IMPORTANT FORMULAE OF TRIGONOMETRY

1 right angle = 90 degrees (= 90°). 1° = 60 minutes (= 60′).
1′ = 60 seconds (= 60″).
$1^c = (180/\pi)^\circ$. $1^\circ = (\pi/180)^c$.

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

Trigonometric ratios

$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$; $\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$;

$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$; $\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}}$;

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$; $\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$.

Reciprocal Relations

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$; $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$;

$\cos\theta = \frac{1}{\sec\theta}$; $\sec\theta = \frac{1}{\cos\theta}$;

$\tan\theta = \frac{1}{\cot\theta}$; $\cot\theta = \frac{1}{\tan\theta}$.

Quotient Relations

$\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

Pythagoren Relations (or Squared Relations)

$\sin^2\theta + \cos^2\theta = 1$. $\sin^2\theta = 1 - \cos^2\theta$. $\cos^2\theta = 1 - \sin^2\theta$.

$\sec^2\theta - \tan^2\theta = 1$. $\tan^2\theta + 1 = \sec^2\theta$. $\tan^2\theta = \sec^2\theta - 1$.
$\operatorname{cosec}^2\theta - \cot^2\theta = 1$. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$. $\cot^2\theta = \operatorname{cosec}^2\theta - 1$.

Trigonometric Functions of some standard angles

θ	0°	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$	$90^\circ = \frac{\pi}{2}$	$180^\circ = 2\pi$	$270^\circ = \frac{3\pi}{2}$	$360^\circ = 2\pi$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	−1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	−1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined	0	not defined	0
$\cot\theta$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0	not defined	0	not defined
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined	−1	not defined	1
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1	not defined	−1	not defined

ASTC Rule

Quadrant →	I	II	III	IV
t-functions which are positive	All t-functions	Sine Cosecant	Tangent Cotangent	Cosine Secant

Addition Formule

$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$.

Subtraction Formulae

Reduction formulae for $(-\theta)$

$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$,
$\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$,
$\sec(-\theta) = \sec\theta$, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$.

Reduction formulae for $(90^\circ - \theta)$

$\sin(90^\circ - \theta) = \cos\theta$, $\cos(90^\circ - \theta) = \sin\theta$,
$\tan(90^\circ - \theta) = \cot\theta$, $\cot(90^\circ - \theta) = \tan\theta$,
$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$, $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$.

Reduction formulae for $(90^\circ + \theta)$

$\sin(90^\circ + \theta) = \cos\theta$, $\cos(90^\circ + \theta) = -\sin\theta$,
$\tan(90^\circ + \theta) = -\cot\theta$, $\cot(90^\circ + \theta) = -\tan\theta$,
$\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$, $\operatorname{cosec}(90^\circ + \theta) = \sec\theta$.

Reduction formulae for $(180^\circ - \theta)$

$\sin(180^\circ - \theta) = \sin\theta$, $\cos(180^\circ - \theta) = -\cos\theta$,
$\tan(180^\circ - \theta) = -\tan\theta$, $\cot(180^\circ - \theta) = -\cot\theta$,
$\sec(180^\circ - \theta) = -\sec\theta$, $\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}\theta$.

Reduction formulae for $(180^\circ + \theta)$

$\sin(180^\circ + \theta) = -\sin\theta$, $\cos(180^\circ + \theta) = -\cos\theta$,
$\tan(180^\circ + \theta) = \tan\theta$, $\cot(180^\circ + \theta) = \cot\theta$,
$\sec(180^\circ + \theta) = -\sec\theta$, $\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$.

Reduction formulae for $(270^\circ - \theta)$

$\sin(270^\circ - \theta) = -\cos\theta$, $\cos(270^\circ - \theta) = -\sin\theta$,
$\tan(270^\circ - \theta) = \cot\theta$, $\cot(270^\circ - \theta) = \tan\theta$,
$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$, $\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$.

Reduction formulae for $(270^\circ + \theta)$

$\sin(270^\circ + \theta) = -\cos\theta$, $\cos(270^\circ + \theta) = \sin\theta$,
$\tan(270^\circ + \theta) = -\cot\theta$, $\cot(270^\circ + \theta) = -\tan\theta$,
$\sec(270^\circ + \theta) = \operatorname{cosec}\theta$, $\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$.

