Modeling Adobe stock price return volatility with a FARIMA-GARCH specification.

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This study applies the $FARIMA-GARCH$ model to Adobe stock returns, focusing on the estimation of conditional volatility. The $FARIMA(3,d,3)-GARCH(1,1)$ model is selected through a grid search based on AIC optimization. The results show that the FARIMA-GARCH model effectively captures both long-memory behavior (with $d = 0.058$) and volatility clustering, as demonstrated by the significance of model parameters and the absence of residual autocorrelation. The model also fits well according to diagnostic tests such as Ljung-Box and ARCH LM tests. Additionally, we extend the $FARIMA-GARCH$ model to squared returns, identifying a similar structure with $FARIMA(3,d,1)-GARCH(2,3)$, capturing both long-memory effects and volatility dynamics. While some minor residual diagnostic concerns are observed, the model is still considered robust.

1 Introduction

The consideration of lag persistence in time series modeling has attracted the attention of many statisticians, particularly with the introduction of the fractional autoregressive model by Granger and Joyeux. This model captures both long-memory effects (in terms of the slow decay of the autocorrelation function) and short-term dynamics. Conditional heteroscedastic variance models, introduced by Engle through the $ARCH$ model and later extended to $GARCH$ models, aim to capture the conditional heteroscedasticity of errors.

The presence of long-memory behavior and/or volatility is observed in many time series, especially in financial data. [Ling et Li] proposed a unification of these two approaches by generalizing the $FARIMA(p,d,q) - GARCH(r,s)$ models, which they applied to the daily returns of the Hang Seng Index (1983-1984). This development has gained widespread popularity, and further extensions have been proposed, such as $SEMIFAR-GARCH$ models ([Feng et al.]), which incorporate deterministic trends, stationarity through differencing, stationarity with short- and long-term dependencies, and heteroscedastic model errors.

In the context of our study, we focus specifically on the $FARIMA(p,d,q) - GARCH(r,s)$ models, which we apply to the returns of Adobe stocks to estimate their conditional volatility.

To achieve this goal, we will first examine the stationarity of the returns and the autocorrelogram of the squared returns. The optimization of maximum orders will be performed through a grid search. The optimal model will minimize the Akaike Information Criterion (AIC), and a residual diagnostic using portmanteau tests will help in selecting the final model.

In the interest of implementing a robust predictive model, we will apply the same approach to the squared returns to estimate their conditional volatility.

2 Specification of the FARIMA-GARCH Model

2.1 Notation

A stationary process $(Y_t)_{t\in\mathbf{Z}}$ that follows a $FARIMA(p,d,q) - GARCH(r,s)$ representation satisfies the following equations:

$$\phi(B)(1-B)^d Y_t = \psi(B)\epsilon_t, \tag{1}$$

$$\epsilon_t = z_t\sqrt{h_t}, \quad h_t = \alpha_0 + \sum_{i=1}^{r}\alpha_i\epsilon_{t-i}^2 + \sum_{j=1}^{s}\beta_j h_{t-j}, \tag{2}$$

with the following parameters and assumptions:

(i) The fractional differencing parameter $d \in]-0.5, 0.5[$,

(ii) z_t are iid standard normal random variables,

(iii) $\alpha_0 > 0$, $\alpha_1, \ldots, \alpha_r \geq 0$, $\beta_1, \ldots, \beta_s \geq 0$,

(iv) B is the backshift operator ($BY_t = Y_{t-1}$),

(v) $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$ and $\psi(B) = 1 + \psi_1 B + \cdots + \psi_q B^q$ are polynomials in B,

(vi) The polynomials $\phi(B)$ and $\psi(B)$ have no common factors, and all their roots lie outside the unit circle.

2.2 Application to Adobe Stock Returns

To estimate the FARIMA-GARCH models, we use the R package `rugarch` proposed by [Ghalanos]. We analyze the series of returns $(Y_t)_t$ as well as the squared returns. The empirical autocorrelations of the series reveal significant persistence (Figure 1(a)), indicating the presence of a long-memory component.

