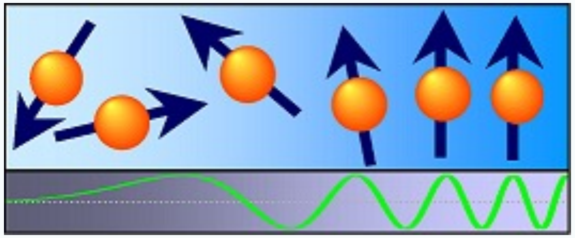
Experimental Physics
EP1 MECHANICS

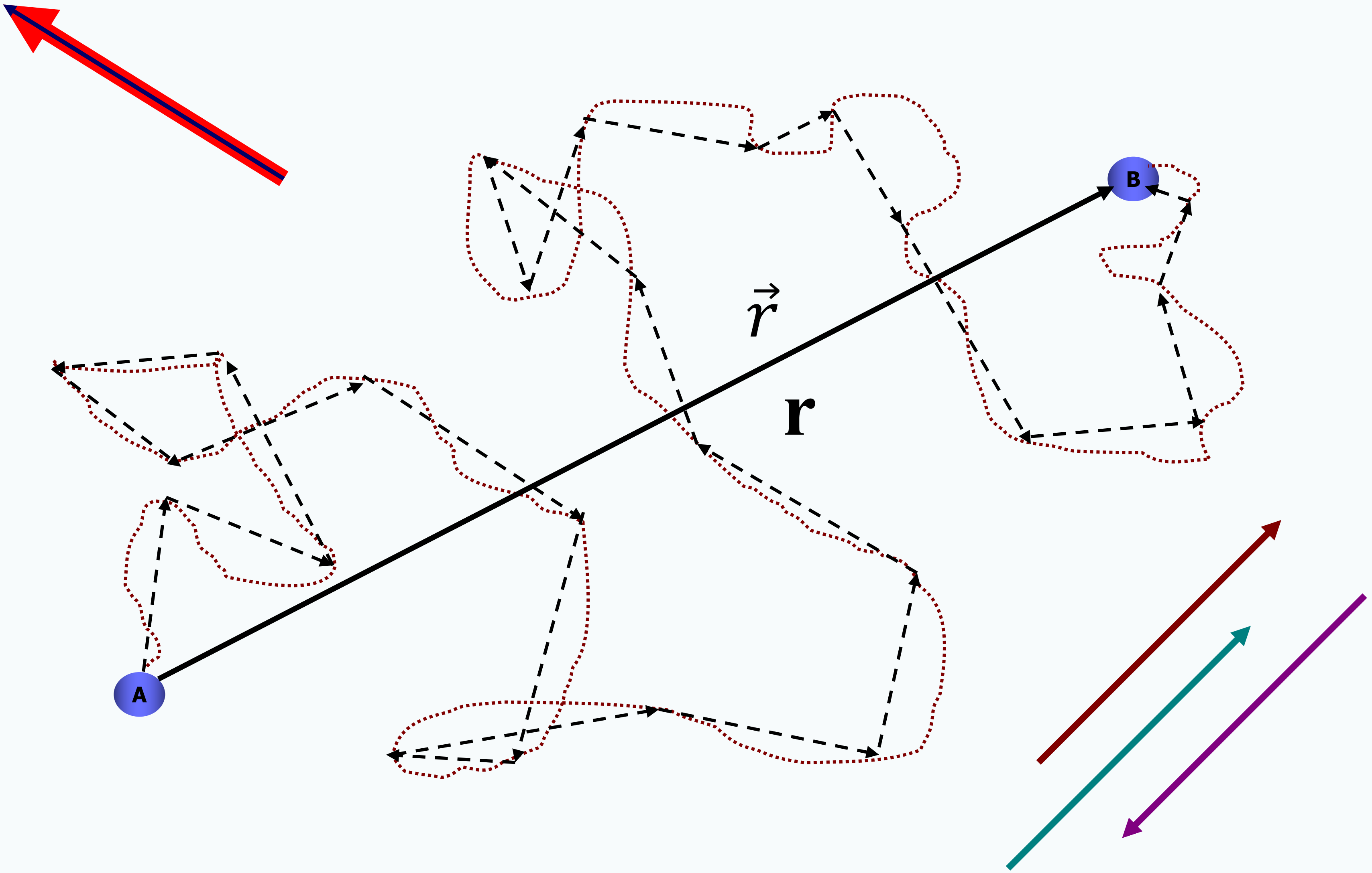
- Vectors and Scalars -



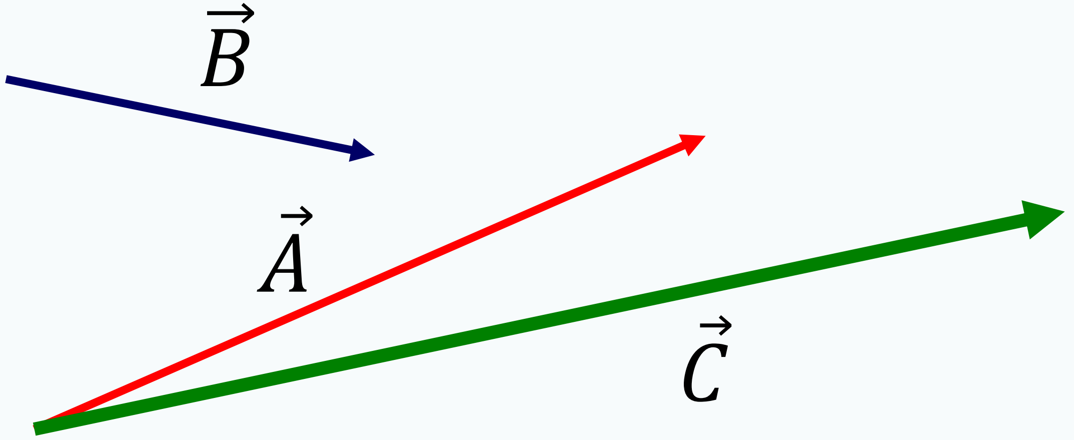
Rustem Valiullin

https://www.physgeo.uni-leipzig.de/en/fbi/applied-magnetic-resonance

Displacement vector



Adding vectors



$$\vec{C} = \vec{A} + \vec{B}$$

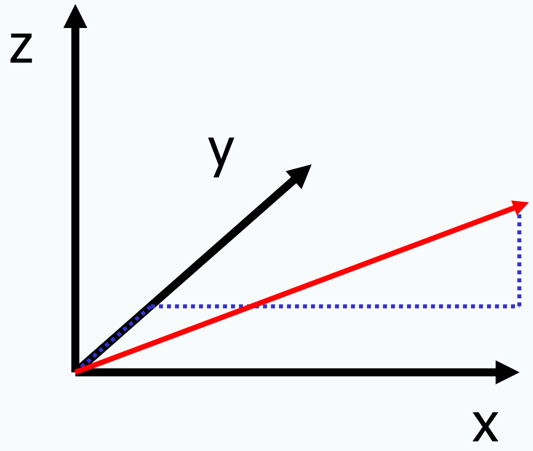
Commutative law: $$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Associative law: $$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$

