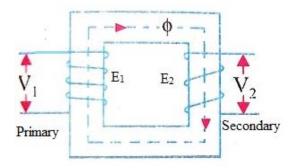
MODULE - I

Principle of Transformer

1.1.0 To explain the working principle of transformer

A transformer is a static piece of apparatus, which transform electric power from one circuit to another without changing the frequency. It can decrease or increase the voltage in a circuit with corresponding increase or decrease in current. The basic principle of transformer is mutual induction between two circuits linked by a common magnetic flux. It consists of an iron core and two inductive coils wound on it. Inductive coils are electrically separated but magnetically linked through a low reluctance path. The two coils posses high mutual inductance.

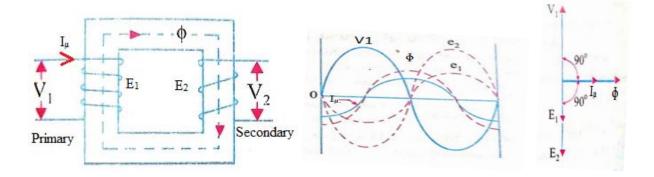


If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of the flux is linked with the other coil in which it produces mutually induced emf according to Faraday's Laws of Electromagnetic induction. If the second coil is closed, a current flows in it and so electric energy is transferred from the first coil to the second coil. The first coil, in which electric energy is fed from the ac supply, it is called primary winding and the other from which energy is drawn out, it is called secondary winding.

1.1.1 To illustrate the concept of ideal transformer

An ideal transformer is one which has no losses. Its windings have no ohmic resistance. It has no magnetic leakage. Hence it has no copper losses and core losses. In other words an ideal transformer consists of two purely inductive coils wound on a loss free core.

Consider an ideal transformer, whose secondary is open and primary is connected to sinusoidal alternating voltage V_1 . This causes an alternating current to flow in the primary. Since the primary coil is purely inductive, primary draws the magnetizing current I_{μ} only. The function



Graphical repesentation

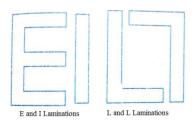
Vector representation

of I_{μ} is to magnetize the core, It is very small in magnitude and lags V_1 by 90^0 . I_{μ} produces an alternating flux φ , which is proportional to the current I_{μ} . This alternating flux is linked with both the primary and secondary windings. It produces self induced emf E_1 in the primary, E_1 equal and opposite to V_1 . It is also called counter emf or back emf of the primary.

Similarly mutually induced emf E_2 is induced in secondary. It is anti phase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

1.2.0 To describe the construction of a single phase transformer

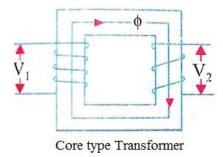
A transformer consists of two coils and a laminated core. The two coils are insulated from each other and the steel core. Suitable container is provided for assembled core and windings. Suitable insulating medium is used for insulating the core and windings from its container. Suitable bushings are used for bringing the terminals from container. In all type of transformers the core is constructed of thin laminations of sheet steel to provide a continuous magnetic path with minimum of air gap included. The steel used for core is of high silicon content to produce high permeability and a low hysteresis loss. The eddy current loss is minimized by laminating the core. The laminations are insulated from each other by a light coat of core plate varnish or by oxide layer on the surface. The thickness of laminations varies from 0.35mm for a frequency of 50 Hz to 0.5mm for a frequency of 25 Hz. Core Laminations are cut in the form of E, I, L shapes



On the basis of construction, transformers are classified in to two types

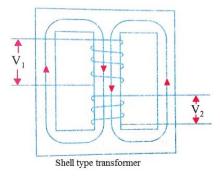
- i) Core type transformer
- ii) Shell type transformer

i) Core type transformer



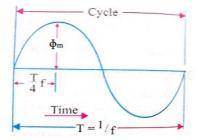
In core type transformers the windings surround a considerable part of the core. Windings are provided on two limbs.

ii) Shell type transformer



In shell type transformers the core surrounds a considerable portion of the core. Both Windings are provided on one limb.

1.3.0 To derive the emf equation of transformer



Let $N_1 = No.$ of turns in primary

 N_2 = No. of turns in secondary

 $\phi_{\rm m}$ = Maximum flux in core in webers

= $B_m \times A$

f = Frequency of a.c. input in Hz

Flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle, ie in 1/4f seconds

Average rate of change of flux
$$= \frac{\phi_{\rm m}}{1/4f}$$
$$= 4f \phi_{\rm m} \text{ Wb/s or volt}$$

Now rate of change of flux per turn means induced emf in volts

Average emf/turn =
$$4f \phi_m \text{ volt}$$

If flux ϕ varies sinusoidally then rms value of induced emf is obtained by multiplying the average value with form factor.

Form factor =
$$\frac{\text{rms value}}{\text{average value}} = 1.11$$

rms value of emf/turn =
$$1.11 \times 4f \phi_m = 4.44f \phi_m$$
 volt.

rms value of the induced emf in the primary winding

= (induced emf/turn) x No of primary turns

$$E_1 = 4.44f N_1 \phi_m = 4.44f N_1 B_m A \text{ volt}$$

Similarly rms value of the induced emf in the secondary winding

$$E_2 = 4.44f N_2 \phi_m = 4.44f N_2 B_m A \text{ volt}$$

1.3.1 To define the voltage and current transformation ratio

$$E_1 = 4.44f N_1 \phi_m = 4.44f N_1 B_m A \text{ volt}$$

$$E_2 = 4.44 f N_2 \phi_m = 4.44 f N_2 B_m A \text{ volt}$$

From the above equations, We get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = K$$

The constant K is known as voltage transformation ratio.

An ideal transformer input VA = Output VA

$$V_1I_1 = V_2I_2$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K}$$

Current is the inverse ratio of voltage transformation ratio

1.3.2 To describe the effect of voltage and frequency variation in transformers

Variations in voltage and frequency affects the iron losses in a transformer. Iron losses is the sum of hysteresis and eddy current losses. The flux variations are sinusoidal hysteresis loss (P_h) and eddy current loss (P_e) varies according to the relation.

$$P_h \propto f(\Phi_m)^x$$

Where x lies between 1.5 and 2.5 depending up on the grade of iron used in transformer core.

$$P_e \propto f^2(\Phi_m)^2$$

If the transformer is operated with the frequency and voltage changed in the same proportion, the flux density will remain unchanged and no load current will also unchanged.

The transformer can be operated safely at frequency and voltage less than its rated value. In this case iron losses will be reduced. But if the transformer is operated with increased frequency and voltage, the core losses may increase. Increase in frequency with constant supply voltage will cause reduction in hysteresis loss and leave the eddy current loss unaffected. Some increase in voltage could be tolerated at higher frequencies, but exactly how much depends on the relative magnitude of the hysteresis loss and eddy current losses and the grade of iron used in the transformer core.

Problem No:1

The maximum flux density in the core of a 250/3000V, 50 Hz single phase transformer is 1.2 Wb/m². If the emf per turn is 8 volt, determine

- (i) Primary and secondary turns
- (ii) Area of the core

Given data

 $E_1 = 250 \text{ V}, E_2 = 3000 \text{ V}, f = 50 \text{Hz}, B_m = 1.2 \text{ Wb/m}^2, \text{ emf per turn} = 8 \text{ V}$

Solution

(i) Primary and secondary turns

 $E_1 = N_1 x$ emf induced per turn

Primary turns $N_1 = E_1$ / emf induced per turn = 250/8 = 31.25 = 32 turns

Secondary turns $N_2 = E_2$ / emf induced per turn = 3000/8 = 375 turns

(ii) Area of the core

Use any one of the emf equation

$$E_1 = 4.44f N_1 \phi_m = 4.44f N_1 B_m A \text{ volt}$$

$$E_2 = 4.44f N_2 \phi_m = 4.44f N_2 B_m A \text{ volt}$$

$$E_2 = 4.44f N_2 B_m A \text{ volt}$$

$$3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$$

Area of the core,
$$A = \frac{3000}{4.44 \times 50 \times 375 \times 1.2} = \underline{0.03 \text{ m}^2}$$

