FUNDAMENTALS OF FLUID MECHANICS

CODE: 3051(21)

MODULE 1

FLUID AND PROPERTIES

Introduction and Importance

Fluid: Fluid is a substance which is capable of flowing. More precisely, a fluid is defined as a substance which deforms continuously when subjected to shear force, however small the shear force may be. Fluid will be liquid or Gas. E.g.:- water, air, mercury, vapour.

Liquids occupy a definite volume and are not compressible. They are not affected by change in temperature. Liquids are not compressible and not affected by temperature.

Gases are fluids which can be compressed and expanded. They are affected by change in temperature. Gases are compressible and affected by temperature.

Fluid mechanics: It is the branch of science which deals with the behavior of the fluids at rest as well as in motion. This branch of science deals with the static, kinematics, dynamics aspects of fluids. The study of fluid at is rest is fluid statics. The study of fluid in motion, where pressure forces are not considered, is called fluid kinematics. The study of fluid in motion, where pressure forces are considered is called fluid dynamics.

Applications of fluid mechanics in following engineering department*:* Irrigation department, Water supply department, bridge department, water power engineering, hydraulic machines.

Properties of fluids

Density or Mass density: It is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (Rho). Unit is kg/m^3.

$$\rho = \frac{mass\ of\ fluid}{volume\ of\ fluid} = \frac{m}{v} = \frac{W/g}{v} = \frac{w}{g} \quad \text{W=weight of fluid}$$

The value of water density is 1 gm/cm^3 or 1000 kg/m^3.

Specific weight or weight density: It is defined as the ratio between the weights of a fluid to its volume. The weight per unit volume of a fluid is called weight density and it is denoted by the symbol ‘w’.

$$\mathbf{w} = \frac{weight\ of\ fluid}{volume\ of\ fluid} = \frac{mass\ of\ fluid * acceleration\ due\ to\ gravity}{volume\ of\ fluid} = \frac{m*g}{v} = \rho g \quad \textbf{...Unit is N/}\boldsymbol{m^3}.$$

Specific weight of water is 9.81*1000 N/m^3.

Specific volume: It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

Specific volume = $\frac{volume\ of\ fluid}{mass\ of\ fluid}$ = $\frac{1}{\rho\rho}$ unit is m^3/kg. It is commonly applied to gases.

Specific Gravity: Specific gravity of a fluid is defined as the ratio of the weight density of a fluid to the weight density of a standard fluid. For liquids water is standard fluid. For gases air is standard fluid. Symbol is 'S'. It has no unit.

S for liquids =$\frac{weight\ density\ of\ liquid}{weight\ density\ of\ water}$, hence weight density of liquid= S*Weight density of water

= S*1000*9.81 N/m^3

= S* Density of water

= S* 1000 kg/m^3.

Viscosity or Dynamic viscosity: It is defined the measure of the resistance of a fluid to deform under shear stress or resistance to shear stress action or resistance to flow. Super fluids have no viscosity. For liquids viscosity decrease with increase in temperature. For gases viscosity increase with increase in temperature.

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance '*dy*' apart, move one over the other at different velocities, say *u* and *u* + *du* as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to *y*. It is denoted by symbol τ called Tau.

Mathematically, $$\tau \propto \frac{du}{dy}$$

or $$\tau = \mu \frac{du}{dy} \qquad ...(1.2)$$

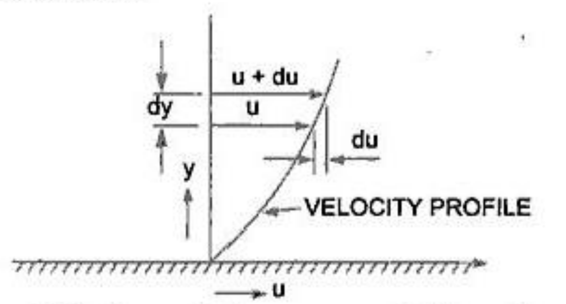


Fig. 1.1 *Velocity variation near a solid boundary.*

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2), we have $\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

SI unit of viscosity = Ns/m^2 = Pa s.

CGS unit of viscosity is poise, and 10 poise = 1Ns/m^2.

Kinematic Viscosity. It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called 'nu'. Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

SI unit of kinematic viscosity is m^2/s. CGS unit of kinematic viscosity stokes.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{ m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$

Newton's Law of Viscosity. It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. Mathematically, it is expressed as given by equation

$$\tau = \mu \frac{du}{dy}.$$

Fluids which obey the above relation are known as **Newtonian** fluids and the fluids which do not obey the above relation are called **Non-newtonian** fluids.

