DESIGN OF MACHINE ELEMENTS

SHAFTS & COUPLINGS
(MODULE II)

SHAFTS

- Shafts are one of the most commonly used rotating machine element which transmits power.
- In order to transmit power from one shaft to another, various transmission elements like gears, pulleys, sprockets etc. are mounted on the shaft by means of keys.
- They carry rotating parts like gears and pulleys, and are subjected to torque due to power transmission and bending moment due to weight of the pulleys and gears.
- They are usually made in circular section and could be either solid or hollow, and are supported on bearings.

SHAFTS

- The hollow transmission shafts offer the following advantages over solid transmission shafts.

1. **Hollow shafts are lighter than solid shafts.**
2. **These shafts are more stronger per kilogram of material.**
3. **They allow internal support or permit other shafts to operate through the interior.**

SHAFTS

- Hollow shafts are preferred under following circumstances.

1. **Where light weight construction is required.**
2. **Where minimum outer diameter of shaft is essential because of functional requirements.**
3. **Where a hole of some minimum diameter is essential in the shaft to accommodate bearings or some other shaft.**

CLASSIFICATION OF SHAFTS

Shafts are given specific names according to their use to which it is put as follows:

1. PRIME MOVER SHAFTS

Prime mover shaft is connected to source of power.

Eg: engine shafts, generator shafts, motor shafts, turbine shafts etc.

2. MACHINE SHAFTS

Machine shafts form an integral part of the machine itself.

Eg: Crank shaft

CLASSIFICATION OF SHAFTS

3. POWER TRANSMISSION SHAFTS

These shafts are used to transmit power between the source and the machines using power.

Eg: Line shafts, counter shafts, jack shafts etc.

SHAFT MATERIALS

The materials used for shafts should have the following desirable properties.

1. It should have sufficient strength.
2. It should have good machinability.
3. It should have good wear resistance.
4. It should have an ability to withstand heat and case hardening treatment.
5. It should have a low sensitivity to stress concentration.

SHAFT MATERIALS

- All commercial shafts are usually made of carbon steel having 0.25 to 0.4 percent carbon.
- For high strength requirements an alloy steel, such as nickel, nickel-chromium or nickel-vanadium steel, is used
- Shafts are formed either by hot rolling or cold rolling processes.

DESIGN FACTORS

Factors considered while designing a shaft

- 1. Material and heat treatment
- 2. Strength to resist torque and bending
- 3. Stiffness to resist deflection
- 4. Weight and space limitation, and
- 5. Stress concentration.

STRESSES IN SHAFTS

Following are the stresses induced in shafts.

1. Shear stresses due to the transmission of torque.
2. Bending stresses due to the forces acting upon by machine elements like gears, pulleys etc. or due to the weight of the shaft.
3. Stresses due to the combined torsional and bending loads.

TORSION EQUATION

- Torsion is the twisting of a structural member when it is loaded by couples that produce rotation about its longitudinal axis. The torsion equation for circular shaft is as follows:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

Where,

T = Torque transmitted by the shaft in N-mm.

J = Polar moment of inertia of the shaft about the axis of rotation in mm^4.

τ = Torsional shear stress induced in the shaft in MPa.

TORSION EQUATION

r = Distance from the neutral axis to the outer most fibre or radius of shaft in mm.

G = Modulus of rigidity for the shaft material in MPa.

θ = Angle of twist in radians in length l.

l = Length of the shaft in mm.

The above torsion equation is based on the following assumptions.

1. The material of the shaft is uniform throughout.
2. The shaft is of circular cross-section remains circular after loading,i.e., diameter of normal cross section remains unchanged.

TORSION EQUATION

3. The plane section of a shaft normal to its axis before loading, remains plane even after the application of torque.

4. The twist along the length of the shaft is uniform throughout.

5. The distance between any two normal cross sections remains same even after the application of torque.

6. Maximum shear stress developed in the shaft under torsion does not exceed the value of its elastic limit.

DESIGN OF SHAFTS

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

- When the shaft is subjected to pure torsional load, the principal stress induced in the shaft will be shear stress.
- The design of shafts subjected to torsion is usually based on the formula for torsional strength which is given as :

$$\frac{T}{J} = \frac{\tau}{r}$$

where,

T = Torque transmitted by the shaft in N-mm.

