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Theoretical Physics I

Lagrange Formalism 2

Exercise 12

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$$\Phi(x) = \begin{cases} -\frac{1}{2}(x-8n)(x-8n+4) & 8n-4 < x \leq 8n \\ \frac{1}{2}(x-8n)(x-8n-4) & 8n < x \leq 8n+4 \end{cases} \quad n \in \mathbb{Z}$$

! $4n-4 < x \leq 4n$ I denote as strip n (strip 1 is $x \in (0,4]$). The idea is to get $\Phi(n)$ in unique way, distinguishing between odd and even potentials.

a) Odd strips: $n = 2k+1$. k is any $\mathbb{Z}$-number. (use $\Phi(x)$ definition mentally replacing n with k)

So $4(2k+1)-4 < x \leq 4(2k+1)$
$8k < x \leq 8k+4$ — then second potential works.

$$(*)\ \underline{\Phi(n)} = \frac{1}{2}(x-8k)(x-8k-4) =$$
$$= \frac{1}{2}\left(x-8\cdot\frac{n-1}{2}\right)\left(x-8\frac{n-1}{2}-4\right) = \frac{1}{2}(x-4(n-1))(x-4(n-1)-4) =$$
$$= \frac{1}{2}(x-4n+4)(x-4n)$$

Even strips: $n=2k$. k is any $\mathbb{Z}$-number. (use $\Phi(x)$ def. mentally replacing $n \to k$)

So $4(2k)-4 < x \leq 4(2k)$
$8k-4 < x \leq 8k$ — then first potential definition works.

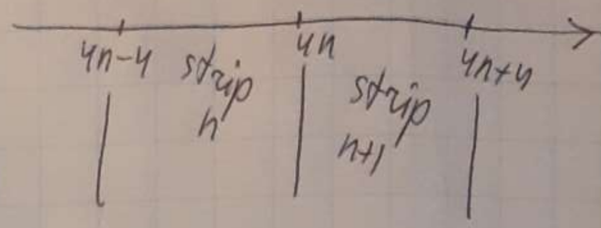
$$(**)\ \underline{\Phi(n)} = -\frac{1}{2}(x-8k)(x-8k+4) = -\frac{1}{2}(x-4n)(x-4n+4).$$

(note $\Phi(\text{even}) = -\Phi(\text{odd})$)

Conclusion
$$\begin{cases} \text{Odd } \cancel{\text{strip}}\ n \text{ value} \to \Phi(n) = \frac{1}{2}(x-4n+4)(x-4n) \\ \text{Even } n \text{ value} \to \Phi(n) = -\frac{1}{2}(x-4n)(x-4n+4) \end{cases}$$

Analyzing continuity, 2 adjacent n values, move from strip $n \to n+1$.

1) Let n be odd ($\varphi_+ \to \varphi_-$):



$$\frac{1}{2}(x-4n+4)(x-4n) \longrightarrow -\frac{1}{2}(x-4(n+1))\cdot(x-4(n+1)+4)$$

$$\frac{1}{2}(x-4n+4)(x-4n) \longrightarrow -\frac{1}{2}(x-4n-4)(x-4n)$$

Border point is $4n$.

$\varphi'(x)$ are:

$$\frac{1}{2}\left[(x-4n+4)+(x-4n)\right] = \frac{1}{2}\left[2x-8n+4\right] = x-4n+2 \longrightarrow$$

$$-\frac{1}{2}\left[(x-4n-4)+(x-4n)\right] = -\frac{1}{2}\left[2x-8n-4\right] = -x+4n+2.$$

$\varphi'(4n) = 2$ in both. Also $\left.\begin{array}{l}\lim\limits_{x\to 4n-}(x-4n+2) = 2, \\ \lim\limits_{x\to 4n+}(-x+4n+2) = 2\end{array}\right\} \varphi'$ is continuous.

2) Let n be even ($\varphi_- \to \varphi_+$):

$$-\frac{1}{2}(x-4n)(x-4n+4) \longrightarrow \frac{1}{2}(x-4(n+1)+4)(x-4(n+1))$$

$$-\frac{1}{2}(x-4n)(x-4n+4) \longrightarrow \frac{1}{2}(x-4n)(x-4n-4)$$

Border point is $4n$.

$\varphi'(x)$ are:

$$-\frac{1}{2}\left[(x-4n)+(x-4n+4)\right] = -\frac{1}{2}\left[2x-8n+4\right] = -x+4n-2 \longrightarrow$$

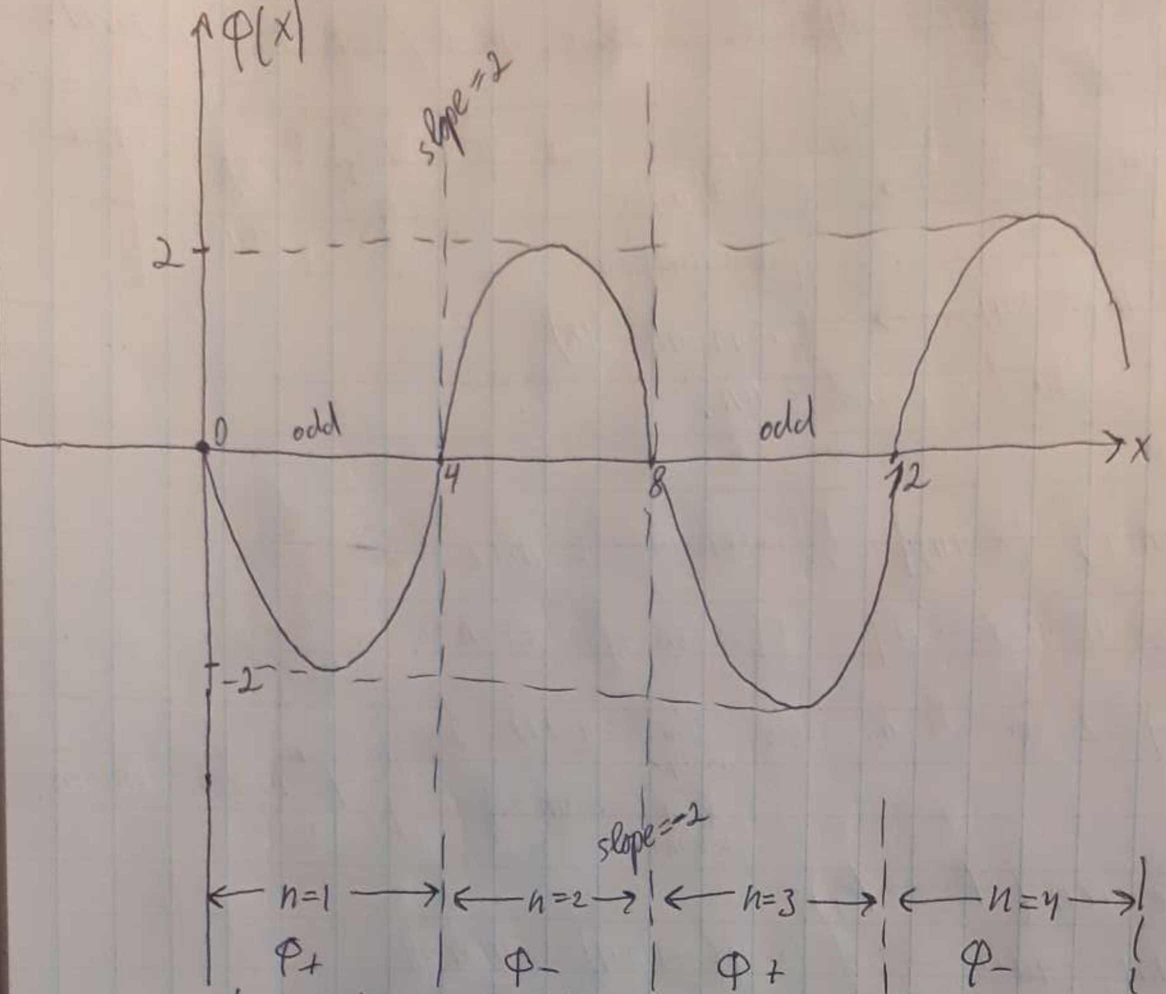
$$\frac{1}{2}\left[(x-4n)+(x-4n-4)\right] = \frac{1}{2}\left[2x-8n-4\right] = x-4n-2$$