Types of Fluids. The fluids may be classified into the following five types :

1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

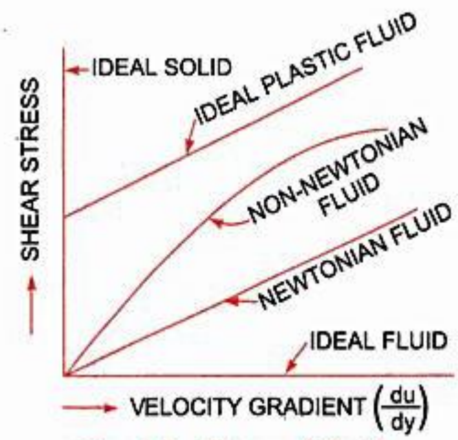


Fig. 1.2 *Types of fluids.*

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

COMPRESSIBILITY AND BULK MODULUS

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in Fig. 1.9.

Let $\forall$ = Volume of a gas enclosed in the cylinder

p = Pressure of gas when volume is $\forall$

Let the pressure is increased to $p + dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$.

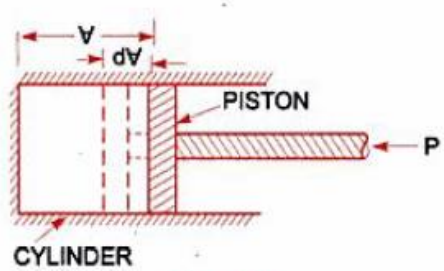


Fig. 1.9

Then increase in pressure $= dp$ kgf/m^2

Decrease in volume $= d\forall$

$\therefore$ Volumetric strain $= -\dfrac{d\forall}{\forall}$

– ve sign means the volume decreases with increase of pressure.

$$\therefore \text{ Bulk modulus } \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}} = \frac{dp}{\frac{-d\forall}{\forall}} = \frac{-dp}{d\forall}\forall$$

Compressibility is given by $= \dfrac{1}{K}$

Compressibility decreases with Increases in Temperature.

Notes

Cohesion:- It is the property of a fluid by which molecules of the same fluid are attracted. This property enables the liqid to resist tensile stress.

Adhesion:-It is the property of fluid by which molecule of different kinds of liquids are attracted to each other or molecules of liquids are attracted to another body.

Surface Tension: Surface tension of a liquid is its property, which enables it resist tensile force. It is due to cohesion between liquid molecules at its surface.

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrance under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

The phenomenon of surface tension is explained by Fig. 1.10. Consider three molecules A, B, C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule A is zero. But the molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule B is acting in the downward direction. The molecule C, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an elastic membrance under tension.

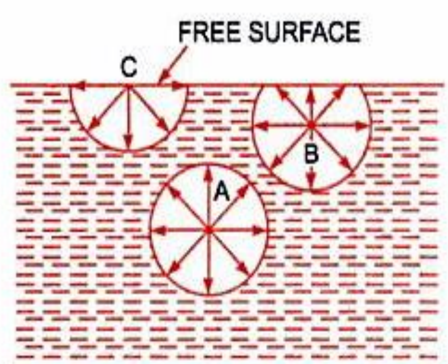


Fig. 1.10 *Surface tension.*

Capillarity

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Expression for Capillary Rise. Consider a glass tube of small diameter '*d*' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

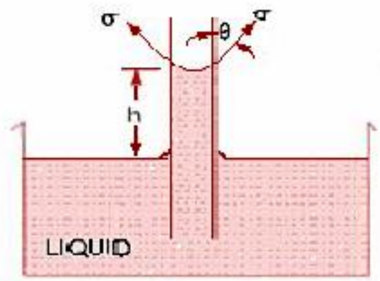


Fig. 1.13 *Capillary rise.*

Expression for Capillary Fall. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig. 1.14.

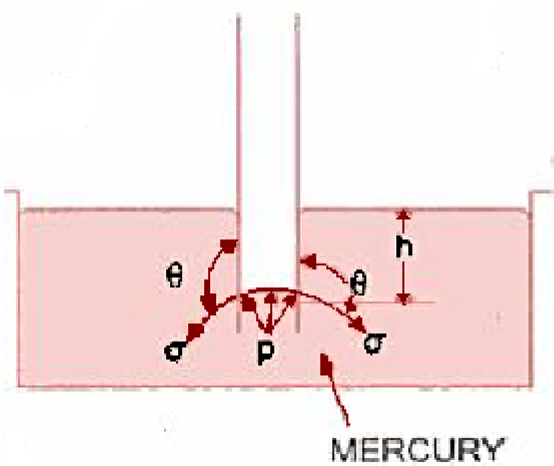


Fig. 1.14

h= capillary rise or fall (depression) in mm or m.

σ= surface tension in N/mm or N/m.

θ=angle of contact between liquid and glass tube in degree.

ρ= density of liquid kg/m^3 or kg/mm^3

d= diameter of glass tube in mm or m.

Then the expression can be obtained by equating lifting force to gravity force.

Upward surface tension = Weight of liquid column

ei; $\pi d.\sigma.\cos\theta = \frac{\pi}{4} d^2 * h * \rho g$

Then expression is $h = \dfrac{4\sigma \cos\theta}{\rho g d}$

The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d}$$

+h implies capillary rise and –h implies capillary depression or fall. And for wetting liquids such as water θ < or = 90 degree. For non-wetting liquids such as mercury θ >90 degree.

