UNIT-I

DC Circuits: Electrical circuit elements (R, L and C), Ohm's Law, Kirchoff's laws (KCL & KVL), series-parallel resistive circuits, Simple numerical problems with Voltage Sources.

AC Circuits: A.C. Fundamentals, Sinusoidal voltages and currents, time period, frequency, amplitude, phase, phase difference, average value, RMS value of sinusoidal waveforms, Phasor representation of Voltages and currents, Concept of Impedance, Impedance of Series R-L, R-C and RLC circuits, Average power, Concept of power factor - Simple Numerical problems.

DC Circuits

Basic Electrical Quantities:

Electric Charge:

. **"Charge is an electrical property of the atomic particles of which matter consist".**

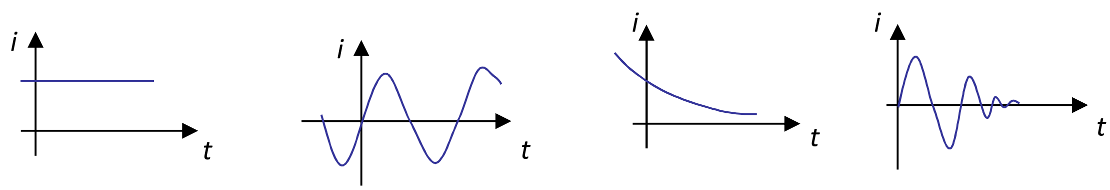
The fundamental unit of charge is Coulomb in MKS system. In terms of this unit, the charge of an electron is – 1.60218 x 10^{-19} C, and 1 coulomb of negative charge represents the combined charge of 6.24 x 10^{18} electrons. Charge is symbolized by the letter Q or q,

Electric Current:

The term electric current describes the phenomenon of transferring charge from one point in a circuit to another. **"An electric current may be defined as the time rate of net motion of electric charge across a cross-sectional boundary"**. The unit of current is the ampere (A), and 1 A corresponds to charge moving at the rate of 1 C/S. Current is symbolized by I or i.

In equation form, $i = \dfrac{dq}{dt}$

In circuits several types of currents like direct current or dc, alternating current or ac etc., will be encountered.



Voltage or Potential difference:

Voltage or Potential difference across a terminal pair of a general circuit element is a measure of the work (or energy) required for moving charge through that element. Specifically, **Voltage across an element is defined as the energy required to move a unit charge through an element from one terminal through the element to the other terminal**. The unit of voltage is the Volt (V) and 1 volt is equal to 1 J/C. A voltage or potential difference can exist between a pair of electrical terminals whether a current is flowing or not.

$$V = \frac{dW}{dQ}$$

Where,

- W is the potential energy and its unit is Joule.
- Q is the charge and its unit is Coloumb.

Power

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as $P = \frac{dW}{dt}$

Where,

- W is the electrical energy and it is measured in terms of Joule.
- t is the time and it is measured in seconds.

We can re-write the above equation a

$$P = \frac{dW}{dt} = \frac{dW}{dQ} * \frac{dQ}{dt} = VI$$

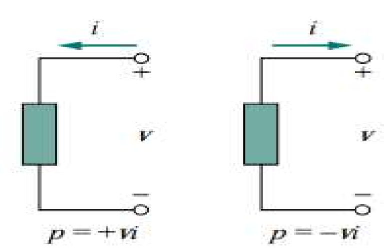
Therefore, power is nothing but the product of voltage V and current I. Its unit is **Watt (W).**

Passive sign convention:

Passive sign convention is used to determine whether the element is absorbing the power or delivering it.

This convention says that if the current arrow and the voltage polarity signs are placed at the terminals of the element so that the current enters the positively marked terminal, and if both the arrow and the sign pair are labeled with the appropriate algebraic quantities, then the power absorbed by the element can be expressed by the algebraic product of these two quantities. If the numerical value of the product is negative, then the element is absorbing negative power or actually generating power and delivering to some external element.

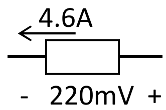
Passive sign convention is satisfied when the current enters through the positive terminal of an element and P=+Vi. If the current enters through the negative terminal ,P= -Vi.

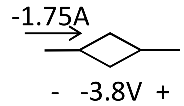


Absorbing power. Supplying power.

Example problems:

1.1. Find the power being supplied by each of the following circuit elements.

(a) 

(b) 

Sol: (a) -1.012W; (b) 6.65W

Electrical circuit elements (R, L and C) & Ohm's Law

Active Element: An element, which is capable of furnishing an average power greater than zero to some external devices, where the average is taken over an infinite time interval. E.g., Voltage sources and Current sources.

Passive Element: An element that can't supply an average power that is greater than zero over an infinite time interval. E.g., Resister, Inductor and capacitor.

The interconnection of two or more simple circuit elements is called an '**Electrical network**'. If the network contains at least one closed path, it is also an '**Electric circuit**'. Every circuit is a network, but not all networks are circuits. A network that contains at least one active element, such as an independent voltage or current source, is an '**Active network**'. A network that doesn't contain any active elements is a '**Passive network**'.

1. Resistor

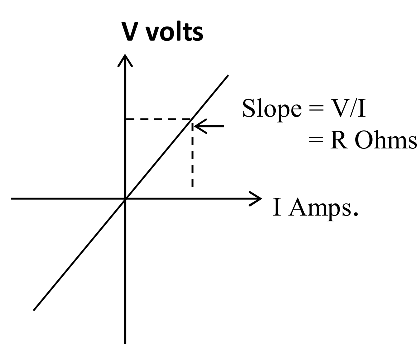
The main functionality of Resistor is either **opposes or restricts the flow of electric current**. Hence, the resistors are used in order to limit the amount of current flow and / or dividing (sharing) voltage.

Ohm's Law

Ohm's Law:

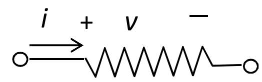
Ohm's law states that, temperature remaining constant the voltage across many types of conducting materials is directly proportional to the current flowing through the material. In equation form,

$$v = Ri.$$



Where the constant of proportionality is called the 'Resistance'. The unit of resistance is ohm (Ω). This resistance is an idealized circuit element. When the above equation is plotted on v-versus-i axes, the graph is a straight line passing through the origin.
In the circuit symbol, the v and i are selected as per the passive sign convention. A resistor is a passive element that can't deliver power or store energy. The absorbed power appears physically as heat and is always positive.

$$p = vi = i^2 R = \frac{v^2}{R}$$



The ratio of current to voltage is also a constant,

$$\frac{i}{v} = \frac{1}{R} = G$$

Where G is called the conductance. The SI unit of conductance is the Seimens (S).

From the above relations, it is obvious that the current through and voltage across a resistor must both vary with time in the same manner.

Resistance may be used as the basis for defining two commonly used terms, short circuit and open circuit.

Short Circuit: A short circuit is a resistance of zero ohms. Since v = Ri, the voltage across a short circuit must be zero, although the current may have any value.

Open Circuit: An open circuit is a resistance of infinity ohms. The current must be zero, regardless of the voltage across the open circuit.

2. Inductor

In general, inductors will have number of turns (N). Hence, they produce magnetic flux when current flows through it. So, the amount of total magnetic flux produced by an inductor depends on the current, I flowing through it and they have linear relationship.
Mathematically, it can be written as

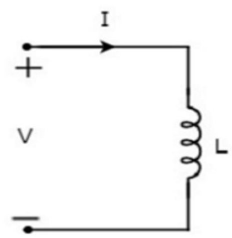
$$\Phi \; \alpha \; I$$

$$\Phi = L\, I$$

Where,

- Ψ is the total magnetic flux
- L is the inductance of an inductor

Let the current flowing through the inductor is I amperes and the voltage across it is V volts. The symbol of inductor along with current I and voltage V are shown in the following figure.



According to Faraday's law, the voltage across the inductor can be written as

$$V = \frac{d\Phi}{dt}$$

Substitute Φ = LI in the above equation.

$$V = \frac{d(LI)}{dt}$$

$$V = L\frac{dI}{dt}$$

$$I = \frac{1}{L}\int V\,dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across inductor and current flowing through it.

We know that power in an electric circuit element can be represented a

$$P = V\,I$$

Substitute $V = L\frac{dI}{dt}$ in the above equation

$$P = (L\frac{dI}{dt})I$$

$$P = LI\frac{dI}{dt}$$

By integrating the above equation, we will get the energy stored in an inductor as

$$W = \frac{1}{2}LI^2$$

So, the inductor stores the energy in the form of magnetic field.

3. Capacitor

In general, a capacitor has two conducting plates, separated by a dielectric medium. If positive voltage is applied across the capacitor, then it stores positive charge. Similarly, if negative voltage is applied across the capacitor, then it stores negative charge.

So, the amount of charge stored in the capacitor depends on the applied voltage V across it and they have linear relationship. Mathematically, it can be written as

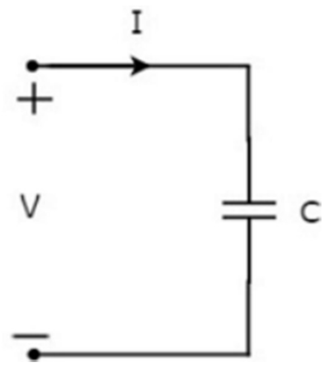
$$Q \,\alpha\, V$$

$$\Rightarrow Q = CV$$

Where,

- Q is the charge stored in the capacitor.
- C is the capacitance of a capacitor.

