MODULE 4

SIMPLE STRESSES AND STRAINS

Definition of rigid bodies:

A body which does not deforms under the action of loads or external forces is known as rigid body.

Definition of elastic body and elasticity

When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed and the body comes back to its origin shape and size the body is known as elastic body. This property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity.

Plastic bodies:

The body which does not have the property of opposing the deforming force, is known as a plastic body. The bodies which remain in deformed state even after removal of the deforming force are defined as plastic bodies.

Deformation of elastic body under various forces: Elastic deformation refers to a temporary deformation of a material's shape that is self-reversing after removing the force or load. Elastic deformation alters the shape of a material upon the application of a force within its elastic limit.

Stress:

The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force.

Mathematically stress is written as, $\sigma = \dfrac{P}{A}$

where σ = Stress (also called intensity of stress),

P = External force or load, and

A = Cross-sectional area.

Strain: When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. Strain is dimensionless.

Strain may be:

1. Tensile strain, 2. Compressive strain,

3. Volumetric strain, and 4. Shear strain.

Tensile strain: If there is some increase in length of a body due to external force, then the ratio of Increase of length to the original length of the body is known as tensile strain.

Compressive strain: If there is some decrease in length of the body, then the ratio of decrease of the length of the body to the original length is known as compressive strain.

Volumetric strain: The ratio of change of volume of the body to the original volume is known as volumetric strain.

Shear strain: The strain produced by shear stress is known as shear strain.

Hooke's Law

Hooke's law states that within elastic limit, the stress is proportional to the strain.

Elastic limit:

The body will regain its previous shape and size only when the deformation caused by the external force, is within a certain limit. Thus there is a limiting value of force up to and within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material.

Modulus of Elasticity (or Young's modulus). The ratio of tensile stress or compressive stress to the corresponding strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity and is denoted by E.

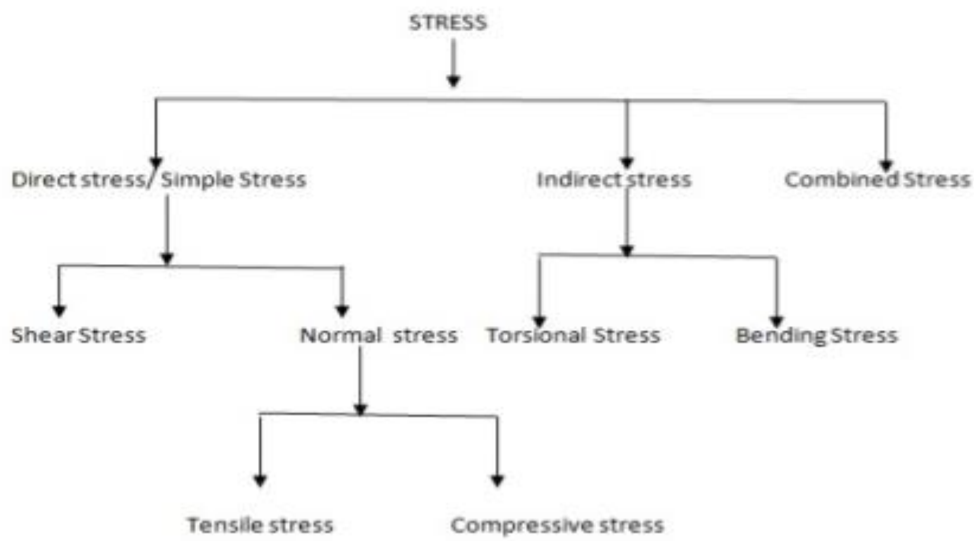
$$\therefore \quad E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \quad \text{or} \quad \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

or

$$E = \frac{\sigma}{e}$$

Type of Stresses:

Basically stresses are classified in to three types.



Direct, Bending, Shear and Torsion - nature of stresses.

Direct stress:

direct stress or simple stress is classified in two type i.e. normal stress and shear stress. As it is also displayed in figure, normal stress will be divided in two type i.e. tensile stress and compressive stress.

Similarly, indirect stress will also be divided in two type i.e. torsion stress and bending stress. Above figure displayed here indicates the brief introduction for the classification of stress in strength of material.

Normal stress

Normal stress is basically defined as the stress acting in a direction perpendicular to the area. Normal stress will be further divided, as we have seen above, in two types of stresses i.e. tensile stress and compressive stress.

Tensile Stress.