Problem No:2

A single phase transformer has 400 primary and 1000 secondary turns. The net cross sectional area of the core is 60 cm². If the primary winding be connected to a 50 Hz supply at 520V. Calculate

- (i) The peak value of flux density in the core
- (ii) The voltage induced in the secondary winding

Given data

$$N_1 = 400$$
, $N_2 = 1000$, $f = 50$ Hz, $A = 60$ cm² = 60 x 10 ⁻⁴ m², $E_1 = 520$ V

Solution

(i) The peak value of flux density in the core

$$E_1 \hspace{0.5cm} = \hspace{0.5cm} 4.44 f \hspace{0.1cm} N_1 B_m A \hspace{0.1cm} volt$$

$$520 = 4.44 \times 50 \times 400 \times B_m \times 60 \times 10^{-4}$$

Peak value of flux density in the core, $B_m = \frac{520}{4.44 \times 50 \times 400 \times 60 \times 10^{-4}} = \underline{\textbf{0.976 Wb/m}^2}$

(ii) The voltage induced in the secondary winding

Voltage transformation ratio, $K = \frac{N_2}{N_1} = \frac{E_2}{E_1}$

$$\frac{1000}{400} = \frac{E_2}{520}$$

Voltage induced in the secondary winding, $E_2 = \frac{1000 \times 520}{400} = \underline{1300 \text{ V}}$

Problem No:3

The core of a 100 kVA, 11000/550 V, 50 Hz, single phase transformer has cross sectional of 20 cm x 20 cm. Assume a stacking factor of 0.9. Find

- (i) The number of turns in HV and LV side
- (ii) The emf per turn if the maximum core density is not to exceed 1.3 Tesla

Given data

$$E_1 = 11000 \text{ V}, E_2 = 550 \text{ V}, f = 50 \text{Hz}, A = 20 \text{ x } 20 \text{ x } 0.9 \text{ cm}^2 = 0.9 \text{ x } 400 \text{ x } 10^{-4} \text{ m}^2 = 0.036 \text{ m}^2$$

Solution

(i) The number of turns in HV and LV side

$$E_1 \hspace{0.5cm} = \hspace{0.5cm} 4.44 f \hspace{0.1cm} N_1 B_m A \hspace{0.1cm} volt$$

$$11000 = 4.44 \times 50 \times N_1 \times 1.3 \times 0.036$$

Number of turns in HV side,
$$N_1 = \frac{11000}{4.44 \times 50 \times 1.3 \times 0.036} = \underline{1060}$$

$$E_2 = 4.44f N_2 B_m A \text{ volt}$$

$$550 = 4.44 \times 50 \times N_2 \times 1.3 \times 0.036$$

Number of turns in LV side,
$$N_2 = \frac{550}{4.44 \times 50 \times 1.3 \times 0.036} = \underline{53}$$

(ii) The emf per turn if the maximum core density is not to exceed 1.3 Tesla

$$Emf/turn = 11000/1060 \text{ or } 550/53 = \underline{10.4 \text{ V}}$$

Problem No:4

A 25 kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000 V, 50 Hz supply. Find

- (i) Full load primary and secondary currents
- (ii) Secondary emf and the maximum flux in the core.

Given data

$$N_1 = 500$$
, $N_2 = 50$, $f = 50$ Hz, $E_1 = 3000$ V, Full load rated capacity = 25 kVA = 25000VA

Solution

(i) Full load primary and secondary currents

Full load primary current,
$$I_1 = \frac{Full\ load\ rated\ capacity\ in\ VA}{E_1} = \frac{25000}{3000} = 8.33\ A$$

Current transformation ratio = $\frac{N_2}{N_1} = \frac{I_1}{I_2}$

$$\frac{50}{500} = \frac{8.33}{I_2}$$

Full load secondary current,
$$I_2 = \frac{500 \times 8.33}{50} = 83.3 \text{ A}$$

(ii) Secondary emf and the maximum flux in the core.

Voltage transformation ratio, $K = \frac{N_2}{N_1} = \frac{E_2}{E_1}$

$$\frac{50}{500} = \frac{E_2}{3000}$$

Voltage induced in the secondary winding, $E_2 = \frac{3000 \times 50}{500} = \mathbf{\underline{300 V}}$

$$E_1 = 4.44f N_1 \phi_m \text{ volt}$$

$$3000 = 4.44 \times 50 \times 500 \times \phi_{\rm m} \text{ Volt}$$

Maximum flux in the core in the core,
$$\phi_m = \frac{3000}{4.44 \times 50 \times 500} = \underline{\textbf{0.027 Wb}}$$

Problem No:5

A 3000/200 V, 50 Hz, single phase transformer has a core of cross sectional area 150 cm² and 80 turns in the low voltage winding. Calculate

- (i) The number of turns in HV side
- (ii) The maximum flux density in the core

Given data

$$E_1 = 3000 \text{ V}, E_2 = 200 \text{ V}, f = 50 \text{Hz}, A = 150 \text{ cm}^2 = 150 \text{ x } 10^{-4} \text{ m}^2, N_2 = 80$$

Solution

(i) The number of turns in HV side

Voltage transformation ratio, $K = \frac{N_2}{N_1} = \frac{E_2}{E_1}$

$$\frac{80}{N_1} = \frac{200}{3000}$$

The number of turns in HV side, $N_1 = \frac{3000 \times 80}{200} = \underline{1200}$

(ii) The maximum flux density in the core

 $E_1 = 4.44f N_1 B_m A \text{ volt}$

$$3000 = 4.44 \times 50 \times 1200 \times B_m \times 150 \times 10^{-4} \text{ Volt}$$

Maximum flux density in the core, $B_m = \frac{3000}{4.44 \times 50 \times 1200 \times 150 \times 10^{-4}} = \underline{0.75 \text{ Wb/m}^2}$

Problem No:6

A 6600/230 V, 50 Hz, single phase transformer has a core of cross sectional area 400 cm^2 and a maximum flux density of 1.18 Wb/m^2 . Calculate the number of turns in primary and secondary windings.

Given data

$$E_1 = 6600 \text{ V}, E_2 = 230 \text{ V}, f = 50 \text{Hz}, A = 400 \text{ cm}^2 = 400 \text{ x } 10^{-4} \text{ m}^2, B_m = \underline{\textbf{1.18 Wb/m}^2}$$

Solution

Number of turns in primary and secondary windings.

$$E_1 \hspace{0.5cm} = \hspace{0.5cm} 4.44 f \hspace{0.1cm} N_1 B_m A \hspace{0.1cm} volt$$

$$6600 = 4.44 \times 50 \times N_1 \times 1.18 \times 400 \times 10^{-4}$$

Number of turns in primary side,
$$N_1 = \frac{6600}{4.44 \times 50 \times 1.18 \times 400 \times 10^{-4}} = \underline{630}$$

$$E_2 = 4.44f N_2 B_m A \text{ volt}$$

$$230 = 4.44 \times 50 \times N_2 \times 1.18 \times 400 \times 10^{-4}$$

Number of turns in secondary side,
$$N_2 = \frac{230}{4.44 \times 50 \times 1.18 \times 400 \times 10^{-4}} = 22$$

Problem No:7

The core of a three phase 11000/550 V, Delta/Star, 300 kVA, 50 Hz core type transformer operates with a flux of 0.05 Wb. Find

- (i) Number of HV and LV turns per phase.
- (ii) emf/turn
- (iii) Full load HV and LV phase currents

Given data

Number of phases = 3

Primary side of the transformer is delta connected, In delta connection $E_L = E_{ph} = 11000 \text{ V}$

Secondary side of the transformer is star connected, In star connection $E_L = \sqrt{3} \; E_{ph}$

Therefore,
$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{550}{\sqrt{3}} = 317.5 \text{ V}$$

f = 50Hz, $\phi_m = 0.05$ Wb, Full load rated capacity = 300 kVA

Solution

(ii) emf/turn

Emf per turn =
$$4.44 \text{ f } \phi_m = 4.44 \text{ x } 50 \text{ x } 0.05 = \underline{\textbf{11.1 V}}$$

(i) Number of HV and LV turns per phase.