Let the current flowing through the capacitor is I amperes and the voltage across it is V volts. The symbol of capacitor along with current I and voltage V are shown in the following figure



$$I = \frac{dQ}{dt}$$

Substitute $Q = CV$ in the above equation.

$$I = \frac{d(CV)}{dt}$$

$$\Rightarrow I = C\frac{dV}{dt}$$

$$\Rightarrow V = \frac{1}{C}\int Idt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across capacitor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $I = C\frac{dV}{dt}$ in the above equation.

$$P = V(C\frac{dV}{dt})$$

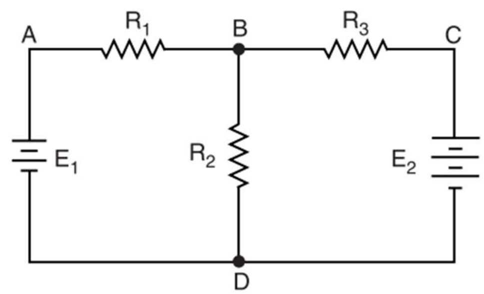
$$\Rightarrow P = CV\frac{dV}{dt}$$

By integrating the above equation, we will get the energy stored in the capacitor as

$$W = \frac{1}{2}CV^2$$

So, the capacitor stores the energy in the form of electric field.

Network Terminology



E_1 and E_2: **Active elements** - Supply energy to the circuit.

R_1, R_2 and R_3: **Passive elements** - Receive energy from the active elements

A, B, C, D: **Nodes** – point at which two or more circuit elements are joined.

B, D: **Junction -** point at which three or more circuit elements are joined.

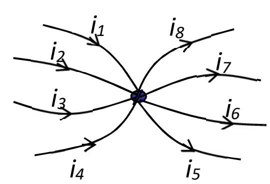
BAD, BCD and BD: **Branch** - lies between two junction points.

Loop: Any closed path of a network - ABDA, BCDB and ABCD.

KIRCHHOFF'S LAWS

Kirchhoff's Laws:
Kirchhoff's Current Law: It states that the algebraic sum of the currents entering or leaving any node is zero.

i.e., $i_1 + i_2 + i_3 + i_4 - i_5 - i_6 - i_7 - i_8 = 0$
(or) $-i_1 - i_2 - i_3 - i_4 + i_5 + i_6 + i_7 + i_8 = 0$.



Kirchhoff's current law may also be stated as under:

The **sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction**. Kirchhoff's current law is also called junction rule.

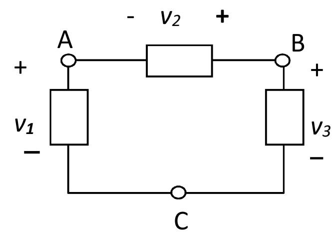
i.e., Sum of incoming currents = Sum of outgoing currents

$$i_1 + i_2 + i_3 + i_4 = i_5 + i_6 + i_7 + i_8$$

Kirchhoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge. Hence, Kirchhoff's current law is based on the law of conservation of charge.

2.Kirchhoff's Voltage Law: It states that the algebraic sum of the voltages around any closed path in a circuit is zero.

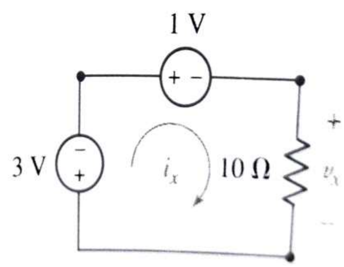
i.e., $-v_1 - v_2 + v_3 = 0$.



Current is variable that is related to the charge flowing through a circuit element, where as voltage is a measure of potential energy difference across the element. There is a single unique value for voltage in circuit theory. Thus, the energy required to move a unit charge from point A to point B in a circuit must have a value that is independent of the path taken from A to B.

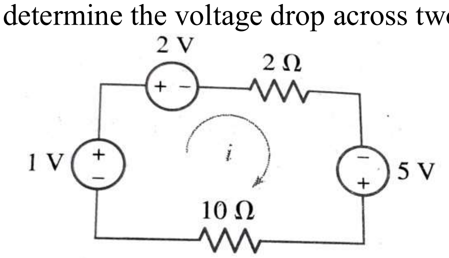
Example Problems:

Problem 1.2:Determine i_x and V_X in the below circuit.



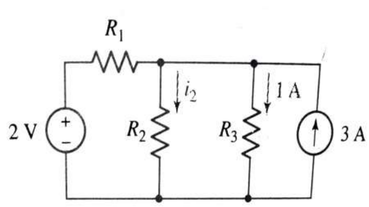
Sol: V_X= - 4V and i_X = - 400mA.

Problem 1.3: Use KVL and determine the voltage drop across two resistances.



Ans: $V_{2\Omega}$=666mv, $V_{10\Omega}$=3.33V.

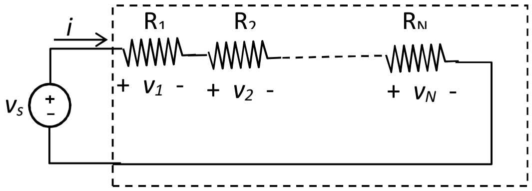
Problem 1.4:Determine the current labeled i_2 in the circuit. The 2V source is supplying 7A of current to the circuit.

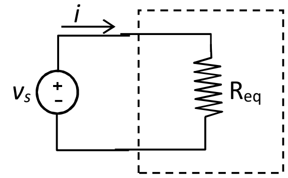


Ans:9A

Series and Parallel Resistors:

Series Connection: All the elements that carry the same current (not equal current) are said to be in series. An equivalent resistance R_{eq} can replace a number of resistors connected in series.

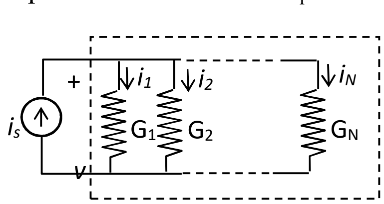


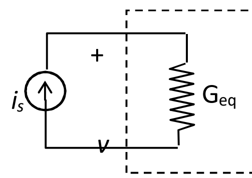


Applying Kirchhoff's Voltage Law to the single loop: $v_s = v_1 + v_2 + \text{-------} + v_N$
Now using Ohm's Law: $vs = R_1\, i + R_2\, i + \text{------} + R_N\, i = (R_1 + R_2 + \text{------} + R_N)\, i$
Now applying Kirchhoff's Voltage Law and using Ohm's Law for the equivalent circuit:
$vs = R_{eq}\, i$.
Thus comparing the above two equations, equivalent resistance of N resistors be

$$\boxed{R_{eq} = R_1 + R_2 + \text{------} + R_N.}$$

Parallel Connection: Elements having a common voltage across them are said to in parallel. An equivalent resistance R_{eq} can replace a number of resistors connected in parallel.





Applying Kirchhoff's current Law to the single node pair: $i_s = i_1 + i_2 + \text{-------} + i_N$
$= G_1\, v + G_2\, v + \text{------} + G_N\, v$
$= (G_1 + G_2 + \text{------} + G_N)\, v$
Now applying Kirchhoff's current Law to the equivalent circuit: $i_s = G_{eq}\, v$.
Thus comparing the above two equations, equivalent conductance of N conductance's connected in parallel be

$$\boxed{G_{eq} = G_1 + G_2 + \text{------} + G_N.}$$

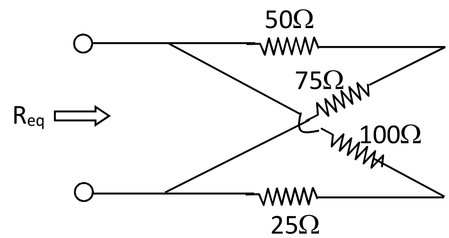
In terms of resistors rather than in conductance's is: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ---- + \frac{1}{R_N}$

Or $$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + ---- + \frac{1}{R_N}}$$

For the special case of two resistors in parallel: $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$

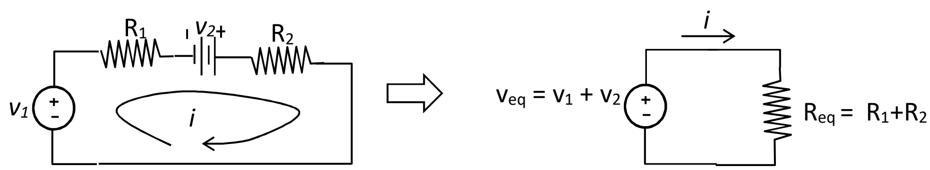
The above resistance equivalents may be used in simplifying circuits. However, Caution must be observed where an element may not be in series or parallel with any other simple circuit element in circuit.
Example problems:

Problem 1.5:Find R_{eq} for the circuits shown below.



Ans: Req=62.5Ω.

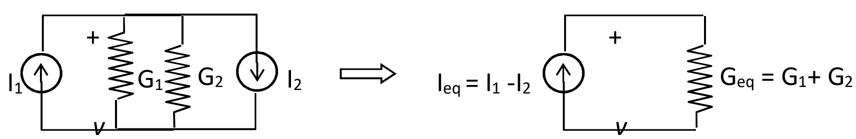
Independent Voltage Sources in Series: Several independent voltage sources in series may be replaced by an equivalent voltage source having a voltage equal to the algebraic sum of the individual source voltages. There is little advantage in including a dependent voltage source in a series combination.



