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TP1-HW8

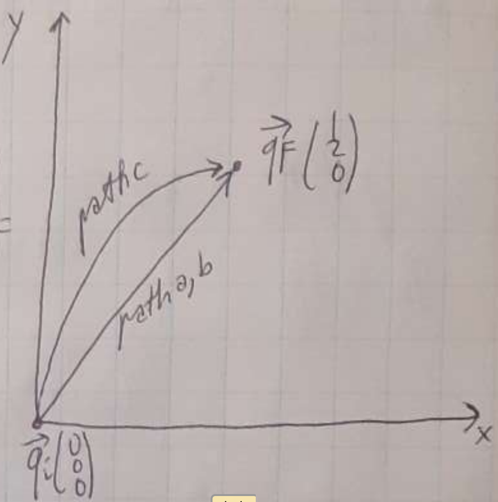
Problem 8.1

$$\vec{K}_1(\vec{q}) = \left(\frac{x}{|\vec{q}|}, \frac{y}{|\vec{q}|}, \frac{z}{|\vec{q}|}\right) \qquad a: \vec{q}_a(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} t/2 \\ t \\ 0 \end{pmatrix}$$

$$\vec{K}_2(\vec{q}) = (x^2+y^2, x^2-y^2, 0) \qquad b: \vec{q}_b(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} t \\ 2t \\ 0 \end{pmatrix}$$

$$\vec{K}_3(\vec{q}) = (x+y+z, x+y+z, x+y+z) \qquad c: \vec{q}_c(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} t^3 \\ 2t^2 \\ 0 \end{pmatrix}$$

$$\vec{q}_I = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \vec{q}_F = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$



Part a)

$$\nabla|\vec{q}| = \nabla\sqrt{x^2+y^2+z^2} = \begin{pmatrix} \partial_x \sqrt{\sim} \\ \partial_y \sqrt{\sim} \\ \partial_z \sqrt{\sim} \end{pmatrix} = \begin{pmatrix} \frac{2x}{2\sqrt{x^2+y^2+z^2}} \\ \frac{2y}{2\sqrt{x^2+y^2+z^2}} \\ \frac{2z}{2\sqrt{x^2+y^2+z^2}} \end{pmatrix} =$$

$$= \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} = \vec{K}_1(x,y,z).$$

Since $\vec{K}_1$ is a gradient of some scalar function, $\vec{K}_1$ is conservative field and

$$-\int d\vec{s}\, \vec{K}_1(\vec{q}) \ \boxed{\text{does not depend on chosen path}}$$

Just check it for fun for these 3 paths

$$W_a = \int_{\vec{q}_i}^{\vec{q}_F} \vec{K}_1 \underbrace{d\vec{q}}_{\vec{q}'(t)dt} = \int_a \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} dt = \int_{t=0}^{t=2} \begin{pmatrix} t/2 \\ t \\ 0 \end{pmatrix} \cdot \frac{2}{\sqrt{5t^2}} \cdot \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} dt =$$

$$= \int_{t=0}^{t=2} \frac{2}{\sqrt{5t^2}} \left(\frac{t}{4}+t\right) dt = \int_{t=0}^{2} \frac{2}{\sqrt{5}} \cdot \left(\frac{1}{4}+1\right) dt = \frac{\sqrt{5}}{2} t \Big|_0^2 = \sqrt{5}.$$

$$W_b = \int_{\vec{q}_i}^{\vec{q}_F} \vec{K}_2\, d\vec{q} = \int_b \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot d\begin{pmatrix} t \\ 2t \\ 0 \end{pmatrix} = \int_{t=0}^{t=1} \begin{pmatrix} t \\ 2t \\ 0 \end{pmatrix} \frac{1}{\sqrt{5}t} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} dt = \int_0^1 \frac{5}{\sqrt{5}} dt = \sqrt{5} t \Big|_0^1 = \sqrt{5}.$$

$$W_c = \int_{\vec{q}_i}^{\vec{q}_F} \vec{K}_3\, d\vec{q} = \int_{t=0}^{t=1} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot d\begin{pmatrix} t^3 \\ 2t^2 \\ 0 \end{pmatrix} = \int_0^1 \begin{pmatrix} t^3 \\ 2t^2 \\ 0 \end{pmatrix} \frac{1}{\sqrt{t^6+4t^4}} \begin{pmatrix} 3t^2 \\ 4t \\ 0 \end{pmatrix} dt = \int_0^1 \frac{1}{t^2\sqrt{4+t^2}} (3t^5+8t^3) dt =$$