J = Polar moment of inertia of the shaft about the axis of rotation in mm^4.

τ = Torsional shear stress induced in the shaft in MPa.

r = Distance from the neutral axis to the outer most fibre or radius of shaft in mm.

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

(a) For solid shaft, polar moment of inertia,

$$\mathbf{J} = \frac{\pi}{32} d^4$$

where

d = Diameter of the shaft in mm.

$\therefore$ Radius of the shaft, $r = \frac{d}{2}$ mm

Consider the torsion equation,

$$\frac{T}{J} = \frac{\tau}{r}$$

$$T = \frac{\tau}{r} \text{ x } J$$

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

- Substitute, J & r in the above equation,

$$T = \frac{\tau}{\frac{d}{2}} \text{ x } \frac{\pi}{32} d^4$$

$$\mathbf{T = \frac{\pi}{16} \tau d^3}$$

where,

T = Twisting moment or maximum torque acting upon the shaft in N-mm.

τ = Maximum torsional shears tress induced in the outermost fibre of the shaft in MPa.

d = Diameter of the solid shaft in mm.

From this equation, we can determine the diameter of the shaft and this equation is known as the ***strength equation for a solid shaft.***

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

(b) For hollow shaft,

Polar moment of inertia, J= $\frac{\pi}{32}\ (d_0^4 - \ d_i^4)$

$= \frac{\pi}{32}\, d_0^4\ (1 - (\frac{d_i}{d_o})^4)$

$= \frac{\pi}{32}\, d_0^4\ (1 - k^4)$

where, d_0 = Outside diameter of hollow shaft in mm.

d_i = Inside diameter of the hollow shaft in mm.

k = Ratio of inside diameter & outside diameter of the hollow shaft or diameter ratio, i.e., $\frac{d_i}{d_o}$

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

- We know that the maximum shears tress induced at the outer surface of the shaft and minimum shear stress induced at the inner surface of the shaft. Therefore distance from the neutral axis to the outermost fibre of the shaft, r = $\frac{d_0}{2}$
- Consider the torsion equation,

$$\frac{T}{J} = \frac{\tau}{r}$$

$$T = \frac{\tau}{r} \times J$$

- Substitute, J & R in the above equation,

$$T = \frac{\tau}{\frac{d_o}{2}} \times \frac{\pi}{32}\, d_0^4(1\text{-}k^4)$$

$$\mathbf{T = \frac{\pi}{16}\tau\, d_0^3(1\text{-}k^4)}$$

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

Where,

T = Twisting moment or maximum torque acting upon the shaft in N-mm.

τ = Maximum torsional shears tress induced in the outermost fibre of the shaft in MPa.

d_0 = Outside diameter of the hollow shaft in mm.

k = Diameter ratio, i.e., $\frac{d_i}{d_o}$ where d_i is the inside diameter of the hollow shaft in mm.

From this equation, we can determine the outside diameter of the hollow shaft and this equation is known as the ***strength equation for a hollow shaft. The inside diameter is then obtained by using the relation for diameter ratio.***

DESIGN OF SHAFTS SUBJECTED TO TWISTING MOMENT OR TORQUE ONLY

- Percentage of saving of material when a solid shaft is replaced by a hollow shaft.

$$= \frac{D^2 - (D_0^2 - d_i^2)}{D^2} \text{ x } 100$$

POWER TRANSMITTED BY THE SHAFT

- The main purpose of the shaft is to transmit power from one shaft to another.
- Consider a rotating shaft which transmits a torque, T by rotating at N revolutions per minute (rpm) by the application of a tangential force F newton acting at the circumference of the shaft of radius r meter. We know that

  Work done on the shaft per minute = Force x Distance moved per minute

  $= F \times 2\pi rN = 2\pi NT$ (Since Torque, $T = F \times r$)

- Power is the rate of doing the work. Since work done on the shaft per second is the power transmitted by the shaft in watts.
- $\therefore$ Power transmitted = Work done in N-m per second

POWER TRANSMITTED BY THE SHAFT

$$P = \frac{2\pi NT}{60}$$

Where,

P = Power transmitted by the shaft in watts (W)

N = Speed of the shaft in rpm

T = Average or mean torque in N-m.

