COURSE CODE: 5021

DESIGN OF MACHINE ELEMENTS

MODULE-1

1

EXPECTED OUTCOMES

- 1.1.0 Understand the factors governing the Design of Machine elements
- 1.1.1 List the factors governing the design
- 1.1.2 Define the general procedure for design
- 1.1.3 Illustrate the methods of design analytical and empirical
- 1.1.4 Explain design stress, working stress and factor of safety
- 1.2.0 Design of Bolts, Nuts and Keys using analytical and empirical methods
- 1.2.1 Define the important terms used in screw threads
- 1.2.2 List the standard dimensions of screw threads and empirical formulae
- 1.2.3 Explain the designation of screw threads
- 1.2.4 Identify the stresses due to static loading, initial stresses and stresses due to external forces, stresses due to combined forces
- 1.2.5 Estimate the size of the screw from above stresses
- 1.2.6 Design cylinder covers
- 1.2.7 Simple problems related to design of cylinder covers
- 1.2.8 Explain the bolts of uniform strength

EXPECTED OUTCOMES

- 1.2.9 Classify the various types of keys
- 1.2.10 Determine the proportions of sunk key
- 1.2.11 Calculate the strength of rectangular sunk key and square sunk key
- 1.2.12 Select the key size using empirical proportions, simple problems
- 1.3.0 Analyze the working efficiency of screw jack
- 1.3.1 Determine the effort, torque required and efficiency of a square threaded screw jack with collar and without collar fiction
- 1.3.2 Calculate the maximum efficiency of a square threaded screw
- 1.3.3 Describe overhauling and self locking
- 1.3.4 Calculate the efficiency of self locking screw jack

TEXT BOOKS AS PER SYLLABUS

TEXT BOOKS

- 1. A text book of Machine Design R.S. Khurmi and J.K. Gupta
- 2. A text book of Theory of Machines R.S. Khurmi and J.K. Gupta
- 3. A text book of Strength of Materials Dr. R.K. Bansal

REFERENCE

- 1. A Text book of Automobile Engineering T.R. Banger and Nathu Singh
- 2. Machine Design Dr. Sadhu Singh.
- 3. Design of Machine elements M.R.Thomas.

INTRODUCTION

Machine is a mechanical device consisting of <u>interrelated parts</u> that <u>modifies force or motion</u> in order to perform useful work by <u>consuming some form of energy</u>. The interrelated parts are called machine elements.

Design of machine element is the process of determining the size, shape and material of machine elements by considering the forces and motions experienced by the elements.

TYPES OF MACHINE DESIGN

The machine design may be classified as follows:

1. Adaptive design.

- Adaptation of existing designs.
- •The designer only makes minor modification in the existing designs.
- •Ordinary technical training is sufficient . No special knowledge or skill needed

2. Development design.

- •Modify the existing designs into a new idea by adopting a new material or different method of manufacture.
- •Starts from the existing design, but the final product may differ from the original product.
- •Needs considerable scientific training and design ability

3. New design.

- Needs lot of research, technical ability and creative thinking.
- •The designs, depending upon the methods used, may be classified as follows:
 - (a) Rational design Depends upon mathematical formulae of principle of mechanics.
 - (b) Empirical design Depends upon empirical formulae based on past experience.

FACTORS GOVERNING THE DESIGN

•Strength •Weight

•Cost •Control

•Reliability •Existing products

•Shape •Maintenance

•Size •Thermal consideration

•Friction •Styling

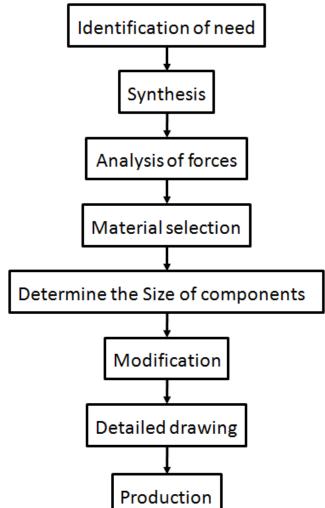
•Corrosion •Stiffness

•Service life •Wear

•Safety •Lubrication

GENERAL PROCEDURE FOR DESIGN

- •Identification of need
- Synthesis (Study of Mechanisms)
- Analysis of forces
- Material selection
- •Determine the Size of components
- Modification
- Detailed drawing
- Production



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DESIGN STRESS, WORKING STRESS AND FACTOR OF SAFETY

Design Stress

It is the maximum stress to which the component is designed. That is it is the stress used in mathematical calculation of the required size of the component.

Working Stress

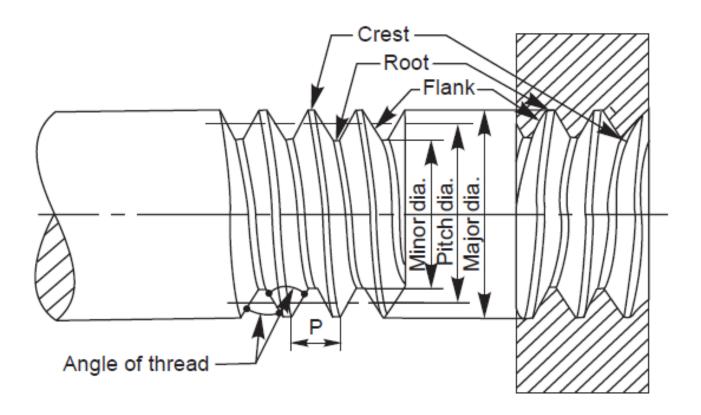
It is the stress actually developed in the component under working condition.

Factor of Safety

It is the ratio of the ultimate stress to the design stress.

$$Factor\ of\ Safety, FS = \frac{\textit{Ultimate Stress}}{\textit{Design Stress}}$$

IMPORTANT TERMS USED IN SCREW THREADS



IMPORTANT TERMS USED IN SCREW THREADS cont...

Major (nominal) diameter

This is the largest diameter of a screw thread, touching the crests on an external thread or the roots of an internal thread.

Minor (core) diameter

This is the smallest diameter of a screw thread, touching the roots or core of an external thread (root or core diameter) thread (root or core diameter) or the crests of an internal thread.

Pitch diameter

This is the diameter of an imaginary cylinder, passing through the threads at the points where the thread width is equal to the space between the threads.

Pitch

It is the distance measured parallel to the axis, between corresponding points on adjacent screwthreads.

IMPORTANT TERMS USED IN SCREW THREADS cont...

Lead

It is the distance a screw advances axially in one turn.

Flank

Flank is the straight portion of the surface, on either side of the screw thread.

Crest

It is the peak edge of a screw thread, that connects the adjacent flanks at the top.

Root

It is the bottom edge of the thread that connects the adjacent flanks at the bottom.

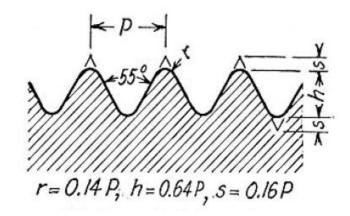
Thread angle

This is the angle included between the flanks of the thread, measured in an axial plane.

FORMS OF SCREW THREADS

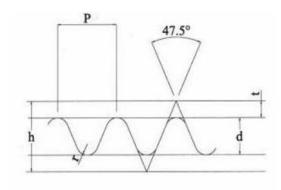
British Standard Whitworth (B.S.W.) thread

This type of thread has V-form and thread angle of 55°. The crest and roots are rounded of to avoid sharp corners which reduces stress concentration. This type of thread is used for general engineering purpose



British association (B.A.) thread

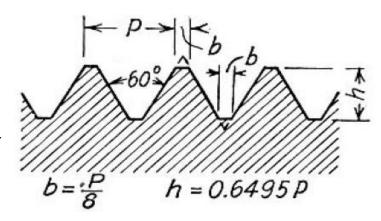
This type of thread has V-form and thread angle of 47.5°. The crest and roots are rounded of to avoid sharp corners which reduces stress concentration. This type of thread is used for precision work such as in electrical fittings and instruments.



FORMS OF SCREW THREADS cont...

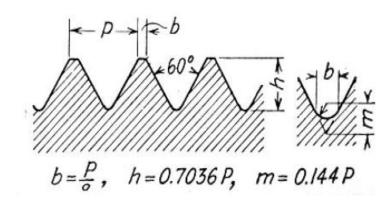
American national standard thread

This thread is also known as Seller's thread. It is similar to V thread and has flat crests and roots. It has thread angle of 60°. The flat crest can withstand more rough usage than sharp V-threads. These threads are used for general purposes e.g. on bolts, nuts, screws and tapped holes.



ISO Metric thread

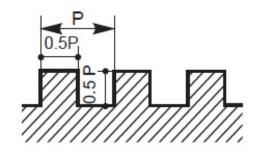
This thread is also known as Unified national thread or Unified thread. It has a V form and thread angle of 60°. It has flat crests and curved roots. The flat crest can withstand more rough usage. These threads are used for general purposes.



FORMS OF SCREW THREADS cont...

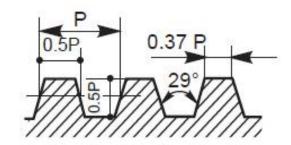
Square thread

This thread has a square form This type of threads are used in machine tools and screw jack etc.



Acme thread

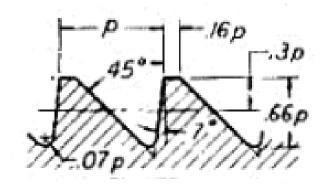
This thread is a modification of square thread and has thread angle of 29°. These threads are stronger than square threads and used in power transmission. These threads are used in lead screw of lathe.



