Applied Programming Lab
Week6 : The Laplace Transform

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March 28, 2022

1 Aim

- Analyze Linear-Time invariant Systems using the library "scipy.signal" in Python
- Finding solutions to three different systems
  1. Forced oscillatory system
  2. Coupled System of Differential Equations
  3. RLC Low-pass Filter

2 Analysis of different systems

2.1 Time response of a spring

Consider the forced oscillatory system given by the equation with initial conditions as $\dot{x}(0) = 0, x(0) = 0$.

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

where

$$f(t) = cos(1.5t)e^{-0.5t}.u(t) \tag{2}$$

with Laplace Transform given by

$$F(s) = \frac{s + 0.5}{(s + 0.5)^2 + 2.25} \tag{3}$$

Solving for $X(s)$ in Laplace domain we get,

$$X(s) = \frac{s + 0.5}{((s + 0.5^2) + 2.25)(s^2 + 2.25)} \tag{4}$$

Now, we use impulse response of $X(s)$ to get its inverse Laplace transform.ie, $x(t)$.

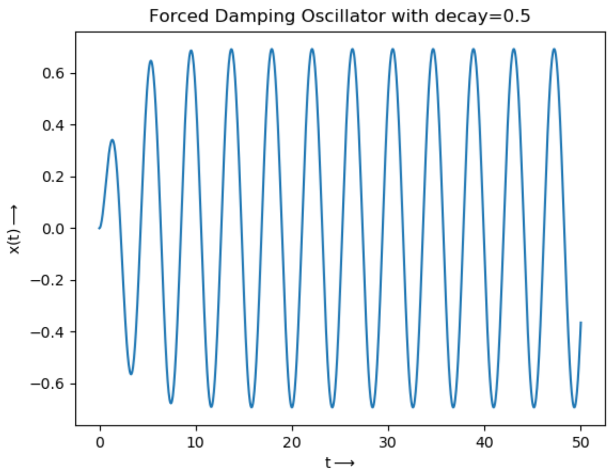

Figure 1: Response of system with Decay = 0.5

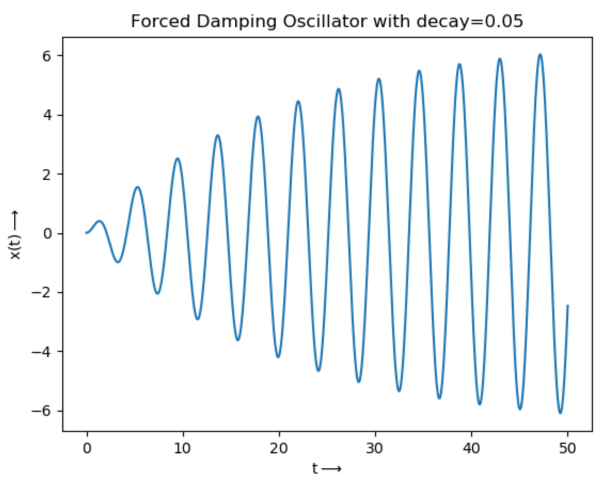

Figure 2: Response of system with Decay = 0.05

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Here, we see that the system takes more time to stabilise for lesser decay

