MODULE 2

TYPES OF BEAMS, SUPPORTS AND SIMPLE TRUSS

TYPES OF SUPPORTS

(a) Simple supports

(b) Roller support

(c) Pin-joint (or hinged) support

(d) Smooth surface support

(e) Fixed or built-in support.

Simple Support

When the end of a beam is kept simply on a smooth flat surface, the support is called simple support. The direction of reaction will be perpendicular to the support and the beam is free to move in the direction of its axis and also it is free to rotate about the support.

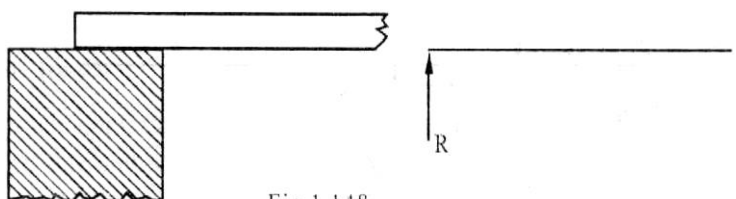


Fig.1.148

Roller Support

Beam is kept on a roller and the roller is supported as shown in fig.The direction of reaction will be perpendicular to the support of the roller.

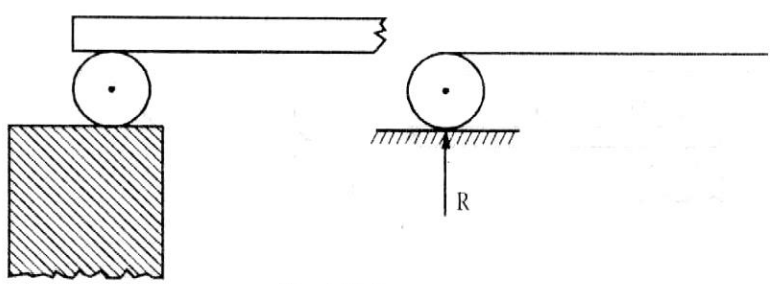


Fig.1.149

Hinged Support

Beam is hinged (pinned) to another rigid member. The beam cannot move in any direction but it can rotate about the hinge. The direction of reaction depends on the direction of external load on the beam. If there is no inclined or horizontal load on the beam, then the reaction at a hinged support will be vertical, otherwise the reaction will be inclined to the vertical.

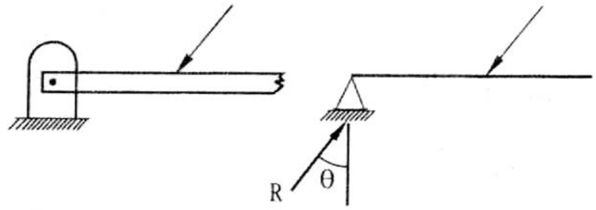


Fig.1.150

Hinged Support with roller

Beam is hinged to a rigid body and the rigid body is then kept on rollers and the rollers are supported. The direction of reaction will be always perpendicular to the support of the rollers.

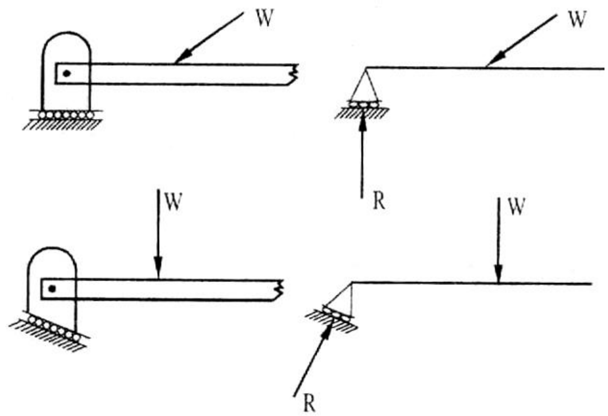


Fig.1.151

Fixed Support

When the end of beam is supported in such a way that it is not free to move or rotate, the support is called fixed support. Movement of beam is prevented by developing reaction in the required direction. Rotation of beam is prevented by developing moment at the support in the required sense.

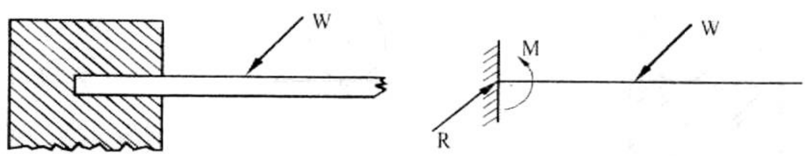


Fig.1.152

Smooth Surface Support: a body in contact with a smooth surface. The reaction will always act normal to the support

TYPES OF LOADING

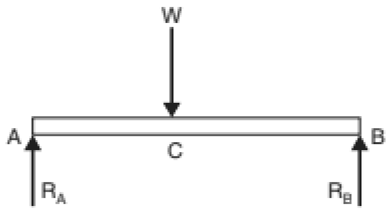
The following are the important types of loading :

(a) Concentrated or point load,

(b) Uniformly distributed load, and

(c) Uniformly varying load.

Concentrated or point load: When loads are applied at certain points in the beam, the loads are said to be point loads or concentrated loads.



Uniformly Distributed Load.

Uniformly Distributed Load (UDL) is a load which has the same intensity of load over a certain length of the beam.The total load will be w X x

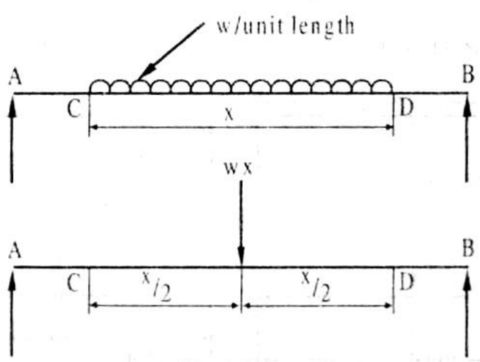


Fig.1.157

where w is the intensity of load and x is the loaded length. For calculating reaction at supports this load can be assumed to be acting as a point load at the middle of the loaded length.

