FUNDAMENTALS OF FLUID MECHANICS

CODE: 3051 (21)

MODULE 2

KINEMATICS OF FLUID FLOW

KINEMATICS

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined.

TYPES OF FLUID FLOW

The fluid flow is classified as :

(*i*) Steady and unsteady flows ;
(*ii*) Uniform and non-uniform flows ;
(*iii*) Laminar and turbulent flows ;
(*iv*) Compressible and incompressible flows ;
(*v*) Rotational and irrotational flows ; and
(*vi*) One, two and three-dimensional flows.

1 Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure of density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity
∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

5 Rotational and Irrotational Flows. Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis that type of flow is called irrotational flow.

6 One, Two and Three-Dimensional Flows. **One-dimensional flow** is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-dimensional flow

$$u = f_1(x, y)\ v = f_2(x, y) \text{ and } w = 0.$$

Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z),\ v = f_2(x, y, z),\ w = f_3(x, y, z).$$

5.4.1. Path line

A path line (Fig. 5.7) is the *path followed by a fluid particle in motion*. A path line shows the direction of particular particle as it moves ahead. In general, this is the curve in three-dimensional space. However, if the conditions are such that the flow is two-dimensional the curve becomes two-dimensional.

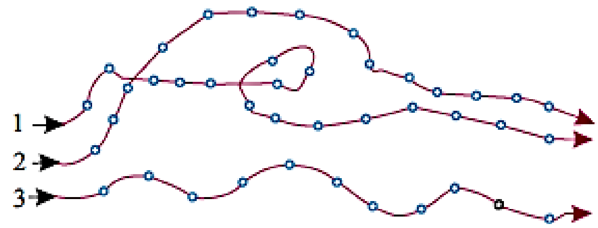


Fig. 5.7. Path lines.

5.4.2. Stream line

A *stream line* way be defined as *an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point*.

Equation of a stream line in a three-dimensional flow is given as:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad ...(5.20)$$

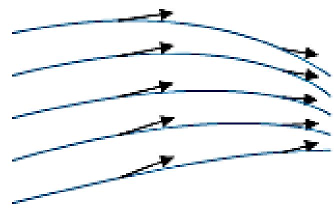


Fig. 5.8. Stream line.

Following *points* about streamlines are *worth noting*:

1. A streamline cannot intersect itself, nor two streamlines can cross.
2. There cannot be any movement of the fluid mass across the streamlines.
3. Streamline spacing varies inversely as the velocity; *converging of streamlines in any particular direction shows accelerated flow in that direction*.
4. Whereas a *path line* gives the path of *one particular particle* at successive instants of time, *a streamline* indicates the direction of a *number of particles* at the same instant.
5. The series of streamlines represent the flow pattern at an instant.

● In *steady flow*, the pattern of streamlines remains invariant with time. The path lines and streamlines will then be identical.

● In *unsteady flow*, the pattern of streamlines may or may not remain the same at the next instant.

RATE OF FLOW OR DISCHARGE (Q)

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(*i*) For liquids the units of Q are m^3/s or litres/s

(*ii*) For gases the units of Q is kgf/s or Newton/s

Consider a liquid flowing through a pipe in which

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

Then discharge $\quad Q = A \times V$.

CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in Fig. 5.1.

Let V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and V_2, ρ_2, A_2 are corresponding valves at section, 2-2.

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(5.2)$$

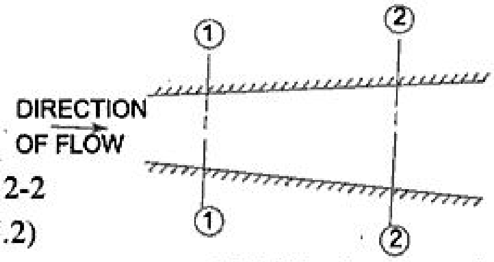


Fig. 5.1 *Fluid flowing through a pipe.*

Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation.** If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2$$

THE ENERGIES POSSESED BY A FLUID PARTICLE

Energy associated with a fluid element may exist in several forms. These are listed here and The method of calculation of their numerical values is also indicated.