2.2 Responses for different frequencies

Model the system as an LTI system with transfer function

$$H(s) = X(s)/F(s) = \frac{1}{s^2 + 2.25} \tag{5}$$

Obtaining the output for different values of input frequencies

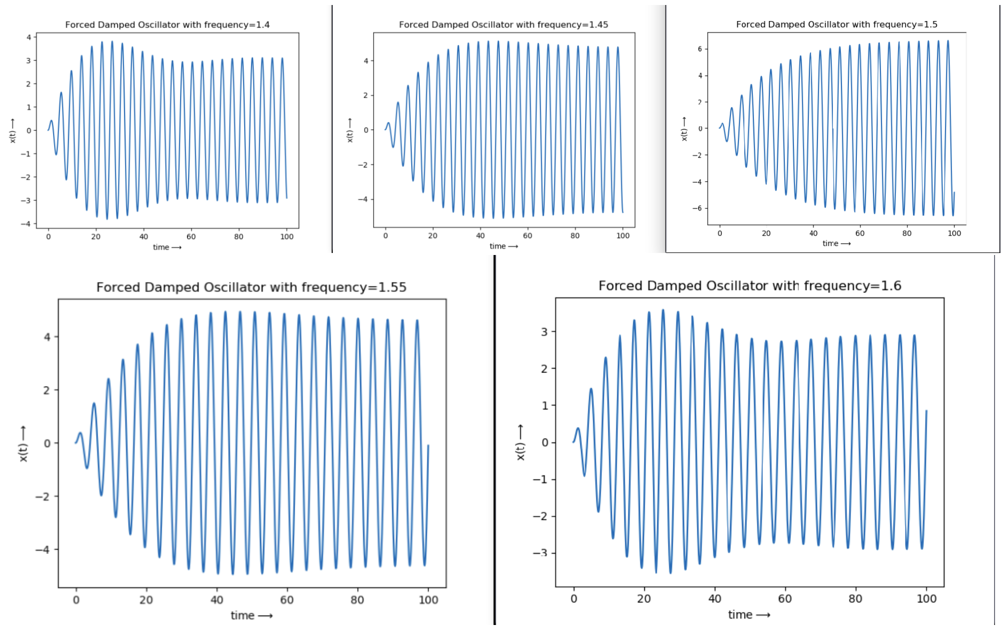

Figure 3: System Response with frequencies from 1.4 to 1.6

From the equation, we see that the frequency of natural response of the system is 1.5 rad/s . Thus, it is obvious that the maximum amplitude of oscillation is obtained when the frequency of $f(t)$ is 1.5 rad/s (resonance) which is clear from the above plots

2.3 The coupled spring system

Consider the coupled Differential system

$$\ddot{x} + (x - y) = 0 \tag{6}$$

and

$$\ddot{y} + 2(y - x) = 0 \tag{7}$$

with the initial conditions: $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$. Taking Laplace Transform and solving for $X(s)$ and $Y(s)$, We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{8}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{9}$$

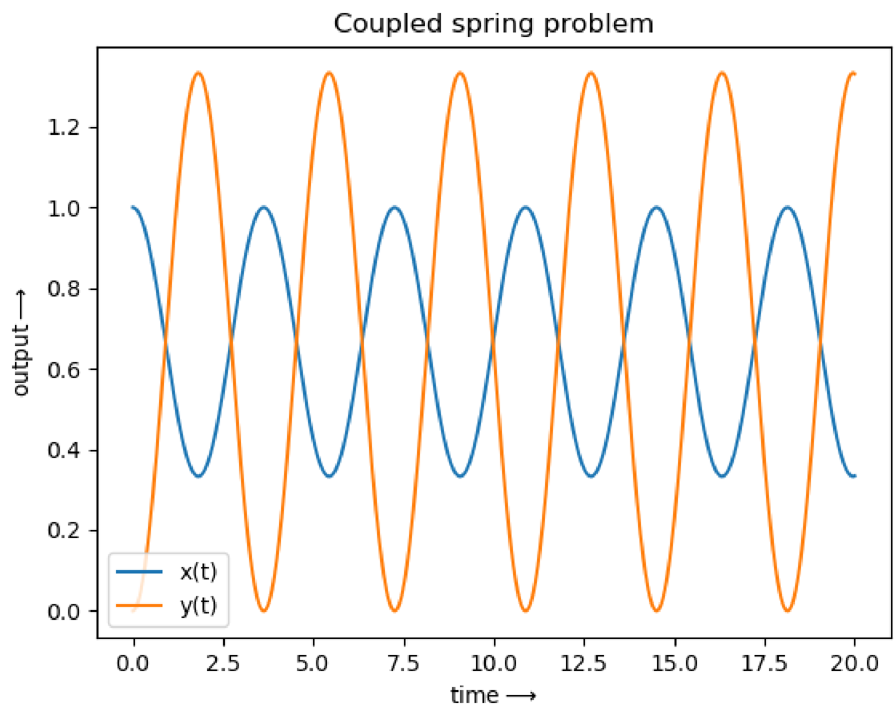

Figure 4: Coupled Oscillations

We see the outputs of this system are two sinusoids which are out of phase and with different magnitudes.The reason for having different magnitudes is that there is no symmetry between x(t) and y(t) in their differential equation

