EE2703 Applied Programming Lab
Week 8 : The Digital Fourier Transform

Niyas Mon P — EE20B094

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1 Introduction

In this assignment, we deal with DFTs, using the `numpy.fft` toolbox in python. We will look into periodic functions and CTFT of Gaussian function.We will analyse with different time range and number of samples

2 Analysis of Periodic signals

2.1 Sinusoidal signal

We calculate the DFT of $sin(5t)$. Then, we plot the phase and magnitude of the DFT.

$$sin(5t) = \frac{1}{2j}(e^{5jt} - e^{-5jt}) \tag{1}$$

is given by :

$$Y(\omega) = \frac{\delta(\omega-1) - \delta(\omega+1)}{2j}$$

So,we expect a spike at w = -5 and w = +5 with $\phi = \pm\frac{\pi}{2}$ rad/sec respectively.

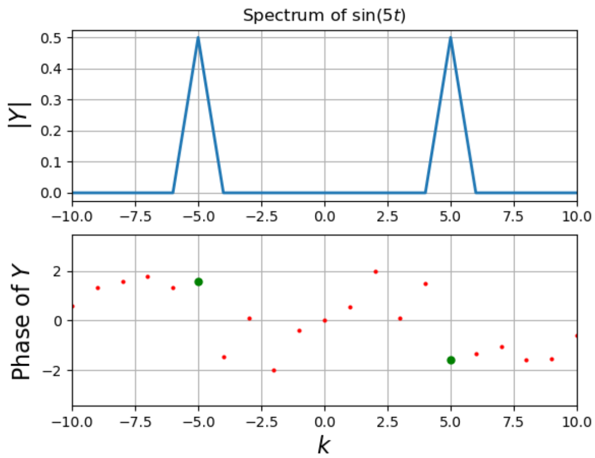

Figure 1: Spectrum of $sin(5t)$

2.2 Amplitude Modulated Signal

The signal is given by:

$$y = (1 + 0.1cos(t))cos(10t) \tag{2}$$

.

$$y = \frac{1}{2}(e^{10jt} + e^{-10jt}) + 0.025(e^{11jt} + e^{-11jt} + e^{9jt} + e^{-9jt}) \tag{3}$$

Here the signal is a combination of three different frequencies (9, 10 and 11) so we expect 6 different impulses in the spectrum

The magnitude and the phase spectrum is shown below

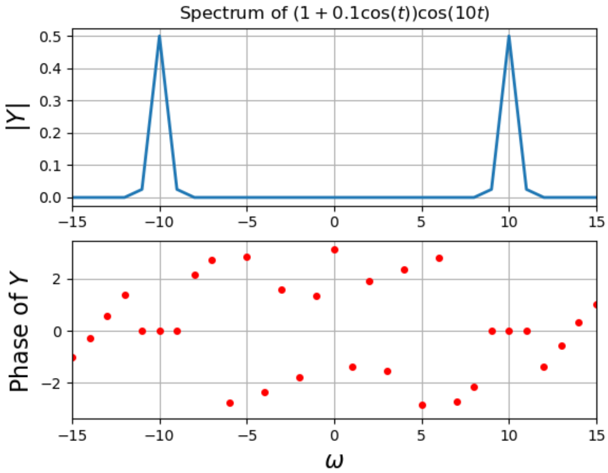

Figure 2: DFT of $(1 + 0.1cos(t))cos(10t)$

But,there are only two peaks instead of six. This is due to small sampling frequency. Now , we are increasing the sampling frequency We increase this by increasing the time range.

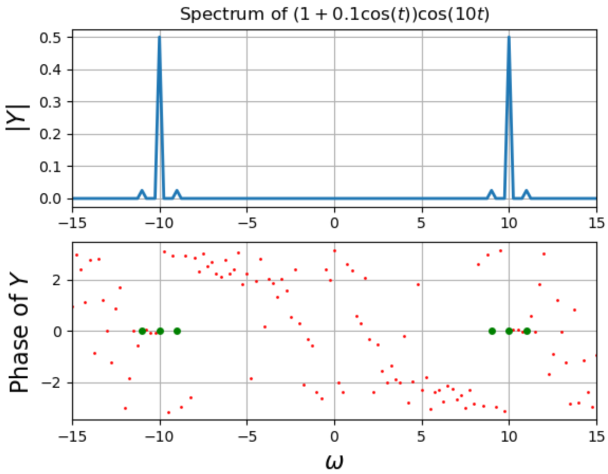

Figure 3: DFT of $(1 + 0.1cos(t))cos(10t)$

Now the graph is perfect and we see the 6 peaks corresponding to different frequencies in the signal. We also see that the response has become more impulsive with higher sampling frequency.