- **Kinetic Energy**
- **Potential Energy**
- **Pressure Energy**

Kinetic Energy

This is the energy due to the motion of the element as a whole. If the velocity is V, then the kinetic energy for m kg is given by

$$KE = \frac{mV^2}{2}$$

The unit in the SI system will be Nm also called Joule (J)

And we know mass m=$\frac{W}{g}$ = $\frac{Weight}{gravity}$. Then KE=$\frac{W\,v^2}{2\,g}$

Hence, kinetic energy / unit weight of fluid = $\frac{v^2}{2\,g}$ in m. And it is also known as kinetic head or velocity head.

Potential Energy

This is the energy possessed by a fluid body by virtue of its position or location in space. And it is the energy due to the position of fluid element in the gravitational field.

Potential Energy with weight of fluid = mgh = Wh = Wz

Hence, potential energy/ unit weight of fluid = z in m and it is also known as potential head or datum head.

Pressure Energy

It is the energy possessed by a fluid body virtue of the pressure at which it is maintained.

The pressure energy with weight of fluid = $\frac{W\,P}{\rho\,g}$ in joule.

Hence pressure energy / unit weight of fluid = $\frac{W\,P}{\rho\,g}$ unit in meters. It is also known as pressure head.

Total Energy

It is the sum of kinetic energy, potential energy and pressure energy.

The total energy with weight of fluid= $\frac{W v^2}{2 g} + Wz + \frac{W P}{\rho g}$**.**

Hence the total energy / unit weight of fluid= $\frac{v^2}{2 g} + z + \frac{P}{\rho g}$**. It is also known total head. Unit is in m.**

Bernoulli's Theorem and Equation

This theorem is a form of the well-known principle of conservation of energy. It is states that "In a steady continuous flow of frictionless incompressible fluid, the sum of the potential head, pressure head and kinetic head will be constant.

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

NOTE: According to the Bernoulli's Equations it can be rearranged into below form for mathematical problems. Considering the two sections of a fluid flow path, then equation will be……

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

ASSUMPTIONS IN BERNOULLI'S THEOREM

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero
(*ii*) The flow is steady
(*iii*) The flow is incompressible
(*iv*) The flow is irrotational.
(v) The flow is one dimensional flow

LIMITATIONS OF BERNOULLI'S THEOREM

1. **In Bernoulli's theorem we assumed, it is steady flow. That is velocity of liquid particle at any cross section must be equal. But in actual case velocity of liquid particle at centre of pipe is maximum and decrease towards walls of pipe due to friction. Hence we take mean velocity.**
2. **In Bernoulli's theorem external forces like as friction force not considered. Actual case friction force must be takes place due to friction of wall of channel.**
3. **In Bernoulli's theorem, it is assumed that there is no loss of energy while flowing. But in actual case there is different energy loss due to friction, for turbulent flow kinetic energy converted to heat energy. In viscous flow energy loss due to viscosity.**
4. **If the fluid flow through curved path there is energy will be formed due to centrifugal force. But in case of normal application of Bernoulli's theorem we neglect the energy due to the centrifugal force.**

PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for Rate of Flow Through Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

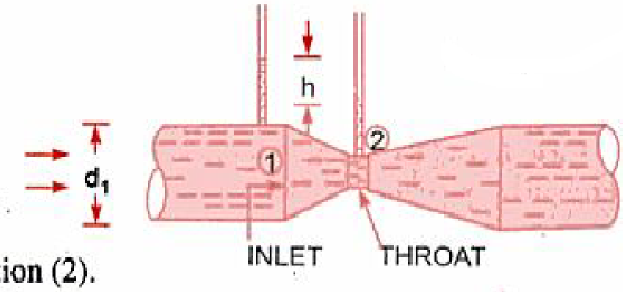


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2{}^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \qquad v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge, $\qquad Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \qquad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid
S_o = Sp. gravity of the liquid flowing through pipe
x = Difference of the heavier liquid column in U-tube

Then $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where S_1 = Sp. gr. of lighter liquid in U-tube
S_0 = Sp. gr. of fluid flowing through pipe
x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$

Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the down stream side from the orifice plate.