- The above equation is known as ***power equation.***
- The power equation may also be written in terms of angular velocity. It can be obtained as follows.

$$\mathbf{P = \frac{2\pi NT}{60} = T \times \frac{2\pi N}{60}}$$

$$\mathbf{P = T\omega}$$

POWER TRANSMITTED BY THE SHAFT

where,

P = Power transmitted in W.

T = Mean or average torque in N-m.

ω = Angular speed in rad/s

Note : Torque in power equation is the average or mean torque and is measured in N-m. While in strength equation it is the maximum torque measured in N-mm. The maximum torque is always less than the mean torque. In design calculations, not specifying the relation between the mean torque and maximum torque then assume both the torque are same.

SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

- The shafts are mostly designed on the basis of strength, but in certain applications the shafts are designed on the basis of stiffness or rigidity of the shaft.
- **Torsional rigidity:** The angle of twist or torsional deflection, θ in radians, may be obtained by using the relation,

$$\frac{T}{J} = \frac{G\theta}{l}$$

where,

T = Twisting moment or torque acting on the shaft in N-mm.

J = Polar moment of inertia of the shaft cross sectional area about the axis of rotation in mm^4.

G = Modulus of rigidity for the shaft material in MPa.

θ = Angle of twist in radians.

l = Length of the shaft in mm.

SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

- **Torsional stiffness:** The torsional stiffness of a shaft is defined as the amount of torque required to twist the shaft through one radians.

  Torsional stiffness, $\mathbf{q} = \frac{T}{\theta} = \frac{JG}{l}$

- The strength of the shaft is measured by the amount of torque it can transmit.

SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

- (a) For solid shaft,

  Polar moment of inertia, J = $\frac{\pi}{32} d^4$.

  Angle of twist, θ = $\frac{584Tl}{Gd^4}$ degrees

where,

θ = Angle of twist in the degrees.

l = Length of the shaft subjected to twisting moment in mm.

T = Twisting or torsional moment in N-mm.

G = Modulus of rigidity in MPa.

d = Diameter of the shaft in mm.

SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

- From the above equation, we can determine the diameter of the shaft. This equation is known as ***rigidity equation.***

- ***Note :*** The permissible angle of twist for machine tool applications is 0.25° per meter length and for line shafts it is 3° per meter length is the limiting value. Modulus of rigidity for steel is approximately 80GPa.

SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

(b) For hollow shaft,

Polar moment of inertia, J = $\frac{\pi}{32}$ $(d_0^4 - d_i^4)$

$$= \frac{\pi}{32} d_0^4 \left(1 - \left(\frac{d_i}{d_o}\right)^4\right)$$

$$= \frac{\pi}{32} d_0^4 (1 - k^4)$$

Angle of twist, θ = $\boldsymbol{\frac{584Tl}{Gd_0^4(1-k^4)}}$ **degrees**

where,

d_0 = Outside diameter of hollow shaft in mm.

d_i = Inside diameter of the hollow shaft in mm.

k = Ratio of inside diameter & outside diameter of the hollow shaft or diameter ratio, i.e., $\frac{d_i}{d_o}$

SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

- From the above equation, we can determine the diameter of the shaft. This equation is known as ***rigidity equation of hollow shaft.***

COUPLINGS

- A coupling is a machine element which can be defined as a mechanical device that permanently connect the shaft of a driving machine to the shaft of a driven machine.

PURPOSE OF COUPLINGS

- Couplings are used to connect one shaft to another to increase the length of the shaft according to the requirement, because shafts are generally available up to 7 meters length, hence where large length is required coupling is used.
- To provide for the connection of shafts of two purchased parts or separate manufactured units together to form a new machine.

