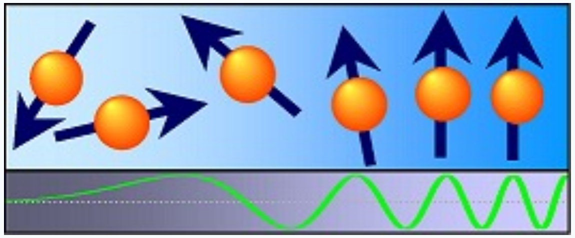
Experimental Physics
EP1 MECHANICS

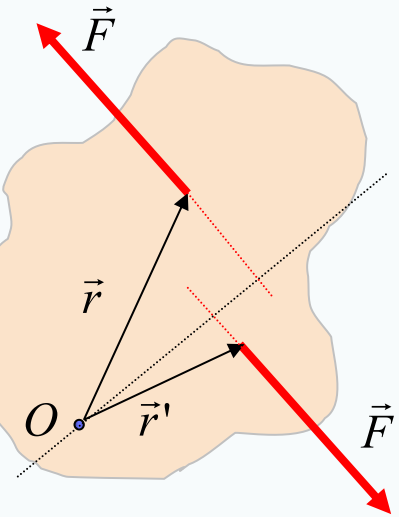
- Static Equilibrium -



Rustem Valiullin

https://www.physgeo.uni-leipzig.de/en/fbi/applied-magnetic-resonance

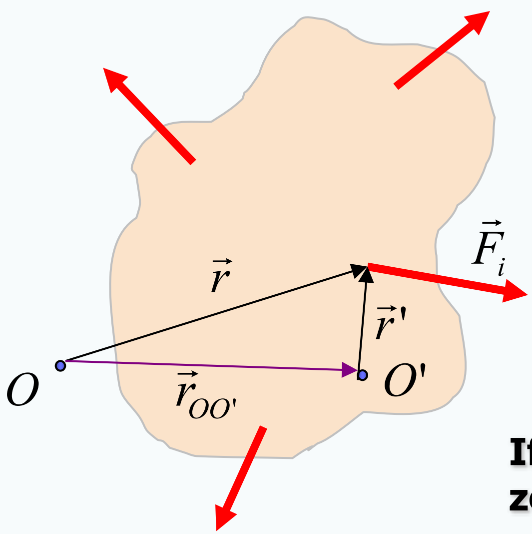
Conditions for equilibrium



The net external force must be zero $\boxed{\vec{F}_{net} = 0}$

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta = Fd$$

The net external torque about any axis must be zero $\boxed{\vec{\tau}_{net} = 0}$



$$\vec{F}_{net} = \sum_i \vec{F}_i = 0 \qquad \vec{\tau}_{net,O} = \sum_i \vec{r}_{iO} \times \vec{F}_i$$

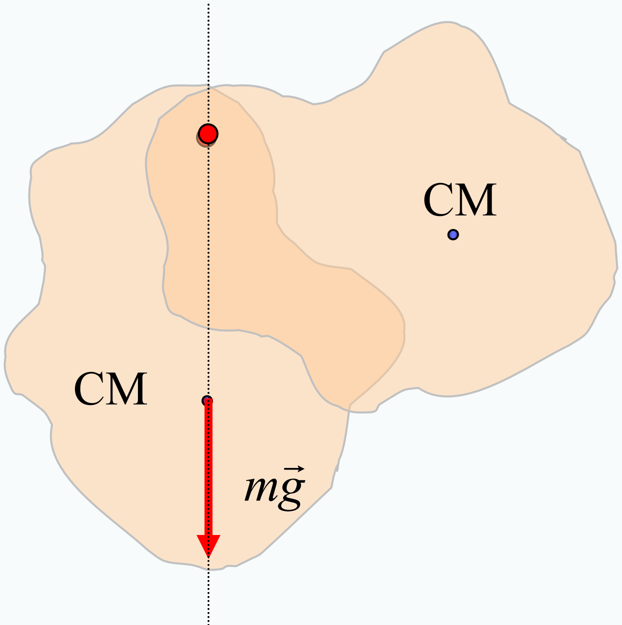
$$\vec{\tau}_{net,O'} = \sum_i \vec{r}\,'_i \times \vec{F}_i \quad = \vec{r}\,'_1 \times \vec{F}_1 + .. + \vec{r}\,'_i \times \vec{F}_i + ..$$

$$= (\vec{r}_1 - \vec{r}_{OO'}) \times \vec{F}_1 + .. + (\vec{r}_i - \vec{r}_{OO'}) \times \vec{F}_i + ..$$

$$= \left(\sum_i \vec{r}_i \times \vec{F}_i \right) - \vec{r}_{OO'} \times \sum_i \vec{F}_i \quad = \vec{\tau}_{net,O}$$

If an object is in translational equilibrium and the net torque is zero about some point, then the net torque is zero about any point.

