MODULE 1

DETERMINANTS AND MATRICES

1. Evaluate $\begin{vmatrix} sinx & cosx \\ -cosx & sinx \end{vmatrix}$

   Ans : $(sinx \times sinx) - (cosx \times -cosx) = sin^2x - -cos^2x = sin^2x + cos^2x = 1$

2. Find A – B, if A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, B = $\begin{pmatrix} 0 & -2 \\ -3 & -3 \end{pmatrix}$

   $A - B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 0 & -2 \\ -3 & -3 \end{pmatrix} = \begin{pmatrix} 1-0 & 2--2 \\ 3--3 & 4--3 \end{pmatrix} = \begin{pmatrix} 1 & 2+2 \\ 3+3 & 4+3 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 6 & 7 \end{pmatrix}, --=+$

3.Evaluate $\begin{vmatrix} sin\theta & cos\theta \\ -cos\theta & sin\theta \end{vmatrix}$

   Ans : $(sin\theta \times sin\theta) - (cos\theta \times -cos\theta) = sin^2\theta + cos^2\theta = 1$

3. Subtract $\begin{pmatrix} 5 & 6 \\ -1 & 2 \end{pmatrix}$ *from* $\begin{pmatrix} 8 & -4 \\ -1 & 0 \end{pmatrix}$

   $\begin{pmatrix} 8 & -4 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 5 & 6 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 8-5 & -4-6 \\ -1--1 & 0-2 \end{pmatrix} = \begin{pmatrix} 3 & -10 \\ -1+1 & -2 \end{pmatrix}$

   $-4-6=-10, -1--1=-1+1=0$

4.*Solve for x if* $\begin{vmatrix} x^2 & 3 \\ 4 & 1 \end{vmatrix} = \begin{vmatrix} 9 & 4 \\ 8 & 5 \end{vmatrix}$

$(x^2 \times 1) - (4 \times 3) = x^2 - 12$ *and* $\begin{vmatrix} 9 & 4 \\ 8 & 5 \end{vmatrix} = (9 \times 5) - (8 \times 4) = 45 - 32 = 13$

Given that $x^2 - 12 = 13, x^2 = 13 + 12 = 25, x^2 = 25, x = \pm 5$

If A = $\begin{pmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & -2 & 2 \\ 4 & 0 & 3 \\ 2 & 1 & 1 \end{pmatrix}$ then find 3A+2B

3A+2B=3$\begin{pmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{pmatrix} + 2\begin{pmatrix} 1 & -2 & 2 \\ 4 & 0 & 3 \\ 2 & 1 & 1 \end{pmatrix} =$

$\begin{pmatrix} 3\times1 & 3\times0 & 3\times5 \\ 3\times-2 & 3\times1 & 3\times6 \\ 3\times3 & 3\times2 & 3\times7 \end{pmatrix} + \begin{pmatrix} 2\times1 & 2\times-2 & 2\times2 \\ 2\times4 & 2\times0 & 2\times3 \\ 2\times2 & 2\times1 & 2\times1 \end{pmatrix} =$

$\begin{pmatrix} 3 & 0 & 15 \\ -6 & 3 & 18 \\ 9 & 6 & 21 \end{pmatrix} + \begin{pmatrix} 2 & -4 & 4 \\ 8 & 0 & 6 \\ 4 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 3+2 & 0+-4 & 15+4 \\ -6+8 & 3+0 & 18+6 \\ 9+4 & 6+2 & 21+2 \end{pmatrix} = \begin{pmatrix} 5 & -4 & 19 \\ 2 & 3 & 24 \\ 13 & 8 & 23 \end{pmatrix}$

$-6+8=2, 3\times-2=-6$

4. If $\begin{bmatrix} a & a+b \\ 2a-c & b+c \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 7 & -2 \end{bmatrix}$ find a,b and c

   $a = 2, \quad a + b = 3, \quad 2 + b = 3, \quad b = 3 - 2 = 1$

   $2a - c = 7, a = 2,$

   $2 \times 2 - c = 7, 4 - c = 7. \quad 4 - 7 = +c, -3 = c\ , \quad c = -3$

5.If A = $\begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$ *then show that* $AA^{-1} = A^{-1}A = I$

$A^{-1} = \frac{AdjA}{|A|}$,AdjA (Adjoint A) is the transpose of the cofactor matrix of A

$Cofactors\ are\ C_{11}, C_{12}, C_{21}, C_{22},$

$C_{11} = (-1)^{1+1} \times 9 = (-1)^2 \times 9 = 9$

$C_{12} = (-1)^{1+2} \times 4 = (-1)^3 \times 4 = -4$

$C_{21} = (-1)^{2+1} \times 2 = (-1)^3 \times 2 = -2 = -2$

$C_{22} = (-1)^{2+2} \times 1 = 1 \times 1 = 1$

$Cofactor\ matrix = \begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$

Adjoint A is the transpose of the cofactor matrix of A = $\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$