Number of turns/phase on HV side,
$$N_1/phase = \frac{E_{ph}}{emf\ per\ turn} = \frac{11000}{11.1} = \underline{991}$$

Number of turns/phase on LV side,
$$N_2/phase = \frac{E_{ph}}{emf per turn} = \frac{317.5}{11.1} = 28.6 = 29$$

(iii) Full load HV and LV phase currents

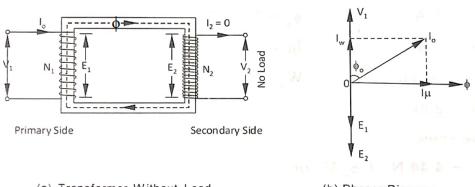
Output per phase = 300/3 = 100 kVA = 100000 VA

High voltage phase current =
$$\frac{Output \ per \ phase}{E_{ph}} = \frac{100000}{11000} = \underline{9.1 \ A}$$

Low voltage phase current =
$$\frac{Output\ per\ phase}{E_{ph}} = \frac{100000}{317.5} = \underline{315\ A}$$

1.3.3 To illustrate the transformer on no load

When transformer on no load, Primary of the transformer is connected to a sinusoidal alternating voltage of V_1 and secondary is open circuited. The primary input current I_0 has to supply iron losses in the core (hysteresis and eddy current losses) and very small amount of copper losses of primary windings. Hence I_0 is not lags the voltage V_1 at 90^0 , by an angle ϕ_0 less than 90^0 .



(a) Transformer Without Load

(b) Phasor Diagram

Let $W_0 = \text{No load input power} = \text{Iron loss} = V_1 I_0 \cos \phi_0$

 V_1 = Primary Voltage

$$I_0$$
 = No load Primary current = $\sqrt{I_w^2 + I_\mu^2}$

 I_w = Active Component = $I_0 \cos \phi_0$

 I_u = Magnetizing component = $I_0 \sin \phi_0$

 $\cos \phi_0 =$ No load power factor.

Primary current I₀ has two components

- i) Active or working or Iron loss component (I_w)
- ii) Magnetizing component (I_{μ})

i) Active or working or Iron loss component (I_w)

It mainly supplies the iron loss and small amount of copper losses of primary windings

$$I_w = I_0 \cos \phi_0$$

ii) Magnetizing component (I_µ)

Its function is to sustain the magnetic flux in the core.

$$I_u = I_0 \sin \phi_0$$

 I_0 is the vector sum of I_w and I_μ , hence $I_0 = \sqrt{I_w^2 + I_\mu^2}$

Problem No:8

The no load current of a transformer is 5 A at 0.3 pf, when supplied at 230 V, 50 Hz. The number of turns on the primary winding is 200. Calculate

- (i) The maximum value of flux in the core
- (ii) The magnetizing current
- (iii) Iron loss in the transformer

Given data

$$E_1 = 230 \text{ V}, I_0 = 5 \text{ A}, f = 50 \text{Hz}, \cos \phi_0 = 0.3, N_1 = 200$$

Solution

(i) The maximum value of flux in the core

$$E_1 \hspace{0.5cm} = \hspace{0.5cm} 4.44 f \hspace{0.1cm} N_1 \hspace{0.1cm} \varphi_m \hspace{0.1cm} volt$$

$$230 = 4.44 \times 50 \times 200 \times \phi_m$$
 Volts

Maximum value of flux in the core,
$$\phi_m = \frac{230}{4.44 \times 50 \times 200} = \underline{\textbf{0.0518 Wb}}$$

(ii) The magnetizing current

$$\cos \phi_0 = 0.3$$

$$\phi_0 = \cos^{-1} 0.3 = 1.27$$

Magnetizing current, $I_{\mu} = I_0 \sin \phi_0 = 5 \text{ x sin } 1.27 = \underline{\textbf{4.77 A}}$

(iii) Iron loss in the transformer

Iron loss in the transformer, $W_0 = V_1 I_0 \cos \phi_0 = 230 \text{ x } 5 \text{ x } 0.3 = 345 \text{ W}$

Problem No:9

The no load current of a transformer is 15 A at 0.2 pf, when connected to 460 V, 50 Hz. Calculate iron loss and magnetizing component of the current.

Given data

$$E_1 = 460 \text{ V}, I_0 = 15 \text{ A}, f = 50 \text{Hz}, \cos \phi_0 = 0.2$$

Solution

Iron loss in the transformer

Iron loss in the transformer, $W_0 = V_1 I_0 \cos \phi_0 = 460 \times 15 \times 0.2 = \underline{\mathbf{1380 W}}$

The magnetizing current

$$cos \; \varphi_0 = 0.2$$

$$\phi_0 = \cos^{-1} 0.2 = 1.37$$

Magnetizing current, $I_{\mu} = I_0 \sin \phi_0 = 15 \text{ x sin } 1.37 = \mathbf{\underline{14.7 A}}$

Problem No:10

A 2200/200V transformer draws a no load primary current of 0.6 A and absorbs 400 watts. Find the magnetizing and iron loss currents.

Given data

$$V_1 = 2200 \text{ V}, \ V_2 = 200 \text{ V}, \ I_0 = 0.6 \text{ A}, \ W_0 = 400 \text{ W}$$

Solution

Iron loss current,
$$I_w = \frac{W_0}{V_1} = \frac{V_1 I_0 Cos \, \phi_0}{V_1} = \frac{400}{2200} = \underline{0.182 \, A}$$

Magnetizing current,
$$I_{\mu} = \sqrt{I_0^2 - I_w^2} = \sqrt{(0.6)^2 - (0.182)^2} = \underline{0.572 \text{ A}}$$

Problem No:11

A 2200/250V transformer takes 0.5 A at a pf of 0.3 on open circuit. Find magnetizing and working components of no load primary current.

Given data

$$V_1 = 2200 \text{ V}, V_2 = 250 \text{ V}, I_0 = 0.5 \text{ A}, \cos \phi_0 = 0.3$$

Solution

Working component, $I_w = I_0 \cos \phi_0 = 0.5 \times 0.3 = \underline{0.15 \text{ A}}$

Magnetizing component, $I_{\mu}\ = I_0\, Sin\, \varphi_0$

 $\cos \phi_0 = 0.3$

$$\phi_0 = \cos^{-1} 0.3 = 1.27$$

Magnetizing current, $I_{\mu} = I_0 \sin \phi_0 = 0.5 \text{ x sin } 1.27 = \underline{\textbf{0.476 A}}$

Problem No:12

A single phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The mean length of the magnetic path in the iron core is 150 cm and the joints are equivalent to an air gap of 0.1 mm. When a potential difference of 3000V is applied to the primary, maximum flux density is 1.2 Wb/m². Given that AT/cm for a flux density of 1.2 Wb/m² in iron to be 5, the corresponding iron loss to be 2 watt/kg at 50 Hz and the density of iron as 7.8 gm/cm³. Calculate

- (i) The cross sectional area of the core
- (ii) No load secondary voltage
- (iii) The no load current drawn by the primary

(iv) Power factor on no load

Given data

$$N_1 = 500 \text{ V}, N_2 = 40 \text{ V}, l = 150 \text{ cm}, \text{ Air gap lengh} = 0.1 \text{ mm} = 0.1 \text{ x } 10^{-3}, E_1 = 3000 \text{ V}, R_2 = 1000 \text{ V}, R_3 = 1000 \text{ V}$$

$$B_m = 1.2 \text{ Wb/m}^2$$
, AT/cm = 5, Iron loss = 2 Watt/kg, f = 50 Hz, Density of iron = 7.8 gm/cm³.