FORMS OF SCREW THREADS cont...

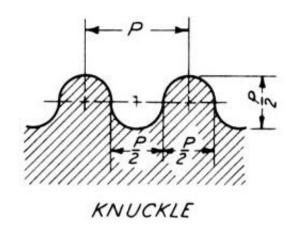
Buttress thread

It is used for transmission of power in one direction only. This thread units the advantage of both square and V-threads. It has a low frictional resistance characteristics of the square thread and have the same strength as that of V-thread. The spindles of bench vices are usually provided with buttress thread.



Knuckle thread

It is a modification of square thread. It has rounded top and bottom. These threads are used for rough and ready work. They are usually found on railway carriage couplings, hydrants, necks of glass bottles and large moulded insulators used in electrical trade.



DESIGNATION OF SCREW THREADS

•The designation of screw thread is based on Indian standards, IS: 4218 (Part IV). The size of the screw thread is designated by the letter `M' followed by the major diameter and pitch in mm, the two being separated by the sign X.

M Diameter X Pitch

Example: M 8 X 1.25

M 12 X 1.75

M 12 X 1.25

•If no pitch is mentioned, it shall mean that coarse pitch is selected.

Example:

M 12

M 12 X 1.25 indicate a fine pitch screw thread.

DESIGNATION OF SCREW THREADS cont...

- •For a complete designation, class of fit should also be mentioned. There are mainly three classes of fits.
 - 1.Close fit 5H/4h
 - 2.Medium fit 6H/6g
 - 3.Free fit -7H/8g

Example:

M 10 X 1.25 -4h M 10 X 1.25 -7H M 14 X 1.5 – 6H/6g Etc

•If no information about class of fit is given a medium fit is assumed.

STANDARD DIMENSIONS OF SCREW THREADS

•The standard dimensions of screw thread are based on Indian standards, IS: 4218 (Part III).

Coarse series			
Designation	Pitch mm	Designation	Pitch mm
M 0.4	0.1	M 10	1.5
M 0.6	0.15	M 12	1.75
M 0.8	0.2	M 14	2
M 1	0.25	M 16	2
M 1.2	0.25	M 18	2.5
M 1.4	0.3	M 20	2.5
M 1.6	0.35	M 22	2.5
M 1.8	0.35	M 24	3
M 2	0.4	M 27	3
M 2.2	0.45	M 30	3.5
M 2.5	0.45	M 33	3.5
M 3	0.5	M 36	4
M 3.5	0.6	M 39	4
M 4	0.7	M 42	4.5
M 4.5	0.75	M 45	4.5
M 5	0.8	M 48	5 5
M 6	1	M 52	5
M 7	1	M 56	5.5
M 8	1.25	M 60	5.5

Fine series		
Designation	Pitch mm	
M8X1	1	
M 10 X 1.25	1.25	
M 12 X 1.25	1.25	
M 14 X 1.5	1.5	
M 16 X 1.5	1.5	
M 18 X 1.5	1.5	
M 20 X 1.5	1.5	
M 22 X 1.5	1.5	
M 24 X 2	2	
M 27 X 2	2	
M 30 X 2	2	
M 33 X 2	2	
M 36 X 3	3	
M 39 X 3	3	

Core diameter, dc = 0.84 d Where, d is the major diameter.

STRESSES IN SCREWED FASTENING DUE TO STATIC LOADING

- •The following stresses in screwed fastening due to static loading are important.
- 1. Initial stresses due to tightening
- 2. Stresses due to external forces
- 3. Stress due to combined forces

The following stresses are developed in a screw fastener while tightening.

- •Tensile stress due to stretching of bolt
- Torsional shear stress during tightening
- Bending stress
- •Shear stress across the threads
- Compression or crushing stress on threads

INITIAL STRESSES DUE TO TIGHTENING

Tensile stress due to stretching of bolt

The initial tensile force in a bolt due to tightening is calculated by following relation. This relation is developed based on experiments.

$$Pi = 2840d N$$

Where,

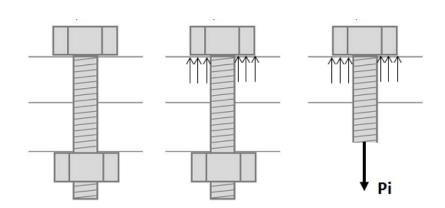
Pi = Initial tension in a bolt, in N d = Nominal diameter of bolt, in mm.

Tensile stress developed, $\sigma_t = \frac{1}{2}$

Where,

$$Area, A = \frac{\pi {d_c}^2}{4}$$

Core diameter, dc = 0.84 d



Torsional shear stress during tightening

The torsional shear stress is developed due to the frictional resistance between the threads of screw and nut during tightening.

The torsional shear stress is calculated using the torsion equation.

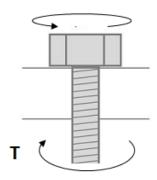
Torsional shear stress,

$$\tau = \frac{16T}{\pi d_c^3}$$

Where,

T = Torque applied

dc = Minor or core diameter of the thread.



Bending stress

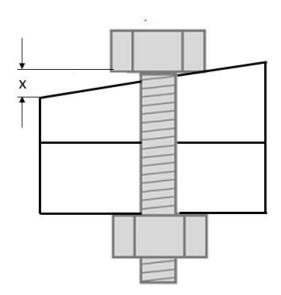
If the surfaces under the head or nut are not perfectly perpendicular to the bolt axis, then the bolt will be subjected to bending action. The bending stress (σ b) induced in the shank of the bolt is given by

$$\sigma_b = \frac{x \cdot E}{2l}$$

x =Difference in height between the extreme corners of the nut or head,

l =Length of the shank of the bolt, and

E =Young's modulus for the material of the bolt.



Shear stress across the threads

The threads tend to shear off about its roots.

The average thread shearing stress for the bolt is

$$\tau_{s} = \frac{P}{\pi d_{c} \times b \times n}$$

The average thread shearing stress for the nut is

$$\tau_s = \frac{P}{\pi d \times b \times n}$$

Where,

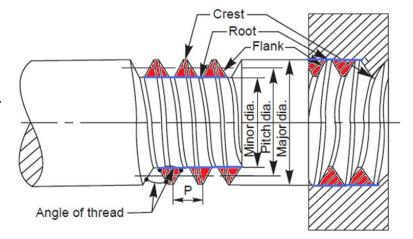
P = applied load

dc = Minor or core diameter of the thread.

d = Nominal diameter of nut.

b = Width of the thread section at the root.

n = number of threads in engagement.



Compression or crushing stress on threads

The compression or crushing stress between the threads (σ c) is obtained by using the relation

$$\sigma_c = \frac{4P}{\pi \left[d^2 - d_c^2\right] n}$$

Where,

P = applied load

dc = Minor or core diameter of the thread.

d = Nominal diameter of nut.

n = number of threads in engagement.

STRESSES IN SCREWED FASTENING DUE TO EXTERNAL LOADING

- •The following stresses in screwed fastening due to static loading are important.
- 1. Tensile stresses
- 2. Shear Stresses
- 3. Combined tensile and shear stress

STRESSES DUE TO EXTERNAL LOADING cont...

Tensile stress due to external load

The tensile stress due to external load in bolt is, $\sigma_t = \frac{P}{A} = \frac{P}{\frac{\pi d_c^2}{A}} = \frac{4P}{\pi d_c^2}$

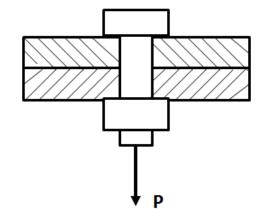
If 'n' number of bolts are resisting the applied load then the tensile stress developed in

each bolt is,

$$\sigma_t = \frac{P}{A} = \frac{P}{n\frac{\pi d_c^2}{4}} = \frac{4P}{n\pi d_c^2}$$

Where,

P = applied external load Core diameter, dc = 0.84 d d = Nominal diameter of bolt



Design of bolt based on material tensile allowable

If P is the applied load, σ_{ta} is the allowable material tensile stress and n is the number of bolts resisting the applied load, then the size of the bolt is calculated as

$$d_c = \sqrt{\frac{4P}{n\pi\sigma_{ta}}}$$

STRESSES DUE TO EXTERNAL LOADING cont...

Shear stress due to external load

The shear stress due to external load in bolt is, $\sigma_s = \frac{P}{A} = \frac{P}{\frac{\pi d^2}{A}} = \frac{4P}{\pi d^2}$

$$\sigma_s = \frac{P}{A} = \frac{P}{\frac{\pi d^2}{4}} = \frac{4P}{\pi d^2}$$

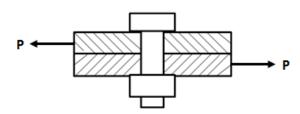
If 'n' number of bolts are resisting the applied load then the tensile stress developed in

each bolt is,
$$\sigma_s = \frac{P}{A} = \frac{P}{n\frac{\pi d^2}{4}} = \frac{4P}{n\pi d^2}$$

Where,

P = applied external load

d = Nominal diameter of bolt



Design of bolt based on material tensile allowable

If P is the applied load, σ_{sa} is the allowable material tensile stress and n is the number of bolts resisting the applied load, then the size of the bolt is calculated as

$$d = \sqrt{\frac{4P}{n\pi\sigma_{sa}}}$$

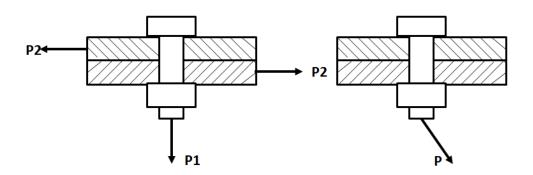
STRESSES DUE TO EXTERNAL LOADING cont...

