



Pi



Notations

Traditional name

p

Traditional notation

p

Mathematica StandardForm notation

Pi

Primary definition

02.03.02.0001.01

$$p \hat{=} \sum_{k=0}^{\infty} \frac{1}{2^{k+1}}$$

Specific values

02.03.03.0001.01

$$p \hat{=} 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534 \frac{1}{4}$$

Above approximate numerical value of p shows 90 decimal digits.

General characteristics

The pi p is a constant. It is irrational and transcendental over \mathbb{Q} positive real number.

Series representations

Generalized power series

Expansions for p

02.03.06.0001.01

$$p \hat{=} \sum_{k=0}^{\infty} \frac{1}{16^k} \left(-\frac{2r+1}{8k+5} - \frac{2r+1}{8k+6} + \frac{8r+4}{8k+1} + \frac{r}{8k+7} - \frac{8r}{8k+2} - \frac{4r}{8k+3} - \frac{8r+2}{8k+4} \right); r \in \mathbb{N}^+$$

02.03.06.0002.01

$$p \tilde{S} 2 \log \mathbb{H} L + 4 \hat{a} \sum_{k=0}^{\infty} \frac{1}{k+1} \mathbb{H} 1 L^{\frac{k+1}{2} \nu}$$

02.03.06.0003.01

$$p \tilde{S} 4 \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{2k+1}$$

02.03.06.0004.01

$$p \tilde{S} \frac{4}{\sqrt{2}} \hat{a} \sum_{k=0}^{\infty} \frac{1}{2k+1} \mathbb{H} 1 L^{\frac{k}{2} \nu}$$

02.03.06.0005.01

$$p \tilde{S} 16 \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{\mathbb{H} k + 1 L 5^{2k+1}} - 4 \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{\mathbb{H} k + 1 L 239^{2k+1}}$$

02.03.06.0006.01

$$p \tilde{S} 3 \sqrt{3} \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{3k+1} - \log \mathbb{H} L \sqrt{3}$$

02.03.06.0007.01

$$p \tilde{S} 4 \sqrt{2} \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{4k+1} - 2 \log L + \sqrt{2} N$$

02.03.06.0008.01

$$p \tilde{S} \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{4^k} \left(\frac{2}{4k+2} + \frac{1}{4k+3} + \frac{2}{4k+1} \right)$$

02.03.06.0009.01

$$p \tilde{S} \hat{a} \sum_{k=1}^{\infty} \frac{1}{k^3} \left(\frac{285}{2 \mathbb{H} k + 1 L} - \frac{667}{32 \mathbb{H} k + 1 L} - \frac{5103}{16 \mathbb{H} k + 3 L} + \frac{35625}{32 \mathbb{H} k + 5 L} - \frac{238}{k+1} \right)$$

02.03.06.0010.01

$$p \tilde{S} \frac{1}{\sqrt{2}} \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} 1 L^k}{8^k} \left(\frac{1}{6k+3} + \frac{1}{6k+5} + \frac{4}{6k+1} \right)$$

02.03.06.0011.01

$$p \tilde{S} \frac{8}{1 + \sqrt{2}} \hat{a} \sum_{k=0}^{\infty} \left(\frac{1}{8k+1} - \frac{1}{8k+7} \right)$$

02.03.06.0012.01

$$p \tilde{S} \hat{a} \sum_{k=0}^{\infty} \frac{1}{16^k} \left(-\frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} + \frac{4}{8k+1} \right)$$

02.03.06.0013.01

$$p \tilde{S} 2 \hat{a} \sum_{k=0}^{\infty} \frac{\mathbb{H} k - 1 L!!}{\mathbb{H} k + 1 L \mathbb{H} k L!!}$$

02.03.06.0014.01

$$p \S \frac{5}{4} \sqrt{5} \hat{a} \sum_{k=0}^{\infty} \binom{2k}{k} \frac{1}{2k+1} F_{2k+1} 16^{-k}$$

02.03.06.0015.01

$$p \S \sqrt{5} \hat{a} \sum_{k=0}^{\infty} \frac{H_1 L^k 2^{2k+3} F_{2k+1}}{H_2 k + 1 L J \sqrt{5} + 3 N} 2^{k+1}$$

02.03.06.0016.01

$$p \S 20 \hat{a} \sum_{k=0}^{\infty} \frac{H_1 L^k F_{2k+1}^2}{H_2 k + 1 L I \sqrt{10} + 3 N} 2^{k+1}$$

02.03.06.0017.01

$$p \S 12 \sqrt{5} \hat{a} \sum_{k=0}^{\infty} \frac{H_1 L^k}{2k+1} \left(\frac{2 I_2 - \sqrt{3} N}{\sqrt{16 I_2 - \sqrt{3} N + 1} + \sqrt{5}} \right)^{2k+1} F_{2k+1}$$

02.03.06.0018.01

$$p \S 4 \hat{a} \sum_{k=1}^{\infty} \tan^{-1} \left(\frac{1}{F_{2k+1}} \right)$$

02.03.06.0019.01

$$p \S \hat{a} \sum_{k=1}^{\infty} \frac{I_3^k - 1 M_2 H + 1 L}{4^k}$$

02.03.06.0044.01

$$p \S -3 \sqrt{3} + \frac{9}{2} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k}{\binom{2k}{k}}$$