Solution

(i) The cross sectional area of the core

$$E_1 = 4.44 \times f \times N_1 \times B_m \times A \text{ volt}$$

$$3000 = 4.44 \times 50 \times 500 \times 1.2 \times A \text{ Volts}$$

cross sectional area of the core,
$$A = \frac{3000}{4.44 \times 50 \times 500 \times 1.2} = \underline{0.0225 \text{ m}^2}$$

(ii) No load secondary voltage

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \; ; \quad \frac{40}{500} = \frac{E_2}{3000}$$

$$E_2 = \frac{3000 \times 40}{500} = 240 \text{ V}$$

(iii) The no load current drawn by the primary

AT/cm = 5

AT for iron core = $150 \times 5 = 750$

AT for air gap =
$$Hl = \frac{B}{\mu_0} \times l = \frac{1.2}{4\pi \times 10^{-7}} \times 0.0001 = 95.5$$

Total AT for given $B_m = 750 + 95.5 = 845.5$

Max. value of magnetizing current drawn by primary = 845.5/500 = 1.691 A

RMS value of
$$I_{\mu} = \frac{1.691}{\sqrt{2}} = 1.196$$

Volume of iron = length x area = $150 \times 225 = 33750 \text{ cm}^3$

Density = 7.8 gm/cm^3

Mass of iron =
$$\frac{33750 \times 7.8}{1000}$$
 = 263.25 kg

Total iron loss, $W_0 = 263.25 \text{ x } 2 = 526.5 \text{ W}$

Iron loss current,
$$I_w = \frac{W_0}{V_1} = \frac{V_1 I_0 \cos \phi_0}{V_1} = \frac{526.5}{3000} = \underline{\mathbf{0.176 A}}$$

No load current,
$$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(1.196)^2 - (0.176)^2} = \underline{0.208 \text{ A}}$$

(iv) Power factor on no load

Power factor on no load,
$$\cos \phi_0 = \frac{I_w}{I_0} = \frac{0.176}{1.208} = \underline{0.1457}$$

Problem No:13

A single phase transformer working on a 2000V, 50Hz circuit has 300 primary turns. The core has a mean magnetic path of 100 cm and cross-section 1000 cm², the iron has a permeability of 1800. The iron loss is 400W. Calculate the primary no load current.

Given data

$$N_1 = 300 \text{ V}, l = 100 \text{ cm} = 100 \text{ x } 10^{-2} \text{ m}, A = 1000 \text{ cm}^2 = 1000 \text{ x } 10^{-4} \text{ m}^2, E_1 = 2000 \text{ V}, E_2 = 1000 \text{ m}$$

$$W_0 = 440 \text{ W}, \, \mu_r = 1800$$

Solution

$$E_1 = 4.44 \times f \times N_1 \times B_m \times A \text{ volt}$$

$$2000 = 4.44 \times 50 \times 300 \times B_m \times 1000 \times 10^{-4} \text{ Volts}$$

Flux density,
$$B_m = \frac{2000}{4.44 \times 50 \times 300 \times 1000 \times 10^{-4}} = 0.3 \text{ Wb/m}^2$$

AT for iron core =
$$Hl = \frac{B}{\mu_0 \mu_r} \times l = \frac{0.3}{4\pi \times 10^{-7} \times 1800} \times 100 \times 10^{-2} = 132.63$$

Max. value of magnetizing current drawn by primary = $\frac{AT}{N_1} = \frac{132.63}{300} = 0.4421$ A

RMS value of
$$I_{\mu} = \frac{0.4421}{\sqrt{2}} = 0.313 \text{ A}$$

Iron loss current,
$$I_w = \frac{W_0}{V_1} = \frac{V_1 I_0 \cos \phi_0}{V_1} = \frac{440}{2000} = 0.2 \text{ A}$$

No load current,
$$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(0.313)^2 - (0.2)^2} = \underline{\textbf{0.371 A}}$$

1.3.4 To illustrate the transformer on load

When the secondary is loaded, the secondary current I_2 is set up. The magnitude and phase angle of I_2 with respect V_2 is determined by the characteristics of the load. Current I_2 is in phase with V_2 if load is resistive, It lags if load is inductive and It leads if load is capacitive.

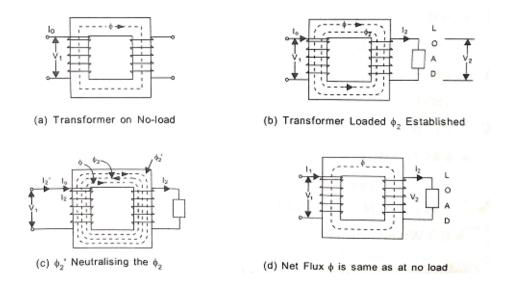
The secondary current sets up its own mmf (N_2I_2) and hence its own flux ϕ_2 which is in opposition to the main primary flux ϕ which is due to I_0 . The secondary ampere-turns (N_2I_2) are called demagnetizing ampere-turns. The opposing secondary flux ϕ_2 weakens the primary flux ϕ momentarily, hence primary back emf E_1 tends to be reduced. For a moment V_1 gains the upper hand over E_1 and hence causes more current to flow in primary which is I_2 .

The additional primary current be I_2 is known as load component of primary current. This current is anti-phase with I_2 . I_2 will setup additional primary mmf (N_1I_2) and its own flux φ_2 . Which is equal in magnitude and opposition to φ_2 but same direction of φ . Hence φ_2 and φ_2 are cancel each other. The magnetic effects of secondary current I_2 are immediately neutralized by the additional primary current I_2 .

Hence whatever the load conditions, the net flux passing through the core is approximately same as at no load, therefore the core loss is practically same as all load conditions.

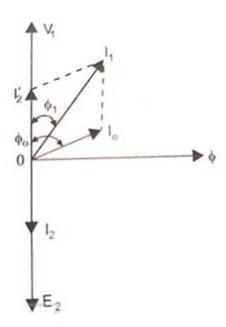
$$\phi_2 = \phi_2$$
, $N_2 I_2 = N_1 I_2$, $I_2 = \frac{N_2}{N_1} \times I_2 = K I_2$

Hence the transformer is on load, The primary current has two components I_0 and I_2 . I_2 is anti phase with I_2 and K times in magnitude. Therefore the primary current I_1 is the vector sum of I_0 and I_2 .



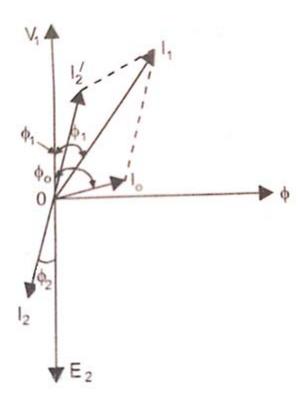
Vector diagram of transformer on resistive load (at unity power factor)

Fig shows the vector diagram of transformer with resistive load. I_2 is in phase with E_2 . I_2 is equal in magnitude and anti phase with I_2 . Primary current I_1 is the vector sum of I_0 and I_2 and lags behind V_1 by an angle φ_1



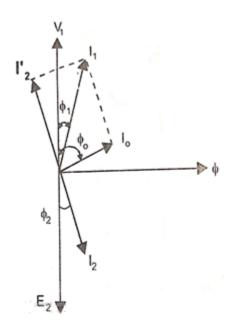
Vector diagram of transformer on inductive load (at lagging power factor)

Fig shows the vector diagram of transformer with inductive load. I_2 lags with E_2 an angle ϕ_2 . I_2 is equal in magnitude and anti phase with I_2 . Primary current I_1 is the vector sum of I_0 and I_2 and lags behind V_1 by an angle ϕ_1



Vector diagram of transformer on capacitive load (at leading power factor)

Fig shows the vector diagram of transformer with capacitive load. I_2 leads with E_2 an angle φ_2 . I_2 is equal in magnitude and anti phase with I_2 . Primary current I_1 is the vector sum of I_0 and I_2 and lags behind V_1 by an angle φ_1



PROBLEM No.14

A single phase transformer has a ratio of 3300/400 V, and takes no load current of 1 A at a pf of 0.2. Estimate the current drawn by the primary when the secondary supplies a current of 60 A at 0.8 pf. Lag.

