EE2703 Applied Programming Lab
Week 7 : Analysis of circuits using sympy

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1 Aim

- Analyze active filters using Laplace Transform
- Solving the equations using Sympy and SciPy modules in Python
- Comparing the output of both high pass and low pass filters for different input frequencies

2 Low Pass Filter

The matrix form of the circuit given in question as follows:

$$\begin{pmatrix} 0 & 0 & 1 & \frac{-1}{G} \\ \frac{-1}{1+sR_2C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -\frac{1}{R_1}-\frac{1}{R_2}-sC_1 & \frac{1}{R_2} & 0 & sC_1 \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_p(s) \\ V_m(s) \\ V_o(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{V_i(s)}{R_1} \end{pmatrix} \tag{1}$$

On solving this we will get $V_o(s)$ as

$$V_o(s) = \frac{G \cdot V_i(s)}{2_1C_2R_1R_2s^2 - C_1GR_1s + 2C_1R_1s + 2C_2R_2s + 2} \tag{2}$$

Which is low-pass filter
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2.1 Magnitude Response

Plotting the Magnitude of transfer function

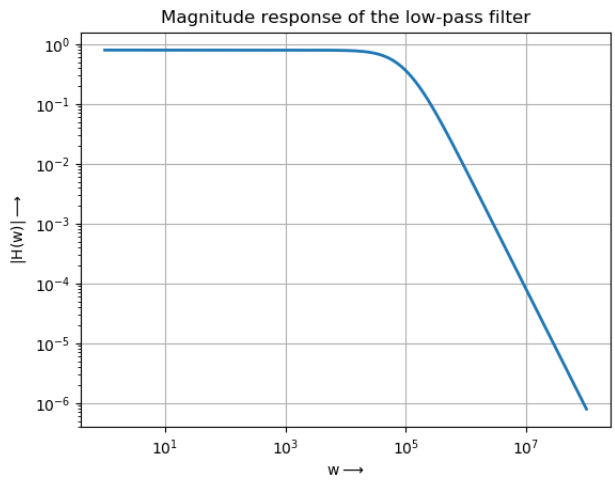

Figure 1: Magnitude response of the LPF

We can see,for high frequencies the output has a very low gain and gets attenuated.

2.2 Step Response

Unit step response of the LPF

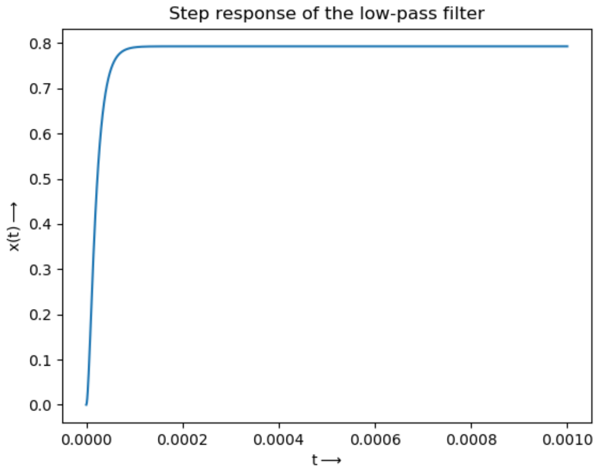

Figure 2: Step response of the LPF

Here we can see the output attains steady state soon and giving constant output as expected as low pass filter allow DC to pass through.
Here the magnitude of steady state output is ≈ 0.79 as the DC gain is 0.79 which is clear from the equation