2.3 Spectrum of $\sin^3 t$ and $\cos^3 t$

The signals are given by

$$sin^3(t) = \frac{3}{4}sin(t) - \frac{1}{4}sin(3t) \tag{4}$$

$$cos^3(t) = \frac{3}{4}cos(t) + \frac{1}{4}cos(3t) \tag{5}$$

So, we expect peaks $\omega = \pm 1\ rad/sec$ and $\omega = \pm 3\ rad/sec$.

DFT Spectrum of $sin^3(t)$:

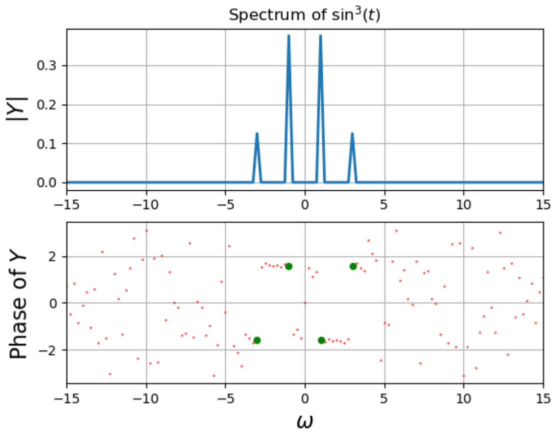

Figure 4: Spectrum of $sin^3(t)$

DFT Spectrum of $cos^3(t)$:

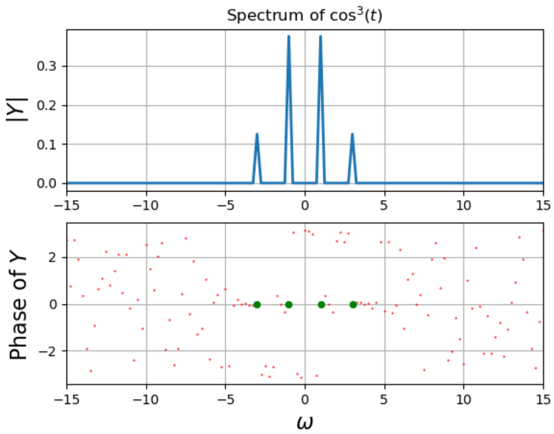

Figure 5: Spectrum of $cos^3(t)$

2.4 Frequency Modulation with $cos(20t + 5cos(t))$

Here we can expect the signal to have impulse responses around 20 and -20 which corresponds to cos(20t). The DFT of $cos(20t + 5cos(t))$ is:

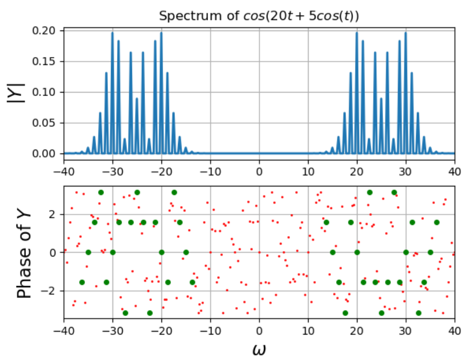

Figure 6: DFT of $cos(20t + 5cos(t))$

There are more side bands as compared to amplitude modulated signal $\omega = \pm 20$ rad/sec As expected all the responses of the system are concentrated around $= 20$ and $= 20$. Regarding phase response,all the responses are either 0,$\pi/2$,$-\pi/2$,π,$-\pi$

3 DFT of a Gaussian

Gaussian function is not “bandlimited” in frequency. This means that it will have some response to any frequency. The DFT of a gaussian is also a gaussian. We need to identify the best time range. So,we are plotting for different time ranges

