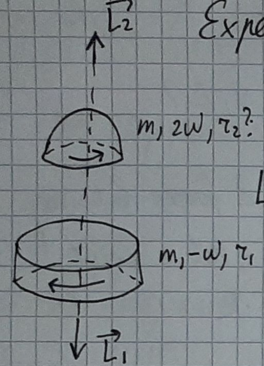
Experimental Physik I

Ex. S. 10

①



$I_2 = \frac{2}{5} m r_2^2$ (already includes $\frac{1}{2}m$ of full sphere $I_S = \frac{2}{5} m r_2^2$)

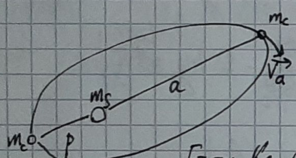
$L_2 = I_2 \cdot \omega_2 = \frac{2}{5} m \cdot 2\omega \cdot r_2^2$

$I_1 = \frac{1}{2} m r_1^2$

$L_1 = -I_1 \omega_1 = -\frac{1}{2} m r_1^2 \cdot \omega$

$L_1 + L_2 = 0 = \frac{4}{5} m\omega r_2^2 - \frac{1}{2} m\omega r_1^2 \Longrightarrow \underline{r_2 = r_1 \sqrt{\frac{5}{8}}}$.

②



$a = 35.4\,AU$, $V_a = 0.91\,km/s$.

$p = 0.59\,AU$

$V_p, T - ?$

From Kepler III, $T = 2\pi \sqrt{\frac{\left(\frac{a+p}{2}\right)^3}{GM}}$ or rather

$$\frac{T_c^2}{T_E^2} = \frac{\left(\frac{a+p}{2}\right)^3}{1AU^3} \Longrightarrow T_c = T_E \left(\frac{a+p}{2\,AU}\right)^{\frac{3}{2}} =$$

$$= 1\text{year} \cdot \left(\frac{35.4+0.59}{2}\right)^{3/2} \approx$$

$$\approx \underline{76\,\text{years}}$$

Velocity – easiest way is Angular Mom. conservation:

$p \cdot m_c \cdot V_p = a \cdot m_c \cdot V_a$

$V_p = \frac{a}{p} V_a = \frac{35.4}{0.59} \cdot 0.91\,\text{km/s} = \underline{54.6\,\text{km/s}}.$

2nd way – energy conserv. (inside Sun frame ≈ inside CM, since $M_S \gg m_c$)

requires data: G, m_S etc.

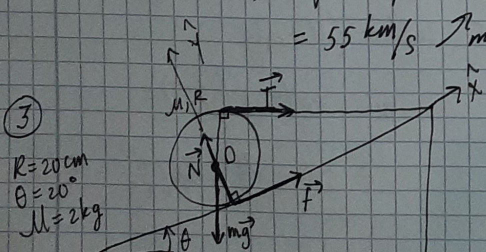
$$\frac{m_c V_p^2}{2} - \frac{G m_c m_S}{p} = \frac{m_c V_a^2}{2} - \frac{G m_c m_S}{a} \Longrightarrow$$

$$V_p = \sqrt{2 G m_S \left(\frac{1}{p} - \frac{1}{a}\right) + V_a^2} = \left[\begin{matrix} G = 6.67 \cdot 10^{-11}[G] \\ m_S = 2 \cdot 10^{30}\,kg \end{matrix}\right]$$

units of G :)

$= 55\,\text{km/s}$ matches.

③



In equilibrium

$(\sum \vec{F}) \cdot \hat{x} = 0$

$(\sum \vec{F}) \cdot \hat{y} = 0$

$(\sum \vec{\tau})_O = \vec{0}$

$$\begin{cases} Rf = RT \Longrightarrow f = T \\ f + T\cos\theta - mg\sin\theta = 0 \\ -mg\cos\theta + N - T\sin\theta = 0 \end{cases} \Longrightarrow T = \frac{mg\sin\theta}{1+\cos\theta} = \frac{mg \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}{1 + \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}} = \frac{mg \cdot 2\tan\frac{\theta}{2}}{2} = mg\tan\frac{\theta}{2}$$

$f = mg\tan\frac{\theta}{2}$

$N = T\sin\theta + mg\cos\theta = mg\left[\cos\theta + \sin\theta \cdot \tan\frac{\theta}{2}\right]$

Values:

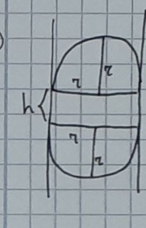
$T = 2\,kg \cdot 10\,\frac{m}{s^2} \cdot \tan 10° \approx 3.53\,N$

$f \approx 3.53\,N$

$N = 2\,kg \cdot 10\,\frac{m}{s^2}\left[\cos 20° + \sin 20° \cdot \tan 10°\right] \approx 21.3\,N$

Note! $mg \neq N$ even with picture symmetry (it is at center, $\vec{N}$ at contact)

④



$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \frac{d^3}{8} = \frac{\pi d^3}{6} \Longrightarrow h = \frac{d^3}{6r^2} - \frac{4r}{3}$

$V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$S = 2\pi r h$, $F_{pressure} = p \cdot S$, $F_{frict\,max} = \mu p S$

Equilibrium: $\mu p S = (M + m) g$

goal

$m = -M + \frac{1}{g} \cdot \mu p \cdot 2\pi r \left(\frac{d^3}{6r^2} - \frac{4r}{3}\right) =$

$= \left[-100\,kg + \frac{1}{10} \cdot 0.2 \cdot 1.2 \cdot 10^5 \cdot 2\pi \cdot \frac{1}{2}\left[\frac{1.04^3}{6 \cdot \left(\frac{1}{2}\right)^2} - \frac{4}{3} \cdot \frac{1}{2}\right]\right] \cdot kg =$

$= 528\,kg$

It will be not easy for him to get free.