$\varphi'(4n) = \underline{-2}$ in both. Also $\lim\limits_{x\to 4n-}(-x+4n-2) = \lim\limits_{x\to 4n+}(x-4n-2) = -2$.

$\to \varphi'$ is continuous.

The continuity of potential slope means force $\to$ and then $\ddot{x}, \dot{x}, x$ are also continuous/smooth.
(and ?)

See sketch of $\varphi(x)$ on the next page.



$\frac{1}{2}x(x-4)$ $\quad -\frac{1}{2}(x-8)(x-4)$ $\quad \frac{1}{2}(x-8)(x-12)$ $\quad -\frac{1}{2}(x-16)(x-12)$

see (*)/(**) to derive actual formulas, periodic from piecewise parabolas.

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b) EOM for particle. Using Lagrange formalism,

$$L = T - V = \frac{m\dot{x}^2}{2} - \varphi(x)$$

<u>Odd strips</u> $\quad L = \frac{m\dot{x}^2}{2} - \frac{1}{2}(x-4n+4)(x-4n)$

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\cdot\left(\frac{\partial L}{\partial \dot{x}}\right)$$

$$m\ddot{x} = -\frac{1}{2}[x-4n+\cancel{x}-4n+4] = -\frac{1}{2}[2x-8n+4] = -x+4n-2$$

$\boxed{m\ddot{x} = -x+4n-2}$ (harmonic after shifting x, see c))

<u>Even strips</u> $\quad L = \frac{m\dot{x}^2}{2} + \frac{1}{2}(x-4n)(x-4n+4)$

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right), \quad m\ddot{x} = \frac{1}{2}[2x-8n+4] = x-4n+2$$

$\boxed{m\ddot{x} = x - 4n + 2}$ (repulsive field, see c))
↳ we covered similar in central field HW also

c) Solutions for EOM:

Set coordinate $\tilde{x} = x - 4n + 2$, $\dot{\tilde{x}} = \dot{x}$, $\ddot{\tilde{x}} = \ddot{x}$,

Then on odd strips:

$$m\ddot{\tilde{x}} = -\tilde{x}$$

$$\tilde{x} = A\cos(\omega t + \varphi_0), \quad \omega = \sqrt{\frac{1}{m}}$$

harmonic oscillations

Then on even strips

$$m\ddot{\tilde{x}} = +\tilde{x}$$

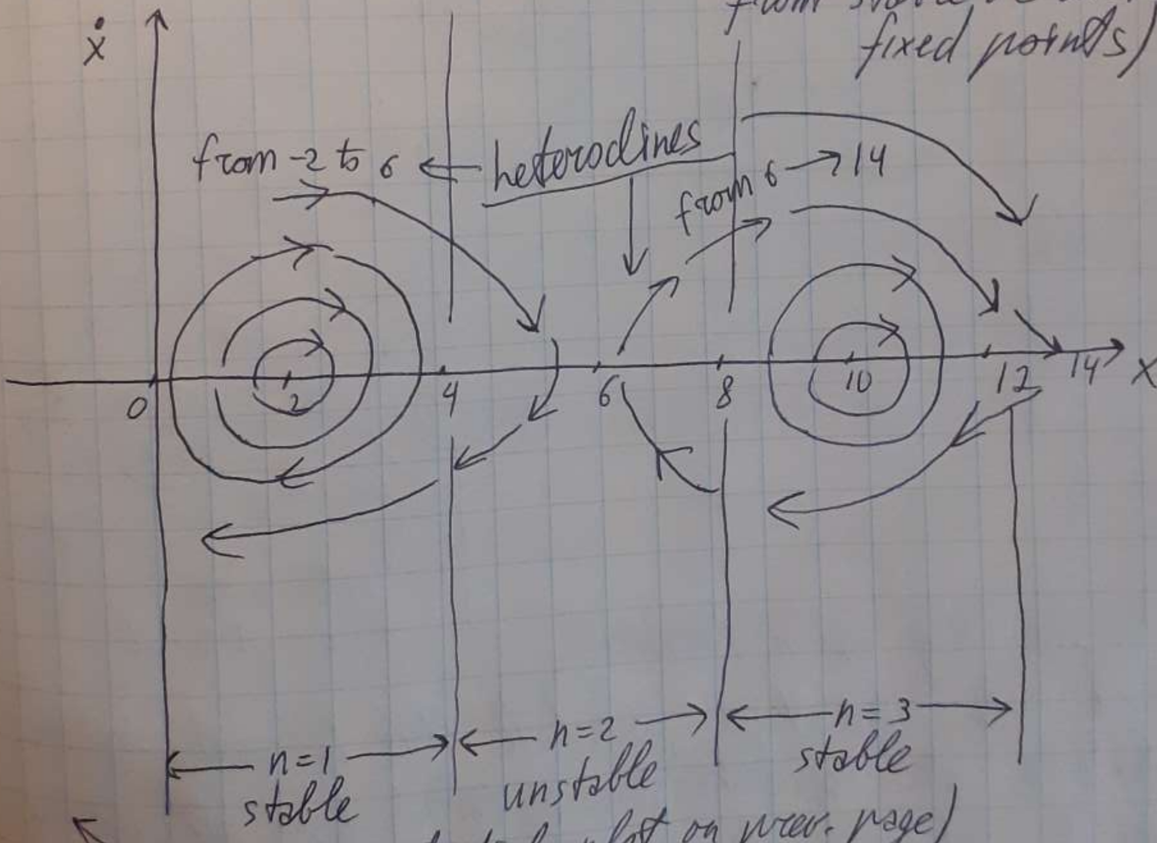
$$m\ddot{\tilde{x}} - \tilde{x} = 0 \qquad m\lambda^2 - 1 = 0$$

