EE2703: Applied Programming Lab
Week9: Spectra of non-periodic signals

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1 Introduction

In this,we analyse the DFT of non-periodic finite length sequence using the `numpy.fft` module. We also look into how to minimise the errors that arise from *Gibbs Phenomenon* by using *hamming windowing*.

2 Spectrum of $sin(\sqrt{2}t)$

Using our previous method, we get the following spectrum for $sin(\sqrt{2}t)$:

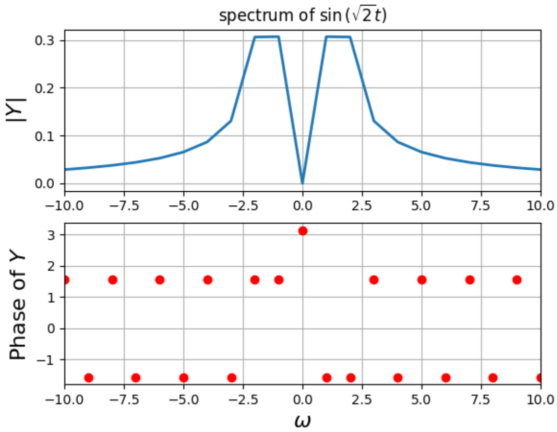

Figure 1: Spectrum of $sin(\sqrt{2}t)$

As we can see in the plot instead of two peaks at $\pm\sqrt{2}$, we got two peaks each with two values and a gradually decaying magnitude. . If we look at the region we have sampled, and construct a periodic function with it we get the following plots:

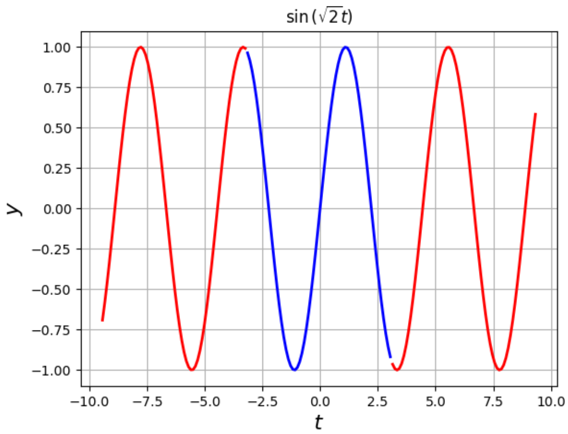

Figure 2: $sin(\sqrt{2}t)$

The plot is discontinous. This leads to a large number of errors because of Gibbs phenomenon. Hence we see significant magnitudes at higher frequencies. Now, we analyse this using the spectrum

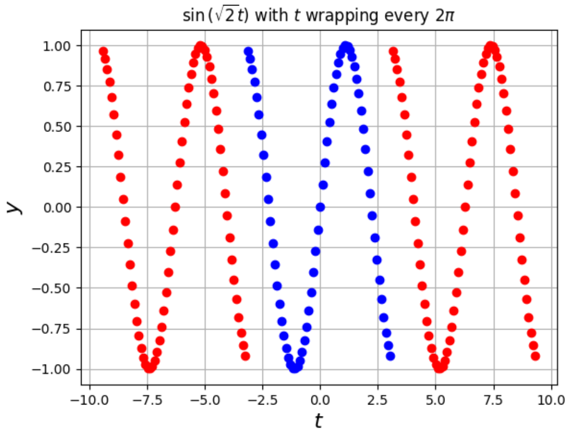

Figure 3: The function for which we are calculating DFT

of ramp function.So here the coefficients are expected to decay very slowly. Plot of the magnitude response for the ramp function. From the plot, we can say that the spectrum decays at the rate of

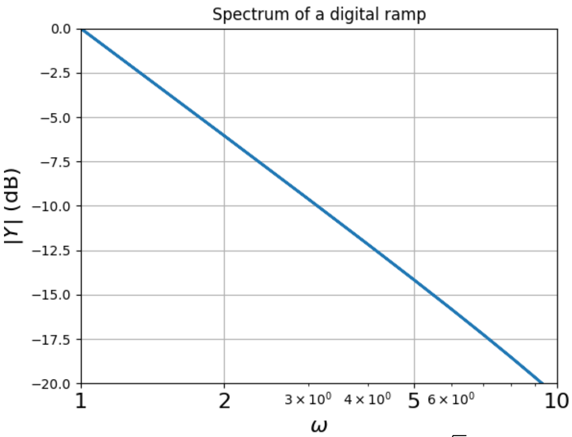

Figure 4: Windowed $sin(\sqrt{2}t)$

20dB per decade (which is due to $\frac{1}{w}$). The big jump at nπ force this slowly decaying spectrum, which is why we don't see the expected spikes for the spectrum of $sin(\sqrt{2}t)$.

