4022 FLUID MECHANICS & HYDRAULIC MACHINERY

Module 2

Course Outline

- **CO2 Apply conservation laws to fluid flow over notches and, through pipes and orifices.**
- M2.01 Explain the types of fluid flow, Rate of discharge, stream lines and path lines.
- M2.02 Explain the equation for continuity of flow without derivation. Derive & State Bernoulli's equation. State the limitations of the Bernoulli's theorem.
- M2.03 Solve fluid flow problems usingBernoulli's equation.
- M2.04 Explain the practical applications of Bernoulli's equation, Venturimeter, Orifice meter, Pitot Tube.
- M2.05 Solve problems related to the practical applications of Bernoulli's Theorem.
- M2.06 Explain the types of notches and orifices. Discuss the equations for discharge over a notch. Mention hydraulic coefficients.
- M2.07 List the losses of head in pipes and identify Major losses and Minor losses. Explain Hagen-Poiseuille equation for laminar flow (no derivation). Derive Darcy's formula and Chezy's formula for loss of head in pipes and explain the terms hydraulic gradient and total gradient line.
- M2.08 Solve Numerical problems to estimate major and minor losses.

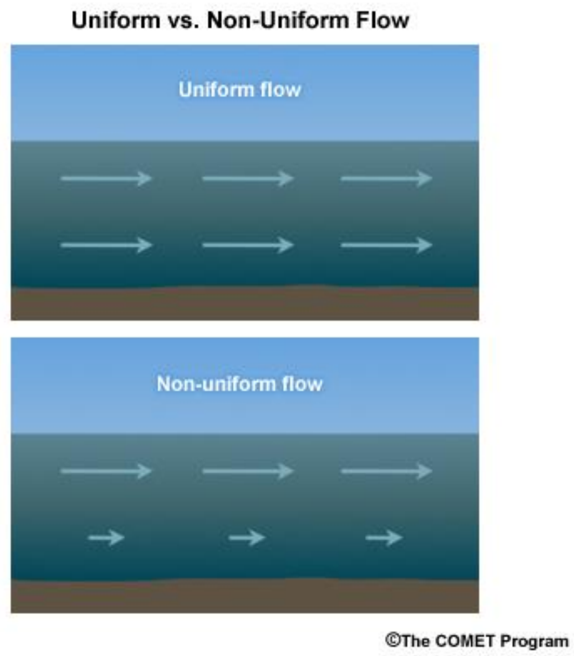
KINEMATICS & DYNAMICS

- FLUID KINEMATICS
- Which deals with motion of fluid without considering the forces which causes the motion.

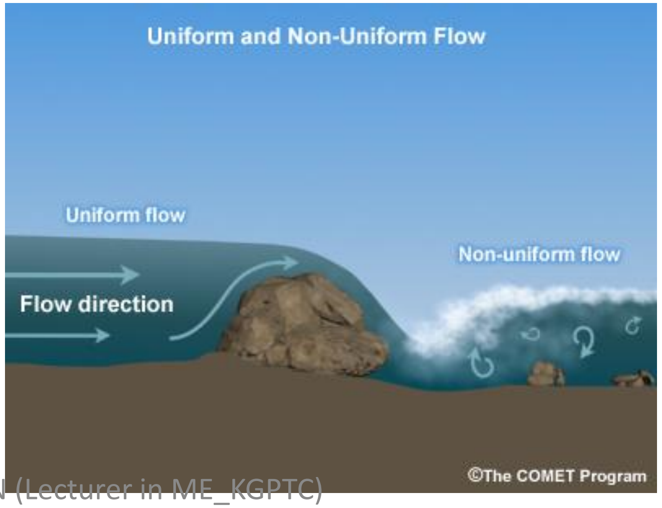
- FLUID DYNAMICS
- Which deals with the motion of fluids with considering the forces which causes the motion.

TYPES OF FLUID FLOW

- UNIFORM FLOW
- A uniform flow is defined as that type of flow in which the velocity of flow of fluid does not change in magnitude and direction from point to point in the flowing fluid for a given instant of time.

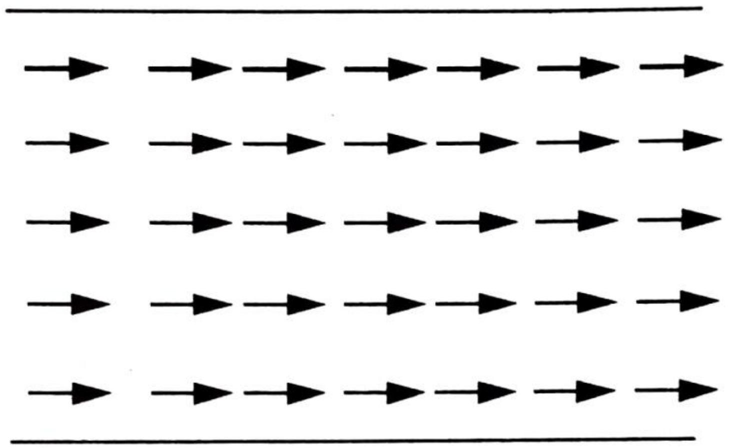


- NON UNIFORM FLOW
- It is defined as that type of flow in which the velocity of flow at any given time changes in magnitude and direction.



LAMINAR FLOW or VISCOUS FLOW or STREAM LINE FLOW

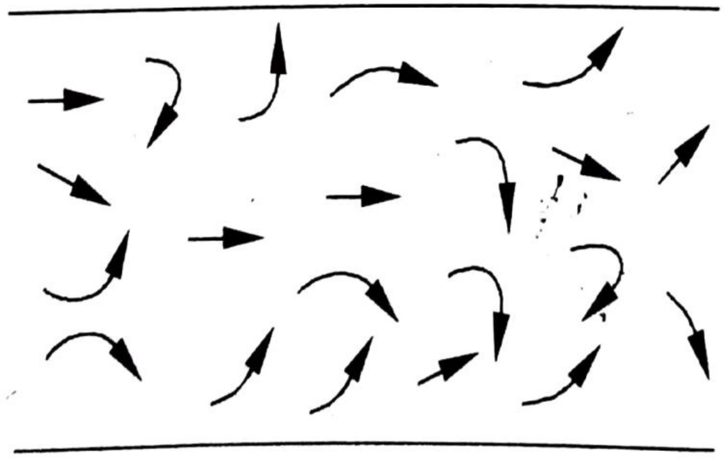
- A laminar flow is one in which the fluid particles moves along smooth regular paths, which can be predicted well in advance.
- The fluid thus move in layers, gliding smoothly over adjacent layers.
- Eg:- Flow of blood in small veins, Oil flow in bearings etc.



Laminar Flow

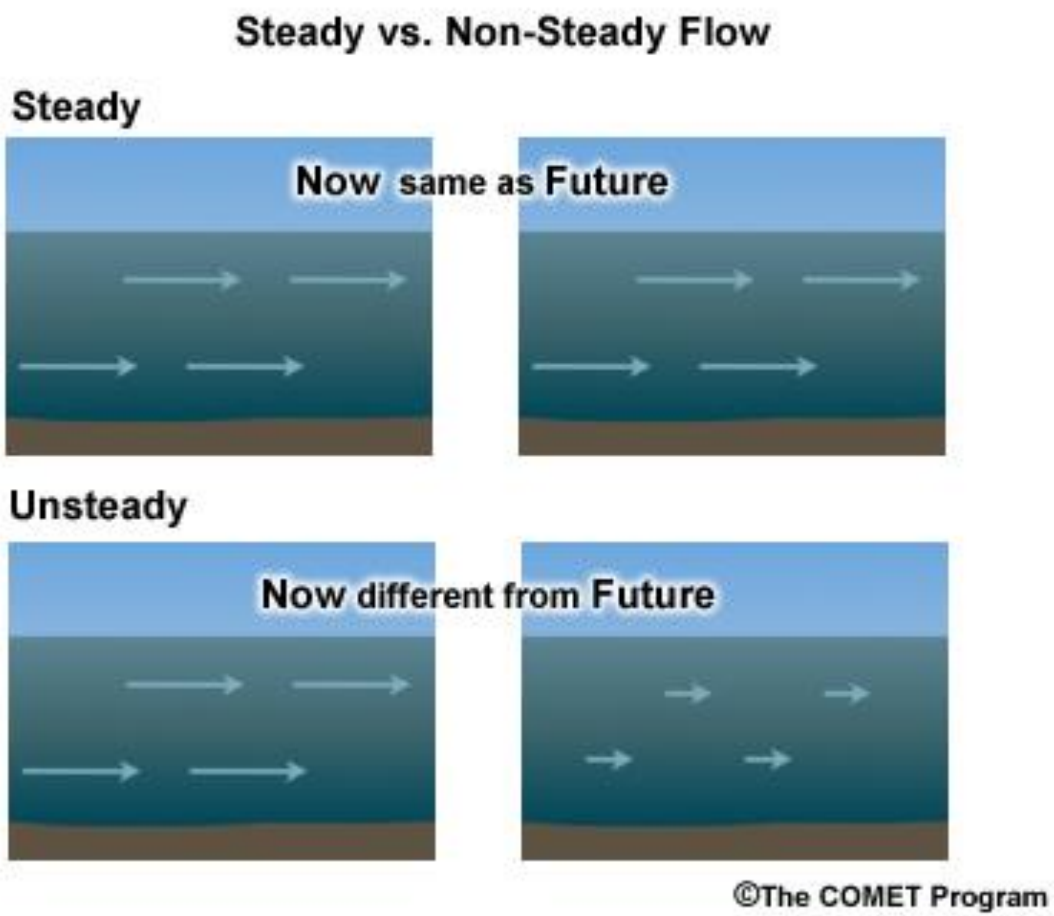
TURBULENT FLOW

- A flow is said to be turbulent when the fluid particles move in very irregular path.
- These results in the formation of eddy’s.



Turbulent flow

- STEADY FLOW
- A flow is said to be steady during which the fluid characteristics like pressure, density, velocity, acceleration etc at any point does not change with time.
- $\frac{dv}{dt} = 0, \qquad \frac{d\rho}{dt} = 0$, etc.
- UNSTEADY FLOW
- A fluid is said to be unsteady during which the fluid characteristics like pressure, density, velocity, acceleration etc at any point changes with time.
- $\frac{dv}{dt} \neq 0, \qquad \frac{d\rho}{dt} \neq 0$, etc.



- COMPRESSIBLE FLOW
- A flow in which the volume and density changes during the flow is called compressible flow.
- INCOMPRESSIBLE FLOW
- A flow in which the volume and density does not changes during the flow is called incompressible flow.
- ROTATIONAL FLOW
- A flow in which, the fluid particle rotate about their own axis while flowing is known as rotational flow.
- IRROTATIONAL FLOW
- A flow in which, the fluid particle do not rotate about their own axis while flowing is known as irrotational flow.

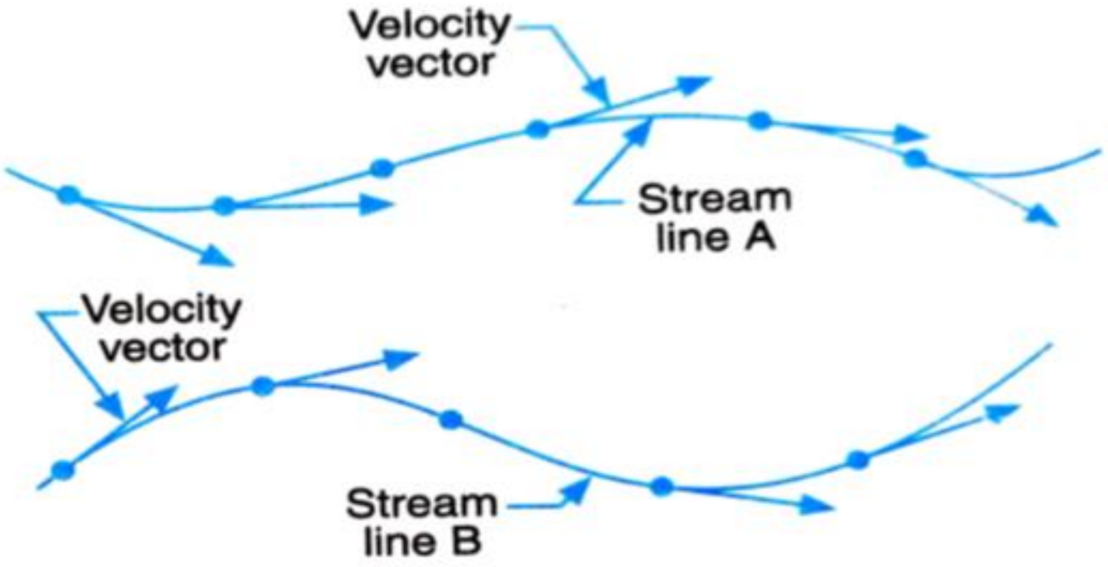
- ONE DIMENSIONAL FLOW
- A flow in which the velocity vector depends on only one space variable and time.
- V = f(x) – for steady flow
- V = f(x , t) – for unsteady flow
- TWO DIMENSIONAL FLOW
- A flow in which the velocity vector depends on two space variables and time.
- V = f(x , y) – for steady flow
- V = f(x , y , t) – for unsteady flow

- THREE DIMENSIONAL FLOW

- A flow in which the velocity vector depends on three space variables and time.
- V = f(x , y , z) – for steady flow
- V = f(x , y , z , t) – for unsteady flow

STREAM LINE

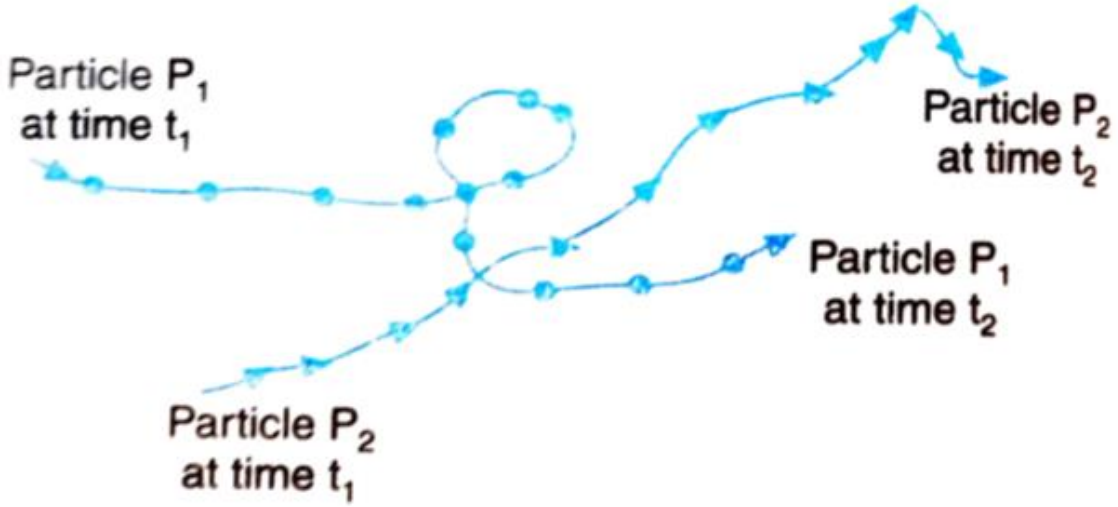
- A stream line is an imaginary line drawn through the flow field in a manner such that the velocity vector of the fluid at each and every point on the stream line is tangent to the stream line at that instant.



