MODULE II

KINEMATICS AND DYNAMICS OF FLUID FLOW

KINEMATICS

Kinematics of fluid flow deals with the motion of fluid particles without considering the forces producing the motion. Kinematics of fluid flow deals with the study of velocity and acceleration of liquid particles in a flow.

HYDRODYNAMICS

Hydrodynamics deals with the motion of liquids and considering forces causing the flow; Its application to flow through pipes, channels and orifices etc.

TYPES OF FLUID FLOW

The fluid flow can be classified as follows

1. Steady and unsteady flow
2. Uniform and non-uniform flow
3. Laminar and turbulent flow
4. Compressible and incompressible flow
5. Rotational and irrotational flow
6. One-, two- and three-dimensional flow.

1-Steady and unsteady flow.

Steady flow is defined as the type of flow in which the fluid characteristics like velocity, density, pressure etc at a point do not change with time.

Unsteady flow

Unsteady flow is defined as the type of flow in which the fluid characteristics like velocity, density, pressure etc at a point change with time.

2- Uniform flow and non-uniform flow

A uniform flow is defined as the type of flow in which the fluid characteristics like velocity, pressure, density etc; at a given instant remains the same at all points. or we can say that velocity, pressure, density etc at any given time has the same value at all points and is dependent of the space position.If the characteristics like velocity, pressure ,density have different values at different points at a given instant of time ,the flow is said to be **non-uniform.**

3- Laminar and turbulent flow.

A laminar flow is defined as the type of flow in which the fluid particles move in layers, gliding smoothly over the adjacent layers. The fluid particles in any layer move along a well- defined path (stream lines) and all the stream lines are straight and parallel. The paths of the individual particles do not cross each other. This type of flow is also called stream line flow or viscous flow.

Turbulent flow is the most common type of flow that occurs in nature. This flow is characterised by random, erratic, unpredictable, zig-zag motion of fluid particles which results in eddies formation which are responsible for high energy loss.

4-Compressible and incompressible flow

A fluid, in which the density of fluid changes from point to point is called compressible flow or in other words the density is not constant for the fluid. All gases are, generally, considered to have compressible flows.

A flow in which the density of fluid is constant during the flow, is called **incompressible flow.** Mathematically for an incompressible flow, p = Constant

All liquids are generally considered to have incompressible flow.

5. Rotational and irrotational flow

Rotational flow is the type of flow in which liquid particles while flowing along a stream line also rotate about their own axis.

if the fluid particles while flowing along a stream line and do not rotate about their own axis that type of flow is called **irrotational flow.**

6-One, two, three-dimensional flow

A flow in which the streamlines of its moving particles may be represents by a strightline,is called one dimensional flow.

A flow in which the flow paths may be represented by a curve, is called two-dimensional flow.A flow in which the flow paths may be represented by three mutually perpendicular directions, is called three dimensional flow, An example of this flow is river flow.

RATE OF FLOW OR DISCHARGE

The quantity of fluid flowing per unit time across any section of a pipe or conduit is called the rate of flow or discharge and is denoted by Q. The rate of flow Q may be expressed mathematically as follows,

$Q = a \times v$

a is the area of cross section of the flow (measured in square metre, i.e., m^2)

v is the average velocity of fluid (measured in metre per second, i.e.,m/s)

S.I.unit of discharge may be measured in the following ways.

1. Weight of fluid flowing per second across a section - N/s.
2. Mass of fluid flowing per second across a section- kg/s.
3. Volume of fluid flowing per second across a section- **m^3/s or litres/s**(lps)

CONTINUITY EQUATION OF A LIQUID FLOW

This is an equation based on the principle of conservation of the mass. Thus considering a fluid flowing through a pipe, at all the cross sections, the quantity of fluid per second is constant.

