MATHEMATICS - I

SUBJECT CODE : 1002 (Revision-2021)

IMPORTANT QUESTIONS AND ANSWERS

Module-1

Chapter :1 Complex Numbers

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1. Find the real part and imaginary part of the complex number $\sqrt{2} - 2i$

   Real part of the complex number $= \sqrt{2}$

   Imaginary part of the complex number $= -2$

2. Find the conjugate of the complex number $-\sqrt{3}i - \sqrt{5}$

   Given $Z = -\sqrt{3}i - \sqrt{5}$

   $Z = -\sqrt{5} - \sqrt{3}i$

   Conjugate of the complex number $Z = \overline{Z} = -\sqrt{5} + \sqrt{3}i$

3. Find the modulus of the complex number $Z = -3 - 2i$

   Modulus of the complex number $Z = |Z| = \sqrt{x^2 + y^2}$

   Here $x = -3$ and $y = -2$

   $|Z| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

4. Find the polar form of the complex number $Z = 1 + i$

   The polar form of the complex number is $Z = r\,(cos\theta + isin\theta)$

   Where $r = |Z| = \sqrt{x^2 + y^2}$, Amplitude $= \theta = tan^{-1}\left(\frac{y}{x}\right)$

   Here $x = 1$ and $y = 1$

   $r = |Z| = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$

   Amplitude$\theta = tan^{-1}\left(\frac{1}{1}\right) = tan^{-1}(1) = 45^o = \frac{\pi}{4}$

   The polar form of the complex number is $Z = r\,(cos\theta + isin\theta) \quad = \sqrt{2}\left(cos\left(\frac{\pi}{4}\right) + isin\left(\frac{\pi}{4}\right)\right)$

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5. Find the polar form of the complex number $Z = 1 + \sqrt{3}i$

The polar form of the complex number is $Z = r\,(cos\theta + isin\theta)$

Where $r = |\,Z\,| = \sqrt{x^2 + y^2}$, Amplitude $= \theta = tan^{-1}\left(\frac{y}{x}\right)$

Here $x = 1$ and $y = \sqrt{3}$

$$r = |\,Z\,| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\text{Amplitude}\theta = tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = tan^{-1}\left(\sqrt{3}\right) = 60^o = \frac{\pi}{3}$$

The polar form of the complex number is $Z = r\,(cos\theta + isin\theta) \quad = 2\left[cos\left(\frac{\pi}{3}\right) + isin\left(\frac{\pi}{3}\right)\right]$

6. Add the complex numbers $2 - 3i$, $-3 + 5i$, $4 + 6i$, $5 + 8i$

Given,

$Z_1 = 2 - 3i$

$Z_2 = -3 + 5i$

$Z_3 = 4 + 6i$

$Z_4 = 5 + 8i$

$Z_1 + Z_2 + Z_3 + Z_4 = 2 - 3i + -3 + 5i + 4 + 6i + 5 + 8i$

Separating the real and imaginary parts and then adding,

$Z_1 + Z_2 + Z_3 + Z_4 = (2 + -3 + 4 + 5) + (-3i + 5i + 6i + 8i)$

$Z_1 + Z_2 + Z_3 + Z_4 = 8 + i(-3 + 5 + 6 + 8)$

$Z_1 + Z_2 + Z_3 + Z_4 = 8 + i(16) = 8 + 16i$

7. Subtract $5 - 8i$ from $6 - 3i$

Given, $Z_1 = 5 - 8i$ and $Z_2 = 6 - 3i$

$Z_2 - Z_1 = 6 - 3i - \left(5 - 8i\right)$

$Z_2 - Z_1 = 6 - 3i - 5 + 8i = (6 + -5) + (-3i + 8i)$

$Z_2 - Z_1 = (6 - 5) + i(-3 + 8) = 1 + i(5) = 1 + 5i$

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8. Find the product of the complex numbers $5+2i$ and $1-2i$

   Given, $Z_1 = 5+2i$ and $Z_2 = 1-2i$

   $Z_1Z_2 = (5+2i)(1-2i) = (5\times 1 + 5\times -2i + 2i\times 1 + 2i\times -2i)$

   $Z_1Z_2 = 5 + -10i + 2i + -4i^2$ we know that $i^2 = -1$

   $Z_1Z_2 = 5 - 10i + 2i - 4\times -1 = 5 - 10i + 2i + 4$

   $Z_1Z_2 = (5+4) + (-10i + 2i) = 9 + i(-10+2) = 9 + i(-8) = 9 - 8i$

9. Divide $3+2i$ by $2+i$

   Given, $Z_1 = 3+2i$ and $Z_2 = 2+i$

   $$\frac{Z_1}{Z_2} = \frac{3+2i}{2+i}$$

   Now consider the conjugate of $2+i = 2-i$

   Multiply Nr. and Dr. By $2-i$

   $$\frac{Z_1}{Z_2} = \frac{(3+2i)}{(2+i)}\times\frac{(2-i)}{(2-i)} = \frac{(3+2i)(2-i)}{(2+i)(2-i)} = \frac{3\times 2 + 3\times -i + 2i\times 2 + 2i\times -i}{2^2 - i^2}$$

   $$\frac{Z_1}{Z_2} = \frac{6-3i+4i-2i^2}{4--1} = \frac{6-3i+4i-2\times -1}{4+1} = \frac{6-3i+4i+2}{5} = \frac{(6+2)+(-3i+4i)}{5}$$

   $$\frac{Z_1}{Z_2} = \frac{(6+2)+i(-3+4)}{5} = \frac{8+i(1)}{5} = \frac{8+i}{5} = \frac{8}{5} + \frac{1}{5}i$$

10. Evaluate $(Z_1+Z_2)Z_1$ if $Z_1 = 2+3i$ and $Z_2 = 1+2i$

    Given, $Z_1 = 2+3i$ and $Z_2 = 1+2i$

    $Z_1 + Z_2 = 2 + 3i + 1 + 2i = (2+1) + (3i+2i)$

    $Z_1 + Z_2 = 3 + i(3+2) = 3 + i(5) = 3 + 5i$

    $(Z_1+Z_2)Z_1 = (3+5i)(2+3i) = 3\times 2 + 3\times 3i + 5i\times 2 + 5i\times 3i$

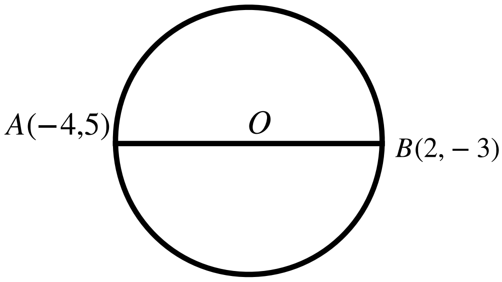
    $(Z_1+Z_2)Z_1 = 6 + 9i + 10i + 15i^2 = 6 + 9i + 10i + 15\times -1$

    $(Z_1+Z_2)Z_1 = 6 + 9i + 10i - 15 = (6-15) + (9i+10i)$

    $(Z_1+Z_2)Z_1 = -9 + i(9+10) = -9 + i(19) = -9 + 19i$

Chapter - 2, Co-Ordinate Geometry

11. The points $(-4,5), (2,-3)$ are at the ends of a diameter of a circle. Find its radius.



Diameter $= AB$

Using the distance formula we can find the length of the diameter

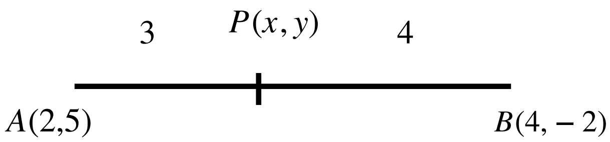
$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(2 - -4)^2 + (-3 - 5)^2} = \sqrt{(2+4)^2 + (-8)^2} = \sqrt{6^2 + (-8)^2}$$

$$AB = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ units}$$

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{AB}{2} = \frac{10}{2} = 5 \text{ units.}$$

