CO2(DAC)

Compare shaft, axle and spindle.

Shaft:

A shaft is a long, slender, and usually cylindrical mechanical component. It is used primarily to transmit rotational motion and torque from one part of a machine to another. Shafts can be stationary or rotate themselves, depending on their specific application. Common examples of shafts include those in engines, transmission systems, and electric motors. Shafts may have various shapes, such as straight, tapered, or splined, to suit their intended function.

Axle:

An axle is a specific type of shaft that is designed to support and transmit the load of a rotating wheel or gear. It is often found in vehicles (e.g., cars, trucks, bicycles) and machinery with wheels. Axles come in pairs (front and rear axles in vehicles) and are critical for load-bearing and maintaining proper alignment of wheels. In vehicles, axles may also contain differentials, allowing the wheels to rotate at different speeds while still receiving power from the engine.

Spindle:

A spindle is a specialized shaft designed for tasks related to machining, cutting, or holding workpieces in place during manufacturing processes. It is commonly used in lathes, milling machines, and similar equipment. Spindles often have precision bearings and are capable of high rotational speeds to ensure accurate and smooth machining operations. Unlike general-purpose shafts or axles, spindles are usually finely balanced and precisely engineered for specific applications.

PURPOSE, TYPES AND FORCES ACTING ON SUNK KEY

A sunk key, also known as a key or keyway, is a common mechanical component used to connect two rotating machine elements, such as a shaft and a hub, to transmit torque while preventing relative axial or radial movement between them. Sunk keys are crucial for maintaining the alignment and proper functioning of rotating parts in various machines and mechanisms. Here, I'll explain the purpose, types, and forces acting on sunk keys:

Purpose of Sunk Keys:

The primary purpose of sunk keys is to ensure a secure and non-slip connection between two rotating components.

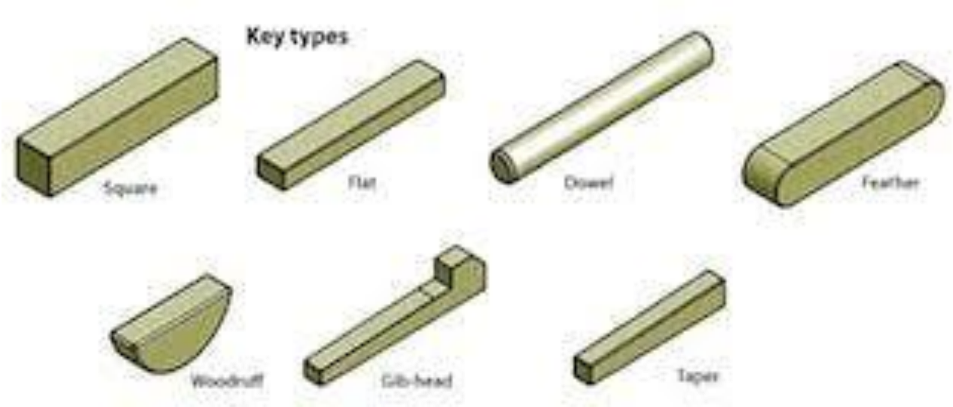
They serve the following key functions:

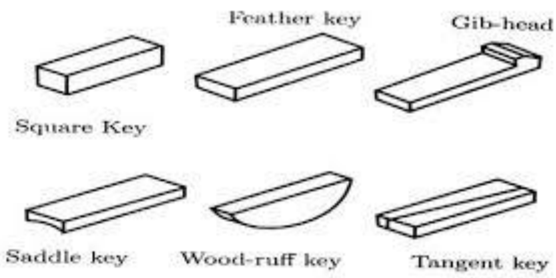
1. **Torque Transmission:** Sunk keys transmit torque from a driver component (e.g., a shaft) to a driven component (e.g., a wheel, gear, or pulley). This allows power to be transferred efficiently from one part to another.

2. **Preventing Relative Motion**: Sunk keys prevent axial (along the shaft) and radial (perpendicular to the shaft) movement between the connected parts. This helps maintain the alignment of machine elements, ensuring smooth and reliable operation.
3. **Easy Disassembly:** In cases where disassembly is required for maintenance or replacement, sunk keys provide a convenient means to connect and disconnect components without damaging them.