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and

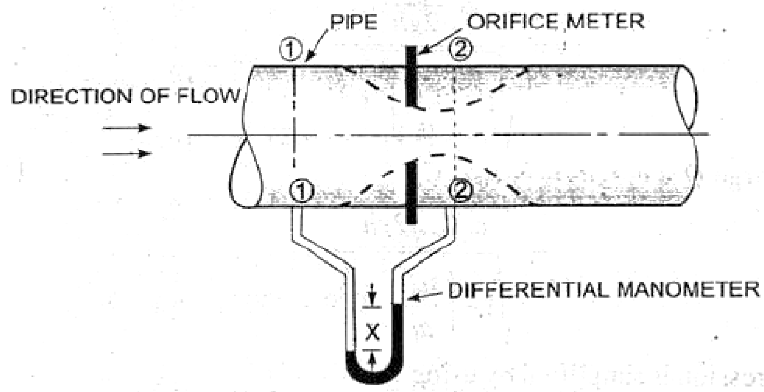


Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$\therefore$
$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = v_2^2 - v_1^2$$

or
$$v_2 = \sqrt{2gh + v_1^2} \qquad \ldots(i)$$

Now section (2) is at the vena contracta and a_2 represents the area at the vena contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$\therefore$
$$a_2 = a_0 \times C_c \qquad \ldots(ii)$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0 C_c}{a_1} v_2 \qquad \ldots(iii)$$

Substituting the value of v_1 in equation (i), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or $$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2 = \left[1-\left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2hg$$

$\therefore$ $$v_2 = \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0 C_c$ $\{\because a_2 = a_0 C_c \text{ from } (ii)\}$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}} \qquad ...(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ $$C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (iv), we get

$$Q = a_0 \times C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}. \qquad ...(6.13)$$

where C_d = Co-efficient of discharge for orifice meter.

6.7.3 Pitot-tube. It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig. 6.13.

The lower end, which is bent through 90° is directed in the upstream direction as shown in Fig. 6.13. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

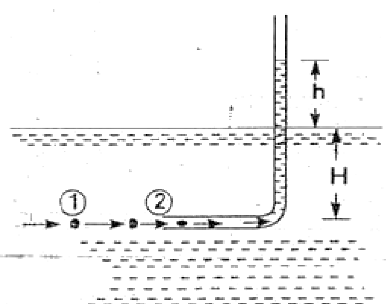


Fig. 6.13 *Pitot-tube.*

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equations at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_2^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $v = C_v \sqrt{2gh}$ ...(6.14)

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig. 6.14.
2. Pitot-tube connected with piezometer tube as shown in Fig. 6.15.
3. Pitot-tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig. 6.16.

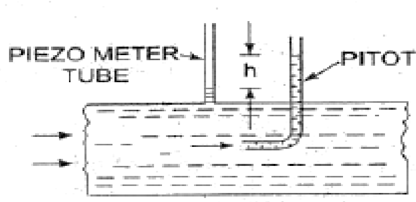


Fig. 6.14

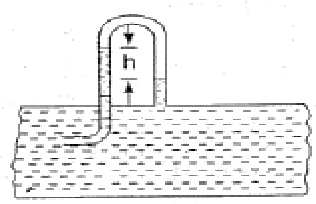


Fig. 6.15

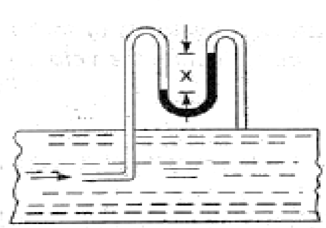


Fig. 6.16

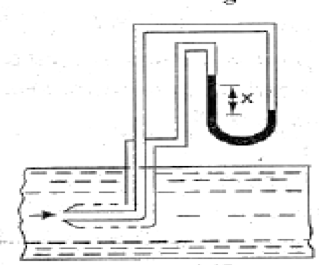


Fig. 6.17

4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. 6.17. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_o} - 1\right]$.

