TOS Module I

Simply supported beam.

A beam is supported or resting freely on the wall or columns at its both ends.

3 Overhanging beam (single overhanging, double over-hanging) A beam in which certain span length is extended beyond the supports on one of its side or both sides.

Single overhanging

Double overhanging.

4 Fixed beam The ends of the beam are rigidly fixed or built in walls.

5 Continuous beam.

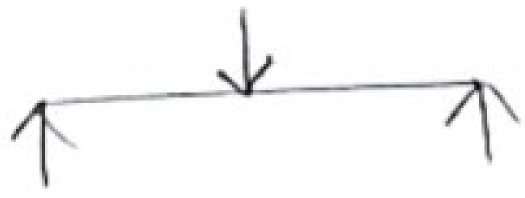
A beam supported over more than two supports is known as continuous beam.

6 Propped cantilever beam.

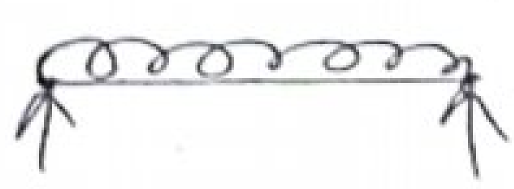
The cantilever beam is supported at free end.

Types of loadings.

(1) Point load or concentrated load :- These loads are considered to be acting at a point.



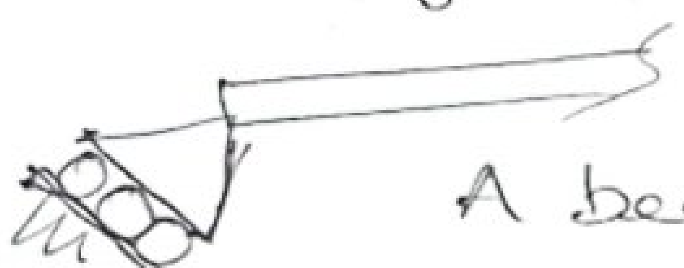
(U.D.L) (2) Uniformly distributed load :- The load on a beam is equally distributed over a length of the beam. ie load per unit length is constant

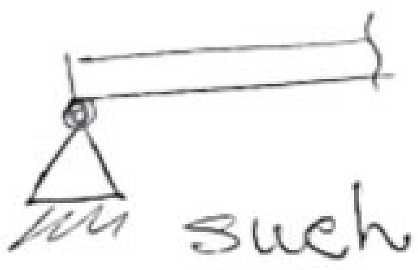


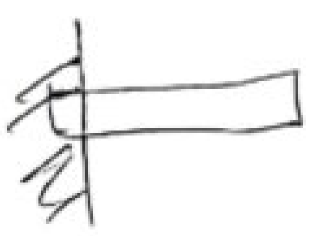
Types of supports.

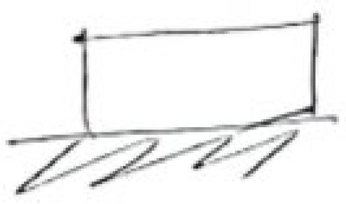
- Simple support or knife edge support
- Roller support
- Hinged support
- Fixed support
- Smooth surface support


A beam rests simply on a support is known as simply supported beam.

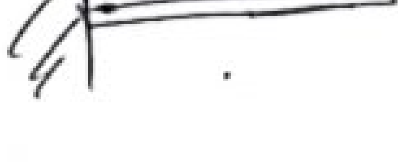

A beam is supported on roller such support is called roller support.


If a beam is supported on hinge or pin then such support is called hinged or pinned support.


The end of beam is fixed or ~~buitd~~ built in such supports are called fixed or built.


If a body is supported ~~on~~ or in contact with a smooth surface then such supports are called smooth surface supports.

Types of beam.

1 Cantilever beam 

One end is fixed and the other end is free

(3) Uniformly varying load (U.V.L)

The load which is spread over a beam in such a manner that the rate of loading varies from point to point along the beam.

(Trapezoidal load).

(4) Gradually varying load or triangular load. :- The load is uniformly varying along the length from zero intensity ~~at~~ at one end to the designated intensity at the other end.

Shear force Is defined as the algebraic sum of all the vertical forces either to left or to right side of the section.

Bending moment.

sum of moments about that section of all external forces acting to one side of that section

Relation b/w load, shearforce and bending moment.

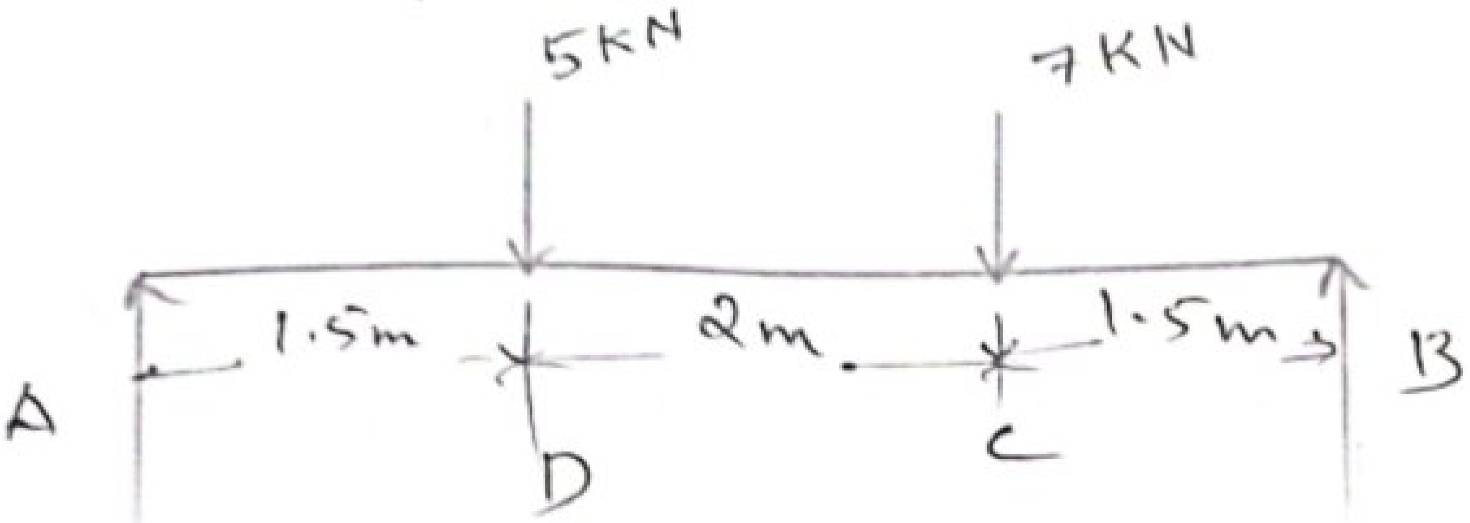
1. The rate of change shear force is equal to the rate of loading.

$$\boxed{\frac{dF}{dx} = -\omega}$$

2. Rate of change of bending moment is equal to the shearforce at the section

$$\boxed{\frac{dM}{dx} = F}$$

Shear force and bending moment diagrams.



