Design of Machine Elements-5021

Module- 4

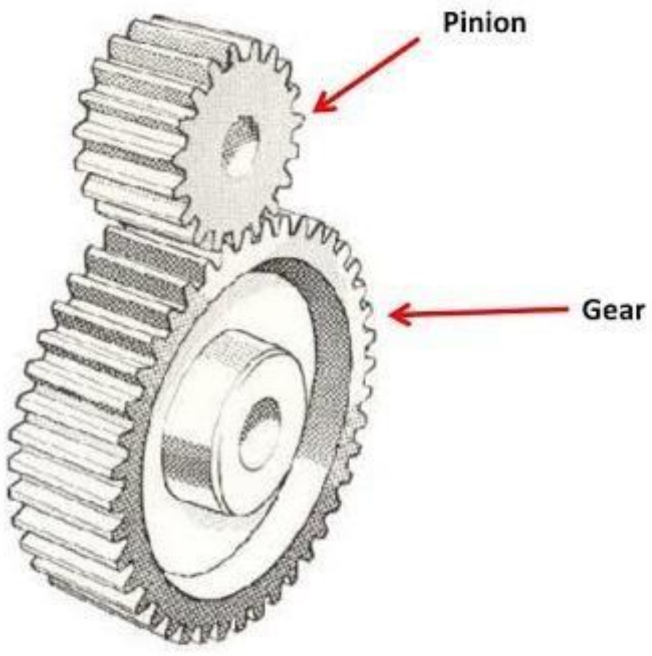
Gears and Gear Trains (Part 2)

SYLLABUS

Gear Drives – Spur gear terminology; Types of gears and gear trains, their selection for different applications; Train value & Velocity ratio for the compound, reverted and simple epicyclic gear train (Simple problems on simple and compound gear train to find the size, teeth and speed of gears)

Gears and Gear drive

- A gear is a wheel having teeth on its periphery.
- It is used to transmit power and it is used in many places like differential, gear boxes, metal cutting machine tools, marine engines, etc.



- When two gears are meshed together, the smaller gear is called a pinion.
- The gear transmitting force is referred to as a drive gear, and the receiving gear is called the driven gear or follower

Advantages of gear drive

1. They give positive drives and constant speed ratio without any slippage.
2. The drive is more compact
3. Can be operated at higher speeds.
4. High efficiency
5. Reliable in service and simple operation.
6. Lighter loads on the shafts and bearings.
7. Wide range of power transmitted.
8. Maintenance is inexpensive.
9. Can be used for non intersecting and non parallel shafts.

Disadvantages of gear drives

- The manufacture of gears require special tools and equipment.
- Not suitable for large center distance shafts.
- The error in cutting teeth may cause vibrations and noise during operation.
- Requires more attention to lubrication.

Classification of gears

The gears or toothed wheels may be classified as follows :

1. According to the position of axes of the shafts.
2. According to the peripheral velocity of the gears
3. According to the type of gearing
4. According to position of teeth on the gear surface

1.According to the position of axes of the shafts.

The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel
(b) Intersecting, and
(c) Non-intersecting and non-parallel.

- Spur, Helical and double helical gear(herringbone gear) are used for connecting parallel shaft
- Bevel gear is used for connecting intersecting shaft
- Worm gear, spiral gear is used for connecting non-parallel non-intersecting shaft


spur gears
(external toothing)
rack
internal toothing
screw gears
(hyperboloid gears)
worm gear
worm
bevel gears
screw bevel gears
(hypoid gears)

2. According to the peripheral velocity of the gears.

According to the peripheral velocity of the gears may be classified as :

(a) Low velocity, (b) Medium velocity, and (c) High velocity.

3. According to the type of gearing.

(a) External gearing, (b) Internal gearing, and (c) Rack and pinion.

4. According to position of teeth on the gear surface.

The teeth on the gear surface may be

(a) straight, (b) inclined, and (c) curved.

INTERNAL GEARING

EXTERNAL GEARING

Some of the important gears used for power transmission are:-

1. Spur gears
2. Helical gears
3. Herringbone gears.(Double Helical gears)
4. Bevel gears.
5. Worm gears.

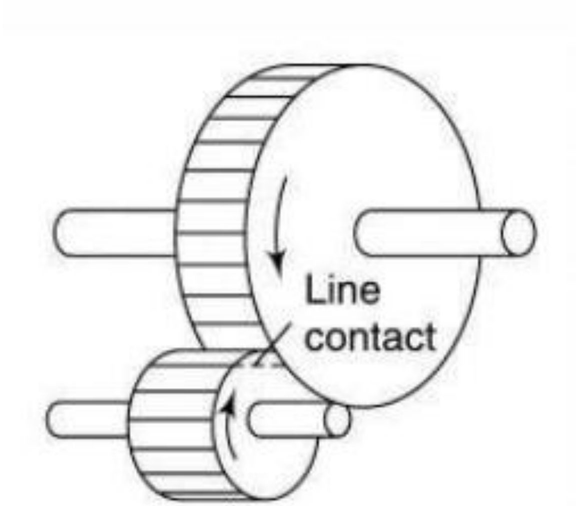
1. Spur gears

- Spur gears are the simplest gear among all types of gears.
- A Spur gear is a cylindrical gear where teeth are straight and teeth are cut parallel to the axis of rotation of the shafts.
- These are used to transmit power between parallel shafts.

Drawback of Spur gear

- Sudden application of load result in high impact stress
- Excessive noise at high speed