FLOW THROUGH ORIFICES, NOTCHES & PIPES

ORIFICES

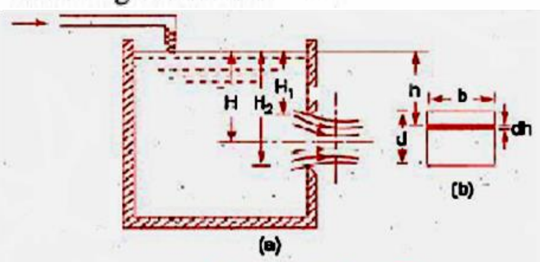
Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

CLASSIFICATIONS OF ORIFICES

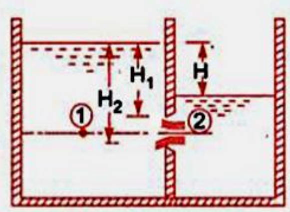
The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upstream edge. The following are the important classifications :

1. The orifices are classified as **small orifice** or **large orifice** depending upon the size of orifice and head of liquid from the centre of the orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice, it is known as large orifice.
2. The orifices are classified as (*i*) Circular orifice, (*ii*) Triangular orifice, (*iii*) Rectangular orifice and (*iv*) Square orifice depending upon their cross-sectional areas.
3. The orifices are classified as (*i*) Sharp-edged orifice and (*ii*) Bell-mouthed orifice depending upon the shape of upstream edge of the orifices.
4. The orifices are classified as (*i*) Free discharging orifices and (*ii*) Drowned or sub-merged orifices depending upon the nature of discharge.

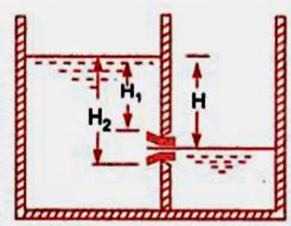
The sub-merged orifices are further classified as (*a*) Fully sub-merged orifices and (*b*) Partially sub-merged orifices.



Large rectangular orifice



Fully sub-merged orifice.



Partially sub-merged orifice.

FLOW THROUGH AN ORIFICE

Consider a tank fitted with a circular orifice in one of its sides as shown in Fig. 7.1. Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section CC, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called **Vena-contracta**. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

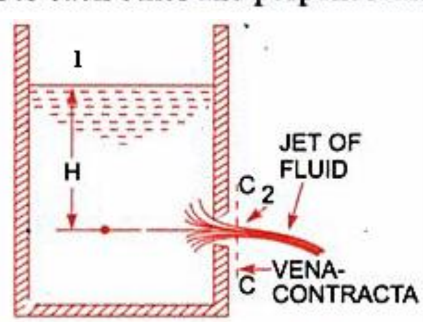


Fig. 7.1 *Tank with an orifice.*

Consider two points 1 and 2 as shown in Fig. 7.1. Point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at points 1 and 2.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $\quad p_1 = p_2 =$ ATMOSPHERE PRESSURE

And $z_1 = H \quad z_2 = 0$

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$$\therefore \quad H + 0 = 0 + \frac{v_2^2}{2g}$$

$$\therefore \quad v_2 = \sqrt{2gH}$$

This is theoretical velocity. Actual velocity will be less than this value.

HYDRAULIC CO-EFFICIENTS

The hydraulic co-efficients are

1. Co-efficient of velocity, C_v
2. Co-efficient of contraction, C_c
3. Co-efficient of discharge, C_d.

1 Co-efficient of Velocity (C_v). It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by C_v and mathematically, C_v is given as

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}, \text{ where } V = \text{actual velocity}, \sqrt{2gH} = \text{Theoretical velocity}$$

The value of C_v varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place. Generally the value of $C_v = 0.98$ is taken for sharp-edged orifices.

2 Co-efficient of Contraction (C_c). It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice and

a_c = area of jet at vena-contracta.

Then
$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken 0.64.

3 Co-efficient of Discharge (C_d). It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$\therefore \quad C_d = C_v \times C_c$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is taken as 0.62.