12. Find the co-ordinates of the point which divides the segment joining the points $(2,5)$ and $(4,-2)$ internally in the ratio $3:4$



Using the section formula

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

We know that $P(x,y)$, so $x = \dfrac{mx_2 + nx_1}{m+n}, y = \dfrac{my_2 + ny_1}{m+n}$

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{3 \times 4 + 4 \times 2}{3+4}, \quad y = \frac{my_2 + ny_1}{m+n} = \frac{3 \times -2 + 4 \times 5}{3+4}$$

$$x = \frac{12+8}{7} = \frac{20}{7}, \quad y = \frac{-6+20}{7} = \frac{14}{7} = 2$$

$$P(x,y) = \left(\frac{20}{7}, 2\right)$$

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13.Find the centroid of a triangle having vertices (2,6), (4,0) and (8,2)

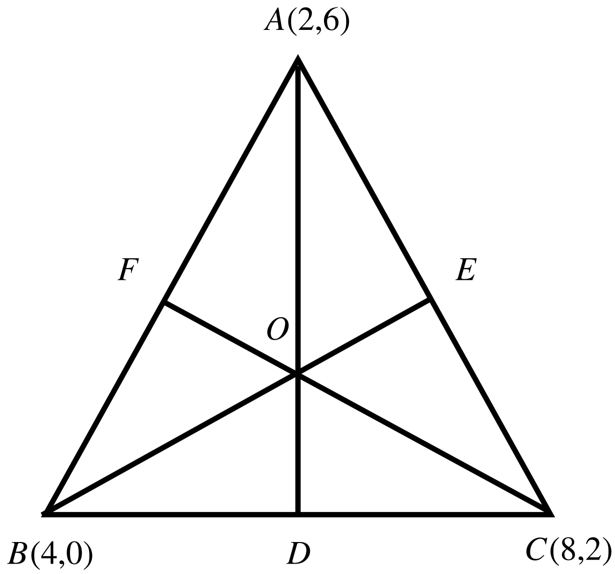
The centroid of a triangle $= O(x, y)$

$$x = \frac{x_1 + x_2 + x_3}{3}, y = \frac{y_1 + y_2 + y_3}{3}$$

$$x = \frac{2+4+8}{3}, y = \frac{6+0+2}{3}$$

$$x = \frac{14}{3}, y = \frac{8}{3}$$

The centroid of a triangle $= O(x, y) = \left(\frac{14}{3}, \frac{8}{3}\right)$



14. Write down the equation of the line with slope $\frac{1}{2}$ and $y-$intercept -1

We know that the slope intercept formula $y = mx + c$

Given, slope $= m = tan\theta = \frac{1}{2}$ and intercept $= c = -1$

$$y = \frac{1}{2}x + -1$$

$$y + 1 = \frac{1}{2}x \Longrightarrow 2(y+1) = 1 \times x \Longrightarrow 2y + 2 = x \Longrightarrow x - 2y - 2 = 0$$

15. Find the equation of a line with angle of inclination 45^o with the $X - axis$ and $y-$intercept -1

We know that the slope intercept formula $y = mx + c$

Given, $\theta = 45^o$, intersept $= c = -1$

Slope $= m = tan\theta = tan 45 = 1$

$$y = 1 \times x + -1 \Longrightarrow y = x - 1 \Longrightarrow x - y - 1 = 0$$

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16. Write down the equation of a line which makes an angle 150^o with the $X - axis$ and cutting the $Y - axis$ at the point $(0, -2)$

Given, $\theta = 150^o$ and $y -$ intercept $= c = -2$

We know that the slope intercept formula $y = mx + c$

Slope $= m = tan\theta = tan150 = tan(2 \times 90 - 30) = -tan30 = -\dfrac{1}{\sqrt{3}}$

$$y = -\frac{1}{\sqrt{3}}x + -2 \implies y = -\frac{1}{\sqrt{3}}x - 2 \implies y + 2 = -\frac{1}{\sqrt{3}}x$$

$$\sqrt{3}\left(y + 2\right) = -1 \times x \implies \sqrt{3}y + 2\sqrt{3} = -x \implies x + \sqrt{3}y + 2\sqrt{3} = 0$$

is the required equation.

17. A straight line is inclined at 135^o with the $X - axis$ and it passes through $(3, -4)$. Find the equation.

Given, $\theta = 135^o$ and $(x_1, y_1) = (3, -4)$

We know that the slope-point form of a straight line $y - y_1 = m(x - x_1)$

Slope $= m = tan\theta = tan135 = tan(2 \times 90 - 45) = -tan45 = -1$

The required equation is $y - -4 = -1(x - 3) \implies y + 4 = -x + 3 \implies x + y + 4 - 3 = 0$

$x + y + 1 = 0$

18. Write down the equation to the lines joining the pairs of points (3,8), (6,12)

We know that the two point form of a straight line $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Here $(x_1, y_1) = (3,8)$ and $(x_2, y_2) = (6,12)$

Slope $= m = tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{12 - 8}{6 - 3} = \dfrac{4}{3}$

The required equation is $y - 8 = \dfrac{4}{3}(x - 3) \implies 3(y - 8) = 4(x - 3)$

$3y - 24 = 4x - 12 \implies 4x - 3y + 24 - 12 = 0 \implies 4x - 3y + 12 = 0$

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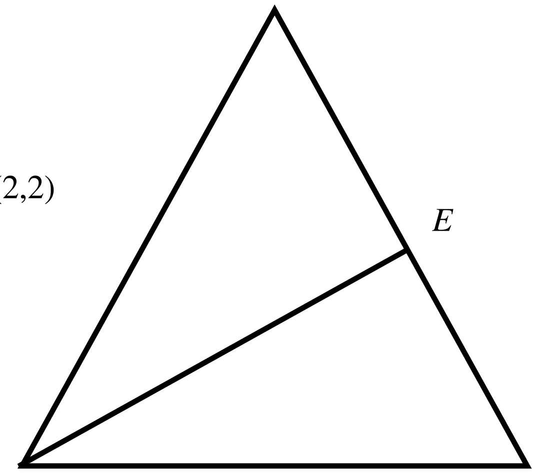
19. If $A(1,-1)$, $B(-2,1)$ and $C(3,5)$ are the vertices of a triangle. Find the equation of the median through $'B'$.



We have to find the equation of $\overline{BE}$.

The co-ordinates of E are $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{1+3}{2},\frac{-1+5}{2}\right)=\left(\frac{4}{2},\frac{4}{2}\right)=(2,2)$$

$\overline{BE}$ passes through $(-2,1)$ and $(2,2)$.

$(x_1,y_1)=(-2,1)$ and $(x_2,y_2)=(2,2)$

The required equation is $y-y_1=\dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$\text{Slope}=m=tan\theta=\frac{y_2-y_1}{x_2-x_1}=\frac{2-1}{2--2}=\frac{1}{4}$$

$$y-1=\frac{1}{4}(x--2) \implies 4(y-1)=1\times(x+2) \implies 4y-4=x+2$$

$\implies x-4y+4+2=0 \implies x-4y+6=0$ is the required equation.