The orders of the FARIMA and GARCH models are selected from the values $\{0, 1, 2, 3\}$ using a grid search where the AIC criterion is optimized. The optimal specification, according to the AIC, is given by the $FARIMA(3, d, 3) - GARCH(1, 1)$ model.

Figure 1(b) highlights that the residuals of the FARIMA model exhibit conditional heteroscedasticity, which is captured by the GARCH component. The estimated conditional variance and the normalized residuals are shown in Figures 1(c) and 1(d), respectively.

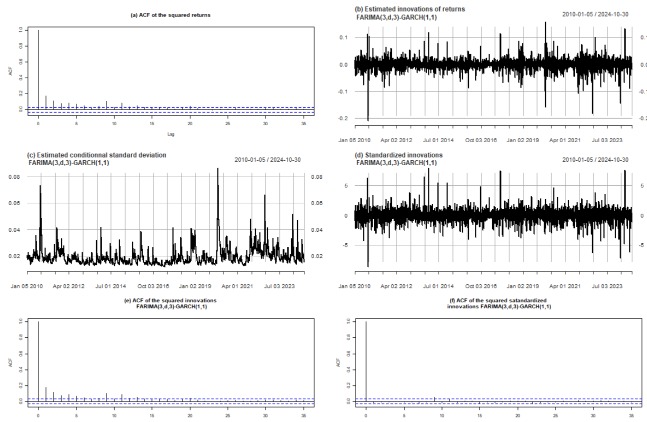

Figure 1: *ACF of the daily squared returns $(Y_t)_t$ of Adobe stock price (a), the innovations of the FARIMA model (b), the GARCH conditional standard deviation (c), the standardized innovations (d), and the ACFs of the squared, non-standardized, and standardized innovations (e and f).*

We observe a significant absence of autocorrelation in the $GARCH(1, 1)$ residuals based on the Ljung-Box white noise test (Tables 3 and 4). The ARCH LM test also supports the absence of residual ARCH effects (Table 5). Additionally, all estimated parameters are significant (Table 1) and stable (Table 6).

All these elements lead to the conclusion of a good fit to the data, which is further confirmed by the goodness-of-fit results (Table 8).

Param.	**Esti.**	**Std. E**	**t value**	**p.value**
μ	0.001	0.000008	154.866	0.000
ar1	-1.016	0.000084	-12044.442	0.000
ar2	0.584	0.000051	11413.499	0.000
ar3	0.602	0.000054	11158.312	0.000
ma1	0.929	0.000016	58256.567	0.000
ma2	-0.710	0.000022	-32091.019	0.000
ma3	-0.639	0.000017	-37807.144	0.000
arfima	0.058	0.014897	3.948	0.000
ω	0.000020	0.000002	13.324	0.000
α_1	0.111	0.005279	20.985	0.000
β_1	0.846	0.005326	158.842	0.000

Table 1: *Optimal Parameters, FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns*

This model effectively captures the dynamics of the conditional variance and the long-memory behavior ($d = 0.058$, significant), which supports the choice of specifying Adobe stock returns using a $FARIMA - GARCH$ representation.

2.3 Application to Adobe Stock Squared Returns Using FARIMA-GARCH Models

We analyzed the series of returns $(Y_t^2)_t$ and their squared. Empirical autocorrelations of the series revealed significant persistence, indicating the presence of long-memory behavior.