NOTE:

In water cohesion between liquid particles is less than the adhesion with the glass tube. So capillary rise is possible. In mercury cohesion between mercury molecules greater than adhesion between the mercury and glass.

Fluid Pressure and measurement

Fluid pressure at a point: Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force dF exerted on the area is will be normal. Then the ratio dF/dA is known as the intensity of pressure or pressure. It is denoted as 'p'.

Pressure $p = \frac{dF}{dA} = \frac{Force}{Area}$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

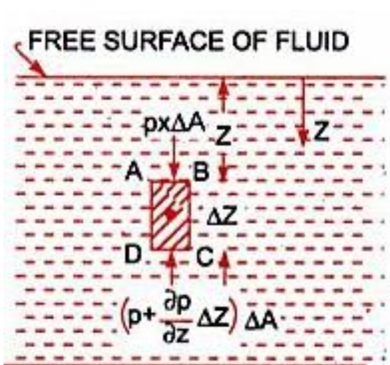
$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

$\therefore$ Force or pressure force, $F = p \times A$.

Unit of pressure is N/m^2 or Pascal (Pa) or Bar.

kPa = kilo pascal = 1000 N/m^2
bar = 100 kPa = 10^5 N/m^2.

Pressure Head: From the equation p =$\rho g h$. 'p' is the pressure above atmosphere pressure and g is acceleration due to gravity and 'h' is the height of the point from free surfaces. And this 'h' is called the Pressure Head. h = $\frac{p}{\rho * g}$

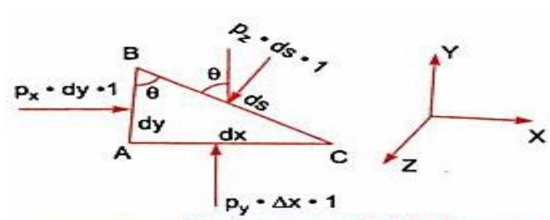


Forces on a fluid element.

Pascal's Law:

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

The fluid element is of very small dimensions *i.e.*, *dx*, *dy* and *ds*.



Forces on a fluid element.

And according to figure forces $p_x = p_y = p_z$

Absolute, Gauge, Atmospheric and Vacuum pressures: The pressure of a fluid can be measured by using two systems. In one system pressure is measured above absolute zero (complete vaccum).In another system, pressure is measured above atmospheric pressure.

1. **Absolute pressure** is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure is masured with th help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. Vacuum pressure is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

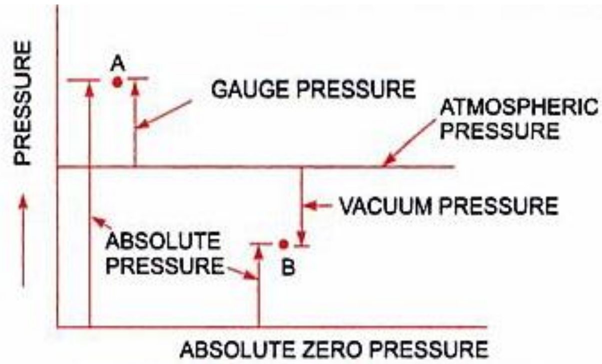
(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure − Absolute pressure.



Relationship between pressures.

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Measurement of fluid pressure

Measurement of Pressure is based on either of the two following principles. First one is by balancing the liquid column whose pressure is to be found out with the same or another column of different liquid. The second one is by balancing the liquid column whose pressure is to be found out with the dead weight or a spring force.

Two types of pressure measuring devices are

1. Manometers
2. Mechanical gauges

1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,`
2. U-tube Manometer, and
3. Single Column Manometer.

1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8.

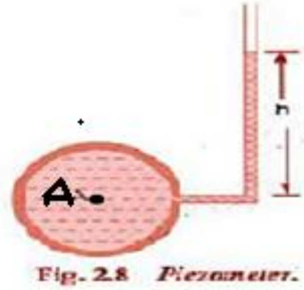


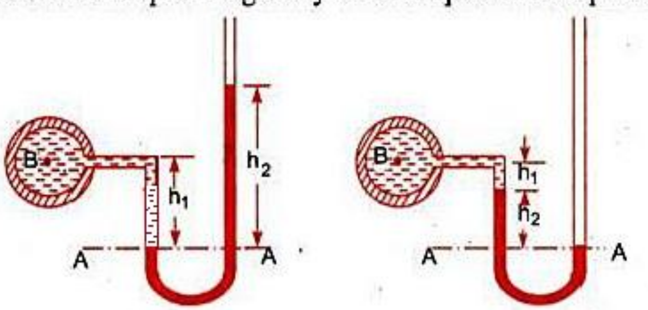
Fig. 2.8 *Piezometer.*

The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A.

$$= \rho \times g \times h \ \frac{N}{m^2}.$$

In this type manometer liquid will rise up into the vertical glass tube. The height of liquid column in the tube gives the pressure head directly. Piezometer is not suitable for measuring negative pressure. The size of tube not less than 10 mm to avoid error due to capillary action. Height of liquid column is to be read up to centre point of meniscus.