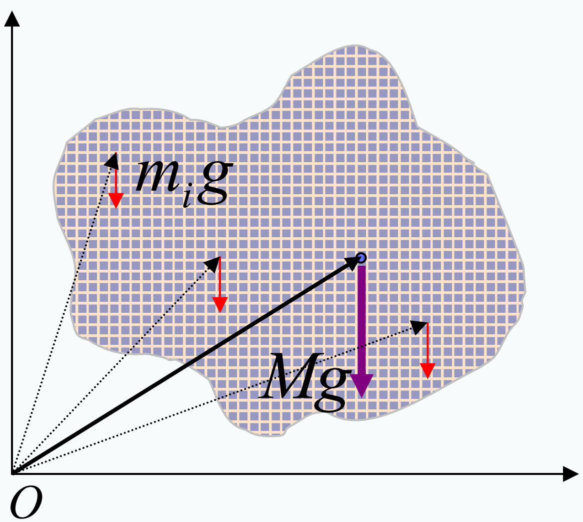
Center of gravity

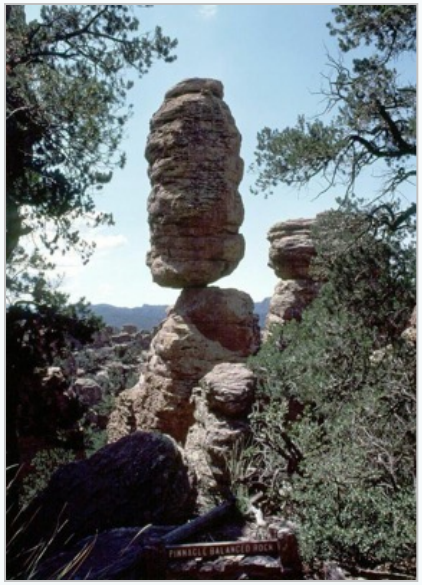


Center of mass: $\vec{r}_{CM} = \dfrac{\sum \vec{r}_i m_i}{\sum m_i} = \dfrac{\int \vec{r}\,dm}{\int dm}$

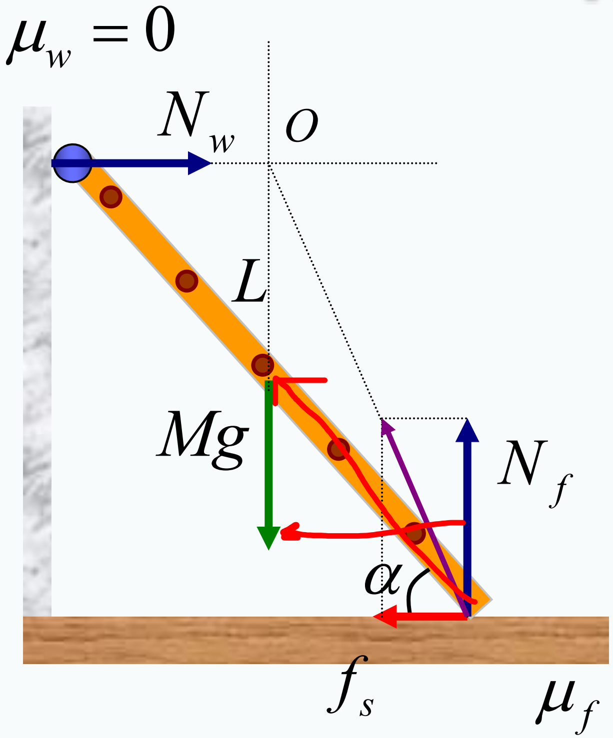
$$\vec{r}_{CG} \sum m_i g = \vec{r}_{CG} M g = \sum \vec{r}_i m_i g$$

If g is constant over an object, then the center of mass and the center of gravity of the object do coincide.





