MODULE II

KINEMATICS OF FLOW

- Fluid mechanics is that branch of science which deals with the behavior of the fluids (Liquid or gases) at rest as well as in motion.
- The branch of science deals with statics, kinematics and dynamics aspects of fluids.
- Statics : Study of fluid at rest.
- Kinematics : Study of fluid in motion, where pressure force are not considered.
- Dynamics : Study of fluid in motion, if the pressure force are also considered.

TYPES OF FLUID FLOW

- The fluid flow is classified as

1) Steady flow and Unsteady flows
2) Uniform flow and Non uniform flows
3) Laminar and Turbulent flows
4) Compressible and Incompressible flows
5) Rotational and Irrotational flows
6) One, two, three dimensional flows

1) STEADY AND UNSTEADY FLOWS

- Steady flow is defined as that type of fluid flow in which the fluid characteristics like velocity, pressure, density etc at a point do not change with time.
- Unsteady flow is defined as that type of fluid flow in which the fluid characteristics like velocity, pressure, density etc at a point changes with time.

2) UNIFORM AND NON UNIFORM FLOWS

- Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space *(i.e.,* length of direction of the flow).
- Non Uniform flow is defined as that type of flow in which the velocity at any given time changes with respect to space.

3) LAMINAR AND TURBULENT FLOWS

- <u>LAMINAR FLOW:</u>
- Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel.
- Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.
- <u>TURBULENT FLOW:</u>
- Turbulent flow is that type of flow in which the fluid particles move in a zigzag way. Due to the movement of fluid particles in a zigzag way, the eddies formation takes place which are responsible for high energy loss.

$\frac{VD}{r}$

Reynolds Number:

- The type of flow is determined by a non dimensional number called Reynolds number.

$$R_e = VD/r$$

D = Diameter of the pipe

V = Mean velocity of flow in pipe

r = Kinematic viscosity of fluid

If the Reynolds number is less than 2000, the flow is called laminar. If the Reynolds number is more than 4000, it is called turbulent flow. If the Reynolds number lies between 2000 and 4000, the flow may be laminar or turbulent in the case of a pipe flow.

4) COMPRESSIBLE AND INCOMPRESSIBLE FLOW

- <u>COMPRESSIBLE FLOW:</u>

  Compressible flow is defined as that type of flow in which the density of the fluid changes from point to point or in other words the density (p) is not constant for the fluid.

- <u>INCOMPRESSIBLE FLOW:</u>

  Incompressible flow is defined as that type of flow in which the density of the fluid is constant for the fluid flow.

5) ROTATIONAL AND IRROTATIONAL FLOW

- Rotational flow is defined as that type of flow in which fluid particles while flowing along stream-lines, also rotate about their own axis.
- And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then that type of flow is called irrotational flow.

6) ONE,TWO AND THREE DIMENSIONAL FLOW

- One dimensional flow is that type of flow in which the flow parameters such as velocity is a function of time and one space coordinate only.
- Two dimensional flow is that type of flow in which the flow parameters such as velocity is a function of time and two rect angular space coordinate x and y.
- Three dimensional flow is that type of flow in which the flow parameters such as velocity is a function of time and three mutually perpendicular direction.

RATE OF FLOW OR DISCHARGE (Q)

- It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.
- For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.
- For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.
- $Q = A \times V$

  where,

  A= Cross sectional area of pipe

V= Average velocity of fluid across the section

CONTINUITY EQUATION

- The equation based on the principle of conservation of mass is called continuity equation.
- Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

and V_2, ρ_2, A_2 are corresponding values at section, 2-2.

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 $= \rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad \text{...(5.2)}$$

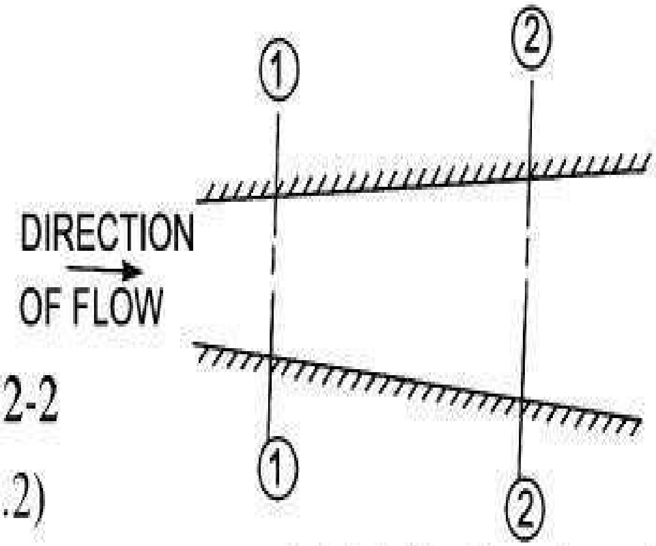


Fig. 5.1 *Fluid flowing through a pipe.*

Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation.** If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2 \qquad \text{...(5.3)}$$

Problem 5.1 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$V_1 = 5$ m/s.