$$\lambda = \pm\frac{1}{\sqrt{m}}$$

$$\tilde{x} = C_1 e^{\frac{1}{\sqrt{m}}t} + C_2 e^{\frac{-1}{\sqrt{m}}t}$$

exponential increase in displacement

d) Phase space for solutions ($\tilde{x}$ above refer to displacements from stable or unstable fixed points)

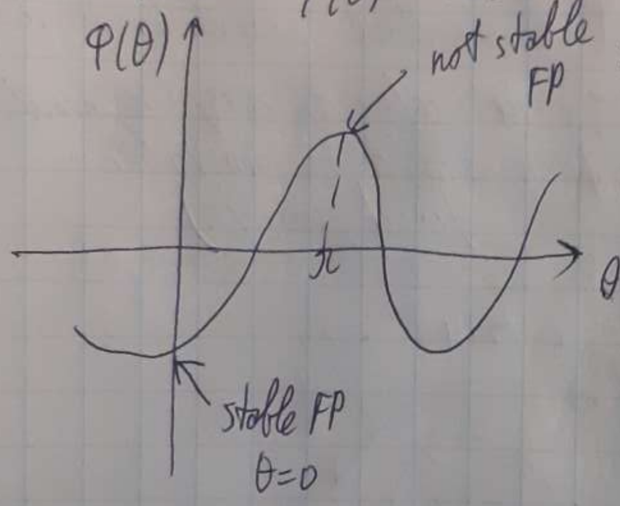


(see potential plot on prev. page)

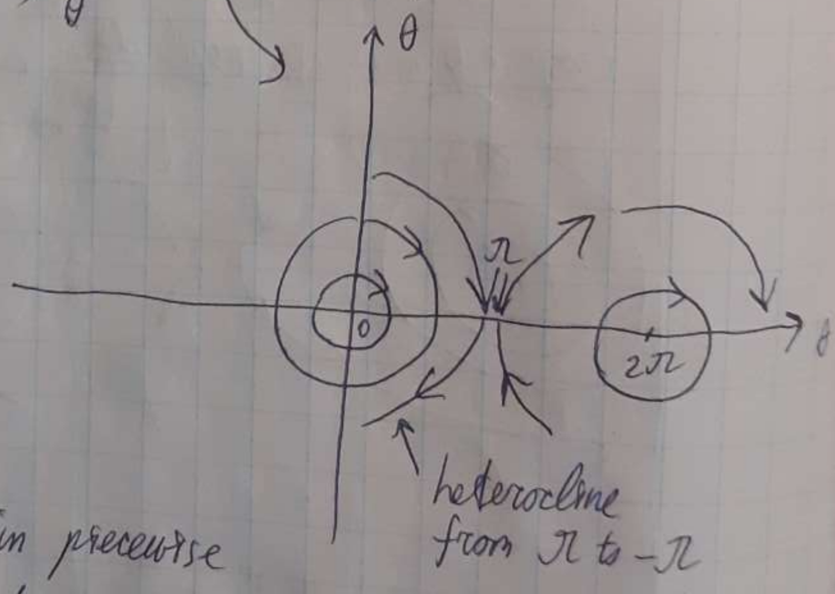
Curves between unstable FP are heteroclines rather than homoclines because same closest potential values are all unstable maxima.

e) In math. pendulum case there is strong similarity. Potential is always $-\cos\theta$ (or $-k\cos\theta$) but it is periodic trig. function unlike parabola, therefore it also has minima and maxima. This corresponds to the fact that $\cos\theta$ can be expanded as Taylor series; with alternating positive & negative components, correspond to the fact that odd/even strips in piecewise parabolic potential switch concavity.

Here: $\varphi(\theta) = -\cos\theta$



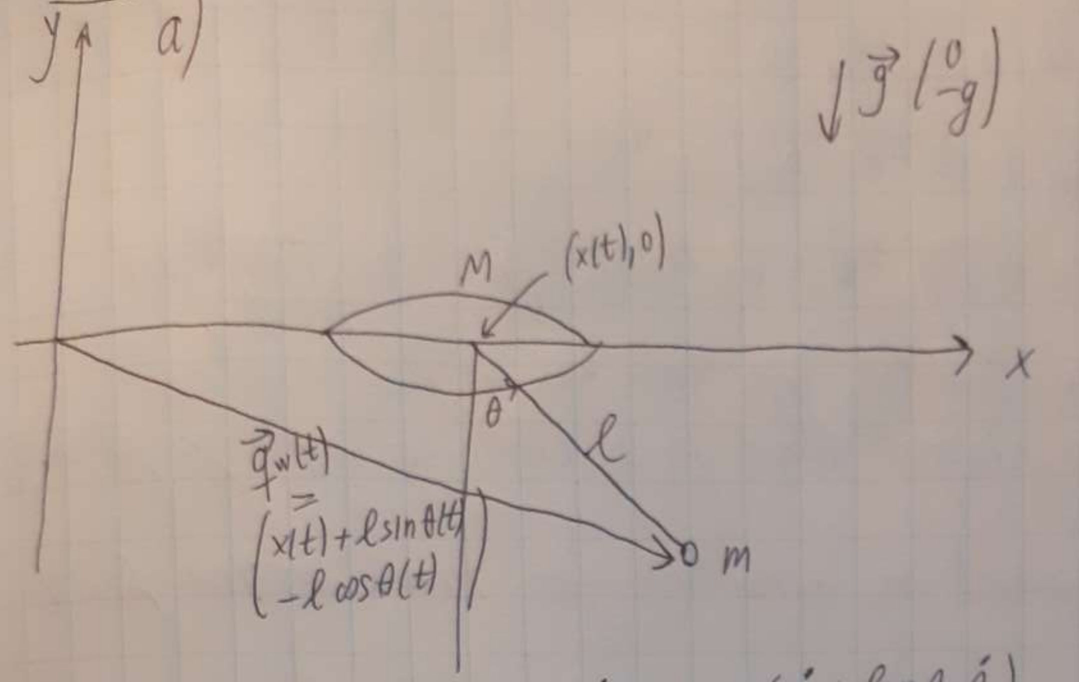
and since same $\varphi(\theta)$ closest is also maxima, the curve is heterocline



