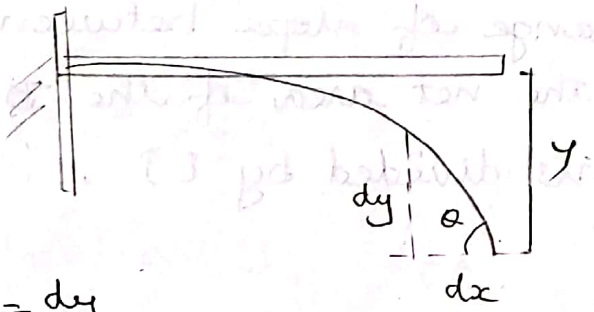
TOS III

Slope

The slope of a beam is the angle between deflected beam to the actual beam at the same point.

Deflection

- Is defined as the vertical displacement of a point on a loaded beam.
- The maximum deflection occurs where the slope is zero.



$$\theta = \frac{dy}{dx}$$

Slope of a beam.

→ Slope at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of the beam.

→ slope of that deflection in the angle between the original position and the deflected position.

Deflection of a beam.

→ The deflection at any point on the axis of the beam is the distance between its position before and after loading.

Methods for finding the slope and deflection of beams.

- Double integration method
- moment area method
- Macaulay's method
- Conjugate beam method
- Strain energy method.

Moment Area Method

Mohr's theorem I

The change of slope between any two points is equal to the net area of the BM diagram between these points divided by EI.

Mohr's theorem II

The total deflection between any two points is equal to the moment of the area of BM diagram between the two points about the last point divided by EI.

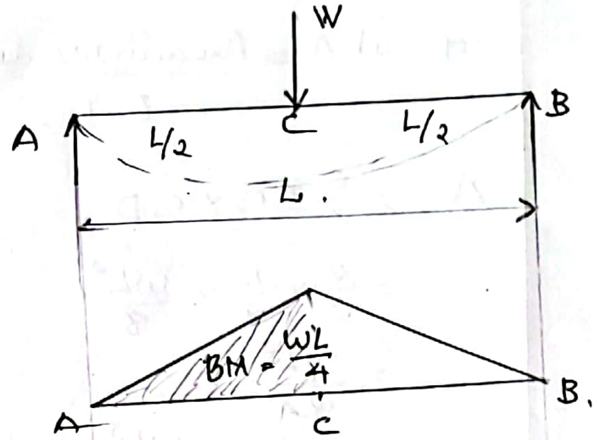
Mohr's theorem is used for problems on

→ Cantilever

→ Simply supported beam carrying symmetrical loading.

→ Fixed beam.

1 Simply supported beam. Length 'L'. point load 'w' at its centre.



Slope at A

$$= \frac{\text{Area of BM b/w A \& C}}{EI}$$

$$A = \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4}$$

$$= \frac{WL^2}{16}$$

$$\text{Slope} = \frac{WL^2}{16} \Big/ EI$$

$$\boxed{\theta = \frac{WL^2}{16EI}}$$

$$\text{Deflection 'y'} = \frac{A\bar{x}}{EI}$$

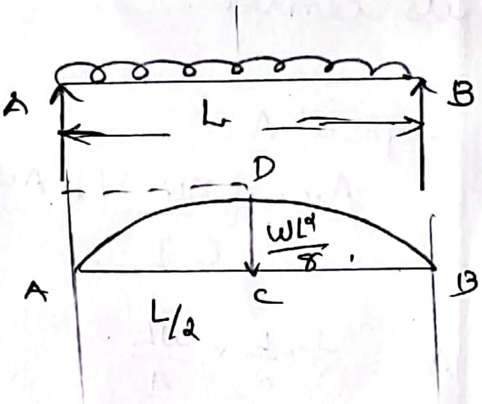
$$\bar{x} = L/3.$$

$$y = \frac{\frac{WL^2}{16} \times \frac{L}{3}}{EI}.$$

$$\boxed{y = \frac{WL^3}{48EI}}$$

simply supported beam with UDL.

slope at A = $\frac{\text{Area of BM diagram}}{EI}$.



$A = \frac{2}{3} AC \times CD$.

$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8}$.

$= \frac{wL^3}{24}$

slope at A $= \frac{wL^3}{24} / EI$

$$\boxed{\text{Slope at A} = \frac{wL^3}{24EI}}$$

$\bar{x} = \frac{5}{8} AC$

$= \frac{5}{8} \, l/2$

$= \frac{5L}{16}$.

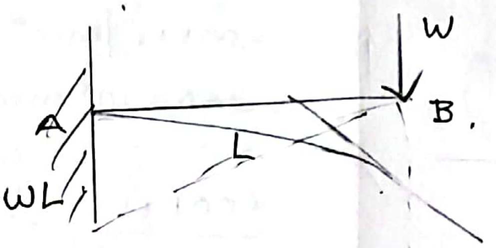
deflection $= \frac{A\bar{x}}{EI}$.

