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Theoretical Physics I – ex. 7

(see note 7.5 – b)

Self Test – Problem 7.5

$\vec{F}(\vec{r})=\vec{\omega}\times(\vec{\omega}\times\vec{r})$

a) Using $\vec{b}(\vec{a}\vec{c})-\vec{c}(\vec{a}\vec{b})$ rule, $\vec{F}(\vec{r})=\vec{\omega}\cdot(\vec{\omega}\cdot\vec{r})-\vec{r}\cdot(\vec{\omega}\cdot\vec{\omega})=$

$=(\vec{\omega}\cdot\vec{r})\cdot\vec{\omega}+(-\omega^2)\vec{r}, = f(\vec{r})\vec{\omega}+c\vec{r}$, where

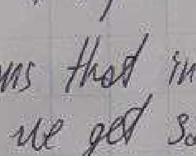
$f(\vec{r})=\vec{\omega}\cdot\vec{r}$, $c=-\omega^2$.

b) In all tasks here and below to show field is conservative I use either:

(*) $\forall i,j$ $\partial_i F_j=\partial_j F_i$ where $i\neq j$, which is equivalent to say

$$\nabla\times\vec{F}=\begin{pmatrix}\partial_y F_z-\partial_z F_y\\ \partial_z F_x-\partial_x F_z\\ \partial_x F_y-\partial_y F_x\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

(the first intuitively means that integrating back with two variables from different components we get same scalar function, the second means that $\oint \vec{F}\,d\vec{r}=0$



which holds iff curl inside the region is zero as it is equal to line integral due to compensation of „rotations" inside in counter-directions – which we will later learn as Stokes theorem and mentioned in EP1 course)

(*) Find (guess or integration) scalar $\varphi(\vec{r})$ so that $F(\vec{r})=-\nabla\varphi(\vec{r})$

(*) Showing somehow that $\int_{q_1}\vec{F}\,dq_1=\int_{q_2}\vec{F}\,dq_2$ for all paths (can be shown for gravitational force).

Now for this task $\vec{F}(\vec{r}) = \vec{\omega}\cdot(\vec{\omega}\cdot\vec{r}) - \vec{r}(\vec{\omega}\,\vec{\omega}) = \vec{\omega}(\vec{\omega}\cdot\vec{r}) - \vec{r}\omega^2 =$

$$= \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \cdot (\omega_x x + \omega_y y + \omega_z z) - \begin{pmatrix} x\omega^2 \\ y\omega^2 \\ z\omega^2 \end{pmatrix} = \begin{pmatrix} \omega_x(\omega_x x + \omega_y y + \omega_z z) - x\omega^2 \\ \omega_y(\omega_x x + \omega_y y + \omega_z z) - y\omega^2 \\ \omega_z(\omega_x x + \omega_y y + \omega_z z) - z\omega^2 \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$$

Table below means differentiate columns by rows

	F_x	F_y	F_z
∂_x	not interested in these values	$\omega_y \omega_x$	$\omega_z \omega_x$
∂_y	$\omega_x \omega_y$		$\omega_z \omega_y$
∂_z	$\omega_x \omega_z$	$\omega_y \omega_z = \omega_z \omega_y$	

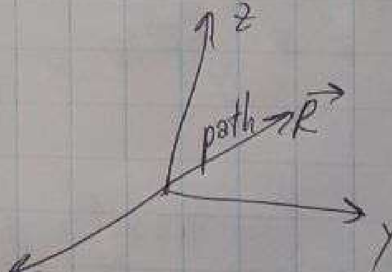
same

same

same

So $\partial_i F_j = \partial_j F_i$ – force is conservative.

c) $\vec{q} = s\vec{R}$ for some given $\vec{R}$ (line from $\vec{0}$ to $\vec{R}$), $s \in [0,1]$.



$$\int_{\vec{0}}^{\vec{R}} \vec{F}\,d\vec{q} = \begin{bmatrix} d\vec{q} = \vec{R}\,ds \\ s\in[0,1] \\ \underbrace{\vec{r} = s\vec{R}}_{\text{current variable position}} \end{bmatrix} = \int_{s=0}^{s=1} \Big[\vec{\omega}(\underbrace{\vec{\omega}\cdot(s\vec{R})}_{\text{scalar out}}) - (s\vec{R})\omega^2\Big]\vec{R}\,ds =$$

distribute

$$= \int_{s=0}^{s=1} s\cdot\vec{\omega}(\underbrace{\vec{\omega}\cdot\vec{R}}_{\text{const, scalar out}})\cdot\vec{R} - s\omega^2\vec{R}\cdot\vec{R}\,ds = \int_{s=0}^{s=1} s\cdot(\vec{\omega}\cdot\vec{R})^2 - s\,\omega^2\vec{R}^2\,ds =$$

$$= (\vec{\omega}\cdot\vec{R})^2 \frac{s^2}{2}\Big|_0^1 - (\omega^2\vec{R}^2)\frac{s^2}{2}\Big|_0^1 = \frac{1}{2}\Big[(\vec{R}\cdot\vec{\omega})^2 - \omega^2\vec{r}^2\Big] =$$

$= \varphi(0) - \varphi(1)$, defining $\varphi(0) = 0$,

$\varphi(s=1) = \varphi(\vec{r}) = -\frac{1}{2}\left[(\vec{r}\cdot\vec{\omega})^2 - \omega^2\vec{r}^{\,2}\right]$, $K = -\frac{1}{2}$ (additive constant $\to 0$)

Check correctness by differentiating, for one component due to symmetry:

$$-\partial_x\left(-\tfrac{1}{2}\left((x\omega_x+y\omega_y+z\omega_z)^2-\omega^2(x^2+y^2+z^2)\right)\right) = \tfrac{1}{2}\partial_x\left[(x\omega_x+y\omega_y+z\omega_z)^2 - \omega^2(x^2+y^2+z^2)\right] = \tfrac{1}{2}\cdot\left(2(x\omega_x+y\omega_y+z\omega_z)\cdot\omega_x - 2x\omega^2\right) =$$