Similarly in piecewise parabolic case, even strips „simulate" this heterocline behaviour with repulsive" force (potential drops with displacement ↑), and push particle back to odd strips etc.

Problem 12.2 Pendulum on rails

a)



b) $T_c = \frac{M}{2}(\dot{x})^2$ $\qquad \dot{\vec{q}}_W(t) = \begin{pmatrix} \dot{x} + \ell\cos\theta\cdot\dot{\theta} \\ \ell\sin\theta\cdot\dot{\theta} \end{pmatrix}$

$V_c = 0$

$$T_W = \frac{m}{2}(\dot{\vec{q}}_W)^2 = \frac{m}{2}\left(\dot{x}^2 + \ell^2(\dot{\theta})^2 + 2\dot{x}\dot{\theta}\,\ell\cos\theta\right)$$

$$V_W = -mg\,\ell\cos\theta \quad (\text{assume } V_W(y=0)=0)$$

Then $L = T_C + T_W - V_W = \frac{M}{2}(\dot{x})^2 + \frac{m}{2}\left(\dot{x}^2 + \ell^2\dot{\theta}^2 + 2\dot{x}\dot{\theta}\,\ell\cos\theta\right) + mg\ell\cos\theta =$

$$= \frac{M+m}{2}(\dot{x})^2 + \frac{m}{2}\left(\ell^2\dot{\theta}^2 + 2\dot{x}\dot{\theta}\,\ell\cos\theta\right) + mg\ell\cos\theta =$$

$$= \frac{M+m}{2}(\dot{x})^2 + \frac{m}{2}\ell^2(\dot{\theta})^2 + m\dot{x}\dot{\theta}\,\ell\cos\theta + mg\ell\cos\theta$$

no x dependence (cyclic v.)

c) Using Euler-Lagrange: $\frac{\partial L}{\partial x} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)$

$$0 = \frac{d}{dt}\left[(M+m)\dot{x} + m\dot{\theta}\ell\cos\theta\right] = (M+m)\ddot{x} + m\ell\,\ddot{\theta}\cos\theta - m\ell\sin\theta(\dot{\theta})^2$$

<u>EOM:</u> $(M+m)\ddot{x} + m\ell\left(\ddot{\theta}\cos\theta - \sin\theta(\dot{\theta})^2\right) = 0$

$(M+m)\dot{x} + m\dot{\theta}\ell\cos\theta = p_x$ is <u>constant of motion</u>

$\underline{\mathcal{L} = M+m}$

$\underline{\beta = m\ell}$

d) $x_{CM} = \frac{M x(t) + m\,\vec{q}_W\cdot\hat{x}}{M+m} = \frac{Mx + m(x+\ell\sin\theta)}{M+m} = x + \ell\frac{m}{M+m}\sin\theta =$

$$= x + \frac{m}{M+m}\ell\sin\theta.$$

Since there is no horizontal force acting, $\dot{x}'_{CM} = 0$.

So $\dot{x}_{CM} = Q$ is COM, so

$$\dot{x} + \frac{m}{M+m} l \cos\theta \cdot \dot{\theta} = Q \;\Big|\cdot (M+m)$$

$$\dot{x}(M+m) + ml\cos\theta\cdot\dot{\theta} = Q(M+m) = p_x \longrightarrow \text{COM from part c)}$$

(Note here Newton approach was used and also showed that p_x is COM, as Lagrange approach in c))

p_x can be viewed as momentum for transl. motion of CM.

e) Given:

$$p_x = 0 \longleftrightarrow (M+m)\dot{x} + m\dot{\theta} l\cos\theta = 0 \quad (\nabla)$$

Find EOM for θ with Euler-Lagrange; note:
(EOM is found with Lagrange, but without using (∇) possible only in CM frame, the resulting simplification to reduced mass etc. description will not work, we <u>need</u> to switch to CM frame)

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\cdot\left(\frac{\partial L}{\partial \dot{\theta}}\right) \text{ for}$$

$$L = \frac{M+m}{2}(\dot{x})^2 + \frac{m}{2}l^2(\dot{\theta})^2 + m\dot{x}\dot{\theta}\, l\cos\theta + mgl\cos\theta$$

$$\frac{\partial L}{\partial \theta} = -m\dot{x}\dot{\theta}\, l\sin\theta - mgl\sin\theta = -ml\sin\theta(\dot{x}\dot{\theta} + g) \quad (*)$$

$$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} + m\dot{x}\, l\cos\theta, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta} + m\ddot{x}\, l\cos\theta - m\dot{x}\, l\sin\theta\dot{\theta} \quad (**)$$

Since $(*) = (**)$

$$-ml\sin\theta(\dot{x}\dot{\theta} + g) = ml^2\ddot{\theta} + m\ddot{x}\, l\cos\theta - m\dot{x}\, l\sin\theta\cdot\dot{\theta} \;\Big|\cdot\frac{-1}{ml}$$

$$\sin\theta(\dot{x}\dot{\theta} + g) = -l\ddot{\theta} - \ddot{x}\cos\theta + \dot{x}\dot{\theta}\sin\theta \;\Big| - \dot{x}\dot{\theta}\sin\theta$$

• is

no

*

*

— *

$$g\sin\theta = -l\ddot{\theta} - \ddot{x}\cos\theta \quad (\square)$$

Now recall $(\triangledown)$: $\dot{x} = -\frac{m\dot{\theta}l\cos\theta}{M+m}$

$$\ddot{x} = -\frac{ml}{M+m}\left(\ddot{\theta}\cos\theta - \sin\theta\cdot(\dot{\theta})^2\right)$$

