Mathematics 1, Homework 10
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Due Date: 21.01.24 until 23:59 on-line
or 22.01.24 until 9:15 am in person

Each problem is estimated by one point. Explain your answers.

1. Use back substitution to solve the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + x_4 &= 5 \\ 3x_2 + x_3 - 2x_4 &= 1 \\ - x_3 + 2x_4 &= -1 \\ 4x_4 &= 4 \end{aligned}$$

2. Write out the system of equations that corresponds to the following augmented matrix:

$$\left(\begin{array}{cccc|c} 4 & -3 & 1 & 2 & 4 \\ 3 & 1 & -5 & 6 & 5 \end{array}\right)$$

3. In each of the following systems, interpret each equation as a line in the plane. For each system, graph the lines and determine geometrically the number of solutions:

(a) $\begin{cases} x_1 + x_2 = 4 \\ x_1 - x_2 = 2 \end{cases}$ (c) $\begin{cases} 2x_1 - x_2 = 3 \\ -4x_1 + 2x_2 = -6 \end{cases}$

(b) $\begin{cases} x_1 + 2x_2 = 4 \\ -2x_1 - 4x_2 = 4 \end{cases}$ (d) $\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 1 \\ -x_1 + 3x_2 = 3 \end{cases}$

4. Solve the following system by the Gaussian elimination method:

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 0 \\ -2x_1 + x_2 - x_3 &= 2 \\ 2x_1 - x_2 + 2x_3 &= -1 \end{aligned}$$

5. Given two linear systems with the same matrix of coefficients and different right-hand sides, construct 3×5 augmented matrix and solve both systems simultaneously by doing Gaussian elimination in the augmented matrix and then performing back substitutions:

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 1 \\ 2x_1 + 5x_2 + x_3 &= 9 \\ x_1 + 3x_2 + 4x_3 &= 9 \end{aligned} \qquad \begin{aligned} x_1 + 2x_2 - 2x_3 &= 9 \\ 2x_1 + 5x_2 + x_3 &= 9 \\ x_1 + 3x_2 + 4x_3 &= -2 \end{aligned}$$

6. Which of the matrices that follow are in row echelon form? Which are in reduced row echelon form? Put your answers in the table:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Row echelon form								
Reduced row echelon form								

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$

(g) $\begin{pmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 & 6 \end{pmatrix}$

(h) $\begin{pmatrix} 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

7. Given an augmented matrix in the row echelon form make a list of the lead variables and a second list of the free variables and white a solution to the corresponding linear system:

$$\left(\begin{array}{cccc|c} 1 & 5 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

8. Decide wether the system is consistent, and if yes, find all solutions to this system:

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 1 \\ x_1 + x_2 + x_3 &= 3 \\ 3x_1 + 4x_2 + 2x_3 &= 4 \end{aligned}$$

9. Decide wether the system is consistent, and if yes, find all solutions to this system:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ 2x_1 + 3x_2 - x_3 &= 1 \\ 7x_1 + 3x_2 + 4x_3 &= 7 \end{aligned}$$

10. Use Gauss–Jordan reduction to find all solutoions of the following system

$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ 3x_1 + x_2 + 2x_3 - x_4 &= -1 \end{aligned}$$