2.3 Response for different frequency inputs

Here , we will analyse for the input

$$v_i(t) = (sin(2\pi \times 10^3 t) + cos(2\pi \times 10^6 t))u(t) \tag{3}$$

The output looks as follows

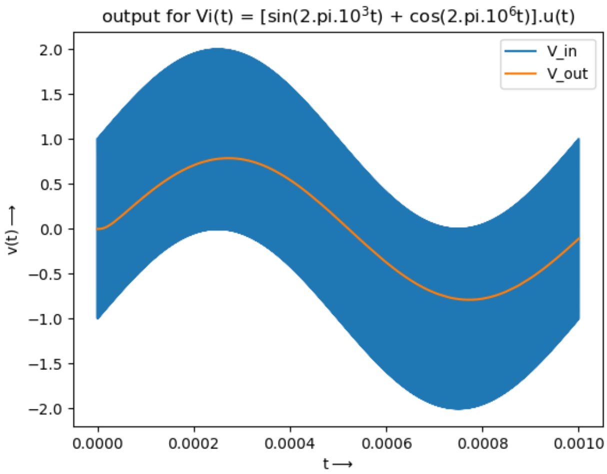

Figure 3: Output Response of LPF for mixed frequency input

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From figure , it is obvious that the LPF has filtered the high frequency component(ie,10^6Hz) and retained the low frequncy component(ie,10^3Hz)

3 High-Pass Filter

The circuit is as follows

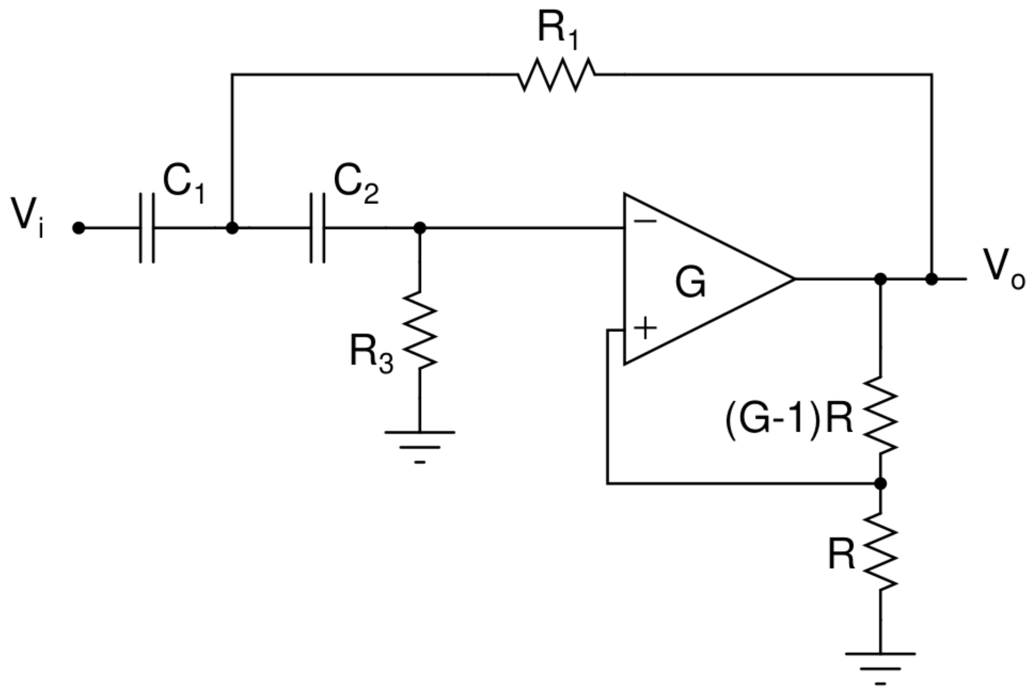

Figure 4: Active high pass filter

The matrix form of the circuit is

$$\begin{pmatrix} \frac{1}{R_1} + sC_2 + sC_1 & -sC_2 & 0 & -\frac{1}{R_1} \\ -sC_2 & \frac{1}{R_3} + sC_2 & 0 & 0 \\ 0 & 0 & G & -1 \\ 0 & -G & G & 1 \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_p(s) \\ V_m(s) \\ V_o(s) \end{pmatrix} = \begin{pmatrix} sC_1V_i(s) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

On solving this we will get $V_o(s)$ as

$$V_o(s) = \frac{sC_1(2C_2R_1R_3s + 2R_1) \cdot V_i(s)}{2_1C_2R_1R_3s^2 + 2C_1R_1s - C_2GR_3S + 2C_2R_1s + 2C_2R_3s + 2} \tag{5}$$