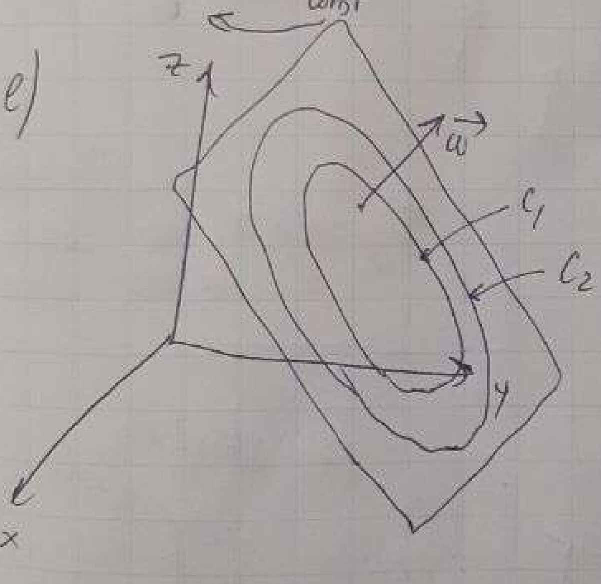
$= (x\omega_x+y\omega_y+z\omega_z)\omega_x - x\omega^2 \to$ exactly F_x!

Similarly $-\partial_y\varphi = F_y$, $-\partial_z\varphi = F_z$.

d) Simplest way: since $\varphi(\vec{r})$ does not change $\uparrow\uparrow\vec{\omega}$, $\pm\nabla\varphi(\vec{r})$ ~~and~~ must be $\perp$ to $\vec{\omega}$. So show $\vec{F}(\vec{r})\cdot\vec{\omega} = 0$.

$$\vec{\omega}\cdot\left(\vec{\omega}\underbrace{(\vec{\omega}\cdot\vec{r})}_{\text{const}} - \vec{r}\omega^2\right) = (\vec{\omega}\cdot\vec{r})\cdot(\vec{\omega}\cdot\vec{\omega}) - (\vec{\omega}\cdot\vec{r})\omega^2 = (\vec{\omega}\vec{r})\omega^2 - (\vec{\omega}\vec{r})\omega^2 = 0.$$

e)



$$\varphi = -\tfrac{1}{2}\left[(\vec{\omega}\vec{r})^2 - \omega^2\vec{r}^{\,2}\right] = C$$
$\downarrow$ contour surface

Solve for $\vec{r}$ to find where C-surface.

$$-\tfrac{1}{2}\left[(\vec{\omega}\vec{r})^2 - \omega^2\vec{r}^{\,2}\right] = C \;\Big|\cdot(-2)$$

$$(\vec{\omega}\vec{r})^2 - \omega^2\vec{r}^{\,2} = C^*$$

$$(\omega_x x + \omega_y y + \omega_z z)^2 - \omega^2(x^2+y^2+z^2) = C^*_{\text{const}}$$

$$\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2 + \underbrace{2\omega_x\omega_y}_{=a}xy + \underbrace{2\omega_x\omega_z}_{=b}xz + \underbrace{2\omega_y\omega_z}_{=c}yz - \omega^2x^2 - \omega^2y^2 - \omega^2z^2 = C^* \;\Big|\cdot(-1)$$

$$\underbrace{(\omega^2-\omega_x^2)}_{\text{positive } \delta}x^2 + \underbrace{(\omega^2-\omega_y)^2}_{\text{pos. } \beta}y^2 + \underbrace{(\omega-\omega_z)^2}_{\text{pos. } \gamma}z^2 + axy + bxz + cyz = A_{\text{const}}$$

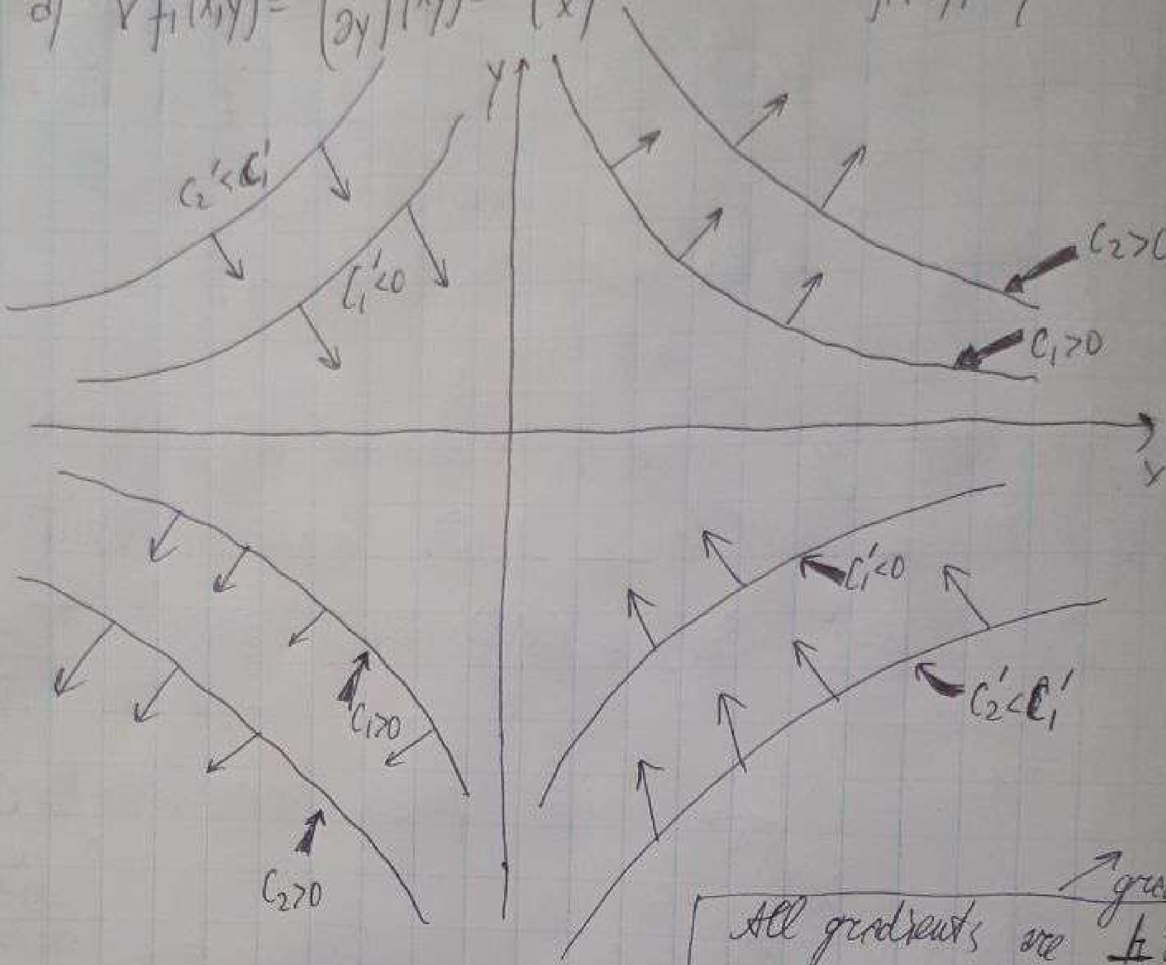
$$\delta x^2 + \beta y^2 + \gamma z^2 + \underbrace{axy + bxz + cyz}_{\text{these can be eliminated}} = A_{\text{const}}$$