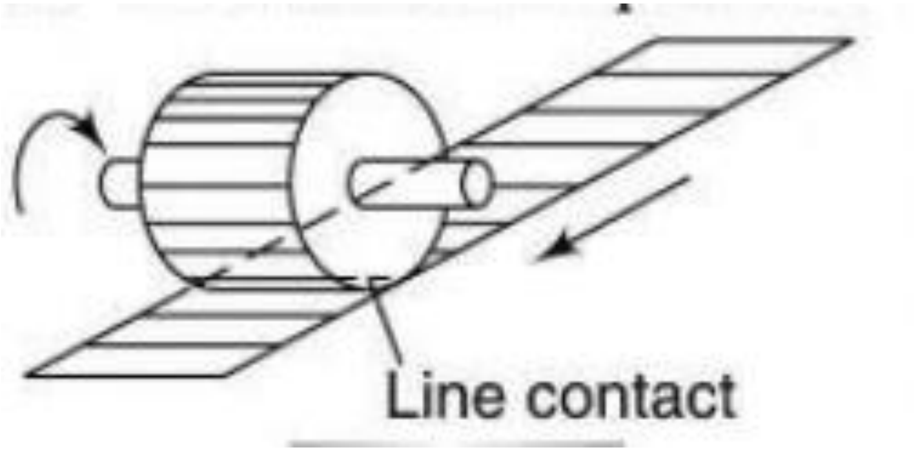
Spur gear



Spur Rack and pinion

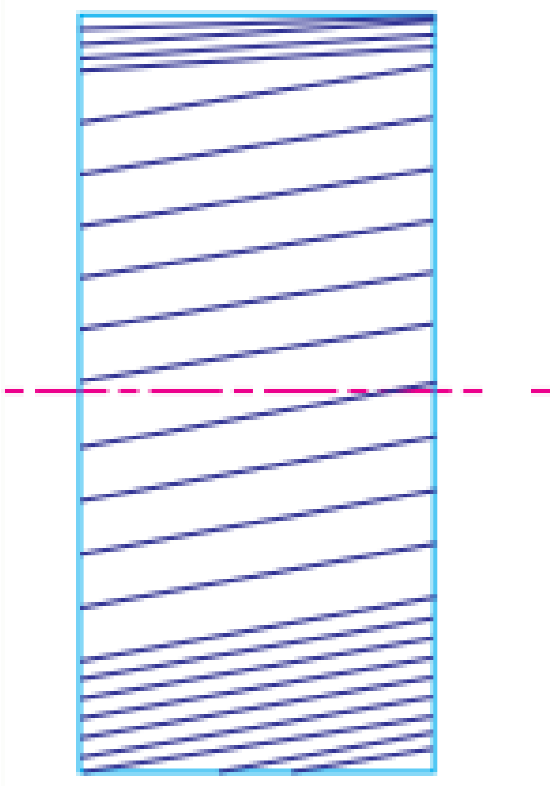
- Spur rack and pinion is a special case of spur gear where gear is made of infinite diameter
- Spur rack and pinion combination converts rotary motion into translatory motion or vice-versa
- It is used in lathe in which rack transmits motion to the saddle

Spur rack and pinion gear



2. Helical gears

- This is similar to spur gears but the teeths cut on the periphery is inclined to the axis of rotation of the shafts.
- Helical gears can be used for the same application as spur gears.
- These gears are not noisy because of the more gradual engagement if teeth during working.
- Some times helical gears are used to transmit power between non parallel shafts.



(a) Single helical gear.

