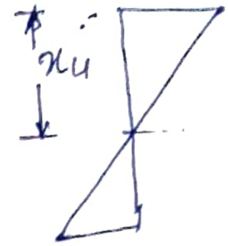
Depth of Neutral axis (x_u)

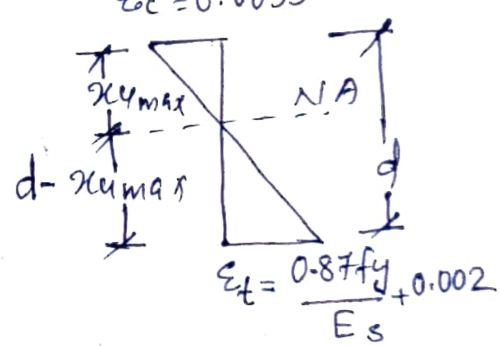
The depth of Neutral axis is defined as the distance of neutral axis from extreme compression fibre.

At neutral axis there is no stress developed.



Limiting depth of NA. (x_{umax})

As per IS:456-2000, Page 69, assumptions (b) & (f) governs the maximum depth of neutral axis (x_{umax}) in members subjected to flexure. from figure considering similar triangles of strain diagram, x_{umax} is obtained as.



$$\frac{x_{umax}}{0.0035} = \frac{d - x_{umax}}{\frac{0.87 f_y}{E_s} + 0.002}$$

$$\therefore \frac{x_{umax}}{d - x_{umax}} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.002}$$

$$\therefore \frac{x_{umax}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

For

Fe 250 grade steel

$f_y = 250\ N/mm^2$.

$E_s = 2\times10^5\ N/mm^2$.

$$\frac{x_{umax}}{d} = \frac{0.0035}{0.0055 + \frac{0.87 f_y}{E_s}}$$

$$= \frac{0.0035}{0.0055 + 0.00108} = \underline{\underline{0.53}}.$$

$$\frac{0.87\times250}{2\times10^5} = 0.00108$$

For fe415

$f_y = 415\ N/mm^2$

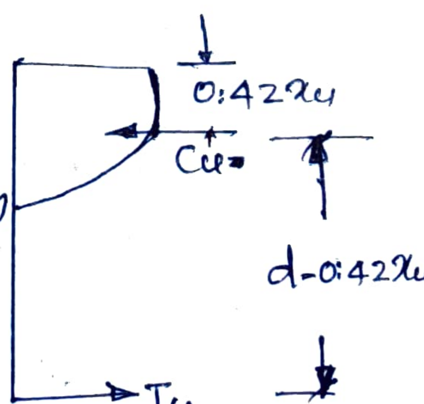
$E_s = 2\times10^5\ N/mm^2$.

$$\frac{x_{umax}}{d} = \frac{0.0035}{0.0055 + 0.0018} = \underline{\underline{0.48}}$$

$$\frac{0.87\times415}{2\times10^5} = 0.0018$$

Lever Arm.

Lever arm is the distance between the point of application of Compressive force and the point of application of Tensile force. It is denoted as 'z'



$\therefore$ Lever arm, $z = d - 0.42x_u$.

Moment of Resistance (M).

ultimate moment of Resistance (Mu).

Mu = Total compression or tension x levera

ie, Moment of resistance is the product of compressive or tensile force and lever arm

$$Mu = 0.87\, fy\, Ast.\, d \left[1 - \frac{Ast\, fy}{fck\, bd}\right].$$ Page 96 of IS:456-2

where,

Mu = ultimate moment of resistance.

fy = characteristic strength of reinforcement

fck = characteristic strength of concrete

d = effective depth.

b = width of the section.

Ast = Area of steel reinforcement

Limiting Moment of Resistance, [Mu lim].

$$Mu\,lim = 0.36 \frac{xumax}{d}\left[1 - 0.42 \frac{xumax}{d}\right] fck\, bd^2.$$ Page 96

where,

x ~~Give steps~~

Mu lim = limiting moment of resistance.

$\frac{xumax}{d}$ = limiting depth of Neutral axis.

fck = characteristic strength of concrete.

* Give steps for determining moment of resistance of a beam.

The moment of resistance of a beam can be obtained as follows (As per IS:456-2000, Annex G. Page 96)

1. For the given grade of concrete and steel (f_{ck} and f_y) are known. find the depth of neutral axis of the given section.

$$\frac{x_u}{d} = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ bd}$$

or $$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b}.$$

2. Find the limiting value of neutral axis, ie, $\frac{x_{u\,max}}{d}$ by the following expression or refer Table 4.2. of Page 70 of IS:456-2000

$$\frac{x_{u\,max}}{d} = \frac{0.0035}{\frac{0.87\ f_y}{E_s} + 0.0055}$$

3. Compare x_u and $x_{u\,max}$.

i) if $\frac{x_u}{d} = \frac{x_{u\,max}}{d}$, the beam is designed as balanced section and moment of resistance of the section is given by following expression

$$M_{u\,lim} = 0.36 f_{ck} \frac{x_{umax}}{d}\left[1 - 0.42\frac{x_{umax}}{d}\right] f_{ck}\, bd^2$$

ii) If $\frac{x_u}{d} < \frac{x_{umax}}{d}$, the beam is unbalanced section i.e, under reinforced section and moment of resistance is calculated by the following equation.