$$= \int_0^1 \frac{3t^3+8t}{\sqrt{t^2+4}} dt = \int_0^1 \frac{t(3t^2+8)}{\sqrt{t^2+4}} dt = \begin{bmatrix} t^2+4=u \\ du = 2t\,dt \\ u(0)=4 \\ u(1)=5 \end{bmatrix} = \frac{1}{2}\int_4^5 \frac{du(3u-4)}{\sqrt{u}} =$$

$$=\frac{1}{2}\int_4^5 3\sqrt{u}-\frac{4}{\sqrt{u}}\,du=\frac{1}{2}\cdot\left[3\frac{u^{3/2}}{3/2}-8\sqrt{u}\right]\Big|_4^5=\frac{1}{2}\left[2\cdot 5^{3/2}-8\sqrt{5}-\cancel{2\cdot 8}+\cancel{16}\right]=\underline{\sqrt{5}}$$

Part B

$$\int_{\vec{q}}\vec{K_2}\,d\vec{q}=\int_{t_i}^{t_f}\begin{pmatrix}K_x(t)\\K_y(t)\\K_z(t)\end{pmatrix}\cdot d\begin{pmatrix}x(t)\\y(t)\\z(t)\end{pmatrix}=\int_{t_i}^{t_f}\begin{pmatrix}K_{2x}(t)\\K_{2y}(t)\\K_{2z}(t)\end{pmatrix}\cdot\begin{pmatrix}\dot{x}(t)\\\dot{y}(t)\\\dot{z}(t)\end{pmatrix}dt$$

<u>path A</u>

$\vec{q}_A=\begin{pmatrix}t/2\\t\\0\end{pmatrix}$

$$\int_0^2\begin{pmatrix}t^2/4+t^2\\t^2/4-t^2\\0\end{pmatrix}\cdot\begin{pmatrix}1/2\\1\\0\end{pmatrix}dt=\int_0^2\frac{5t^2}{8}-\frac{6t^2}{8}\,dt=\int_0^2-\frac{t^2}{8}\,dt=-\frac{t^3}{24}\Big|_0^2=$$

$$=\underline{-\frac{1}{3}}$$

<u>path B</u> → note, it is same path but traversed more quickly (2 times).

$\vec{q}_B=\begin{pmatrix}t\\2t\\0\end{pmatrix}$

$$\int_0^1\begin{pmatrix}t^2+4t^2\\t^2-4t^2\\0\end{pmatrix}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}dt=\int_0^1 t^2+4t^2-6t^2\,dt=\int_0^1-t^2\,dt=-\frac{t^3}{3}\Big|_0^1=$$

$$=\underline{-\frac{1}{3}}$$

<u>This implies:</u> the work on the given path does not depend on parametrization („how fast we walk") if the force field depends only on coordinates (not velocity/time). This is true <u>independently</u> of whether force is conservative or not.

<u>path C</u>

$\vec{q}_C=\begin{pmatrix}t^3\\2t^2\\0\end{pmatrix}$

$$\int_0^1\begin{pmatrix}t^6+4t^4\\t^6-4t^4\\0\end{pmatrix}\cdot\begin{pmatrix}3t^2\\4t\\0\end{pmatrix}dt=\int_0^1 3t^8+12t^6+4t^7-16t^5\,dt=$$

$$\frac{3t^9}{9}+\frac{12t^7}{7}+\frac{4t^8}{8}-\frac{16t^6}{6}\Big|_0^1=\frac{1}{3}+\frac{12}{7}+\frac{1}{2}-\frac{8}{3}=\frac{14+72+21-112}{42}=$$

$$=\underline{-\frac{5}{42}}$$

$W_C\neq W_A=W_B.$

Therefore $\vec{K_2}$ is <u>not</u> conservative.

c) $W_A = \int_{t=0}^{2} \begin{pmatrix} t/2+t+0 \\ t/2+t+0 \\ t/2+t+0 \end{pmatrix} \cdot \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} dt = \int_{t=0}^{2} \frac{3t}{4} + \frac{3t}{2}\, dt = \frac{9}{4} \cdot \frac{t^2}{2} \Big|_0^2 = \underline{\frac{9}{2}}$

