EXPERIMENTAL PHYSICS 1

TEST EXAM

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Lecturer: Prof. Käs
Tutor: Dimitrij Tschodu,
Salvador Rivera
E-Mail: dimitrijtschodu@googlemail.com,
chava_rm@outlook.com

Bonus exercises / Test exam - Solutions

1. General knowledge $1 + 1 + 2$ Points

a. Name Newton's 3rd law!

Actio = Reactio

b. In which field of physics does the Bernoulli equation find its application?

Fluid dynamics

c. Write down Kepler's 3rd law!

$$mr\omega = G\frac{mM}{r^2}$$

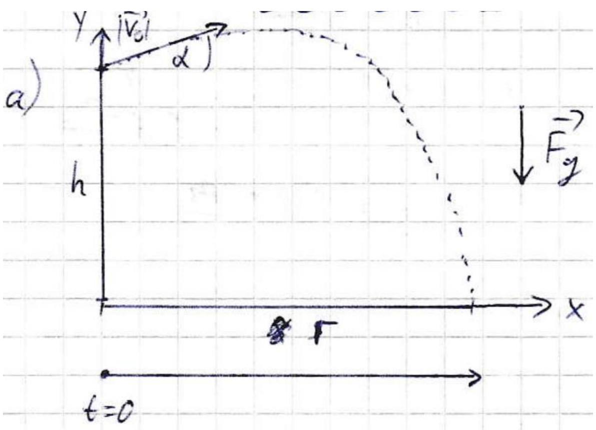
or

The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.

2. Point mass $1 + 2 + 3$ Points

A point mass is located at the place $(0, h)$ at time $t = 0$. The mass is shot with an initial velocity $|\vec{v}_0|$ under the angle α to the x-axis so that it moves in positive x-direction. The gravitational force $\vec{F}_G$ acts in the (-y)-direction (on Earth). Air resistance is neglected.

a. Sketch the situation and draw in all the given quantities!



b. Give the velocity $\vec{v}(t)$ and the position vector $\vec{r}(t)$ (vectorial equation)!

$$\vec{v}(t) = \begin{pmatrix} |\vec{v}_0|\cos(\alpha) \\ -gt + |\vec{v}_0|\sin(\alpha) \end{pmatrix}$$

$$\vec{r}(t) = \begin{pmatrix} |\vec{v}_0|\cos(\alpha)\cdot t \\ -\frac{g}{2}t^2 + |\vec{v}_0|\sin(\alpha)\cdot t + h \end{pmatrix}$$

c. Derive the formula for the calculation of angle α, under which the mass must be launched, so that the mass with given $|\vec{v}_0|$ and $h = 0$ has covered the maximum distance in x-direction, when it hits the ground ($y = 0$)!

$$t = \frac{x}{|\vec{v}_0|\cos(\alpha)}$$

$$y(t) = -\frac{g}{2}t^2 + |\vec{v}_0|\sin(\alpha)\cdot t + h$$

$$y(x) = -\frac{g}{2}\left(\frac{x}{|\vec{v}_0|\cos(\alpha)}\right)^2 + |\vec{v}_0|\sin(\alpha)\cdot\frac{x}{|\vec{v}_0|\cos(\alpha)}$$

$$= x\left(-\frac{g}{2}\frac{x}{|\vec{v}_0|^2\cos(\alpha)^2} + \tan(\alpha)\right)$$

$$x_{y=0} = \frac{2|\vec{v}_0|^2}{g}\sin(\alpha)\cos(\alpha) = \frac{|\vec{v}_0|^2}{g}\sin(2\alpha)$$

$$\frac{dx}{d\alpha} = \frac{|\vec{v}_0|^2}{g}(2\cos(2\alpha)) = 0$$

$$\cos(2\alpha) = 0$$

$$\alpha = 45^\circ$$

3. Energy and forces $\quad 1 + 3 + 6$ Points

The potential energy of an object constrained to the x-axis is given by $U(x) = 3x^2 - 2x^2$ **(typo in the task, correct: $3x^2 - 2x^3$)**, where U is in joules and x is in meters.

a. Determine the force F_x associated with this potential energy function.
b. Assuming no other forces act on the object, at what positions is this object in equilibrium?
c. Which of these equilibrium positions are stable and which are unstable?