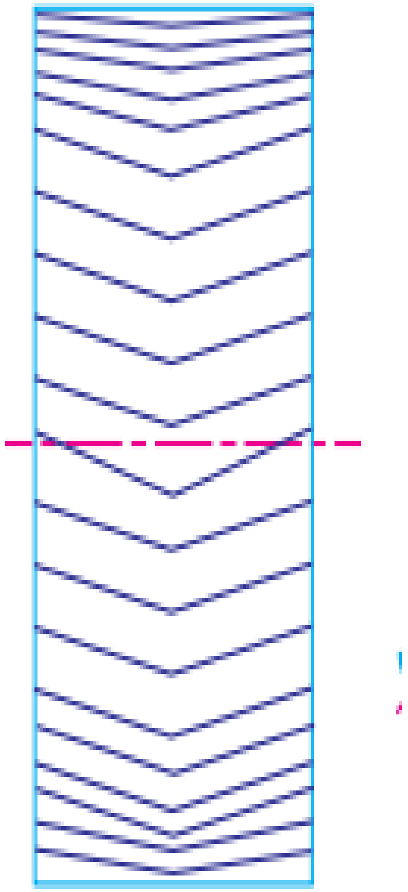
Drawback of Helical Gear

- Presence of end thrust load

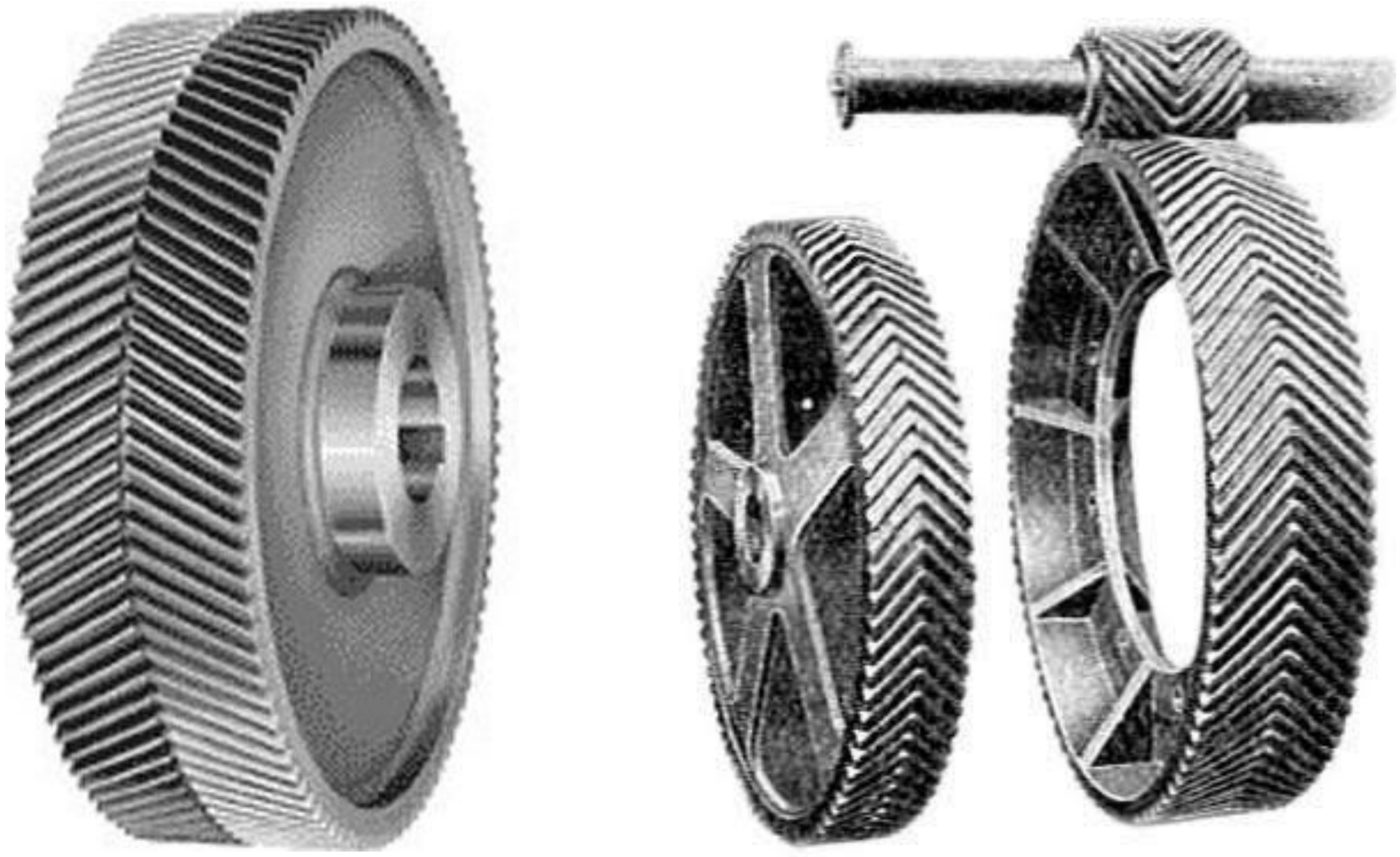
HELICAL GEAR

3. Herringbone gears

- The double helical gears are called herringbone gears.
- This type have teeth on the part of the face width as a right hand helix and the other a left hand helix, with or without a gap between them.
- Axial thrust which occurs in case of single helical gears is eliminated in double helical gears.
- This is because the axial thrusts of the two rows of teeth cancel each other.
- These gears are run at high speeds with less noise and vibrations.



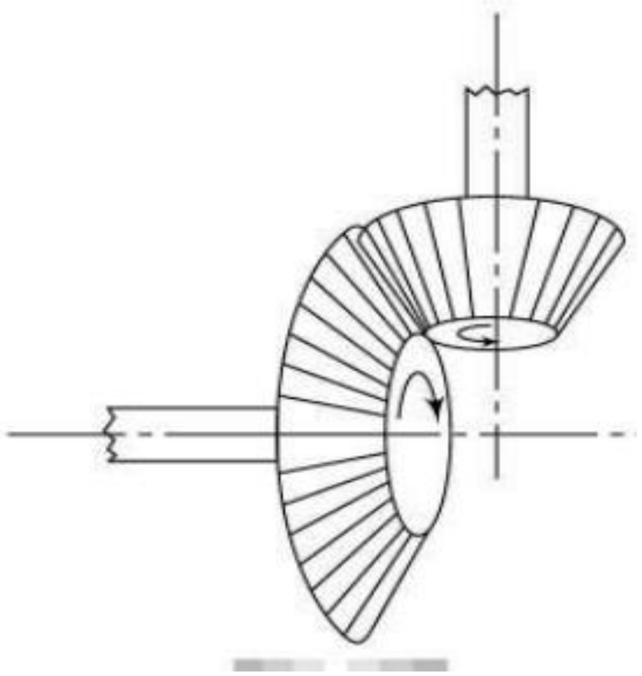
Double helical gear.



DOUBLE HELICAL GEAR

4. Bevel gears

- In bevel gears, the teeths are cut on conical surface and are used for transmitting power between intersecting shafts.
- If the connecting shafts are right angles to each other it is known as ***mitre gears.***