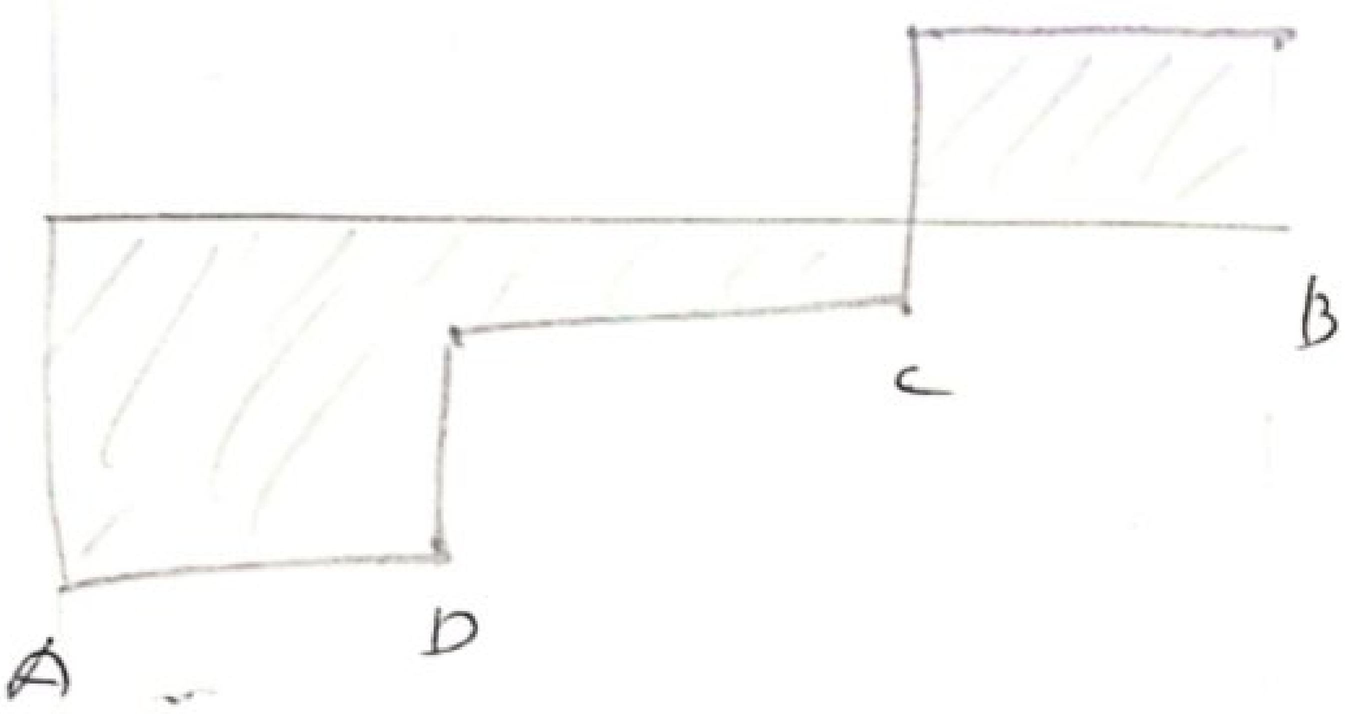
$\Sigma H = 0 \quad \Sigma V = 0 \quad \Sigma M = 0$

$\Sigma V = R_A + R_B - 7 - 5 = 0$

$R_A + R_B = 12 \quad —(1)$

$M_A = 5B - 24.5 - 7.5$

$5B = 32 \quad \boxed{B = 6.4\ KN. \quad A = 5.6\ KN}$



$SF_B = 6.4\ KN.$

$SF_C = -7 + 6.4 = \underline{-0.6\ KN}$

$SF_D = -5 + -0.6 = \underline{-5.6\ KN}$

$SF_A = -5.6 + 5.6 = \underline{0}$

$M_A = M_B = 0$.

$M_C = 9.6\ KN \quad M_D = 22.4 - 14$

$= \underline{8.4\ KN}$

Point of c
bending
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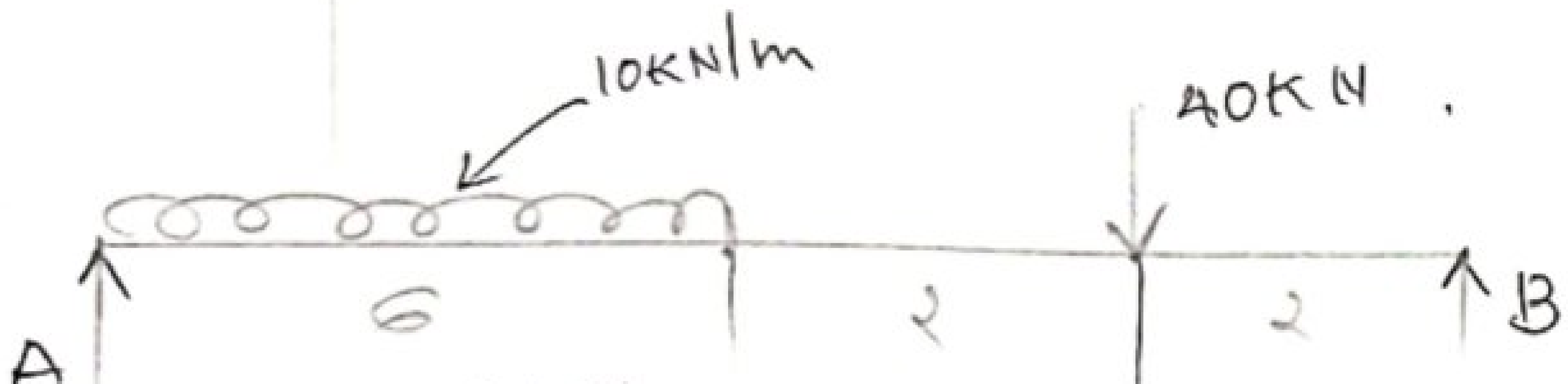
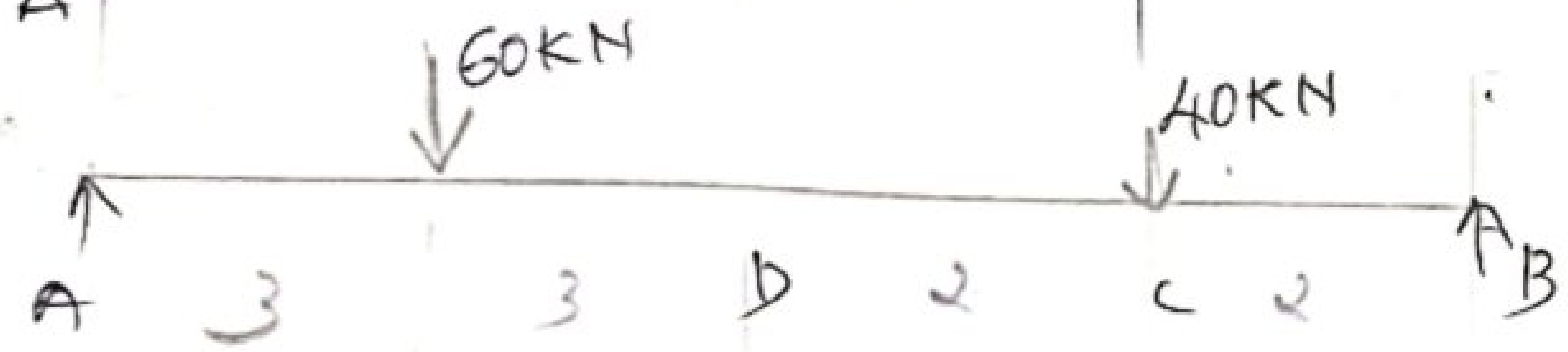
neg





$\Sigma V = 0 \quad \Sigma H = 0 \quad \Sigma M = 0$.

