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Theoretical Physics 1

Problem 6.1 Properties of cross product

a) $\forall \lambda \in \mathbb{R}, a, b \in \mathbb{R}^3$: $\vec{b} = \lambda \vec{a} \longrightarrow \vec{a} \times \vec{b} = \vec{0}$

$$\vec{a} \times \vec{b} = \vec{a} \times (\lambda \vec{a}) = \lambda \cdot (\vec{a} \times \vec{a}) = \lambda \cdot \vec{0} = \vec{0}.$$

Geometrically: $\vec{b}, \vec{a}$ on same line, area of parallelogram is 0.

b) Geometrically: when 3 vectors are in plane/line $\to$ the volume of paralelepiped is 0.

It is enough to prove 1, because if $\vec{c} = \alpha \vec{a} + \beta \vec{b} \to \vec{a} = \underbrace{-\frac{1}{\alpha}}_{\gamma}\vec{c} - \underbrace{\frac{\beta}{\alpha}}_{\delta}\vec{b}$,

or $\vec{b} = \underbrace{-\frac{1}{\beta}}_{\varepsilon}\vec{c} - \underbrace{\frac{\alpha}{\beta}}_{\varphi}\vec{a} = \vec{b}$. (cases where α or $\beta = 0$ are trivial)

So without gener. loss, $\underline{\vec{c} = \alpha \vec{a} + \beta \vec{b}}$.

Then $(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{a} \times \vec{b}) \cdot (\alpha \vec{a} + \beta \vec{b}) = \left[\vec{v} = \vec{a} \times \vec{b}\right] =$

$$= \vec{v} \cdot (\alpha \vec{a} + \beta \vec{b}) = \alpha(\vec{v}\,\vec{a}) + \beta(\vec{b}\,\vec{v}) = \alpha(\vec{a} \cdot \vec{v}) + \beta(\vec{b} \cdot \vec{v}) =$$

$$= \alpha(\underbrace{\vec{a} \cdot (\vec{a} \times \vec{b})) + \beta(\vec{b} \cdot (\vec{a} \times \vec{b})}_{\text{Jacobi cyclic permutations}}) = \alpha(\vec{b} \cdot \underbrace{(\vec{a} \times \vec{a})}_{\vec{0}}) + \beta(\vec{a} \cdot \underbrace{(\vec{b} \times \vec{b})}_{\vec{0}}) = \vec{0}.$$

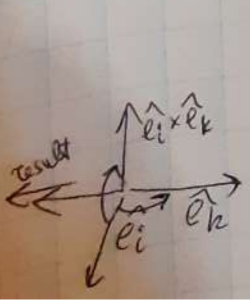
c) $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) =$

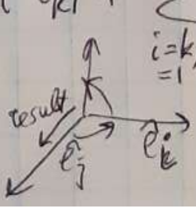
$$\vec{a} \times (\vec{b} \times \vec{c}) = \sum_{i=1}^{3} a_i \hat{e}_i \times \Big(\sum_{j=1}^{3} b_j \hat{e}_j \times \sum_{k=1}^{3} c_k \hat{e}_k\Big) = \sum_{i=1}^{3} a_i \hat{e}_i \times \Big(\sum_{j,k=1}^{3} b_j c_k \hat{e}_j \times \hat{e}_k\Big)$$

expanding everything to 9 values

$= \left(\begin{array}{l}\text{expanding to 27 combinations}\\ \text{using distributivity}\end{array}\right) = \sum_{i,j,k=1}^{3} a_i b_j c_k (\hat{e}_i \times (\hat{e}_j \times \hat{e}_k)) = \left[\begin{array}{l}\text{in cases } j = k \to 0\\ i \neq j \wedge i \neq k \to 0\\ \text{similar reasoning}\\ \text{as in textbook}\end{array}\right] =$

$$= \sum_{i=j,\,k=1}^{3} a_i b_i c_k(-\hat{e}_k) + \sum_{i=k,\,j=1}^{3} a_i c_i b_j(\hat{e}_j) = \underbrace{\sum_{i=1}^{3}(a_i b_i)}_{\vec{a}\cdot\vec{b}} \underbrace{\sum_{k=1}^{3} c_k(-\hat{e}_k)}_{-\vec{c}} +$$