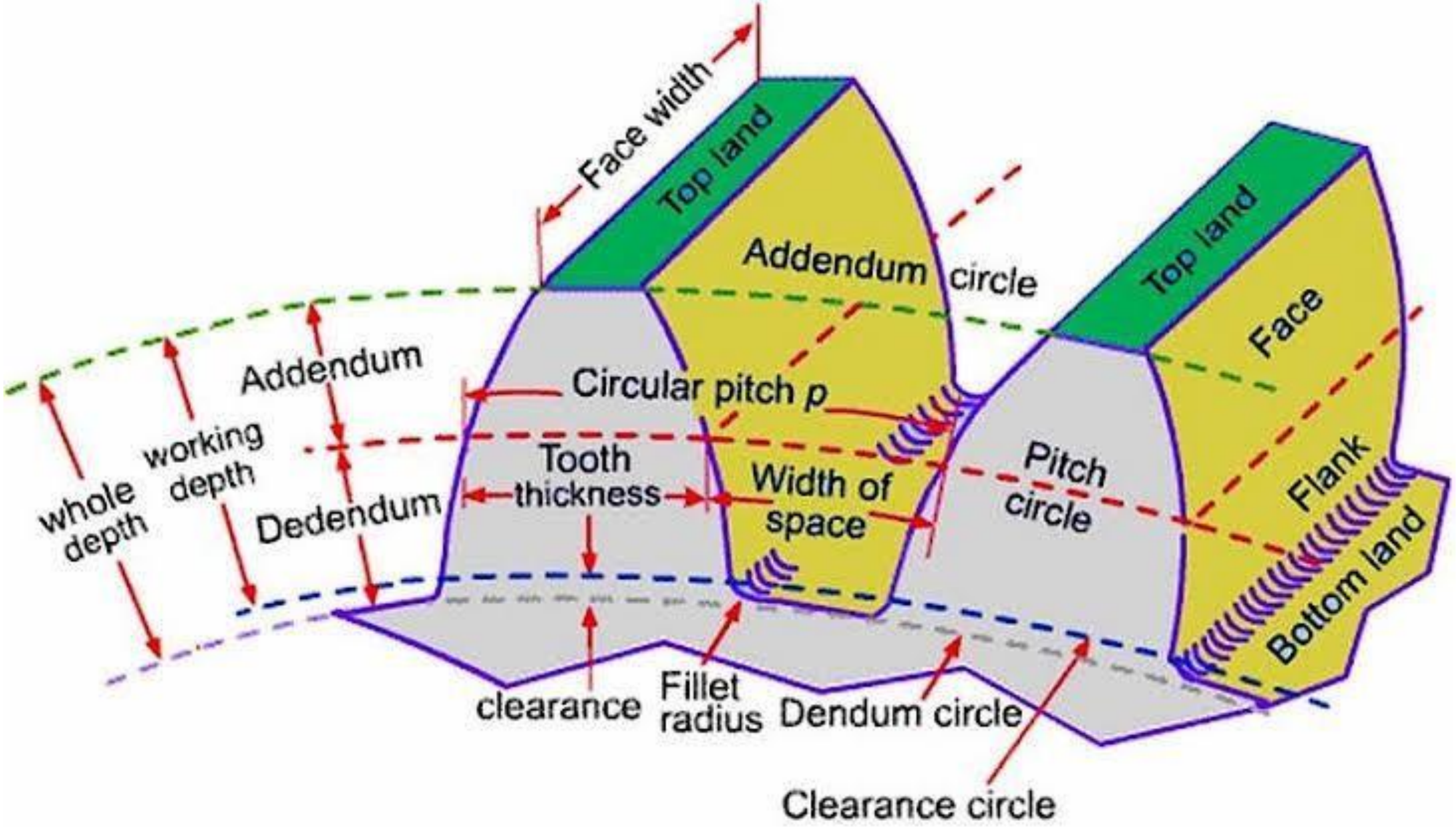


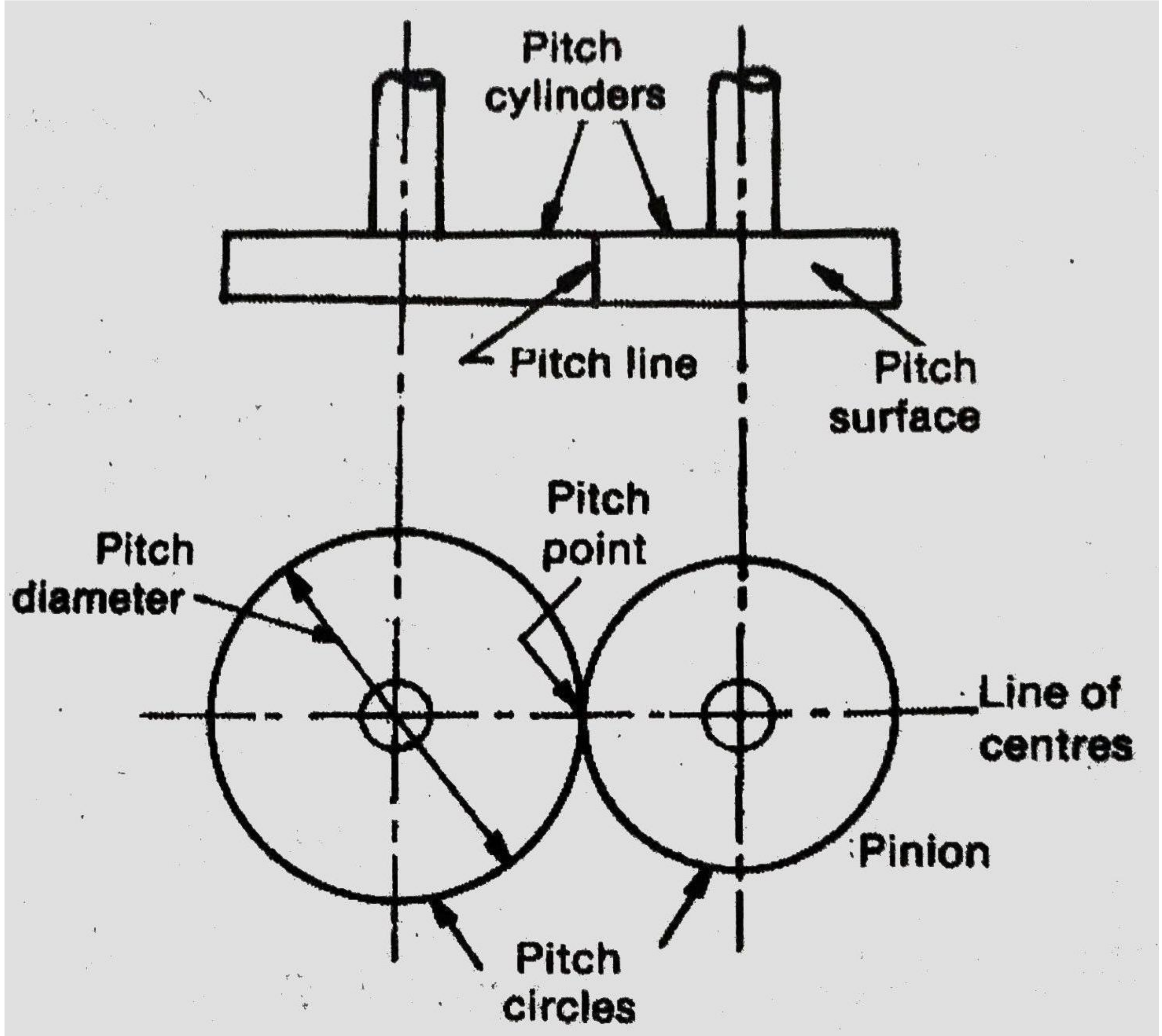
5. Worm gears

- In this gears, the worm resembles as a screw.
- A worm gear is a gear consisting of a shaft with a spiral thread that engages with and drives a toothed wheel.
- The worm wheel rotate in straight or in any angular or other direction also.
- Generally the worm gears are used for changing the direction of two shafts in cross ways or on inclined position.

WORM GEAR

Gear nomenclature




Pitch cylinders
Pitch line
Pitch surface
Pitch point
Pitch diameter
Line of centres
Pinion
Pitch circles

1. **Pitch cylinders**

- Pitch cylinders of a pair of gears in mesh are the imaginary friction cylinders which by pure rolling together to transmit the same motion as the pair of gears.

2. **Pitch circle**

- It is an imaginary circle with radius equal to the radius of pitch cylinder.

3. **Pitch circle diameter**

- It is the diameter of the imaginary pitch circle.

4. **Pitch point**

- It is the point of the contact of two pitch circles of meshing gears.

5. Circular pitch(p_c)

- This is the distance between the corresponding points if successive teeth measured along the circumference of the pitch circle.
- The circular pitch is equal to the sum of the tooth thickness and width of space.
- It is denoted by p_c and, $p_c = \pi d/T$

Where d – Pitch circle diameter

T – Number of teeth.