$\vec{q}_A = \begin{pmatrix} t/2 \\ t \\ 0 \end{pmatrix}$

$W_B = \int_{t=0}^{1} \begin{pmatrix} t+2t \\ t+2t \\ 3t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} dt = \int_{t=0}^{1} 3t + 6t\, dt = \frac{9t^2}{2} \Big|_0^1 = \underline{\frac{9}{2}}$

$\vec{q}_B = \begin{pmatrix} t \\ 2t \\ 0 \end{pmatrix}$

$W_C = \int_{t=0}^{1} \begin{pmatrix} t^3+2t^2 \\ t^3+2t^2 \\ t^3+2t^2 \end{pmatrix} \cdot \begin{pmatrix} 3t^2 \\ 4t \\ 0 \end{pmatrix} dt = \int_0^1 3t^5 + 6t^4 + 4t^4 + 8t^3\, dt =$

$\vec{q}_C = \begin{pmatrix} t^3 \\ 2t^2 \\ 0 \end{pmatrix}$

$= \int_0^1 3t^5 + 10t^4 + 8t^3\, dt = \left(\frac{3t^6}{6} + \frac{10t^5}{5} + \frac{8t^4}{4}\right)\Big|_0^1 = \frac{1}{2} + 2 + 2 = \underline{\frac{9}{2}}$

$\underbrace{W_A = W_B}_{\text{should always be}} = W_C \longrightarrow$ but this is not guarantee of conservativeness.

Check $\nabla \times \vec{K}_3 = \begin{pmatrix} \partial x \\ \partial y \\ \partial z \end{pmatrix} \times \begin{pmatrix} x+y+z \\ x+y+z \\ x+y+z \end{pmatrix} = \begin{pmatrix} 1-1 \\ 1-1 \\ 1-1 \end{pmatrix} = \vec{0}$ —

3.1

$\vec{K}_3$ is conservative indeed.

Problem 8.2

$\vec{F}(\vec{r}) = -k\vec{r}$

a) $k > 0$. Attractive force. If $\vec{F} \perp \dot{\vec{q}}$, the particle may circle, otherwise radial component of $\dot{\vec{q}}$ will decrease (so it will tend toward center).

3.2

In both cases there is distance bound.

Only this works for <u>all</u> conditions $(\vec{q}_0, \dot{\vec{q}}_0)$.

b) $E = T + \Phi = \frac{m v^2}{2} + \Phi(-k\vec{r})$.

3.3

$\Phi(-k\vec{r}) = \frac{kr^2}{2} + C$, since $-\nabla\left(\frac{kr^2}{2} + C\right) = -\nabla\left(\frac{kr^2}{2}\right) = -\frac{k}{2}\nabla(x^2+y^2+z^2) =$

$-\frac{k}{2}\begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} = -k\vec{r} = \underline{\vec{F}}$

So $E = \frac{m\vec{v}\cdot\vec{v}}{2} + \frac{kr^2}{2} + C$ up to some constant.

$\frac{dE}{dt} = \frac{m}{2} \cdot 2\vec{v} \cdot \dot{\vec{v}} + \frac{k}{2} \cdot 2\vec{r} \cdot \dot{\vec{r}} = m\vec{v}\dot{\vec{v}} + k\vec{r}\cdot\vec{v} = \vec{v}(m\dot{\vec{v}} + k\vec{r}) = 0$

means (if particle moves) $m\dot{\vec{v}} = -k\vec{r}$, $m\vec{a} = \vec{F} \rightarrow$ entails Newton's law.

