INTEGRATION

1]$\int(2x+3)dx = \int 2xdx + \int 3dx = 2\int xdx + 3\int 1dx = 2\frac{x^2}{2} + 3x$ +C , $\int xdx = \frac{x^2}{2}, \int 1dx = x + C$

2]$\int_0^1 xdx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$

3]Find the order and degree of the differential equation, $\left(\frac{d^3y}{dx^3}\right)+\left(\frac{d^2y}{dx^2}\right)^2+5\frac{dy}{dx}=y$

Highest order derivative = $\frac{d^3y}{dx^3}$,Its order =3, its degree = 1, So order of differential equation = 3, degree of differential equation = 1.

4]Solve $\frac{dy}{dx} = \frac{x}{y}$, $\frac{dy}{dx} = \frac{x}{y}$, $y\,dy = xdx$,

$Integrating, \int ydy = \int xdx\ , \frac{y^2}{2} = \frac{x^2}{2} + C.$

5]$\int \frac{sin^{-1}2x}{\sqrt{1-4x^2}}dx$, $u = sin^{-1}2x, \frac{du}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \times \frac{d}{dx}(2x)$

$\frac{du}{dx} = \frac{1}{\sqrt{1-4x^2}} \times 2\ , du = \frac{1}{\sqrt{1-4x^2}}dx \times 2\ , \frac{du}{2} = \frac{1}{\sqrt{1-4x^2}}dx,$

$\int \frac{sin^{-1}2x}{\sqrt{1-4x^2}}dx = \int u\frac{du}{2} = \frac{1}{2}\int u\,du = \frac{1}{2} \times \frac{u^2}{2} = \frac{(sin^{-1}2x)^2}{4}$ + C

$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}, \frac{d}{dx}(sin^{-1}2x) = \frac{1}{\sqrt{1-(2x)^2}} \times \frac{d}{dx}(2x),$

6] $\int secx(secx + tanx)dx = \int (sec^2x + secxtanx)dx$

$\int sec^2xdx + \int secxtanxdx = tanx + secx + C$,

7]$\int x.sinxdx = first \times \int second - \int [\frac{d}{dx}(first) \times \int second]$

$first = x, \;\; second = sinx$

$\int x.sinxdx = x \times \int sinxdx - \int [\frac{d}{dx}(x) \times \int sinxdx]dx$

$= x \times -cosx - \int 1 \times -cosxdx = -xcosx + \int cosxdx$

$= -xcosx + sinx + C$, $\int sinxdx = -cosx + C, \int cosxdx = sinx + C$

8]$\int_0^{\frac{\pi}{2}} cos4x.cosxdx$, $cosAcosB = \frac{1}{2}[\cos(A + B) + cos(A - B)]$

$A = 4x, B = x, cos4x.cosx = \frac{1}{2}[\cos(4x + x) + cos(4x - x)]$

$\frac{1}{2}(\cos 5\mathrm{x} + cos3x)$

$$\int_0^{\frac{\pi}{2}} cos4x.cosxdx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos 5\mathrm{x} + cos3x)dx$$

$\frac{1}{2}\int_0^{\frac{\pi}{2}}(cos5x + cos3x)dx = \frac{1}{2}[\int_0^{\frac{\pi}{2}} cos5xdx + \int_0^{\frac{\pi}{2}} cos3xdx]$

$= \frac{1}{2}\left\{\left[\frac{sin5x}{5}\right]_0^{\frac{\pi}{2}} + \left[\frac{sin3x}{3}\right]_0^{\frac{\pi}{2}}\right\} = \frac{1}{2}\left\{\left[\frac{sin5\frac{\pi}{2}}{5} + \frac{sin3\frac{\pi}{2}}{3}\right] - \left[\frac{sin5\times 0}{5} + \frac{sin3\times 0}{3}\right]\right\}$

$=\frac{1}{2}\left\{\left[\frac{1}{5}+\frac{-1}{3}\right]-\left[\frac{0}{5}+\frac{0}{3}\right]\right\}$, $\left(sin5\frac{\pi}{2}=1, sin3\frac{\pi}{2}\right)$= -1

$=\frac{1}{2}\left\{\frac{1\times3+5\times-1}{5\times3}\right\}=\frac{1}{2}\left(\frac{3-5}{15}\right)=\frac{1}{2}\times\frac{-2}{15}=\frac{-1}{15}$

9]Evaluate $\int secx\ dx$

Multiply numerator and denominator with $secx+tanx$

$$\int secx\ dx=\int\frac{secx(secx+tanx)}{secx+tanx}\ dx=\int\frac{(sec^2x+secxtanx)}{secx+tanx}\ dx$$

