

# INTRODUCTION TO FLEXURAL MEMBERS: BEAMS

#### **INTRODUCTION**

- Flexural members or bending members are commonly called BEAMS.
- A beam is a structural member subjected to transverse loads i.e., loads perpendicular to the longitudinal axis.
- The load produce Bending moment & Shear forces.



I-sections is the most efficient and economical and therefore, most commonly used section as a beam member.

$$\sigma = \frac{M}{I}y$$

$$I = \frac{1}{12}bd^3$$

#### **DIFFERENT TYPES OF BEAMS**

- **JOIST:** A closely spaced beams supporting floors or roofs of building but not supporting the other beams.
- GIRDER: A large beam, used for supporting a number of joists.
- PURLIN: Purlins are used to carry roof loads in trusses.
- **STRINGER:** In building, beams supporting stair steps; in bridges a longitudinal beam supporting deck floor & supported by floor beam.
- **FLOOR BEAM:** A major beam supporting other beams in a building; also the transverse beam in bridge floors.

#### **DIFFERENT TYPES OF BEAMS**

- **SPANDREL BEAM:** In a building, a beam on the outside perimeter of a floor, supporting the exterior walls and outside edge of the floor
- GIRT: A horizontal beam spanning the wall columns of industrial buildings used to support wall coverings is called a GIRT.
- **RAFTER:** A roof beam usually supported by purlins.
- LINTELS: This type of beams are used to support the loads from the masonry over the openings.

#### NATURE OF FORCES ACTING ON BEAMS

- It is assumed that the beam is subjected to only transverse loading.
- All the loads and sections lie in the plane of symmetry.
- It follows that such a beam will be primarily subjected to bending accompanied by shear in the loading plane with no external torsion and axial force.

#### NATURE OF FORCES ACTING ON BEAMS

- The problem of torsion can not completely be avoided in a beam even if the beam shape is symmetrical and loads are in the plane of symmetry.
- The reason is the instability caused by compressive stresses.
   Such instability is defined as LATERAL BUCKLING.
   When it is involving only local components of a beam it is called LOCAL BUCKLING.
- Local buckling is a function of width-thickness ratio.

#### **MODES OF FAILURE**

Primary modes of failure of beams are as follows:

- 1. Bending failure
- 2. Shear failure
- 3. Deflection failure
- 1. Bending failure: Bending failure generally occurs due to crushing of compression flange or fracture of tension flange of the beam.
- 2. Shear failure: This occurs due to buckling of web of the beam near location of high shear forces. The beam can fail locally due to crushing or buckling of the web near the reaction of concentrated loads.
- **3. Deflection failure:** A beam designed to have adequate strength may become unsuitable if it is not able to support its load without excessive deflections.

## CRITERION OF SELECTING A BEAM SECTION

- The usual method of selecting a beam section is by using a section modulus.
- The criterion of economy is weight rather than the section modulus.
- Sometimes deflection and occasionally shear may be the necessary criterion for selection of section.
- It is desirable to choose a light beam furnishing the required modulus of section.

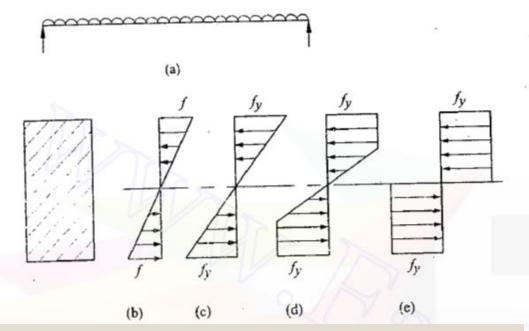
#### LIMITING DEFLECTION

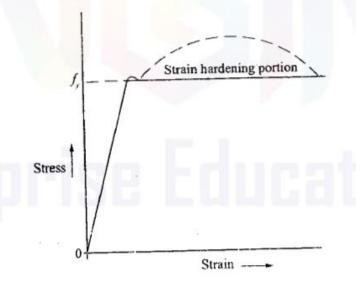
The deflection of a member shall not be such that as to impair the strength or efficiency of the structure & lead to damage to finishing. Generally, the maximum deflection should not exceed the limit recommended by IS 800-2007 in Table 6.