Concept of a streamline

PATH LINE

- A path line represents the trace or trajectory of a fluid particle over a period of time.



Pathline of a fluid particle

RATE OF FLOW or DISCHARGE (Q)

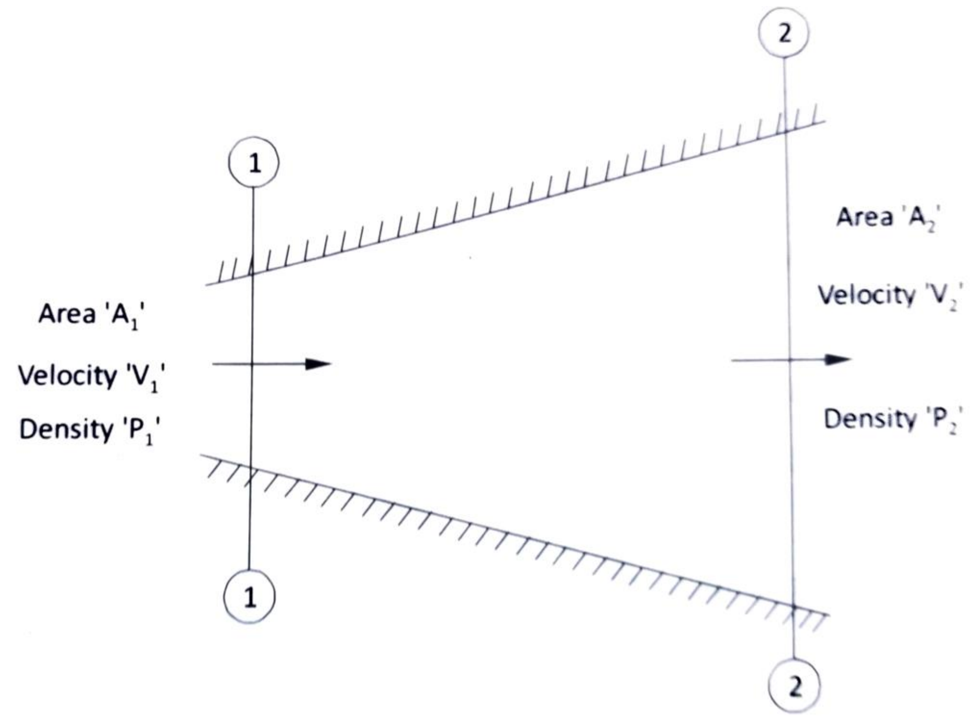
- It is defined as the quantity of fluid flowing through a section of pipe or channel per second is known as discharge.
- Discharge through a pipe, $Q = AV$
- Where, A = Area of pipe in m^2.
- $A = \frac{\pi}{4} d^2$
- d = Diameter of the pipe in meter (m).
- V = Average velocity of the fluid in m/s.
- Unit of discharge = m^3/s or cumec
- 1 cumec = 1 m^3/s = 1000 litre/s
- 1 litre/s = 10^{-3} m^3/s

CONTINUITY EQUATION

- It is based on the principle of conservation of mass.

- According to continuity equation, when a fluid flowing through a pipe or channel at all cross sections, the quantity of fluid flowing per second is constant.

- Consider a taper pipe in which some fluid is flowing.
- Let A_1 = Area of pipe at section 1-1
- V_1 = Velocity of liquid at section 1-1
- ρ_1 = Density of liquid at section 1-1
- A_2, V_2, ρ_2 are the corresponding values at section 2-2
- According to continuity equation,
- The quantity of fluid flowing per second at section 1-1 = The quantity of fluid flowing per second at section 2-2.



- The quantity of fluid flowing per second at section 1-1 = $\rho_1 A_1 V_1$
- The quantity of fluid flowing per second at section 2-2 = $\rho_2 A_2 V_2$
- Ie, $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ -for compressible fluids
- This equation is known as continuity equation.
- For an incompressible fluid, density is same.
- Ie, $\rho_1 = \rho_2$
- The continuity equation for incompressible fluid is given by,
- $A_1 V_1 = A_2 V_2$

ENERGY IN FLUID MOTION

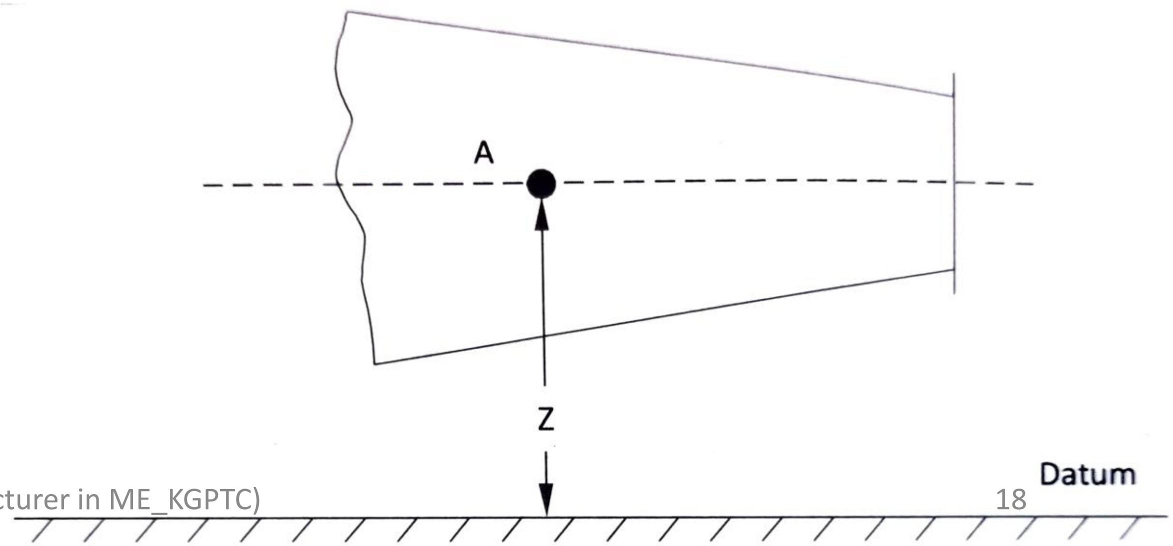
- Energy in a fluid is defined as the ability of a fluid to do work.

Types

- Potential energy
- Pressure energy
- Kinetic energy
- Total energy

POTENTIAL ENERGY

- The energy possessed by a liquid particle by virtue of its position is called potential energy (PE).
- Consider a liquid particle "A" at "Z" metre above the datum.
- Then the potential energy of the liquid particle will be,
- PE = m g Z (in Nm or Joule)
- Potential energy per unit mass = g Z (in Nm/kg)
- Potential energy is also expressed in terms of potential head or datum head
- ie, potential head or datum head
- = Z (in m of liquid)



PRESSURE ENERGY

- The energy possessed by a liquid particle by virtue of its pressure is called pressure energy.
- It is also expressed in pressure head.
- Pressure energy per unit mass = $\frac{P}{\rho}$ (in Nm/kg)
- Pressure head, h = $\frac{P}{\rho g} = \frac{P}{\omega}$ (in meter of liquid)

KINETIC ENERGY

- The energy possessed by a liquid particle by virtue of its velocity is called kinetic energy.
- We know, K.E = $\frac{1}{2}$ mV^2
- Where, m = mass of liquid in kg,
- V = velocity of the liquid in m/s
- K.E per unit mass = $\frac{1}{2}$ V^2 (in Nm/kg)
- Kinetic energy is also expressed in kinetic head.
- Kinetic head = $\frac{V^2}{2g}$ (in m of liquid)

TOTAL ENERGY OF LIQUID IN MOTION

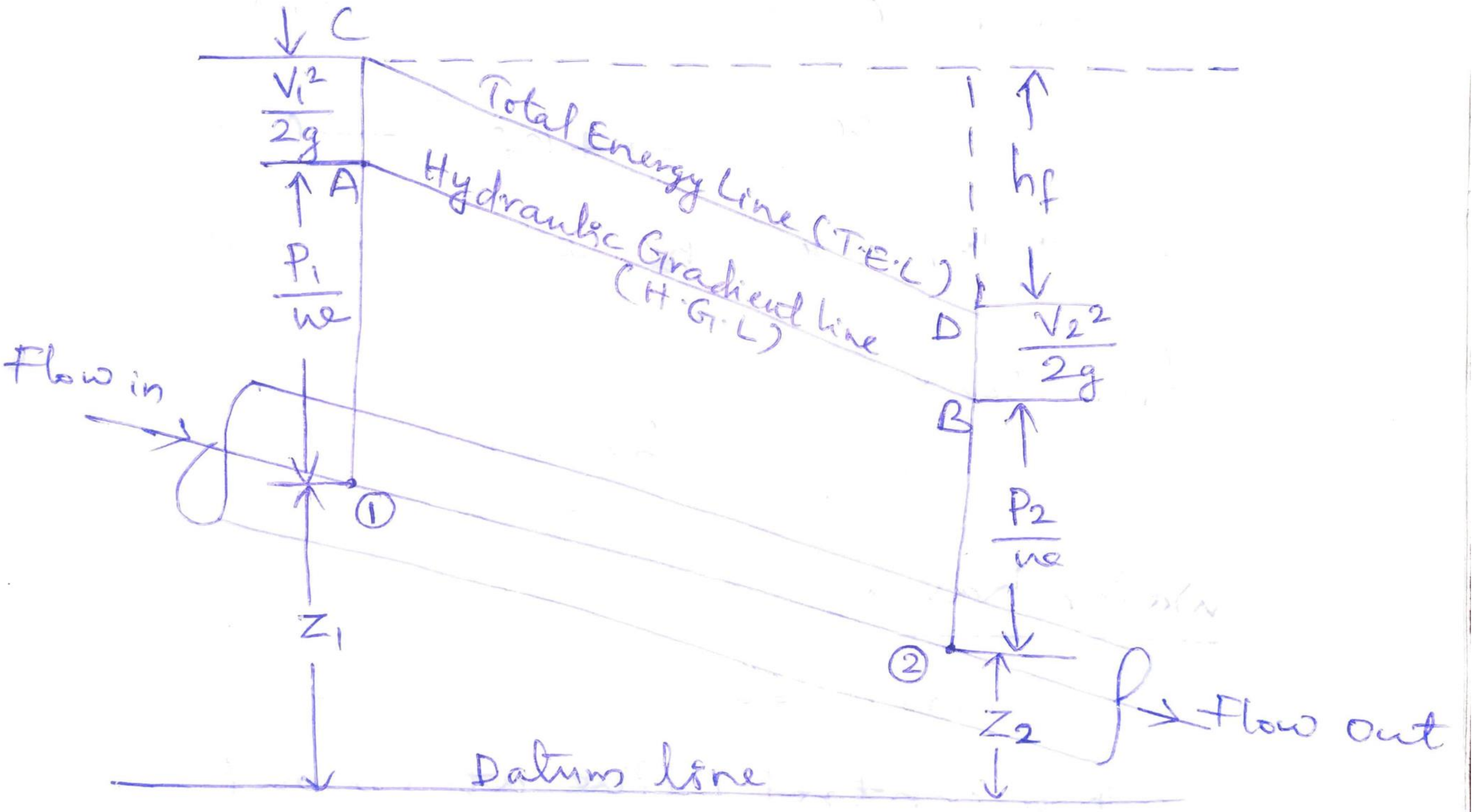
- It is the sum of pressure energy, kinetic energy and potential energy of the liquid particle.

- Total energy $= \frac{P}{\rho} + \frac{V^2}{2} + gZ$ in Nm/kg

- Total head $= \frac{P}{\omega} + \frac{V^2}{2g} + Z$ in m

HYDRAULIC GRADIENT LINE AND TOTAL ENERGY LINE

- These are the graphical representation of the piezometric head and the total energy along the length of the pipe respectively.
- The line joining the sum of the pressure head and datum head, $[\frac{P}{\omega} + Z]$ plotted along the length of the pipe line is known as hydraulic gradient line (HGL) or piezometric head line.
- The line joining the sum of the pressure head, datum head and velocity head $[\frac{P}{\omega} + \frac{V^2}{2g} + Z]$ plotted along the length of the pipe is known as total energy line (TEL).


C
$\frac{V_1^2}{2g}$
A
$\frac{P_1}{w}$
Total Energy Line (T.E.L)
Hydraulic Gradient line
(H.G.L)
h_f
D
$\frac{V_2^2}{2g}$
B
$\frac{P_2}{w}$
Flow in
①
Z_1
②
Z_2
Flow out
Datum line

EULER'S EQUATION

- Euler's equation for steady flow of an ideal fluid along a stream line is given by
- $\frac{dP}{\rho} + \mathrm{v}\ \mathrm{dv} + \mathrm{g}\ \mathrm{dz} = 0$

BERNOULLI'S THEOREM

- It states that "for a steady flow of frictionless incompressible fluid, the total energy of a particle remains constant at every points in its path of flow".

DERIVATION OF BERNOULLI'S EQUATION

- We know, Euler's equation for steady flow of an ideal fluid along a stream line is given by
- $\frac{dP}{\rho} + \mathrm{v\ dv} + \mathrm{g\ dz} = 0$
- Now integrating the above equation we get Bernoulli's equation as follows.
- $\int (\frac{dP}{\rho} + \mathrm{vdv} + \mathrm{gdz}) = 0$
- $\int \frac{dP}{\rho} + \int \mathrm{vdv} + \int \mathrm{gdz} = 0$

- $\frac{1}{\rho} \int dP + \int vdv + g \int dz = Constant$
- $\frac{P}{\rho} + \frac{v^2}{2}$ + gz = Constant
- Dividing all the terms by g, we have
- $\frac{P}{\rho g} + \frac{v^2}{2g}$ + z = Constant
- $\frac{P}{\omega} + \frac{v^2}{2g}$ + z = Constant
- The above equation is known as Bernoulli's equation.
- $\frac{\mathrm{P}}{\omega}$ = Pressure head
- $\frac{\mathrm{V}^2}{2\mathrm{g}}$ = Kinetic head
- Z = Datum head

ASSUMPTIONS IN BERNOULLI'S THEOREM

1. Flow is steady and continuous.
2. Flow is incompressible and homogenous.
3. Flow is irrotational.
4. The velocity of flow is uniform over the section.
5. No force except gravitational force is acting on the fluid.
6. Bernoulli's equation is derived on the assumption that the fluid is non viscous or frictionless.

