# Module III

## Moment of Inertia.

### Strain energy

Strain energy is defined as the energy stored in a body due to deformation. The strain energy per unit volume is known as strain energy density and the area under the stress-strain curve towards the point of deformation when the applied force is released.

### Resilience

The total strain energy stored in a body is commonly known as resilience. Hence resilience also defined as the capacity of a strained body for doing work on the removal of strained force.

### Proof resilience

* Maximum strain energy stored in body is known as proof resilience.
* strain energy stored in body will be maximum when the body is stressed up to elastic limit.
* Hence proof resilience is quantity of strain energy stored in a body when strained up to elastic limit.

Modulus of Resilience

* proof resilience of material per unit volume
* It is important property of a material

Modulus of resilience = $\frac{\text{proof resilience}}{\text{volume of the body}}$

Strain energy stored in a Body when the load is gradually applied.

U = strain energy, V = volume of body, E
E = modulus of elasticity
$\sigma$ = stress

when the load is suddenly applied

$\sigma = 2 \times P/A$

$\sigma$ = stress
P = load applied suddenly
A = area of cross section.

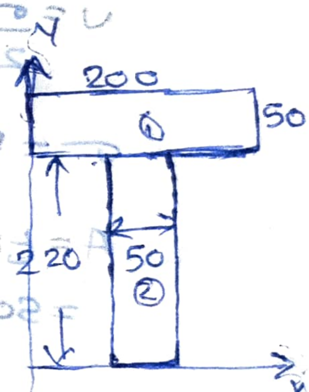
Center of gravity (COG)

* Center of gravity is an imaginary balancing point where the body weight can be assumed to be concentrated and equaually distributed. Its symbol is COG or CG.

# ~~Centroid~~ and centre of gravity

| Centroid | Center of gravity |
|---|---|
| It is defined as a point about which the entire line area or volume is assumed to be concentrated. | It is defined as a point about which the entire weight of the body is assumed to be concentrated |
| * It is related to distribution of length, area and volume | * It is ~~related~~ to distribution of mass. |

Q Find the centroid of T section as shown in the figure.




An: Here T section is symmetrical about ~~capital~~ Y-Y axis therefore centroid will be somewhere at Y axis

Rectangle ①

$A_1 = 200 \times 50 = 10000\,mm^2$

$Y_1 = 220 + \frac{50}{2} = 245\ mm$

Rectangle ②

$A_2 = 220 \times 50 = 11000\,mm^2$

$Y_2 = \frac{220}{2} = 110\,mm$

∴ Coordinate the centroid of T section

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2} = \frac{10000 \times 245 + 11000 \times 110}{10000 + 11000}$$

Calculate strain energy stored to a bar 2m long. 50mm wide and 40mm thickness when it is subject to tensile load of 60kN take modulus of elasticity 200Gpa.

An:

$L = 2m = 2000mm$

$w = 50mm$

$t = 40mm$

$P = 60kN$

$= 60 \times 10^3 N$

$E = 200 Gpa = 200 \times 10^9 N/m^2$

$= 200 \times 10^3 N/mm^2$

$U = \frac{\sigma^2}{2E} \times V$

$\sigma = P/A$

$A = t \times w$

$= 50 \times 40 = 2000 mm^2$

$\sigma = \frac{60 \times 10^3}{2000}$

$\sigma = \frac{60 \times 10^3}{2000} = 30 N/mm^2$

$V = A \times L$

$= 2000 \times 2000$

$= 4 \times 10^6 mm^3$

$$U = \frac{30^2}{2 \times 200 \times 10^3} \times 4 \times 10^6$$

$$= 9 \times 10^3 \text{ N.mm}$$

Q An axle pull of 20 kN is suddenly applied on steel rod 2.5m long and 1000mm² in crosection calculate the strain energy which can be absorbed in the rod. E = 200Gpa.

An: We know that

Suddenly load

$$\sigma = 2 \times \frac{P}{A}$$

$L = 2.5m = 2500mm$

$A = 1000mm^2$

$E = 200Gpa$

$$= \frac{200 \times 10^9}{10^6} = 200 \times 10^3 \text{ N/mm}^2$$

m = mm = $10^3$

m² = mm² = $10^6$

$$\cancel{U = \frac{\sigma^2}{2E}} \times V$$

$$\sigma = 2 \times \frac{20 \times 10^3}{1000}$$

$$\sigma = 40 \text{N/mm}^2$$

$$U = \frac{\sigma^2}{2E} \times V$$ → strain energy stored in body

$$U = \frac{40^2}{2 \times 200 \times 10^3} \times 1000 \times 2500$$

$$U = 10000 \text{ N/mm}$$

Q A tensile load of 60kN is gradually applied load to a circular bar of 4cm diameter and 5m long. If the value of $E = 2\times10^5 N/mm^2$. Determine

1) Stretch in the rod
2) Stress in the rod
3) Strain energy absorbed by the rod.