$$\underbrace{\sum_{i=1}^{3}(a_i c_i)}_{\vec a\vec c}\ \underbrace{\sum_{j=1}^{3} b_j \hat e_j}_{\vec b} = (\vec a\vec c)\vec b + (\vec a\vec b)(-\vec c) = \vec b(\vec a\cdot\vec c) - \vec c(\vec a\cdot\vec b).$$

d) $\forall\ \vec a, \vec b, \vec c, \vec d \in \mathbb{R}^3$:

$$\underbrace{(\vec a\times\vec b)\cdot(\vec c\times\vec d)}_{\text{cyclic permut. of these three vectors}} = \vec c\cdot\underbrace{(\overset{\text{"a"}}{\vec d}\times(\overset{\text{"b"}}{\vec a}\times\overset{\text{"c"}}{\vec b}))}_{\text{bac-cab rule}} = \vec c\cdot(\vec a(\vec d\vec b) - \vec b(\vec d\vec a)) =$$

$$= (\vec c\cdot\vec a)(\vec d\cdot\vec b) - (\vec b\cdot\vec c)(\vec d\cdot\vec a) = (\vec a\cdot\vec c)(\vec b\cdot\vec d) - (\vec a\cdot\vec d)(\vec b\cdot\vec c).$$

Problem 6.2. Pauli Matrices — 4DVS of matrices over $\mathbb{R}$ field

$$\sigma_0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\quad \sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

form basis for 4D vector space of 2x2 matrices: $A = \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}$, where $a_{ij} \in \mathbb{C}$, $a_{ij} = a_{ji}^*$ — meaning $a_{11} = a_{11}^*$, $a_{22} = a_{22}^*$ are <u>real</u> elements.

a) Linear Independence.

$$\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} = \sum_i c_i\sigma_i = \begin{pmatrix}c_0 & 0\\ 0 & c_0\end{pmatrix} + \begin{pmatrix}0 & c_1\\ c_1 & 0\end{pmatrix} + \begin{pmatrix}0 & -c_2 i\\ c_2 i & 0\end{pmatrix} + \begin{pmatrix}c_3 & 0\\ 0 & -c_3\end{pmatrix} =$$

$$= \begin{pmatrix}c_0+c_3 & c_1-c_2 i\\ c_1+c_2 i & c_0-c_3\end{pmatrix},\ \text{solving}\ \underset{c_1=c_2=0}{\left\{\begin{array}{l} c_0+c_3=0\\ c_1-c_2 i=0\\ c_1+c_2 i=0\\ c_0-c_3=0\end{array}\right\}}_{c_0=0,\ c_3=0}\ \text{the only solutions are}$$

$$c_0 = c_1 = c_2 = c_3 = 0.$$

There is unique $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} \longrightarrow$ matrices are independent.

b) Show that all their combinations are matrices like A.

$$\sum_i x_i\sigma_i = \begin{pmatrix}x_0 & 0\\ 0 & x_0\end{pmatrix} + \begin{pmatrix}0 & x_1\\ x_1 & 0\end{pmatrix} + \begin{pmatrix}0 & -x_2 i\\ x_2 i & 0\end{pmatrix} + \begin{pmatrix}x_3 & 0\\ 0 & -x_3\end{pmatrix} = \begin{pmatrix}x_0+x_3 & x_1-x_2 i\\ x_1+x_2 i & x_0-x_3\end{pmatrix},$$

exactly of type of A ($x_0 \pm x_3$ are their own $*$, and $x_1 \pm x_2 i = (x_1 \mp x_2 i)^*$). $\longrightarrow\ \in \mathbb{H}$.

c) Show that all matrices in $\mathbb{H}$ are "reachable" by Pauli m.

