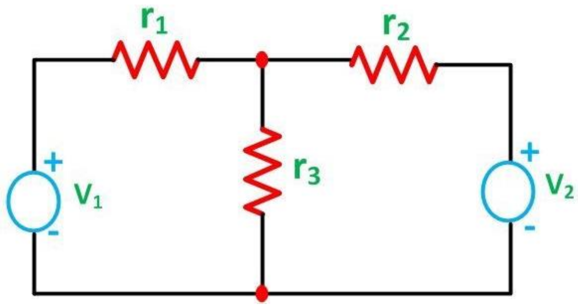
MODULE-I
NETWORK THEOREMS

Superposition Theorem

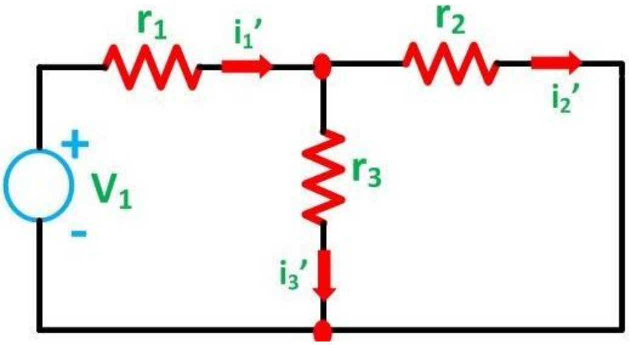
Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected.

In other words, it can be stated as if a number of voltage or current sources are acting in a linear network, the resulting current in any branch is the algebraic sum of all the currents that would be produced in it when each source acts alone while all the other independent sources are replaced by their internal resistances. It is only applicable to the circuit which is valid for the ohm's law (i.e., for the linear circuit).

Explanation of Superposition Theorem



Let us understand the superposition theorem with the help of an example. The circuit diagram is shown below consists of two voltage sources V_1 and V_2. First, take the source V_1 alone and short circuit the V2 Source as below:



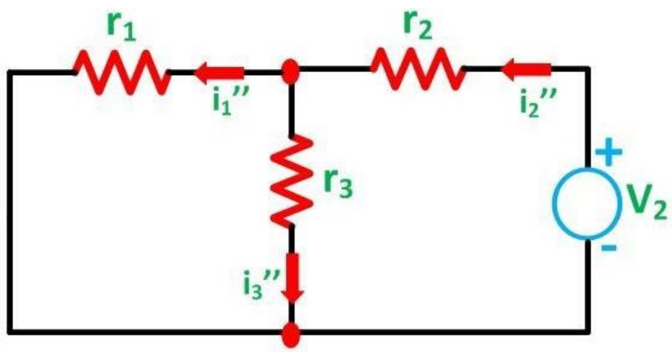
Here, the value of current flowing in each branch, i.e. i_1', i_2' and i_3' is calculated by the following equations.

$$i_1' = \frac{V_1}{\left(\frac{r_2 r_3}{r_2 + r_3}\right)} \ldots\ldots\ldots\ldots 1$$

$i_2^{'} = i_1^{'}\frac{r_3}{r_2 + r_3}$ similarly, $i_3^{'} = i_1^{'}\frac{r_2}{r_2 + r_3}$ or

The difference between the above two equations gives the value of the current

$i_3^{'}$, $i_3^{'} = i_1^{'} - i_2^{'}$



Now, activating the voltage source V_2 and deactivating the voltage source V_1 by short-circuiting it, find the various currents, i.e. i_1'', i_2'', i_3'' flowing in the circuit diagram shown left.

