FUNDAMENTALS OF FLUID MECHANICS

CO1

Introduction To Fluid Mechanics

Fluid mechanics is the branch of physics and engineering that deals with the behaviour of fluids—liquids and gases—when they are in motion or at rest. It is a fundamental field of study that plays a crucial role in understanding and solving real-world problems in various industries, including aerospace, civil engineering, chemical engineering, environmental science, and more.

Appreciate the properties of fluids.

Density

Density, often referred to as mass density, is a fundamental physical property that measures how much mass is contained in a given volume of a substance. It is typically denoted by the symbol "ρ" (rho) and is expressed in units such as kilograms per cubic meter **(kg/m³)** in the International System of Units (SI).

Mathematically, density is defined as:

Density (ρ)=Mass (m)/Volume (V)

(ρ)= (m)/ (V)

Where:

- ρ is the density.
- m is the mass of the substance.
- V is the volume occupied by the substance.

SPECIFIC WEIGHT,

Specific weight, also known as weight density, is a physical property that measures the weight of a substance per unit volume. It quantifies the gravitational force acting on a specific volume of a material and is often used in fluid mechanics and engineering. Specific weight is denoted by the symbol "γ" (gamma) and is typically expressed in units such as newtons per cubic meter **(N/m³)** in the International System of Units (SI).

Mathematically, specific weight is defined as:

Specific Weight (γ)= Weight (W)/ Volume (V)

=W/V

Where:

- γ is the specific weight.
- W is the weight of the substance (force due to gravity acting on the mass).
- V is the volume occupied by the substance.

In SI units, weight is typically measured in newtons (N), and volume is measured in cubic meters (m^3). Therefore, the unit of specific weight, **N/m^3,** reflects the force per unit volume.

Specific Gravity: Specific gravity, often denoted as "SG," is a dimensionless ratio that compares the density of a substance to the density of a reference substance, typically water. It is a measure of how much denser or lighter a substance is compared to water at a specified temperature and pressure. The formula for specific gravity is:

Specific Gravity (SG)=Density of SubstanceDensity of WaterSpecific Gravity (SG)=Density of WaterDensity of Substance

Since it is a ratio, specific gravity has no units.

Specific Volume: Specific volume, denoted as "v," is the reciprocal of density. It is the volume occupied by a unit mass of a substance. Specific volume is typically expressed in units like cubic meters per kilogram **(m^3/kg)** in the International System of Units (SI).

V=volume /mass

Dynamic Viscosity: Dynamic viscosity, often denoted as "μ" (mu), is a measure of a fluid's resistance to flow when subjected to an applied force or stress. It is also known as absolute viscosity and is expressed in units like pascal-seconds (Pa·s) in the SI system. Common units in non-SI systems include poise (P) and centipoise (cP).

Units: In SI units, dynamic viscosity is measured in **pascal-seconds (Pa·s),** which is equivalent to newton-seconds per square meter ($N \cdot s/m^2$). In simpler terms, it represents the force (in newtons) required to move one square meter of a fluid at a speed of one meter per second.

Kinematic Viscosity: Kinematic viscosity, denoted as **"ν" (nu),** is the ratio of dynamic viscosity to density. It represents the relative ease with which a fluid flows under the influence of gravity. Kinematic viscosity is expressed in units like **square meters per second (m^2/s)** in SI.

Kinematic Viscosity (ν)=Dynamic Viscosity (μ)/Density

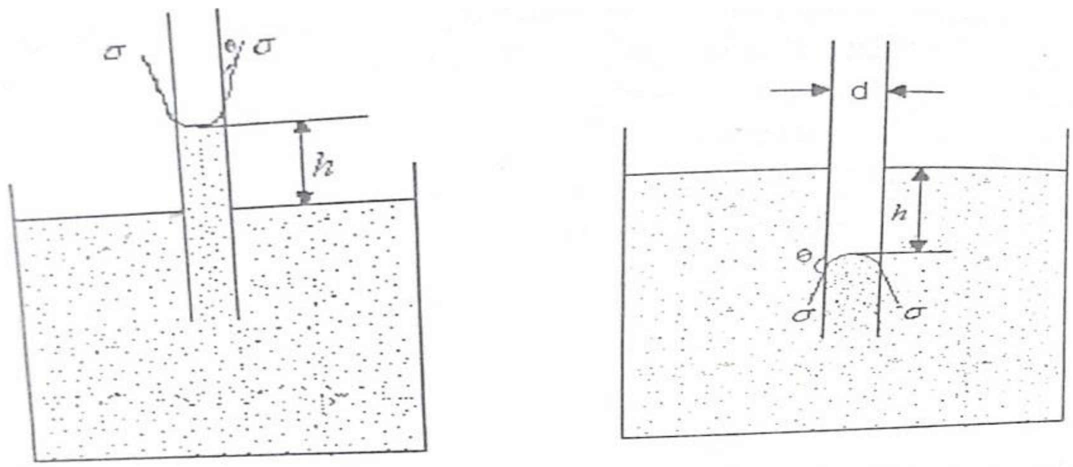
Surface Tension: Surface tension is a property of liquids that causes the surface of a liquid to behave like a stretched elastic membrane. It is measured in units of force per unit length (e.g., **newtons per meter, N/m).** Surface tension is responsible for phenomena like capillary action and the formation of droplets.

Capillarity: Capillarity is the ability of a liquid to flow in narrow spaces or tubes against the force of gravity. It occurs due to the combination of surface tension and the adhesive and cohesive properties of liquids. Capillary action is responsible for the rise or fall of liquids in small-diameter tubes.

Adhesion: Adhesion is the attraction of molecules at the liquid-solid interface. In capillarity, the liquid wets the solid surface of the capillary tube, which means the liquid molecules are attracted to and form bonds with the solid surface. Adhesive forces tend to pull the liquid upward in the tube.

Cohesion: Cohesion is the attraction between molecules of the same substance. In a liquid, cohesive forces cause the liquid molecules to stick together. These forces tend to pull the liquid

Capillary Rise: When the adhesive forces between the liquid and the solid are stronger than the cohesive forces within the liquid, the liquid rises in the capillary tube. The height to which the liquid rises is determined by the competition between these forces and can be described using the Jurin's Law equation:



a)Glass tube is dipped in water

(b)Glass tube is dipped in mercury

$$h = \frac{4\sigma\cos\theta}{wd} = \frac{4\sigma\cos\theta}{\rho g d}$$

Capillary Fall: In some cases, the cohesive forces within the liquid are stronger than the adhesive forces, causing the liquid to fall in the capillary tube. This typically happens when the liquid does not wet the solid surface. Upward as a cohesive column.

Vapor Pressure: Vapor pressure is the pressure exerted by the vapor (gas phase) of a substance in equilibrium with its liquid or solid phase at a given temperature. It is a measure of a substance's tendency to evaporate. Vapor pressure is typically expressed in units like **pascals (Pa) or millimeters of mercury (mmHg).**

Compressibility: Compressibility is a measure of how much a substance's volume changes in response to changes in pressure. It quantifies the degree to which a substance can be compressed or made denser under pressure. It is often expressed as the coefficient of volume expansion.

Compressibility is typically expressed as the coefficient of compressibility, often denoted as "β" (beta) or "κ" (kappa), and it is defined as the fractional change in volume (ΔV/V) per unit change in pressure (ΔP):

Mathematically,

$$\text{Bulk modulus of elasticity}, K = \frac{1}{\text{Compressibility}}$$

Or

$$K = \frac{\text{Compressive stress}}{\text{Change in volume / unit volume}}$$

Fluid Pressure& Pressure Measurement:

Fluid pressure is a fundamental concept in fluid mechanics and describes the force exerted by a fluid on its surroundings, typically a surface. It is essential in understanding the behaviour of fluids in various contexts, including engineering, physics, and environmental science. Here are definitions for fluid pressure, pressure head, and pressure intensity:

Fluid Pressure:

- Fluid pressure, often denoted as "P," is the force per unit area exerted by a fluid at a given point within the fluid or on a solid surface immersed in the fluid.
- Mathematically, fluid pressure is defined as:
- **Fluid Pressure (P)=Force (F)/Area (A)**
- The SI unit of pressure is the **pascal (Pa),** which is equal to **one newton per square meter (N/m^2).**
- Fluid pressure depends on the depth within the fluid (hydrostatic pressure), fluid density, and gravitational acceleration. It is often calculated using the hydrostatic pressure equation:
    - $P=\rho \cdot g \cdot h$

    Where:

        - P is the pressure.
        - ρ is the fluid density.
        - g is the acceleration due to gravity.
        - h is the height or depth of the fluid column.