When the load varies uniformly from one point to another point on the beam, the load is called uniformly varying load. When the intensity of load varies from zero at one end to w/unit length at the other end, over a length x, then the total load will be $\frac{1}{2} \times x \times w$ = area of the triangle. For the calculation of reaction at the supports, this total load can be considered as a point load acting at the centre of gravity of the triangle.

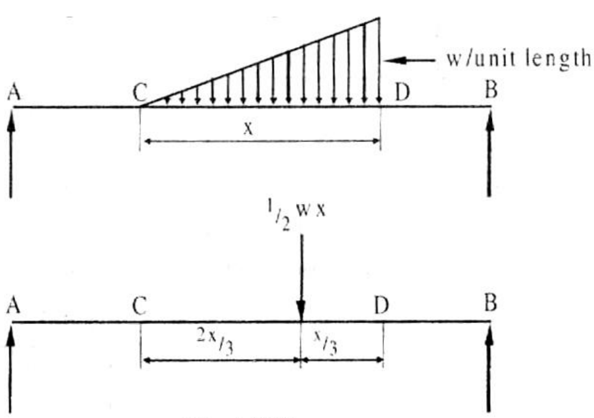
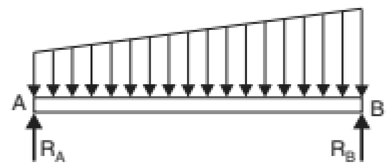


Fig.1.158

Uniformly Varying Load : a beam AB, which carries load in such a way that the rate of loading on each unit length of the beam varies uniformly. This type of load is known as uniformly varying load. The total load on the beam is equal to the area of the load diagram. The total load acts at the C.G. of the load diagram.



Reactions at supports of beams and frames

Due to the applied loads, reactions are developed at the supports and the system of forces consisting of applied loads and reactions keep the beam in equilibrium. The nature of reaction depends upon the type of supports and type of loads. The equations for equilibrium, $\sum F_H = 0,\ \sum F_V = 0,$ and $\sum M = 0$, can be applied to the beams and frames for calculating the reactions at the supports.

Example 1

A beam 6 m long is loaded as shown in fig. 1.160. Calculate the reactions at A and B.

Solution.

Consider the free-body diagram of beam as shown in fig. 1.161.

For $\sum F_v = 0,$

$R_A - 10 - 20 + R_B = 0$

$R_A + R_B = 30$ ....(i)

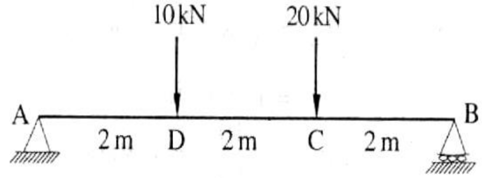


Fig.1.160

For $\sum M = 0$, taking moments about A,

$10 \times 2 + 20 \times 4 - R_B \times 6 = 0$.

$$R_B = \frac{100}{6} = 16.67\text{kN}$$

$R_A + R_B = 30$

$R_A + 16.67 = 30$

$R_A = 30 - 16.67$

$= 13.33$ kN

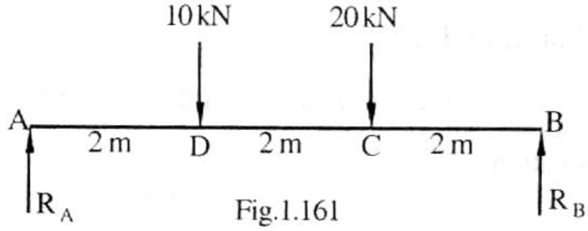


Fig.1.161

Example 2

A beam 6 m long is loaded as shown in fig. 1.162. Calculate the reactions at A and B.

Solution.

Consider the free-body diagram of beam as shown in fig. 1.163. The reaction at A is replaced by its horizontal and vertical components, R_{AH} and R_{AV} respectively.

For $\sum F_H = 0$

$$R_{AH} - 20\cos 60 = 0$$

$$R_{AH} = 20\cos 60 = 10\text{kN}$$

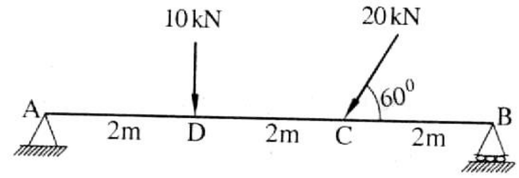


Fig.1.162

For $\sum F_V = 0$

$$R_{AV} - 10 - 20\sin 60 + R_B = 0$$

$$R_{AV} + R_B = 10 + 20\sin 60$$

$$= 10 + 17.32 = 27.32$$

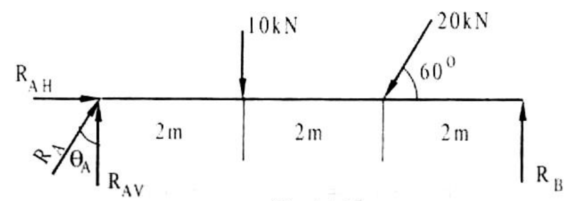


Fig.1.163

For $\sum M = 0$,

taking moments about A,

$$10 \times 2 + (20\sin 60) \times 4 - R_B \times 6 = 0$$

Example 3

A beam 6 m long is loaded as shown in fig. 1.166. Calculate the reactions at supports A and B.

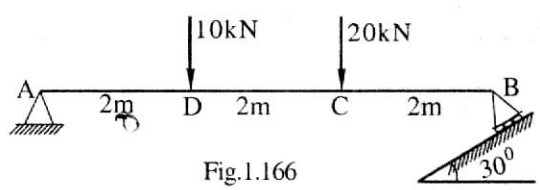


Fig.1.166

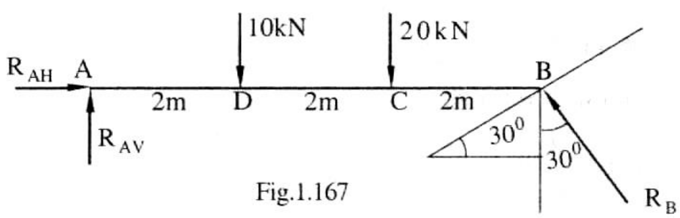


Fig.1.167

Solution.