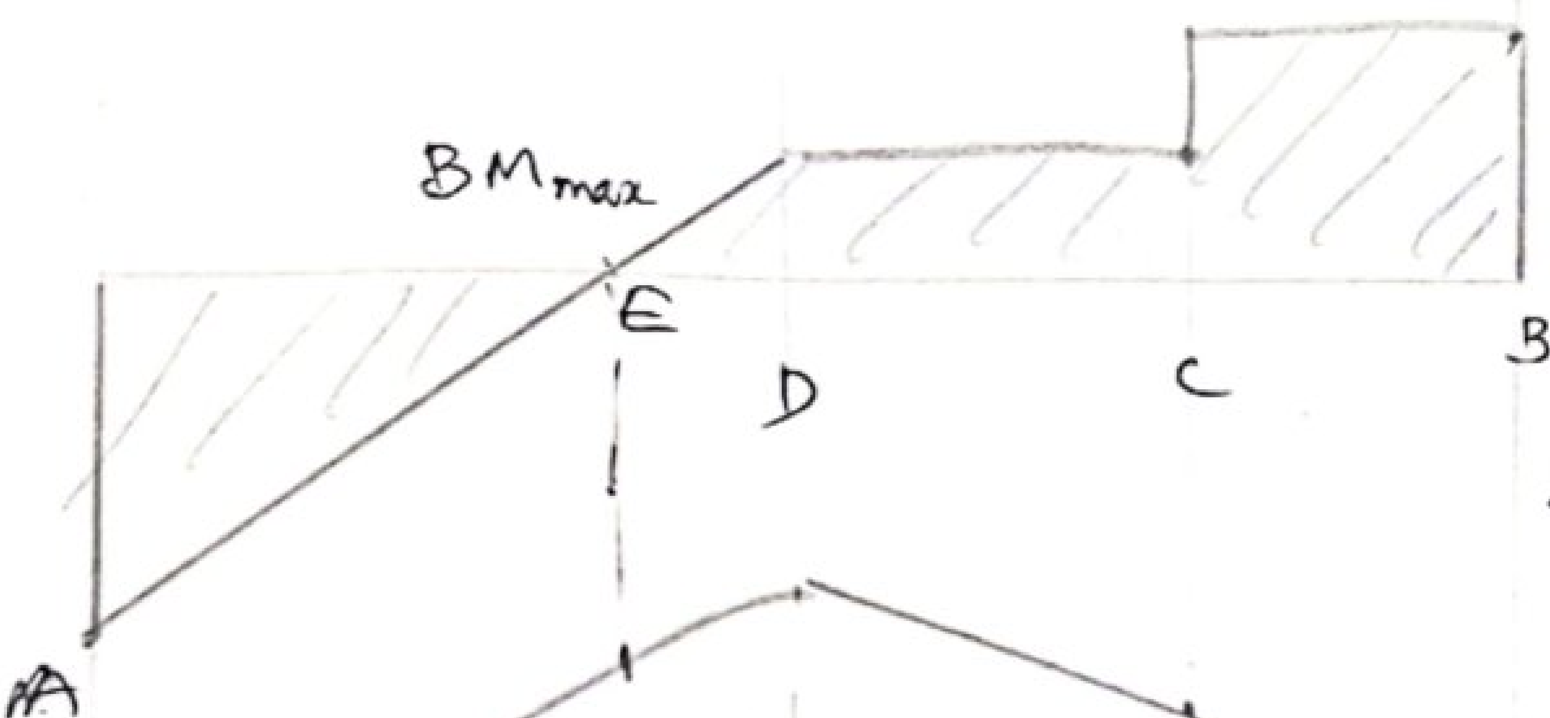
$\Sigma V = R_A + R_B - 40 - 60 = 0$.

$R_A + R_B = 100 \quad —(1)$.

$\Sigma M = 0$.

$= 10B - 320 - 180$.

$10B = 500 \quad \boxed{B = \underline{50\ KN} \quad A = 50\ KN}$



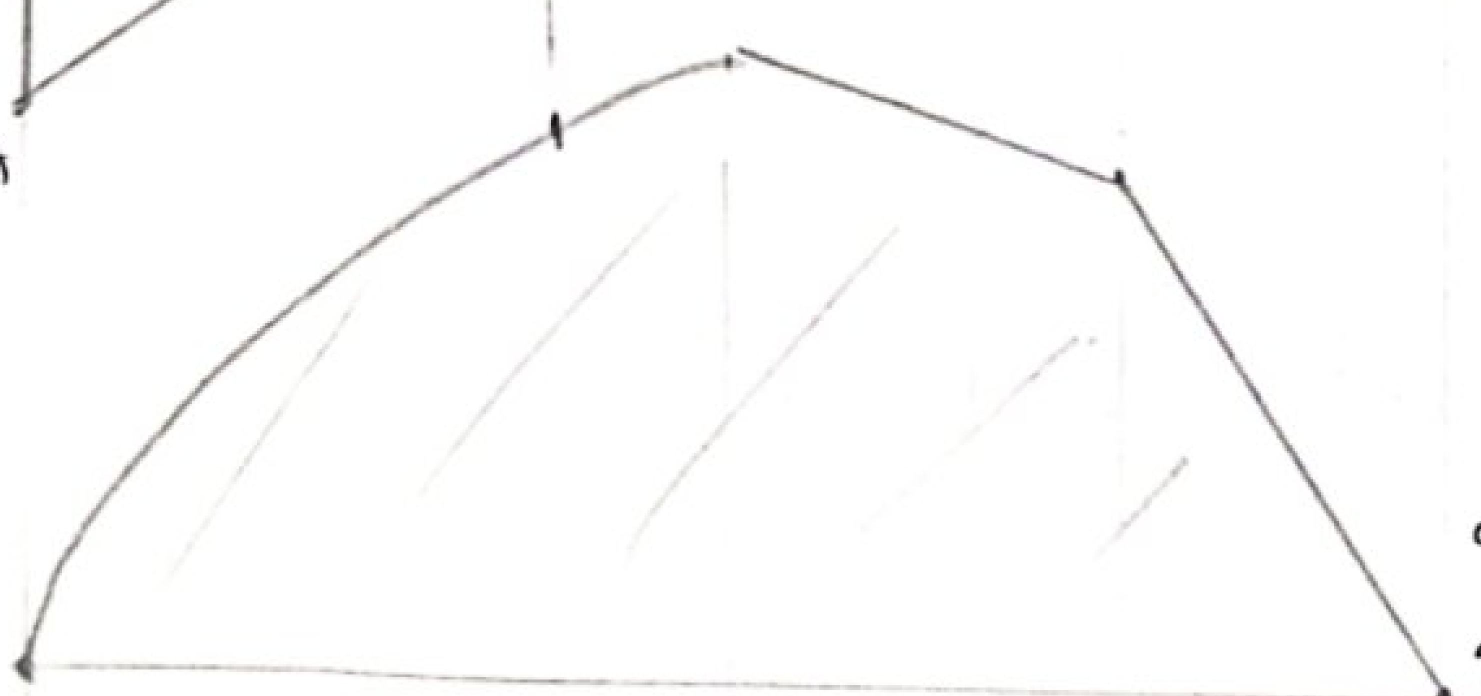
$SF_B = \underline{50\ KN}. \qquad M_A = M_B = 0$.

$SF_C = 50 - 40 = 10 \qquad M_C = \underline{100\ KNm}$

$SF_D = \underline{10}. \qquad M_D = 200 - 80 = \underline{120\ KNm}$

$SF_A = 10 - 60$.

$= \underline{-50}$.

