**Module -II**

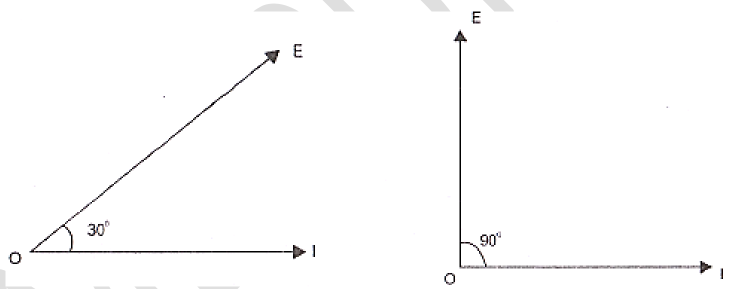
**ALTERNATING CURRENT THROUGH SERIES CIRCUITS**

## 1. INTRODUCTION

1. Resistance: - It is the opposition offered by a substance or body to the flow of and electric current through it. It is represented by letter R and is measured in ohm ( Ω ).
2. Inductance: It is the property of a coil due to which it opposes any increase or decrease of current or flux through it. It is represented by letter L and is measured in henry (H).
3. Capacitance:- It is the property of a capacitor to store electricity or the amount of charge required to create a unit potential difference between plates. It is represented by letter C and is measured in farad (F).

**Phasors**

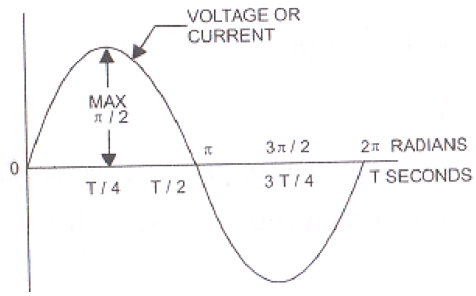
Vectors representing alternating voltages and currents also show phase relationships, they sometimes are called phasors.

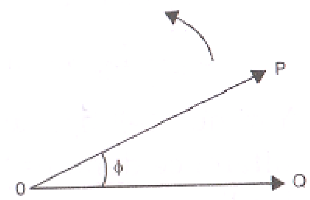



**Phase and Phase Difference**

The angle turned /moved/rotated through by an alternating current or voltage from a given instant is called phase.

The phase of an alternating quantity can be defined as the fraction of a time period or cycle through which the alternating quantity has moved from given instant. The angle between the phases of two alternating quantity of the same frequency as measured in degrees is known as phase difference. Vector diagrams of sine waves of same frequency




In the above figure the voltage (e) and current (i) of same frequency is shown. The voltage advances the current by $90^0$ or current lag voltage by $90^0$.

## Addition and Subtraction of Alternating quantities by Vector Method

### Addition

For addition and subtraction of alternating quantities, best suited method is rectangular or complex form. The general form of an alternating quantity can be expressed in vector form as follows $E_1 = a_1 + jb_1$ and $E_2 = a_2 + jb_2$. For adding the quantities add the constant terms together and add the imaginary terms together (imaginary term is the term which contains *i* or *j* term).

$$E = E_1 + E_2$$

$$= (a_1 + jb_1) + (a_2 + jb_2)$$

$$= (a_1 + a_2) + j(b_1 + b_2)$$

The magnitude of the resultant vector can be find out using following equation

$$E = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$$

The position of E with respect to x-axis is

$$\theta = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$$

**General formula for finding the magnitude of a vector is as follows**

**Magnitude, E=** $\sqrt{a^2 + b^2}$

**The position (angle) of E with respect to x-axis is,**

**Angle,** $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

**Subtraction**

For subtraction of the quantities subtract the constant terms and subtract the imaginary terms together (imaginary term is the term which contains *i* or *j* term).

E = $E_1$-$E_2$

$= (a_1 + j\,b_1) - (a_2 + j\,b_2)$

$= (a_1 - a_2) + j(b_1 - b_2)$

The magnitude of E = $\sqrt{(a_1a_2 - b_1b_2)^2 + (a_1b_2 + b_1a_2)^2}$

$$\theta = \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right)$$

**Multiplication and Division of Alternating Quantities**

Exponential and Polar forms are best suited for multiplication and division.

