Course Title: **Engineering Mechanics** Course Code : **2021**

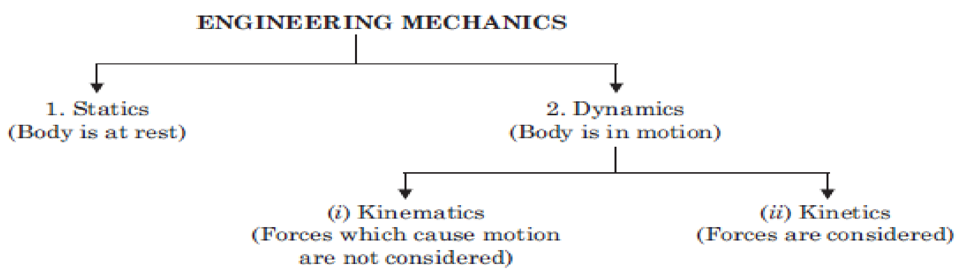
MODULE 1

Basic Concepts of Engineering Mechanics: Engineering mechanics is that branch of science which deals with the behaviour of a body when the body is at rest or in motion.

The engineering mechanics may be divided into Statics and Dynamics.

The branch of science, which deals with the study of a body when the body is at rest, is known as Statics while the branch of science which deals with the study of a body when the body is in motion, is known as Dynamics.

Dynamics is further divided into kinematics and kinetics. The study of a body in motion, when the forces which cause the motion are not considered, is called kinematics and if the forces are also considered for the body in motion, that branch of science is called kinetics.



Significance and relevance of Mechanics:

Modern engineering practice demands a high level of analytical capability and the study of engineering mechanics is to build a foundation of analytical capability for the solution of a variety of engineering problems. In fact no other physical science plays a greater role in engineering than does mechanics. In engineering mechanics we learn to construct and solve mathematical models which describe the effects of forces and motion on a variety of structures and mechanisms that are concern to engineers.

Basic concepts:

The basic concepts used in mechanics are space, time, mass and force.

The concept of space is associated with motion of the position of a point. The position of the point may be defined by three lengths measured from certain reference lines, in three given directions. These lengths are known as the co-ordinates of the point.

The concept for ordering the flow of events is the time. It is the measure of succession of events.

The concept of mass is used to characterize and compare bodies on the basis of the response to a mechanical disturbance. Two bodies of the same mass will be attracted by the earth in the same manner. They will also offer the same resistance to a change in translational motion. Thus mass is the quantitative measure of the resistance to change in motion of a body.

Force represents the action of one body on another. It may be exerted by actual contact or at a distance as in the case of gravitational force and magnetic force. A force is characterised by its point of application, magnitude and direction.

A particle is a body of negligible dimensions and hence may be assumed to occupy a single point in space. When the dimensions of a body are irrelevant to the description of its motion or the action of forces on it, the body may be treated as a particle. For example, even an aeroplane can be treated as a particle for the description of its flight path.

A rigid body is a body which does not deform when subjected to external forces. However, bodies are never absolutely rigid and hence deform under the forces to which they are subjected. When the deformation is negligibly small compared with the overall dimension of the body, the body can be treated as rigid.

SCALAR AND VECTOR QUANTITIES

Vector Quantity. A quantity which is completely specified by magnitude and direction, is known as a vector quantity. Some examples of vector quantities are : velocity, acceleration, force and momentum. A vector quantity is represented by means of a straight line with an arrow as shown in Fig. 1.2. The length of the straight line (i.e., AB) represents the magnitude and arrow represents the direction of the vector. The symbol AB → also represents this vector, which means it is acting from A to B.

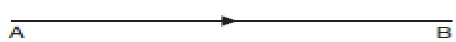


Fig. 1.2. Vector Quantity.

Scalar Quantity. A quantity, which is completely specified by magnitude only, is known as a scalar quantity. Some examples of scalar quantity are : mass, length, time and temperature (4 kg mass, 3.2 m length, 2 second etc.).

RESOLUTION OF A FORCE

Resolution of a force means "finding the components of a given force in two given directions."

Let a given force be R which makes an angle θ with X-axis as shown in Fig. 1.14. It is required to find the components of the force R along X-axis and Y-axis.

Components of R along X-axis $= R \cos \theta$.

Components of R along Y-axis $= R \sin \theta$.

Hence, the resolution of forces is the process of finding components of forces in specified directions.

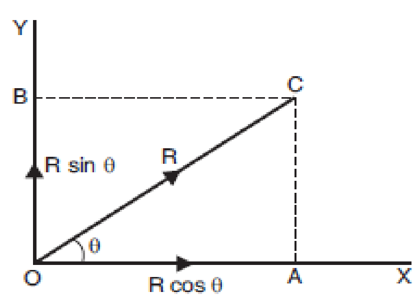


Fig. 1.14

MOMENT OF A FORCE

The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

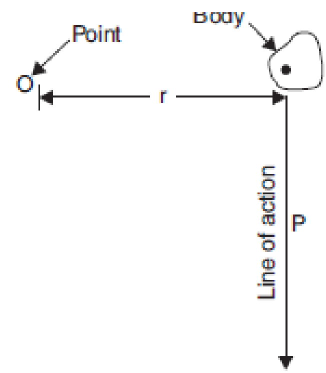
P = A force acting on a body as shown in Fig. 1.20.

r = Perpendicular distance between the point O and line of action of the force P.

The moment of the force P about $O = P \times r$

The tendency of the moment $P \times r$ is to rotate the body in the clockwise direction about O.

Hence this moment is called clockwise moment. If the tendency of rotation is anti-clockwise, the moment is called anti-clockwise moments.



Problem 1.13. *A beam of span 10 m is carrying a point load of 200 N at a distance 4 m from A. Determine the beam reactions.*

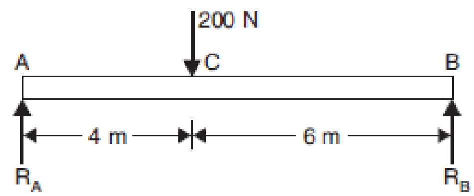


Fig. 1.21

Sol. Given :

Span $AB = 10$ m

Load at C, $W = 200$ N

Distance, $AC = 4$ m

Distance, $BC = 10 - 4 = 6$ m

Let R_A = Reaction at A and

R_B = Reaction at B

As the beam is in equilibrium, the clockwise moments of all forces about any point must be equal to anti-clockwise moments about that point. Also the resultant force in any direction must be zero.

Taking moments about A,

Clockwise moment = Anti-clockwise moments

$$200 \times 4 = R_B \times 10$$

$$\therefore \quad R_B = \frac{200 \times 4}{10} = \mathbf{80\ N.} \quad \textbf{Ans.}$$

Also $R_A + R_B = 200$ N

$$\therefore \quad R_A = 200 - R_B = 200 - 80 = \mathbf{120\ N.} \quad \textbf{Ans.}$$

Problem 1.14. *Four forces of magnitudes 10 N, 20 N, 30 N and 40 N are acting respectively along the four sides of a square ABCD as shown in Fig. 1.22. Determine the magnitude, direction and position of the resultant force.*

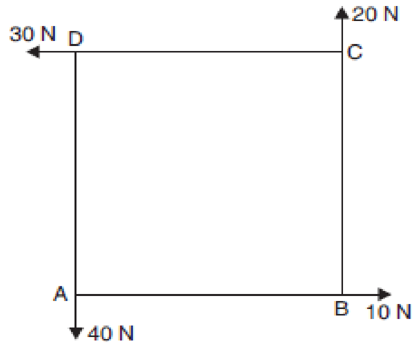


Fig. 1.22

Sol. Given :

Force along $AB = 10$ N

Force along $BC = 20$ N

Force along $CD = 30$ N

Force along $DA = 40$ N

(*i*) *Magnitude and direction of the resultant force*

The net force in the horizontal direction is given as,

$$H = 10 - 30 = -20 \text{ N}$$

The net force in the vertical direction given as,

$$V = 20 - 40 = -20 \text{ N}$$

The resultant force is given by equation (1.8) as

$$R = \sqrt{H^2 + V^2} = \sqrt{(-20)^2 + (-20)^2}$$

$$= \sqrt{400 + 400} = \sqrt{2 \times 400}$$

$$= 20 \times \sqrt{2} \text{ N.} \quad \textbf{Ans.}$$

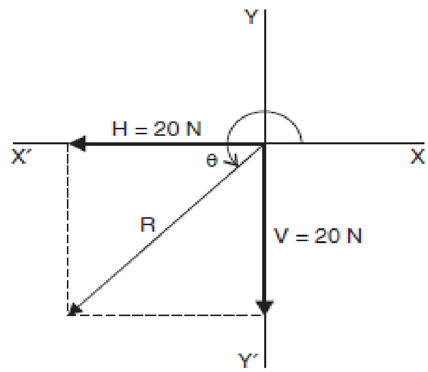
The direction of the resultant force is given by equation (1.9) as

$$\tan\theta = \frac{V}{H} = \frac{-20}{-20} = 1$$

$$\therefore \quad \theta = 45°.$$

Since H and V are –ve, hence θ lies between 180° and 270°. Hence from Fig. 1.23, it is clear that actual

$$\theta = 180 + 45 = 225°. \quad \textbf{Ans.}$$



THE PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states that if a force, acting at a point on a rigid body, is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