General matrix in $\mathbb{H}$ is of type $\begin{pmatrix} a & c-di \\ c+di & b \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} +$

$+ x_1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \left[\begin{matrix} \text{need find} \\ x_i \end{matrix} \right].$

Take $x_0 = \frac{a+b}{2}$, $x_3 = \frac{a-b}{2}$, $x_1 = c$, $x_2 = d$, then indeed

$$\begin{pmatrix} a & c-di \\ c+di & b \end{pmatrix} = \frac{a+b}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + c\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + d\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \frac{a-b}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

indeed, each matrix in H can be reachable by Pauli matrices if taking coefficients like this. So the coordinate of such matrix ~~in~~ $\begin{pmatrix} a & c-di \\ c+di & b \end{pmatrix}$ in Pauli base will be $\begin{pmatrix} \frac{a+b}{2} \\ c \\ d \\ \frac{a-b}{2} \end{pmatrix}$.

d) $\sum_{i=0}^{3} z_i \sigma_i \in \mathbb{H}$? where $z_i \in \mathbb{C}$

No! Let me show it in generic way:

$* \; M = z_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + z_1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + z_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + z_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} z_0+z_3 & z_1-z_2 i \\ z_1+z_2 i & z_0-z_3 \end{pmatrix}$

Neither element pairs have to be conjugates.

Example: $z_1 = i$, $z_2 = 0$, $i \neq i^*$, $z_1 - z_2 i \neq (z_1+z_2 i)^*$.

No.

But they do form their own vector space (if defining $+$, $\cdot$ as usual):

* Show that $(V, +)$ is commutative group.

** Closure $\begin{pmatrix} z_0+z_3 & z_1-z_2 i \\ z_1+z_2 i & z_0-z_3 \end{pmatrix} + \begin{pmatrix} \tilde{z}_0+\tilde{z}_3 & \tilde{z}_1-\tilde{z}_2 i \\ \tilde{z}_1+\tilde{z}_2 i & \tilde{z}_0-\tilde{z}_3 \end{pmatrix} =$ just some complex number, not z^*

$= \begin{pmatrix} (z_0+\tilde{z}_0)+(z_3+\tilde{z}_3) & (z_1+\tilde{z}_1)-(z_2+\tilde{z}_2)i \\ (z_1+\tilde{z}_1)+(z_2+\tilde{z}_2)i & (z_0+\tilde{z}_0)-(z_3+\tilde{z}_3) \end{pmatrix} \in \mathbb{H}$

of same kind (see $(*)$)

** Neutral is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ which is trivially checked

** Inverse for $\begin{pmatrix} z_0+z_3 & z_1-z_2 i \\ z_1+z_2 i & z_0-z_3 \end{pmatrix}$ is $\begin{pmatrix} -z_0-z_3 & -z_1+z_2 i \\ -z_1-z_2 i & -z_0+z_3 \end{pmatrix} \in \mathbb{H}$

** Associativity/ Commutativity follow from the respective properties of general 2x2 matrices.

* Show $\alpha \cdot (\beta \cdot M) = (\alpha \cdot \beta) M$ where M is matrix of such type. This also follows from respective property of all 2x2 matrices.

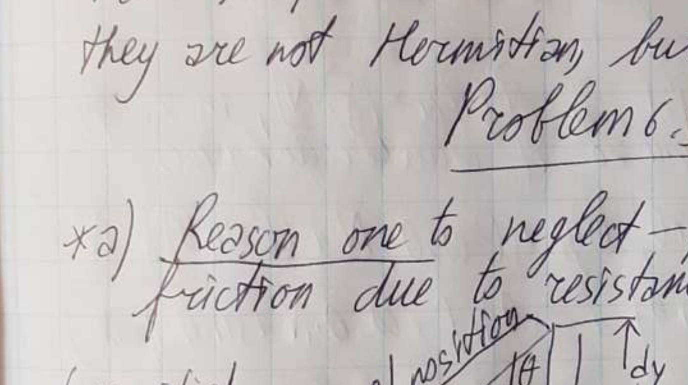
* $(\alpha+\beta)M = \alpha M + \beta M$ also directly results from 2x2 matrices.