3 Windowing

We can resolve this issue by **Windowing**. Here, we multiply the time-domain function with an appropriate window function. Here, we the *Hamming Window*, given as:

$$W_N[n] = \begin{cases} 0.54 + 0.46\ cos(\frac{2\pi n}{N-1}), & |n| < N \\ 0, & otherwise \end{cases}$$

By doing this we can reduce the jumb in the time domain, thus minimizing the effect of Gibb's phenomenon in the frequency domain:

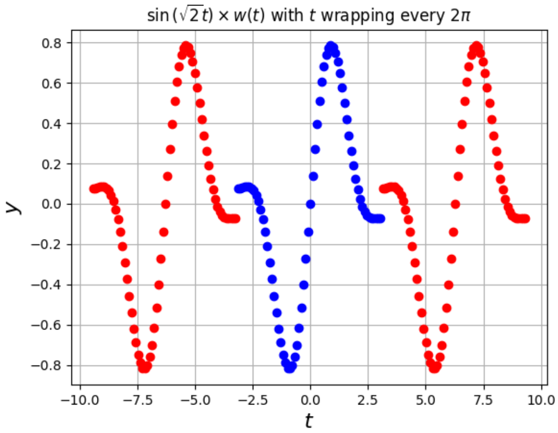

Figure 5: Windowed $sin(\sqrt{2}t)$

The discontinuity is reduced significantly. The corresponding DFT plots are:

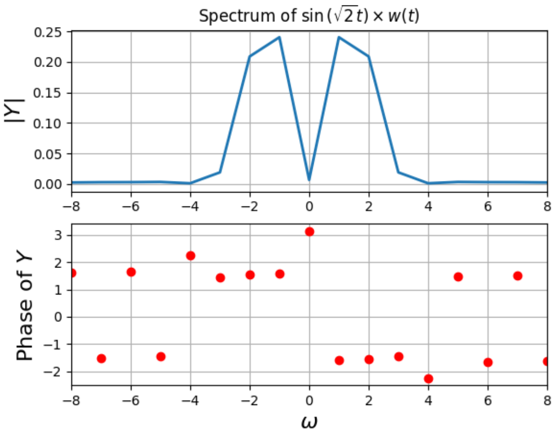

Figure 6: Spectrum of windowed $sin(\sqrt{2}t)$

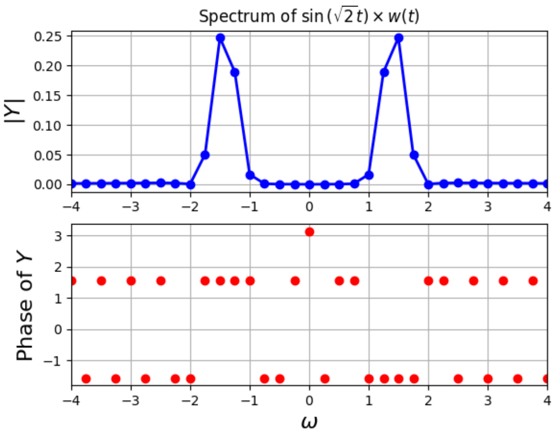

Figure 7: Spectrum of windowed $sin(\sqrt{2}t)$ with high resolution

Now we will increase the number of samples to get a better result(given above). So , we understand that the peaks are more well defined and the magnitude response falls more rapidly for higher values with windowing.