Here,

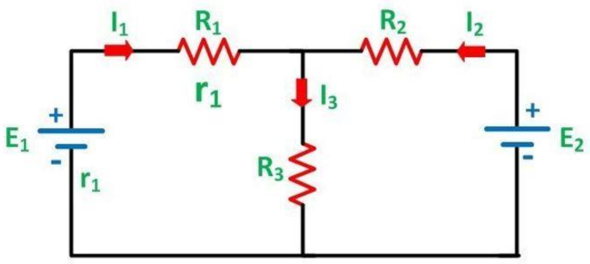
$$i_2^{''} = \frac{V_2}{\left(\frac{r_1 r_3}{r_1 + r_3}\right) + r_2} \text{ and } i_1^{''} = i_2^{''}\frac{r_3}{r_1 + r_3}$$

And the value of the current i_3'' will be calculated by the equation shown below: $i_3^{''} = i_2^{''} - i_1^{''}$. As per the superposition theorem, the value of current i_1, i_2, i_3 is now calculated as:

$i_1 = i'_1 - i''_1$ $\qquad$ $i_2 = i'_2 - i''_2$ $\qquad$ $i_3 = i'_3 + i''_3$

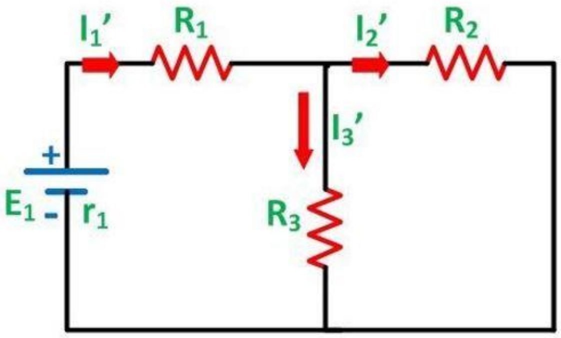
The direction of the current should be taken care of while finding the current in the various branches.

Steps for Solving network by Superposition Theorem

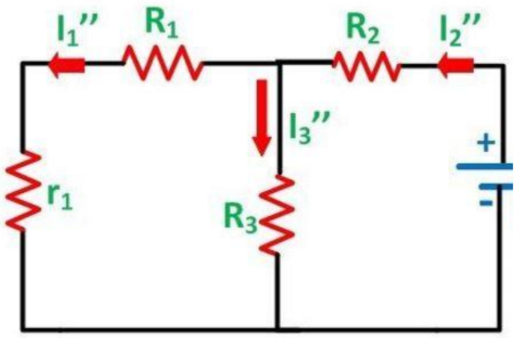


Circuit Diagram A

Considering the circuit diagram, A, let us see the various steps to solve the superposition theorem



Circuit Diagram B



Circuit Diagram C

Step 1 – Take only one independent source of voltage or current and deactivate the other sources.

Step 2 – In the circuit diagram B shown above, consider the source E_1 and replace the other source E_2 by its internal resistance. If its internal resistance is not given, then it is taken as zero and the source is short-circuited.

Step 3 – If there is a voltage source than short circuit it and if there is a current source then just open circuit it.

Step 4 – Thus, by activating one source and deactivating the other source find the current in each branch of the network. Taking the above example find the current I_1', I_2'and I_3'.

Step 5 – Now consider the other source E_2 and replace the source E_1 by its internal resistance r_1 as shown in the circuit diagram C.

Step 6 – Determine the current in various sections, I_1'', I_2'' and I_3''.

Step 7 – Now to determine the net branch current utilizing the superposition theorem, add the currents obtained from each individual source for each branch.

Step 8 – If the current obtained by each branch is in the same direction then add them and if it is in the opposite direction, subtract them to obtain the net current in each branch.

The actual flow of current in the circuit C will be given by the equations shown below:

$$I_1 = I_1' - I_1''$$

$$I_2 = I_2' - I_2''$$

$$I_3 = I_3' - I_3''$$

Thevenin's Theorem

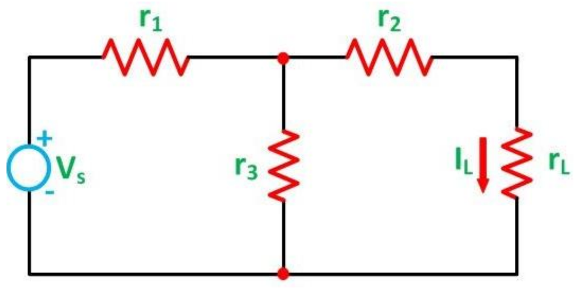
Thevenin's Theorem states that any complicated network across its load terminals can be substituted by a voltage source with one resistance in series. This theorem helps in the study of the variation of current in a particular branch when the resistance of the branch is varied while the remaining network remains the same.