Determinant A = $|A|$

$\begin{vmatrix} 1 & 2 \\ 4 & 9 \end{vmatrix} = (1 \times 9) - (2 \times 4) = 9 - 8 = 1$

$$A^{-1} = \frac{AdjA}{|A|} = \frac{\begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}}{1} = \begin{bmatrix} \frac{9}{1} & \frac{-2}{1} \\ \frac{-4}{1} & \frac{1}{1} \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$$

$to\ prove\ AA^{-1} = A^{-1}A = I$

$$AA^{-1} = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 9 + 2 \times -4 & 1 \times -2 + 2 \times 1 \\ 4 \times 9 + 9 \times -4 & 4 \times -2 + 9 \times 1 \end{bmatrix} = \begin{bmatrix} 9 + -8 & -2 + 2 \\ 36 + -36 & -8 + 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

6.If $\begin{vmatrix} x & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \\ -1 & 0 & 2 \end{vmatrix}$ find x

$\begin{vmatrix} x & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 0 & 3 \end{vmatrix} = x \times \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} - 1 \times \begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix} + 3 \times \begin{vmatrix} 4 & 1 \\ 2 & 0 \end{vmatrix}$,change the sign of 1

= x [(1×3) - (0 × -1)] – 1 [(4×3) – (2× - 1)+3 [(4× 0) - (2×1)] = $x[3 - 0] - 1[12 - -2] + 3[0 - 2]$

= x [3] -1 [12+2] +3 [- 2] = 3 $x - 1 \times 14 + -6,$ $-14 - 6 = -20$

= $3x - 14 - 6 = 3x - 20$ ……………(1)

$\begin{vmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 2 \times \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + 1 \times \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} + 1 \times \begin{vmatrix} 3 & 0 \\ -1 & 0 \end{vmatrix}$, $change\ the\ sign\ of\ -1$

= 2 [(0×2) - (0 × 1)] + 1 [(3×2) – (1× - 1)+1 [(3× 0) - (0×-1)] = $2[0 - 0] + 1[6 - -1] + 1[0 - 0]$

=2×0+1×(6+1)+1(0) = 0 +1×7+1×0=0+7+0=7…………(2)

From (1) and (2), $3x - 20 = 7,\quad (given)\quad , 3x = 7 + 20,\ 3x = 27, x = \frac{27}{3} = 9$

7.Solve for x if, $\begin{vmatrix} 2 & 1 & x \\ 3 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 4 & x \\ 3 & 2 \end{vmatrix}$

$\begin{vmatrix} 2 & 1 & x \\ 3 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix} = 2 \times \begin{vmatrix} -1 & 2 \\ 1 & 6 \end{vmatrix} - 1 \times \begin{vmatrix} 3 & 2 \\ 1 & 6 \end{vmatrix} + x \times \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix}$, $change\ the\ sign\ of\ 1$

= 2 [(-1×6) - (2 × 1)] -1 [(3×6) – (1× 2)+ x [(3× 1) - (1×-1)] = $2[-6 - 2] - 1[18 - 2] + x[3 - -1]$

$=2[-8]-1[16]+x[4]=-16-16+4x=4x-32$

$$\begin{vmatrix}4 & x\\ 3 & 2\end{vmatrix}=(4\times 2)-(3\times x)=8-3x$$

Given that $4x-32=8-3x$

$4x+3x=8+32$,$7x=40, x=\frac{40}{7}$

8.If A = $\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}$ B=$\begin{bmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ -3 & 1 & 2\end{bmatrix}$. Compute AB and BA

AB=$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ -3 & 1 & 2\end{bmatrix}=$

$\begin{bmatrix}1\times 1+0\times 2+2\times -3 & 1\times -2+0\times 3+2\times 1 & 1\times 3+0\times -1+2\times 2\\ 0\times 1+1\times 2+2\times -3 & 0\times -2+1\times 3+2\times 1 & 0\times 3+1\times -1+2\times 2\\ 1\times 1+2\times 2+0\times -3 & 1\times -2+2\times 3+0\times 1 & 1\times 3+2\times -1+0\times 2\end{bmatrix}$

= $\begin{bmatrix}1+0-6 & -2+0+2 & 3+0+4\\ 0+2-6 & 0+3+2 & 0-1+4\\ 1+4+0 & -2+6+0 & 3-2+0\end{bmatrix}=\begin{bmatrix}-5 & 0 & 7\\ -4 & 5 & 3\\ 5 & 4 & 1\end{bmatrix}$

BA=$\begin{bmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ -3 & 1 & 2\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}=$

$\begin{bmatrix}1\times 1-2\times 0+3\times 1 & 1\times 0-2\times 1+3\times 2 & 1\times 2-2\times 2+3\times 0\\ 2\times 1+3\times 0-1\times 1 & 2\times 0+3\times 1-1\times 2 & 2\times 2+3\times 2-1\times 0\\ -3\times 1+1\times 0+2\times 1 & -3\times 0+1\times 1+2\times 2 & -3\times 2+1\times 2+2\times 0\end{bmatrix}=$