Types of Sunk Keys:

There are several types of sunk keys, each designed for specific applications and load requirements. The main types include:





1. **Rectangular Key**: Rectangular keys are the most common type and have a rectangular cross-section. They come in various sizes and are suitable for a wide range of applications. Rectangular keys are easy to manufacture and install.
2. **Square Key:** Square keys are similar to rectangular keys but have a square cross-section. They are often used when additional strength and rigidity are required.
3. **Woodruff Key:** Woodruff keys are semicircular keys that fit into a keyseat with a matching semicircular shape in the shaft. They are commonly used in applications with limited axial space or where a key might interfere with other components.
4. **Gib Head Key:** Gib head keys have an enlarged head on one end, which prevents the key from moving out of the keyseat. They are used when axial movement of the key needs to be restricted.

5. **Tapered Key (Spline Key):** Tapered keys have a tapered shape, providing a wedging action when driven into the keyseat. They are used when a very secure and non-slip connection is required.

Forces Acting on Sunk Keys:

The forces acting on sunk keys primarily include:

1. **Torque (T):** The key must transmit the torque generated by the rotating shaft to the connected component. The key experiences shear stress due to this torque.
2. **Shear Stress (τ):** Shear stress occurs in the key as it resists the twisting motion applied by the torque. The key material and dimensions must be selected to ensure that the shear stress does not exceed the material's allowable shear strength.
3. **Compressive Stress:** In some cases, especially with rectangular or square keys, there can be compressive stress on the key when it is clamped between the shaft and the hub. The key material must be able to withstand this compressive force without deformation or failure.
4. **Bearing Stress:** The bearing stress occurs where the key makes contact with the keyseat. It's important to ensure that the bearing stress is within the allowable limits for both the key and the shaft/hub materials to prevent wear or deformation.

FUNCTIONS, REQUIREMENTS AND TYPES OF VARIOUS COUPLINGS

Couplings are mechanical devices used to connect two rotating shafts in order to transmit torque and motion from one shaft to another. They serve several important functions in machinery and equipment, including accommodating misalignment, reducing shock loads, and dampening vibrations. Couplings come in various types to suit different requirements and applications.

Functions of Couplings:

1. **Torque Transmission:** The primary function of a coupling is to transmit torque from one shaft to another, allowing mechanical power to be transferred efficiently.
2. **Misalignment Compensation:** Couplings can accommodate angular, parallel, and axial misalignment between the connected shafts, ensuring smooth operation and reducing wear on bearings and other components.
3. **Shock Load Absorption:** Couplings can absorb and dampen shock loads and torsional vibrations that may occur during operation, protecting the connected components from damage.
4. **Isolation of Vibrations:** In some cases, couplings are designed to isolate and reduce vibrations, preventing them from propagating through the system and causing noise or discomfort.

5. **Torsional Stiffness:** Couplings can provide torsional stiffness to maintain the synchronization of shafts in precision applications.

Requirements for Couplings:

When selecting a coupling for a specific application, it's essential to consider the following requirements:

1. **Torque Capacity:** The coupling must be able to handle the maximum torque generated in the system without failure.
2. **Misalignment Tolerance:** The coupling should accommodate the anticipated misalignment between shafts, whether angular, parallel, or axial.
3. **Vibration Damping:** In applications where vibration damping is critical, the coupling should provide this function effectively.
4. **Stiffness:** In precision machinery, the coupling's torsional stiffness may be important to maintain accurate synchronization.
5. **Speed and RPM Range:** The coupling must be suitable for the required speed and RPM range of the system.
6. **Environmental Considerations:** Consider factors such as temperature, humidity, and exposure to corrosive substances when selecting a coupling material.
7. **Ease of Installation and Maintenance:** Choose a coupling that is easy to install and maintain, reducing downtime and maintenance costs.

Types of Couplings:

1. Flexible Couplings:

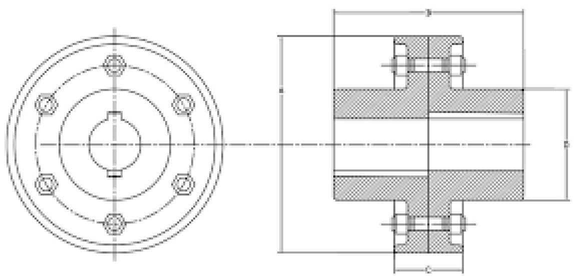
These couplings are designed to accommodate misalignment and vibration damping.

Types include elastomeric (rubber), jaw, and grid couplings.

