$

$= C \geq \alpha_i$

$0 \leq \alpha_i \leq C$

Max L.D

s.t $0 \leq \alpha_i \leq C$, $\sum_{i=1}^{m} \alpha_i y^{(i)} = 0$