TOS II.

Column.

- A vertical compression member which is mainly subjected to axial loads and effective length of which exceeds three times its least lateral dimension.
- The compression member whose effective length is less than three times its least lateral dimension is called pedestal.
- The compression member which is inclined or horizondal is subjected to axial loads is called strut. (They are used in trusses).
- Load carrying capacity of a compression member depends not only its cross sectional area but also on its length. and the manner in which the ends of a column are held.

Compression members are structural elements that are pushed together or carry a load. (They are subjected only to axial compressive force

Euler's theory of column.

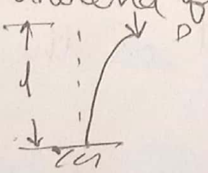
I = least moment of inertia

Both end hinged

$$P = \frac{\pi^2 EI}{l^2}$$

$L_e = l$

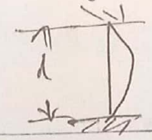
One end fixed other end free.



$$P = \frac{\pi^2 EI}{4l^2}$$

$L_e = 2l$.

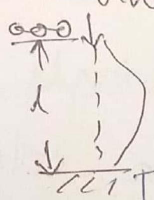
Both end fixed.



$$P = \frac{4\pi^2 EI}{l^2}$$

$L_e = \frac{l}{2}$.

One end fixed other end hinged



$$P = \frac{2\pi^2 EI}{l^2}$$

$L_e = \frac{l}{\sqrt{2}}$

(effective length. L_e)

~~The force~~

Column :- A column may be defined as an element used to support axial compressive loads and with a height of at least three times its lateral dimension.

A column may be short or long column depending on its effective slenderness ratio. A short column has a maximum sledesness ratio of '12' Long column has a slederness ratio greater than '12' (The maximum slenderness ratio should not exceed 60)

Short column	long column.
• Length / least dimension. less than 12.	• Length / least demension ~~less~~ greater than 12.
• Slenderness ratio less than 12	• Slederness ratio greater than 12.
• Fails by crushing	• Fails by buckling
• Radius of Gyration is more	• Radius of gyration is less.
• More load capacity	• Less load capacity.

effective length of a column is the distance b/w the points of zeromoment or the inflection points of the column.

The effective length factor 'K' lies b/w 0.5 to 1 (represents the ratio of the effective of a column to its actual length.

<u>Radius of gyration</u> is a measure of the elastic stability of a cross section against buckling

It can be defined as the imaginary distance from the ~~cr~~ centroid at which the region of the cross section is imagined to be concentrated at a point in order to achieve the same moment of inertia.

The radius of gyration of a body is always about an axis of rotation. (K)

radius of gyration $\boxed{K = \sqrt{\frac{I}{M}}}$ OR $\boxed{K = \sqrt{\frac{I}{A}}}$

I = moment of inertia

m = mass of the body.

End conditions for long column.

- Both ends are hinged or pinned.
- One end free and the other end fixed
- Both ends are fixed
- One end fixed and other end is pinned.

buckling is a sudden lateral failure of an ~~axi~~ axially loaded member in compression

Euler buckling formula for a perfectly elastic column

$$\boxed{N_{euler}^{(F)} = \frac{\pi^2 EI}{Kle^2}}$$

F = ~~critical~~ Critical force

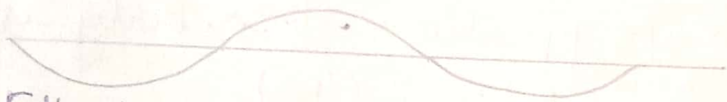
E = modulus of elasticity

I = moment of inertia

L = Unsupported length of column.

K = column effective length factor.

sinuous wave.



Effective length.

Usupported length of column which is required for forming full sinuous wave. Denoted by (KL)

L original.

$L_e = 2L$ → cantilever. $K = 2$

$L_e = L$ → Both end hinged. $K = 1$

$L_e = \frac{L}{2}$ → Both end fixed $K = 0.5$

$L_e = \frac{L}{\sqrt{2}}$ → One end fixed, other hinged. K=

Sledesness ratio is defined as the ratio of length 'l' to the radius of gyration 'k' (l/k)

Long column → S more than 120.