- But all the real fluids are viscous and there are always some losses occurred due to friction. This losses have to be taken into consideration.
- Thus the Bernoulli's equation for the real fluids between section 1-1 and 2-2 is given by, $\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2 + h_L$
- Where, h_L = Head loss due to friction between section 1-1 and 2-2.

LIMITATIONS OF BERNOULLI’S THEOREM

- The velocity of liquid particle across any cross section of pipe is uniform, is not practical.

✓In actual practice, the velocity of liquid particle in the centre of a pipe is maximum and gradually decreases towards the wall of the pipe due to friction.

✓Thus while using Bernoulli’s equation, the mean velocity of liquid is considered.

- The external forces except the gravity acts on the liquid is also not practical.

✓There are always some external forces like pipe friction acts on the liquid and affects its flow.

- There is no loss of energy of the liquid particle while flowing is not practical.

✓ But in turbulent flow, some kinetic energy is converted into heat energy and in viscous flow, some energy is lost due to shear force.

- Bernoulli’s theorem does not taken into account, the energy due to centrifugal forces for a liquid flowing in curved path.

Q.1. A pipe of diameter 200 mm conveys 2500 litre/min of water and has a pressure of 20 kN/m^2 at a certain section. Find the total energy head with respect to a datum 5 m below the pipe?

Given data:

- d = 200 mm = 0.2 m
- Q = 2500 litre/min
- P = 20 kN/m^2 = 20 x 10^3 N/m^2
- Z = 5 m
- Total head = ?
- Q = $\frac{2500}{60}$ Litre/s
- $= \frac{2500 \times 10^{-3}}{60}$ m^3/s
- = 0.0416 m^3/s

- Total head = $\frac{P}{\omega} + \frac{V^2}{2g} + Z$
- To find the velocity, V = Q/A (Q = AV)
- Area , A = $\frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\ m^2$
- V = 0.0416 / 0.0314 = 1.32 m/s
- Total head = $\frac{20 \times 10^3}{9810} + \frac{(1.32)^2}{2 \times 9.81} + 5$
- = 7.127 m
-

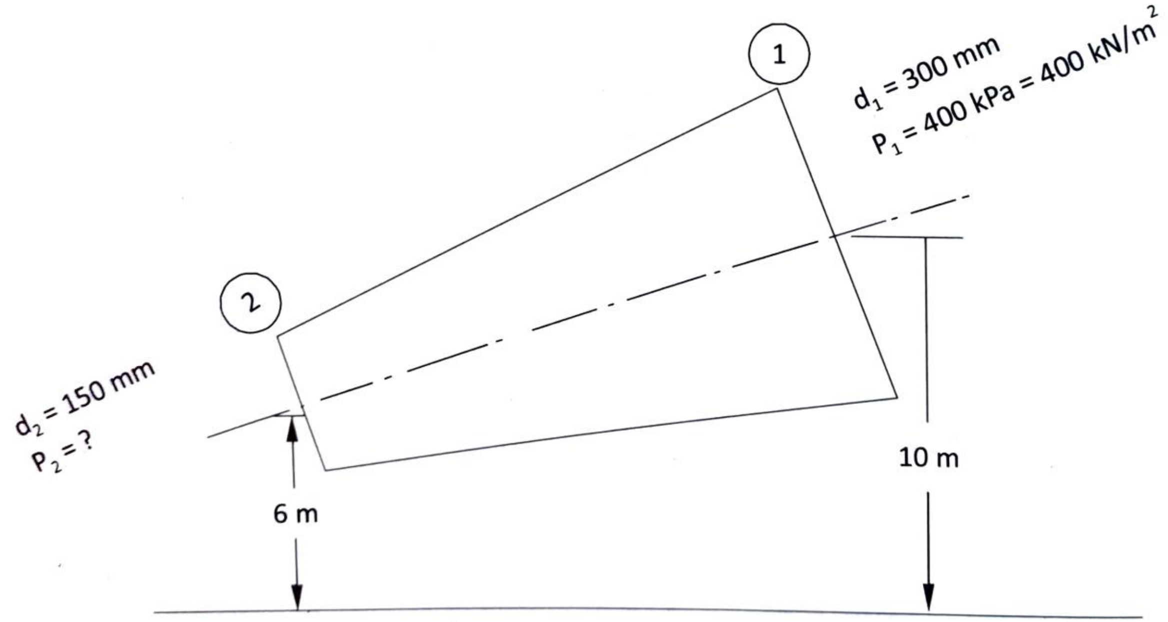
Q.2. Find the total energy of 3 kg of water flowing with a velocity of 5 m/s under a pressure of 4 bar at a height of 10 m above the ground level.

Given data:

- m = 3 kg, V = 5 m/s
- P = 4 bar = 4×10^5 N/m^2
- Z = 10 m, Total energy = ?
- Total energy = $\frac{P}{\rho} + \frac{V^2}{2} + g\,Z$ in Nm/kg
- Total energy = $\frac{4 \times 10^5}{1000} + \frac{5^2}{2} + 9.81 \times 10$
- Total energy per unit mass = 510.6 Nm/kg
- Total energy of 3 kg of water = 510.6 x 3
- = 1531.8 Nm or Joule

Q.3. The water is flowing through a tapering pipe having diameter 300 mm and 150 mm at section 1-1 and 2-2 respectively. The discharge through the pipe is 40 litre/s, the section 1 is 10 m above the datum and section 2 is 6 m above the datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kPa?

- Given data:
- d_1 = 300 mm = 0.3 m
- d_2 = 150 mm = 0.15 m
- $Q = Q_1 = Q_2$ = 40 litre/s
- $= 40 \times 10^{-3}\ m^3/s$
- Z_1 = 10 m
- Z_2 = 6 m
- $P_1 = 400\ kN/m^2 = 400 \times 10^3\ N/m^2$
- P_2 = ?



- $\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2$

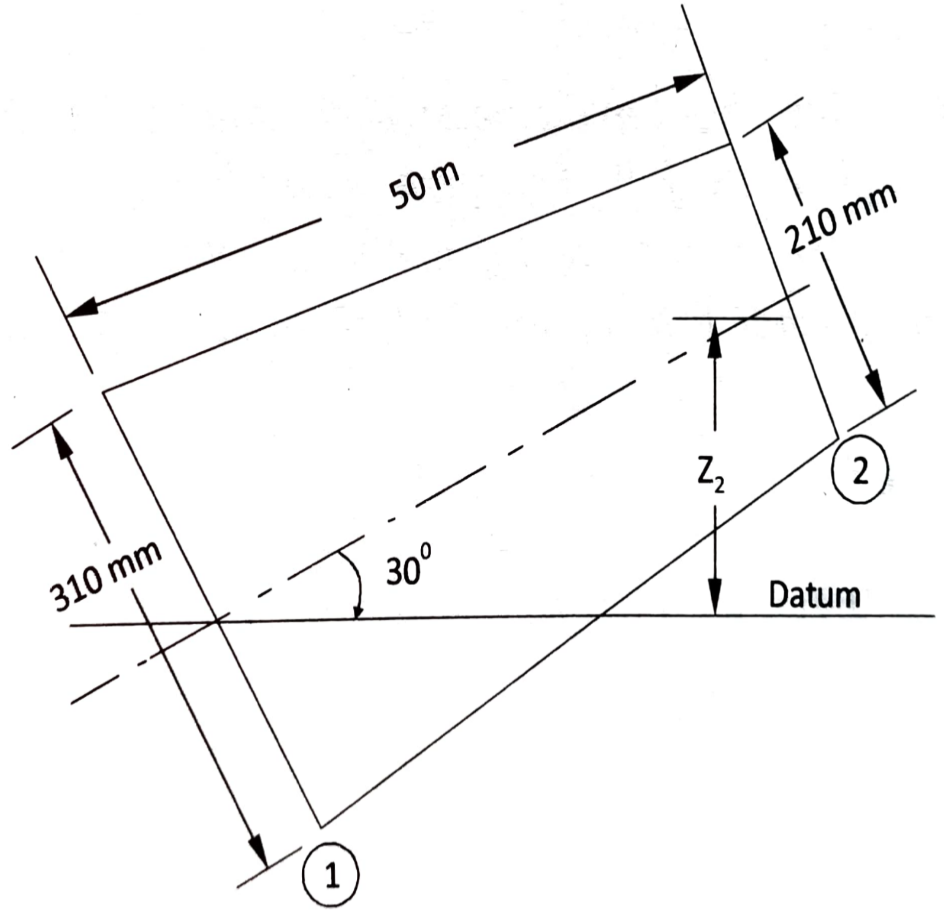
- $A_1 = \frac{\pi}{4} d_1^2 \quad = \frac{\pi}{4} \text{ x } (0.3)^2 = 0.0706 \text{ m}^2$
- $A_2 = \frac{\pi}{4} d_2^2 \quad = \frac{\pi}{4} \text{ x } (0.15)^2 = 0.0176 \text{ m}^2$
- $Q_1 = A_1 V_1$
- $40 \text{ x } 10^{-3} = 0.0706 \text{ x } V_1$
- $V_1 \quad = 0.566 \text{ m/s}$
- $Q_2 = A_2 V_2$
- $40 \text{ x } 10^{-3} = 0.0176 \text{ x } V_2$
- $V_2 \quad = 2.272 \text{ m/s}$

- $\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2$
- $\omega = 9810 \text{ N/m}^3$
- $\frac{400 \text{ x } 10^3}{9810} + \frac{(0.566)^2}{2\ x\ 9.81} + 10$
- $= \frac{P_2}{9810} + \frac{(2.272)^2}{2\ x\ 9.81} + 6$
- $40.77 + 0.0163 + 10 = \frac{P_2}{9810} + 0.263 + 6$
- $50.78 = \frac{P_2}{9810} + 6.263$
- $\frac{P_2}{9810} = 50.78 - 6.263$
- $= 44.517$
- $P_2 = 44.517 \text{ x } 9810$
- $= 436711.77 \text{ N/m}^2$
- $= 436.71 \text{ k Pa}$

Q.4. A 50 m long pipe is placed at an angle of 30^0 with the horizontal at the higher end. The diameter of pipe is 210 mm which is gradually increases to 310 mm at the lower end where the pressure is 260 kPa. If the discharge through the pipe is 160 litre/s of water. Find the pressure at the upper end?

Given data:

- $L = 50$ m
- $\theta = 30^0$
- $d_1 = 310$ mm $= 0.31$ m
- $d_2 = 210$ mm $= 0.21$ m
- $P_1 = 260$ kPa $= 260 \times 10^3$ N/m^2
- $Q = Q_1 = Q_2 = 160$ litre/s
- $= 160 \times 10^{-3}$ m^3/s
- $Z_1 = 0$ m
- $P_2 = ?$
- $\sin\theta = \frac{Z_2}{50}$
- $Z_2 = 50 \times \sin 30 = 25$ m

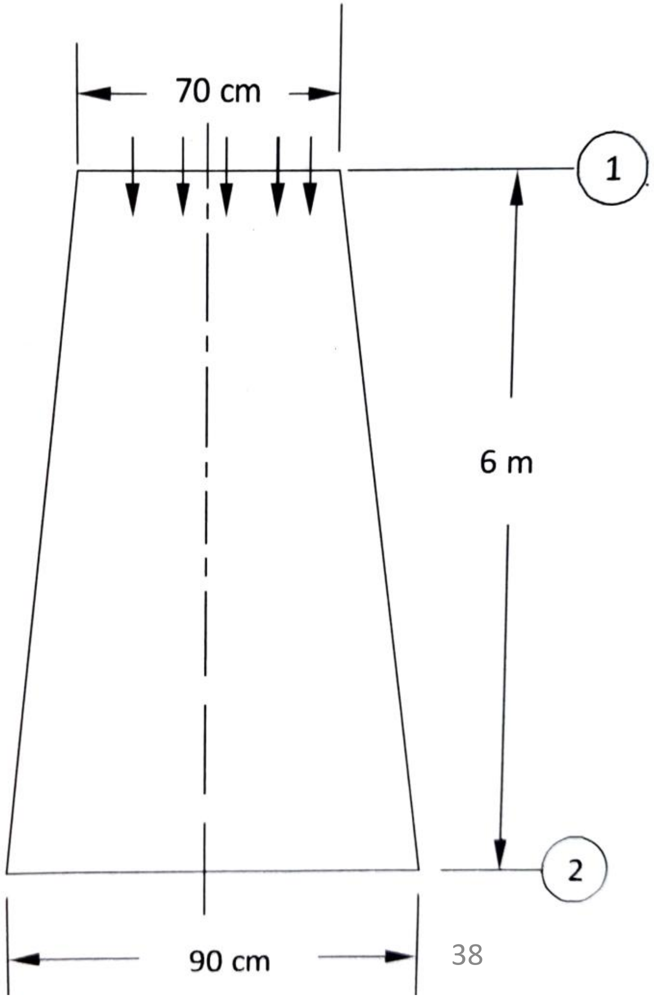


- $A_1 = \frac{\pi}{4} d_1^2 \quad = \frac{\pi}{4} \times (0.31)^2 = 0.0754\ m^2$
- $A_2 = \frac{\pi}{4} d_2^2 \quad = \frac{\pi}{4} \times (0.21)^2 = 0.0346\ m^2$
- $Q_1 = A_1 V_1$
- $160 \times 10^{-3} = 0.0754 \times V_1$
- $V_1 \quad = 2.12\ m/s$
- $Q_2 = A_2 V_2$
- $160 \times 10^{-3} = 0.0346 \times V_2$
- $V_2 \quad = 4.624\ m/s$
- $\frac{P_1}{\omega} + \frac{V_1^{\ 2}}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^{\ 2}}{2g} + Z_2$
- $\omega = 9810\ N/m^3$
- Substituting
- $\frac{260 \times 10^3}{9810} + \frac{(2.12)^2}{2 \times 9.81} + 0$
- $= \frac{P_2}{9810} + \frac{(4.624)^2}{2 \times 9.81} + 25$
- $26.50 + 0.229 = \frac{P_2}{9810} + 1.09 + 25$
- $26.729 = \frac{P_2}{9810} + 26.09$
- $\frac{P_2}{9810} = 26.729 - 26.09$
- $= 0.639$
- $P_2 = 0.639 \times 9810$
- $= 6268.59\ N/m^2$
- $= 6.26\ k\ Pa$

Q.5. A vertical tapering pipe has top diameter 70 cm and bottom diameter 90 cm, the water is flowing down in full. The pipe is 6 m long. The velocity at inlet is 6 m/s. Determine the pressure at top in N/mm^2. when the pressure head at the bottom is 8.8 m of water. Friction loss is 15 % of velocity head at inlet.