**Exponential Form**

$E_1 = E_1 e^{j\alpha}$

$E_2 = E_2 e^{j\beta}$

$E = E_1 \times E_2$

$E = E_1 e^{j\alpha} E_2 e^{j\beta}$

$= E_1 E_2 e^{j(\alpha - \beta)}$

**Polar Form**

$E_1 = E_1 \angle \alpha$

$E_2 = E_2 \angle_{\beta}$

$E = E_1 \times E_2$

$E = E_1 \angle \alpha \times E_2 \angle \beta$

$E = E_1\, E_2 \angle(\alpha + \beta)$

**Division**

**Exponential Form**

$$E = \frac{E_1}{E_2} = \frac{E_1 e^{j\alpha}}{E_2 e^{j\beta}}$$

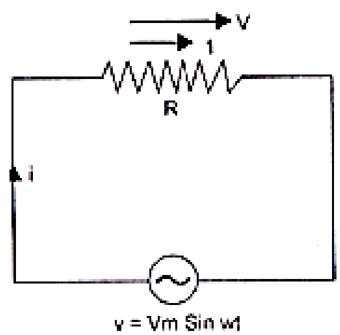
$$= \frac{E_1}{E_2} e^{j(\alpha-\beta)}$$

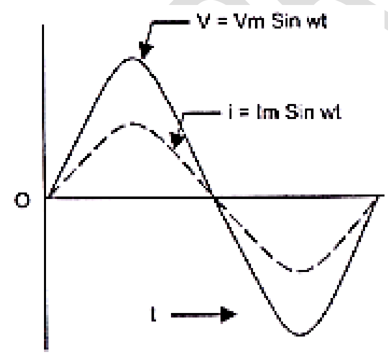
**Polar Form**

$$E = \frac{E_1}{E_2} = \frac{E_1\angle\alpha}{E_2\angle\beta}$$

$$= \frac{E_1}{E_2}\angle(\alpha-\beta)$$

## 2. AC THROUGH PURE RESITANCE




Let a pure resistor (R) is connected across an AC voltage source, $v = V_m \sin\omega t$ ………….. (1)

Let '$R$' be the ohmic resistance and '$i$' be the current flowing through it. When voltage is applied, there is a small voltage drop or loss occurs across the resistance, called as ohmic drop. This will be directly proportional to the current flowing through the resistor R. It is denoted by the expression, $v = iR$ ……..(2)

Equate the equations 1 and 2 we get, $iR = V_m \sin\omega t$

$$i = \frac{V_m \sin\omega t}{R} = i = \frac{V_m}{R} \times \sin\omega t \text{ ……………} \quad (3)$$

Current 'i' is maximum when $\sin\omega t$ is unity (or $\omega t = 90^0$)

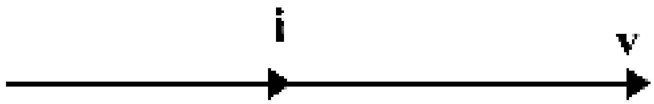
$$\therefore I_m = \frac{V_m}{R} \text{ ………………..} \quad (4)$$

Substitute equation (4) in (3), we get

$$i = I_m \sin\omega t \text{ ……………..} \quad (5)$$

Comparing equations 1and 5, we find that the voltage and current are in phase, because the angle $\theta$ is same for both instantaneous voltage equation and current equations, ie, the angle between voltage and current is 0. At any instant the voltage and current will be on the same phase. It is shown in the figure.

**2.1. Vector diagram**




**2.2. Instantaneous Power**

Equation for power $P = v \times i\ watt$

Substitute instantaneous values for *v* and *i* in the above equation

$$P = V_m \sin \omega t \times I_m \sin \omega t$$

$$P = V_m I_m \sin^2 \omega t \qquad \left[\sin^2 \theta = \frac{1-\cos 2\theta}{2}\right]$$