COUPLINGS

- To provide for misalignment of shafts or to introduce mechanical flexibility.
- To alter the vibration and minimize shock characteristics of rotating units throughout the length.
- To transmit power from one shaft to other.

REQUIREMENTS OF COUPLINGS

- It should be capable of transmitting torque from the driving machine shaft to the driven machine shaft without any loss.
- It should permit easy connection and disconnection of the shafts for the purpose of repairs and alterations.

COUPLINGS

- It should keep the perfect alignment of the two shafts.
- It should be safe from protecting parts.

TYPES OF COUPLINGS

Shafts are available in different lengths, if longer shafts are required, they are joined by various types of couplings. Couplings may be classified according to the use into two groups.

(i) Rigid couplings

- These couplings are simple and inexpensive.
- Rigid couplings has no flexibility or resilience, hence it cannot tolerate misalignment between the axes of the shafts.

COUPLINGS

- Rigid couplings can be used only when there is precise alignment between two shafts.
- It is suitable for accurately aligned shafts having low speeds.
- The commonly used rigid couplings are:

1. ***Sleeve or muff coupling***
2. ***Clamp or compression coupling***
3. ***Flange coupling***
4. ***Solid or marine flange coupling***

COUPLINGS

(ii) Flexible couplings

These couplings are costlier due to additional parts compared to rigid couplings. Flexible couplings has flexibility and resilience, hence tolerate misalignment between the axes of the shafts. Therefore it is used to provide a small amount of lateral and angular misalignment. Most commonly used flexible couplings are :

1. *Oldham flexible coupling*
2. *Universal flexible coupling*
3. *Bushed pin type flexible coupling*

COUPLINGS

- **SLEEVE OR MUFF COUPLING**

It is the simplest form of rigid coupling used for connecting the smaller sizes of shaft and also called box coupling.

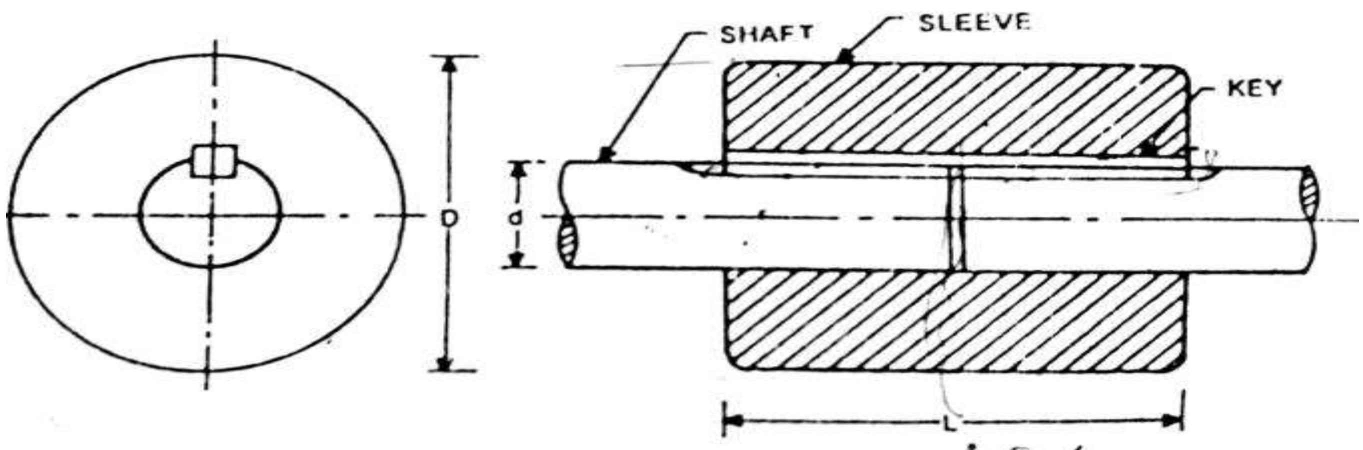

Fig. 5.1 MUFF COUPLING

COUPLINGS

- It consists of a hollow cast iron cylinder whose inner diameter is the same as that of the shaft called sleeve or muff.
- The muff enveloping the butting ends of the input and output shafts by means of a single rectangular sunk key or a gib head key.
- The torque is transmitted from input shaft to sleeve through the key and is transmitted from sleeve to output shaft through the key.