Given data:

- d_1 = 70 cm = 0.7 m
- d_2 = 90 cm = 0.9 m
- L = 6 m
- V_1 = 6 m/s
- P_1 = ?
- $\frac{P_2}{\omega}$ = 8.8 m
- Z_1 = 6 m
- Z_2 = 0 m
- Friction loss = 15 % of velocity head at inlet



- Velocity head at inlet = $\frac{V_1^{\,2}}{2g}$
- $= \frac{6^2}{2x\ 9.81}$ = 1.83 m of water
- Friction loss = $\frac{15}{100}$ x 1.83
- = 0.2752 m of water
- $A_1 = \frac{\pi}{4} d_1^{\,2} \quad = \frac{\pi}{4} \times (0.7)^2 = 0.3848\ m^2$
- $A_2 = \frac{\pi}{4} d_2^{\,2} \quad = \frac{\pi}{4} \times (0.9)^2 = 0.636\ m^2$
- $Q_1 = A_1V_1$
- = 0.3848 x 6 = 2.31 m^3/s
- $Q_2 = A_2V_2$
- 2.31 = 0.636 x V_2
- $V_2 \quad = 3.63$ m/s

- $\frac{P_1}{\omega} + \frac{V_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^{\,2}}{2g} + Z_2 + h_L$
- $\frac{P_1}{\omega} + \frac{(6)^2}{2\ x\ 9.81} + 6$
- $= 8.8 + \frac{(3.63)^2}{2\ x\ 9.81} + 0 + 0.2752$
- $\frac{P_1}{\omega} + 1.83 + 6 \ = 8.8 + 0.671 + 0.2752$
- $\frac{P_1}{9810} = 9.746 - 7.83$
- = 1.916
- P_1 = 1.916 x 9810
- = 18795.96 N/m^2
- = 0 .0187 N/mm^2

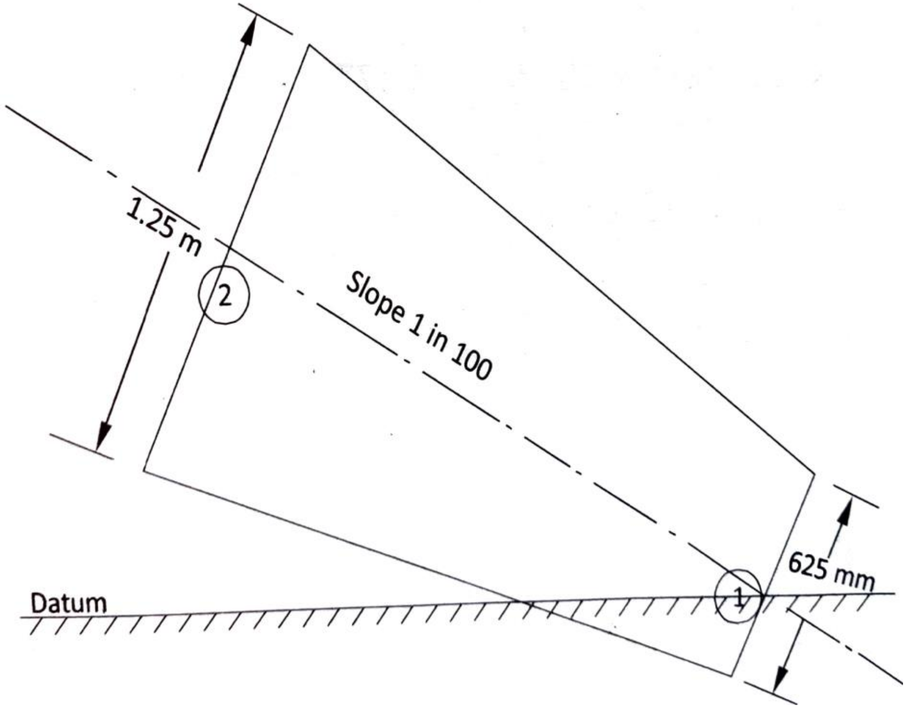
Q.6. A pipe 300 m long has a slop of 1 in 100 and tapers from 1.25 m diameter at the higher end and of 625 mm diameter at the lower end. Find the pressure at the lower end if the pressure at higher end is 0.1 N/mm^2 and discharge through the pipe is 100 litre/s of water.

Given data:

- $L = 300$ m
- $d_1 = 625$ mm $= 0.625$ m
- $d_2 = 1.25$ m
- $P_1 = ?$
- $P_2 = 0.1$ N/mm^2 $= 0.1 \times 10^6$ N/m^2
- $Q = 100$ litre/s

  $= 100 \times 10^{-3}$ m^3/s

  $= 0.1$ m^3/s
- $Z_1 = 0$ m
- $Z_2 = \frac{1}{100} \times 300 = 3$ m



- $A_1 = \frac{\pi}{4} d_1^2 \quad = \frac{\pi}{4} x\ (0.625)^2 = 0.306\ m^2$
- $A_2 = \frac{\pi}{4} d_2^2 \quad = \frac{\pi}{4} x\ (1.25)^2 = 1.227\ m^2$
- $Q_1 = A_1V_1$
- $0.1 = 0.306\ x\ V_1$
- $V_1 = 0.326\ m/s$
- $Q_2 = A_2V_2$
- $0.1 = 1.227\ x\ V_2$
- $V_2 = 0.0814\ m/s$
- $\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2$
- $\omega = 9810\ N/m^3$
- Substituting
- $\frac{P_1}{\omega} + \frac{(0.326)^2}{2\ x\ 9.81} + 0$
- $= \frac{0.1\ x\ 10^6}{9810} + \frac{(0.0814)^2}{2\ x\ 9.81} + 3$
- $\frac{P_1}{\omega} + 5.416\ x\ 10^{-3} = 10.193 + 3.377\ x\ 10^{-4} + 3$
- $\frac{P_1}{9810} + 5.416\ x\ 10^{-3} = 13.193$
- $\frac{P_1}{9810} = 13.187$
- $P_1 = 13.187\ x\ 9810$
- $= 129364.47\ N/m^2$
- $= 129.36\ k\ Pa$

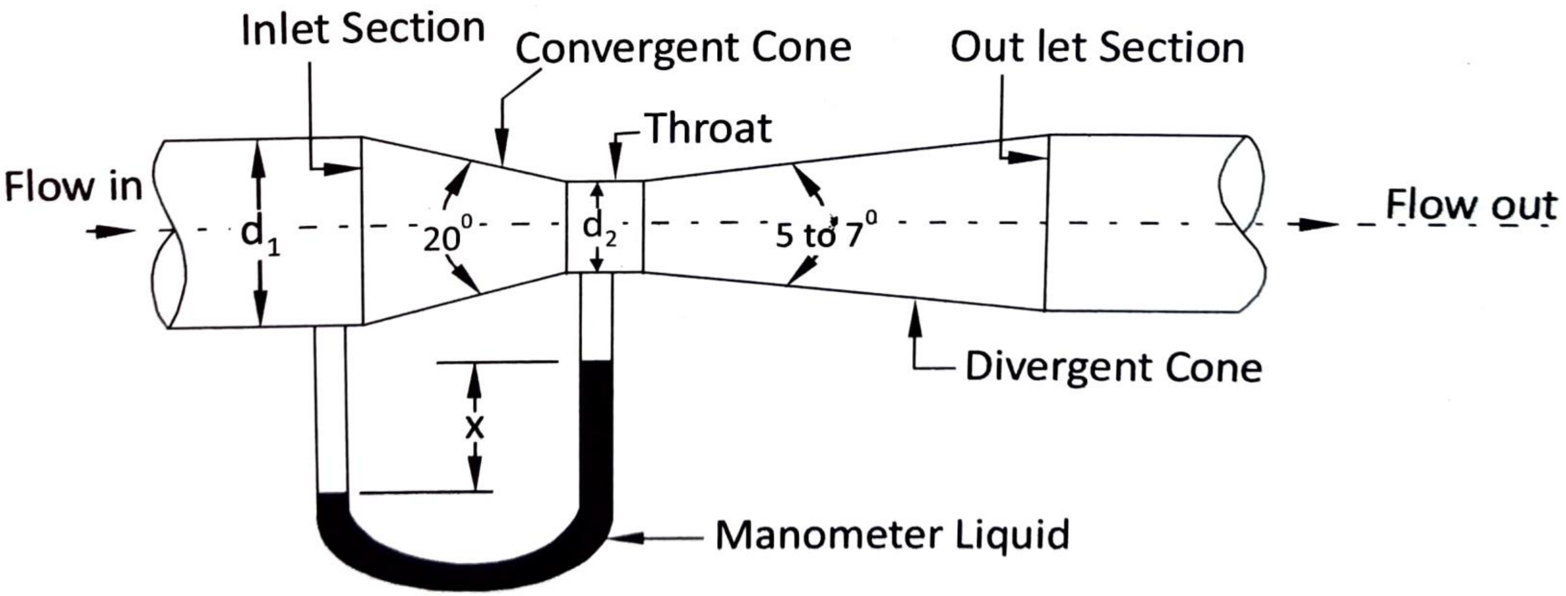
APPLICATIONS OF BERNOULLI’S THEOREM

- The following measuring devices utilises the principle of Bernoulli’s equation.

1. Venturimeter
2. Pitot tube
3. Orifice meter

VENTURIMETER

- It is a device for measuring discharge or rate of flow of fluid flowing in a pipe. It consists of three components.
- Convergent cone
- Throat
- Divergent cone



CONVERGENT CONE

- It is a short taper pipe which converges from the pipe diameter to throat diameter.
- The velocity of fluid increases as it passes through the convergent section.
- The angle of the convergent cone varies from 20^0 to 22^0.

THROAT

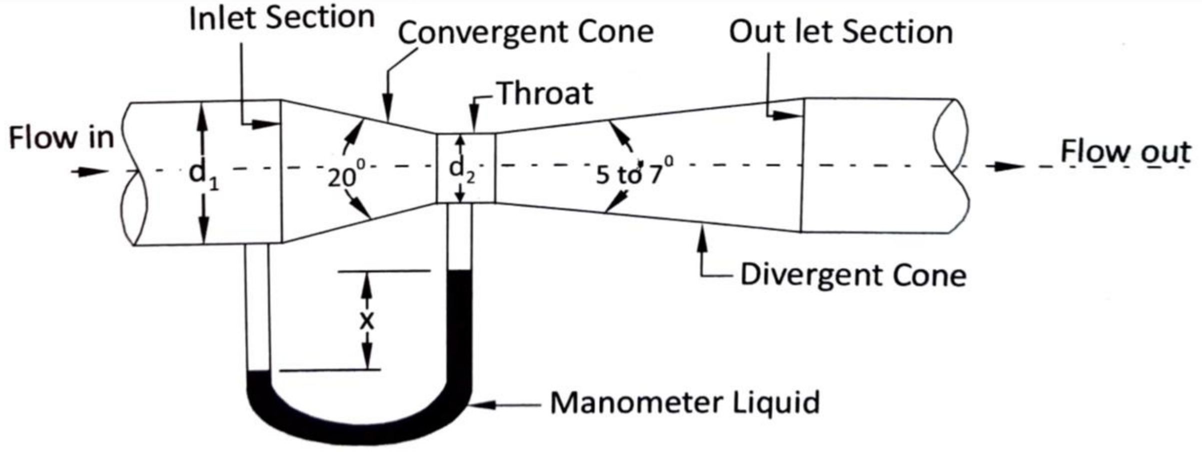
- It is a short circular pipe in which the diameter is kept constant.
- At this section the velocity is maximum and the pressure is minimum.
- The throat diameter is usually ½ to ¼ of inlet diameter and the length of throat is equal to its diameter.

DIVERGENT CONE

- It is a tapered pipe of gradually diverging cross sectional area from that of throat to the original size of the pipe.
- The divergent cone is also known as outlet of venturimeter.
- The angle of divergent cone varies from 5^0 to 7^0.

DISCHARGE THROUGH VENTURIMETER

- Applying Bernoullis equation at inlet and throat, we get
- $\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g} + Z_2$
- Since the pipe is horizontal, there fore the datum heads at section 1 and 2 are same, ie, $Z_1 = Z_2$



- $\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g}$
- $\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g}$
- $\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$
- $\frac{P_1 - P_2}{\rho g}$ = Difference of pressure head at section- 1 and section-2 and is equal to h.

- $\frac{P_1 - P_2}{\rho g} = h$
- $h = \frac{v_2^2 - v_1^2}{2g}$
- Now applying continuity equation at section 1 and 2, we have
- $Q = a_1 v_1 = a_2\, v_2$
- $v_1 = \frac{a_2\, v_2}{a_1}$
- $h = \frac{V_2^2 - (\frac{a_2 v_2}{a_1})^2}{2g} = \frac{V_2^2}{2g}(1 - \frac{a_2^2}{a_1^2})$
- $= \frac{V_2^2}{2g}(\frac{a_1^2 - a_2^2}{a_1^2})$
- $v_2^2 = 2gh \times \frac{a_1^2}{a_1^2 - a_2^2}$

- $v_2 = \sqrt{2gh\ x\ \frac{a_1^2}{a_1^2 - a_2^2}}$
- $= \sqrt{2gh}$ x $\frac{a_1}{\sqrt{a_1^2 - a_2^2}}$
- We know that discharge Q = $a_2 v_2$, substituting the value of v_2 in this equation,
- Q = $\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}$ x $\sqrt{2gh}$
- The above equation gives the theoretical discharge under ideal conditions. Actual discharge will be less than the theoretical discharge.
- Actual discharge, $Q_{act} = C_d\ x\ \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}$ x $\sqrt{2gh}$

- The actual discharge through a venturimeter is given by,
- $$Q_{act} = C_d \frac{a_1 \text{ x } a_2 \text{ x } \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$
- Where, $\frac{a_1 \text{ x } a_2 \text{ x } \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}} = Q_t$ = Theoretical discharge
- $Q_{act} = C_d \times Q_t$
- C_d = Coefficient of discharge (standard value ranges from 0.90 -0.99)
- $= \frac{Q_{act}}{Q_t}$
- a_1 = Area of inlet pipe in m^2 ($a_1 = \frac{\pi}{4} {d_1}^2$, d_1= diameter of the pipe at inlet)
- a_2 = Throat area of the Venturimeter in m^2 ($a_2 = \frac{\pi}{4} {d_2}^2$, d_2= diameter of the pipe at throat)
- h = Manometric pressure head (Difference between the pressure head at inlet and throat)

‘h’ can be calculated by

CASE 1:

- If the liquid in the differential manometer is heavier than the liquid flowing through the pipe.
- $h = x\left[\frac{S_m}{S_f} - 1\right]$ for $S_m > S_f$
- Where, x = Manometer deflection
- S_m = Specific gravity of manometric liquid
- S_f = Specific gravity of flowing fluid

CASE 2:

- If the liquid in the differential manometer is lighter than the liquid flowing through the pipe.
- $h = x\,[1 - \frac{S_m}{S_f}]$ for $S_m < S_f$
- Where, x = Manometer deflection
- S_m = Specific gravity of manometric liquid
- Sf = Specific gravity of flowing fluid

Q.1. A horizontal venturimeter has a diameter of 300 mm and a throat diameter of 200 mm. If the discharge is 10000 litres of water/min. When the difference of pressure head between inlet and throat is 1.5 m of water. Find the coefficient of discharge for venturimeter.