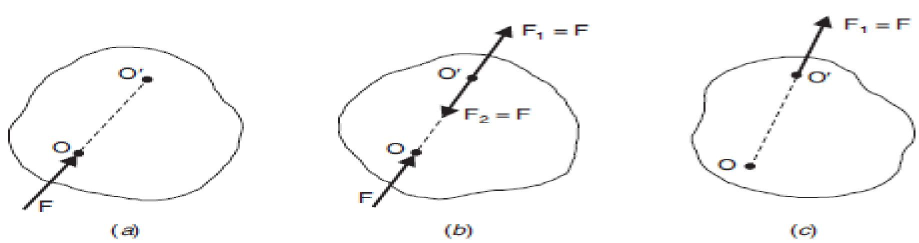


Fig. 1.25

any force acting at a point on a rigid body can be transmitted to act at any other point along its line of action without changing its effect on the rigid body. This proves the principle of transmissibility of a force.

LAMI'S THEOREM

It states that, "If there forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces."

Suppose the three forces *P*, *Q* and *R* are acting at a point *O* and they are in equilibrium as shown in Fig. 1.6.

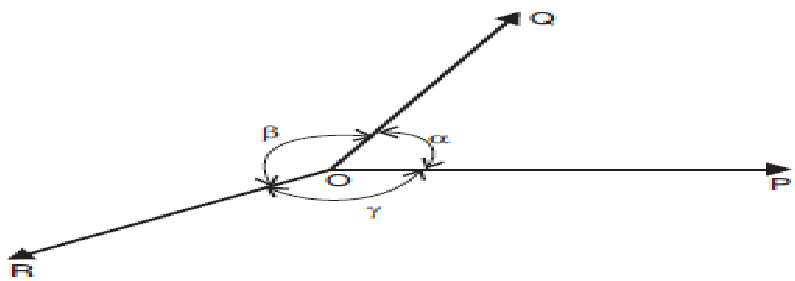


Fig. 1.6

Let α = Angle between force P and Q.

β = Angle between force Q and R.

γ = Angle between force R and P.

Then according to Lami's theorem,

$P \propto$ sine of angle between Q and $R \propto \sin\beta$.

$$\therefore \quad \frac{P}{\sin\beta} = \text{constant}$$

Similarly
$$\frac{Q}{\sin\gamma} = \text{constant and } \frac{R}{\sin\alpha} = \text{constant}$$

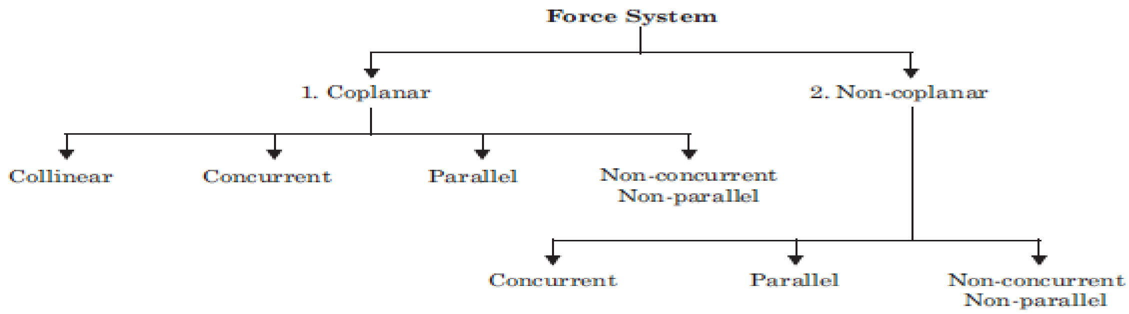
$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}.$$

FORCE SYSTEM

When several forces act on a body, then they are called *a force system* or *a system of forces*. In a system in which all the forces lie in the same plane, it is known as *coplanar force system*.

CLASSIFICATION OF A FORCE SYSTEM

A force system may be coplanar or non-coplanar. If in a system all the forces lie in the same plane then the force system is known as coplanar. But if in a system all the forces lie in different planes, then the force system is known as non-coplanar.



coplanar force system, in which the forces may be :

(*i*) Collinear
(*ii*) Concurrent
(*iii*) Parallel
(*iv*) Non-concurrent, non-parallel

2.2.1. Coplanar Collinear. Fig. 2.2 shows three forces F_1, F_2 and F_3 acting in a plane. These three forces are in the same line, *i.e.*, these three forces are having a common line of action. This system of forces is known as coplanar collinear force system. Hence in coplanar collinear system of forces, all the forces act in the same plane and have a common line of action.

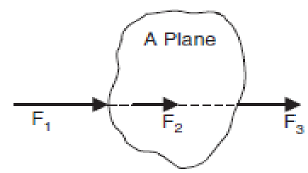


Fig. 2.2. Coplanar Collinear Forces.

2.2.2. Coplanar Concurrent. Fig. 2.3 shows three forces F_1, F_2 and F_3 acting in a plane and these forces intersect or meet at a common point O. This system of forces is known as coplanar concurrent force system. Hence in coplanar concurrent system of forces, all the forces act in the same plane and they intersect at a common point.

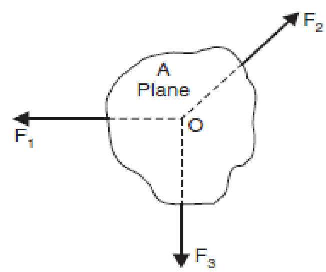


Fig. 2.3. Concurrent Coplanar Forces.

2.2.3. Coplanar Parallel. Fig. 2.4 shows three forces F_1, F_2 and F_3 acting in a plane and these forces are parallel. This system of forces is known as coplanar parallel force system. Hence in coplanar parallel system of forces, all the forces act in the same plane and are parallel.

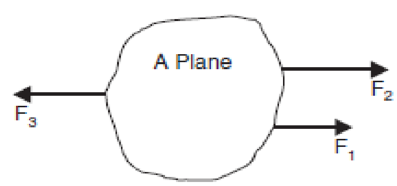


Fig. 2.4. Coplanar Parallel Forces.

2.2.4. Coplanar Non-concurrent Non-parallel. Fig. 2.5 shows four forces F_1, F_2, F_3 and F_4 acting in a plane. The lines of action of these forces lie in the same plane but they are neither parallel nor meet or intersect at a common point. This system of forces is known as coplanar non-concurrent non-parallel force system. Hence in coplanar non-concurrent non-parallel system of forces, all the forces act in the same plane but the forces are neither parallel nor meet at a common point. This force system is also known as *general system of forces*.

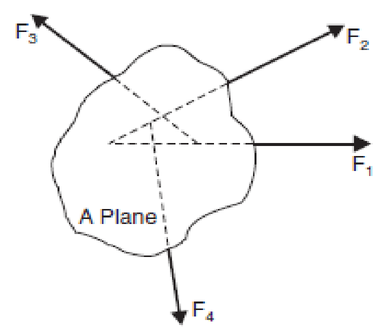


Fig. 2.5. Non-concurrent Non-parallel.

RESULTANT FORCE

A single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body, is known as the *resultant force*.

RESULTANT OF COLLINEAR COPLANAR FORCES

collinear coplanar forces are those forces which act in the same plane and have a common line of action.

Analytical Method. The resultant is obtained by adding all the forces if they are acting in the same direction. If any one of the forces is acting in the opposite direction, then resultant is obtained by subtracting that force.

