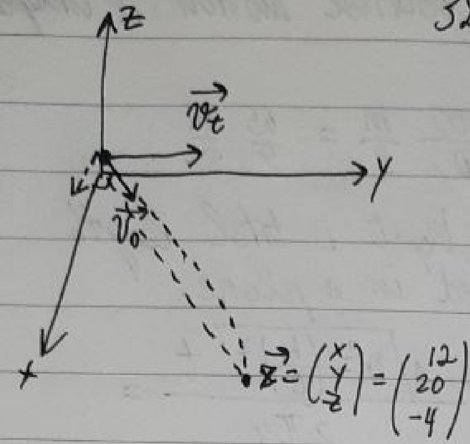
Stanislav 3720433 Exercise sheet 3

3D motion

1.




EOM for bottle:

$$\vec{r}(t) = \begin{pmatrix} v_{0x} \cdot t \\ v_t \cdot t \\ -g\frac{t^2}{2} \end{pmatrix}$$ because:

1) $v_{0y} = v_t$ since thrown $\perp$ to train + 1st Newton law

2) $v_{0z} = 0$ since thrown horizontally.

So $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} v_{0x} \cdot t \\ v_t \cdot t \\ -g\frac{t^2}{2} \end{pmatrix}$ with 3 unknowns $v_{0x}, v_t, t$.

a) $v_t = \frac{y}{t} = \frac{y}{\sqrt{-2z/g}} = \frac{20}{\sqrt{8/10}} \frac{m}{s} \approx 22.4 \frac{m}{s} = 10\sqrt{5} \frac{m}{s}$.

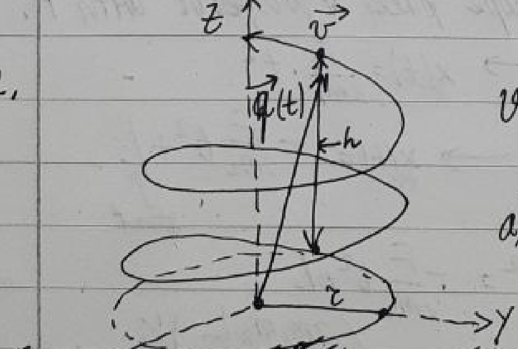
b) $\vec{v}_0 = \begin{pmatrix} v_{0x} \\ v_t \\ 0 \end{pmatrix}$, $v_{0x} = \frac{x}{t} = \frac{x}{\sqrt{2z/g}} = \frac{12}{\sqrt{8/10}} \frac{m}{s} = 6\sqrt{5} \frac{m}{s} \approx 13.4 \frac{m}{s}$.

$\vec{v}_0 = \begin{pmatrix} 6\sqrt{5} \\ 10\sqrt{5} \\ 0 \end{pmatrix} \frac{m}{s}$, $|\vec{v}_0| = \sqrt{180+500} \frac{m}{s} = \sqrt{680} \frac{m}{s} \approx 26 \frac{m}{s}$.

c) $\vec{v}(t) = \begin{pmatrix} v_{0x} \\ v_t \\ -gt \end{pmatrix} = \begin{pmatrix} 6\sqrt{5} \cdot \frac{m}{s} \\ 10\sqrt{5} \cdot \frac{m}{s} \\ -10 \cdot \sqrt{-2z/g} \end{pmatrix} = \begin{pmatrix} 6\sqrt{5} \\ 10\sqrt{5} \\ -10 \cdot \sqrt{8/10} \end{pmatrix} \frac{m}{s} = \begin{pmatrix} 6\sqrt{5} \\ 10\sqrt{5} \\ -4\sqrt{5} \end{pmatrix} \frac{m}{s}$,

$|\vec{v}(t)| = \sqrt{5} \cdot \sqrt{6^2+10^2+4^2} \frac{m}{s} = \sqrt{5}\sqrt{152} \frac{m}{s} \approx 27.6 \frac{m}{s}$.

2.




$v = 250 \frac{km}{h}$, $r = 300 m$, $h(t = 3 min) = 1300 m$,

a) $l = v \cdot t = 250 \frac{km}{h} \cdot 3 \, min =$

$= \frac{250}{20} km = 12.5 \, km$

b) Consider motion on helix as

$$\vec{q}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} r\cos(\omega t) \\ r\sin(\omega t) \\ v_z \cdot t \end{pmatrix} = \begin{pmatrix} r\cos\left(\frac{v_{xy}}{r} t\right) \\ r\sin\left(\frac{v_{xy}}{r} t\right) \\ v_z \cdot t \end{pmatrix}$$

where $\vec{v}_{xy}$ – constant component of $\vec{v}$ in XoY plane ($\vec{v}_{xy} = v_x \hat{e}_x + v_y \hat{e}_y$), $v_{xy} = |\vec{v}_{xy}|$, $\omega = \frac{v_{xy}}{r}$ – angular speed of uniform rotation in circle $r$ in XoY plane.