- Given data:
- d_1 = 300 mm = 0.3 m
- d_2 = 200 mm = 0.2 m
- Q_{act} = 10000 litres/min
- = 10000 x 10^{-3} / 60
- = 0.167 m^3/s
- h = 1.5 m
- C_d = ?
- $Q_{act} = C_d \dfrac{a_1 \text{ x } a_2 \text{ x } \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

- $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \text{ x } (0.3)^2 = 0.0706 \text{ m}^2$
- $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \text{ x } (0.2)^2 = 0.0314 \text{ m}^2$
- Substituting,
- $Q_{act} = C_d \dfrac{a_1 \text{ x } a_2 \text{ x } \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$
- $0.167 = C_d \dfrac{0.0706 \text{ x } 0.0314 \text{ x } \sqrt{2\text{x } 9.81\text{x } 1.5}}{\sqrt{(0.0706)^2 - (0.0314)^2}}$
- $0.167 = C_d \text{ x } \dfrac{0.012}{0.063}$
- $0.167 = C_d$ x 0.190
- $C_d = 0.878$

Q.2. Petrol of specific gravity 0.8 flows through a pipe of 300 mm diameter and the pipe is inclined at 30^0 to the horizontal. The venturimeter connected to the mains has its throat of 100 mm diameter. The mercury differential manometer connected between the mains and the throat reads 60 mm. Calculate the discharge? Assume the coefficient of discharge is to be unity.

- Given data:
- $S_p = 0.8 = S_f$
- $d_1 = 300$ mm $= 0.3$ m
- $\theta = 30^0$
- $d_2 = 100$ mm $= 0.1$ m
- x = 60 mm of mercury
-   = 0.06 m of mercury
- $S_m = 13.6$
- $C_d = 1$
- $Q_{act} = ?$

- $h = x\left[\frac{S_m}{S_f} - 1\right] = 0.06\left[\frac{13.6}{0.8} - 1\right]$
- $= 0.96$ m
- $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0706\ m^2$
- $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.1)^2 = 7.853 \times 10^{-3}\ m^2$
- $Q_{act} = C_d \frac{a_1 \times a_2 \times \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$
- $Q_{act} = 1 \times \frac{0.0706 \times 7.853 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 0.96}}{\sqrt{(0.0706)^2 - (7.853 \times 10^{-3})^2}}$
- $Q_{act} = \frac{2.406 \times 10^{-3}}{0.0701}$
- $Q_{act} = 0.0343\ m^3/s$

Q.3. A horizontal venturimeter is provided in a pipe line 300 mm diameter carrying water. The throat diameter is 150 mm. If the pressure in pipe is 160 kPa and that of the throat is 350 mm of mercury. Find the discharge in litre/s, if $C_d = 0.98$?

Given data:

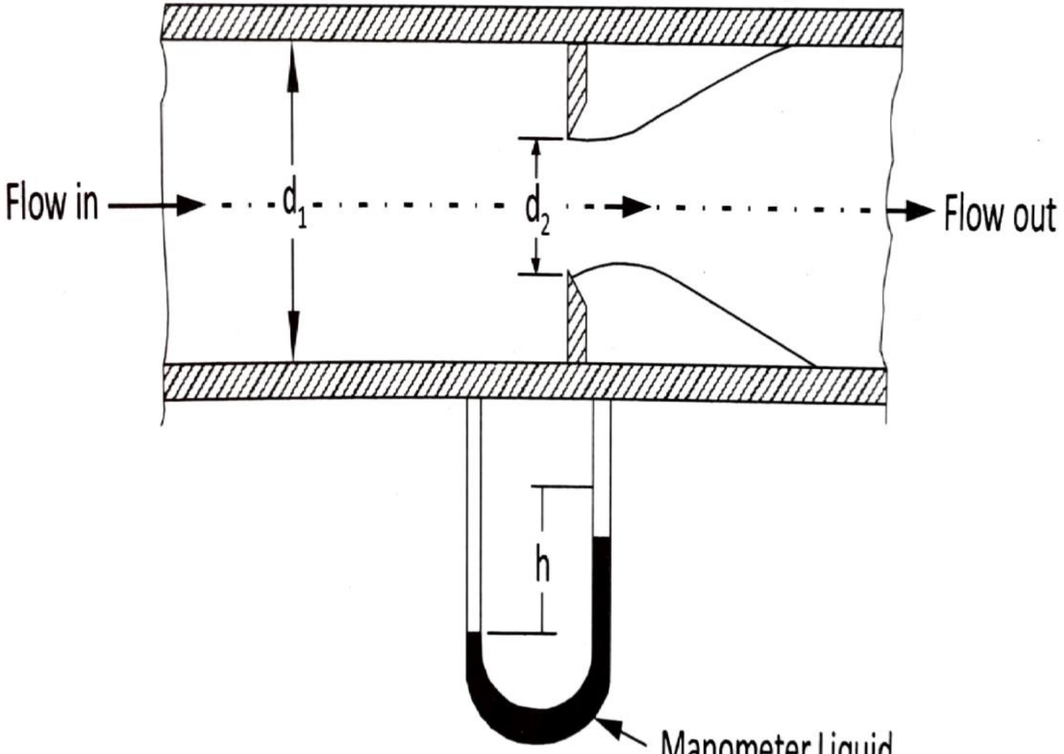
- $d_1 = 300$ mm = 0.3 m
- $d_2 = 150$ mm = 0.15 m
- $P_1 = 160$ kPa = 160×10^3 Pa
- $\frac{P_2}{\omega} = 350$ mm of mercury = h_m
- = 0.35 m of mercury
- $C_d = 0.98$
- $Q_{act} = ?$
- $a_1 = \frac{\pi}{4} d_1^2 \quad = \frac{\pi}{4} \times (0.3)^2 = 0.0706 \text{ m}^2$
- $a_2 = \frac{\pi}{4} d_2^2 \quad = \frac{\pi}{4} \times (0.15)^2 = 0.0176 \text{ m}^2$

- $h_1 = \frac{P_1}{\omega} = \frac{160 \times 103}{9810}$
- = 16.31 m of water
- To convert the head in terms of Hg into m of water,
- $S_w h_w = S_m h_m$
- $1 \times h_w = 13.6 \times 0.35$
- $h_w = 4.76$ m of water = h_2
- $h = h_1 - h_2$
- = 16.31 – 4.76 = 11.5 m of water
- $Q_{act} = C_d \frac{a_1 \times a_2 \times \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

- $Q_{act} = 0.98 \times \dfrac{0.0706 \times 0.0176 \times \sqrt{2 \times 9.81 \times 11.5}}{\sqrt{(0.0706)^2 - (0.0176)^2}}$
- $= \dfrac{0.98 \, x \, 0.0186}{0.0684}$
- $= 0.26649$ m^3/s
- $= 0.26649 \times 10^3$ litre/s
- $= 266.49$ litre/s

ORIFICE METER

- It is a device used for measuring discharge or the rate of flow through pipes.
- It also works on the same principle as venturimeter.
- An orifice meter consists of a flat circular plate with a circular hole called orifice.
- Which is concentric with the pipe as shown in figure.

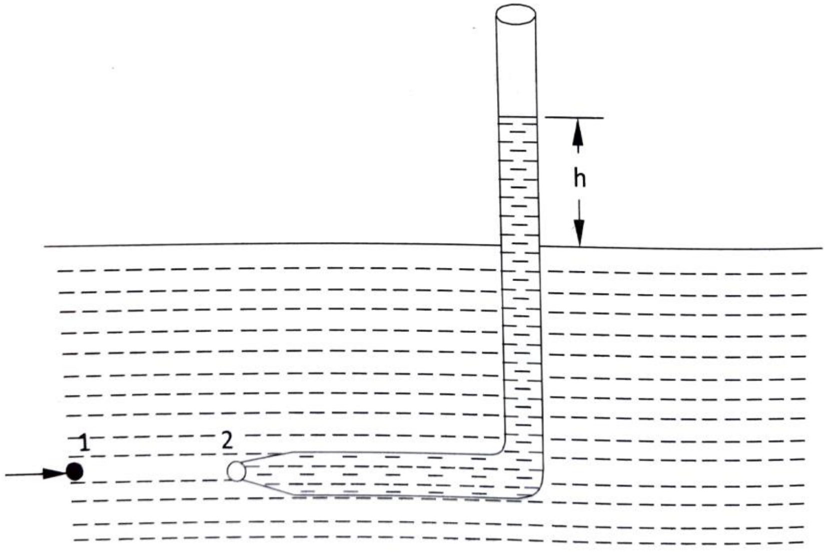


Orifice Meter

- Actual discharge through an orifice is given by,
- $Q_{act} = C_d \frac{a_1 \text{ x } a_2 \text{ x } \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$
- C_d = coefficient of discharge of orifice meter.
- (its value ranges from 0.6 to 0.65)
- a_1 = Area of the pipe at section 1-1 in m^2
- ($a_1 = \frac{\pi}{4} {d_1}^2$, d_1= diameter of the pipe at section 1-1)
- a_2 = Area of the pipe at section 2-2 in m^2
- ($a_2 = \frac{\pi}{4} {d_2}^2$, d_2= diameter of the pipe at section 2-2)
- h = Difference between the pressure head at section 1 and 2.
- $h = \frac{P_1}{\omega} - \frac{P_2}{\omega}$

PITOT TUBE

- It is a device used to determine the velocity of flow at any point in a pipe or a channel.
- It consists of a glass tube bend at right angle as shown in figure.
- The lower end of tube which is bend through 90^0 is faced to the direction of flow.
- The liquid rises up in the tube due to pressure exerted by the flowing liquid.
- The velocity of liquid is determined by measuring the rise of liquid in the tube.



Pitot Tube

- Actual velocity of flow is given by,
- $V_{act} = C_v \times \sqrt{2gh}$
- $V_{act} = C_v \times V_t$
- Where, C_v = Coefficient of velocity or coefficient of pitot tube
- (Its value ranges from 0.97 to 1)
- V_t = Theoretical velocity
- $V_t = \sqrt{2gh}$
- h = Difference between the pressure head at section 1 and 2.
- $h = \frac{P_1}{\omega} - \frac{P_2}{\omega}$
- P_1 = Pressure at section 1
- P_2 = Pressure at section 2
- ω = Specific weight of water

Q.1. An orifice meter of diameter 120 mm is inserted in a pipe of diameter 240 mm, to measure a rate of flow of a fluid of specific gravity 0.88. The differential manometer connected between the upstream and downstream side shows a mercury level difference of 400 mm. The coefficient of discharge of meter is 0.65. Determine the rate of flow of oil.

Given data:

- d_1 = 240 mm = 0.24 m
- d_2 = 120 mm = 0.12 m
- S_f = 0.88 , S_m = 13.6
- x = 400 mm of mercury
- = 0.4 m of mercury
- C_d = 0.65
- Q_{act} =?
- $Q_{act} = C_d \dfrac{a_1 \text{ x } a_2 \text{ x} \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

- $h = x\ [\frac{S_m}{S_f} - 1] = 0.4\ [\frac{13.6}{0.88} - 1]$
- = 5.78 m
- $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \text{ x } (0.24)^2 = 0.0452 \text{ m}^2$
- $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \text{ x } (0.12)^2 = 0.0113 \text{ m}^2$
- $Q_{act} = C_d \dfrac{a_1 \text{ x } a_2 \text{ x} \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$
- $Q_{act} = 0.65 \text{ x } \dfrac{0.0452 \text{x} 0.0113 \text{x} \sqrt{2 \text{x} 9.81 x 5.78}}{\sqrt{(0.0452)^2 - (0.0113)^2}} = 0.65 \text{ x} \dfrac{0.005439}{0.04376}$
- $Q_{act} = 0.0807 \text{ m}^3/\text{s}$

Q.2. An orifice meter of diameter 100mm is inserted in a pipe of 200 mm diameter. The pressure gauge fitted upstream and downstream of the orifice meter gives a reading of 200 kPa and 100 kPa respectively. Coefficient of discharge of meter is given by 0.6. Find the discharge of water through pipes?

Given data:

- d_1 = 200 mm = 0.2 m
- d_2 = 100 mm = 0.1 m
- P_1 = 200 kPa = 200 x 10^3 Pa
- P_2 = 100 kPa = 100 x 10^3 Pa
- C_d = 0.6
- Q_{act} = ?
- $Q_{act} = C_d \dfrac{a_1 \text{ x } a_2 \text{ x} \sqrt{2gh}}{\sqrt{a_1{}^2 - a_2{}^2}}$

- $h = \dfrac{P_1}{\omega} - \dfrac{P_2}{\omega} = \dfrac{200 \text{ x } 10^3}{9810} - \dfrac{100 \text{ x} 10^3}{9810}$
- $= 10.19$ m
- $a_1 = \dfrac{\pi}{4} d_1{}^2 = \dfrac{\pi}{4} \text{x } (0.2)^2 = 0.0314 \text{ m}^2$
- $a_2 = \dfrac{\pi}{4} d_2{}^2 = \dfrac{\pi}{4} \text{x } (0.1)^2 = 7.853 \text{ x } 10^{-3} \text{ m}^2$
- $Q_{act} = C_d \dfrac{a_1 \text{ x } a_2 \text{ x} \sqrt{2gh}}{\sqrt{a_1{}^2 - a_2{}^2}}$
- $Q_{act} = 0.6 \text{ x } \dfrac{0.0314 \text{x} 7.853 \text{ x} 10^{-3} \text{x} \sqrt{2\text{x}9.81\text{x}10.19}}{\sqrt{(0.0314)^2 - (7.853 \text{ x} 10^{-3})^2}}$
- $Q_{act} = 0.068 \text{ m}^3/\text{s}$

Q.3. Find the velocity of flow of oil through a pipe when the difference of mercury level in a differential U-tube manometer connected to the two tapings of the pitot tube is 200 mm, the coefficient of pitot tube is 0.98 and specific gravity of oil is 0.8.