$\begin{bmatrix}1+0+3 & 0-2+6 & 2-4+0\\ 2+0-1 & 0+3-2 & 4+6+0\\ -3+0+2 & 0+1+4 & -6+2+0\end{bmatrix}=\begin{bmatrix}4 & 4 & -2\\ 1 & 1 & 10\\ -1 & 5 & -4\end{bmatrix}$

AB $\neq$BA, A and B do not commute[A and B,commute if, AB=BA]

8.Find the values of a,b and c that satisfy

$\begin{bmatrix}a+3 & 3a-2b\\ 3a-c & a+b+c\end{bmatrix}=\begin{bmatrix}2 & -7+2b\\ b+4 & 8a\end{bmatrix}$

$a+3=2$, $a=2-3=-1$

$3a-2b=-7+2b$, 3 × -1 - 2b = - 7 + 2b, -3 + 7 = 2b + 2b , 4 =4b , $\frac{4}{4}$ = b , 1 = b ,b =1

$3a-c=\ b+4$

$3a-c=1+4,\ \ 3\times -1-c=5,\quad -3-c=5,\quad -c=5+3,\quad -c=8,\quad c=\ -8$

9.Solve by determinant method $x+2y-z=\ -3, 3x+y+z-4=0, x-y+2z=6$

The equations as $1x+2y-1z=\ -3$, ,$3x+1y+1z=4$, $1x-1y+2z=6$

|A |= $\begin{vmatrix}1 & 2 & -1\\ 3 & 1 & 1\\ 1 & -1 & 2\end{vmatrix}$ = Δ = [*change the sign of 2*] $=1\times\begin{vmatrix}1 & 1\\ -1 & 2\end{vmatrix}-2\times\begin{vmatrix}3 & 1\\ 1 & 2\end{vmatrix}-1\times\begin{vmatrix}3 & 1\\ 1 & -1\end{vmatrix}$

= 1 [(1×2) - (1 × - 1)] - 2 [(3×2) − (1× 1) - 1 × [(3× - 1) - (1×1)] $=1[2--\ 1]-2\ [6-1]-1[-3-1]$

$=1[2+\ 1]-2\,[5]-1[-4] = 1\times 3\ - 10 + 4 = 3 + 4 - 10 = 7 - 10\ = -3$, $-3-1=\ -4$

$x=\frac{\Delta_1}{\Delta}, y=\frac{\Delta_2}{\Delta}, z=\frac{\Delta_3}{\Delta}, \Delta_1 = change\ the\ first\ column\ of\ \Delta$ by $-3, 4, 6 = \Delta_1 = \begin{vmatrix} -3 & 2 & -1 \\ 4 & 1 & 1 \\ 6 & -1 & 2 \end{vmatrix}$

$= -3\times\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - 2\times\begin{vmatrix} 4 & 1 \\ 6 & 2 \end{vmatrix} - 1\times\begin{vmatrix} 4 & 1 \\ 6 & -1 \end{vmatrix} = -3$ [(1×2) - (1 × - 1)] - 2 [(4×2) – (6× 1) - 1 × [(4× -1) -(6×1)]

$=\ -3[2--\ 1]-2\,[8-6]-1[-4-6] == \ -3[2+1]-2\,[2]-1[-10] = -3[3]-2\,[2]+10 =$

$$-9-4+10=-13+10=-3$$

$\Delta_2 =\ change\ the\ second\ column\ of\ \Delta$ by $-3, 4, 6,\ \Delta_2 = \begin{vmatrix} 1 & -3 & -1 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{vmatrix}$

$= 1\times\begin{vmatrix} 4 & 1 \\ 6 & 2 \end{vmatrix} + 3\times\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 1\times\begin{vmatrix} 3 & 4 \\ 1 & 6 \end{vmatrix}$ = 1 [(4×2) - (6 × 1)] + 3 [(3×2) – (1× 1) - 1 × [(3× 6) - (1×4)]

$= 1[8 -\ 6] + 3\,[6-1] - 1[18-4] = 1[2\,] + 3\,[5] - 1[14] = 2+15-14 = 17-14 = 3$

$\Delta_3 = change\ the\ \ third\ column\ of\ \Delta$ by $-3, 4, 6\ \ \Delta_3 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 1 & 4 \\ 1 & -1 & 6 \end{vmatrix}$

$= 1\times\begin{vmatrix} 1 & 4 \\ -1 & 6 \end{vmatrix} - 2\times\begin{vmatrix} 3 & 4 \\ 1 & 6 \end{vmatrix} - 3\times\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = 1$ [(1×6) - (4 × - 1)] - 2 [(3×6) – (4× 1) - 3 × [(3× -1) - (1×1)]

$= 1[6\ -\ -4] - 2\,[18-4] - 3[-3-1] = 1[10] - 2\,[14] - 3[-4] = 10 - 28\ + 12 = 10 + 12 - 28 = 22 - 28 = -6$

$x=\frac{\Delta_1}{\Delta}=\frac{-3}{-3}=1, y=\frac{\Delta_2}{\Delta}=\frac{3}{-3}=\ -1,, z=\frac{\Delta_3}{\Delta}=\frac{-6}{-3}=2$