28 •• The potential energy of an object constrained to the x axis is given by $U(x) = 3x^2 - 2x^3$, where U is in joules and x is in meters. (*a*) Determine the force F_x associated with this potential energy function. (*b*) Assuming no other forces act on the object, at what positions is this object in equilibrium? (*c*) Which of these equilibrium positions are stable and which are unstable?

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(3x^2 - 2x^3\right) = \boxed{6x(x-1)}$$

(*b*) We know that, at equilibrium, $F_x = 0$:

When $F_x = 0$, $6x(x-1) = 0$. Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 1\,\text{m}.}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{dU}{dx} = \frac{d}{dx}\left(3x^2 - 2x^3\right) = 6x - 6x^2$$

and

$$\frac{d^2U}{dx^2} = 6 - 12x$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 6 > 0$$

$$\Rightarrow \boxed{\text{stable equilibrium at } x = 0}$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 1$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=1\,\text{m}} = 6 - 12 < 0$$

$$\Rightarrow \boxed{\text{unstable equilibrium at } x = 1\,\text{m}}$$

4. Orbiting space station **2 + 1 + 3 Points**

A space station with a mass of 100 t is moving around the Earth on a circular orbit at a height of 100 km above the Earth's surface.

a. What is the velocity v (in m·s^{-1}) of the station?

$$F_G = F_R$$

$$G\frac{m \cdot M_E}{r^2} = \frac{m \cdot v^2}{r}$$

$$v = \sqrt{\frac{G \cdot M_E}{r}}$$

$$r = R_E + h$$

$$v = \sqrt{\frac{G \cdot M_E}{R_E + h}} \approx 7843{,}9\ \frac{m}{s}$$

b. How long does the satellite need for one orbit?

$$s = v \cdot t$$

$$t = \frac{s}{v} = \frac{2\pi r}{v} = \frac{2\pi(R_E + h)}{v} \approx 5189 \text{ s}$$

c. What is the kinetic, potential and total energy of the station with a reference point for the potential energy at infinity ($E_{pot}(\infty) = 0$)?

$$E_{kin} = \frac{m}{2}v^2 = 3{,}07 \cdot 10^{12} \text{ J}$$

$$E_{pot} = -G\frac{m \cdot M_E}{r} = -G\frac{m \cdot M_E}{R_E + h} = -6{,}1 \cdot 10^{12} \text{ J}$$

$$E_{tot} = E_{kin} + E_{pot} = -3{,}07 \cdot 10^{12} \text{ J}$$

5. Rotating cylinder **1 + 4 + 1 Points**

A solid cylinder with a diameter of 60 cm is rotated by a thread wound around its circumference, from which a 2 kg mass is suspended. 12 seconds after the start of the movement, the mass has passed through a height of fall of 5.3 m and has reached a velocity of 0.88 m·s^{-1}.

a. What is the angular velocity of the cylinder after 12 s?

$$s = \varphi \cdot R$$

$$\dot{s} = \dot{\varphi} \cdot R$$

$$v = \omega \cdot R$$

$$\omega = \frac{v}{R} = 2{,}93 \text{ s}^{-1}$$

b. Determine the moment of inertia and the mass of the solid cylinder!

Hint: Use the law of conservation of energy.