Reduction formulae for $(360^\circ - \theta)$

$\sin(360^\circ - \theta) = -\sin\theta$, $\cos(360^\circ - \theta) = \cos\theta$,
$\tan(360^\circ - \theta) = -\tan\theta$, $\cot(360^\circ - \theta) = -\cot\theta$,
$\sec(360^\circ - \theta) = \sec\theta$, $\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$.

Reduction formulae for $(360^\circ + \theta)$

$\sin(360^\circ + \theta) = \sin\theta$, $\cos(360^\circ + \theta) = \cos\theta$,
$\tan(360^\circ + \theta) = \tan\theta$, $\cot(360^\circ + \theta) = \cot\theta$,
$\sec(360^\circ + \theta) = \sec\theta$, $\operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec}\theta$.

Multiple and Submultiple Angles

$\sin 2A = 2\sin A\cos A$. $\cos 2A = \cos^2 A - \sin^2 A$.
$\cos 2A = 1 - 2\sin^2 A$. $\cos 2A = 2\cos^2 A - 1$.
$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$. $\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$.
$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$. $\sin^2 A = \frac{1 - \cos 2A}{2}$.
$\cos^2 A = \frac{1 + \cos 2A}{2}$. $\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$.
$\sin 3A = 3\sin A - 4\sin^3 A$. $\cos 3A = 4\cos^3 A - 3\cos A$.
$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$. $\sin\frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$.
$\cos\frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}}$. $\tan\frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$.

Product Formulae

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$.
$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$.
$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$.
$\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.
$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.

Converse of Product formulae

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$.
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$.
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$.
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$.

CHAPTER SUMMARY

- Vertical line : Equation of a vertical line at a distance h from y-axis is $x = h$.
- Horizontal line : Equation of a horizontal line at a distance k from x-axis is $y = k$.
- Slope-Intercept form : Equation of a line having slope m and y-intercept c is $y = mx + c$.
- Point-Slope form : Equation of a line having slope m and passing through (x_1, y_1) is $y - y_1 = m(x - x_1)$.
- Two-Point form : Equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}.$$
- Intercept form : Equation of a straight line having x-intercept a and y-intercept b is
$$\frac{x}{a} + \frac{y}{b} = 1$$
- General Equation to a line : $ax + by = c$.
General equation in the slope-intercept form :
$$y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right).$$
General equation in the intercept form :
$$\frac{x}{-c/a} + \frac{y}{-c/b} = 1.$$
- Angle between two lines : If m_1 and m_2 are the slopes of two lines and if θ is the angle between the lines, then
$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$
Two lines are parallel if $m_1 = m_2$ and perpendicular if $m_1 m_2 = -1$.
The equation to a line parallel to a given line can be obtained by changing the constant term.
The equation to a line perpendicular to a given line can be obtained by interchanging the coefficient of x and y with a change of sign between them and changing (not essential) the constant term.
- The point of intersection : The point of intersection of two lines is obtained by solving their equations.
- Perpendicular Distance Formula : Perpendicular distance of the point (x_1, y_1) from the straight line $ax + by + c = 0$ is
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

CHAPTER SUMMARY

- Function : If to each value of a variable x (within a certain range), there corresponds one definite value of another variable y, then y is function of x, or in functional notation $y = f(x)$.
- Properties of Limits : Let $f(x)$ and $g(x)$ be two functions of x, such that
$$\lim_{x\to a} f(x) = A \quad \text{and} \quad \lim_{x\to a} g(x) = B.$$
Then (i) $\lim_{x\to a}\left[f(x) + g(x)\right] = A + B = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.
(ii) $\lim_{x\to a}\left[f(x) - g(x)\right] = A - B = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$.
(iii) $\lim_{x\to a}\left[f(x)\cdot g(x)\right] = A\cdot B = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$.
(iv) $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{A}{B} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided $B \neq 0$.
- Algebraic Limit : $\lim_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$, for all rational values of n.
- Trigonometrical Limit : $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$, θ being in radian measure.

CHAPTER SUMMARY

Derivative : $\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x + \Delta x) - f(x)}{\Delta x}$.

Standard Results of Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$
e^x	e^x
$\log x$	$\frac{1}{x}$

Function	Derivative
$\sin^{-1} x$	$\frac{1}{\sqrt{1 - x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1 - x^2}}$
$\tan^{-1} x$	$\frac{1}{1 + x^2}$
$\cot^{-1} x$	$\frac{-1}{1 + x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2 - 1}}$
$\operatorname{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2 - 1}}$

Rules of Differentiation

Rule 1. If c is a constant, then $\frac{d}{dx}c = 0$.

