MOD II - SHAFTS

- Rotating machine element which is used to transmit power from one place to another
- Power is delivered to the shaft by some tangential force and the resultant torque setup within permits the power to be transferred to various machines
- SHAFT is used for the transmission of **TORQUE** and **BENDING MOMENT**

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12. The mechanical properties of these grades of carbon steel are given in the following table.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Types of Shafts

The following two types of shafts are important from the subject point of view:

1. *Transmission shafts*. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. *Machine shafts*. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Difference between shaft, axle and spindle

A shaft is a general term for a rotating component, usually circular in cross section, which is used to transmit power from one place to another.

A spindle is a type of shaft with specific features for mounting or driving parts of a machine such as lathes, mills, hard disc drives, etc.

An axle is a shaft that is central to a wheel or gear and often carries the load of the vehicle or machine. In most automobiles, the axle is an integral part of the drivetrain and helps transmit power from the engine to the wheels.

Stresses in Shafts

The following stresses are induced in the shafts:

i. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).
ii. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
iii. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength **2.** Rigidity and stiffness.

Design on the Basis of Strength

The following cases may be considered:

(*a*) Shafts subjected to twisting moment or torque only,
(*b*) Shafts subjected to bending moment only,
(*c*) Shafts subjected to combined twisting and bending moments,
(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

(a) Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation.

We know that,

$$\frac{T}{J} = \frac{\tau}{r} \quad \ldots(i)$$

Where

T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
τ = Torsional shear stress, and
r = Distance from neutral axis to the outer most fibre = d/2; where d is the diameter of the shaft in mm.

Polar moment of inertia J

It is the measure of a circular beam's ability to resist torsion. It is required to calculate the twist of a beam subjected to a Torque.

$J = \pi d^4/32$

Substituting J and r in eqn(i)

- **Torque** $\mathbf{T = \frac{\pi}{16} \times \tau \times d^3}$

Also, for hollow shaft, polar moment of inertia,

$J = \frac{\pi}{32} \times [(d_o)^4 - (d_i)^4]$

where do and di = Outside and inside diameter of the shaft, and $r = do / 2$.

i.e, for a hollow shaft,

- **Torque** $\mathbf{T = \frac{\pi}{16} \times \tau \times (d_o)^3 (1 - k^4)}$ Where $k = d_i/d_o$

For Belt Drives,

$$\mathbf{T = (T_1 - T_2) \times R}$$

- T_1 -Tension in the tight side
- T_2 -Tension in the slack side
- R - Radius of the pulley

Problems:

1. A line shaft rotating at 200rpm is to transmit 20kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42MPa. Determine the diameter of the shaft assuming only torsional effect.
2. A solid shaft is transmitting 1MW at 240rpm. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60MPa.
3. Find the diameter of a solid steel shaft to transmit 20kW at 200rpm. The ultimate shear stress for the steel may be taken as 360MPa and the factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

(b) Shafts Subjected To Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation.

$$\frac{M}{I} = \frac{\sigma b}{y} \quad \text{……………………………………}(1)$$

M = Bending moment,
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation
σb = Bending stress, and
y = Distance from neutral axis to the outer-most fibre.
for a round solid shaft, moment of inertia

$$I = \frac{\pi}{64} \times d^4$$

For a hollow shaft,

$$I = \frac{\pi}{64} \times [(do)^4 - (d_i)^4]$$

And for solid shaft, $y = d/2$, for hollow shaft, $y = d_o/2$
Substituting I and y in eqn (1),

For solid shaft, $$\mathbf{M = \frac{\pi}{32} \times \sigma_b \times d^3}$$

For hollow shaft, $$\mathbf{M = \frac{\pi}{32} \times \sigma_b \times (d_o)^3 (1-k^4)}$$ Where $k = d_i/d_o$

Problem:

1. A pair of wheels of a railway wagon carries a load of 50kN on each axle box, acting at a distance of 100mm outside the wheel base. The gauge of the rails is 1.4m. Find the diameter of the axle if the stress is not to exceed 100MPa.

(c) Shafts Subjected To Combined Twisting and Bending Moments

Based on:

- Maximum shear stress theory or Guest's theory
- Maximum normal stress theory or Rankine's theory

τ = Shear stress induced due to twisting moment

σ_b = Bending stress induced due to bending moment

Equivalent Twisting Moment T_e

$$Te = \sqrt{(M^2 + T^2)}$$

Also, $Te = \frac{\pi}{16} \times \tau \times d^3$

Equivalent Bending Moment M_e

$$M_e = \frac{1}{2} \times [M + \sqrt{(M^2 + T^2)}]$$

Also, $M_e = \frac{\pi}{32} \times \sigma_b \times d^3$

For hollow shaft:

$Te = \sqrt{(M^2 + T^2)} = \frac{\pi}{16} \times \tau \times (d_o)^3(1-k^4)$

$M_e = \frac{1}{2} \times [M + \sqrt{(M^2 + T^2)}] = \frac{\pi}{32} \times \sigma_b \times (d_o)^3(1-k^4)$

Problems:

1. A solid circular shaft is subjected to a bending moment of 3000N-m and a torque of 10,000N-m. The shaft is made of 45C8 steel having ultimate tensile stress of 700MPa and an ultimate shear stress of 500MPa. Assuming a factor of safety of 6, determine the diameter of the shaft.
2. A shaft made of mild steel is required to transmit 100kW at 300rpm. The supported length of the shaft is 3m. It carries two pulleys each weighing 1500N supported at a distance of 1m from each ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.
3. A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5m in diameter and has belt tensions 5.4kN and 1.8kN on the tight side and slack side of the belt respectively. If the pulley is overhung from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400mm, find the diameter of the shaft. Assume maximum allowable shear stress of 42MPa.