# EFFECTIVE LENGTH FOR LATERAL TORSIONAL BUCKLING

Effective length  $L_{LT}$  for lateral torsional buckling shall be calculated as given in **Table 15** 

# PLASTIC MOMENT CARRYING CAPACITY





Let the area of the section in compression be  $A_c$ , in tension be  $A_t$  and total area A. Equating the horizontal forces for the equilibrium condition, we get

$$A_c f_y = A_t f_y$$

$$A_c f_y = A_t f_y$$

$$\therefore A_c = A_t = \frac{A}{2}$$

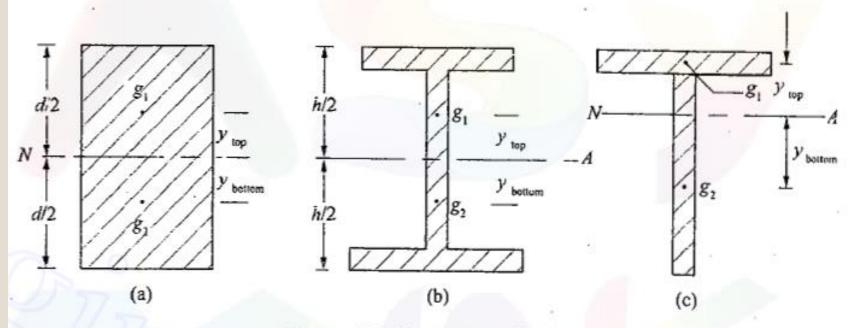


Figure 7.3 Plastic neutral axis.

# SHAPE FACTOR

The ratio of the plastic moment to the yield moment is known as the *shape factor* since it depends on the shape of the cross section.

Let Mp= plastic moment capacity=fy.Zp 
$$Zp = A/2*(\overline{y1}+\overline{y2})$$

My= Elastic moment capacity=fy.Ze Ze= I/y

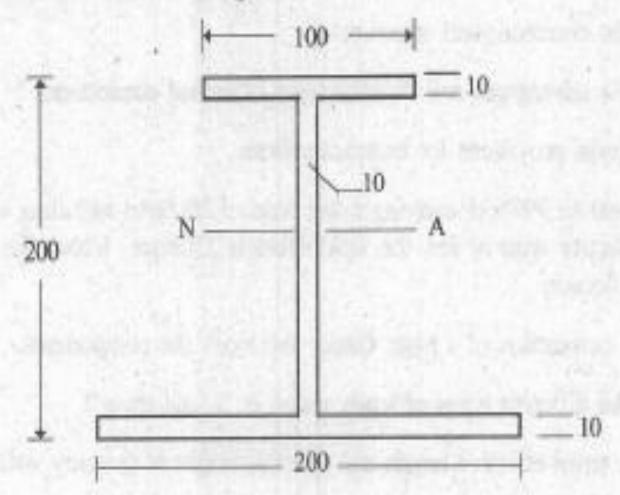
SF = Mp/My

For rectangular section section, SF = 1.5

I- section SF= 1.125 to 1.4

Channel section = 1.7 to 1.8

- (a) Determine the plastic moment capacity and plastic section Modulus of a rectangular beam section of size b x d about z - z axis.
- (b) Determine the plastic moment capacity and plastic modulus of section of the Unsymmetric section shown in figure.



