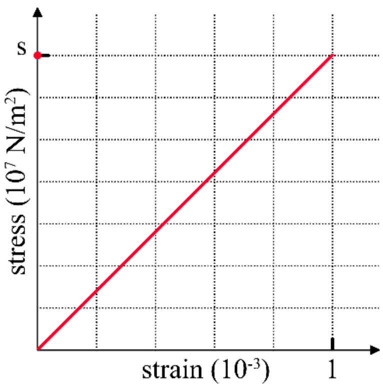
Problem 1: Stress and Strain **(X Points)**

The figure shows the stress versus strain plot for a wire that is stretched by a machine pulling in opposite directions at the two ends of the wire. The scale of the stress axis is set by $s = 7$, in units of $10^7\ \mathrm{N/m^2}$. The wire has an initial length of $l = 0.3\ \mathrm{m}$ and an initial cross-sectional area of $A = 6 \cdot 10^{-6}\mathrm{m}^2$.



a. Calculate Young's modulus.
b. How much work does the force from the machine do on the wire to produce a strain of $1 \cdot 10^{-3} = 0.1\ \%$?

Solution

(a)

Starting with the definition of stress-strain relation for purely elastic materials,

$$\gamma = \frac{1}{E}\,\sigma\,,$$

one can immediately calculate the Young's modulus for the wire

$$E = \frac{\sigma}{\gamma} = \frac{7 \cdot 10^7\ \mathrm{N/m^2}}{1 \cdot 10^{-3}} = 70\ \mathrm{GPa}\,,$$

which is a reasonable size for a metal.

(b)

Starting with the definition of the work done,

$$W = \int \vec{F} \cdot \mathrm{d}\vec{s}\,,$$

one can either calculate the forces and elongation first and then the work, or one can substitute the units in the integral:

$$\begin{aligned} W &= \int \vec{F} \cdot \mathrm{d}\vec{s} \overset{\vec{F}||\vec{s}}{=} \int F\mathrm{d}s = \int F\frac{A}{A}\frac{s_0}{s_0}\mathrm{d}s = \\ &= \int \sigma V_0 \frac{\mathrm{d}s}{s_0} = V_0 \int \sigma \mathrm{d}\gamma = V_0 E \int \gamma \mathrm{d}\gamma = \frac{1}{2} V_0 E \gamma^2 = \\ &= \frac{1}{2} \cdot (6 \cdot 10^{-6} \cdot 3 \cdot 10^{-1}) \cdot (7 \cdot 10^{10}) \cdot (1 \cdot 10^{-3})^2 = \\ &= 63 \cdot 10^{-3}\ \mathrm{J} = 63\ \mathrm{mJ}\,. \end{aligned}$$

Problem 2: Elasticity of Wire $\mathbf{1+1+1+2}$ **Points**

A wire has an original length of $l = 12\ \mathrm{m}$. It is firmly clamped at one end and then set under tension along its length with a force of $205\ \mathrm{N}$, whereby experiencing a length change of $4.3\ \mathrm{mm}$. The wire has a Young's modulus of $1.98 \cdot 10^{11}\ \mathrm{Pa}$ and a shear modulus of $7.4 \cdot 10^{10}\ \mathrm{Pa}$.

a. Determine the original diameter of the wire.
b. What is the Poisson Ratio of this material?
c. How much does the diameter of the wire decrease?
d. By how much does the density of the wire change?

Solution

(a)
According to Hooks law,

$$F = E \cdot A \cdot \frac{\Delta L}{L}$$

In this case, $A = \frac{\pi d^2}{4}$, so (1P):

$$d = \sqrt{\frac{4F}{E\pi}\frac{L}{\Delta L}} = \sqrt{\frac{4 \cdot 205\ \mathrm{N}}{1.98 \cdot 10^{11}\ \mathrm{Pa} \cdot \pi} \cdot \frac{12\ \mathrm{m}}{0.0043\ \mathrm{m}}} = 1.92\ \mathrm{mm}$$

(b)
The Poisson ratio can be calculated by:

$$\frac{E}{2G} = 1 + \mu$$

$$\mu = \frac{E}{2G} - 1 = \frac{1.98 \cdot 10^{11}\ \mathrm{Pa}}{2 \cdot 7.4 \cdot 10^{10}\ \mathrm{Pa}} - 1$$
$$= 0.338\ (1\mathrm{P})$$

