NOTES Dates

$$A = \begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix}$$

$$AX = \lambda X$$

where X is eigen vector

λ is eigen value

$$(AX - \lambda X) = 0$$

$$(A - \lambda I)X = 0.$$

$$\det(A - \lambda I) = 0.$$

L&D

NOTES

Dates

$$\begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det\left[\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix}\right] = 0$$

$$4-\lambda \begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8 \begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix}$$

$$+ (-1) \begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = 0$$

NOTES Dates

$4-\lambda\left[(-9-\lambda)(5-\lambda)+20\right]-8(-10+2\lambda)$

$+20 = 0$

$(4-\lambda)\left[-45+9\lambda-5\lambda+\lambda^2+20\right]$

$+80-16\lambda+20 = 0$

$(4-\lambda)(\lambda^2+4\lambda-25)-16\lambda+100=0$

[illegible]

$\cancel{4\lambda^2}+\cancel{16\lambda}-\cancel{100}-\lambda^3-\cancel{4\lambda^2}+25\lambda$

$-\cancel{16\lambda}+\cancel{100}=0$

$-\lambda^3+25\lambda=0$

$\lambda_1 = -5$

$\lambda_2 = 5$

$\lambda_3 = 0.$

NOTES

Dates

$$\lambda(-\lambda^2 + 25) = 0$$

$$\lambda_1 = 0 \quad , \quad -\lambda^2 + 25 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$k =$

$$\lambda = \frac{\pm\sqrt{-4(-1 \times 25)}}{-2}$$

$$\lambda = \frac{\pm\sqrt{100}}{-2}$$

$$\lambda = \pm 5$$

$$\lambda_2 = 5 \quad , \quad \lambda_3 = -5$$

L&D

NOTES

Dates

$(A - \lambda I)X = 0$

Given.

$$\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

For $\lambda_1 = 0$.

$$\begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By using Echelon form.

NOTES Dates

Augmented matrix

$$\left(\begin{array}{ccc|c} 4 & 8 & -1 & 0 \\ -2 & -9 & -2 & 0 \\ 0 & 10 & 5 & 0 \end{array}\right) \quad r_2 \leftrightarrow r_3.$$

$$\left(\begin{array}{ccc|c} 4 & 8 & -1 & 0 \\ 0 & 10 & 5 & 0 \\ -2 & -9 & -2 & 0 \end{array}\right) \quad 2r_3 + r_1$$

$$\left(\begin{array}{ccc|c} 4 & 8 & -1 & 0 \\ 0 & 10 & 5 & 0 \\ 0 & -10 & -5 & 0 \end{array}\right) \quad r_3 + r_2.$$

NOTES

Dates

$$\begin{pmatrix} 4 & 8 & -1 & 10 \\ 0 & 10 & 5 & 10 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 8 & -1 \\ 0 & 10 & 5 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$4x + 8y - z = 0$

$10y + 5z = 0$

$0z = 0$

let $z = 1$, $10y + 5 = 0$

$10y = -5$

$y = -1/2$.

L&D

NOTES Dates

$4x + 8y - z = 0.$

$4x - 8/2 - 1 = 0$

$4x - 4 - 1 = 0$

$4x = 5$

$x = 5/4$

Therefore, eigen vektor.

$\Psi_1 = (5/4, -1/2, 1).$

$$\Psi_1 = \begin{pmatrix} 5/4 \\ -1/2 \\ 1 \end{pmatrix}.$$

for $\lambda_2 = 5$

NOTES

Dates

$$\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\begin{pmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By using Gaussian elimination.

Augmented matrix.

$$\left(\begin{array}{ccc|c} -1 & 8 & -1 & 0 \\ -2 & -14 & -2 & 0 \\ 0 & 10 & 0 & 0 \end{array}\right). \quad r_2 \leftrightarrow r_3.$$

NOTES Dates

$$\left(\begin{array}{ccc|c} -1 & 8 & -1 & 0 \\ & 10 & 0 & 0 \\ -2 & -14 & -2 & 0 \end{array}\right) \quad 2r_1 + r_3.$$

$$\left(\begin{array}{ccc|c} -1 & 8 & -1 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 30 & 0 & 0 \end{array}\right) \quad 3r_2 - r_3.$$

$$\left(\begin{array}{ccc|c} -1 & 8 & -1 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix} -1 & 8 & -1 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

NOTES

Dates

$-x + 8y - z = 0$

$10y = 0$

$0z = 0$

let $z = 1$, $y = 0$.

$-x + 8(0) - 1 = 0$

$-x = 1$

$x = -1$.

Therefore eigen vector

$$v_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

Date:

For Eigen value $L_3 = -5$

$$(A - \lambda)(x) = 0$$

$$\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 8 & -1 \\ -2 & -4 & -2 \\ 0 & 10 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By using Gaussian Elimination

Date:

Augmented matrix

$$\begin{pmatrix} 9 & 8 & -1 & 10 \\ -2 & -4 & -2 & 10 \\ 0 & 10 & 10 & 10 \end{pmatrix} \quad r_2 \leftrightarrow r_3$$

$$\begin{pmatrix} 9 & 8 & -1 & 10 \\ 0 & 10 & 10 & 10 \\ -2 & -4 & -2 & 10 \end{pmatrix} \quad 9/2\, r_3 + r_1$$

$$\begin{pmatrix} 9 & 8 & -1 & 10 \\ 0 & 10 & 10 & 10 \\ 0 & -10 & -10 & 10 \end{pmatrix} \quad r_2 + r_3$$

$$\begin{pmatrix} 9 & 8 & -1 & 10 \\ 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

Date:

$$\begin{pmatrix} 9 & 8 & -1 \\ 0 & 10 & 10 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$9x + 8y - z = 0$$
$$10y + 10z = 0$$
$$0z = 0$$

let $z = 1$

$$10y + 10 = 0$$
$$y = -1$$

$$9x - 8 - 1 = 0$$
$$9x - 9 = 0$$
$$x = 1$$

Date:

Therefore eigen vector

$$v_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Importance of eigen values

$$\left(\frac{\lambda i}{\text{sum}\lambda}\right) \times 100$$

$$\text{Sum} = |0| + |-5| + |5|$$
$$= 10$$

For $\lambda_1 = 0$

$$\frac{|0|}{10} \times 100 = 0\%$$

Date:

For $\lambda_2 = 5$

$$\frac{|5|}{10} \times 100 = 50\%$$

For $\lambda_3 = -5$

$$\frac{|-5|}{10} \times 100 = 50\%$$