$$P = V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}$$

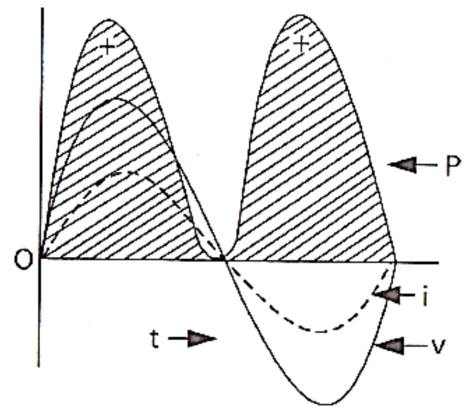
in the above equation we have two parts, a constant part and a fluctuating part. $\frac{V_m I_m}{2}$ is the constant part and $\frac{V_m I_m \cos 2\omega t}{2}$ is the fluctuating or varying part, this part depends on the frequency. Normally this fluctuating part is the double of the frequency of that of voltage and current.

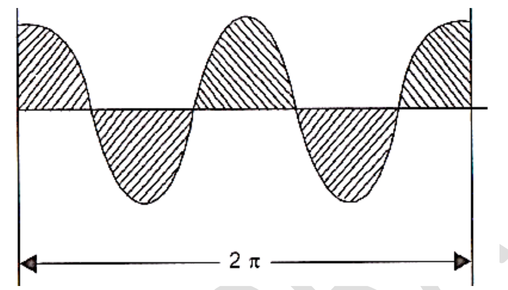
From the figure, we can see that the areas above and below the axis are equal, hence average value of the variable portion is zero. Thus the power of the cycle is the **constant portion** of the power expression only. Therefore the power for the whole cycle is,

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

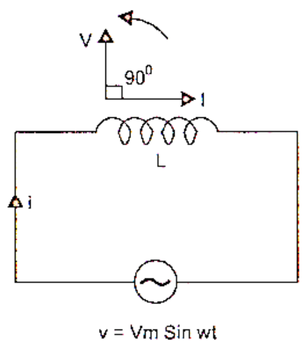
$$P = V_{rms} \times I_{rms}$$

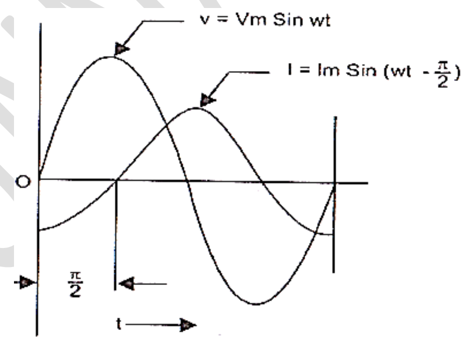
$$P = V \times I\ watt$$




## 3. AC THROGH PURE INDUCTOR




Consider a circuit containing a coil of pure inductance. This is denoted by *L*. if an alternating current is applied across the inductor in the circuit, a magnetic flux will be set up when the current flows through it. Due to the inference of the magnetic flux an alternating emf is induced in the coil. It is called back emf, and is due to self-induction. The back emf will oppose the rise or fall of current through it, and is equal and opposite to the applied emf.

The instantaneous voltage equation, $v = L.\frac{di}{dt}$ ……………… (1)

where, $v = V_m \sin \omega t$ , L= self-inductance.

Equate both voltage equations, we get, $L.\frac{di}{dt} = V_m \sin \omega t = v$ ……………………. (2)

$$di = \frac{V_m}{L} \sin \omega t.dt \text{ ……………….. } (3)$$

Integrating both sides, $\int di = \int \frac{V_m}{L} \sin \omega t.dt$ ……………… (4)

$$\int di = \frac{V_m}{L}\int \sin \omega t.dt \text{ ………………..} \quad (5)$$

$$i = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right) \text{ ……………………….} \quad (6)$$

$$i = \frac{V_m}{\omega L}(-\cos \omega t) \text{ …………………….} \quad (7)$$

$$i = \frac{V_m}{\omega L}(\sin \omega t - 90^0) \text{ ……………….} \quad (8)$$

Current ‘i’ is max $I_m$, when $(\sin \omega t - 90^0)$ is unity.

$$I_m = \frac{V_m}{\omega L} \text{ …………………………………..} \quad (9)$$

Substitute eqn 9 in 8 we get, $i = I_m(\sin \omega t - 90^0)$

By analyzing the above equation, we can see that the current lags behind the applied voltage by $90^0$ or the phase difference between the two is $\pi/2$ radians as shown in figure below. The quantity $\omega L$ is known as inductive reactance and denoted by $X_L$ and is measured in $\Omega$.