$$A = \text{area} = \text{bd}.$$

$$V_1 = V_2 = \frac{d}{4}$$

$$Z_p = \frac{A}{2} \left( V_1 + V_2 \right)$$

$$= \frac{bd}{4} \left( \frac{d}{4} \right) \left( \frac{d}{4} \right)$$

$$= \frac{bd}{4} \left( \frac{d}{4} \right) \left( \frac{d}{4} \right)$$

$$= \frac{bd^2}{4}$$

$$= \frac{bd^2}{$$

- 3.7.2 On basis of the above, four classes of sections are defined as follows:
  - can develop plastic hinges and have the rotation capacity required for failure of the structure by formation of plastic mechanism. The width to thickness ratio of plate elements shall be less than that specified under Class 1 (Plastic), in Table 2.
- b) Class 2 (Compact) Cross-sections, which can develop plastic moment of resistance, but have inadequate plastic hinge rotation capacity for formation of plastic mechanism, due to local buckling. The width to thickness ratio of plate elements shall be less than that specified under Class 2 (Compact), but greater than that specified under Class 1 (Plastic), in Table 2.
- c) Class 3 (Semi-compact) Cross-sections, in which the extreme fiber in compression can reach yield stress, but cannot develop the plastic moment of resistance, due to local buckling. The width to thickness ratio of plate elements shall be less than that specified under Class 3 (Semi-compact), but greater than that specified under Class 2 (Compact), in Table 2.
- d) Class 4 (Slender) Cross-sections in which the elements buckle locally even before reaching yield stress. The width to thickness ratio of plate elements shall be greater than that specified under Class 3 (Semi-compact), in Table 2. In such cases, the effective sections for design shall be calculated either by following the provisions of IS 801 to account for the post-local-buckling strength or by deducting width of the compression plate element in excess of the semi-compact section limit.

# Laterally Supported Beam (Cl. 8.2.1, IS 800: 2007)

#### **Design Bending Strength**

$$ightharpoonup$$
 If  $V < 0.6 V_d$ 

Where,

V is the factored design shear force and  $V_d$  is the design shear strength of the cross-section

The design bending strength,  $M_{\rm d}$  shall be taken as:

$$M_d = \beta_b Z_p f_v / \Upsilon_{m0}$$

To avoid irreversible deformation under serviceability loads, following conditions are to be satisfied.

$$M_d \le 1.2 Z_e f_y / Y_{m0}$$
 for simply supported beams  $M_d \le 1.5 Z_e f_y / Y_{m0}$  for cantilever beams;

Where,

 $\beta_b = 1.0$  for plastic and compact sections;

 $\beta_b = Z_e / Z_p$  for semi-compact sections;

 $Z_p$ ,  $Z_e$  = plastic and elastic section moduli of the cross-section, respectively;

 $f_v$  = yield stress of the material; and

 $Y_{m0}$  = partial safety factor

### Design for Shear (Cl. 8.4, IS 800: 2007)

The factored design shear force V in a beam should satisfy,

$$V \le \frac{V_n}{\gamma_{m0}}$$

Where  $V_n$  = nominal shear strength of a section

$$V_n = \frac{A_v f_{yw}}{\sqrt{3}}$$

Where  $A_v$  = shear area  $f_{vw}$  = yield strength of the web

# Shear Areas of different Sections (Cl. 8.4.1.1, IS 800: 2007):

Section	Shear Area A <sub>v</sub>
Hot rolled (major axis	$Dt_w$
bending)	
Welded (major axis bending)	$dt_w$
Hot rolled or Welded	$2bt_f$
(minor axis bending)	
Rectangular hollow Sections	AD/(b+D)
(loaded parallel to height)	
Rectangular hollow Sections	Ab/(b+D)
(loaded parallel to width)	
Circular hollow tubes	$2A/\pi$
Plates & solid bars	A

# Web Buckling

- The web behaves like a column if placed under concentrated load.
- The Web is quite thin and therefore is subjected to buckling.
- Web buckling occurs when the intensity of vertical compressive stress near the center of section becomes greater than the critical buckling stress for the web acting as column.

Example 3.4: Determine the design bending of a laterally restrained beam ISMB 300 @ 442 N/m, the yield stress of steel is 250 MPa.