10.If A = $\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -2 & 2 \\ 4 & 0 & 3 \\ 2 & 1 & 1 \end{bmatrix}$ find 3A+2B

3A+2B=

$$3\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix} + 2\begin{bmatrix} 1 & -2 & 2 \\ 4 & 0 & 3 \\ 2 & 1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 3\times 1 & 3\times 0 & 3\times 5 \\ 3\times -2 & 3\times 1 & 3\times 6 \\ 3\times 3 & 3\times 2 & 3\times 7 \end{bmatrix} + \begin{bmatrix} 2\times 1 & 2\times -2 & 2\times 2 \\ 2\times 4 & 2\times 0 & 2\times 3 \\ 2\times 2 & 2\times 1 & 2\times 1 \end{bmatrix} = \begin{bmatrix} 3+2 & 0+\ -4 & 15+4 \\ -6+8 & 3+0 & 18+6 \\ 9+4 & 6+2 & 21+2 \end{bmatrix} = \begin{bmatrix} 5 & -4 & 19 \\ 2 & 3 & 24 \\ 13 & 8 & 23 \end{bmatrix}$$

11.Solve $5x+2y=4, 2x-y=7, The\ equations\ are\ \ 5x+2y=4, 2x-1y=7$

A = $\begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}$ $B=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \end{bmatrix}$

X = $A^{-1}.B$, and $A^{-1} = \frac{Adj\ A}{|A|}$

Adjoint A is the transpose of the cofactor matrix of A.

A = $\begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}$, $Cofactors\ are\ C_{11}, C_{12}, C_{21}, C_{22},$

$C_{11} = (-1)^{1+1} \times -1 = (-1)^2 \times -1 = -1$

$C_{12} = (-1)^{1+2} \times 2 = (-1)^3 \times 2 = -2$

$C_{21} = (-1)^{2+1} \times 2 = (-1)^3 \times 2 = -2 = -2$

$C_{22} = (-1)^{2+2} \times 5 = 1 \times 5 = 5$

$Cofactor\ matrix = \begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}$

Adjoint A is the transpose of the cofactor matrix of A = $\begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}^T = \begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}$

Determinant A = $|A| = \begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = (5 \times -1) - (2 \times 2) = -5 - 4 = -9, A^{-1} = \frac{Adj\ A}{|A|} = \frac{\begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}}{-9} = \begin{bmatrix} \frac{-1}{-9} & \frac{-2}{-9} \\ \frac{-2}{-9} & \frac{5}{-9} \end{bmatrix}$

X = $A^{-1}.B = \begin{bmatrix} \frac{1}{9} & \frac{-2}{-9} \\ \frac{-2}{-9} & \frac{5}{-9} \end{bmatrix} . \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{9} \times 4 + \frac{2}{9} \times 7 \\ \frac{2}{9} \times 4 + \frac{5}{-9} \times 7 \end{bmatrix} = \begin{bmatrix} \frac{4}{9} + \frac{14}{9} \\ \frac{8}{9} + \frac{-35}{9} \end{bmatrix} = \begin{bmatrix} \frac{18}{9} \\ \frac{8-35}{9} \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{-27}{9} \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ $x = 2$, y = - 3

12.Find AB, A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ -1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -2 & 1 \end{bmatrix}$

AB = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ -1 & 2 \end{bmatrix} . \begin{bmatrix} 1 & 0 & -2 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + -1 \times 3 & 1 \times 0 - 1 \times -2 & 1 \times -2 - 1 \times 1 \\ 2 \times 1 + 3 \times 3 & 2 \times 0 + 3 \times -2 & 2 \times -2 + 3 \times 1 \\ -1 \times 1 + 2 \times 3 & -1 \times 0 + 2 \times -2 & -1 \times -2 + 2 \times 1 \end{bmatrix} =$

$\begin{bmatrix} 1 + -3 & 0 + 2 & -2 + -1 \\ 2 + 9 & 0 + -6 & -4 + 3 \\ -1 + 6 & 0 + -4 & 2 + 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 & -3 \\ 11 & -6 & -1 \\ 5 & -4 & 4 \end{bmatrix}$

MODULE 2

VECTOR ALGEBRA

1.Find the sum of the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $2\hat{\imath} - 3\hat{\jmath} + \hat{k}$, $-\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.

$(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + (2\hat{\imath} - 3\hat{\jmath} + \hat{k}) + (-\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = (\hat{\imath} + 2\hat{\imath} + -\hat{\imath}) + (-2\hat{\jmath} + -3\hat{\jmath} + 2\hat{\jmath}) + (3\hat{k} + \hat{k} + -3\hat{k})$

$= 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$, $\hat{\imath} - \hat{\imath} = 0, -2\hat{\jmath} + 2\hat{\jmath} = 0, 3\hat{k} - 3\hat{k} = 0$

2.Find the length of the vector, $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, The length of the vector = $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$\vec{a} = 1\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, $x = 1, y = -2, z = 2$, length $= \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$, $(-2)^2 = 4$

3.If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, find $\vec{a} . \vec{b}$