(c)
Using the definition of Poisson Ratio:

$$\mu = -\frac{\Delta d}{d} / \frac{\Delta L}{L}$$

$$\Delta d = -\mu \frac{\Delta L}{L} d = -0.33784 \cdot \frac{0.0043}{12} \cdot 1.92\ \mathrm{mm}$$
$$= -2.32 \cdot 10^{-4}\ \mathrm{mm} = -0.232\ \mu\mathrm{m}\ (1P)$$

(d)
Since the mass of the wire is constant during the change, we got

$$\Delta\rho = \frac{M}{V} - \frac{M}{V - \Delta V} = -\frac{M}{V^2}\Delta V = -\rho\frac{\Delta V}{V}$$

This yields (1P):

$$\frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V}$$

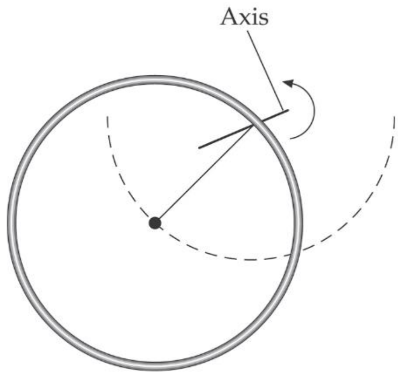
With $V = \frac{\pi}{4} d^2 L$

$$\frac{\Delta V}{V} = \frac{\left[\frac{\pi}{4}(d+\Delta d)^2(L+\Delta L) - \frac{\pi}{4} d^2 L\right]}{\frac{\pi}{4} d^2 L} = 2\frac{\Delta d}{d} + \frac{\Delta L}{L} = (1-2\mu)\frac{\Delta \mathrm{L}}{L}$$

$$\frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V} = (2\mu - 1)\frac{\Delta \mathrm{L}}{L} = (2 * 0.33784 - 1) \cdot \frac{0.0043}{12}$$
$$= -1.16 \cdot 10^{-4}\ (1P)$$

Angular speed of a pivoted ring **(3 + 3 Points)**

A uniform 1,5-m-diameter ring is pivoted at a point on its perimeter so that it is free to rotate about a horizontal axis that is perpendicular to the plane of the ring. The ring is released with the center of the ring at the same height as the axis (see figure, right).



(a) If the ring was released from rest, what was its maximum angular speed?

(b) What minimum angular speed must it be given at release if it is to rotate a full 360° ?

69 •• A uniform 1.5-m-diameter ring is pivoted at a point on its perimeter so that it is free to rotate about a horizontal axis that is perpendicular to the plane of the ring. The ring is released with the center of the ring at the same height as the axis (Figure 9-54). (*a*) If the ring was released from rest, what was its maximum angular speed? (*b*) What minimum angular speed must it be given at release if it is to rotate a full 360°?

Picture the Problem Let the zero of gravitational potential energy be at the center of mass of the ring when it is directly below the point of support. We'll use conservation of energy to relate the maximum angular speed and the initial angular speed required for a complete revolution to the changes in the potential energy of the ring.

(*a*) Use conservation of energy to relate the initial potential energy of the ring to its rotational kinetic energy when its center of mass is directly below the point of support:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2} I_P \omega_{max}^2 - mg\Delta h = 0 \qquad (1)$$

Use the parallel axis theorem and Table 9-1 to express the moment of inertia of the ring with respect to its pivot point P:

$$I_P = I_{cm} + mR^2$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_{max}^2 - mgR = 0$$

Solving for ω_{max} yields:

$$\omega_{max} = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_{max}:

$$\omega_{max} = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.6\,\text{rad/s}}$$

(*b*) Use conservation of energy to relate the final potential energy of the ring to its initial rotational kinetic energy:

$$\Delta K + \Delta U = 0$$

or, because $U_i = K_f = 0$,

$$-\tfrac{1}{2} I_P \omega_i^2 + mg\Delta h = 0$$

Noting that the center of mass must rise a distance R if the ring is to make a complete revolution, substitute for I_P and Δh to obtain:

$$-\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_i^2 + mgR = 0$$

Solving for ω_i yields:

$$\omega_i = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_i:

$$\omega_i = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.6\,\text{rad/s}}$$