(April/May 2012)

Given Data:

ISMB300 and 
$$f_y = 250 \,\mathrm{N/mm}^2$$

#### Properties:

$$h = 300 \text{ mm}$$
  $d = \text{depth of web}$   
 $b_f = 140 \text{ mm}$   $= h - 2t_f$   
 $t_f = 12.4 \text{ mm}$   $= 300 - 2 \times 12.4$   
 $t_w = 7.5 \text{ mm}$   $= 275.2 \text{ mm}$   
 $z_{xx} = 573.6 \times 10^3 \text{ mm}^3$ 

$$M_{d} = \frac{B_{b}z_{p}f_{y}}{r_{mo}}$$

 $\beta_b$  can be obtained from section classification (table 2, page 18)

$$\frac{b}{t_f} = \frac{\left(\frac{b_f}{2}\right)}{t_f} = \frac{\frac{140}{2}}{12.4} = 5.64 < 9.4\epsilon \qquad \epsilon = \sqrt{\frac{250}{f_y}} \quad \epsilon = \sqrt{\frac{250}{250}} \quad \epsilon = 1$$

$$\frac{d}{t_{yy}} = \frac{275.2}{7.5} = 36.69 < 84\epsilon$$

The given section is "plastic" for plastic section  $\beta_b = 1$ 

$$S = \frac{z_p}{z_e} = 1.14 \rightarrow \text{for symmetrical I-section}$$
,

$$z_p = 1.14 \times z_{xx}$$

$$z_p = 1.14 \! \times \! 473.6 \! \times \! 10^3$$

$$=653.90\times10^3 \text{ mm}^3$$

$$\therefore \quad M_d = \frac{\beta_b \ z_p \ f_y}{r_{mo}} = \frac{1 \times 653.90 \times 10^3 \times 250}{1.1}$$

$$M_d = 148.61 \text{ kNm}$$

Example 3.5: An ISMB 600 @ 1226 N/m is used as simple beam over an effective span of 6m, the beam carries a udl of 24 kN/m including self weight  $f_y = 250 \text{ N/m}$  mm² check the safety of beam in deflection.

$$W = 24 \frac{kN}{m}$$
 (Including self weight)

l = 6 m = 6000 mm

$$f_y = 250 \, \text{N/mm}^2$$

ISMB 600 @ 1226 N/m,  $I_{xx} = 91813 \times 10^4 \text{ mm}^4$ 

Check for deflection condition ( $\delta_{max} < \delta_{per}$ )

$$W = 24 \text{ kN/m}$$

 $\delta_{max}$  for with SS beam udl

$$= \frac{5WI^4}{384 \text{ E I}} = \frac{5 \times 24 \times 6000^4}{384 \times 2 \times 10^5 \times 91813 \times 10^4}$$

$$\delta_{\text{max}} = 2.20 \text{ mm}$$

$$\delta_{per}$$
: SS beam  $\delta_{per} = \frac{1}{240}$ 

$$\delta_{per} = \frac{6000}{240} = 25 \text{ mm}$$

$$\delta_{\text{max}} < \delta_{\text{per}}$$
 $2.20 < 25$ 

Hence given beam is safe against deflection.

Example 3.2: An ISLB 600 @ 995 N/m carrying an Imposed load (or) live load of 20kN/m (or) excluding self weight over an effective span of 6m, the yield stress of steel is 250 Mpa, check the safety of beam in shear.

#### Given data:

Shear check condition  $(0.6 \text{ V}_d > \text{V}_u)$ Imposed load =  $W_1 = 20 \text{ kN/m} = \text{live load}$ dead load = self weight = 995 N/m = 0.995 kN/m =  $W_2$ 

Total load = dead load + live load

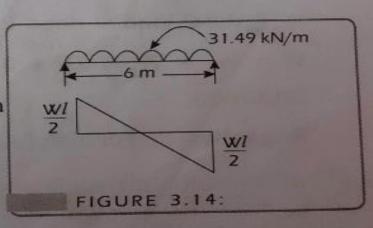
 $W = W_1 + W_2 = 20 + 0.995$ = 20.995 kN/m