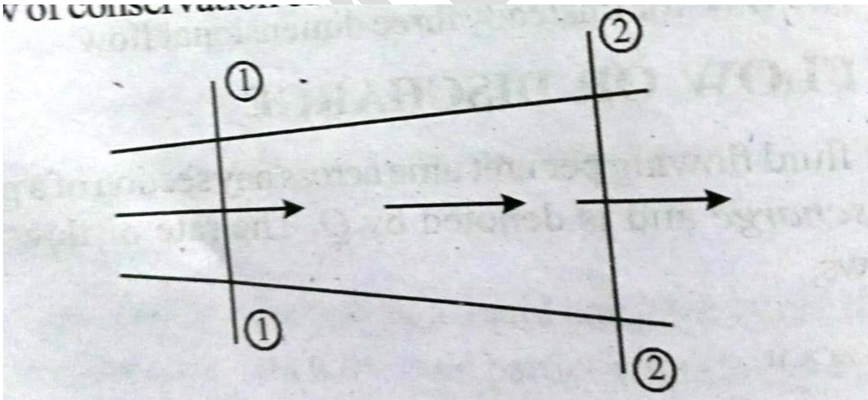


Fig -2.1.1

Problem 2.1.1 Water is flowing into a horizontal tapered pipe of diameter 200 mm at the larger end and 100 mm at the smaller end. The velocity of water at the larger end is 4.5 m/s. Determine the rate of discharge and velocity at the smaller end.

Solution : Given

Larger diameter of tapered pipe, $d_1 = 200 \text{ mm} = 0.2 \text{ m}$

Smaller diameter of tapered pipe, $d_2 = 100 \text{ mm} = 0.1\text{m}$

Velocity of flow at larger end, $v_1 = 4.5 \text{ m/sec}$

Larger cross sectional area, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$

Smaller cross sectional area, $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 \text{ m}^2$

Discharge through the pipe, $Q = a_1 v_1 = 0.0314 \times 4.5 =$ **$0.1413 \text{ m}^3/\text{s}$** ***(Ans)***

Using continuity equation, *i.e.,* $a_1 v_1 = a_2 v_2$

$\therefore$ Velocity at the smaller end of the pipe, $v_2 = \frac{a_1 v_1}{a_2} = \frac{0.1413}{0.00785}$ **$= 18 \text{ m/s}$** ***(Ans)***

Problem 2.1.2 The diameter of a pipe is gradually reduced from 500

BENOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's Theorem

This theorem is a form of the well known principle of conservation of energy. The theorem is stated as follows. "In a steady continuous flow of frictionless incompressible fluid, the sum of the potential head, pressure head and kinetic head is the same at all points". Hence it is mathematically represented as

ASSUMPTIONS IN BERNOULLIS THEOREM

The following are the assumptions made in the Bernoullis equation
(i) The fluid is ideal i.e., viscosity is zero.
(ii) The flow is steady
(iii) The flow is incompressible
(iv) The flow is irrotational, and
(v) The flow is one dimensional.

LIMITATIONS OF BERNOULLIS THEOREM

1. In Bernoullis theorem, it is assumed that the velocity of every liquid particle at any cross-section is uniform. But actually, the velocity of liquid particle at the centre of the pipe is maximum and gradually decreases towards the walls of the pipe due to friction. Thus, while using Bernaullis equation, we take only mean velocity
2. In Bernoullis theorem, it is also assumed that no external force except the gravity force is acting on the liquid. But actually, there are some external forces like pipe friction is acting on the liquid, which may effect the flow of liquid.
3. In Bernoullis theorem, it is assumed that there is no loss of energy while flowing. But actually, it is not true. For example, in turbulant flow, some of the kinetic energy is converted into heat energy and in a viscous flow, some energy is lost due to shear forces.
4. If the fluid is flowing in a curved path, the energy due to centrifugal force should also be taken into account. But normally we neglect the energy due to centrifugal force.

