EE2703: Assignment 5
Week 5 : The Resistor Problem

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1 Aim

- Finding the flow of currents in a resistor in a conductor
- visualise how much heating takes place in the conductor
- solve the Laplace equation numerically and plot the 2D and 3D plots of potential distribution and current flow

2 Theory

- A wire is soldered to the middle of a copper plate and its voltage is held at 1 Volt. One side of the plate is rounded, while the remaining are floating. The plate is 1 cm by 1 cm in size.
- To solve for currents in resistor,we use following equations
- Conductivity

$$\vec{J} = \sigma \vec{E} \tag{1}$$

- Electric field

$$\vec{E} = -\nabla \phi \tag{2}$$

- Continuity equation

$$\nabla . \vec{J} = -\frac{\partial \rho}{\partial t} \tag{3}$$

- From above equations above, we get

$$\nabla .(-\sigma \nabla \phi) = -\frac{\partial \rho}{\partial t} \tag{4}$$

- Assuming constant conductivity

$$\nabla^2 \phi = \frac{1}{\sigma} \frac{\partial \rho}{\partial t} \tag{5}$$

- For DC currents, the right side is zero, and we obtain

$$\nabla^2 \phi = 0 \tag{6}$$

- Considering 2D plate

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{7}$$

- we get

$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \tag{8}$$

- So the potential at any point is the average of its neighbours.At boundaries where there is no electrode the gradient of potential is tangential and where the electrode is present the potential is zero.
- The equations for the current densities are:

$$J_x = -\frac{\partial \phi}{\partial x} \tag{9}$$

$$J_y = -\frac{\partial \phi}{\partial y} \tag{10}$$

$$J_{x,ij} = \frac{1}{2}(\phi_{i,j-1} - \phi_{i,j+1}) \tag{11}$$

$$J_{y,ij} = \frac{1}{2}(\phi_{i-1,j} - \phi_{i+1,j}) \tag{12}$$

3 Plots

3.1 Contour Plot of Initial Potentials

We initialise all the potentials as 0 and then make the potential of the wire portion to be 1 V.This refers to all the points which satisfy the condition:

$$X^2 + Y^2 \leq (radius)^2 \tag{13}$$

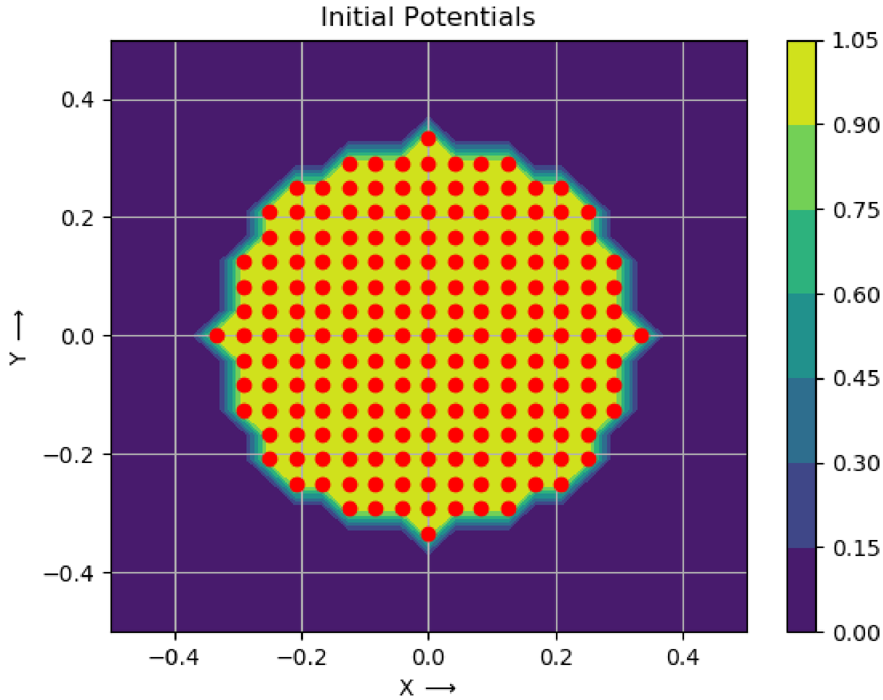

Figure 1: Contour plot of initial potential

3.1.1 Updating potential and plotting errors

Update the potential ϕ according to Equation

$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \tag{14}$$

Apply Boundary Conditions and maintain potential of central bead as 1V.

