Exercises in Introductory Physics I

Exercise Sheet 9
due to 18.12.23, 11:59 AM

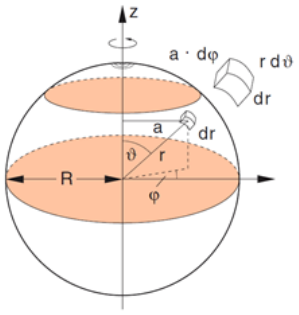
Rotation

1. The moment of inertia of a sphere (mass M, radius R) as depicted in the figure below (a) is given by:

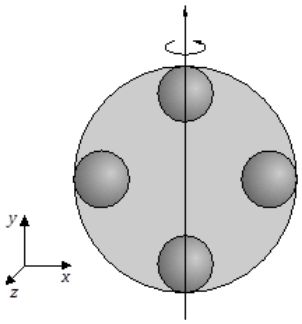
$$I = \int a^2 \rho(\vec{r}) \, \mathrm{d}V = \int a^2 \, \mathrm{d}m$$

For a sphere with a constant density ρ this can be expressed in spherical coordinates:

$$I = \rho \int_0^R \int_0^\pi \int_0^{2\pi} a^3 r \, \mathrm{d}\varphi \, \mathrm{d}\vartheta \, \mathrm{d}r$$



(a) Moment of inertia of a sphere



(b) Moment of inertia of a sphere with holes (dark grey)

a) Show that the Moment of Inertia I of an isotropic sphere is (2P)

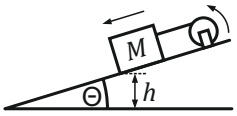
$$I = \frac{2}{5} M R^2$$

Hint: Show all steps of the calculation. While there are multiple ways to solve this, one might encounter the following integral, for which the solution is given:

$$\int_0^\pi (\sin(\vartheta))^3 \, \mathrm{d}\vartheta = \frac{4}{3}$$

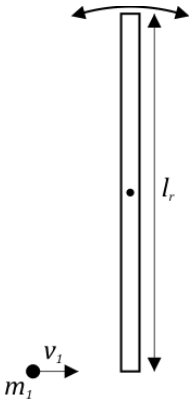
b) In the figure (b), a sphere (radius R, density ρ) with 4 spherical holes (radius $r = \frac{R}{4}$, the holes are displaced from the centre of the sphere in x- and y-direction) is shown. Calculate the moment of inertia for a rotation around the central y-axis for this object (1P).

2. A wheel of radius r and moment of inertia I about its axis is fixed at the top of an inclined plane (inclination Θ) as shown in the figure below. A string is wrapped around the wheel and its free end supports a block of mass M which can slide on the plane without friction. Given that at $t = 0$ the system is at rest at a height h, how long does it take until it reaches the bottom ?(2P)



3. A thin rod with a length $l_\mathrm{r} = 0.4\,\mathrm{m}$ and a mass $m_\mathrm{r} = 1\,\mathrm{kg}$ is allowed to turn around its centre. At the beginning it is not moving. Then it is hit at one end by a projectile (mass $m_1 = 0.01\,\mathrm{kg}$, speed $v_1 = 200\,\mathrm{m \cdot s^{-1}}$) which is going perpendicular to the rod and the rotation axis. The projectile sticks to the rod.

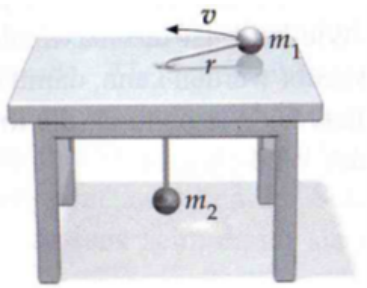
   Hint: The mass of the projectile should not be neglected.



   a) Calculate the angular speed with which the rod starts to move immediately after being hit by the projectile. (1P)

   b) Calculate the energy that is dissipated during the collision. (1P)

4. A tethered sphere (mass $m_1 = 1\,\text{kg}$) slides frictionless on a horizontal table surface on a circular trajectory of radius r_0 and period $T = 3\,\text{s}$ as depicted in the figure above. The sphere is connected via a massless string to a second sphere (mass $m_2 = 2\,\text{kg}$) through a small hole in the center of the circular trajectory of the first sphere.



a) For the equilibrium situation at which $r_0 = \text{const}$, calculate the radius r_0. (1P)

b) Now the mass of the second sphere is increased by 1 kg. What will happen to the velocity of the first sphere? Explain your answer with respect to the acting forces. (1P)

c) What are the final radius r_{final} and angular velocity ω_{final} of the sphere on the table when the system reaches equilibrium again? (1P)

Note: Neglect friction. Assume that the second sphere does not reach the ground.