Consider the free-body diagram of beam shown in fig. 1.167.

For $\sum F_H = 0$,

$$R_{AH} - R_B \sin 30 = 0$$

$$R_{AH} = R_B \sin 30 \quad ....(i)$$

For $\sum F_V = 0$,

$$R_{AV} - 10 - 20 + R_B \cos 30 = 0$$

$$R_{AV} + R_B \cos 30 = 30 \quad ....(ii)$$

For $\sum M = 0$, taking moments about A,

$$10 \times 2 + 20 \times 4 - (R_B \cos 30) \times 6 = 0$$

$$20 + 80 = 6 \ R_B \cos 30$$

$$R_B = \frac{100}{6\cos 30} = 19.25\text{kN}$$

From eqn (i),

$$R_{AH} = R_B \sin 30$$

$$= 19.25 \times 0.5$$

$$= 9.625 \text{ kN}$$

From eqn (ii),

$$R_{AV} + 19.25\cos 30 = 30$$

$$R_{AV} = 30 - 19.25\cos 30$$

$$= 13.33 \text{ kN.}$$

$$R_A = \sqrt{(R_{AH})^2 + (R_{AV})^2}$$

$$= \sqrt{9.625^2 + 13.33^2}$$

$$= 16.44 \text{ kN}$$

Inclination of resultant with vertical,

$$\theta_A = \tan^{-1}\frac{R_{AH}}{R_{AV}}$$

$$= \tan^{-1}(\frac{9.625}{13.33})$$

$$= 35.83^0$$

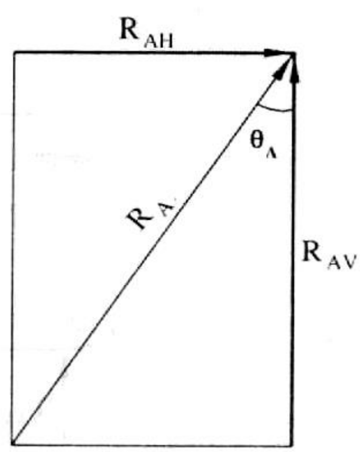


Fig. 1.168

Example 4

A beam 6 m long is loaded as shown in fig. 1.173. Calculate the reactions at A and B.

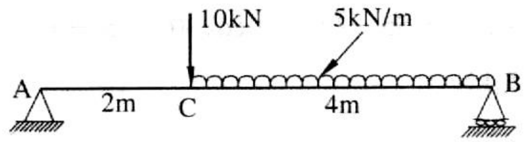


Fig.1.173

Solution:

For calculating the support reactions, the UDL can be replaced by a concentrated load of magnitude 5 × 4 = 20 kN at the mid point of BC are shown in fig. 1.174.

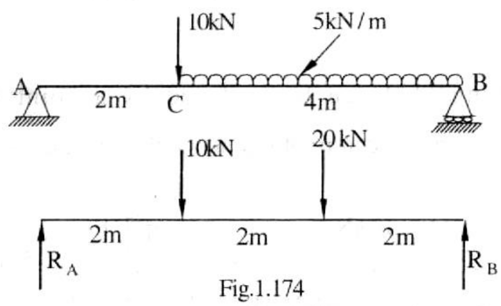


Fig.1.174

Problem 5.1. *A simply supported beam AB of span 6 m carries point loads of 3 kN and 6 kN at a distance of 2 m and 4 m from the left end A as shown in Fig. 5.12. Find the reactions at A and B analytically and graphically.*

Sol. Given :

Span of beam = 6 m

Let R_A = Reaction at A

R_B = Reaction at B

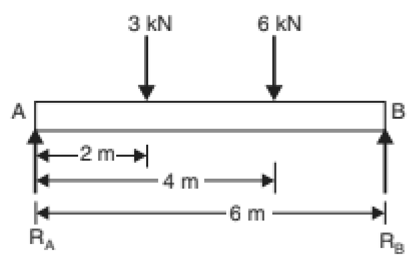


Fig. 5.12

(*a*) **Analytical Method.** As the beam is in equilibrium, the moments of all the forces about any point should be zero.

Now taking the moment of all forces about A, and equating the resultant moment to zero, we get

$$R_B \times 6 - 3 \times 2 - 6 \times 4 = 0$$

or $$6R_B = 6 + 24 = 30$$

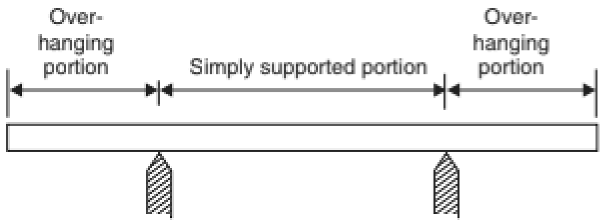
or $$R_B = \frac{30}{6} = \mathbf{5\ kN.} \quad \textbf{Ans.}$$

Also for equilibrium, $\Sigma F_y = 0$

$\therefore$ $$R_A + R_B = 3 + 6 = 9$$

$\therefore$ $$R_A = 9 - R_B = 9 - 5 = \mathbf{4\ kN.} \quad \textbf{Ans.}$$

Overhanging Beams. If the end portion of a beam is extended beyond the support, then the beam is known as overhanging beam. Overhanging portion may be at one end of the beam or at both ends of the beam as shown in Fig. 5.18.



Problem 5.2. *A simply supported beam AB of length 9 m, carries a uniformly distributed load of 10 kN/m for a distance of 6 m from the left end. Calculate the reactions at A and B.*

Sol. Given :

Length of beam = 9 m

Rate of U.D.L. = 10 kN/m

Length of U.D.L. = 6 m

Total load due to U.D.L.

= (Length of U.D.L.) × Rate of U.D.L.

