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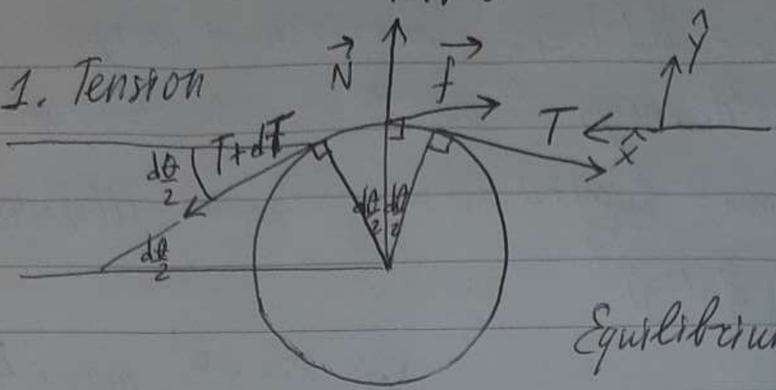
HW-6

1. Tension



Equilibrium:

$$\vec{T_L}+\vec{T_H}+\vec{N}+\vec{f}=\vec{0},\quad |\vec{T_L}|=T+dT,\quad |\vec{T_H}|=T.$$

$$\begin{cases}(T+dT)\cos\frac{d\theta}{2}-T\cos\frac{d\theta}{2}-f=0\\ f=\mu N\\ N=(T+dT)\sin\frac{d\theta}{2}+T\sin\frac{d\theta}{2}\end{cases} \Longrightarrow (T+dT)\cos\frac{d\theta}{2}-T\cos\frac{d\theta}{2}-\mu\sin\frac{d\theta}{2}[2T+dT]=0$$

$$\cos\frac{d\theta}{2}\xrightarrow{d\theta/2\to 0}1-\left(\frac{d\theta}{2}\right)^2\Big/2=1-\frac{d\theta^2}{8}$$

$$\sin\frac{d\theta}{2}\xrightarrow{d\theta/2\to 0}\frac{d\theta}{2}$$

Note: $\underset{d\theta\to 0}{\delta=o(d\theta)}\Longleftrightarrow\lim_{d\theta\to 0}\frac{\delta}{d\theta}=0$

Therefore $\cancel{T+d}\ dT\cos\frac{d\theta}{2}=\mu\frac{d\theta}{2}[2T+dT]$

$$dT[1-\underbrace{\frac{d\theta^2}{8}}_{o(d\theta)}]=\mu(d\theta)T+\underbrace{\frac{\mu}{2}d\theta dT}_{o(d\theta)}$$

$$dT=\mu(d\theta)T$$

$$\frac{dT}{d\theta}=\mu T,\quad \int\frac{dT}{T}=\int\mu d\theta\Longrightarrow \ln T=\mu\theta+C,$$

$$T=Ce^{\mu\theta},$$

$$\boxed{T(\theta)=T(0)e^{\mu\theta}}$$

2. Lennard-Jones potential for molecules

$$U(r)=4\varepsilon\left[\left(\frac{r_m}{r}\right)^{12}-2\left(\frac{r_m}{r}\right)^6\right]$$

a) Equilibrium is reached when U is minima (= zero force).

$$-\frac{\partial U}{\partial r}=0\Longrightarrow 4\varepsilon\left[12\left(\frac{r_m}{r}\right)^{11}\cdot\frac{(-r_m)}{r^2}-2\cdot 6\left(\frac{r_m}{r}\right)^5\left(-\frac{r_m}{r^2}\right)\right]=$$

$$=48\varepsilon\left[-\frac{r_m^{12}}{r^{13}}+\frac{r_m^6}{r^7}\right]=0$$

$$-r_m^{12}+r_m^6r^6=0\Longrightarrow r^6-r_m^6=0\Longrightarrow\boxed{r=r_m},\ U(r_m)\text{ is minima.}$$

$U'(r)$: $-$ | $+$

$U(r)$: $\searrow$ r_m $\nearrow$

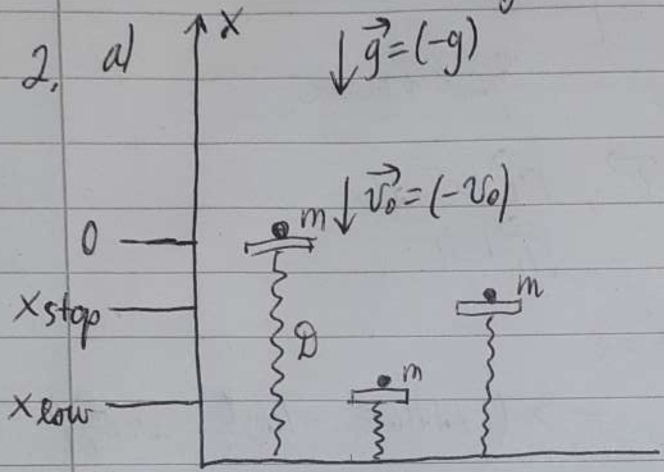
b) To evaporate (similar idea as of escape velocity in gravity):