VENTURIMETER

This is an instrument in which the practical application of Bernoullis theorem is applied. It is used for measuring the discharge or rate of flow of fluid. It consists of three parts.

i. A short convergent part

ii. A throat; and

iii. A long divergent part

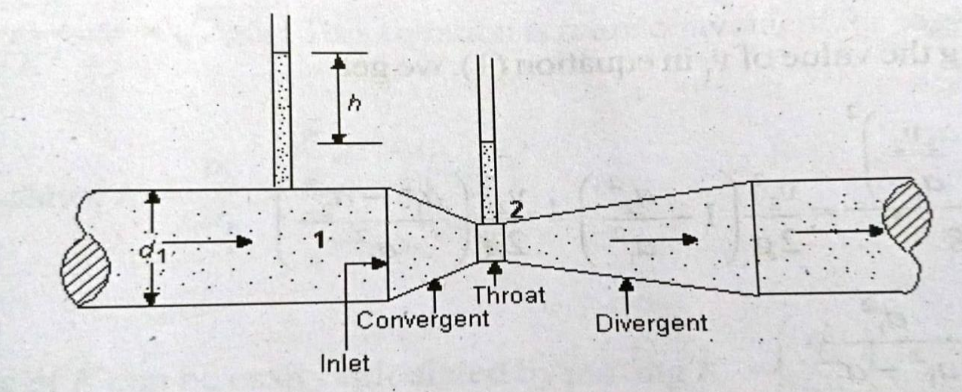


Fig -2.1.12 Venturimeter

The discharge throught the venturimeter $Q = C_d \times \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

ORIFICE METER OR ORIFICE PLATE

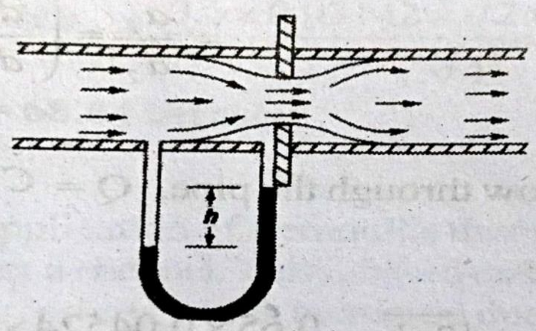


Fig 2.1.13 Orifice meter or orifice plate

This is another devise to measure the rate of flow of fluid through a pipe. This is also based on the priciple of Bernoullis theorem. It consists of a plate having sharp edged circular hole known as orifice. This plate can be fitted to the pipe by flanged joint.

i.e., Discharge through the orifice meter

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = C_d \frac{a_1}{\sqrt{K^2 - 1}} \sqrt{2gh}$$

Where,

Area ratio, $K = \frac{a_1}{a_2} = \left(\frac{d_1}{d_2}\right)^2$ and C_d is the coefficient of discharge.

discharge.

Problem 2.1.33. An orifice meter of diameter 120mm is inserted in a pipe of diameter 240mm to measure the rate of flow of a liquid of specific gravity 0.88. The differential manometer connected between the upstream and downstream sides shows a mercury level difference of 400mm. If the coefficient of discharge of meter is 0.65, determine the rate of flow of oil.

(Diploma Examination Question April 2004)

Solution : Given

Diameter of orifice, $d_2 = 120\text{mm} = 0.12\text{ m}$

Diameter of pipe, $d_1 = 240\text{mm} = 0.24\text{m}$

Specific gravity of liquid in the pipe line, $S_p = 0.88$

Differential mercury manometer reading, $x = 400\text{mm} = 0.4\text{m}$

Coefficient of discharge, $C_d = 0.65$

Assume specific gravity of mercury, $S_h = 13.6$

Area of the pipe, $a_1 = \frac{\pi}{4} d_1^2$

$= \frac{\pi}{4} \times (0.24)^2$

$= 0.04524\text{ m}^2$

Differential head, $h = x\left[\frac{S_h}{S_p} - 1\right]$

$= 0.4\left[\frac{13.6}{0.88} - 1\right]$

$= 5.7818$ m of liquid

Area ratio, $K = \frac{a_1}{a_2} = \left(\frac{d_1}{d_2}\right)^2$

$= \left(\frac{0.24}{0.12}\right)^2 = 4$

$\therefore$ Discharge or rate of flow through the pipe,

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= C_d \times \frac{a_1}{\sqrt{K^2 - 1}} \times \sqrt{2gh}$$

$$= \frac{0.65 \times 0.04524 \times \sqrt{2 \times 9.81 \times 5.7818}}{\sqrt{4^2 - 1}}$$