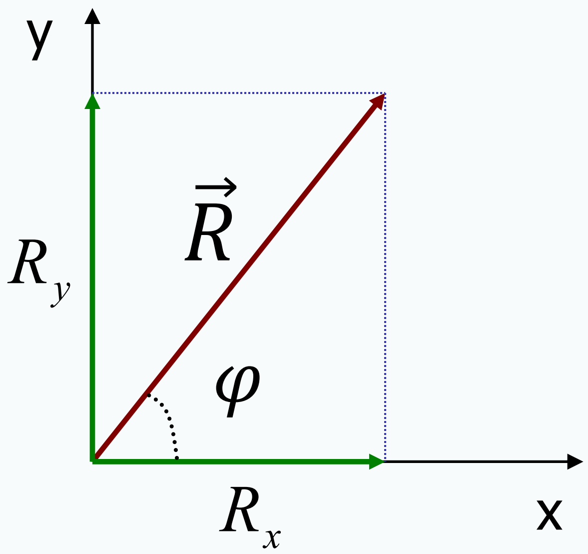
Vector subtraction: $$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

Cartesian coordinate system

- A Cartesian coordinate system consists of three mutually perpendicular axes, the x-, y-, and z-axes.
- By convention, the orientation of these axes is such that when the index finger , the middle finger, and the thumb of the right-hand are configured so as to be mutually perpendicular.
- The index finger , the middle finger , and the thumb now give the alignments of the x-, y-, and z-axes, respectively.
- This is a so-called right-handed coordinate system.



Vector components (2D)

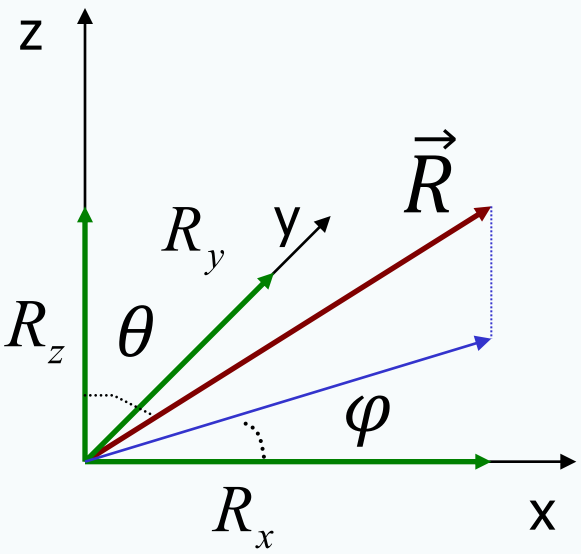


φ – azimuthal angle

$$\begin{cases} R_x = R\cos(\varphi) \\ R_y = R\sin(\varphi) \end{cases}$$

$$\begin{cases} R = \sqrt{R_x^2 + R_y^2} \\ \tan(\varphi) = \dfrac{\sin(\varphi)}{\cos(\varphi)} = \dfrac{R_y}{R_y} \end{cases}$$

Vector components (3D)