NOTCHES

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

CLASSIFICATION OF NOTCHES

The notches are classified as :

1. According to the shape of the opening :
   - (*a*) Rectangular notch,
   - (*b*) Triangular notch,
   - (*c*) Trapezoidal notch, and
   - (*d*) Stepped notch.
2. According to the effect of the sides on the nappe :
   - (*a*) Notch with end contraction.
   - (*b*) Notch without end contraction or suppressed notch.

DISCHARGE OVER A RECTANGULAR NOTCH

The expression for discharge over a rectangular notch

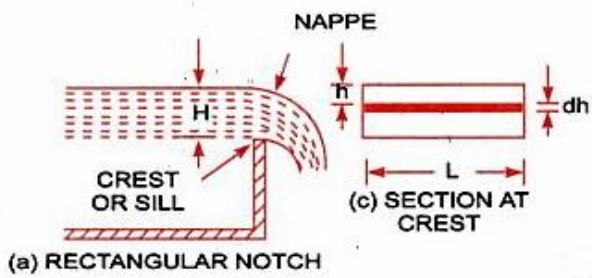


Fig. 8.1 *Rectangular notch and weir.*

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h form the free surface of water as shown in Fig. 8.1(*c*).

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \qquad ...(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (*i*) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \, . \, L \, . \, \sqrt{2gh} \; . \, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} \, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g} \qquad (1)$$

Cd value of rectangular notch between 0.6 to 0.65

Assume Cd = 0.623, then equation 1 will be

Q=1.84 × L × $[H]^{3/2}$.

DISCHARGE OVER A TRIANGULAR NOTCH

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (b), we have

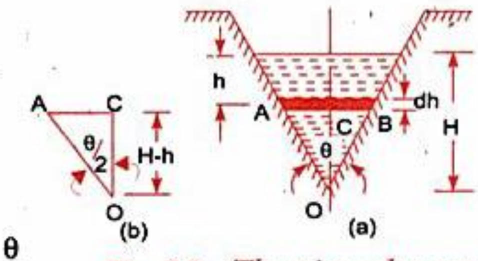


Fig. 8.3 *The triangular notch.*

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$\therefore$ $$AC = (H-h)\tan\frac{\theta}{2}$$

Width of strip $$= AB = 2AC = 2\,(H-h)\tan\frac{\theta}{2}$$

$\therefore$ Area of strip $$= 2\,(H-h)\tan\frac{\theta}{2}\times dh$$

The theoretical velocity of water through strip = $\sqrt{2gh}$

$\therefore$ Discharge, dQ, through the strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2}\times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times gh$$

$\therefore$ Total discharge, Q is $$Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times dh$$

$$= 2C_d \times \tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (Hh^{1/2}-h^{3/2})\,dh$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{Hh^{3/2}}{3/2}-\frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H.H^{3/2}-\frac{2}{5}H^{5/2}\right]$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H^{5/2}-\frac{2}{5}H^{5/2}\right]$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

$$Q = \frac{8}{15}C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90^\circ, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge $$Q = \frac{8}{15}\times 0.6\times 1\times\sqrt{2\times 9.81}\times H^{5/2}$$

$$= 1.417\,H^{5/2}.$$

DISCHARGE OVER A TRAPEZOIDAL NOTCH

As shown in Fig. 8.4, a trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

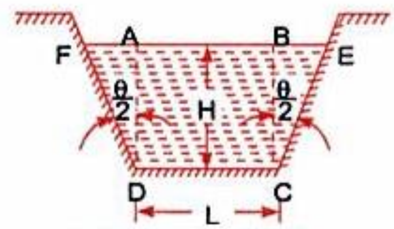


Fig. 8.4 *The trapezoidal notch.*

Let H = Height of water over the notch

L = Length of the crest of the notch

C_{d_1} = Co-efficient or discharge for rectangular portion $ABCD$ of Fig. 8.4.