20. Write down the equation of a line which has $y-$intercept -2 and passing through $(2,-4)$.

We know that the intercept form of a straight line $\dfrac{x}{a}+\dfrac{y}{b}=1$..........(1)

Given, $b=-2$ here $'a'$ is unknown

$\dfrac{x}{a}+\dfrac{y}{-2}=1$ also given that $(x,y)=(2,-4)$

$$\frac{2}{a}+\frac{-4}{-2}=1 \implies \frac{2}{a}+2=1 \implies \frac{2}{a}=1-2 \implies \frac{2}{a}=-1 \implies 2=-1\times a$$

$$\implies 2=-a \implies a=-2$$

Substitute the values of $a=-2$ and $b=-2$ in equation (1)

$$\frac{x}{-2}+\frac{y}{-2}=1 \implies \frac{x+y}{-2}=1 \implies x+y=1\times-2 \implies x+y=-2$$

$\implies x+y+2=0$ is the required equation.

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21. Find the slope and intercept of the straight line $2x - 3y + 5 = 0$

We know that the slope intercept formula $y = mx + c$

We can rewrite the given equation in the above form

$$2x - 3y + 5 = 0 \implies -3y = -2x + -5$$

$$y = \frac{-2x + -5}{-3} \implies y = \frac{-2}{-3}x + -5 \implies y = \frac{2}{3}x + -5$$

Here slope=Gradient $= m = \dfrac{2}{3}$

Next we want to find out the intercept of the straight line, so we rewrite the above equation in the form $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$$2x - 3y + 5 = 0 \implies 2x + -3y = -5$$

Dividing the above equation by $'-5'$ on both sides

$$\frac{2x + -3y}{-5} = \frac{-5}{-5} \implies \frac{2x}{-5} + \frac{-3y}{-5} = 1 \implies \frac{x}{\frac{-5}{2}} + \frac{y}{\frac{-5}{-3}} = 1$$

We now that $a = x-$intercept $= \dfrac{-5}{2}$, $b = y-$intercept $= \dfrac{-5}{-3} = \dfrac{5}{3}$

22. Find the angle between two lines with slopes $\sqrt{3}$ and $\dfrac{1}{\sqrt{3}}$

Angle between two lines $= \theta = tan^{-1}\left[\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}\right]$

Where $m_1 =$ slope of the first straight line.

$m_2 =$ slope of the second straight line.

Given, $m_1 = \sqrt{3}$ and $m_2 = \dfrac{1}{\sqrt{3}}$

$$\text{Angle} = \theta = tan^{-1}\left[\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}\right] = tan^{-1}\left[\frac{\frac{\sqrt{3}}{1} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}\right] = tan^{-1}\left[\frac{\frac{3-1}{\sqrt{3}}}{1 + 1}\right]$$

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$$\text{Angle} = \theta = tan^{-1}\left[\frac{\frac{2}{\sqrt{3}}}{\frac{2}{1}}\right] = tan^{-1}\left[\frac{2}{\sqrt{3}} \cdot \frac{1}{2}\right] = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^o$$

23. Find the equation of a straight line passing through (4,5) which is parallel to $2x + 3y = 4$

Any line parallel to $ax + by + c = 0$ has the form $ax + by + k = 0$ where k is new constant.

The given line is $2x + 3y = 4 \implies 2x + 3y - 4 = 0$

Any line parallel to $2x + 3y - 4 = 0$ has the form $2x + 3y + k = 0$ also given this line passes through (4,5).

$\therefore 2 \times 4 + 3 \times 5 + k = 0 \implies 8 + 15 + k = 0 \implies 23 + k = 0$

$k = -23$

The required parallel line is $2x + 3y + -23 = 0 \implies 2x + 3y - 23 = 0$

24. Find the equation of a straight line passing through (4,5) and perpendicular to $2x + 3y = 4$

Any line perpendicular to $ax + by + c = 0$ has the form $bx - ay + k = 0$ where k is new constant

The given line is $2x + 3y = 4 \implies 2x + 3y - 4 = 0$

Any line perpendicular to $2x + 3y - 4 = 0$ has the form $3x - 2y + k = 0$ also given this line passes through (4,5).

$\therefore 3 \times 4 - 2 \times 5 + k = 0 \implies 12 - 10 + k = 0 \implies 2 + k = 0$

$k = -2$

The required perpendicular line is $3x - 2y + -2 = 0 \implies 3x - 2y - 2 = 0$

25. Find the point of intersection of $3x - y + 5 = 0$ and $x + 3y - 2 = 0$

Given, $3x - y + 5 = 0$ and $x + 3y - 2 = 0$

We can rewrite the above equation in the form

$3x - y = -5$

$x + 3y = 2$

We can open a two by two determinant in the way $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c = ad - bc$

From the above equation we can find three determinants Δ, Δ_1 and Δ_2

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$$\Delta = \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = 3 \times 3 - 1 \times -1 = 9 + 1 = 10$$

$$\Delta_1 = \begin{vmatrix} -5 & -1 \\ 2 & 3 \end{vmatrix} = -5 \times 3 - 2 \times -1 = -15 + 2 = -13$$

$$\Delta_2 = \begin{vmatrix} 3 & -5 \\ 1 & 2 \end{vmatrix} = 3 \times 2 - 1 \times -5 = 6 + 5 = 11$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-13}{10}, \qquad y = \frac{\Delta_2}{\Delta} = \frac{11}{10}$$

$\therefore \left(\frac{-13}{10}, \frac{11}{10}\right)$ is the point of intersection of the above two lines.

26. Prove that the lines $2x - 3y - 7 = 0$, $3x - 4y - 10 = 0$ and $8x + 11y - 5 = 0$ are concurrent.

Solve any two equation, $2x - 3y - 7 = 0$, $3x - 4y - 10 = 0$

Rewriting the equations in the form $2x - 3y = 7$ and $3x - 4y = 10$

Next we want to find the point of intersection of the above two lines

$$\Delta = \begin{vmatrix} 2 & -3 \\ 3 & -4 \end{vmatrix} = 2 \times -4 - 3 \times -3 = -8 + 9 = 1$$

$$\Delta_1 = \begin{vmatrix} 7 & -3 \\ 10 & -4 \end{vmatrix} = 7 \times -4 - 10 \times -3 = -28 + 30 = 2$$

$$\Delta_2 = \begin{vmatrix} 2 & 7 \\ 3 & 10 \end{vmatrix} = 2 \times 10 - 3 \times 7 = 20 - 21 = -1$$

$$x = \frac{\Delta_1}{\Delta} = \frac{2}{1} = 2, \qquad y = \frac{\Delta_2}{\Delta} = \frac{-1}{1} = -1$$

The point of intersection $(2, -1)$ substitute in the third equation.

$ie;\ 8x + 11y - 5 = 0$

$8 \times 2 + 11 \times -1 - 5 = 16 - 11 - 5 = 16 - 16 = 0$

$\therefore$ The given lines are concurrent.

27. For what value of k shall the three lines $5x + 2y - 4 = 0$, $2x + ky + 11 = 0$ and $3x - 4y - 18 = 0$ are concurrent.