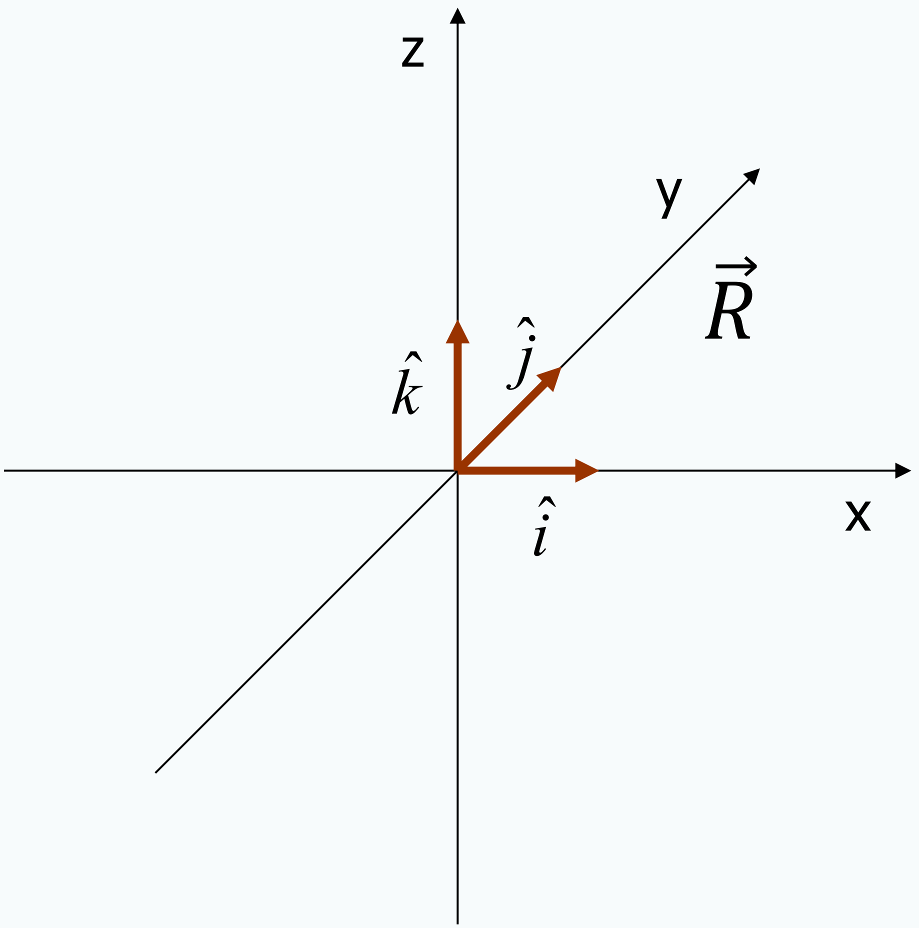
***φ* – azimuthal angle**

***θ* – polar angle**

$$\begin{cases} R_x = R\sin(\theta)\cos(\varphi) \\ R_y = R\sin(\theta)\sin(\varphi) \\ R_z = R\cos(\theta) \end{cases}$$

$$\begin{cases} R = \sqrt{R_x^2 + R_y^2 + R_z^2} \\ \tan(\varphi) = \dfrac{\sin(\varphi)}{\cos(\varphi)} = \dfrac{R_y}{R_y} \\ \cos(\theta) = \dfrac{R_z}{R} \end{cases}$$