The stress induced in a body, when subjected to two equal and opposite pulls as a result of which there is an increase in length, is known as tensile stress. The ratio of increase in length to the original length is known as tensile strain.

$$\therefore \quad \text{Tensile stress,} \quad \sigma = \frac{\text{Resisting force } (R)}{\text{Cross-sectional area}} = \frac{\text{Tensile load } (P)}{\text{A}}$$

$$\sigma = \frac{P}{A}$$

Compressive Stress. The stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a decrease in length of the body, is known as compressive stress. And the ratio of decrease in length to the original length is known as compressive strain.

Then compressive stress is given by,

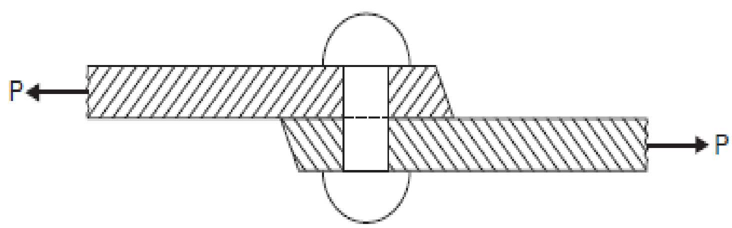
$$\sigma = \frac{\text{Resisting Force } (R)}{\text{Area } (A)} = \frac{\text{Push } (P)}{\text{Area } (A)} = \frac{P}{A}.$$

And compressive strain is given by,

$$e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dL}{L}.$$

Bending stress: Bending stress is the resistance of an object to bend. In bending, the force is a normal load and applied at a specific spot of the object. In other words, a bending moment is a measure of the bending effect which is developed internally due to external loads.

Shear Stress. The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in Fig. as a result of which the body tends to shear off across the section, is known as shear stress. The corresponding strain is known as *shear strain*. The shear stress is the stress which acts tangential to the area. It is represented by τ.



$$\therefore \quad \text{Shear stress, } \tau = \frac{\text{Shear resistance}}{\text{Shear area}} = \frac{R}{A}$$

Torsional stress: Torsional stress is shear stress caused due to twisting. In other words, it may be described as the angular deformation of a body.

Problem *A rod 150 cm long and of diameter 2.0 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm^2 ; determine :*

(i) the stress,

(ii) the strain, and

(iii) the elongation of the rod.

Sol. Given : Length of the rod, $L = 150$ cm

Diameter of the rod, $D = 2.0 \text{ cm} = 20 \text{ mm}$

$\therefore$ Area, $A = \frac{\pi}{4}(20)^2 = 100\pi \text{ mm}^2$

Axial pull, $P = 20 \text{ kN} = 20{,}000 \text{ N}$

Modulus of elasticity, $E = 2.0 \times 10^5 \text{ N/mm}^2$

(*i*) *The stress* (σ) is given by equation (12.1) as

$$\sigma = \frac{P}{A} = \frac{20000}{100\pi} = \mathbf{63.662\ N/mm^2}. \quad \textbf{Ans.}$$

(*ii*) Using equation (12.5), the *strain is obtained* as

$$E = \frac{\sigma}{e}.$$

$\therefore$ Strain,
$$e = \frac{\sigma}{E} = \frac{63.662}{2 \times 10^5} = \mathbf{0.000318}. \quad \textbf{Ans.}$$

(*iii*) *Elongation is obtained* by using equation (12.2) as

$$e = \frac{dL}{L}.$$

$\therefore$ Elongation, $dL = e \times L$

$$= 0.000318 \times 150 = \mathbf{0.0477\ cm}. \quad \textbf{Ans.}$$

Problem .2. *Find the minimum diameter of a steel wire, which is used to raise a load of 4000 N if the stress in the rod is not to exceed 95 MN/m².*

Sol. Given : Load, $P = 4000$ N

Stress, $\sigma = 95\ \text{MN/m}^2 = 95 \times 10^6\ \text{N/m}^2$ $(\because \quad \text{M = Mega} = 10^6)$

$= 95\ \text{N/mm}^2$ $(\because \quad 10^6\ \text{N/m}^2 = 1\ \text{N/mm}^2)$

Let D = Diameter of wire in mm

$\therefore$ Area, $A = \frac{\pi}{4} D^2$

Now, $\text{stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$

$$95 = \frac{4000}{\frac{\pi}{4} D^2} = \frac{4000 \times 4}{\pi D^2} \quad \text{or} \quad D^2 = \frac{4000 \times 4}{\pi \times 95} = 53.61$$