2. Rigid Couplings:

Rigid couplings do not accommodate misalignment and are used when shafts must be precisely aligned.

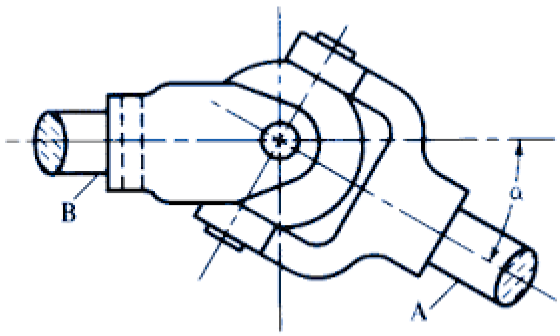
Types include sleeve, clamp, and flange couplings.



3. Universal Joint Couplings:

Universal joints (U-joints) are used to transmit torque between shafts at an angle.

Common in automotive and industrial applications.



Gear Couplings:

Gear couplings are robust and capable of transmitting high torques, often used in heavy machinery.

They are known for their high torsional stiffness.

Disc Couplings:

Disc couplings offer high torque capacity and are often used in precision applications.

They provide excellent misalignment accommodation and vibration damping.

Chain Couplings:

Chain couplings are suitable for high torque, shock-load applications and are often used in conveyors and industrial equipment.

Oldham Couplings:

Oldham couplings allow axial and radial misalignment compensation through sliding elements.

Bearings:

Bearings are mechanical components that play a vital role in reducing friction and facilitating smooth relative motion between two or more parts of a machine or system. They support loads, guide moving parts, and enable the rotation of shafts. Bearings can be classified into various types based on their function and design. Here, we'll discuss the functions and classifications of bearings:

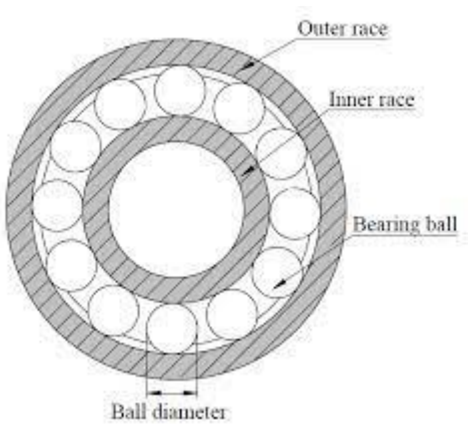
Functions of Bearings:

1. Reduce Friction: The primary function of a bearing is to reduce friction between moving parts. This minimizes wear and heat generation, improving the efficiency and longevity of mechanical systems.
2. Support Loads: Bearings provide support to loads, allowing machines to carry and distribute forces while maintaining stability and alignment.
3. Facilitate Motion: Bearings enable smooth, controlled relative motion between machine components, such as shafts, gears, and wheels.
4. Alignment: Bearings help maintain proper alignment between rotating and stationary parts, ensuring that the components work together as intended.
5. Shock Absorption: In some applications, bearings can absorb and dampen shocks and vibrations, protecting the machinery from damage.
6. Sealing: Bearings can be integrated with seals or shields to prevent contaminants, such as dust and moisture, from entering the bearing housing.
7. Load Distribution: Bearings distribute loads evenly over a larger surface area, reducing stress and wear on individual components.

Classification of Bearings:

Bearings can be classified into several categories based on various factors, including their design, load-carrying capacity, and application. Common classifications include:

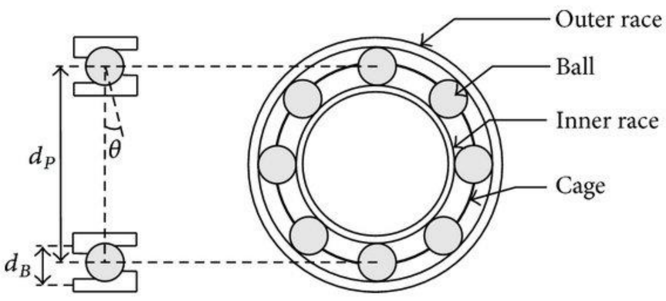
1. **Ball Bearings:**

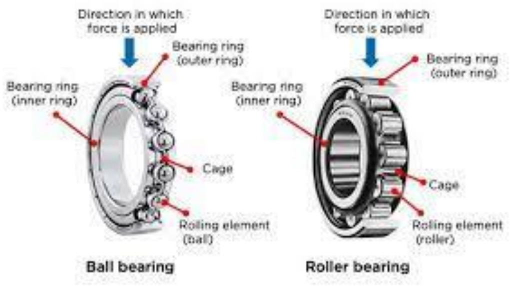


These bearings use balls to reduce friction and support radial and axial loads.