* $\alpha(M_1+M_2) \stackrel{?}{=} \alpha M_1 + \alpha M_2$ also directly results from 2x2 matrices.

Hence, if we scale Pauli matrices by complex numbers, they are not Hermitian, but they form vector space.

Problem 6.3 Forces/Torques on ladder

*a) Reason one to neglect — purely geometrical + nature of friction due to resistance of irregularities of surface.



L of Ladder is parameter, constant.

$$L^2 = \left(\frac{H}{\cos\theta}\right)^2 = (dx + H\tan\theta)^2 + (H-dy)^2$$

$$\frac{H^2}{\cos^2\theta} = dx^2 + H^2\tan^2\theta + 2dx\,H\tan\theta + H^2 + dy^2 - 2Hdy$$

(since $\frac{1}{\cos^2\theta} = 1+\tan^2\theta$)

$$0 = dx^2 + 2dx\,H\tan\theta + dy^2 - 2Hdy$$

dx^2, dy^2 are second order $\to 0$,

$$0 = 2dx\,H\tan\theta - 2H\,dy \quad \Big| \cdot \frac{1}{2H}$$

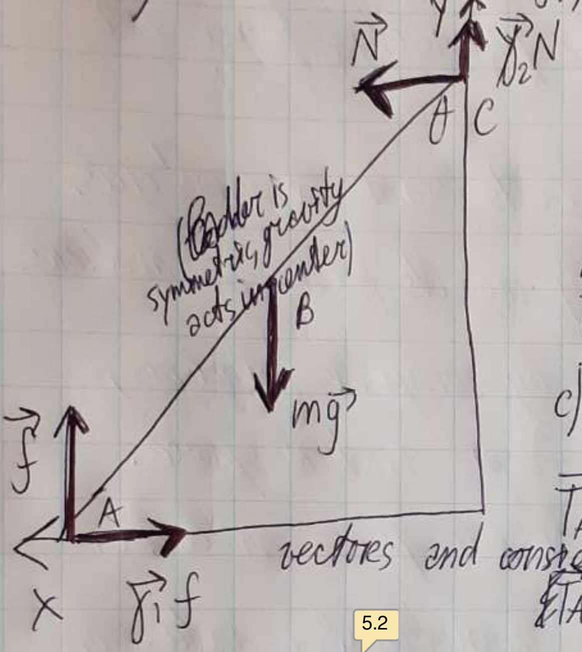
$$dx\tan\theta = dy$$

Since it is $\boxed{\text{tiny-tiny}}$ shift (which friction opposes), $d\theta \to 0$, $\tan\theta \to d\theta$, $dx\,d\theta = dy$, so dy is much-much smaller than dx, it does not cause such resistance to motion from wall irregularities as bottom part has from ground. It is negligible.

Reason 2 to neglect γ_2 is very small compared to γ_1 due to less irregularities in wall than in ground. Also (will be shown below) $N=\gamma_1 f$, so $\gamma_2 N = \underbrace{\gamma_2\gamma_1}_{\text{2 small values.}} f$. (~~But for this claim actual c~~

This effect can differ widely if being honest.

b) I consider γ_1, γ_2 also changing in some limits ($\gamma_1 \le \mu_s$



$$\left.\begin{array}{l}\text{Horizontal: } N=\gamma_1 f \\ \text{Vertical: } mg = f + \gamma_2 N\end{array}\right\}\text{ two many solutions}$$

Need to consider angles and torques.

c) Add torque in consideration:

$\vec{T}_A = \vec{r}_{AB} \times m\vec{g} + \vec{r}_{AC} \times \vec{N} + \vec{r}_{AC} \times (\vec{\gamma_2} N)$, in 2 dropping vectors and considering torque >0 if counterclockwise,

$$\Sigma T_A = -\left(\frac{l}{2}\cdot \sin\theta\right)\cdot mg + (l\cos\theta)N + (l \sin\theta)\gamma_2 N. \quad (*)$$