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

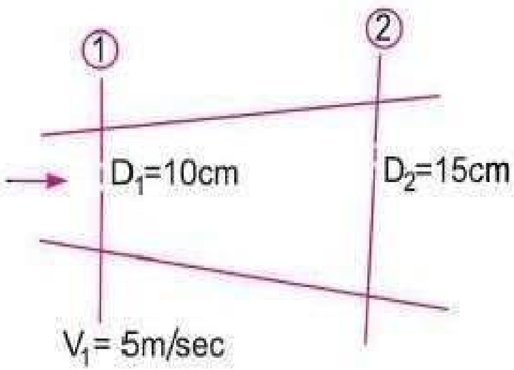


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or $Q = A_1 \times V_1$

$= 0.007854 \times 5 =$ **0.03927 m³/s. Ans.**

Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore$ $$V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22 \text{ m/s.}}$$ **Ans.**

Problem 5.2 *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Solution. Given :

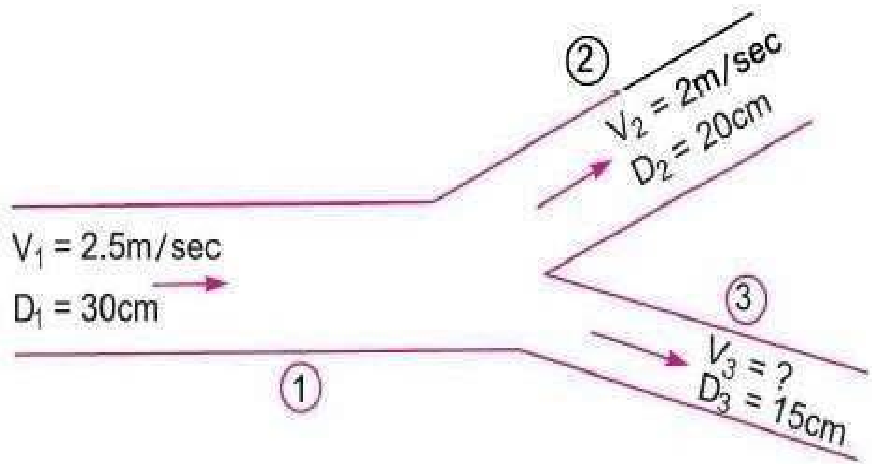


Fig. 5.3

Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3 \qquad \text{...(1)}$$

(*i*) **The discharge $\mathbf{Q_1}$** in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767 \text{ m}^3/\text{s. Ans.}}$$

(*ii*) **Value of $\mathbf{V_3}$**

$$Q_2 = A_2V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3/\text{s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$\therefore$ $$Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3/\text{s}$$

But $$Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$$

$\therefore$ $$V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44 \text{ m/s. Ans.}}$$

DYNAMICS OF FLUID FLOW

- In the previous chapter, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration of the forces causing the flow
- This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow.
- The dynamic behaviour of the fluid flow is analysed by the Newton′s second law of motion, which relates the acceleration with the forces.

EQUATIONS OF MOTION

- In the fluid flow the following forces are present

1) Force due to gravity
2) The pressure force
3) Force due to viscosity
4) Force due to turbulence
5) Force due to compressibility

The net force,

$$F_X = (F_g)_X + (F_P)_X + (F_V)_X + (F_t)_x + (F_c)_x$$

The above equation is known as Reynolds equation of motion.

$$F_X = (F_g)_X + (F_P)_X$$

The above equation is Euler's equation of motion.

EULERS EQUATION OF MOTION

$$\frac{dp}{\rho} + g\,dz + v\,dv = 0$$

BERNOULLIS EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

ASSUMPTIONS OF BERNOULLIS EQUATION

- The following are the assumptions made in the derivation of Bernoulli's equation.

1) The fluid is ideal
2) The flow is steady
3) The flow is incompressible
4) The flow is irrotational

Problem 6.1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head = pressure head + kinetic head + datum head

Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{ \rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

HOME WORK

- 1) Problem 6.2(Dr. R K Bansal, Page: 261)
- 2) Problem 6.4v(Dr. R K Bansal, Page: 263)

BERNOULLIS EQUATION FOR REAL FLUID

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \quad \ldots(6.5)$$

where h_L is loss of energy between points 1 and 2.