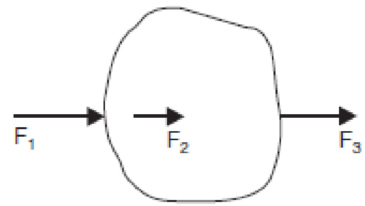


Fig. 2.6

Fig. 2.6 shows three collinear coplanar forces F_1, F_2 and F_3 acting on a rigid body in the same direction. Their resultant R will be sum of these forces.

$$\therefore \quad R = F_1 + F_2 + F_3 \quad \text{...(2.1)}$$

If any one of these forces (say force F_2) is acting in the opposite direction, as shown in Fig. 2.7, then their resultant will be given by

$$R = F_1 - F_2 + F_3 \quad \text{...(2.2)}$$

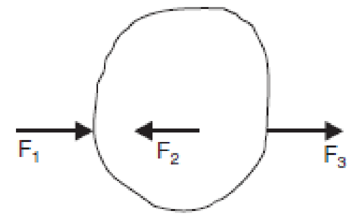


Fig. 2.7

Problem 2.1. *Three collinear horizontal forces of magnitude 200 N, 100 N and 300 N are acting on a rigid body. Determine the resultant of the forces analytically and graphically when*

(*i*) *all the forces are acting in the same direction,*

(*ii*) *the force 100 N acts in the opposite direction.*

Sol. Given : F_1 = 200 N, F_2 = 100 N and F_3 = 300 N

(*a*) *Analytical method*

(*i*) When all the forces are acting in the same direction, then resultant is given by equation (2.1) as

$$R = F_1 + F_2 + F_3 = 200 + 100 + 300 = \mathbf{600\ N}. \quad \textbf{Ans.}$$

(*ii*) When the force 100 N acts in the opposite direction, then resultant is given by equation (2.2) as

$$R = F_1 + F_2 + F_3 = 200 - 100 + 300 = \mathbf{400\ N}. \quad \textbf{Ans.}$$

RESULTANT OF CONCURRENT COPLANAR FORCES

concurrent coplanar forces are those forces which act in the same plane and they intersect or meet at a common point.

(*i*) When two forces act at a point

(*ii*) When more than two forces act at a point.

2.6.1. When Two Forces Act at a Point

(*a*) *Analytical method*

In Art. 1.4, we have mentioned that when two forces act at a point, their resultant is found by the law of parallelogram of forces. The magnitude of resultant is obtained from equation (1.1) and the direction of resultant with one of the forces is obtained from equation (1.2).

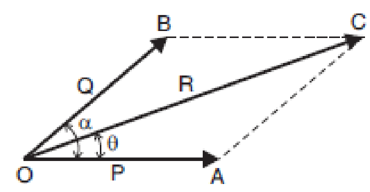


Fig. 2.11

Suppose two forces P and Q act at point O as shown in Fig. 2.11 and α is the angle between them. Let θ is the angle made by the resultant R with the direction of force P.

Forces P and Q form two sides of a parallelogram and according to the law, the diagonal through the point O gives the resultant R as shown.

The magnitude* of resultant is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

The above method of determining the resultant is also known as the *cosine law method*.

The direction* of the resultant with the force P is given by

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

When More than Two Forces Act at a Point

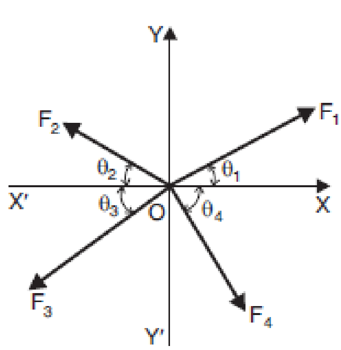
According to this method all the forces acting at a point are resolved into horizontal and vertical components and then *algebraic summation** of horizontal and vertical components is done separately. The summation of horizontal component is written as ΣH and that of vertical as ΣV. Then resultant R is given by

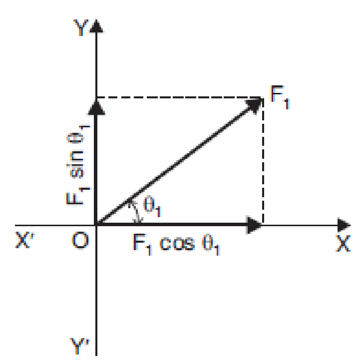
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}.$$

The angle made by the resultant with horizontal is given by

$$\tan\theta = \frac{(\Sigma V)}{(\Sigma H)}$$

$\therefore$ Let four forces F_1, F_2, F_3 and F_4 act at a point O as shown in Fig. 2.14.





Problem 2.2. *Two forces of magnitude 240 N and 200 N are acting at a point O as shown in Fig. 2.16. If the angle between the forces is 60°, determine the magnitude of the resultant force. Also determine the angle β and γ as shown in the figure.*

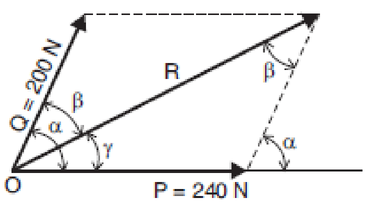


Fig. 2.16

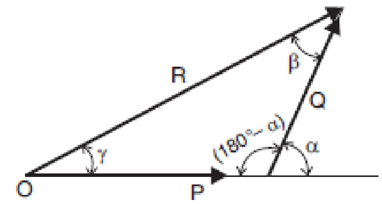


Fig. 2.16(*a*)

Sol. Given :

Force $P = 240$ N, $Q = 200$ N

Angle between the forces, $\alpha = 60°$

The magnitude of resultant R is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{240^2 + 200^2 + 2 \times 240 \times 200 \times \cos 60°}$$

$$= \sqrt{57600 + 40000 + 48000} = \mathbf{381.57\ N.} \quad \textbf{Ans.}$$

Now refer to Fig. 2.16 (*a*). Using sine formula, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)} \qquad \ldots(i)$$

or

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180° - \alpha)}$$

$$\therefore \quad \sin\beta = \frac{P\sin(180° - \alpha)}{R} = \frac{240\sin(180 - 60)}{381.57}$$

$$(\because \; P = 240 \text{ N}, \alpha = 60°, R = 381.57 \text{ N})$$

$$= \frac{240 \times \sin 120°}{381.57} = 0.5447$$

$$\therefore \quad \beta = \sin^{-1} 0.5447 = \mathbf{33°.} \quad \textbf{Ans.}$$

From equation (*i*), also we have $\dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin(180 - \alpha)}$

$$\therefore \quad \sin\gamma = \frac{Q\sin(180 - \alpha)}{R}$$

$$= \frac{200\sin(180 - 60)}{381.57} = \frac{200\sin 120°}{381.57} = 0.4539$$

$$\therefore \quad \gamma = \sin^{-1} 0.4539 = \mathbf{26.966°.} \quad \textbf{Ans.}$$

Problem 2.3. *Two forces P and Q are acting at a point Q as shown in Fig. 2.17. The resultant force is 400 N and angles β and γ are 35° and 25° respectively. Find the two forces P and Q.*

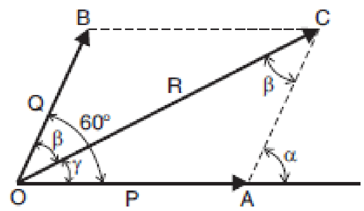


Fig. 2.17

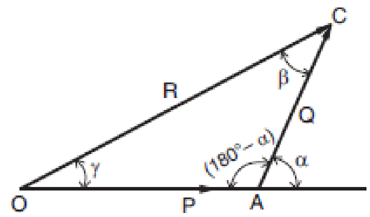


Fig. 2.17(*a*)

Sol. Given :

Resultant, $R = 400$ N

Angles, $\beta = 35°, \gamma = 25°$

∴ Angle between the two forces, $\alpha = \beta + \gamma = 35° + 25° = 60°$

Refer to Fig. 2.17 (*a*). Using sine formula for ΔOAC, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180-\alpha)} \qquad \text{...}(i)$$

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180-\alpha)}$$

$$\therefore \quad P = \frac{R\sin\beta}{\sin(180-\alpha)} = \frac{400\times\sin 35°}{\sin(180-60)} \qquad (\because \quad R = 400, \beta = 35, \alpha = 60°)$$

$$= \frac{400\times 0.5736}{0.866} = \mathbf{264.93\ N.} \quad \textbf{Ans.}$$

Also from equation (*i*), we have $\dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin(180-\alpha)}$

$$\therefore \quad Q = \frac{R\sin\gamma}{\sin(180-\alpha)} = \frac{400\times\sin 25°}{\sin(180-60)} = \frac{400\times 0.4226}{0.866} = \mathbf{195.19\ N.} \quad \textbf{Ans.}$$

Problem 2.4. *Two forces P and Q are acting at a point O as shown in Fig. 2.18. The force P = 240 N and force Q = 200 N. If the resultant of the forces is equal to 400 N, then find the values of angles β, γ and α.*

Sol. Given :

Forces, $P = 240$ N, $Q = 200$ N

Resultant, $R = 400$ N

Let β = Angle between R and Q,

γ = Angle between R and P.