Short column → S less than 32.

Higher the 'S' greater the load bearing Capacity.

Types of column end conditions.

- Both ends fixed
- Both ends hinged
- One end fixed and other hinged.
- One end fixed and other free.

Both ends hinged

This is the standard column end condition. Effective length in this condition is equal to the length of the column.

$l_e = l$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

$l_e = l/2$

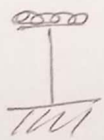
$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

Both ends-fixed

- Column load bearing capacity increases with the decrease in column equivalent length.
- This is the strongest column end condition - carries the maximum load.
- Effective length for this condition is considered as half of the total column length.

One end fixed and other hinged

One end of column is stronger, while the other end is very weak. $l_e = l/\sqrt{2}$.



$l_e = l/\sqrt{2}$.

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

I = least moment of inertia

One end fixed and other free

This end condition makes columns to bear smallest load than all other end condition.

$l_e = 2l$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

Euler's theory of column

According to Euler's theory of column, failure of a column occurs either due to the buckling load or due to the buckling criteria.

Assumptions of Euler's column theory.

- Axis of column is perfectly straight when unloaded
- Material is isotropic and homogeneous.
- The column is perfectly straight, cross-section of the column is uniform throughout its length.
- The load is axial and passes through the centroid of the section.
- Failure of the column occurs buckling only.
- Length of column is large compared to its cross-sectional dimensions.

Limitations

- The possibility of crookedness (വളവ്) in column is not accounted for in this theory.
- It is less accurate and numerically unstable unstable.
- Euler's formula is applied only for long column.
- As the slenderness ratio decreases the crippling stress increases. consiquently if the slenderness ratio reaches to zero, then the crippling stress reaches infinity. praticallly which is not possible.

Rankine's Formula

This formula can be used for any type of column. According to Rankine's formula.

$$\boxed{\frac{1}{P_R} = \frac{1}{P_{cs}} + \frac{1}{P_E}}$$

P_R = crippling load by Rankine

P_{cs} = Ultimate crushing load.

$P_E = \dfrac{\pi^2 EI}{l^2}$ (Euler's crippling load).

Conditions

(a) Short column: Rankine's formula will give the value of crippling load approximately equal to ultimate crushing load. $\boxed{\frac{1}{P_R} = \frac{1}{P_{cs}}}$

(b) long column : The value of P_E is considered is long column.

$$\boxed{\frac{1}{P_R} = \frac{1}{P_{cs}} + \frac{1}{P_e}}$$

$$\boxed{P_R = \frac{\sigma_{cs} \cdot A}{1 + a \times \left(\frac{l_e}{k}\right)^2}}$$

OR

$$\boxed{P_R = \frac{P_{cs}}{1 + a \times \left(\frac{l_e}{k}\right)^2}}$$

σ_{cs} = Ultimate crushing stress

A = Area of cross section

l_e = effective length.

K = radius of gyration

a = Rankin's constant.

1 External dia = 5 cm ⟶ 50 mm.

Internal dia = 4 cm ⟶ 40 mm.

length = 3 m ⟶ 3×10^3 mm.

Both end fixed $l_e = l/2$.

$$l_e = \frac{3 \times 10^3}{2}.$$

Ultimate crushing stress $\sigma_{cs} = 550 \text{ N/mm}^2$.

Rankin's constant $a = \frac{1}{1600}$.

find the crippling load ?

$$P_R = \frac{\sigma_{cs} \cdot A}{1 + \left[a \times \left(\frac{l_e}{k}\right)^2\right]}$$

=

$$A = \frac{\pi D^2}{4} = \frac{\pi (D^2 - d^2)}{4}$$

$$= \frac{3.14 (50^2 - 40^2)}{4}$$

$$= 706.86 \text{ mm}^2$$

$$P_R = \frac{550 \times 706.86}{1 + \frac{1}{1600} \times \left(\frac{1500}{16}\right)^2}$$

$$= \frac{388773}{1 + 5.493}$$

$$= 59875 \text{ N}.$$

$$K = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi D^4}{64} = \frac{\pi (50^4 - 40^4)}{64}$$

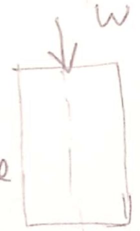
$$= \sqrt{\frac{18104.625}{706.5}}$$

$$= 16.0078$$

Factor of safety (FOS).