Rule 2. If u is a differentiable function of x and c is a constant, then
$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

Rule 3. ***(Sum Rule)*** If u and v are differentiable functions of x,
i. e., $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$.
In general, $\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$.

Rule 4. ***(Product Rule)*** If u, v and w are differentiable functions of x,
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$
$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}.$$

Rule 5. ***(Quotient Rule)*** If u and v are differentiable functions of x,
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

Rule 6. ***(Function of a function Rule)*** If $y = f(u)$ where $u = \phi(x)$,
$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}.$$
If $y = f(u)$ where $u = \phi(v)$ and $v = \psi(x)$, then
$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dv}\times\frac{dv}{dx}.$$
$$\frac{dx}{dy} = 1\Big/\frac{dy}{dx}.$$
$$\frac{d}{dx}\left\{[f(x)]^n\right\} = n[f(x)]^{n-1}\frac{d}{dx}[f(x)].$$
$$\frac{d}{dx}\left\{g(f(x))\right\} = g'(f(x))\times f'(x).$$
If $x = f(t)$ and $y = g(t)$, where t is the parameter,
$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{f'(t)}{g'(t)}.$$

Successive Differentiation :

Second derivative : $y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$.

Third derivative : $y''' = \frac{dy''}{dx} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$.

n^{th} derivative : $y^{(n)} = \frac{d}{dx}y^{(n-1)} = \frac{d^ny}{dx^n}$.

CHAPTER SUMMARY

- A number of the form $z = x + iy$, where x and y are real numbers and i is the imaginary unit is called a ***complex number***. The real number x in $z = x + iy$ is called the ***real part*** of z and the real number y is called the ***imaginary part*** of z, written $x = \text{Re}(z)$ and $y = \text{Im}(z)$.
- Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Then
(i) $z_1 = z_2 \Leftrightarrow x_1 = x_2$ & $y_1 = y_2$.
(ii) $z_1 + z_2 = x_1 + x_2 + i(y_1 + y_2)$.
(iii) $z_1 z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$.
(iv) $\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$.
- For any non-zero complex number $z = x + iy$, there exists a complex number denoted by z^{-1} or $1/z$ called its ***multiplicative inverse*** such that
$$z^{-1} = \frac{x}{x^2 + y^2} + i\frac{-y}{x^2 + y^2} \quad\text{and}\quad z^{-1}z = 1 + i\,0 = 1.$$
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$ and $i^{4k+3} = -i$.
- The ***complex conjugate*** or simply the ***conjugate***, of a complex number $z = x + iy$ is defined as the complex number $\bar{z} = x - iy$.
- The conjugate of a complex number satisfies the properties :
(i) $\frac{z_1 + \bar{z}_1}{2} = \text{Re}\,z_1$; $\frac{z_1 - \bar{z}_1}{2i} = \text{Im}\,z_1$.
(ii) $\overline{(\bar{z}_1)} = z_1$. (iii) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$.
(iv) $\overline{z_1 z_2} = \bar{z}_1\,\bar{z}_2$. (v) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$.
- The ***modulus*** of a complex number $z = x + iy$ is defined by
$$|z| = \sqrt{x^2 + y^2}.$$
- The modulus of a complex number satisfies the properties :
(i) $|\bar{z}_1| = |z_1|$. (ii) $|z_1 z_2| = |z_1|\cdot|z_2|$.
(iii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ (iv) $|z_1 + z_2| \le |z_1| + |z_2|$
(v) $|z_1 + z_2| \ge |z_1| - |z_2|$ (vi) $|z_1 - z_2| \ge \big||z_1| - |z_2|\big|$.
- The polar form of the complex number $z = x + iy$ is $r(\cos\theta + i\sin\theta)$, where $r = \sqrt{x^2 + y^2}$, $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$ and $\theta = \tan^{-1}(y/x)$.
The positive number r is the ***modulus*** $|z|$ of z and θ is known as the ***argument*** of z and is denoted by $\arg z$. The unique value of θ that lies in the interval $-\pi < \theta \le \pi$ is called the ***principal value*** of $\arg z$, denoted by $\text{Arg}\,z$.