Combined tension and shear stress

When the bolt is subjected to both tension and shear loads, then the diameter of the shank of the bolt is obtained from the shear load and that of threaded part from the tensile load. Out of these the largest diameter is considered and stresses due to combined load should be checked for the following principal stresses.

Maximum principal shear stress, $\tau_{max} = \frac{1}{2} \sqrt{(\sigma_t)^2 + 4\tau^2}$

Maximum principal tensile stress, $\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2} \sqrt{(\sigma_t)^2 + 4\tau^2}$



PROBLEMS

M1.P1) Two machine components are connected together tightly by a M24 bolt. Calculate the tensile stress induces in the bolt due to initial tightening.

Nominal diameter of the bolt, d = 24mmInitial load, Pi = 2840d N = 2840 x 24 = 68160 N

Core diameter of the bolt, $dc = 0.84d = 0.84 \times 24 = 20.16 \text{ mm}$

Tensile stress developed,
$$\sigma_t = \frac{P_i}{A}$$
 where, $Area, A = \frac{\pi d_c^2}{4}$

$$\sigma_t = \frac{4 P_i}{\pi d_c^2}$$

$$\sigma_t = \frac{4 \times 68160}{\pi \times 20.16^2} = 213.4 \, N/mm^2$$

M1.P2) A machine weighing 9kN has to be provided with an eye bolt for lifting purpose. Find the diameter of the bolt if the permissible stress on the bolt is 20N/mm².

Weight of the machine, P Allowable tensile stress, σ_t Core diameter of the bolt, dc = 9kN = 9000N $= 20N/mm^{2}$ $= \sqrt{\frac{4P}{\pi \sigma_{t}}}$

Nominal diameter, d

 $= \sqrt{\frac{4 \times 9000}{\pi \times 20}} = 23.94mm$ $= \frac{d_c}{0.84}$

$$=\frac{23.94}{0.84}=28.5\ mm$$

Size of the bolt

= M 30

M1.P3) A machine weighing 25kN has to be provided with an steel eye bolt for lifting purpose. Find the diameter of the bolt if the ultimate tensile strength of steel is 480N/mm². Consider a factor of safety of 6 in the design. (2018 OCTOBER 7 Marks)

Weight of the machine, P Ultimate tensile stress, σ_u Factor of safety ,FS

Allowable tensile stress, σ_t

Core diameter of the bolt, dc

Nominal diameter, d

Size of the bolt

= 25kN = 25000N = 480 N/mm²

= 6

 $=\frac{\sigma_u}{FS}$

 $=\frac{480}{6}=80\ N/mm^2$

 $=\sqrt{\frac{4P}{\pi\sigma_t}}$

 $= \sqrt{\frac{4 \times 25000}{\pi \times 80}} = 19.95 mm$

 $=\frac{d_c}{0.84}=\frac{19.95}{0.84}=23.75 mm$

= M 24

32

M1.P4) Two screws for a pipe hanger must hold a tensile load of 10kN. Calculate the most suitable size of screw. Take the working stress in tension is 45MPa. (2019 APRIL 6 Marks)

Applied load, P Allowable tensile stress, σ_t Number of screws, n Core diameter of the bolt, dc

= 10kN = 10000N = 45MPa = 45 N/mm² =2 = $\sqrt{\frac{4P}{n\pi\sigma_t}}$

Nominal diameter, d

 $= \sqrt{\frac{4 \times 10000}{2 \times \pi \times 45}} = 11.9 mm$ $= \frac{d_c}{0.84}$

$$=\frac{11.9}{0.84}=14.17\ mm$$

Size of the screw

= M 16

M1.P5) A generator weighing 20kN is to be provided with an eye bolt in the housing for lifting purposes. Find the size of bolts if it is made of C- 40 steel. If the ultimate tensile strength of C- 40 steel is 600 MPa and the factor of safety is 6? (2018 APRIL 6 Marks)

M1.P5) Answer

Weight of the machine, P Ultimate tensile stress, σ_u Factor of safety ,FS

Allowable tensile stress, σ_t

Core diameter of the bolt, dc

Nominal diameter, d

Size of the bolt

= 20kN = 20000N = 600 N/mm²

=6

 $=\frac{\sigma_u}{FS}$

 $=\frac{600}{6}=100\ N/mm^2$

 $= \sqrt{\frac{4P}{\pi\sigma_t}}$

 $= \sqrt{\frac{4 \times 20000}{\pi \times 100}} = 15.96mm$

 $=\frac{d_c}{0.84}=\frac{15.96}{0.84}=19.00\,mm$

= M 20

M1.P6) An eye bolt carries a tensile load of 18kN. Find the size of the bolt, if the tensile stress is not to exceed 100 MPa.? (2018 APRIL 7 Marks)

M1.P6) Answer

Applied load, P Allowable tensile stress, σ_t Number of screws, n Core diameter of the bolt, dc

Nominal diameter, d

Size of the screw

= 18kN = 18000N
= 100MPa = 100 N/mm²
=1
=
$$\sqrt{\frac{4P}{n\pi\sigma_t}}$$

$$= \sqrt{\frac{4 \times 18000}{1 \times \pi \times 100}} = 15.14mm$$
$$= \frac{d_c}{0.84}$$

$$=\frac{15.14}{0.84}=18.02\ mm$$

= M 20

M1.P7) Two shafts are connected by means of a flange coupling to transmit torque of 210 N-m. The flange of the coupling are fastened by four bolts of the same material at a radius of 50 mm. Find the size of the bolts, if the allowable shear stress for the bolt material is 40 MPa. (2018 APRIL 7 marks)

M1.P7)

Torque transmitted, T Number of bolts ,n Allowable shear stress, τ

Bolt circle radius, r Torque transmitted, T Where, P is the shearing force acting on the bolts shearing force, P

shearing force, P

Where, d is the nominal diameter of the bolt. Nominal diameter of the bolt, d = 210 N - m = $210 \times 10^3 N - mm$ = 4= $40 Mpa = 40 N/mm^2$

= 50 mm $= P \times r$

 $= \frac{T}{r}$ $= \frac{210 \times 10^3}{50} = 4200 N$ $= \frac{\pi}{4} d^2 \tau n$

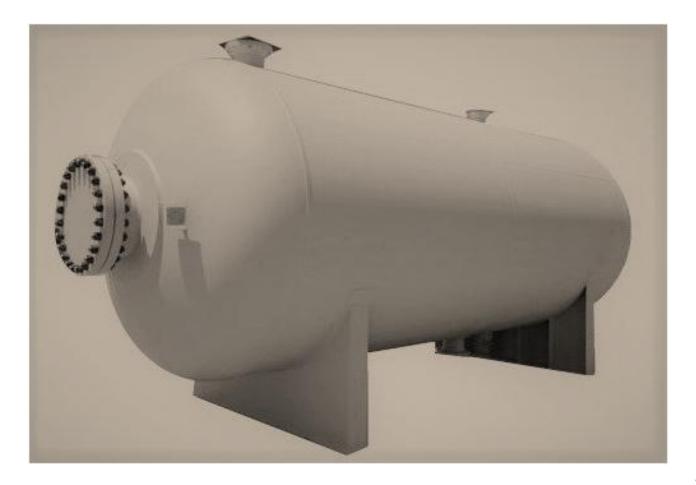
 $= \sqrt{\frac{4P}{\pi \tau n}}$

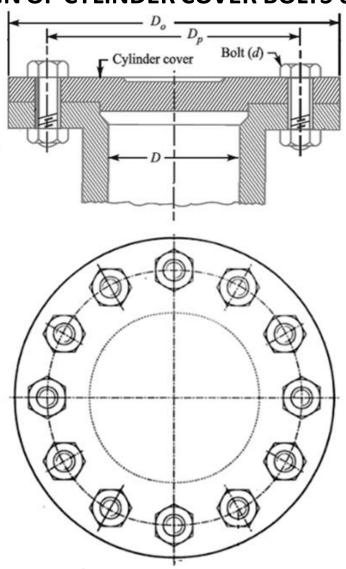
 $= \sqrt{\frac{4 \times 4200}{\pi \times 40 \times 4}} = 10.25 \ mm$

Size of the bolt = M 12

DESIGN OF CYLINDER COVER BOLTS

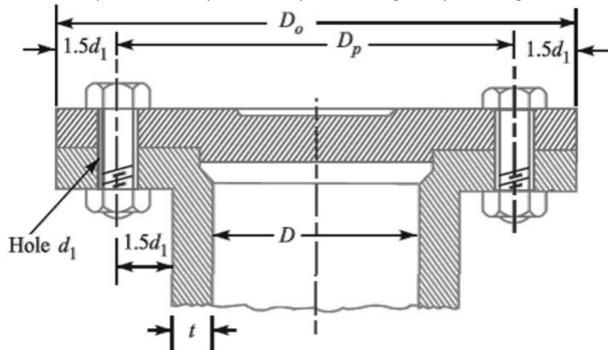
The cylinder covers may be secured by means of bolts or studs.





The bolts or studs, cylinder cover plate and cylinder flange may be designed as discussed

below:



Pitch circle diameter, $D_p = D + 2t + 3d_1$ Outside diameter of the cover, $D_o = D_p + 3d_1 = D + 2t + 6d_1$ Where,

D = Diameter of the cylinder

d1 = Diameter of the hole for bolt or stud

t = Thickness of the cylinder wall

Circular pitch $=\frac{\pi D_p}{n}$

Where,

n = number of bolts

Assumptions for design of cylinder cover bolt

- •Initial stresses in bolts due to tightening are neglected.
- •The effect of gasket is neglected
- •Total load due to the pressure inside the cylinder is equally shared by the bolts.
- •There is no stress concentration on the threads.