= 6 × 10 = 60 kN

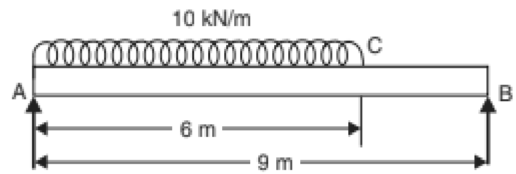


Fig. 5.14

This load of 60 kN will be acting at the middle point of *AC i.e.*, at a distance of $\frac{6}{2} = 3$ m from *A*.

Let R_A = Reaction at *A* and R_B = Reaction at *B*

Taking the moments of all forces about point *A*, and equating the resultant moment to zero, we get

$$R_B \times 9 - (6 \times 10) \times 3 = 0 \quad \text{or} \quad 9R_B - 180 = 0$$

$$\therefore \quad R_B = \frac{180}{9} = \mathbf{20\ kN.} \quad \textbf{Ans.}$$

Also for equilibrium, $\Sigma F_y = 0$

or $$R_A + R_B = 6 \times 10 = 60$$

$$\therefore \quad R_A = 60 - R_B = 60 - 20 = \mathbf{40\ kN.} \quad \textbf{Ans.}$$

Problem 5.3. *A simply supported beam of length 10 m, carries the uniformly distributed load and two point loads as shown in Fig. 5.15. Calculate the reactions R_A and R_B.*

Sol. Given :

Length of beam = 10 m

Length of U.D.L. = 4 m

Rate of U.D.L. = 10 kN/m

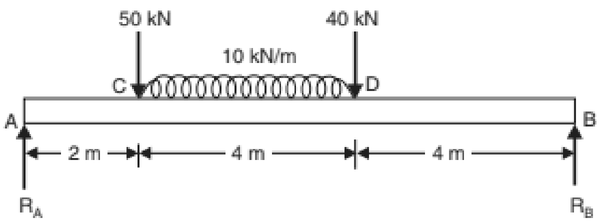


Fig. 5.15

$\therefore$ Total load due to U.D.L. = 4 × 10 = 40 kN

This load of 40 kN due to U.D.L. will be acting at the middle point of *CD, i.e.,* at a distance of $\frac{4}{2}$ = 2 m from *C* (or at a distance of 2 + 2 = 4 m from point *A*).

Let R_A = Reaction at *A*

and R_B = Reaction at *B*

Taking the moments of all forces about point *A* and equating the resultant moment to zero, we get

$$R_B \times 10 - 50 \times 2 - 40 \times (2 + 4) - (10 \times 4)\left(2 + \frac{4}{2}\right) = 0$$

or $$10R_B - 100 - 240 - 160 = 0$$

or $$10R_B = 100 + 240 + 160 = 500$$

$$\therefore \quad R_B = \frac{500}{10} = \mathbf{50\ kN.} \quad \textbf{Ans.}$$

Also for equilibrium of the beam, $\Sigma F_y = 0$

$$\therefore \quad R_A + R_B = \text{Total load on the beam} = 50 + 10 \times 4 + 40 = 130$$

$$\therefore \quad R_A = 130 - R_B = 130 - 50 = \mathbf{80\ kN.} \quad \textbf{Ans.}$$

Problem 5.4. *A simply supported beam of span 9 m carries a uniformly varying load from zero at end A to 900 N/m at end B. Calculate the reactions at the two ends of the support.*

Sol. Given :

Span of beam $= 9$ m

Load at end $A = 0$

Load at end $B = 900$ N/m

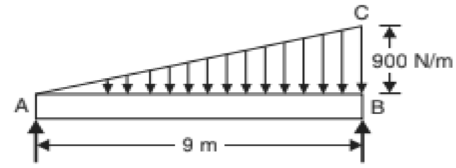


Fig. 5.16

Total load on the beam

= Area of right-angled triangle ABC

$$= \frac{AB \times BC}{2} = \frac{9 \times 900}{2} = 4050 \text{ N}$$

This load will be acting at the C.G. of the ΔABC, *i.e.*, at a distance of $\frac{2}{3} \times AB = \frac{2}{3} \times 9 = 6$ m from end A.

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking the moments of all forces about point A and equating the resultant moment to zero, we get

$$R_B \times 9 = (\text{Total load on beam}) \times \text{Distance of total load from } A$$
$$= 4050 \times 6$$

$$\therefore \quad R_B = \frac{4050 \times 6}{9} = \mathbf{2700\ N.} \quad \textbf{Ans.}$$

Also for equilibrium of the beam, $\Sigma F_y = 0$

or $R_A + R_B$ = Total load on beam = 4050

$$\therefore \quad R_A = 4050 - R_B = 4050 - 2700 = \mathbf{1350\ N.} \quad \textbf{Ans.}$$

Problem . . *A beam AB of span 8 m, overhanging on both sides, is loaded as shown in Fig. 5.19. Calculate the reactions at both ends.*

Sol. Given :

Span of beam $= 8$ m

Let R_A = Reaction at A

and R_B = Reaction at B.

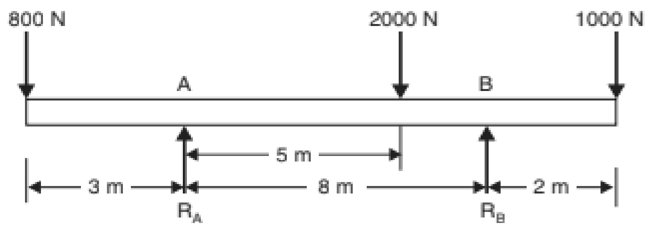


Fig. 5.19

Taking the moments of all forces about point A and equating the resultant moment to zero, we get

$$R_B \times 8 + 800 \times 3 - 2000 \times 5 - 1000 \times (8 + 2) = 0$$

or $$8R_B + 2400 - 10000 - 10000 = 0$$

or $$8R_B = 20000 - 2400 = 17600$$

$$\therefore \quad R_B = \frac{17600}{8} = \mathbf{2200\ N.} \quad \textbf{Ans.}$$

Also for the equilibrium of the beam, we have

$$R_A + R_B = 800 + 2000 + 1000 = 3800$$

$$\therefore \quad R_A = 3800 - R_B = 3800 - 2200 = \mathbf{1600\ N.} \quad \textbf{Ans.}$$

Problem . *A beam AB of span 4 m, overhanging on one side upto a length of 2 m, carries a uniformly distributed load of 2 kN/m over the entire length of 6 m and a point load of 2 kN/m as shown in Fig. 5.20. Calculate the reactions at A and B.*

Sol. Given :

Span of beam = 4 m

Total length = 6 m

Rate of U.D.L. = 2 kN/m

Total load due to U.D.L. = 2 × 6 = 12 kN

The load of 12 kN (*i.e.*, due to U.D.L.) will act at the middle point of *AC*, *i.e.*, at a distance of 3 m from *A*.