COUPLINGS

- The standard proportions used in practice for the sleeve or muff coupling are as follows.

$D = 2d + 13$

$L = 3.5d$

where,

d = Diameter of the shaft in mm.

D = Outer diameter of the sleeve in mm.

L = Axial length of the sleeve in mm.

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

- In the design of sleeve or muff coupling, the basic procedure for finding out the dimensions of the coupling consists of following steps:

Step 1 : Design of shaft

Calculate the diameter of the shaft by the following equations.

(i) If the torque is not given, using power equation determine the mean or average torque. Mathematically,

$$T_{mean} = \frac{60P}{2\pi N}$$

(ii) Compute maximum torque as per the given condition.

(a) If T_{max} is x % greater than T_{mean}, then,

$$T_{max} = (100 + x)\ \%\ \text{of}\ T_{mean}$$

(b) If the condition is not specified assume mean torque is equal to maximum torque, i.e.,

$$T_{max} = T_{mean}$$

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

(iii) Convert the maximum torque in N-mm.

(iv) Using strength equation determine the diameter of each shaft. Mathematically,

$$d = \sqrt[3]{\frac{16T_{max}}{\pi\tau_s}}$$

where,

τ_s = Maximum shears stress for the shaft material in MPa.

(v) Adopt standard size dimension for the transmission shaft.

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

Step 2 : Design of sleeve or muff

Calculate the dimensions of the sleeve by the standard empirical proportions.

(i) Outer diameter of sleeve or muff, D = 2d + 13

(ii) Axial length of sleeve or muff, L = 3.5 d

(iii) Check the torsional shear stress induced in the sleeve or muff by considering it to be as a hollow shaft. Using strength equation for hollow shaft,i.e.,

$$T_{max} = \frac{\pi}{16} \tau_m D^3 (1\text{-}k^4)$$

$$\tau_m = \frac{16 T_{max}}{\pi D^3 (1-k^4)}$$

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

where,

τ_m = Shears tress induced in the muff or sleeve material in MPa.

k = Diameter ratio, that is equal to $\frac{d}{D}$.

The calculated value of induced shear stress for muff τ_m is less than the given permissible or safe value of torsional shear stress for the muff material, design of sleeve is ***safe*** otherwise it is ***not safe.***

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

- **Step 3 : Design of key**

Calculate the standard cross-section of key and the length of the key in each shaft by the following empirical equations.

(i) Compare the given crushing stress σ_{ck} and shearing stress τ_k for the key material.

If $\sigma_{ck} = 2\tau_k$; adopt square key otherwise adopt rectangular key.

(ii) Width of the key, w = $\frac{d}{4}$

If possible adopt standard key size or adopt next whole number.

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

(iii) Thickness of the key

t = w (For square key)

t = $\frac{d}{6}$ (For rectangular key)

If possible adopt standard key size or adopt the next whole number.

(iv) Length of the key in each shaft, l = $\frac{L}{2} = \frac{3.5d}{2}$

(v) Check the torsional shear stress τ_k and compressive or crushing stress σ_{ck} for the key material by using the strength equations for key.

$$T_{max} = \text{lw}\tau_k \text{ x } \frac{d}{2}$$

$$\tau_k = \frac{2T_{max}}{lwd}$$

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

where,

∴ τ_k = Shear stress induced in the key material in Mpa.

$$T_{max} = l\,\frac{t}{2}\,\sigma_{ck} \times \frac{d}{2}$$

$$\sigma_{ck} = \frac{4T_{max}}{ltd}$$

where,

σ_{ck} = Crushing stress or compressive stress induced in the key material in MPa.

DESIGN PROCEDURE FOR SLEEVE OR MUFF COUPLING

- The calculated value of induced shear stress τ_k and crushing stress σ_{ck} are less than the given permissible value of shear stress and crushing stress for the key material, design is safe otherwise not safe.