$\therefore$ $D = \mathbf{7.32\ mm}.$ **Ans.**

Problem .3. *Find the Young's Modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50 kN when the extension of the rod is equal to 0.3 mm.*

Sol. Given : Dia. of rod, $D = 25$ mm

$\therefore$ Area of rod, $A = \frac{\pi}{4} (25)^2 = 490.87\ \text{mm}^2$

Tensile load, $P = 50\ \text{kN} = 50 \times 1000 = 50{,}000$ N

Extension of rod, $dL = 0.3$ mm

Length of rod, $L = 250$ mm

Stress (σ) is given by equation (12.1), as

$$\sigma = \frac{P}{A} = \frac{50{,}000}{490.87} = 101.86\ \text{N/mm}^2.$$

Strain (e) is given by equation (12.2), as

$$e = \frac{dL}{L} = \frac{0.3}{250} = 0.0012.$$

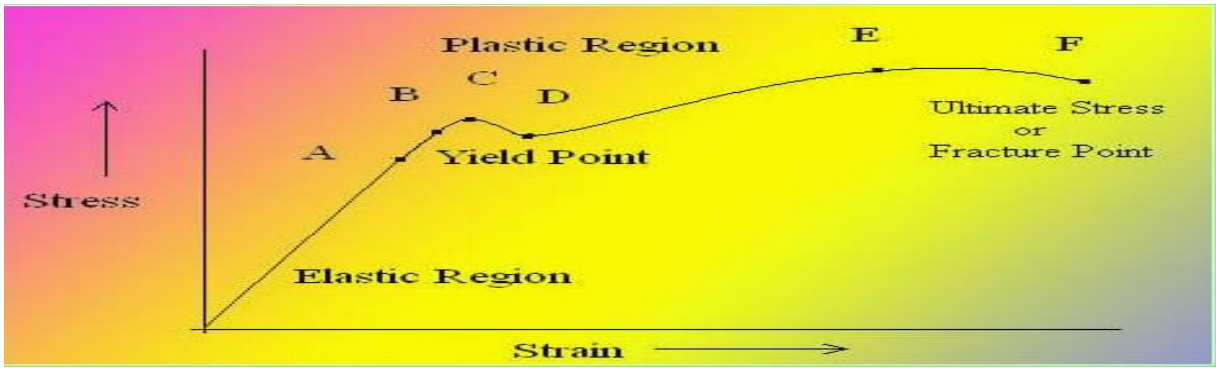
Using equation (12.5), the Young's Modulus (E) is obtained, as

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{101.86\ \text{N/mm}^2}{0.0012} = 84883.33\ \text{N/mm}^2$$

$= \mathbf{84883.33 \times 10^6\ N/m^2}.$ **Ans.** $(\because \quad 1\ \text{N/mm}^2 = 10^6\ \text{N/m}^2)$

$= 84.883 \times 10^9\ \text{N/m}^2 = \mathbf{84.883\ GN/m^2}.$ **Ans.** $(\because \quad 10^9 = \text{G})$

Stress strain curve for mild steel bar under tension



Here is the list of different stages when ductile material subjected to force till its failure.

1. Proportional limit (point A)
2. Elastic limit (point B)
3. Yield point (upper yield point C and lower yield point D)
4. Ultimate stress point (point E)
5. Breaking point (point F)

Proportional limit: up to the point A, stress directly followed the strain. This means ratio of stress and strain remains constant

Elastic limit: Up to this limit (point B), is material will regain its original shape is unloaded. Point B is known as elastic point.

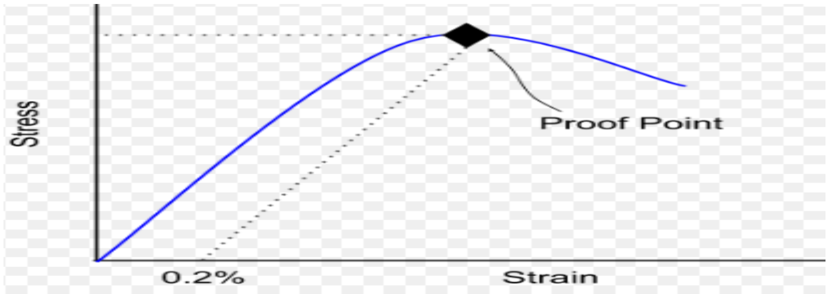
Yield limit: When material is loaded beyond its elastic limit, it will not regain its original shape. There will be always some deformation.

Ultimate stress: This is the maximum stress a material can bear. Value of stress correspond to peak point on stress strain curve for mild steel is the ultimate stress. It is denoted by point E in diagram.

