SPRINGS

6.6 INTRODUCTION :

A spring is a device used to absorb or store energy and release it when required. "A spring is defined as an elastic machine element that deflects under the action of the load and returns to its original shape when the load is removed". The deformation produced in the spring is not permanent nature and regain their original shape when load is removed. It finds application in railway carriages, motor cars, motor cycles, watches, governors etc.

6.7 MATERIAL FOR SPRINGS :

The springs are usually made of either high carbon steel or medium carbon alloy steels, brass, bronze, stainless steel, phosphor bronze and other alloy metals are used.

6.8 FUNCTIONS AND APPLICATIONS OF SPRINGS :

According to their uses, the springs perform the following functions.

1) To absorb shock or impact loading as in carriage springs.

   Eg : Vehicle suspension springs.

2) To apply forces and to control motions as in brakes and clutches.

3) To measure forces as in spring balances.

4) To store energy as in clocks, toys, circuit breakers etc.

6.9 TYPES OF SPRINGS :

Various types of springs are employed for different applications. Some of the them are as follows.

1) Helical springs

2) Leaf springs

3) Torsion springs

4) Circular springs

5) Flat springs

6.10 HELICAL SPRINGS :

A length of wire wound into helix is known as a helical spring. There are two types of helical springs.

(a) Closed-Coiled Helical Springs (b) Open-Coiled Helical Springs

(a) Closed-Coiled Helical Springs :

In closed-coiled helical springs the wire is wound quite closely such that the pitch distance between any two adjacent turns is small. In this case the helical angle less than 10^0.

(b) Open-Coiled Helical Springs :

Open-coiled helical spring is that in which the plane containing each coil is inclined to the axis of the helix and the helix angle is more than 10^0.

6.11 IMPORTANT DEFINITIONS USED IN HELICAL SPRINGS :

1) Spring Index (C) :

The ratio of mean coil diameter to the diameter of wire of spring is called the spring index and is denoted by 'C'.

$$\text{Spring index,} \qquad C = \frac{\text{Mean diameter of the coil}}{\text{diameter of the spring wire}} = \frac{D}{d}$$

2) Stiffness :

The spring stiffness or spring constant is defined as the load required per unit deflection of the spring.

$$K = \frac{\text{load}}{\text{deflection}} = \frac{W}{\delta}$$

3) Solid length :

The length spring under the maximum compression is called solid length of the spring :

$$\text{Solid length} = n \cdot d$$

where, n = total number of turns

d = diameter of the spring wire

4) **Free length :**

It is the length of spring measured along the axis when the spring is unloaded.

Free length = solid length + maximum compression + clearance

$$= nd + \delta_{max} + 0.15\,\delta_{max}$$

5) **Pitch :**

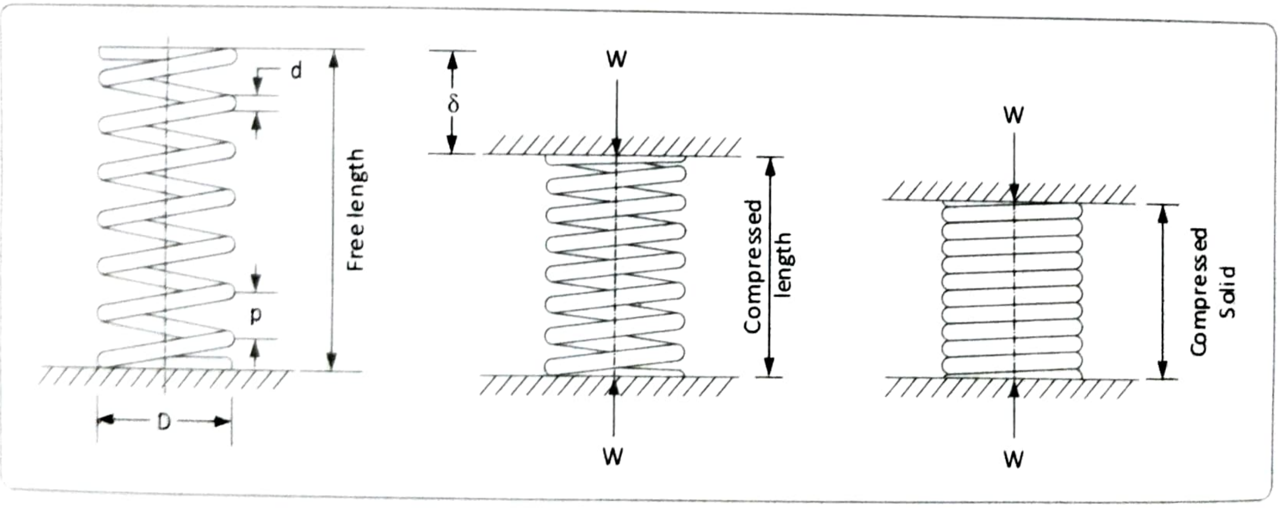
Pitch of the spring is defined as the axial distance between adjacent coils in uncompressed state.