HOME WORK

- 1) Problem 6.7 (Dr. R K Bansal, Page: 266)

PRACTICAL APPLICATIONS OF BERNOULLIS EQUATION

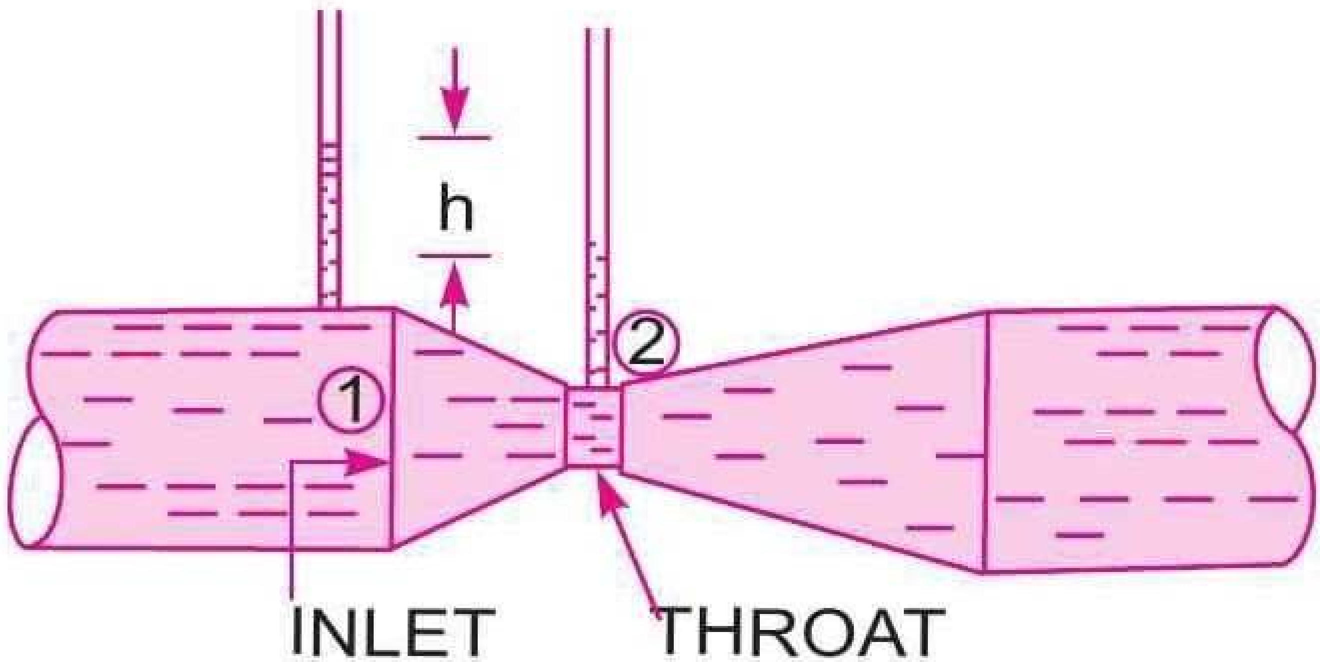
Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

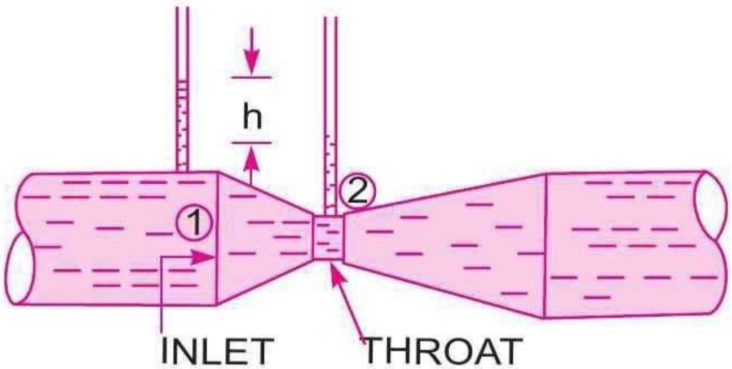
1).VENTURIMETER

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.



h
1
2
INLET
THROAT



$$\therefore \qquad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

VALUE OF "h" GIVEN BY DIFFERENTIAL U TUBE MANOMETER

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid

S_o = Sp. gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Then

$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad ...(6.9)$$

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where S_l = Sp. gr. of lighter liquid in U-tube

S_o = Sp. gr. of fluid flowing through pipe

x = Difference of the lighter liquid columns in U-tube.

Problem 6.10 *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take* $C_d = 0.98$.