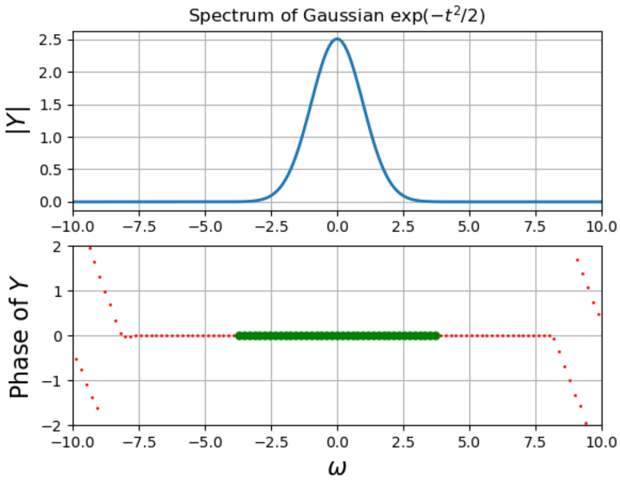

Figure 7: Gaussian Spectrum

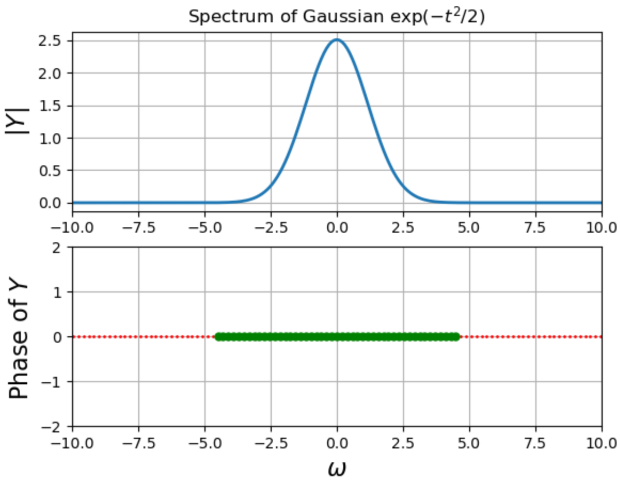

Figure 8: Gaussian Spectrum

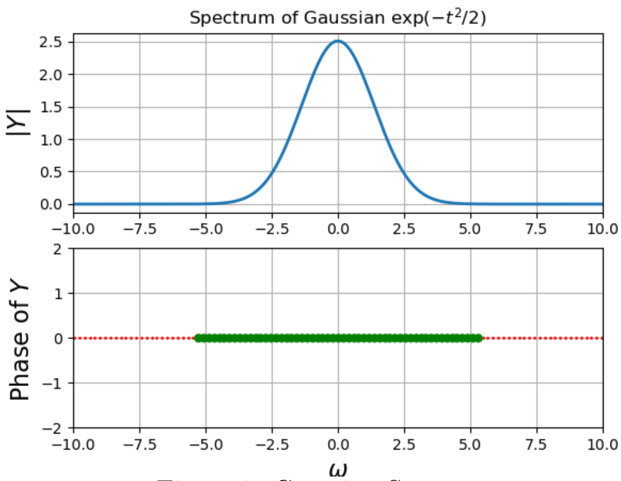

Figure 9: Gaussian Spectrum

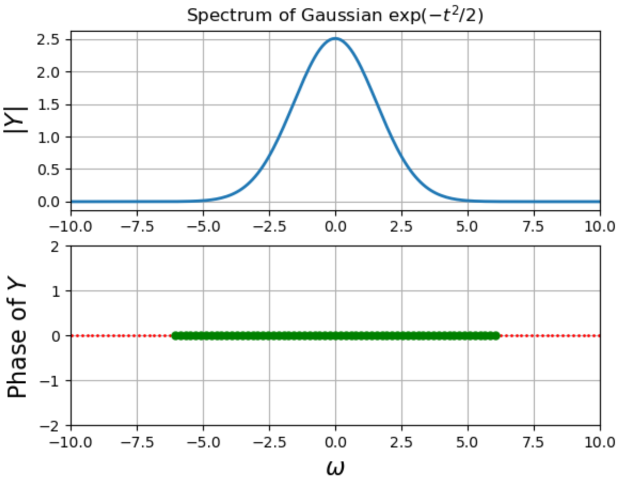

Figure 10: Gaussian Spectrum

4 Conclusion

We analysed the Digital Fourier Transform of simple sinusoids, multiple sinusoids and non-sinusoidal signals like Gaussian using python functions like fft,fftshif etc. We analysed the effects of Time range and Number of samples on the DFT and plotted graphs for the same.