C_{d_2} = Co-efficient of discharge for triangular portion [FAD and BCE]

The discharge through rectangular portion $ABCD$ is given by

or
$$Q_1 = \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches FDA and BCE is equal to the discharge through a single triangular notch of angle θ and it is given by equation

$$Q_2 = \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir $FDCEF = Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}.$$

ADVANTAGES OF TRIANGULAR NOTCH OVER RECTANGULAR NOTCH

A triangular notch or weir is preferred to a rectangular weir or notch due to following reasons :

1. The expression for discharge for a right-angled V-notch or weir is very simple.
2. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.
3. In case of triangular notch, only one reading, *i.e.*, (H) is required for the computation of discharge.
4. Ventilation of a triangular notch is not necessary.

FLOW THROUGH PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

1) **Major Energy losses due to the friction**
2) **Minor Energy losses due to**
   a) **Sudden expansion of pipe**
   b) **Sudden contraction of pipe**
   c) **Entrance of pipe**
   d) **Exit of pipe**
   e) **Bend in pipe**
   f) **Pipe fittings**
   g) **An obstruction in pipe**

FRICTIONAL LOSS IN PIPE FLOW

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.

On the basis of his experiments, William Froude gave the following laws of fluid fraction for turbulent flow.

The friction resistance for turbulent flow is :

(*i*) proportional to V^n, where n varies from 1.5 to 2.0,

(*ii*) proportional to the density of fluid,

(*iii*) proportional to the area of surface in contact,

(*iv*) independent of pressure,

(*v*) dependent on the nature of the surface in contact.

Darcy-Weisbach Equation

Expression for Loss of Head Due to Friction in Pipes. Consider a uniform horizontal pipe, having steady flow as shown in Fig. 10.3. Let 1-1 and 2-2 are two sections of pipe.

Let p_1 = pressure intensity at section 1-1,

V_1 = velocity of flow at section 1-1,

L = length of the pipe between sections 1-1 and 2-2,

d = diameter of pipe,

f' = frictional resistance per unit wetted area per unit velocity,

h_f = loss of head due to friction,

p_2, V_2 = are values of pressure intensity and velocity at section 2-2.

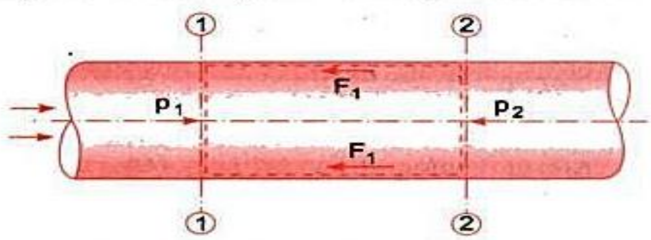


Fig. 10.3 *Uniform horizontal pipe.*

Then equation will be

$$\therefore \quad h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d} \quad \text{...}(iv)$$

Putting $\frac{f'}{\rho g} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (*iv*), becomes as $h_f = \frac{4 . f}{2g} . \frac{LV^2}{d} = \frac{4f . L . V^2}{d \times 2g}$ ... (*v*)

Equation (*v*) is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to friction in pipes.

Sometimes equation (*v*) is written as

$$h_f = \frac{f . L . V^2}{d \times 2g}$$

Then f is known as friction factor.

In Darcy-Wiesbach equation if we substitute $V = \frac{4Q}{\pi d^2}$ **, then** $h_f = \frac{4.f.L.(\frac{4Q}{\pi d^2})^2}{2.d.g} = \frac{fLQ^2}{3.d^5}$.

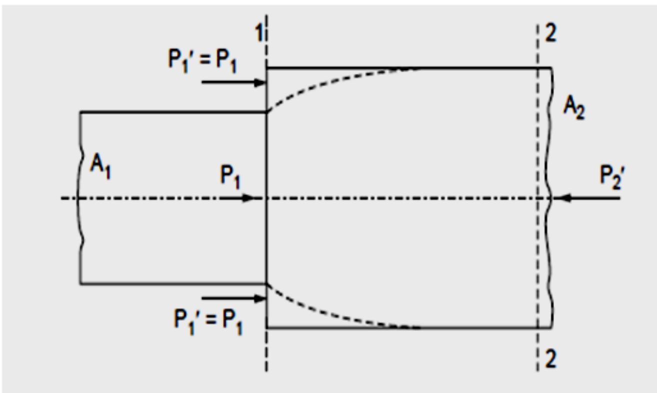
Chezy's Formula

V = C $\sqrt{mi}$

And here

C=$\sqrt{\frac{\rho g}{f'}}$, m=$\frac{d}{4}$, i =$\frac{hf}{L}$ hf is head loss due to the friction, L is the length of the pipe, d is diameter of the pipe, ρ is the density, *f'* is the frictional resistance. And the m is called hydraulic mean depth or hydraulic radius, which is the ratio of area of flow per wetted perimeter of the pipe.