02.03.06.0045.01

$$p \S -\frac{18 \sqrt{3}}{5} + \frac{27}{10} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^2}{\binom{2k}{k}}$$

02.03.06.0046.01

$$p \S -\frac{135 \sqrt{3}}{37} + \frac{81}{74} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^3}{\binom{2k}{k}}$$

02.03.06.0047.01

$$p \S -\frac{432 \sqrt{3}}{119} + \frac{81}{238} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^4}{\binom{2k}{k}}$$

02.03.06.0048.01

$$p \tilde{S} - \frac{243 \sqrt{3}}{67} + \frac{81}{938} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^5}{\binom{2k}{k}}$$

02.03.06.0049.01

$$p \tilde{S} - \frac{23814 \sqrt{3}}{6565} + \frac{243}{13130} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^6}{\binom{2k}{k}}$$

02.03.06.0050.01

$$p \tilde{S} - \frac{42795 \sqrt{3}}{11797} + \frac{81}{23594} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^7}{\binom{2k}{k}}$$

02.03.06.0051.01

$$p \tilde{S} - \frac{2355156 \sqrt{3}}{649231} + \frac{729}{1298462} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^8}{\binom{2k}{k}}$$

02.03.06.0052.01

$$p \tilde{S} - \frac{48314475 \sqrt{3}}{13318583} + \frac{2187}{26637166} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^9}{\binom{2k}{k}}$$

02.03.06.0053.01

$$p \tilde{S} - \frac{365306274 \sqrt{3}}{100701965} + \frac{2187}{201403930} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^{10}}{\binom{2k}{k}}$$

02.03.06.0054.01

$$p \tilde{S} - \frac{99760005 \sqrt{3}}{27500287} + \frac{6561}{5005052234} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^{11}}{\binom{2k}{k}}$$

02.03.06.0055.01

$$p \tilde{S} - \frac{245273327208 \sqrt{3}}{67613135957} + \frac{19683}{135226271914} \sqrt{3} \hat{a} \sum_{k=1}^{\infty} \frac{k^{12}}{\binom{2k}{k}}$$

02.03.06.0056.01

$$p \ddagger 4 - 2 \hat{a} \sum_{k=0}^{\infty} \frac{k!}{\hat{U}_{j=0}^k \mathbb{H} j + 3L}$$

Candido Otero Ramos (2007)

Expansions for $1 \bullet p$

02.03.06.0020.01

$$\frac{1}{p} \tilde{S} - \frac{2 \sqrt{2}}{9801} \hat{a} \sum_{k=0}^{\infty} \frac{k! \mathbb{H} 6390k + 1103L}{k!^4 396^4 k}$$

$$02.03.06.0021.01$$

$$\frac{1}{p} \tilde{S} \hat{a} \sum_{k=0}^{\infty} \frac{42k+5}{2^{12k+4}} \binom{2k}{k}^3$$

$$02.03.06.0022.01$$

$$\frac{1}{p} \tilde{S} \hat{a} \sum_{k=0}^{\infty} \frac{H_1 L^k H k! H 45 140 134 k + 13 591 409 L}{k!^3 H k! I 640 320^3 M^{\frac{k+1}{2}}}$$

The above Chudnovsky's formula is used for the numerical computation of p in *Mathematica*.

Expansions for p^2

$$02.03.06.0023.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$02.03.06.0024.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=0}^{\infty} \frac{1}{H k + 1 L^2}$$

$$02.03.06.0025.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=1}^{\infty} \frac{H_1 L^{k-1}}{k^2}$$

$$02.03.06.0026.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=1}^{\infty} \frac{k!^2}{k^2 H k!}$$

$$02.03.06.0027.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=0}^{\infty} \frac{H k!!}{H k + 1 L!! 2^{2k+2} H k + 1 L}$$

$$02.03.06.0028.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=1}^{\infty} \frac{1}{k^3} \left(\frac{45}{2 H k + 1 L} + \frac{384}{k+2} - \frac{1215}{2 H k + 3 L} - \frac{12}{k+1} \right)$$

$$02.03.06.0029.01$$

$$p^2 \tilde{S} \hat{a} \sum_{k=0}^{\infty} \left(-\frac{3}{2 H k + 2 L^2} - \frac{1}{2 H k + 3 L^2} - \frac{3}{8 H k + 4 L^2} + \frac{1}{16 H k + 5 L^2} + \frac{1}{H k + 1 L^2} \right) 64^{-k}$$

Expansions for p^3

$$02.03.06.0030.01$$

$$p^3 \tilde{S} \hat{a} \sum_{k=1}^{\infty} \frac{H_1 L^{k+1}}{H k - 1 L^3}$$

02.03.06.0057.01

$$p^3 \ddagger \frac{1}{16} \hat{a} \sum_{k=0}^{\infty} \frac{1}{1024^k} \left(\frac{8}{\mathbb{H} k + 2\mathbb{L}^3} + \frac{1}{\mathbb{H} k + 3\mathbb{L}^3} + \frac{32}{\mathbb{H} k + 1\mathbb{L}^3} \right) +$$

$$\frac{5}{2} \hat{a} \sum_{k=0}^{\infty} \frac{1}{64^k} \left(-\frac{192}{\mathbb{H} 2k + 2\mathbb{L}^3} + \frac{88}{\mathbb{H} 2k + 3\mathbb{L}^3} - \frac{8}{\mathbb{H} 2k + 5\mathbb{L}^3} + \frac{84}{\mathbb{H} 2k + 6\mathbb{L}^3} - \right.$$

$$\left. \frac{4}{\mathbb{H} 2k + 7\mathbb{L}^3} + \frac{11}{\mathbb{H} 2k + 9\mathbb{L}^3} - \frac{12}{\mathbb{H} 2k + 10\mathbb{L}^3} + \frac{1}{\mathbb{H} 2k + 11\mathbb{L}^3} + \frac{32}{\mathbb{H} 2k + 1\mathbb{L}^3} \right)$$

