HW8-Collisions

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(1) Experiment 1 – $\vec{v}_{CM} = \vec{0}$, so after fusion $V_{A after} = V_{B after} = 0$, so $E_{k before} - \underbrace{E_{k after}}_{0} = E_{tot}$ – released.

Experiment 2 – 1) after fusion move as one, 2) total momentum $= p_A$ (conserved).

Needed release $E_{tot} = \underbrace{\frac{p_A^2}{2m_A}}_{E_{kA}} - \frac{p_A^2}{2(m_A+m_B)} =$

$$= \frac{p_A^2}{2}\left[\frac{1}{m_A} - \frac{1}{m_A+m_B}\right] =$$

$$= \frac{p_A^2}{2} \cdot \frac{m_B}{m_A(m_A+m_B)} = E_{kA} \cdot \frac{m_B}{m_A+m_B} = E_{tot} \Longrightarrow \underline{E_{kA} = E_{tot}\left(1 + \frac{m_A}{m_B}\right)}$$

(m_A, m_B depend on particles, cannot be found in this task uniquely)

(2) a) For elastic collisions (these formulas were derived in last HWs):

$$v_0' = \frac{(m_0 - m_1)v_0 + 2m_1 v_1}{m_0+m_1}, \quad v_1' = \frac{(m_1 - m_0)v_1 + 2m_0 v_0}{m_0+m_1}$$

Since $m_1 = 4m_0$, $v_1 = 0$, $v_0 = 10$ m/s,

$$v_0' = \frac{-3m_0 v_0 + 0}{5m_0} = -\frac{3}{5}v_0 = -6 \text{ m/s}$$

$$v_1' = \frac{3m_0 \cdot 0 + 2m_0 v_0}{5m_0} = \frac{2}{5}v_0 = 4 \text{ m/s} \quad \text{(note that mom. is cons.)}$$

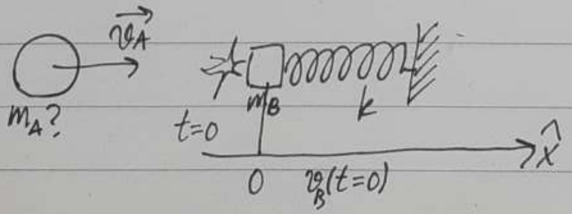
b) Using same reasoning $v_{i+1}' = \frac{2}{5}v_i'$, so

$$v_3' = \left(\frac{2}{5}\right)^3 v_0 = \frac{8}{125}v_0 = \frac{16}{25} \text{ m/s}.$$

(3) v_{CM} does not change (momentum conserved):

$$v_{CM} = \frac{m(-2v) + 2mv}{3m} = \underline{0}.$$

(4)



a) From equation for elastic collision (see 2.a):

$$v_B(0) = \frac{(m_B - m_A)v_{B before} + 2m_A v_{A before}}{m_A + m_B} =$$

$$= \frac{0 + 2m_A v_A}{m_A+m_B} = v \Longrightarrow \frac{2v_A}{1 + \frac{m_B}{m_A}} = v \Longrightarrow \boxed{m_A = \frac{m_B}{-1 + \frac{2v_A}{v}}} = \frac{60g}{-1 + \frac{2\cdot 50}{20}} = 15g$$

b) EOM: $m_B \ddot{x} = -kx$

$\ddot{x} = -\underbrace{\frac{k}{m_B}}_{\omega^2} x \Longrightarrow$ solved by $x = A\sin(\omega t + \varphi_0)$, find A, φ_0:

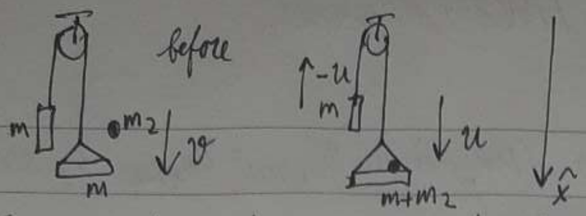
1) $x(t=0) = 0 = A\sin\varphi_0 \Rightarrow \varphi_0 = 0$.

2) $v = \dot{x} = A\omega\cos(\omega t)$, $v(0) = v = A\omega \Longrightarrow A = \frac{v}{\omega}$.

So EOM is: $\boxed{x(t) = \frac{v}{\omega}\sin(\omega t)}$ where $\omega = \sqrt{\frac{k}{m_B}} = \sqrt{\frac{10}{60\cdot 10^{-3}}} \cdot \frac{1}{s} \approx 13\ s^{-1}$.

In SI units $x(t) = \frac{20 \cdot 0.01\,m}{13}\sin(13t \cdot s^{-1}) \approx 0.015\, \underset{\text{meter}}{m} \cdot \sin(13t \cdot s^{-1})$.