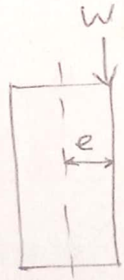
= Ratio b/w critical load & Safe load.

$$\boxed{\frac{\text{Critical load}}{\text{Safe load}} = \text{factor of safety}}$$



Direct stress (σ_0)

Line of action of load coincide with the axis of the column the stress produced is uniform direct compressive stress or direct stress (f_a).



Bending stress (σ_b)

e = eccentricity

w = eccentric load.

The load is acted at a point away from its axis. Due to eccentric load bending stress is developed (f_b)

when the line of action of load does not coinside with the axis of the column, this load is known as eccentric load.

? length of column $L = 4\,m \rightarrow 4 \times 10^3\,mm$.

breadth $= 200\,mm$.

depth $= 100\,mm$.

$E = 200\,KN/mm^2 \rightarrow 200 \times 10^3\,N/mm^2$,

Both end is hinged $le = l$.

$$P = \frac{\pi^2 E I}{Le^2}$$

I = ~~minimum~~ least moment of inertia

$$I_{xx} = \frac{bd^3}{12} = 16.67 \times 10^6\,mm^4$$

$$I_{yy} = \frac{db^3}{12} = 66.67 \times 10^6\,mm^4$$

$I_{min} = 16.67 \times 10^6\,mm^4$.

$$= \frac{\pi^2 \times 200 \times 10^3 \times 16.67 \times 10^6}{(4 \times 10^3)^2}$$

$$= \frac{3.287 \times 10^{13}}{16 \times 10^6}$$

$$= 2.05 \times 10^6\,N$$

Resultant stress $\rightarrow$ Algebraic sum of direct stress and bending stress.

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right) \text{ or } \boxed{\frac{P}{A} + \frac{M}{Z}}$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right) \text{ or } \boxed{\frac{P}{A} - \frac{M}{Z}}$$

A rectangular column.

width = 200mm.

Thickness = 150mm

Point load = 240 KN $\rightarrow 240 \times 10^3$ N.

eccentricity e = 10mm.

Find the maximum and minimum stress.

Z = section modulus

$= \frac{I}{y}$ moment of inertia

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right) \text{ or } \frac{P}{A} + \frac{M}{Z},$$

$$\sigma_{mini} = \frac{P}{A}\left(1 - \frac{6e}{b}\right) \text{ or } \frac{P}{A} - \frac{M}{Z}.$$

$P = 240 \times 10^3$ N.

$A = 200 \times 150$.

$e = 10$ mm.

$b = 200$ mm.

$$\sigma_{max} = \frac{240 \times 10^3}{(200 \times 150)}\left(1 + \frac{6 \times 10}{200}\right).$$

$$= \underline{\underline{10.4 \text{ N/mm}^2}}$$

$$\sigma_{mini} = \frac{240 \times 10^3}{(200 \times 150)}\left(1 - \frac{6 \times 10}{200}\right).$$

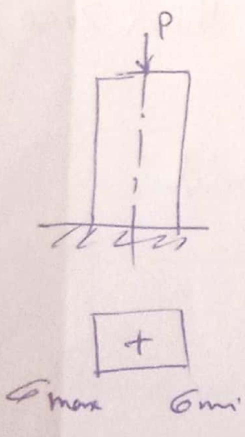
$$= \underline{\underline{5.6 \text{ N/mm}^2}}$$

σ_0 (direct stress) $= \frac{P}{A}$.

