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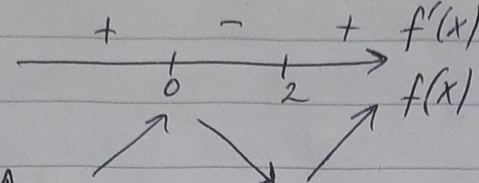
HW- MA-1

$$f(x)=(x+1)(x-2)^2$$

1. $f'(x)=(x-2)^2+2(x-2)(x+1)=(x-2)[x-2+2x+2]=3x(x-2)=3x^2-6x$



$x\in(-\infty,0)$ $f(x)\uparrow$

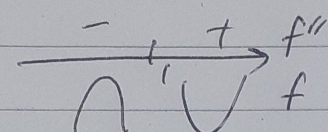
$f(0)=4$ — local max

$x\in(0,2)$ $f(x)\downarrow$

$f(2)=0$ — local min

$x\in(2,+\infty)$ $f(x)\uparrow$

2. $f''(x)=6x-6=6(x-1)$

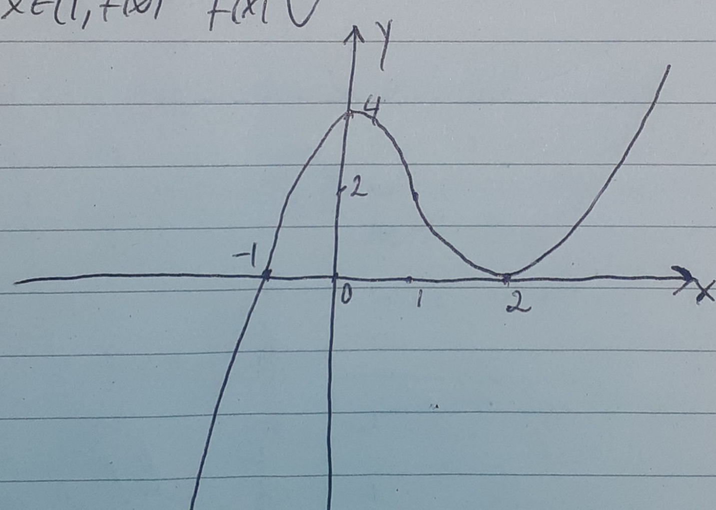


$x\in(-\infty,1)$ $f(x)\cap$

$x=1$, $f(1)=2$, inflection

$x\in(1,+\infty)$ $f(x)\cup$

3.

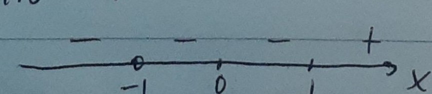


$$f(x)=\frac{x^2(x-1)}{(x+1)^2}$$

4. $D(f)$: $x\in(-\infty,-1)\cup(-1,+\infty)$

$$\lim_{x\to-1-0} f(x)=\lim_{x\to-1-0}\frac{x^2(x-1)}{(x+1)^2}=-\infty$$
$$\lim_{x\to-1+0} f(x)=\lim_{x\to-1+0}\frac{x^2(x-1)}{(x+1)^2}=-\infty$$
$\Rightarrow x=-1$ vertical asymptote

5. $f(x)=0$, $x\in\{0,1\}$



$x\in(-\infty,-1)\cup(-1,0)\cup(0,1)$ — $f(x)<0$

$f(-1)$ Not defined, $f(0)=f(1)=0$

$x\in(1,+\infty)$ — $f(x)>0$

6. $$f'(x)=\left(\frac{x^3-x^2}{(x+1)^2}\right)'=\frac{(3x^2-2x)(x+1)^2-2(x+1)(x^3-x^2)}{(x+1)^4}=\frac{(3x^2-2x)(x+1)-2(x^3-x^2)}{(x+1)^3}=$$

$$=\frac{\cancel{x^3}+3x^2-\cancel{2x^2}-2x-\cancel{2x^3}+\cancel{2x^2}}{(x+1)^3}=\frac{x^3+3x^2-2x}{(x+1)^3}=\frac{x(x^2+3x-2)}{(x+1)^3}.$$