Now from $(\square)$: $g\sin\theta = -l\ddot{\theta} + \frac{ml}{M+m}\cos\theta\left(\ddot{\theta}\cos\theta - \sin\theta\cdot(\dot{\theta})^2\right)$

to get rid of $x, \dot{x}, \ddot{x}$ etc.,

$$\ddot{\theta}\left(-l + \frac{ml}{M+m}\cos^2\theta\right) = g\sin\theta + \frac{ml}{M+m}\sin\theta\cos\theta(\dot{\theta})^2$$

$$\ddot{\theta} = -\sin\theta\,\frac{g + \frac{ml}{M+m}\cos\theta(\dot{\theta})^2}{l - \frac{ml}{M+m}\cos^2\theta}$$

$$\ddot{\theta} = -\sin\theta\cdot\frac{\frac{g}{l} + \frac{m}{M+m}\cos\theta(\dot{\theta})^2}{1 - \frac{m}{M+m}\cos^2(\theta)},$$

$\boxed{\mu = \frac{m}{M+m}}$ — not reduced mass here, but ratio

$$\ddot{\theta} = -\sin\theta\cdot\frac{g/l + \cancel{\frac{m}{M+m}}\mu\cos\theta(\dot{\theta})^2}{1 - \mu\cos^2(\theta)}$$

$[g/l] = \frac{1}{s^2}$, $[g/l] = \frac{1}{[t]^2}$, $[g/l]^{\frac{1}{2}} = \frac{1}{[t]}$, $[t] = \left[\frac{l}{g}\right]^{\frac{1}{2}}$

Dim. time $\tau = \frac{t}{\sqrt{l/g}}$, $\boxed{t = \sqrt{\frac{l}{g}}\cdot\tau}$ is time scale.

Then indeed: $$\frac{d^2\theta}{d\left(\sqrt{\frac{l}{g}}\tau\right)^2} = \frac{(-\sin\theta)\left(g/l + \mu\cos\theta\left(\frac{d\theta}{d\left(\tau\sqrt{\frac{l}{g}}\right)}\right)^2\right)}{1-\mu\cos^2\theta}$$

$$\cancel{\frac{g}{l}}\frac{d^2\theta}{d\tau^2} = -\sin\theta\cdot\frac{\cancel{\frac{g}{l}}\left(1 + \mu\cos\theta\left(\frac{d\theta}{d\tau}\right)^2\right)}{1-\mu\cos^2\theta} \quad \Big|\cdot\frac{l}{g}$$

$\cdot$ is $\frac{d}{d\tau}$ now $\rightarrow$ $\boxed{\ddot{\theta} = -\sin\theta\cdot\frac{1+\mu\cos\theta(\dot{\theta})^2}{1-\mu\cos^2\theta}}$ is EOM for θ in CM frame

*f) because of unit analysis (at least)

*g) because $\mu \to 0 \longleftrightarrow m \ll M$ and CM is basically same as CM of cart only, thus relative to it only pendulum oscillations are left.

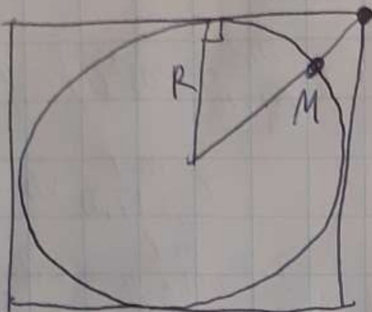
— *h, i) are important to cover when time allows.

Problem 12.3 Particles on wheel and pulley

[Assumptions! and important considerations]

#1) Mass m_2 is always „tied" to wheel.

#2) Since it is stated in b) about linearity at $|\theta| \geq \frac{\pi}{4}$,



pulley is in right up edge of square surrounding the wheel

#3) Since it is stated in the task that rope has fixed length always.

#4) For $\theta = 0$ m_2 stays in point M.

~~#5~~ All these allow to clearly specify everything including $z(\theta, |\theta| < \frac{\pi}{4})$ if only knowing one number

- z_0 or $z(\frac{\pi}{4}) = z(-\frac{\pi}{4})$, shown below
- or length of rope L.

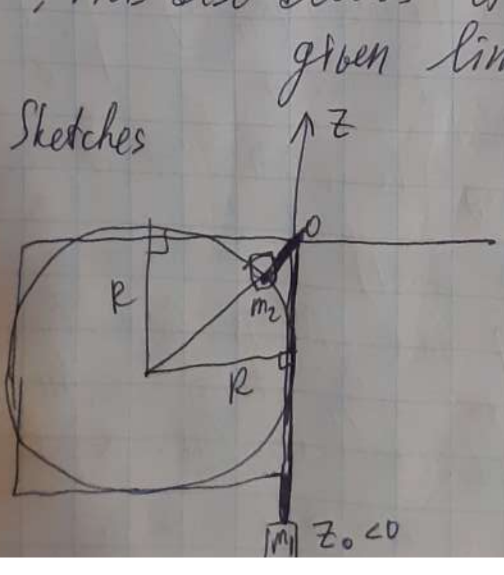
For this task I chose z_0 as reference point.

After that can get $z(\pm\frac{\pi}{4})$, L etc,

! This also allows to specify domain of $z(\theta)$ given limitations on rope length L.

a) Sketches

Case 1, $\theta = 0$



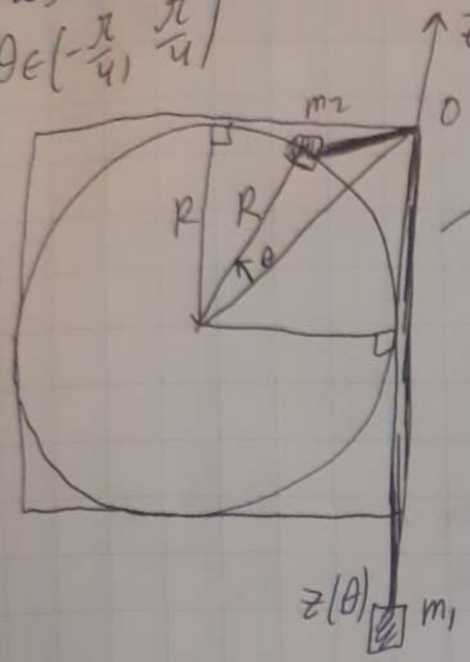
Let z be negative, and $V(0) = 0$.