FLANGE COUPLING

- Flange couplings are widely used for heavy power transmission at low speeds.
- The flange coupling consists of two hubs keyed to the two shafts as in the Fig. 5.2 (a).
- The hubs extend into flanges whose faces are brought and held together by a series of bolts arranged concentrically about the shaft so that their axes are parallel to the shaft axis.
- To ensure correct alignment, one of the hubs has a circular projection (A) which fits into a corresponding depression in the other hub.
- The bolts transmit torque from one flange to the other flange, and then to shaft.
- Design of flange coupling includes the design of hubs, flanges, key and bolts.

FLANGE COUPLING

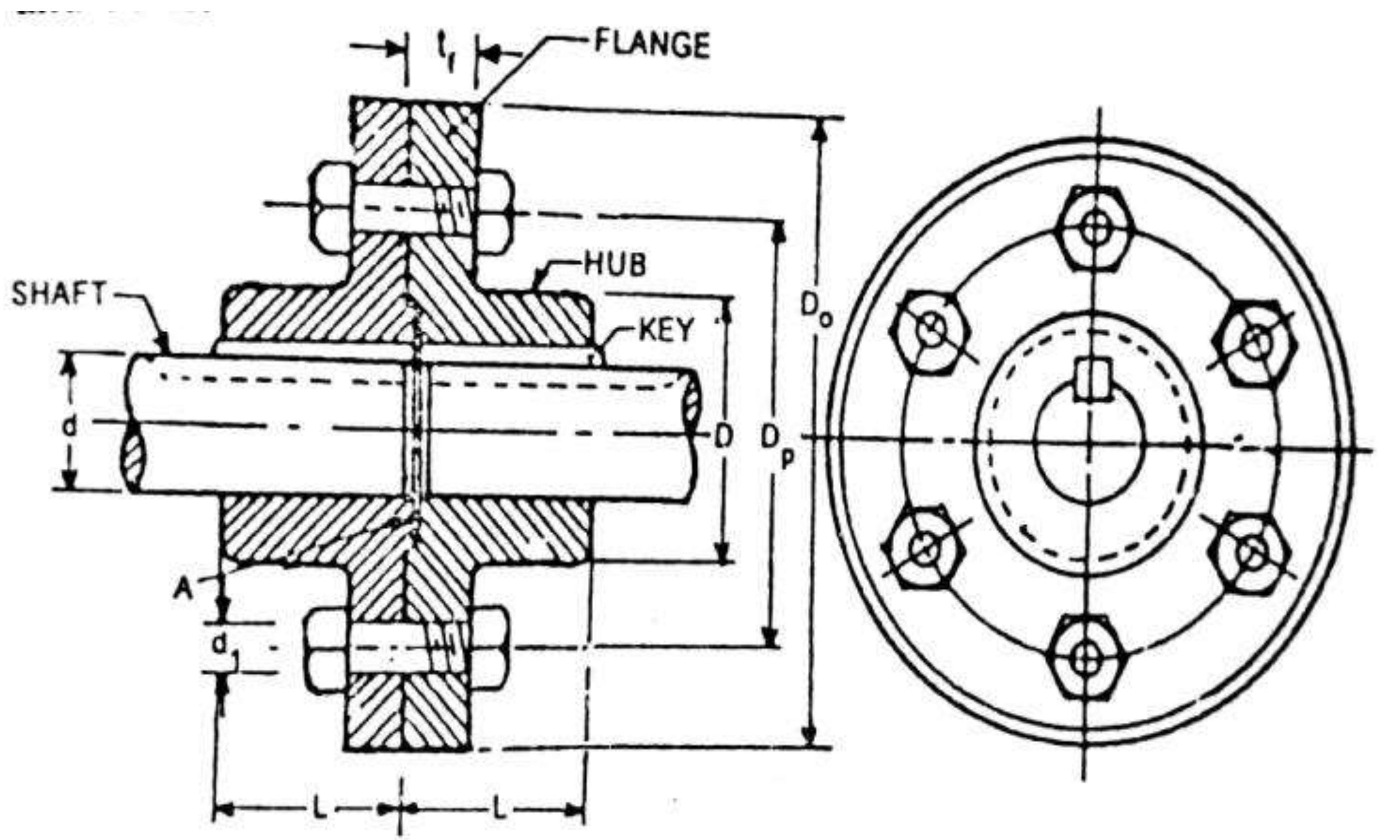

Fig. 5.2 (a) Flange Coupling

FLANGE COUPLING

There are three types of rigid flange couplings.