Design of bolts or studs

Let,

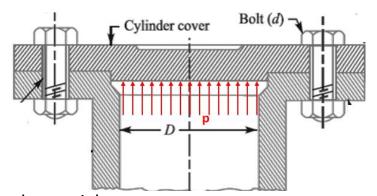
D = Diameter of the cylinder

p = Pressure in the cylinder

dc = Core diameter of the bolts or studs

n = Number of bolts or studs

 σ_{tb} = Permissible tensile stress for the bolt or stud material



Then,

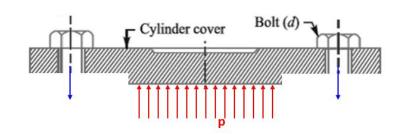
Upward force acting on the cylinder cover, $P = \frac{\pi}{4} (D^2) p$

This force is resisted by n number of bolts or studs provided on the cover

$$P = \frac{\pi}{4} \left(d_c \right)^2 \sigma_{tb} \times n$$

From the above two equations

$$\frac{\pi}{4} (D^2) p = \frac{\pi}{4} (d_c)^2 \sigma_{tb} \times n$$



- •From the above equation, the number of bolts or studs may be obtained, if the size of the bolt or stud is known and vice-versa.
- •If the value of n as obtained from the above relation is odd or a fraction, then next higher even number is adopted.
- •A bolt or a stud less than 16mm diameter should never be used.
- •The circular pitch should be between $20\sqrt{d_1}$ and $30\sqrt{d_1}$

M1.P8) The effective diameter of a cylinder is 0.5m and the highest pressure of steam acting on the cylinder head is 1.2MPa. Allowable stress in tension of bolt material is 35MPa. If the cylinder head is held by 10 bolts, find the size of the bolts. (2018 APRIL 8 Marks)

= 0.5 m = 500 mm

 $= 35 Mpa = 35 N/mm^{2}$

= 1.2Mpa = 1.2 N/mm²

Diameter of cylinder, D Number of studs, n Allowable stress, σ_t Steam pressure, p Force acting on cylinder cover

Resistance force offered by studs

Equating Eq[1] and Eq[2] Core diameter of the bolt, dc $= \frac{\pi D^{2}}{4} p - \text{Eq[1]}$ $= \frac{\pi d_{c}^{2}}{4} \sigma_{t} n - \text{Eq[2]}$ $= \sqrt{\frac{D^{2} p}{\sigma_{c}^{2}}}$

 $=\sqrt{\frac{500^2 \times 1.2}{35 \times 10}} = 29.28mm$

Nominal diameter, d

 $=\frac{u_c}{0.84}$

 $=\frac{29.28}{0.84}=34.86\ mm$

Size of the bolt

= M36

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M1.P9) A cylinder cover of a steam engine is secured by 12 studs. The cylinder is 0.3m diameter and has a steam pressure of 1.2 N/mm²gauge. Calculate the diameter of the studs, assuming the permissible stress to be 30 N/mm². (2017 OCTOBER 8 Marks)

M1.P9) Answer

Diameter of cylinder, D Number of studs, n Allowable stress, σ_t Steam pressure, p_g Absolute steam pressure, p

Force acting on cylinder cover

Resistance force offered by studs

Equating Eq[1] and Eq[2] Core diameter of the bolt, dc

Nominal diameter, d

Size of the stud

Atmospheric pressure =
$$1.013 \text{ bar}$$

= $0.3\text{m} = 300\text{mm}$
= 12
= 30 N/mm^2
= $\frac{1.013 \times 10^5 \text{ N/m}^2}{10^3 \times 10^3 \text{ N/mm}^2}$
= 0.1013 N/mm^2

= gauge pressure + atmospheric pressure

= 1.2 + 0.1013 N/mm²

= 1.2 N/mm² gauge

=1.3013 N/mm²
=
$$\frac{\pi D^2}{1}$$
 p -----Eq[1

$$= \frac{\pi D^2}{4} p - \text{Eq[1]}$$

$$= \frac{\pi d_c^2}{4} \sigma_t n - \text{Eq[2]}$$

$$= \sqrt{\frac{D^2 p}{\sigma_t n}}$$

$$=\sqrt{\frac{300^2 \times 1.3013}{30 \times 12}} = 18.04mm$$

$$=\frac{d_c}{0.84}$$

$$=\frac{18.04}{0.84}=21.48\,mm$$

M1.P10) In a steam engine the maximum steam pressure is 1 N/mm² absolute and back pressure is 0.015 N/mm² absolute. The cylinder diameter is 300 mm. Determine the diameter of the screwed end of the piston rod, when the allowable stress is 45 N/mm² in tension. (2018 OCTOBER 8 marks)

M1.P10) Answer

Diameter of cylinder, D

Number of bolts, n

Allowable stress, σ_t

Steam pressure, p_s

Back pressure, p_h

Steam pressure, p

Force acting on cylinder cover

Resistance force offered by studs

Equating Eq[1] and Eq[2]

Core diameter of the bolt, dc

Nominal diameter, d

Size of the piston rod at screwed end

= 300 mm

=1

 $= 45 \text{ N/mm}^2$

= 1 N/mm²

=0.015 N/mm²

 $= p_s - p_b = 1.0 - 0.015 = 0.985 \text{N/mm}^2$ $= \frac{\pi D^2}{4} p - \text{Eq[1]}$ $= \frac{\pi d_c^2}{4} \sigma_t n - \text{Eq[2]}$

 $=\sqrt{\frac{300^2 \times 0.985}{45}} = 44.38 \ mm$

 $=\frac{44.38}{0.84}=52.83 mm$

= 53 mm

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M1.P11) A steam engine cylinder has an effective diameter of 340mm and the maximum steam pressure acting on the cylinder cover is 1.25 N/mm². Calculate the number and size of studs required to fix the cylinder cover, assuming the permissible stress in the studs as 30Mpa. (2019 OCTOBER 8 marks)

$$= 0.84 \times 16 = 13.44 \ mm$$

Equating Eq[1] and Eq[2]

$$\frac{\pi d_c^2}{4} \sigma_t n = \frac{\pi D^2}{4}$$

Number of bolts, n
$$= \frac{D^2 p}{\sigma_t d_c^2}$$

$$= \frac{340^2 \times 1.25}{30 \times 13.44^2} = 26.67$$
$$= 28$$

Hole diameter,
$$d_1 = d + 1 = 17 mm$$

Pitch circle diameter,
$$Q_0$$
 = D + 2t + 3d₁

The circular pitch is not between $20\sqrt{d1}$ and $30\sqrt{d1}$. So the assumed bolt is wrong.

M1.P11cont.)

Since bolt size and number of bots to be used are unknown, assume bolt of size M18 is used.

Nominal diameter, d $= 18 \ mm$ Core diameter of the bolt, dc = 0.84d $= 0.84 \times 18 = 15.12 \, mm$ Equating Eq[1] and Eq[2] $=\frac{\pi D^2}{4} p$ $\frac{\pi d_c^2}{4} \sigma_t n$ $= \frac{D^2 p}{\sigma_t d_c^2}$ Number of bolts, n $=\frac{340^2\times1.25}{30\times15.12^2}=21.06$ = 22Hole diameter, d1 = d + 1 = 19 mmPitch circle diameter, Dp $= D + 2t + 3d_1$ By assuming thickness of cylinder as 10mm $D_p = 340 + 20 + 3 \times 19 = 417 \ mm$ $=\frac{\pi D_p}{n}=\frac{\pi \times 417}{22}$ Circular pitch = 59.55 mm= 87.18 $20\sqrt{d1}$

The circular pitch is not between $20\sqrt{d1}$ and $30\sqrt{d1}$. So the assumed bolt is wrong.

 $30\sqrt{d1}$

= 130.77

M1.P11 cont.)

Since bolt size and number of bots to be used are unknown, assume bolt of size M20 is used.

 $\begin{array}{ll} \mbox{Nominal diameter, d} & = 20 \ mm \\ \mbox{Core diameter of the bolt, dc} & = 0.84 d \end{array}$

 $= 0.84 \times 20 = 16.8 \, mm$

Equating Eq[1] and Eq[2]

 $\frac{\pi d_c^2}{4} \sigma_t n = \frac{\pi D^2}{4} p$

Number of bolts, n $= \frac{D^2 p}{\sigma_t d_c^2}$

 $=\frac{340^2 \times 1.25}{30 \times 16.8^2} = 17.06$

= 18

Hole diameter, $d_1 = d + 1 = 21 \, mm$

Pitch circle diameter, D_0 = D + 2t + 3d₁

By assuming thickness of cylinder as 10mm

n 18 = 73.83 mm

 $20\sqrt{d1}$ = 91.65 $30\sqrt{d1}$ = 137.48

The circular pitch is not between $20\sqrt{d1}$ and $30\sqrt{d1}$. So the assumed bolt is wrong.

M1.P11 cont.)

Since bolt size and number of bots to be used are unknown, assume bolt of size M22 is used.