### 3.1. Instantaneous Power

We know that power $P = v \times i \; watt$ where, $v = V_m \sin \omega t$ and $i = I_m(\sin \omega t - 90^0)$. Substitute the values of v and i in power equation, we get $P = V_m \sin \omega t \times I_m(\sin \omega t - 90^0)$ …………… (10)

$$P = V_m I_m \sin \omega t \times (-\cos \omega t) \text{ …………………} (11)$$

$$P = -V_m I_m \sin \omega t \times \cos \omega t \text{ ………………….} \quad (12)$$

Multiply and divide the equation 12 by 2, we get

$$P = \frac{-V_m I_m}{2} \times 2\sin \omega t \cos \omega t \text{ ………………….} (13)$$

$$P = \frac{-V_m I_m}{2} \times \sin 2\omega t \text{ ………………….} \quad (14)$$

$$P = -\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} \sin 2\omega t \text{ ……………….} \quad (15)$$

$$P = -V_{rms} I_{rms} \sin 2\omega t \text{ ……………….} \quad (16)$$

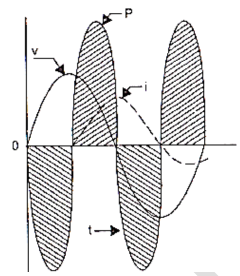
This is the instantaneous power equation.

Total power for the whole cycle is

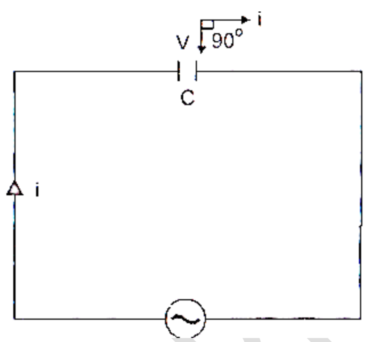
$$P = \int_0^{2\pi} -\frac{V_m I_m}{2}(\sin 2\omega t)dt = 0,$$

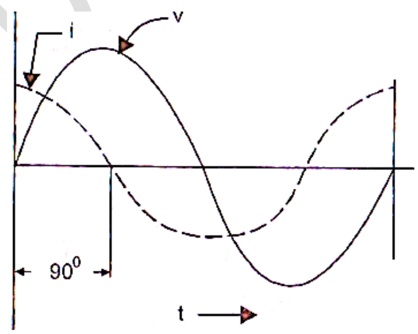
Here power has no constant term. The power term exists of the double the frequency whose average value over a whole cycle is zero. Thus P=0.

$$P = \int_0^{2\pi} -\frac{V_m I_m}{2}(\sin 2\omega t)dt = 0$$




## 4. CURRENT THROUGH PURE CAPACITOR




Consider a circuit consist of pure capacitor C connected across a voltage $v = V_m \sin \omega t$. When an alternating emf is applied across the capacitor, the potential difference between the plates is given $v = \frac{q}{c}$ where q is the charge and c is the capacitance of the capacitor. Substitute the instantaneous value of voltage in the above equation,

$$V_m \sin \omega t = \frac{q}{c} \quad \text{.................................................(1)}$$

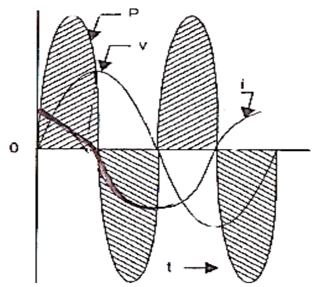
$$q = CV_m \sin \omega t$$

Now, current $i$ is given by the rate of flow of charge. ie, $i = \frac{dq}{dt}$ ………………(2)

Substitute the value of q in the above equation, $i = \frac{d(CV_m \sin \omega t)}{dt}$, ……… (3)