1. Unprotected type or ordinary rigid flange couplings
2. Protected type flange coupling
3. Solid flange or marine type flange coupling

Sl. No:	PARAMETER	EMPIRICAL RELATION
1	Inside diameter of hub	d
2	Outside diameter of hub, D	2d
3	Length of hub or effective length of key, L	1.5d
4	Pitch circle diameter of bolts, D_p	3d
5	Outside diameter of flange, D_0	4d
6	Thickness of flange, t_f	0.5d
7	Number of bolts, n	$\frac{4}{150}$d + 3
8	Allowable shear stress for shaft , key and bolt material	τ_{shaft} , τ_{key} , τ_{bolt}
9	Allowable shear stress for Hub and Flange material	τ_{hub} , τ_{flange}
10	Allowable crushing stress for bolt and key material	σ_{cb} , σ_{ck}

DESIGN OF FLANGE COUPLING

Sl.No:	DESIGN PART
1	**Design of shaft** (i) Power equation, $P = \frac{2\pi NT}{60}$ (ii) Strength equation of solid shaft, $T_{max} = \frac{\pi}{16} \tau\, d^3$
2	**Design of hub** (i) Inside diameter of the hub = d (ii) Outside diameter of the hub, D = 2d (iii) Axial length of the hub, L = 1.5d Checking safety of the hub by considering it as a hollow shaft, $T_{max} = \frac{\pi}{16} \tau_h\, D^3\, (1-k^4)$

3	**Design of flanges** (i) Overall or outside diameter of flange, D_0 = 4d (ii) Thickness of flanges, t_f = 0.5d (iii) Checking the safety of these dimensions from strength point of view. $$T_{max} = \pi \text{ D x } t_f \text{ x } \tau_f \text{ x } \frac{D}{2}$$
4	**Design of key** (i) Compare the given crushing stress σ_{ck} and shearing stress τ_k for the key material . If $\sigma_{ck} = 2\tau_k$; adopt square key otherwise adopt rectangular key. (ii) Width of the key, w = $\frac{d}{4}$ Thickness of the key, t = w ……..For square key Thickness of the key, t = $\frac{d}{6}$ ………For rectangular key (iii) Length of the key in each shaft, l = 1.5d

(iv) Check the safety of dimensions with the torsional shear stress τ_k and compressive or crushing stress σ_{ck} for the key material by using the strength equation.

Checking shearing stress, $T_{max} = lw\tau_k \times \frac{d}{2}$

Checking crushing stress, $T_{max} = l\,\frac{t}{2}\,\sigma_{ck} \times \frac{d}{2}$

The computed value of induced shear stress τ_k and crushing stress σ_{ck} are less than the given permissible value of shear stress and crushing stress for the key material, design is safe otherwise it is not safe.

5

Design of bolts

- No: of bolts, $\mathbf{n} = \frac{4}{150}\mathbf{d} + 3$
- Pitch circle diameter of bolts, $D_p = 3\mathrm{d}$
- Compute the nominal diameter of bolts from strength equation

$$T_{max} = \frac{\pi}{4}\ d^2\ \mathrm{n\ x}\ \tau_b\ \mathrm{x}\ \frac{D_p}{2}$$

∴ **Diameter of the bolt,** $d_1 = \sqrt{\frac{8T_{max}}{\pi n D_p \tau_b}}$

- Check the safety of nominal diameter of the bolt by crushing stress in the bolt

$$T_{max} = d_1 t_f\ \mathrm{n\ x}\ \sigma_{cb}\ \mathrm{x}\ \frac{D_p}{2}$$

Computed value of crushing stress in the bolt σ_{cb} is less than the permissible value of crushing or compressive stress for the bolt material, design is safe otherwise not safe