6. Diametral pitch(p_d)

- It is the number of teeth per unit length of the pitch circle diameter in mm.
- It is dedenoted by p_d, and $p_d = T/d$

7. Module

- It is the ratio of pitch circle diameter in mm to the number of teeth.
- It is denoted by m and is given by, $m = d/T = 1/p_d$.
- Module and circular pitch must be the same for the gears in mesh.
- Also it can be seen that,

$$p_c p_d = \frac{\pi d}{T} \times \frac{T}{d} = \pi$$

And

$$p_c = \pi \frac{d}{T} = \pi m$$

8. Addendum circle

- The circle passing through the tips of teeth.

9. Dedendum circle

- It is the circle passing through the roots of teeth.

10. Addendum

- The radial distance between the pitch circle to the addendum circle is called addendum.

11. Dedendum.

- The radial distance between the pitch circle to the dedendum circle is called dedendum.

Velocity ratio of simple gear drive

- The velocity ratio of a gear drive is defined as the ratio of the angular velocity of the driver gear to the driven gear. $V.R = N_1/N_2$
- The linear velocity of meshing gears will be same. ie:- $V_1 = V_2$

$$\pi d_1 N_1 = \pi d_2 N_2$$

Then, $N_1/N_2 = d_2/d_1$

Since the circular pitch of each gear must be same for correct meshing.

So $p_{c1} = p_{c2}$

$\pi d_1/T_1 = \pi d_2/T_2$. ie:- $d_1/T_1 = d_2/T_2$. or $d_2/d_1 = T_2/T_1$

So velocity ratio becomes, $\mathbf{V.R = N_1/N_2 = d_2/d_1 = T_2/T_1}$

Example 10.1 : A toothed wheel has 108 teeth. Its module is 1.25 mm. Find the circular pitch, pitch diameter and the diametral pitch. (***November* 2002**)

***Solution*:**

Given :

Number of teeth, $T = 108$

Module of the toothed wheel, $m = 1.25$ mm

***Analysis*:**

Using the relation for circular pitch .

$$p_c = \frac{\pi d}{T} \quad = \pi m \quad = \pi \times 1.25 \quad = 3.93 \text{ mm}$$

Using the relation for module i.e., $m = \dfrac{d}{T}$

$\therefore$ Pitch diameter, $d = mT \quad = 1.25 \times 108 \quad = 135$ mm

Using the relation for diametral pitch i.e.,

$$p_d = \frac{T}{d} \quad = \frac{108}{135} \quad = 0.8 \text{ tooth/mm}$$

Example 10.3 : A gear wheel of 48 teeth has a pitch circle diameter of 367 mm. Find the module, diametral pitch and the circular pitch.

Solution :

Given :

Number of teeth, $T = 48$

Pitch circle diameter, $d = 367$ mm.

Analysis :

Using the relation for module of the gear wheel.

$$m = \frac{d}{T} = \frac{367}{48} = 7.65 \text{ mm/tooth} \text{ or simply } 7.65 \text{ mm}$$

Using the relation for diametral pitch of the gear wheel

$$p_d = \frac{T}{d} = \frac{48}{367} = 0.13 \text{ tooth/mm}$$

Using the relation for circular pitch of the gear wheel

$$p_c = \frac{\pi d}{T} = \frac{\pi \times 367}{48} = 24.02 \text{ mm}$$

Example 10.6 : Two spur gears have a velocity ratio $\frac{30}{13}$. The driven gear has 60 teeth of 5 mm module and rotates at 390 rpm. Calculate the number of teeth and the speed of the driver. What will be the pitch circle diameter of gears and the pitch line velocity? (**October 2009**)

Solution :

Given :

$$\text{Velocity ratio}, VR = \frac{30}{13}$$

$$\text{Number of teeth on driven gear}, T_2 = 60$$

$$\text{Module of the gears}, \; m = 5 \text{ mm}$$

$$\text{Speed of the driven gear}, \; N_2 = 390 \text{ rpm}$$

Using the expression for velocity ratio of gear drive

$$VR = \frac{N_1}{N_2} = \frac{T_2}{T_1} = \frac{30}{13}$$

Using $\frac{N_1}{N_2} = \frac{30}{13}$

Speed of the driver gear, $N_1 = N_2 \times \frac{30}{13} = 390 \times \frac{30}{13} = 900$ rpm

Using $\frac{T_2}{T_1} = \frac{30}{13}$ or $T_1 = \frac{13}{30} \times T_2$

Number of teeth on driver gear, $T_1 = \frac{13}{30} \times T_2 = \frac{13}{30} \times 60 = 26$

Using the relation for module of the gear in mesh.

Module, $m = \frac{d_1}{T_1} = \frac{d_2}{T_2}$

Pitch circle diameter, $d_1 = mT_1 = 5 \times 26 = 130$ mm

Pitch line velocity, $v = \omega_1 r_1 = \pi d_1 N_1 = \pi \times 130 \times 900$

$$= 367566.34 \text{ mm / min} = \frac{367566.34 \times 10^{-3}}{60} = 6.13 \text{ m/s}$$

Example 10.11: Two parallel shafts, about 600 mm apart, are to be connected by spur wheels. One shaft is to run at 120 rpm and the other at 360 rpm. Design the wheels, if the diametral pitch of the teeth is to be 0.25 mm (***March* 2007, *November* 2005, *April* 2001**)

Solution **:**

Given :

Distance between parallel shafts, $C = 600$ mm

Speed of first gear, $N_1 = 120$ rpm

Speed of second gear, $N_2 = 360$ rpm

Diametral pitch of gear, $p_d = 0.25$ mm

Using the relation for velocity ratio of gear drive

$$\frac{d_2}{d_1} = \frac{N_1}{N_2} = \frac{120}{360} = \frac{1}{3} \quad \text{or} \quad d_1 = 3d_2 .$$

Using the expression for the centre distance between gears.

$$C = \frac{d_1 + d_2}{2} \quad \text{Or,} \quad d_1 + d_2 = 2C = 2 \times 600 = 1200$$

From equation (*i*) and (*ii*)

Pitch circle diameter of first gear, $d_1 = 900$ mm

Pitch circle diameter of second gear, $d_2 = 300$ mm

Using the relation for diametral pitch of mating gears.