2.4 The Two Port Network (LCR low-pass filter)

2.4.1 Bode plot

The Steady-State transfer function of the given circuit is given by

$$H(s) = \frac{1/LC}{s^2 + sR/L + 1} \tag{10}$$

The magnitude and phase response are given by

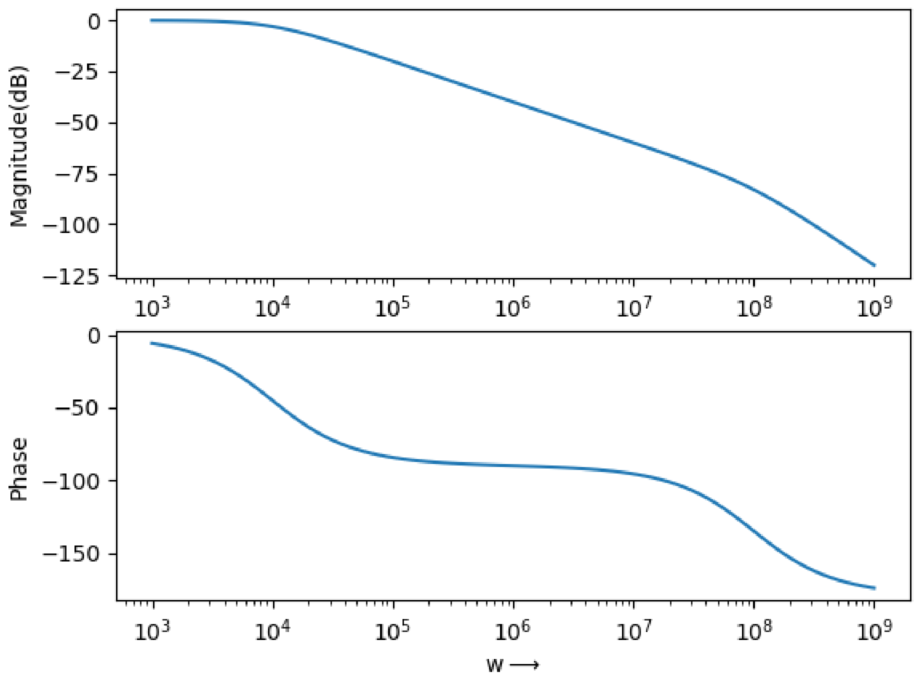

Figure 5: Bode Plots For RLC Low pass filter

2.4.2 Plot the response of the low pass filter to the input

$$V_i(t) = (cos(10^3 t) - cos(10^6 t))u(t)$$

for $0 < t < 30\mu s$ and $0 < t < 30ms$

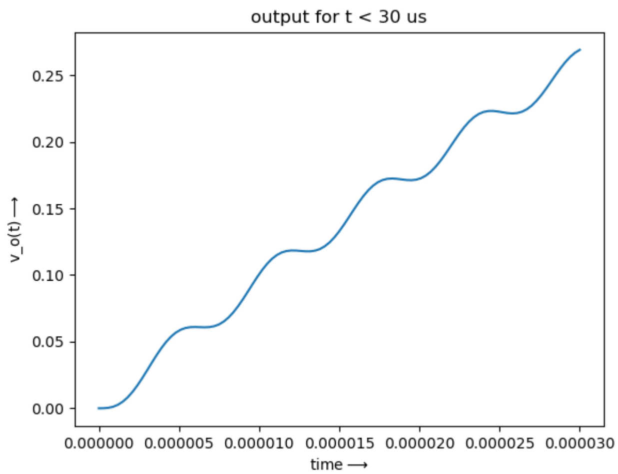

Figure 6: System response for t < 30us

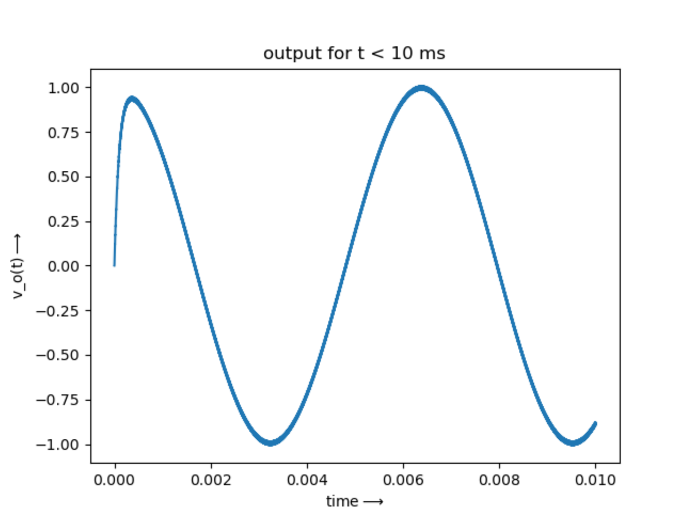

Figure 7: System response for t < 10ms

From the Bode plot of $H(s)$ we can see that the system is a low pass filter. It provides unity gain for frequency less than 10^3 rad/s. So the low frequency component remains as such while the high frequency components are filtered off.We can see that frequency of output function is nearly 10^3rad/s. But when we zoom in the plot, we can see ripple like behaviour which is actually high frequency components with very less magnitude.As our filter is not an ideal filter, the high frequency components are present in very less magnitude(eg: the frequency 10^6 rad/s is still present but with magnitude of dB ≈ -40 which means its magnitude diminished to 1/100th of original value)

3 Conclusions

- The scipy.signal library is very useful for analysis of different LTI systems in different domains like mechanics and circuits.
- Forced response of a simple spring body system was obtained for different frequencies and decays of the applied force, and highest amplitude was observed at resonant frequency.
- A coupled differential problem was solved using the tools and the functions obtained were analysed.
- A two-port network, behaving as a low-pass filter was analysed and the output behaviour was analysed for the given input frequencies.