Differentiate RHS of the equation, $i = \omega C V_m \cos \omega t$ ……… (4)

$$i = \frac{V_m}{1/\omega C} \cos \omega t \quad \text{………} \quad (5)$$




$$i = \frac{V_m}{1/\omega C} \sin(\omega t + 90^0) \text{ ………………}(6)$$

Current 'i' is maximum when $\sin(\omega t + 90^0)$ is unity

$$I_m = \frac{V_m}{1/\omega C}$$

$$\therefore i = I_m \sin(\omega t + 90^0) \text{ ………} \quad (7)$$

The factor $1/\omega C$ corresponds to the resistance of the capacitor and is known as its reactance usually called **capacitive reactance** and is denoted by $X_c$, $X_c = \frac{1}{2\pi fC}$

Thus when the applied emf is given by v=$V_m \sin \omega t$

The current in the capacitive circuit is given by $i = I_m \sin(\omega t + 90^0)$,

from the above equations we can see that the current in a capacitive circuit leads the applied voltage by $90^0$ or $\frac{\pi}{2}$ radians.

**4.1Power in a pure Capacitor**

We know that $P = vi$ where

$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t + 90)$$
$$P = V_m \sin \omega t I_m \sin(\omega t + 90)$$
$$P = V_m I_m (\sin \omega t \times \cos \omega t)$$

Divide and multiply the above eqn by 2,

$$P = \frac{V_m I_m (2 \times \sin \omega t \times \cos \omega t)}{2}$$

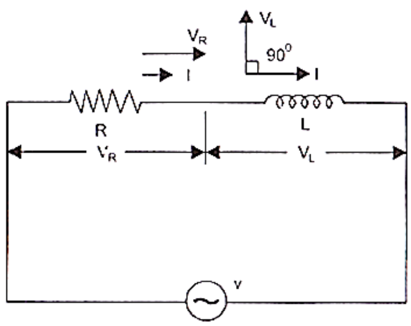
$$P = \frac{V_m I_m}{2} \sin 2\omega t \qquad P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

$$P = V_{rms} I_{rms} \sin 2\omega t$$

The total power for the whole cycle in pure capacitor is zero, P = 0

## 5. AC THROUGH R-L SERIES CIRCUIT

Consider a circuit consisting of a pure resistance R ohm and a pure inductance L henry connected in series.



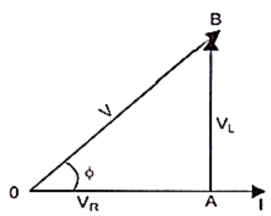

Let I = rms value of total current

V = rms value of applied voltage

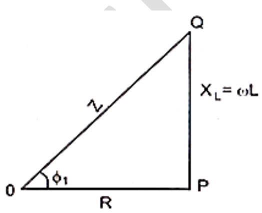
$V_R$ = voltage drop across R

$V_L = I\omega L = IX_L$ = voltage drop across L

The vector diagram represents the voltage drops. Vector OA represents $V_R$, and AB represents $V_L$. The applied voltage V is the vector sum of $V_L$ and $V_R$




Vector diagram




Impedance triangle

$$OB = \sqrt{(OA)^2 + (AB)^2}$$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= \sqrt{I^2(R^2 + X_L{}^2)} = I\sqrt{(R^2 + X_L{}^2)}$$

$$I = \frac{V}{\sqrt{(R^2 + X_L{}^2)}}$$

The quantity $\sqrt{(R^2 + X_L{}^2)}$ is expressed in ohm and is known as impedance of the circuit and is denoted by the letter Z, therefore, impedance Z=$\sqrt{(R^2 + X_L{}^2)}$.

Thus $I = \frac{V}{z}$ or $V = IZ$. From the figure we can find that current I lags behind the applied voltage by and angle $\theta$. The cosine of the angle between voltage and current is known as power factor.

$$pf = \cos\theta = \frac{R}{Z}$$

## 5.1 Power in R-L Circuit

We know that $P = vi$ where

$$v = V_m \sin\omega t$$
$$i = I_m \sin(\omega t - \phi)$$
$$P = V_m \sin\omega t I_m \sin(\omega t - \phi)$$
$$P = V_m I_m \sin\omega t \times \sin(\omega t - \phi) \quad [2\,\mathrm{Sin}\, A\,\mathrm{Sin}\, B = \mathrm{Cos}(A - B) - \cos(A + B)]$$