From Fig. 2.18, it is clear that, $\alpha = \beta + \gamma$.

Let us first calculate the angle α (*i.e.*, angle between the two forces).

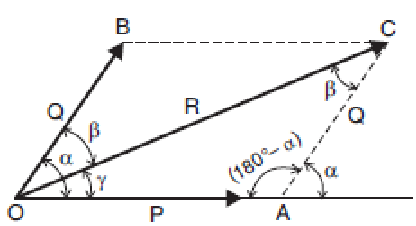


Fig. 2.18

Using the relation,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \text{or} \quad R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$400^2 = 240^2 + 200^2 + 2 \times 240 \times 200 \times \cos\alpha$$

$$16000 = 57600 + 40000 + 96000 \times \cos\alpha$$

$$\therefore \quad \cos\alpha = \frac{16000 - 57600 - 40000}{96000} = 0.65$$

$$\therefore \quad \alpha = \cos^{-1} 0.65 = 49.458° = 49° \; (0.458 \times 60') = 49° \; 27.5'$$

Now using sine formula for ΔOAC of Fig. 2.18, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180 - \alpha)} \qquad \ldots(i)$$

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180 - \alpha)}$$

$$\therefore \quad \sin\beta = \frac{P\sin(180 - \alpha)}{R} = \frac{240\sin(180 - 49.458)}{400} \qquad (\because \; P = 240, \alpha = 49.458°)$$

$$= \frac{240\sin(130.542°)}{400} = 0.4559$$

$$\therefore \quad \beta = \sin^{-1} 0.4559 = \mathbf{27.12°.} \quad \textbf{Ans.}$$

Also from equation (i), we have $\dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin(180 - \alpha)}$

$$\therefore \quad \sin\gamma = \frac{Q\sin(180 - \alpha)}{R} = \frac{200 \times \sin(180 - 49.458)}{400}$$

$$= \frac{200 \times \sin(130.542°)}{400} = 0.3799$$

$$\therefore \quad \gamma = \sin^{-1} 0.3799 = \mathbf{22.33°.} \quad \textbf{Ans.}$$

Problem 2.7. *The four coplanar forces are acting at a point as shown in Fig. 2.21. Determine the resultant in magnitude and direction analytically and graphically.*

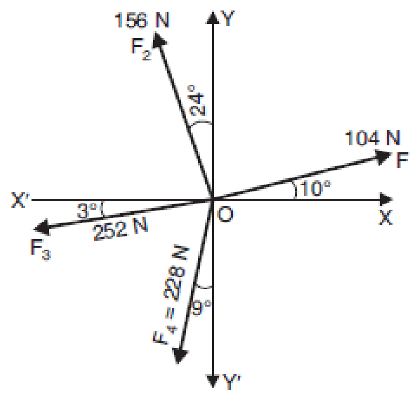


Fig. 2.21

Sol. Given :

Forces, $F_1 = 104$ N,

$F_2 = 156$ N,

$F_3 = 252$ N, and

$F_4 = 228$ N.

(*a*) **Analytical method.** Resolve each force along horizontal and vertical axes. The horizontal components along *OX* will be considered as +ve whereas along *OX′* as –ve. Similarly, vertical components in upward direction will be +ve whereas in downward direction as –ve.

(*i*) *Consider force* $F_1 = 104\ N$. Horizontal and vertical components are shown in Fig. 2.21 (*a*).

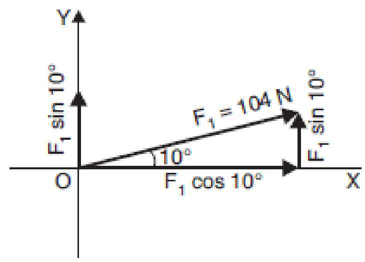


Fig. 2.21 (*a*)

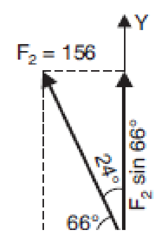
Horizontal component,

$$Fx_1 = F_1 \cos 10° = 104 \times 0.9848$$
$$= 102.42 \text{ N}$$

Vertical component,

$$Fy_1 = F_1 \sin 10° = 104 \times 0.1736$$
$$= 18.06 \text{ N}.$$

(*ii*) *Consider force* $F_2 = 156\ N$. Horizontal and vertical components are shown in Fig. 2.21 (*b*).



Angle made by F_2 with horizontal axis

$$OX' = 90 - 24 = 66°$$

∴ Horizontal components,

$$Fx_2 = F_2 \cos 66° = 156 \times 0.4067$$
$$= 63.44 \text{ N}.$$

It is negative as it is acting along *OX′*.

Vertical component,

$$Fy_2 = F_2 \sin 66° = 156 \times 0.9135$$
$$= 142.50 \text{ N}.$$

(*iii*) *Consider force* $F_3 = 252$ *N*. Horizontal and vertical components are shown in Fig. 2.21 (*c*).

Horizontal component,

$$Fx_3 = F_3 \cos 3° = 252 \times 0.9986$$
$$= 251.64 \text{ N.} \qquad (-\text{ve})$$

Vertical component,

$$Fy_3 = F_3 \sin 3° = 252 \times 0.0523$$
$$= 13.18 \text{ N.} \qquad (-\text{ve})$$

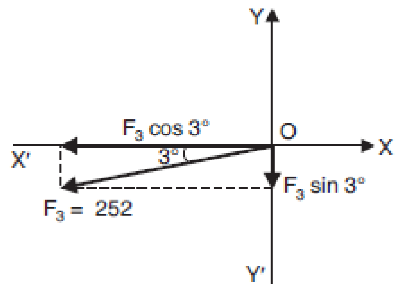


Fig. 2.21 (*c*)

(*iv*) *Consider force* $F_4 = 228$ *N*. Horizontal and vertical components are shown in Fig. 2.21 (*d*).

Angle made by F_4 with horizontal axis

$$OX' = 90 - 9 = 81°.$$

$\therefore$ Horizontal component,

$$Fx_4 = F_4 \cos 81° = 228 \times 0.1564$$
$$= 35.66 \text{ N} \qquad (-\text{ve})$$

Vertical component,

$$Fy_4 = F_4 \sin 81° = 228 \times 0.9877$$
$$= 225.2 \text{ N.} \qquad (-\text{ve})$$

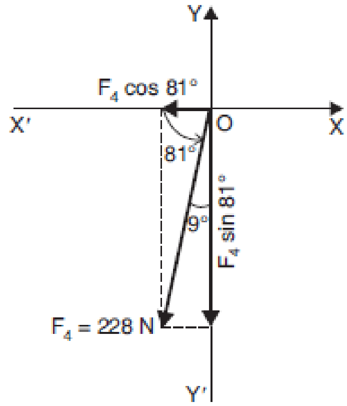


Fig. 2.21 (*d*)

Now algebraic sum of horizontal components is given by,

$$\Sigma H = Fx_1 - Fx_2 - Fx_3 - Fx_4$$
$$= 102.4 - 63.44 - 251.64 - 35.66$$
$$= -248.32 \text{ N.}$$

–ve sign means that ΣH is acting along OX' as shown in Fig. 2.21 (*e*).

Similarly, the algebraic sum of vertical components is given by,

$$\Sigma V = 18.06 = 142.50 + 13.18 - 225.2$$
$$= -77.82 \text{ N.}$$

–ve sign means that ΣV is acting along OY' as shown in Fig. 2.21 (*e*).