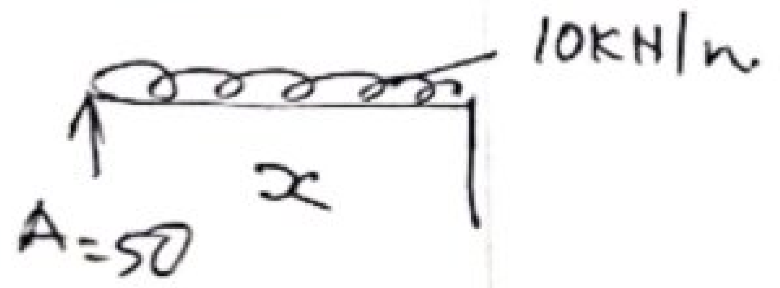


maximum bending moment at point E

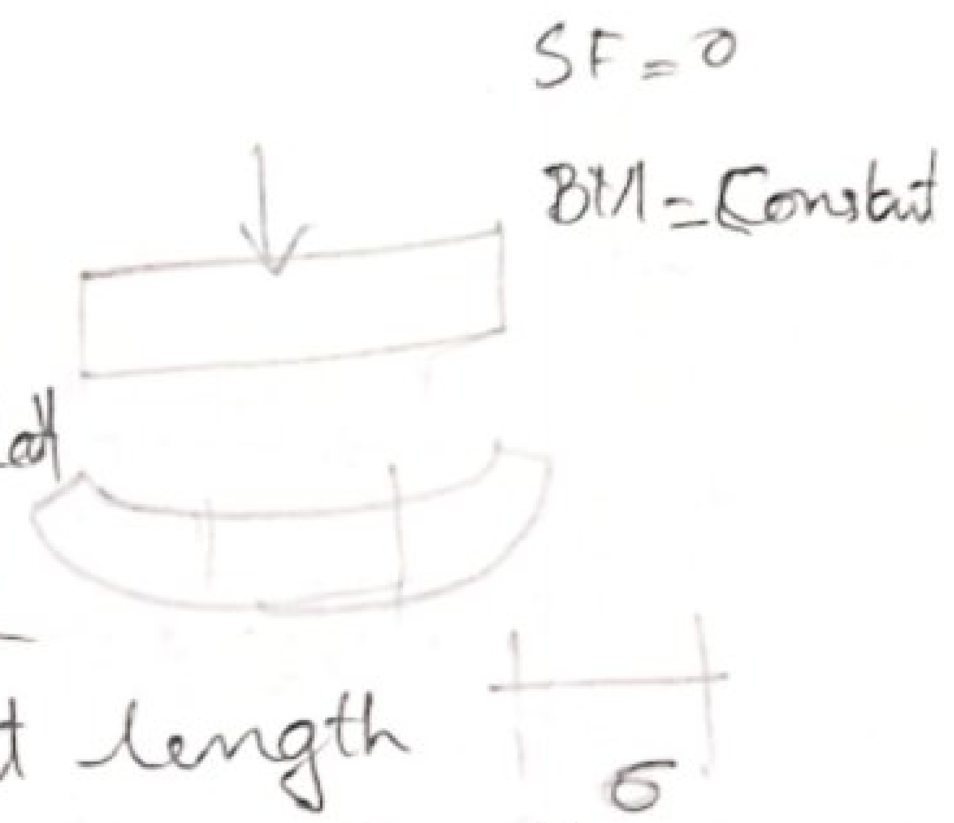
$\Sigma V = 50 - 10x = 0$.

$x = 50/10 = 5\ \underline{m}$

$BM_E = 50 \times 2.5 = \underline{125\ KNm}$



Point of contraflexure is a point where there is zero bending moment, at that point the direction of bending changes its sign from positive to negative or from negative to positive.



Pure bending or simple bending.

If a length of beam is subjected to a constant BM and no shear force Then the stresses will be setup in that length of beam due to BM only and that length of the beam is said to be in pure bending or simple bending.

OR

A beam or a part of it is said to be in a state of pure bending / simple bending when it bends under the action of uniform or constant bending moment without any shear force.

Assumptions used for the analysis of the beam under pure bending

→ The material of the beam is homogeneous and isotropic.

→ Each layer of beam is free to expand or contract

→ Young's modulus is considered as same for the compression and tension.

→ The radius of curvature of the beam is comparatively higher than the width or the depth of the beam.

→ The beam is loaded within the elastic

→ The plane section remains plane after

defarmation also.

$$\boxed{\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}}$$

Flescural Equation.

Bending equation

I = moment of inertia

σ = stress

M = Bending moment.

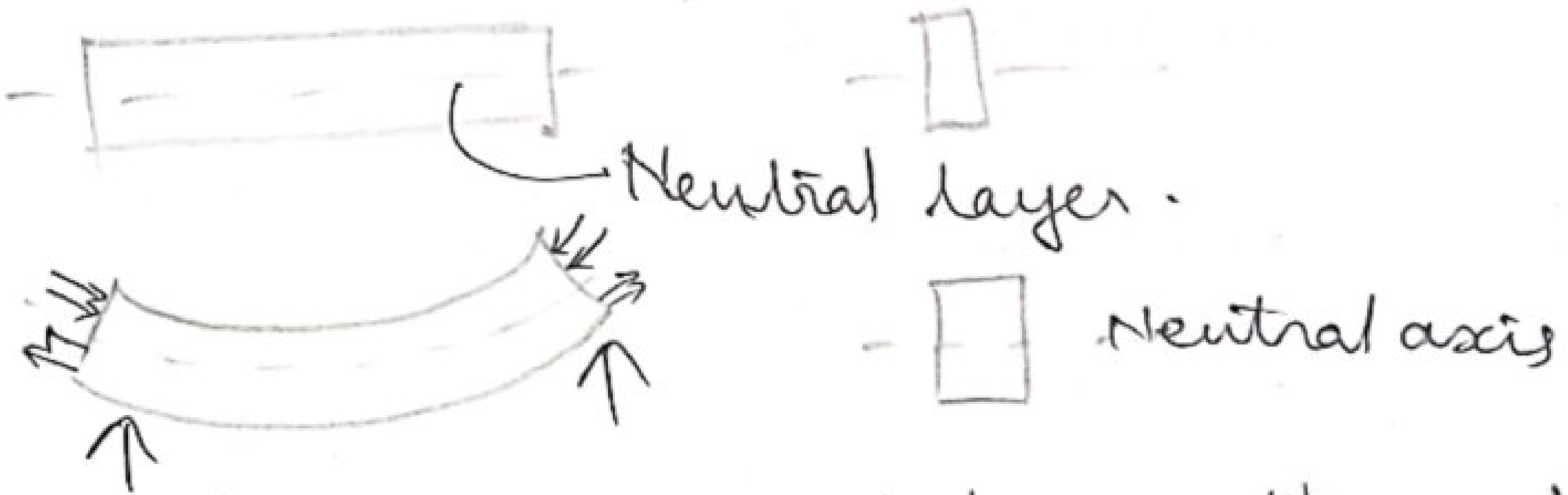
E = Modulus of elasticity

R = Radius of curvature.

$y = d/2$ (distance from neutral layer.

Nature of bending stress

Due to bending of the beam its upper layer are compressed and the lower layer are stretched. There fare longitudinal compressive stress are induced in the upper layer and longitudinal tensile stress are induced in the lower layer. These stresses are bending stress.