G.Huvent (2006)

Expansions for p^4

02.03.06.0031.01

$$p^4 \S 90 \hat{a} \sum_{k=1}^{\infty} \frac{1}{k^4}$$

02.03.06.0032.01

$$p^4 \S 96 \hat{a} \sum_{k=0}^{\infty} \frac{1}{\mathbb{H} k + 1\mathbb{L}^4}$$

02.03.06.0058.01

$$p^4 \ddagger \frac{27}{164} \hat{a} \sum_{k=0}^{\infty} \frac{1}{2^{12k}} \left(-\frac{38912}{\mathbb{H} 4k + 2\mathbb{L}^4} + \frac{81920}{\mathbb{H} 4k + 3\mathbb{L}^4} - \frac{2048}{\mathbb{H} 4k + 4\mathbb{L}^4} - \frac{512}{\mathbb{H} 4k + 5\mathbb{L}^4} - \frac{23552}{\mathbb{H} 4k + 6\mathbb{L}^4} + \frac{256}{\mathbb{H} 4k + 7\mathbb{L}^4} - \frac{27648}{\mathbb{H} 4k + 8\mathbb{L}^4} - \frac{10240}{\mathbb{H} 4k + 9\mathbb{L}^4} - \right.$$

$$\frac{2432}{\mathbb{H} 4k + 10\mathbb{L}^4} - \frac{64}{\mathbb{H} 4k + 11\mathbb{L}^4} - \frac{3584}{\mathbb{H} 4k + 12\mathbb{L}^4} - \frac{32}{\mathbb{H} 4k + 13\mathbb{L}^4} - \frac{608}{\mathbb{H} 4k + 14\mathbb{L}^4} -$$

$$\frac{1280}{\mathbb{H} 4k + 15\mathbb{L}^4} - \frac{1728}{\mathbb{H} 4k + 16\mathbb{L}^4} + \frac{8}{\mathbb{H} 4k + 17\mathbb{L}^4} - \frac{368}{\mathbb{H} 4k + 18\mathbb{L}^4} - \frac{4}{\mathbb{H} 4k + 19\mathbb{L}^4} -$$

$$\left. \frac{8}{\mathbb{H} 4k + 20\mathbb{L}^4} + \frac{160}{\mathbb{H} 4k + 21\mathbb{L}^4} - \frac{38}{\mathbb{H} 4k + 22\mathbb{L}^4} + \frac{1}{\mathbb{H} 4k + 23\mathbb{L}^4} + \frac{2048}{\mathbb{H} 4k + 1\mathbb{L}^4} \right)$$

G.Huvent (2006)

02.03.06.0059.01

$$p^4 \S \frac{3240}{17} \hat{a} \sum_{k=1}^{\infty} \frac{1}{k^4} \binom{2k}{k}$$

Expansions for p^6

02.03.06.0033.01

$$p^6 \S 945 \hat{a} \sum_{k=1}^{\infty} \frac{1}{k^6}$$

02.03.06.0034.01

$$p^6 \S 960 \hat{a} \sum_{k=0}^{\infty} \frac{1}{\mathbb{H} k + 1\mathbb{L}^6}$$

Expansions for p^{2n}

02.03.06.0035.01

$$p^{2n} \sim \frac{H(1L^{n-1}) H(nL)!}{2^{2n-1} B_{2n}} \hat{a} \frac{1}{k^{2n}}; n \in \mathbb{N}^+$$

02.03.06.0060.01

$$p^{2n} \sim \frac{H(1L^{n-2}) H(nL)!}{B_{2n} \Gamma(\frac{1}{2})} \hat{a} \frac{H(1L^{k-1})}{k^{2n}}; n \in \mathbb{N}^+$$

02.03.06.0036.01

$$p^{2n} \sim \frac{H(1L^{n-1}) H(nL)!}{H^n - 1 B_{2n}} \hat{a} \frac{1}{H(k+1)L^{2n}}; n \in \mathbb{N}^+$$

Expansions for p^{2n-1}

02.03.06.0061.01

$$p^{2n-1} \sim \frac{H(1L^{n-1}) 2^{2n} H(n-2L)!}{E_{2n-2}} \hat{a} \frac{H(1L^k)}{H(k+1)L^{2n-1}}; n \in \mathbb{N}^+$$

Exponential Fourier series

02.03.06.0042.01

$$p \sim x + 2 \hat{a} \sum_{k=1}^{\infty} \frac{\sin kxL}{k}; x \in \mathbb{R}, x > 0$$

02.03.06.0043.01

$$p \sim 4 \hat{a} \sum_{k=0}^{\infty} \frac{H(1L^k) \cos H(k+1)LxL}{2k+1}; x \in \mathbb{R}$$