The ladder problem



$$\vec{F}_{net} = 0 \qquad \begin{cases} N_w - f_s = 0 \\ N_f - Mg = 0 \end{cases}$$

$$\vec{\tau}_{net} = 0 \qquad N_w L \sin\alpha - Mg\frac{L}{2}\cos\alpha = 0$$

$$N_w = \frac{Mg}{2}\cot\alpha = f_s \qquad \boxed{f_s \le f_{s,\max} = \mu_f N_f}$$

$$\frac{Mg}{2}\cot\alpha \le \mu_f N_f = \mu_f Mg \Rightarrow \cot\alpha \le 2\mu_f$$

$$\boxed{\tan\alpha \ge \frac{1}{2\mu_f}}$$

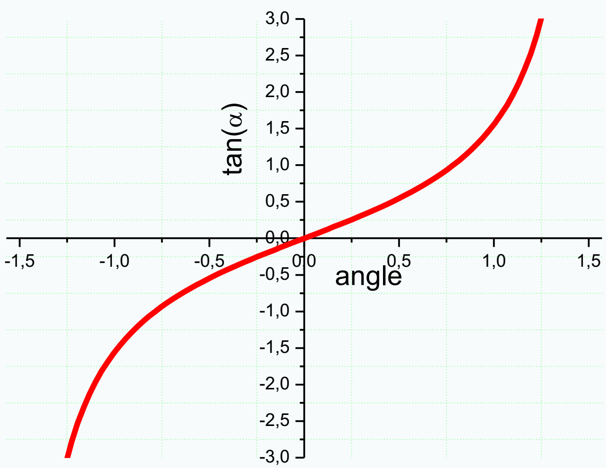


TABLE 5.2 Coefficients of Friction*

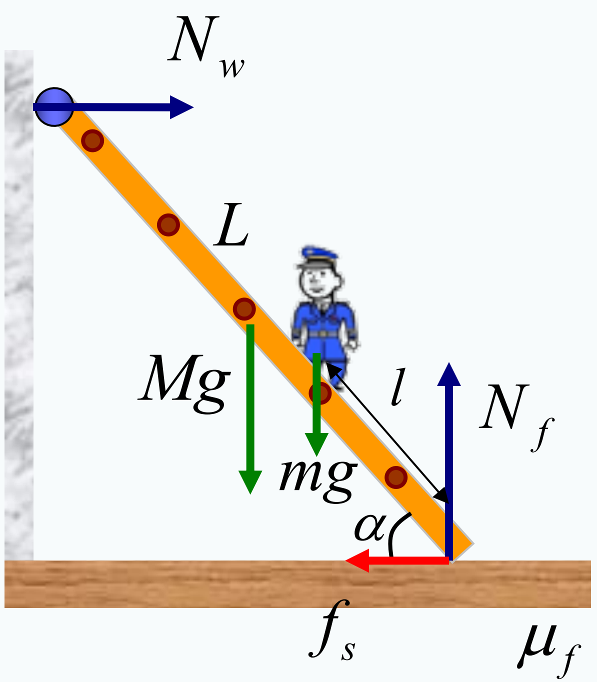
	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4

$\boxed{\alpha_{cr} \approx 25^\circ}$

$\boxed{\alpha_{cr} \approx 60^\circ - 45^\circ}$

The ladder climbing problem

$$\mu_w = 0$$



$$\vec{F}_{net} = 0 \qquad \begin{cases} N_w - f_s = 0 \\ N_f - (M+m)g = 0 \end{cases}$$

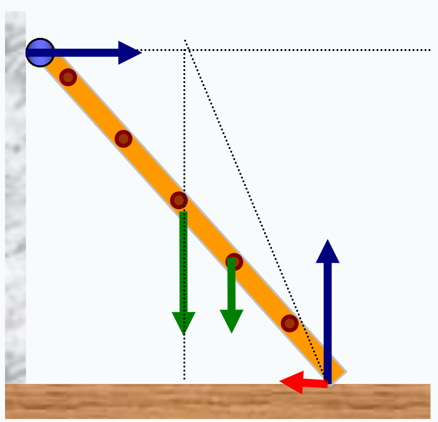
$$\vec{\tau}_{net} = 0 \qquad N_w L \sin\alpha - \left(Mg\frac{L}{2} + mgl \right)\cos\alpha = 0$$