3.1 Magnitude Response

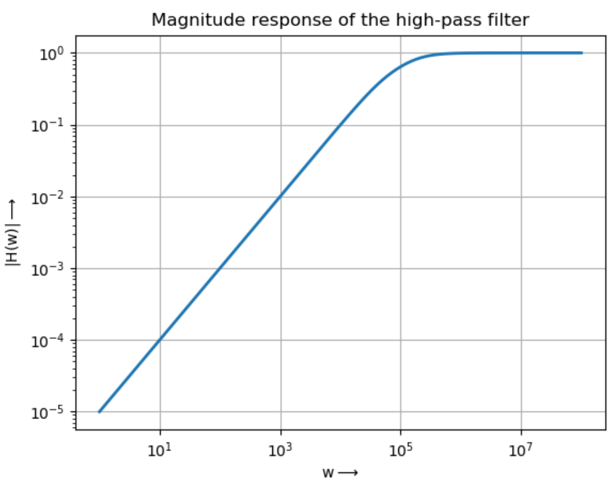

Figure 5: Magnitude Response of HPF

3.2 Step Response

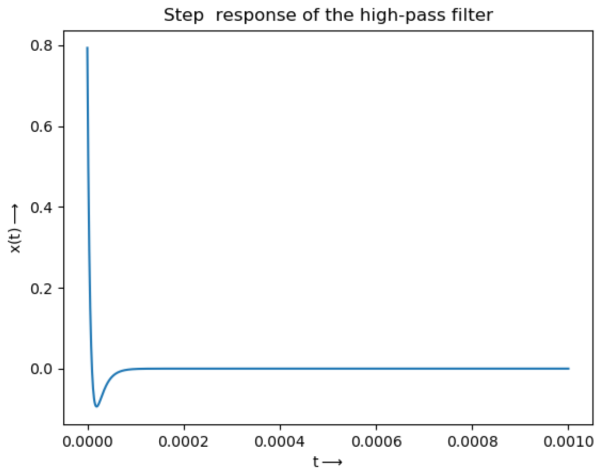

Figure 6: Unit step response of HPF

The output quickly becomes Zero after an initial impulse which occurs cause of the nature of step function.

3.3 Response for different frequency inputs

Here , we will analyse for the input

$$v_i(t) = e^{-500t}cos(2\pi \times 10^3 t))u(t) \tag{6}$$

The response is as follows

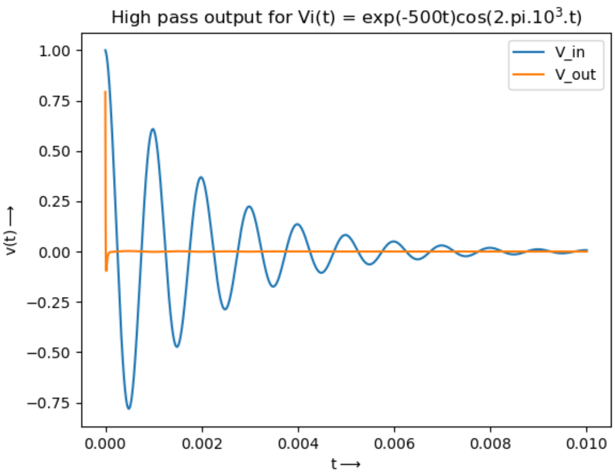

Figure 7: Response for the given input

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Here ,we can see from the response that the lower frequency(ie,10^3Hz) is getting attenuated from the HPF.