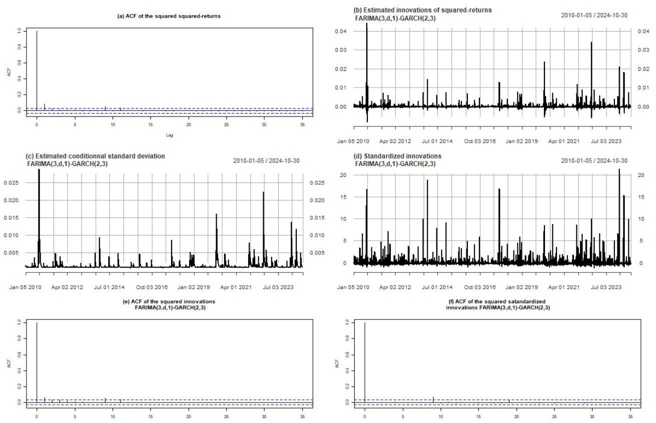

Figure 2: *ACF of the daily squared of $(Y_t^2)_t(a)$, the innovations of the FARIMA model (b), the GARCH conditional standard deviation (c), the standardized innovations (d), and the ACFs of the squared, non-standardized, and standardized innovations (e and f).*

Through a grid search across orders $\{0, 1, 2, 3\}$, the optimal model was selected based on the Akaike Information Criterion (AIC). The optimal specification was determined to be $FARIMA(3, d, 1) - GARCH(2, 3)$.

The significant parameters of the model (at a 5% level, Table 2) include:

- **Mean equation**: The AFRIMA parameters $(ar1, ar2, ar3, ma1, d)$ are significant, indicating robust long-memory dynamics. Fractional differentiation $d = 0.141$ was found to be significant, supporting the presence of long-memory behavior.

- **Variance equation**: Among the GARCH parameters, α_2, β_1, and β_3 are significant , capturing the conditional variance dynamics effectively.

- **Residual Diagnostics**:
  - The Ljung-Box test on standardized residuals (Table 9)indicates some remaining serial correlation at specific lags (e.g., Lag[11]), though other lags (Lag[1]) suggest minimal serial dependence. This suggests potential model improvements.
  - For squared residuals, the Weighted Ljung-Box test (Table 10) shows no significant autocorrelation, confirming that the model captures volatility clustering effectively.

- **ARCH Effects**:
  - Weighted ARCH LM tests (Table 11) indicate no significant residual ARCH effects at most lags, suggesting the GARCH specification is well-fitted.

- **Parameter Stability**:
  - The Nyblom stability test (Table 12) shows stability for most individual parameters, with the joint statistic slightly exceeding the critical value, indicating mild concerns over overall stability.

- **Goodness-of-Fit**:
  - Adjusted Pearson Goodness-of-Fit statistics reveal some misfit, possibly due to model misspecification or extreme events in the data.

The $FARIMA(3, d, 1) - GARCH(2, 3)$ model effectively captures the dynamics of Adobe stock squared returns, as evidenced by significant fractional differentiation ($d = 0.141$) and key variance equation parameters. Diagnostics confirm that the model explains the conditional heteroskedasticity and long-memory behavior in the data, making it a robust choice for volatility modeling. However, residual diagnostics highlight areas for potential refinement.

Param.	Estim.	Std. E	t value	Pvalue
μ	0.000	0.000	6.388	0.000
ar_1	0.593	0.116	5.130	0.000
ar_2	-0.186	0.032	-5.777	0.000
ar_3	0.248	0.032	7.674	0.000
ma_1	-0.651	0.124	-5.231	0.000
$arfima$	0.141	0.024	5.881	0.000
ω	0.000	0.000	58.183	0.000
α_1	0.006	0.006	0.997	0.319
α_2	0.425	0.023	18.095	0.000
β_1	0.144	0.013	10.999	0.000
β_2	0.000	0.018	0.000	1.000
β_3	0.424	0.009	45.237	0.000

Table 2: *Optimal Parameter, FARIMA(3,1)-GARCH(2,3) estimated on adobe stock adjusted closed price squared returns*

3 Conclusion

The $FARIMA - GARCH$ model provides a comprehensive and reliable approach for modeling the conditional volatility of Adobe stock returns. By capturing both long-memory dependencies and volatility clustering, the model fits well and passes necessary diagnostic tests. Although minor concerns were found in the residuals, the model remains a strong choice for volatility forecasting. Further studies may explore additional model variations and alternative specifications for enhanced accuracy.