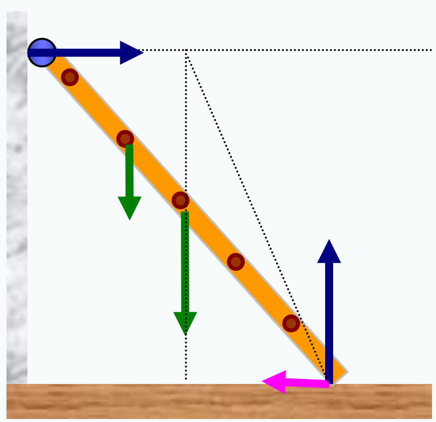
$$N_w = g\cot\alpha\left(\frac{M}{2} + \frac{ml}{L}\right) = f_s$$

$$\boxed{f_s \le f_{s,\max} = \mu_f N_f}$$

$$f_{s,\max} = \mu_f N_f = \mu_f (M+m)g$$

$$g2\mu_f\left(\frac{M}{2} + \frac{ml}{L}\right) \le \mu_f (M+m)g \Rightarrow l \le \frac{L}{2}$$





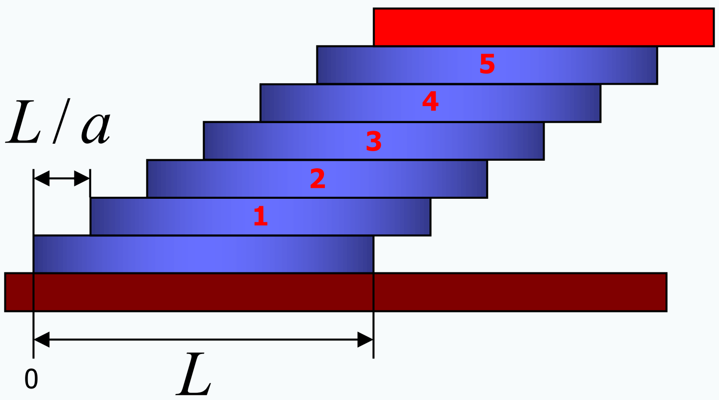
general case

$$g\cot\alpha\left(\frac{M}{2} + \frac{ml}{L}\right) \le \mu_f (M+m)g$$

$$l \le \frac{\mu_f (M+m)L}{m}\tan\alpha - \frac{LM}{2m}$$

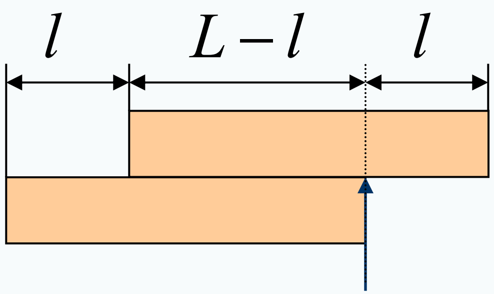
$$m >> M: \quad l \le \mu_f L \tan\alpha$$

Inclined domino set



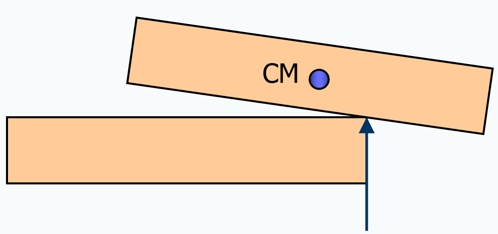
$$\rho A g \frac{L-l_{cr}}{2} - \rho A g \frac{l_{cr}}{2} = 0 \qquad l_{cr} = \frac{L}{2}$$

$$x_1 = \frac{L}{a} + \frac{L}{2} \qquad x_2 = 2\frac{L}{a} + \frac{L}{2} \qquad x_i = i\frac{L}{a} + \frac{L}{2}$$



$$X_{CM,n} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{1}{n}\sum_{i=1}^{n}\left[i\frac{L}{a} + \frac{L}{2}\right]$$

$$X_{CM,n} \approx \frac{1}{n} n \frac{L}{2} + \frac{1}{n}\frac{L}{a}(n+1)\frac{n}{2} = \frac{L}{2} + (n+1)\frac{L}{2a}$$



$$\boxed{X_{CM,n} \le L}$$

$$n + 1 \le a$$

$$a = 6 \Rightarrow n = 5$$

The sixth domino will crush the system.

To remember!

➢ **The two <u>conditions</u> have to be fulfilled for a body to be in static equilibrium:**

- the net external force must be zero;

- the net external torque must be zero.

➢ **If an object is in static equilibrium under action of three non-parallel forces, the <u>lines of action</u> of these forces must intersect at one point.**

➢ **The force of gravity can be replaced by single force acting at the <u>center of gravity</u>.**