(Neutral Surface)

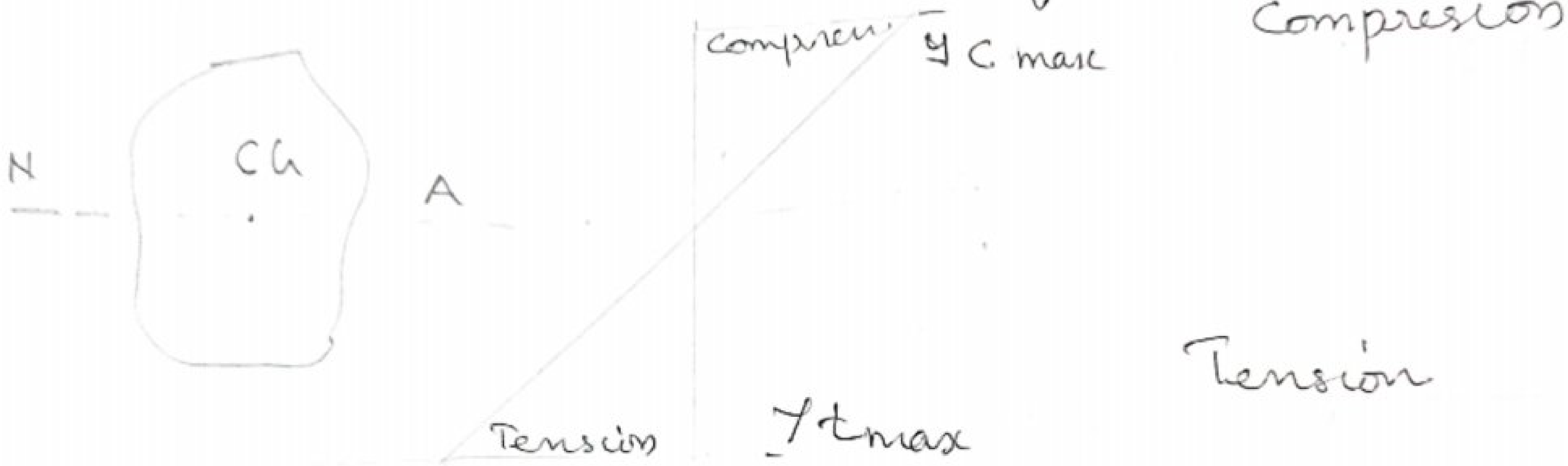
Neutral layer → In a beam or cantilever there is one layer which retain its original length even after bending. So in this layer nether tensile stress nor compressive stress is setup, this layer is called netral layer.

Neutral axis → Is the line of intersection of the neutral layer with any normal (transverse section) section of the beam no bending stress is setup in neutral axis

Bending stress

The resistance against bending offered by internal stress induced when a bending moment tries to bend a beam is bending stress.

- Bending stress increases as the distance from the centroid increases.
  - zero at the neutral axis
  - maximum at the extreme fibres



maximum compressive stress fm

$$\sigma_{max} = \frac{M}{I} y_{c\,max}$$

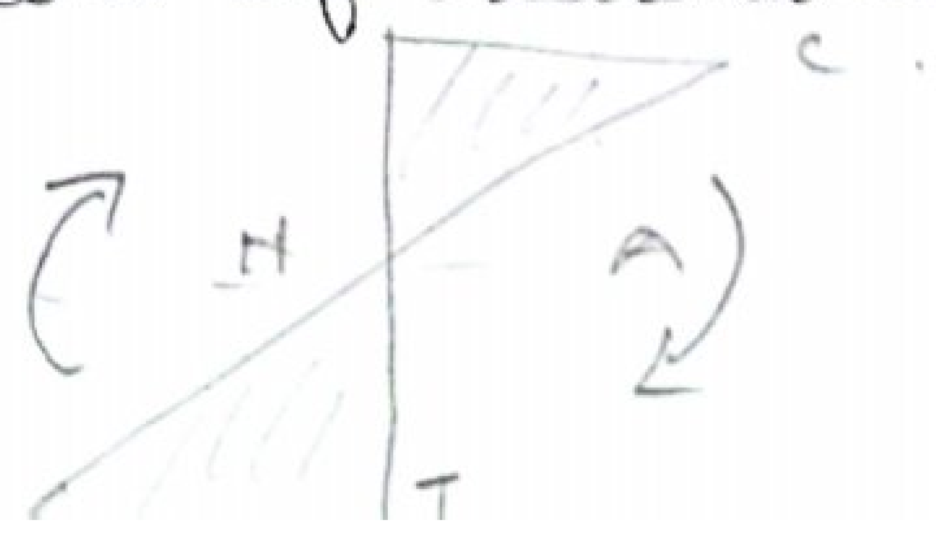
stress = $\frac{F}{A}$.

$\boxed{F = \text{stress} \times A}$

maximum tensile stress = $\frac{M}{I} y_{T\,max}$.

Moment of Resistance

- Due to pure bending compressive stress are developed on the fibres above neutral axis and tensile stress are developed on fibres below ~~neta~~ neutral axis
- The stress causes forces on fibres.
- Total moment of these forces about the neutral axis is known as moment of resistance



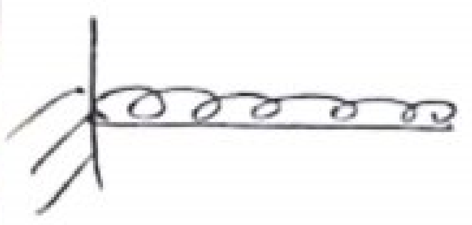
Moment of resistance is the resistance offered by the against the applied moment.

Cantilever beam.

 Pointed load.

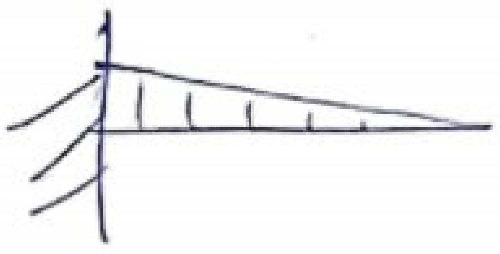
$SF = w$

$BM = wl$

 U.D.L

$SF = wl$

$BM = \frac{wl^2}{2}$

 U.V.L

$SF = \frac{wl}{2}$

$BM = \frac{wl^2}{6}$

Simply supported beam.

Pointed load $SF = \frac{w}{2}$

$BM = \frac{wl}{4}$

U.D.L $SF = \frac{wl}{2}$

$BM = \frac{wl^2}{8}$

U.V.L $SF = \frac{wl}{6}$

$BM = 0.064\, wl^2$.

Shear stress $\tau = \frac{F}{A}$

F = applied force

A = Cross sectional area

τ = Shear stress in pascal or N/m^2

Shear stress is the ~~amound~~ amount of force per unit area perpendicular to the axial of the member. Bending stress, also called flexural stress is parallel to the axial of the member.

Shear stress is perpendicular to the axial of the member

where F = shear force

$A\bar{y}$ = moment of area

I = moment of inertia

b = width of the section.

The ratio of maximum and average shear stress on a rectangular section is $3/2$ or 1.5

circular section is $4/3$ or 1.33

1 Algebraic sum of all vertical forces to left or right of the section is called.

Shear force.