σ_b (bending stress) $= \frac{M}{Z}$

Section modulus

M = moment (Pe)

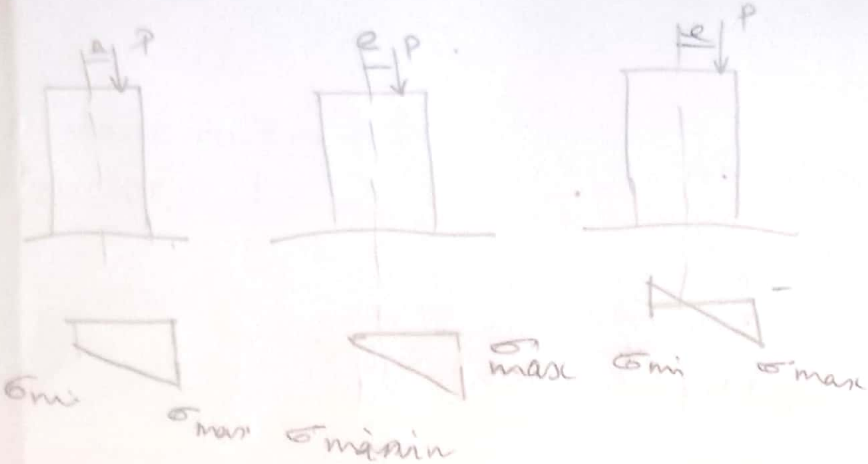


σ_{max} = +v (compression)

σ_{mi} = +ve compression

σ_{mini} = −ve tensile.

$\left\{ \sigma_0 - \sigma_b \right\}$.



Limit of eccentricity (e - limit)

The maximum distance of load from centre of column, such that if load act within the distance there is no tension in the column. This maximum distance is called limit of eccentricity.

$\boxed{\sigma_{mini} \neq -ve}$

when the load is acting within the e limit σ minimum will positive (load will be compression)

when the load is acting at the point of e limit. σ minimum will be zero.

No tension condition

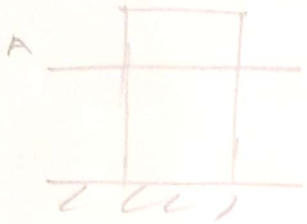
* $\sigma_{min} \geq 0$ (equal or greater than zero)

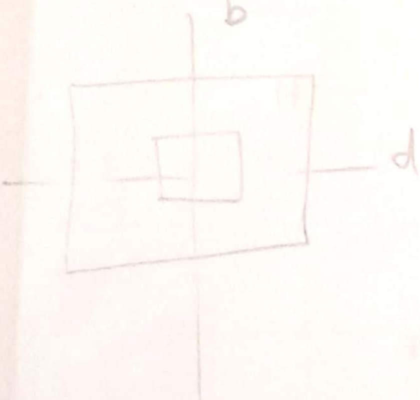
* Load acting within e - limit

$$\boxed{e\ limit = \frac{Z}{A}}$$

Core or Kernal of Section

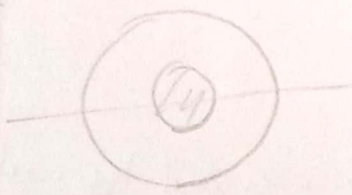
The central part of the cross section of column joining the points of e limit, such that if load acts within this part there will be no tension induced in the column, This central part is known as core or kernal of the section