Factored load (or) ultimate load = 1.5 × working load

$$W_u = 1.5 \times W$$
  
 $W_u = 1.5 \times 20.995$   
 $= 31.49 \text{ kN/m}, l = 6\text{m}$ 

$$\therefore V_{\rm u} = \frac{W_{\rm u}l}{2} = \frac{31.49 \times 6}{2}$$

$$V_{..} = 94.47 \text{ kN}$$



$$V_d : V_d = \text{design shear} = \frac{A_v f_y}{\sqrt{3} r_{mo}}$$

ISLB 600 
$$A_v = \text{shear area} = 600 \times 10.5$$

Properties 
$$V_d = \frac{(h \times t_w) f_y}{\sqrt{3} \times r_{mo}} = \frac{600 \times 10.5 \times 250}{\sqrt{3} \times 1.1}$$

$$h = 600 \text{ mm} \text{ V}_d = 826.66 \text{ kN}$$

$$t_{w} = 10.5 \text{ mm}$$

$$60\% \text{ of V}_d = 0.6 \text{ V}_d = 0.6 \times 826.66 = 496 \text{ kN}$$

$$0.6 V_d > V_u$$
 The beam is safe in shear

# Example:

A cantilever beam of length 4.5 m supports a dead load (including self weight) of 18 kN/m and a live load of 12 kN/m. Assume a bearing length of 100 mm. Design the beam.

#### Solution:

# Step 1: Calculation of load Dead load = 18 kN/m Live load = 12 kN/m

Total load = 
$$(18 + 12) = 30 \text{ kN/m}$$
  
Total factored load =  $1.5 (18 + 12) = 45 \text{ kN/m}$ 

Step 2: Calculation of BM and SF  

$$BM = \frac{wl^{2}}{2} = \frac{45 \times 4.5^{2}}{2} = 456 \text{ kN-m}$$

$$SF = w \times l = 45 \times 4.5 = 202.5 \text{ kN}$$

$$Z_{p,reqd} = \frac{M \times \gamma_{m0}}{f_y} = \frac{456 \times 10^6 \times 1.1}{250} = 2006.4 \times 10^3 \text{ mm}^3$$

Let us select the section ISLB 550 @ 0.846 kN/m  $Z_{pz} = 2228.16 \times 10^3 \text{ mm}^3$ 

$$Z_{ez} = 1933.2 \times 10^3 \text{ mm}^3$$

$$h = 550 \text{ mm}, b_f = 190 \text{ mm}, t_f = 15 \text{ mm}, t_w = 9.9 \text{ mm}, R = 18$$
  
 $d = 550 - 2 \times (15 + 18)$   
 $= 484 \text{ mm}$ 

$$I_{zz} = 53161.6 \times 10^4 \text{ mm}^4$$

Section classification

$$\frac{{}^{b}f/_{2}}{t_{f}} = \frac{95}{15} = 6.33 < 9.4 \qquad \qquad \frac{d}{t_{w}} = \frac{484}{9.9} = 48.9 < 84$$

Hence, the section is plastic

## Step 4: Calculation of shear capacity of the section

$$V_d = \frac{f_y}{\gamma_{m0} \times \sqrt{3}} \times h \times t_w = \frac{250}{1.1 \times \sqrt{3}} \times 550 \times 9.9$$

$$= 714.47 \text{ kN}$$

$$0.6V_d = 0.6 \times 714.47 = 428.68 \text{ kN} > 202.5 \text{ kN}$$
Hence, Low shear

Step 5: Design capacity of the section

$$M_d = \frac{Z_p \times f_y}{\gamma_{m0}} = \frac{2228.16 \times 10^3}{1.1} \times 250$$
  
= 506.4 kNm

$$\leq \frac{1.5 \times Z_e \times f_y}{\gamma_{m0}} = \frac{1.5 \times 1933.2 \times 10^3 \times 250}{1.1}$$
$$= 659.04 \text{ kNm}$$

Step 6: Check for deflection

$$\delta = \frac{wl^4}{8EI} = \frac{30 \times 4500^4}{8 \times 2 \times 10^5 \times 53161.6 \times 10^4} = 14.5 \text{ mm}$$

Allowable deflection = L/150 = 4500/150 = 30 mmOK **Example 3.7**: Design a rolled steel beam using I-section for simply supported beam of span 6m, carries a UDL of 15 kN/m excluding self weight. The compression flange of beam is laterally restrained. Take  $f_y = 250 \text{ Mpa}$ .