$$E_{pot} = E_{trans} + E_{rot}$$

$$m \cdot g \cdot s = \frac{m}{s}v^2 + \frac{I}{2}\omega^2$$

$$I = \frac{m(2gs - v^2)}{\omega^2} = 24{,}04 \text{ kg m}^2$$

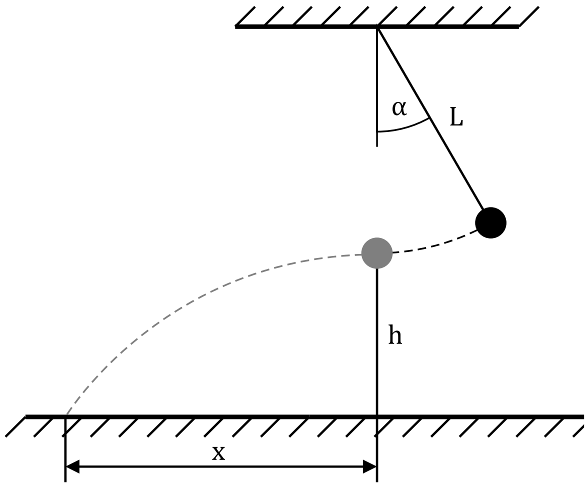
$$I = \frac{M}{2}R^2$$

$$M = \frac{2I}{R^2} = 534{,}2 \text{ kg}$$

c. What is the angular momentum of the cylinder after 12 s?

$$L = I \cdot \omega = 70{,}44 \text{ kg}\frac{\text{m}^2}{\text{s}}$$

6. Pendulum collision **$1+2+1$ Points**



A pendulum with a length L and mass m starts at a time $t = 0$ with a velocity $v = 0$, displaced by an angle α. At the lowest point it hits another mass which sits on a pole, and an elastic collision happens.

a. How much time passes from the beginning of the movement unto the collision?
b. What are the speeds of the pendulum and the mass on the pole before and after the collision?
c. How far does the mass on the pole fly until it hits the ground?

Solution

(a)

It is quarter of the full pendulum period:

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}}$$

They should be able to take the formula from the lecture.

(b)

Speeds before the collision are 0 for the pole mass and $v_1 = \sqrt{2 \cdot g \cdot L \cdot (1 - \cos\alpha)}$ for the pendulum mass (calculated form energy conservation).

Energy conservation law for the collision: $\frac{m_1 \cdot v_1^2}{2} = \frac{m_1 \cdot {v'}_1^2}{2} + \frac{m_2 \cdot {v'}_2^2}{2}$, where v' is speed after collision for each of the bodies.

Momentum conservation law for the collision:

$$m_1 v_1 = m_1 v_1' + m_2 v_2', \qquad \frac{m_2}{m_1} = p$$

Then:

$$\begin{cases} v_1^2 = {v'}_1^2 + p \cdot {v'}_2^2 \\ v_1 = v_1' + p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases} v_1^2 - {v'}_1^2 = p \cdot {v'}_2^2 \\ v_1 - v_1' = p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases}(v_1 - v_1') \cdot (v_1 + v_1') = p \cdot {v'}_2^2 \\ v_1 - v_1' = p \cdot v_2'\end{cases}$$
$$\downarrow$$
$$\begin{cases}p \cdot v_2' \cdot (v_1 + v_1') = p \cdot {v'}_2^2 \\ v_1 - v_1' = p \cdot v_2'\end{cases}$$
$$\downarrow$$
$$\begin{cases}v_1 + v_1' = v_2' \\ v_1 - v_1' = p \cdot v_2'\end{cases}$$
$$\downarrow$$
$$\begin{cases}v_1' = v_1 \cdot \dfrac{1-p}{1+p} = v_1 \cdot \dfrac{m_1 - m_2}{m_1 + m_2} \\ v_2' = v_1 \cdot \dfrac{2}{1+p} = v_1 \cdot \dfrac{2 \cdot m_1}{m_1 + m_2}\end{cases}$$

That means that if $m_1 > m_2$ then the pendulum will continue motion in the same direction; if $m_1 < m_2$ then the pendulum will bounce back to ($v_1' < 0$) case; if $m_1 = m_2$ then all motion will be transferred to the mass on the pole and pendulum will stop.

It does not matter how students derive the equations, if they use energy and momentum conservation laws. I think it's reasonable to give them 1 point if they put these laws into the right equations (see above). And another point if they come up with the correct final formulas for v_1' *and* v_2'.