$= 0.08087\text{ m}^3/\text{s}$

$= 80.87$ litres/s (Ans)

PITOT TUBE

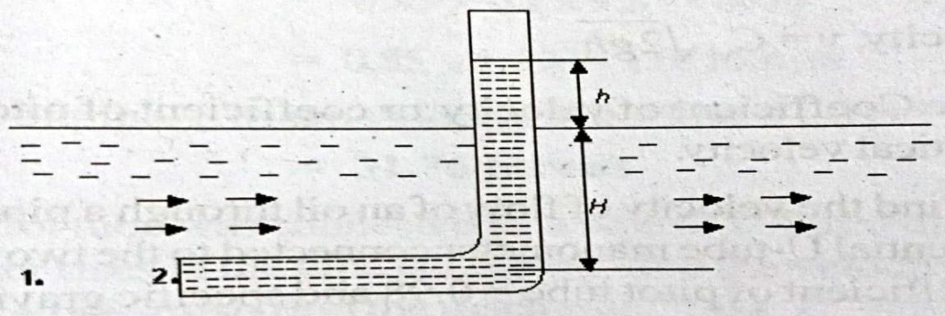


Figure 2.1.14 Pitot tube

Pitot tube is another application of Bernoulli’s theorem. It is used to find the velocity of flow at any point in a pipe line or a channel. This is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of kinetic energy into pressure energy. The simplest form of pitot tube is shown in Fig 2.1.14, which is a glass tube, bent at right angles.

The lower end of the tube faces the direction of flow as shown in Fig 2.1.14. The liquid rises up in tube due to the conversion of kinetic energy in to pressure energy. The velocity of fluid is determined by measuring the rise of fluid in the tube.

$$\text{velocity, } v = C_v \sqrt{2gh}$$

Problem 2.1.39 A pitot tube is used to determine the velocity of a boat Find the speed of the boat if the water level in the tube is 0.75 m above the surface. Take the $C_v = 0.98$

Solution : Given

Water level in the pitot tube, h $= 0.75$ m

Coefficient of velocity, C_v $= 0.98$

Velocity of the boat, v $= C_v\sqrt{2gh}$

$= 0.98\times\sqrt{2\times 9.81\times 0.75}$

$=$ **3.7593 m/s.** (Ans)

Speed of the boat $= \dfrac{3.7593\times 60\times 60}{1000}$

$=$**13.533 km / hr.** (Ans)

NOTCHES

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a ank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