$\delta, \beta, \gamma \geq 0$

3D-Isosurface is <u>ellipsoid</u>, and contour lines where it intersects the plane is ellipse.

Problem 7.1

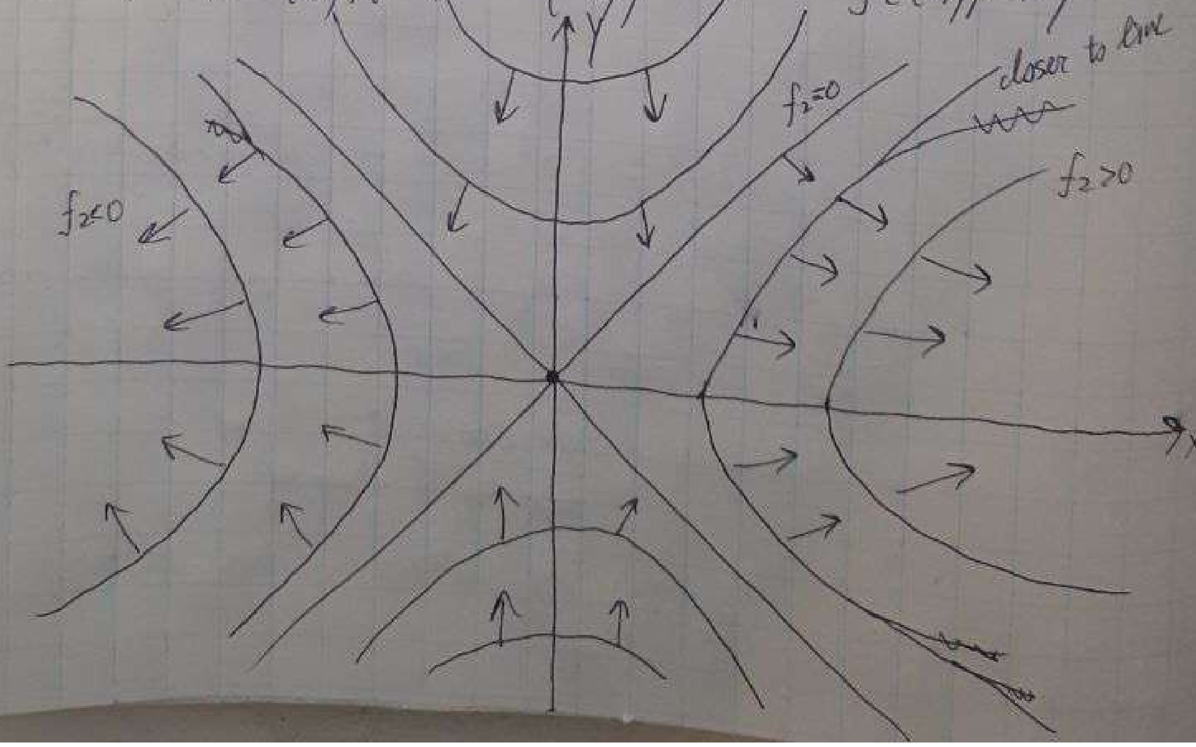
a) $\nabla f_1(x,y) = \begin{pmatrix} \partial x \\ \partial y \end{pmatrix} (xy) = \begin{pmatrix} y \\ x \end{pmatrix}$ $\qquad f_1(x,y) = xy$



$\nearrow$ gradient

All gradients are $\perp$ to contour lines.

b) $\nabla f_2(x,y) = \begin{pmatrix} \partial x \\ \partial y \end{pmatrix} (x^2 - y^2) = \begin{pmatrix} 2x \\ -2y \end{pmatrix}$ $\qquad f_2(x,y) = x^2 - y^2$



c) $f_1(x,y) = xy = R\cos\theta \, R\sin\theta = \frac{1}{2}R^2\sin 2\theta$
$f_2(x,y) = R^2(\cos^2\theta - \sin^2\theta) = R^2\cos 2\theta$

Can always find different R_1, R_2 so that

$$f_1(x,y) = \frac{1}{2}R_1^2\sin(2\theta) = R^{*2}\sin(2\theta)$$
$$f_2(x,y) = R_2^2\sin\left(2\theta + \underline{\frac{\pi}{2}}\right) = R^{*2}\sin\left(2\theta+\frac{\pi}{2}\right)$$

rotated by $\frac{\pi}{2}$

All contour lines are rotated and gradients are perpendicular to each other.