$$p_d = \frac{T_1}{d_1} = \frac{T_2}{d_2} = 0.25$$

Number of teeth on first gear, $T_1 = 0.25 d_1 = 0.25 \times 900 = 225$

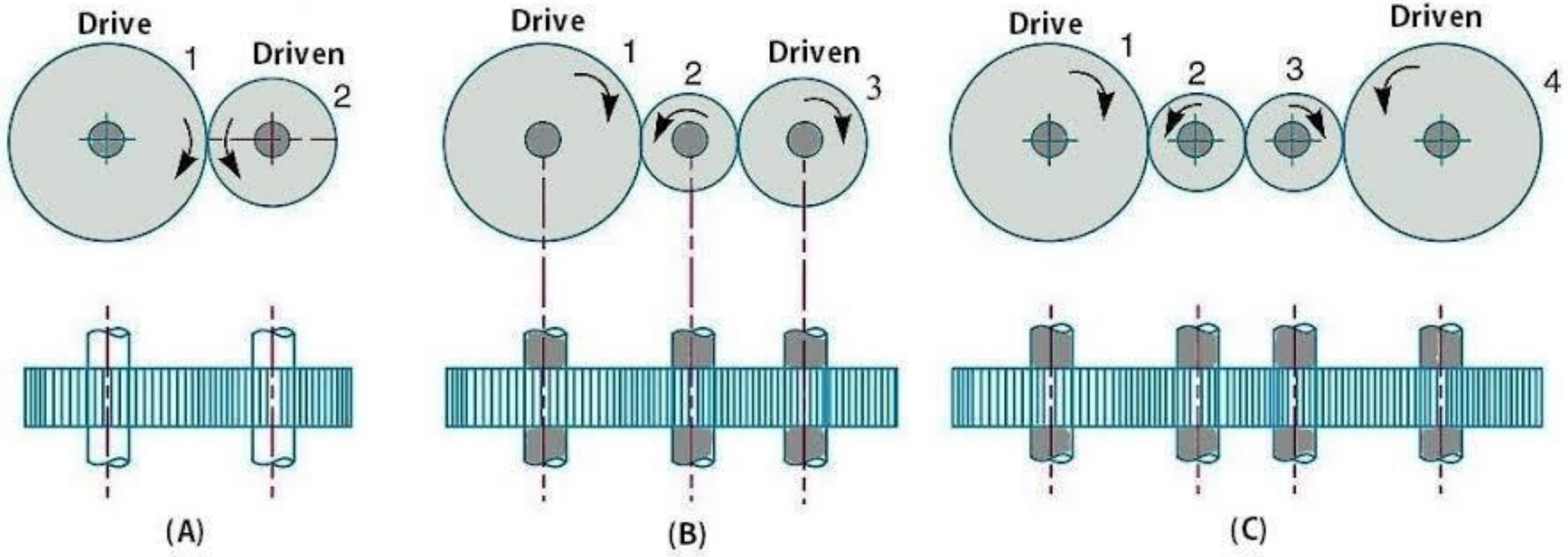
Number of teeth on second gear, $T_2 = 0.25 d_2 = 0.25 \times 300 = 75$

Gear trains

- When two or more gears in mesh is used to transmit motion or power from one shaft to another which are at a considerable distance apart is called gear train.
- Following are the different gear trains.

1. Simple gear train.
2. Compound gear train.
3. Reverted gear train.
4. Epicyclic gear train.

1. Simple gear train



1. Simple gear train

- In this type, each shaft carries only one gear.
- The odd numbered simple gear train, the driver and driven gears rotates in one direction and all even numbered simple gear train, the driver and driven gears rotates in the opposite direction.
- Intermediate gears are called idle gears(no effect on velocity ratio).

$$\text{Train value} = \frac{\text{Speed of follower}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

$$\text{Speed ratio} = \frac{1}{\text{Train value}}$$

Compound gear train

- In this type, each intermediate gear shaft carries two gears which are fastened together rigidly.
- If the number of intermediate compound gear shaft is even then the driver and driven is rotated in opposite directions.
- Similerly, if the number of intermediate compound gear shaft is odd then the driven is rotated in same direction of driver.
- In this type, the velocity ratio is depends upon intermediate gears.
- The main advantage of compound gear train is that it can provide a larger velocity ratio in a limited space and the drive is compact.

$$\text{Train value} = \frac{\text{Speed of follower}}{\text{Speed of driver}} = \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}} \quad \frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4}$$