Solve the equations $5x + 2y - 4 = 0$ and $3x - 4y - 18 = 0$

Rewriting the equations in the form $5x + 2y = 4$ and $3x - 4y = 18$

Next we want to find the point of intersection of these two lines

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$$\Delta = \begin{vmatrix} 5 & 2 \\ 3 & -4 \end{vmatrix} = 5 \times -4 - 3 \times 2 = -20 - 6 = -26$$

$$\Delta_1 = \begin{vmatrix} 4 & 2 \\ 18 & -4 \end{vmatrix} = 4 \times -4 - 18 \times 2 = -16 - 36 = -52$$

$$\Delta_2 = \begin{vmatrix} 5 & 4 \\ 3 & 18 \end{vmatrix} = 5 \times 18 - 3 \times 4 = 90 - 12 = 78$$

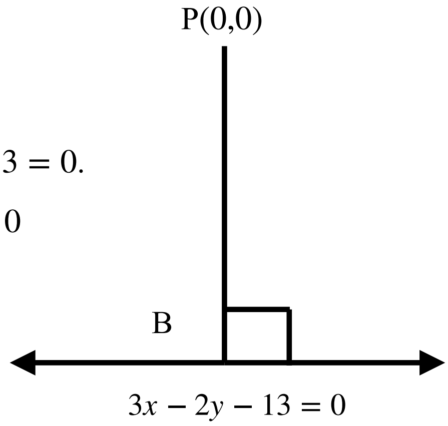
$$x = \frac{\Delta_1}{\Delta} = \frac{-52}{-26} = 2, \quad y = \frac{\Delta_2}{\Delta} = \frac{78}{-26} = -3$$

$(2, -3)$ is the point of intersection of the above straight line. Substitute this in second equation.

$$2x + ky + 11 = 0 \implies 2 \times 2 + k \times -3 + 11 = 0 \implies 2 - 3k + 11 = 0$$

$$15 - 3k = 0 \implies -3k = -15 \implies k = \frac{-15}{-3} = 5$$

28. Find the foot of the perpendicular from the origin to the line $3x - 2y - 13 = 0$



We have to find $'B'$ (foot of the $\perp^{er}$). B is the point of intersection of $\overline{PB}$ and the line $3x - 2y - 13 = 0$.

Now we have to find the straight line $\overline{PB}$. It is $\perp^{er}$ to $3x - 2y - 13 = 0$.

Any line $\perp^{er}$ to $3x - 2y - 13 = 0$ has the form $-2x - 3y + k = 0$

Since it passes through (0,0),

$$-2 \times 0 - 3 \times 0 + k = 0 \implies 0 - 0 + k = 0 \implies k = 0$$

The equation of $\overline{PB}$ is $-2x - 3y + 0 = 0 \implies -2x - 3y = 0$

Next we have to solve $3x - 2y - 13 = 0$ and $-2x - 3y = 0$ to get the foot of the $\perp^{er}$.

$$3x - 2y = 13$$

$$-2x - 3y = 0$$

$$\Delta = \begin{vmatrix} 3 & -2 \\ -2 & -3 \end{vmatrix} = 3 \times -3 - -2 \times -2 = -9 - 4 = -13$$

$$\Delta_1 = \begin{vmatrix} 13 & -2 \\ 0 & -3 \end{vmatrix} = 13 \times -3 - 0 \times -2 = -39 - 0 = -39$$

$$\Delta_2 = \begin{vmatrix} 3 & 13 \\ -2 & 0 \end{vmatrix} = 3 \times 0 - -2 \times 13 = 0 + 26 = 26$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-39}{-13} = 3 \quad \text{and} \quad y = \frac{\Delta_2}{\Delta} = \frac{26}{-13} = -2$$ $\therefore$ the foot of the $\perp^{er}$ is $(3, -2)$

MATHEMATICS - I

SUBJECT CODE : 1002

Module-2, Trigonometry

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1. Prove that $\frac{cosec\theta}{cosec\theta - 1} + \frac{cosec\theta}{cosec\theta + 1} = 2sec^2\theta$

$$\text{LHS} = \frac{cosec\theta}{(cosec\theta - 1)} + \frac{cosec\theta}{(cosec\theta + 1)}$$

$$= \frac{cosec\theta(cosec\theta + 1) + cosec\theta(cosec\theta - 1)}{(cosec\theta - 1)(cosec\theta + 1)}$$

$$= \frac{cosec^2\theta + cosec\theta + cosec^2\theta - cosec\theta}{cosec^2\theta - 1^2}$$

$$= \frac{2cosec^2\theta}{cosec^2\theta - 1} = \frac{2cosec^2\theta}{cot^2\theta}$$

$$= \frac{2\frac{1}{sin^2\theta}}{\frac{cos^2\theta}{sin^2\theta}} = 2\frac{1}{cos^2\theta} = 2sec^2\theta.$$

2. Prove that $\frac{tan45 - tan30}{1 + tan45tan30} = 2 - \sqrt{3}$

$$\text{LHS} = \frac{tan45 - tan30}{1 + tan45tan30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Multiply Nr and Dr by '$\sqrt{3} - 1$'

$$= \frac{(\sqrt{3} - 1) \cdot (\sqrt{3} - 1)}{(\sqrt{3} + 1) \cdot (\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{(\sqrt{3})^2 - 2 \cdot \sqrt{3} \cdot 1 + 1^2}{3 - 1}$$

$$= \frac{3 - 2\sqrt{3} + 1}{2} = \frac{4 - 2\sqrt{3}}{2} = \frac{2(2 - \sqrt{3})}{2} = 2 - \sqrt{3}.$$

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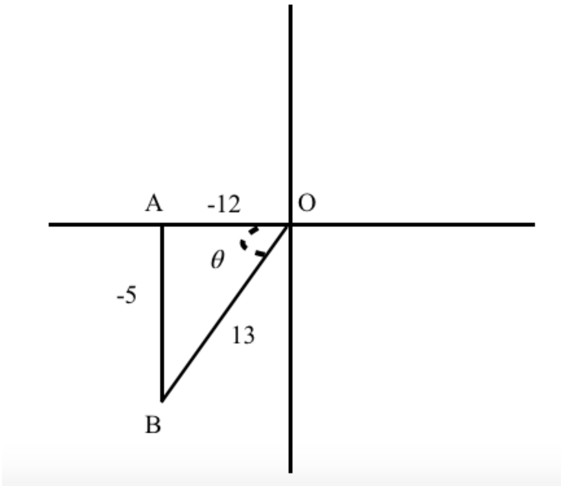
3. If $tan\theta = \frac{5}{12}$, 'θ' lies in third quadrant, find all other t-functions.