Applying Kirchhoff's Voltage law to the simple loop :

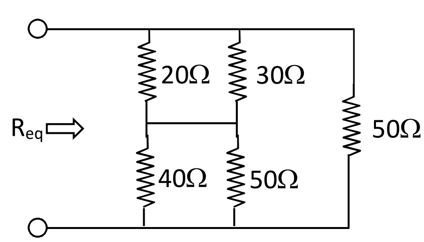
$\Rightarrow \quad -v_1 + i\,R1 - v_2 + i\,R_2 = 0 \qquad \Rightarrow v_1 + v_2 = (R_1+R_2)\, i \qquad \Rightarrow v_{eq} = R_{eq}\, i$

Independent Current Sources in Parallel: Parallel current sources may also be combined by algebraically adding the individual currents, and the order of the parallel elements may be arranged as desired.



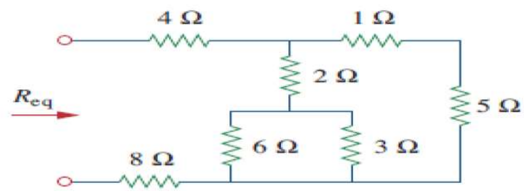
Ideal voltage sources in parallel are permissible only when each has the same terminal voltage at every instant. However, practical voltage sources may be combined in parallel without any theoretical difficulty. In a similar way, two current sources may not be placed in series unless each has the same current, including sign, for every instant of time.

Problems 1.6.Find the equivalent resistance for the network shown below.



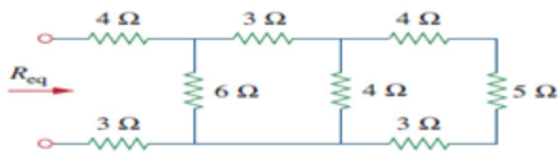
Ans: Req=20.3 Ω

Find the Req for the circuit shown in below figure.



Ans:14.4 ohms

.. Find the Req in the circuit Shown in fig below



Ans:10 ohms

AC CIRCUITS

Sinusoid:

A sinusoid is a function that has a sin or cosine function.

Consider the sinusoidal voltage $V(t) = V_m \, sin \, \omega t$

Where V_m is the amplitude of the sinusoid, ω is the angular frequncy and ωt is the argument of the sinusoid.

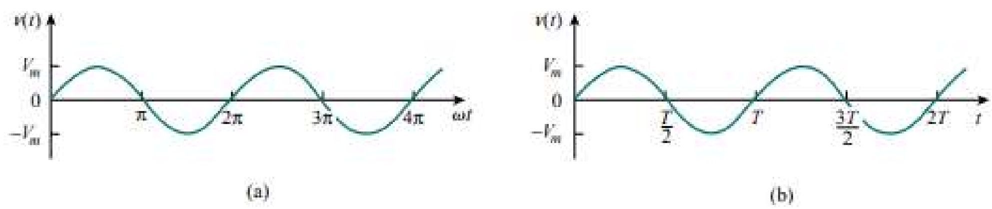


Fig (a) Sketch of Vm sinωt as a function of ωt   Fig(b) Sketch of Vm sinωt as a function of t.

An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time. Important terms associated with an alternating quantity are defined below.

Cycle: It is defined as one complete set of positive, negative and zero values of an alternating quantity.

Instantaneous value: It is defined as the value of an alternating quantity at a particular instant of given time. Generally denoted by small letters.

e.g. i= Instantaneous value of current   v= Instantaneous value of voltage

Amplitude/ Peak value/ Crest value/ Maximum value: It is defined as the maximum value (either positive or negative) attained by an alternating quantity in one cycle. Generally denoted by capital letters.

e.g. Im= Maximum Value of current

Vm= Maximum value of voltage

Frequency :It is defined as number of cycles completed by an alternating quantity per second. Symbol is f. Unit is Hertz (Hz).

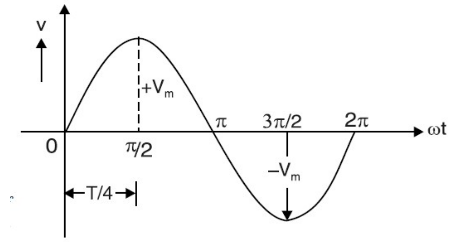
$$f = \frac{1}{T}$$

Time period: It is defined as time taken to complete one cycle. Symbol is T. Unit is seconds.

$$T = \frac{2\Pi}{\omega}$$

Phase: Phase is defined as the fractional part of time period or cycle through which The quantity has advanced from selected zero position of reference

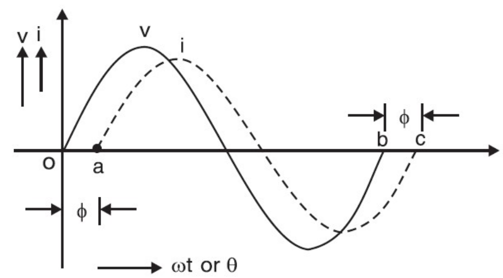
Phase of +Vm is π/2 rad or T/4 sec
Phase of -Vm is 3π/2 rad or 3T/4 sec



Phase Difference:
When two alternating quantities of of the same frequency have the different zero points they are said to have a phase difference. angle between the two zero points is the angle of phase difference.

The generalized mathematical expression to define these two sinusoidal quantities will be written as: $V = V_m \sin \omega t$

$$i = I_m \sin(\omega t - \Phi)$$



Example 1: *Write the mathematical expression for a 50 Hz sinusoidal voltage of peak value 80 V. Sketch the waveform versus time t.*
Ans:* v = 80 sin 314 *t

Example 2: *The maximum current in a sinusoidal a.c. circuit is 10A. What is the instantaneous current at 45°?*
Ans:i = 10 × sin 45° = **7·07 A**

Example 3 : *An alternating current i is given by ; i = 141·4 sin 314 t Find (i) the maximum value (ii) frequency (iii) time period and (iv) the instantaneous value when t is 3 ms.*
Solution. Comparing the given equation of alternating current with the standard form *i* = *Im* sinω*t*, we have,
(i) Maximum value, *Im* = **141·4 A**
(ii) Frequency, *f* = ω/2π = 314/2π = **50 Hz**
(iii) Time period, *T* = 1/*f* = 1/50 = **0·02 s**
(iv) *i* = 141·4 sin 314 *t*
When *t* = 3 m s = 3 × 10−3 s,
i = 141·4 sin 314 × 3 × 10−3 = **114· 35 A**

Values of Alternating Voltage and Current

In a d.c. system, the voltage and current are constant so that there is no problem of specifying their magnitudes. However, an alternating voltage or current varies from instant to instant. A natural question arises how to express the magnitude of an alternating voltage or current. There are four ways of expressing it, namely ;
(*i*) Peak value (*ii*) Average value or mean value
(*iii*) R.M.S. value or effective value (*iv*) Peak-to-peak value
Although peak, average and peak-to-peak values may be important in some engineering applications, it is the r.m.s. or effective value which is used to express the magnitude of an alternating voltage or current.

Average Value of Sinusoidal Current

Average Value: The arithmetic average of all the values of an alternating quantity over one cycle is called its average value.

$$\text{Average value}= \frac{Area\ under\ the\ curve}{Base\ length}$$

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v\, d(\omega t)$$

The average value of alternating current (or voltage) over one cycle is zero. It is because the waveform is symmetrical about time axis and positive area exactly cancels the negative area. However,the average value over a half-cycle (positive or negative) is not zero. *Therefore, average value of alternating current (or voltage) means half-cycle average value unless stated otherwise*.

The **half-cycle average value** *of a.c. is that value of steady current (d.c.) which would send thesame amount of charge through a circuit for half the time period of a.c. as is sent by the a.c. through the same circuit in the same time. It is represented by Iavg*. This can be obtained by integrating the instantaneous value of current over one half cycle (*i.e.* area over half-cycle) and dividing the result by base length of half-cycle (π).

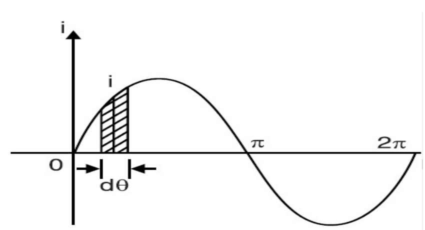
The equation of an alternating current varying sinusoidally is given by ;

$i = I_m \sin\theta$

Consider an elementary strip of thickness $d\theta$ in the first half-cycle of current wave as shown in Fig.Let i be the mid-ordinate of this strip.

Area of strip = $i\, d\theta$

$$\text{Area of half-cycle} = \int_0^{\pi} i\, d\theta = \int_0^{\pi} I_m \sin\theta\, d\theta = I_m[-\cos\theta]_0^{\pi} = 2I_m$$



$$\therefore\quad \text{Average value, } I_{av} = \frac{\text{Area of half-cycle}}{\text{Base length of half-cycle}} = \frac{2I_m}{\pi}$$

$$\text{or}\qquad \boldsymbol{I_{av} = 0{\cdot}637\, I_m}$$

Hence, the half-cycle average value of a.c. is 0·637 times the peak value of a.c.