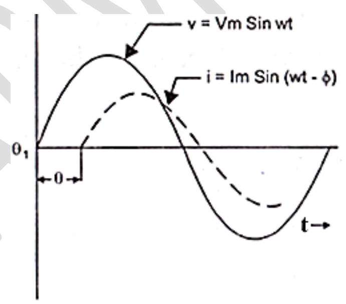
Divide and multiply the above eqn by 2,

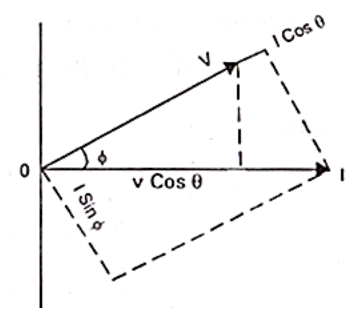
$$P = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t - \phi]$$

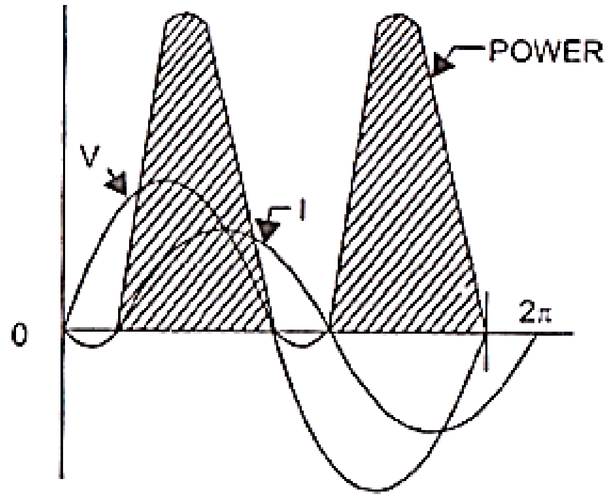
Thus power consists of a constant part $\frac{V_m I_m}{2}$ $\cos\phi$ and a variable part $\frac{V_m I_m}{2}[\cos(2\omega t - \phi]$ of double frequency whose average value over a cycle is zero.

Hence average power = $P = \frac{V_m I_m}{2}[\cos\phi] = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}[\cos\phi]$ $= V_{rms} I_{rms} \cos\phi$

$$P = VI\cos\phi \;\; watt$$






Hence, the power is no longer a product of V and I. The mean power is given by the product of voltage V and that part of current I which is in phase with V and the power factor.

$$P = VI\cos\theta$$

$$= voltage \times current \times power\ factor$$

$$kW = kVA \times p.f.$$

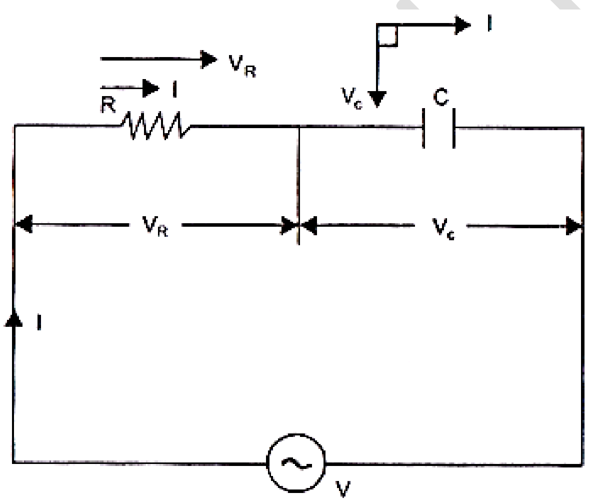
$$p.f. = \frac{kW}{kVA}$$

## 6 AC THROUGH R-C SERIES CIRCUITS

Consider a circuit consisting of R and C in series as in figure. Let $V_R$ ($V_R = IR$ )be the resistance drop across the resistance and $V_C$ ($V_C = IX_C$) be the capacitive drop across capacitor . As $X_C$ is taken negative, $V_C$ also taken as negative, as in figure.