Given. $tan\theta = \frac{5}{12} = \frac{opposite\,side}{adjacent\,side}$



By Pythagorean theorem $OB^2 = OA^2 + AB^2$

$OB^2 = (-12)^2 + (-5)^2$

$OB^2 = 144 + 25 = 169$

$OB = \sqrt{169} = \pm 13$

$OB = 13$ $(Hypotenuse\,always\,positive)$

$sin\theta = \frac{opposite\,side}{Hypotenuse} = \frac{-5}{13}$, $\quad cosec\theta = \frac{Hypotenuse}{opposite\,side} = \frac{13}{-5} = \frac{-13}{5}$

$cos\theta = \frac{adjacent\,side}{Hypotenuse} = \frac{-12}{13}$, $\quad sec\theta = \frac{Hypotenuse}{adjacent\,side} = \frac{13}{-12} = \frac{-13}{12}$

$tan\theta = \frac{opposite\,side}{adjacent\,side} = \frac{-5}{-12} = \frac{5}{12}$, $\quad cot\theta = \frac{adjacent\,side}{opposite\,side} = \frac{-12}{-5} = \frac{12}{5}$

4. Prove that $\frac{cos(90+A)sec(360+A)tan(180-A)}{sec(A-720)sin(540+A)cot(A-90)} = 1$

$$LHS = \frac{cos(90+A)sec(360+A)tan(180-A)}{sec(A-720)sin(540+A)cot(A-90)}$$

$cos(90+A) = cos(1\cdot 90+A) = -\,sinA$

$sec(360+A) = sec(4\cdot 90+A) = secA$

$tan(180-A) = tan(2\cdot 90-A) = -\,tanA$

$sec(A-720) = sec\left(-(720-A)\right) = sec(720-A) = sec(8\cdot 90-A) = secA$

$sin(540+A) = sin(6\cdot 90+A) = -\,sinA$

$cot(A-90) = cot\left(-(90-A)\right) = -\,cot(90-A) = -\,cot(1\cdot 90-A) = -\,tanA$

$$LHS = \frac{cos(90+A)sec(360+A)tan(180-A)}{sec(A-720)sin(540+A)cot(A-90)} = \frac{-sinA\cdot secA\cdot -tanA}{secA\cdot -sinA\cdot -tanA} = 1.$$

5. If $tanA = \frac{3}{4}, sinB = \frac{5}{13}$, A lies in third quadrant and B lies in second quadrant find $sin(A-B), cos(A+B)$.

Given $tanA = \frac{3}{4}, sinB = \frac{5}{13}$, (A lies in third quadrant B lies in second quadrant)

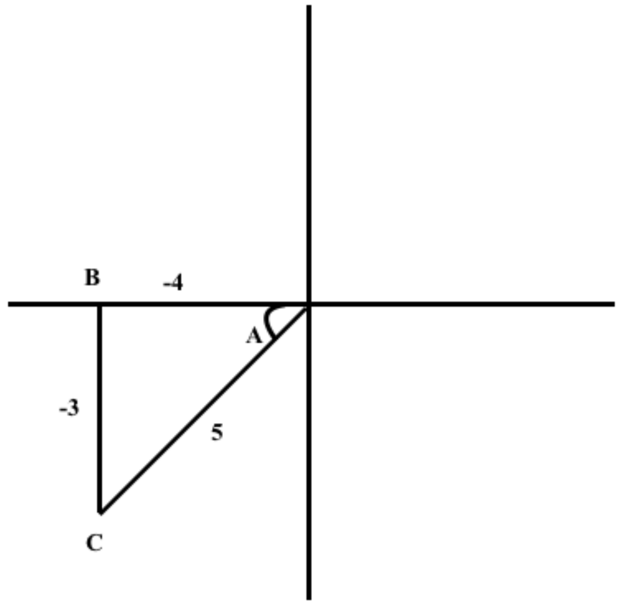
By Pythagorean theorem $AC^2 = AB^2 + BC^2$

$AC^2 = (-4)^2 + (-3)^2$

$AC^2 = 16 + 9 \implies AC^2 = 25$

$AC = \sqrt{25} \implies AC = \pm 5$

$AC = 5$ (since it is hypotenuse)



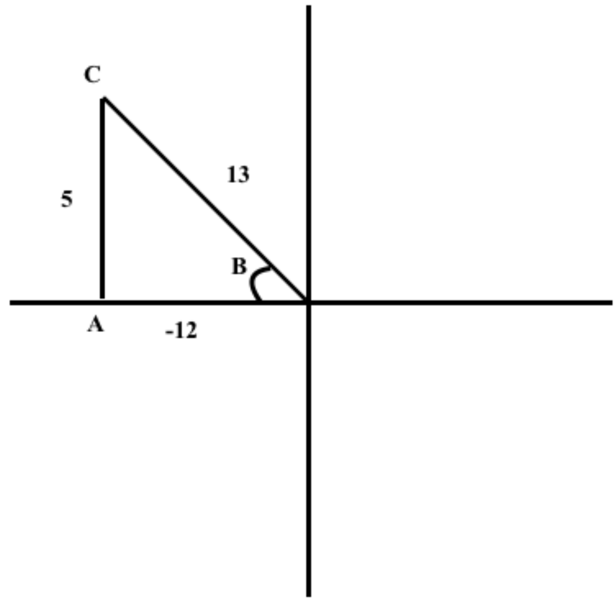
By Pythagorean theorem $BC^2 = AB^2 + AC^2$

$13^2 = AB^2 + 5^2$

$169 = AB^2 + 25 \implies AB^2 = 169 - 25 = 144$

$AB = \sqrt{144} \implies AB = \pm 12$

$AB = -12$ (since B lies in second quadrant)



From the above diagrams,

$$sinA = \frac{BC}{AC} = \frac{-3}{5}, \quad cosA = \frac{AB}{AC} = \frac{-4}{5}$$

$$sinB = \frac{AC}{BC} = \frac{5}{13}, \quad cosB = \frac{AB}{BC} = \frac{-12}{13}$$

$sin(A-B) = sinAcosB - cosAsinB$

$$sin(A-B) = \frac{-3}{5} \times \frac{-12}{13} - \frac{-4}{5} \times \frac{5}{13} = \frac{36}{65} + \frac{20}{65} = \frac{56}{65}$$

$cos(A+B) = cosAcosB - sinAsinB$

$$cos(A+B) = \frac{-4}{5} \times \frac{-12}{13} - \frac{-3}{5} \times \frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$

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6. If $tan A = \frac{18}{17}, tan B = \frac{1}{35}$, prove that $A - B = 45^o$.

$$tan(A - B) = \frac{tan A - tan B}{1 + tan A tan B}$$

$$= \frac{\frac{18}{17} - \frac{1}{35}}{1 + \frac{18}{17} \times \frac{1}{35}} = \frac{\frac{18 \times 35 - 1 \times 17}{17 \times 35}}{1 + \frac{18}{595}} = \frac{\frac{613}{595}}{\frac{613}{595}} = 1$$

$$tan(A - B) = 1 \implies A - B = tan^{-1}(1) = 45^o$$

7. Find the value of $tan 75$ without using tables and show that $tan 75 + cot 75 = 4$.

We have, $tan 75 = tan(45 + 30)$

$$= \frac{tan 45 + tan 30}{1 - tan 45 tan 30} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$tan 75 = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \dots\dots\dots (1)$$

$$cot 75 = \frac{\sqrt{3}-1}{\sqrt{3}+1} \dots\dots\dots\dots\dots\dots\dots (2)$$

$$tan 75 + cot 75 = \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + 1^2 + \sqrt{3})^2 - 2 \times \sqrt{3} \times 1 + 1^2}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} + 1}{3 - 1} = \frac{8}{2} = 4.$$

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8. If $A+B=45^o$ show that $(1+tanA)(1+tanB)=2$.

We have $A+B=45^o$

$tan(A+B)=tan45$

$\frac{tanA+tanB}{1-tanAtanB}=1$

$tanA+tanB=1\times(1-tanAtanB)$

$tanA+tanB=1-tanAtanB$

$tanA+tanB+tanAtanB=1$

Adding'1' both sides of the above equation

$1+tanA+tanB+tanAtanB=1+1$

$(1+tanA)+tanB+tanAtanB=2$

$(1+tanA)+tanB(1+tanA)=2$

$(1+tanA)[1+tanB]=2.$

9. Express $\sqrt{3}cosx+sinx$ in the form $Rsin(x+\alpha)$ where α is acute.

For expressing $\sqrt{3}cosx+sinx$ in the form $Rsin(x+\alpha)$ we have to find out R and α.