$$M_u = 0.87 f_y A_{st}\, d\left[1 - \frac{A_{st} f_y}{f_{ck}\, bd}\right].$$

or

$$M_u = 0.87 f_y A_{st}\, d\left[1 - \frac{0.42 x_u}{d}\right].$$

iii) if $\frac{x_u}{d} > \frac{x_{umax}}{d}$, the moment of resistance of the section is equal to $M_{u\,lim}$, but the IS: 456-2000 code recommends that the section is to be redesigned as it is a case of unbalanced section, i.e, over reinforced section.

Limiting percentage of steel.

The limiting percentage of tensile steel corresponding to the limiting moment of resistance, limiting percentage of steel, $P_{t\,lim} = \frac{100\, A_{st\,lim}}{bd}$.

from stress block diagram,

Maximum compressive force, $C_{u\,lim} = 0.36 f_{ck}\, b\, x_{umax}$

maximum Tensile force, $T_{u\,lim} = 0.87 f_y A_{st\,lim}$.

To find P_t lim,

equating $T_{u\,lim} = C_{u\,lim}$.

$$0.87\, f_y\, A_{st\,lim} = 0.36\, f_{ck}\, b\, x_{umax}.$$

dividing by bd.

$$\frac{0.87\, f_y\, A_{st\,lim}}{bd} = \frac{0.036\, f_{ck}\, b\, x_{umax}}{bd}.$$

$$\frac{A_{st\,lim}}{bd} = \frac{0.36\, f_{ck}\, x_{umax}}{0.87\, f_y\, d}$$

$$P_{t\,lim} = \frac{A_{st\,lim} \times 100}{bd}.$$

$$\therefore P_{t\,lim} = \frac{0.36\, f_{ck}\, x_{umax}}{0.87\, f_y\, d} \times 100.$$

- For Fe 250 grade steel.-

$$P_{t\,lim} = \frac{0.36\, f_{ck}\, x_{umax}}{0.87\, f_y \cdot d} \times 100$$

Fe 250, $\frac{x_{umax}}{d} = 0.53$. (Page 70)

$$\therefore P_{t\,lim} = \frac{0.36\, f_{ck}}{0.87\, f_y} \times 0.53 \times 100 = 21.97 \frac{f_{ck}}{f_y}.$$

- For Fe 415 grade steel.

$$\frac{x_{umax}}{d} = 0.48.$$

$$P_{t\,lim} = \frac{0.36\, f_{ck}}{0.87\, f_y} \times 0.48 \times 100 = 19.86 \frac{f_{ck}}{f_y}$$

- For Fe 500 grade steel.

$$\frac{x_{umax}}{d} = 0.46.$$

$$P_t = \frac{0.36\ f_{ck} \cdot x_{umax} \times 100}{0.87 \cdot f_y \quad d}$$

$$= \frac{0.36\ f_{ck} \times 0.46 \times 100}{0.87 \times f_y} = \underline{18.87 \frac{f_{ck}}{f_y}}$$

* Limiting moment of resistance.

For Fe 250 grade steel

$$M_{ulim} = 0.36 \frac{x_{umax}}{d}\left[1 - 0.42 \frac{x_{umax}}{d}\right] bd^2 f_{ck}.$$

$$\frac{x_{umax}}{d} = 0.53.$$

$$M_{ulim} = 0.36 \times 0.53 \left[1 - (0.42 \times 0.53)\right] bd^2 f_{ck}.$$

$$= 0.\underline{149}\ f_{ck}\ bd^2$$

* For Fe 415 grade steel.

$$\frac{x_{umax}}{d} = 0.48$$

$$M_{ulim} = 0.36 \times 0.48 \left[1 - (0.42 \times 0.48)\right] f_{ck}\ bd^2$$

$$= 0.\underline{138}\ f_{ck}\ bd^2$$

- For Fe 500 grade steel.

$$\frac{x_{umax}}{d} = 0.46.$$

$$M_{ulim} = 0.36 \times 0.46 \left[1 - (0.42 \times 0.46)\right] f_{ck}\ bd^2$$

$$= 0.\underline{133}\ f_{ck}\ bd^2.$$

Grade of Steel	$\frac{x_{u\,max}}{d}$	$M_{u\,lim}$	$P_{t\,lim}$
Fe 250	0.53	$0.149\, f_{ck}\, bd^2$	$21.97\, \frac{f_{ck}}{f_y}$
Fe 415	0.48	$0.138\, f_{ck}\, bd^2$	$19.86\, \frac{f_{ck}}{f_y}$
Fe 500	0.46	$0.133\, f_{ck}\, bd^2$	$18.87\, \frac{f_{ck}}{f_y}$.

Types of Section.

There are two types of section.
i) Balanced section
ii) Unbalanced section.

Balanced section.