$= \frac{\frac{wL^3}{24E} \times \frac{5L}{16}}{EI}$

$$\boxed{\text{deflection} = \frac{5wL^4}{384EI}}$$

Simply supported			
Point load $\longrightarrow$	slope	$= \frac{wL^2}{16EI}$	$\frac{wL^2}{16EI}$
	Deflection	$= \frac{wL^3}{48EI}$.	$\frac{wL^3}{48EI}$
UDL $\longrightarrow$	slope	$= \frac{wL^3}{24EI}$	$\frac{wL^3}{24EI}$
	deflection	$= \frac{5wL^4}{384EI}$	$\frac{5wL^4}{384EI}$

Simply ~~supported~~ cantilever beam with pointed load.



change in slope = $\theta_B - \theta_A = \frac{A}{EI}$

$= \frac{1}{2} L \times WL$

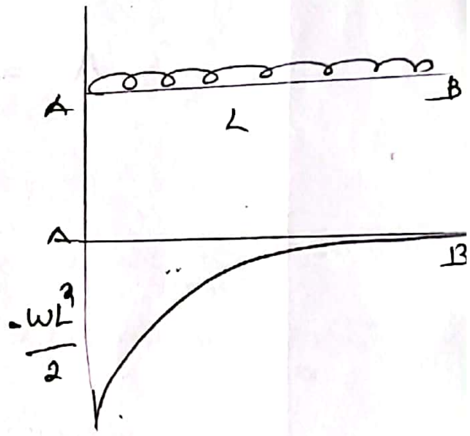
$$\boxed{\theta = \frac{WL^2}{2EI}}$$

Deflection @ B $= \frac{A\bar{x}}{EI}$

$$= \frac{\frac{1}{2} L \times WL \times \frac{2}{3} \times L}{EI}$$

$$\boxed{y = \frac{WL^3}{3EI}}$$

Simply ~~supported~~ cantilever beam with UDL



slope $\theta = \frac{\text{Area of BM diagram}}{EI}$

Area $= \frac{1}{3} bh$

$= \frac{1}{3} L \times \frac{WL^2}{2}$

$= \frac{WL^3}{6}$

$$\boxed{\theta(\text{slope}) = \frac{WL^3}{6EI}}$$

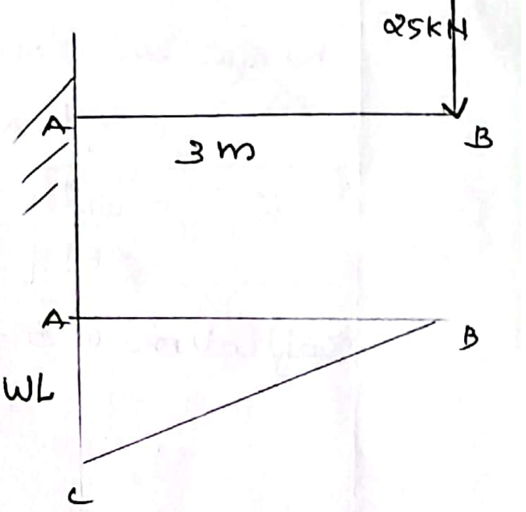
Deflection $= \frac{A\bar{x}}{EI}$

$\bar{x} = \frac{3}{4} L$

$$y = \frac{\frac{WL^3}{6} \times \frac{3}{4} L}{EI}$$

$$\boxed{\text{deflection } y = \frac{WL^4}{8EI}}$$

? Cantilever beam span 3m, point load 25KN at free end. find the slope and deflection



$E = 200 KN/mm^2$

$I = 360 \times 10^6 mm^4$

$$\frac{200 KN}{(10^{-3})^2} = \underline{\underline{200 \times 10^6 \, KN/m^2}}.$$

$$\frac{360 \times 10^6 \, mm^4}{10^{12}} = \underline{\underline{360 \times 10^{-6} m^4}}$$

$$\text{Slope } \theta = \frac{\text{Area of BM diagram}}{EI}.$$

$$A = \tfrac{1}{2} bh \rightarrow \frac{1}{2} 3 \times WL.$$

$$= \frac{1}{2} 3 \times 75$$

$$= \frac{112.5}{200 \times 10^6 \times 360 \times 10^{-6}}$$

$$= \underline{\underline{0.0015 \text{ radian}}}$$

$$\text{Deflection} = \frac{A\bar{x}}{EI}.$$

$$\bar{x} = \frac{2}{3} \times 3.$$
$$= 2$$

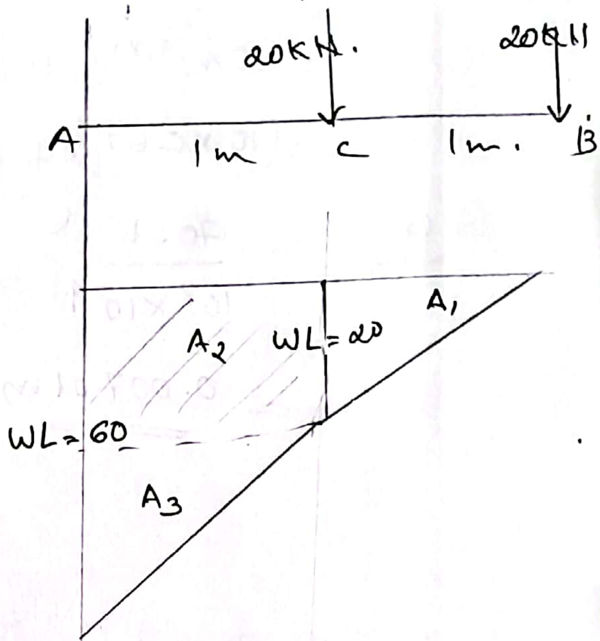
$$= \frac{112.5 \times 2}{200 \times 10^6 \times 360 \times 10^{-6}}.$$