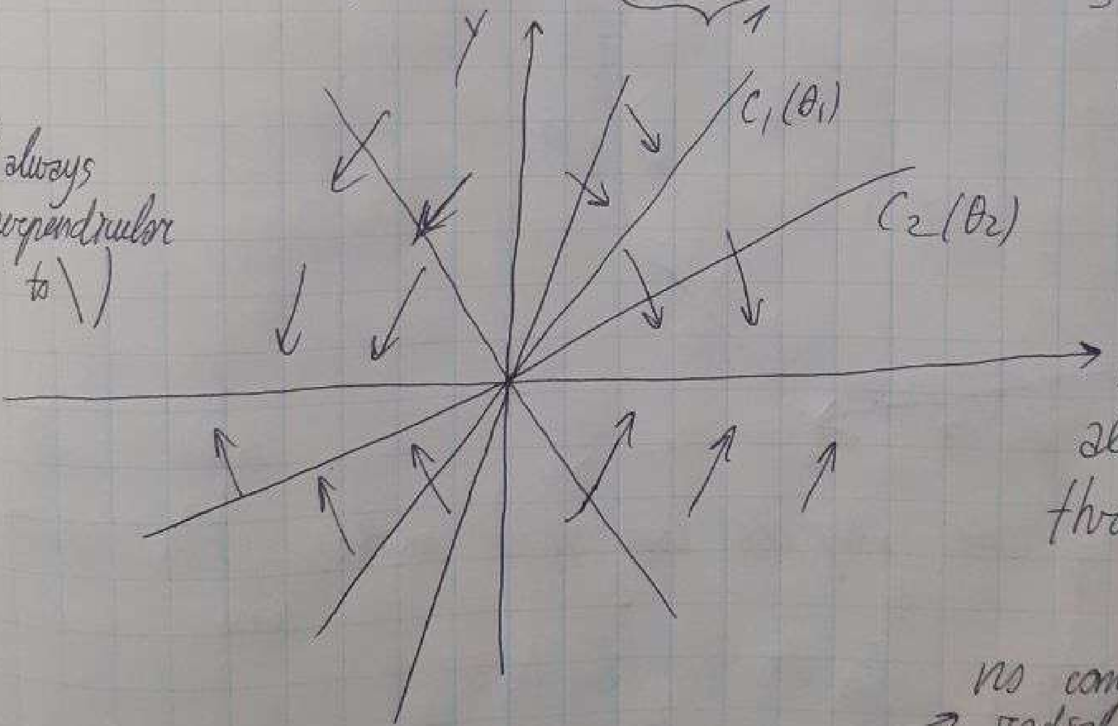
Can also verify in cartesian:

$$\nabla f_1 \cdot \nabla f_2 = \begin{pmatrix} y \\ x \end{pmatrix} \cdot \begin{pmatrix} 2x \\ -2y \end{pmatrix} = \underline{2yx - 2xy = 0}.$$

d) $$f_3(x,y) = \frac{x^2 - y^2}{x^2+y^2} = \frac{R^2(\cos^2\theta - \sin^2\theta)}{R^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{1}} = \underline{\cos 2\theta}.$$

for each angles values are same. Contour lines are all lines through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(↑ always perpendicular to \)



$$\nabla f_3 = \begin{bmatrix} \partial_R \\ \frac{1}{R}\partial_\theta \end{bmatrix} \cos 2\theta = \begin{bmatrix} \partial_R(\cos 2\theta) \\ \frac{1}{R}(-2\sin 2\theta) \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{2}{R}\sin 2\theta \end{bmatrix} = -\frac{2}{R}\sin 2\theta \cdot \hat{\theta}$$

no component in radial direction. as expected

When $\theta \in (0, \frac{\pi}{2})$, is negative (clockwise gradient)
When $\theta \in (\frac{\pi}{2}, \pi)$ positive (counterclockwise gradient)
When $\theta \in (\pi, \frac{3\pi}{2})$, negative (clockwise)
When $\theta \in (\frac{3\pi}{2}, 2\pi)$, positive (counterclockwise)

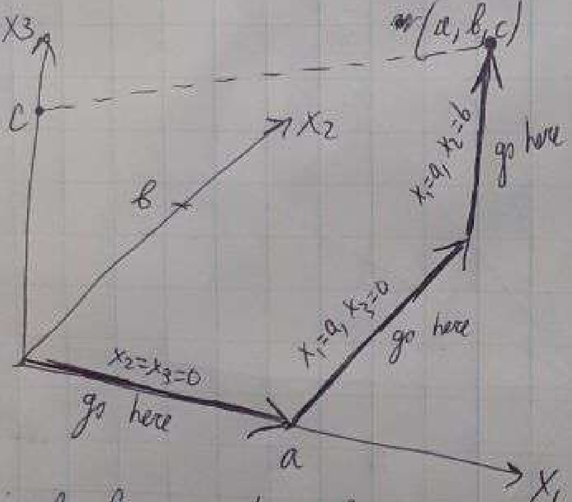
Problem 7.2 Forces, potentials and line integrals

$$\vec{F}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1+x_2x_3\\x_2+x_1x_3\\x_3+x_1x_2\end{pmatrix} \qquad \vec{s} = \begin{pmatrix}at\\bt\\ct\end{pmatrix}, \quad \vec{s}\,'(t) = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

a) $$\int_\gamma \vec{F}\,d\vec{s} = \int_{t=0}^{t=1} \begin{pmatrix}at+bct^2\\bt+act^2\\ct+abt^2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}dt = \int_{t=0}^{t=1}\left(a^2t+abct^2+b^2t+abct^2+c^2t+abct^2\right)dt =$$

$$= \int_{t=0}^{t=1} (a^2+b^2+c^2)t+(3abc)t^2\,dt = \frac{(a^2+b^2+c^2)t^2}{2}\Big|_0^1 + \frac{\cancel{3}abc\,t^3}{\cancel{3}}\Big|_0^1 =$$

$$= \frac{a^2+b^2+c^2}{2} + abc$$

b) 