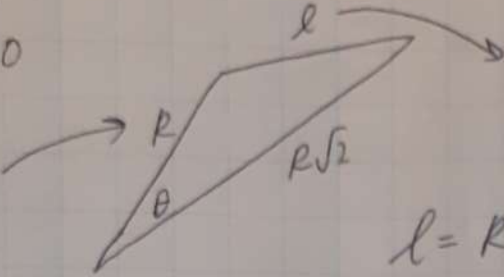
Then length of rope L =

$$\boxed{= R\sqrt{2} - R + |z_0|}$$

Case 2

$\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$





(cosine law)

$$l^2 = 2R^2 + R^2 - 2\sqrt{2}R^2\cos\theta = R^2(3-2\sqrt{2}\cos\theta)$$

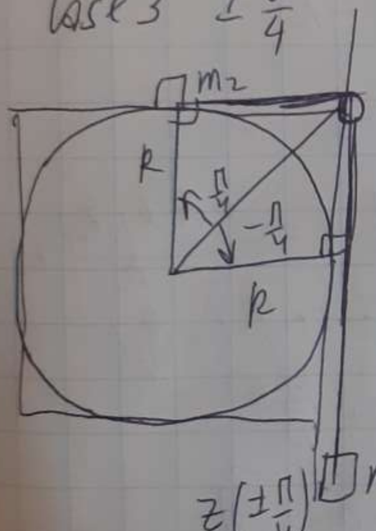
$$l = R\sqrt{3-2\sqrt{2}\cos\theta}$$ (when $\theta=0$, $R\sqrt{3-2\sqrt{2}} = (\sqrt{2}-1)R$ as expected)

Then $|z(\theta)| = R\sqrt{2} - R + |z_0| - R\sqrt{3-2\sqrt{2}\cos\theta}$,

$$\boxed{z(\theta) = -|z(\theta)| = R\left(\sqrt{3-2\sqrt{2}\cos\theta} - \sqrt{2}+1\right) - |z_0|}$$

most generic, if known z_0

Case 3 $\pm\frac{\pi}{4}$

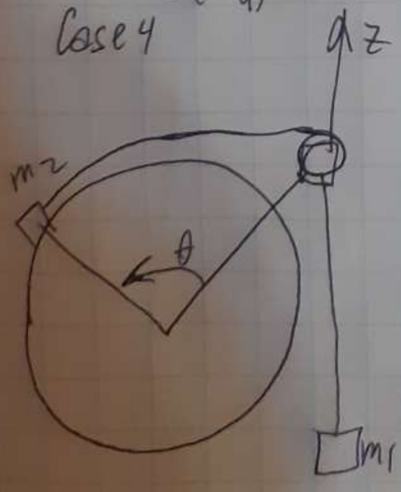


$z\left(\frac{\pi}{4}\right) = z\left(-\frac{\pi}{4}\right)$ due to symmetry (length outside $|z|$ is always R)

$$z\left(\pm\frac{\pi}{4}\right) = -(l-R) = (2-\sqrt{2})R - |z_0|$$

also matches special case for $z(\theta)$!

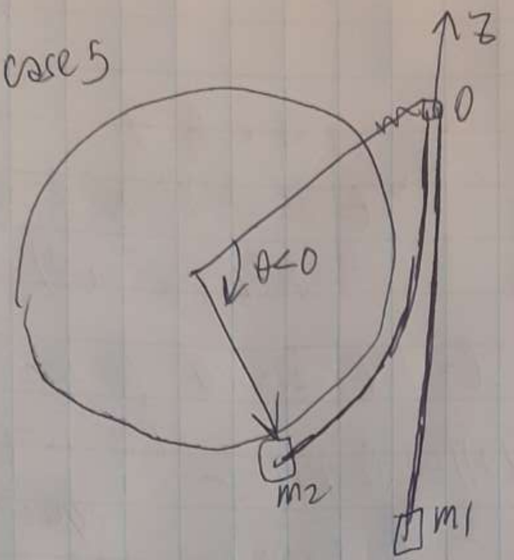
Case 4



(Here triangle approach does not work, dependency will be linear)

$$z\left(\theta > \frac{\pi}{4}\right) = z\left(\frac{\pi}{4}\right) + \left(\theta - \frac{\pi}{4}\right)R = (2-\sqrt{2})R - |z_0| + \left(\theta - \frac{\pi}{4}\right)R = \theta R + \underbrace{\left(2-\sqrt{2}-\frac{\pi}{4}\right)R - |z_0|}_{const}$$

case 5



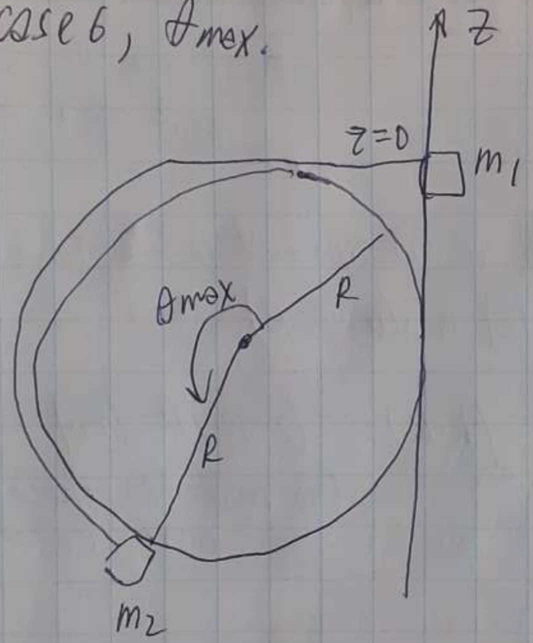
$$z(\theta<-\tfrac{\pi}{4}) = z(-\tfrac{\pi}{4}) + (-\tfrac{\pi}{4}-\theta)R =$$

$$= (2-\sqrt{2})R - |z_0| + (-\tfrac{\pi}{4}-\theta)R =$$

$$= \underbrace{(2-\sqrt{2}-\tfrac{\pi}{4})R - |z_0|}_{\text{same cond as before}} - \theta\cdot R$$

Now can find maximum/minima allowed angles!

case 6, θ_{max}.