We have, $\sqrt{3}cosx+sinx=Rsin(x+\alpha)\ldots\ldots\ldots\ldots(1)$

$\sqrt{3}cosx+sinx=R[sinxcos\alpha+cosxsin\alpha]$

$\sqrt{3}cosx+sinx=Rsinxcos\alpha+Rcosxsin\alpha$

Equating the similar terms on both sides,

$\sqrt{3}cosx=Rcosxsin\alpha$

$\sqrt{3}=Rsin\alpha\ldots\ldots\ldots\ldots(2)$

$sinx=Rsinxcos\alpha$

$1=Rcos\alpha\ldots\ldots\ldots\ldots\ldots(3)$

Squaring and adding (2) and (3)

$(\sqrt{3})^2+1^2=R^2sin^2\alpha+R^2cos^2\alpha$

$3+1=R^2[sin^2\alpha+cos^2\alpha]$

$4=R^2\times1 \implies 4=R^2 \implies R=\pm2$

Dividing equation (2) by equation (3)

ie; $\frac{\sqrt{3}}{1}=\frac{Rsin\alpha}{Rcos\alpha} \implies \sqrt{3}=tan\alpha$

$\alpha=tan^{-1}(\sqrt{3})=60^o$

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Substitute R and α in the first equation to get, the expression

$\sqrt{3}cosx + sinx = \pm 2sin(x + 60^o)$

10. Prove that $\dfrac{1 + cos2A}{sin2A} = cotA$ and deduce the value of $cot15$.

$LHS = \dfrac{1 + cos2A}{sin2A}$,

$cos2A = 2cos^2A - 1 \implies 1 + cos2A = 2cos^2A$

$sin2A = 2sinAcosA$

$$LHS = \frac{2cos^2A}{2sinAcosA} = \frac{cosA}{sinA} = cotA$$

Put $A = 15$ in $\dfrac{1 + cos2A}{sin2A} = cotA$ then it becomes

$$\frac{1 + cos(2 \times 15)}{sin(2 \times 15)} = cot15 \implies \frac{1 + cos30}{sin30} = cot15$$

$$\frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}} = cot15 \implies \frac{\frac{2 + \sqrt{3}}{2}}{\frac{1}{2}} = cot15$$

$$cot15 = \frac{2 + \sqrt{3}}{1} = 2 + \sqrt{3}.$$

11. Prove that $cos4\theta = 1 - 8sin^2\theta cos^2\theta$

We know that,

$cos2\theta = 1 - 2sin^2\theta$

Multiply both sides the angle by ‘2’

$cos(2\theta \times 2) = 1 - 2sin^2(\theta \times 2)$

$cos4\theta = 1 - 2sin^22\theta$

$cos4\theta = 1 - 2sin2\theta sin2\theta$ $\qquad sin2\theta = 2sin\theta cos\theta$

$cos4\theta = 1 - 2 \times 2sin\theta cos\theta \times 2sin\theta cos\theta$

$cos4\theta = 1 - 8sin^2\theta cos^2\theta$

12. Prove that $\dfrac{cos3A + cosA}{sin3A - sinA} = cotA$

$$LHS = \frac{cos3A + cosA}{sin3A - sinA}$$

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$$= \frac{4cos^3A - 3cosA + cosA}{3sinA - 4sin^3A - sinA} = \frac{4cos^3A - 2cosA}{2sinA - 4sin^3A}$$

$$= \frac{2cosA(2cos^2A - 1)}{2sinA(1 - 2sin^2A)} = \frac{2cosA \times cos2A}{2sinA \times cos2A} = \frac{cosA}{sinA} = cotA$$

13. Prove that $\frac{sin3x}{sinx} - \frac{cos3x}{cosx} = 2$

$$LHS = \frac{sin3x}{sinx} - \frac{cos3x}{cosx}$$

$$= \frac{3sinx - 4sin^3x}{sinx} - \frac{4cos^3x - 3cosx}{cosx} = \frac{sinx(3 - 4sin^2x)}{sinx} - \frac{cosx(4cos^2x - 3)}{cosx}$$

$$= 3 - 4sin^2x - (4cos^2x - 3) = 3 - 4sin^2x - 4cos^2x + 3$$

$$= 3 + 3 - 4sin^2x - 4cos^2x = 6 - 4(sin^2x + cos^2x)$$

$$= 6 - 4 \times 1 = 6 - 4 = 2$$

14. Prove that $\frac{sin3A}{sinA} + \frac{cos3A}{cosA} = 4cos2A$

$$LHS = \frac{sin3A}{sinA} + \frac{cos3A}{cosA}$$

$$= \frac{3sinA - 4sin^3A}{sinA} + \frac{4cos^3A - 3cosA}{cosA} = \frac{sinA(3 - 4sin^2A)}{sinA} + \frac{cosA(4cos^2A - 3)}{cosA}$$

$$= 3 - 4sin^2A + 4cos^2A - 3 = 4cos^2A - 4sin^2A$$

$= 4(cos^2A - sin^2A) = 4cos2A.$ $cos2A = cos^2A - sin^2A.$

15. Prove that $cos^4A - sin^4A = cos2A$

$$LHS = cos^4A - sin^4A$$

$= \left(cos^2A\right)^2 - \left(sin^2A\right)^2$ $a^2 - b^2 = (a + b)(a - b)$

$$= \left(cos^2A + sin^2A\right)\left(cos^2A - sin^2A\right)$$

$= 1 \times \left(cos^2A - sin^2A\right) = cos2A$ $cos2A = cos^2A - sin^2A$

16. Prove that $\frac{sin2\alpha + sin5\alpha - sin\alpha}{cos2\alpha + cos5\alpha + cos\alpha} = tan2\alpha$

$sinC - sinD = 2cos\frac{C+D}{2}sin\frac{C-D}{2},$ $cosC + cosD = 2cos\frac{C+D}{2}cos\frac{C-D}{2}$

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$$LHS = \frac{sin2\alpha + (sin5\alpha - sin\alpha)}{cos2\alpha + (cos5\alpha + cos\alpha)}$$

$$= \frac{sin2\alpha + 2cos\frac{5\alpha+\alpha}{2}sin\frac{5\alpha-\alpha}{2}}{cos2\alpha + 2cos\frac{5\alpha+\alpha}{2}cos\frac{5\alpha-\alpha}{2}} = \frac{sin2\alpha + 2cos\frac{6\alpha}{2}sin\frac{4\alpha}{2}}{cos2\alpha + 2cos\frac{6\alpha}{2}cos\frac{4\alpha}{2}}$$

$$= \frac{sin2\alpha + 2cos3\alpha sin2\alpha}{cos2\alpha + 2cos3\alpha cos2\alpha} = \frac{sin2\alpha(1+2cos3\alpha)}{cos2\alpha(1+2cos3\alpha)} = \frac{sin2\alpha}{cos2\alpha} = tan2\alpha$$