$$e_x = \frac{b}{6}$$

$$e_y = \frac{b}{6}$$



$$e_y = \frac{D}{8}$$

$$e_x = \frac{D}{8}$$

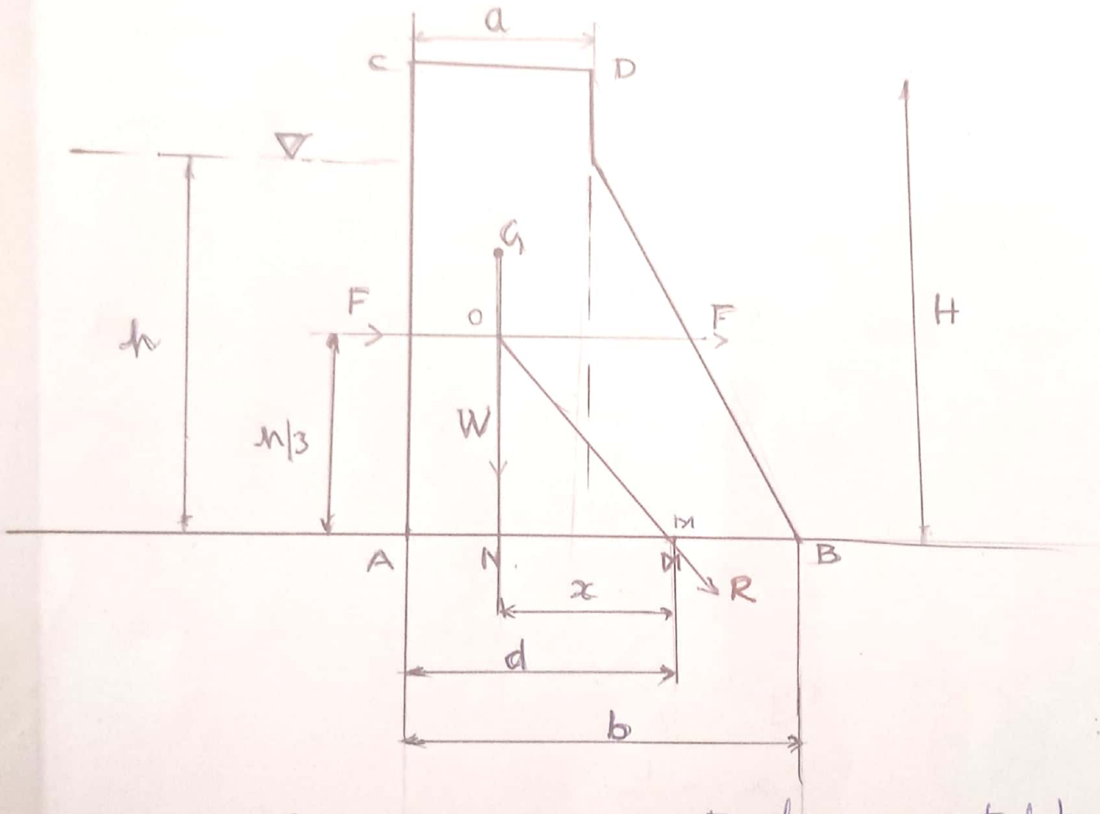
Middle third rule

In case of rectangular cross section, if the load is applied at location along the middle third part of both mutually perpendicular axis then the stress produced are wholly of compressive

Dam

- Main purpose of dam is to store water.
- Front side of dam is toe level

Rare side, where the water is stored → Heel level.



H = height of dam.

h = height of water.

a = Top width of dam.

b = Bottom width of dam.

W_0 = weight density of dam masonry.

F = force exerted by water.

W = weight of dam.

ω = weight density of water.

G = Centre of gravity

R = resultant force

Density of water $= 1000 kg/m^3$.

$10 KN/m^3 \rightarrow 9810 N/m^3$.

Force exerted by water / water pressure

$$\boxed{P/F = \frac{1}{2} wh^2 \left(\frac{wh^2}{2}\right)}$$

w = density of water.

weight of dam $\boxed{W = W_0 \cdot \left(\frac{a+b}{2}\right)H}$

W_0 = weight density of dam

$$\boxed{\text{Resultant } R = \sqrt{F^2 + W^2}}$$

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

$$e = d - \frac{b}{2}$$

$$(d = AN + x)$$

$$AN = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$x = \frac{F}{W} \times \frac{h}{3}$$

? $H = 18m$

$h = 15m$.