2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



(a) For gauge pressure (b) For vacuum pressure

Fig. 2.9 *U-tube Manometer.*

(*a*) For Gauge Pressure. Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = Height of light liquid above the datum line

h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = Density of light liquid = $1000 \times S_1$

S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1)$.

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A = 0

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1)$.

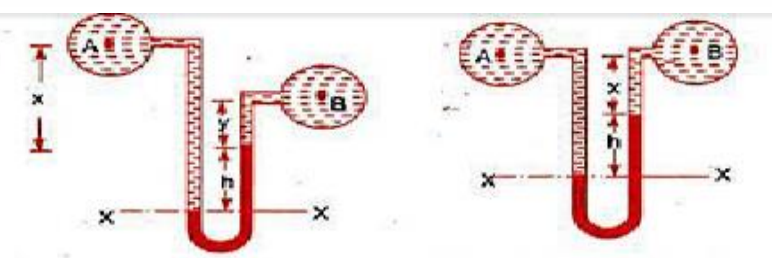
Note: According to U-tube manometer two sides of the manometer is called limbs. U-tube manometer can be used for measuring both vaccum pressure and gauge pressure. According to our syllabus U-tube manometer is also means simple u-tube manometer or simple manometer.

DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

1 U-tube Differential Manometer. Fig. 2.18 shows the differential manometers of U-tube type.



(a)Two pipes at different levels (b) A and B are at the same level

Fig. 2.18 *U-tube differential manometers.*

Fig. 2.18 (*a*). Let the two points *A* and *B* are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at *A* and *B* are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of *B*, from the mercury level in the right limb.

x = Distance of the centre of *A*, from the mercury level in the right limb.

ρ_1 = Density of liquid at *A*.

ρ_2 = Density of liquid at *B*.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$$

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

Fig. 2.18 (*b*). A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1).$$

2 Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let
h_1 = Height of liquid in left limb below the datum line X-X
h_2 = Height of liquid in right limb
h = Difference of light liquid
ρ_1 = Density of liquid at A
ρ_2 = Density of liquid at B
ρ_s = Density of light liquid
p_A = Pressure at A
p_B = Pressure at B.

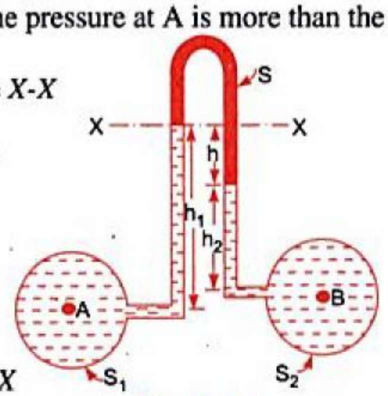


Fig. 2.21

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

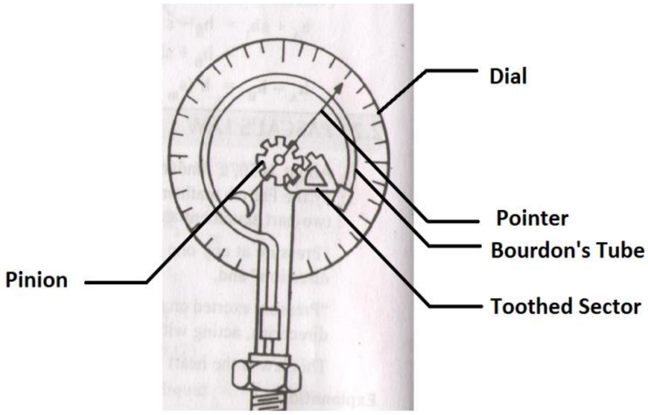
or

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

Inverted U- tube differential manometers are employed for more accurate measurement of small pressure difference between two points.

Bourdon's tube pressure gauge

Bourdon's Pressure gauge is a mechanical gauge. Mechanical gauges are used to measure high fluid pressures. Bourdon's pressure gauge is used extensively in steam boilers. The given figure



Shows Bourdon's tube pressure gauge in its simplest form. It essentially consists of a steam less phosphorous bronze tube of elliptical cross section bent into an arc of a circle. When the gauge is connected to the fluid whose pressure is to be found out, that fluid flows into the tube. The pressure of fluid in the tube makes its cross section more nearly circular and this change of cross section is accompanied by flattening of the curve in the other direction. This outward movement of tube actuates the toothed sector which in turn rotates a pinion meshing with the toothed sector. A pointer attached to the pinion moves on the dial and shows the reading of fluid pressure. As the tube is surrounded by atmospheric pressure, the movement of the tube is against atmospheric pressure and hence the gauge registers the pressure above that of atmospheric pressure.

Problem 1.2.17 : The left limb of a manometer is connected to a pipe in which a fluid of specific gravity 0.8 is flowing. The right limb containing mercury is open to atmosphere. The centre of the pipe is 100 mm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 250 mm.