$f'(x)=0$, $x=0$ or $x^2+3x-2=0$

$D=9+8=17$, $x_{1,2}=\frac{-3\pm\sqrt{17}}{2}$

$+\quad-\quad+\quad-\quad+$ $\;f'$

$\frac{-3-\sqrt{17}}{2}\quad -1\quad 0\quad \frac{-3+\sqrt{17}}{2}$ $\;f$

$\nearrow\;\searrow\;\nearrow\;\searrow\;\nearrow$

$x\in\left(-\infty,\frac{-3-\sqrt{17}}{2}\right)$ $f(x)\uparrow$

$x=\frac{-3-\sqrt{17}}{2}$ – local max

$x\in\left(\frac{-3-\sqrt{17}}{2},-1\right)$ $f(x)\downarrow$

$x\in(-1,0)$ $f(x)\uparrow$

$x=0$ – local max

$x\in\left(0,-\frac{3+\sqrt{17}}{2}\right)$ $f(x)\downarrow$

$x=\frac{-3+\sqrt{17}}{2}$ – local min

$x\in\left(\frac{-3+\sqrt{17}}{2},+\infty\right)$ $f(x)\uparrow$

7. $$f''(x)=\frac{(3x^2+6x-2)(x+1)^3-(x^3+3x^2-2x)\cdot 3(x+1)^2}{(x+1)^6}=\frac{(3x^2+6x-2)(x+1)-3(x^3+3x^2-2x)}{(x+1)^4}=$$

$$=\frac{\cancel{3x^3}+\cancel{3x^2}+\cancel{6x^2}+6x-2x-2-\cancel{3x^3}-\cancel{9x^2}+6x}{(x+1)^4}=\frac{10x-2}{(x+1)^4}$$

$-\quad-\quad+$ $\;f''$

$-1\quad\frac{1}{5}$

$\cap\quad\cap\quad\cup$ $\;f$

$f(x)\cap$ on $x\in(-\infty,-1)\cup\left(-1,\frac{1}{5}\right)$

$f(x)\cup$ on $x\in\left(\frac{1}{5},+\infty\right)$

8. $$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\frac{x^3-x^2}{x^2+2x+1}=\underline{+\infty}=\lim_{-\infty}\frac{x^3-x^2}{x^2+2x+1}=\lim_{-\infty}f(x)\Longrightarrow$$

no horizontal asymptotes.

$$\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{x^3-x^2}{(x^2+2x+1)x}=\lim_{x\to+\infty}\frac{x^3-x^2}{x^3+2x^2+x}=1,$$

$$\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{x^3-x^2}{(x^3+2x^2+x)}=1$$

$\Longrightarrow k=1$

$$\lim_{x\to+\infty}[f(x)-kx]=\lim_{x\to+\infty}\frac{\cancel{x^3}-x^2-\cancel{x^3}-2x^2-x}{x^2+2x+1}=\lim_{x\to+\infty}\frac{-3x^2-x}{x^2+2x+1}=-3$$

$$\lim_{x\to-\infty}[f(x)-kx]=\lim_{x\to-\infty}\frac{-3x^2-x}{x^2+2x+1}=-3$$

$\Longrightarrow b=-3$

$y=x-3$ – oblique asymptote as $x\to\pm\infty$.

9. Important points: $-1, 0, 2, 1, \frac{-3-\sqrt{17}}{2}, \frac{-3+\sqrt{17}}{2}, \frac{1}{5}$ (ND $\equiv$ not defined)

Values of x	$(-\infty, \frac{-3-\sqrt{17}}{2})$	$\frac{-3-\sqrt{17}}{2}$	$(\frac{-3-\sqrt{17}}{2}, -1)$	-1	$(-1,0)$	0	$(0,\frac{1}{5})$	$\frac{1}{5}$	$(\frac{1}{5}, \frac{-3+\sqrt{17}}{2})$	$\frac{-3+\sqrt{17}}{2}$	$(\frac{-3+\sqrt{17}}{2}, 1)$	1	$(1,2)$	2	$(2,+\infty)$
Values of $f(x)$	$-$	≈ -9	$-$	ND	$-$	0	$-$	$\approx -\frac{1}{45}$	$-$	$\approx -\frac{1}{18}$	$-$	0	$+$	$\approx \frac{4}{9}$	$+$
Sign $f'(x)$	$+$	0	$-$	ND	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
Monotonicity f	↑	max	↓	ND	↑	max	↓	↓	↓	MIN	↑	↑	↑	↑	↑
Sign of f''	$-$	$-$	$-$	ND	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$
Convexity f	$\frown$	$\frown$	$\frown$	ND	$\frown$	$\frown$	$\frown$	Inf. point	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$

Also $y = x-3$ is oblique asymptote.

10. Graph of $f(x)$ (not scaled)