Given data

$$E_1 = 3300 \text{ V}, E_2 = 400 \text{ V}, I_0 = 1 \text{ A}, \cos \phi_0 = 0.2, I_2 = 60 \text{ A}, \cos \phi_2 = 0.8$$

Solution

Primary current, $I_1 = \text{Vector sum of } I_0 \text{ and } I_2^{/}$

$$I_2^{\prime} = K \times I_2$$

$$K = E_2/E_1 = 400/3300 = 0.1212$$

$$I_2^{\prime} = \text{K} \times I_2 = 0.1212 \times 60 = 7.27 \text{ A}$$

Method I [Vector addition using parallelogram method]

$$I_1 = \sqrt{I_0^2 + I_2^{/2} + 2 x I_0 x I_2^{/2} x \cos \theta}$$

$$\theta = \phi_0 - \phi_2$$

$$\phi_0 = \cos^{-1}(\cos\phi_0) = \cos^{-1}(0.2) = 78.46^0$$

$$\phi_2 = \cos^{-1}(\cos\phi_2) = \cos^{-1}(0.8) = 36.87^0$$

$$\theta = \phi_0 - \phi_2 = 78.46^0 - 36.87^0 = 41.59^0$$

$$I_1 = \sqrt{I_0^2 + I_2^2 + 2 x I_0 x I_2^2 x \cos \theta}$$

$$I_1 = \sqrt{1^2 + 7.27^2 + 2 \times 1 \times 7.27 \times \cos 41.59} = 8.04 \text{ A}$$

Phase angle $\phi_1 = \phi_0 - \phi$

$$\Phi = \tan^{-1} \left[\frac{I_2' \sin \theta}{I_0 + I_2' \cos \theta} \right] = \tan^{-1} \left[\frac{7.27 \sin 41.59}{1 + 7.27 \cos 41.59} \right] = 36.85^0$$

Phase angle $\phi_1 = 78.46 - 36.85 = 41.61^{\circ}$

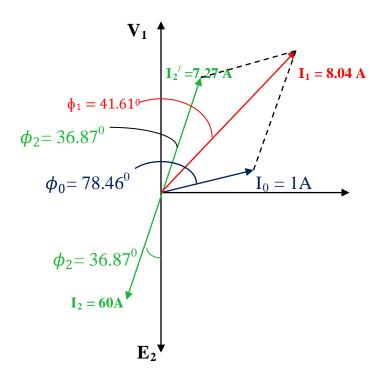
Method II [Vector addition using method of components]

X component = $I_0 \cos \phi_0 + I_2' \cos \phi_2 = 1 \times 0.2 + 7.27 \times 0.8 = 6.016$

Y component = $I_0 \sin \phi_0 + I_2^{\prime} \sin \phi_2 = 1 \times 0.98 + 7.27 \times 0.6 = 5.342$

$$I_1 = \sqrt{X^2 + Y^2} = \sqrt{6.016^2 + 5.342^2} = 8.04 \text{ A}$$

Phase angle
$$\phi_1 = \tan^{-1} \left[\frac{Y}{X} \right] = \tan^{-1} \left[\frac{5.342}{6.016} \right] = \underline{41.61}^{0}$$



PROBLEM No.15

A 400/200 V, Single phase transformer is supplying a load of 25A at a pf of 0.866 lag. On no load the current and pf are 2A and 0.208 respectively. Calculate the current taken from the supply.

Given data

$$E_1$$
 = 400 V, E_2 = 200 V, I_0 = 2 A, $Cos\ \varphi_0$ = 0.208, I_2 = 25 A, $Cos\ \varphi_2$ = 0.866

Solution

Primary current, $I_1 = \text{Vector sum of } I_0 \text{ and } I_2^{/}$

$$I_2^{\prime} = K \times I_2$$

$$K = E_2/E_1 = 200/400 = 0.5$$

$$I_2^{\prime} = \text{K x } I_2 = 0.5 \text{ x } 25 = 12.5 \text{ A}$$

Method I [Vector addition using parallelogram method]

$$I_1 = \sqrt{I_0^2 + I_2^{\prime 2} + 2 x I_0 x I_2^{\prime} x \cos \theta}$$

$$\theta = \phi_0 - \phi_2$$

$$\phi_0 = \cos^{-1}(\cos \phi_0) = \cos^{-1}(0.208) = 77.90^0 = 78^0$$

$$\phi_2 = \cos^{-1}(\cos\phi_2) = \cos^{-1}(0.866) = 30^0$$

$$\theta = \phi_0 - \phi_2 = 78^0 - 30^0 = 48^0$$

$$I_1 = \sqrt{I_0^2 + I_2^{\prime 2} + 2 x I_0 x I_2^{\prime} x \cos \theta}$$

$$I_1 = \sqrt{2^2 + 12.5^2 + 2 \times 2 \times 12.5 \times \cos 48} = 13.92 \text{ A}$$

Phase angle $\phi_1 = \phi_0 - \phi$

$$\Phi = \tan^{-1} \left[\frac{I_2' \sin \theta}{I_0 + I_2' \cos \theta} \right] = \tan^{-1} \left[\frac{12.5 \sin 48}{2 + 12.5 \cos 48} \right] = 41.86^0$$

Phase angle $\phi_1 = 78 - 41.86 = \underline{36.14}^0$

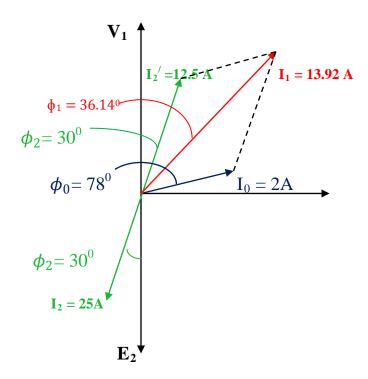
Method II [Vector addition using method of components]

X component = $I_0 \cos \phi_0 + I_2' \cos \phi_2 = 2 \times 0.208 + 12.5 \times 0.866 = 11.241$

Y component = $I_0 \sin \phi_0 + I_2^{\prime} \sin \phi_2 = 2 \times 0.98 + 12.5 \times 0.5 = 8.21$

$$I_1 = \sqrt{X^2 + Y^2} = \sqrt{11.241^2 + 8.21^2} =$$
13.92 A

Phase angle
$$\phi_1 = \tan^{-1} \left[\frac{Y}{X} \right] = \tan^{-1} \left[\frac{8.21}{11.241} \right] = \underline{36.14^0}$$



PROBLEM No.16

A 2200/200 V, Single phase transformer takes 1A on the HT side on no load at a power factor of 0.385 lagging. Calculate the iron losses. If a load of 50A at a pf of 0.8 lagging is taken from the secondary of the transformer, calculate the actual primary current and its power factor.

Given data

$$E_1$$
 =2200 V, E_2 = 200 V, I_0 = 1 A, $Cos \phi_0$ = 0.385, I_2 = 50 A, $Cos \phi_2$ = 0.8

Solution

Iron loss =
$$V_1I_0Cos \phi_0 = 2200 \times 1 \times 0.385 = 847 \text{ W}$$

Primary current, $I_1 = \text{Vector sum of } I_0 \text{ and } I_2^{\prime}$

$$I_2^{\prime} = K \times I_2$$

$$K = E_2/E_1 = 200/2200 = 0.09$$

$$I_2^{\prime} = \text{K x } I_2 = 0.09 \text{ x } 50 = 4.5 \text{ A}$$

Method I [Vector addition using parallelogram method]

$$I_1 = \sqrt{I_0^2 + I_2^{/2} + 2 x I_0 x I_2^{/2} x \cos \theta}$$

$$\theta = \phi_0 - \phi_2$$

$$\phi_0 = \cos^{-1}(\cos\phi_0) = \cos^{-1}(0.385) = 67.36^0$$

$$\phi_2 = \cos^{-1}(\cos\phi_2) = \cos^{-1}(0.8) = 36.87^0$$

$$\theta = \phi_0 - \phi_2 = 67.36^0 - 36.87^0 = 30.49^0$$

$$I_1 = \sqrt{I_0^2 + I_2'^2 + 2 x I_0 x I_2' x \cos \theta}$$

$$I_1 = \sqrt{1^2 + 4.5^2 + 2 \times 1 \times 4.5 \times \cos 30.49} = 5.39 \text{ A}$$

Phase angle $\phi_1 = \phi_0 - \phi$

$$\Phi = \tan^{-1} \left[\frac{l_2' \sin \theta}{l_0 + l_2' \cos \theta} \right] = \tan^{-1} \left[\frac{4.5 \sin 30.49}{1 + 4.5 \cos 30.49} \right] = 25.08^0$$