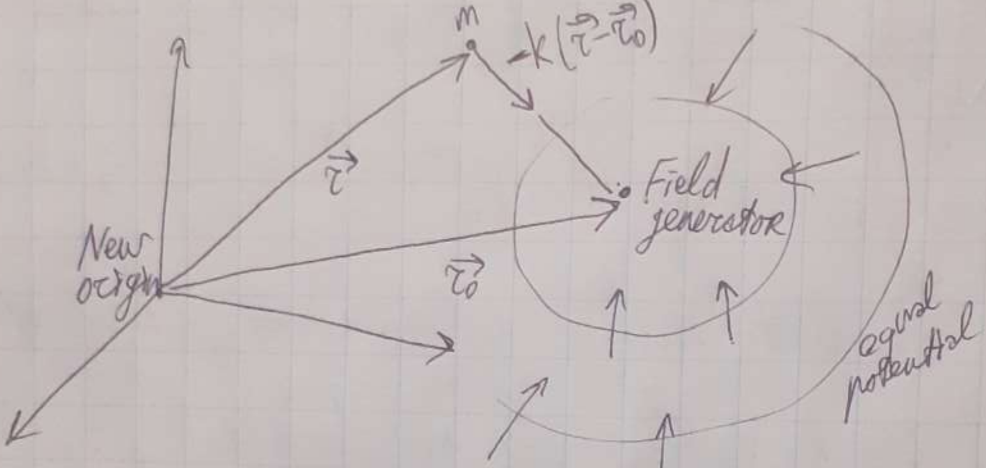
4.1

c) $$\frac{dL}{dt} = \frac{d}{dt}\left(\vec{r}\times(m\dot{\vec{r}})\right) = m\frac{d}{dt}\left(\vec{r}\times\dot{\vec{r}}\right) = m\left(\underbrace{\dot{\vec{r}}\times\dot{\vec{r}}}_{\vec{0}} + \vec{r}\times\ddot{\vec{r}}\right) =$$

$$= m(\vec{r}\times\ddot{\vec{r}}) = \vec{r}\times(\underbrace{m\ddot{\vec{r}}}_{\vec{F}}) = \vec{r}\times(-k\vec{r}) =$$

$$= -k(\vec{r}\times\vec{r}) = \underline{\vec{0}}.$$

If using other origin it is in general not true, here is why:



$\vec{r}_0$ is where centre of field is fixed,
Relative position is $\vec{r}-\vec{r}_0$ ($\vec{r}$ – particle position in <u>new</u> system).
So $\vec{F} = -k(\vec{r}-\vec{r}_0)$.

$\vec{L} = \vec{r}\times(m\dot{\vec{r}})$, $\quad \frac{d\vec{L}}{dt} = m\left(\underbrace{\dot{\vec{r}}\times\dot{\vec{r}}}_{0} + \vec{r}\times\ddot{\vec{r}}\right) = \vec{r}\times(\underbrace{m\ddot{\vec{r}}}_{\vec{F}}) =$

relative to new origin

$$= \vec{r}\times\left((-k)\cdot(\vec{r}-\vec{r}_0)\right) = \vec{r}\times\left(k(\vec{r}_0-\vec{r})\right) = k\left(\vec{r}\times\vec{r}_0 - \underbrace{\vec{r}\times\vec{r}}_{\vec{0}}\right) =$$

$$= \underline{k(\vec{r}\times\vec{r}_0)}$$

It can still be 0 (so $\vec{L} = const$) iff
$\underline{\vec{r}_0 = \alpha\vec{r}}$, then $\dot{\vec{L}} = k(\vec{r}\times(\alpha\vec{r})) = k\alpha\cdot\vec{0} = \vec{0}$.

So it is conserved iff particle is always on the line connecting field centre and origin.
When particle shifts away, it becomes changing until it again moves radially w.r.t. field center.

d) Initial equation: $m\ddot{\vec{q}} = -k\vec{q}$

5.1 $m\ddot{\vec{q}} + k\vec{q} = 0.$

$$m\frac{d^2\vec{q}}{dt^2} + k\vec{q} = 0$$

Take dimensionless

$\vec{x} = \frac{\vec{q}}{B}$ → some of bounds from a)

$\tau = \sqrt{\frac{k}{m}}\cdot(t-t_0)$, $t = \tau\sqrt{\frac{m}{k}} + t_0$

Then

$$\frac{m}{k}\cdot\frac{d^2(B\vec{x})}{d\left(\tau\sqrt{\frac{m}{k}}+t_0\right)^2} + \vec{q} = 0$$

$$B\cdot\frac{m}{k}\cdot\frac{d^2(\vec{x})}{\left(\sqrt{\frac{m}{k}}\right)^2 d\tau^2} + \underbrace{\vec{q}}_{\to B\vec{x}} = 0$$

$$B\cdot\frac{m}{k}\cdot\frac{k}{m}\frac{d^2\vec{x}}{d\tau^2} + B\vec{x} = 0 \;\Big|\cdot\frac{1}{B} \longrightarrow \frac{d^2\vec{x}}{d\tau^2} + \vec{x} = 0$$

$\ddot{\vec{x}} + \vec{x} = 0$, all dimensionless

now $\frac{\partial}{\partial\tau}$

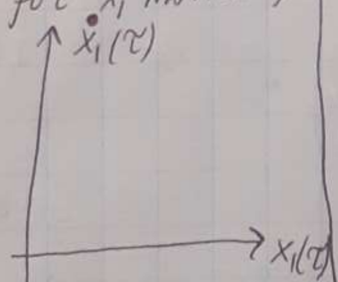
$\vec{x} = (x_1, x_2)$, so $\ddot{x}_k + x_k = 0$