Types of ball bearings include deep groove, angular contact, and thrust bearings.

2. **Roller Bearings:**





Roller bearings use cylindrical, tapered, or spherical rollers to support loads.

Types include cylindrical roller, tapered roller, spherical roller, and needle roller bearings.

3. **Plain Bearings (Sleeve Bearings):**

Plain bearings have a sliding surface between the shaft and the bearing material.

Lubrication is essential to minimize friction and wear.

Types include bushings and sleeve bearings.

4. **Thrust Bearings:**

Thrust bearings are designed to support axial loads, often with minimal radial load capacity.

They include thrust ball bearings, thrust roller bearings, and thrust washers.

5. **Needle Bearings:**

Needle bearings are specialized roller bearings with a high load-carrying capacity in a compact design.

They are commonly used in applications with limited space.

6. **Spherical Bearings:**

Spherical bearings can accommodate misalignment and angular movement. They are often used in applications where shafts may not be perfectly aligned.

7. **Linear Bearings:**

Linear bearings facilitate linear motion along a shaft or guide rail. Types include linear ball bearings, linear roller bearings, and slide bearings.

Mounted Bearings:

Mounted bearings come pre-assembled in housings or flanges for easy installation.

Examples include pillow block bearings, flanged bearings, and take-up bearings.

8. **Rolling-Element Bearings:**

This category includes ball and roller bearings, where rolling elements reduce friction.

Rolling-element bearings are widely used in various applications.

9. **Sliding Contact Bearings:**
10. **Journal Bearings:** Also known as sleeve bearings or plain bearings, these are cylindrical bushings that provide a sliding surface for a shaft to rotate within. They rely on a film of lubricant (usually oil) to reduce friction and wear. Journal bearings are commonly used in engines, gearboxes, and other machinery.
11. **Bushed Bearings:** These are similar to journal bearings but may have a flanged design for better stability and alignment. Bushed bearings are often used in automotive applications, such as suspension systems.
12. **Plummer Block:** A plummer block, also known as a pillow block, is a housing that contains a bearing. It is used to support a rotating shaft and provide stability. Plummer blocks are commonly found in conveyor systems, fans, and other machinery.
13. **Thrust Bearings:**

Footstep Bearing: Footstep bearings are a type of thrust bearing designed to support vertical axial loads. They consist of a stationary pad (the footstep) and a rotating disk. These bearings are often used in vertical shaft applications, such as hydroelectric turbines.

Collar Bearing: Collar bearings, also known as collar thrust bearings or collar bushings, are used to support axial loads by using a collar that fits around the shaft. They are common in machinery with rotating shafts.

Rolling Contact Bearings:

Ball Bearings: Ball bearings use rolling elements (balls) to reduce friction and support both radial and axial loads. They are versatile and widely used in various applications, including electric motors, automotive wheels, and industrial machinery.

Roller Bearings: Roller bearings use cylindrical or tapered rollers to support radial and axial loads. They come in various designs, such as cylindrical roller bearings and tapered roller bearings, each suited to specific load and speed requirements

Needle Bearings: Needle bearings are a type of roller bearing with long, thin cylindrical rollers. They have a high load-carrying capacity in a compact design, making them suitable for applications with limited space, such as automotive transmissions and cam followers.

Antifriction bearings, also known as rolling contact bearings, have several advantages over sliding contact bearings, but they also come with some disadvantages. Let's explore the pros and cons of antifriction bearings compared to sliding contact bearings:

Advantages of Antifriction Bearings (Rolling Contact Bearings):

1. **Lower Friction:** Rolling contact bearings significantly reduce friction compared to sliding contact bearings. This results in less heat generation and energy loss, contributing to improved efficiency in machines and reduced wear.