- To plot the errors in semilog and log-log and observe how the errors are evolving.

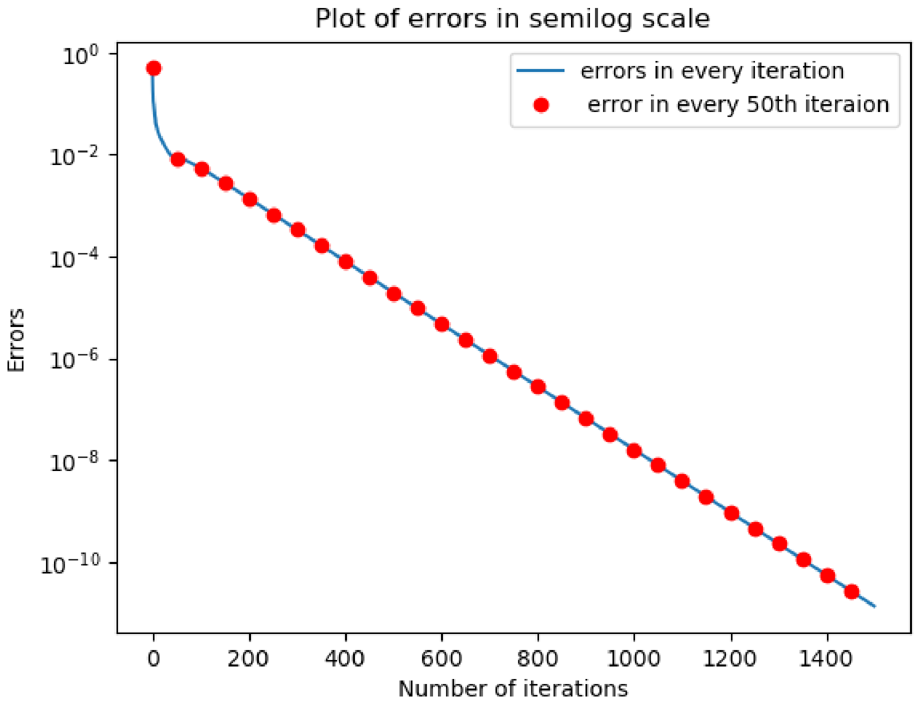

Figure 2: Semilog plot of Error vs No.of Iterations

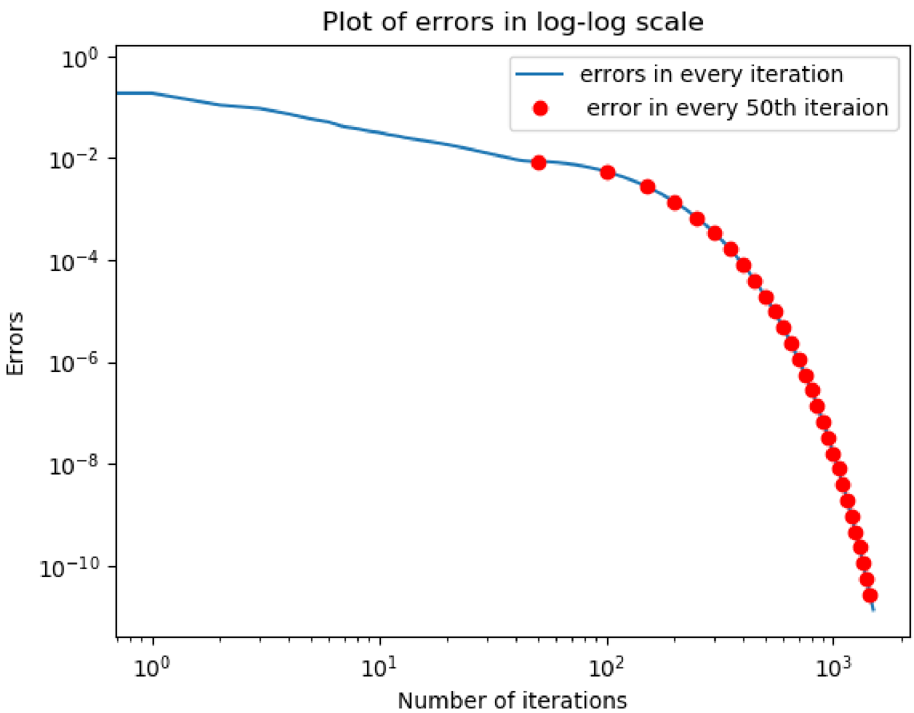

Figure 3: Log-Log plots of Error vs No.of Iterations

- We observe that error decreases linearly in semilog plot for higher no of iterations,so for large iterations error decreases exponentially with No of iterations i.e it follows Ae^{Bx}
And if we observe log-log plot the error is almost linearly decreasing So to conclude the error follows Ae^{Bx} for higher no of iterations($\approx$ 500)

3.1.2 Fitting errors

- To find the fit using Least squares for all iterations named as fit1 and for iterations ≥ 500 named as fit2
- The error follows Ae^{Bx} at large iterations, we use equation given below to fit the errors using least squares