Sum of line integrals over three paths: $\int_{(0,0,0)}^{(a,b,c)} \vec{F}(x_1,x_2,x_3)\,d\vec{x} =$

$$\int_0^a F_{x_1}dx_1 + \int_0^b F_{x_2}\,dx_2 + \int_0^c F_{x_3}\,dx_3 =$$

$$= \int_0^a (x_1+\underbrace{x_2x_3}_{0})\,dx_1 + \int_0^b (x_2+\underbrace{x_1x_3}_{0})\,dx_2 + \int_0^c (x_3+\underbrace{x_1x_2}_{ab})\,dx_3 =$$

$$= \int_0^a x_1\,dx_1 + \int_0^b x_2\,dx_2 + \int_0^c (x_3+ab)\,dx_3 = \frac{x_1^2}{2}\Big|_0^a + \frac{x_2^2}{2}\Big|_0^b + \left(\frac{x_3^2}{2}+abx_3\right)\Big|_0^c =$$

$= \frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2} + abc$ — same value as on direct line $\begin{pmatrix}a\\b\\c\end{pmatrix}$!

c) $$\nabla \times \vec{F} = \sum_{i,j,k}^{3} \varepsilon_{ijk}\, \hat{e}_i \frac{\partial}{\partial x_j} F_k = \begin{pmatrix} \frac{\partial F_3}{\partial x_2} - \frac{\partial F_2}{\partial x_3} \\ \frac{\partial F_1}{\partial x_3} - \frac{\partial F_3}{\partial x_1} \\ \frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2} \end{pmatrix} = \begin{pmatrix} x_1 - x_1 \\ x_2 - x_2 \\ x_3 - x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} =$$

$= \vec{0}.$

rule above

d) $$\nabla \times \nabla S(\vec{q}) = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} \partial_x S \\ \partial_y S \\ \partial_z S \end{pmatrix} = \begin{pmatrix} \partial_y \partial_z S - \partial_z \partial_y S \\ \partial_z \partial_x S - \partial_x \partial_z S \\ \partial_x \partial_y S - \partial_y \partial_x S \end{pmatrix} =$$ [because we deal with smooth functions with continuous 2nd derivatives, $\partial_i \partial_j = \partial_j \partial_i$] =

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0}.$$

e) $\vec{F}$ is conservative $\longrightarrow$ „hint" is that a), b) give same result, for 2 paths proof is d).

Find potential.

$$\Phi(x_1, x_2, x_3) = -\frac{1}{2}(x_1^2 + x_2^2 + x_3^2) - x_1 x_2 x_3 + C.$$

Then $-\nabla \Phi = \cancel{x_1 + x_2 + x} \begin{bmatrix} x_1 + x_2 x_3 \\ x_2 + x_1 x_3 \\ x_3 + x_1 x_2 \end{bmatrix} = \vec{F}$ — exactly the force.

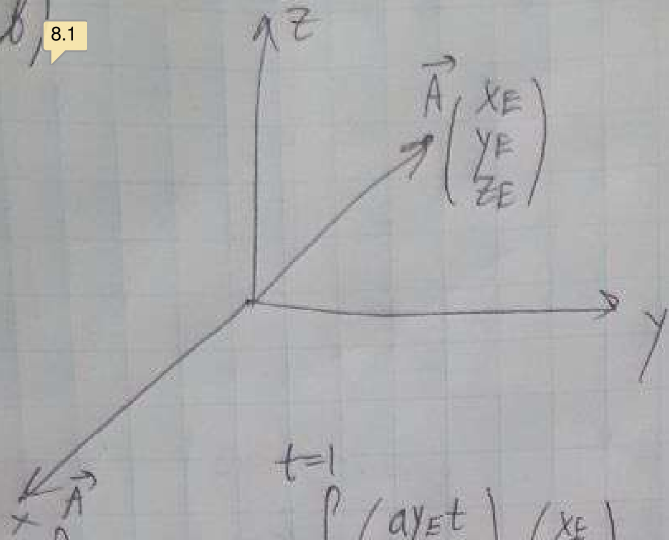
Problem 7.3 Conservative forces

$\vec{F} = \begin{pmatrix} ay \\ bx \\ f(z) \end{pmatrix}$ $\quad f: \mathbb{R} \to \mathbb{R},\ a, b \in \mathbb{R}$

a) $\left.\begin{array}{l} \partial_z F_x = \partial_x F_z = 0 \\ \partial_z F_y = \partial_y F_z = 0 \end{array}\right\}$ F_z does not depend on x or y, only z.

Suppose $\begin{pmatrix} ay \\ bx \\ \sin(z) \end{pmatrix}$ or $\begin{pmatrix} ay \\ bx \\ 3 + e^{z^2} \end{pmatrix}$.

b)



$t \in [0,1]$

path $\vec{s} = \begin{pmatrix} x_E t \\ y_E t \\ z_E t \end{pmatrix}$, $\left(\vec{0} \xrightarrow{\text{goes}} \begin{pmatrix} x_E \\ y_E \\ z_E \end{pmatrix}\right)$

then $\vec{s}\,'(t) = \begin{pmatrix} x_E \\ y_E \\ z_E \end{pmatrix}$

$\vec{F}(\vec{s}) = \begin{pmatrix} ay \\ bx \\ f(z) \end{pmatrix} = \begin{pmatrix} a y_E t \\ b x_E t \\ f(z_E t) \end{pmatrix}$

$$\int_{\vec{0}}^{\vec{A}} \vec{F}\, d\vec{s} = \int_{t=0}^{t=1} \begin{pmatrix} a y_E t \\ b x_E t \\ f(z_E t) \end{pmatrix} \cdot \begin{pmatrix} x_E \\ y_E \\ z_E \end{pmatrix} dt =$$

$$= \int_{t=0}^{t=1} a x_E y_E t + b x_E y_E t + f(z_E t) z_E \, dt = \int_{t=0}^{t=1} (a x_E y_E + b x_E y_E) t \, dt + \int_{t=0}^{t=1} f(z_E t) z_E \, dt =$$

$$= \frac{(a x_E y_E + b x_E y_E) t^2}{2} \Big|_0^1 + F(z_E t) \Big|_0^1 = \boxed{\frac{(a+b) x_E y_E}{2} + F(z_E)} =$$

note antiderivative of f.