Put $u=\ secx+tanx, \frac{du}{dx}=sec^2x+secxtanx,$

$du=(sec^2x+secxtanx)dx$

$\int\frac{(sec^2x+secxtanx)}{secx+tanx}\ dx=\int\frac{du}{u}=logu=\log(\ secx+tanx)+C$

10] Evaluate $\int cosecx\ dx$

Multiply numerator and denominator with $cosecx-cotx$

$\int cosecx\ dx=\int\frac{cosecx(cosecx-cotx)}{cosecx-cotx}\ dx=$
$\int\frac{(cosec^2x-cosecxcotx)}{cosecx-cotx}\ dx$

Put $u=\ cosecx-cotx, \frac{du}{dx}=cosec^2x-cosecxcotx,$

$du=(cosec^2x-cosecxcotx)dx$

$\int\frac{(cosec^2x-cosecxotx)}{secx+tanx}\ dx=\int\frac{du}{u}=logu=\log(\ cosecx-cotx)+C$

11]$Evaluate \int \quad \int \quad \int tanxdx\ , tanx = \frac{sinx}{cosx}$

$\int tanxdx$ =$\int \frac{sinx}{cosx} dx, u = cosx, \frac{du}{dx} = -sinx, du = -sinxdx,$

$\int tanxdx$ =$\int \frac{sinx}{cosx} dx = \int \frac{-du}{u}$,$(-du = sinxdx)$

=$-logu = -\log(cosx) = \log(secx)$

12] $Evaluate \int cotxdx\ , cotx = \frac{cosx}{sinx}$

$\int cotxdx\ , cotx = \frac{cosx}{sinx}, u = sinx, \frac{du}{dx} = cosx, du = cosxdx$

$\int cotxdx = \int \frac{cosx}{sinx} dx = \int \frac{du}{u}$ = $logu = \log(sinx) + C$

13]Find the area between one arch of $y = sinx\ and\ X\ axis$

Area=$\int_a^b y\, dx = \int_a^b sinx\, dx$

On the X axis $y = 0, sinx = 0, x = 0. x = \pi$

$a = 0, b = \pi\ , \int_a^b sinx\, dx = \int_0^\pi sinx\, dx = [-cosx]_0^\pi$=

$[-cos\pi] - [cos0] = --1 - -1 = 1 + 1 = 2$ sq.units.

14] Find the area between $y = x^2 - x - 2, and\ X\ axis$.

Area=$\int_a^b y\, dx = \int_a^b (x^2 - x - 2)dx\ , a = 1, b = -1, c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $x = -1; x = 2$,$a = -1, b = 2$

$\int_a^b (x^2 - x - 2)dx = \int_{-1}^{2} (x^2 - x - 2)dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2} =$

$\left[\frac{2^3}{3} - \frac{2^2}{2} - 2 \times 2\right] - \left[\frac{1^3}{3} - \frac{1^2}{2} - 2 \times 1\right]$

$Solve\ \frac{dy}{dx} + ycotx = cosecx$,Compare with linear differential equation, $\frac{dy}{dx} + Py = Q, P = cotx, Q = cosecx$

$Integrating\ factor = IF = e^{\int Pdx}$ $= e^{\int Pdx} = e^{\int cotxdx} = e^{logsinx} = sinx$

$Solution\ , y.I\ F = \int Q.IF$

$y.sinx = \int cosecx.sinx\ dx$

$y.sinx = \int \frac{1}{sinx}.sinx\ dx\ , y.sinx = \int 1\,.dx, y.sinx = x + C$

LIST OF INTEGRALS

$\int 1dx = x + C, \int xdx = \frac{x^2}{2} + C, \int x^2dx = \frac{x^3}{3} + C, \int x^3dx = \frac{x^4}{4} + C,$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \int e^x dx = e^x + C, \int \frac{1}{x}dx = logx + C,$

$\int xdx = \frac{x^2}{2}, \int 1dx = x + C, \int cosxdx = sinx + C,$

$\int sindx = -cosx + C\ , \int sec^2 xdx = tanx + C,$

$\int cosec^2 xdx = -cotx + C\ , \int secxtanxdx = secx + C,$

$\int cosecxcotxdx = -cosecx + C$, $\int tanxdx = logsecx + C$

$\int cotxdx = logsinx + C$,$\int \frac{1}{\sqrt{1-x^2}} dx = sin^{-1}x + C,$

$\int \frac{1}{1+x^2} dx = tan^{-1}x + C$, $\int 2dx$=$2x + C$, $\int 2xdx = 2\frac{x^2}{2}$+C,

$\int cos2xdx = \frac{sin2x}{2} + C, \int sin3xdx = \frac{-cos3x}{3} + C,$

$\int e^{2x+3}dx = \frac{e^{2x+3}}{2}$+ C