Nominal diameter, d $= 22 \ mm$ Core diameter of the bolt, dc = 0.84d $= 0.84 \times 22 = 18.48 \ mm$ Equating Eq[1] and Eq[2] $\frac{\pi \ d_c^2}{4} \ \sigma_t \ n = \frac{\pi \ D^2}{4} \ p$

Number of bolts, n
$$= \frac{D^2 p}{\sigma_t d_c^2}$$

$$=\frac{340^2\times1.25}{30\times18.48^2}=14.1$$

Hole diameter, $d_1 = d + 1 = 23 \, mm$

Pitch circle diameter, D_p = D + 2t + 3d₁ By assuming thickness of cylinder as 10mm

Circular pitch $D_p = 340 + 20 + 3 \times 23 = 429 \ mm$ $= \frac{\pi D_p}{n} = \frac{\pi \times 429}{12}$

The circular pitch is not between $20\sqrt{d1}$ and $30\sqrt{d1}$. So the assumed bolt is wrong.

M1.P11 cont.)

Since bolt size and number of bots to be used are unknown, assume bolt of size M24 is used.

Nominal diameter, d = 24 mmCore diameter of the bolt, dc = 0.84d

 $= 0.84 \times 24 = 20.16 \, mm$

Equating Eq[1] and Eq[2]

$$\frac{\pi d_c^2}{4} \sigma_t n = \frac{\pi D^2}{4} p$$

Number of bolts, n = $\frac{I}{I}$

$$= \frac{D^2 p}{\sigma_t d_c^2}$$

$$=\frac{340^2\times1.25}{30\times20.16^2}=11.85$$

$$= 12$$

Hole diameter, $d_1 = d + 1 = 25 \, mm$

Pitch circle diameter, D_0 = D + 2t + 3d₁

By assuming thickness of cylinder as 10mm

 $= 113.88 \ mm$ = 100.00 = 150.00

The circular pitch is between $20\sqrt{d1}$ and $30\sqrt{d1}$. So the assumed bolt can be used.

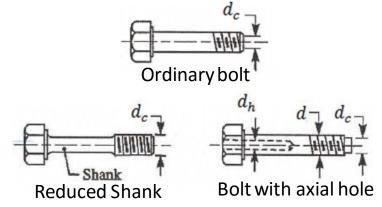
Size of the bolt = M 24 Number of bolt = 12

BOLTS OF UNIFORM STRENGTH

- •In an ordinary bolt, the stress in the threaded part of the bolt will be higher than that in the shank since the core diameter is less than the shank diameter. Hence the threaded portion may fracture because of its small length.
- •If the shank of the bolt is turned down to a diameter equal or even slightly less than the core diameter of the thread (dc), then shank of the bolt will undergo a higher stress.
- •The bolt, in this way, becomes stronger and lighter and known as bolts of uniform strength.
- •An alternative method of obtaining the bolts of uniform strength is, an axial hole is drilled through the head as far as the thread portion such that the area of the shank becomes equal to the root area of the thread.

Let, d_h = Diameter of the hole d = Major diameter of the thread d_c = Root or core diameter of the thread $\pi d_c^2 = \pi [d^2 + d^2]$

$$\begin{array}{ll} \frac{\pi {d_h}^2}{4} &= \frac{\pi}{4} \big[d^2 - {d_c}^2 \big] \\ {d_h}^2 &= d^2 - {d_c}^2 \\ d_h &= \sqrt{d^2 - {d_c}^2} \end{array}$$



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STRESSED DUE TO COMBINED INITIAL AND EXTERNAL LOAD

•The resultant load due to initial load and external load is given by

$$P = P_1 + kP_2$$

Where,

P₁ = Initial load

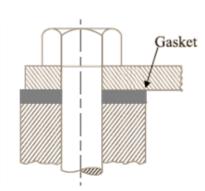
P₂ = External load

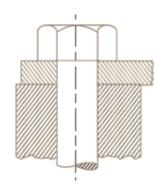
 $k = stiffness constant, k = \frac{a}{1+a}$

$$a = \frac{K_b}{K_p}$$

K_b = stiffness of bolt

 K_p = stiffness of connected parts





- •For gasketed joints with soft material the value of k =1
- •For metal to metal contact joints the value of k = 0
- •For all other types of joints the value of k is between 0 and 1

KEYS

- •A key is a machine element inserted between the shaft and pulley or shaft and gear to connect these together in order to prevent relative motion between them.
- •Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.
- •A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.
- •The following types of keys are important
 - 1. Sunk keys
 - 2. Saddle keys
 - 3. Tangent keys
 - 4. Round keys
 - 5. Splines

SUNK KEYS

•The sunk keys requires keyway in the shaft and in the hub. The sunk keys are of the following types:

1.Rectangular sunk key

The usual proportions of this key are:

Width of key, w = d/4

thickness of key, t = 2w/3 = d/6

Where,

d = Diameter of the shaft or diameter of the hole in the hub.

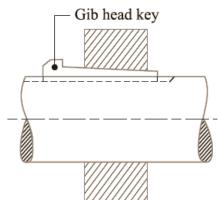


The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

$$w = t = d / 4$$

3.Gib-head key

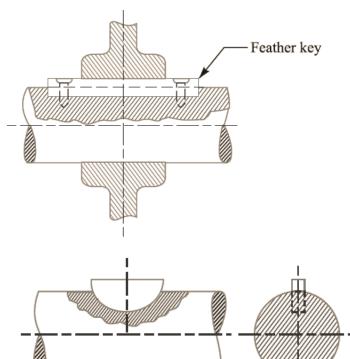
It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key.



SUNK KEYS cont...

4.Feather key

A key attached to one member of the connected torque transmitting members and which permits relative axial movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub.



5.Woodruff key

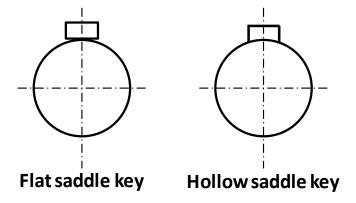
It is a semi circular disc having similar keyway. A woodruff key is capable of tilting in the keyway so can be easily adjusted. This key is largely used in machine tool and automobile construction.

SADDLE KEYS

- •The saddle keys are of the following two types:
- 1. Flat saddle key
- 2. Hollow saddle key

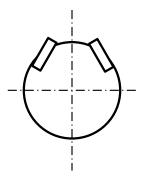
A flat saddle key is a key which needs a keyway in the hub and is flat on the shaft. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

A hollow saddle key's face which touches shaft is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, these are suitable for light loads.



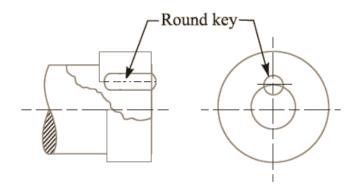
TANGENT KEYS

The tangent keys are fitted in pair at an angle. Each key is to withstand torsion in one direction only. These are used in heavy duty shafts.



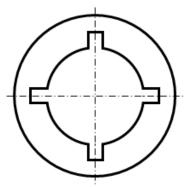
ROUND KEYS

The round keys, are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



SPLINES

Sometimes, keys are made integral with the shaft. Such shafts are known as splined shafts. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway. The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions.



Let

T = Torque transmitted by the shaft

F = Tangential force acting at the circumference of the shaft

d = Diameter of shaft

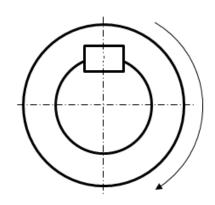
I = Length of key

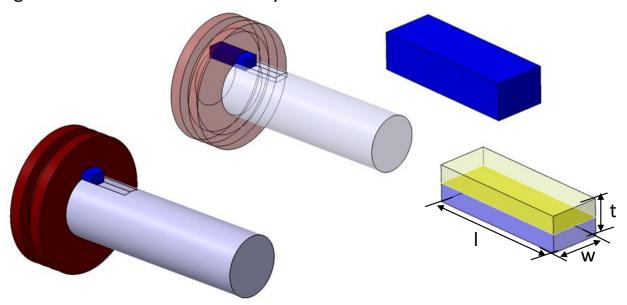
w = Width of key

t = Thickness of key

 τ = Shear stresses for the material of key

 $\sigma_{c}\,$ = crushing stresses for the material of key





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Due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

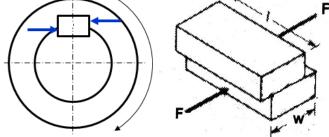
Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

 $F = Area resisting shearing \times Shear stress = I \times w \times \tau$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2}$$

$$\tau = \frac{2T}{l \times w \times d}$$



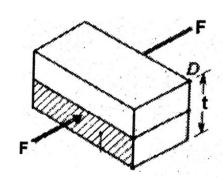
Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\sigma_c = \frac{4T}{l \times t \times d}$$



The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\frac{w}{t} = \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the key material should be atleast twice the permissible shearing stress for equal width and thickness.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

Shearing strength of key,
$$T = l \times w \times \tau \times \frac{d}{2}$$

Torsional shear strength of the shaft, $T = \frac{\pi}{16} \tau_1 d^3$

Equating these two equations

By taking
$$w = d/4$$

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \tau_1 d^3$$

$$l \times w = \frac{\pi}{8} \frac{\tau_1}{\tau} d^2$$

$$l \times \frac{d}{4} = \frac{\pi}{8} \frac{\tau_1}{\tau} d^2$$

$$l = \frac{\pi}{2} \frac{\tau_1}{\tau} d$$

If the key material is same as that of the shaft, $\tau_1 = \tau$

Length of key,

$$l = 1.571d$$

STANDARD DIMENSIONS OF RECTANGULAR AND SQUARE SUNK KEY

Shaft Diameter		Key Section
(mm)		(mm x mm)
Above	Up to & including	wxt
6	8	2x2
8	10	3x3
10	12	4x4
12	17	5x5
17	22	6x6
22	30	8x7
30	38	10x8
38	44	12x8
44	50	14x9
50	58	16x10
58	65	18x11
65	75	20x12
75	85	22x14
85	95	25x14
95	110	28x16
110	130	32x18
130	150	36x20
150	170	40x22
170	200	45x25
200	230	50x28
230	260	56x32
260	290	63x32
290	330	70x36
330	380	80x40
380	440	90x45
440	500	100x50