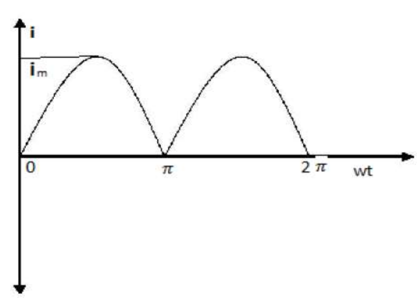
For positive half-cycle, $I_{av} = +\,0{\cdot}637\, I_m$

For negative half-cycle, $I_{av} = -\,0{\cdot}637\, I_m$

Clearly, average value of a.c. over a complete cycle is zero. Similarly, it can be proved that for alternating voltage varying sinusoidally, $\boldsymbol{V_{av} = 0{\cdot}637\, V_m}$.

Average value of a full wave rectifier output

$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t)\, d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$



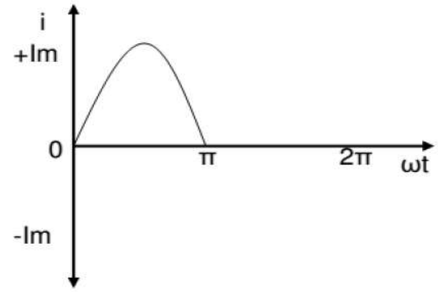
Average value of a half wave rectifier output

$$i = i_m \sin(\omega t)$$

$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i \, d(\omega t)$$

$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i_m \sin(\omega t) d(\omega t)$$

$$i_{avg} = \frac{2 i_m}{2\pi} = 0.318 i_m$$



RMS(Root Mean Square) or Effective value:

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

The r.m.s. value of symmetrical wave can also be expressed as = $\sqrt{\dfrac{area\ of\ half-cycl\ \ of\ squared\ wave}{Half\ cycle\ base}}$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta}$$

R.M.S. Value of Sinusoidal Current

The equation of the alternating current varying sinusoidally is given by ;

$i = Im \sin \theta$

Consider an elementary strip of thickness $d\theta$ in first half-cycle of the squared current wave (shown dotted in Fig. 11.21). Let i^2 be the mid-ordinate of this strip.

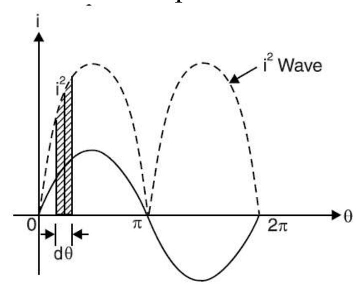
Area of strip $= i^2 d\theta$

Area of half-cycle of the squared wave

$$= \int_0^{\pi} i^2 d\theta$$

$$= \int_0^{\pi} I_m^2 \sin^2 \theta d\theta$$

$$= I_m^2 \int_0^{\pi} \sin^2 \theta d\theta = \frac{{}^*\pi I_m^2}{2}$$



$$\therefore \qquad I_{r.m.s.} = \sqrt{\frac{\text{Area of half-cycle squared wave}}{\text{Half-cycle base}}}$$

$$= \sqrt{\frac{\pi I_m^2/2}{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$\therefore \qquad \boldsymbol{I_{r.m.s.} = 0{\cdot}707\, I_m}$$

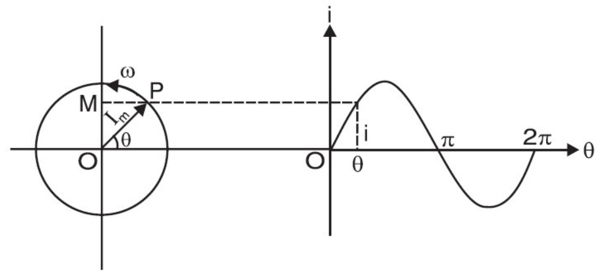
Similarly, it can be proved that for alternating voltage varying sinusoidally, $\boldsymbol{V_{r.m.s.} = 0{\cdot}707\, V_m}$.

- **Phasor Representation of Alternating Quantities**

An alternating quantity can be represented using
(i) Waveform
(ii) Equations
(iii) Phasor

Consider an alternating current represented by the equation $i = Im \sin \omega t$. Take a line *OP* to represent to scale the maximum value *Im*. Imagine the line *OP* (or *phasor,* as it is called) to be rotating in anticlockwise direction at an angular velocity ω rad/sec about the point *O*. Measuring the time from the instant when *OP* is horizontal, let *OP* rotate through an angle θ (= ω*t*) in the anticlockwise direction. The projection of *OP* on the *Y*-axis is *OM*.

$OM = OP \sin \theta$
$= Im \sin \omega t$
$= i$, the value of currentat that instant



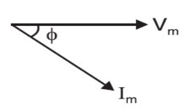
Hence the projection of the phasor *OP* on the *Y*-axis at any instant gives the value of current at that instant. Thus when θ = 90º, the projection on *Y*-axis is *OP* (= *Im*) itself. That the value of current at this instant (*i.e.* at θ or ω*t* = 90º) is *Im* can be readily established if we put θ = 90º in the current equation. If we plot the projections of the phasor on the *Y*-axis *versus* its angular position point-by-point, a sinusoidal alternating current wave is generated as shown in Fig. Thus the phasor represents the sine wave for every instant of time.

Phasor Diagram of Sine Waves of Same Frequency:

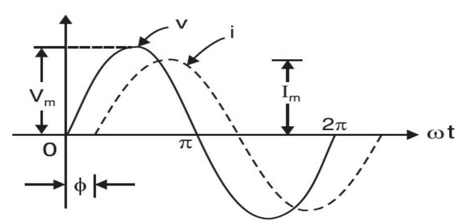
Consider a sinusoidal voltage wave *v* and sinusoidal current wave *i* of the same frequency. Suppose the current lags behind the voltage by φº. The two alternating quantities can be represented on the same phasor diagram because the phasors *Vm* and *Im* [See Fig (*i*)] rotate at the same angular velocity ω and hence phase difference φ between them remains the same at all times.When each phasor completes one revolution, it generates the corresponding cycle [See Fig.(ii)].

The equations of the two waves can be represented as :

$v = Vm \sin \omega t$
$i = Im \sin (\omega t - \varphi)$



(*i*)



(*ii*)

The following points may be noted carefully :

(i) The wave diagram and the phasor diagram convey the same information. However, it is more difficult to draw the waves than to sketch the phasor diagram.

(ii) Since the two phasors have the same angular velocity (ω) and there is no relative motion between them, they can be displayed in a stationary diagram, the common angular rotation (ω*t*) being disregarded.

Analysis of AC Circuits

A.C. Circuit Containing Resistance Only:

When an alternating voltage is applied across pure resistance, then free electrons flow (*i.e.* current) in one direction for the first half-cycle of the supply and then flow in the opposite directionduring the next half-cycle, thus constituting alternating current in the circuit.

Consider a circuit containing a pure resistance of $R\ \Omega$ connected across an alternating voltage source [See Fig. Let the alternating voltage be given by the equation :

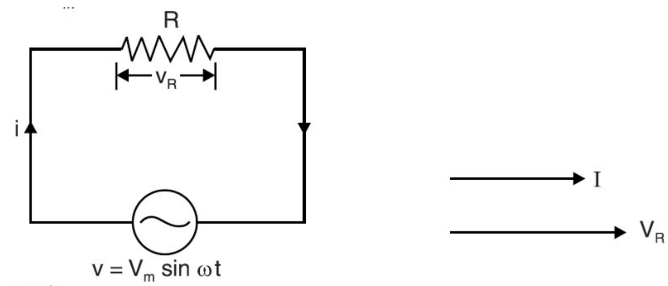
$$v_t = V_m \sin \omega t$$

As a result of this voltage, an alternating current i will flow in the circuit. The applied voltage has to overcome the drop in the resistance only *i.e.*

$$v = i\,R$$

$$\text{or } i = V/R$$

Substituting the value of v, we get,



$$i = \frac{V_m \sin \omega t}{R}$$

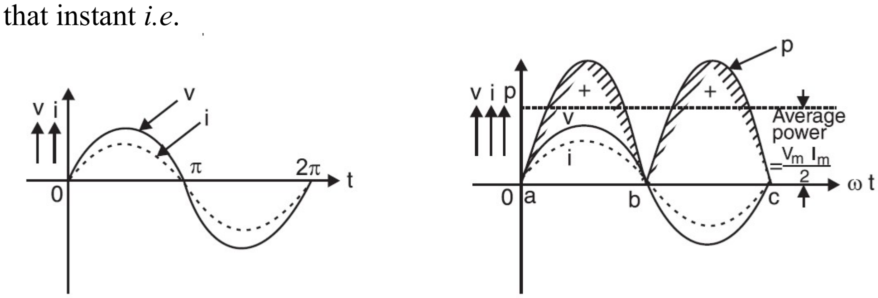
The value of i will be maximum (*i.e. Im*) when $\sin \omega t = 1$.

$$i = I_m \sin \omega t$$

$$I_m = V_m/R$$

Phase angle: From above equations it is clear the current is in phase with voltage for purely resistive circuit. i.e phase angle between voltage and current is zero.