A more general statement of Thevenin's Theorem is that any linear active network consisting of independent or dependent voltage and current source and the network elements can be replaced by an equivalent circuit having a voltage source in series with a resistance.

Where the voltage source being the open-circuited voltage across the open-circuited load terminals and the resistance being the internal resistance of the source.

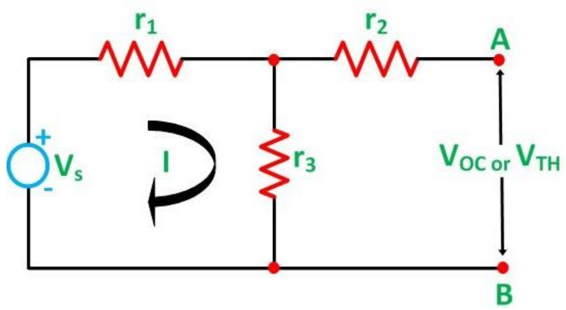
In other words, the current flowing through a resistor connected across any two terminals of a network by an equivalent circuit having a voltage source E_{th} in series with a resistor R_{th}. Where E_{th} is the open-circuit voltage between the required two terminals called the Thevenin voltage and the R_{th} is the equivalent resistance of the network as seen from the two-terminal with all other sources replaced by their internal resistances called Thevenin resistance.

The Thevenin's statement is explained with the help of a circuit shown below:



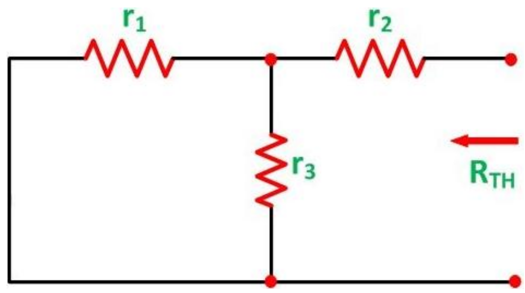
Let us consider a simple DC circuit as shown in the figure above, where we have to find the load current **I_L** by the Thevenin's theorem. In order to find the equivalent voltage source, **r_L** is removed from the circuit as shown in the figure below

and **V_{oc}** or **V_{TH}** is calculated.

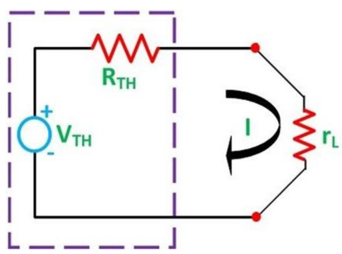


$$V_{OC} = I\, r_3 = \frac{V_S}{r_1 + r_3}\, r_3$$

So,Now, to find the internal resistance of the network (Thevenin's resistance or equivalent resistance) in series with the open-circuit voltage **V_{OC}**, also known as Thevenin's voltage **V_{TH}**, the voltage source is removed or we can say it is deactivated by a short circuit (as the source does not have any internal resistance) as shown in the figure below:



Equivalent Circuit of Thevenin's Theorem



So, As per Thevenin's Statement, the load current is determined by the circuit shown above and the equivalent Thevenin's circuit is obtained.

The load current **I_L** is given as: $I_L = \dfrac{V_{TH}}{R_{TH} + r_L}$

Where,

V_{TH} is the Thevenin's equivalent voltage. It is an open circuit voltage across the terminal AB known as **load terminal. R_{TH}** is the Thevenin's equivalent resistance, as seen from the load terminals where all the sources are replaced by their internal impedance. **r_L** is the **load resistance**

Steps for Solving Thevenin's Theorem

Step 1 – First of all remove the load resistance **r_L** of the given circuit.

Step 2 – Replace all the sources by their internal resistance.

Step 3 – If sources are ideal then short circuit the voltage source and open circuit the current source.

Step 4 – Now find the equivalent resistance at the load terminals, known as Thevenin's Resistance (R_{TH}).

Step 5 – Draw the Thevenin's equivalent circuit by connecting the load resistance and after that determine the desired response.

This theorem is possibly the most extensively used networks theorem. It is applicable where it is desired to determine the current through or voltage across any one element in a network. Thevenin's Theorem is an easy way to solve a complicated network.