Phase angle $\phi_1 = 67.36 - 25.08 = 42.28^{\circ}$

Power factor $\cos \phi_1 = \cos 42.28^0 = \underline{\textbf{0.74 lag}}$

Method II [Vector addition using method of components]

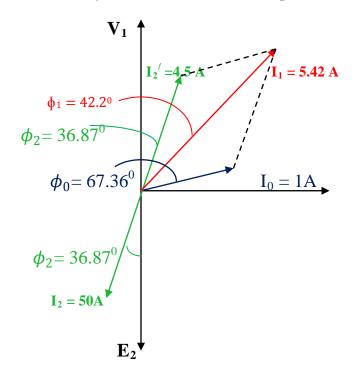
X component = $I_0 \cos \phi_0 + I_2^{\prime} \cos \phi_2 = 1 \times 0.385 + 4.545 \times 0.8 = 4.021$

Y component = $I_0 \sin \phi_0 + I_2^{\prime} \sin \phi_2 = 1 \times 0.92 + 4.545 \times 0.6 = 3.647$

$$I_1 = \sqrt{X^2 + Y^2} = \sqrt{4.021^2 + 3.647^2} =$$
5.42 A

Phase angle
$$\phi_1 = \tan^{-1} \left[\frac{Y}{X} \right] = \tan^{-1} \left[\frac{3.647}{4.021} \right] = \underline{42.20^0}$$

Power factor $\cos \phi_1 = \cos 42.20^0 = 0.74 \text{ lag}$



PROBLEM No.17

A Single phase transformer has a turn ratio of 8:1. The load current on secondary is 100 A at a pf of 0.78 lag and the corresponding primary current is 20A at a pf of 0.707A lag. Find the no load current and its power factor.

Given data

Transformer turn ratio = 8:1, I_1 = 20 A, $Cos\ \varphi_1$ = 0.707, I_2 = 100 A, $Cos\ \varphi_2$ = 0.78

Solution

$$K = 1/8 = 0.125$$

$$I_2^{\prime} = K \times I_2$$

$$I_2^{\prime} = \text{K x I}_2 = 0.125 \text{ x } 100 = 12.5 \text{ A}$$

$$\phi_2 = \cos^{-1}(\cos\phi_2) = \cos^{-1}(0.78) = 38.74^0$$

$$\phi_1 = \cos^{-1}(\cos\phi_1) = \cos^{-1}(0.707) = 45^0$$

X component of I_1 vector, $I_1 \cos \phi_1 = I_0 \cos \phi_0 + I_2^{\prime} \cos \phi_2$

X component of I_0 vector, $I_0 \cos \phi_0 = I_1 \cos \phi_1 - I_2^{\prime} \cos \phi_2$

$$= 20 \times 0.707 - 12.5 \times 0.78$$

$$=4.39$$

Y component of I_1 vector, $I_1 \sin \phi_1 = I_0 \sin \phi_0 + I_2 \sin \phi_2$

Y component of I_0 vector, $I_0 \sin \phi_0 = I_1 \sin \phi_1 - I_2^{\prime} \sin \phi_2$

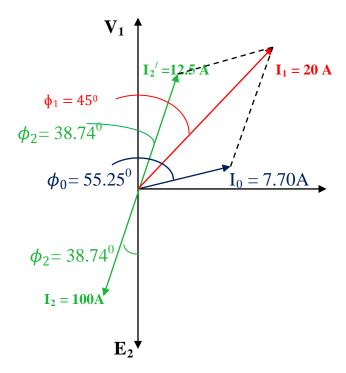
$$= 20 \times 0.707 - 12.5 \times 0.625$$

$$= 6.33$$

$$I_0 = \sqrt{X^2 + Y^2} = \sqrt{4.39^2 + 6.33^2} =$$
7.70 A

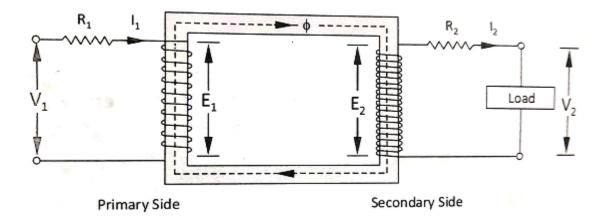
Phase angle
$$\phi_0 = \tan^{-1} \left[\frac{Y}{X} \right] = \tan^{-1} \left[\frac{6.33}{4.39} \right] = \underline{55.25^0}$$

Power factor $\cos \phi_0 = \cos 55.25^0 = \underline{0.57 \text{ lag}}$



Transformer with winding resistance

In preceding sections we considered an ideal transformer. But in actual transformer both primary and secondary windings have some resistance. This resistance causes some voltage drop and power loss in two windings.



Let R_1 = Primary winding resistance

 R_2 = Secondary winding resistance

 E_1 = Primary induced emf

 E_2 = Secondary induced emf

 V_1 = Primary supply voltage

 V_2 = Secondary terminal voltage

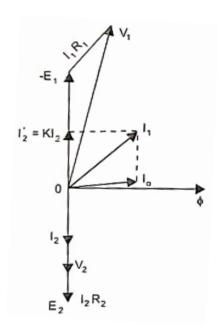
 $V_2 = E_2 - I_2 R_2$ (Vector difference)

 $E_1 = V_1 - I_1 R_1$ (Vector difference)

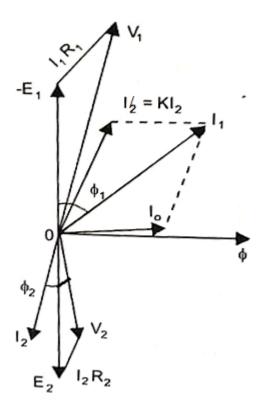
 I_1R_1 = Primary voltage drop due to primary winding resistance

 I_2R_2 = Secondary voltage drop due to secondary winding resistance

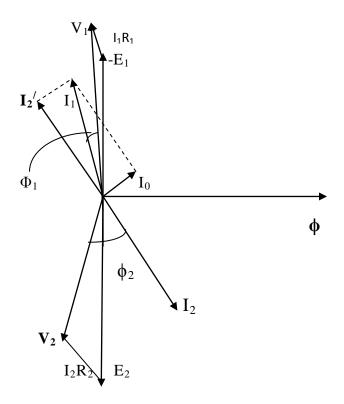
Vector diagram of transformer on resistive load (with winding resistance)



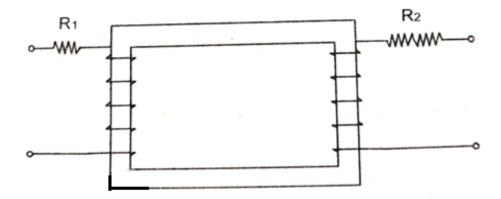
Vector diagram of transformer on inductive load(with winding resistance)



Vector diagram of transformer on capacitive load(with winding resistance)



Equivalent Resistance



The primary winding resistance R₁ and secondary winding resistance R₂ is shown externally to the winding. Which can be transferred from one side to other side and vice-versa, it makes calculations simple and easy.

 R_1' = Primary winding resistance referred to secondary Let

 R_2' = Secondary winding resistance referred to primary

 R_{01} = Equivalent resistance of the transformer referred to primary

 $R_{02} = Equivalent$ resistance of the transformer referred to secondary

The transfer of resistance from one side to other side on the basis of 'equal power loss'.