Other torques $\Sigma T_B = \frac{l}{2}\left(-\sin\theta f + \cos\theta \gamma_1 f + \cos\theta \cdot N + \sin\theta \cdot \gamma_2 N\right)$

and

$T_C = mg\frac{l}{2}\sin\theta - fl\sin\theta + \gamma_1 f\, l\cos\theta$ give the same values (this is also a generic rule)

d) Mass of the ladder is included in force balance, and in torque calculation — assumed uniform distribution, so center of mass is in the middle — and this ~~rule~~ used in torque calculation. It is not clear whether I understood the question...

e) Now use three equations: (there could be way to draw lines to intersection but still with γ_1, γ_2 varying there are too many parameters)

$$\begin{cases} N = \gamma_1 f \\ mg = f + \gamma_2 N \\ (*)\ 0 = N\cos\theta + \gamma_2 N \sin\theta - \frac{mg\sin\theta}{2}\end{cases}$$

Since many unknowns and due to a) drop $\gamma_2 = 0$.

Simplified system:

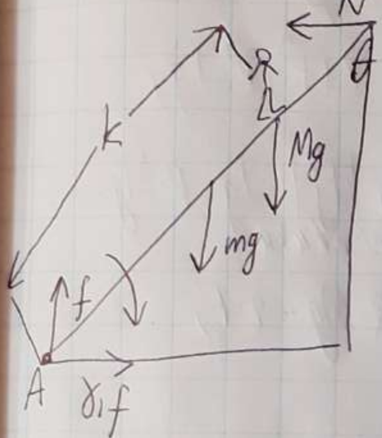
$$\begin{cases} \left.\begin{matrix} N = \gamma_1 f \\ mg = f \end{matrix}\right\} N = \gamma_1 mg \\ 0 = N\cos\theta - \frac{mg\sin\theta}{2} \end{cases} \Bigg\} \; 0 = \gamma_1 mg\cos\theta - \frac{mg\sin\theta}{2} \rightarrow \gamma_1\cos\theta = \frac{\sin\theta}{2}$$

$$2\gamma_1 = \tan\theta,$$

so $\theta = \operatorname{atan}(2\gamma_1)$.

When we consider critical value, $\theta = \operatorname{atan}(2\mu_s) \approx \operatorname{atan} 4 \approx 76°$.

* f) To analyze impact of walking man, consider simplified version ($\gamma_2 = 0$).



$k \in [0,1]$ Equations:

$$\begin{cases} (M+m)g = f \\ N = \gamma_1 f = \mu f \text{ (take critical case)} \\ -mg\sin\theta\frac{l}{2} - kl\,Mg\sin\theta + Nl\cos\theta = 0 \end{cases}$$

$$\mu(M+m)\cancel{g}\,\cancel{l}\cos\theta - m\cancel{g}\sin\theta\frac{\cancel{l}}{2} = k\cancel{l}M\cancel{g}\sin\theta$$

$$\mu M\cos\theta + \mu m\cos\theta - \frac{1}{2}m\sin\theta = kM\sin\theta$$

If k (relative position on ladder) increases, μ must also increase to maintain equilibrium ~~for~~.

Other way to say: $\mu M + \mu m = \frac{1}{2}m\tan\theta + kM\tan\theta$,

$$\tan\theta\left(\frac{m}{2} + kM\right) = \mu(M+m)$$

$$\theta = \operatorname{atan}\frac{\mu(M+m)}{\frac{m}{2}+kM} \rightarrow \operatorname{atan} 2\mu \text{ in old case when no man}$$

If $k\uparrow$, θ must become smaller as possible.