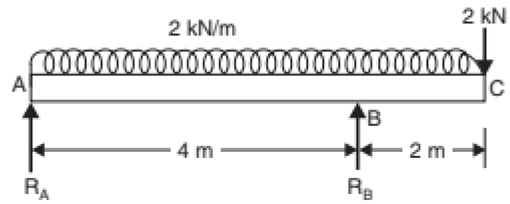


Fig. 5.20

Let R_A = Reaction at *A*

and R_B = Reaction at *B*.

Taking the moments of all forces about point *A* and equating the resultant moment to zero, we get

$$R_B \times 4 - (2 \times 6) \times 3 - 2 \times (4 + 2) = 0$$

or $$4R_B - 36 - 12 = 0$$

or $$4R_B = 36 + 12 = 48$$

$$\therefore \quad R_B = \frac{48}{4} = \mathbf{12\ kN.} \quad \textbf{Ans.}$$

Also for equilibrium, $\Sigma F_y = 0$ or $R_A + R_B = 12 + 2 = 14$

$$\therefore \quad R_A = 14 - R_B = 14 - 12 = \mathbf{2\ kN.} \quad \textbf{Ans.}$$

FRICTION

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called the force of friction and is always acting in the direction opposite to the direction of motion.

Let W = Weight of body acting through C.G. downward,

R = Normal reaction of body acting through C.G. upward,

P = Force acting on the body through C.G. and parallel to the horizontal surface.

If P is small, the body will not move as the force of friction acting on the body in the direction opposite to P will be more than P. But if the magnitude of P goes on increasing, a stage comes, when the solid body is on the point of motion. At this stage, the force of friction acting on the body is called *limiting force of friction*. The limiting force of friction is denoted by F.

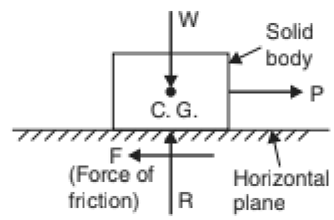


Fig. 6.1. Solid body on horizontal surface.

Resolving the forces on the body vertically and horizontally, we get

$$R = W$$
$$F = P.$$

If the magnitude of P is further increased the body will start moving. The force of friction, acting on the body when the body is moving, is called *kinetic friction*.

CO-EFFICIENT OF FRICTION (μ)

It is defined as the ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies. It is denoted by the symbol μ. Thus

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}\ .$$

$$\therefore \qquad F = \mu R$$

ANGLE OF FRICTION (ϕ)

It is defined as the angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction (R). It is denoted by ϕ. Fig. 6.2 shows a solid body resting on a rough horizontal plane.

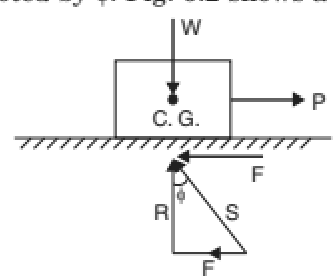


Fig. 6.2. Angle of friction.

TYPES OF FRICTION

The friction is divided into following two types depending upon the nature of the two surfaces in contact :

1. Static and dynamic friction
2. Wet and dry friction.

6.5.1. Static and Dynamic Friction. If the two surfaces, which are in contact, are at rest, the force experienced by one surface is called **static friction.** But if one surface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction.

6.5.2. Wet and Dry Friction. If between two surfaces, which are in contact, lubrication (oil or grease) is used, the friction, that exists between two surfaces is known wet friction. But if no lubrication (oil or grease) is used, then the friction between two surfaces is called **Dry Friction** or **Solid Friction.**

COULOMB'S LAWS OF FRICTION

The friction, that exists between two surfaces which are not lubricated, is known as solid friction. The two surfaces may be at rest or one of the surface is moving and other surface is at rest. The following are the laws of solid friction :

1. The force of friction acts in the opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. When the surface is on the point of motion, the force of friction is maximum and this maximum frictional force is called the limiting friction force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
6. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.
7. The force of friction is independent of the velocity of sliding.

The above laws of solid friction are also called laws of static and dynamic friction.

Problem 1. *A body of weight 100 Newtons is placed on a rough horizontal plane. Determine the co-efficient of friction if a horizontal force of 60 Newtons just causes the body to slide over the horizontal plane.*

Sol. Given :

Weight of body, $W = 100$ N

Horizontal force applied,

$$P = 60 \text{ N}$$

∴ Limiting force of friction,

$$F = P = 60 \text{ N}$$

Let μ = Co-efficient of friction.

The normal reaction of the body is given as

$$R = W = 100 \text{ N}$$

Using equation (6.1),

$$F = \mu R$$

or

$$\mu = \frac{F}{R} = \frac{60}{100} = \mathbf{0.6.} \quad \textbf{Ans.}$$

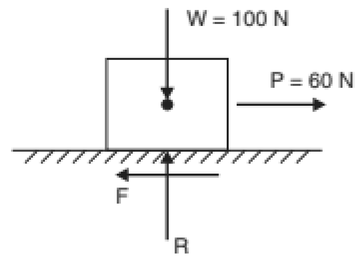


Fig. 6.5

Problem .2. *A body of weight 200 N is placed on a rough horizontal plane. If the co-efficient of friction between the body and the horizontal plane is 0.3, determine the horizontal force required to just slide the body on the plane.*

Sol. Given :

Weight of body, $W = 200$ N

Co-efficient of friction, $\mu = 0.3$

Normal reaction, $R = W = 200$ N

Let F = horizontal force which causes the body to just slide over the plane.