Then

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

$$R_2' = (\frac{I_2}{I_1})^2 R_2$$
 Since $\frac{I_1}{I_2} = K$, $\frac{I_2}{I_1} = \frac{1}{K}$

$$R_2' = \frac{R_2}{K^2}$$

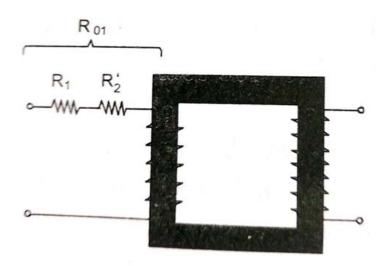
Similarly $I_1^2 R_1 = I_2^2 R_1^{'}$

$$R_1' = \left(\frac{I_1}{I_2}\right)^2 R_1$$
 Since $\frac{I_1}{I_2} = K$

$$R_1' = \mathbf{K}^2 \mathbf{R}_1$$

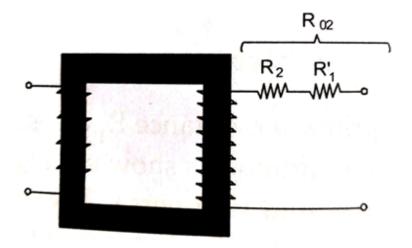
Equivalent Resistance of transformer referred to primary $[R_{01}]$

$$R_{01} = R_1 + R_2^{\ \ \prime}$$

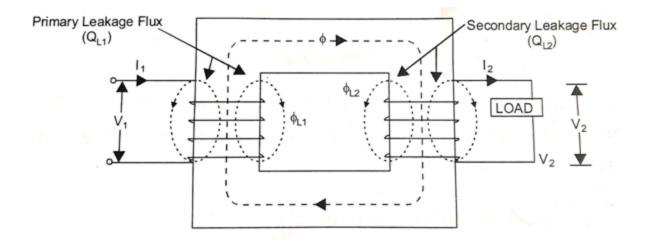


Equivalent Resistance of transformer referred to secondary $[R_{02}]$

$$R_{02} = R_2 + {R_1}'$$



Transformer with Magnetic Leakage



The total flux created in the transformer does not link the primary and secondary but it is divided into three parts

- 1) The main flux ϕ linking both primary and secondary windings
- 2) Primary leakage flux ϕ_{L1} linking with primary winding only
- 3) Secondary leakage flux ϕ_{L2} linking with secondary winding only

Let $L_1 = Self$ inductance of the primary winding produced by primary leakage $\label{eq:local_local_local} flux \; \varphi_{L1}$

Primary leakage reactance $X_1 = 2\pi f L_1$

Similarly $L_2 = \text{Self inductance of the secondary winding produced by secondary}$ $\text{leakage flux } \phi_{L2}$

Secondary leakage reactance $X_2 = 2\pi f L_2$

Equivalent Reactance

A transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected externally in both primary and secondary circuits.

This leakage flux causes some voltage drop. Which can be transferred from one side to other side and vice-versa, it makes calculations simple and easy.



 X_1' = Primary leakage reactance referred to secondary

 X_2' = Secondary leakage reactance referred to primary

 X_{01} = Equivalent leakage reactance of the transformer referred to primary

 X_{02} = Equivalent leakage reactance of the transformer referred to secondary

The transfer of reactance from one side to other side on the basis of 'equal percentage drop'.

Then

$$\frac{I_2 X_2}{V_2} \times 100 = \frac{I_1 X_2'}{V_1} \times 100$$

$$X_2' = \frac{V_1}{V_2} \times \frac{I_2}{I_1} \times X_2$$

Since
$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$
, $\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$

$$X_2' = \frac{X_2}{K^2}$$

Similarly
$$\frac{I_1 X_1}{V_1} \times 100 = \frac{I_2 X_1'}{V_2} \times 100$$

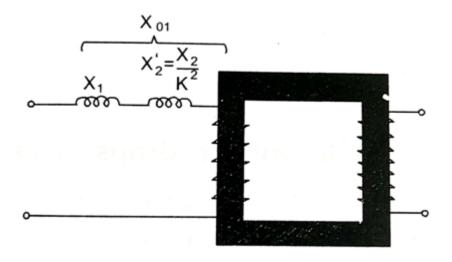
$$\mathbf{X_1}' = \frac{\mathbf{V_2}}{\mathbf{V_1}} \mathbf{x} \frac{\mathbf{I_1}}{\mathbf{I_2}} \mathbf{x} \mathbf{X_2}$$

$$X_1' = \mathbf{K}^2 \mathbf{X_1}$$

Since
$$\frac{V_2}{V_1} = \frac{I_1}{I_2} =$$

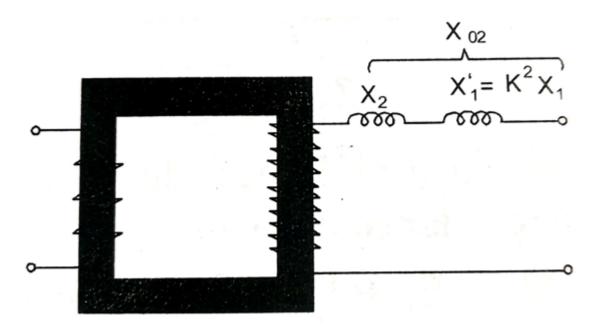
Equivalent leakage reactance of transformer referred to primary $[X_{01}]$

$$X_{01} = X_1 + X_2'$$

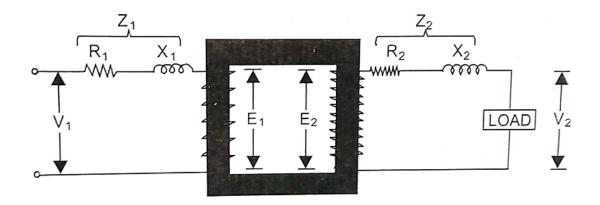


Equivalent leakage reactance of transformer referred to secondary $[X_{02}]$

$$X_{02} = X_2 + {X_1}'$$



Transformer with resistance and leakage reactance



Primary impedance, $Z_1 = \sqrt{R_1^2 + X_1^2} = R_1 + jX_1$

Secondary impedance,
$$Z_2 = \sqrt{R_2^2 + X_2^2} = R_2 + jX_2$$

The primary and secondary impedance cause some voltage drops in each winding.

Hence
$$V_1 = E_1 + I_1Z_1 = E_1 + I_1 (R_1 + jX_1)$$

$$E_2 = V_2 + I_2Z_2 = V_2 + I_2 (R_2 + jX_2)$$

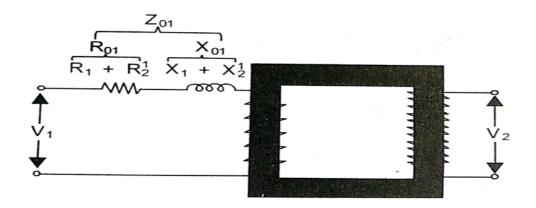
Equivalent impedance of transformer referred to primary $[Z_{\underline{01}}]$

 R_{01} = Equivalent resistance of the transformer referred to primary = $R_1 + R_2$

 X_{01} = Equivalent leakage reactance of the transformer referred to primary= $X_1 + X_2$

 Z_{01} = Equivalent impedance of transformer referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$



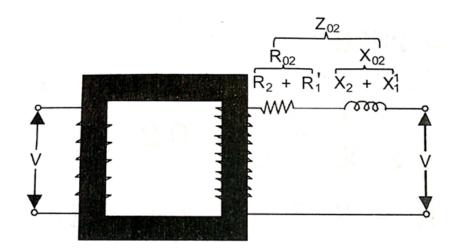
Equivalent impedance of transformer referred to secondary [Z₀₂]

 R_{02} = Equivalent resistance of the transformer referred to secondary = $R_2 + R_1^{\prime}$

 X_{02} =Equivalent leakage reactance of the transformer referred to secondary= $X_2 + {X_1}^\prime$

 Z_{02} = Equivalent impedance of transformer referred to secondary

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$



Steps for draw the Vector diagram of transformer with winding resistance and leakage reactances

Draw E2 vector along with the negative Y axis

Draw - E_1 vector in opposite to E_2

Draw Flux ϕ vector along the positive X axis

Draw V₂ and I₂ vector, according to the nature of load

Draw I₂R₂ Vector in phase or parallel with I₂ Vector from V₂ vector

Draw I₂X₂ Vector perpendicular to I₂R₂ Vector

Vector from V₂ to E₂ represents I₂Z₂ Vector

Draw I_0 Vector by an angle ϕ_0 from ϕ axis

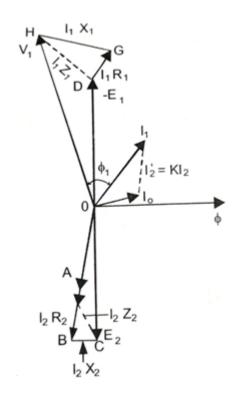
Draw $I_2^{\ /}$ Vector in parallel to I_2 Vector from I_0 vector and gives I_1 vector