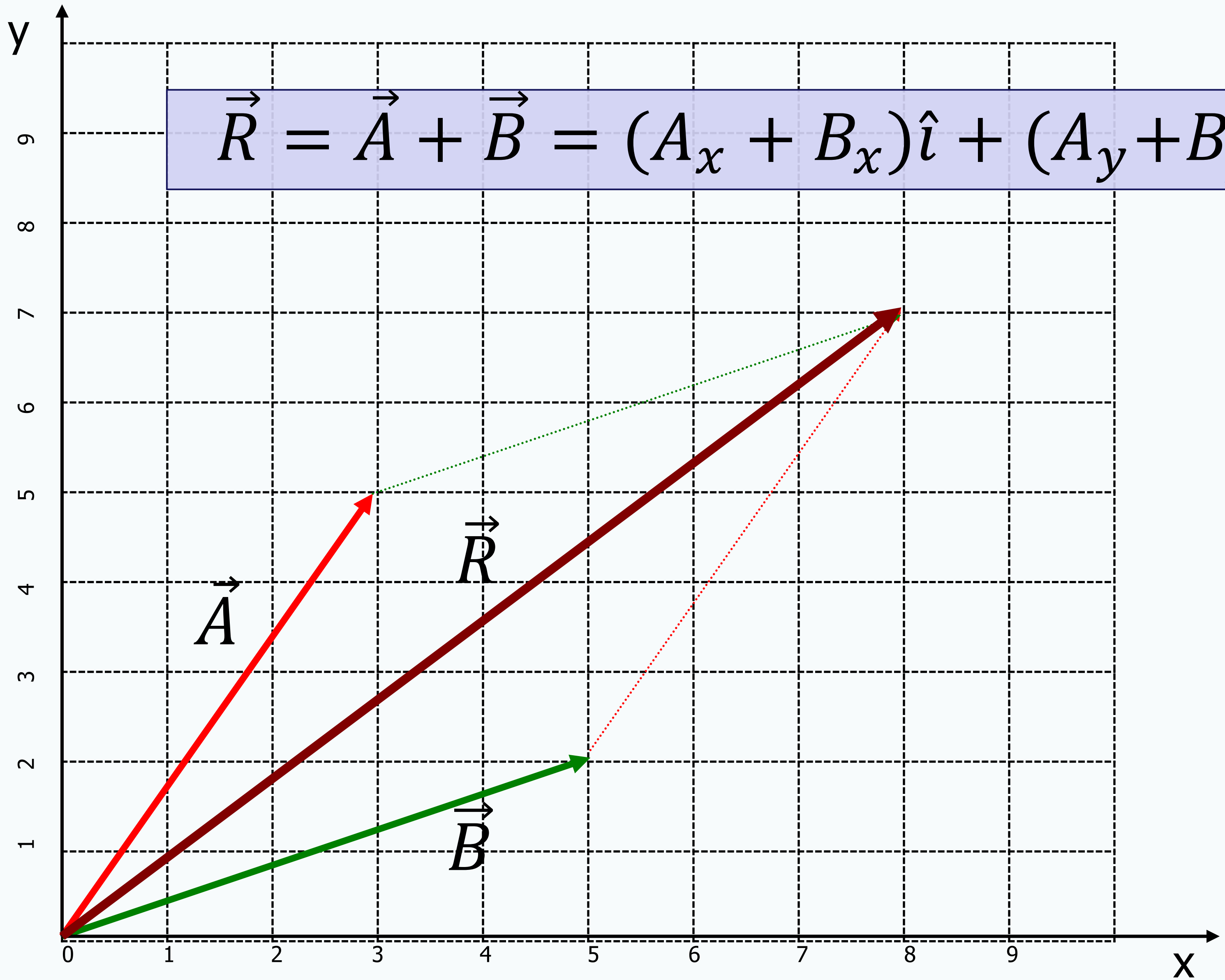
Unit vectors



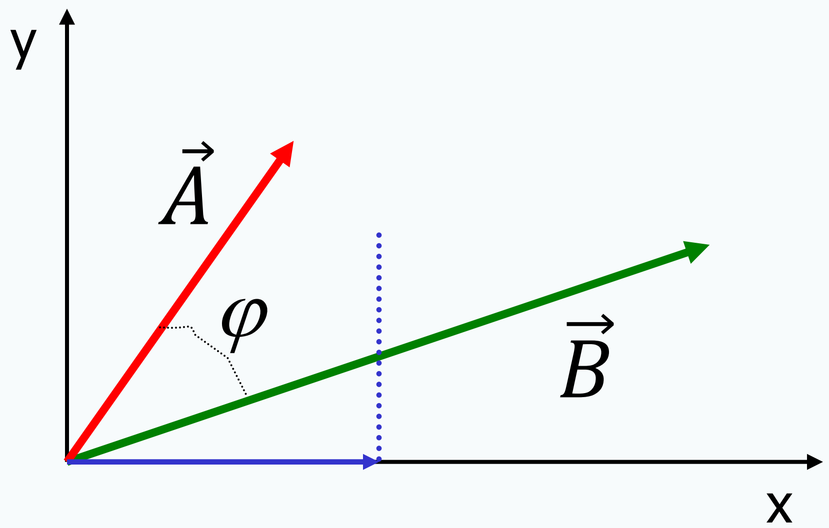
$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$