17. Prove that $cos3A + cos5A + cos9A + cos17A = 4cos4Acos6Acos7A$

$LHS = (cos3A + cos5A) + (cos9A + cos17A)$

$$cosC + cosD = 2cos\frac{C+D}{2}cos\frac{C-D}{2}$$

$$= 2cos\frac{3A+5A}{2}cos\frac{3A-5A}{2} + 2cos\frac{9A+17A}{2}cos\frac{9A-17A}{2}$$

$$= 2cos\frac{8A}{2}cos\frac{-2A}{2} + 2cos\frac{26A}{2}cos\frac{-8A}{2}$$

$$= 2cos4Acos(-A) + 2cos13Acos(-4A) \qquad cos(-\theta) = cos\theta$$

$$= 2cos4AcosA + 2cos13Acos4A$$

$$= 2cos4A(cosA + cos13A)$$

$$cosC + cosD = 2cos\frac{C+D}{2}cos\frac{C-D}{2}$$

$$= 2cos4A\left(2cos\frac{A+13A}{2}cos\frac{A-13A}{2}\right)$$

$$= 4cos4Acos\frac{14A}{2}cos\frac{-12A}{2} = 4cos4Acos7Acos(-6A) \qquad cos(-\theta) = cos\theta$$

$$= 4cos4Acos7Acos6A = 4cos4Acos6Acos7A$$

18. Show that $sin10sin50sin70 = \frac{1}{8}$

$LHS = sin10(sin50sin70)$

$$sinAsinB = \frac{-1}{2}\left[cos(A+B) - cos(A-B)\right]$$

$$= sin10 \times \frac{-1}{2}\left[cos(50+70) - cos(50-70)\right] \qquad cos(-\theta) = cos\theta$$

$$= \frac{-1}{2}sin10\left[cos(120) - cos(-20)\right] \qquad cos(n \cdot 90 + \theta) = \pm sin\theta \ if\, n\, is\, odd$$

$$= \frac{-1}{2} sin 10 \left[cos(1 \times 90 + 30) - cos 20\right] = \frac{-1}{2} sin 10 \left[-sin 30 - cos 20\right]$$

$$= \frac{-1}{2} sin 10 \left[\frac{-1}{2} - cos 20\right] = \frac{1}{4} sin 10 + \frac{1}{2} sin 10 cos 20$$

$$sin A cos B = \frac{1}{2}\left[sin(A+B) + sin(A-B)\right]$$

$$= \frac{1}{4} sin 10 + \frac{1}{2} \times \frac{1}{2}\left[sin(10+20) + sin(10-20)\right]$$

$$= \frac{1}{4} sin 10 + \frac{1}{4}\left[sin(30) + sin(-10)\right]$$

$$sin(-\theta) = -sin\theta$$

$$= \frac{1}{4} sin 10 + \frac{1}{4}\left[\frac{1}{2} + -sin 10\right] = \frac{1}{4} sin 10 + \frac{1}{4}\left[\frac{1}{2} - sin 10\right]$$

$$= \frac{1}{4} sin 10 + \frac{1}{8} - \frac{1}{4} sin 10 = \frac{1}{8}$$

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MATHEMATICS - I

SUBJECT CODE : 1002

Module-3, Differentiation-I

1. Calculate $\lim_{x \to \infty} \left(\frac{x^2 + 2x + 1}{x^2 + x - 3} \right)$

$$\lim_{x \to \infty} \left(\frac{x^2 + 2x + 1}{x^2 + x - 3} \right) = \lim_{x \to \infty} \frac{x^2 \left(1 + \frac{2}{x} + \frac{1}{x^2} \right)}{x^2 \left(1 + \frac{1}{x} - \frac{3}{x^2} \right)}$$

$$= \lim_{x \to \infty} \frac{1 + \frac{2}{x} + \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{3}{x^2}}$$

we can use the formula $\lim_{x \to \infty} \frac{1}{x} = 0$

$$= \frac{1 + \frac{2}{\infty} + \frac{1}{\infty^2}}{1 + \frac{1}{\infty} - \frac{3}{\infty^2}} = \frac{1 + 0 + 0}{1 + 0 - 0} = \frac{1}{1} = 1$$

2. Evaluate $\lim_{x \to 1} \frac{x^2 + 4x - 5}{x^2 + x - 2}$

Here we can use the factorisation method

$$\lim_{x \to 1} \frac{x^2 + 4x - 5}{x^2 + x - 2} = \lim_{x \to 1} \frac{(x+5)(x-1)}{(x+2)(x-1)} = \lim_{x \to 1} \frac{x+5}{x+2} = \frac{1+5}{1+2} = \frac{6}{3} = 2$$

3. Evaluate $\lim_{x \to 4} \left(\frac{x^3 - 64}{x^2 - 16} \right)$

$$\lim_{x \to 4} \left(\frac{x^3 - 64}{x^2 - 16} \right) = \lim_{x \to 4} \left(\frac{x^3 - 4^3}{x^2 - 4^2} \right) = \lim_{x \to 4} \left(\frac{\frac{x^3 - 4^3}{x - 4}}{\frac{x^2 - 4^2}{x - 4}} \right)$$

$$= \frac{\lim_{x \to 4} \frac{x^3 - 4^3}{x - 4}}{\lim_{x \to 4} \frac{x^2 - 4^2}{x - 4}}$$

Now use the algebraical limit formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$

both numerator and denominator

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$$= \frac{3 \times 4^{3-1}}{2 \times 4^{2-1}} = \frac{3 \times 4^2}{2 \times 4^1} = \frac{3 \times 16}{2 \times 4} = \frac{3 \times 4}{2 \times 1} = \frac{12}{2} = 6$$

4. Evaluate $\lim_{x \to 0} \frac{tanx}{x}$

Here we are going to use the formula $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1,\qquad \lim_{\theta \to 0} cos\theta = 1$

$$\lim_{x \to 0} \frac{tanx}{x} = \lim_{x \to 0} \frac{\frac{sinx}{cosx}}{\frac{x}{1}} = \lim_{x \to 0} \frac{sinx}{cosx} \times \frac{1}{x} = \lim_{x \to 0} \frac{sinx}{x} \times \frac{1}{cosx}$$

$$= \lim_{x \to 0} \frac{sinx}{x} \times \lim_{x \to 0} \frac{1}{cosx} = \lim_{x \to 0} \frac{sinx}{x} \times \frac{\lim_{x \to 0} 1}{\lim_{x \to 0} cosx} = 1 \times \frac{1}{1} = 1$$

5. Evaluate $\lim_{\theta \to 0} \left[\frac{sin4\theta + sin2\theta}{6\theta}\right]$

$$\lim_{\theta \to 0} \left[\frac{sin4\theta + sin2\theta}{6\theta}\right] = \lim_{\theta \to 0} \left[\frac{sin4\theta}{6\theta} + \frac{sin2\theta}{6\theta}\right] = \frac{1}{6} \lim_{\theta \to 0} \left[\frac{sin4\theta}{\theta} + \frac{sin2\theta}{\theta}\right]$$

$$= \frac{1}{6} \lim_{\theta \to 0} \left[\frac{sin4\theta \times 4}{4\theta} + \frac{sin2\theta \times 2}{2\theta}\right] = \frac{1}{6} \left[\lim_{\theta \to 0} \frac{sin4\theta \times 4}{4\theta} + \lim_{\theta \to 0} \frac{sin2\theta \times 2}{2\theta}\right]$$

$$= \frac{1}{6} \left[4 \lim_{4\theta \to 0} \frac{sin4\theta}{4\theta} + 2 \lim_{2\theta \to 0} \frac{sin2\theta}{2\theta}\right]$$ Here we use the formula $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$

$$= \frac{1}{6} [4 \times 1 + 2 \times 1] = \frac{1}{6} [4 + 2] = \frac{1}{6} \times 6 = 1$$