If the area of tensile reinforcement is provided is equal to the area of steel reinforcement required in a section is called Balanced section.

Unbalanced Section.

If the area of tensile reinforcement provided is more or less than, whatever required for a balanced section is known as unbalanced section.

Unbalanced section further classified as
i) Under reinforced section.
ii) Over reinforced Section.

Under reinforced section.

If the steel area provided is less than what is required for a balanced section is known as under reinforced section.

Over reinforced section.

If the area of steel provided is more than what is required for a balanced section. Section is known as Over reinforced section.

Section.	x_u	M_u	Ast.
Balanced section	$x_u = x_{umax}$	$M_u = M_{ulim}$	$A_{st\,prov.} = A_{st\,req.}$
Under reinforced section	$x_u < x_{umax}$	$M_u < M_{ulim}$	$A_{st\,prov.} < A_{st\,req.}$
Over reinforced section	$x_u > x_{umax}$	$M_u > M_{ulim}$	$A_{st\,prov.} > A_{st\,req.}$

Moment of resistance of under reinforced section.

Depth of neutral axis, $\frac{x_u}{d} = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b}$

If $\frac{x_u}{d} < \frac{x_{umax}}{d}$

$$M_u = 0.87\ f_y\ A_{st} \cdot d \left[1 - \frac{A_{st}\ f_y}{f_{ck}\ bd}\right].$$

OR.

$$M_u = 0.87\ f_y\ A_{st}\ (d - 0.42 x_u).$$

Moment of resistance of over reinforced section

Depth of neutral axis, $\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$.

if $\frac{x_u}{d} > \frac{x_{umax}}{d}$, $\frac{x_u}{d}$ is limited to $\frac{x_{umax}}{d}$.

$\therefore M_u = M_{ulim} = 0.36 \frac{x_{umax}}{d} \left[1 - 0.42 \frac{x_{umax}}{d}\right] f_{ck} bd^2$

OR

$M_{ulim} = 0.36 f_{ck} b x_{umax} (d - 0.42 x_{umax})$.

1. Determine the moment of resistance of a beam of dimension 250 mm x 350 mm. The area of steel consists of 3 bars of 12 mm diameter placed at a distance of 40 mm from bottom of beam. Use M20 concrete and Fe 415 grade steel.

Given,

$b = 250 mm$.

$D = 350 mm$.

$d_c = 40 mm$.

$d = D - d_c = 350 - 40 = 310 mm$.

$A_{st} = 3 \times \frac{\pi}{4} (12)^2 = 339 mm^2$.

M_{20} concrete, $f_{ck} = 20 N/mm^2$

Fe415 steel, $f_y = 415 N/mm^2$.

Depth of neutral axis, (x_u).

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 339}{0.36 \times 20 \times 250 \times 310} = \cancel{0.40}\ 0.219$$

Limiting depth of neutral axis ($x_{u\,max}$).

for Fe415 grade steel.

$$\frac{x_{u\,max}}{d} = 0.48.$$

Compare $\frac{x_u}{d}$ and $\frac{x_{u\,max}}{d}$.

$$\frac{x_u}{d} = \cancel{0.40}\ 0.219$$

$$\frac{x_{u\,max}}{d} = 0.48.$$

$\therefore \frac{x_u}{d} < \frac{x_{u\,max}}{d}$, $\frac{0.219}{\cancel{0.40}} < 0.48$. Hence it is an

under reinforced section.

$$\therefore M_u = 0.87 f_y A_{st}\, d \left(1 - \frac{A_{st} f_y}{f_{ck} b d}\right).$$

$$= 0.87 \times 415 \times 339 \times 310 \left(1 - \frac{339 \times 415}{20 \times 250 \times 310}\right).$$

$$= \cancel{34415759\ N mm} =$$

$$= 34497889.66\ N mm = \underline{\underline{34.49}}\ kN.m.$$

2 Determine the ultimate moment of resistance of a singly reinforced simply supported beam having size 200 mm × 400 mm (effective), reinforced with 3 numbers of 16 mm diameter bars, Concrete is of M_{20} grade and Fe 415 grade steel.

Given,

$$b = 200\,mm$$

$$d = 400\,mm.$$

$$A_{st} = 3 \times \frac{\pi}{4}(16)^2 = 602.88\ mm^2.$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b\, d.} = \frac{0.87 \times 415 \times 602.88}{0.36 \times 20 \times 200 \times 400} = 0.377.$$

For Fe 415

$$\frac{x_{umax}}{d} = 0.48$$

Compare $\frac{x_u}{d}$ and $\frac{x_{umax}}{d}$.

$0.377 < 0.48 \therefore \frac{x_u}{d} < \frac{x_{umax}}{d} \therefore$ Under reinforced

Section i.e,

$$M_u = 0.87 f_y A_{st} \, d \left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right).$$

$$= 0.87 \times 602.88 \times 415 \times 400 \left(1 - \frac{602.88 \times 415}{200 \times 400 \times 20}\right).$$

$$= \underline{73.45 \text{ KNm}}.$$

... of resistance of a