2 At point of contra flexure value of bending moment is zero.

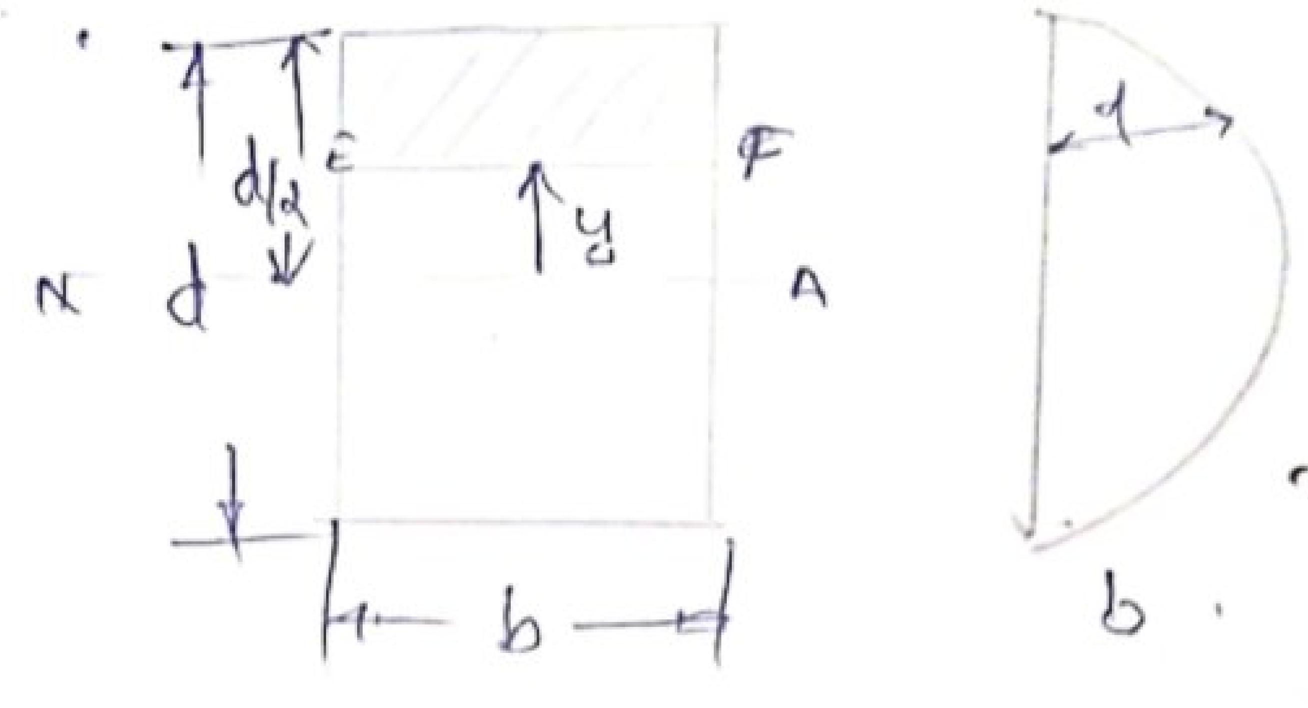
3 In a rectangular homogeneous section the ratio of maximum shear stress to average shear stress.

$3/2$ or 1.5

B

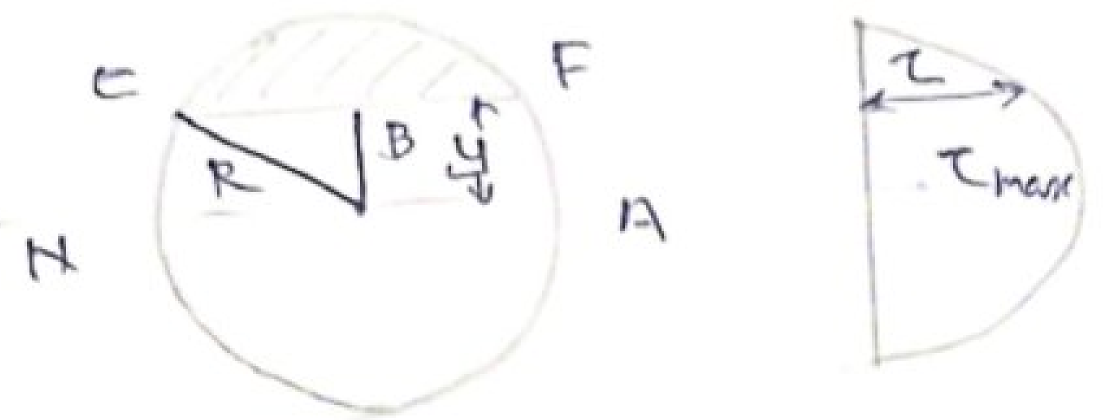
1 Sketch the typical shear stress distribution for following beam.

(1) rectangular



$$\boxed{\tau = \frac{F A \bar{y}}{z I}}$$

(2) Circular



(3) I Section

2 Write down the assumptions in theory of simple bending

→ The material of the beam is homogeneous and isotropic

→ The value of young's modulus of elasticity is same in tension and compression

→ The transverse sections which were plane before bending, remain plane after bending also.

→ beam is initially straight and all longitudinal filaments bend ½ each into circular arc with a common centre of curvature.

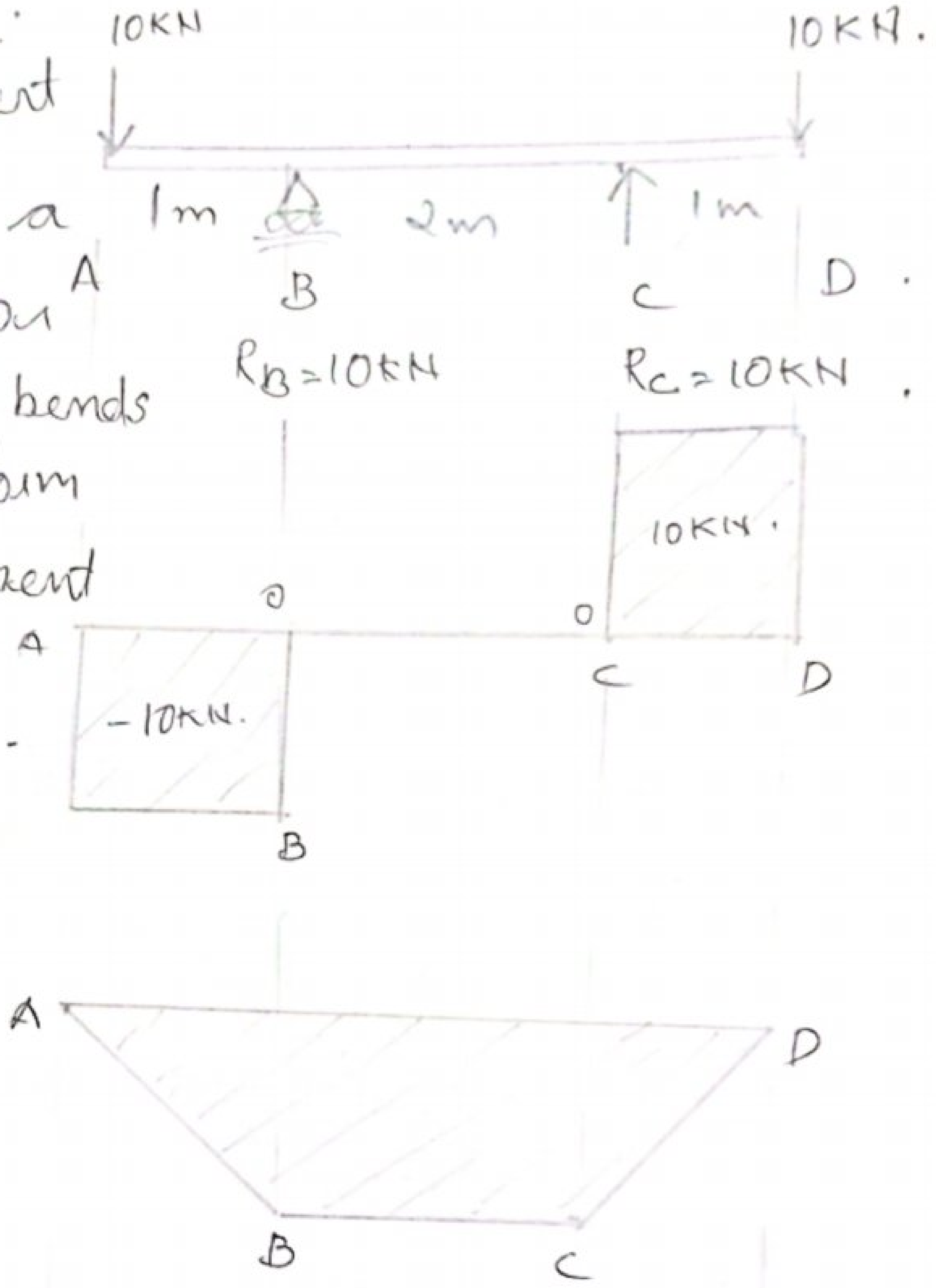
→ The radius of curvature is large as compared to the dimensions of cross sections.