$\vec{a} = 1\ \hat{\imath} + 1\hat{\jmath} + 1\ \hat{k}, \vec{b} = 2\ \hat{\imath} - 1\hat{\jmath} + 3\ \hat{k}$, $\vec{a} . \vec{b} = a_1 a_2 + b_1 b_2 + c_1 c_2$

$a_1 = 1, b_1 = 1, c_1 = 1, a_2 = 2, b_2 = -1, c_2 = 3,$

$\vec{a} . \vec{b} = 1 \times 2 + 1 \times -1 + 1 \times 3 = 2 + -1 + 3 = 5 - 1 = 4$

3.Find the unit vector in the direction of $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$|\vec{a}| = \text{length} = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$, $(-1)^2 = 1$

unit vector = $\frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{\imath}-\hat{\jmath}+4\hat{k}}{\sqrt{21}}$

4.Find a unit vector perpendicular to the vectors $2\hat{\imath}+3\hat{\jmath}+4\hat{k}$, $\hat{\imath}+\hat{\jmath}+\hat{k}$

Vector perpendicular to $\vec{a}$ *and* $\vec{b}$ *is* $\vec{a}\times\vec{b}$,

unit vector perpendicular to $\vec{a}$ *and* $\vec{b}$ *is* $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$\vec{a} = 2\hat{\imath}+3\hat{\jmath}+4\hat{k}$, $\vec{b} = \hat{\imath}+\hat{\jmath}+\hat{k} = 1\,\hat{\imath}+1\,\hat{\jmath}+1\,\hat{k}$

$\vec{a}\times\vec{b} = (2\hat{\imath}+3\hat{\jmath}+4\hat{k})\times(1\,\hat{\imath}+1\,\hat{\jmath}+1\,\hat{k}) = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 4\\ 1 & 1 & 1\end{vmatrix} = \hat{\imath}\begin{vmatrix}3 & 4\\ 1 & 1\end{vmatrix} - \hat{\jmath}\begin{vmatrix}2 & 4\\ 1 & 1\end{vmatrix} + \hat{k}\begin{vmatrix}2 & 3\\ 1 & 1\end{vmatrix}$

$= \hat{\imath}\,[(3\times1) - (4\times1)] - \hat{\jmath}\,[(2\times1) - (4\times1)] + \hat{k}\,[(2\times1) - (3\times1)] = \hat{\imath}\,[3-4] - \hat{\jmath}[2-4] + \hat{k}[2-3]$

$= \hat{\imath}\,[-1] - \hat{\jmath}[-2] + \hat{k}[-1] = -1\hat{\imath}+2\hat{\jmath}-1\hat{k}$.

unit vector perpendicular to $\vec{a}$ *and* $\vec{b}$ *is* $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-1\hat{\imath}+2\hat{\jmath}-1\hat{k}.}{|-1\hat{\imath}+2\hat{\jmath}-1\hat{k}.|} =$

$\frac{-1\hat{\imath}+2\hat{\jmath}-1\hat{k}}{\sqrt{(-1)^2+(2)^2+(-1)^2}} = \frac{-1\hat{\imath}+2\hat{\jmath}-1\hat{k}}{\sqrt{1+4+1}} = \frac{-1\hat{\imath}+2\hat{\jmath}-1\hat{k}}{\sqrt{6}}$, $|-1\hat{\imath}+2\hat{\jmath}-1\hat{k}| = \sqrt{(-1)^2+(2)^2+(-1)^2}$

5.Find the angle between the vectors $6\hat{\imath}-3\hat{\jmath}+2\hat{k}$ *and* $2\hat{\imath}+2\hat{\jmath}-\hat{k}$.

Angle between the vectors , $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a} = (6\hat{i}-3\hat{j}+2\hat{k})$, $\vec{b} = (2\hat{i}+2\hat{j}-1\hat{k})$

$\vec{a}\cdot\vec{b} = (6\hat{i}-3\hat{j}+2\hat{k}).(2\hat{i}+2\hat{j}-1\hat{k}) = 6\times2-3\times2+2\times-1 = 12-6-2 = 12-8 = 4$, $-6-2=-8$

$|\vec{a}| = \sqrt{6^2+(-3)^2+2^2} = \sqrt{36+9+4} = \sqrt{49} = 7$

$|\vec{b}| = \sqrt{2^2+(2)^2+(-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

$\cos\theta = \frac{4}{7\times3} = \frac{4}{21}$, $\theta = \cos^{-1}(\frac{4}{21})$

6 .Find the work done by a force $\vec{F} = \hat{\imath}+2\hat{\jmath}+\hat{k} = 1\hat{\imath}+2\hat{\jmath}+1\hat{k}$ *which is displaced from a point with position vector* $2\hat{\imath}+\hat{\jmath}+\hat{k}$ to the point with position vector $3\hat{\imath}+2\hat{\jmath}+4\hat{k}$.