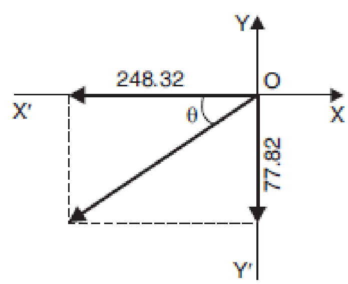


Fig. 2.21 (*e*)

The magnitude of resultant (*i.e.*, *R*) is obtained by using equation (2.1).

$$\therefore \qquad R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(248.32)^2 + (77.82)^2} = \textbf{260.2 N.} \quad \textbf{Ans.}$$

The direction of resultant is given by equation (2.2).

$$\therefore \qquad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{77.82}{248.32} = 0.3134$$
$$\therefore \qquad \theta = \tan^{-1} 0.3134 = \textbf{17.4°}. \quad \textbf{Ans.}$$

Problem 2.8. *The resultant of four forces which are acting at a point O as shown in Fig. 2.23, is along Y-axis. The magnitude of forces F_1, F_3 and F_4 are 10 kN, 20 kN and 40 kN respectively. The angles made by 10 kN, 20 kN and 40 kN with X-axis are 30°, 90° and 120° respectively. Find the magnitude and direction of force F_2 if resultant is 72 kN.*

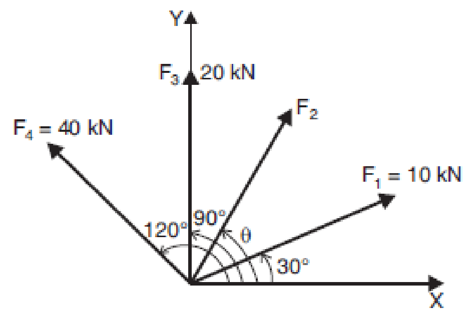


Fig. 2.23

Sol. Given :

$$F_1 = 10 \text{ kN}, \theta_1 = 30°$$
$$F_2 = ?, \quad \theta_2 = \theta$$
$$F_3 = 20 \text{ kN}, \theta_3 = 90°$$
$$F_4 = 40 \text{ kN}, \theta_4 = 120°$$

Resultant, $R = 72$ kN

Resultant is along Y-axis.

Hence the algebraic sum of horizontal component should be zero and algebraic sum of vertical components should be equal to the resultant.

$\therefore \quad \Sigma H = 0 \quad$ and $\Sigma V = R = 72$ kN

But $\quad \Sigma H = F_1 \cos 30° + F_2 \cos \theta + F_3 \cos 90° + F_4 \cos 120°$

$$= 10 \times 0.866 + F_2 \cos \theta + 20 \times 0 + 40 \times (-\tfrac{1}{2})$$
$$= 8.66 + F_2 \cos \theta + 0 - 20$$
$$= F_2 \cos \theta - 11.34$$

$\therefore \quad \Sigma H = 0 \quad$ or $\quad F_2 \cos \theta - 11.34 = 0$

or $\quad F_2 \cos \theta = 11.34 \quad \ldots(i)$

Now $\quad \Sigma V = F_1 \sin 30° + F_2 \sin \theta + F_3 \sin 90° + F_4 \sin 120°$

$$= 10 \times \frac{1}{2} + F_2 \sin \theta + 20 \times 1 + 40 \times 0.866$$
$$= 5 + F_2 \sin \theta + 20 + 34.64$$
$$= F_2 \sin \theta + 59.64$$

But $\quad \Sigma V = R$

$\therefore \quad F_2 \sin \theta + 59.64 = 72$

$\therefore \quad F_2 \sin \theta = 72 - 59.64 = 12.36 \quad \ldots(ii)$

Dividing equation (*ii*) and (*i*),

$$\frac{F_2 \sin \theta}{F_2 \cos \theta} = \frac{12.36}{11.34} \quad \text{or} \quad \tan \theta = 1.0899$$

$\therefore \quad \theta = \tan^{-1} 1.0899 = 47.46°$. **Ans.**

Substituting the value of θ in equation (*ii*), we get $F_2 \sin (47.46°) = 12.36$

Dividing equation (*ii*) and (*i*),

$$\frac{F_2 \sin\theta}{F_2 \cos\theta} = \frac{12.36}{11.34} \quad \text{or} \quad \tan\theta = 1.0899$$

$$\therefore \qquad \theta = \tan^{-1} 1.0899 = \mathbf{47.46°.} \quad \textbf{Ans.}$$

Substituting the value of θ in equation (*ii*), we get $F_2 \sin(47.46°) = 12.36$

$$F_2 = \frac{12.36}{\sin(47.46°)} = \frac{12.36}{0.7368} = \mathbf{16.77\ kN.} \quad \textbf{Ans.}$$

Problem 1.1. *Two forces of magnitude 10 N and 8 N are acting at a point. If the angle between the two forces is 60°, determine the magnitude of the resultant force.*

Sol. Given :

Force $P = 10$ N

Force $Q = 8$ N

Angle between the two forces, $\alpha = 60°$

The magnitude of the resultant force (R) is given by equation (1.1)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{10^2 + 8^2 + 2 \times 10 \times 8 \times \cos 60°}$$

$$= \sqrt{100 + 64 + 2 \times 10 \times 8 \times \tfrac{1}{2}} \qquad \left(\because \cos 60° = \tfrac{1}{2}\right)$$

$$= \sqrt{100 + 64 + 80} = \sqrt{244} = \mathbf{15.62\ N.} \quad \textbf{Ans.}$$

Problem 1.2. *Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to 20 × $\sqrt{3}$ N, find magnitude of each force.*

Sol. Given : Angle between the force, $\alpha = 60°$

Resultant, $R = 20 \times \sqrt{3}$

The forces are equal. Let P is the magnitude of each force.

Using equation (1.3), we have

$$R = 2P\cos\frac{\alpha}{2} \quad \text{or} \quad 20 \times \sqrt{3} = 2P \times \cos\left(\frac{60°}{2}\right) = 2P\cos 30°$$

$$= 2P \times \frac{\sqrt{3}}{2} = P \times \sqrt{3} \qquad \left(\because \cos 30° = \frac{\sqrt{3}}{2}\right)$$

$$\therefore \qquad P = \frac{20 \times \sqrt{3}}{\sqrt{3}} = 20\ \text{N}.$$

∴ Magnitude of each force = **20 N.** **Ans.**

3. Two forces are acting at a point O as shown in Fig. 1.8. Determine the resultant in magnitude and direction.

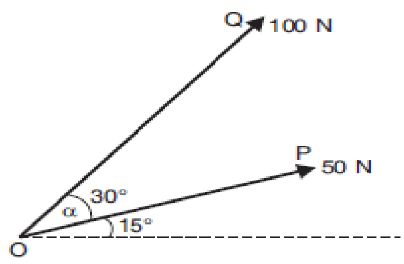


Fig. 1.8

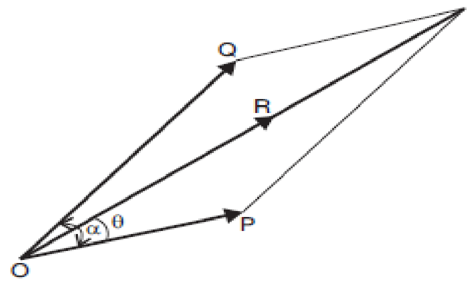


Fig. 1.9

Sol. Given :

Force $P = 50$ N, Force $Q = 100$ N

Angle between the two forces, $\alpha = 30°$

The magnitude of the resultant R is given by equation (1.1) as

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{50^2 + 100^2 + 2 \times 50 \times 100 \times \cos 30°}$$

$$= \sqrt{2500 + 10000 + 8660} = \sqrt{21160} = 145.46 \text{ N. Ans.}$$

The resultant R is shown in Fig. 1.9.

The angle made by the resultant with the direction of P is given by equation (1.2) as

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\left(\frac{100 \times \sin 30°}{50 + 100\cos 30°}\right)$$

$$= \tan^{-1} 0.366 = 20.10°$$

$\therefore$ Angle made by resultant with x-axis $= \theta + 15° = 20.10 + 15 = 35.10°$. **Ans.**

Problem 1.5. *The resultant of two concurrent forces is 1500 N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the force. Find the magnitude of each force.*

Sol. Given :

Resultant, $R = 1500$ N

Angle between the forces, $\alpha = 90°$

Angle made by resultant with one force, $\theta = 36°$

Let P and Q are two forces.

Using equation (1.2), $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha}$

or $$\tan 36° = \frac{Q\sin 90°}{P + Q\cos 90°} = \frac{Q \times 1}{P + Q \times 0} = \frac{Q}{P} \quad \text{or} \quad 0.726 = \frac{Q}{P}$$

or $$Q = 0.726\,P \qquad \ldots(i)$$

Using equation (1.1), $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

or $$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

or $$1500^2 = P^2 + (0.726P)^2 + 2P(0.726P) \times \cos 90° \quad (\because \; Q = 0.726P)$$

$$1500^2 = P^2 + 0.527P^2 + 0 \qquad (\because \quad \cos 90° = 0)$$
$$= 1.527\ P^2$$

$$\therefore \qquad P = \sqrt{\frac{1500^2}{1.527}} = \frac{1500}{1.2357} = 1213.86 \text{ N}$$

Substituting the value of P in equation (i), we get

$$Q = 0.726 \times 1213.86 = \mathbf{881.26 \text{ N.}} \quad \textbf{Ans.}$$

Problem 1.10. *Two forces are acting at a point O as shown in Fig. 1.16. Determine the resultant in magnitude and direction.*

Sol. The above problem has been solved earlier. Hence it will be solved by resolution of forces.