02.03.06.0062.01

$$p \sim 4 - 2 \hat{a} \sum_{k=0}^{\infty} \frac{k!}{\Gamma(j+3L)}$$

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02.03.06.0063.01

$$p \sim \lim_{n \rightarrow \infty} 2^{n+2} f(HL); f(HL) \sim 1 + \frac{f(HL) - 1L}{1 + \sqrt{1 + f(HL) - 1L^2}}; n \in \mathbb{N}^+$$

Candido Otero Ramos (2007)

Other series representations

02.03.06.0040.01

$$p^{2n} \sim H(n+1L)! \hat{a}^{-1/4} \hat{a}^{-n} \frac{1}{\prod_{l=1}^n k_l \Gamma(k_l)}; n \in \mathbb{N}^+$$

Integral representations

On the real axis

Of the direct function

02.03.07.0001.01

$$\int_0^x \frac{1}{t^2 + 1} dt$$

02.03.07.0002.01

$$\int_0^x \sqrt{1 - t^2} dt$$

02.03.07.0003.01

$$\int_0^x \frac{1}{\sqrt{1 - t^2}} dt$$

02.03.07.0004.01

$$\int_0^x \frac{\sin t}{t} dt$$

02.03.07.0005.01

$$\int_0^x \frac{\sin^2 t}{t^2} dt$$

02.03.07.0006.01

$$\int_0^x \frac{\sin^3 t}{t^3} dt$$

02.03.07.0007.01

$$\int_0^x \frac{\sin^4 t}{t^4} dt$$

02.03.07.0008.01

$$\int_0^x \frac{\sin^5 t}{t^5} dt$$

02.03.07.0009.01

$$\int_0^x \frac{\sin^6 t}{t^6} dt$$

02.03.07.0010.01

$$\int_0^x \frac{\sin^n t}{t^n} dt = \sum_{k=0}^{n-1} \frac{(-1)^k}{k!} \frac{\Gamma(n-k)}{\Gamma(n)} x^{n-k} + \frac{(-1)^{n-1}}{(n-1)!} \int_0^x \frac{\sin t}{t} dt$$

Involving the direct function

02.03.07.0011.01

$$\int_0^x \frac{1}{\sqrt{1 - t^2}} dt$$

Gaussian probability density integral

02.03.07.0012.01

$$\sqrt{p} \sim 2\sqrt{2} \int_0^{\infty} \sin t^2 \, dt$$

Fresnel integral

02.03.07.0013.01

$$\sqrt{p} \sim 2\sqrt{2} \int_0^{\infty} \cos t^2 \, dt$$

Fresnel integral

02.03.07.0014.01

$$\sqrt{p} \sim 2 \int_0^1 \log^{\frac{1}{2}}\left(\frac{1}{t}\right) \, dt$$

02.03.07.0015.01

$$\sqrt{p} \sim \int_0^1 \frac{1}{\log^{\frac{1}{2}} J^{-1} N} \, dt$$

Involving related functions

02.03.07.0016.01

$$p \sim 2 \int_0^{\infty} \frac{\cos t}{t^2 + 1} \, dt$$

Product representations

02.03.08.0001.01

$$p \sim 2 \prod_{k=1}^{\infty} \frac{4k^2}{k^2 - 1} \frac{k+1}{k}$$

02.03.08.0002.01

$$p \sim \frac{4}{\sqrt{2}} \prod_{k=1}^{\infty} \frac{4k^{\frac{k+1}{2}}}{2k+1}$$

02.03.08.0003.01

$$p \sim 2 \prod_{k=2}^{\infty} \sec\left(\frac{p}{2^k}\right)$$

02.03.08.0004.01

$$p \sim 3 \prod_{k=0}^{\infty} \sec\left(\frac{p}{12 \cdot 2^k}\right)$$

02.03.08.0008.01

$$p \sim 2 \prod_{k=1}^{\infty} \left(1 + \frac{2}{k}\right)^{H_{k+1}}$$

02.03.08.0009.01

$$p \sim \frac{6}{\prod_{k=2}^{\infty} \left(1 + \frac{2}{k}\right)^{H_k}}$$

02.03.08.0005.01

$$\frac{6}{p^2} \sum_{k=1}^{\infty} \left(1 - \frac{1}{p_k^2} \right) \cdot p_k \hat{I} P$$

02.03.08.0006.01

$$\frac{2}{p} \sum_{k=1}^{\infty} \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2} \cdot \frac{1}{4}$$

02.03.08.0007.01

$$\frac{2}{p} \sum_{k=1}^{\infty} \lim_{n \rightarrow \infty} \frac{1}{2} \text{Nest} \sqrt{2 + \frac{1}{n}} \quad \&, 0, k \in \mathbb{N}$$

Limit representations

02.03.09.0001.01

$$p \sum_{k=1}^{\infty} \left(2^{4n} \cdot \left(n \binom{2n}{n} \right)^2 \right)$$

02.03.09.0002.01

$$p \sum_{k=0}^{\infty} \frac{4^n}{n^2} \hat{a} \sqrt{n^2 - k^2}$$

02.03.09.0003.01

$$p \sum_{k=1}^{\infty} \frac{4 \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n}}{\hat{a} \sqrt{n - k^2}}$$

02.03.09.0004.01

$$p \sum_{k=1}^{\infty} \frac{2^{4n+1} n!^4}{\mathbb{H}_{n+1} \mathbb{H}_{n!}^2}$$