Main Idea of everything

(We can associate phase space with complex plane in general, and in f) we find specific representation of motion for our case → then solve it easily → plot phase space trajectories for one coordinate, and the other in plane will behave similarly → only ~~some constants differ~~ & amplitude of oscillations & possible time shift → show trajectory in real plane is ellipse from 2 independent oscillations (obtained from phase space in complex plane)

e) $Z(\tau) = x_1(\tau) + i\dot{x}_1(\tau)$

5.2 („representing phase space for x_1 motion")



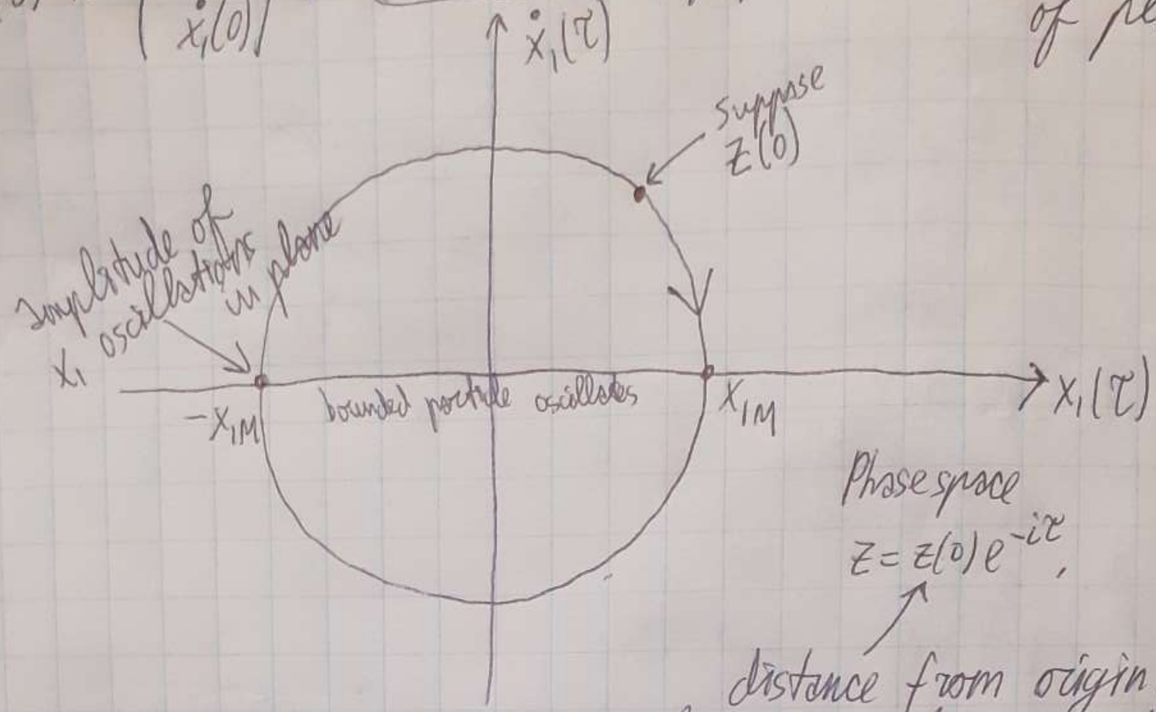
$$\frac{dz}{d\tau} = \dot{x}_1 + i\ddot{x}_1 = [\text{in our case}] = \dot{x}_1 + i(-x_1) = (-i)x_1 + \dot{x}_1 = (-i)\underbrace{(x_1 + i\dot{x}_1)}_{z} = -iz,$$

So $\boxed{\dot{z} = -iz}$ (*) is our equation for x_1 in complex plane (or phase space).

5.3 f) Solve it. $z(\tau) = z(0)\cdot e^{-i\tau}$ is solution: ~~z(0) is phase state of~~

$\dot{z} = -i\,z(0)e^{-i\tau} = -iz$ which is (*).

$z(0)$ is $\begin{pmatrix} x_1(0) \\ \dot{x}_1(0) \end{pmatrix}$ or <u>initial state</u> of particle in x_1 direction of plane.