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

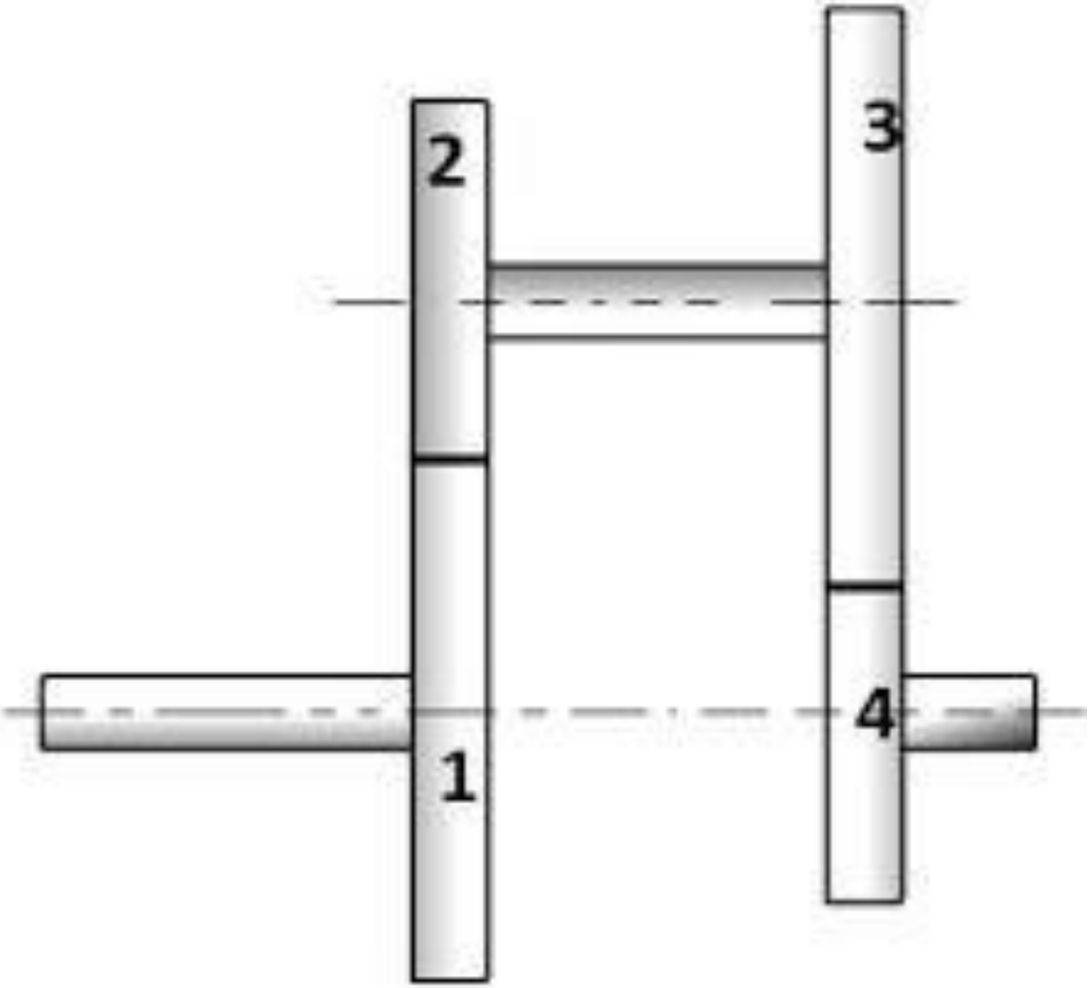
Reverted gear train

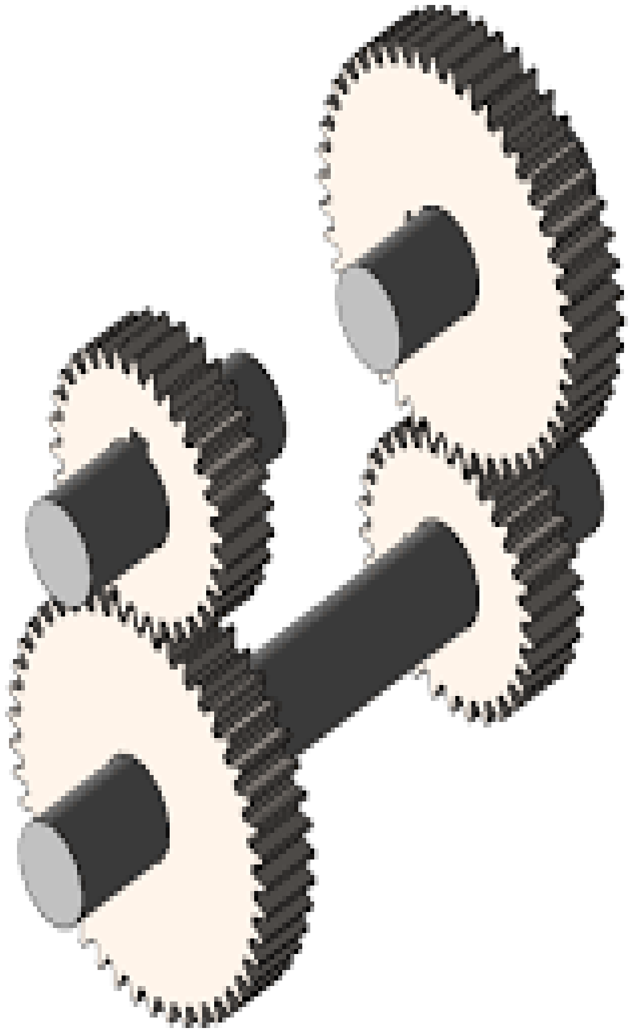
- It is a special type of compound gear train in which the driver and driven gears are mounted on co-axial shafts and they are rotated in same direction.
- Main advantage of this gear train is reduced the space occupied.
- Eg: back gear of lathe, sliding gearbox of automobiles and in clock works.

$$\text{Train value} = \frac{\text{Speed of follower}}{\text{Speed of driver}} = \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}} \qquad \frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4}$$

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

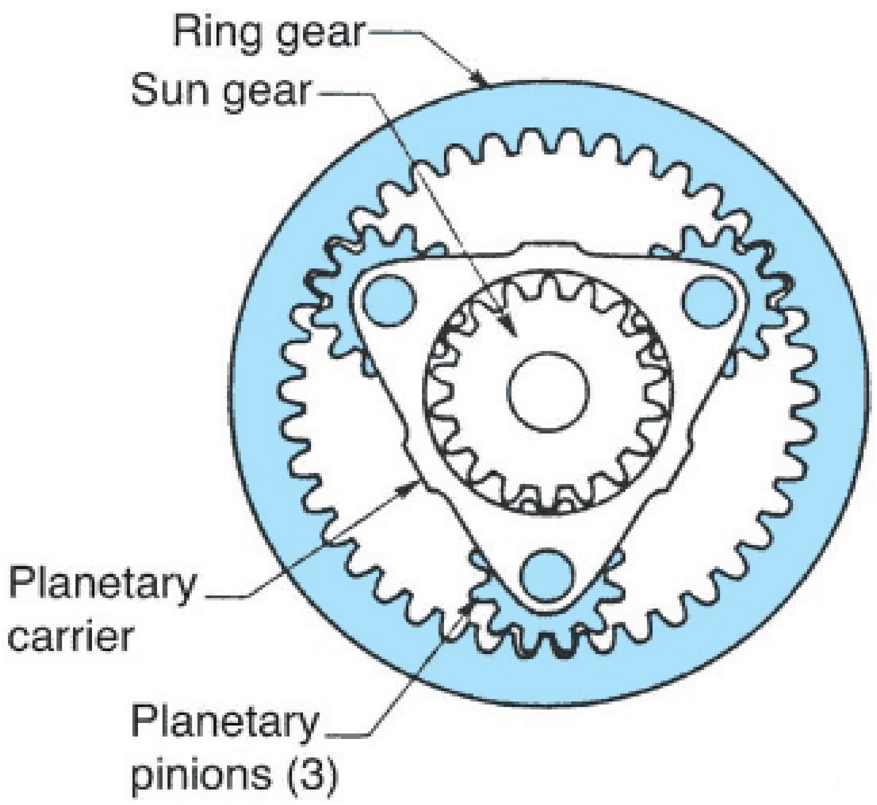
$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$





Reverted gear train

Planetary or Epicyclic gear train



Parts Of Epicyclic Gear train

Planetary or Epicyclic gear train

- An epicyclic gear train (also known as a planetary geartrain) consists of two gears mounted so that the centre of one gear revolves around the centre of the other.
- A carrier connects the centres of the two gears and rotates the planet and sun gears
- The four basic components of the epicyclic gear are:

1)Sun gear: The central gear

2)Carrier frame: Holds one or more planetary gear(s) symmetrically and separated, all meshed with the sun gear

3)Planet gear(s): Usually two to four peripheral gears, all of the same size, that mesh between the sun gear and the ring gear

4)Ring gear or Annulus gear: An outer ring with inward-facing teeth that mesh with the planetary gear(s)

Advantages of Epicyclic gear train

- High speed reduction ratios
- Compact and lightweight with high torque transmission
- High radial loads on the output shaft
- It is quieter in operation
- Uniform distribution of load over all gears having greater tooth contact.
- All gears are constantly in mesh, so a change of one gear to another is possible without any loss.