02.03.09.0006.01

$$p \sum_{k=1}^{\infty} \frac{2 \mathbb{H}_{n!}^2}{\mathbb{H}_{n+1} \mathbb{H}_{n-1}!^2}$$

02.03.09.0012.01

$$p \sum_{k=0}^{\infty} \mathbb{H}_{1L^n} 2^{2-2n} \hat{a} \mathbb{H}_{1L^k} \binom{4n}{2k+1} H_{2k+1} \cdot n \hat{I} n$$

02.03.09.0013.01

$$p \sum_{k=1}^{\infty} \frac{n!^2 \mathbb{H}_n + 1L^{2n^2+n}}{2n^{2n^2+3n+1}}$$

Pete Koupriyanov

02.03.09.0007.01

$$p \sum_{k=1}^{\infty} \mathbb{H}_k + 1L \hat{a} \frac{k^2}{\mathbb{H}_{k+1} 1L^2}$$

02.03.09.0014.01

$$p \S \lim_{n \otimes \Psi} \sqrt{6 \frac{\log I \hat{U}_{k=1}^n F_k M}{\log H_{\text{cm}} F_1, F_2, \frac{1}{4}, F_n L}}; n \hat{I} N$$

02.03.09.0008.01

$$p \S - 12 \lim_{n \otimes \Psi} \frac{1}{n} \hat{a} \cot H a \log H \cos H a \log H; a \hat{I} R$$

02.03.09.0009.01

$$p \S \lim_{n \otimes \Psi} \left(a n \left(\hat{a} \left(\sum_{k=0}^n d_{\text{sgn} H \cos H a L, - \text{sgn} H \cos H + 1 L a L} \right) \right) \right); 0 \leq a \leq p$$

02.03.09.0010.01

$$p \S \lim_{n \otimes \Psi} \frac{2 a_k^2}{s_k};$$

$$a_k \S \frac{1}{2} H_{k-1} + b_{k-1} L \hat{I} \quad b_k \S \sqrt{a_{k-1} b_{k-1}} \hat{I} \quad s_k \S s_{k-1} - 2^k c_k \hat{I} \quad c_k \S a_k^2 - b_k^2 \hat{I} \quad a_0 \S 1 \hat{I} \quad b_0 \S \frac{1}{\sqrt{2}} \hat{I} \quad s_0 \S \frac{1}{2}$$

02.03.09.0011.01

$$p \S \lim_{n \otimes \Psi} \frac{1}{a_n};$$

$$a_{n+1} \S I b^{n+1} + 1 M a_n - 2^{2n+3} b_{n+1} I b_{n+1}^2 + b_{n+1} + 1 M \quad b_{n+1} \S \frac{1 - \sqrt[4]{1 - b_n^4}}{1 + \sqrt[4]{1 - b_n^4}} \hat{I} \quad a_0 \S 6 - 4 \sqrt{2} \hat{I} \quad b_0 \S \sqrt{2} - 1$$

02.03.09.0015.01

$$p \S \lim_{n \otimes \Psi} \frac{2^{n+1}}{2 - b_1} \left(\frac{b_n}{2} \sqrt{2 + b_{n-1} \sqrt{2 + b_{n-2} \sqrt{2 + \frac{1}{4} + b_2 \sqrt{2 + \sin\left(\frac{p b_1}{4}\right)}}}} \right);$$

$$b_n \S 1 \hat{I} \quad b_{n-1} \S - 1 \hat{I} \quad H_k \S 1 \hat{I} \quad 2 \leq k \leq n - 2 \hat{I} \quad k \hat{I} N \hat{I} \quad b_1 \hat{I} R \hat{I} \quad - 2 \leq b_1 \leq 2$$

L. D. Servi: Nested Square Roots of 2 American Mathematical Monthly 110, 326-329 (2003)

02.03.09.0016.01

$$p \S \lim_{n \otimes \Psi} A H L; A H L \S 4 \hat{I} \quad B H L \S \frac{1}{\sqrt{2}} \hat{I} \quad A H L \S \frac{2 A H - 1 L B H - 1 L}{B H - 1 L + 1} \hat{I} \quad B H L \S \sqrt{\frac{1}{2} B H - 1 L + 1 L} \hat{I} \quad n \hat{I} N^+$$

Candido Otero Ramos (2007)

Continued fraction representations

$$\left(\begin{array}{l} \text{p Š 3 + 1 " } \\ 7 + 1 " \\ 15 + 1 " \\ 1 + 1 " \\ 292 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{14 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4}}}}}}}}}}}}}} \end{array} \right)$$
$$\begin{array}{r} \text{p Š 3+} \frac{1}{9} \\ 6+ \frac{25}{49} \\ 6+ \frac{81}{121} \\ 6+ \frac{121}{6+1/4} \end{array}$$
$$p \tilde{S}^3 + K_k \mathbb{H}^k - 1 L^2, 6M^{\mathbb{Y}}$$

02.03.10.0004.01

$$\frac{p}{2} \tilde{S} 1 - \frac{1}{3 - \frac{6}{1 - \frac{2}{3 - \frac{20}{1 - \frac{12}{3 - \frac{42}{1 - \frac{30}{3 - \frac{1}{4}}}}}}}}$$

02.03.10.0005.01

$$\frac{p}{2} \tilde{S} 1 - \frac{1}{3 + K_k I - I k - H 1 L^k M k - H 1 L^k + 1 M 2 + H 1 L^k M^{\tilde{Y}}}$$