#### Given data

load is excluding self weight, then it is live load

$$W_1 = 15 \text{ kN/m} = 15 \text{ N/mm}$$
  
 $span = l = 6m = 6000 \text{ mm}$   
 $f_y = 250 \text{ N/mm}^2$ 

(1) Given load

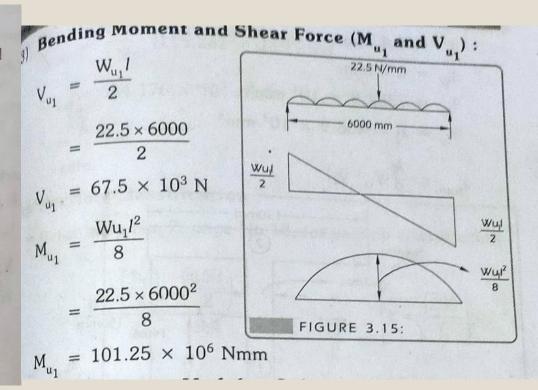
$$W_1 = 15 \text{ N/mm}$$

(2) Ultimate load  $(W_{u_1})$ :

$$1.5 = \frac{W_{u_1}}{W_1}$$

$$W_{u_1} = 1.5 \times W_1 = 1.5 \times 15$$

$$W_{u_1} = 22.5 \text{ N/mm}$$



#### plastic Section Modulus Calculated (zp cal):

$$Z_{p \text{ cal}} = \frac{Mu_1 \times r_{mo}}{f_y} = \frac{101.25 \times 10^6 \times 1.1}{250}$$

$$Z_{p cal} = 445.5 \times 10^3 \text{ mm}^3$$

#### 5) Providing of section :

Choose a section of I-from steel tables which has section modulus more than plastic section modulus calculated  $(z_{p,cal})$ .

Provide ISLB 300 @ 377 N/m

#### roperties :

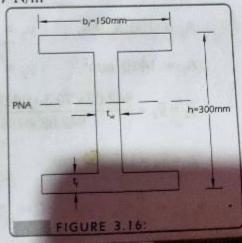
$$W_2 = 377 \text{ N/m}$$
  
= 0.377 N/mm

$$W_{u_2} = 1.5 \times 0.377$$

= 0.565 N/mm

$$A = 4808 \text{ mm}^2$$

h = 300 mm



$$t_{\rm s} = 9.4 \, \text{mm}$$

$$t_w = 6.7 \text{ mm}$$

$$I_{xx} = 7332.9 \times 10^4 \text{ mm}^4$$

$$z_{xx} = z_{e} = 488.9 \times 10^{3} \text{ mm}^{3}$$

# Section Classification :

Refer table No. 2, page No 18, for section classification

$$\frac{b}{t_f} = \frac{\left(\frac{b_f}{2}\right)}{t_f} = \frac{\left(\frac{150}{2}\right)}{9.4}$$

$$\left[\epsilon = \sqrt{\frac{250}{f_y}}\right]$$

$$\frac{b}{t_f} = 7.97 < 9.4 \epsilon$$

$$\frac{b}{t_{\ell}} = 7.97 < 9.4$$

$$\frac{d}{t_w} = \frac{(h - 2t_f)}{t_w} = \frac{300 - 2 \times 9.4}{6.7}$$

$$\frac{d}{t_{uv}} = 41.97 < 84 \epsilon$$

$$\frac{d}{t_w} = 41.97 < 84$$

$$\frac{b}{t_f}$$
 < 9.4  $\epsilon$  and  $\frac{d}{t_w}$  < 84  $\epsilon$ 

: Section is plastic

for plastic section  $\beta_p = 1$ 

(7) Check for Shear 
$$(0.6 V_d > V_v)$$
:

$$V_d = \frac{A_v f_v}{\sqrt{3} r_{mo}}$$

$$V_{d} = \frac{(h \times t_{w}) f_{y}}{\sqrt{3} \times r_{ma}}$$

$$V_d = \frac{300 \times 6.7 \times 250}{\sqrt{3} \times 1.1}$$

$$V_d = 263.74 \times 10^3 \ N$$

$$0.6V_d = 158.24 \times 10^3 \text{ N}$$

$$V_{u} = V_{u_{1}} + V_{u_{2}}$$

$$V_{u_2} = \frac{Wu_2l}{2} = \frac{0.565 \times 6000}{2}$$

$$V_{u_2} = 1695 \text{ N}$$

$$V_u = 67.5 \times 10^3 + 1695$$

$$V_u = 69.195 \times 10^3$$

$$0.6 V_d > V_u$$

The beam is safe against shear.