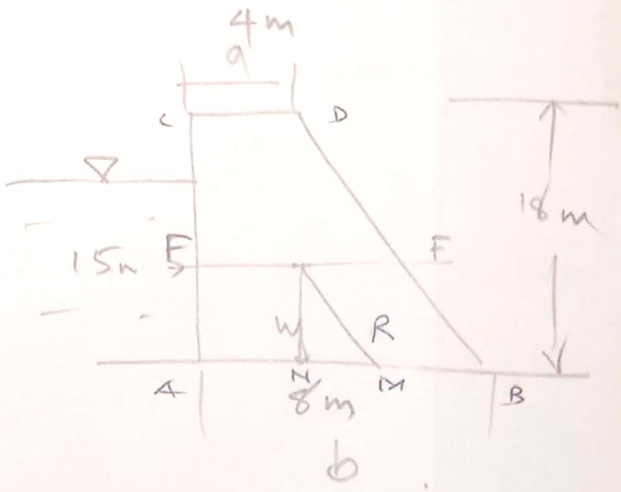
$a = 4m$.

$b = 8m$.

$W_0 = 19.62 KN/m^3$

$= 19.62 \times 10^3 N$

$w = 9810$.



(i) $F = \frac{wh^2}{2}$

$= \frac{9810 \times 15^2}{2}$

$= 1103625 N$

(ii) $W = \omega_0 \left(\frac{a+b}{2}\right) H$

$= 19620 \left(\frac{4+8}{2}\right) 18$

$= \underline{2118960\,N}$

$e = d - \left(\frac{b}{2}\right)$

$= 5.715 - \frac{8}{2}$

$= \underline{1.715}$

$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

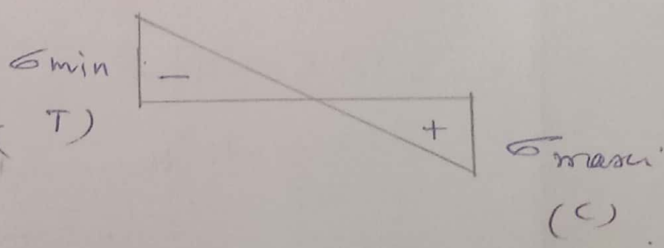
$= \frac{2118960}{8}\left(1 + \frac{6 \times 1.715}{8}\right)$

$= \underline{605492.82\,N/m^2}$ (Compressive)

$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

$= \frac{218960}{8}\left(1 - \frac{6 \times 1.715}{8}\right)$

$= \underline{-75752.82}\ N/m^2$ (Tensile)



$R = \sqrt{F^2 + W^2}$

$= \sqrt{(1103625)^2 + (2118960)^2}$

$= \underline{2389137.84\,N}$

$d = AN + x$

$AN = \frac{a^2 + ab + b^2}{3(a+b)}$

$= \frac{4^2 + (8 \times 4) + 8^2}{3(8+4)}$

$AN = \underline{3.11\,m}$

$x = \frac{F}{W} \times \frac{h}{3}$

$= \frac{1103625}{2118960} \times \frac{15}{3}$

$x = \underline{2.604\,m}$

$d = 3.11 + 2.604$

$= \underline{5.715\,m}$

Angle of repose.

A dam may fail.

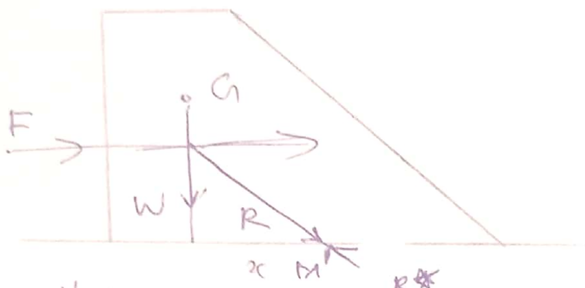
→ by sliding on the soil on which it rest

→ by overturning

→ Due to tensile stress developed.

→ Due to excessive compressive stress.

(i) Condition to prevent the sliding of the dam.



The dam will be in equilibrium if or for R^* equal to 'R' is applied at a point M in the opposite direction of 'R'

R^* → reaction of dam.

The max force of friction:

$$F_{max} = \mu \times W$$

μ. co-efficient of friction b/w the base of dam & soil

If F_{max} > force due to water pressure (F) the dam will be safe against sliding.

(ii) Condition to prevent the overturning of dam.

If the resultant R strike the base within the width, there will be no overturning.