Pressure Head:

- Pressure head, also known as hydraulic head or piezometric head, is a concept used in fluid mechanics and hydrogeology to describe the potential energy associated with fluid pressure at a specific point.
- It is defined as the height to which a fluid could be raised by its own pressure in a column, assuming no energy losses. Pressure head is often represented by the symbol "H."

Mathematically, pressure head is related to fluid pressure by the equation:

$$H=\rho/\dot{\rho}g$$

Pressure head is a useful concept in groundwater flow and hydraulic engineering, where it helps describe the energy status of water in wells, aquifers, and pipes.

Pressure Intensity:

- Pressure intensity refers to the magnitude of fluid pressure at a specific point in a fluid or on a solid surface.
- It is a scalar quantity that represents the strength of the pressure at that location.
- Pressure intensity is often used to compare pressure values at different points within a fluid or in various parts of a fluid system.
- It is measured in units of pressure, such as pascals (Pa) or any other appropriate unit depending on the context.

1. **Atmospheric Pressure:**
   - Atmospheric pressure is the pressure exerted by the Earth's atmosphere on objects at or near the Earth's surface.
   - It is primarily caused by the weight of the air above a given point in the atmosphere.
   - At sea level, standard atmospheric pressure is approximately 101.3 kilopascals (kPa), 1 atmosphere (atm), or 760 millimeters of mercury (mmHg).
   - The actual atmospheric pressure can vary with altitude, weather conditions, and location but is typically close to these standard values at sea level.
2. **Absolute Pressure:**
   - Absolute pressure is the total pressure at a given point in a fluid and includes both atmospheric pressure and the pressure exerted by the fluid itself.
   - It is measured relative to a perfect vacuum, meaning that absolute pressure is always positive or zero.
   - Mathematically, absolute pressure (P_abs) is defined as: abs=atm+gauge$P_{abs}=P_{atm}+P_{gauge}$ Where:
     - absP_{abs} is the absolute pressure.
     - atmP_{atm} is the atmospheric pressure.
     - gaugeP_{gauge} is the gauge pressure (discussed below).
3. **Gauge Pressure:**
   - Gauge pressure is the pressure above atmospheric pressure (or above ambient pressure) at a given point in a fluid. It is the pressure relative to atmospheric pressure.
   - Gauge pressure can be positive or negative, depending on whether it is above or below atmospheric pressure.

- Most pressure gauges used in everyday applications, such as tire pressure gauges, measure gauge pressure.
- Mathematically, gauge pressure (P_gauge) is defined as: gauge=abs−atm $P_{gauge} = P_{abs} - P_{atm}$

4. **Vacuum Pressure:**

- Vacuum pressure is a type of gauge pressure that occurs when the pressure in a closed system is lower than atmospheric pressure.
- Vacuum is often measured in units like torr or pascals (Pa), where a perfect vacuum (complete absence of gas molecules) is defined as zero pressure.
- Negative gauge pressures are used to represent vacuum pressures. For example, if the gauge pressure is -50 kPa, it means the pressure is 50 kPa below atmospheric pressure, which indicates a partial vacuum.

A piezometer

A piezometer is a simple device used to measure the static pressure (hydrostatic pressure) of a fluid at a specific point in a fluid column, typically a liquid like water. It's a type of pressure measurement tool commonly employed in fields such as hydrogeology, civil engineering, and fluid mechanics to determine the pressure head or hydraulic head of a fluid in a particular location.

Here's how a piezometer works and some key points about its usage:

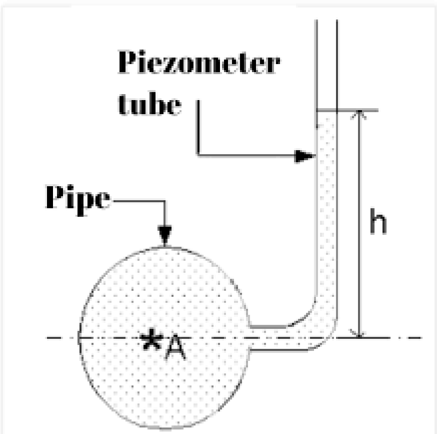
1. **Design and Components:**

- A typical piezometer consists of a narrow tube or pipe that is inserted vertically into the fluid (e.g., groundwater or a liquid in a tank) at the point where pressure is to be measured.
- The open end of the tube is submerged in the fluid, allowing the fluid to rise inside the tube to a certain height due to hydrostatic pressure.
- There may be a filter or screen at the bottom of the tube to prevent the entry of solid particles while allowing fluid to flow freely.

2. **Measurement Principle:**

- The height to which the fluid rises inside the piezometer tube is directly proportional to the hydrostatic pressure at that point in the fluid column.
- This rise in fluid level is caused by the balance between the fluid's weight and the hydrostatic pressure it experiences.
- The fluid level within the piezometer stabilizes at a certain height, representing the pressure head or hydraulic head.

3. **Pressure Head Measurement:**
   - The pressure head (H) can be calculated using the equation: $\boldsymbol{H=\rho/\dot{\rho}g}$
   - Where:
     - H is the pressure head (in meters).
     - P is the pressure (in pascals or newtons per square meter).
     - ρ is the fluid density (in kilograms per cubic meter).
     - g is the acceleration due to gravity (approximately 9.81 m/s^2).
4. **Applications:**
   - Piezometers are commonly used to monitor groundwater levels in wells and boreholes, helping assess water table fluctuations and groundwater flow patterns.
   - They are also used in geotechnical engineering to measure pore water pressures in soil and rock formations.
   - In civil engineering, piezometers are utilized in the design and monitoring of dams, embankments, and retaining walls.
   - In fluid mechanics research, they aid in studying pressure distributions in closed conduits and pipelines.



SIMPLE MANOMETER

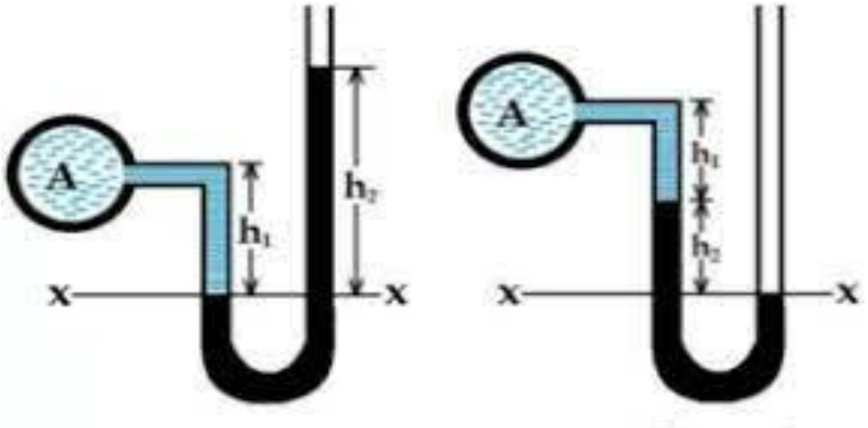


Figure 1 Figure 2

A simple manometer, also known as a single-tube manometer, is a basic device used to measure the pressure of a single fluid (usually a gas or liquid) at a specific point in a system. It operates on the principle of balancing the pressure of the fluid with the weight of a column of liquid (often mercury or water) to determine the pressure.

Pressure Calculation: To calculate the pressure of the fluid in the system, you can use the hydrostatic pressure formula: $P=\rho \cdot g \cdot h$

- P is the pressure of the fluid in the system.
- ρ is the density of the liquid in the U-tube.
- g is the acceleration due to gravity (approximately 9.81 m/s²).
- h is the difference in height between the two liquid columns in the U-tube.