$$(\theta_{max}-\tfrac{\pi}{4})R + R = L$$

$$R(1+\theta_{max}-\tfrac{\pi}{4}) = L$$

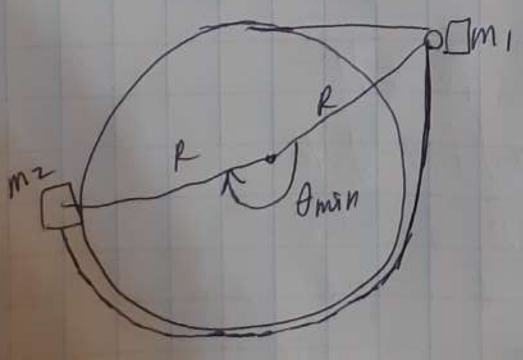
$$\theta_{max} = \frac{L}{R} + \frac{\pi}{4} - 1 = \frac{R\sqrt{2}-R+|z_0|}{R} +$$

$$+\frac{\pi}{4} - 1 = \sqrt{2}-1+\frac{\pi}{4}-1+\frac{|z_0|}{R} =$$

$$= \sqrt{2}+\frac{\pi}{4}-2+\frac{|z_0|}{R}$$

(clear >0)

case 7, θ_{min}.



$$(-\theta_{min}-\tfrac{\pi}{4})R + R = L$$

$$R(1-\tfrac{\pi}{4}-\theta_{min}) = L$$

$-\theta_{min} = \theta_{max}$ then,

$$\theta_{min} = 2-\sqrt{2}-\frac{\pi}{4}-\frac{|z_0|}{R}$$

Everything specified based on knowing just $\underline{z_0}$.

b) The slopes are R and $-R$ respectively (inverses of each other)

just see cases 4/5 in a).

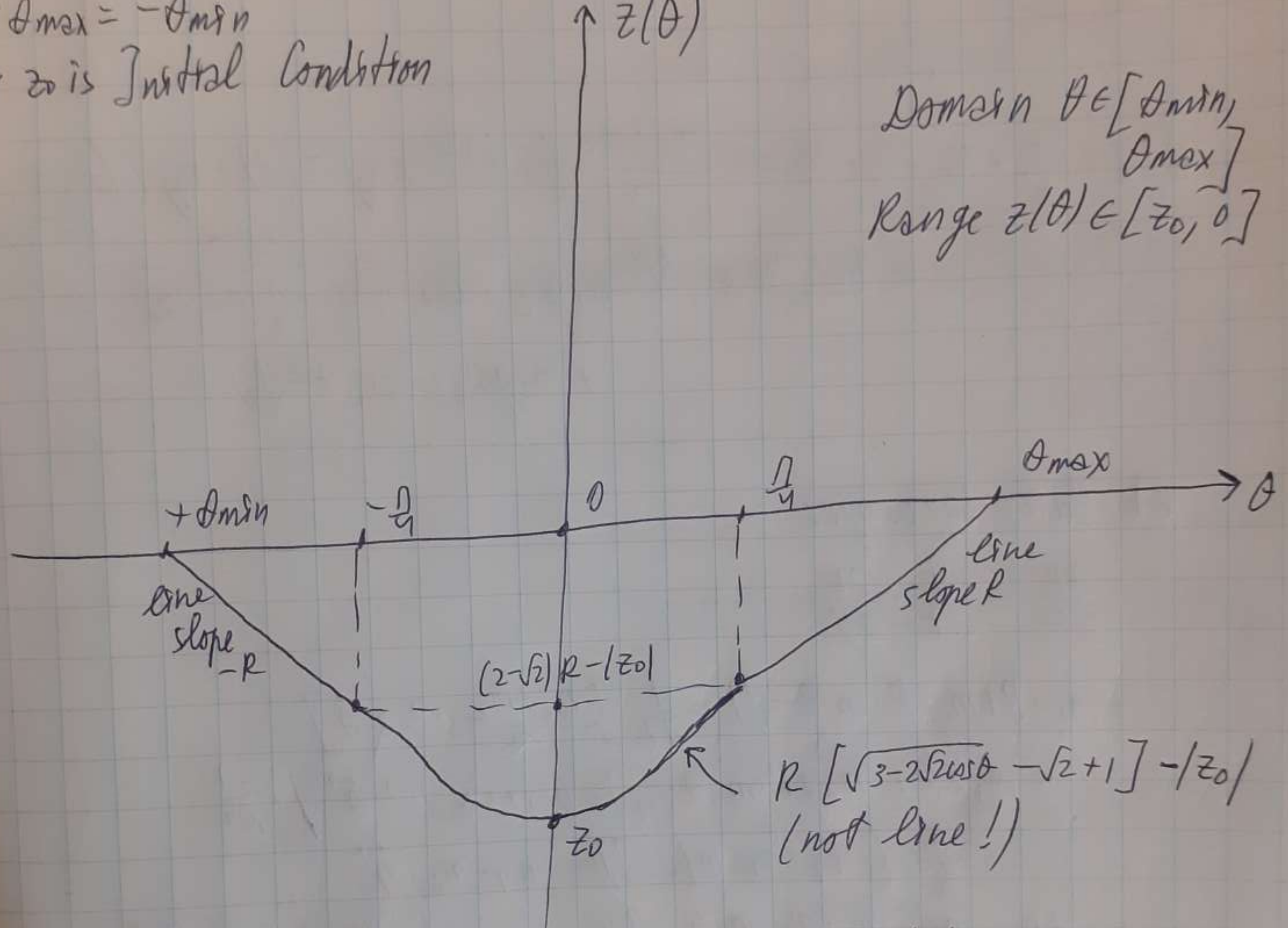
Now using a), full description of $z(\theta)$:

(assuming $z_0 < -R$, otherwise nothing interesting)

* $\theta_{min} = 2 - \sqrt{2} - \frac{\pi}{4} - \frac{|z_0|}{R}$
* $\theta_{max} = -\theta_{min}$
* z_0 is Initial Condition

Domain $\theta \in [\theta_{min}, \theta_{max}]$

Range $z(\theta) \in [z_0, 0]$



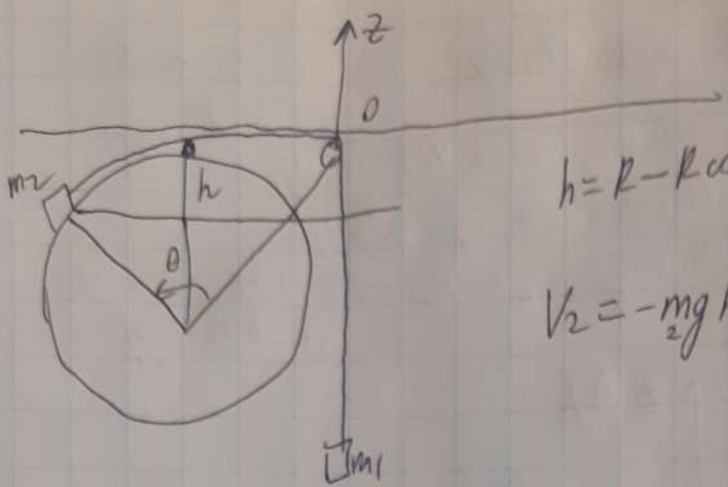
c) For this & later tasks, since EOMs and Lagrangian will differ in θ intervals, assuming $\theta \geq \frac{\pi}{4}$ not get mad.