Suppose $m = M$, $\mu = 2$,

$$2m + 2m = \frac{m}{2}\tan\theta + km\tan\theta,$$

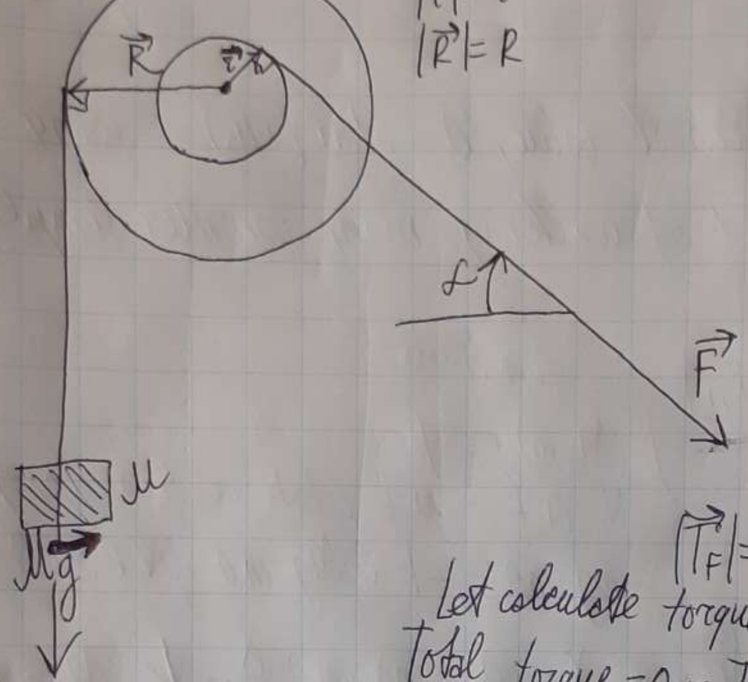
$$4 - \frac{\tan\theta}{2} = k\tan\theta$$

$$\frac{4}{\tan\theta} - \frac{1}{2} = k$$ – knowing the vertical angle how to put ladder, so that man can climb for the given k [0 to 1] without fall.

It is not just urban legend.

Problem 6.4. Differential pulley

a)



$|\vec r| = r$
$|\vec R| = R$

7.1

Calculating relative to axis.

b) Torque
$\vec{T_F} = \vec r \times \vec F$ by man pulling

Torque
$\vec{T_g} = \vec R \times (M\vec g)$ by gravity.

Since $\vec r \perp \vec F$, $\vec R \perp \vec g$,

$|\vec{T_F}| = T_F = rF$, $|\vec{T_g}| = T_g = R\cdot Mg$.

Let calculate torque counterclockwise.

Total torque $= 0 = T_F + T_g = R\cdot Mg - r\cdot F$,

$$\underline{F = \frac{R}{r} Mg.}$$

7.2

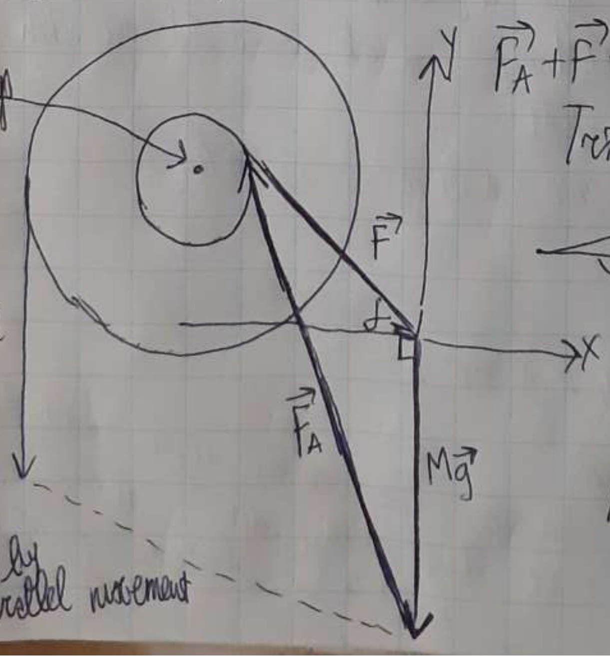
c) $F = 3\cdot 30kg\cdot 10\,\frac{m}{s^2} = 900\,N >$ my weight, no I wouldn't.
It also can create large pressure on skin etc.

7.3

d) 1) Geometric approach to find $\vec{F_A}$.