Differential manometer

(a)Two pipes at different levels (b) A and B are at the same level

1. **Pressure Calculation:** To calculate the pressure difference between the two points in the system, you can use the hydrostatic pressure formula for each column of liquid:

$$P1=\rho 1\cdot g\cdot h1$$

$$P2=\rho 2\cdot g\cdot h2$$

Where:

- $P1$ and $P2$ are the pressures at the two points in the system.
- ρ is the density of the manometric liquid.
- g is the acceleration due to gravity.
- $h1$ and $h2$ are the heights of the liquid columns on the respective sides of the U-tube.

2. Pressure Difference: The pressure difference (ΔP) is calculated as:

$$\Delta P=P1-P2=(\rho 1\cdot g\cdot h1)-(\rho 2\cdot g\cdot h2)$$

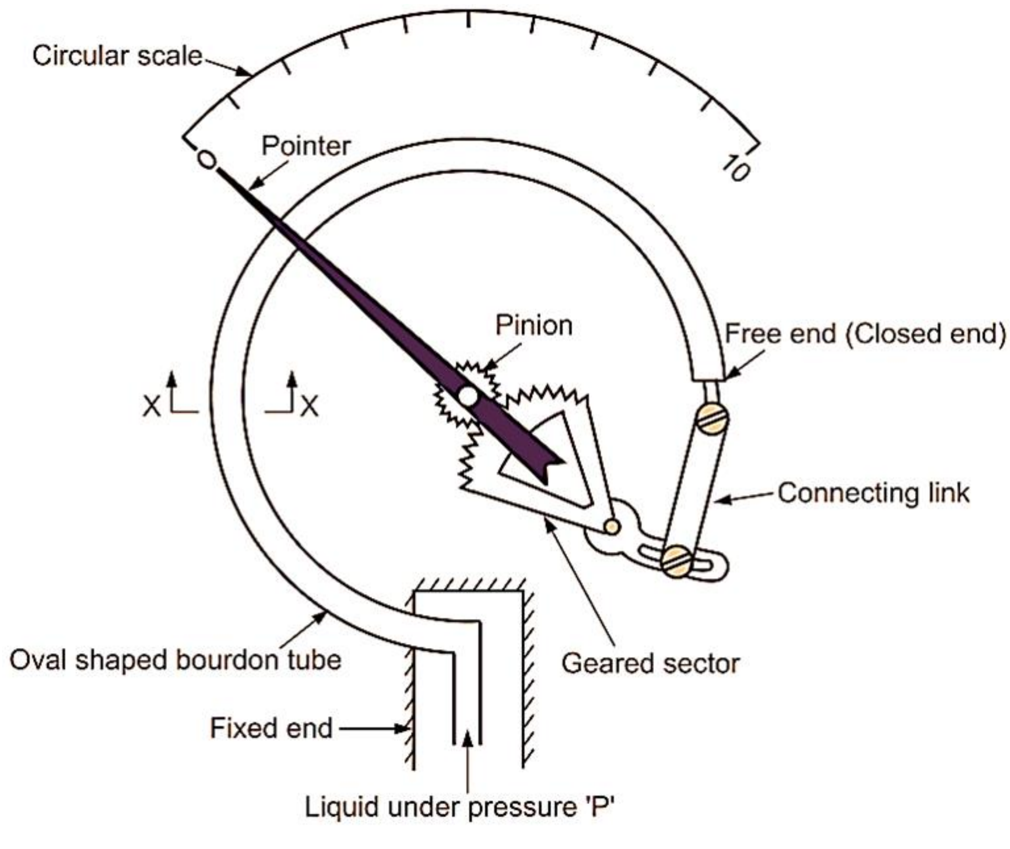
Bourdon tube pressure gauge

A Bourdon pressure gauge is a mechanical device used to measure the pressure of a fluid (liquid or gas) within a closed system. It is one of the most common types of pressure gauges and is named after its inventor, Eugène Bourdon. Bourdon pressure gauges are widely used in various industries, including manufacturing, petrochemical, HVAC (heating, ventilation, and air conditioning), and more.

Here's how a Bourdon pressure gauge typically works:

1. Bourdon Tube: The heart of the Bourdon pressure gauge is the Bourdon tube, which is a curved, hollow tube usually made of metal (commonly phosphor bronze, stainless steel, or brass). This tube is oval or C-shaped in cross-section.
2. Attachment: One end of the Bourdon tube is sealed and attached to the pressure source through a connector. The other end is free to move.
3. Pressure Application: When the pressure inside the system being measured increases, it applies force to the inside of the Bourdon tube. This causes the tube to straighten or expand slightly, changing its shape.

4. Pointer and Dial: The free end of the Bourdon tube is connected to a linkage mechanism, which is connected to a pointer. As the Bourdon tube changes shape due to the pressure, the pointer moves along a calibrated dial.
5. Measurement Display: The calibrated dial is marked with pressure readings in various units such as PSI (pounds per square inch), kPa (kilopascals), bar, etc. The position of the pointer on the dial indicates the pressure within the system.



Problem 1.1.1. Calculate the specific weight, specific mass, specific volume, and specific gravity of one litre of a liquid which weighs 7 N.

(Diploma Examination Question, April-2005)

Solution: Given

Volume of given liquid, $V = 1$ litre $= 1 \times 10^{-3}$ m^3

($\because$ 1 litre = 1/ 1000 m^3)

Weight of liquid, $W = 7$ N.

(i) Specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V} = \dfrac{7}{1\times10^{-3}}$

$= \mathbf{7000\ N / m^3}$ *(Ans)*

(ii) Specific mass or density, $\rho = \dfrac{\text{Specific weight}}{g}$

$= \dfrac{w}{g} = \dfrac{7000}{9.81}$

$= \mathbf{713.56\ kg / m^3}$ *(Ans)*

(iii) Specific volume, $v = \dfrac{1}{\rho} = \dfrac{1}{713.56}$

$= \mathbf{1.40^{-}\ 10^{-3}\ m^3 / kg}$ *(Ans)*

(iv) Specific gravity, $S = \dfrac{\text{Density of liquid}}{\text{Density of water}}$

$= \dfrac{\rho_{liquid}}{\rho_{water}} = \dfrac{713.56}{1000}$

$= \mathbf{0.7136}$ *(Ans)*

Problem 1.1.2. Two litre of petrol weighs 14N, calculate the specific weight, mass density, and specific gravity of petrol.

(Diploma Examination Question, November-2003)

Solution: Given

Volume of petrol, $V = 2$ litre $= 2 \times 10^{-3}$ m^3

Weight of petrol, $W = 14$ N.

(i) Specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V} = \dfrac{14}{2\times10^{-3}}$

$= \mathbf{7000\ N / m^3}$ *(Ans)*

(ii) Mass density, $\rho = \dfrac{\text{Specific weight}}{g}$

$= \dfrac{w}{g} = \dfrac{7000}{9.81}$

$= \mathbf{713.56\ kg/m^3}$ (Ans

(iv) Specific gravity, $S = \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{\rho_{liquid}}{\rho_{water}}$

$= \dfrac{713.56}{1000} = \mathbf{0.7136}$ (Ans

Problem 1.1.3. Calculate the specific weight, specific mas and specific gravity of a liquid having a volume of 4 cubic meter an weighting 30 kN.