Solution: Given

Specific gravity of fluid in the pipe, $S_1 = 0.8$

$\therefore$ Density of the fluid, $\rho_1 = S_1 \times 1000 = 0.8 \times 1000 = 800 \text{ kg/m}^3$

Specific gravity of mercury, $S_2 = 13.6$ (Assumed data)

$\therefore$ Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000 = 13600 \text{ kg/m}^3$

Height of mercury level in the right limb above XX or

Difference of mercury level, $h_2 = 250 \text{ mm} = 0.25 \text{ m}$

Height of liquid in the left limb from X-X, $= 250 - 100 = 150 \text{ mm} = 0.15 \text{ m}$

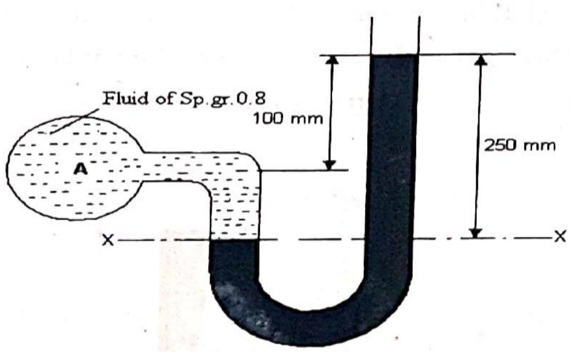


Fig 1.2.9

Let p be the intensity pressure of fluid flowing in pipe

Equating the pressure at the left and right limb above the datum X-X, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

Substituting the values in the above equation

$$p + 800 \times 9.81 \times 0.15 = 13600 \times 9.81 \times 0.25$$

$$p + 1177.2 = 33354$$

$$p = 33354 - 1177.2 = 32176.8 \text{ N/m}^2 = \mathbf{32.176 \text{ kN/m}^2} \quad \textit{(Ans)}$$

Problem 1.2.18 A simple manometer containing mercury is used to determine the pressure of oil of specific gravity 0.8 flowing in a pipe. Its right limb is open to atmosphere and left limb is connected to the pipe. The centre of the pipe is 90mm below the level of mercury in the right limb. If the difference of mercury level in the two limbs is 150 mm, find the pressure of oil in the pipe. **(Diploma Examination Question, April-2005)**

Solution: Given

Specific gravity of oil in the pipe, $S_1 = 0.8$

$\therefore$ Density of the oil, $\rho_1 = S_1 \times 1000 = 0.8 \times 1000 = 800 \text{ kg/m}^3$

Specific gravity of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000 = 13600 \text{ kg/m}^3$

Height of liquid in the left limb from X-X, $h_1 = 150 - 90 = 60 \text{ mm} = 0.06\text{m}$

Height of mercury in the right limb above X-X or difference of mercury level,

$h_2 = 150\text{mm} = 0.15\text{m}$

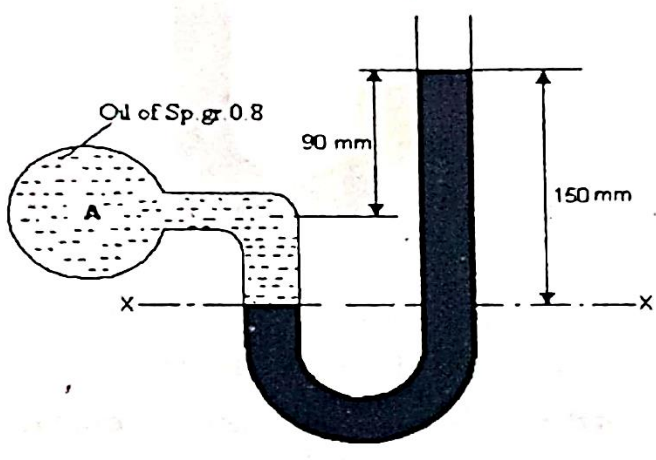


Fig.1.2.10

Let p be the intensity pressure of oil flowing in the pipe

Equating the pressure at the left and right limb above datum X-X, we get.

$$p + 800 \times 9.81 \times 0.06 = 13600 \times 9.81 \times 0.15$$

$$p + 470.88 = 20012.4$$

$$p = 20012.4 - 470.88 = \mathbf{19541.52 \ N/m^2} \quad \textit{(Ans)}$$

$$= \mathbf{19.541 \ kN/m^2 \text{ or } kPa} \quad \textit{(Ans)}$$

Problem 1.2.19 A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of specificgravity 0.8 and having a vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 400 mm and the height of fluid in the left from the centre of pipe is 150 mm below. **(Diploma Examination Question ,April-2005).**

Solution : Given

Specific gravity of fluid in the pipe, $S_1 = 0.8$

$\therefore$ Density of the fluid, $\rho_1 = S_1 \times 1000 = 0.8 \times 1000 = 800\ \text{kg/m}^3$

