問題2.2

1. 標本空間 $-\infty < x < \infty$

母数空間 $\mu \in \mathbb{R}, \sigma^2 > 0$

2. 正規分布の定義より

期待値 $E[X] = \mu$

3. 正規分布の定義より

分散 $V[X] = \sigma^2$

4.

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x;\mu,\sigma^2)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(tx - \frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{1}{2\sigma^2}\left(2\sigma^2 tx - x^2 + 2\mu x - \mu^2\right)\right\} dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\left(x^2 - 2(\mu+\sigma^2 t)x + \mu^2\right)\right\} dx$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\left((x-(\mu+\sigma^2 t))^2 - (\mu+\sigma^2 t)^2 + \mu^2\right)\right\} dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\left((x-(\mu+\sigma^2 t))^2 - 2\mu\sigma^2 t - (\sigma^2 t)^2\right)\right\} dx$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-(\mu+\sigma^2 t))^2}{2\sigma^2} + \mu t + \frac{\sigma^2 t^2}{2}\right) dx$$