(5)



just after collision, impulse of gravity/tension out of picture

Momentum conserved, string not stretching —

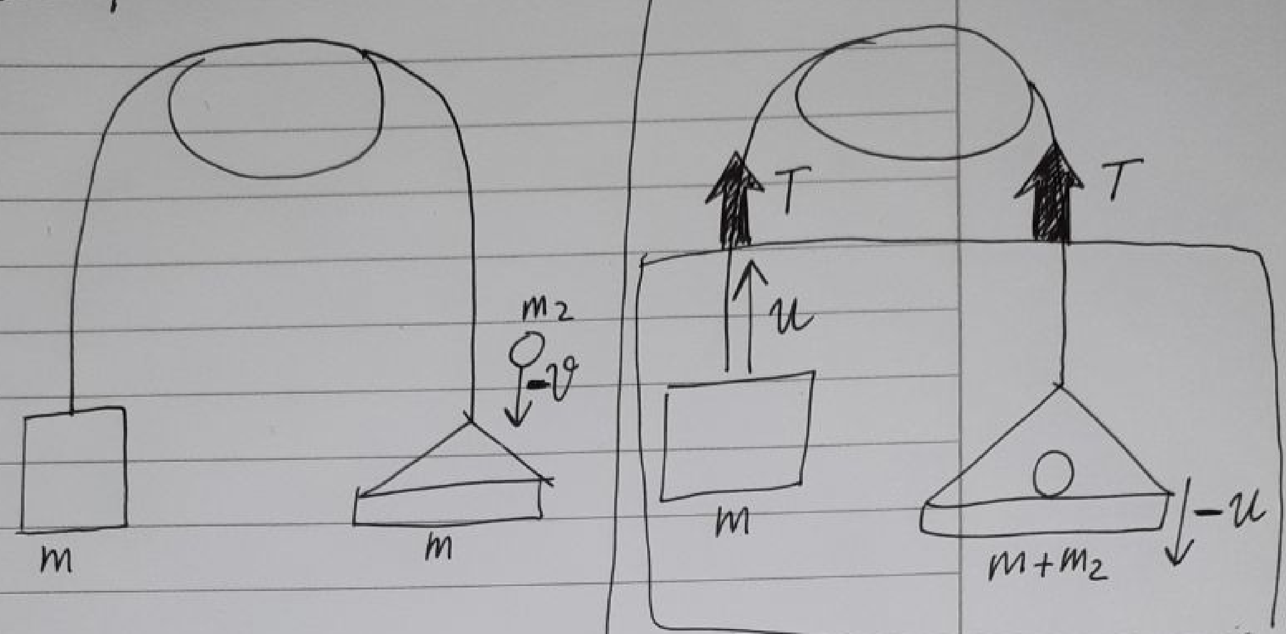
$$m_2 v = (m+m_2)u - mu \Rightarrow \boxed{u=v}.$$

(oppositely moving equal masses do not contribute to mom.)

Long way to show the same:

$$m_2(u-v) = \Delta t\,(m_2 g - N)$$
$$mu = \Delta t\,(mg - T + N)$$
$$-mu = \Delta t\,(T - mg)$$
$$\Rightarrow \Delta t \cdot N = 0,\quad m_2(u-v) = \Delta t \cdot m_2 g \approx 0\ [\Delta t \to 0],\quad u \approx v.$$

Way 1



Why I am wrong — $p_x(\text{before}) \neq p_x(\text{after})$ — tension increases, is <u>not</u> closed system.

1) $\Delta p_x = \underbrace{[mu + (m+m_2)(-u)]}_{p_2} - \underbrace{[-m_2\vartheta]}_{p_1} =$

$= 2T\Delta t \Rightarrow m_2(\vartheta - u) = 2T\Delta t$

2) $mu - 0 = T\Delta t$ (for left only)

$\Rightarrow m_2\vartheta = (2m+m_2)u$

$u = \dfrac{m_2\vartheta}{m_2+2m}$

Way 2 — Why we can think about "bent" axis →

So mom. along such "axis" <u>is</u> conserved

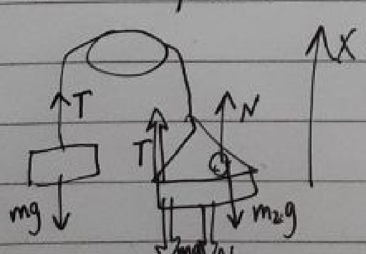
$\rightarrow (2m+m_2)u = m_2\vartheta$.

will vanish $(T+(-T))$

Way 3 — Long way to convince everyone

N. laws for all bodies:

$$\begin{cases} m_2(-u) - m_2(-\vartheta) = \Delta t(-m_2 g + N) & \text{(ball)} \\ mu = \Delta t(T - mg) & \text{(left)} \\ -mu = \Delta t(-mg + T - N) & \text{(right)} \end{cases}$$



$$\begin{cases} 2mu = \Delta t N \\ m_2 u - m_2\vartheta = \underbrace{m_2 g \cdot \Delta t}_{\to 0} - \underbrace{\Delta t N}_{2mu} \end{cases} \Rightarrow u = \frac{m_2\vartheta}{m_2+2m}$$

same