(Diploma Examination Question, April-2002

Solution: Given

Volume of given liquid, $V = 4\ m^3$

Weight of liquid, $W = 30\ kN = 30 \times 10^3\ N$

(i) Specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V} = \dfrac{30 \times 10^3}{4}$

$= \mathbf{7500\ N/m^3}$ (Ans

(ii) Specific mass or density, $\rho = \dfrac{\text{Specific weight}}{g}$

$= \dfrac{w}{g} = \dfrac{7500}{9.81}$

$= \mathbf{764.53\ kg/m^3}$ (Ans

(iii) Specific volume, $v = \dfrac{1}{\rho} = \dfrac{1}{764.53}$

$= 1.308 \times 10^{-3}\ m^3/kg$ (Ans

(iv) Relative density or specific gravity $S = \dfrac{\text{Density of liquid}}{\text{Density of water}}$

$$= \frac{\rho_{liquid}}{\rho_{water}} = \frac{764.53}{1000}$$

$$= \mathbf{0.7645} \qquad \textbf{(Ans)}$$

Problem 1.1.4. If the specific gravity of one litre of a liquid is 0.8 calculate its mass density, specific weight and weight .

Solution: Given

Specific gravity, $S = 0.8$

Volume of liquid , $V = 1$ litre $= 1 \times 10^{-3}$ m^3

(i) We know that, specific gravity , $S = \dfrac{\text{Density of liquid}}{\text{Density of water}}$

$$= \frac{\rho_{liquid}}{\rho_{water}}$$

$\therefore$ Density of liquid, ρ_{liquid} = Specific gravity $\times$ Density of water

$$= S \times \rho_{water}$$

$$= 0.8 \times 1000 \text{ kg/m}^3$$

$$= \mathbf{800\ kg/m^3}. \qquad \textbf{(Ans)}$$

(ii) Specific weight, w $= \rho g = 800 \times 9.81$

$$= \mathbf{7848\ N/m^3}. \qquad \textbf{(Ans)}$$

We know that , specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V}$

$\therefore$ Weight of liquid , W = Specific weight $\times$ Volume

$$= 7848 \times 1 \times 10^{-3}$$

$$= \mathbf{7.848\ N} \qquad \textbf{(Ans)}$$

Problem 1.1.5. Determine the mass density of an oil if 3 tonnes of the oil occupies a volume of 4 m³, and also find the specific weight of the oil.

(Diploma Examination Question ,April- 2005)

Solution: Given

Weight of oil, $W = 3 \text{ tonnes} = 3 \times 10^{3} \text{kg}$

Volume of oil, $V = 4 \text{ m}^3$

(i) Mass density, $\rho = \dfrac{\text{Mass}}{\text{Volume}}$

$$= \frac{m}{V} = \frac{3\times 10^3}{4}$$

$= \mathbf{750 \ kg / m^3}$. *(Ans)*

(ii) Specific weight, $w = \rho g = 750 \times 9.81$

$= \mathbf{7357.5 \ N / m^3}$ *(Ans)*

Problem 1.1.6. Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of 6m³ and weight of 44 kN.

(Diploma Examination Question, April-2004)

Solution: Given

Volume of liquid, $V = 6 \text{ m}^3$

Weight of liquid, $W = 44 \text{ kN} = 44 \times 10^3 \text{ N}$

(i) Specific weight, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{W}{V} = \dfrac{44\times10^3}{6}$

$= \mathbf{7333.33 \ N / m^3}$. *(Ans)*

(ii) Specific mass or mass density, $\rho = \dfrac{w}{g}$

$$= \frac{7333.33}{9.81}$$

$$= \mathbf{747.54\ kg/m^3}. \qquad \textbf{(Ans)}$$

(iii) Specific volume, $v = \frac{1}{\rho} = \frac{1}{747.54}$

$$= \mathbf{1.3377 \times 10^{-3}\ m^3/kg}. \qquad \textbf{(Ans)}$$

(iv) Specific gravity, $S = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{\rho_{liquid}}{\rho_{water}}$

$$= \frac{747.54}{1000} = \mathbf{0.74754} \qquad \textbf{(Ans)}$$

Fig - 1.2.10 - U-tube Manometer for negative pressure

Problem 1.2.21 : The left limb of a manometer is connected to a pipe in which a fluid of specific gravity 0.8 is flowing. The right limb containing mercury is open to atmosphere. The centre of the pipe is 100 mm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 250 mm.

Solution: Given

Specific gravity of fluid in the pipe, $S_1 = 0.8$

$\therefore$ Density of the fluid, $\rho_1 = S_1 \times 1000$

$$= 0.8 \times 1000$$

$$= 800\ kg/m^3$$

Specific gravity of mercury, $S_2 = 13.6$ (Assumed data)

$\therefore$ Density of mercury, $\rho_2 = S_2 \times 1000$

$= 13.6 \times 1000$

$= 13600 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 250 \text{ mm} = 0.25 \text{ m}$

Height of liquid in the left limb from X-X,

$h_1 = 250 - 100 = 150 \text{ mm}$

$= 0.15 \text{ m}$

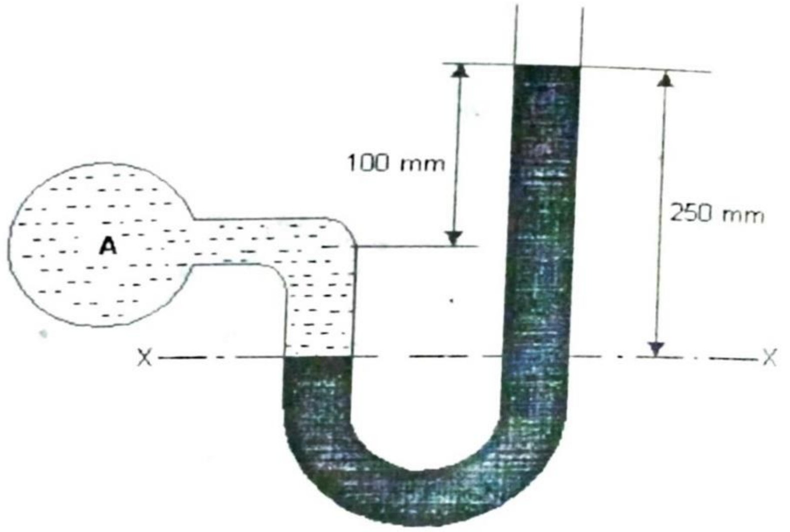


Fig 1.2.11

Let p be the intensity pressure of fluid flowing in pipe

Equating the pressure at the left and right limb above the datum X-X , we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

Substituting the values in the above equation

$$p + 800 \times 9.81 \times 0.15 = 13600 \times 9.81 \times 0.25$$

$$p + 1177.2 = 33354$$

$$p = 33354 - 1177.2$$
$$= 32176.8\ N/m^2$$
$$= \mathbf{32.176\ kN/m^2} \quad (Ans)$$

Mr-20/3

Problem 1.2.22 A simple manometer containing mercury is used to determine the pressure of oil of specific gravity 0.8 flowing in a pipe .Its right limb is open to atmosphere and left limb is connected to the pipe . The centre of the pipe is 90mm below the level of mercury in the right limb. If the difference of mercury level in the two limbs is 150 mm, find the pressure of oil in the pipe.

(Diploma Examination Question, April -2005)

Solution: Given

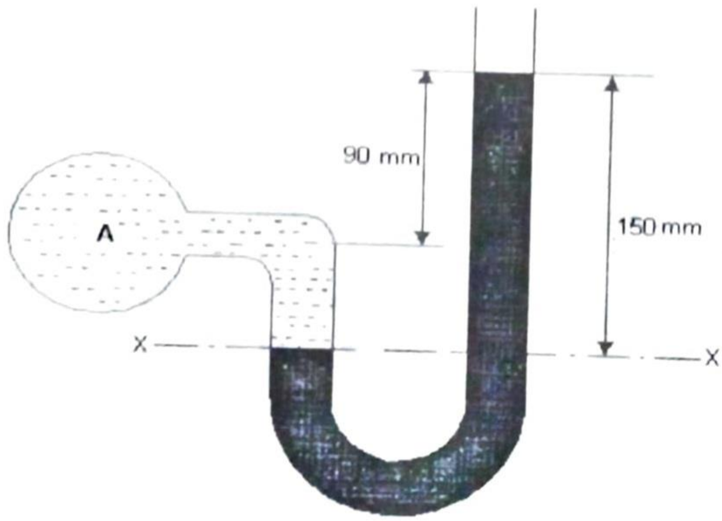


Fig.1.2.12

Specific gravity of oil in the pipe, $S_1 = 0.8$

$\therefore$ Density of the oil, $\rho_1 = S_1 \times 1000$

$= 0.8 \times 1000$

$= 800\ kg/m^3$

Specific gravity of mercury, $S_2 = 13.6$

Problem 1.2.29 A differential manometer is connected at the two points M and N of two pipes as shown in the Fig-1.2.20 . The pipe M contains carbon tetrachloride of specific gravity 1.594 under a pressure of 103 kPa and pipe N contains oil of specific gravity 0.8 under a pressure of 172 kPa. If the manometric fluid is mercury, find the difference in mercury level in the manometer.

