

**Module -III****ALTERNATING CURRENT THROUGH PARALLEL CIRCUITS****1. INTRODUCTION**

1. Resistance: - It is the opposition offered by a substance or body to the flow of and electric current through it. It is represented by letter R and is measured in ohm ( $\Omega$ ).
2. Inductance: It is the property of a coil due to which it opposes any increase or decrease of current or flux through it. It is represented by letter L and is measured in henry (H).
3. Capacitance:- It is the property of a capacitor to store electricity or the amount of charge required to create a unit potential difference between plates. It is represented by letter C and is measured in farad (F).

**PARALLEL CIRCUITS**

Parallel circuits are those in which two or more branches have the same impressed voltage. When calculating the total current taken by a number of parallel paths joined in parallel having the same voltage, calculate separately the branch currents and then sum up vectorially. All the rule, applicable to dc parallel circuits, can be applied to ac parallel circuits, the only difference that the phase angles must be considered in the case of ac parallel circuits. When impedances are joined in parallel, then there are three methods for solving such circuits:

1. Vector method
2. Admittance method
3. Symbolic or Complex algebra or “j” method

Here we are considering vector or phasor method only.

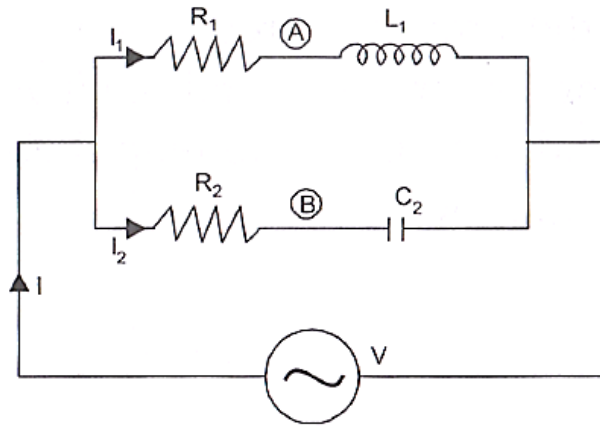
**Vector Method**

The following points should be kept in mind in solving parallel circuit is by vector method

1. Since voltage across each parallel path is same, therefore it is taken as reference vector
2. Each branch current is determined separately, and its angle (lead or lag)
3. The resultant current is obtained by adding the branch vectorially.

Consider a circuit having two branches A and B are connected in parallel across a voltage source V. The voltage across branches A and B will same and the current will be different  $I_1$

$$Z_1 = \sqrt{R_1^2 + (X_1)^2}$$



and  $I_2$  as in figure.

Consider branch A,

Since the branch A consisting of inductance, the current will lag the applied voltage by an  $\phi_1$ .

Consider the branch B,

$$X_C = \frac{1}{C\omega}$$

$$Z_2 = \sqrt{R_2^2 + (X_2)^2}$$

$$I_2 = \frac{V}{Z_2}$$

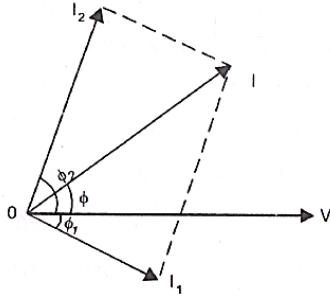
$$\cos \phi_2 = \frac{R_2}{Z_2} \quad \text{and} \quad \phi_2 = \cos^{-1} \left( \frac{R_2}{Z_2} \right)$$

Since the branch B is more capacitive in nature, the current  $I_2$  will lead the applied voltage by an angle  $\phi_2$ .

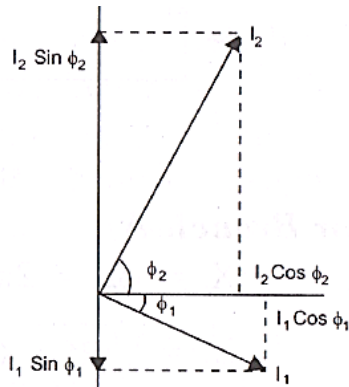
Resultant current  $I = I_1 + I_2$ . The resultant circuit current  $I$  is the vector sum of the branch current  $I_1$  and  $I_2$ . This can be found out using the parallelogram.