$$logy = logA + Bx \tag{15}$$

- To plot the two fits obtained and observe them

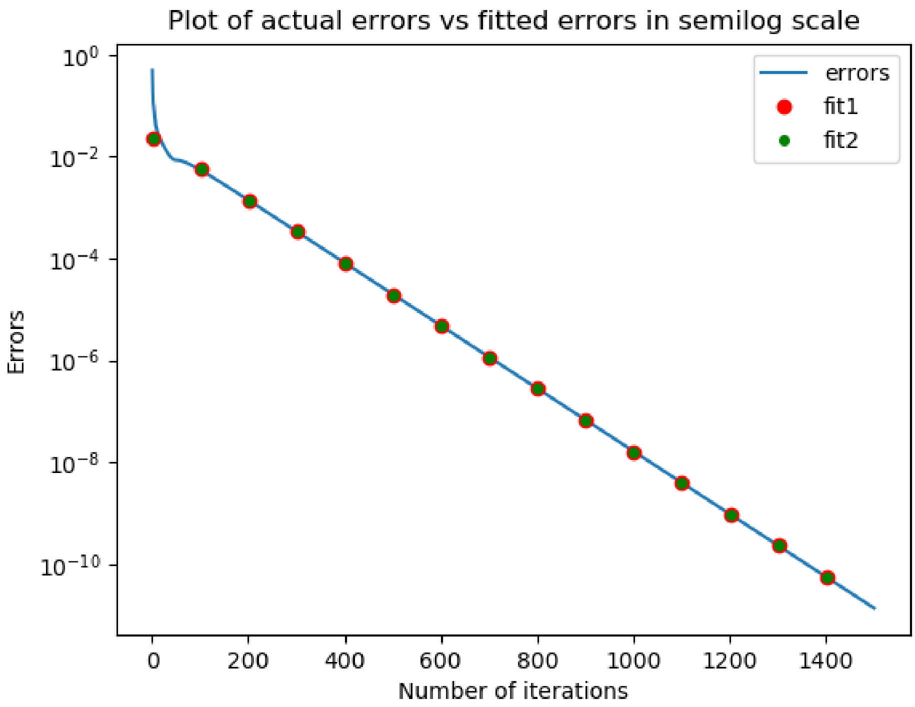

Figure 4: Semilog plot of Error vs No.of Iterations

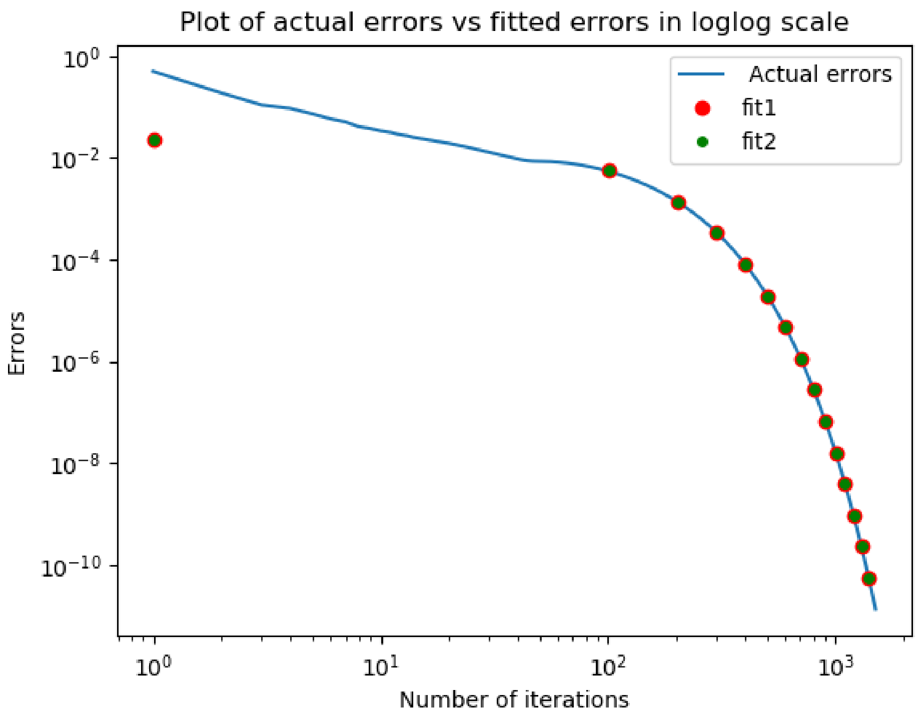

Figure 5: log-log plot of Error vs No.of Iterations

- Fit1 : log A = -3.74207 and B = -0.0142
  Fit2 : log A = -3.73857 and B = -0.0142