Solution: Given

Specific gravity of liquid at M, $S_1 = 1.594$

$\therefore$ Density of liquid at point M, $\rho_M = S_M \times 1000$

$= 1.594 \times 1000$

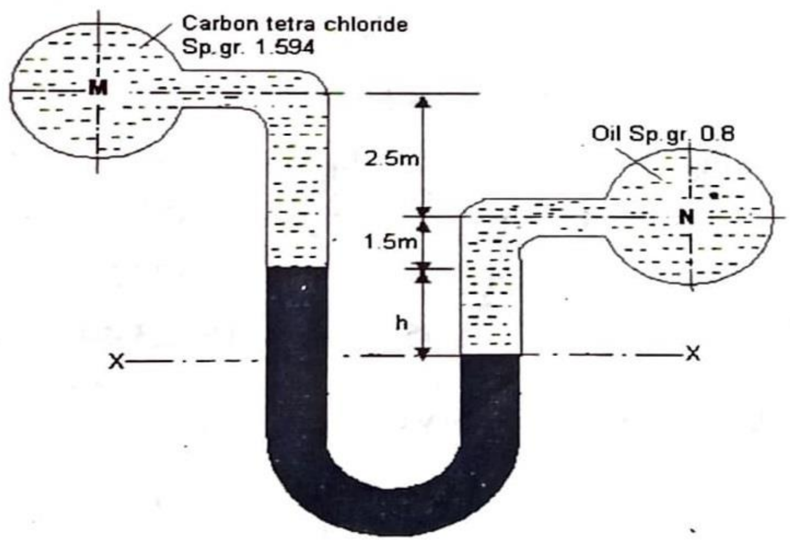
$= 1594 \text{ kg/m}^3$

Specific gravity of liquid at N, $S_2 = 0.8$

$\therefore$ Density of liquid at point N, $\rho_N = S_2 \times 1000$

$= 0.8 \times 1000 = 800 \text{ kg/m}^3$

Pressure at M, $p_M = 103 \text{ kPa} = 103 \text{ kN/m}^2 = 103 \times 10^3 \text{N/m}^2$



Pressure at N , p_N $= 172\ kPa = 172\ kN/m^2$

$= 172 \times 10^3\ N/m^2.$

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, ρ_{Hg} $= S_{Hg} \times 1000 = 13.6 \times 1000$

$= 13600\ kg/m^3.$

Taking X-X as datum line

Pressure above X-X in the left limb

$$= p_M + \rho_M\ g\ (2.5 + 1.5) + \rho_{Hg}\ g\ h$$

$$= 103 \times 10^3 + 1594 \times 9.81 \times 4 + 13600 \times 9.81\ h$$

$$= 103000 + 62548.56 + 133416\ h$$

$$= 165548.56 + 133416\ h$$

Pressure above X-X in the right limb

$$= p_N + \rho_N\ g\ (1.5 + h)$$

$$= 172 \times 10^3 + 800 \times 9.81\ (1.5 + h)$$

$$= 172000 + 11772 + 7848\ h$$

$$= 183772 + 7848\ h$$

Equating the left limb and right limb pressure.

$$165548.56 + 133416\ h = 183772 + 7848\ h$$

$$133416\ h - 7848\ h = 183772 - 165548.56$$

$$125568\ h = 18223.44$$

$\therefore$ Difference in mercury level, $h = \dfrac{18223.44}{125568} = 0.145\ m$

$= 145\ mm$ **(*Ans*)**

Problem 1.2.30 A differential U-tube manometer is used to measure pressure in two pipes A and B . The left limb of U-tube is connected to pipe A which is 2.6 m above the pipe B. The pipe A contains a liquid of specific gravity 1.6 under a pressure of 110 kN/m^2. The pipe B contains oil of specific gravity 0.8 under a pressure of 200 kN/m^2. The level of mercury in the left limb is 1.0m below the level of pipe B and level of mercury in the right limb is below to that in left limb. Find the difference between the mercury levels.

(Diploma Examination Question, April-2004)

Solution: Given

Specific gravity of liquid at A, $S_A = 1.6$

$\therefore$ Density of liquid at point A, $\rho_A = S_A \times 1000 = 1.6 \times 1000$
$= 1600 \text{ kg/m}^3$.

Specific gravity of liquid at B, $S_B = 0.8$

$\therefore$ Density of liquid at B, $\rho_B = S_B \times 1000 = 0.8 \times 1000$
$= 800 \text{ kg/m}^3$.

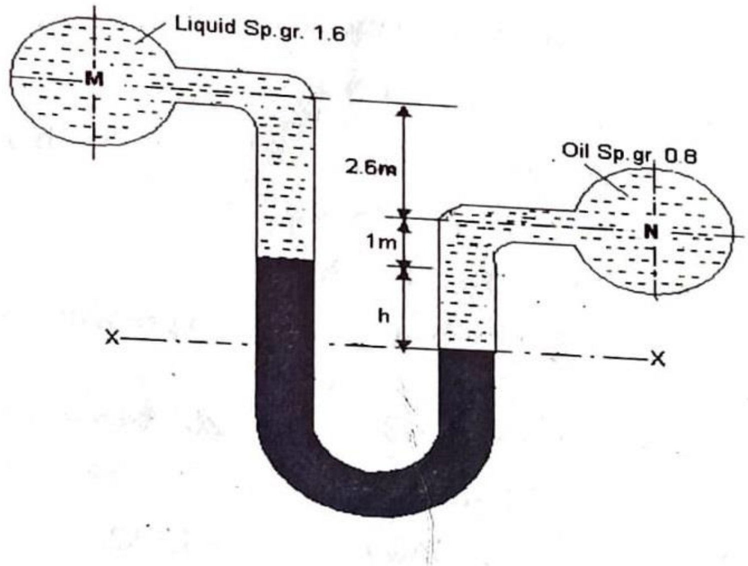


Fig. 1.2.21

Pressure at A, $p_A = 110 \text{ kN/m}^2$
$= 110 \times 10^3 \text{ N/m}^2$

Pressure at B =, p_B $= 200\ \text{kN/m}^2$

$= 200 \times 10^3\ \text{N/m}^2$

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, ρ_{Hg} $= S_{Hg} \times 1000$

$= 13.6 \times 1000$

$= 13600\ \text{kg/m}^3.$

Taking X - X as datum line

Pressure above X -X in the left limb

$$= p_A + \rho_A\, g\, (2.6 + 1.0) + \rho_{Hg}\, g\, h$$

$$= 110000 + 1600 \times 9.81 \times 3.6 + 13600 \times 9.81 \times h$$

$$= 110000 + 56505.6 + 133416\, h$$

$$= 166505.6 + 133416\, h$$

Pressure above X -X in the right limb

$$= p_B + \rho_B\, g\, (1.0 + h)$$

$$= 200000 + 800 \times 9.81 \times (1.0 + h)$$

$$= 200000 + 7848 + 7848\, h$$

$$= 207848 + 7848\, h$$

Equating the left limb and right limb pressure above the datum X-X, we get.

$$166505.6 + 133416\, h = 207848 + 7848\, h$$

$$133416\, h - 7848\, h = 207848 - 166505.6$$

$$125568\, h = 41342.4$$

$\therefore$ Difference in mercury level, $h = \dfrac{41342.4}{125568} = 0.3292\text{m}$

$= \mathbf{329.2\ mm}.$ *(Ans)*

Problem 1.2.31 A Differential U-tube manometer connects two pressure pipes A and B . The pipe A contains carbon tetrachloride having a specific gravity 1.6 under a pressure of 120 kPa. The pipe B contains oil of specific gravity 0.8 under a pressure of 200kPa. The pipe A lies 2.5m above pipe B .Find the difference in mercury level in the manometers when assuming the left limb mercury level is coinciding with the centre of the pipe B.