Phase space
$z = z(0) e^{-i\tau}$.

distance from origin does <u>not</u> change, so phase space diagram is a circle, determined by $z(0)$.

<u>So:</u> Phase space diagrams for x_1 and x_2 are both circles which size is determined by initial conditions (and therefore initial conditions determine x_{1M}, x_{2M}, or amplitude of oscillations).

* g) Returning from phase space $\begin{pmatrix} x_k \\ \dot{x}_k \end{pmatrix}$ for x_k to space $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ we then have that:
(for x_1 example)

$$z = \underbrace{z(0)}_{x_{1M}} e^{-i\tau} = [\text{Euler formula}] = x_{1M}(\overset{\text{coordinate}}{\cos\tau} - i\overset{\text{speed}}{\sin\tau}),$$

So x_1 (which is real part):

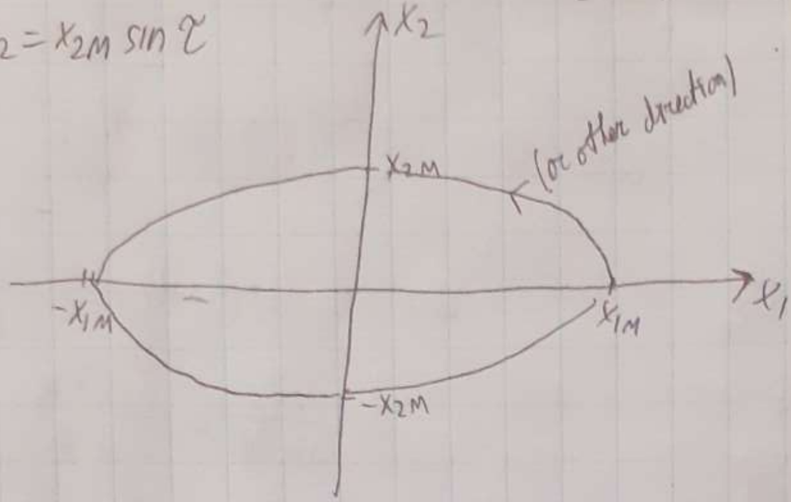
$$\begin{cases} x_1 = x_{1M}\cos\tau \\ x_2 = x_{2M}\cos(\tau + \underbrace{\varphi}_{}) \end{cases}$$

$\to$ possible shift since, for example, they do not have to oscillate in same phase.

Such equations <u>always</u> describe ellipse. If realigning axis properly, it is possible to make the following system:

$$\begin{cases} x_1 = x_{1M}\cos\tau \\ x_2 = x_{2M}\sin\tau \end{cases}$$

(I cannot prove that though strictly, only feel it; this will prepare for seminars):



The shape such as amplitudes x_{KM} are determined from radius $z(0)$ in phase space — initial conditions.

It is not system parameter.

* h) However, period of trajectory is dependent exactly on system parameters.

Returning to dimensions: $x_1 = x_{1M}\cos\tau \longrightarrow$

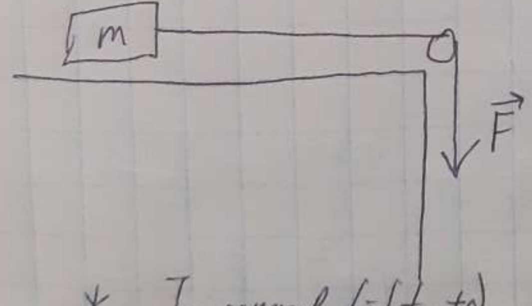
$$q_1 = Bx_{1M}\cos\left(\sqrt{\frac{k}{m}}(t-t_0)\right), \text{ so}$$

$$\underline{T = 2\pi\sqrt{\frac{m}{k}}.}$$

For same mass and field constant k, depending on initial conditions, will be different ellipses, traversed in same time.