02.03.10.0006.01

$$\frac{4}{p} \tilde{S} 1 + \frac{1}{3 + \frac{4}{5 + \frac{9}{7 + \frac{16}{9 + \frac{25}{11 + \frac{36}{13 + \frac{1}{4}}}}}}}}$$

02.03.10.0007.01

$$\frac{4}{p} \tilde{S} 1 + K_k I k^2, 2 k + 1 M^{\tilde{Y}}$$

02.03.10.0008.01

$$\frac{4}{p} \tilde{S} 1 + \frac{1}{2 + \frac{9}{2 + \frac{25}{2 + \frac{49}{2 + \frac{81}{2 + \frac{121}{2 + \frac{1}{4}}}}}}}}$$

02.03.10.0009.01

$$\frac{4}{p} \tilde{S} 1 + K_k I H k - 1 L^2, 2 M^{\tilde{Y}}$$

02.03.10.0010.01

$$\frac{12}{p^2} \tilde{S} 1 + \frac{1}{3 + \frac{16}{5 + \frac{81}{7 + \frac{256}{9 + \frac{625}{11 + \frac{1296}{13 + \frac{1}{4}}}}}}}}$$

02.03.10.0011.01

$$\frac{12}{p^2} \tilde{S} \left(1 + K_k I k^4, 2k + 1 \right) \tilde{M}$$

02.03.10.0012.01

$$\frac{6}{p^2 - 6} \tilde{S} \left(1 + \frac{1}{1 + \frac{2}{1 + \frac{4}{1 + \frac{6}{1 + \frac{9}{1 + \frac{12}{1 + \frac{1}{1 + \frac{1}{4}}}}}}}} \right)$$

02.03.10.0013.01

$$\frac{6}{p^2 - 6} \tilde{S} \left(1 + K_k \left(\left\lfloor \frac{k+1}{2} \right\rfloor \left\lfloor \frac{k+2}{2} \right\rfloor, 1 \right) \right) \tilde{Y}$$

Complex characteristics

Real part

02.03.19.0001.01

$$\operatorname{Re} \tilde{H} L \tilde{S} p$$

Imaginary part

02.03.19.0002.01

$$\operatorname{Im} \tilde{H} L \tilde{S} 0$$

Absolute value

02.03.19.0003.01

$$p^{\alpha} \tilde{S} p$$

Argument

02.03.19.0004.01

$$\arg \tilde{H} L \tilde{S} 0$$

Conjugate value

02.03.19.0005.01

$$\bar{p} \tilde{S} p$$

Signum value

02.03.19.0006.01

$$\operatorname{sgn} \tilde{H} L \tilde{S} 1$$

Differentiation

Low-order differentiation

02.03.20.0001.01

$$\frac{\mathbb{D}^p}{\mathbb{D}z} \mathbb{S} 0$$

Fractional integro-differentiation

02.03.20.0002.01

$$\frac{\mathbb{D}^a p}{\mathbb{D}z^a} \mathbb{S} \frac{z^{-a} p}{\Gamma(a)}$$

Integration

Indefinite integration

02.03.21.0001.01

$$\int p \, dz \mathbb{S} p z$$

02.03.21.0002.01

$$\int z^{a-1} p \, dz \mathbb{S} \frac{z^a p}{a}$$

Integral transforms

Fourier exp transforms

02.03.22.0001.01

$$\mathcal{F}_t \mathbb{D} \mathbb{S} \sqrt{2} p^{3/2} d\mathbb{H}$$

Inverse Fourier exp transforms

02.03.22.0002.01

$$\mathcal{F}_t^{-1} \mathbb{D} \mathbb{S} \sqrt{2} p^{3/2} d\mathbb{H}$$

Fourier cos transforms

02.03.22.0003.01

$$\mathcal{F}_c \mathbb{D} \mathbb{S} \frac{p^{3/2}}{\sqrt{2}} d\mathbb{H}$$

Fourier sin transforms

02.03.22.0004.01

$$\mathcal{F}_s \mathbb{D} \mathbb{S} \frac{\sqrt{2} p}{z}$$

Laplace transforms

02.03.22.0005.01

$$\mathcal{L}_t^{-1} \mathcal{P} \frac{p}{z}$$

Inverse Laplace transforms

02.03.22.0006.01

$$\mathcal{L}_t^{-1} \mathcal{P} \mathcal{D} \mathcal{H} \mathcal{L} \mathcal{S} \mathcal{P} \mathcal{d} \mathcal{H} \mathcal{L}$$

Representations through more general functions

Through Meijer G

02.03.26.0014.01

$$\mathcal{P} \mathcal{F} \mathcal{P} G_{0,1}^{1,0} \mathcal{H} \mathcal{E} \mathcal{O} \mathcal{L} + \mathcal{P} G_{1,2}^{1,1} \left(z \left| \begin{matrix} 1 \\ 1, 0 \end{matrix} \right. \right)$$

Through other functions

02.03.26.0001.01

$$\mathcal{P} \mathcal{S} 4 \left(4 \tan^{-1} \left(\frac{1}{5} \right) - \tan^{-1} \left(\frac{1}{239} \right) \right)$$

02.03.26.0008.01

$$\mathcal{P} \mathcal{S} 4 \tan^{-1} \left(\frac{1}{2} \right) + 4 \tan^{-1} \left(\frac{1}{3} \right)$$