2. **Lower Maintenance:** Antifriction bearings require less maintenance because they experience lower wear and have longer service life due to reduced friction. This leads to cost savings in terms of maintenance and replacement.
3. **Higher Load Capacity:** Rolling contact bearings can often support higher radial and axial loads than sliding contact bearings of similar size. This makes them suitable for heavy-duty applications.
4. **Higher Speeds**: Antifriction bearings are capable of operating at higher rotational speeds without excessive wear or overheating. This makes them ideal for high-speed machinery.
5. **Precision and Accuracy:** Rolling contact bearings offer precise and consistent motion control, making them suitable for applications requiring precision, such as machine tools and automotive components.
6. **Lower Start-Up Torque:** Rolling contact bearings generally require lower start-up torque, which can be advantageous in applications with frequent starts and stops.
7. **Wide Variety:** There is a wide range of rolling contact bearing types, including ball bearings, roller bearings, and needle bearings, each designed for specific applications and load conditions.

Disadvantages of Antifriction Bearings (Rolling Contact Bearings):

1. **Limited Misalignment Tolerance:** Rolling contact bearings have limited tolerance for misalignment compared to sliding contact bearings. Proper alignment is crucial to prevent premature wear and failure.
2. **Higher Initial Cost:** In some cases, antifriction bearings may have a higher initial cost than sliding contact bearings, although this cost difference is often offset by longer service life and reduced maintenance.
3. **Sensitive to Contaminants:** Rolling contact bearings can be sensitive to contamination by dirt, dust, and foreign particles. Contaminants can accelerate wear and reduce bearing life.
4. **Noise and Vibration:** Depending on the application and type of rolling contact bearing, they may produce more noise and vibration compared to sliding contact bearings.
5. **Temperature Sensitivity:** Rolling contact bearings can be sensitive to temperature variations, which may affect their performance. Special high-temperature or low-temperature bearings may be required for extreme conditions.
6. **Complex Lubrication:**

Proper lubrication is essential for rolling contact bearings. Inadequate lubrication or lubrication failure can lead to premature wear and failure.

PROBLEMS

1.A-line shaft rotating at 200 rpm is to transmit 20 kW power. The allowable shear stress for the shaft material is 42 N/mm2. Determine the diameter of the shaft.

Given :

P = 20 kW = 20×10^3 W; τ= 42 MPa = 42 N/mm^2

N = 200 r.p.m. ;

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$\text{Power, } P = \frac{2\pi NT}{60} \text{ watts}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (*T*),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.25 = 115\ 733 \text{ or } d = 48.7 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

2.A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Solution.

Given : ***P = 1 MW = 1 × 106 W ;***

N = 240 r.p.m. ;

Tmax = 1.2 Tmean ;

τ = 60 MPa = 60 N/mm2

Let *d = Diameter of the shaft.*

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

Maximum torque transmitted,

Tmax = 1.2 Tmean = 1.2 × 39 784 × 10 3= 47 741 × 103 N-mm

We know that maximum torque transmitted (*Tmax),*

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

$$d = 159.4 \text{ say } 160 \text{ mm } \textbf{Ans.}$$

4.Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : *P = 20 kW = 20 × 103 W ;*

N = 200 r.p.m. ;

τu = 360 MPa = 360 N/mm2

; F.S. = 8 ;

k = di / do = 0.5

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let *d = Diameter of the solid shaft.*

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (*T),*

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$$d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

Diameter of hollow shaft

Let *di = Inside diameter, and*

do = Outside diameter.

We know that the torque transmitted by the hollow shaft (*T),*

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\,(d_o)^3\,[1 - (0.5)^4] = 8.3\,(d_o)^3$$

$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

$d_i = 0.5\,d_o = 0.5 \times 50 = 25$ mm **Ans.**

Example 5.

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\,000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\,d^3$$

$\therefore$ $d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6$ or $d = 86$ mm

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}\,(M + T_e)$$

$$= \frac{1}{2}\,(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 86 \text{ say } 90 \text{ mm}$$ **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

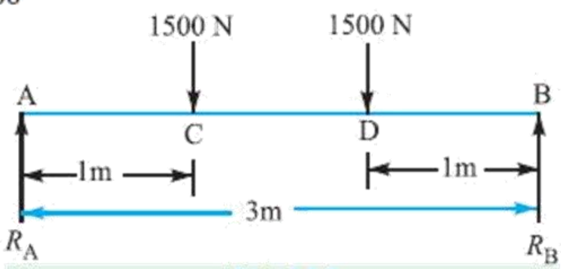


Fig. 14.3

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

$\therefore$ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let $\quad d$ = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$

$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

$$\therefore \quad d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3 \text{ or } d = 66.8 \text{ say } 70 \text{ mm}$$ **Ans.**