M1.P12) A 40 mm diameter shaft is subjected to a tangential force of 20kN around it's circumference. Determine the size of key. The allowable shear stress in key is 60 N/mm2. (2018 OCTOBER 7 marks)

M1.P12)

Diameter of the shaft, d Tangential force, F Allowable shear stress, τ Width of the key, w

Thickness of the key, t

Take thickness as 8mm, t = 8 mm Stress

Force Considering shear strength, Tangential force, F Length, I

Size of the key

= 40 mm = 20kN = 20000N = 60 N/mm² = $\frac{d}{4} = \frac{40}{4} = 10 mm$ = $\frac{d}{6} = \frac{40}{6} = 6.67 = 7 mm$

= $\frac{force}{area}$ = Stress x area

 $= \frac{l \times w \times \tau}{F}$ $= \frac{F}{w \times \tau}$

 $= \frac{20000}{10 \times 60} = 33.33$ $= 34 \ mm$

width = 10 mm thickness = 8 mm length = 34 mm

M1.P13) A steel shaft has a diameter, of 25 mm. The shaft rotates at a speed of 600 rpm and transmits 7 kW through a gear. Design a suitable key for the gear. The allowable shear and crushing stress for the material of the key can be taken as 60MPa and 120MPa respectively. Assume the same material for both shaft and key. (2019 OCTOBER 8 marks)

M1.P13) Diameter of the shaft, d

Allowable shear stress, τ Allowable crushing stress, σ_c Power transmitted by shaft, P shaft rotational speed, N

Power, P

Where, N is shaft rotational speed in rpm Tis torque in N-m

From the above equation,

Torque transmitted by shaft, T

= 60Mpa = 60 N/mm² =120Mpa = 120 N/mm²

=7kW = 7000 W

$$=\frac{2\pi NT}{60}$$

$$=\frac{60P}{2\pi N}$$

$$=\frac{60\times7000}{2\pi\times600}$$

$$= 111.4 N - m$$

= 111.4 \times 10^3 N - mm

Since Allowable crushing stress (σ_c) is two times of Allowable shear stress(τ) square key can be adopted.

Width of the key, w

$$= \frac{d}{4} = \frac{25}{4} = 6.25 \, mm$$
$$= 8 \, mm$$

$$= \frac{d}{4} = \frac{25}{4} = 6.25 \, mm$$

M1.P13 cont.)

Considering shear strength of the key,

Length of the key, I

$$= l \times w \times \tau \times \frac{d}{2}$$

$$= \frac{2T}{w \times \tau \times d}$$

$$= \frac{2 \times 111.4 \times 10^3}{8 \times 60 \times 25} = 18.57 \ mm$$
$$= 19 \ mm$$

Considering crushing strength of the key,

Torque, T

Length of the key, I

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$= \frac{4T}{t \times \sigma_c \times d}$$

$$= \frac{4 \times 111.4 \times 10^3}{8 \times 120 \times 25} = 18.57 \ mm$$
$$= 19 \ mm$$

Length, I Size of the key = 19 mm width = 8 mm thickness = 8 mm length = 19 mm

M1.P14) A rectangular sunk key of 75 mm long , 14mm wide and 10 mm thick has to transmit 1200 N-m torque from a 50mm diameter solid shaft. Determine if the designed length is sufficient to transmit the torque if the allowable shear strength and crushing strength of the key material are 80Mpa and 180Mpa respectively.

M1.P14) Diameter of the shaft, d

Length of the key, I Width of the key, w Thickness of the key, t Torque transmitted by shaft, T

Allowable shear stress Allowable crushing stress

Considering shear strength of the key,

Torque, T

Induced shear stress, τ

=50 mm

 $= 75 \, mm$

= 14 mm= 10 mm

= 1200 N - m

 $= 1200 \times 10^3 N - mm$

= 80Mpa = 80 N/mm²

=180Mpa =180 N/mm²

$$= l \times w \times \tau \times \frac{d}{2}$$
$$= \frac{2T}{}$$

$$= \frac{2 \times 1200 \times 10^3}{75 \times 14 \times 50} = 46 \text{ N/mm}^2$$

Considering crushing strength of the key,

Torque, T

Induced crushing stress, σ_c

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$= \frac{4T}{l \times t \times d}$$

$$= \frac{4 \times 1200 \times 10^3}{75 \times 10 \times 50} = 128 \text{ N/mm}^2$$

$$= 128 Mpa$$

Since the induced stresses in the key are less than the allowable material strength of the key, the designed length of key is sufficient.

M1.P15) A shaft of 50mm diameter is transmitting 150 kW at 3000 rpm. If a key of length 75mm, width 16mm and thickness 10mm is used, calculate induced shear stress and crushing stress in the key.

M1.P15)

Diameter of the shaft, d = 50 mm Length of the key, I = 75 mm Width of the key, w = 16 mm Thickness of the key, t = 10 mm Power transmitted by shaft, P = $150 \times 10^3 \text{ W}$ shaft rotational speed, N = 3000 rpmPower, P = $\frac{2\pi NT}{60}$

Where,
N is shaft rotational speed in rpm
T is torque in N-m
From the above equation,
Torque transmitted by shaft, T

Considering shear strength of the key,

Torque, T Induced shear stress, τ

Considering crushing strength of the key,

Torque, T Induæd crushing stress, σ_c

$$= \frac{60P}{2\pi N} = \frac{60 \times 150 \times 10^3}{2\pi \times 3000} = 477.5 N - m$$

= 477.5 \times 10^3 N - mm

$$= l \times w \times \tau \times \frac{d}{2}$$

$$= \frac{2T}{l \times w \times d} = \frac{2 \times 477.5 \times 10^3}{75 \times 16 \times 50} = 16 \text{ N/mm}^2$$

$$= 16 \text{ Mpa}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$= \frac{4T}{l \times t \times d} = \frac{4 \times 477.5 \times 10^3}{75 \times 10 \times 50} = 51 \text{ N/mm}^2$$

$$= 51 \text{ Mpa}$$

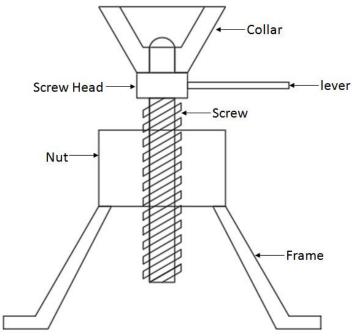
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POWER SCREWS

- •The power screws are used to convert rotary motion into translatory motion. Power screws are used in lathes, screw jacks, vices, testing machines, presses etc.
- •In power screws, the relative motion between screw and nut can be following types.
 - 1. The nut moves axially against the resisting axial force while the screw rotates in its bearings.
 - 2. The screw rotates and moves axially against the resisting force while the nut is stationary.
 - 3. The nut rotates while the screw moves axially with no rotation.
- •Following are the three types of screw threads mostly used for power screws
 - 1.Square thread
 - 2.Acme thread
 - 3.Buttress thread

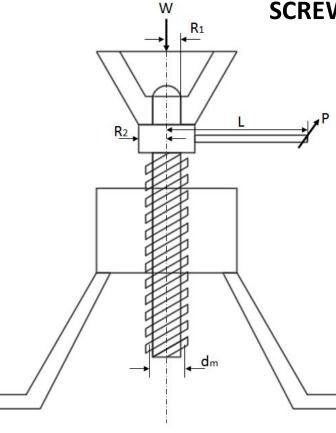
SCREW JACK

- •Screw jack is a device used wherever there is a need to lift or hold heavy loads.
- •Screw jack convert rotational motion into linear motion.
- •A screw jack consists of a vertical screw with a table mounted on its top, which screws into a threaded hole (act as nut) in a stationary support frame.
- •It works on the principle same as that of incline plane.
- •It utilizes the property of a screw thread providing a mechanical advantage i.e. it can be used to amplify force.



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SCREW JACK cont...



Effort required to raise the load

If T is the torque required to raise the load then,

$$T = P \times L$$

$$PxL = P_s x d_m/2$$

Where,

P = Effort applied at the end of the handle

L = length of handle

P_s = imaginary force acting at mean

circumference of the screw.

 d_m = Mean diameter of the screw

$$d_{\rm m} = (d+d_{\rm c})/2$$

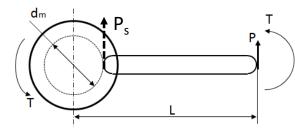
$$d_{\rm m} = d_{\rm c} + 0.5 p$$

$$d_{m} = d-0.5p$$

d = Nominal diameter of the screw

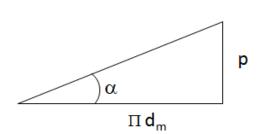
d_c = Core diameter of the screw

p = Pitch of the screw



SCREW JACK cont...

Development of a screw thread



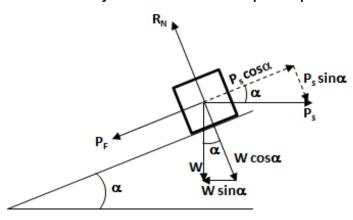
 α = Helix angle tan α = n p / π d_m

Where, p = Pitch of the screw n = number of starts

For single start thread, n = 1 For double start thread, n = 2 etc..

SCREW JACK- TORQUE REQUIRED TO RAISE LOAD

•Screw jack works on the principle same as that of incline plane.