for example if f is cos, $F = \sin$

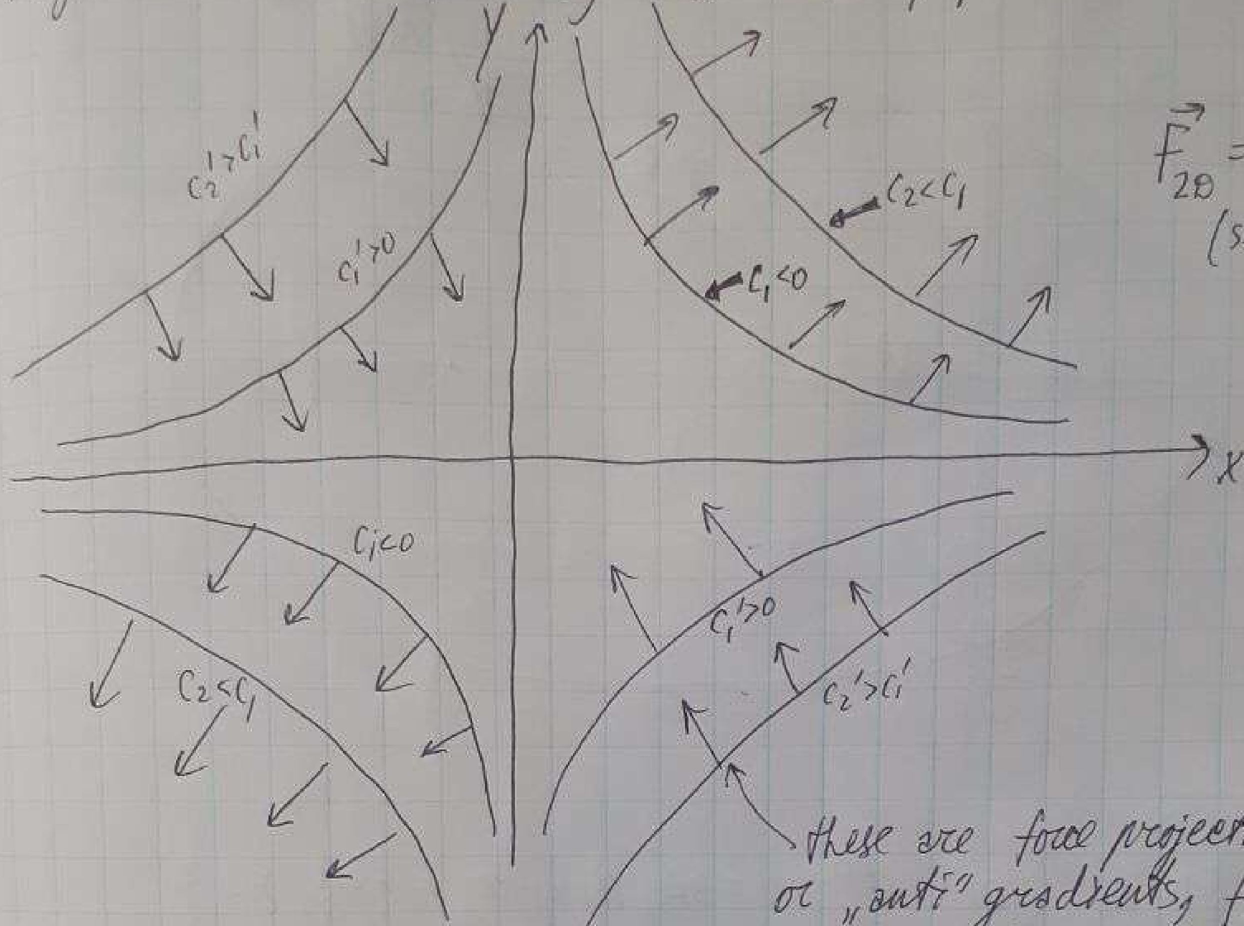
$= \varphi(0) - \varphi(\vec{s}_E)$ for conserv.

$\partial_t F(z_E t) = \partial_t (\sin(z_E t)) = z_E \cos(z_E t) = z_E f(z_E t)$

c) For conservative case ($f(z)$ is independ. from x,y) potential, if defined as 0 in origin, is then ~~$-\frac{(a+b)xy}{2}$~~

$$-\frac{(a+b)}{2} xy - F(z).$$

d) In x,y plane, since $z=0$, contour lines of $a=b=1$ will be exactly like plotted for $-xy$ (but in 3D corresponding heights will be shifted by $F(z)$). In xy-plane:



$\vec{F}_{2D} = \begin{pmatrix} y \\ x \end{pmatrix}$ (see below)

these are force projections, or „anti" gradients, for potential $-xy + F(z)$.