CLASSIFICATION OF NOTCHES

According to the shape of the opening

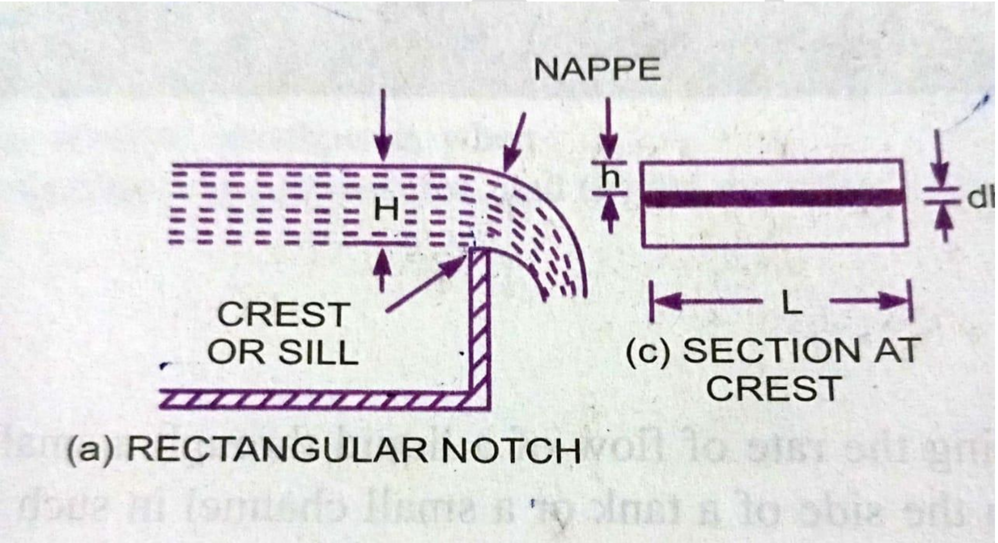
(a) Rectangular notch,

(b) Triangular notch,

(c) Trapezoidal notch, and

(d) Stepped notch.

Rectangular notch



H= Head of water over the crest.
L=Length of the notch
Cd= Co-efficient of discharge.
Discharge equation

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\ [H]^{3/2}.$$

Problem 8.1 *Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.*

Solution. Given :

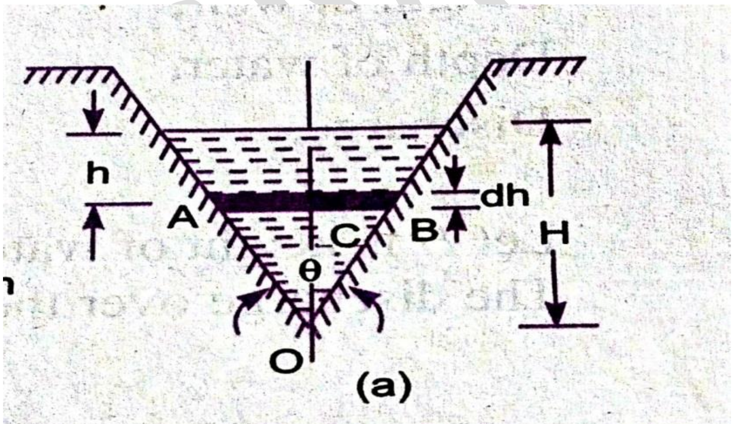
Length of the notch, $L = 2.0$ m

Head over notch, $H = 300 \text{ m} = 0.30 \text{ m}$

$C_d = 0.60$

Discharge,
$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\left[H^{3/2}\right]$$
$$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times [0.30]^{1.5} \text{ m}^3/\text{s}$$
$$= 3.5435 \times 0.1643 = \mathbf{0.582\ m^3/s.\ Ans.}$$

TRIANGULAR NOTCH



.3 *The triangular notch.*

Let H = Head of water above the v notch

ɵ = Angle of notch

Discharge equation

$$= \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= 1.417\ H^{5/2}.$$

Problem 8.4 *Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$.*

Solution. Given :

Angle of V-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.6$

Discharge, Q over a V-notch is given by equation (8.2)

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \tan \frac{60°}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.8182 \times 0.0493 = \mathbf{0.040\ m^3/s.\ Ans.}$$

TRAPEZOIDAL NOTCH

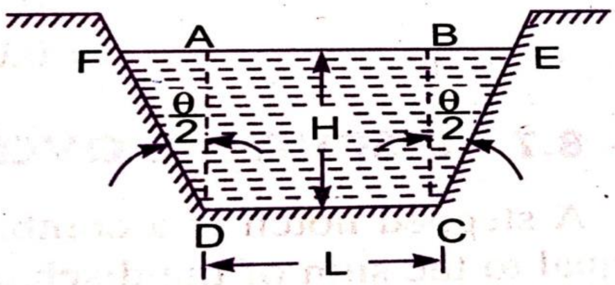


Fig. 8.4 *The trapezoidal notch.*

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan \theta/2 \times \sqrt{2g} \times H^{5/2}.$$

H=Height of water over the notch
L= Length of the crest of the notch
Cd1=Co-efficient of discharge for rectangular portion. ABCD
Cd2=Co-efficient of discharge for triangular portion. FAD and BCE.