Disadvantages of planetary gear systems

- Complexity
- Assembly of gears is limited to specific teeth per gear ratios
- Efficiency calculations are difficult

Applications of Epicyclic Gear Train

- These gear trains are used in the automatic gear transmission system of cars, mopeds, and electric vehicles.
- They are used in lathe machines and wristwatches.
- They are used in robotic systems in robotic arms, grippers, and other mechanisms.

PROBLEMS ON GEAR TRAIN

Example. 10.16* :** A set of spur gear wheels are arranged as follows: *A* drives *B*, *B* and *C* is a compound wheel and *C* drives *D*. When $T_A = 25, T_B = 50, T_C = 35$ and $T_D = 70$ teeth. If *A* rotates at 300 rpm, find the rpm of wheel *D*. (November* 2000**)

***Solution* :**

Given :

Number of teeth on gear *A*, $T_A = 25$

Number of teeth on gear *B*, $T_B = 50$

Number of teeth on gear *C*, $T_C = 35$

Number of teeth on gear *D*, $T_D = 70$

Speed of spur gear wheel *A*, $N_A = 300$ rpm

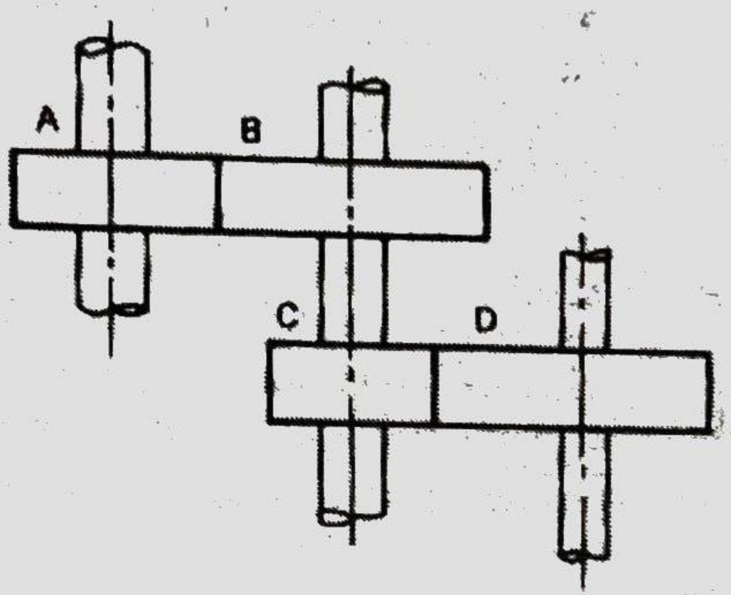


Fig 10.13 Arrangement of gear wheels

Using the relation for train value of compound gear train.

$$\frac{N_D}{N_A} = \frac{T_A T_C}{T_B T_D} \qquad \text{Or,} \quad \text{Speed of spur gear wheel } D, N_D = \frac{T_A T_C}{T_B T_D} \times N_A$$

$$= \frac{25 \times 35}{50 \times 70} \times 300 \quad = 75 \text{ rpm}$$

Result :

Speed of the wheel D is **75 rpm** .

Example 10.17 : A set of spur wheels for the gearing of a machine are arranged as follows : *A* drives *B*, *C* drives *D* and *E* drives *F*. Gears *B* and *C* and gears *D* and *E* are the compounds wheels. When $T_A = 20, T_B = 50, T_C = 25, T_D = 75, T_E = 26$ and $T_F = 65$ teeth. If gear *A* rotates in clockwise direction at 975 rpm. Find the speed and direction of rotation of follower gear *F*.

Solution :

Given :

Teeth of driver gears, $T_A = 20, T_C = 25$ and $T_E = 26$.

Teeth on driven gears, $T_B = 50, T_D = 75$ and $T_F = 65$

Speed of first driver gear, $N_A = 975$ rpm(Clock wise)

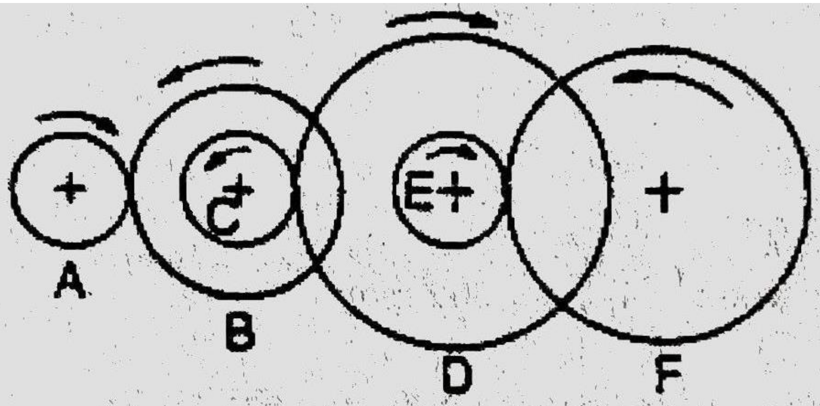


Fig 10.14 Arrangement of gear wheels

Using the relation , $\frac{N_F}{N_A} = \frac{T_A T_C T_E}{T_B T_D T_F}$

Or, Speed of follower or last driven, $N_F = \frac{T_A T_C T_E}{T_B T_D T_F} \times N_A = \frac{20 \times 25 \times 26}{50 \times 75 \times 65} \times 975 = 52$ rpm

Since the number of compound gears in the train is even (or the total number of shafts), therefore the first driver and the last driven are rotates in opposite directions.

$\therefore$ Direction of rotation of follower gear in *Anti-clock wise*

Result :

The speed of follower gear F is **52 rpm** and its rotation is in ***anti clockwise direction***.