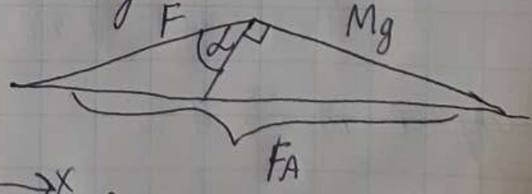
It's badly written, it should go through axle

$\vec{F_A}$ is applied there since axle resists,
triangle is built by parallel movement



$\vec{F_A} + \vec F + M\vec g = \vec 0$.

Triangle



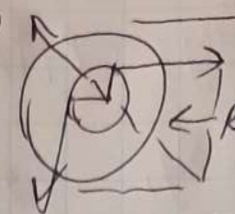
Cosine law:

$F_A^2 = F^2 + (Mg)^2 - 2F\cdot Mg\cos(\frac{\pi}{2}+\alpha)$
$= F^2 + (Mg)^2 + 2F\cdot Mg\sin\alpha \rightarrow$

$$\boxed{F_A = \sqrt{F^2 + (Mg)^2 + 2FMg\sin\alpha}}$$

Test special cases

— $\sin\alpha = 0 \longrightarrow F_A = \sqrt{F^2 + (Mg)^2}$



— $\sin\alpha = \frac{\pi}{2} \longrightarrow F_A = \sqrt{F^2 + (Mg)^2 + 2F \cdot Mg} = \underline{F + Mg}$

$\vec{F_A}$ is applied at axle ~~and~~ and does not create torque.

2) Algebraic approach to find $\vec{F_A}$.

$$\vec{F} = \begin{pmatrix} F\cos\alpha \\ -F\sin\alpha \end{pmatrix} \quad M\vec{g} = \begin{pmatrix} 0 \\ -Mg \end{pmatrix} \quad \vec{F_A} = -(\vec{F} + M\vec{g}) = \begin{pmatrix} -F\cos\alpha \\ F\sin\alpha + Mg \end{pmatrix},$$

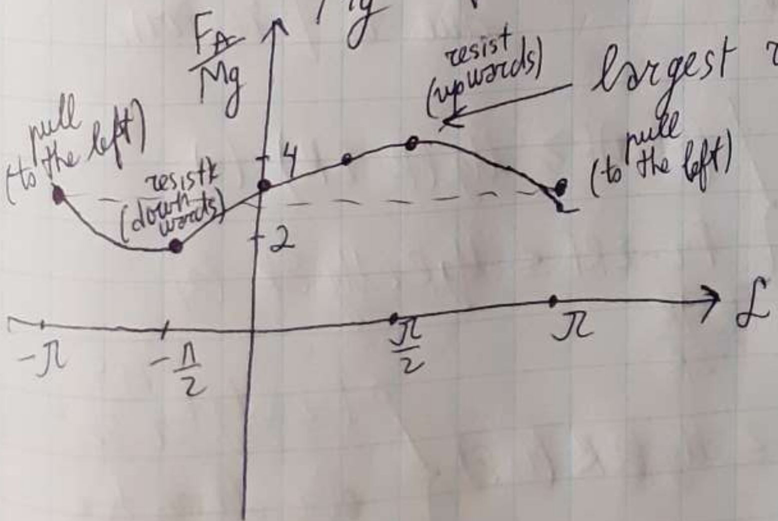
then direction is $\pi + \operatorname{atan}\left(\frac{-F\cos\alpha}{F\sin\alpha + Mg}\right)$ ↖,

and value is $\sqrt{(F\cos\alpha)^2 + (F\sin\alpha + Mg)^2} = \underline{\sqrt{F^2 + (Mg)^2 + 2MgF\sin\alpha}}$ which matches geometric way.