But $h = v_z \cdot t$ (assume it is on same $(x, y)$ position on a loop ~~xx~~ is not necessary since vertical motion is independent).

So $v_z = \frac{h}{t} = \frac{1300\,m}{3\,mins} = \frac{1300}{180}\,\frac{m}{s} = \frac{65}{9}\,\frac{m}{s}$.

But then $v_{xy} = \sqrt{v^2 - v_z^2}$, & $v_{xy} \cdot t$ is total length moved in a plane.

$$N_{loops} = \frac{v_{xy} \cdot t}{2\pi r} = \frac{\sqrt{v^2 - v_z^2}\, t}{2\pi r} = \frac{\sqrt{v^2 - \left(\frac{h}{t}\right)^2}\, t}{2\pi r} =$$

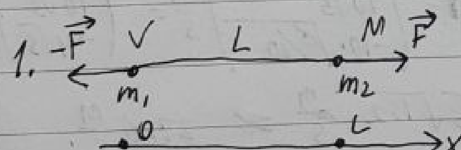
$$= \frac{\sqrt{\left(\frac{250}{3.6}\right)^2 - \left(\frac{65}{9}\right)^2} \cdot 180}{2\pi \cdot 300} \cdot 1 = \boxed{6.6}.$$

! Note – $\frac{v_{xy} \cdot t}{2\pi r} \neq \frac{v \cdot t}{2\pi r}$, even though in this case the answers would be close.

c) $T = \frac{t}{N_{loops}} \approx 27.3$ seconds.

$h_e = v_z \cdot T \approx 197\,m.$
(or $\frac{h}{N_{loops}}$)

Forces

1. 


I consider F as total force, and it's same in rope. By 3rd Newton's law, rope pulls ~~&~~ professor with F.

a) EOM for V: $\ddot{x}_V = \frac{F}{m_1} \longrightarrow x_V(t) = \frac{F}{2m_1} \cdot t^2$

EOM for M: $\ddot{x}_M = \frac{-F}{m_2} \longrightarrow x_M(t) = -\frac{F}{2m_2} t^2 + \underset{\text{start}}{L}$.

Meeting: $x_V = x_M$, $\frac{F}{2m_1} t^2 = \frac{-F}{2m_2} t^2 + L$

$$\frac{F}{2}\left[\frac{1}{m_1} + \frac{1}{m_2}\right] t^2 = L \longrightarrow t = \left(\frac{2m_1 m_2 L}{F(m_1 + m_2)}\right)^{1/2}$$

Then $x = \frac{F}{2m_1} \cdot \frac{2m_1 m_2 L}{F(m_1 + m_2)} = \boxed{\frac{m_2 L}{m_1 + m_2}}$ (also, it is possible to find x as CM)

Then $x = \frac{105}{165} \cdot 20\,m \approx \boxed{12.7\,m}$

b) $\dot{x}_V = \frac{Ft}{m_1}$, $\dot{x}_M = \frac{-Ft}{m_2}$, $\dot{x}_V(t_{meet}) = \frac{F}{m_1} \sqrt{\frac{2m_1 m_2 L}{F(m_1 + m_2)}} =$

$$= \sqrt{\frac{2\cancel{m_1} m_2 L F}{m_1^{\cancel{2}}(m_1+m_2)}} = \sqrt{\frac{LF \cdot 2m_2}{m_1(m_1+m_2)}}, \text{ and } \dot{x}_M(\text{meet}) = -\frac{F}{m_2}\sqrt{\frac{2m_1 m_2 L}{F(m_1+m_2)}} =$$

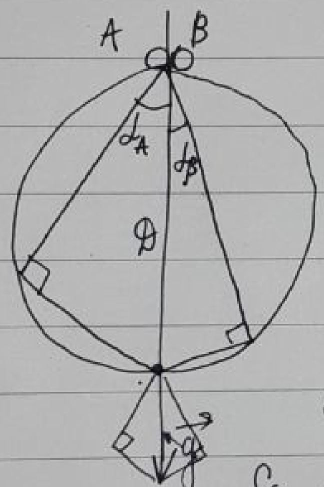
$$= -\sqrt{\frac{LF \cdot 2m_1}{m_2(m_1+m_2)}}.$$

Speeds are $|\dot{x}_V| = \sqrt{\frac{2LF m_2}{m_1(m_1+m_2)}} = 2.06 \frac{m}{s}$

$$|\dot{x}_M| = \sqrt{\frac{2LF m_1}{m_2(m_1+m_2)}} = 1.18 \frac{m}{s}.$$

## Circle

1.




The motion will be with const. acceleration, consider case of A:

$l = \frac{at^2}{2}$, $t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2 \cdot D \cancel{\cos\alpha_A}}{g \cancel{\cos\alpha_A}}} = \sqrt{\frac{2D}{g}}$.

That means $t$ does not depend on $\alpha$!

So both balls reach at same time.