4 Spectrum of $cos^3(w_0t)$

We get the following spectra before and after windowing for $cos^3(0.86t)$

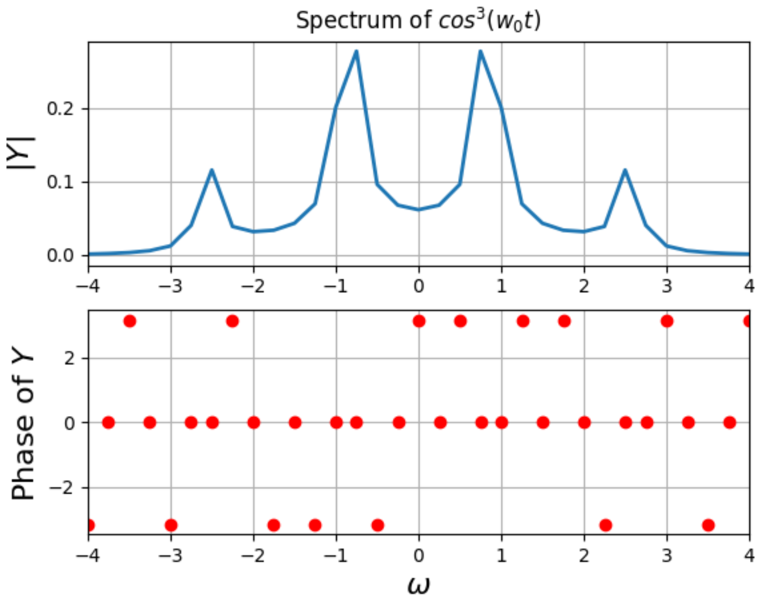

Figure 8: Spectrum of $cos^3(0.86t)$ without windowing

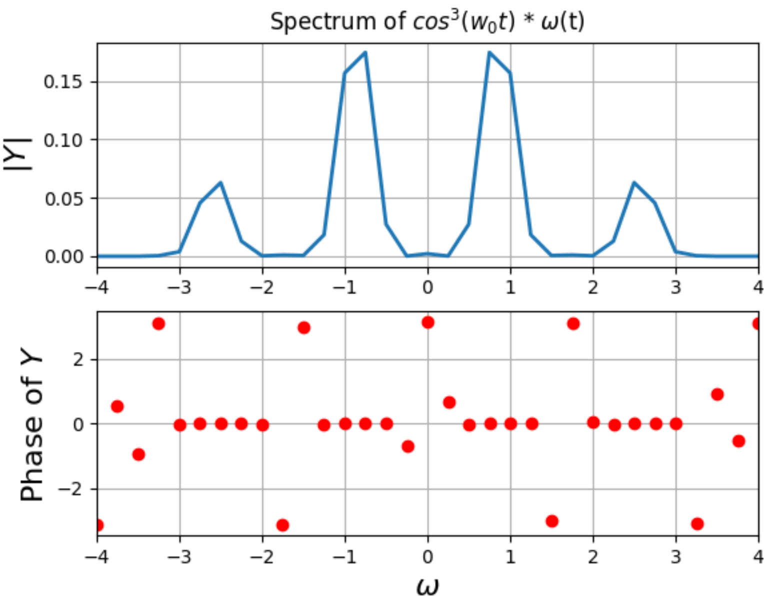

Figure 9: Spectrum of $cos^3(0.86t)$ after windowing

We can see more thin and sharper peaks at the frequencies that are present in the signal.

5 DFT Analysis of $\cos(w_0t + \delta)$:

5.1 Without noise

We expect frequency spectrum of $\cos(w_0t + \delta)$ to be located at $\pm w_0$. Let's analyse by taking w_0 = 1.4 and δ = 0.5