Given data:

- x = 200 mm of mercury
- = 0.2 m of mercury
- C_v = 0.98
- S_{oil} = 0.8
- V_{act} =?
- $V_{act} = C_v \times \sqrt{2gh}$

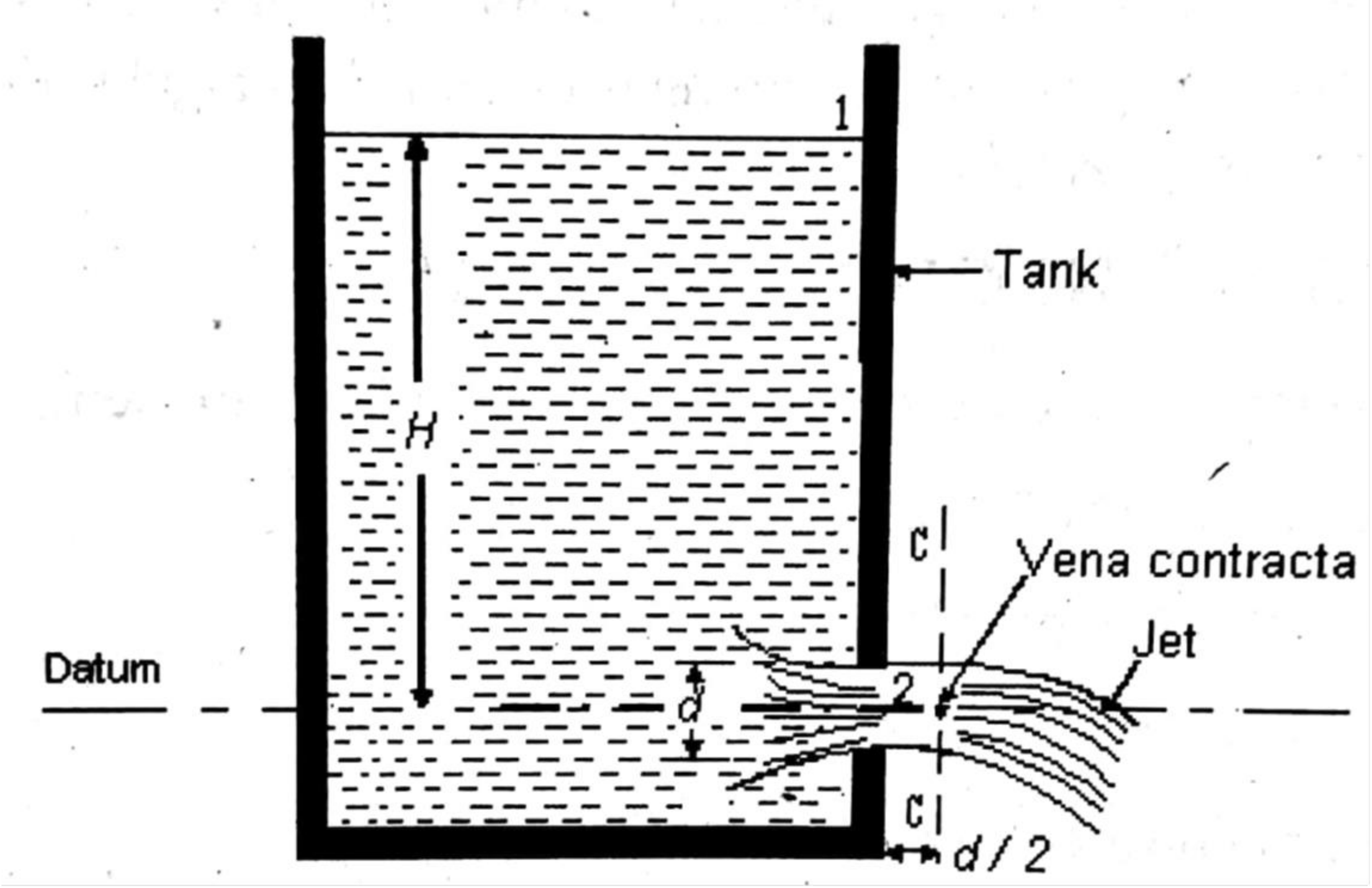
- $h = x\ [\frac{S_m}{S_f} - 1]$
- $= 0.2\ [\frac{13.6}{0.8} - 1]$
- = 3.2 m
- $V_{act} = C_v \times \sqrt{2gh}$
- $= 0.98 \times \sqrt{2x9.81x3.2}$
- V_{act}= 7.765 m/s

Q.4. A pitot tube is placed in a pipe of 200 mm diameter to measure the velocity of water. If the pitot tubes one orifice pointing upstream and other perpendicular to it. The difference of pressure between the two orifice is 80 mm of water. Find the discharge through the pipe , if C_v = 0.95 and mean velocity is equal to 0.85 x Central velocity.

- Given data:
- d = 200 mm = 0.2 m
- h = 80 mm of water
- = 0.08 m of water
- $C_v = 0.95$
- Q = ?
- $V_{act} = C_v \times \sqrt{2gh}$
- $= 0.95 \times \sqrt{2 \times 9.81 \times 0.08}$
- = 1.19 m/s (Central velocity)
- Mean velocity, V = 0.85 x Central velocity
- V = 0.85 x 1.19
- = 1.0115 m/s
- Q = AV
- $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\ m^2$
- Discharge,
- Q = 0.0314 x 1.0115
- $Q = 0.03176\ m^3/s$

ORIFICE

- It is a small opening of any cross section on the side or at the bottom of a tank, through which a fluid is flowing.



H=Head of liquid from the centre of orifice.
d= depth of orifice

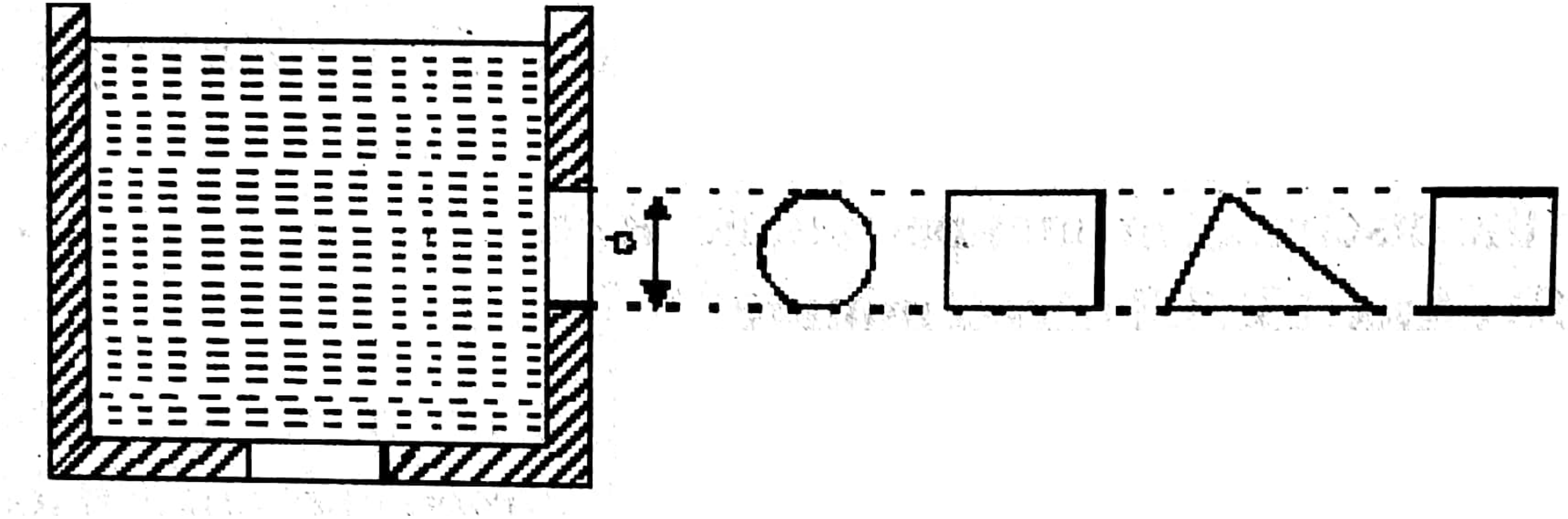
TYPES OF ORIFICES

- According to Size

1. Small orifice- $H>5d$
2. Large orifice- $H<5d$

- According to shape

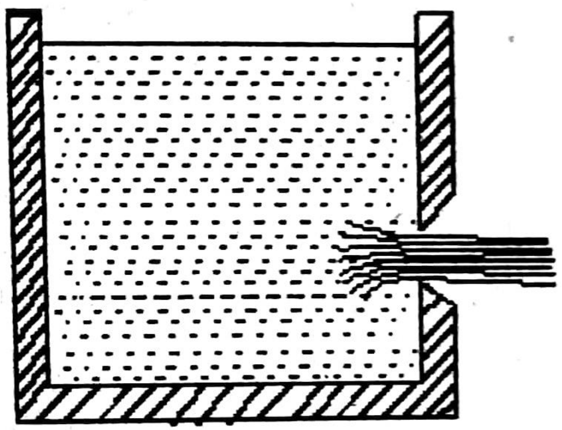
1. Circular
2. Triangular
3. Rectangular
4. Square cross section



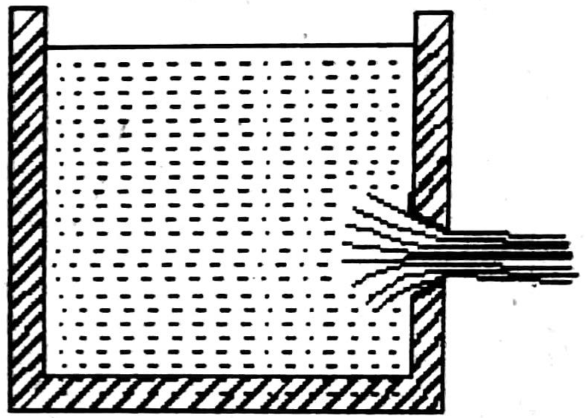
d

SHAPE OF UPSTREAM EDGE

1. Sharp edged orifices
2. Bell-mouthed orifices



Sharp -edged orifice
or Standard orifice



Bell- mouthed orifice

SHARP EDGED ORIFICES

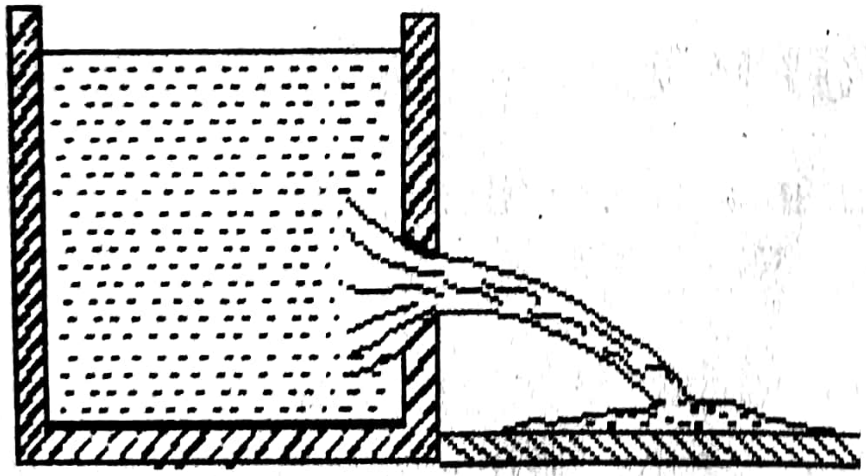
- When liquid flows out has to pass through sharp line-boundary of the orifice.
- This is made in thin plate.

BELL-MOUTHED ORIFICES

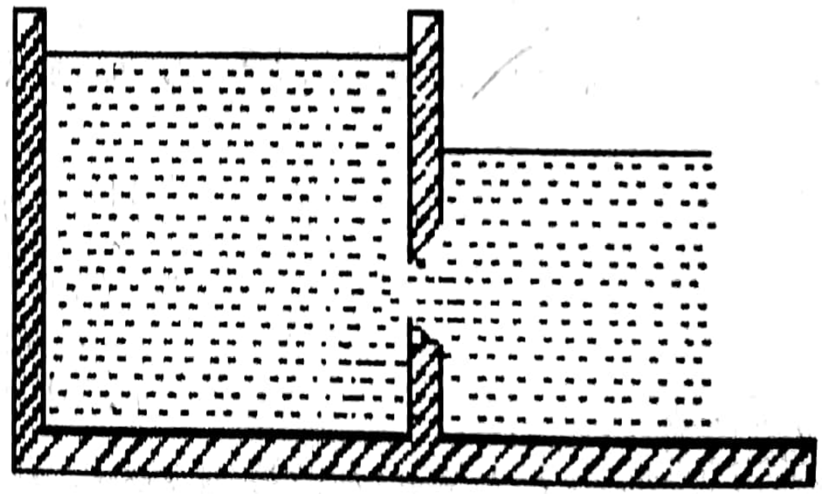
- When it is in the form of a church-bell or like a frustum of a cone and the liquid flows out has to contact with the entire inside surface of the orifices.
- This is made in thick plate.

NATURE OF DISCHARGE

1. Free discharge orifice
2. Fully submerged orifice
3. Partially submerged orifice

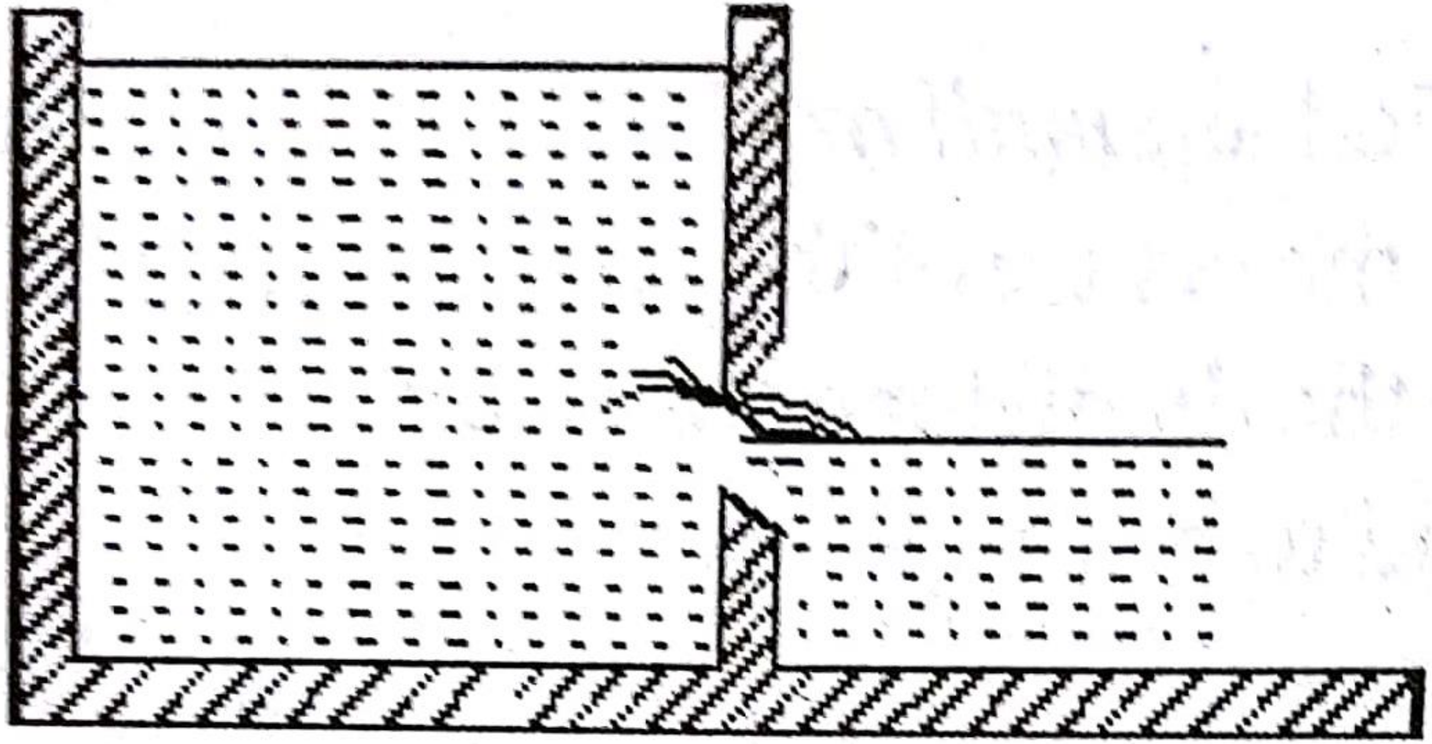


Free discharge orifice



Fully submerged orifice

Partially submerged orifice



Partially submerged orifice

FREE DISCHARGE ORIFICE

- If there is no liquid on the downstream side of the orifice is called free orifice and its discharge is called free discharge.