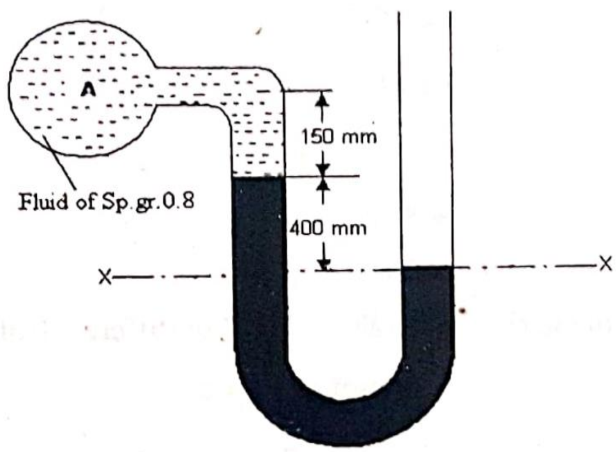


Fig.1.2.11

Specific gravity of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000 = 13600\ \text{kg/m}^3$

Height of liquid in the left limb $h_1 = 150\ \text{mm} = 0.15\ \text{m}$

Difference of mercury level, $h_2 = 400\ \text{mm} = 0.4\ \text{m}$

Let p be the intensity pressure of fluid flowing in the pipe

Equating pressure at the left and right limb above datum line X-X, we get

$$p + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

Or, $p = -(\rho_1 g h_1 + \rho_2 g h_2) = -(800 \times 9.81 \times 0.15 + 13600 \times 9.81 \times 0.4)$

$= -(1177.2 + 53366.4) =$ **- 54543. 6 N / m²** ***(Ans)***

$=$ **- 54.54 kN / m² or kPa** ***(Ans)***

Problem 1.2.22 A Simple U-tube manometer containing mercury is connected to a pipe in which a fluid of specific gravity 0.85 and having a vacuum pressure is flow.. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in two limbs is 500mm and the height of fluid in the left from the centre of pipe is 150mm below. **(Diploma Examination Question ,March-2006)**

Solution: Given

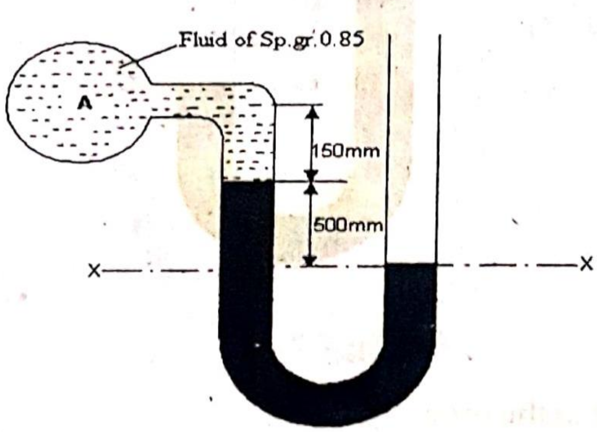


Fig. 1.2.14

Specific gravity of fluid in the pipe, $S_1 = 0.85$

$\therefore$ Density of the fluid, $\rho_1 = S_1 \times 1000 = 0.85 \times 1000 = \mathbf{850\ kg/m^3}$

Specific gravity of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000 = \mathbf{13600\ kg/m^3}$

Difference of mercury level, $h_2 = 500\text{ mm} = 0.5\text{ m}$

Height of liquid in the left limb, $h_1 = 150\text{mm} = 0.15\text{ m}$

Let p is the vacuum pressure of fluid in pipe $p + \rho_1 g h_1 + \rho_2 g h_2 = 0$

Or, $p = -(\rho_1 g h_1 + \rho_2 g h_2)$

$= -(850 \times 9.81 \times 0.15 + 13600 \times 9.81 \times 0.5) = -(1250.775 + 66708)$

$= -67958.775\text{ N/m}^2 = \mathbf{-67.96\text{k N/m}^2 \text{ or kPa}}$ ***(Ans)***

Problem 1.2.25 A differential manometer is connected at the two points M and N of two pipes as shown in the Fig-1.2.18. The pipe M contains carbon tetrachloride of specific gravity 1.594 under a pressure of 103 kPa and pipe N contains oil of specific gravity 0.8 under a pressure of 172 kPa. If the manometric fluid is mercury, find the difference in mercury level in the manometer.

Solution: Given

Specific gravity of liquid at M, $S_1 = 1.594$

$\therefore$ Density of liquid at point M, $\rho_M = S_M \times 1000 = 1.594 \times 1000 = 1594\ kg/m^3$.

Specific gravity of liquid at N, $S_2 = 0.8$

$\therefore$ Density of liquid at point N, $\rho_N = S_2 \times 1000 = 0.8 \times 1000 = 800\ kg/m^3$.