The total current  $I$  and power factor can be found by

- i. Parallelogram law of forces



- ii. Resolving the branch current  $I_1$  and  $I_2$  into their X and Y components (or active and reactive components) respectively and they by combining these components as in figure.



$$\text{X-components of } I_1 \text{ and } I_2 = I_1 \cos \phi_1 + I_2 \cos \phi_2$$

$$\text{Y-components of } I_1 \text{ and } I_2 = -I_1 \sin \phi_1 + I_2 \sin \phi_2$$

$$\text{Therefore, total current, } I = \sqrt{(X\text{-component})^2 + (Y\text{-component})^2}$$

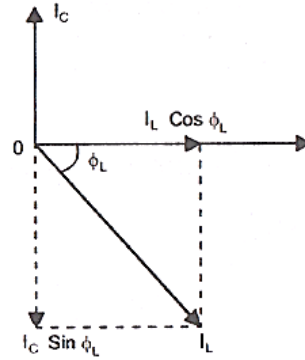
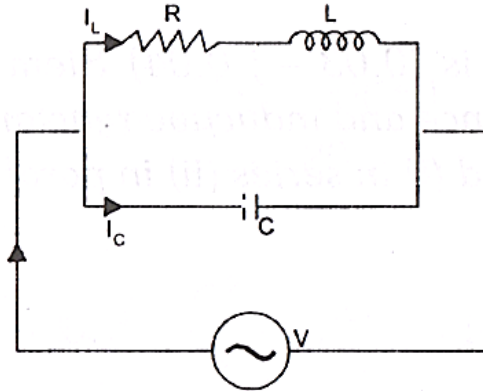
$$= \sqrt{(I_1 \cos \phi_1 + I_2 \cos \phi_2)^2 + (-I_1 \sin \phi_1 + I_2 \sin \phi_2)^2}$$

$$\tan \phi = \frac{Y\text{-component}}{X\text{-component}}$$

$$\phi = \tan^{-1} \left( \frac{Y\text{-component}}{X\text{-component}} \right), \quad \phi = \tan^{-1} \left( \frac{-I_1 \sin \phi_1 + I_2 \sin \phi_2}{I_1 \cos \phi_1 + I_2 \cos \phi_2} \right)$$

$$\text{Then power factor} = \cos \phi$$

## 11.1 RESONANCE IN PARALLEL CIRCUITS



A parallel circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as resonant frequency. Let us consider a circuit consisting of a coil in parallel with a capacitor as in figure.

$$\text{Net reactive component} = I_C - I_L \sin \phi_L$$

As at resonance its value is zero,  $I_C - I_L \sin \phi_L = 0$ .

$$\text{Now } \frac{I_L}{Z} = \frac{V}{X_L} \sin \phi_L, \quad \frac{I_C}{Z} = \frac{V}{X_C} \quad \text{and} \quad I_C = \frac{V}{X_C}$$

Substituting the above values in the equation,

$$I_C + I_L \sin \phi_L$$

$$\frac{V}{X_C} = \frac{V}{Z} \cdot \frac{X_L}{Z}$$

$$Z^2 = X_L \cdot X_C \quad X_L = L\omega \quad \text{and} \quad X_C = \frac{1}{C\omega}$$

$$Z^2 = \frac{L\omega}{C\omega} \quad \text{or} \quad Z^2 = \frac{L}{C} \quad \text{or} \quad Z^2 = \frac{L}{C} = R^2 + X_L^2$$

$$\frac{L}{C} = R^2 + (2\pi f_r L)^2$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

This is the resonant frequency and is given in Hz if R is in ohm, L in henry and C in farad. If R is negligibly small then.

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

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The circuit current  $I = I_L \cos \phi_L$ ,  $\therefore$  wattles component is zero.

$$I = \frac{V}{Z} \cdot \frac{R}{Z} = \frac{VR}{Z^2}$$

Substituting the value  $Z^2 = \frac{L}{C}$ , we get

$$I = \frac{VR}{L/C}$$

$$I = \frac{V}{L/CR}$$

The denominator  $L/CR$  is known as the dynamic impedance of the parallel circuit at resonance, and it will be maximum. Hence the current will be minimum. At resonance, the parallel RLC circuit has high impedance and hence low current. When such a circuit used in radio stations, it is called rejector. Because it rejects or takes minimum current. It forms the basis for tuned circuits in electronics.

## 12 Q-FACTOR OF A PARALLEL CIRCUIT

It is the voltage magnification in the series circuit at resonance. The formula for finding Q-factor as follows

$$Q - factor = \frac{1}{R} \sqrt{\frac{L}{C}}$$