Mathematics 1, Homework 12
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Due Date: 04.02.24 until 23:59 on-line
Only on-line submission is available!

Each problem is estimated by one point. Explain your answers.

1. Evaluate the following determinant:

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & 3 & 2 \\ 5 & 1 & 6 \end{vmatrix}$$

2. Evaluate the following determinant:

$$\begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{vmatrix}$$

3. Find all possible choices of $x \in \mathbb{R}$ that would make the following matrix singular:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & x \\ 1 & x & 3 \end{pmatrix}$$

4. Let A and B be 3×3 matrices with $\det A = 4$ and $\det B = 5$. Find the value of

$$\det\left(2A^{-1}B^2\right)$$

5. Using properties of determinants, compute the following determinant:

$$\begin{vmatrix} 1001 & 1002 & 1003 & 1004 \\ 1002 & 1003 & 1001 & 1002 \\ 1001 & 1001 & 1001 & 999 \\ 1001 & 1000 & 998 & 999 \end{vmatrix}$$

6. Find the matrix A^{-1} inverse to the matrix A which is a product of elementary matrices:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

7. Find the 2×2 matrix X such that

$$\begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

8. Given the matrix A, compute $\det A$, $\operatorname{Cof} A$ and A^{-1}. Verify the identiny $A^{-1}A = \mathbb{I}$.

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{pmatrix}$$

9. Apply the inverse matrix A^{-1} computed in the previous problem to solve the linear systems

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 3 \\ 2x_1 + x_2 + x_3 &= 6 \\ -2x_1 + 2x_2 - x_3 &= 9 \end{aligned} \qquad \begin{aligned} x_1 + 3x_2 + x_3 &= 9 \\ 2x_1 + x_2 + x_3 &= 3 \\ -2x_1 + 2x_2 - x_3 &= 6 \end{aligned} \qquad \begin{aligned} x_1 + 3x_2 + x_3 &= 6 \\ 2x_1 + x_2 + x_3 &= 9 \\ -2x_1 + 2x_2 - x_3 &= 3 \end{aligned}$$

10. Use Cramer's rule to solve the following system:

$$\begin{gathered} 2x_1 + x_2 - 3x_3 = 0 \\ 4x_1 + 5x_2 + x_3 = 8 \\ -2x_1 - x_2 + 4x_3 = 2 \end{gathered}$$