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Vectors and Matrices.

1. Inner product of y and z.

$$y = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$y^T z = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 11.$$

2. $$XY = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 10 \end{bmatrix}$$

3. $\because$ $\det(x) \neq 0$. it is invertible.

$$x^{-1} = \frac{1}{|x|} \text{Adj} |x|$$

$$= \frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}$$

4. Rank of the matrix = 2.

$\because$ This is a 2x2 square matrix and its det $\neq 0$, the order of matrix = rank of the matrix.

Calculus.

1. $y = x^3 + x - 5.$

$$\frac{dy}{dx} = 3x^2 + 1$$

2. $f(x_1, x_2) = x_1 \sin(x_2) e^{-x_1}$

$$\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{bmatrix} \sin x_2 e^{-x_1} - x_1 \sin(x_2) e^{-x_1} \\ x_1 \cos(x_2) e^{-x_1} \end{bmatrix} = \begin{bmatrix} \sin(x_2) e^{-x_1}(1 - x_1) \\ x_1 \cos x_2 e^{-x_1} \end{bmatrix}$$

Probability and statistics.

1. $S.\ Mean = \frac{\Sigma x_i}{N} = \frac{3}{5}$

2. $S.\ Variance = \frac{\Sigma (X_i - \bar{x})^2}{n-1}$

(~~Note~~) Taking n-1 to get unbiased estimate of variability.

$$= \frac{1}{4}\left[\frac{4}{25} + \frac{4}{25} + \frac{9}{25} + \frac{4}{25} + \frac{9}{25}\right]$$

$$= \frac{10}{25}.$$

$12+$
$18+$
$\frac{12}{40}$

3. Probability of sample $= (0.5)^5 = 1/32$.

4. The log likelihood function

$\ln L(x_1, x_2 \dots x_n ; \theta)$ is given as:

$$L(x:\theta) = \sum_{i=1}^{5} x_i \log p + \left(n - \sum_{i=1}^{5} x_i\right) \log(1-p).$$

To find where p maximises. derivative wrt p. equate to 0.

$$\frac{dL}{dp} = \frac{1}{p}\sum_{i=1}^{5} x_i - \frac{1}{1-p}\left(n - \sum_{i=1}^{5} x_i\right) = 0$$

$$= \frac{(1-p)\Sigma x_i - p(n - \Sigma x_i)}{p(1-p)}$$

$$= \frac{\Sigma x_i - pn}{p(1-p)} = 0.$$

$$pn = \sum_{i=1}^{5} x_i$$

$$p = \frac{\sum_{i=1}^{5} x_i}{n}$$

$\therefore p = 3/5$ maximises

This p = 0 or 1 is much less are both head and tails are their with eq probability.

- $P(x=T \text{ and } y=b) = 0.1$.

- $P(x=T \mid y=b) = \frac{P(x=T \cap x=b)}{P(y=b)} = \frac{0.1}{0.28} = \underline{0.4}$.

Big-O Notation

1. Both true

The difference is only on the base number. which is multiplicative

2. $g(n) = O(f(n))$.

$f(n)$ grows faster as n ~~grows~~ is in exponential.

3.) $g(n) = O(f(n))$

$f(n)$ grows faster

(4) $f(n) = O(g(n))$

n^3 grows faster than n^2.

Probability and Random Variable.

(a) False

(b) True.

(c). False.

(d) False.

(e). True.

Discrete and Continuous distribution:

(a) — (h)

(b) — (e)

(c) — (f)

(d) — (g).

Mean Variance and Entropy.

(a) $Var(X) = \mathbb{E}[(X - \mathbb{E}(X))^2]$

$= \mathbb{E}[X^2 - 2X\mathbb{E}(X) + \mathbb{E}(X)^2]$

$= \mathbb{E}[X^2] - \mathbb{E}[2X\mathbb{E}[X]] + \mathbb{E}[\mathbb{E}[X]^2]$

$= \mathbb{E}[X^2] - 2\mathbb{E}(X^2) + (\mathbb{E}(X))^2$.

$= \mathbb{E}[X]^2 - \mathbb{E}[X^2]$

(b) Mean :- p.

Varian :- $p(1-p)$

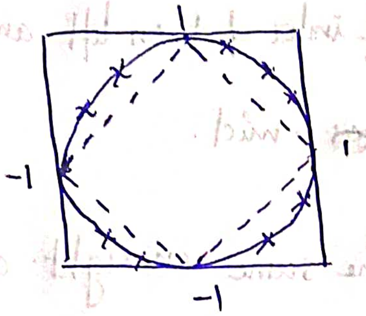
Entropy :- $-(1-p)\log(1-p) - p\log p$

Law of Large Numbers and central limit theorem.

(a) Law of Large Numbers theorem:

(b). Central Limit theorem as $n \to \infty$.

Linear Algebra.



(a) $\times\!\!-\!\!\times$

(b) - - -

(c) —

Geometry.

(a) Consider two points x_1, x_2 that lie on the line.

To show w is orthogonal to $w^T x + b$.

$$w^T x_1 + b = 0.$$

$$w^T x_2 + b = 0$$

$$w^T (x_1 - x_2) = 0.$$

$\therefore$ w^T is orthogonal to the line.

(b) Taking $w^T x + b = 0$ hyperplane and a point x.

The distance by taking projection onto normal vector w.

The distance $= \frac{|w^T x|}{\|w\|^2} = \frac{-b}{\|w\|^2} = \frac{|b|}{\|w\|^2}$

Algorithms.

1) Take 2 variables l and k, $l=0$ and $k = len(array) - 1$.
p variable to ans.

2) Check, if the element in middle index is 1 or 0

$mid = (l+k)/2$

3) If 1: check min of index of 1 on left and update k

$k = mid - 1$, $p = mid$.

4) If 0: check for the same on right and update l

$l = m+1$

5) repeat from step 2 till $l < k$.

Here the array is split to two and some linear time arithmetic.
As this recurse, we get running time

$$T(n) = 2T(n/2) + O(n) \equiv O(n \log n).$$