Power: In any circuit, electric power consumed at any instant is the product of voltage and current at that instant *i.e.*



$$\text{Instantaneous power, } p = v\,i = (V_m \sin \omega t)(I_m \sin \omega t) = V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{(1 - \cos 2\omega t)}{2} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Thus power consists of two parts *viz*. a constant part $(V_m I_m/2)$ and a fluctuating part $(V_m I_m/2)\cos 2\omega t$. Since power is a scalar quantity, average power over a complete cycle is to be considered.

$$\therefore \quad \text{Power consumed, } P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} d(\omega t) + \frac{1}{2\pi} * \int_0^{2\pi} \frac{V_m I_m}{2} \cos 2\omega t\, d(\omega t)$$

$$= \frac{V_m I_m}{2} + 0 = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad P = V_R I = V I$$

where $V = V_R$ = r.m.s. value of the applied voltage

I = r.m.s. value of the circuit current

Problems: *An a.c. circuit consists of a pure resistance of 10 Ω and is connected across an a.c. supply of 230 V, 50 Hz. Calculate (i) current (ii) power consumed and (iii) equations for voltage and current.*

Solution. (*i*) Current, $I = V/R = 230/10 =$ **23 A**

(*ii*) Power, $P = VI = 230 \times 23 =$ **5290 W**

(*iii*) Now, $Vm = \sqrt{2}V = \sqrt{2} \times 230 = 325.27$ volts

$Im = \sqrt{2}I = \sqrt{2} \times 23 = 32.52$ A

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/s

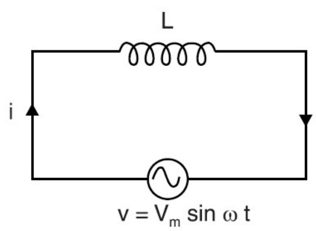
Equations of voltage and current are :

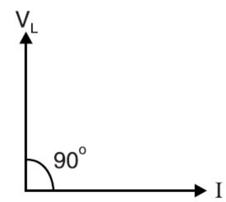
- $\boldsymbol{v = 325{\cdot}27 \sin 314\, t\ ;\ i = 32{\cdot}52 \sin 314\, t}$

A.C. Circuit Containing Pure Inductance Only

When an alternating current flows through a pure inductive coil, a back e.m.f. (= $L\, di/dt$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. Since there is no ohmic drop, the applied voltage has to overcome the back e.m.f. only.

Applied alternating voltage = Back e.m.f.





Equations for Voltage & Current

Consider an alternating voltage applied to a pure inductance of L henry as shown in Fig.
Let the equation of the applied alternating voltage be

$$v = V_m \sin \omega t \qquad \ldots(i)$$

Clearly, $$V_m \sin \omega t = L \frac{di}{dt}$$

or $$di = \frac{V_m}{L} \sin \omega t\, dt$$

Integrating both sides, we get, $i = \dfrac{V_m}{L} \displaystyle\int \sin \omega t\, dt = \dfrac{V_m}{\omega L}(-\cos \omega t)$

$$\therefore \quad i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2) \qquad \ldots(ii)$$

The value of i will be maximum (*i.e.* I_m) when $\sin(\omega t - \pi/2)$ is unity.

$$\therefore \quad I_m = V_m/\omega L$$

Substituting the value of $V_m/\omega L = I_m$ in eq. (*ii*), we get,

$$i = I_m \sin(\omega t - \pi/2) \qquad \ldots(iii)$$

Note that Ohm's law for an inductor states that peak current (Im) through the inductor equals the peak voltage (Vm) across the inductor divided by the inductive reactance ($X_L = \omega L$).

Phase angle: It is clear from eqs. (*i*) and (*iii*) that current lags behind the voltage by π/2 radians or 90º. *Hence in a pure inductance, current lags the voltage by 90º.* This is also indicated by the phasor diagram shown in Fig.

Inductive reactance: Inductance not only causes the current to lag behind the voltage but it also limits the magnitude of current in the circuit. We have seen above that :

$$Im = Vm/\omega L$$

$$\text{Or } \frac{V_m}{I_m} = \omega L$$

Clearly, the opposition offered by inductance to current flow is ω *L*. This quantity ω*L* is called the *inductive reactance XL* of the coil. It has the same dimensions as resistance and is, therefore,measured in Ω.

inductive reactance $XL = \omega L = 2\pi f L$

(*iv*) Power

$$\text{Instantaneous power, } p = v\,i = V_m \sin \omega t \times I_m \sin(\omega t - \pi/2)$$

$$= -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t$$

$$\therefore \quad \text{Average power, } P = \text{Average of } p \text{ over one cycle}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

Hence power absorbed in pure inductance is zero.

Problem: *A pure inductive coil allows a current of 10 A to flow from a 230 V, 50 Hz supply. Find (i) inductive reactance (ii) inductance of the coil (iii) power absorbed. Write down the equations for voltage and current.*

Solution. (*i*) Circuit current, $I = V/X_L$ ($VL = V$)

\ Inductive reactance, $X_L = V/I = 230/10 =$ **23 Ω**

(*ii*) Now, $X_L = 2\pi f L$

$L = X_L/2\pi f =$ **0.073 H**

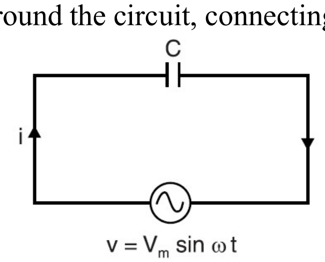
(*iii*) Power absorbed = **Zero**

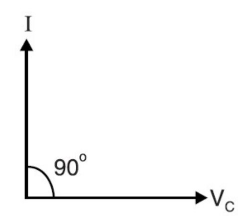
$Vm = 230 \times 2 = 325.27$ V ; $Im = 10 \times 2 = 14.14$ A ; $\omega = 2\pi \times 50 = 314$ rad/s

Since in a pure inductive circuit, current lags behind the voltage by π/2 radians, the equations are : $\boldsymbol{v = 325.27 \sin 314\, t \; ; \; i = 14.14 \sin (314\, t - \pi/2)}$

A.C. Circuit Containing Capacitance Only

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electrons move to and fro around the circuit, connecting the plates, thus constituting alternating current.





Equations for Voltage & Current

Consider an alternating voltage applied to a capacitor of capacitance *C* farad as shown in Fig. Let the equation of the applied alternating voltage be :

$$v = V_m \sin \omega t \qquad ...(i)$$

As a result of this alternating voltage, alternating current will flow through the circuit. Let at any instant i be the current and q be the charge on the plates.

$$\text{Charge on capacitor, } q = C\,v = C\,V_m \sin \omega t$$

$$\therefore \quad \text{Circuit current, } i = \frac{d}{dt}(q) = \frac{d}{dt}(C\,V_m \sin \omega t) = \omega\,C\,V_m \cos \omega\,t$$

$$\therefore \quad i = \omega\,C\,V_m \sin(\omega\,t + \pi/2) \qquad ...(ii)$$

The value of i will be maximum ($i..e.$ I_m) when $\sin(\omega\,t + \pi/2)$ is unity.

$$\therefore \quad I_m = \omega\,C\,V_m$$

Substituting the value $\omega C\,V_m = I_m$ in eq. (ii), we get,

$$i = I_m \sin(\omega\,t + \pi/2) \qquad ...(iii)$$

Phase angle. It is clear from eqs. (i) and (iii) that current leads the voltage by π/2 radians or 90º. *Hence in a pure capacitance, current leads the voltage by 90º.* This is also indicated in the phasor diagram shown in Fig. above

Capacitive reactance. Capacitance not only causes the voltage to lag behind current but it also limits the magnitude of current in the circuit. We have seen above that :

$$I_m = \omega\ C\,V_m$$

$$\frac{V_m}{I_m} = \frac{1}{\omega\ c}$$

Clearly, the opposition offered by capacitance to current flow is $1/\omega C$. This quantity $1/\omega C$ is called the *capacitive reactance XC* of the capacitor. It has the same dimensions as resistance and is, therefore, measured in Ω.

capacitive reactance is $X_C = \frac{1}{\omega\ c} = \frac{1}{2\pi\,f C}$

Note that X_C will be in Ω if C is in farad and f in Hz.

(iv) Power. Instantaneous power is given by ;

$$p = v\,i = V_m \sin \omega\,t \times I_m \sin(\omega\,t + \pi/2) = V_m I_m \sin \omega\,t \cos \omega\,t$$

$$\therefore \quad p = \frac{V_m I_m}{2} \sin 2\omega t$$

$$\therefore \quad \text{Average power, } P = \text{Average of } p \text{ over one cycle}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega\,t\; d(\omega\,t) = 0$$

Hence power absorbed in a pure capacitance is zero.

