Mathematics 1, Homework 05
Leipzig University, WiSe 2023/24, Tim Shilkin
Due Date: 26.11.23 until 23:59 on-line
or 27.11.23 until 9:15 am in person

Each problem is estimated by one point. Explain your answers.

Given a function

$$f(x) = (x+1)(x-2)^2$$

1. Determine intervals of monotonicity and local extreme of the function $f(x)$
2. Determine intervals of convexity and inflection points of the function $f(x)$
3. Sketch the graph $y = f(x)$

Given a function

$$f(x) = \frac{x^2(x-1)}{(x+1)^2}$$

4. Find the admissible set of f and points of discontinuity of f. Compute the left and the right limits of f at each discontinuity point and determine the type of each discontinuity (jump discontinuity or vertical asymptote).
5. Solve the equation $f(x) = 0$, find the roots of f and intervals of constant sign of f.
6. Solve the equation $f'(x) = 0$, find the extremum points of f and determine the intervals of monotonicity of f.
7. Solve the equation $f''(x) = 0$, find the inflection points of f and determine the intervals of the convexity/concavity of f.
8. Investigate the behavior of f as $x \to \pm\infty$. Find the horizontal and oblique asymptotes for $x \to +\infty$ and $x \to -\infty$ if they exist.
9. List all specific points of the function (points of discontinuity, roots of the function, local extreme and inflection points)

   $$x_1 < x_2 < x_3 < \ldots < x_k$$

   Compute the (approximate) values of x_j and $f(x_j)$ for all these points and fill in the table containing all the above information:

Values of x	$(-\infty, x_1)$	x_1	(x_1, x_2)	x_2	$\ldots$
Values of f	write $+$ if f is positive on $(-\infty, x_1)$ write $-$ if f is negative on $(-\infty, x_1)$	$f(x_1)$	$\ldots$	$\ldots$	$\ldots$
Sign of f'	write $+$ if f' is positive on $(-\infty, x_1)$ write $-$ if f' is negative on $(-\infty, x_1)$	$f'(x_1)$	$\ldots$	$\ldots$	$\ldots$
Monotonicity f	write $\uparrow$ if f is increasing on $(-\infty, x_1)$ write $\downarrow$ if f is decreasing on $(-\infty, x_1)$	max or min or nothing	$\ldots$	$\ldots$	$\ldots$
Sign of f''	write $+$ if f'' is positive on $(-\infty, x_1)$ write $-$ if f'' is negative on $(-\infty, x_1)$	$f''(x_1)$	$\ldots$	$\ldots$	$\ldots$
Convexity of f	write $\smile$ if f is convex on $(-\infty, x_1)$ write $\frown$ if f is concave on $(-\infty, x_1)$	inflection or nothing	$\ldots$	$\ldots$	$\ldots$

10. Sketch the graph $y = f(x)$.