Work done = $\vec{F}.\vec{AB}$, B$(3\hat{\imath}+2\hat{\jmath}+4\hat{k})$, $A(2\hat{\imath}+\hat{\jmath}+\hat{k})$

$\vec{AB}$ = position vector of B – position vector of A = $(3\hat{\imath}+2\hat{\jmath}+4\hat{k}) - (2\hat{\imath}+\hat{\jmath}+\hat{k})$

$= 3\hat{\imath}+2\hat{\jmath}+4\hat{k} - 2\hat{\imath}-\hat{\jmath}-\hat{k} = (3\hat{\imath}-2\hat{\imath}) + (2\hat{\jmath}-\hat{\jmath}) + (4\hat{k}-\hat{k}) = 1\hat{\imath}+1\hat{\jmath}+3\hat{k}$

Work done = $\vec{F}.\vec{AB} = (1\hat{\imath}+2\hat{\jmath}+1\hat{k}).(1\hat{\imath}+1\hat{\jmath}+3\hat{k}) = 1\times1+2\times1+1\times3 = 1+2+3 = 6$ *units*.

7.The constant forces $2\hat{\imath} - 5\,\hat{\jmath} + 6\hat{k}$, $-\hat{\imath} + 2\hat{\jmath} - \hat{k}$ *and* $2\hat{\imath} + 7\,\hat{\jmath}$ act on a particle from the position $4\hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k}$ to $6\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$. *Find the total work done.*

Ans: A($4\hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k}$) , B($6\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$)

Work done = $\vec{F}\,.\,\overrightarrow{AB}$ $\quad \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$, $\vec{F_1} = 2\hat{\imath} - 5\,\hat{\jmath} + 6\,\hat{k}$, $\vec{F_2} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$, $\vec{F_3} = 2\hat{\imath} + 7\,\hat{\jmath}$

$\vec{F_1} + \vec{F_2} + \vec{F_3}$ = $(2\hat{\imath} - 5\,\hat{\jmath} + 6\hat{k}) + (-\hat{\imath} + 2\hat{\jmath} - \hat{k}\,) + (2\hat{\imath} + 7\,\hat{\jmath})$ =

$(2\hat{\imath} + -\hat{\imath} + 2\hat{\imath}) + (-5\,\hat{\jmath} + 2\hat{\jmath} + 7\,\hat{\jmath}) + (6\hat{k} + -\hat{k})$ = $(4\hat{\imath} - \hat{\imath}) + (9\hat{\jmath} - 5\,\hat{\jmath}) + (6\hat{k} - \hat{k})$ = $3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

$\overrightarrow{AB}$ = B$(6\hat{\imath} + \hat{\jmath} - 3\,\hat{k})$ – A$(4\hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k})$ $= 6\hat{\imath} + \hat{\jmath} - 3\,\hat{k} - \left(4\hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k}\right) =$

$6\hat{\imath} + \hat{\jmath} - 3\,\hat{k} - 4\hat{\imath} - -3\,\hat{\jmath} - -2\,\hat{k}$ = $6\hat{\imath} + \hat{\jmath} - 3\,\hat{k} - 4\hat{\imath} + 3\,\hat{\jmath} + 2\,\hat{k}$ =$(6\hat{\imath} - 4\hat{\imath}) + (\hat{\jmath} + 3\,\hat{\jmath}) + (-3\hat{k} + 2\hat{k})$= $2\hat{\imath} + 4\,\hat{\jmath} - 1\,\hat{k}$, $-3\hat{k} + 2\hat{k} = -1k$

Work done = $(\vec{F_1} + \vec{F_2} + \vec{F_3}).\,\overrightarrow{AB} = (3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}).(\,2\hat{\imath} + 4\,\hat{\jmath} - 1\,\hat{k}) = 3 \times 2 + 4 \times 4 + 5 \times -1$

$$= 6 + 16 + -5 = 22 - 5 = 17 units.$$

8.Find a vector perpendicular to the vectors $2\,\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$

Vector perpendicular to $\vec{a}$ *and* $\vec{b}$ *is* $\vec{a} \times \vec{b}$

$2\hat{\imath} + 3\hat{\jmath} + 4\hat{k} = \vec{a}$ ' $\qquad \hat{\imath} + \hat{\jmath} - \hat{k} = \vec{b}$ = $1\hat{\imath} + 1\hat{\jmath} - 1\hat{k}$

$$\vec{a} \times \vec{b} = \left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) \times \left(1\hat{\imath} + 1\hat{\jmath} - 1\hat{k}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 1 & -1 \end{vmatrix} = \hat{\imath}\begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

= $\hat{\imath}$ [(3×-1) - (4 × 1)] $-\hat{\jmath}$ [(2×-1) – (4× 1)] + $\hat{k}$ [(2× 1) - (3×1)] =

= $\hat{\imath}$ [-3 - 4] $-\hat{\jmath}[-2 - 4] + \hat{k}[2 - 3]$ = $\hat{\imath}$ [-7] $-\hat{\jmath}[-6] + \hat{k}[-1]$ = -7$\hat{\imath}$ + 6$\hat{\jmath}$ -1$\hat{k}$,

$2 - 3 = -1$ -3 - 4 = -7, $-2 - 4 = -6$

9. A Force $4\hat{\imath} - 3\hat{k}$ passes through the point A whose position vector is $2\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$. Find the moment of the force about the point B whose position vector is $\hat{\imath} - 3\hat{\jmath} + \hat{k}$.