Adding vectors by components

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$



The scalar product



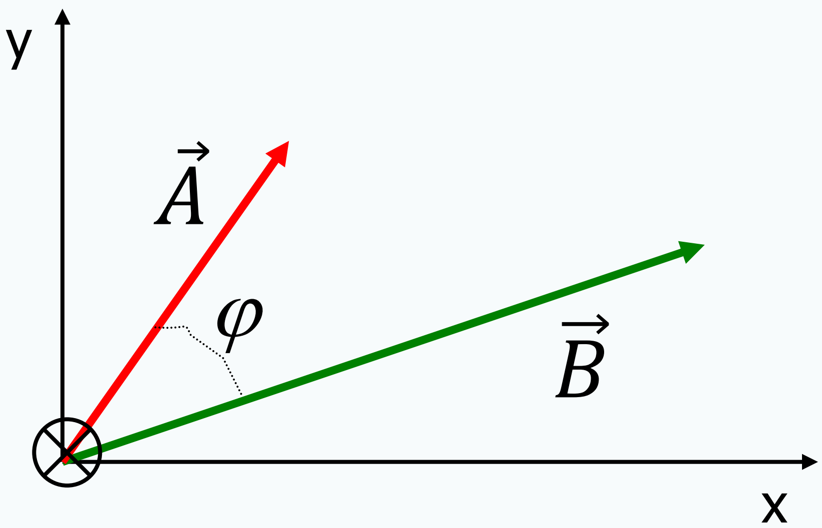
$$\vec{A} \cdot \vec{B} = ABcos(\varphi)$$

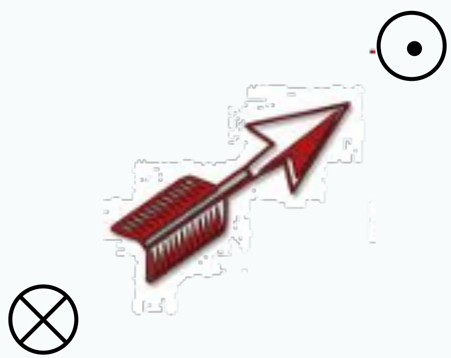
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