References

[Feng et al.] Feng, Yuanhua and Beran, Jan and Yu, Keming (2006): Modelling financial time series with SEMIFAR-GARCH model, `https://mpra.ub.uni-muenchen.de/1593/1.haslightboxThumbnailVersion/MPRA_paper_1593.pdf`

[Ling et Li] Ling, S. and Li, W.K. (1997). *On fractional integrated autoregressive moving-average time series models with conditional heteroskedasticity.* J. Amer. Statist. Assoc., 92, 1184–1194.

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4 Annexes

4.1 FARIMA-GARCH Model Fit on adobe stock adjusted closed price returns

Lag	Statistic	p-value
Lag[1]	0.008	0.928
Lag[2*(p+q)+(p+q)-1][17]	4.545	1.000
Lag[4*(p+q)+(p+q)-1][29]	11.445	0.899

Table 3: *Weighted Ljung-Box Test on Standardized Residuals, FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns.*

4.2 FARIMA-GARCH Model Fit on adobe stock adjusted closed price squared returns

Lag	Statistic	p-value
Lag[1]	0.909	0.340
Lag[2*(p+q)+(p+q)-1][5]	1.276	0.794
Lag[4*(p+q)+(p+q)-1][9]	3.755	0.630

Table 4: *Weighted Ljung-Box Test on Standardized Squared Residuals,FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns.*

Lag	Statistic	Shape	Scale	Pvalue
ARCH Lag[3]	0.3226	0.500	2.000	0.570
ARCH Lag[5]	0.5507	1.440	1.667	0.868
ARCH Lag[7]	1.8307	2.315	1.543	0.753

Table 5: *Weighted ARCH LM Tests, FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns.*

Statistic	Value
Joint Statistic	4.0016
Individual Statistics:	
μ	0.017
ar1	0.018
ar2	0.018
ar3	0.018
ma1	0.016
ma2	0.016
ma3	0.017
arfima	0.556
ω	0.340
α_1	0.289
β_1	0.333

Table 6: *Nyblom Stability Test, FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns.*

Test	t-value	Prob
Sign Bias	1.821	0.068
Negative Sign Bias	0.764	0.444
Positive Sign Bias	0.355	0.722
Joint Effect	5.791	0.122

Table 7: *Sign Bias Test, FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns.*

Group	**Statistic**	**p-value(g-1)**
1	247.3	0.00
2	261.4	0.00
3	274.8	0.00
4	288.0	0.00

Table 8: *Adjusted Pearson Goodness-of-Fit Test, FARIMA(3,3)-GARCH(1,1) estimated on adobe stock adjusted closed price returns.*

Lag	Statistic	p-value
Lag[1]	1.141	0.285
Lag[11]	24.188	0.000
Lag[19]	30.033	0.000

Table 9: *Weighted Ljung-Box Test on Standardized Residuals, FARIMA(3,1)-GARCH(2,3) estimated on adobe stock adjusted closed price squared returns*

Lag	Statistic	p-value
Lag[1]	0.000	0.987
Lag[14]	7.711	0.418
Lag[24]	12.820	0.426

Table 10: *Weighted Ljung-Box Test on Standardized Squared Residuals, FARIMA(3,1)-GARCH(2,3) estimated on adobe stock adjusted closed price squared returns*

Lag	Statistic	Shape	P-Value
ARCH Lag[6]	0.081	0.500	0.777
ARCH Lag[8]	0.195	1.480	0.973
ARCH Lag[10]	12.285	2.424	0.010

Table 11: *Weighted ARCH LM Tests, FARIMA(3,1)-GARCH(2,3) estimated on adobe stock adjusted closed price squared returns*

Parameter	Statistic
μ	0.185
ar_1	0.066
ar_2	0.298
ar_3	0.130
ma_1	0.063
$arfima$	0.064
ω	1.064
α_1	0.255
α_2	0.176
β_1	0.080
β_2	0.235
β_3	0.087

Table 12: *Nyblom Stability Test Individual Statistics, FARIMA(3,1)-GARCH(2,3) estimated on adobe stock adjusted closed price squared returns*