Force $= \vec{F} = 4\hat{\imath} - 3\hat{k}$= $4\hat{\imath} + 0\hat{\jmath} - 3\hat{k}$

$\vec{r}$ = Through the point A – about the point B = $(2\hat{\imath} - 2\hat{\jmath} + 5\hat{k}\,) - (\hat{\imath} - 3\hat{\jmath} + \hat{k}\,) =$

= $2\hat{\imath} - 2\hat{\jmath} + 5\hat{k} - \hat{\imath} - -3\hat{\jmath} - \hat{k}$ =$2\hat{\imath} - \hat{\imath} - 2\hat{\jmath} + 3\hat{\jmath} + 5\hat{k} - \hat{k} = 1\hat{\imath} + 1\hat{\jmath} + 4\hat{k}$

Moment of the force = $|\,\vec{r} \times \vec{F}\,|$

$$\vec{r} \times \vec{F} = (1\hat{\imath} + 1\hat{\jmath} + 4\hat{k}\,) \times (4\hat{\imath} + 0\hat{\jmath} - 3\hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 4 \\ 4 & 0 & -3 \end{vmatrix} = \hat{\imath}\begin{vmatrix} 1 & 4 \\ 0 & -3 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & 4 \\ 4 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix}$$

= $\hat{\imath}$ [(1×-3) - (4 × 0)] $-\hat{\jmath}$ [(1×-3) – (4× 4)] + $\hat{k}$ [(1× 0) - (4×1)] = $\hat{\imath}$ [-3 - 0] $-\hat{\jmath}[-3 - 16] + \hat{k}[0 - 4]$

=$\hat{\imath}$ [-3] $-\hat{\jmath}[-19] + \hat{k}[-4]$ $= -3\hat{\imath} + 19\hat{\jmath} - 4\hat{k}$

Moment of the force = $|\vec{r} \times \vec{F}|$ = $|-3\hat{\imath} + 19\hat{\jmath} - 4\hat{k}|$ = $\sqrt{(-3)^2 + (19)^2 + (-4)^2}$ = $\sqrt{9 + 361 + 16}$ = $\sqrt{386}$

10.Find the angle between $7\hat{\imath} - \hat{\jmath} + 11\hat{k}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$.

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$, $\vec{a} = (7\hat{\imath} - 1\hat{\jmath} + 11\,\hat{k})$, $\vec{b} = (1\hat{\imath} + 1\,\hat{\jmath} + 1\,\hat{k})$

$\vec{a} \cdot \vec{b}$ = 7 × 1 − 1 × 1 + 11 × 1 = 7 − 1 + 11 = 7 + 11 − 1 = 18 − 1 = 17

$|\vec{a}| = \sqrt{7^2 + (-1)^2 + 11^2}$ = $\sqrt{49 + 1 + 121}$ = $\sqrt{171}$

$|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$ = $\sqrt{3}$

Cos θ = $\frac{17}{\sqrt{171}\times\sqrt{3}} = \frac{17}{\sqrt{513}}$, θ $= \cos^{-1}\left(\frac{17}{\sqrt{513}}\right)$

11.Find the area of a parallelogram whose adjacent sides are determined by the vectors

$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k} = 1\hat{\imath} - 1\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\,\hat{\jmath} + \hat{k} = 2\hat{\imath} - 7\,\hat{\jmath} + 1\,\hat{k}$

Area of the parallelogram = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b}$ = $(1\hat{\imath} - 1\hat{\jmath} + 3\hat{k}) \times (2\hat{\imath} - 7\hat{\jmath} + 1\hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{\imath}\begin{vmatrix} -1 & 3 \\ -7 & 1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -1 \\ 2 & -7 \end{vmatrix}$

= $\hat{\imath}$ [(-1×1) - (3 × -7)] $-\hat{\jmath}$ [(1×1) − (2× 3)] + $\hat{k}$ [(1× -7) - (2×-1)] = $\hat{\imath}$ [-1 +21] $-\hat{\jmath}[1 - 6] + \hat{k}[-7 + 2]$

= $\hat{\imath}$ [20] $-\hat{\jmath}[-5] + \hat{k}[5]$ = $20\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$

Area of the parallelogram = $|\vec{a} \times \vec{b}| = \sqrt{(20)^2 + (5)^2 + (5)^2}$ = $\sqrt{450}$ sq. units

12.Find the area of a triangle with vertices A(1,0,-1), B(2,1,5) and C(0,1,2).