FLOW THROUGH PIPES

LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

1-Major Energy Losses

2-Minor Energy Losses

1-Major Energy Losses

This is due to friction and it is calculated by the following formulae:

(a) Darcy-Weisbach Formula

(b) Chezy's Formula

2-Minor Energy Losses

(a) Sudden expansion of pipe

(b) Sudden contraction of pipe

(d) Pipe fittings etc.

(e) An obstruction in pipe.

Darcy-Weisbach Formula

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$= \frac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \frac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe,
V = mean velocity of flow,
d = diameter of pipe.

Problem 2.3.1. The velocity of water in a pipe of 200mm diameter is 5 m/s. The length of the pipe is 600 metre. Find the loss of head due to friction. If $f = 0.008$

(Diploma Examination Question, November 2005 , April -2000)

Solution : Given

Diameter of pipe, d $= 200\text{mm} = 0.2\text{ m}$

Velocity of flow, v $= 5\text{ m/s}$

Length of pipe, l $= 600\text{ m}$

Friction factor, f $= 0.008$

Using the equation for the loss of head due to friction,

$$h_f = \frac{4flv^2}{2gd} = \frac{4\times0.008\times600\times5^2}{2\times9.81\times0.2} = \mathbf{122.3\text{ m of water}} \qquad (Ans)$$

$2\times9.81\times0.5$

Problem 2.3.3. A pipe 500 m long is conveying water with a velocity of 1 m/s. Find the suitable diameter of the pipe, if the loss of head due to friction is 3.4 m. Take $f = 0.01$

(Diploma Exam Question, April - 2005)

Solution : Given

Length of pipe, l $= 500\text{m}$

Velocity of flow, v $= 1\text{m/s}$

Loss of head due to friction, $h_f = 3.4\text{m}$

Friction factor, f $= 0.01$

Using the equation of loss of head due to friction, i.e., $h_f = \dfrac{4flv^2}{2gd}$

$$\therefore \text{ Diameter of pipe, } d = \frac{4flv^2}{2gh_f} = \frac{4\times0.01\times500\times1^2}{2\times9.81\times3.4} = \mathbf{0.2998\text{ m}} \text{ say } \mathbf{300\text{mm}} \quad (Ans)$$

CHEZY'S FORMULA

$$v = C\sqrt{mi}$$

Problem 11.1 *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.*

Take ν for water = 0.01 stoke.

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow, $V = 3 \text{ m/s}$

Chezy's constant, $C = 60$

Kinematic viscosity, $\nu = 0.01 \text{ stoke} = 0.01 \text{ cm}^2/\text{s}$

$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}.$

(*i*) **Darcy Formula** is given by equation (11.1) as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by $$R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$$

$\therefore$ Value of $$f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$$

$\therefore$ Head lost, $$h_f = \frac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} = \mathbf{.7828 \text{ m. Ans.}}$$

(*ii*) **Chezy's Formula.** Using equation (11.4)

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \dfrac{d}{4} = \dfrac{0.30}{4} = 0.075 \text{ m}$

$\therefore$ $$3 = 60\sqrt{.075 \times i} \text{ or } i = \left(\frac{3}{60}\right)^2 \times \frac{1}{.075} = 0.0333$$

But $$i = \frac{h_f}{L} = \frac{h_f}{50}$$

Equating the two values of i, we have $\dfrac{h_f}{50} = .0333$

$\therefore$ $h_f = 50 \times .0333 = \mathbf{1.665 \text{ m. Ans.}}$

... of a pipe of length 2000 m when the rate of flow of water throu