Moment due to F. $F \times h/3$.

Moment due to W. $W \times x$

For the equilibrium of dam [Two moments should be equal]

$$\boxed{F \times \frac{h}{3} = Wx}$$

Restoring moment $= W \times NB$,

There will be no overturning about point 'B' if restoring moment about B is greater than the overturning moment about 'B'.

(iii) Conditions to avoid tension at the base.

The maximum distance between A and the point through which 'R' meets the base $\leq 2/3$ of the base width there will be no tension.

$$\boxed{d \leq \frac{2}{3} b}$$

(iv) Condition to avoid the excessive compressive stress at the base.

Maximum stress in the masonry should be less than the permissible stress in the masonry.

$$\sigma_{max} < \sigma_{permissible}.$$

? $a = 4m$

$b = 8m$

$H = 12m$

$h = 10m$

$W_0 = 2000\ kg/m^3$

$= 2000 \times 9.81$

$= 19620\ N/m^3$

$w = 9810\ N/m^3$

$\mu = 0.55$

$\sigma_{permissible} = 343350\ N/m^2$

$F = \frac{wh^2}{2}$

$= 9810 \times \frac{(10)^2}{2}$

$= 490500\ N$

$W = w_0 \left(\frac{a+b}{2}\right) H$

$= 19620 \left(\frac{4+8}{2}\right) \times 12$

$= 1412640\ N$

$e = d - \frac{b}{2}$

$= 4.267 - \frac{8}{2}$

$= 0.267$

$d = AN + x$

$AN = \frac{a^2 + ab + b^2}{3(a+b)}$

$= \frac{4^2 + (4 \times 8) + 8^2}{3(4+8)}$

$= 3.11\ m$

$x = \frac{F}{W} \times \frac{h}{3}$

$= \frac{490500}{1412640} \times \frac{10}{3}$

$= 1.157\ m$

$d = 3.11 + 1.157$

$= 4.267\ m$

(1) Check for sliding

$F_{max} = \mu \times W$

$= 0.5 \times 1412640$

$= 776952\ N > 490500$

$F_{max} > F$

(2) Check for overturning

R is passing through the base.

(iii) Check for tension. $(d < \frac{2}{3} b)$.

$d = 4.267 m$.

$\frac{2}{3} \times 8 = 5.33 m$.

$d < \frac{2}{3} b$. no tension

(IV) Check for compression. $\sigma_{max} < \sigma_{permissible}$

$\sigma_{max} = \frac{W}{b} \left(1 + \frac{6e}{b} \right)$

$= \frac{1412640}{8} \left(1 + \frac{6 \times 0.267}{8} \right)$

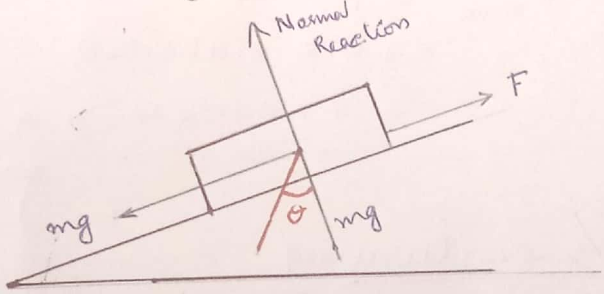
$\sigma_{max} = \underline{211940} < 343350 N$.

Retaining wall

The wall which are used for retaining soil or earth are known as retaining wall. The earth retained by a retaining wall exerts a pressure on the retaining wall in the same way as water exerts pressure on the dam.