Ans: $L = 5m = 5\times10^3 mm$

$P = 60 kN = 60\times10^3 N$

$E = 2\times10^5 N/mm^2$

$d = 4cm = 4\times10 mm$

~~Stre~~ $A = \pi/4 D^2 = \frac{3.14}{4}\times(40)^2$

$= 125600 mm^2$

Stress in rod

$\sigma = P/A$

$= \frac{60\times10^3}{125600} = 47.746 N/mm^2$

Stretch $x = \frac{\sigma}{E}\times L$

$x = \frac{47.746}{2\times10^5}\times5\times10^3 = 1.19mm$

Strain energy, $U = \frac{\sigma^2}{2E}\times V$

$U = \frac{47.746^2}{2\times2\times10^5}\times125600\times5\times10^3$

$= 35810 N/mm$

Q. A tensile load of 60kN is suddenly load applied a circular bar of 4 cm dia, and 5 cm long. If value of $E = 2\times10^5 N/mm^2$ determine stress, stretch, stain energy absorbed by the rod.

An: $P = 60kN = 60\times10^3 N$ $\quad E = 2\times10^5 N/mm^2$

$d = 4cm = 4\times10 \, mm$

$L = 5cm = 5\times10 \, mm$

$A = \frac{\pi}{4}\times D^2$

$= 1256$ ~~00~~

suddenly load $= \sigma = 2\times\frac{P}{A}$

$= \frac{2\times60\times10^3}{1256}$

$= \underline{95.54 \, N/mm^2}$

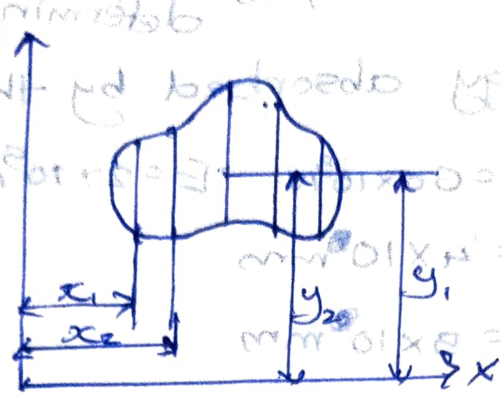
$x = \frac{\sigma}{E}\times L$

$x = \frac{95.54}{2\times10^5}\times(5\times10^5) = \underline{2.38}$

strain

$U = \frac{\sigma^2}{2E}\times V = \frac{95.54^2}{2\times(2\times10^5)}\times1256\times5\times10^3$

# Moment of inertia of a plain area (I)

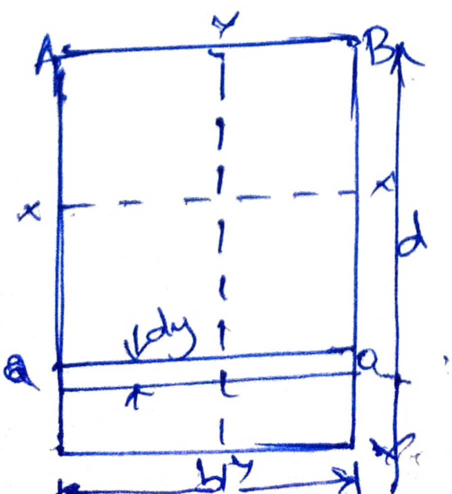



Consider a plain area whose moment of inertia required to be found out. Split up the whole area into a number of small elements let $a_1$, $a_2$, $a_3$ = area of small elements and $x_1, x_2, x_3$ etc = corresponding distance of element from the y axis $y_1, y_2, y_3$ = corresponding distance of x axis there for moment of inertia

$$I = a_1 x_1^2 + a_2 x_2^2 + \dots$$

$$I = a_1 y^2 + a_2 y_2^2 + \dots$$

## Moment of inertia of rectangular section



Consider a rectangular section 'ABCD' as shown in the figure let $b$ = width of the section $d$ = depth of the section. Now we consider a strip PQ of thickness '$dy$' parallel to the x axis and at a distance $y$ from the x axis

Area of strip = $b \times dy$

moment of inertia of strip about xx axis

$$= \text{Area} \times y^2$$

$$\therefore = b \times dy \times y^2$$

$$I_{xx} = b \times y^2 \times dy \quad \text{—(1)}$$

Now moment of inertia of all section may be found by integrating eqn (1) for the whole length of rectangle laminar from $-\frac{d}{2}$ to $\frac{d}{2}$

$$\int_{-d/2}^{d/2} b \times dy^2 \times dy$$

$$= b \int_{-d/2}^{d/2}$$

$$= b \left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b \left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right]$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