Problem: *A 318 μF capacitor is connected across a 230 V, 50 Hz system. Determine (i) the capacitive reactance (ii) r.m.s. value of current and (iii) equations for voltage and current.*

Solution. (i) Capacitive reactance, $X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 318} = \mathbf{10\Omega}$

(ii) R.M.S. value of current, $I = V/X_C = 230/10 = \mathbf{23\ A}$

(iii) $V_m = 230 \times \sqrt{2} = 325{\cdot}27$ volts ; $I_m = \sqrt{2} \times 23 = 32{\cdot}53$ A ; $\omega = 2\pi \times 50 = 314$ rad/s

∴ Equations for voltage and current are :

$$\mathbf{v = 325{\cdot}27 \sin 314\,t \quad ; \quad i = 32{\cdot}53 \sin(314\,t + \pi/2)}$$

concept of Impedance:

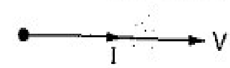
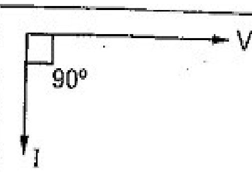
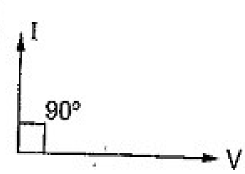
In the alternating circuits alongwith the resistances, inductances and capacitances also play an important role. The inductances are represented by inductive reactances in AC Fundamentals circuits. An inductive reactance is the ohmic representation of an inductance denoted as X_L and given by, $X_L = \omega L = 2\pi f L$ ohms

The capacitances are represented by capacitive reactances in a.c. circuits. A capacitive reactance is the ohmic representation of a capacitance denoted as X_C and given by,

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f c}$ ohms

The combination of R, X_L and X_C present in the circuit is called an impedance of the circuit. The impedance is denoted by letter Z. But the behaviour of R, L and C is different from each other in a.c. circuits hence R, X_L and X_C cannot be algebraically added to find total impedance of the circuit.

Let us summarize the behaviour of R, L and C in the tabular form.

Parameter	Characterisitics	Impedance in rectangular form	Impedance in polar form	Phasor diagram
Pure resistance R	V and I are in phase	$Z = R + j0$	$Z = R\angle 0°$	
Pure inductance L	I lags V by 90°	$Z = 0 + j X_L$	$Z = X_L \angle +90°$	
Pure capacitance C	I leads V by 90°	$Z = 0 - j X_C$	$Z = X_C \angle -90°$	

Inductive reactances are represented by positive sign + X_L in the impedance while capacitive reactances are represented by negative sign – X_C in the impedance.

Thus for R – L series circuit, the impedance is represented as,

$$Z = R + j X_L = |Z| \angle \theta° \Omega$$

where $$|Z| = \sqrt{R^2 + (X_L)^2} \quad \text{and } \theta = \tan^{-1} \frac{X_L}{R}$$

In such circuit, current lags voltage by angle θ.

For R-C series circuit, the impedance is represented as,

$$Z = R - j X_C = |Z| \angle \theta°$$

$$|Z| = \sqrt{R^2 + (X_C)^2} \quad \text{and } \theta = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

In this case θ is negative and current leads voltage by angle θ.

Power Factor:

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It is also defined as the ratio of resistance to the impedance. It is denoted as cos φ.

$$\cos\phi = \text{p.f.} = \frac{R}{Z}$$

For pure L and C, $\phi = 90°$ hence the p.f. is zero.

For other combinations, the p.f. is defined as lagging or leading i.e. whether the resultant current lags or leads the supply voltage.

R-L Series A.C. Circuit:

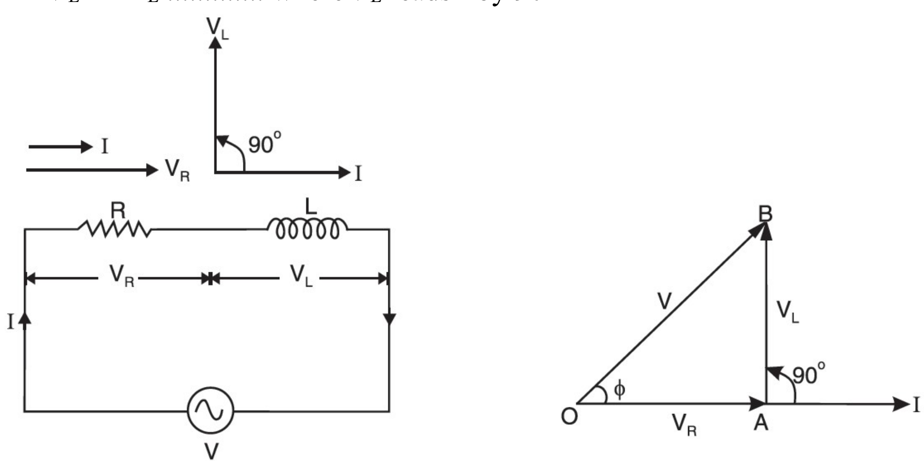
This is the most general case met in practice as nearly all a.c. circuits contain both resistance and inductance. Fig. 12.1 (*i*) shows a pure resistance of *R* ohms connected in series with a coil of pure inductance *L* henry.

Let V = r.m.s. value of the applied voltage

I = r.m.s. value of the circuit current

then $V_R = I R$ .... where V_R is in phase with I

$V_L = I X_L$ .... where V_L leads I by 90°



Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 12.1 (*ii*). The voltage drop V_R ($= I R$) is in phase with current and is represented in magnitude and direction by the phasor *OA*. The voltage drop V_L ($= I X_L$) leads the current by 90° and is represented in magnitude and direction by the phasor *AB*. The applied voltage *V* is the phasor sum of these two drops *i.e.*

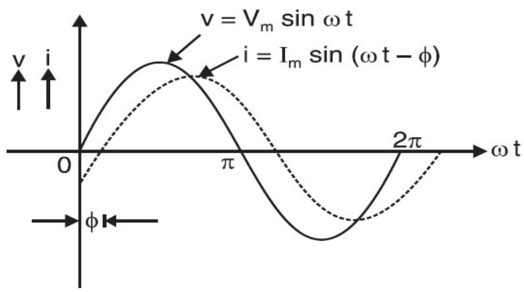
$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called **impedance** of the circuit. It is represented by Z and is measured in ohms (Ω).

$$\therefore \qquad I = \frac{V}{Z} \quad \text{where } Z = \sqrt{R^2 + X_L^2}$$

Phase angle. It is clear from the phasor diagram that circuit current I lags behind the applied voltage V by f°. This fact is also illustrated in the wave diagram shown in Fig. . The value of phase angle Φ can be determined from the phasor diagram.



$$\tan\phi = \frac{V_L}{V_R} = \frac{I X_L}{I R} = \frac{X_L}{R}$$

Since X_L and R are known, f can be calculated.If the applied voltage is $v = V_m \sin \omega t$, then equation for the circuit current will be :

$$i = Im \sin(\omega t - \Phi) \qquad \text{where } Im = Vm/Z$$

We arrive at a very important conclusion that *in an inductive circuit, current lags behind the applied voltage*.

Impedance. *The total opposition offered to the flow of alternating current by a circuit is called* **impedance** *Z of the circuit*. In *R-L* series circuit,

Impedance, $Z = \sqrt{R^2 + X_L{}^2}$ where $X_L = 2\pi f L$

The magnitude of impedance in *R-L* series circuit depends upon the values of *R, L* and the supply frequency *f.*

Power

Instantaneous power, $p = v\,i = Vm \sin wt \times Im \sin(wt - \Phi)$

$$= \frac{1}{2} V_m I_m \,[2 \sin \omega t \sin(\omega t - \phi)]$$

$$= \frac{1}{2} V_m I_m \,[\cos\phi - \cos(2\omega t - \phi)]$$

$$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

Thus instantaneous power consists of two parts :

(*a*) Constant part $\frac{1}{2} V_m I_m \cos\phi$ whose average value over a cycle is the same.

(*b*) A pulsating component $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ whose average value over one complete cycle is zero.

$$\therefore \qquad \text{Average power, } P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$$

or

$$P = V I \cos\phi$$

where V and I are the r.m.s. values of voltage and current. The term $\cos\phi$ is called **power factor** of the circuit and its value is given by (from phasor diagram) ;

$$\text{Power factor, } \cos\phi = \frac{IR}{IZ} = \frac{R}{Z}$$

Problem. *A coil having a resistance of 7* W *and an inductance of 31·8 mH is connected to 230 V, 50 Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed and (v) voltage drop across resistor and inductor.*

Solution. (*i*) Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Coil impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$

$\therefore$ Circuit current, $I = V/Z = 230/12.2 =$ **18·85 A**

(*ii*) $\tan\phi = X_L/R = 10/7$

$\therefore$ Phase angle, $\phi = \tan^{-1}(10/7) =$ **55° lag**

(*iii*) Power factor $= \cos\phi = \cos 55° =$ **0.573 lag**

(*iv*) Power consumed, $P = VI\cos\phi = 230 \times 18.85 \times 0.573 =$ **2484·24 W**

(*v*) Voltage drop across $R = IR = 18.85 \times 7 =$ **131.95V**

Voltage drop across $L = IX_L = 18.85 \times 10 =$ **188.5 V**

Problem.*An inductor coil is connected to a supply of 250 V at 50 Hz and takes a current of 5 A. The coil dissipates 750 W. Calculate (i) power factor (ii) resistance of coil and (iii) inductance of coil.*

Solution. (*i*) Power consumed, $P = VI\cos\phi$

$\therefore$ Power factor, $\cos\phi = \dfrac{P}{VI} = \dfrac{750}{250\times 5} =$ **0.6 lag**

(*ii*) Impedance of coil, $Z = V/I = 250/5 = 50\ \Omega$

Resistance of coil, $R = Z\cos\phi = 50 \times 0.6 =$ **30 Ω**

(*iii*) Reactance of coil, $X_L = \sqrt{Z^2 - R^2} = \sqrt{(50)^2 - (30)^2} = 40\ \Omega$

$\therefore$ Inductance of coil, $L = \dfrac{X_L}{2\pi f} = \dfrac{40}{2\pi\times 50} =$ **0.127 H**

Problem.. *A pure inductance of 318 mH is connected in series with a pure resistance of 75*W. *The circuit is supplied from 50 Hz source and the voltage across 75 ohms resistor is found to be 150 V. Calculate the supply voltage and the phase angle.*

Solution. The circuit diagram and the phasor diagram are shown in Fig. 12.8.