Pressure at M, $p_M = 103\ kPa = 103\ kN/m^2 = 103 \times 10^3\ N/m^2$

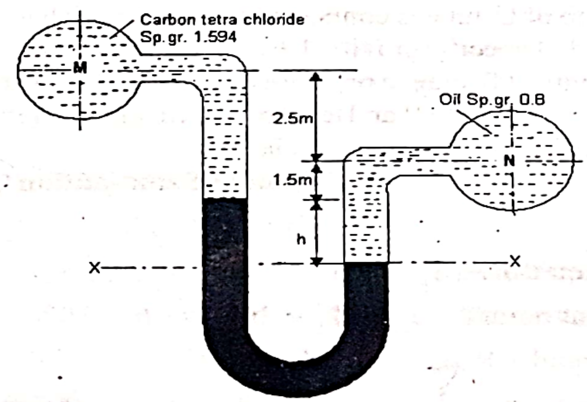


Fig. 1.2.18

Pressure at N, $p_N = 172\ kPa = 172\ kN/m^2 = 172 \times 10^3\ N/m^2$.

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, $\rho_{Hg} = S_{Hg} \times 1000 = 13.6 \times 1000 = 13600\ kg/m^3$.

Taking X-X as datum line

Pressure above X-X in the left limb $= p_M + \rho_M\, g\,(2.5 + 1.5) + \rho_{Hg}\, g\, h$

$= 103 \times 10^3 + 1594 \times 9.81 \times 4 + 13600 \times 9.81\, h$

$= 103000 + 62548.56 + 133416\, h = 165548.56 + 133416\, h$

Pressure above X-X in the right limb

$= p_N + \rho_N\, g\,(1.5 + h) = 172 \times 10^3 + 800 \times 9.81\,(1.5 + h)$

$= 172000 + 11772 + 7848\, h = 183772 + 7848\, h$

Equating the left limb and right limb pressure.

$165548.56 + 133416\, h = 183772 + 7848\, h$

$133416\, h - 7848\, h = 183772 - 165548.56$

$125568\, h = 18223.44$

$\therefore$ Difference in mercury level, $h = \dfrac{18223.44}{125568} = 0.145\ m$ **= 145 mm** ***(Ans)***

Problem 1.2.28 A U-tube Mercury manometer is connected to two pipes A &B . Pipe B is 60 mm. below pipe A . The specific gravity of liquid in pipe A and B is 1.6 and 0.85 respectively. Mercury level in the left limb is 80 mm. below the centre of pipe A. Find the pressure difference between two pipes in kN / m² if the level difference of mercury in the two limbs of the manometer is 120mm. **(Diploma Examination Question, Nov-2001)**

Solution: Given

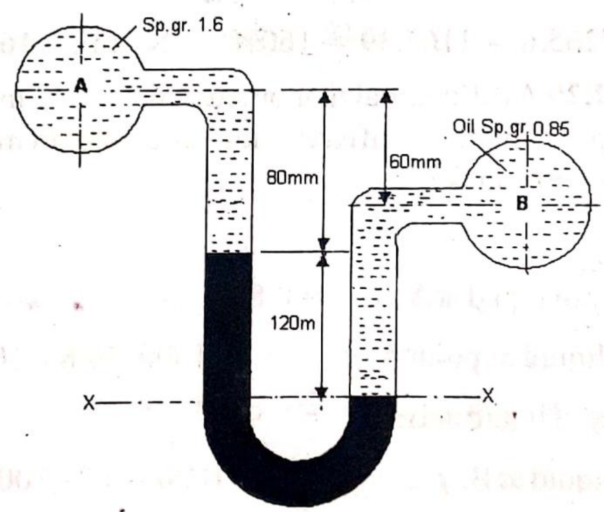


Fig. 1.2.21

Specific gravity of liquid at A, S_A = 1.6

∴ Density of liquid at point A, $\rho_A = S_A \times 1000 = 1.6 \times 1000 = 1600$ kg/ m³.

Specific gravity of liquid at B, $S_B = 0.85$

∴ Density of liquid at B, $\rho_B = S_B \times 1000 = 0.85 \times 1000 = 850$ kg/ m³

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, $\rho_{Hg} = S_{Hg} \times 1000 = 13.6 \times 1000 = 13600$ kg / m³

Taking X - X as datum line

Pressure above X-X in the left limb

$$= p_A + \rho_A g \times 0.08 + \rho_{Hg}\, g \times 0.12$$

$$= p_A + 1600 \times 9.81 \times 0.08 + 13600 \times 9.81 \times 0.12$$

$$= p_A + 1255.68 + 16009.92 = p_A + 17265.6$$

Pressure above X -X in the right limb

$$= p_B + \rho_B\, g\,(0.08 + 0.12 - 0.06) = p_B + 850 \times 9.81 \times (0.14)$$

$$= p_B + 1167.39$$

Equating the two pressures, we get $p_A + 17265.6 = p_B + 1167.39$

∴ Pressure difference between two pipes, $p_B - p_A$

= 17265.6 – 1167.39 = 16098.21 N / m². = **16.098 kN/ m²** ***(Ans)***

Problem 1.2.31 An inverted U-tube manometer is connected with two pipes M and N which carries an oil of specific gravity 1.2 and 0.8 respectively. The fluid in the manometer is an oil of specific gravity 0.7. For the manometer reading shown in the Fig-1.2.25, find the pressure difference between M and N.