FULLY SUBMERGED ORIFICE

- If the liquid level on downstream side is above the bottom edge of orifice is known as drowned or submerged orifice and its discharge is called submerged discharge.
- If the liquid level on downstream side is at the top edge of the orifice or above it, the orifice is said to be fully submerged orifice.

PARTIALLY SUBMERGED ORIFICE

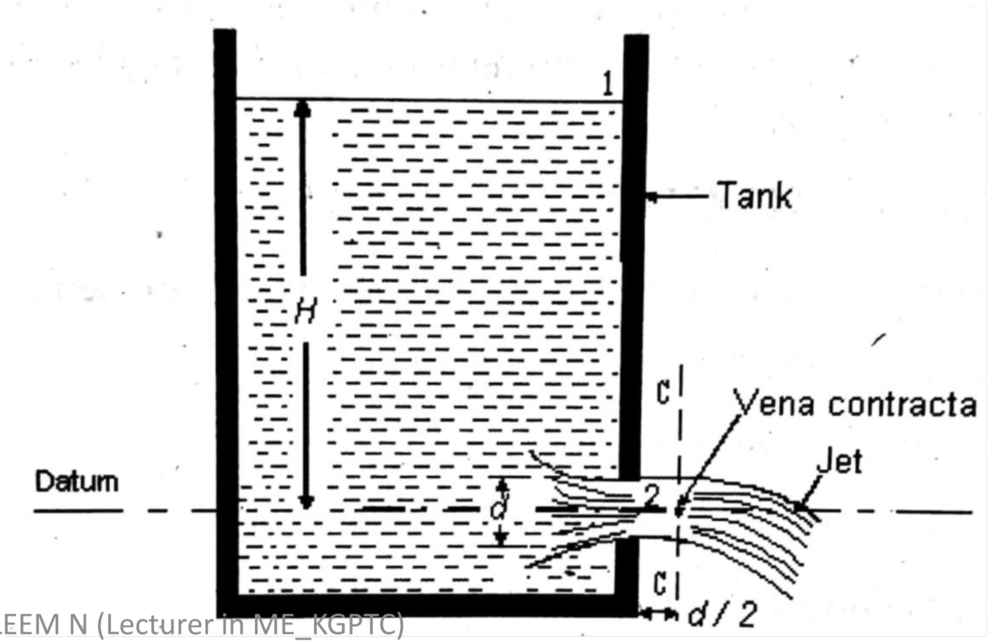
- If the liquid level on the downstream side is above the bottom edge and below the top edge of the orifice, the orifice is said to be partially submerged orifice.

VENA CONTRACTA

- The continuous stream of liquid that comes on the downstream side of an orifice is known as jet.
- The jet comes out of an orifice gets contracted from the mouth of orifice to a distance of about half the orifice diameter.
- The section at which maximum contraction is reached is known as vena contracta.
- The velocity at this section is maximum.

FLOW THROUGH AN ORIFICE

- Consider a tank fitted with a small circular orifice as shown in figure.
- Considering points 1 and 2 and assume datum line pass through the horizontal axis of orifice.
- Point 1 is at the top of upstream and point 2 is at vena contracta.



- Applying Bernoulli's theorem to points 1 and 2.
- $\frac{P_1}{\omega} + \frac{V_1^{\ 2}}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^{\ 2}}{2g} + Z_2$
- But $P_1 = P_2 = P_a$, (P_a = Atmospheric pressure)
- $\frac{V_1^{\ 2}}{2g} + Z_1 = \frac{V_2^{\ 2}}{2g} + Z_2$
- The cross sectional area of the tank is very large, the liquid at point 1 is partially at rest as compared to the point 2 at vena contracta. Hence $V_1=0$

- $Z_1 = \frac{{V_2}^2}{2g} + Z_2$
- Datum line is along the centre of the orifice. Hence, Z_1 =H, Z_2= 0
- $H = \frac{{V_2}^2}{2g}$
- $V_2 = \sqrt{2gH}$
- This is theoretical velocity and actual velocity will be less than the theoretical value.

HYDRAULIC CO-EFFICIENTS

- Coefficient of contraction, C_c
- Coefficient of velocity, C_v
- Coefficient of discharge, C_d

Coefficient of contraction, C_c

- Defined as the area of the jet at vena contracta to the area of the orifice.
- $C_c = \dfrac{\text{Area of jet at vena contracta}}{\text{Area of orifice}} = \dfrac{a_c}{a}$
- Value of C_c varies from 0.61 - 0.69
- General value of C_c may be taken as 0.64

Coefficient of velocity, C_v

- Defined as the ratio of the actual velocity of a jet of liquid at vena contracta to the theoretical velocity of jet.
- $C_v = \dfrac{\text{Actual velocity of a jet of liquid at vena contracta}}{\text{Theoretical velocity of jet}}$
- $C_v = \dfrac{V}{\sqrt{2gH}}$
- Value of C_v varies from 0.95 - 0.99
- General value may be taken as 0.98

Coefficient of discharge, C_d

- Defined as the ratio of actual discharge from an orifice to the theoretical discharge from an orifice.
- $C_d = \dfrac{\text{Actual discharge}}{\text{Theoretical discharge}} = \dfrac{Q_{act}}{Q_{th}}$
- Q_{th} = Theoretical velocity of jet x Theoretical area of jet
- $Q_{th} = a\sqrt{2gH}$
- $C_d = \dfrac{Q_{act}}{a\sqrt{2gH}}$
- Value of C_d varies from 0.61 - 0.65
- General value may be taken as 0.62

Relation between C_c , C_v and C_d

- Q_{act} = Actual velocity of jet at vena contracta x Actual area of jet at vena contracta.

- Q_{th} = Theoretical velocity of jet x Theoretical area of jet

- $C_d = \frac{Q_{act}}{Q_{th}} = \frac{\text{Actual velocity x Actual area}}{\text{Theoretical velocity x Theoretical area}}$

- $C_d = C_v \times C_c$

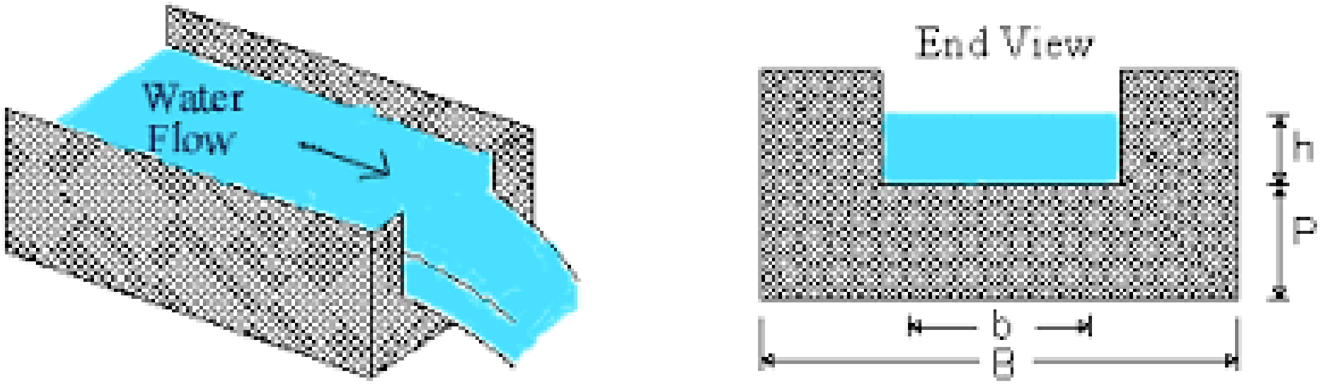
NOTCHES

- It may be defined as an opening in the side of tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.
- It is used for measuring the rate of flow of liquid through a small channel or a tank.
- It is generally made with metallic plates.
- The bottom edge over which the liquid flows is called the sill or crest of notch.
- The layer of liquid flowing over a notch is called the nappe or vein.
- The height of water above sill of the notch is called as head on notch.

TYPES OF NOTCHES

- According to shape of opening

1. Rectangular notch
2. Triangular notch
3. Trapezoidal notch
4. Stepped notch



- According to the effect of the sides on the nappe

Notch without end contraction

- The notch width or length extends over the full width of the channel in which it is placed.

Notch with end contraction

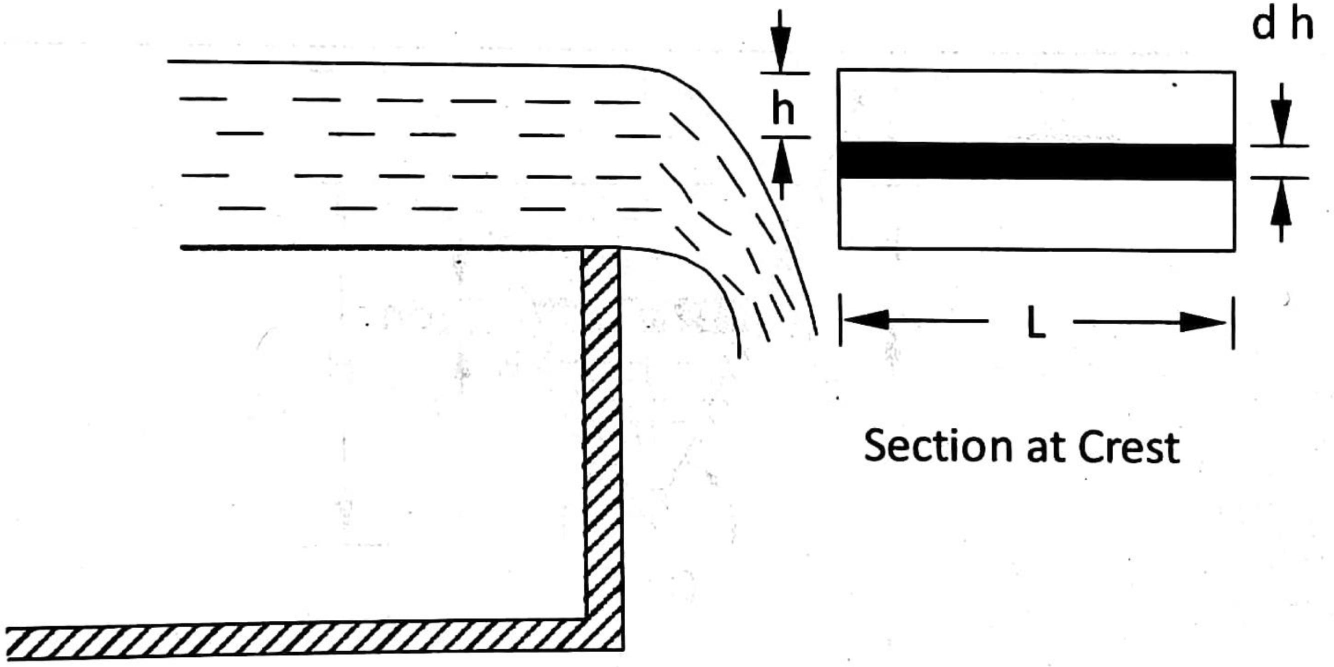
- The notch width or length is less than the width of the channel.

Difference between Orifice and Notch

ORIFICE	NOTCH
Small opening in the side of a tank through which the fluid flows.	Large opening in the side of a tank or dam over which the fluid flows.
The upper edge of the orifice is below the free surface of the water in the tank.	The upstream water level is below the upper edge of the notch.
The stream of water flowing through orifice is called jet.	The layer of water flowing over a notch is called nappe or vein.
Head of water compared to the orifice dimension is large.	Head of water over the sill of the notch is small compared to the notch dimensions.
Pressure on the upstream side of the orifice is more than the downstream side pressure which is atmospheric.	Pressure on the upstream as well as downstream side of the notch is atmospheric.

Discharge over a rectangular notch

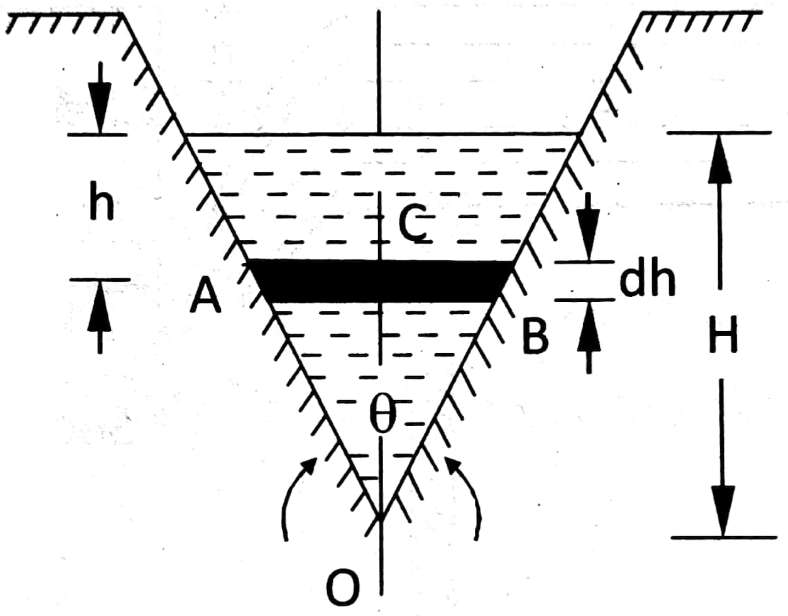
$$Q = \frac{2}{3}\, Cd \times L \times \sqrt{2g} \times (H)^{3/2}$$



Rectangular Notch

Discharge over a triangular notch (V-NOTCH)

$$Q = \frac{8}{15}\ Cd \times \tan(\frac{\theta}{2}) \times \sqrt{2g} \times (H)^{5/2}$$



Triangular Notch

FLOW THROUGH PIPES

- The fluid flowing in pipe is always subjected to resistance due to shear forces between fluid particles and boundary walls of the pipe.
- The resistant to flow of fluid particles is known as frictional resistance.
- Due to frictional resistance, there is always some loss of energy in the direction of flow.