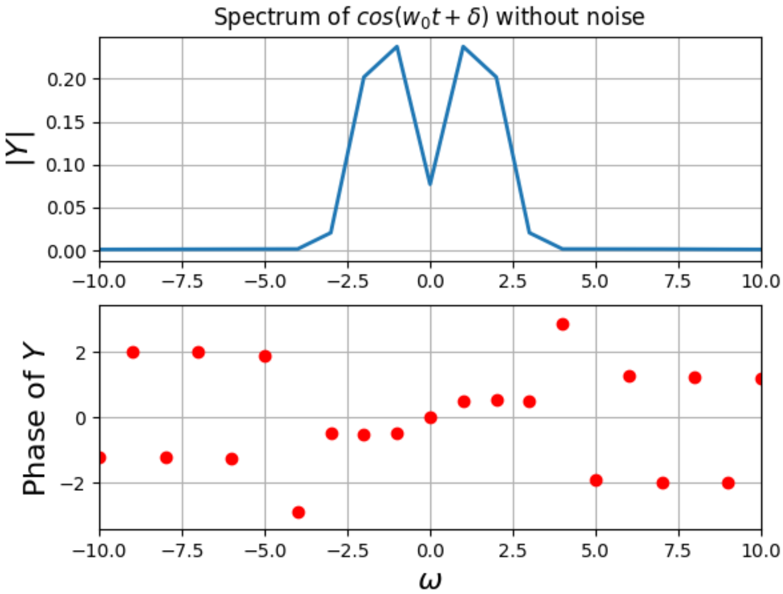

Figure 10: Spectrum of $\cos(1.4t + 0.5)$ after windowing

Now try to estimate more accurately the values by performing a weighted average with the magnitude squared being the weight for all the frequencies under the peak. The values obtained are

```
Estimations for cos(1.4t+0.5):
Estimated w_0: (1.400454-0j)
Estimated delta: 0.508469
```

The estimated values are very close to actual values.

5.2 With Gaussian noise added

We add Gaussian noise to the function $\cos(w_0t + \delta)$ and analyze the results and estimations

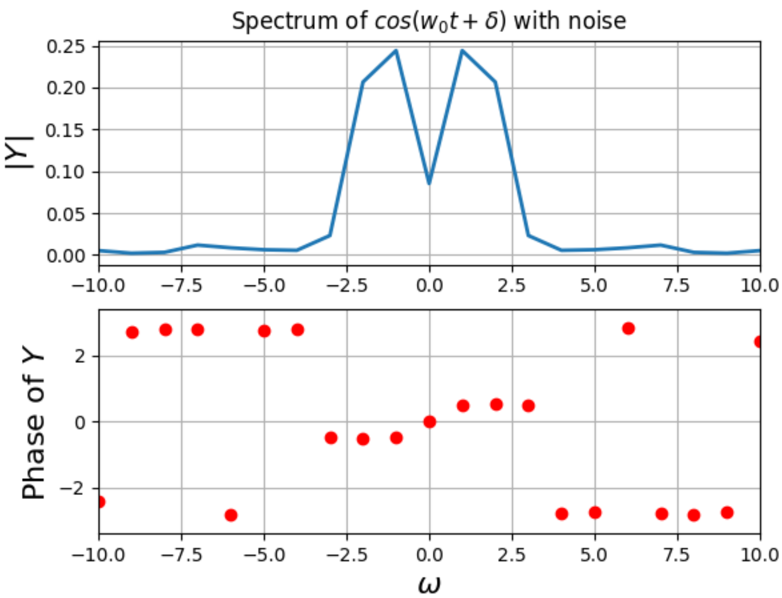

Figure 11: Spectrum of $\cos(1.4t + 0.5) + n(t)$ after windowing

The estimated values are

```
Estimations for cos(1.4t+0.5)+n(t):
Estimated w_0: (1.407807-0j)
Estimated delta: 0.51164
```

The values estimated are very close to the actual values. There is very less effect due to the noise.