Using equation (6.1),

$$F = \mu R = 0.3 \times 200 = \mathbf{60 \text{ N.}} \quad \textbf{Ans.}$$

Problem .3. *The force required to pull a body of weight 50 N on a rough horizontal plane is 15 N. Determine the co-efficient of friction if the force is applied at an angle of 15° with the horizontal.*

Sol. Given :

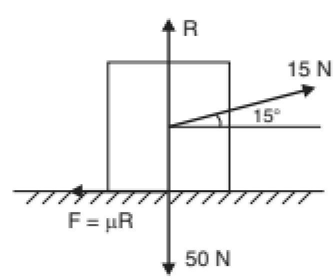
Weight of the body, $W = 50$ N

Force applied, $p = 15$ N

Angle made by the force P, with horizontal, $\theta = 15°$

Let the co-efficient of friction $= \mu$

Normal reaction $= R$



Resolving the forces along the plane, $\mu R = 15 \cos 15°$

Resolving the forces normal to the plane

$$R + 15 \sin 15° = 50$$

$$\therefore \quad R = 50 - 15 \sin 15° = 50 - 15 \times 0.2588$$

$$= 46.12 \text{ N}$$

Substituting the value of R in equation (i), we get

$$\mu \times 46.12 = 15 \cos 15°$$

$$\therefore \quad \mu = \frac{15 \cos 15}{46.12} = \frac{15 \times 0.9659}{46.12} = \mathbf{0.314.} \quad \textbf{Ans.}$$

Problem ˆ.5. *A body of weight 70 N is placed on a rough horizontal plane. To just move the body on the horizontal plane, a push of 20 N inclined at 20° to the horizontal plane is required. Find the co-efficient of friction.*

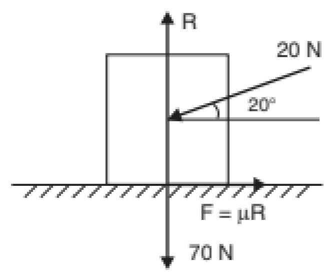


Fig. 6.7

Sol. Given :

Weight of body, $W = 70$ N

Force applied, $P = 20$ N

Inclination of P, $\theta = 20°$.

Let μ = Co-efficient of friction

R = Normal reaction

F = Force of friction = μR.

When a push of 20 N at an angle 20° to the horizontal is applied to the body, the body is just to move towards left. Hence a force of friction $F = \mu R$, will be acting towards right as shown in Fig. 6.7.

Resolving forces along the plane, $\mu R = 20 \cos 20°$ ...(i)

Resolving forces normal to the plane, $R = 70 + 20 \sin 20°$

$$= 70 + 20 \times 0.342 = 70 + 6.84 = 76.84$$

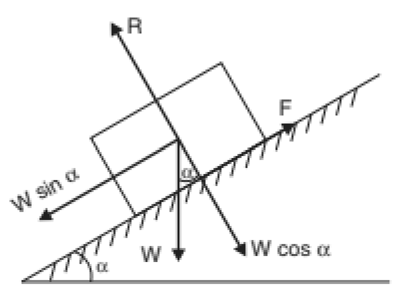
Substituting the value of R in equation (i),

$$\mu \times 76.84 = 20 \cos 20$$

$$\therefore \quad \mu = \frac{20 \cos 20}{76.84} = \frac{20 \times 0.9397}{76.84} = \mathbf{0.244.} \quad \textbf{Ans.}$$

ANGLE OF REPOSE

The angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.



Problem 6.10. *A body of weight 500 N is pulled up an inclined plane, by a force of 350 N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the co-efficient of friction.*

Sol. Given :

Weight of body, $W = 500$ N

Force applied, $P = 350$ N

Inclination, $\alpha = 30°$

Let μ = Co-efficient of Q friction

R = Normal reaction

F = Force of friction = μR.

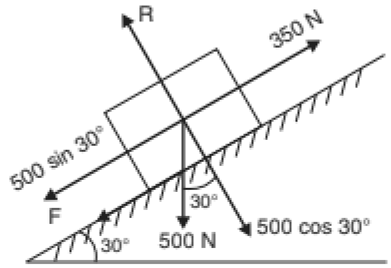


Fig. 6.14. Body moving up.

The body is in equilibrium under the action of the forces shown in Fig. 6.14.

Resolving the forces along the plane,

$$500 \sin 30° + F = 350$$

or $$500 \sin 30° + \mu R = 350 \qquad (\because \quad F = \mu R) \qquad ...(i)$$

Resolving forces normal to the plane,

$$R = 500 \cos 30° = 500 \times .866 = 433 \text{ N}$$

Substituting the value of R in equation (i), we get

$$500 \sin 30° + \mu \times 433 = 350$$

or $$500 \times 0.5 + 433\,\mu = 350$$

or $$433\,\mu = 350 - 500 \times 0.5 = 350 - 250 = 100$$

$$\therefore \quad \mu = \frac{100}{433} = \mathbf{0.23.} \quad \textbf{Ans.}$$

Example 2.55

A body of weight 90 N is placed on a rough horizontal plane. Determine the coefficient of friction if a horizontal force of 63 N just cause the body to slide over the horizontal plane.

Solution

Since there is no motion in the vertical direction, net force in the vertical direction is zero.

$$\therefore R_N - W = 0$$

$$R_N = W = 90\text{N}$$

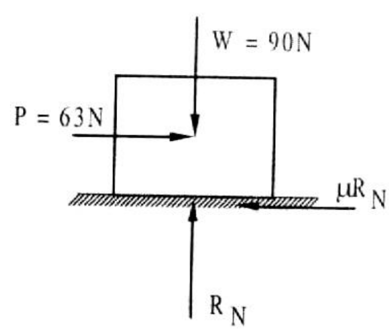


Fig.2.130

Since P just causes motion,

$$\mu R_N = P = 63\text{N}$$

$$\therefore \mu = \frac{63}{R_N} = \frac{63}{90}$$

$$= 0.7$$

Example 2.56

A block of weight 200 N is placed on a rough horizontal floor. If $\mu = 0.25$, find the pull P required to move the block if P is inclined upwards at 30^0 to the horizontal.