W = Load to be lifted

P_s = Effort applied at the circumference of the screw

 P_F = Frictional load

 α = Helix angle

 μ = Coefficient of friction between the screw and nut

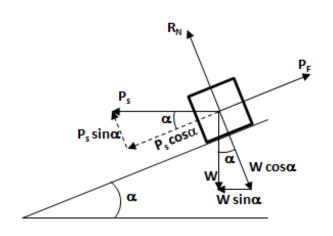
 μ = tan ϕ , where ϕ is the friction angle

Effort to be applied at circumference of the screw to lift the load, $P_s = W \tan(\phi + \alpha)$

Torque to be applied at circumference of the screw to lift the load, $T = P_s d_m / 2$

 $T = W \tan(\phi + \alpha) d_m/2$

SCREW JACK- TORQUE REQUIRED TO LOWER LOAD



W = Load to be lifted

P_s = Effort applied at the circumference of the screw

 P_F = Frictional load

 α = Helix angle

 μ = Coefficient of friction between the screw and nut

 μ = tan ϕ , where ϕ is the friction angle

Effort to be applied at circumference of the screw to lower the load, $P_s = W \tan(\phi - \alpha)$

Torque to be applied at circumference of the screw to lower the load, $T = P_s d_m/2$ $T = W \tan(\phi - \alpha) d_m/2$

SCREW JACK- TORQUE REQUIRED CONSIDERING COLLAR FRICTION

To prevent the rotation of the load a collar is provided at the top of the screw head. Collar friction takes place at the contact surface between screw head and collar.

Let,

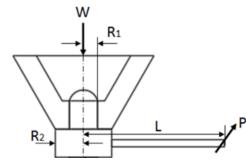
 μ_c = Coefficient of friction between the screw head and collar

 R_1 = Internal radius of bearing surface of collar in mm

 R_2 = External radius of bearing surface of collar in mm

Mean radius considering uniform wear, $R = \frac{1}{2}(R_1 + R_2)$

Mean radius considering uniform pressure, $R = \frac{2}{3} \left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \right)$



Collar friction torque, $T_c = \mu_c WR$

Torque to be applied at circumference of the screw to lift the load considering collar friction

$$T = P_s d_m / 2 + T_c$$

$$T = W \tan(\phi + \alpha)(d_m/2) + \mu_c WR$$

Torque to be applied at circumference of the screw to lower the load considering collar friction

$$T = P_s d_m / 2 + T_c$$

$$T = W \tan(\phi - \alpha)(d_m/2) + \mu_c WR$$

SCREW JACK- EFFICIENCY

The efficiency of screw jack is defined as the ratio between the ideal effort (i.e. the effort required to move the load, neglecting friction) to the actual effort (i.e. the effort required to move the load taking friction into account).

Efficiency,
$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}}$$

Actual effort, $P = W \tan(\phi + \alpha)$

Where,

 α = Helix angle, ϕ = friction angle

Ideal effort is the effort required to move the load, neglecting friction. Ideal effort, P_{α} = W tan α

$$Efficiency, \eta = \frac{Ideal\,effort}{Actual\,effort} = \frac{P_o}{P} = \frac{W\,\tan\alpha}{W\,\tan(\phi + \alpha)} = \frac{\tan\alpha}{\tan(\phi + \alpha)}$$

The efficiency of a screw jack is independent of the load raised.

SCREW JACK- EFFICIENCY CONSIDERING COLLAR FRICTION

Considering collar friction,

$$Efficiency, \eta = \frac{Ideal\:effort}{Actual\:effort} = \frac{Ideal\:torque}{Actual\:torque}$$

Actual torque,
$$T = P d_m/2 + \mu_c WR$$

Ideal torque, $T_o = P_o d_m/2$

Where,

P = Actual effort, P_o = Ideal effort, d_m = Mean diameter of the screw μ_c = Coefficient of friction between the screw head and collar R = Mean radius

Efficiency,
$$\eta = \frac{Ideal\ torque}{Actual\ torque} = \frac{T_o}{T} = \frac{P_o \frac{a_m}{2}}{P \frac{d_m}{2} + \mu_c WR}$$

SCREW JACK- MAXIMUM EFFICIENCY

The efficiency of screw jack is given as

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{\sin \alpha / \cos \alpha}{\sin (\alpha + \phi) / \cos (\alpha + \phi)} = \frac{\sin \alpha \times \cos (\alpha + \phi)}{\cos \alpha \times \sin (\alpha + \phi)}$$

Multiplying the numerator and denominator by 2,

$$\eta = \frac{2 \sin \alpha \times \cos (\alpha + \phi)}{2 \cos \alpha \times \sin (\alpha + \phi)} = \frac{\sin (2\alpha + \phi) - \sin \phi}{\sin (2\alpha + \phi) + \sin \phi}$$

The efficiency given by the above equation will be maximum when sin $(2\alpha + \phi)$ is maximum, i.e. when

$$\sin(2\alpha + \phi) = 1$$

Means, $2\alpha + \phi = 90^{\circ}$

$$\therefore 2\alpha = 90^{\circ} - \Phi$$

$$\alpha = 45^{\circ} - \phi / 2$$

$$\eta_{max} = \frac{\sin (90^{\circ} - \phi + \phi) - \sin \phi}{\sin (90^{\circ} - \phi + \phi) + \sin \phi} = \frac{\sin 90^{\circ} - \sin \phi}{\sin 90^{\circ} + \sin \phi} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Maximum efficiency of screw jack, $\eta_{max} = \frac{1 - \sin \phi}{1 + \sin \phi}$

SCREW JACK- OVER HAULING AND SELF LOCKING

The effort to be applied at circumference of the screw to lower the load, $P_s = W \tan(\phi - \alpha)$ The torque to be applied at circumference of screw to lower the load, $T = W \tan(\phi - \alpha) d_m/2$

Where, W = Load to be lifted, d_m = Mean diameter of the screw α = Helix angle, ϕ = friction angle

In the above expression, if $\phi < \alpha$, then torque required to lower the load will be negative. In other words, the load will start moving downward without the application of any torque. Such a condition is known as over hauling of screws.

If however, $\phi > \alpha$, the torque required to lower the load will be positive, indicating that an effort is applied to lower the load. Such a screw is known as self locking screw. In other words, a screw will be self locking if the friction angle is greater than helix angle.

The efficiency of a self locking screw is less than 50%. If the efficiency of a screw jack is 50% the screw is at the point of reversal. If the efficiency is more than 50% the screw jack is overhauling.

M1.P16) A screw jack having square threads of 50 mm mean diameter and 12.5 mm pitch is operated by a 500 mm long hand lever Coefficient of friction at the thread is 0.1.

- 1. Find the Efficiency of screw jack.
- 2. Check Whether it is self locking or overhauling
- 3. Determine the force needed to be applied at the end of the lever to lift the load of 20kN. (2017 OCTOBER 8 marks)

M1.P16)

Mean diameter of the screw, d_m $= 50 \, mm$ Pitch of the screw, p = 12.5 mmCoefficient of friction at thread, μ = 0.1Length of lever, :L $= 500 \ mm$ Load to be lifted, W $= 20kN = 20 \times 10^3 N$ Number of thread starts, n $= tan^{-1} \left(\frac{n p}{\pi d_m} \right)$ Helix angle, α $= tan^{-1} \left(\frac{12.5}{\pi \times 50} \right) = tan^{-1} (0.0796)$ $= 4.55^{\circ}$ Friction angle, φ $= tan^{-1}\mu$ $= tan^{-1}0.1$ $= 5.71^{\circ}$ $=\frac{\tan\alpha}{\tan(\phi+\alpha)}$ Efficiency of screw jack, η $=\frac{\tan(5.71 + 4.55)}{\tan(5.71 + 4.55)}$ = 0.439= 43.9%

Since Friction angle (ϕ)> Helix angle (α) or Efficiency, $\eta < 50\%$ the screw jack is self locking.

M1.P16 Cont.)

Force to be applied at circumference of the screw to lift the load, P,

=
$$W \tan(\phi + \alpha)$$

= $20 \times 10^3 \times \tan(5.71 + 4.55)$
= 3621 N

Torque required to raise the load, T

$$= P_s \times \frac{d_m}{2}$$

Same torque has to be generated if the load is applied at the end of the lever

$$= P \times L$$

$$= \frac{T}{L}$$

$$= \frac{P_s \times \frac{d_m}{2}}{L} = \frac{3621 \times \frac{50}{2}}{500}$$

$$= 182 N$$

M1.P17) Find the force to be applied at the end of one meter long handle of a screw jack so that a load of 7 kN is lifted with constant velocity. The screw is square threaded having a pitch of 16mm and root diameter 50mm. The coefficient of friction between the screw and nut is 0.16. (2018 APRIL 8 marks)

M1.P17)

Root diameter of the screw, d_c
Pitch of the screw, p
Mean diameter of the screw, d_m
Coefficient of friction at thread, µ
Length of lever, L
Load to be lifted, W
Number of thread starts, n
Helix angle, α

Friction angle, φ

Force to be applied at circumference of the screw to lift the load, P.