02.03.26.0009.01

$$\mathcal{P} \mathcal{S} 8 \tan^{-1} \left(\frac{1}{3} \right) + 4 \tan^{-1} \left(\frac{1}{7} \right)$$

02.03.26.0010.01

$$\mathcal{P} \mathcal{S} 4 \tan^{-1} \left(\frac{1}{2} \right) + 4 \tan^{-1} \left(\frac{1}{5} \right) + 4 \tan^{-1} \left(\frac{1}{8} \right)$$

02.03.26.0013.01

$$\mathcal{P} \mathcal{S} 4 \left(6 \tan^{-1} \left(\frac{1}{8} \right) + 2 \tan^{-1} \left(\frac{1}{57} \right) + \tan^{-1} \left(\frac{1}{239} \right) \right)$$

Jeff Reid

02.03.26.0015.01

$$\mathcal{P} \mathcal{S} 4 \left(\tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{1}{47} \right) \right)$$

Adam Bui (2007)

02.03.26.0016.01

$$\mathcal{P} \mathcal{F} 4 \left(\tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{1}{13} \right) \right)$$

Adam Bui (2007)

02.03.26.0017.01

$$p \nmid - \frac{4 \left(\cot^{-1} \frac{a-1}{a+\sqrt{2}\sqrt{a^2+1}} + 2 \tan^{-1} \left(\frac{a-1}{a+\sqrt{2}\sqrt{a^2+1}} \right) \right)}{\frac{2\sqrt{a^2}}{a} - 2\sqrt{\frac{a}{a+\tilde{a}}} \sqrt{\frac{a+\tilde{a}}{a}} + 2\sqrt{\frac{a-\tilde{a}}{a}} \sqrt{\frac{a}{a-\tilde{a}}} - 1} \bullet; a^2 - 1$$

Adam Bui & O.I. Marichev (2007)

02.03.26.0002.01

$$p \nmid 88 \tan^{-1} \left(\frac{1}{28} \right) + 8 \tan^{-1} \left(\frac{1}{443} \right) - 20 \tan^{-1} \left(\frac{1}{1393} \right) - 40 \tan^{-1} \left(\frac{1}{11018} \right)$$

02.03.26.0011.01

$$p \nmid 48 \tan^{-1} \left(\frac{1}{18} \right) + 12 \tan^{-1} \left(\frac{1}{70} \right) + 20 \tan^{-1} \left(\frac{1}{99} \right) + 32 \tan^{-1} \left(\frac{1}{307} \right)$$

02.03.26.0012.01

$$p \nmid 640 \tan^{-1} \left(\frac{1}{200} \right) - 4 \tan^{-1} \left(\frac{1}{239} \right) - 16 \tan^{-1} \left(\frac{1}{515} \right) - 32 \tan^{-1} \left(\frac{1}{4030} \right) - 64 \tan^{-1} \left(\frac{1}{50105} \right) - 64 \tan^{-1} \left(\frac{1}{62575} \right) - 128 \tan^{-1} \left(\frac{1}{500150} \right) - 320 \tan^{-1} \left(\frac{1}{4000300} \right)$$

02.03.26.0003.01

$$p \nmid 4 \left(\tan^{-1} \left(\frac{p}{q} \right) + \tan^{-1} \left(\frac{q-p}{p+q} \right) \right) \bullet; p \hat{I} N^+ \hat{I} q \hat{I} N^+$$

02.03.26.0004.01

$$p \nmid 2 K \hat{H} L$$

02.03.26.0005.01

$$p \nmid 2 E \hat{H} L$$

02.03.26.0006.01

$$p \nmid \sqrt{6 \text{Li}_2} \hat{H} L$$

02.03.26.0007.01

$$p \nmid G \left(\frac{1}{2} \right)^2$$

Representations through equivalent functions

02.03.27.0001.01

$$p \nmid 180^\circ$$

02.03.27.0002.01

$$p \nmid -\tilde{a} \log \hat{H} L$$

02.03.27.0003.01

$$p \nmid 2 \tilde{a} \log \left(\frac{1-\tilde{a}}{1+\tilde{a}} \right)$$

02.03.27.0004.01

$$\tilde{a}^{p\tilde{a}} \nmid \tilde{S} - 1$$

identity due to L. Euler

02.03.27.0005.01

$$\tilde{a}^{2p\tilde{a}} \tilde{S} 1$$

02.03.27.0006.01

$$\tilde{a}^{p\tilde{a}k} \tilde{S} \mathbb{H} 1\mathbb{L}^k \bullet; k \hat{\mathbb{I}} \mathbb{Z}$$

02.03.27.0007.01

$$\tilde{a}^{-\frac{p}{2}} \tilde{S} \tilde{a}^{\tilde{a}}$$

02.03.27.0008.01

$$\tilde{a}^{\frac{p}{2}\tilde{a}k} \tilde{S} \tilde{a}^k \bullet; k \hat{\mathbb{I}} \mathbb{Z}$$

Inequalities

02.03.29.0001.01

$$3 + \frac{10}{71} < p < 3 + \frac{1}{7}$$

B.C. Archimedes

02.03.29.0002.01

$$\left| p - \frac{q}{p} \right| > \frac{1}{q^{14.65}} \bullet; p \hat{\mathbb{I}} \mathbb{N}^+ \mathbb{I} q \hat{\mathbb{I}} \mathbb{N}^+$$

02.03.29.0003.01

$$\tilde{a}^p \ni p^{\tilde{a}}$$

Theorems

Volume of an n -dimensional sphere

Volume V_n of an n -dimensional sphere of radius r :

$$V_{2k} \tilde{S} \frac{p^k}{k!} r^{2k}; \quad V_{2k+1} \tilde{S} \frac{p^k 2^{2k+1} k!}{\mathbb{H} k + 1\mathbb{L}!} r^{2k+1}.$$

For instance, the area of a circle with radius r is $p r^2$ and the volume of a sphere with radius r is $\frac{4p}{3} r^3$.