$b = 1903\,mm$

$d = 2b \rightarrow \underline{3806\,mm}$

Pure bending of beam.

A beam or a part of beam is said to be in a state of pure bending or simple bending when it bends under the action of uniform or constant bending moment without any shear force.

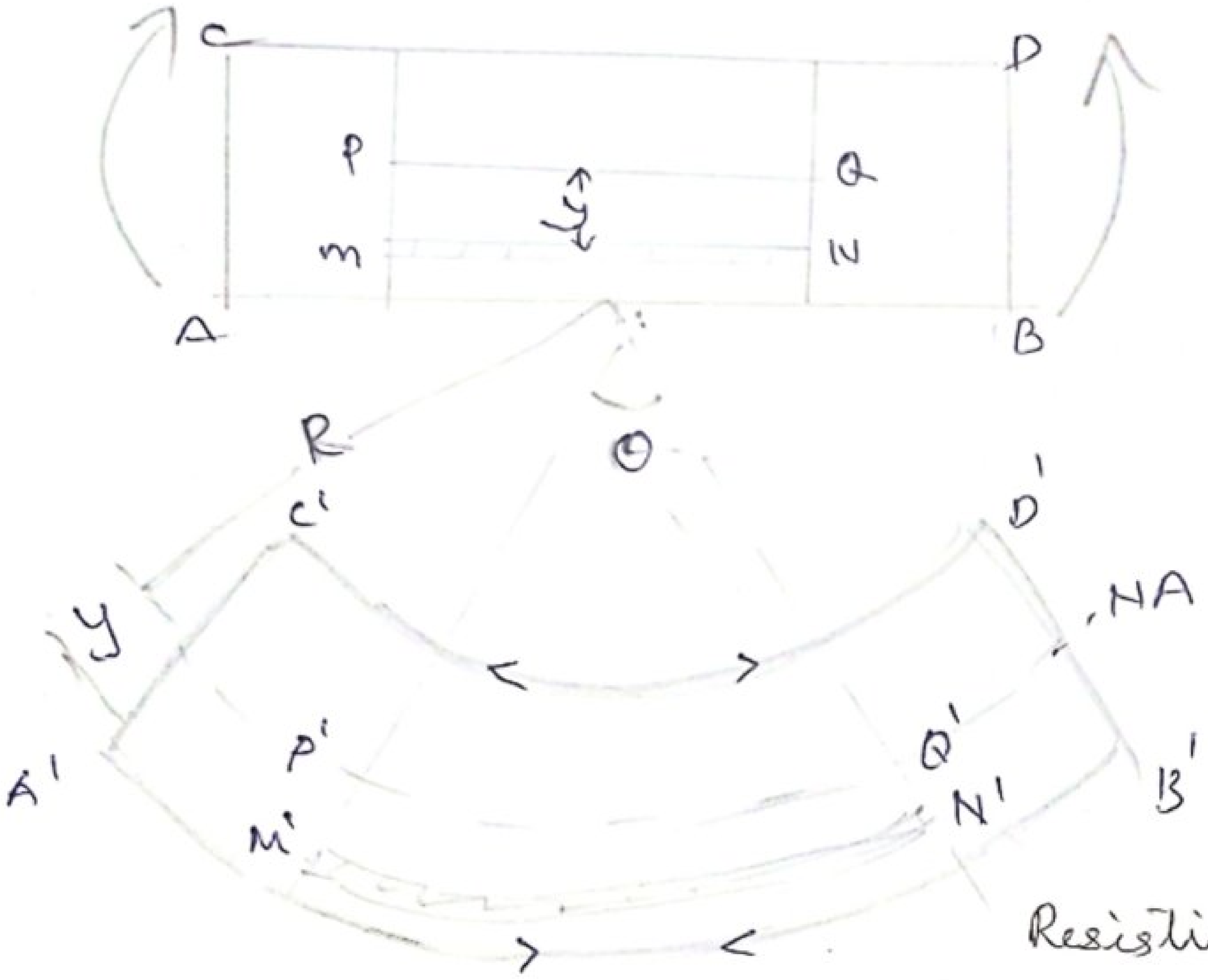


Assumptions

Derive the bending equations

$$\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$$

consider a beam part subjected to simple bending



$PQ = MN = P'Q'$

$M'N' = (R+Y)\,\theta$

$P'Q' = R\theta$

$\therefore MN = R\theta$

$$\text{strain} = \frac{M'N' - MN}{MN}$$

$$e = \frac{(R+Y)\theta - R\theta}{R\theta}$$

$$= \frac{R\theta + y\theta - R\theta}{R\theta}$$

$$= \frac{Y\theta}{R\theta}$$

$$e = \frac{Y}{R} \text{ or } \frac{\sigma}{E}$$

$$\frac{\sigma}{E} = \frac{Y}{R} \longrightarrow \boxed{\frac{\sigma}{Y} = \frac{E}{R}}$$

Resistive force developed by strip.

~~R.F x distance~~ (stress x Area)

$= \sigma \times dA \Rightarrow \frac{E}{R} y\, dA$.

Resistive moment developed by strip

$= R.F \times$ distance.

$= \frac{E}{R} Y\, dA \times Y \Rightarrow \frac{E}{R} y^2 dA$.

Total resistant moment $= \frac{E}{R} \int y^2 dA$.

$\because \int y^2 dA = I$ as second moment of area

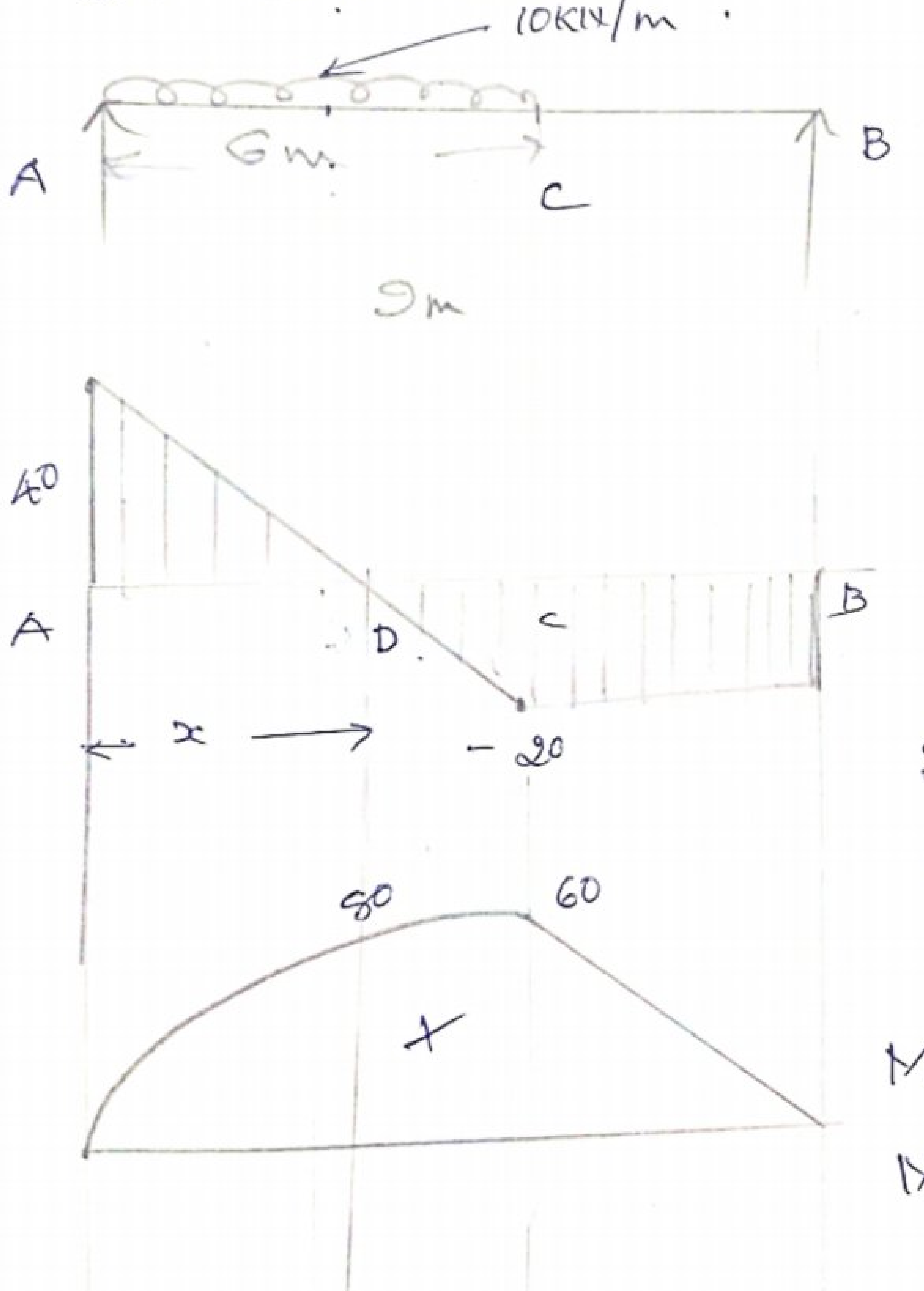
$$M = \frac{E}{R} I \longrightarrow \boxed{\frac{M}{I} = \frac{E}{R}}$$