Torque required to raise the load, T

Same torque has to be generated if the load is applied at the end of the lever

Torque required to raise the load, T Force to be applied on lever to lift the load, P = 50 mm = 16 mm = dc + 0.5p = 58 mm = 0.16 = 1 m = 1000 mm = 7 kN = 7 × 10³ N = 1 = $tan^{-1} \left(\frac{n p}{\pi d_m}\right)$ = $tan^{-1} \left(\frac{16}{\pi \times 58}\right) = tan^{-1}(0.0878)$ = 5.02° = $tan^{-1} \mu$ = $tan^{-1}0.16$

= $W \tan(\phi + \alpha)$ = $7 \times 10^3 \times \tan(9.09 + 5.02)$ = 1760 N

 $= P_s \times \frac{d_m}{2}$

 $= 9.09^{\circ}$

 $= P \times L$

 $= \frac{\frac{1}{L}}{L}$ $= \frac{P_s \times \frac{d_m}{2}}{L} = \frac{1760 \times \frac{58}{2}}{1000}$ = 51.1 N

M1.P18) A load of 2500 N is to be raised by a screw jack with a screw of 75 mm mean diameter and pitch of 12 mm. Find the efficiency of the screw jack, if the coefficient of friction of screw and nut is 0.075. (2018 OCTOBER 8 marks)

M1.P18)

Mean diameter of the screw, d_m = 75 mmPitch of the screw, p $= 12 \, mm$ Coefficient of friction at thread, μ = 0.075Load to be lifted, W = 2500 NNumber of thread starts, n $= tan^{-1} \left(\frac{n p}{\pi d_m} \right)$ Helix angle, α $= tan^{-1} \left(\frac{12}{\pi \times 75} \right) = tan^{-1} (0.0509)$ $= 2.92^{\circ}$ Friction angle, φ $= tan^{-1}\mu$ $= tan^{-1}0.075$ $= 4.29^{\circ}$ Efficiency of screw jack, η $=\frac{1}{\tan(\phi+\alpha)}$ tan 2.92 $=\frac{1}{tan(4.29+2.92)}$ = 0.403= 40.3%

M1.P19) A load of 10000 N is to be raised by a screw jack with a square threaded double start screw of 75mm mean diameter and pitch of 10 mm. Find the efficiency of the screw jack, if the coefficient of friction of screw and nut is 0.1.

 $= 50 \, mm$

= 0.455= 45.5%

M1.P19)

Mean diameter of the screw, d_m
Pitch of the screw, p
Coefficient of friction at thread, μ
Load to be lifted, W
Number of thread starts, n
Helix angle, α

Friction angle, $\boldsymbol{\varphi}$

Efficiency of screw jack, η

$$= 10 mm$$

$$= 0.1$$

$$= 10000 N$$

$$= 2$$

$$= tan^{-1} \left(\frac{n p}{\pi d_m}\right)$$

$$= tan^{-1} \left(\frac{2 \times 10}{\pi \times 75}\right) = tan^{-1} (0.0849)$$

$$= 4.85^{\circ}$$

$$= tan^{-1} \mu$$

$$= tan^{-1} 0.1$$

$$= 5.71^{\circ}$$

$$= \frac{tan \alpha}{tan(\phi + \alpha)}$$

$$= \frac{tan 4.85}{tan(5.71 + 4.85)}$$

M1.P20) A turn buckle is used to tighten a wire rope. The threads are single right and left hand and square in section. The outside diameter of the screw is 40 mm and the pitch is 8.5 mm. The coefficient of friction between the screws and nuts is 0.15. What torque on the turn buckle is necessary, if the rope is to be tightened to a tension of 8 kN? (2018 APRIL 9 marks)



 $= 40 \, mm$

 $= 8.5 \, mm$

= 0.15

= d - 0.5p = 35.75 mm

 $M1.P20) \\ {\color{red}Nominal diameter of the screw, d}$ Pitch of the screw, p Mean diameter of the screw, d_m Coefficient of friction at thread, μ Load to be lifted, W Number of thread starts, n Helix angle, α

 $= 8 \text{ kN} = 8 \times 10^3 \text{ N}$ $= tan^{-1} \left(\frac{n p}{\pi d_m} \right)$ $= tan^{-1} \left(\frac{8.5}{\pi \times 35.75} \right) = tan^{-1} (0.0757)$ $= tan^{-1}\mu$ $= tan^{-1}0.15$ $= 8.53^{\circ}$

Friction angle, φ

Force to be applied at circumference of the screw to lift the load, P.

=
$$W \tan(\phi + \alpha)$$

= $8 \times 10^3 \times \tan(8.53 + 4.33)$
= 1826.19 N

Torque required for one screw, T

$$= P_s \times \frac{d_m}{2}$$

$$= 1826.19 \times \frac{35.75}{2} = 32643.15N - mm$$

$$= 32.65 N - m$$

Same torque has to be generated if the load is applied at the end of the lever

Total torque required to tighten the rope

$$= 2 \times T$$
$$= 65.3 N - m$$

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M1.P21) An electric motor driven power screw moves a nut in a horizontal plane against a force of 75kN at a speed of 300mm/min. The screw has a single square thread of 6mm pitch on a major diameter of 40 mm. The coefficient of friction at screw threads is 0.1. Estimate power of the motor. (2019 OCTOBER 7 marks)

M1.P21)

Nominal diameter of the screw, d Pitch of the screw, p Mean diameter of the screw, d_m Coefficient of friction at thread, μ Load to be lifted, W Number of thread starts, n Helix angle, α

Friction angle, φ

Force to be applied at circumference of the screw to move the load, P_s

Torque required to move the load, T

=
$$40 mm$$

= $6 mm$
= 0.1
= $75 kN = 75 \times 10^3 N$
= 1
= $tan^{-1} \left(\frac{n p}{\pi d_m}\right)$
= $tan^{-1} \left(\frac{6}{\pi \times 37}\right) = tan^{-1}(0.0516)$
= 2.96°
= $tan^{-1}\mu$
= $tan^{-1}0.1$
= 5.71°

= 11431 N
=
$$P_s \times \frac{d_m}{2}$$

= 11431 $\times \frac{37}{2}$ = 211474 N - mm

 $= 75 \times 10^3 \times tan(5.71 + 2.96)$

 $= W \tan(\phi + \alpha)$

= 212 N - m

M1.P21 cont.)

In one revolution, the screw moves one lead distance axially.

Lead of the screw = 6 mm

Speed of axial travel of screw $= 300 \ mm/min$

Revolution of screw per min, N Speed of axial travel of screw

lead of of the screw

 $=\frac{300}{6}$

= 50 rpm

Power, P = $\frac{2\pi NT}{60}$

 $=\frac{2\times\pi\times50\times212}{2}$

= 1111 W = **1.11** kW

M1.P22) A screw jack having square threaded screw of pitch of 16mm and nominal diameter 50mm. The coefficient of friction between the screw and nut is 0.16 and screw and collar is 0.18. The outside and inside diameter of collar is 80 mm and 50 mm respectively. Find the force to be applied at the end of one meter long handle of a screw jack so that a load of 7 kN is lifted with constant velocity considering

- I. Uniform wear at collar
- II. Uniform pressure at collar

M1.P22) $_{Nominal\,diameter\,of\,the\,screw,\,d}$ Pitch of the screw, p Mean diameter of the screw, d_m Coefficient of friction at thread, μ Coefficient of friction at collar, µ Inside diameter of collar, d1 Outside diameter of collar, d2 Inside radius of collar, R1 Outside radius of collar, R2 Length of lever, L Load to be lifted, W Number of thread starts, n

Helix angle, α

Friction angle, φ

Torque to be applied at circumference of the screw to lift the load to overcome screw friction, Т,

 $= 50 \, mm$ = 16 mm

= d - 0.5p = 42 mm

= 0.16= 0.18 $= 50 \, mm$ $= 80 \, mm$ = 25 mm

 $= 40 \, mm$ = 1 m = 1000 mm $= 7 \text{ kN} = 7 \times 10^3 \text{ N}$

= 1

 $= tan^{-1} \left(\frac{n p}{\pi d_m} \right)$ $= tan^{-1} \left(\frac{16}{\pi \times 42} \right) = tan^{-1} (0.1213)$ $= 6.91^{\circ}$

 $= tan^{-1}\mu$ $= tan^{-1}0.16$ $= 9.09^{\circ}$

=
$$W \tan(\phi + \alpha) \times \frac{d_m}{2}$$

= $7 \times 10^3 \times \tan(9.09 + 6.91) \times \frac{42}{2}$
= $42168 \text{ N} - \text{mm}$

M1.P22 cont.)

i) considering uniform wear at collar

Mean radius of collar, R

$$= \frac{1}{2}(R_1 + R_2)$$
$$= \frac{1}{2}(25 + 40)$$
$$= 32.5 mm$$

Torque due to collar friction, T_c

$$= \mu_c WR$$

= 0.18 × 7000 × 32.5

= 40950 N - mm

Total torque required to overcome friction, T = $T_s + T_c$

Same torque has to be generated if the load is applied at the end of the lever

Torque required to raise the load, T

Force to be applied on lever to lift the load, P

$$= \frac{P \times L}{T}$$
$$= \frac{\frac{T}{L}}{1000}$$

= 83.12 N

M1.P22 cont.)

ii) considering uniform pressure at collar

Mean radius of collar, R $= \frac{2}{3} \frac{(R_2{}^3 - R_1{}^3)}{(R_2{}^2 - R_1{}^2)}$ $= \frac{2}{3} \frac{(40^3 - 25^3)}{(40^2 - 25^2)}$ $= 33.08 \ mm$ $= \mu_c WR$ $= 0.18 \times 7000 \times 33.08$

Total torque required to overcome friction, T $= 0.18 \times 7000 \times 33.08$ = 41681 N - mm $= T_s + T_c$ = 83849 N - mm

Same torque has to be generated if the load is applied at the end of the lever

Torque required to raise the load, T $= P \times L$ Force to be applied on lever to lift the load, P $= \frac{T}{L}$ $= \frac{83849}{1000}$ = 83.85 N