3.1.3 Cumulative

- To find the cumulative error we use the following equation

$$Error = \sum_{N+1}^{\infty} error_k \tag{16}$$

- The above error is approximated to

$$Error \approx -\frac{A}{B} exp(B(N+0.5)) \tag{17}$$

where N is no of iteration

3.1.4 Surface Plot of Potential

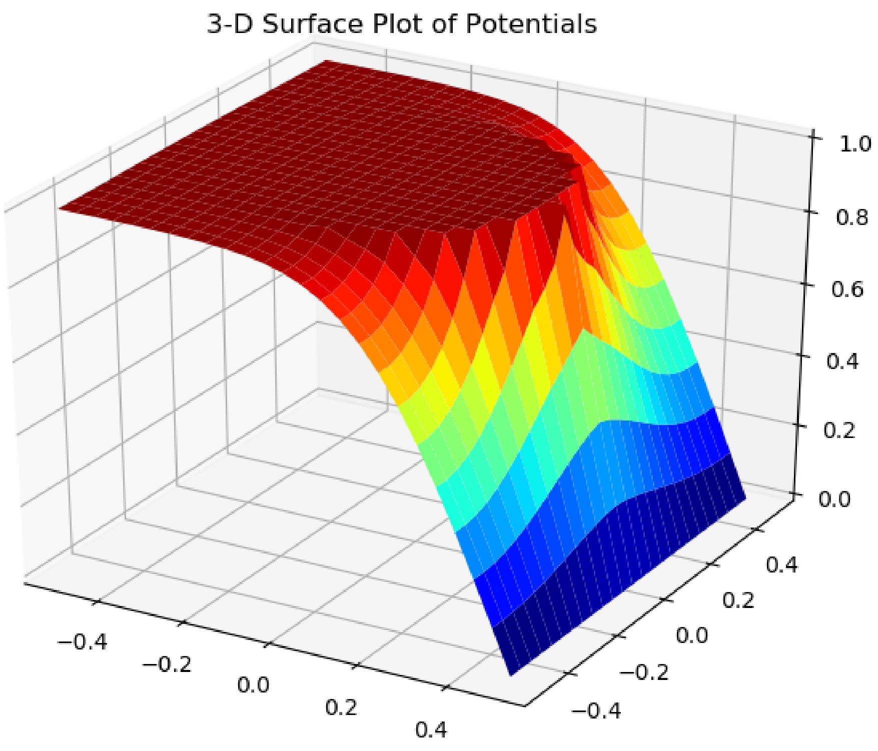

Figure 6: 3-D Surface plot of potential plot

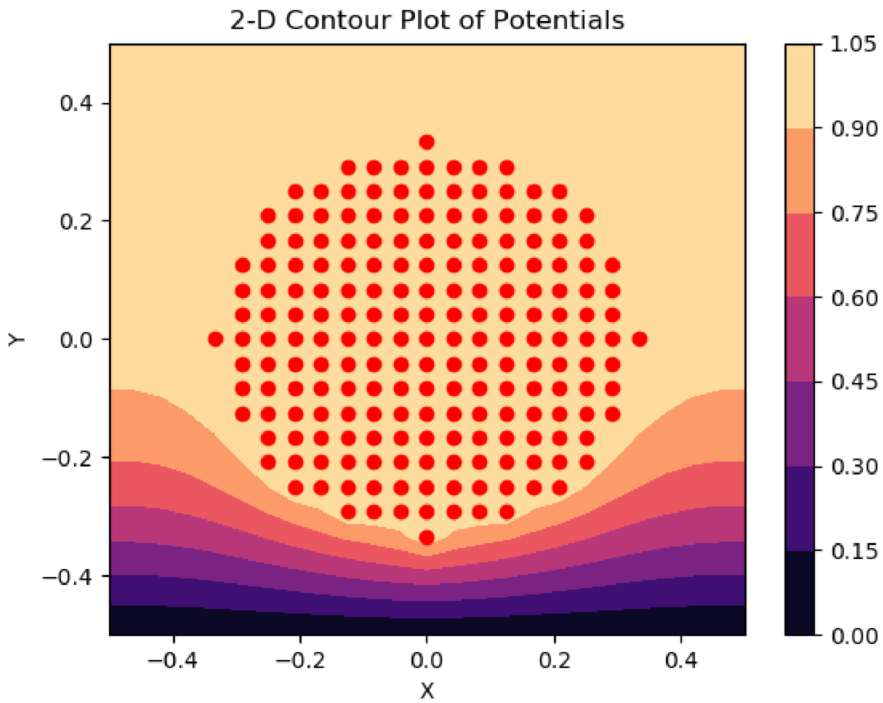

Figure 7: Contour plot of potential

3.1.5 Vector Plot of Currents :

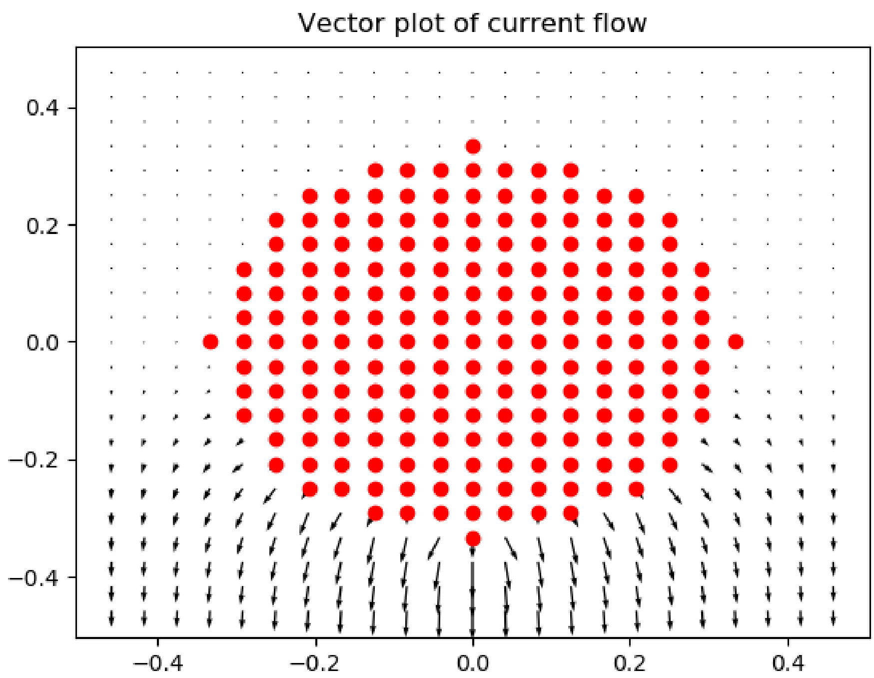

Figure 8: Vector plot of current