$U_1(r) + E_{K1} = U_2(r) + E_{K2}$ (conservation of energy)

Minimum kinetic energy to move to ∞ with $v=0$ after:

$$U(r) + E_{k\,min} = \underbrace{U(\infty)}_{0} + 0 = 0 \Longrightarrow E_{k\,min} = -U(r) = 4\varepsilon\left[2\left(\frac{r_m}{r}\right)^6 - \left(\frac{r_m}{r}\right)^{12}\right]$$

2. a)



Set $E = \frac{mv^2}{2} + mgx + \frac{Dx^2}{2}$

where x is spring compres.

Energy cons.

b) $\frac{mv_0^2}{2} = mgx_{low} + \frac{Dx_{low}^2}{2}$

$$v_0 = \sqrt{\frac{2}{m}\left[mgx_{low} + \frac{Dx_{low}^2}{2}\right]} = \sqrt{2gx_{low} + \frac{D}{m}x_{low}^2} =$$

$$= \sqrt{2\cdot 10\frac{m}{s^2}\cdot(-0.2m) + 270\frac{N}{m}\cdot\frac{0.2^2m^2}{0.275kg}} \approx 5.9\ \frac{m}{s}$$

c) $\frac{(2m)v_0^2}{2} = (2m)g\,x_{low} + \frac{Dx_{low}^2}{2}$

$mv_0^2 = 2mg\,x_{low} + \frac{Dx_{low}^2}{2} \,\Big|\cdot\frac{2}{D}$

$x_{low}^2 + \frac{4mg\,x_{low}}{D} - \frac{2mv_0^2}{D} = 0$, $\mathcal{D} = \left(\frac{4mg}{D}\right)^2 + \left(\frac{8mv_0^2}{D}\right)$, $x_{low1,2} =$

$$= \frac{-\frac{4mg}{D} \pm \sqrt{16\left(\frac{mg}{D}\right)^2 + \frac{8mv_0^2}{D}}}{2}$$ (only negative solution!) $$= -\frac{2mg}{D} - \sqrt{4\left(\frac{mg}{D}\right)^2 + \frac{2mv_0^2}{D}} =$$

$$= \frac{-2\cdot0.275\,kg\cdot10\frac{m}{s^2}}{270\ N/m} - \sqrt{4\left(\frac{0.275kg\cdot10\frac{m}{s^2}}{270\frac{N}{m}}\right)^2 + \frac{2\cdot0.275kg\cdot5.9^2\frac{m^2}{s^2}}{270\ N/m}} \approx -0.29\ m.$$

(1.5 more)

d) Cannot consider energy conservation due to friction — Newton Laws:

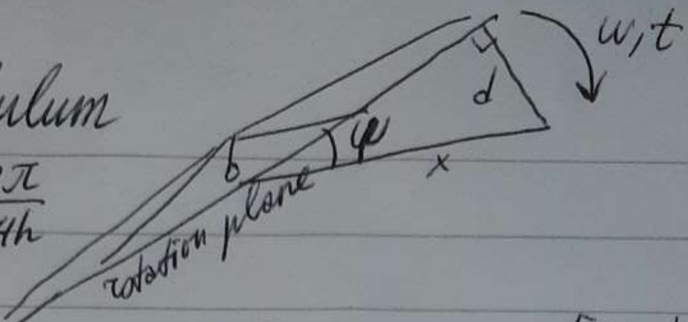
$m\vec{g} + D\vec{x} = \vec{0}$

$-mg - Dx = 0$

$$x = -\frac{mg}{D} = \frac{-0.275\,kg\cdot10\frac{m}{s^2}}{270\frac{N}{m}} = \underline{-0.01\ m} \approx 0$$ -1cm (almost all energy lost).

~~Alternative~~

3. Foucalt Pendulum



$$\omega = \sin(\varphi)\frac{2\pi}{24h}$$

$$\psi = \underline{\operatorname{asin}\frac{d}{x}} = \omega t = \underline{\sin(\varphi)\frac{2\pi}{24h}t} \Longrightarrow \varphi = \operatorname{asin}\left[\left(\operatorname{asin}\frac{d}{x}\right)\cdot\frac{24h}{2\pi t}\right] =$$

$$= \operatorname{asin}\left[\frac{24\cdot 60\,\text{min}}{2\pi\cdot 5.5\,\text{min}}\operatorname{asin}\left(\frac{3\,\text{cm}}{163\,\text{cm}}\right)\right] \approx 0.87 \approx 50°$$ which is Leipzig latitude.