6. Find $\frac{d}{dx} \left(\sqrt{x} cotx\right)$

Use the product formula $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

$$\frac{d}{dx} \left(\sqrt{x} cotx\right) = \sqrt{x} \frac{d}{dx}(cotx) + \frac{d}{dx} \left(\sqrt{x}\right) cotx$$

$$= \sqrt{x} \times -cosec^2x + \frac{1}{2\sqrt{x}} cotx = -\sqrt{x} cosec^2x + \frac{cotx}{2\sqrt{x}}$$

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7. Find the derivative of $tanx$ using quotient rule

Use the quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - \frac{dv}{dx}u}{v^2}$

$$\frac{d}{dx}(tanx) = \frac{d}{dx}\left(\frac{sinx}{cosx}\right) = \frac{cosx\frac{d}{dx}(sinx) - \frac{d}{dx}(cosx)sinx}{cos^2x}$$

$$= \frac{cosxcosx - -sinxsinx}{cos^2x} = \frac{cos^2x + sin^2x}{cos^2x} = \frac{1}{cos^2x} = sec^2x$$

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MATHEMATICS - I

SUBJECT CODE : 1002

Module-4, Differentiation-II

1. If $y = \sqrt{2x-3}$ find $\dfrac{dy}{dx}$

We know that the derivative of $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$ using this formula we can write it as

$$\frac{d}{dx}\left(\sqrt{2x-3}\right) = \frac{1}{2\sqrt{2x-3}} \times \frac{d}{dx}(2x-3)$$

$$= \frac{1}{2\sqrt{2x-3}} \times \left[\frac{d}{dx}(2x) - \frac{d}{dx}(3)\right] = \frac{1}{2\sqrt{2x-3}} \times \left[2 \times 1 - 0\right]$$

$$= \frac{1}{2\sqrt{2x-3}} \times 2 = \frac{1}{\sqrt{2x-3}}$$

2. If $y = log(secx + tanx)$ find $\dfrac{dy}{dx}$

We know that the derivative of $\dfrac{d}{dx}\left(logx\right) = \dfrac{1}{x}$ using this formula we can write it as

$$\frac{d}{dx}\left(log(secx + tanx)\right) = \frac{1}{secx + tanx} \times \frac{d}{dx}(secx + tanx)$$

$$= \frac{1}{secx + tanx} \times \left[\frac{d}{dx}(secx) + \frac{d}{dx}(tanx)\right]$$

$$= \frac{1}{secx + tanx} \times \left[secxtanx + sec^2x\right] = \frac{1}{secx + tanx} \times secx\,[tanx + secx]$$

$$= secx$$

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3. If $y = log\left(x+\sqrt{1+x^2}\right)$ find $\frac{dy}{dx}$

$$\frac{d}{dx}\left(log\left(x+\sqrt{1+x^2}\right)\right) = \frac{1}{x+\sqrt{1+x^2}} \times \frac{d}{dx}\left(x+\sqrt{1+x^2}\right)$$

$$= \frac{1}{x+\sqrt{1+x^2}} \times \left[\frac{d}{dx}(x) + \frac{d}{dx}\left(\sqrt{1+x^2}\right)\right]$$

$$= \frac{1}{x+\sqrt{1+x^2}} \times \left[1 + \frac{1}{2\sqrt{1+x^2}} \times \frac{d}{dx}\left(1+x^2\right)\right]$$

$$= \frac{1}{x+\sqrt{1+x^2}} \times \left[1 + \frac{1}{2\sqrt{1+x^2}} \times (0+2x)\right]$$

$$= \frac{1}{x+\sqrt{1+x^2}} \times \left[1 + \frac{2x}{2\sqrt{1+x^2}}\right] = \frac{1}{x+\sqrt{1+x^2}} \times \left[1 + \frac{x}{\sqrt{1+x^2}}\right]$$

$$= \frac{1}{x+\sqrt{1+x^2}} \times \left[\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right] = \frac{1}{\sqrt{1+x^2}}$$

4. Find $\frac{dy}{dx}$ when x and y are connected by the relation $ax^2 + 2hxy + by^2 = 0$

Given $ax^2 + 2hxy + by^2 = 0$

differentiate the above equation both sides with respect to ‘x’

$$\frac{d}{dx}\left(ax^2 + 2hxy + by^2\right) = \frac{d}{dx}(0)$$

$$\frac{d}{dx}\left(ax^2\right) + \frac{d}{dx}\left(2hxy\right) + \frac{d}{dx}\left(by^2\right) = \frac{d}{dx}(0)$$

$$a \times 2x + 2h\left[x\frac{dy}{dx} + \frac{dx}{dx}y\right] + b \times 2y\frac{dy}{dx} = 0$$

$$2ax + 2hx\frac{dy}{dx} + 2h \times 1 \times y + 2by\frac{dy}{dx} = 0$$

$$2hx\frac{dy}{dx} + 2by\frac{dy}{dx} = -2ax - 2hy$$

$$\frac{dy}{dx}\left[2hx + 2by\right] = -2ax - 2hy$$

$$\frac{dy}{dx} = \frac{-2ax-2hy}{\left(2hx+2by\right)} = \frac{-2(ax+hy)}{2\left(hx+by\right)} = \frac{-(ax+hy)}{\left(hx+by\right)}$$

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5. If $x = asec\theta, y = btan\theta$ Find $\dfrac{dy}{dx}$

Given $x = asec\theta$ $\quad\quad$ $y = btan\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(asec\theta) \qquad \frac{dy}{d\theta} = \frac{d}{d\theta}(btan\theta)$$

$$\frac{dx}{d\theta} = asec\theta tan\theta \qquad \frac{dy}{d\theta} = bsec^2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{bsec^2\theta}{asec\theta tan\theta} = \frac{b}{a} \times \frac{sec\theta}{tan\theta} = \frac{b}{a} \times \frac{\frac{1}{cos\theta}}{\frac{sin\theta}{cos\theta}} = \frac{b}{a} \times \frac{1}{sin\theta} = \frac{b}{a}cosec\theta$$

6. If $y = xsinx$, Prove that $\dfrac{d^2y}{dx^2} + y = 2cosx$

Given $y = xsinx$

Use the product formula $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + \dfrac{du}{dx}v$

$$\frac{dy}{dx} = \frac{d}{dx}(xsinx) = x\frac{d}{dx}(sinx) + \frac{d}{dx}(x)sinx$$

$$\frac{dy}{dx} = xcosx + 1 \times sinx = xcosx + sinx$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(xcosx + sinx) = \frac{d}{dx}(xcosx) + \frac{d}{dx}(sinx)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(xcosx) + \frac{d}{dx}(sinx)$$

$$\frac{d^2y}{dx^2} = = x\frac{d}{dx}(cosx) + \frac{d}{dx}(x)cosx + \frac{d}{dx}(sinx)$$

$$\frac{d^2y}{dx^2} = = x \times -sinx + 1 \times cosx + cosx$$

$$= -xsinx + cosx + cosx = -xsinx + 2cosx$$

$$\frac{d^2y}{dx^2} + y = -xsinx + 2cosx + xsinx = 2cosx$$

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