Breaking stress: Point on the stress strain curve where material fails, is known as breaking point. Stress correspond to this point is known as breaking stress.

Yield stress: Yield stress, marking the transition from elastic to plastic behaviour, is the minimum stress at which a solid will undergo permanent deformation or plastic flow without a significant increase in the load or external force.

The proof stress: The proof stress of a material is defined as the amount of stress it can endure until it undergoes a relatively small amount of plastic deformation. Specifically, proof stress is the point at which the material exhibits 0.2% of plastic deformation.



Ultimate Stress: It is the stress obtained by dividing maximum load (i.e., load corresponding to point D) by the initial cross-sectional area of the specimen.

The percentage elongation is obtained as,

Percentage elongation

$$= \frac{\text{Total increase in length}}{\text{Original length (or Gauge length)}} \times 100$$

Stiffness: The product of modulus of rigidity and polar moment of inertia of a shaft is known as torsional rigidity or stiffness of the shaft. Mathematically,

Torsional rigidity = C × J

Longitudinal strain: When a body is subjected to an axial tensile or compressive load, there is an axial deformation in the length of the body. The ratio of axial deformation to the original length of the body is known as longitudinal (or linear) strain

Let L = Length of the body,

P = Tensile force acting on the body,

δL = Increase in the length of the body in the direction of P.

Then, longitudinal strain $= \dfrac{\delta L}{L}$.

Lateral strain: The strain at right angles to the direction of applied load is known as lateral strain. The length of the bar will increase while the breadth and depth will decrease.

Let δL = Increase in length,

δb = Decrease in breadth, and

δd = Decrease in depth.

Then longitudinal strain $= \dfrac{\delta L}{L}$

lateral strain $= \dfrac{\delta b}{b}$ or $\dfrac{\delta d}{d}$

Modulus of Rigidity: the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as Modulus of Elasticity or Modulus of Rigidity or Elastic Modulii.

Poisson's ratio:

The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called **Poisson's ratio** and it is generally denoted by μ. Hence mathematically,

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

or

$$\text{Lateral strain} = \mu \times \text{longitudinal strain}$$

VOLUMETRIC STRAIN: The ratio of change in volume to the original volume of a body is called volumetric strain. It is denoted by e_v.

Mathematically, volumetric strain is given by

$$e_v = \frac{\delta V}{V}$$

where δV = Change in volume, and

V = Original volume.

BULK MODULUS

When a body is subjected to the mutually perpendicular like and equal direct stresses, the ratio of direct stress to the corresponding volumetric strain is found to be constant for a given material when the deformation is within a certain limit. This ratio is known as bulk modulus and is usually denoted by K. Mathematically bulk modulus is given by

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

The relation between Young's modulus and bulk modulus is given by,

$$E = 3K(1 - 2\mu).$$

The relation between modulus of elasticity and modulus of rigidity is given by

$$E = 2C(1 + \mu) \quad \text{or} \quad C = \frac{E}{2(1 + \mu)}.$$

Problem 1. *Determine the changes in length, breadth and thickness of a steel bar which is 4 m long, 30 mm wide and 20 mm thick and is subjected to an axial pull of 30 kN in the direction of its length. Take $E = 2 \times 10^5$ N/mm^2 and Poisson's ratio = 0.3.*

Sol. Given :

Length of the bar, $L = 4$ m $= 4000$ mm

Breadth of the bar, $b = 30$ mm

Thickness of the bar, $t = 20$ mm

$\therefore$ Area of cross-section, $A = b \times t = 30 \times 20 = 600$ mm^2

Axial pull, $P = 30$ kN $= 30000$ N

Young's modulus, $E = 2 \times 10^5$ N/mm^2

Poisson's ratio, $\mu = 0.3$.

Now strain in the direction of load (or longitudinal strain),

$$= \frac{\text{Stress}}{E} = \frac{\text{Load}}{\text{Area} \times E} \qquad \left(\because \ \text{Stress} = \frac{\text{Load}}{\text{Area}}\right)$$

$$= \frac{P}{A.E.} = \frac{30000}{600 \times 2 \times 10^5} = 0.00025.$$

But longitudinal strain $= \dfrac{\delta L}{L}$.

$$\therefore \quad \frac{\delta L}{L} = 0.00025.$$

$\therefore$ δL (or change in length) $= 0.00025 \times L$

$= 0.00025 \times 4000 =$ **1.0 mm. Ans.**

Using equation (13.3),

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$0.3 = \frac{\text{Lateral strain}}{0.00025}$$

$\therefore$ Lateral strain $= 0.3 \times 0.00025 = 0.000075$.