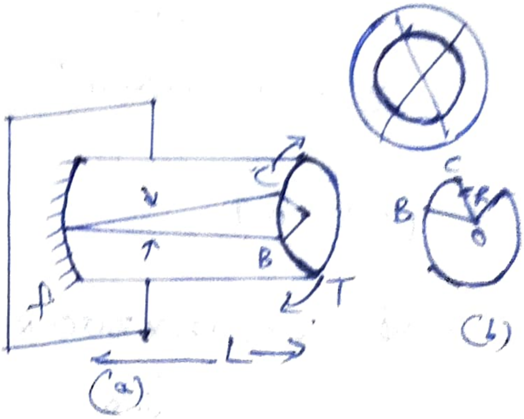
20/20

Module - 4

# Torsion of shaft

A tangential force acting on a machine parts tongsten twist. The twisting of turning effect is called torque.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$




where,

T. Torque in Nm or Nmm

J- polar moment of inertia $m^4$ or $mm^4$

$\tau$ = shearstress $N/m^2$ or $N/mm^2$

$\theta$ = Angle of twist = in Radianes

R = Radius of shaft in m or $mm^2$

G = Torsional Rigidity in $N/m^2$ or $N/mm^2$

Torque transmitted by the shaft (solid)

| | |
|---|---|
| $T = \tau \times \frac{\pi d^3}{16}$ | for solid |
| $T = \tau \times \pi \frac{(D^4 - d^4)}{16D}$ | hollow shaft |

Power transmitted by the shaft

$$\text{Power } (P) = \frac{2\pi NT}{60}$$

T = Torque = N/mm

N = Speed = rpm

P = Power = W

Q A solid circular shaft running at 300rpm transmit 200kW. Corresponding shearstress produced is 100 N/mm$^2$. Calculate the suitable diameter of the shaft.

An:

$N = 300\,rpm$

$P = 200\,kW$

$= 200 \times 10^3\,W$

$\tau = 100\,N/mm^2$

$$P = \frac{2\pi N T}{60}$$

$$200\times10^3 = \frac{2\times3.14\times300\times T}{60}$$

$$T = \frac{2\times\cancel{3.14\times300\times200\times10^3}}{\cancel{60}}$$

$$1884T = 200\times10^3\times60$$

$$T = \frac{12\times10^6}{1884}$$

$$= 6369.4\,N/m$$

$$T = \frac{\tau\times\pi d^3}{16}$$

$$6369.4\times10 = 100\times\frac{3.14d^3}{16}$$

$$63694 = \frac{3.14d^3}{16}$$

$$d^3 = \frac{63694\times16}{3.14}$$

$$d = \sqrt[3]{324555} = \underline{68.7\,mm}$$

Q find the power transmitted by a circular shaft of 50mm dia at 120 rpm. The maximum shear stress of the shaft is not exceeded is 60 N/mm²

~~find the~~

50mm = dia

120 = rpm

$\tau = 60\ N/mm^2$

$\cancel{P = \frac{2\pi N T}{60}}$

$\cancel{P = 2 \times 3.14 \times 120 \times}$

$T = \tau \times \frac{\pi d^3}{16}$

$= \frac{60 \times 3.14 \times 50^3}{16}$

$= 1471875\ N/mm$

$P = \frac{2\pi N T}{60}$

$= \frac{2 \times 3.14 \times 120 \times 1471875}{60}$

$= 18\ kW$

Q A hollow shaft is transmit 200kW at 80 rpm. If the the shear stress is not to exceed. An iternal diameter is 0.6" of external diameter, find the diameter of shaft.

Ans:

$P = 200kN$

$= 200 \times 10^3 N$

$N = 80rpm$

$\tau = 60\,Mpa = 60\,N/mm^2$

$d = 0.6D$

$\cancel{T = \frac{\tau \times \pi D^3}{16}}$

$\cancel{= 60 \times 3.14}$

$\cancel{T = \frac{\tau \pi (D^4 - d^4)}{16D}}$

$= 60 \times \pi \left[\frac{D^4 - (0.6D)^4}{16D}\right]$

$P = \frac{2\pi NT}{60}$

$200 \times 10^3 = \frac{2 \times 3.14 \times 80 \times T}{60}$

$T = \cancel{8.37} \frac{200 \times 10^3}{8.37}$

$= 2388 \times 10^6$

$T = \frac{\tau \pi (D^4 - d^4)}{16D}$

$23.88 \times 10^6 = 60 \times \pi \times \left[\frac{D^4 - (0.6D)^4}{16D}\right]$

$23.88 \times 10^6 = 60 \times \pi \times D^{4\,3} \left[\frac{1 - 0.6^4}{16D}\right]$

$23.88 \times 10^6 = 60 \pi D^3 \left[\frac{1 - 0.6^4}{16}\right]$

$D^3 = \frac{23.88 \times 10^6}{10.254}$

$D = \sqrt[3]{2328847.27}$

$= 132.55\,mm$

$\therefore d = 0.6D$

$d = 79.5\,mm$