#### (8) Check for deflection $(\delta_{max} < \delta_{per})$ :

$$\delta_{\text{max}} = \frac{5(W_1 + W_2)l^4}{384EI}$$

[Partial safety factor

for serviceability is '1

$$= \frac{5(15 + 0.377) \times 6000^4}{384 \times 2 \times 10^5 \times 7332.9 \times 10^4}$$

$$= 17.69 \text{ mm}$$

$$\delta_{\text{pet}} = \frac{1}{300} = \frac{6000}{300}$$

$$\delta_{\rm max} < \delta_{\rm per}$$

The beam is safe against deflection.

# Check for design capacity of section :

$$M_d < \frac{1.2 z_e f_y}{r_{mo}}$$

$$M_d = \frac{\beta_b z_{p pro} f_y}{r_{mo}}$$

$$= \frac{1 \times 554.176 \times 10^3 \times 250}{1.1}$$

$$M_d = 125.94 \times 10^6 \text{ Nmm}$$

$$\frac{1.2 z_e f_y}{r} = \frac{1.2 \times 488.9 \times 10^3 \times 250}{1.1}$$

$$= 133.33 \times 10^6 \text{ Nmm}$$

$$M_d < \frac{1.2 z_e f_y}{r_{mo}}$$

Safe.

# DESIGN STEPS FOR LATERALLY SUPPORTED BEAMS

- 1) The loads acting on the beam are calculated by multiplying the appropriate partial load factors.
- 2) The distribution of B.M. & S.F. along the length of the beam is determined. The maximum B.M. & S.F. is calculated
- 3) A trial plastic section for the beam is worked out from the following equation:

$$Z_p = \frac{M_d}{f_y / \gamma_{m0}}$$

4) A suitable section is selected which has plastic section modulus greater than the calculated value. ISMB, ISLB, ISWB sections are in general preferred.

- 5) The section is classified as plastic, compact or semi compact depending upon the specified limits of  $b/t_f$  and  $d/t_w$  as specified in **Table 2, IS 800: 2007**.
- 6) Calculate the design shear strength ( $V_d$ ) from the relation:

$$V_d = \frac{f_y}{\sqrt{3}\gamma_{m0}} h t_w$$

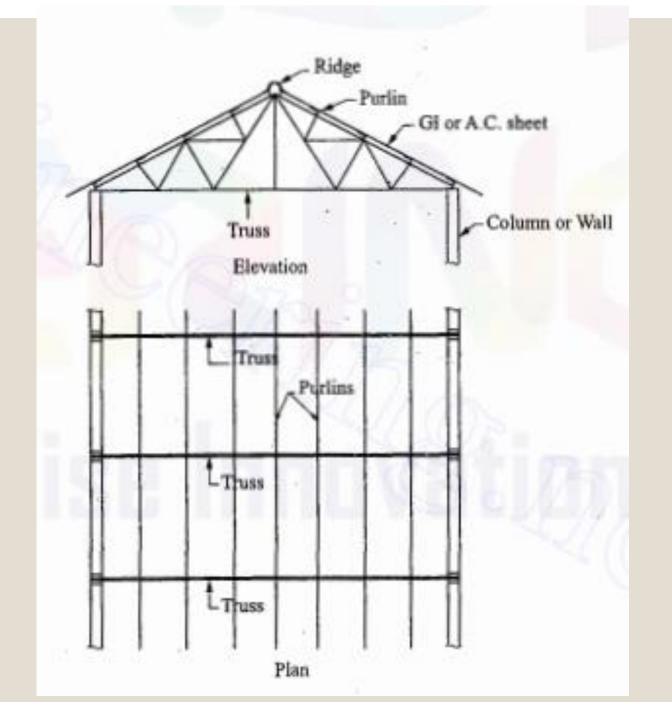
- 7) The beam is checked for high/low shear. If  $V < 0.6 \ V_d$ , the beam will be low shear and if  $V > 0.6 \ V_d$ , the beam will be high shear.
- 8) The trial section is checked for design bending strength For low shear:

$$M_d = \beta_b Z_p f_y / \Upsilon_{m0}$$
  
 $\leq 1.2 Z_o f_y / \Upsilon_{m0}$  (for simply supported beams)  
 $\leq 1.5 Z_o f_y / \Upsilon_{m0}$  (for cantilever beams)