Solution : Given

Specific gravity of fluid in the pipe M, $S_M = 1.2$

$\therefore$ Density of of fluid in the pipe M, $\rho_M = S_M \times 1000 = 1.2 \times 1000 = 1200\ \text{kg/m}^3$.

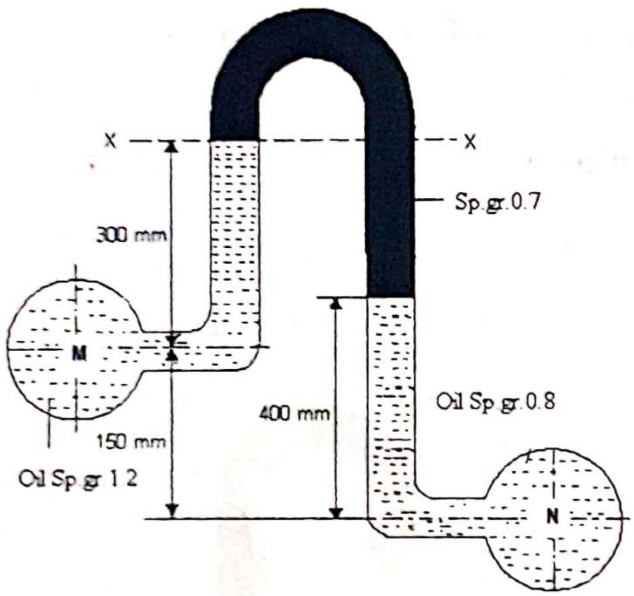


Fig 1.2.25

Specific gravity of fluid in the pipe N, $S_N = 0.8$

$\therefore$ Density of of fluid in the pipe N, $\rho_N = S_N \times 1000 = 0.8 \times 1000 = 800\ \text{kg/m}^3$.

Specific gravity of oil U-tube, $S_l = 0.7$

Density of oil in the U-tube, $\rho_l = S_l \times 1000 = 0.7 \times 1000 = 700\ \text{kg/m}^3$.

Difference of oil in theU-tube, $h = (300 + 150) - 400 = 50\ \text{mm} = 0.5\text{m}$

Taking datum line X-X

Pressure in the left limb below X-X $= p_M - 1200 \times 9.81 \times 0.30 = p_M - 3531.6$

Pressure in the right limb below X-X

$$= p_N - 800 \times 9.81 \times 0.4 - 700 \times 9.81 \times 0.05$$

$$= p_N - 3139.2 - 343.35 = p_N - 3482.55$$

Equating two pressures $p_M - 3531.6 = p_N - 3482.55$

$\therefore$ Pressure difference between M and N, $p_M - p_N = 3531.6 - 3482.55$

$$= \mathbf{49.05\ N/m^2} = \mathbf{0.049\ kN/m^2} \qquad \textbf{\textit{(Ans)}}$$

Problem 1.1.1. Calculate the specific weight, specific mass, specific volume, and specific gravity of one litre of a liquid which weighs 7 N.

(Diploma Examination Question, April-2005)

Solution: Given

Volume of given liquid, $V = 1$ litre $= 1 \times 10^{-3}$ m^3

($\because$ 1 litre = 1/ 1000 m^3)

Weight of liquid, $W = 7$ N.

(i) Specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V} = \dfrac{7}{1\times10^{-3}}$ $= \mathbf{7000\ N / m^3}$ *(Ans)*

(ii) Specific mass or density, $\rho = \dfrac{\text{Specific weight}}{g} = \dfrac{w}{g} = \dfrac{7000}{9.81} = \mathbf{713.56\ kg / m^3}$*(Ans)*

(iii) Specific volume, $v = \dfrac{1}{\rho} = \dfrac{1}{713.56}$ $= \mathbf{1.40 \times 10^{-3}\ m^3 / kg}$ *(Ans)*

(iv) Specific gravity, $S = \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{\rho_{liquid}}{\rho_{water}} = \dfrac{713.56}{1000} = \mathbf{0.7136}$ *(Ans)*

Problem 1.1.2. Two litre of petrol weighs 14N, calculate the specific weight, mass density, and specific gravity of petrol. **(Diploma Examination Question, November-2003)**

Solution: Given

Volume of petrol, $V = 2$ litre $= 2 \times 10^{-3}$ m^3

Weight of petrol, $W = 14$ N.

(i) Specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V} = \dfrac{14}{2\times10^{-3}} = \mathbf{7000\ N / m^3}$ *(Ans)*

(ii) Mass density, $\rho = \dfrac{\text{Specific weight}}{g} = \dfrac{w}{g} = \dfrac{7000}{9.81} = \mathbf{713.56\ kg / m^3}$ *(Ans)*

(iii) Specific gravity, $S = \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{\rho_{liquid}}{\rho_{water}} = \dfrac{713.56}{1000} = \mathbf{0.7136}$ *(Ans)*