From the voltage triangle, $V^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2$

$$= I^2(R^2 + X_C^2)$$




$$V = I\sqrt{(R^2 + X_C^2)}$$

$$I = \frac{V}{\sqrt{(R^2 + X_C^2)}} = \frac{V}{Z} \qquad \sqrt{(R^2 + X_C^2)} = Z$$

From the figure we can see that, the current I leads the voltage V by angle $\phi$ and the two quantities are represented by the equations as follows

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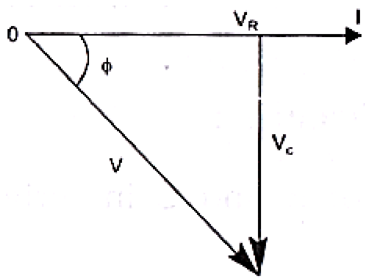
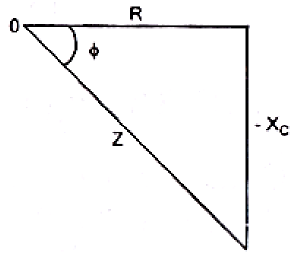



$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \phi)$$

Or $v = V_m \sin(\omega t - \phi)$

$$i = I_m \sin \omega t$$



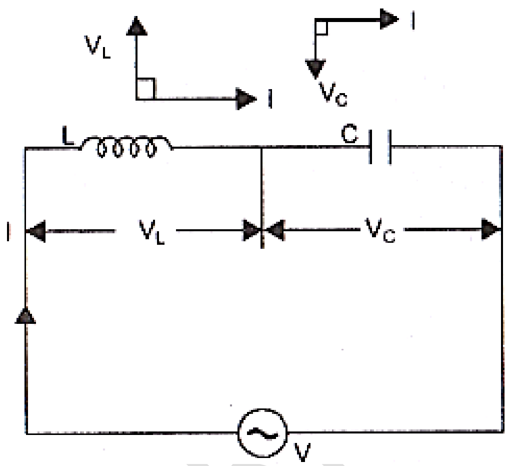

From the impedance triangle

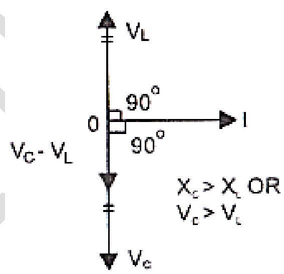
Power factor = $\cos\phi = \dfrac{R}{Z}$

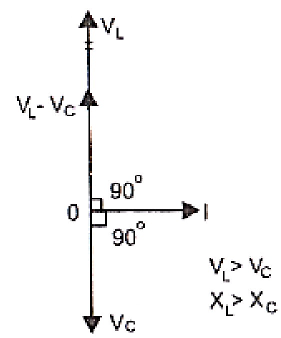
$Power\ consumed = v \times i$

$= VI\cos\phi$

## 7 AC THROUGH L-C SERIES CIRCUIT




Consider a circuit consisting of Inductor (L) and Capacitor(C) connected in series and the applied voltage is v.

Here the only opposite to current flow is the net reactance X which is given by the formula $X = X_L - X_C$ *OR* $X = X_C - X_L$ If $X_L$ is more the resulting reactance has the characteristics of **inductive reactance.** If $X_C$ is more the resulting reactance has characteristics of **capacitive reactance.**

Thus Z= X and

$$I = \frac{V}{Z} = \frac{V}{X}$$

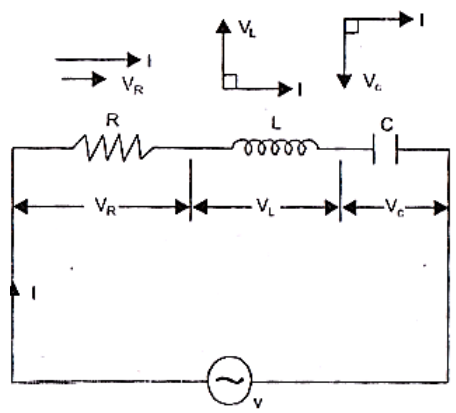
$$\cos\phi = 0 \quad (\phi = 90^0)$$

Power consumed =vi $\cos\phi$ .

Hence power consumed = 0.