6

→ each layer of the beam is free to expand or contract independently of the layer above or below it.

- c -

1 Draw the SFD and BMD of simply supported beam of length 9m and carrying a uniformly distributed load of 10KN/m for a distance of 6m from the left end. Also calculate the maximum B.M on the section



$\Sigma V = 0 \quad \Sigma M = 0$.

$\Sigma V = R_A + R_B - 60 = 0$.

$R_A + R_B = 60 \text{ KN} \quad -(1)$

$\Sigma M = 0$.

$M_A = 9B - (60 \times 3) = 0$.

$9B = 180 \rightarrow B = 20 \quad -(2)$

$A = 40 \text{KN} \quad B = 20 \text{ KN}$.

$SF_A = +40 \text{KN} \ . \ SF_D =$

$SF_C = -60 + 40 = -20$.

$SF_B = -20 + 20 = 0$.

$M_A = \cancel{(20 \times 9) - (60 \times 3)} \quad M_B = 0$.

$M_C = (20 \times 3) = 60 \text{ KNm}$.

consider AC, max BM at 'D'.

where SF = 0. $40 + (-10 \times x) = 0$

$x = 4 \text{ m}$

$BM_D = (20 \times 5) - (20 \times 1)$.

$= 100 - 20$

$= 80 \text{ KNm}$

maximum BM = 80 KNm

* A timber beam of rectangular section supports a [illegible] of 40KN uniformly distributed over a span of 3.6m. The depth of the beam section is twice the width and maximum bending stress is not exceed 7 N/mm² find the dimension of the beam section.

load $w = 40KN \longrightarrow 40 \times 10^3 N$.

length $l = 3.6m \longrightarrow 3.6 \times 10^3 mm$

depth $d = 2b$.

M bending stress $\cdot F_{max} = 7 N/mm^2$

BM for UDL (Simply supported beam).

$$\frac{wl^2}{8} = \frac{40 \times 10^3 N \times (3.6 \times 10^3)^2}{8}$$

$$= 6.48 \times 10^{10} Nmm$$

$y =$

$$I = \frac{bd^3}{12}.$$

$$\boxed{\frac{M}{I} = \frac{\sigma F}{y} = \frac{E}{R}}$$

$$= \frac{b(2b)^3}{12}.$$

$$= \frac{8b^4}{12} \qquad \frac{6.48 \times 10^{10}}{\frac{8b^4}{12}} = \frac{7}{\frac{2b}{2}}.$$

$$\frac{6.48 \times 10^{10}}{7} = \frac{8b^4}{\cancel{12}\,6} \times \cancel{2}. \frac{\cancel{2}}{2b}. = \frac{\overset{4}{\cancel{8}}b^4}{_3\cancel{6} \times 2b}. \quad \frac{4b^3}{3}.$$

$9257142857 .=$

~~92.5×10~~

$$92 \times 10^8 = \frac{4b^3}{3}. \longrightarrow 4b^3 = 2.76 \times 10^{10}.$$

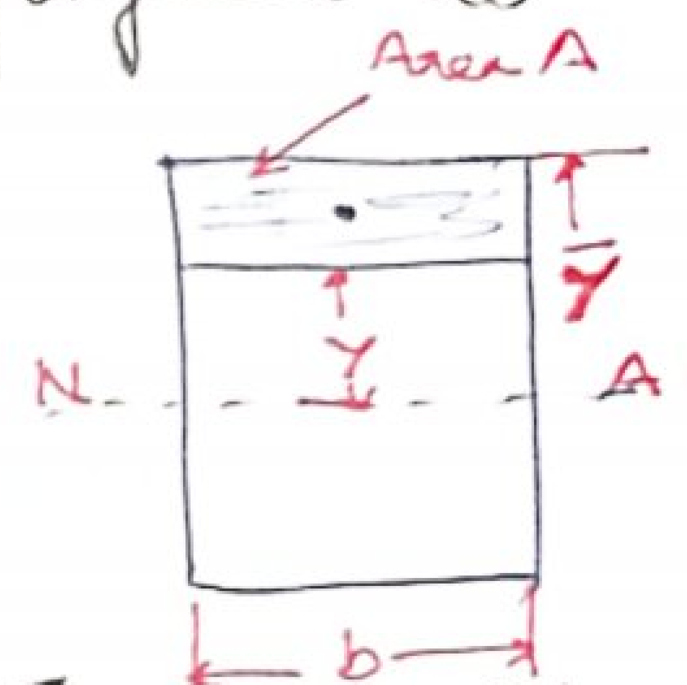
$$b^3 = 69 \times 10^8 \longrightarrow b = \underline{1903 \text{ mm}}.$$

Shear stress in beam.

• Incase of bending of beams ie the beam subjected to constant bending moment only and no shear force and hence no shear stress.

• But in actual practice when the beam is loaded it is subjected to a bending moment, which varies from section to section and hence is subjected to shear force which also varies from section to section.

• Due to these shear forces, the beam is subjected to transverse shear stress, which produce complimentary horizondal shear stress. These shear stresses act on longitudinal layers of the beam. Effect of shear stresses is negligible as compared to the bending stresses.



Cross section of beam

Shear stress equation

$$\boxed{\tau = \frac{FA\bar{Y}}{Ib}}$$

$$\boxed{\tau = \frac{SA\bar{Y}}{Ib}}$$

$$\boxed{\text{where } \frac{\partial M}{\partial x} = S}$$

τ = shear stress on a layer Y distance from N-A. N/mm²

S = shear force at the section (N).

A = Area of section above 'Y' distance from N-A (mm²)

$\bar{Y}$ = Distance of C.G of area 'A' from N-A. (mm)

$A\bar{Y}$ = Moment of area 'A' about N-A. (mm³)

b = width of section 'Y' distance from N-A (mm)

I = moment of inertia the whole section

small extent of survey and detailed design work

contour interval should be small (for important work

... the contour interval

moment of inertia is the property to oppose rotational motion.

$$I = mr^2 \left(\begin{array}{l} m = \text{mass of the object} \\ r^2 = \text{the distance from the axis of rotation} \end{array} \right.$$