$$\text{Pitch} = \frac{\text{free length}}{n-1}$$

where, n = total number coils

6) **Helix angle :**

It is the angle which the axis of the spring wire makes with horizontal line perpendicular to the axis of the spring.



6.12 CLOSED COILED HELICAL SPRINGS :

In closed coiled Helical springs the wire is wound quite closely such that the pitch distance between any two adjacent turns is small. In closed coiled helical springs, the helix angle is less than 10^0.

6.12.1 *Expression for Maximum Shear Stress Induced in Wire :*

Fig. 6.3 shows a closed coiled helical spring.

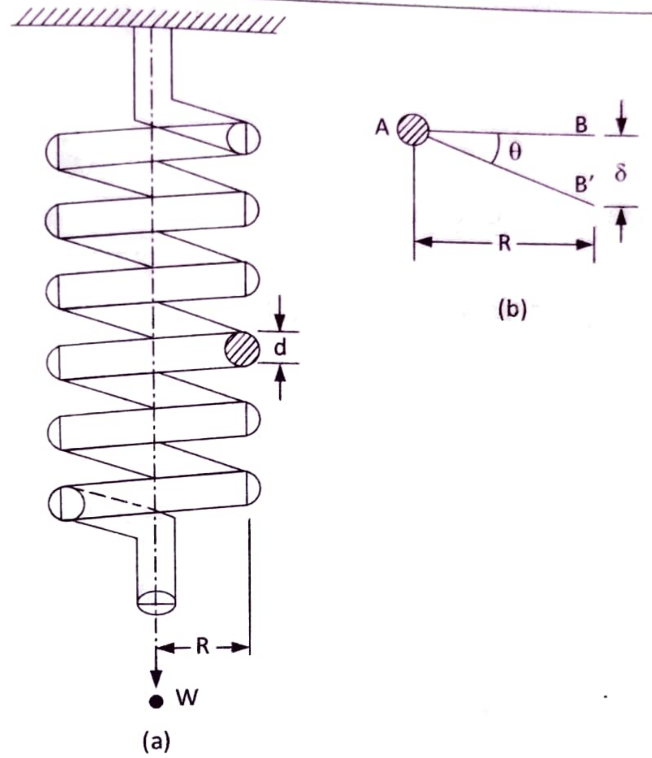


Fig. 6.3

Subjected to an axial load 'W'

let R = mean radius of coil

d = diameter of spring wire

n = number of coil or turns

θ = angle of twist

l = length of wire = $2\pi R n$

τ = shear stress

G = modulus of rigidity

δ = deflection of coil under load 'W'

W = axial load on spring

Now, torque on the spring wire,

$$T = W \times R \qquad \text{...... (i)}$$

But Torque, T is also given by, $T = \frac{\pi}{16}\tau (d)^3$ ...... (ii)

From the equations (i) and (ii)

$$W \times R = \frac{\pi}{16}\tau (d)^3$$

$$\text{or} \qquad \tau = \frac{16W \times R}{\pi(d)^3}$$

$$\therefore \quad \boxed{\text{Maximum shear induced in the wire, } \tau = \frac{16W \times R}{\pi(d)^3}}$$

6.12.2 Expression for Deflection of Spring :

From the torsion equation, $\frac{T}{J} = \frac{G\theta}{L}$

$$\theta = \frac{T \times L}{J \times G} = \frac{W \times R \times 2\pi Rn}{\frac{\pi}{32}(d)^4 \times G}$$

$$\theta = \frac{64\, W\, R^2 n}{Gd^4}$$

From the Fig. deflection of spring is given by,

$$\delta = R \times \theta$$

$$\delta = \frac{R \cdot 64WR^2n}{Gd^4}$$

$$\delta = \frac{64WR^3n}{Gd^4}$$

$$\therefore \quad \boxed{\text{Deflection of spring, } \delta = \frac{64WR^3n}{G(d)^4}}$$