Norton's Theorem

Norton's Theorem states that – A linear active network consisting of the independent or dependent voltage source and current sources and the various circuit elements can be substituted by an equivalent circuit consisting of a current source in parallel with a resistance. The current source being the short-circuited current across the load terminal and the resistance being the internal resistance of the source network.

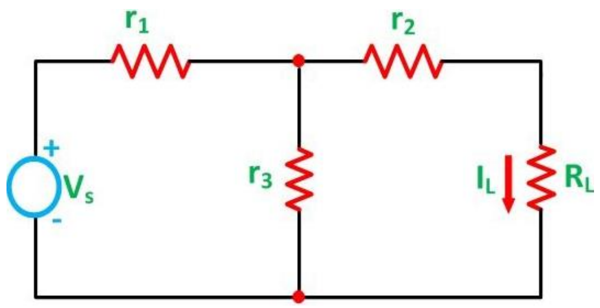
The Norton's theorems reduce the networks equivalent to the circuit having one current source, parallel resistance and load**. Norton's theorem** is the converse of Thevenin's Theorem. It consists of the equivalent current source instead of an equivalent voltage source as in Thevenin's theorem.

The determination of internal resistance of the source network is identical in both the theorems.

In the final stage that is in the equivalent circuit, the current is placed in parallel to the internal resistance in Norton's Theorem whereas in Thevenin's Theorem the equivalent voltage source is placed in series with the internal resistance.

Explanation of Norton's Theorem

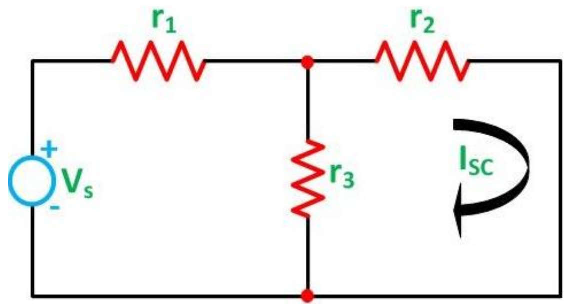
To understand Norton's Theorem in detail, let us consider a circuit diagram given below



In order to find the current through the load resistance I_L as shown in the circuit diagram above, the load resistance has to be short-circuited as shown in the diagram below:

Now, the value of current I flowing in the circuit is found out by the equation

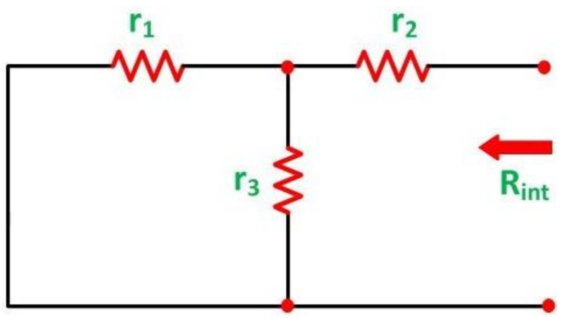
$$I = \frac{V_S}{r_1 + \left(\frac{r_2 r_3}{r_2 + r_3}\right)}$$



And the short-circuit current I_{SC} is given by the equation shown below:

$$I_{SC} = I \frac{r_3}{r_3 + r_2}$$

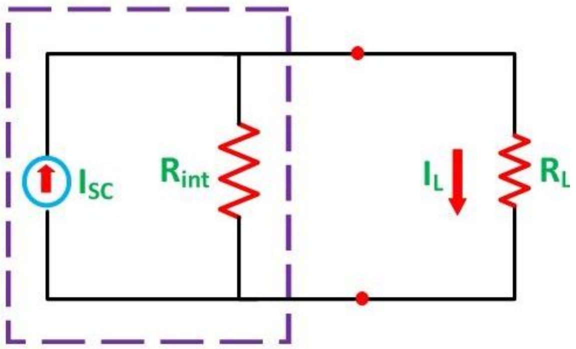
Now the short circuit is removed, and the independent source is deactivated as shown in the circuit diagram below and



circuit diagram on the left and the value of the internal resistance is calculated by:

So, $R_{int} = r_2 + \dfrac{r_1 r_3}{r_1 + r_3}$

As per Norton's Theorem, the equivalent source circuit would contain a current source in parallel to the internal resistance, the current source being the short-circuited current across the shorted terminals of the load resistor. The Norton's Equivalent circuit is represented as



Finally, the load current I_L calculated by the equation shown below

$$I_L = I_{SC} \frac{R_{int}}{R_{int} + R_L}$$

Where,

- I_L is the load current
- I_{sc} is the short circuit current
- R_{int} is the internal resistance of the circuit
- R_L is the load resistance of the circuit

Steps for Solving a Network Utilizing Norton's Theorem

Step 1 – Remove the load resistance of the circuit.