Example 14.22. *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation *(i)* by equation *(ii)*, we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \quad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \quad ...(iv)$$

Dividing equation *(iii)* by equation *(iv)*, we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \quad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

$\therefore$ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \quad ...(vi)$$

Dividing equation *(v)* by equation *(vi)*, we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \quad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : $d = 50$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2; $\sigma_c = 70$ MPa $= 70$ N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \qquad \text{...}(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \qquad \text{...}(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$l = 117.7$ say 120 mm **Ans.**

Example 13.2. *A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.*

Solution. Given : $d = 45$ mm ; σ_{yt} for shaft $= 400$ MPa $= 400$ N/mm^2 ; $w = 14$ mm ; $t = 9$ mm ; σ_{yt} for key $= 340$ MPa $= 340$ N/mm^2; $F.S. = 2$

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$$\therefore \quad l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\, l$$

$$\text{...}\left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$$\therefore \quad l = 1.8 \times 10^6 / 17\,213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

$l = 104.6$ say 105 mm **Ans.**

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : $P = 40$ kW $= 40 \times 10^3$ W; $N = 350$ r.p.m.; $\tau_s = 40$ MPa $= 40$ N/mm^2; $\sigma_{cs} = 80$ MPa $=$ 80 N/mm^2; $\tau_c = 15$ MPa $= 15$ N/mm^2

A type of muff couplings.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$
$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm} \quad \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$
$$= 370 \times 10^3\, \tau_c$$

$\therefore$ $\tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97$ N/mm^2

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, $w = 18$ mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm} \textbf{ Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \, \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

Example 3.17 : *A 40 mm diameter shaft is subjected to a tangentia force of 20 kN around its circumference. Determine the sizes of key. The allowable shear stress in key is 60 N/mm².*

Solution :

Given : Diameter of shaft, $D = 40$ mm

Tangential force, $F = 20$ kN $= 20 \times 10^3$ N

For 40 mm diameter shaft, sizes of key ;

width , $w = \frac{D}{4} = \frac{40}{4} = 10$ mm

thickness, $t = \frac{D}{6} = \frac{40}{6} = 6.67 = 7$ mm

Adopt, $t = 8$ mm

Consider shear strength of key

$$F = (w \times l).\tau$$

or $$l = \frac{F}{w.\tau} = \frac{20 \times 10^3}{10 \times 60} = 33.3 = 34 \text{ mm}$$

Size of key : Width = 10 mm ; thickness = **8 mm ;**

Length = **34 mm ; Ans.**

Example 3.18 : *A rectangular sunk key 14 mm wide 10 mm thick and 70 mm long is required to transmit 1200 Nm torque from a 50 mm diameter solid shaft. Determine, whether the length is sufficient, if the permissible shear stress and crushing stresses are limited to 56 and 168 N/mm² respectively.* **[March, 2006]**

Solution :

Given : Sizes of key; width, $w = 14$ mm

Thickness, $t = 10$ mm

Length , $l = 70$ mm

Torque, $t = 1200$ Nm $= 1200 \times 10^3$ Nmm

Permissible shear stress, $\tau = 56$ N/mm²

Permissible crushing stress, $\sigma_c = 168$ N/mm²

Consider shear strength of key ;

$$T = w.l.\tau.\ \frac{D}{2}$$

or $$\tau = \frac{2\ T}{w.l.D} = \frac{2 \times 1200 \times 10^3}{14 \times 70 \times 50}$$

$$= 48.98\ \text{N/mm}^2$$

Consider crushing strength of key ;

$$T = \frac{t}{2}\ .\ l.\sigma_c.\ \frac{D}{2}$$

or $$\sigma_c = \frac{4\ T}{t.l.D} = \frac{4 \times 1200 \times 10^3}{10 \times 70 \times 50}$$

$$= 137.14\ \text{N/mm}^2$$

Since the induced stresses are less than the permissible value, the length of the key is sufficient.

Example 3.19 : *A shaft of 50 mm diameter is transmitting 150 kW at 2000 rpm. A square key having 12 mm side and 75 mm long is used for the shaft. Determine the induced shear stress and compression stress in the key.*

Solution :

Given : Diameter of shaft, D = 50 mm

Power, P = 150 kW = 150 × 10^3 W

Speed, N = 2000 rpm.