Above general formulas can be joined into one $V_n \tilde{S} \frac{p^{n/2}}{\mathbb{G}\mathbb{I} \frac{n+2}{2}\mathbb{N}} r^n$.

Surface area of an n -dimensional sphere

Surface area S_n of n -dimensional sphere of radius r :

$$S_{2k} \tilde{S} \frac{2p^k}{\mathbb{H} k - 1\mathbb{L}!} r^{2k-1}; \quad S_{2k+1} \tilde{S} \frac{p^k 2^{2k+1} k!}{\mathbb{H} k\mathbb{L}!} r^{2k}.$$

For instance, the circumference of circle with radius r is $2p r$, and the surface area of a sphere with radius r is $4p r^2$.

Above general formulas can be joined into one $S_n \approx \frac{2p^{n^2}}{G \frac{n+1}{2} N} r^{n-1}$.

Volume of an n -dimensional cylinder ??

Volume V_n of an n -dimensional cylinder of radius r and height h :

$$V_{2k} \approx \frac{p^k}{k!} r^{2k} h; \quad V_{2k+1} \approx \frac{p^k 2^{2k+1} k!}{H k + 1L!} r^{2k+1} h.$$

For instance, the volume of a cylinder with radius r and height h is $\frac{4}{3} p r^3 h$.

Above general formulas can be joined into one $V_n \approx \frac{p^{\frac{n-1}{2}}}{n G \frac{n+1}{2} N} r^{n-1} h$.

Surface area of an n -dimensional cylinder ??

Surface area S_n of n -dimensional cone of radius r and height h :

$$S_{2k} \approx \frac{2p^k}{k!} r^{2k-1} H k + rL; \quad S_{2k+1} \approx \frac{2^{2k+1} p^k k!}{H k + 1L!} r^{2k} H H k + 1L + 2 rL.$$

For instance, the volume of a cylinder with radius r and height h is $\frac{1}{3} p r^2 h$.

For instance, the surface area of a cylinder with radius r and height h is $p r \left(r + \sqrt{h^2 + r^2} \right)$.

Above general formulas can be joined into one $S_n \approx \frac{p^{\frac{n-1}{2}}}{G \frac{n+1}{2} N} \left(r + \sqrt{h^2 + r^2} \right) r^{n-2}$.

Volume of an n -dimensional cone

Volume V_n of an n -dimensional cone of radius r and height h :

$$V_{2k} \approx \frac{2^{2k-1} p^{k-1} H k - 1L!}{H k L!} r^{2k-1} h; \quad V_{2k+1} \approx \frac{p^k}{H k + 1L k!} r^{2k} h.$$

For instance, the volume of a cone with radius r and height h is $\frac{1}{3} p r^2 h$.

Above general formulas can be joined into one $V_n \approx \frac{p^{\frac{n-1}{2}}}{n \Gamma(\frac{n+1}{2})} r^{n-1} h$.

Surface area of an n -dimensional cone

Surface area S_n of n -dimensional cone of radius r and height h :

$$S_{2k} \approx \frac{4^k p^{k-1} k!}{\pi k!} \left(r + \sqrt{h^2 + r^2} \right) r^{2k-2}; \quad S_{2k+1} \approx \frac{p^k \left(r + \sqrt{h^2 + r^2} \right)}{k!} r^{2k-1}.$$

For instance, the volume of a cone with radius r and height h is $\frac{1}{3} \pi r^2 h$.

For instance, the surface area of a cone with radius r and height h is $\pi r \left(r + \sqrt{h^2 + r^2} \right)$.

Above general formulas can be joined into one $S_n \approx \frac{p^{\frac{n-1}{2}}}{n \Gamma(\frac{n+1}{2})} \left(r + \sqrt{h^2 + r^2} \right) r^{n-2}$.

Probability of two random integers being relatively prime

The probability that two integers picked at random are relatively prime is $\frac{6}{\pi^2}$.

History

- The design of Egyptian pyramids (c. 3000 BC) incorporated π as $3 + \frac{1}{6} \pi \approx 3.142857\frac{1}{4}$;
- Egyptians (Rhind Papyrus, c. 2000 BC) gave π as $\frac{256}{81} \approx 3.16045\frac{1}{4}$
- China (c. 1200 BC) gave π as 3
- The Biblical verse I Kings 7:23 (c. 950 BC) gave π as $30 \cdot 10 \approx 3.0$
- Archimedes (Greece, c. 240 BC) knew that $3 + \frac{10}{71} < \pi < 3 + \frac{1}{7}$ and gave π as 3.1418...
- W. Jones (1706) introduced the symbol π
- C. Goldbach (1742) also used the symbol π
- J. H. Lambert (1761) established that π is an irrational number
- F. Lindemann (1882) proved that π is transcendental

The constant π is the most frequently encountered classical constant in mathematics and the natural sciences.

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