6.13 The Energy Stored in Spring :

$$U = \frac{1}{2} W \cdot \delta = \frac{1}{2} W \cdot R \cdot \theta$$

$$= \frac{1}{2} T \cdot \theta$$

$$= \frac{1}{2}\left[\frac{\pi}{16}\tau(d)^3\right] \times \left[\frac{2\tau L}{G \cdot d}\right] \qquad \left(\because T = \frac{\pi}{16}\tau(d)^3\right)$$

$$\therefore \quad U = \frac{\tau^2}{4G} \times \text{volume of the wire}$$

6.14 EXPRESSION FOR STIFFNESS SPRING (K) :

$$K = \frac{W}{\delta} = \frac{W}{\dfrac{64\,WR^3n}{Gd^4}} = \frac{Gd^4}{64\,R^3n}$$

$$\therefore \quad \boxed{K = \frac{Gd^4}{64\,R^3n}}$$

Worked Examples

Example 6.15 :

A close-coiled helical spring is to carry a load of 120 N and the mean coil diameter is 10 times the diameter of the wire. Find the diameter of wire if the maximum shear stress is to be 90 N/mm².

Solution :

Given : Axial load, $W = 120$ N

Mean coil diameter, $D = 10$ d, Where d = dia. of wire

$\therefore \quad R = 5$ d

Max. shear stress, $\tau = 90$ N/mm^2

Max. shear stress, $$\tau = \frac{16WR}{\pi d^3} = \frac{16 \times 120 \times 5d}{\pi d^3}$$

$$\therefore \quad d = \sqrt{\frac{16 \times 120 \times 5}{90 \times \pi}} = \textbf{5.8mm} \quad \textbf{Ans.}$$

Example 6.16 :

A closely coiled helical spring is made of 6 mm wire. The maximum shear stress and deflection under a 200 N load is not to exceed 80 MPa and 11 mm respectively. Determine the number of coils and their mean diameter. Take modulus of rigidity for the spring material as 84 MPa.

Solution :

Given : Diameter of wire, d = 6 mm

Axial load, W = 200 N

Max. shear stress, τ = 80 MPa = 80 N/mm^2

Modulus of rigidity, G = 84 MPa = 0.84×10^5 N/mm^2

$$\tau = \frac{16WR}{\pi d^3}$$

$$\text{or} \quad R = \frac{80 \times \pi \times 6^3}{16 \times 200} = 17 \text{ mm}$$

$\therefore$ Mean Diameter, D = 17×2 = **34 mm Ans.**

$$\text{Deflection, } \delta = \frac{64WR^3n}{G.d^4} = 11$$

$$\text{or} \quad n = \frac{11 \times 0.84 \times 10^5 \times 6^4}{64 \times 200 \times 17^3} = 19.04 \approx 20$$

Number coil, n = **20 Ans.**

Example 6.17 :

A close coiled helical spring of round steel wire 6 mm in diameter having 12 complete coils of 60 mm mean diameter is subjected to an axial load of 125 N. Find the deflection of the spring and the maximum shear stress in the material. G = 0.8 × 10⁵ N/mm².

Solution :

Mean coil diameter, D = 60 mm

$\therefore$ R = 30 mm

Diameter of spring wire, d = 6 m

Number of turns (coils), n = 12

Axial load, W = 125 N

G = 0.8×10^5 N/mm^2

Deflection of the spring,

$$\delta = \frac{64WR^3.n}{G.d^4} = \frac{64 \times 125 \times 30^3 \times 12}{0.8 \times 10^5 \times 6^4} = \textbf{25 mm Ans}$$

Maximum shear stress

$$\tau = \frac{16WR}{\pi d^3} = \frac{16 \times 125 \times 30}{\pi \times 6^3} = \textbf{88.42 N/mm}^2 \textbf{ Ans.}$$

Example 6.18 :

It is required to design a close-coiled helical spring which shall deflect 16 mm under an axial load of 175 N with a shear stress of 9 × 10⁴ kN/m². The spring is to be made out of a round wire having modulus of rigidity of 0.8 × 10⁵ N/mm². The mean diameter of the circle is 11 times the diameter of wire. Find the diameter and length of the wire necessary to form the spring.