Draw I₁R₁ Vector in parallel with I₁ Vector from E₁ vector

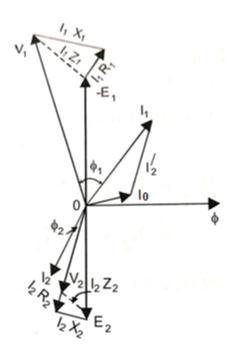
Draw I_1X_1 Vector perpendicular to I_1R_1 Vector and gives V_1 vector

Vector from V_1 to E_1 represents I_1Z_1 Vector

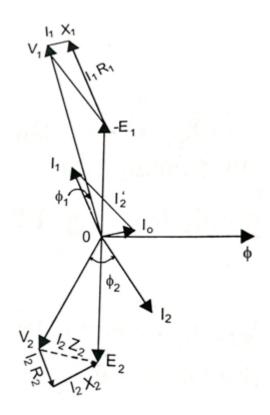
<u>Vector diagram of transformer on resistive load (with winding resistance and leakage reactances</u>



<u>Vector diagram of transformer on inductive load(with winding resistance and leakage reactances</u>



Vector diagram of transformer on capacitive load



PROBLEM No.18

A 10 kVA Single phase transformer 2000/400V at no load has $R_1 = 5.5 \Omega$, $X_1 = 12 \Omega$ and $R_2 = 0.2 \Omega$, $X_2 = 0.45 \Omega$. Determine (i) Total resistance referred to secondary (ii) Total reactance referred to secondary

Given data

$$V_1 = 2000$$
, $V_2 = 400$, $R_1 = 5.5 \Omega$, $X_1 = 12 \Omega$, $R_2 = 0.2 \Omega$, $X_2 = 0.45 \Omega$.

Solution

(i) Total resistance of the transformer referred to secondary $R_{02}=R_2+{R_1}^\prime$

$$\mathbf{R}_1 = \mathbf{K}^2 \mathbf{R}_1$$

$$K = \frac{V_2}{V_1} = \frac{400}{2000} = 0.2$$

$$R_1 = K^2 R_1 = 0.2^2 \text{ x } 5.5 = 0.22 \Omega$$

$$R_{02} = R_2 + R_1^{\ /} = 0.2 + 0.22 =$$
0.42 Ω

(ii) Total reactance of the transformer referred to secondary $X_{02} = X_2 + {X_1}'$

$$X_1 = K^2 X_1 = 0.2^2 \text{ x } 12 = 0.48 \Omega$$

$$X_{02} = X_2 + X_1^{\ /} = 0.45 + 0.48 = \underline{\textbf{0.93}\ \Omega}$$

PROBLEM No.19

A 500 kVA, 3300/230V Single phase transformer has $R_1 = 3.45 \Omega$, $X_1 = 5.4 \Omega$ and $R_2 = 0.0085 \Omega$, $X_2 = 0.034 \Omega$. Determine (i) Total impedance referred to primary and secondary (ii) copper loss

Given data

$$V_1 = 3300, \ V_2 = 230, \ R_1 = 3.45 \ \Omega, \ \ X_1 = 5.4 \ \Omega, \ R_2 = 0.0085 \ \Omega, \ \ X_2 = 0.034 \ \Omega.$$

Solution

(i) Total impedance of the transformer referred to primary $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

$$R_{01} = R_1 + {R_2}'$$

$$X_{01} = X_1 + X_2'$$

$$R_2 = R_2 / K^2$$

$$K = \frac{V_2}{V_1} = \frac{230}{3300} = 0.0697$$

$$R_2^{\prime} = R_2 / K^2 = 0.0085 / 0.0697^2 = 1.75 \Omega$$

$$R_{01} = R_1 + R_2' = 3.45 + 1.75 =$$
5.2 Ω

$$X_{01} = X_1 + X_2'$$

$$X_2 = X_2 / K^2 = 0.034 / 0.0697^2 = 7 \Omega$$

$$X_{01} = X_1 + X_2^{\ /} = 5.4 + 7 = 12.4 \ \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{5.2^2 + 12.4^2} = \underline{\mathbf{13.45}\ \Omega}$$

Total impedance of the transformer referred to secondary $Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$

$$R_{02} = R_2 + {R_1}'$$

$$X_{02} = X_2 + {X_1}'$$

$$R_1 = K^2 R_1 = 0.0697^2 \text{ x } 3.45 = 0.017 \Omega$$

$$R_{02} = R_2 + {R_1}' = 0.0085 + 0.017 = \textbf{0.0255} \ \boldsymbol{\Omega}$$

$$X_{02} = X_2 + {X_1}'$$

$$X_1' = \mathbf{K}^2 \mathbf{X_1} = 0.0697^2 \text{ x } 5.4 = 0.0262 \Omega$$

$$X_{02} = X_2 + X_1^{\ /} = 0.034 + 0.0262 =$$
0.0602 Ω

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0255^2 + 0.0602^2} = 0.0653 \ \Omega$$

(ii) copper loss =
$$I_1^2 \times R_{01}$$

$$I_1 = \frac{\textit{Rating of the transformer}}{\textit{Primary voltage}} = \frac{500 \text{ x } 1000}{3300} = 151.515 \text{ A}$$

copper loss =
$$I_1^2 \times R_{01} = 151.515^2 \times 5.2 = \mathbf{119375 W}$$

PROBLEM No.20

A 30 kVA, 2400/120V, 50Hz, Single phase transformer has a high voltage winding resistance of 0.1Ω and a leakage reactance of $0.22~\Omega$. The low voltage winding resistance is $0.035~\Omega$ and the leakage reactance is $0.012~\Omega$. Find the equivalent winding resistance, reactance and impedance referred to high voltage side and low voltage side.

Given data

$$V_1 = 2400, \ V_2 = 120, \ R_1 = 0.1 \ \Omega, \ \ X_1 = 0.22 \ \Omega, \ R_2 = 0.035 \ \Omega, \ \ X_2 = 0.012 \ \Omega.$$

Solution

Here high voltage side is primary and low voltage side is secondary

Equivalent resistance of the transformer referred to primary $R_{01} = R_1 + R_2^{1}$

Equivalent reactance of the transformer referred to primary $X_{01} = X_1 + X_2^{\prime}$

Equivalent impedance of the transformer referred to primary $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

$$R_{01} = R_1 + R_2'$$

$$R_2' = R_2 / K^2$$

$$K = \frac{V_2}{V_1} = \frac{120}{2400} = 0.05$$

$$R_2 = R_2 / K^2 = 0.035 / 0.05^2 = 14 \Omega$$

$$R_{01} = R_1 + R_2' = 0.1 + 14 = \underline{14.1 \Omega}$$

$$X_{01} = X_1 + X_2'$$

$$X_2 = X_2 / K^2 = 0.012 / 0.05^2 = 4.8 \Omega$$

$$X_{01} = X_1 + X_2^{\prime} = 0.22 + 4.8 = 5.02 \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = 15 \Omega$$

Equivalent resistance of the transformer referred to secondary $R_{02} = R_2 + R_1^{\ \ \prime}$

Equivalent reactance of the transformer referred to secondary $X_{02} = X_2 + X_1^{\prime}$

Equivalent impedance of the transformer referred to secondary $Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$

$$R_{02} = R_2 + {R_1}'$$

$$R_1 = K^2 R_1 = 0.05^2 \text{ x } 0.1 = 0.00025 \Omega$$

$$R_{02} = R_2 + R_1^{\ /} = 0.035 + 0.00025 = \underline{\textbf{0.03525}} \ \Omega$$

$$X_{02} = X_2 + {X_1}'$$

$$X_1' = \mathbf{K}^2 \mathbf{X_1} = 0.05^2 \times 0.22 = 0.00055 \Omega$$

$$X_{02} = X_2 + X_1^{\ /} = 0.012 + 0.00055 =$$
0.01255 Ω

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.03525^2 + 0.01255^2} = \underline{0.0374} \ \Omega$$