Step 2 – Find the internal resistance R_{int} of the source network by deactivating the constant sources.

Step 3 – Now short the load terminals and find the short circuit current I_{SC} flowing through the shorted load terminals using conventional network analysis methods.

Step 4 – Norton's equivalent circuit is drawn by keeping the internal resistance R_{int} in parallel with the short circuit current I_{SC}.

Step 5 – Reconnect the load resistance R_L of the circuit across the load terminals and find the current through it known as load current I_L.

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem states that – A resistive load, being connected to a DC network, receives maximum power when the load resistance is equal to the internal resistance known as (Thevenin's equivalent resistance) of the source network as seen from the load terminals. The Maximum Power Transfer theorem is used to find the load resistance for which there would be the maximum amount of power transfer from the source to the load.

Explanation of Maximum Power Transfer Theorem

A variable resistance R_L is connected to a DC source network as shown in the circuit diagram in figure A below and the figure B represents the Thevenin's voltage V_{TH} and Thevenin's resistance R_{TH} of the source network.

The aim of the Maximum Power Transfer theorem is to determine the value of load resistance R_L, such that it receives maximum power from the DC source.

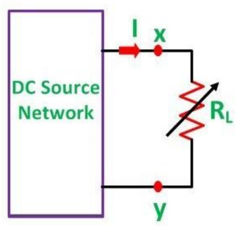


Figure A

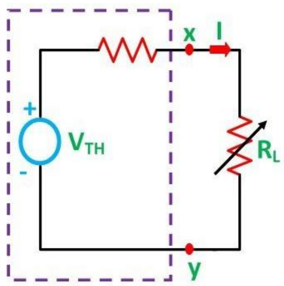


Figure B

Considering figure B the value of current will be calculated by the equation shown below

$$I = \frac{V_{TH}}{R_{TH} + R_L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

While the power delivered to the resistive load is given by the equation

$P_L = I^2 R_L$......................(2)

Putting the value of I from the equation (1) in the equation (2) we will get $P_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 \times R_L$

P_L can be maximized by varying R_L and hence, maximum power can be delivered when

$$\left(\frac{dP_L}{dR_L} \right) = 0$$

However,

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{TH} + R_L)^2]^2} \left[(R_{TH} + R_L)^2 \frac{d}{dR_L}(V_{TH}^2 R_L) - V_{TH}^2 R_L \frac{d}{dR_L}(R_{TH} + R_L)^2 \right]$$

$$\frac{dP_L}{dR_L} = \frac{1}{(R_{TH} + R_L)^4} [(R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \text{ x } 2(R_{TH} + R_L)]$$

$$\frac{dP_L}{dR_L} = \frac{V_{TH}^2 (R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} = \frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2}$$

But as we know, $\left(\frac{dP_L}{dR_L} \right) = 0$

Therefore,

$$\frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2} = 0$$

Which gives

$$(R_{TH} - R_L) = 0 \quad \text{or} \quad R_{TH} = R_L$$

Hence, it is proved that power transfer from a DC source network to a resistive network is maximum when the internal resistance of the DC source network is equal to the load resistance.

Again, with $R_{TH} = R_L$, the system is perfectly matched to the load and the source, thus, the power transfer becomes maximum, and this amount of power Pmax can be obtained by the equation shown below:

$$P_{\max} = \frac{V_{TH}^2 R_{TH}}{(R_{TH} + R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}} \text{..........................(3)}$$

Equation (3) gives the power which is consumed by the load. The power transfer by the source will also be the same as the power consumed by the load, i.e. equation (3), as the load power and the source power being the same.