$$= \underline{\underline{0.0031 m}}$$

2. A cantilever span 2m point load 20kN at free end 20kN at midspan. find slope and deflection.

$E = 10^5 N/mm^2$

$I = 10^8 mm^4$



$E = 10^5 N/mm^2 \rightarrow \frac{10^5 \times 10^{-3}}{(10^{-3})^2}$

$= 10^5 \times 10^{-3} \times 10^6$

$= 10^8 kN/m^2$

$I = 10^8 mm^4 \rightarrow 10^8 \times (10^{-3})^4$

$= 10^{-4} m^4$

Area $A_1 = \frac{1}{2} bh$

$= \frac{1}{2} 1 \times 20$

$= 20 kNm$

Slope $\theta = \frac{A}{EI}$

$A_2 = 20 \times 1 = 20 kNm^2$

$A_3 = \frac{1}{2} \times 1 \times 40 = 20 kNm^2$

Total area $= 50 kNm^2$

$\theta_B = \frac{\text{Area of BMD}}{EI}$

$= \frac{50}{10^8 \times 10^{-4}}$

$= 0.005$ radian

deflection $y_B = \frac{A\bar{x}}{EI}$.

$A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$,

$= 10 \times (\frac{2}{3} \times 1) + 20 (1 + \frac{1}{2}) + 20 (1 + \frac{2}{3} \times 1)$.

$= (10 \times 0.67) + (20 \times 1.5) + (20 \times 1.67)$

$y_B = \frac{70.1}{10^8 \times 10^{-4}}$

$= 0.00701 \, m$

Torsion of shaft

A shaft is said to be in torsion when equal and opposite torques are applied at the two ends of the shaft. The torque is equal to the product of the force applied (tangentially) and the radius of the shaft.

Torque (twisting moment) $T = F \times r$ (radius of shaft)

Nm

Torsion is a moment that twist / deforms a member about its longitudinal axis.

Assumptions.

- Plane section remains plane and perpendicular to the torsional axis.
- Material of the shaft is uniform.
- Twist along the shaft is uniform
- Axis remains straight and inextensible.

Application of the torques the shaft is subjected to a twisting moment. This causes shear stress and shear strain in the material of the shaft.

- cross section is uniform throught
- Plane before and after twist remains same

shear stress produced in a circular shaft subjected to torsion.

$$\boxed{\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r} = \frac{T}{J}}$$

τ = shear stress induced at the surface of the shaft due to torque T

R = radius of the shaft.

L = length of the shaft.

T = Torque applied

C = Modulus of rigidity.

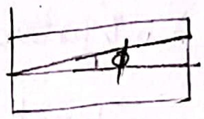
J = Polar moment of inertia

J → Solid shaft $\frac{\pi}{32} D^4$

hollow shaft $\frac{\pi}{32}\left[D_o^4 - D_i^4\right]$

ϕ = shear strain ($\tan\phi = \phi$) $= \frac{DD'}{L}$

θ = angle of twist.



q = Shear stress.

r = radius from the centre of shaft to the shear stress induced.

Maximum torque transmitted by a circular solid shaft.

$$\boxed{T = \frac{\pi}{16}\tau D^3}$$

- Torque transmitted by a hollow shaft.

$$\boxed{T = \frac{\pi}{16}\tau\left(\frac{D_o^4 - D_i^4}{D_o}\right)}$$

Torque transmitted by a solid shaft.

$$\boxed{T = \frac{\pi}{16}\tau D^3}$$

$P = T\omega$ $\quad \omega = \frac{2\pi N}{60}$ $\quad N =$ rpm speed of shaft.

$$\boxed{P = T\frac{2\pi N}{60}}$$

$T =$ Torque in Nm

$$\boxed{P = \frac{2\pi NT}{60} \rightarrow KW}$$ in SI $\boxed{T = KNm}$

$$\boxed{P = \frac{2\pi NT}{4500}}$$ MKS :- $\boxed{T = kg-m}$

shear stress

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{T}{J} = \frac{r}{R}$$

Max. torque

$$T = \frac{\pi}{16} \tau D^3$$

hollow shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

Power $P = \frac{2\pi N T}{60}$.

$1^\circ = \pi/180$.

Polar moment of inertia $J = \frac{\pi}{32} D^4$ (solid)

$$= \frac{\pi}{32} \left[D_o^4 - D_i^4 \right]$$

(hollow)