Plotting $F_A(\alpha) = \sqrt{F^2 + (Mg)^2 + 2FMg\sin\alpha}$, $\frac{R}{r} = 3 \rightarrow F = Mg\frac{R}{r} = 3Mg$,

$$|\vec{F_A}|(\alpha) = F_A(\alpha) = \sqrt{10(Mg)^2 + 6(Mg)^2\sin\alpha} = Mg\sqrt{10 + 6\sin\alpha},$$

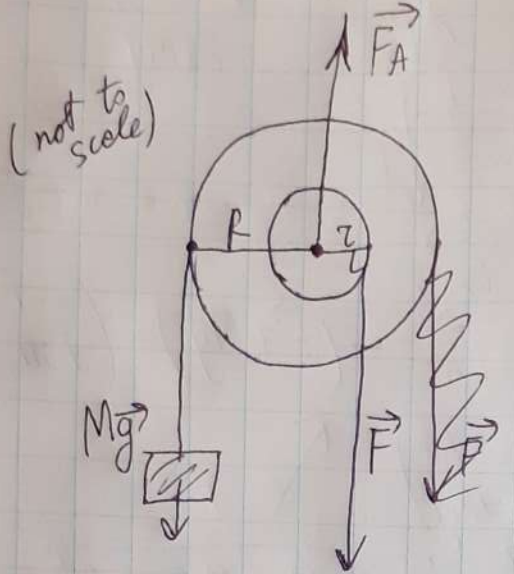
$$\frac{F_A(\alpha)}{Mg} = \sqrt{10 + 6\sin\alpha}$$ — note it is possible to use any angle.



$T = 2\pi$

*e) When not pulling, $\varepsilon R|\vec{F_A}| = MgR$ (compensate gravity torque)

$F_A = \sqrt{(Mg)^2+0} = Mg$, $\quad \varepsilon R Mg = MgR$, $\underline{\varepsilon \geq 1}$



If $\alpha = \frac{\pi}{2}$,
The net torque is $|MgR - rF|$, if it is not enough large — roll does not rotate in any direction.

Condition:

$$|MgR - rF| \le \varepsilon R|\vec{F_A}|,$$

Where $F_A = \sqrt{(Mg)^2 + F^2 + 2MgF\underbrace{\sin\alpha}_{1}} = \sqrt{(Mg+F)^2} = Mg + F.$

Therefore

$$|MgR - Fr| \le \varepsilon R(Mg+F).$$

or

$$-\varepsilon R(F+Mg) \overset{(2)}{\le} MgR - rF \overset{(1)}{\le} \varepsilon R(Mg+F) \rightarrow$$

F lies in solution range, and roll does not rotate due to friction.

$$\begin{cases} \cancel{-rF + RMg \le \varepsilon R(Mg+F)} \\ \cancel{-r\ldots} \end{cases}$$

$$\begin{cases} (\varepsilon R + r)F \geq Mg(R - \varepsilon R) & (1) \\ Fr - \varepsilon RF \le MgR + Mg\,\varepsilon R & (2) \end{cases}$$

$$\begin{cases} F \geq Mg\dfrac{(1-\varepsilon)R}{r+\varepsilon R} \\ F(r - \varepsilon R) \le Mg R(1+\varepsilon) \end{cases} \begin{cases} F \le MgR\dfrac{1+\varepsilon}{r-\varepsilon R} \text{ if } r > \varepsilon R \\ \text{else always true (any force fails)} \end{cases}$$

<u>Case 1</u> $F \in \left[Mg\dfrac{(1-\varepsilon)R}{r+\varepsilon R};\ Mg\dfrac{R(1+\varepsilon)}{r-\varepsilon R}\right]$ if $r > \varepsilon R$ $\rightarrow$ if the force

is less, it rolls counterclockwise due to gravity, if more — clockwise due to this force, otherwise stuck by friction.

Case 2 if $r \le \varepsilon R$, $F \in \left[\frac{(1-\varepsilon)R}{r+\varepsilon R}, +\infty\right)$ - so if the force is less (e.g. absent) it can roll counterclockwise due to gravity, otherwise however strong pull is, due to small lever and large friction, roll does not rotate.

$\boxed{\text{If } \varepsilon \to 1, \text{ case 1 is not working, } F \in [0, +\infty).}$ So no force can roll this. Intuitively: whatever force is applied and even if on distance R, „friction compensation" is $\varepsilon R |\vec{F_A}| = R(F+Mg)$, that is full compensation.

Bonus problems: to be covered with the professor, or later.

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