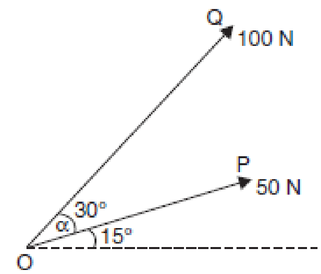


Fig. 1.16

Force $P = 50$ N and force $Q = 100$ N.

Let us first find the angles made by each force with X-axis.

Angle made by P with x-axis = 15°

Angle made by Q with x-axis = 15 + 30 = 45°

Let H = Sum of components of all forces along X-axis.

V = Sum of components of all forces along Y-axis.

The sum of components of all forces along X-axis is given by,

$$H = P \cos 15° + Q \cos 45°$$
$$= 50 \times \cos 15° + 100 \cos 45° = 119 \text{ N}$$

The sum of components of all forces along Y-axis is given by,

$$V = P \sin 15° + Q \sin 45°$$
$$= 50 \sin 15° + 100 \sin 45° = 83.64 \text{ N}$$

The magnitude of the resultant force is given by equation (1.8),

$$R = \sqrt{H^2 + V^2} = \sqrt{119^2 + 83.64^2} = \mathbf{145.46 \text{ N.}} \quad \textbf{Ans.}$$

The direction of the resultant force is given by equation (1.9), $\tan \theta = \dfrac{V}{H} = \dfrac{83.64}{119}$

$$\therefore \qquad \theta = \tan^{-1} \frac{83.64}{119} = \mathbf{35.10°.} \quad \textbf{Ans.}$$

Here θ is the angle made by resultant R with x-axis.

Problem 1.11. *Three forces of magnitude 40 kN, 15 kN and 20 kN are acting at a point O as shown in Fig. 1.17. The angles made by 40 kN, 15 kN and 20 kN forces with X-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.*

Sol. Given :

$R_1 = 10$ kN, $\theta_1 = 60°$

$R_2 = 15$ kN, $\theta_2 = 120°$

$R_3 = 20$ kN, $\theta_3 = 240°$

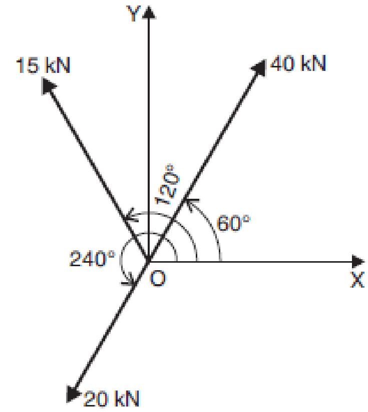


Fig. 1.17

The sum of components of all forces along X-axis is given by equation (1.6) as

$$H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3$$
$$= 40 \times \cos 60° + 15 \times \cos 120° + 20 \times \cos 240°$$
$$= 40 \times \frac{1}{2} + 15 \times \left(-\frac{1}{2}\right) + 20 \times \left(-\frac{1}{2}\right)$$
$$= 20 - 7.5 - 10 = 2.5 \text{ kN.}$$

The resultant component along Y-axis is given by equation (1.7) as

$$V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3$$
$$= 40 \times \sin (60°) + 15 \times \sin (120°) + 20 \times \sin (240°)$$
$$= 40 + \frac{\sqrt{3}}{3} + 15 \times \frac{\sqrt{3}}{2} + 20 \times \left(\frac{-\sqrt{3}}{2}\right)$$
$$= 20 \times \sqrt{3} + 7.5 \times \sqrt{3} - 10 \times \sqrt{3} = 17.5 \times \sqrt{3} \text{ kN} = 30.31 \text{ kN.}$$

The magnitude of the resultant force is given by equation (1.8)

$$R = \sqrt{H^2 + V^2} = \sqrt{2.5^2 + 30.31^2} = \mathbf{30.41 \text{ kN.}} \quad \textbf{Ans.}$$

The direction of the resultant force is given by equation (1.9)

$$\tan \theta = \frac{V}{H} = \frac{30.31}{2.5} = 12.124 = \tan 85.28°$$

Problem 1.12. *Four forces of magnitude 10 kN , 15 kN, 20 kN and 40 kN are acting at a point O as shown in Fig. 1.18. The angles made by 10 kN, 15 kN, 20 kN and 40 kN with X-axis are 30°, 60°, 90° and 120° respectively. Find the magnitude and direction of the resultant force.*

Sol. Given :

$R_1 = 10$ kN and $\theta_1 = 30°$

$R_2 = 15$ kN and $\theta_2 = 60°$

$R_3 = 20$ kN and $\theta_3 = 90°$

$R_4 = 40$ kN and $\theta_4 = 120°$

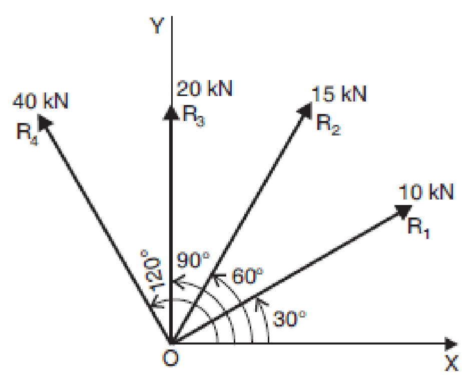


Fig. 1.18

The resultant components along X-axis is given by (1.6) as

$$H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3 + R_4 \cos \theta_4$$
$$= 10 \times \cos 30° + 15 \cos 60° + 20 \cos 90° + 40 \cos 120°$$

$$= 10 \times \frac{\sqrt{3}}{2} + 15 \times \frac{1}{2} + 20 \times 0 + 40 \times \left(-\frac{1}{2}\right) \qquad (\because \cos 90° = 0 \text{ and } \cos 120° = -\tfrac{1}{2})$$

$$= 5 \times \sqrt{3} + 7.5 - 20 = 8.66 + 7.5 - 20 = -3.84 \text{ kN}.$$

Negative sign means that H is acting along OX' as shown in Fig. 1.19.

The resultant component along Y-axis is given by equation (1.7) as

$$V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3 + R_4 \sin \theta_4$$
$$= 10 \sin 30° + 15 \sin 60° + 20 \sin 90° + 40 \sin 120°$$
$$= 10 \times \frac{1}{2} + 15 \times \frac{\sqrt{3}}{2} + 20 \times 1 + 40 \times \frac{\sqrt{3}}{2}$$
$$= 5 + 7.5 \times \sqrt{3} + 20 + 20 \times \sqrt{3}$$
$$= 25 + 27.5 \times \sqrt{3} = 72.63 \text{ kN}.$$

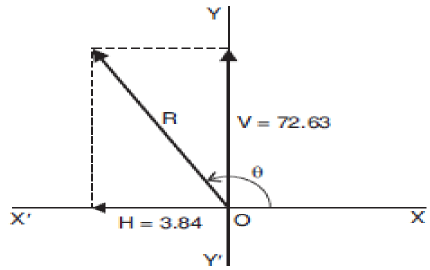


Fig. 1.19

Positive sign means that V is acting along OY as shown in Fig. 1.19.

The magnitude of the resultant force is given by equation (1.8) as

$$R = \sqrt{H^2 + V^2} = \sqrt{(-3.84)^2 + 72.63^2}$$
$$= \sqrt{14.745 + 5275.117} = \mathbf{72.73 \text{ kN}}. \quad \textbf{Ans.}$$

The direction of the resultant force is given by equation (1.9) as

$$\tan \theta = \frac{V}{H} = \frac{72.63}{-3.84} = -18.91.$$

From Fig. 1.19 it is clear that θ lies between 90° and 180°.