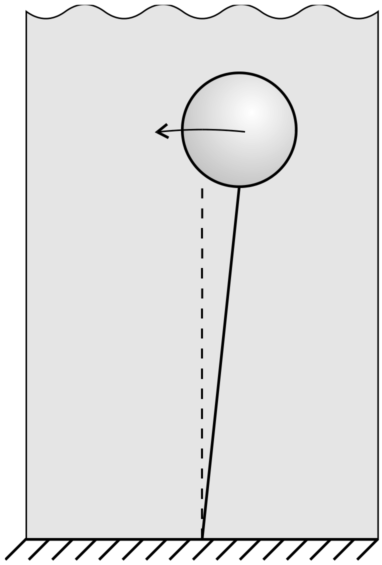
(c)

$$t_{flight} = \sqrt{\frac{2h}{g}} \rightarrow x = \sqrt{\frac{2h}{g}} \cdot v_1 \cdot \frac{2 \cdot m_1}{m_1 + m_2} = \sqrt{\frac{2h}{g}} \cdot \sqrt{2 \cdot g \cdot L \cdot (1 - \cos\alpha)} \cdot \frac{2 \cdot m_1}{m_1 + m_2}$$
$$= \sqrt{h \cdot L \cdot (1 - \cos\alpha)} \cdot \frac{4 \cdot m_1}{m_1 + m_2}$$

That's cool. It means that this experiment on Earth and Mars would provide the same value x for the mass flight distance.

Any form of these answers should count for the score. Students don't have to simplify, if they mentioned before what t_{flight} *or* v_1 *are.*

7. Inverted Pendulum **$2+2+2$ Points**



An isotropic sphere ($m = 10\mathrm{g}$) is submerged in an aqueous solution at 20°C (e.g. it can be treated as water so its viscosity is $\eta = 10^{-3}\frac{\mathrm{kg}}{\mathrm{m \cdot s}}$). Due to buoyancy a lifting force acts on the sphere which is connected to one end of a string. The other end of the string is attached to the bottom. At $t = 0$ the sphere has a speed $v = 0$ and is displaced by an angle of 2°. It then oscillates with a frequency of $f = 0.5\mathrm{Hz}$. After $t_1 = 20\mathrm{s}$ the amplitude is reduced by a factor of 15.

a. What is the radius of the sphere?
b. What is the length of the tether?
c. What would change if the temperature is increased to 37°C. (Describe qualitatively. No calculations necessary.)

Hint: Assume Stokes friction for the sphere, small angles for the amplitude and neglect friction and weight for the string.

Solution

Generally the solution is similar to the dampened oscillation of an object on a spring which was discussed in the lecture. The general equation to describe this motion in that case was:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{\gamma}{m}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{k}{m}x = 0$$

Using the information from the lecture about the viscous drag and a pendulum one can derive the equation for the case here:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\gamma}{m}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{F_B}{mL}\theta = 0,$$

where γ is the viscous drag, m the mass of the sphere, F_B the uplifting force due to buoyancy and L the length of the tether.

From this one can derive (in a similar way as in the lecture):

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2m}\right)^2} = \sqrt{\frac{F_B}{mL} - \left(\frac{\gamma}{2m}\right)^2}$$

Where here:

$$\omega_0^2 = \frac{F_B}{mL}$$

For the amplitude of the oscillation:

$$A(t) = A(0) \cdot e^{-\frac{\gamma t}{2m}}$$

(a)

Using the equation above one can write:

$$\frac{A(t)}{A(0)} = e^{-\frac{\gamma t}{2m}} \rightarrow \ln\left(\frac{A(t)}{A(0)}\right) = -\frac{\gamma t}{2m} \rightarrow \ln\left(\frac{A(0)}{A(t)}\right) = \frac{\gamma t}{2m}$$

For a sphere we can use Stoke's formula to get the radius from the drag coefficient:

$$\gamma = 6\pi\eta R$$

where η is the viscosity of the liquid and R the radius of the sphere. So now we have (1P):

$$\ln\left(\frac{A(0)}{A(t)}\right) = \frac{6\pi\eta R t}{2m}$$

Which leads to

$$R = \frac{\ln\left(\frac{A(0)}{A(t)}\right) m}{3\pi\eta t} = \frac{\ln(15) \cdot 0.01\ \mathrm{kg}}{3\pi \cdot 10^{-3}\ \frac{\mathrm{kg}}{\mathrm{s \cdot m}} \cdot 20\ \mathrm{s}}$$
$$= 0.143\ \mathrm{m}\ (1P)$$