(Diploma Examination Question, Nov-2000).

Solution : Given

Specific gravity of carbon tetrachlorideat A, $S_A = 1.6$

$\therefore$ Density of carbon tetrachloride at point A, $\rho_A = S_A \times 1000$

$= 1.6 \times 1000$

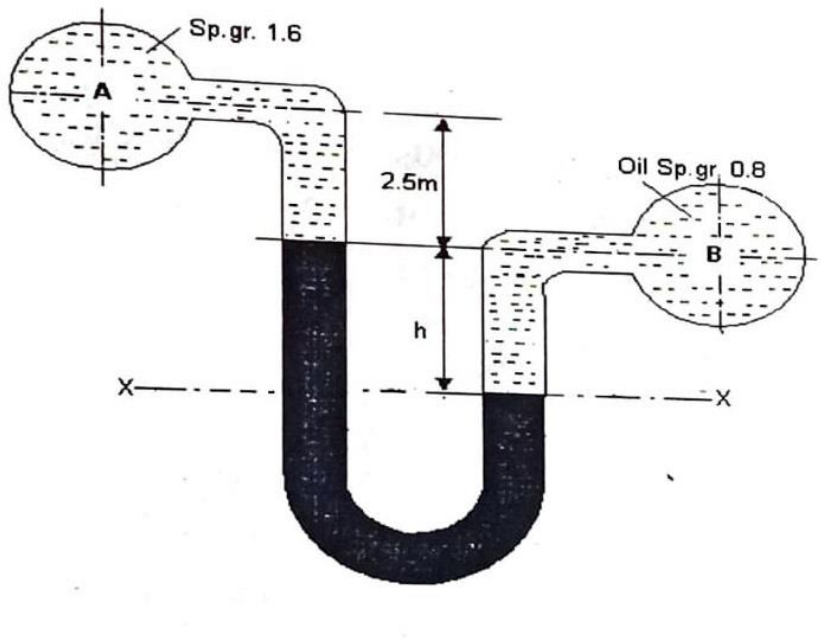


Fig. 1.2.22

$= 1600$ kg/ m^3.

Specific gravity of liquid at B, $S_B = 0.8$

$\therefore$ Density of liquid at B, $\rho_B = S_B \times 1000$

$= 0.8 \times 1000 = 800 \text{ kg/m}^3$

Pressure at A, p_A $= 120 \text{ kN/m}^2$

$= 120 \times 10^3 \text{ N/m}^2$

Pressure at B =, p_B $= 200 \text{ kN/m}^2$

$= 200 \times 10^3 \text{ N/m}^2$

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, ρ_{Hg} $= S_{Hg} \times 1000 = 13.6 \times 1000$

$= 13600 \text{ kg/m}^3$.

Taking X - X as datum line

Pressure above X-X in the left limb

$$= p_A + \rho_A g \times 2.5 + \rho_{Hg} g h$$

$$= 120000 + 1600 \times 9.81 \times 2.5 + 13600 \times 9.81 \times h$$

$$= 120000 + 39240 + 133416 h$$

$$= 159240 + 133416 h$$

Pressure above X -X in the right limb

$$= p_B + \rho_B g h$$

$$= 200000 + 800 \times 9.81 \times h$$

$$= 200000 + 7848 h$$

$$= 200000 + 7848 h$$

Equating the left limb and right limb pressure above the datum X-X, we get.

$$159240 + 133416 h = 200000 + 7848 h$$

$$133416 h - 7848 h = 200000 - 159240$$

$$125568 h = 40760$$

$\therefore$ Difference of mercury level, $h = \dfrac{40760}{125568} = 0.3246$ m

$= \mathbf{324.6\ mm}$ ***(Ans)***

Problem 1.2.32 A U-tube Mercury manometer is connected to two pipes A &B . Pipe B is 60 mm. below pipe A . The specific gravity of liquid in pipe A and B is 1.6 and 0.85 respectively. Mercury level in the left limb is 80 mm. below the centre of pipe A. Find the pressure difference between two pipes in kN / m² if the level difference of mercury in the two limbs of the manometer is 120mm.

(Diploma Examination Question, Nov-2001)

Solution: Given

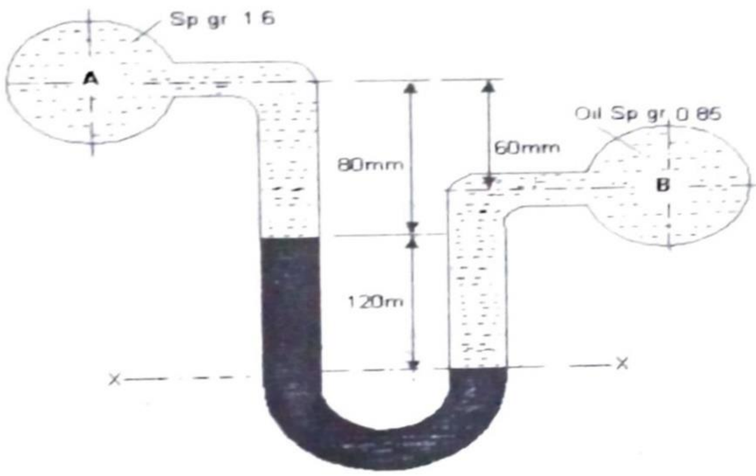


Fig. 1.2.23

Specific gravity of liquid at A, $S_A = 1.6$

$\therefore$ Density of liquid at point A, $\rho_A = S_A \times 1000 = 1.6 \times 1000$

$= 1600$ kg/ m³

Specific gravity of liquid at B, $S_B = 0.85$

$\therefore$ Density of liquid at B, $\rho_B = S_B \times 1000 = 0.85 \times 1000$

$= 850 \text{ kg/ m}^3$

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, ρ_{Hg} $= S_{Hg} \times 1000 = 13.6 \times 1000$
$= 13600 \text{ kg / m}^3$

Taking X - X as datum line

Pressure above X-X in the left limb

$$= p_A + \rho_A g \times 0.08 + \rho_{Hg} g \times 0.12$$

$$= p_A + 1600 \times 9.81 \times 0.08 + 13600 \times 9.81 \times 0.12$$

$$= p_A + 1255.68 + 16009.92$$

$$= p_A + 17265.6$$

Pressure above X -X in the right limb

$$= p_B + \rho_B g (0.08 + 0.12 - 0.06)$$

$$= p_B + 850 \times 9.81 \times (0.14)$$

$$= p_B + 1167.39$$

Equating the two pressures, we get

$$p_A + 17265.6 = p_B + 1167.39$$

$\therefore$ Pressure difference between two pipes, $p_B - p_A$

$= 17265.6 - 1167.39$

$= 16098.21 \text{ N / m}^2.$

$= \mathbf{16.098 \text{ kN/ m}^2}$ ***(Ans)***

Problem 1.2.33 A differential manometer containing mercury was used to measure difference in two pipes containing different liquids, as shown in the Fig.1.2.24. Find out the pressure difference in terms of kPa.