Solution

Since there is no motion in the vertical direction, net force in the vertical direction is zero.

$$R_N + P\sin 30 - 200 = 0$$

$$R_N = 200 - P\sin 30$$

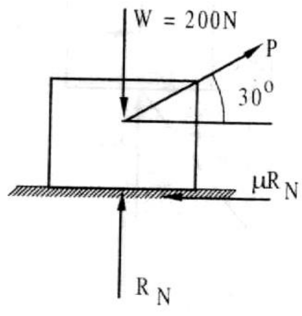


Fig. 2.131

Since P is the limiting force causing motion of the block.

$$P\cos 30 - \mu R_N = 0$$

$$P\cos 30 = 0.25(200 - P\sin 30)$$

$$= 50 - 0.25P\sin 30$$

$$P(\cos 30 + 0.25\sin 30) = 50$$

P =50.45 N

TRUSS :A structure made up of several bars (or members) riveted or welded together is known as truss.

The different types of frames are :

(i) Perfect frame, and

(ii) Imperfect frame.

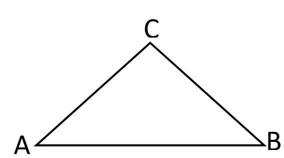
Perfect Frame. The frame which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, is known as perfect frame.

Mathematically,

m = number of members

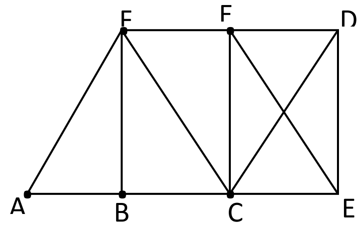
j = number of joints

then m = 2j – 3 is the condition for the frame to be a perfect frame.



(b) Imperfect frames:- (a) When the numbers are less than that required by equation m = 2j – 3 then frame is called as imperfect frame. Such frame, can not resist geometrical distortion under the action of loads.

(c) Redundant frames: If the number of members are more than that required by equation m = 2j – 3, then such frames will be called as redundant frames.



ASSUMPTION

Following assumptions are made in finding out the forces in a frame.

(a) Frame is a perfect frame.

(b) Load is applied at joints only.

(c) All members are hinged or pin-jointed. (It means members will have only axial force and there will be no moment due to pin, because at a pin moment becomes zero.)

ANALYSIS OF A FRAME

(a) Reaction at the supports are first determined due to applied external loads at joints of a perfect frame, considering conditions of equilibrium.

(b) Forces in the members of the frame are determined by considering conditions of equilibrium.

Problem 7.1. *Find the forces in the members AB, AC and BC of the truss shown in Fig. 7.5.*

Sol. First determine the reactions R_B and R_C. The line of action of load of 20 kN acting at A is vertical. This load is at a distance of $AB \times \cos 60°$ from the point B. Now let us find the distance AB.

The triangle ABC is a right-angled triangle with angle $BAC = 90°$. Hence AB will be equal to $BC \times \cos 60°$.

$\therefore \quad AB = 5 \times \cos 60° = 5 \times \frac{1}{2} = 2.5$ m

Now the distance of line of action of 20 kN from B is

$$AB \times \cos 60° \text{ or } 2.5 \times \frac{1}{2} = 1.25 \text{ m.}$$

Taking the moments about B, we get

$$R_C \times 5 = 20 \times 1.25 = 25$$

$$\therefore \quad R_C = \frac{25}{5} = 5 \text{ kN}$$

and $\quad R_B = \text{Total load} - R_C = 20 - 5 = 15$ kN

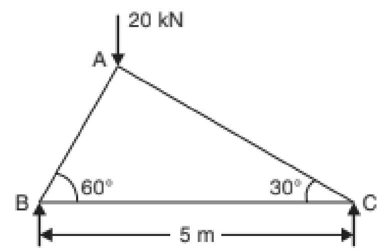


Fig. 7.5

Now let us consider the equilibrium of the various joints.

Joint B

Let F_1 = Force in member AB

F_2 = Force in member BC

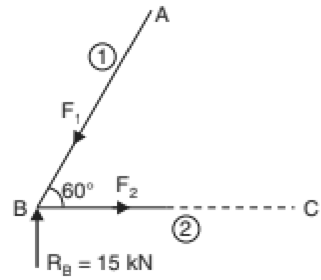


Fig. 7.6

Let the force F_1 is acting towards the joint B and the force F_2 is acting away* from the joint B as shown in Fig. 7.6. (The reaction R_B is acting vertically up. The force F_2 is horizontal. The reaction R_B will be balanced by the vertical component of F_1. The vertical component of F_1 must act downwards to balance R_B. Hence F_1 must act towards the joint B so that its vertical component is downward. Now the horizontal component of F_1 is towards the joint B. Hence force F_2 must act away from the joint to balance the horizontal component of F_1).

Resolving the forces acting on the joint B, vertically

$$F_1 \sin 60° = 15$$

$$\therefore \quad F_1 = \frac{15}{\sin 60°} = \frac{15}{0.866} = 17.32 \text{ kN (Compressive)}$$

As F_1 is pushing the joint B, hence this force will be compressive. Now resolving the forces horizontally, we get

$$F_2 = F_1 \cos 60° = 17.32 \times \tfrac{1}{2} = 8.66 \text{ kN}$$
(tensile)

As F_2 is pulling the joint B, hence this force will be tensile.

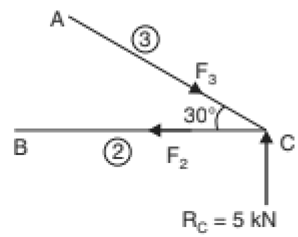


Fig. 7.7

Joint C

Let F_3 = Force in the member AC

F_2 = Force in the member BC

The force F_2 has already been calculated in magnitude and direction. We have seen that force F_2 is tensile and hence it will pull the joint C. Hence it must act away from the joint C as shown in Fig. 7.7.