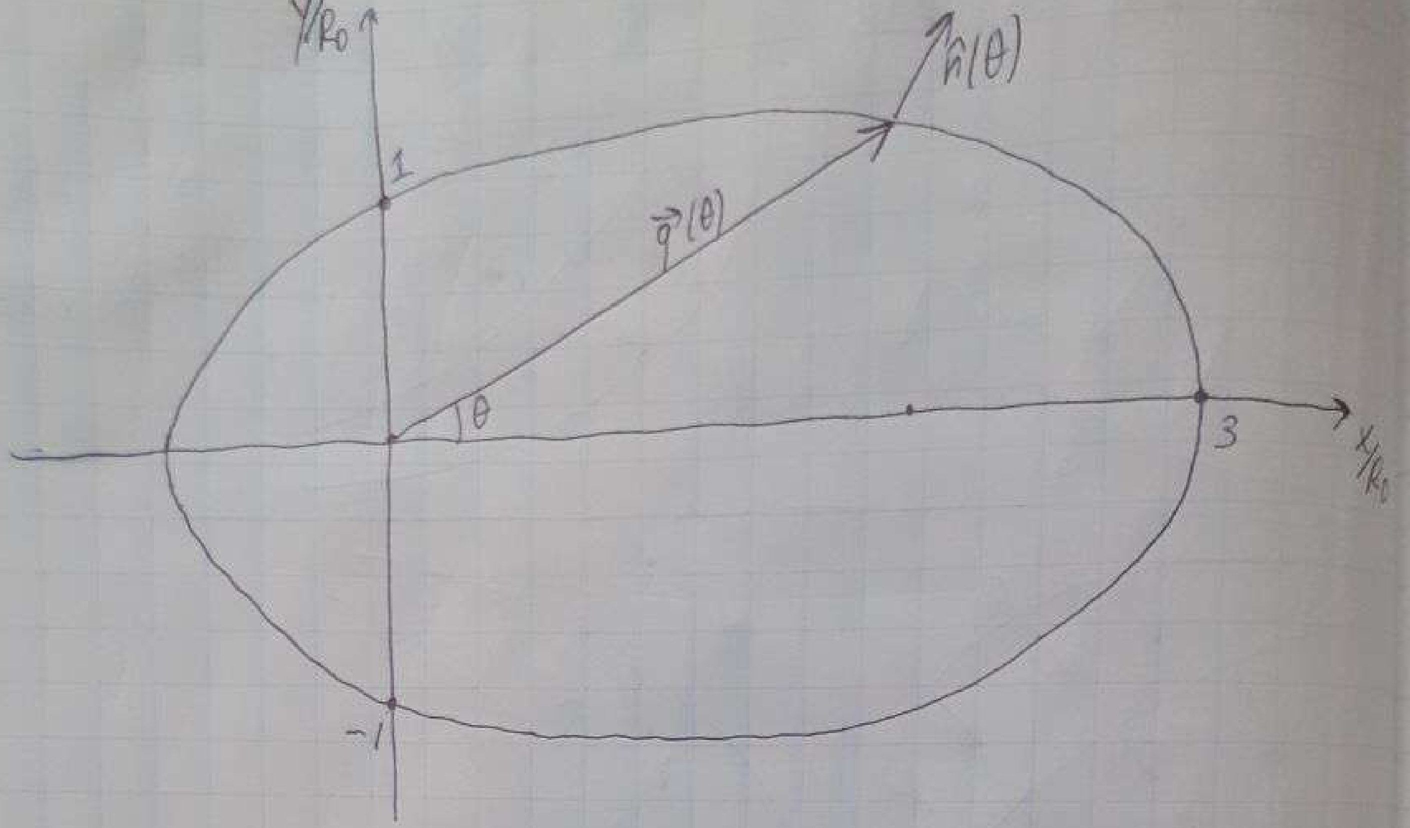
Then $F_x = -\partial_x(-xy + F(z)) = y$

$F_y = -\partial_y(-xy + F(z)) = x$

Problem 7.4 Wobblying wheel

$\vec{q} = R(\theta)\,\hat{r}(\theta)$ in polar form.

See picture and solution on next page.



a) $R(\frac{\pi}{2}) = R_0$, $R(0) = \frac{R_0}{1-\varepsilon} = 3R_0$, therefore $1-\varepsilon = \frac{1}{3}$, $\underline{\varepsilon = \frac{2}{3}}$.

Circle will be when $\varepsilon = 0 \rightarrow R = \frac{R_0}{1-0} = R_0$, same distance from origin for all θ.

For ellipses $\underline{\varepsilon \in (0,1)}$ → for when $\varepsilon = 1$,
$R(0)$ increases $\rightarrow \infty$, ellipse „opens" to parabola.
$\varepsilon > 1$ – hyperbola (further opening to axes).

[Note if measure angle θ from other focal point (so that $\theta = 0$ is smallest and not largest distance), $\varepsilon \in (-1, 0)$, for ellipse.

For our case $\varepsilon \in (0,1)$ gives ellipse, in this task $\varepsilon = \frac{2}{3}$.

b) $\vec{n}(\theta)$ orthogonal to surface (not necessarily unit vector) →

main idea: means it is a ± gradient for scaler function which contour map ellipse is. Find this scaler.

$R(\theta) = \frac{R_0}{1-\varepsilon\cos\theta}$, Rearranging $R_0 = \underbrace{R}_{\text{variables}} \cdot (1-\varepsilon\cos\theta)$.

So the function is $f(R,\theta) = R(1-\varepsilon\cos\theta)$, and ellipse is contour map $f(R,\theta) = R_0$. In dimensionless units of R_0,

$f(R,\theta) = R(1-\varepsilon\cos\theta)$, $f(R,\theta) = 1$ is contour map (ellipse).

$$\nabla f(R,\theta) = \begin{pmatrix} \partial/\partial R \\ \frac{1}{R}\cdot\partial/\partial\theta \end{pmatrix} R(1-\varepsilon\cos\theta) = \begin{pmatrix} 1-\varepsilon\cos\theta \\ \varepsilon\sin\theta \end{pmatrix}.$$

see first problem

! Since $\hat{n}(\theta)$ points outwards, for positive θ,

it is $\boxed{\begin{pmatrix} 1-\varepsilon\cos\theta \\ \varepsilon\sin\theta \end{pmatrix}}$, not $\begin{pmatrix} \varepsilon\cos\theta - 1 \\ -\varepsilon\sin\theta \end{pmatrix}$

$\boxed{\hat{n}(\theta) = \mathcal{L}\cdot\left[(1-\varepsilon\cos\theta)\,\hat{r}(\theta) + \varepsilon\sin\theta\,\hat{\theta}(\theta)\right]}$ where $\mathcal{L}$ is any positive constant.

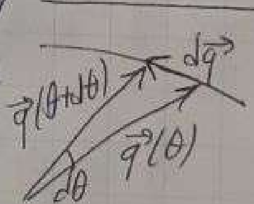
$$\mathcal{L}\left[(1-\varepsilon\cos\theta)\,\hat{r}(\theta) + \varepsilon\sin\theta\cdot\hat{\theta}(\theta)\right] = L\left(\hat{r}(\theta) + \frac{c\sin\theta}{1-\varepsilon\cos\theta}\,\hat{\theta}(\theta)\right),$$

Then $\underline{L = \mathcal{L}(1-\varepsilon\cos\theta)}$ $\forall \mathcal{L} \geq 0$. ($\mathcal{L}=1$, $L = 1-\varepsilon\cos\theta$).