Solution: Given

Specific gravity of liquid at A, $S_A = 0.8$

$\therefore$ Density of liquid at point A, $\rho_A = S_A \times 1000 = 0.8 \times 1000$

$= 800\ kg/m^3$

Specific gravity of liquid at B, $S_B = 0.9$

$\therefore$ Density of liquid at B, $\rho_B = S_B \times 1000 = 0.9 \times 1000$

$= 900\ kg/m^3$

Specific gravity of mercury, $S_{Hg} = 13.6$

Density of mercury, $\rho_{Hg} = S_{Hg} \times 1000 = 13.6 \times 1000$

$= 13600\ kg/m^3$

Taking **X - X** as datum line

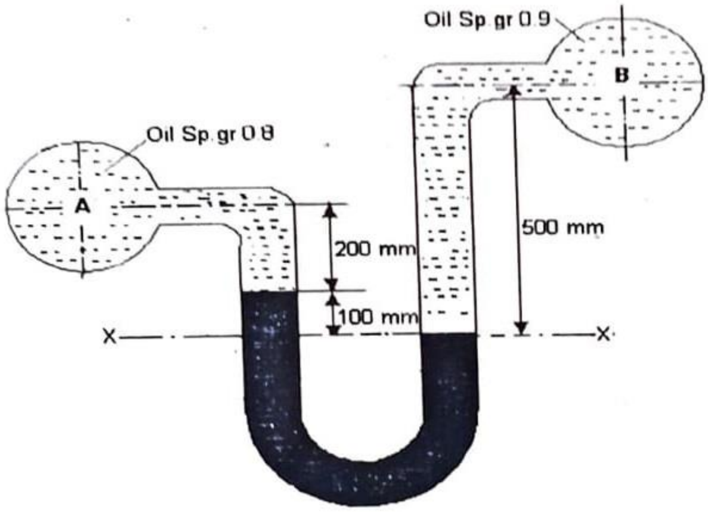


Fig.1.2.24

Pressure above **X-X** in the left limb

$$= p_A + \rho_A\, g \times 0.20 + \rho_{Hg}\, g \times 0.1$$

$$= p_A + 800 \times 9.81 \times 0.20 + 13600 \times 9.81 \times 0.1$$

$= p_A + 1569.6 + 13341.6$

$= p_A + 14911.2$

Pressure above X-X in the right limb

$= p_B + \rho_B\, g \times 0.5$

$= p_B + 900 \times 9.81 \times 0.5$

$= p_B + 4414.5$

Equating the pressure in the two limbs

$$p_A + 14911.2 = p_B + 4414.5$$

Difference in pressure, $p_B - p_A = 14911.2 - 4414.5$

$= 10496.7\ N/m^2$

$= \mathbf{10.4967\ kN/m^2}$ *(Ans)*

2. Inverted U-tube differential manometer.

It consists of an inverted U-tube, containing a light liquid. It is used for measuring the difference of low pressures between two points. Accuracy is the main characteristic of this type of manometer. The two limbs of the manometer are connected to two different points whose pressure difference is to be measured.

Consider a U-tube connected to two different points as shown in Fig-1.2.25

Let

h_1 is the height of liquid in the left limb above the datum line X-X(measured in metres of liquid, i.e., m)

h_2 is the height of liquid in the right limb(measured in metres of liquid, i.e., m)

The pressure in the right limb below the datum X-X

$$p_N - \rho_N \times g \times h_2 - \rho_l \times g \times h$$

Equating the two pressures

$$p_M - \rho_M \times g \times h_1 = p_N - \rho_N \times g \times h_2 - \rho_l \times g \times h$$

$$p_M - p_N = \rho_M \times g \times h_1 - \rho_N \times g \times h_2 - \rho_l \times g \times h$$

Problem 1.2.34 An inverted differential manometer is connected to two pipes M and N which carries water. The fluid in the manometer is oil of specific gravity 0.9. For the manometer readings shown in the Fig- 1.2.26, find the pressure difference between M and N.

Solution: Given

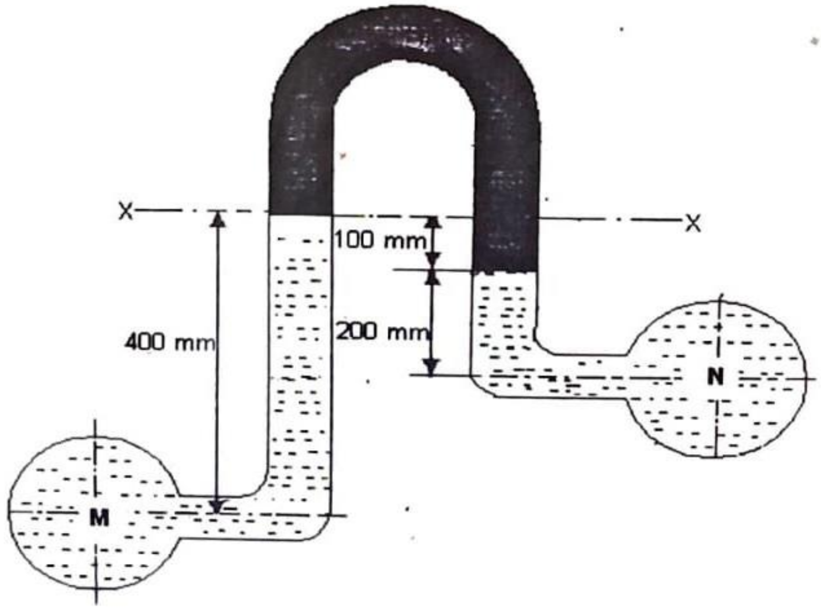


Fig. 1.2.26

Specific gravity of oil, $S_l = 0.9$

$\therefore$ Density of oil, $\rho_l \quad = S_l \times 1000 = 0.9 \times 1000 = 900\ \text{kg/m}^3$.

Difference of oil in the two limbs, $h = 100\ \text{mm} = 0.1\ \text{m}$

Taking datum line X-X

Pressure in the left limb below X-X

$$= p_M - 1000 \times 9.81 \times 0.40$$

$$= p_M - 3924$$

Pressure in the right limb below X-X

$$= p_N - 1000 \times 9.81 \times 0.20 - 900 \times 9.81 \times 0.1$$

$$= p_N - 1962 - 882.9$$

$$= p_N - 2844.9$$

Equating two pressures

$$p_M - 3924 = p_N - 2844.9$$

$\therefore$ Pressure difference between M and N,

$$p_M - p_N = 3924 - 2844.9$$

$$= \mathbf{1079.1\ N/m^2} = \mathbf{1.08\ kN/m^2} \qquad (Ans)$$

Problem 1.2.35 An inverted U-tube manometer is connected with two pipes M and N which carries an oil of specific gravity 1.2 and 0.8 respectively. The fluid in the manometer is an oil of specific gravity 0.7. For the manometer reading shown in the Fig-1.2.27 , find the pressure difference between M and N.