Resolving forces vertically, we get

$$F_3 \sin 30° = 5 \text{ kN}$$

$$\therefore \quad F_3 = \frac{5}{\sin 30°} = 10 \text{ kN (Compressive)}$$

As the force F_3 is pushing the joint C, hence it will be compressive.

hence it will be compressive.

Problem 7.2. *A truss of span 7.5 m carries a point load of 1 kN at joint D as shown in Fig. 7.8. Find the reactions and forces in the members of the truss.*

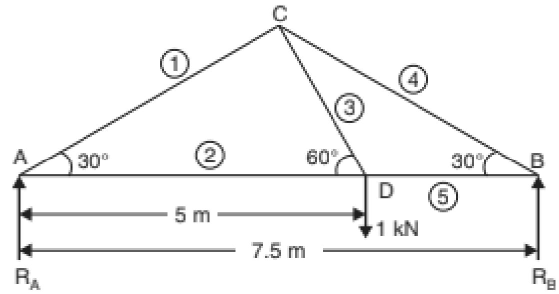


Fig. 7.8

Sol. Let us first determine the reactions R_A and R_B

Taking moments about A, we get

$$R_B \times 7.5 = 5 \times 1$$

$$\therefore \quad R_B = \frac{5}{7.5} = \frac{2}{3} = 0.667 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load} - R_B$$
$$= 1 - 0.667 = 0.333 \text{ kN}$$

Now consider the equilibrium of the various joints.

Joint A

Let F_1 = Force in member AC

F_2 = Force in member AD.

Let the force F_1 is acting towards the joint A and F_2 is acting away from the joint A as shown in Fig. 7.9.

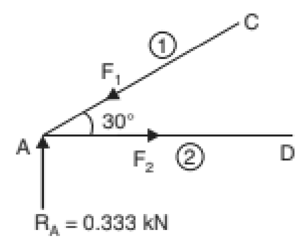


Fig. 7.9

Resolving the forces vertically, we get

$$F_1 \sin 30° = R_A$$

or $$F_1 = \frac{R_A}{\sin 30°} = \frac{0.333}{0.5}$$
$$= 0.666 \text{ kN (Compressive)}$$

Resolving the forces horizontally, we get

$$F_2 = F_1 \times \cos 30°$$
$$= 0.666 \times 0.866 = 0.5767 \text{ kN (Tensile)}$$

Joint B

Let F_4 = Force in member BC

F_5 = Force in member BD

Let the direction of F_4 and F_5 are assumed as shown in Fig. 7.10.

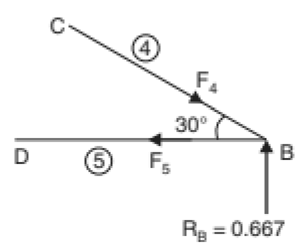


Fig. 7.10

Resolving the forces vertically, we get

$$F_4 \sin 30° = R_B = 0.667$$

or $$F_4 = \frac{0.667}{\sin 30°} = 1.334 \text{ kN (Compressive)}$$

Resolving the forces horizontally, we get

$$F_5 = F_4 \cos 30° = 1.334 \times 0.866 = 1.155 \text{ kN (Tensile)}$$

Joint D

Let F_3 = Force in member CD. The forces F_2 and F_5 have been already calculated in magnitude and direction. The forces F_2 and F_5 are tensile and hence they will be pulling the joint D as shown in Fig. 7.11. Let the direction* of F_3 is assumed as shown in Fig. 7.11.

Resolving the forces vertically, we get

$$F_3 \sin 60° = 1$$

$$\therefore \quad F_3 = \frac{1}{\sin 60°} = \frac{1}{0.866}$$

$$= 1.1547 \text{ kN (Tensile)}$$

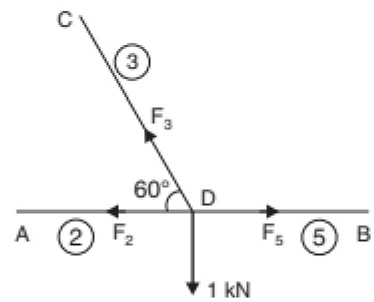


Fig. 7.11

Hence the forces in the members are :

F_1 = 0.666 kN (Compressive)

F_2 = 0.5767 kN (Tensile)

F_3 = 1.1547 kN (Tensile)

F_4 = 1.334 kN (Compressive)

F_5 = **1.155 kN** (Tensile). **Ans.**

METHOD OF SECTIONS

When the forces in a few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

In this method, a section line is passed through the members, in which forces are to be determined as shown in Fig. 7.25. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown. The part of the truss, on any one side of the section line, is treated as a free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line. The unknown forces in the members are then determined by using equations of equilibrium as

$$\Sigma F_x = 0, \quad \Sigma F_y = 0 \text{ and } \Sigma M = 0.$$

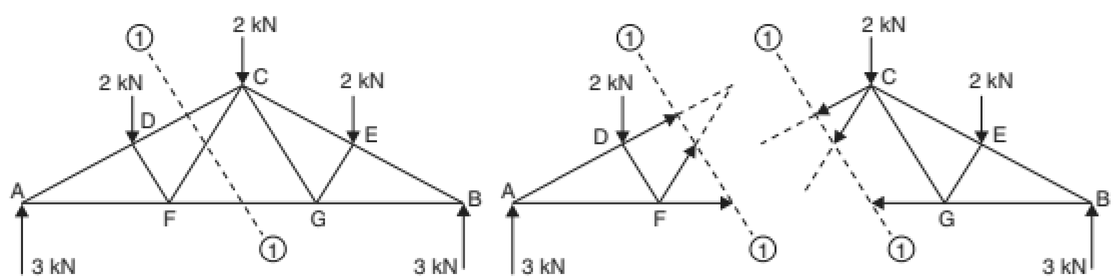


Fig. 7.25

If the magnitude of the forces, in the members cut by a section line, is positive then the assumed direction is correct. If magnitude of a force is negative, then reverse the direction of that force.