LAMINAR FLOW and TURBULENT FLOW

- A laminar flow is one in which the fluid particles moves along smooth regular paths, which can be predicted well in advance.
- The fluid thus move in layers, gliding smoothly over adjacent layers.
- Eg:- Flow of blood in small veins, Oil flow in bearings etc.
- A flow is said to be turbulent when the fluid particles move in very irregular path.
- These results in the formation of eddy's.

FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE

- The flow through the circular pipe will be viscous or laminar, if the Reynolds number (Re) is less than 2000. The expression for Reynolds number is given by,
- $R_e = \frac{\rho V D}{\mu}$
- Where, ρ = Density of fluid flowing through pipe.
- V = Average velocity of fluid.
- D = Diameter of pipe.
- μ = Viscosity of fluid

- In case of circular pipe, if $R_e < 2000$, the flow is said to be laminar,
- If $R_e > 4000$, the flow is said to be turbulent.
- If R_e lies between 2000 to 4000, the flow changes from laminar to turbulent

- For the flow of viscous fluid through circular pipe, the drop of pressure for a given length of pipe is given by Hagen Poiseuille equation.
- $\frac{P_1 - P_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$
- h_f = Loss of pressure head
- μ = Viscosity of fluid
- $\bar{u}$ = Average velocity
- L = Length of the pipe
- ρ = Density of fluid flowing through pipe
- D = Diameter of pipe.

LOSS OF ENERGY IN PIPES

- When a fluid is flowing through a pipe, the fluid is subjected to some resistance, due to some of energy of fluid is lost.
- This loss of energy or head is classified as follows.

- Minor energy losses

- Major energy losses

MINOR ENERGY LOSSES

It is classified into four

1. Loss of head at entrance or inlet or entry.

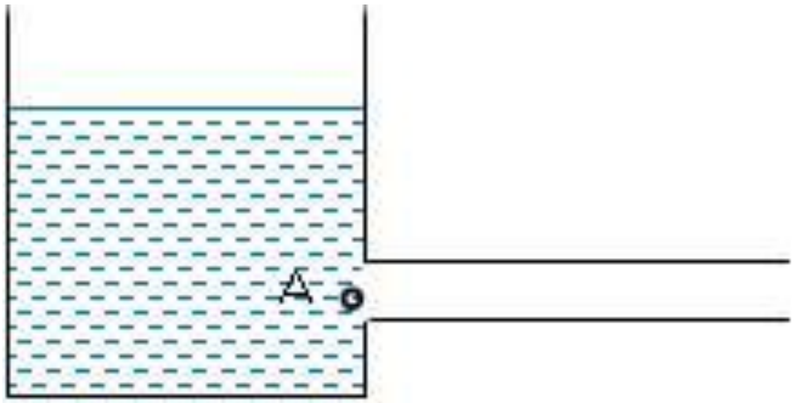
2. Loss of head due to sudden enlargement.

3. Loss of head due to sudden contraction.

4. Loss of head at exit or outlet.

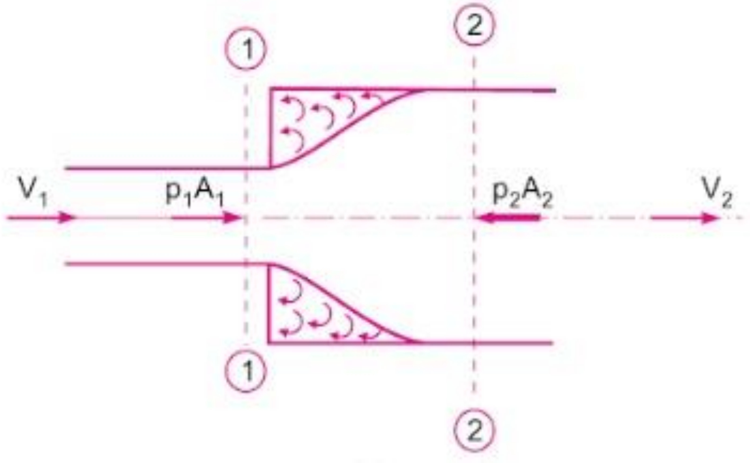
LOSS OF HEAD AT ENTRANCE or INLET or ENTRY

- Figure shows a pipe line receiving liquid from reservoir.
- While the liquid enters the pipe, it gets contracted to a narrow neck and again expanded to the full bore of the pipe.
- The loss of energy is due to expansion of liquid from neck to the full bore.
- $h_i \approx 0.5 \frac{V^2}{2g}$
- Where h_i = Loss of head at entrance or inlet.
- V = Velocity of the liquid at the pipe in m/s.



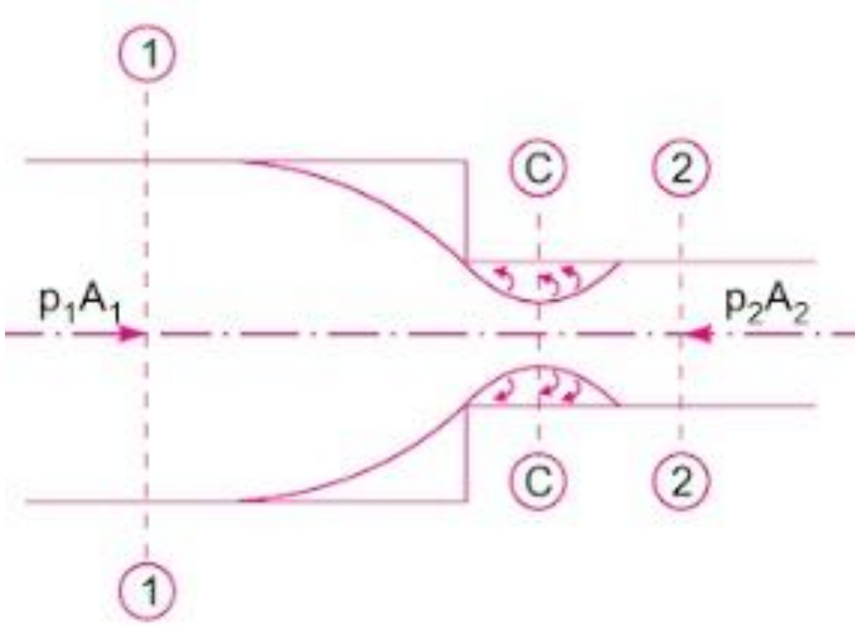
LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT

- Consider a pipe of cross section suddenly enlarges as shown in figure.
- Imagine a flowing fluid is flowing through a pipe at pressure P_1, velocity V_1 and area A_1 at section 1-1.
- The corresponding values at section 2-2 are P_2, V_2 and A_2.
- Loss occurs at sudden enlargement due to formation of eddy's.
- The loss of head due to sudden enlargement is,
- $h_e = \frac{(V_1 - V_2)^2}{2g}$



LOSS OF HEAD DUE TO SUDDEN CONTRACTION

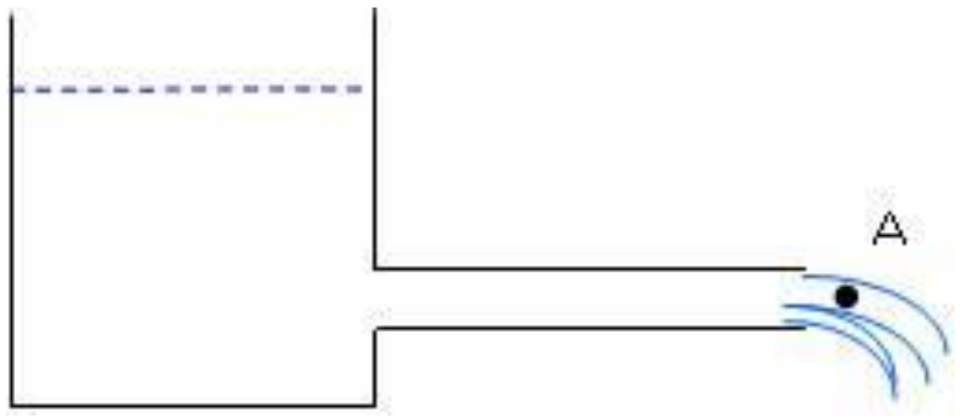
- Figure shows flow of liquid at pressure P_1, velocity V_1 and area A_1 through a pipe which suddenly contracts to a smaller section.
- The corresponding values at section 2-2 are P_2, V_2 and A_2.
- The loss of head due to sudden contraction is given by,
- $h_c = \frac{(Vc - V_2)^2}{2g}$
- Where, V_c = Velocity at venacontracta.
- Applying continuity equation,
- $A_c V_c = A_2 V_2$
- $V_c = \frac{A_2}{A_c} V_2$ (But $C_c = \frac{A_c}{A_2}$)



- $V_c = \frac{1}{C_c} V_2$
- $V_c = \frac{V_2}{C_c}$
- $h_c = \frac{(\frac{V_2}{C_c} - V_2)^2}{2g}$
- $h_c = \frac{{V_2}^2}{2g} \left(\frac{1}{C_c} - 1 \right)^2$
- Normally $C_c = 0.62$, substituting in the above equation,
- $h_c = 0.376 \frac{{V_2}^2}{2g}$
- $h_c \approx 0.5 \frac{V^2}{2g}$ for gradual contraction.

LOSS OF HEAD AT EXIT OR OUTLET

- When the pipe is at exit discharges liquid into another reservoir and the loss of head is given by,
- $h_o = \frac{V^2}{2g}$
- Where, V = Velocity of liquid at the pipe in m/s.



Q.1. A pipe of 250 mm diameter is suddenly reduced to 150 mm. If the discharge of water through the pipe is 25 liters/s. Calculate the head loss due to sudden contraction.

Given data:

- d_1 = 250 mm = 0.25 m
- d_2 = 150 mm = 0.15 m
- Q = 25 liters/s
- = 25×10^{-3} m^3/s
- = 0.025 m^3/s
- h_c = ?
- $h_c = 0.376 \frac{V_2^2}{2g}$

- $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.0177\ m^2$
- $Q = a_2 V_2$
- $0.025 = 0.0177 \times V_2$
- V_2 = 1.412 m/s
- $h_c = 0.376 \frac{V_2^2}{2g}$
- $= 0.376 \frac{(1.412)^2}{2 \times 9.81}$
- = 0.038 m of water

Q.2. A pipe of 240 mm in diameter is enlarged to 480 mm in diameter. Determine the loss of head, when the discharge is 32.75 liters/s.

- Given data:
- d_1 = 240 mm = 0.24 m
- d_2 = 480 mm = 0.48 m
- Q = 32.75 liters/s
- = 32.75 x 10^{-3} m^3/s
- = 0.03275 m^3/s
- h_e = ?
- $a_1 = \frac{\pi}{4} d_1^2 \quad = \frac{\pi}{4} \times (0.24)^2 = 0.0452\ m^2$
- $a_2 = \frac{\pi}{4} d_2^2 \quad = \frac{\pi}{4} \times (0.48)^2 = 0.181\ m^2$

- $h_e = \frac{(V_1 - V_2)^2}{2g}$
- $Q = a_1 V_1$
- $0.03275 = 0.0452 \times V_1$
- $V_1 = 0.724$ m/s
- $Q = a_2 V_2$
- $0.03275 = 0.181 \times V_2$
- $V_2 = 0.181$ m/s
- $h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(0.724 - 0.181)^2}{2 \, x \, 9.81} = 0.015$ m of water

MAJOR LOSSES IN PIPES

- Major energy losses are caused due to friction and is calculated by the following formulas.

- Darcy-Weisbach formula

- Chezy's formula

DARCY-WEISBACH FORMULA

- Due to frictional force in the pipe flow, the available head gradually reduces due to loss of head.
- Darcy-Weisbach equation gives head losses due to friction and is given by,
- $h_f = \frac{4flV^2}{2gd}$
- Where, h_f = Head loss due to friction in meter.
- f = Darcy's friction factor or coefficient of friction.
- l = Length of the pipe in metre
- V = Velocity of flow in m/s.
- d = Diameter of pipe in metre.

- Head loss due to friction in terms of discharge is given by,

- $h_f = \dfrac{flQ^2}{3d^5}$

- Where, Q = Discharge in m^3/s

CHEZY'S FORMULA

- Velocity of flow,
- V = C $\sqrt{mi}$ in m/s
- Where, C = Chezy's constant
- m = Hydraulic mean depth
- i = Loss of head per unit length
- Chezy's constant , C = $\frac{\sqrt{2g}}{f}$
- Where, f = coefficient of friction.

- Hydraulic mean depth is defined as the area of the flow section divided by the top water surface width.
- Hydraulic mean depth, $m = \frac{A}{P}$
- Where, A = Area of pipe/cross section
- P = Perimeter of wetted region
- Loss of head per unit length, $i = \frac{h_f}{l}$
- Where, h_f = head loss due to friction
- l = Length of the pipe

Q.1. Water is flowing through a pipe of 250 mm in diameter and 100 mm long with a velocity of 2.5 m/s. Find the head loss due to friction using Darcy's formula and Chezy's formula. Assume, f = 0.005 , C = 55

Given data:

- d = 250 mm = 0.25 m
- l = 100 mm = 0.1 m
- V = 2.5 m/s
- h_f =?
- f = 0.005
- C = 55

- Using Darcy's formula
- $h_f = \frac{4flV^2}{2gd}$
- $h_f = \frac{4\ x\ 0.005\ x\ 0.1\ x\ 2.5^2}{2\ x\ 9.81\ x\ 0.25}$
- $= 2.54 \times 10^{-3}$ m

- Chezy's formula, $V = C\sqrt{mi}$
- Hydraulic mean depth, $m = \frac{A}{P}$
- A = Area of pipe = $\frac{\pi}{4} d^2$
- $= \frac{\pi}{4} (0.25)^2 = 0.04908 \text{ m}^2$
- P = Perimeter of wetted region = πd
- $m = \frac{A}{P}$
- $= \frac{0.04908}{\pi \, x \, 0.25} = 0.062 \text{ m}$

- Loss of head per unit length, $i = \frac{h_f}{l}$
- $V = C\sqrt{mi}$
- $2.5 = 55\sqrt{0.062 \, x \, i}$
- Squaring on both sides
- 6.25 = 3025 x 0.062 x i
- i = 0.0333
- $i = \frac{h_f}{l}$
- $0.0333 = \frac{h_f}{0.1}$
- $h_f = 0.1 \times 0.0333$
- $h_f = 3.33 \times 10^{-3}\text{m}$

Q.2. When the difference of pressure head between the ends of the pipe is 500 m of water. And length of the pipe is 6 m. Diameter of pipe is 150 mm. Take f = 0.008. Calculate the discharge?

Given data:

- $h_f = 500$ m
- $l = 6$ m
- $d = 150$ mm $= 0.15$ m
- $f = 0.008$
- $Q = ?$

- $h_f = \frac{flQ^2}{3d^5}$
- $500 = \frac{0.008 \text{ x } 6 \text{ x } Q^2}{3 \text{ x } (0.15)^5}$
- $Q^2 = 2.37$
- $Q = 1.54$ m^3/s