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENTS

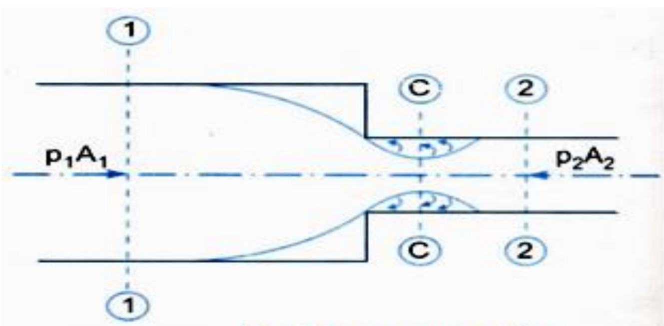


Sudden Expansion

$$h_f = \frac{(u_1 - u_2)^2}{2g}.$$

u1 and u2 are the velocities at sections

LOSS OF HEAD DUE TO SUDDEN CONTRACTION



Sudden contraction.

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$= \frac{kV_2^2}{2g}, \text{ where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

If the value of C_c is not given then the value k=0.5. And here C_c is called coefficient of contraction. It means C_c is the ratio of area of vena contracta to area of section 2. 'v$_2$' is velocity of section 2.

12·4·3 Loss of Head due to Obstruction in Pipe

Refer to Fig. 12·7. The loss of energy due to an obstruction in pipe takes place on account of the reduction in the cross-sectional area of the pipe by the presence of obstruction which is followed by an abrupt enlargement of the stream beyond the obstruction.

Head loss due to obstruction ($h_{obs.}$) is given by the relation :

$$h_{obs} = \left[\frac{A}{C_c(A-a)}\right]^2 \frac{V^2}{2g} \qquad ...(12.5)$$

where, A = Area of the pipe,

a = Maximum area of obstruction, and

V = Velocity of liquid in pipe.

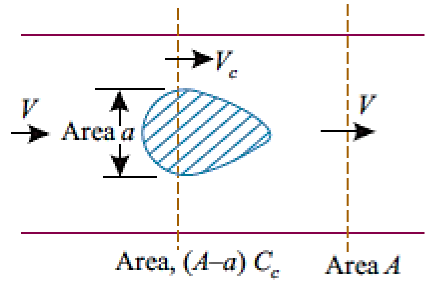


Fig. 12.7

12·4·4 Loss of Head at the Entrance to Pipe

Loss of head at the entrance to pipe (h_i) is given by the relation :

$$h_i = 0.5\frac{V^2}{2g} \qquad ...(12.6)$$

where, V = Velocity of liquid in pipe.

12·4·5 Loss of Dead at the Exit of a Pipe

Loss of head at the exit of a pipe is denoted by h_0 and is given by the relation:

$$h_0 = \frac{V^2}{2g} \qquad ...(12.7)$$

where, V = Velocity at outlet of pipe.

12·4·6 Loss of Head due to Bend in the Pipe

In general the loss of head in bends (h_b) provided in pipes may be expressed as :

$$h_0 = k\frac{V^2}{2g} \qquad ...(12.8)$$

where, V = Mean velocity of flow of fluid, and

and, k = Co-efficient of bend; it depends upon *angle of bend, radius of curvature of bend* and *diameter of pipe.*

12·4·7 Loss of Head in Various Pipe Fittings

The loss of head in the various pipe fittings (such as valves, couplings, etc.) may also be represented as :

$$h_{\text{fittings}} = k\frac{V^2}{2g}$$

where, V = Mean velocity flow in the pipe, and k = value of the co-efficient; it depends on the type of the pipe fitting.