Problem 8.3 Car.

a)



b) $a = \frac{F}{m}$,

$$v(t) = v(0) + \int_0^t a\,dt' = v(0) + \frac{F}{m}t, \quad \left(\text{consider } t_0 = 0\right)$$

$$x(t) = x(0) + \int_0^t v(t')dt' = x(0) + \int_0^t \left(v(0) + \frac{F}{m}t'\right)dt' =$$

$$= x(0) + v_0(t) + \frac{F}{m}\frac{t^2}{2}.$$

* In general (if $t_0 \neq 0$), $v(t) = v(t_0) + \frac{F}{m}(t-t_0)$,

$$x(t) = x(t_0) + v(t_0)\cdot(t-t_0) + \frac{F}{m}\cdot\frac{(t-t_0)^2}{2}.$$

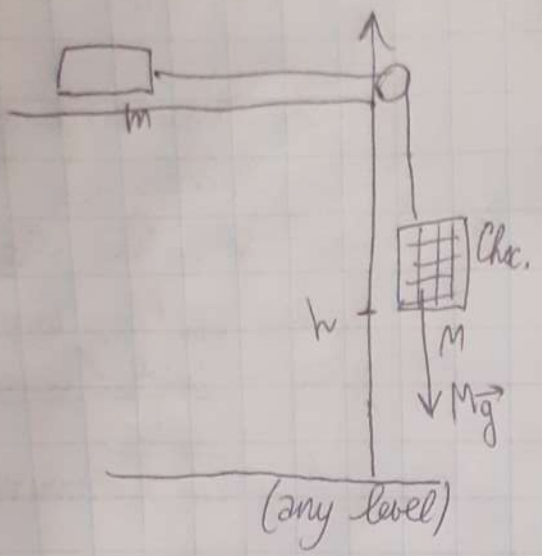
* Actual numbers (suppose $t_0 = 0\,s$, $v(t_0) = 0\,\frac{m}{s}$, $x(t_0) = 0\,\frac{m}{s}$):

$$\boxed{a = \frac{2N}{20\cdot 10^{-3}kg} = 10^2\,\frac{m}{s^2}}$$

$$v(t) = 100\,\frac{m}{s^2}\cdot t, \quad x(t) = 50t^2\cdot\frac{m}{s^2}.$$

c) $F = mg = 0.2\,kg \cdot 10\,\frac{m}{s^2} = 2\,N$ – exactly as before by

d) $E = E_{kin} + E_{pot} = \frac{(m+M)\,v^2}{2} + Mgh = E$



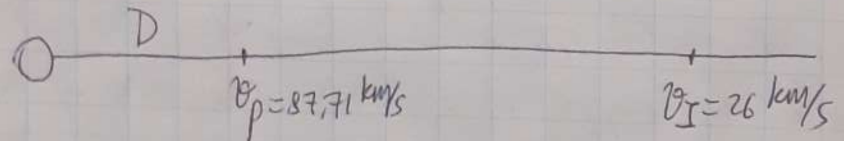
$$\frac{dE}{dt} = 0 = \frac{(m+M)}{\cancel{2}} \cdot \cancel{2} v \cdot \dot{v} + Mg\underbrace{\dot{h}}_{=-v} \quad \Big| \cdot \frac{1}{v} \quad = -\frac{1}{\dot{h}}$$

$$0 = (m+M)\dot{v} + Mg$$

$$\dot{v} = \frac{Mg}{m+M} = \frac{0.2 \cdot 10\,m}{0.22\,s} \approx 9\,\frac{m}{s^2},$$

The acceleration almost 10 times smaller, as the same force now has to accelerate ≈ 10 times larger mass (including chocolate).

Problem 8.4 – Oumuamua



~~Considering~~ Neglecting initial potential energy,

$$-\frac{G M_S m}{D} + \frac{m v_p^2}{2} = \frac{m v_I^2}{2}$$

$$\frac{v_p^2 - v_I^2}{2} \simeq \frac{M_S G}{D} \quad \text{(comet)}$$

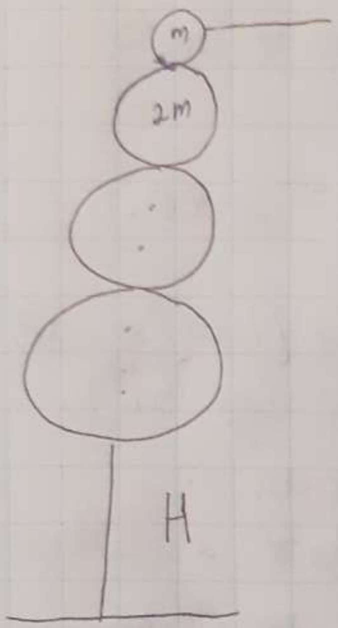
$$\frac{4\pi^2 R^3}{T^2} \simeq \frac{M_S G}{R^2} \quad \text{(Earth), or} \quad \frac{4\pi^2 R^2}{T^2} \simeq \frac{M_S G}{R} = \left(\frac{2\pi R}{T}\right)^2 = v_{ave,\,Earth}$$