6 DFT of chirp $cos(16t(1.5 + \frac{t}{2\pi}))$

The plot of the signal is given by

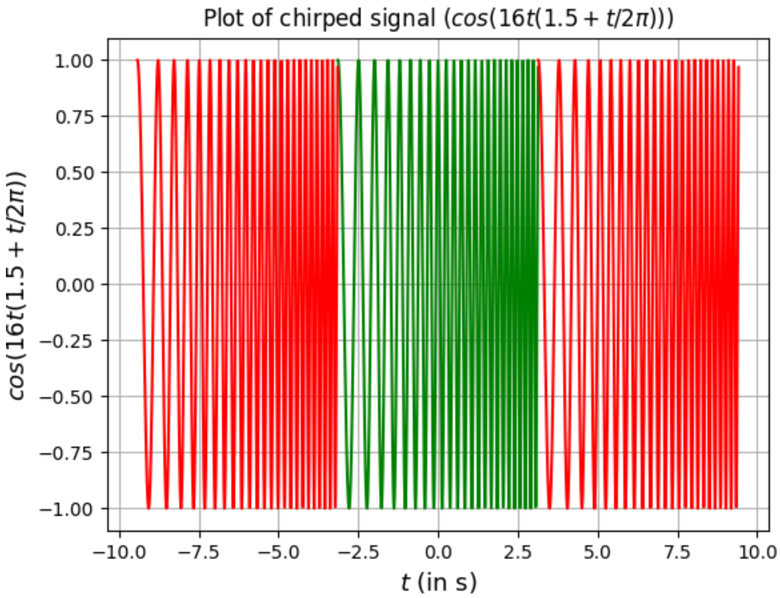

Figure 12: $cos(16t(1.5 + \frac{t}{2\pi}))$

We see that the frequency varies from 16 rad/sec to 32 rad/sec as t goes from $-\pi$ sec to π sec.

On finding the DFT of the above signal, we get:

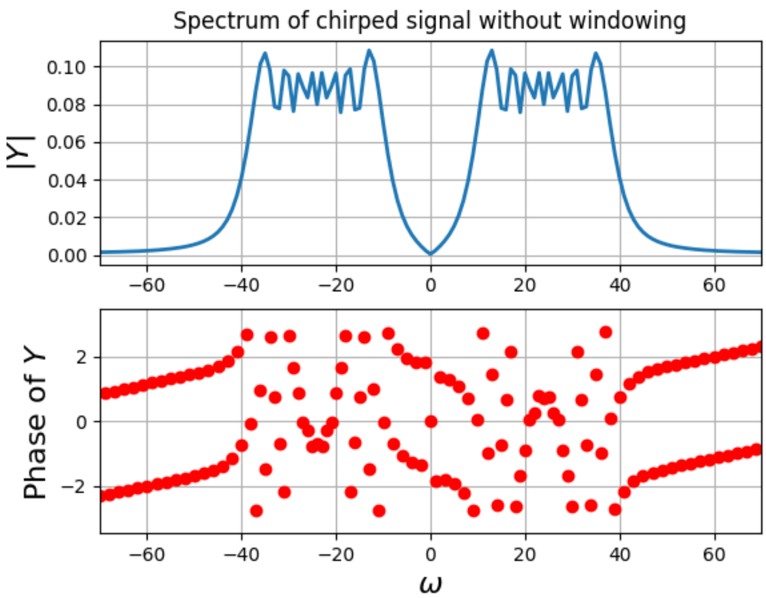

Figure 13: Spectrum of $cos(16t(1.5 + \frac{t}{2\pi}))$

Applying the Hamming Window to the chirp:

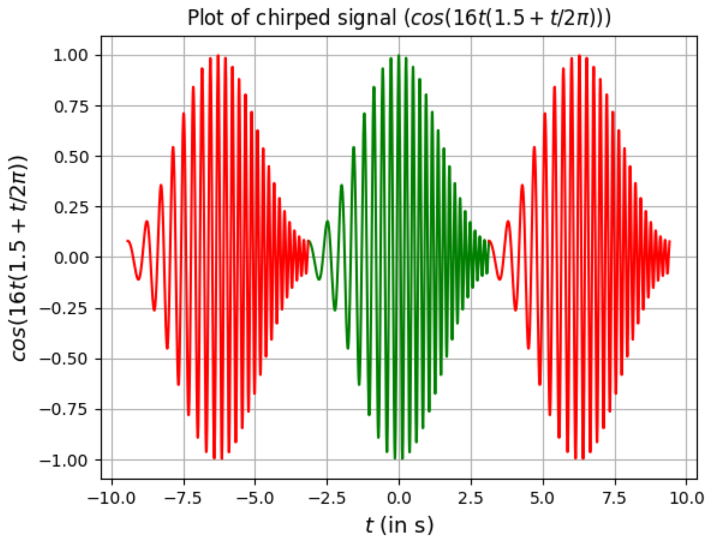

Figure 14: Windowed $cos(16t(1.5 + \frac{t}{2\pi}))$

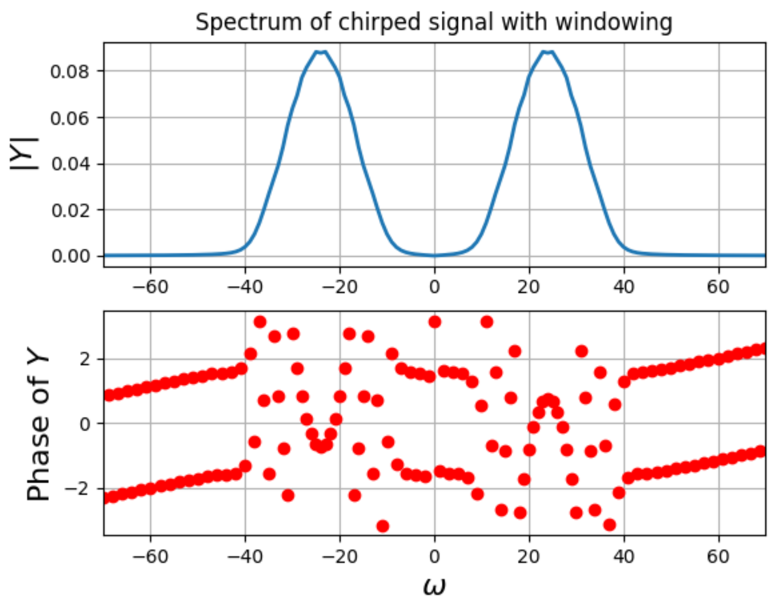

Figure 15: Spectrum of windowed $cos(16t(1.5 + \frac{t}{2\pi}))$

The variations in the frequency have been smoothed out by the window, and also, we can see that the frequencies are more accurately confined to the range 16-32 rad/sec.

7 Time-frequency plot of $cos(16t(1.5 + \frac{t}{2\pi}))$

We shall split the chirp in the time interval $[-\pi,\ \pi]$ into smaller intervals of time, and observe how the frequency of the signal varies with time.

We shall break 1024 samples from $[\pi,\ \pi)$ into 16 contiguous pieces of 64 samples each and find DFT for each interval and plot a time-frequency surface plot to observe the variation of the frequency with time.

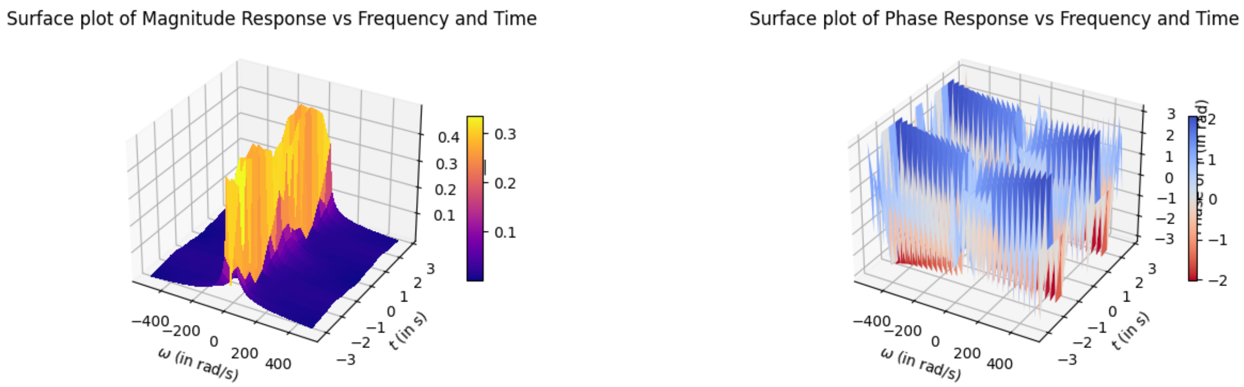

Figure 16: Spectrum of windowed $cos(16t(1.5 + \frac{t}{2\pi}))$

8 Conclusions

Frequency Spectrum of some non-periodic signals were analysed and plotted.DFT was obtained using a 2π periodic extension of the signal.But it was not what we expected.It happens due to the discontinuity in time domain and correspondingly Gibbs phenomenon . It was resolved by using Hamming window. Hamming Window increases the accuracy of the peaks. We also extracted the frequency and phase shift using weighted average method. We also analysed chirped signal at different time ranges and understood their time variation of DFT.