Solution :

Given : Deflection, $\delta = 16$ mm

Axial load, $W = 175$ N

Shear stress, $\tau = 9 \times 10^4$ kN/m²

$$= \frac{9 \times 10^4 \times 10^3}{10^6} = 90 \text{ N/mm}^2$$

Modulus of rigidity, $G = 0.8 \times 10^5$ N/mm²

Mean dia. of coil, $D = 11d$, d = dia. of wire

∴ Radius, $R = 5.5\,d$

The shear stress, $$\tau = \frac{16WR}{\pi d^3} = \frac{16W(5.5d)}{\pi d^3}$$

or $$= \frac{16WR}{\pi d^3} = \frac{16W(5.5d)}{\pi d^3}$$

and Mean radius of coil, $R = 5.5 \times 7.38 = 40.6$ mm

The deflection, $$\delta = \frac{64\,WR^3 n}{G.\,d^4}$$

or $$n = \frac{\delta.\,G.\,d^4}{64\,W\,R^3} = \frac{16 \times 0.8 \times 10^5 (7.38)^4}{64 \times 175\,(40.6)^3} = 5.06$$

∴ Length of spring, $l = 2\pi R.n$

$$= 2\pi \times 40.6 \times 5.06 = \textbf{1290.14 mm Ans.}$$

Example 6.19 :

A close-helical spring is required to exert a force of 2.5 kN and to have stiffness of 80 kN/m. If the mean diameter of the coil is to be 90 mm and the working stress 220 N/mm². Find the number of turns and the diameter of steel rod of which it is made. Assume G = 0.8 × 10⁵ N/mm².

Solution :

Given : Force (load), $W = 2.5 \text{ kN} = 2.5 \times 10^3 \text{ N}$

Stiffness, $s = 80 \times 10^3 \text{ N/m} = 80 \text{ N/mm}$

Working stress, $\tau = 220 \text{ N/mm}^2$

Dia. of coil, $D = 90 \text{ mm}$

$\therefore$ Radius, $R = 45 \text{ mm}$

Modulus of rigidity, $G = 0.8 \times 10^5 \text{ N/mm}^2$

Shear stress, $$\tau = \frac{16\,WR}{\pi d^3}$$

or $$d = \sqrt[3]{\frac{16 \times 32.5 \times 10^3 \times 45}{220 \times \pi}} = 13.758 \text{ mm Ans.}$$

Stiffness of spring, $$K = \frac{G.d^4}{64\,R^3 n}$$

$\therefore$ Number of springs, $$n = \frac{G.d^4}{64\,R^3 s} = \frac{0.8 \times 10^5 \times 13.758^4}{64 \times 45^3 \times 80}$$

$$= \mathbf{6.14, \text{ say } 7 \text{ Ans.}}$$

Example 6.20 :

A helical spring 150 mm mean diameter is required to absorb 30 kJ of energy with a maximum shear stress of 470 N/mm². Determine the diameter of steel rod and the number of coils if maximum compression is to be 150 mm. Take G = 84 kN/mm².

Solution :

Given : Mean dia. of spring, $D = 150 \text{ mm}$

$\therefore$ Radius, $R = 75 \text{ mm}$

Energy absorbed, $E = 30\ kJ = 30 \times 10^3\ J$

Max. shear stress $\tau = 470\ N/mm^2$

Maximum compression, $\delta = 150\ mm$

Modulus of rigidity, $G = 84\ kN/mm^2$

$= 0.84 \times 10^5\ N/mm^2$

W D on the spring $= \dfrac{1}{2}\ W.\ \delta$

Energy stored, $E = 30 \times 10^6\ Nmm$

$$\therefore \quad \frac{1}{2}\ W\ \delta = 30 \times 10^6$$

$$W = \frac{30 \times 10^6 \times 2}{150} = 0.4 \times 10^6\ N$$

Shear stress, $\tau = \dfrac{16\ WR}{\pi d^3}$

or $$d = \sqrt[3]{\frac{16 \times 0.4 \times 10^6 \times 75}{\pi \times 470}}$$

$= \mathbf{68.76\ mm}$ **Ans.**

Deflection (compression) of spring,

$$\delta = \frac{64\ W\ R^3.n}{G.\ d^4}$$

$\therefore$ Number of springs,

$$n = \frac{\delta.\ G.d^4}{64\ W\ R^3} = \frac{150 \times 0.84 \times 10^3 \times (68.76)^4}{64 \times 0.4 \times 10^6 \times (75)^3}$$

$= \mathbf{26.06\ say\ 27}$ **Ans.**

Example 6.21 :

A helical spring is made of 12 mm diameter steel wire winding it on a 120 mm diameter mandrel. If there are 10 active coils, what is the spring constant ? Take $G = 0.82 \times 10^5$ N/mm^2. What force must be applied to the spring to elongate it by 40 mm.