Circuit current, $I = V_R/R = 150/75 = 2\ \text{A}$

Reactance of coil, $X_L = 2\pi f L = 2\pi \times 50 \times 318 \times 10^{-3} = 100\ \Omega$

Voltage across L, $V_L = IX_L = 2 \times 100 = 200\ \text{V}$

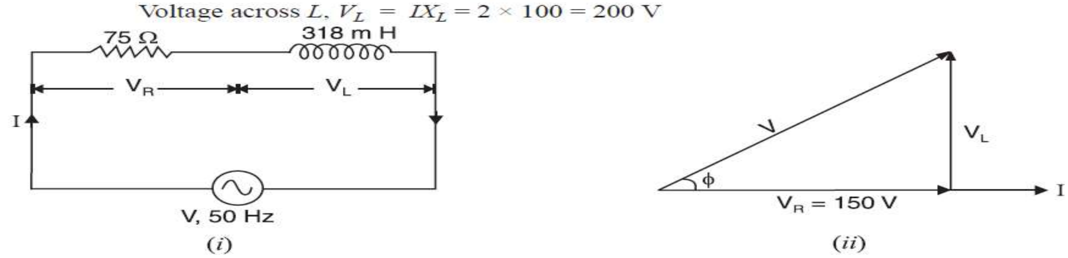


Fig. 12.8

Referring to the phasor diagram of the circuit in Fig. 12.8 (*ii*),

Supply voltage, $V = \sqrt{V_R^2 + V_L^2} = \sqrt{150^2 + 200^2} =$ **250 V**

R-C Series A.C. Circuit

Below Fig. shows a resistance of *R* ohms connected in series with a capacitor of *C* farad.

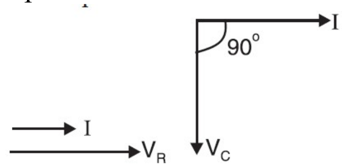
Let V = r.m.s. value of applied voltage

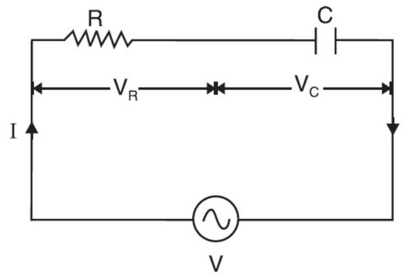
I = r.m.s. value of the circuit current

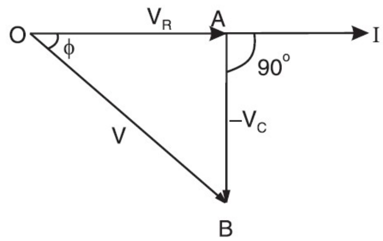
$V_R = IR$ .... where V_R is in phase with I

$V_C = IX_C$ .... where V_C lags I by 90°

Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 12.19. The voltage drop V_R (= IR) is in phase with current and is represented in magnitude and direction by the phasor OA. The voltage drop V_C (= IX_C) lags behind the current by 90° and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drops *i.e.*





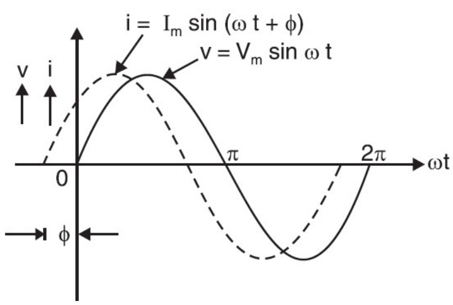


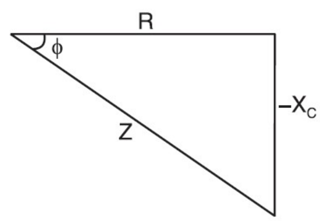
$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2 + X_c{}^2}$ offers opposition to current flow and is called **impedance** of the circuit.

I=V/Z   Where Z= $\sqrt{R^2 + X_c{}^2}$





Phase angle: It is clear from the phasor diagram that circuit current I leads the applied voltage V by f°. This fact is also illustrated in the wave diagram (See Fig. 12.20) and impedance triangle (See Fig. 12.21) of the circuit. The value of the phase angle can be determined as under :

$$\tan\phi = -\frac{V_C}{V_R} = -\frac{IX_C}{IR} = -\frac{X_C}{R}$$

Since current is taken as the reference phasor, negative phase angle implies that voltage lags behind the current. This is the same thing as current leads the voltage.

If the applied voltage is $v = V_m \sin \omega t$, then equation for the circuit current will be :
$i = Im \sin(\omega t - \Phi)$ where $Im = Vm/Z$

Power: The equations for voltage and current are :

$v = Vm \sin wt$; $i = Im \sin(wt + \Phi)$

therefore , Average power, P = Average of vi

$= VI \cos \Phi$

Problem: *A capacitor of capacitance 79·5 μ F is connected in series with a non-inductive resistance of 30 Ω across 100 V, 50 Hz supply. Find (i) impedance (ii) current (iii) phase angle and (iv) equation for the instantaneous value of current.*

Solution. *(i)* Capacitive reactance, $X_C = \dfrac{1}{2\pi f C} = \dfrac{10^6}{2\pi \times 50 \times 79 \cdot 5} = 40\ \Omega$

Circuit impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \mathbf{50\ \Omega}$

(ii) Circuit current, $I = V/Z = 100/50 = \mathbf{2\ A}$

(iii) $\tan\phi = X_C/R = 40/30 = 1.33$

$\therefore$ Phase angle, $\phi = \tan^{-1} 1.33 = \mathbf{53^\circ}$ ***lead***

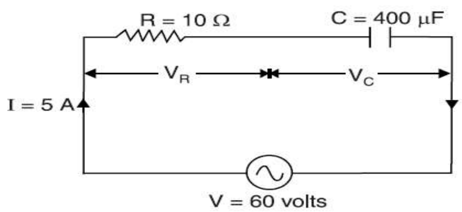
(iv) $I_m = 2 \times \sqrt{2} = 2.828$ A

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/sec.

$\therefore$ $i = \mathbf{2.828 \sin (314\, t + 53^\circ)}$

Problem: *A 10 Ω resistor and 400 μF capacitor are connected in series to a 60-V sinusoidal supply. The circuit current is 5 A. Calculate the supply frequency and phase angle between the current and voltage.*

Solution. Fig. 12.22 (*i*) shows the circuit diagram whereas Fig. 12.22 (*ii*) shows phasor diagram.



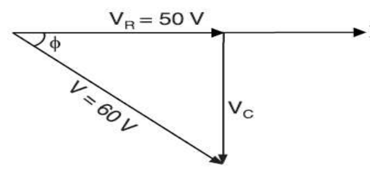


Fig. 12.22

Voltage across R, $V_R = IR = 5 \times 10 = 50$ V

Voltage across C, $V_C = \sqrt{V^2 - V_R^2} = \sqrt{60^2 - 50^2} = 33.17$ V

Reactance of capacitor, $X_C = V_C/I = 33 \cdot 17/5 = 6 \cdot 634\ \Omega$

$\therefore$ Supply frequency, $f = \dfrac{1}{2\pi C X_C} = \dfrac{10^6}{2\pi \times 400 \times 6 \cdot 634} = \mathbf{60\ Hz}$

$\tan\phi = V_C/V_R = 33.17/50 = 0.6634$

$\therefore$ Phase angle, $\phi = \tan^{-1} 0.6634 = \mathbf{33.6^\circ}$ **lead**

R-L-C Series A.C. Circuit:

This is a general series a.c. circuit. Below Fig. shows R, L and C connected in series across a supply voltage V (*r.m.s.*). The resulting circuit current is I (*r.m.s.*).

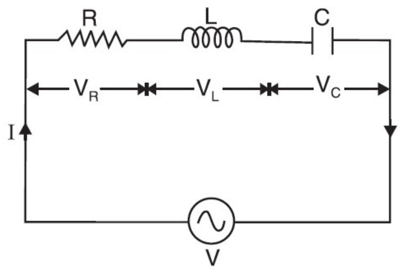
therefore Voltage across R, $V_R = I R$ ... VR is in phase with I

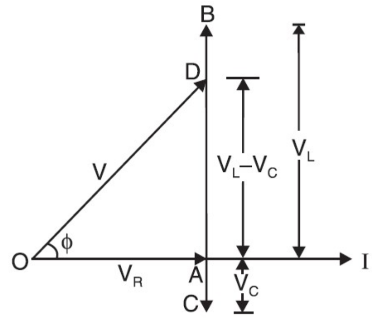
Voltage across L, $V_L = I X_L$ ... where V_L leads I by 90°

Voltage across C, $V_C = I X_C$ ... where V_C lags I by 90°

As before, the phasor diagram is drawn taking current as the reference phasor. In the phasor diagram (See Fig. 12.33), OA represents V_R, AB represents V_L and AC represents V_C. It may be seen that V_L is in phase opposition to V_C. It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. For the case considered, $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ and is represented by AD.

Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and is represented by OD.





$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$
$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The quantity $\sqrt{R^2 + (X_L - X_c)^2}$ offers opposition to current flow and is called **impedance** of the circuit.

Circuit power factor, $\cos \Phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_c)^2}}$

Also, $\tan \Phi = \frac{v_L - v_c}{R} = \frac{X_L - X_c}{R}$

Since X_L, X_C and R are known, phase angle Φ of the circuit can be determined.

Power consumed, $P = VI\cos\Phi = I^2R$

Three cases of *R-L-C* series circuit. We have seen that the impedance of a *R-L-C* series circuit is given by ;

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

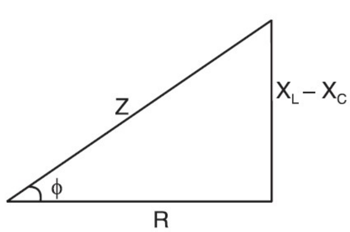
(*i*) When $X_L - X_C$ is positive (*i.e.* $X_L > X_C$), phase angle Φ is positive and the circuit will be inductive. In other words, in such a case, the circuit current *I* will lag behind the applied voltage *V* by Φ ; the value of f being given by eq. (*ii*) above.

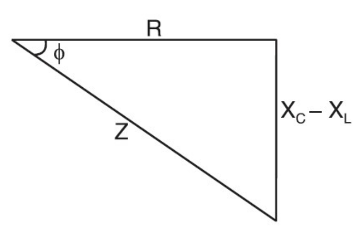
(*ii*) When $X_L - X_C$ is negative (*i.e.* $X_C > X_L$), phase angle Φ is negative and the circuit is capacitive. That is to say the circuit current *I* leads the applied voltage *V* by Φ ; the value of f being given by eq. (*ii*) above.

(*iii*) When $X_L - X_C$ is zero (*i.e.* $X_L = X_C$), the circuit is purely resistive. In other words, circuit current *I* and applied voltage *V* will be in phase *i.e.* Φ = 0°. The circuit will then have unity power factor.

If the equation for the applied voltage is $v = V_m \sin \omega t$, then equation for the circuit current will be : $i = I_m \sin(\omega t \pm \Phi)$ where $I_m = V_m/Z$

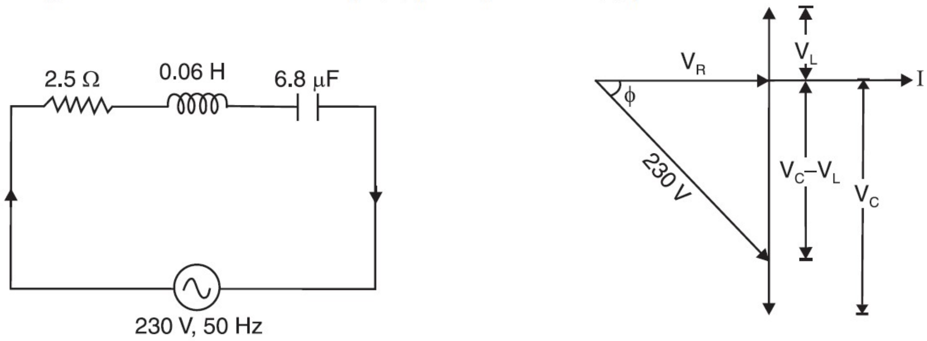
The value of f will be positive or negative depending upon which reactance (X_L or X_C) predominates.





Above Fig. (i) shows the impedance triangle of the circuit for the case when $X_L > X_C$ whereas impedance triangle in Fig. (*ii*) is for the case when $X_C > X_L$.

Problem *:A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5* W *resistance connected in series with a 6·8* μ*F capacitor. Calculate (i) impedance (ii) current (iii)phase angle between current and voltage (iv) power factor and (v) power consumed*.



(*i*) (*ii*)

Solution. Fig. 12.35 (*i*) shows the conditions of the problem.

$$X_L = 2\,\pi f L = 2\,\pi \times 50 \times 0.06 = 18.85\ \Omega$$

$$X_C = \frac{1}{2\pi\, f\, C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\ \Omega$$

(*i*) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.5)^2 + (18.85 - 468)^2} =$ **449.2 Ω**

(*ii*) Circuit current, $I = V/Z = 230/449.2 =$ **0.512 A**

(*iii*) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{18.85 - 468}{2.5} = -\,179.66$

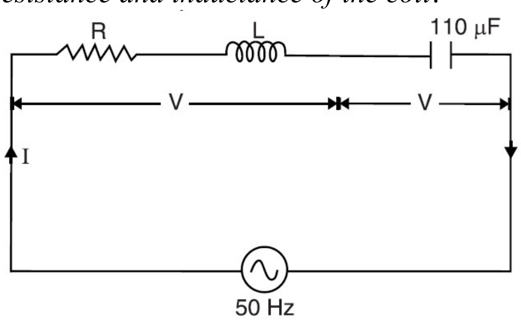
∴ Phase angle, $\phi = \tan^{-1} - 179.66 = -\,89.7° =$ **89.7° *lead***

The negative sign with ϕ shows that current is leading the voltage [See the phasor diagram in Fig. 12.35 (*ii*)].

(*iv*) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{2.5}{449.2} =$ **0.00557 *lead***

(*v*) Power consumed, $P = VI\cos\phi = 230 \times 0.512 \times 0.00557 =$ **0.656 W**

Problem: *A coil of p.f. 0.8 is connected in series with a 110 μF capacitor. The supply frequency is 50 Hz. The p.d. across the coil is found to be equal to the p.d. across the capacitor.Calculate the resistance and inductance of the coil*.



Solution. Fig. 12.36 shows the conditions of the problem.

Reactance of capacitor, $X_C = \dfrac{1}{2\pi\, f\, C} = \dfrac{10^6}{2\pi \times 50 \times 110} = 29\ \Omega$

Now, $I\,Z_{coil} = IX_C \ \therefore\ Z_{coil} = X_C = 29\ \Omega$

For the coil, $\cos\phi = R/Z_{coil} \ \therefore\ R = Z_{coil}\cos\phi = 29 \times 0.8 =$ **23.2 Ω**

Reactance of coil, $*X_L = Z_{coil}\sin\phi = 29 \times 0.6 = 17.4\ \Omega$

∴ Inductance of coil, $L = \dfrac{X_L}{2\pi f} = \dfrac{17.4}{2\pi \times 50} =$ **0.055 H**

Apparent, True and Reactive Powers
Consider an inductive circuit in which circuit current I lags behind the applied voltage V by $\Phi°$. The phasor diagram of the circuit is shown in Fig. . The current I can be resolved into two rectangular components *viz*.

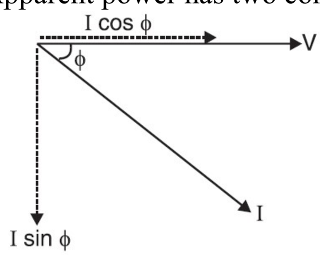
(*i*) $I \cos \Phi$ in phase with V.

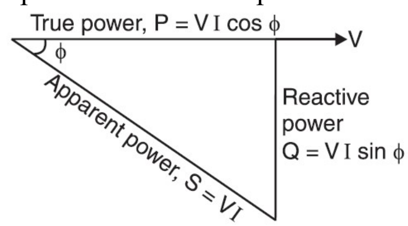
(*ii*) $I \sin \Phi$; 90° out of phase with V.

1. Apparent power. *The total power that appears to be transferred between the source and load is called* **apparent power.** It is equal to the product of applied voltage (V) and circuit current (I) *i.e.* Apparent power, $S = V \times I = VI$

It is measured in volt-ampers (VA).

Apparent power has two components *viz* true power and reactive power.





2. True power. The power which is actually consumed in the circuit is called **true power** or **active power.** We know that power is consumed in resistance only since neither pure inductor (L) norpure capacitor (C) consumes any active power. Now, current and voltage are in phase in a resistance.

Therefore, current in phase with voltage produces true or active power. It is the useful component of apparent power.

The product of voltage (V) and component of total current in phase with voltage (I cos f) *is equal to* **true power** *i.e.*

True power, P = Voltage × Component of total current in phase with voltage

$$= V \times I \cos \Phi$$

$$P = VI \cos \Phi$$

It is measured in watts (W).

3. Reactive power. The component of apparent power which is neither consumed nor does any useful work in the circuit is called **reactive power.** The power consumed (or true power) in L and C is zero because all the power received from the source in one quarter-cycle is returned to the source in the next quarter-cycle. This circulating power is called *reactive power. Now, current and voltage in L or C are 90° out of phase. Therefore, current 90° out of phase with voltage contributes to reactive power.

The product of voltage (V) and component of total current 90° out of phase with voltage (I sin f) *is equal to* **reactive power** *i.e.*

Reactive power, Q = Voltage × Component of total current 90° out of phase with voltage

$$= V \times I \sin \Phi$$

$$Q = VI \sin \Phi$$

It is measured in volt-amperes reactive (VAR).