Now we will analyse with high frequency signal. The input is

$$v_i(t) = e^{-5000t}cos(2\pi \times 10^6 t)u(t) \tag{7}$$

The response is as follows

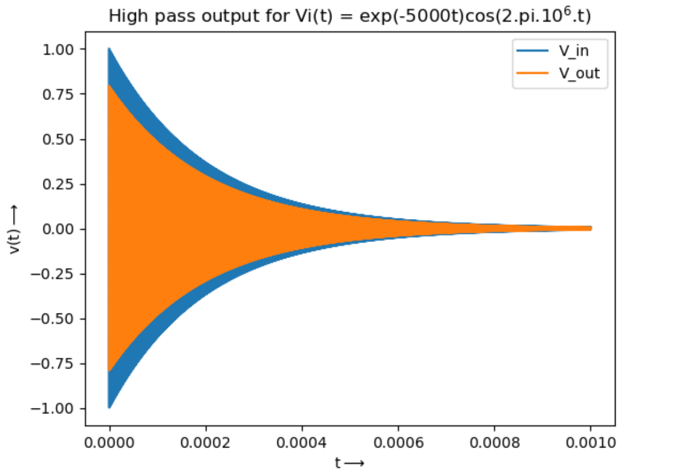


Figure 8: Response for the given input

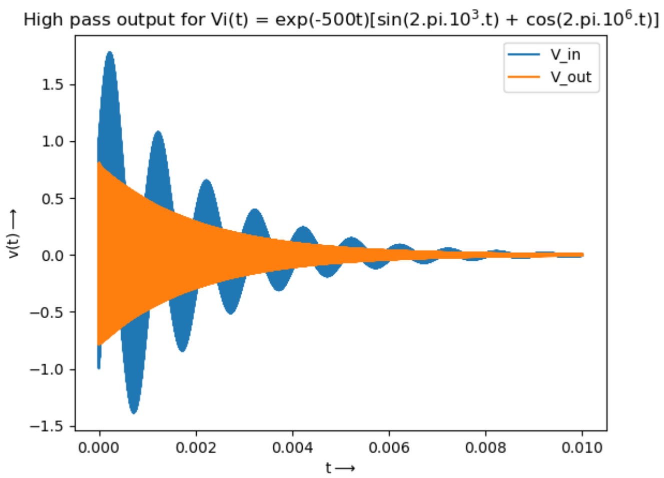
Here we can see that the high frequency component (ie,10^6Hz) is preserved in the output

4 Comparing LPF and HPF responses for same input signal

Now , we are going to compare the output of both HPF and LPF for the same input signal consisting of both high and low frequencies

The input is

$$v_i(t) = e^{-500t}(sin(2\pi \times 10^3 t) + cos(2\pi \times 10^6 t))u(t) \tag{8}$$



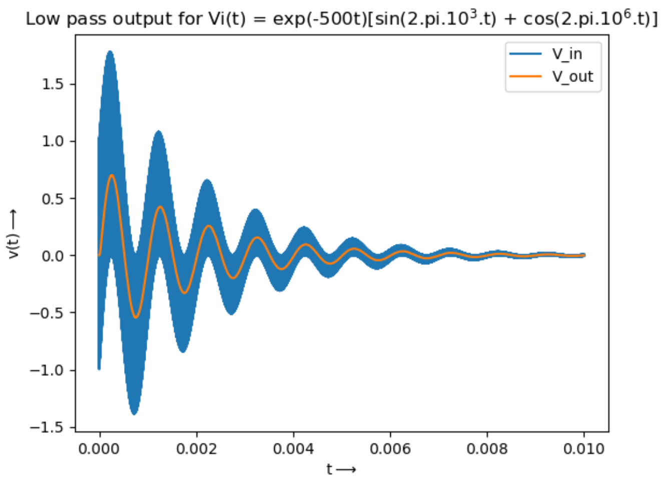


Figure 9: *Left*:HPF output
Right:LPF output

It's very clear from the above two plots that high pass filter preserves high frequency and filters off low frequency components while LPF preserves low frequency and filters off high frequency components

5 Conclusion

- Sympy provides a convenient way to analyse LTI systems using their Laplace Transforms.
- We converted the symbolic representation of the transfer functions to a LTI class which can be used with SciPy's signal toolbox.
- For a mixed frequency sinusoid inputs,the LPF suppressed the high frequencies while allowing the low frequency components and HPF suppressed the low frequencies while allowing the high frequency components.