$$T_1 = \frac{m_2}{2}(\dot{z}) = \frac{m_2}{2} R^2 (\dot{\theta})^2$$

$$V_1 = m_1 g z$$

$$T_2 = \frac{m_1}{2} (\underbrace{\dot{\theta} R}_{\text{linear velocity around } \circlearrowleft})^2 = \frac{m_1}{2} R^2 (\dot{\theta})^2$$

Now find V_2:



$$h = R - R\cos\left(\theta - \frac{\pi}{4}\right) = R\left(1-\cos\left(\theta - \frac{\pi}{4}\right)\right)$$

$$V_2 = -m_2 g h = -m_2 g R\left(1-\cos\left(\theta - \frac{\pi}{4}\right)\right)$$

d) $$L = T_1 + T_2 - V_1 - V_2 = \frac{m_1+m_2}{2}(R\dot\theta)^2 - m_1 g\, z(\theta) + m_2 g R\left(1-\cos\left(\theta - \frac{\pi}{4}\right)\right)$$

$$= \frac{m_1+m_2}{2} R^2 (\dot\theta)^2 - m_1 g\left[\theta R + \overbrace{\left(2-\sqrt{2}-\frac{\pi}{4}\right)R - |z_0|}^{const}\right] + m_2 g R\left(1-\cos\left(\theta-\frac{\pi}{4}\right)\right)$$

e) Euler-Lagrange:

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\cdot\frac{\partial L}{\partial \dot\theta}$$

$$-m_1 g R + m_2 g R \sin\left(\theta - \frac{\pi}{4}\right) = \frac{d}{dt}\left[\frac{m_1+m_2}{\cancel{2}} R^2 \cdot \cancel{2}\dot\theta\right]$$

$$-m_1 g R + m_2 g R\sin\left(\theta-\frac{\pi}{4}\right) = (m_1+m_2)R^2\ddot\theta \quad \Big| \cdot \frac{1}{R^2}$$

$$\frac{g}{R}\left(-m_1 + m_2\sin\left(\theta - \frac{\pi}{4}\right)\right) = (m_1+m_2)\ddot\theta$$

$$\ddot\theta = \frac{g\left(-m_1+m_2\sin\left(\theta-\frac{\pi}{4}\right)\right)}{R(m_1+m_2)} = \frac{g}{R}\,\frac{\left[-\frac{m_1}{m_2} + \sin\left(\theta - \frac{\pi}{4}\right)\right]}{\frac{m_1}{m_2}+1} =$$

$$= \underbrace{-\frac{g}{R}\,\frac{m_2}{m_1+m_2}}_{K_2}\left(+\frac{m_1}{m_2} - \sin\left(\theta + \left(-\frac{\pi}{4}\right)\right)\right)$$

K_1 $s(\theta)=1$ in this case

θ_2 in this case

$[K_1]=1$

$[K_2]=s^{-2}$, time scale can be done

? f) Show single fixed point for $\kappa >> 1$.

Fixed points mean $\vec{V_\theta} = \begin{pmatrix}\dot{\theta} \\ \ddot{\theta}\end{pmatrix} = \begin{pmatrix}0 \\ 0\end{pmatrix}$

$$\ddot{\theta} = \tilde{K}(\kappa s(\theta) - \sin(\theta+\theta_2))$$

$\boxed{\text{Set } \ddot{\theta}=0,}$ $\kappa s(\theta) - \sin(\theta+\theta_2) = 0$, assume $s(\theta)=1$

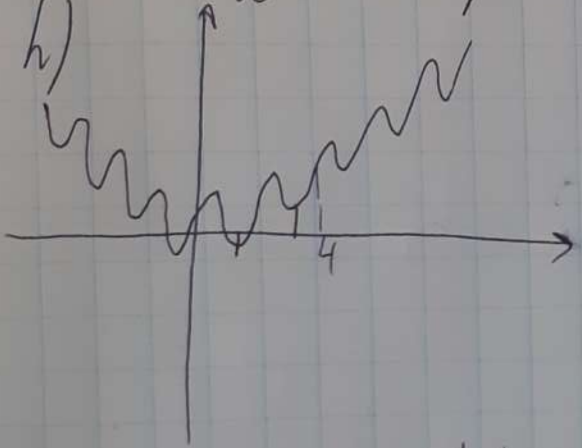
Then $\underbrace{\sin(\theta+\theta_2)}_{=\kappa} >> 1$ which is not possible

Then $\tilde{K} = 0$, $\frac{m_2}{m_1+m_2} \to 0$, $m_1 >> m_2$ (as should be since $\kappa >> 1$)

(not sure how to proceed here, finding <u>one</u> solution)

? Perhaps I should analyze $s(\theta)$, not say it is 1.

h)



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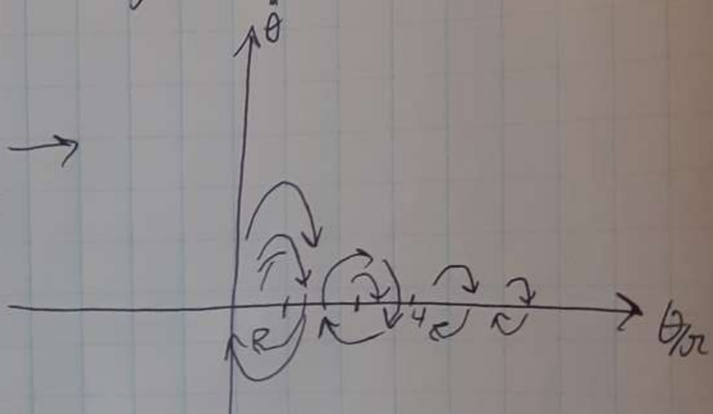

Like in task 1, but difference:

1) All curves are <u>homocline</u>, because the same closest potential values near unstable FP are not extrema

2) Since potential grows, the velocity absolute value drops, while in piecewise harmonic $U(x)$ was periodic, and velocity did not drop in phase space.