$\underline{c = \varepsilon.}$

— Proof ~~that $\hat{n}(\theta)$ must~~ for „hint" part —

1. $\vec{v}(\theta) = \frac{d\vec{q}}{d\theta}$ must be tangential because $d\vec{q}$ becomes tangent when $d\theta \to 0$.



2. $\hat{n}(\theta)\cdot\vec{v}(\theta) = 0$, because $\vec{v}(\theta) = \frac{d\vec{q}}{d\theta} = \frac{d}{d\theta}(\hat{r}(\theta)R(\theta)) = \hat{\theta}R(\theta) +$

$+ \hat{r}(\theta)\frac{d}{d\theta}\left(\frac{1}{1-\varepsilon\cos\theta}\right) = \hat{\theta}R(\theta) + \hat{r}(\theta)\frac{d}{d\theta}(1-\varepsilon\cos\theta)^{-1} =$

$= \hat{\theta}R(\theta) + \hat{r}(\theta)(-1)(1-\varepsilon\cos\theta)^{-2}(\varepsilon\sin\theta) =$

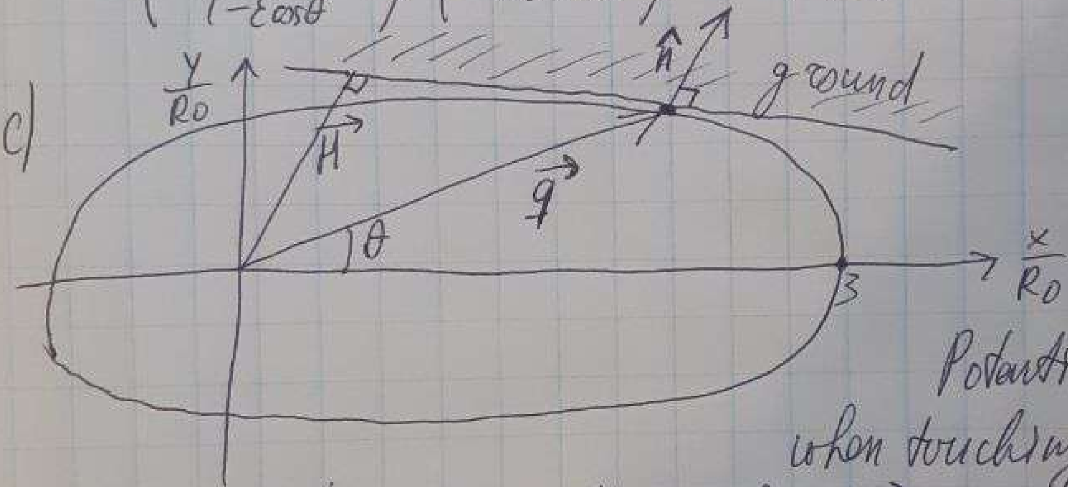
$= \hat{\theta}\underbrace{R(\theta)}_{\frac{1}{1-\varepsilon\cos\theta}} - \hat{r}(\theta)\frac{\varepsilon\sin\theta}{(1-\varepsilon\cos\theta)^2} = \begin{pmatrix} \frac{-\varepsilon\sin\theta}{(1-\varepsilon\cos\theta)^2} \\ \frac{1}{1-\varepsilon\cos\theta} \end{pmatrix} = \vec{v}(\theta).$

$$\begin{bmatrix} \mathcal{L}(1-\varepsilon\cos\theta) \\ \mathcal{L}\varepsilon\sin\theta \end{bmatrix}$$

Now $\vec{v}(\theta)\cdot\hat{n}(\theta) =$ [we can calculate sc product like this because $\hat{\theta}$ and $\hat{r}$ form ONB] =

$$= \begin{pmatrix} \frac{-\varepsilon\sin\theta}{(1-\varepsilon\cos\theta)^2} \\ \frac{1}{1-\varepsilon\cos\theta} \end{pmatrix} \cdot \begin{pmatrix} \mathcal{L}(1-\varepsilon\cos\theta) \\ \mathcal{L}\varepsilon\sin\theta \end{pmatrix} = \frac{-\varepsilon\mathcal{L}\sin\theta}{1-\varepsilon\cos\theta} + \frac{\mathcal{L}\varepsilon\sin\theta}{1-\varepsilon\cos\theta} = \underline{0}.$$

c)



Potential energy when touching the ground in position θ is MgH, where $\vec{H}$ as seen from picture $\uparrow\uparrow \hat{n}$ because bouth are perpendicular to ground.

Then to find H — calculate component of $\vec{q}$ in direction of $\hat{n}$, meaning $\vec{q}\cdot\frac{\hat{n}}{|\hat{n}|}$. Take $\hat{n} = \begin{pmatrix} 1-\varepsilon\cos\theta \\ \varepsilon\sin\theta \end{pmatrix}$, $|\hat{n}| = \sqrt{1+\varepsilon^2-2\varepsilon\cos\theta}$,

12.1 Then $\frac{\vec{q}\cdot\hat{n}}{|\hat{n}|} = \begin{pmatrix} \frac{1}{1-\varepsilon\cos\theta} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1-\varepsilon\cos\theta \\ \varepsilon\sin\theta \end{pmatrix} \cdot \frac{1}{\sqrt{1+\varepsilon^2-2\varepsilon\cos\theta}} = \underline{\frac{1}{\sqrt{1+\varepsilon^2-2\varepsilon\cos\theta}}} = H$

Check special case $\theta = 0$: $H = \frac{1}{\sqrt{1+\varepsilon^2-2\varepsilon}} = \frac{1}{1-\varepsilon} = \left(\varepsilon = \frac{2}{3}\right) = 3$, which is exactly what should be.

Potential energy is $\underline{\frac{MgR_0}{\sqrt{1+\varepsilon^2-2\varepsilon\cos\theta}}}$ then, if using R_0 ~~units~~ again.

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