$$\begin{cases} v_E^2 \simeq \dfrac{M_S G}{R} \\ \dfrac{v_p^2 - v_I^2}{2} = \dfrac{M_S G}{D} \end{cases} \quad \text{Then} \quad \frac{D}{R} = \frac{2 v_E^2}{v_p^2 - v_I^2}$$

$$D = \frac{2 v_E^2 R}{v_p^2 - v_I^2} = \frac{1\,AU \cdot 2 \cdot \left(\frac{2\pi \cdot 1\,AU}{365 \cdot 24 \cdot 3600\,s}\right)^2}{87.71^2 \cdot 10^6\,\frac{m^2}{s^2} - 26.6^2 \cdot 10^6\,\frac{m^2}{s^2}} = AU \cdot \frac{2\left(\frac{2\pi \cdot 1.44 \cdot 10^{11}}{365 \cdot 24 \cdot 3600\,s}\right)^2}{87.71^2 \cdot 10^6 - 26.6^2 \cdot 10^6} \approx$$

≈ 0.235670 AU (with calculator due to given precision).

Problem 8.5 – Galilean Balls



General collision rules: (elastic)

$m_1 \xrightarrow{} v_1 \quad \xleftarrow{} m_2\, v_2 \Longrightarrow m_1\ v_1' \quad m_2\ v_2'$

Side notes

$$\begin{cases} m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \\ m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2 \end{cases}$$

$$\begin{cases} m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2 \\ m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 - m_2 v_2^2 \end{cases}$$

$$v_1 + v_1' = v_2' + v_2,$$

$$v_2' = v_1 - v_2 + v_1' \rightarrow$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 (v_1 - v_2 + v_1')$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_1 - m_2 v_2 + m_2 v_1'$$

$$v_1' = \frac{v_1(m_1 - m_2) + 2 m_2 v_2}{m_1 + m_2}, \text{ by symmetry}$$

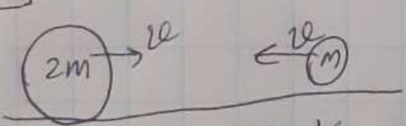
$$v_2' = \frac{v_2(m_2 - m_1) + 2 m_1 v_1}{m_1 + m_2}$$

1) When colliding with large stationary ground ($v_2 = 0$, $m_2 = \infty$),

$$v_1' = \frac{v_1(0-1)}{0+1} = -v_1 \text{ just reflected back,}$$

2) After lower ball reflects back → it will sequentially push any of balls falling on it with the same speed (speed is same because all objects in uniform gravity field have same accel, $\vec{g}$).

Take $\boxed{\text{any}}$ of such collisions:



$$\text{Then } v_2' = \frac{(-v)(m-2m) + 2\cdot 2m \cdot v}{3m} = \frac{mv + 4mv}{3m} = \frac{5}{3} v.$$

For n's ball the speed will be $\left(\frac{5}{3}\right)^{n-1} v$.

3) From energy conservation v (the very first) is $\sqrt{2gH}$, so

$v_n = \left(\frac{5}{3}\right)^{n-1} \sqrt{2gH}$. Then again using energy conservation,

~~In our case~~ $\cancel{v_4 = \left(\frac{5}{3}\right)^3 \sqrt{2 \cdot 10 \frac{m}{s^2} \cdot 1m} = \frac{125}{27}\sqrt{20}\, \frac{m}{s}.}$

the largest height will be: ~~H~~

$$\frac{m v_n^2}{2} = mg\Delta H, \quad \Delta H = \frac{v_n^2}{2g}$$

$$H_{max} = -H + \frac{1}{2g} \cdot 2gH \cdot \left(\left(\frac{5}{3}\right)^{n-1}\right)^2 = -H + H\left(\frac{5}{3}\right)^{2n-2} = H\left(\left(\frac{5}{3}\right)^{2n-2} - 1\right).$$

(If $H=0$ where highest ball was initially, since we need to know the increase).

In our case of $n=4$: $H_{max} = 1m\left(\left(\frac{5}{3}\right)^6 - 1\right) \approx 20.4m.$

Note: if the task asks to find $H_{max}(i)$ for $i \in [1, n-1]$ as well, the solution is much more complicated because say ball 3 after striking ball 4 again strikes ball 2 and so on.
For this situation in general I have not developed a solution.
But for 4 balls this can be ugly calculated, one by one.

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