The angle whose tangent is 18.91° is 86.97.

$$\therefore \qquad \theta = (180° - 86.97°) = \mathbf{93.03°}. \quad \textbf{Ans.}$$

Problem 2.1. *Three collinear horizontal forces of magnitude 200 N, 100 N and 300 N are acting on a rigid body. Determine the resultant of the forces analytically and graphically when*

(*i*) *all the forces are acting in the same direction,*

(*ii*) *the force 100 N acts in the opposite direction.*

Sol. Given : F_1 = 200 N, F_2 = 100 N and F_3 = 300 N

(*a*) *Analytical method*

(*i*) When all the forces are acting in the same direction, then resultant is given by equation (2.1) as

$$R = F_1 + F_2 + F_3 = 200 + 100 + 300 = \mathbf{600 \text{ N}}. \quad \textbf{Ans.}$$

(*ii*) When the force 100 N acts in the opposite direction, then resultant is given by equation (2.2) as

$$R = F_1 + F_2 + F_3 = 200 - 100 + 300 = \mathbf{400 \text{ N}}. \quad \textbf{Ans.}$$

(*b*) *Graphical method*

Select a suitable scale. Suppose 100 N = 1 cm. Then to this scale, we have

$$F_1 = \frac{200}{100} = 2 \text{ cm},$$

$$F_2 = \frac{100}{100} = 1 \text{ cm},$$

and $$F_3 = \frac{300}{100} = 3 \text{ cm}.$$

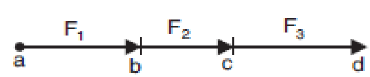


Fig. 2.10

(*i*) When all the forces act in the same direction.

Draw vectors ab = 2 cm to represent F_1,

vector bc = 1 cm to represent F_2 and

vector cd = 3 cm to represent F_3 as shown in Fig. 2.10.

Measure vector ad which represents the resultant.

By measurement, length ad = 6 cm

$\therefore$ Resultant = Length ad × chosen scale (∵ Chosen scale is 1 cm = 100 N)

= 6 × 100 = **600 N. Ans.**

(*ii*) When force 100 N = F_2, acts in the opposite direction

Draw length ab = 2 cm to represent force F_1.

From b, draw bc = 1 cm in the opposite direction to represent F_2. From c, draw cd = 3 cm to represent F_3 as shown in Fig. 2.10 (a).

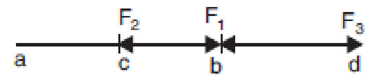


Fig. 2.10 (*a*)

Measure length ad. This gives the resultant.

By measurement, length ad = 4 cm

$\therefore$ Resultant = Length ad × chosen scale

= 4 × 100 = **400 N. Ans.**

Problem 2.10. *Determine the magnitude, direction and position of a single force P, which keeps in equilibrium the system of forces acting on the corners of a rectangular block as shown in Fig. 2.25. The position of force P may be stated by reference to axes with origin O and coinciding with the edges of the block.*

Sol. Given :

Length OC = 4 m, Length BC = 3 m

Force at O = 20 N (←), Force at C = 35 N (↓)

Force at B = 25 N (→), Force at A = 50 N (↓)

Let O be the origin and OX and OY be the reference axes as shown in Fig. 2.26.

All these forces are neither concurrent nor-parallel. Some forces are parallel while others are concurrent.

Forces 50 N and 20 N form a concurrent system and their line of action intersect at O. These forces are at right angles.

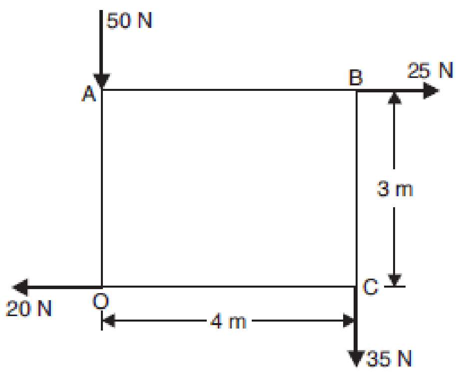


Fig. 2.25

The resultant of these forces

$$R_1 = \sqrt{50^2 + 20^2} = \sqrt{2900} = 53.85 \text{ N}$$

and $$\theta_1 = \tan^{-1}\left(\frac{20}{50}\right) = 21.8° \text{ with vertical axis.}$$

Similarly the forces 35 N and 25 N form a concurrent system and their line of action intersect at *B*. These forces are also at right angles.

The resultant of these forces

$$R_2 = \sqrt{25^2 + 35^2} = \sqrt{1850} = 43.01 \text{ N}$$

and $$\theta_2 = \tan^{-1}\left(\frac{25}{35}\right) = 35.53° \text{ with } BC \text{ i.e., with vertical line.}$$

These two forces R_1 and R_2 intersect at D. The angle between these forces is $\theta_1 + \theta_2$ *i.e.*, angle $R_1DR_2 = \theta_1 + \theta_2 = 21.8° + 35.53° = 57.33°$. The resultant of R_1 and R_2 can be obtained by using equations (1.1) and (1.2).

Let P be the resultant of the forces R_1 and R_2.

$$\therefore \quad P = \sqrt{R_1^2 + R_2^2 + 2R_1 \times R_2 \times \cos(57.33°)}$$

$$= \sqrt{53.85^2 + 43.01^2 + 2 \times 53.85 \times 43.01 \times \cos 57.33°}$$

$$= \sqrt{2900 + 1850 + 4632.17 \times 0.5398} = 85.15 \text{ N.} \quad \textbf{Ans.}$$

The angle made by the resultant P with R_1 is given by [See equation (1.2)]

$$\tan\alpha = \frac{R_2 \sin 57.33°}{R_1 + R_2 \cos 57.33°} = \frac{43.01 \sin 57.33°}{53.85 + 43.01 \times \cos 57.33°}$$

$$= \frac{43.01 \times 0.8418}{53.85 + 23.21} = \frac{36.2058}{77.06} = 0.4698$$

$$\therefore \quad \alpha = \tan^{-1} 0.4698 = 25.16°$$

Hence the resultant P makes $(\alpha - \theta_1)$ angle with vertical in anti-clockwise direction *i.e.*, P makes $(25.16 - 21.8 = 3.36°)$. **Ans.**

MOMENT OF A FORCE

The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Let F = A force acting on a body as shown in Fig. 3.1.

r = Perpendicular distance from the point O on the line of action of force F.

Then moment (M) of the force F about O is given by,

$$M = F \times r$$

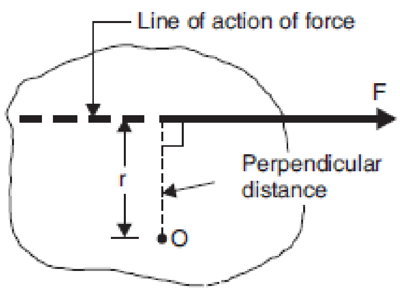


Fig. 3.1

The tendency of this moment is to rotate the body in the clockwise direction about O. Hence this moment is called *clockwise moment*. If the tendency of a moment is to rotate the body in anti-clockwise direction, then that moment is known as *anti-clockwise moment*. If clockwise moment is taken –ve then anti-clockwise moment will be +ve.

Problem 3.1. *Four forces of magnitude 10 N, 20 N, 30 N and 40 N are acting respectively along the four sides of a square ABCD as shown in Fig. 3.3. Determine the resultant moment about the point A. Each side of the square is given 2 m.*

Sol. Given :

Length $AB = BC = CD$
$= DA = 2$ m

Force at $B = 10$ N,
Force at $C = 20$ N,
Force at $D = 30$ N,
Force at $A = 40$ N,

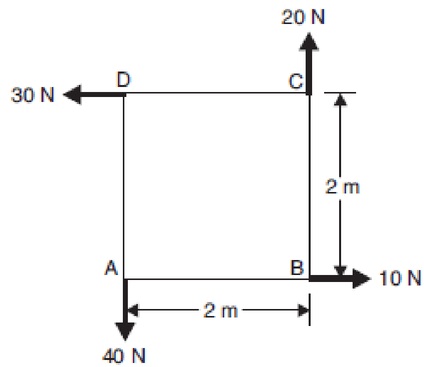


Fig. 3.3

The resultant moment about point A is to be determined.

The forces at A and B passes through point A. Hence perpendicular distance from A on the lines of action of these forces will be zero.

Hence their moments about A will be zero. The moment of the force at C about point A.

= Force at C × ⊥ distance from A on the line of action of force at C.
= (20 N) × (Length AB).
= 20 × 2 Nm = 40 Nm (anti-clockwise).

The moment of force at D about point A.

= Force at D × ⊥ distance from A on the line of action of force at D.
= (30 N) × (Length AD).
= 30 × 2 Nm = 60 Nm (anti-clockwise).

∴ Resultant moment of all forces about A.

= 40 + 60 = 100 Nm (anti-clockwise). **Ans.**

VARIGNON'S THEOREM

Varignon's Theorem states that the moment of a force about any point is equal to the *algebraic sum* of the moments of its components about *that point*.

$$= F_1 \times r_1 + F_2 \times r_2$$

= Moment of F_1 about O' + Moment of F_2 about O'.