Solution : Given

Specific gravity of fluid in the pipe M, $S_M = 1.2$

$\therefore$ Density of of fluid in the pipe M, $\rho_M = S_M \times 1000$

$$= 1.2 \times 1000$$
$$= 1200\ kg/m^3.$$

Specific gravity of fluid in the pipe N, $S_N = 0.8$

$\therefore$ Density of of fluid in the pipe N, $\rho_N \quad = S_N \times 1000$

$= 0.8 \times 1000$

$= 800 \text{ kg/m}^3.$

Specific gravity of oil U-tube, $S_l \quad = 0.7$

Density of oil in the U-tube, $\rho_l \quad = S_l \times 1000$

$= 0.7 \times 1000$

$= 700 \text{ kg / m}^3.$

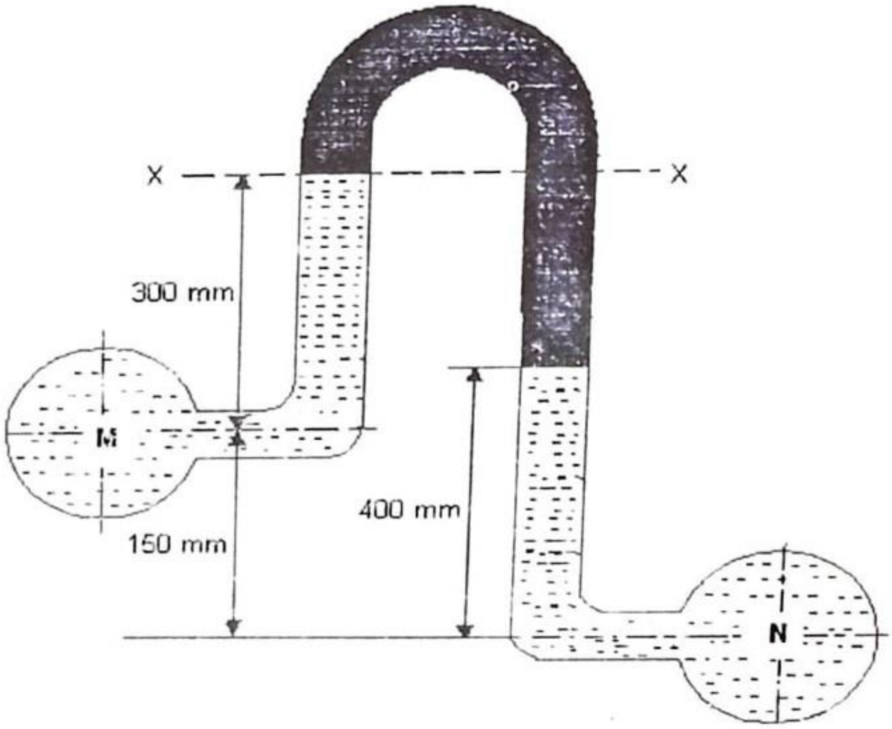


Fig 1.2.27

Difference of oil in theU-tube, $h = (300 + 150) - 400 = 50$ mı

$= 0.5\text{m}$

Taking datum line X-X

Pressure in the left limb below X-X

$$= p_M - 1200 \times 9.81 \times 0.30$$

$$= p_M - 3531.6$$

Pressure in the right limb below X-X

$$= p_N - 800 \times 9.81 \times 0.4 - 700 \times 9.81 \times 0.05$$

$$= p_N - 3139.2 - 343.35$$

$$= p_N - 3482.55$$

Equating two pressures

$$p_M - 3531.6 = p_N - 3482.55$$

$\therefore$ Pressure difference between M and N, $p_M - p_N$

$$= 3531.6 - 3482.55 = \mathbf{49.05\ N / m^2}$$

$$= \mathbf{0.049\ kN / m^2} \qquad \textit{(Ans)}$$

Problem 1.2.36 The pipes M and N containing oil of Specific gravity 0.9 and 0.8 respectively , which is connected to a U-tube manometer as shown in Fig- 1.2.28. The measuring fluid in the manometer is oil having specific gravity 0.7. Determine the difference of pressure between the centres of fluids in the pipes.

Solution: Given

Specific gravity of fluid in the pipe M, $S_M = 0.9$

$\therefore$ Density of of fluid in the pipe M, $\rho_M = S_M \times 1000$

$$= 0.9 \times 1000$$
$$= 900\ kg/m^3.$$

Specific gravity of fluid in the pipe N, $S_N = 0.8$

$\therefore$ Density of of fluid in the pipe N, $\rho_N = S_N \times 1000$

$$= 0.8 \times 1000$$
$$= 800\ kg/m^3.$$

Specific gravity of oil U-tube, $S_l = 0.7$

$\therefore$ Density of oil in the U-tube, $\rho_l = S_l \times 1000$

$$= 0.7 \times 1000$$
$$= 700 \text{ kg}/\text{m}^3.$$

Taking datum line X-X

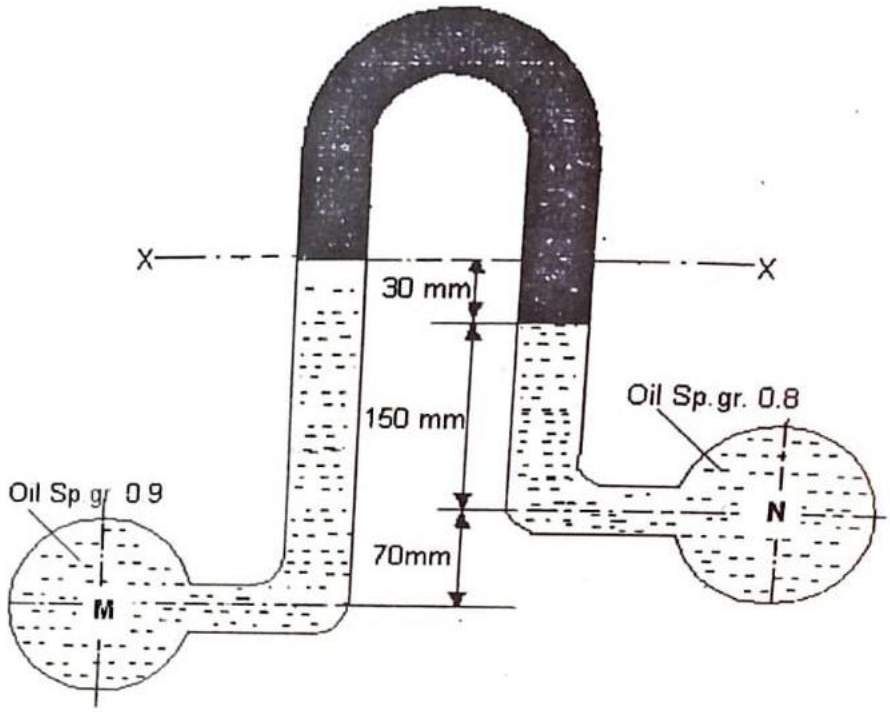


Fig. 1.2.28

Pressure in the left limb below X-X

$$= p_M - 900 \times 9.81 \times (0.07 + 0.15 + 0.03)$$

$$= p_M - 900 \times 9.81 \times 0.25$$

$$= p_M - 2207.25$$

Pressure in the right limb below X-X

$$= p_N - 800 \times 9.81 \times 0.15 - 700 \times 9.81 \times 0.03$$

$$= p_N - 1177.2 - 206.01$$

$$= p_N - 1383.21$$

Equating two pressures

$$p_M - 2207.25 = p_N - 1383.21$$

$\therefore$ Difference in pressures, $p_M - p_N$

$$= 2207.25 - 1383.21$$

$$= \mathbf{824.04\ N/m^2 = 0.824\ kPa} \qquad (\textit{Ans})$$

Problem 1.2.37 Find the difference in reading '*h*' of an inverted U-tube manometer containing oil of specific gravity 0.9 as the manometric fluid when connected across pipes M and N as shown in Fig 1.2.29. pipe M contains an oil of specific gravity 1.3 and pipe N contains water of specificgravity 1.pipes M and N are located at the same level and assume the pressure at M and N are equal.

Solution: Given

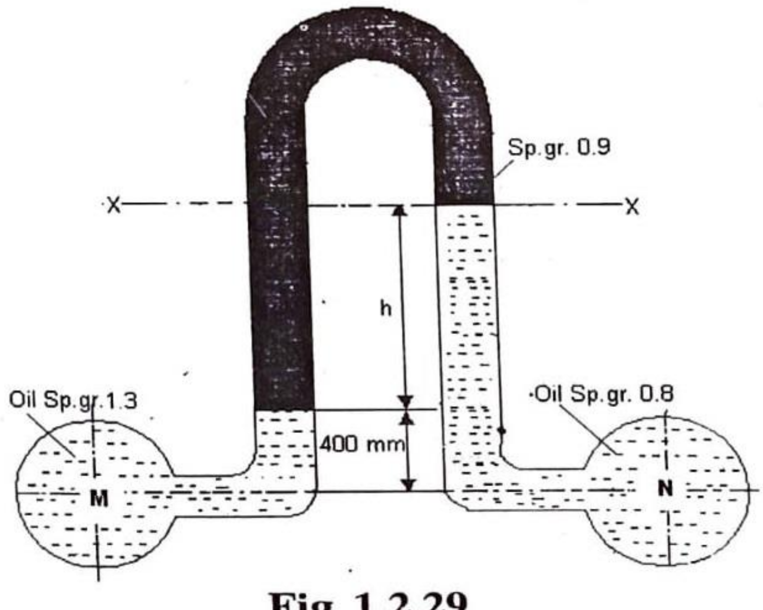


Fig. 1.2.29

Specific gravity of oil in the pipe M, $S_M = 1.3$

$\therefore$ Density of of oil in the pipe M, $\rho_M = S_M \times 1000$