Sizes of key :

width, w = thickness, t = 12 mm.

length, l = 75 mm

$$\text{Torque,}\quad T = \frac{60 \times P}{2\pi N} = \frac{60 \times 150 \times 10^3}{2 \times \pi \times 2000}$$

$$= 716.56\ \text{Nm.} = 716560\ \text{Nmm.}$$

Consider shear strength of key ;

$$\text{Shear stress induced, } \tau = \frac{2\ T}{w.l.D} = \frac{2 \times 716560}{12 \times 75 \times 50}$$

$$= \mathbf{31.8\ N/mm^2}\quad \textbf{Ans.}$$

(Chapter-3)

Consider shear strength of key ;

$$T = w.l.\tau.\ \frac{D}{2}$$

or

$$\tau = \frac{2\ T}{w.l.D} = \frac{2 \times 1200 \times 10^3}{14 \times 70 \times 50}$$

$$= 48.98\ \text{N/mm}^2$$

Consider crushing strength of key ;

$$T = \frac{t}{2}\ .\ l.\sigma_c.\ \frac{D}{2}$$

or

$$\sigma_c = \frac{4\ T}{t.l.D} = \frac{4 \times 1200 \times 10^3}{10 \times 70 \times 50}$$

$$= 137.14\ \text{N/mm}^2$$

Since the induced stresses are less than the permissible value, the length of the key is sufficient.

Example 3.19 : *A shaft of 50 mm diameter is transmitting 150 kW at 2000 rpm. A square key having 12 mm side and 75 mm long is used for the shaft. Determine the induced shear stress and compression stress in the key.*

Solution :

Given : Diameter of shaft, D = 50 mm

Power, P = 150 kW = 150 × 10^3 W

Speed, N = 2000 rpm.

Sizes of key :

width, w = thickness, t = 12 mm.

length, l = 75 mm

$$\text{Torque,}\ T = \frac{60 \times P}{2\pi N} = \frac{60 \times 150 \times 10^3}{2 \times \pi \times 2000}$$

$$= 716.56\ \text{Nm.} = 716560\ \text{Nmm.}$$

Consider shear strength of key ;

$$\text{Shear stress induced,}\ \tau = \frac{2\ T}{w.l.D} = \frac{2 \times 716560}{12 \times 75 \times 50}$$

$$= \mathbf{31.8\ N/mm^2}\quad \textbf{Ans.}$$

(Chapter-3)

Thus the length of key,

l = 132 mm (larger of the two values)

Sizes of key,

width, w = 15 mm ; thickness, t = 10 mm ;

length , l = **132 mm** **Ans.**

Example 3.21 : *A motor shaft of 50 mm diameter transmits a torque of 150 Nm. It has an extension of 75 mm. The permissible shear and crushing stresses for m.s, key are 55 N/mm² and 110 N/mm². Determine the sizes of key way in motor shaft extension. Check the shear strength of key against the normal strength of the shaft.*

Solution :

Given : Diameter of shaft, $D = 50$ mm

Torque, $T = 150$ N m $= 150 \times 10^3$ Nmm

Extension of shaft, $= 75$ mm

Permissible shear stress, $\tau = 55\ \text{N/mm}^2$

Permissible crushing stress, $\sigma_c = 110\ \text{N/mm}^2$.

The size of key is equal to the size of key way ; and the length of key way (i.e., key) is equal to the extension of the shaft.

Consider the shear strength of key ;

$$T = w.l.\tau, \frac{D}{2}$$

or

$$w = \frac{2T}{l.\tau.D} = \frac{2 \times 150 \times 10^3}{75 \times 55 \times 50}$$

$$= 1.45 \text{ mm}$$

The width is too small, Adopt $w = \frac{D}{4} = \frac{50}{4} = 12.5$

$= 13$ mm

Also $\sigma_c = 2\tau$, therefore the square key may be used.

Sizes of key way :

width, w = 13 mm ; thickness (depth), t = 13 mm,

length, $l = 75$ mm

Normal strength of shaft

$$T = \frac{\pi D^3}{16} . \tau$$

Check for shear strength of key against normal strength of shaft :

$$\frac{\text{Shear strength of key}}{\text{Normal strength of key}} = \frac{w.l.\tau.D/2}{\frac{\pi D^3}{16}.\tau} = \frac{8\ w\ l}{\pi\ D^2}$$

$$= \frac{8 \times 13 \times 75}{\pi \times 50^2} = 0.99$$