We know that

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}\left(\text{or } \frac{\delta t}{t}\right)$$

$\therefore$ $\delta b = b \times$ Lateral strain

$= 30 \times 0.000075 =$ **0.00225 mm. Ans.**

Similarly, $\delta t = t \times$ Lateral strain

$= 20 \times 0.000075 =$ **0.0015 mm. Ans.**

Problem 13.2. *Determine the value of Young's modulus and Poisson's ratio of a metallic bar of length 30 cm, breadth 4 cm and depth 4 cm when the bar is subjected to an axial compressive load of 400 kN. The decrease in length is given as 0.075 cm and increase in breadth is 0.003 cm.*

Sol. Given :

Length, $L = 30$ cm ; Breadth, $b = 4$ cm ; and Depth, $d = 4$ cm.

$\therefore$ Area of cross-section, $A = b \times d = 4 \times 4$

$= 16\text{ cm}^2 = 16 \times 100 = 1600\text{ mm}^2$

Axial compressive load, $P = 400\text{ kN} = 400 \times 1000\text{ N}$

Decrease in length, $\delta L = 0.075$ cm

Increase in breadth, $\delta b = 0.003$ cm

Longitudinal strain $= \dfrac{\delta L}{L} = \dfrac{0.075}{30} = 0.0025$

Lateral strain $= \dfrac{\delta b}{b} = \dfrac{0.003}{4} = 0.00075.$

Using equation (13.3),

Poisson's ratio $= \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}} = \dfrac{0.00075}{0.0025} = \mathbf{0.3}.$ **Ans.**

Longitudinal strain $= \dfrac{\text{Stress}}{E} = \dfrac{P}{A \times E}$ $\left(\because \text{ Stress} = \dfrac{\text{Load}}{\text{Area}} = \dfrac{P}{A}\right)$

or $$0.0025 = \frac{400000}{1600 \times E}$$

$$E = \frac{400000}{1600 \times 0.0025} = \mathbf{1 \times 10^5\ N/mm^2}. \quad \textbf{Ans.}$$

The **mechanical properties of a material** are those which affect the mechanical strength and ability of a material to be molded in suitable shape. Some of the typical mechanical properties of a material include:

Elasticity, stiffness, plasticity, toughness, brittleness, ductility, Malleability and hardness.

1. **Elasticity**: property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity
2. **Stiffness** is the capacity of a mechanical system to sustain external loads without excessive changes of its geometry (deformations). It is one of the most important design criteria for mechanical components and systems.

3**. Plasticity** is a mechanical property of materials that shows the ability to deform under stress without breaking, while retaining the deformed shape after the load is lifted.

4. **Toughness:** It is the ability of a material to absorb the energy and gets plastically deformed without fracturing. Its numerical value is determined by the amount of energy per unit volume. For good toughness, materials should have good strength as well as ductility.

5. Brittleness

Brittleness of a material indicates that how easily it gets fractured when it is subjected to a force or load. When a brittle material is subjected to a stress it observes very less energy and gets fractures without significant strain. Brittleness is converse to ductility of material. Brittleness of material is temperature dependent. Some metals which are ductile at normal temperature become brittle at low temperature.

6. Ductility

Ductility is a property of a solid material which indicates that how easily a material gets deformed under tensile stress. Ductility is often categorized by the ability of material to get stretched into a wire by pulling or drawing. This mechanical property is also an aspect of plasticity of material and is temperature dependent. With rise in temperature, the ductility of material increases.

7. Malleability

Malleability is a property of solid materials which indicates that how easily a material gets deformed under compressive stress. Malleability is often categorized by the ability of material to be formed in the form of a thin sheet by hammering or rolling. This mechanical property is an aspect of plasticity of material. Malleability of material is temperature dependent. With rise in temperature, the malleability of material increases.

8**. Hardness:** It is the ability of a material to resist to permanent shape change due to external stress. There are various measure of hardness – Scratch Hardness, Indentation Hardness and Rebound

Scratch Hardness: Scratch Hardness is the ability of materials to the oppose the scratches to outer surface layer due to external force.

Indentation Hardness: It is the ability of materials to oppose the dent due to punch of external hard and sharp objects.

Rebound Hardness

Rebound hardness is also called as dynamic hardness. It is determined by the height of “bounce” of a diamond tipped hammer dropped from a fixed height on the material.