Solution :

Given,

Diameter of wire, $d = 12$ mm

Internal dia. of coil (Dia. of mandrel),

$$D_i = 120 \text{ mm}$$

$\therefore$ Mean radius, $R = \left(\dfrac{120+12}{2}\right) = 66$ mm

Elongation (deflection), $\delta = 40$ mm

Number of coils, $n = 10$

Spring constant (Stiffness), $K = \dfrac{G.d^4}{64\,R^3.n}$

$$= \frac{0.2 \times 10^5 \times (12^4)}{64 \times (66)^3 \times 10}$$

$$= 9.24 \text{ N/mm}$$

$$= \mathbf{9240\ N/m}\ \textbf{Ans.}$$

and Deflection, $\delta = \dfrac{64W.R^3.n}{G.d^4}$

$$\therefore \quad W = \frac{\delta.G.d^4}{64.R^3.n} = \frac{40 \times 0.82 \times 10^5\ (12)^4}{64 \times 66^3 \times 10}$$

$$= \mathbf{369.64\ N}\ \textbf{Ans.}$$

Example 6.22 :

A weight of 260 N is dropped on a close-coiled helical spring consisting of 20 coils. The diameter of wire is 16 mm and mean diameter of coil is 150 mm. Find the height of drop so that the spring may be compressed by 120 mm. Take $G = 0.85 \times 10^5$ N/mm².

Solution :

Given : Dropped weight, $W_g = 260$ N

Number of coils, $n = 20$

Diameter of spring wire, $d = 16$ mm

Mean coil diameter, $D = 150$ mm

∴ Radius, $R = 75$ mm

Compression of spring, $\delta = 120$ mm

Modulus of rigidity, $G = 0.8 \times 10^5$ N/mm²

Let W be the gradually applied load to produce same deflection (compression) of 120 mm, using the relation,

Deflection, $$\delta = \frac{64\,W\,R^3.n}{G.d^4}$$

or $$W = \frac{\delta.G.d^4}{64\,R^3.n}$$

$$= \frac{120 \times 0.85 \times 10^5 \times (16)^4}{64 \times (75)^3 \times 20}$$

$$= 1237.9 \text{ N}$$

Energy stored, $$E = \frac{1}{2} W \delta = \frac{1}{2} \times 1237.9 \times 120$$

$$= 74274 \text{ Nmm}$$

Workdone by the load $= W\,(h + \delta)$

∴ $W_g\,(h + \delta) = 74274$

(∴ W.D by load = Energy stored)

$$260\,(h + 120) = 74274$$

or $h =$ **165.67 mm Ans.**

Example 6.23 :

A wagon weighing 30 kN moving at 7.2 kmph. How many spring each of 18 coils will be required in a buffer stop to absorb the energy of motion during a compression of 250 mm. The mean diameter of coil is 200 mm and the wire diameter is 25 mm. Take G = 0.9 × 10^5 N/mm^2.

Solution :

Given,

Weight of wagon, $W = 30 \text{ kN}$

$\therefore$ Mass, $m = \dfrac{30 \times 10^3}{9.81} = 3.05 \times 10^3 \text{ kg}$

Speed, $V = 7.2 \text{ kmph}$

$$= \frac{7.2 \times 1000}{3600} = 2 \text{ m/s}$$

$\therefore$ K.E of wagon, $\dfrac{1}{2} mV^2 = \dfrac{1}{2} \times 3.058 \times 10^3 \times 2^2$

$$= 6.116 \times 10^3 \text{ Nm}$$

$$= 6.116 \times 10^6 \text{ Nmm}$$

Let W - be the gradually applied load to produce same compression,

$$\therefore \quad \delta = \frac{64 \, W \, R^3.n}{G.d^4}$$

$$\therefore \quad W = \frac{250 \times 0.9 \times 10^5 \times 25^4}{64 \times 100^3 \times 18}$$

$$= 7629.4 \text{ N}$$

Energy absorbed by each spring $= \dfrac{1}{2} W \delta = \dfrac{1}{2} \times 7629.4 \times 250$

$$= 953675 \text{ Nmm}$$

$\therefore$ Number of springs $= \dfrac{\text{K. E of wagon}}{\text{Energy absorbed per spring}} = \dfrac{6.116 \times 10^6}{953675}$

$= 6.4$, say 7 Ans.