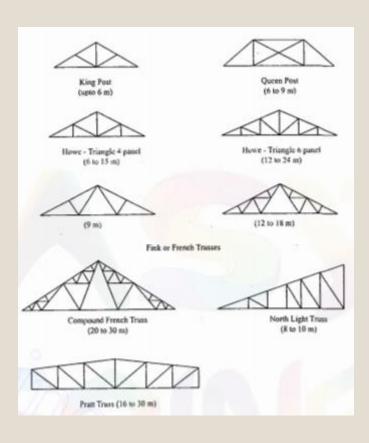
- 8) If  $M > M_d$ , increase the section size and repeat from step 5.
- 9) The design shear strength  $(V_d)$  should be greater than the maximum factored shear force developed due to external load. If  $V > V_d$ , redesign the section by increasing the section size.
- 10) The beam is checked for deflection as per Table 6, IS 800: 2007.

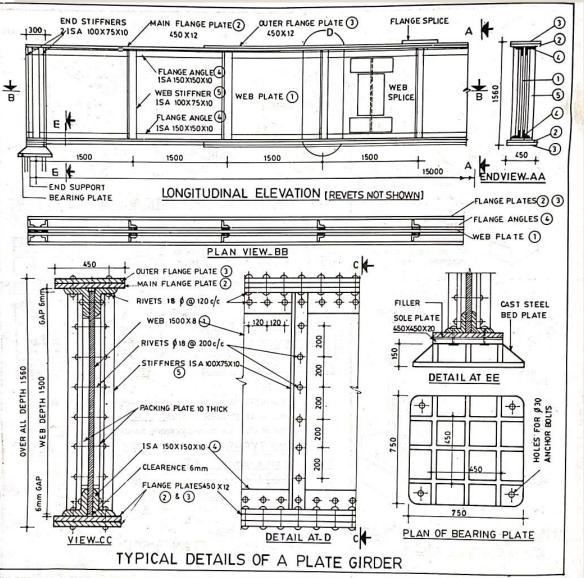
# ROOF TRUSS

 A framed structure composed of several members which are bolted or welded together at their ends.



# Type of Roof Trusses





#### Component parts of plate girder are: (Oct./Nov. 2011, April/May 2019

- (a) Web plates
- (b) Flange angles with (or) without flange cover plates
- (c) Bearing stiffners
- (d) Vertical stiffners
- (e) Horizontal stiffners
- (f) Web splice
- (g) Flange splice
- Web plate: The web of a plate girder resists the entire shear force, the web plate sometimes buckles due to excess load, to avoid buckling vertical and horizontal stiffeners are provided.
- (b) Flange angles: Flanges are essentially consisting of flange angles, it si designed for resisting the maximum bending moment over the beam.
- (c) Bearing stiffners: bearing stiffners are provided at the points of concentrated loads and at supports, the ends of the load bearing stiffners should be milled to fit tightly.
- (d) Vertical stiffners: They should be provided with the length of girder at distance not greater than '1.5 d,' and not less than '0.33 d,'.
  - d, = clear distance between the flange angles.
- (e) Horizontal stiffners:

The horizontal stiffners are extended between the vertice stiffners but need not be continuous over the beam.

- (f) Web splice: Splices in web plates are designed to rest shear and moment at the spliced section.
- (g) Flange splice: A joint in the flange element provided increase the length of flange plate is called flange splice

#### Different types of stiffners:

The webs are stiffned by providing stiffners called w stiffners types.

- (a) Horizontal stiffners
- (b) Vertical stiffners