I study **arithmetic geometry** and **arithmetic statistics**. The fundamental question is to understand all *integral* or *rational* solutions to $f = 0$, where f is a polynomial in n variables. That is, find all tuples $(a_1, \ldots, a_n)$ of integers or rationals (fractions) such that $f(a_1, \ldots, a_n) = 0$. Surprisingly, this is closely related to studying the geometry of the associated space $X(\mathbb{C}) \coloneqq \{\vec{a} = (a_1, \ldots, a_n) \in \mathbb{C}^n : f(\vec{a}) = 0\}$ of *complex* solutions. While I have studied such questions when X is a high-dimensional space/shape [AM23], we will focus on the case of curves in this proposal, i.e. on polynomials $f(x, y)$ in two variables. Studying the rational solutions to $f = 0$ is completely understood when f is of degree 1 or 2, but much remains open for f of degree 3. This case boils down to studying *elliptic curves* which are the curves $E_{a,b}$ cut out by equations of the form $F_{a,b}(x, y) = y^2 - (x^3 + ax + b)$, where a, b are integers. I propose to study these and related objects along the following two themes.

(1) Geometry of the family $\{E_{a,b}\}$ of all elliptic curves, i.e. **Brauer groups of moduli stacks**.

Remarkably, the *collection of all $E_{a,b}$'s* itself forms a geometric object, the 'moduli space of elliptic curves' $Y(1)$. A *rational point* on $Y(1)$ is, equivalently, a choice of some $E_{a,b}$.[1] Moduli spaces (spaces which parameterize other geometric objects of interest) are often studied in mathematics. In [ABJ+24, Ach24], we study a particular geometric invariant (the 'Brauer group') of spaces like $Y(1)$, building on earlier work of Antieau–Meier [AM20]. Interests in this invariants stems from its ability to obstruct points on these spaces. *I propose to both expand the techniques of* [Ach24] *so they apply to more general classes of moduli spaces and to use these Brauer groups to prove that $Y(1)$ has no integral points*[2] This is well-known via other means, but I hope a Brauer obstruction-theoretic proof may better generalize to studying integral points on some currently less well understood moduli spaces.

(2) Statistics on elliptic curves, i.e. **arithmetic statistics of Selmer groups**.

One special feature of elliptic curves is that there is a procedure which takes any two *rational points* (x_1, y_1) and (x_2, y_2) on $E_{a,b}$ and produces a third one (x_3, y_3). Often, repeating this procedure allows one to construct infinitely many rational points on this curve. This naturally leads one to ask, "How many points on $E_{a,b}$ must one start with in order to generate all of them, up to finitely many exceptions?" This number is called the *rank* of $E_{a,b}$. There is a conjectural algorithm for computing these ranks, but proving it always works remains an open problem. Instead, it is more tractable to obtain unconditional results about the *distribution* of their ranks. There is a deep conjectural prediction [BKL+15] for this and related distributions. The first step in studying this distribution is bounding the *average rank* of elliptic curves (see e.g. [dJ02, BS15]), which is initially not even obviously finite. This problem is studied not just over the rational numbers, but also over other "number systems" such as *global function fields*, where ratios of polynomials play the role of ratios of integers. In this setting, I [Ach23] gave the first proof that this average rank is finite over every global function field. *I propose to both improve the average rank bound I obtained and to study theoretic limitations of the "parameterize-and-count" strategy typically employed in this area.*[3]

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[1] Up to *isomorphism*; if there is a nice matching between points of $E_{a,b}$ and $E_{c,d}$, they correspond to the same raional point on $Y(1)$.

[2] For the experts, I hope to show that the integral étale-Brauer obstruction set for the moduli *stack* of elliptic curves is empty.

[3] For the experts, I would like to find an explicit value of n for which n-Selmer elements (thought of as genus 1, degree n curves in $\mathbb{P}^{n-1}$) provable cannot be parameterized in a way analogous to the known parameterizations for $n \leq 5$.