(b)

To calculate the length of the pendulum we can use:

$$\omega = \sqrt{\frac{F_B}{mL} - \left(\frac{\gamma}{2m}\right)^2}$$

Using the radius which we computed above the density of the sphere can be computed which allows us to calculate the buoyancy force:

$$F_B = \rho_{H_2O} V g - mg$$

V for the sphere is:

$$V = \frac{4}{3}\pi R^3 = 0.0124\ \mathrm{m}^3$$

With this the equation above (1P):

$$\omega^2 = \frac{F_B}{mL} - \left(\frac{\gamma}{2m}\right)^2$$
$$L = \frac{F_B}{m\left(\omega^2 + \left(\frac{\gamma}{2m}\right)^2\right)}$$
$$= 1.23\ \mathrm{km}\ (1P)$$

Unfortunately, the numbers didn't work out as planned. It's of course not realistic.

(c)

At 37°C two things change: The density of water and the viscosity $(\mathbf{1P})$. The change to the density of water is small. The change to the viscosity however is significant. This means that in the equation for the frequency, the second expression will be smaller, which means the frequency will go up. $(\mathbf{1P})$

8. Spring Oscillator $\mathbf{2+1+1+1}$ **Points**

A massless spring hangs from the ceiling with a small object attached to its lower end. The object is initially held at rest in a position y_i such that the spring is at its rest length. The object is then released from y_i and oscillates up and down, with its lowest position being $10\ \mathrm{cm}$ below y_i.

a. What is the frequency of the oscillation?
b. What is the speed of the object when it is $80\ \mathrm{cm}$ below the initial position?
c. An object of mass $300\ \mathrm{g}$ is attached to the first object, after which the system oscillates with half the original frequency. What is the mass of the first object?
d. How far below y_i is the new equilibrium (rest) position with both objects attached to the spring?

Solution

(a)
The equation of motion is

$$m\ddot{y} = -D(y - y_i) - mg$$

where y is the displacement and y_i the rest position. Introducing $x = y - y_i$ leads to

$$\ddot{x} + \frac{D}{m}x = -g$$

Trying an ansatz of the form

$$x = x_0 + A\cos(\omega t + \alpha)$$

yields $x_0 = -mg/D = -g/\omega^2$. With the initial conditions $x(0) = 0$ and $v(0) = 0$ one obtains $\alpha = 0$ and $A = g/\omega^2$ such that the solution can be written as

$$x = \frac{g}{\omega^2}(-1 + \cos(\omega t))$$

The lowest point with $x_{\text{low}} = -0.1\text{m}$ is obtained at $\omega t = \pi + 2\,\pi n$ with $x_{low} = -2g/\omega^2$. This yields an angular frequency

$$\omega = \sqrt{\frac{-2g}{x_{\text{low}}}} = 14\text{s}^{-1}$$

corresponding to a frequency

$$\nu = \omega/(2\pi) = 2.23\text{Hz}$$

(b)

The velocity is given by

$$v = -\frac{g}{\omega}\sin(\omega t)$$

and the phase by

$$\omega t = 1 + \frac{x\omega^2}{g}$$

With $x = -0.08\text{m}$ one obtains a velocity

$$v = -\frac{g}{\omega}\sin\left(\arccos\left(1 + \frac{x\omega^2}{g}\right)\right) = -0.56\text{m/s}$$

(c)

The new angular frequency is given by

$$\Omega = \sqrt{\frac{D}{m+M}} = \frac{1}{2}\omega = \frac{1}{2}\sqrt{\frac{D}{m}}$$

such that

$$m + M = 4\,m$$

and

$$m = M/3 = 100\text{g}$$

(d)

The equilibrium position changes from

$$x_0 = -\frac{g}{\omega^2} = -\frac{mg}{D} = -5\text{cm}$$

to

$$x_0' = -(m+M)g/D = -4mg/D = 4x_0 = -20\text{ cm}$$