Area of the triangle = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$\overrightarrow{AB}$ = position vector B – position vector A = B(2,1,5) - A(1,0,-1) = $2\hat{\imath} + 1\hat{\jmath} + 5\hat{k} - (1\hat{\imath} + 0\hat{\jmath} - 1\hat{k})$

= $2\hat{\imath} + 1\hat{\jmath} + 5\hat{k} - 1\hat{\imath} - 0\hat{\jmath} + 1\hat{k} = 2\hat{\imath} - 1\hat{\imath} + 1\hat{\jmath} - 0\hat{\jmath} + 5\hat{k} + 1\hat{k} = 1\hat{\imath} + 1\hat{\jmath} + 6\hat{k}$

$\overrightarrow{AC}$ = position vector C – position vector A = C(0,1,2) - A(1,0,-1) = $0\hat{\imath} + 1\hat{\jmath} + 2\hat{k} - (1\hat{\imath} + 0\hat{\jmath} - 1\hat{k})$

= $0\hat{\imath} + 1\hat{\jmath} + 2\hat{k} - 1\hat{\imath} - 0\hat{\jmath} + 1\hat{k}$ = $0\hat{\imath} - 1\hat{\imath} + 1\hat{\jmath} - 0\hat{\jmath} + 2\hat{k} + 1\hat{k}$ = $-1\hat{\imath} + 1\hat{\jmath} + 3\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC}$ = $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 6 \\ -1 & 1 & 3 \end{vmatrix} = \hat{\imath}\begin{vmatrix} 1 & 6 \\ 1 & 3 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & 6 \\ -1 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$

= $\hat{\imath}$ [(1×3) - (6 × 1)] $-\hat{\jmath}$ [(1×3) − (6× -1)] + $\hat{k}$ [(1× 1) - (1×-1)] = $\hat{\imath}$ [3- 6] $-\hat{\jmath}[3 - -6] + \hat{k}[1 - -1]$

= $\hat{\imath}$ [-3] $-\hat{\jmath}[3 + 6] + \hat{k}[1 + 1]$ = $-3\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = |-3\hat{\imath} - 9\hat{\jmath} + 2\hat{k}| = \sqrt{(-3)^2 + (-9)^2 + (2)^2} = \sqrt{9 + 81 + 4} = \sqrt{94}$

Area of the triangle $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{94}\,\text{sq. units}$

13.Find the unit vector in the direction of $3\vec{a} + 4\vec{b}$.$\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k} = 1\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$,$\vec{b} = 2\hat{\imath} - 3\hat{\jmath} + 1\hat{k}$

unit vector in the direction of $3\vec{a} + 4\vec{b} = \frac{3\vec{a}+4\vec{b}}{|3\vec{a}+4\vec{b}|}$

$3\vec{a} + 4\vec{b} = 3(1\hat{\imath} + 2\hat{\jmath} + 1\hat{k}) + 4(2\hat{\imath} - 3\hat{\jmath} + 1\hat{k}) = 3\hat{\imath} + 6\hat{\jmath} + 3\hat{k} + 8\hat{\imath} - 12\hat{\jmath} + 4\hat{k}$

$= 3\hat{\imath} + 8\hat{\imath} + 6\hat{\jmath} - 12\hat{\jmath} + 3\hat{k} + 4\hat{k} = 11\hat{\imath} - 6\hat{\jmath} + 7\hat{k}$

$|3\vec{a} + 4\vec{b}| = |11\hat{\imath} - 6\hat{\jmath} + 7\hat{k}| = \sqrt{(11)^2 + (-6)^2 + (7)^2} = \sqrt{121 + 36 + 49} = \sqrt{206}$

unit vector in the direction of $3\vec{a} + 4\vec{b} = \frac{3\vec{a}+4\vec{b}}{|3\vec{a}+4\vec{b}|} = \frac{11\hat{\imath}-6\hat{\jmath}+7\hat{k}}{\sqrt{206}}$

14.Find the value of p such that $2\hat{\imath} - 3\hat{\jmath} - \hat{k}$, $4\hat{\imath} - p\hat{\jmath} - 2\hat{k}$ are perpendicular.

Two vectors are perpendicular when $\vec{a}.\vec{b} = 0$

$\vec{a}.\vec{b} = (2\hat{\imath} - 3\hat{\jmath} - \hat{k}).(4\hat{\imath} - p\hat{\jmath} - 2\hat{k}) = 0$,

$\vec{a} \cdot \vec{b} = 2 \times 4 - 3 \times -p - 1 \times -2$ = 8 +3p +2=0,10+3p = 0, 3p = -10, p $=\frac{-10}{3}$

15.Find the dot product of the vectors $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - \hat{k}$.

Dot product $\vec{a}.\vec{b} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k}).(1\hat{\imath} - 2\hat{\jmath} - 1\hat{k})$

$= 2 \times 1 + 3 \times -2 - 1 \times -1 = 2 - 6 + 1 = 3 - 6 = -3$

15. Find the unit vector in the direction of $\hat{\imath} - 2\hat{\jmath} - \hat{k}$.

Unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1\hat{\imath}-2\hat{\jmath}-1\hat{k}}{\sqrt{(1)^2+(-2)^2+(-1)^2}} = \frac{1\hat{\imath}-2\hat{\jmath}-1\hat{k}}{\sqrt{1+4+1}} = \frac{\hat{\imath}-2\hat{\jmath}-\hat{k}}{\sqrt{6}}$

16. Find the length of the vector $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

Length of the vector $2\hat{\imath} + 3\hat{\jmath} - \hat{k} = |\vec{a}| = \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$