## 8 AC THROUGH R-L-C SERIES CIRCUIT

Consider a circuit consisting of Resistor (R), Inductor (L) and Capacitor(C) connected in series and the applied voltage is v.



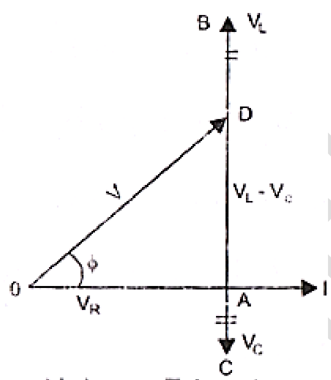

Let,

$V_R = IR$

$V_L = IX_L$

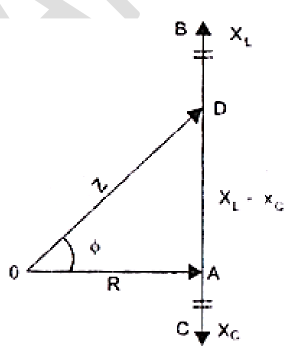
$V_C = IX_C$

$v = V_m \sin \omega t$ .



Voltage Triangle

In voltage triangle $V_L, V_C$ are $180^0$ out of phase with each other. It has been assumed that $V_L > V_C$



Impedance Triangle

The net reactive drop = AB-BD= $V_L - V_C$

from voltage triangle

$$OD^2 = OA^2 + AD^2$$

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$= (IR)^2 + (IX_L - IX_C)^2$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where Z is the impedance of the circuit

$Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$

$(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Net reactance})^2$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (X)^2}}$

The voltage and current equations in RLC circuit are,

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t \pm \phi)$$

+ve sign is to be used when $X_C > X_L$ (or current leads)

-ve sign is to be used when $X_L > X_C$ (or current lags)

**8.1 POWER IN RLC SERIES CIRCUIT**

$$Power\ consumed = v \times i$$

$$= VI \cos\phi$$

**9 RESONANCE IN R-L-C SERIES CIRCUIT**

A series circuit is said to be in electrical resonance when its net reactance is zero.ie, $X_L - X_C = 0$ where $X_L = X_C$. The impedance of the circuit can be found out from the equation,

$Z = \sqrt{R^2 + (X)^2}$. Here $(X)^2$ be the net reactance.

The supply voltage is kept constant, the frequency of the supply varying from zero to infinity, at a certain frequency of applied voltage the inductive reactance ( $X_L$ ) becomes equal to the capacitive reactance ( $X_C$ ) in magnitude. This situation is known as resonance. The frequency at which $X_L = X_C$ is known as **resonance frequency** and is represented by $f_r$. At resonance condition the net reactance X=0. $\therefore$ The impedance will be equal to resistance of the circuit 'R'. Therefore the circuit will act as a purely resistive circuit and current is in phase with the applied voltage v.

We know that, $V_L = IX_L$ and $V_C = IX_C$. From the figure, we can see that $V_L = V_C$ and in opposite. Therefore they are opposite in direction (phase). Hence both the voltages are cancel each other.

The two reactances ( $X_L, X_C$ ) taken together act as a short circuit since no voltage develops across them. The whole of the applied voltage drops across the resistance.

ie, $V = IR$.

## 9.1 CALCULATION OF RESONACE FREQUENCY

The frequency at which $X_L = X_C$ is known as **resonance frequency** and is represented by $f_r$.

$$\text{Where,}\quad X_L = L\omega \quad and \quad X_C = \frac{1}{C\omega}$$

At resonant condition, $L\omega = \dfrac{1}{C\omega}$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$(2\pi f_r)^2 = \frac{1}{LC}$$

$$(f_r)^2 = \frac{1}{(2\pi)^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

This is the formula for finding resonance frequency. At resonance condition, the RLC circuit possess minimum impedance because Z=R. therefore, the current is flowing through the circuit is maximum. It produces large voltage drop across L and C. but these drops are equal and opposite. Therefore they can each other.

## 10 Q-FACTOR OF A SERIES CIRCUIT

It is the voltage magnification in the series circuit at resonance. The formula for finding Q-factor as follows

$$Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$