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s.}} \text{ Ans.}$$

Problem 6.11 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} d_1^{\,2} = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s.}}$$ **Ans.**

HOME WORK

- 1) Problem 6.12 (Dr. R K Bansal, Page: 271)
- 2) Problem 6.13 (Dr. R K Bansal, Page: 272)

FLOW THROUGH PIPES

In chapters 9 and 10, laminar flow and turbulent flow have been discussed. We have seen that when the Reynolds number is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynolds number is more than 4000, the flow is known as turbulent flow. In this chapter, the turbulent flow of fluids through pipes running full will be considered. If the pipes are partially full as in the case of sewer lines, the pressure inside the pipe is same and equal to atmospheric pressure. Then the flow of fluid in the pipe is not under pressure. This case will be taken in the chapter of flow of water through open channels. Here we will consider flow of fluids through pipes under pressure only.

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

   This is due to friction and it is calculated by the following formulae :
   - (*a*) Darcy-Weisbach Formula
   - (*b*) Chezy's Formula

2. Minor Energy Losses

   This is due to
   - (*a*) Sudden expansion of pipe
   - (*b*) Sudden contraction of pipe
   - (*c*) Bend in pipe
   - (*d*) Pipe fittings etc.
   - (*e*) An obstruction in pipe.

DARCY WEISBACH FORMULA

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g} \qquad \text{...(11.1)}$$

where h_f = loss of head due to friction

CHEZYS FORMULA

$$V = C\sqrt{mi} \quad \text{...(11.4)}$$

Equation (11.4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

PROBLEMS

Problem 11.5 *Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 'f' = 0.009 in the formula* $h_f = \dfrac{4 . f . L . V^2}{d \times 2g}$.

Solution. Given :

Dia. of pipe, $d = 200 \text{ mm} = 0.20 \text{ m}$

Length of pipe, $L = 500 \text{ m}$

Difference of pressure head, $h_f = 4 \text{ m of water}$

$f = .009$

Using equation (11.1), we have $h_f = \dfrac{4 \times f \times L \times V^2}{d \times 2g}$

or $$4.0 = \frac{4 \times .009 \times 500 \times V^2}{0.2 \times 2 \times 9.81} \text{ or } V^2 = \frac{4.0 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

$\therefore$ $$V = \sqrt{0.872} = 0.9338 \simeq 0.934 \text{ m/s}$$

$\therefore$ Discharge, $$Q = \text{velocity} \times \text{area}$$

$$= 0.934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} (0.2)^2$$

$$= 0.0293 \text{ m}^3/\text{s} = \mathbf{29.3\ litres/s.\ Ans.}$$

HOME WORK

- 1) Problem 11.1(Dr. R K Bansal, Page: 467)
- 2) Problem 11.2(Dr. R K Bansal, Page: 468)

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

11.4.1 Loss of Head Due to Sudden Enlargement. Consider a liquid flowing through a pipe which has sudden enlargement as shown in Fig. 11.1. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

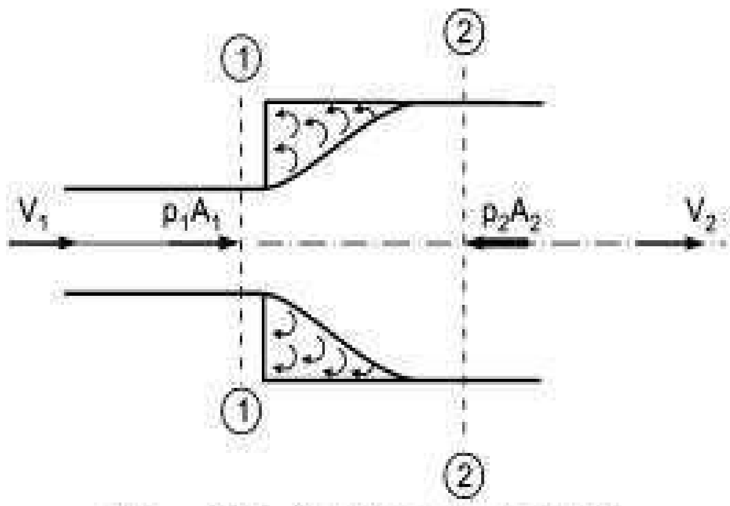


Fig. 11.1 *Sudden enlargement.*

$$h_e = \frac{(V_1 - V_2)^2}{2g}.$$

Problem 11.8 *Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.*

Solution. Given :

Dia. of smaller pipe, $D_1 = 200$ mm $= 0.20$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400$ mm $= 0.4$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250$ litres/s $= 0.25 \text{ m}^3/\text{s}$

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96$ m/s

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99$ m/s

Loss of head due to enlargement is given by equation (11.5) as

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \textbf{1.816 m of water. Ans.}$$