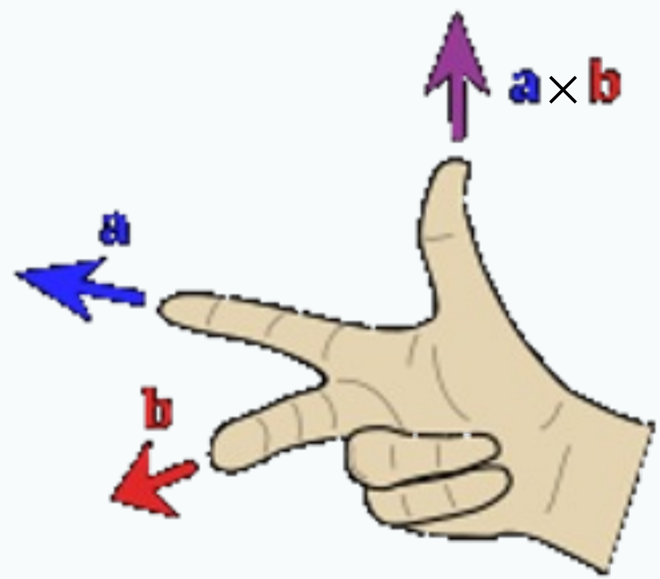
The vector product

$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = AB\sin(\varphi)$$

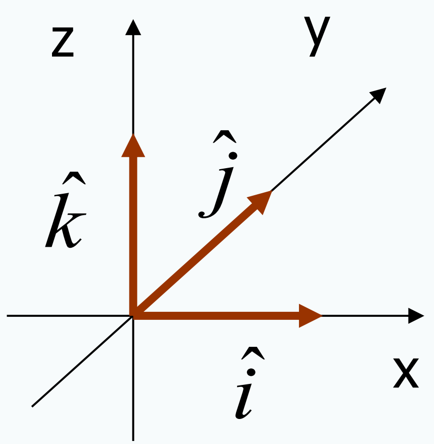


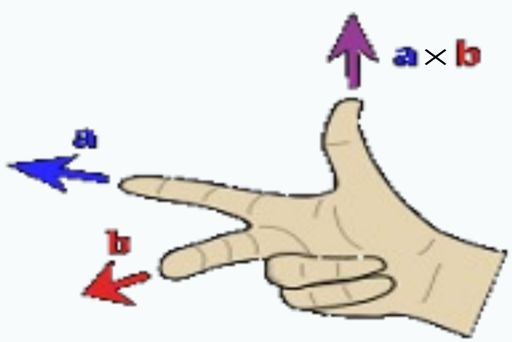




Some properties of vector product

$$\begin{cases} \hat{i} \times \hat{i} = 0 \\ \hat{j} \times \hat{j} = 0 \\ \hat{k} \times \hat{k} = 0 \end{cases} \qquad \begin{cases} \hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{i} = \hat{j} \end{cases} \qquad \begin{cases} \hat{j} \times \hat{i} = -\hat{k} \\ \hat{k} \times \hat{j} = -\hat{i} \\ \hat{i} \times \hat{k} = -\hat{j} \end{cases}$$



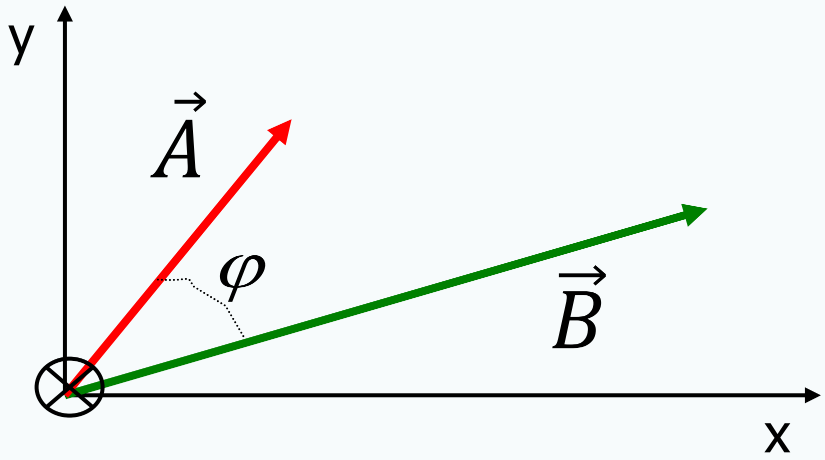


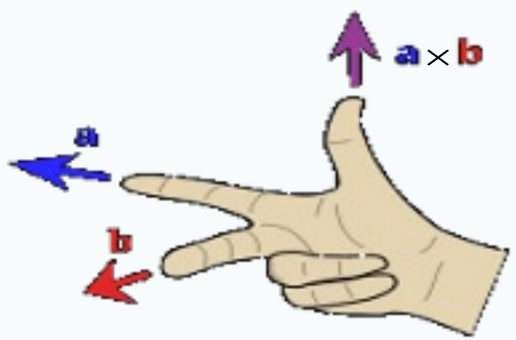
Some properties of vector product

Anticommutative: $$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

Distributive over addition: $$\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$$

$$\begin{aligned}\vec{A}\times\vec{B} = {} & \left(A_yB_z - A_zB_y\right)\hat{\imath} + \\ & \left(A_zB_x - A_xB_z\right)\hat{\jmath} + \\ & \left(A_xB_y - A_yB_x\right)\hat{k}\end{aligned}$$





To remember!

- **There are <u>scalar</u> and <u>vector</u> quantities.**
- **Vectors can be added <u>geometrically,</u> but it is more convenient to do it in a <u>component form</u>.**
- **The <u>scalar components</u> of a vector are its projections onto the axes of a Cartesian coordinate system.**
- **<u>Unit vectors</u> are dimensionless, they are pointing along axes of a right-handed coordinate system.**
- **Two different types of vector products: the <u>scalar</u> (dot) and <u>vector</u> (cross) products.**