Angle of repose

The maximum inclination of a plane at which a body remains the equilibrium over the inclined plane by the assistance of friction only



CamScanner

Rankine's theory of earth pressure

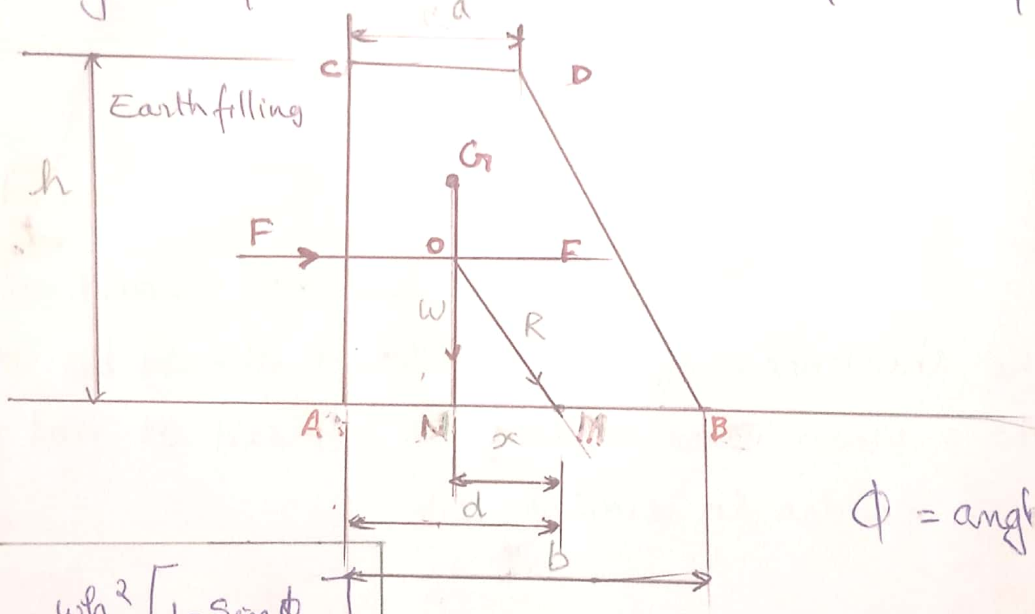
It is used to determine the pressure exerted by the earth or soil on the retaining wall.

Assumptions

1. The earth or soil retained by a retaining wall is cohesionless

2. Frictional resistance between the retaining wall and retained material is neglected.

3. The failure of the retained material takes place along a plane known as rupture plane

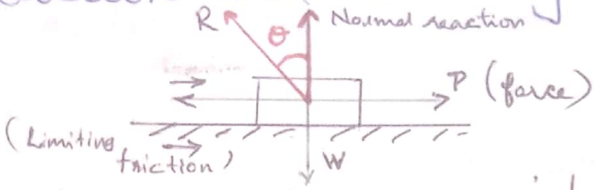


ϕ = angle of repose

$$\text{Force } F = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$$

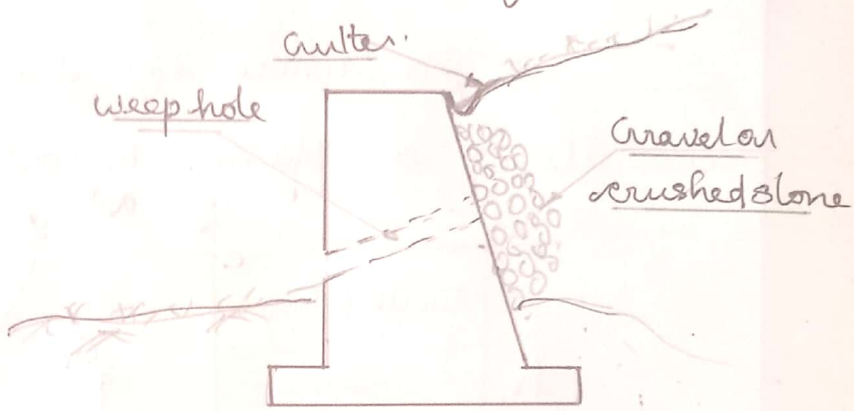
$$\text{Pressure intensity } P = wh\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

• Angle of friction :- The angle made by the resultant of the normal reaction and limiting friction.



• Purpose of weep holes .

weep holes are the openings provided to drain off accumulated water from the retained side. The additional hydrostatic pressure can be reduced by providing weep hole at the bottom of these structure.



The length of weep hole should not be less than the thickness of the wall and should be at least 50 mm dia. weep holes should be spaced at not more than 2 m center to center.