Thus, the total power supplied is given by the equation

$$P = 2\frac{V_{TH}^2}{4R_{TH}} = \frac{V_{TH}^2}{2R_{TH}}$$

During Maximum power Transfer the efficiency becomes: $\eta = \frac{P_{\max}}{P} x100 = 50\%$

The concept of Maximum Power Transfer theorem is that by making the source resistance equal to the load resistance, which has wide application in communication circuits where the magnitude of power transfer is sufficiently small. To achieve maximum power transfer, the source and the load resistance are matched and with this, efficiency becomes 50% with the flow of maximum power from the source to the load.

In the Electrical Power Transmission system, the load resistance being sufficiently greater than the source resistance, it is difficult to achieve the condition of maximum power transfer.

In power system emphasis is given to keep the voltage drops and the line losses to a minimum value and hence the operation of the power system, operating with bulk power transmission capability, becomes uneconomical if it is operating with only **50%** efficiency just for achieving maximum power transfer. Hence, in the electrical power transmission system, the criterion of maximum power transfer is very rarely used.

Steps for Solving Network Using Maximum Power Transfer Theorem

Following steps are used to solve the problem by Maximum Power Transfer theorem

Step 1 – Remove the load resistance of the circuit.

Step 2 – Find the Thevenin's resistance (R_{TH}) of the source network looking through the open-circuited load terminals.

Step 3 – As per the maximum power transfer theorem, this R_{TH} is the load resistance of the network, i.e., $R_L = R_{TH}$ that allows maximum power transfer.

Step 4 – Maximum Power Transfer is calculated by the equation shown below

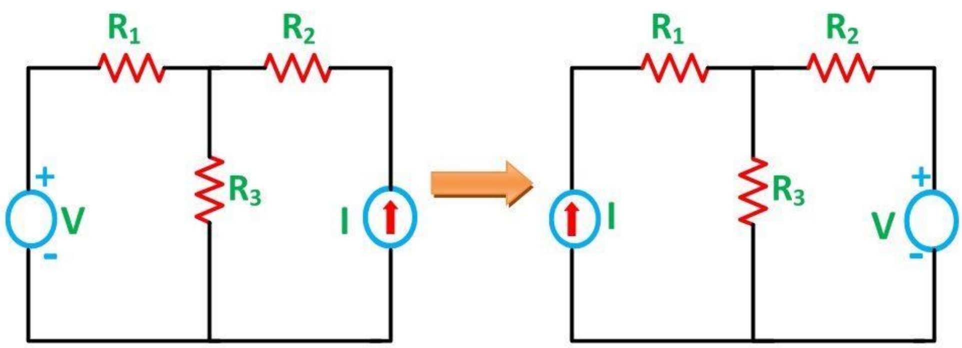
$$P_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

This is all about Maximum Power Transfer Theorem.

Reciprocity Theorem

Reciprocity Theorem states that – In any branch of a network or circuit, the current due to a single source of voltage (V) in the network is equal to the current through that branch in which the source was originally placed when the source is again put in the branch in which the current was originally obtained. This theorem is used in the bilateral linear network which consists of bilateral components.

The Reciprocity Theorem is explained with the help of the circuit diagram shown below



The various resistances R_1, R_2, R_3 is connected in the circuit diagram above with a voltage source (V) and a current source (I). It is clear from the figure above that the voltage source and current sources are interchanged for solving the network with the help of Reciprocity Theorem.

The limitation of this theorem is that it is applicable only to single-source networks and not in the multi-source network. The network where reciprocity theorem is applied should be linear and consist of resistors, inductors, capacitors and coupled circuits. The circuit should not have any time-varying elements.

Steps for Solving a Network Utilizing Reciprocity Theorem

Step 1 – Firstly, select the branches between which reciprocity has to be established.

Step 2 – The current in the branch is obtained using any conventional network analysis method.

Step 3 – The voltage source is interchanged between the branch which is selected.

Step 4 – The current in the branch where the voltage source was existing earlier is calculated.

Step 5 – Now, it is seen that the current obtained in the previous connection, i.e., in step 2 and the current which is calculated when the source is interchanged, i.e., in step 4 are identical to each other.