Problem 3.3. *Three like parallel forces 100 N, 200 N and 300 N are acting at points A, B and C respectively on a straight line ABC as shown in Fig. 3.12. The distances are AB = 30 cm and BC = 40 cm. Find the resultant and also the distance of the resultant from point A on line ABC.*

Sol. Given :

Force at $A = 100$ N

Force at $B = 200$ N

Force at $C = 300$ N

Distance $AB = 30$ cm, $BC = 40$ cm. As all the forces are parallel and acting in the same direction, their resultant R is given by

$$R = 100 + 200 + 300 = 600 \text{ N}$$

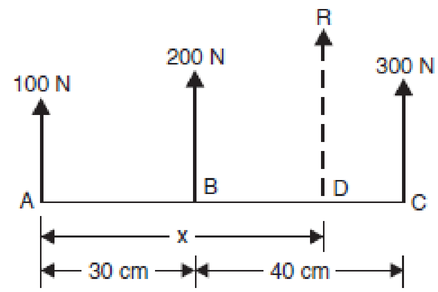


Fig. 3.12

Let the resultant is acting at a distance of x cm from the point A as shown in Fig. 3.12.

Now take the moments of all forces about point A. The force 100 N is passing A, hence its moment about A will be zero.

$\therefore$ Moment of 100 N force about $A = 0$

Moment of 200 N force about $A = 200 \times 30 = 6000$ N cm (anti-clockwise)

Moment of 300 N force about $A = 300 \times AC$

$= 300 \times 70 = 21000$ N cm (anti-clockwise)

Algebraic sum of moments of all forces about A

$= 0 + 6000 + 21000 = 27000$ N cm (anti-clockwise)

Moment of resultant R about $A = R \times x$

$= 600 \times x$ N cm $\qquad (\because \; R = 600)$

But algebraic sum of moments of all forces about A

= Moment of resultant about A

or $$27000 = 600 \times x \quad \text{or} \quad x = \frac{27000}{600} = \mathbf{45 \text{ cm.}} \quad \textbf{Ans.}$$

Problem 4.1. *Two forces F_1 and F_2 are acting on a body and the body is in equilibrium. If the magnitude of the force F_1 is 100 N and its acting at O along x-axis as shown in Fig. 4.4, then determine the magnitude and direction of force F_2.*

Sol. Given :

Force, $F_1 = 100$ N

The body is in equilibrium under the action of two forces F_1 and F_2.

When two forces are acting on a body and the body is in equilibrium, then the two forces should be collinear, equal and opposite.

$\therefore$ $F_2 = F_1 = 100$ N

The force F_2 should pass through O, and would be acting in the opposite direction of F_1.

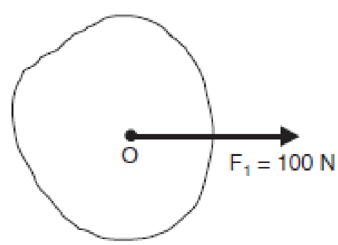


Fig. 4.4

Problem 4.2. *Three forces F_1, F_2 and F_3 are acting on a body as shown in Fig. 4.5 and the body is in equilibrium. If the magnitude of force F_3 is 400 N, find the magnitudes of force F_1 and F_2.*

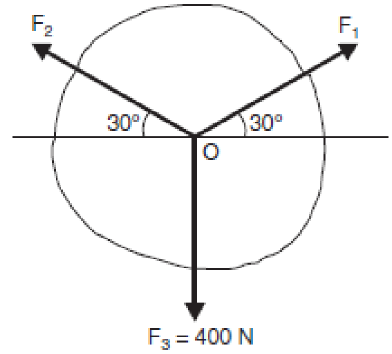


Fig. 4.5

Sol. Given :

Force, $F_3 = 400$ N.

As the body is in equilibrium, the resultant force in x-direction should be zero and also the resultant force in y-direction should be zero.

1st Method

(*i*) For $\Sigma F_x = 0$, we get

$$F_1 \cos 30° - F_2 \cos 30° = 0$$

or $$F_1 - F_2 = 0$$

or $$F_1 = F_2$$

(*ii*) For $\Sigma F_y = 0$, we get

$$F_1 \sin 30° + F_2 \sin 30° - 400 = 0$$

or $$F_1 \times 0.5 + F_2 \times 0.5 = 400$$

or $$F_1 \times 0.5 + F_1 \times 0.5 = 400 \quad (\because \; F_2 = F_1)$$

or $$F_1 = 400 \text{ N. } \quad \textbf{Ans.}$$

Also $$F_2 = F_1 = 400 \text{ N. } \quad \textbf{Ans.}$$

FREE BODY DIAGRAMS

Free body diagram of a body is a diagram in which the body is completely isolated from its support and the supports are replaced by the reactions which these supports exert on the body.

Problem 4.8. *Draw the free body diagram of ball of weight W supported by a string AB and resting against a smooth vertical wall at C as shown in Fig. 4.11 (a).*

Sol. Given :

Weight of ball = W

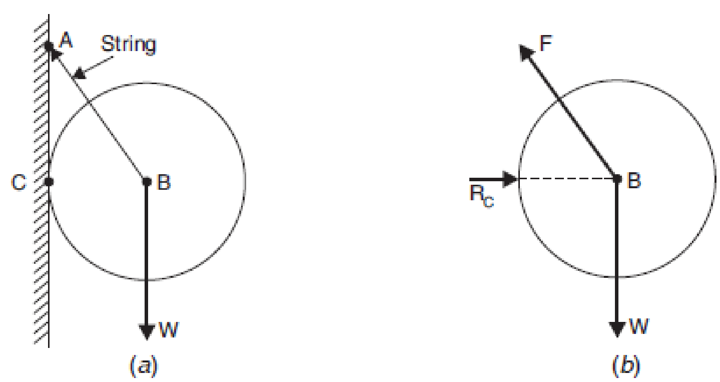


Fig. 4.11

Problem 4.10. *Draw the free-body diagram of a ball of weight W, supported by a string AB and resting against a smooth vertical wall at C and also resting against a smooth horizontal floor at D as shown in Fig. 4.13 (a).*

Sol. Given :

To draw the free-body diagram of the ball, the ball should be isolated completely from the vertical support, horizontal support and string AB. Then the forces acting on the isolated ball as shown in Fig. 4.13 (*b*), will be :

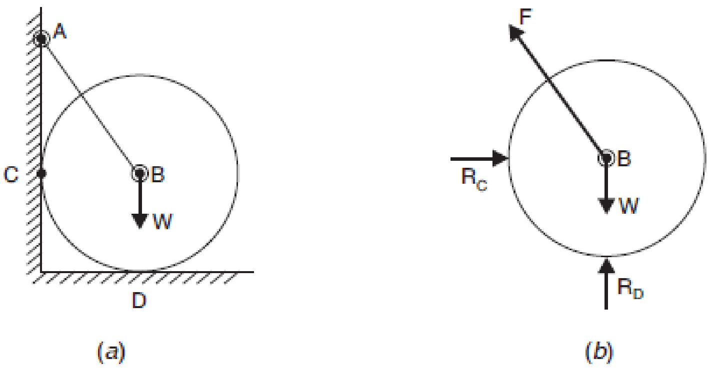


Fig. 4.13

(*i*) Reaction R_C at point C, normal to AC.

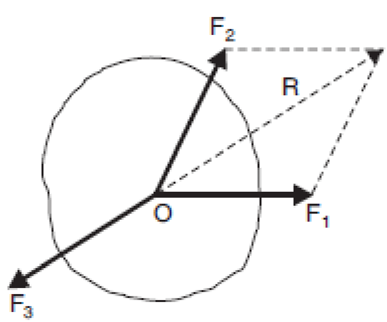
(*ii*) Force F in the direction of string.

(*iii*) Weight W of the ball.

(*iv*) Reaction R_D at point D, normal to horizontal surface.

The reactions R_C and R_D will pass through the centre of the ball *i.e.*, through point B.

Equilibrant: The three concurrent forces $F1$, $F2$ and $F3$ are acting on a body at point O and the body is in equilibrium. The resultant of $F1$ and $F2$ is given by R. If the force $F3$ is collinear, equal and opposite to the resultant R, then the body will be in equilibrium. The force $F3$ which is equal and opposite to the resultant R is known as *equilibrant.*



Equilibrium: a state in which opposing forces or influences are balanced is called Equilibrium