Stanislav 3720433 Exercise Sheet 2

1. For the first part of motion

$v_0 = 0,\ h_0 = 0,$

$v(t) = a_1 t,\quad h(t) = a_1 \frac{t^2}{2}.$

At $h(t_1) = \frac{a_1 t_1^2}{2},\ v(t_1) = a_1 t_1$ accel. vanished.

Then (using t from 0) $v(t) = a_1 t_1,$

$h(t) = h_0 + v(t)\cdot t = \frac{a_1 t_1^2}{2} + a_1 t_1 t.$

Helmet falls at $v(t_2) = a_1 t_1,$

$h(t_2) = \frac{a_1 t_1^2}{2} + a_1 t_1 t_2.$

Then (using t from 0) for helmet which is in free fall

$v(t) = a_1 t_1 - gt$

$h(t) = \left[\frac{a_1 t_1^2}{2} + a_1 t_1 t_2\right] + \cancel{[a_1 t_1 - gt]}\, a_1 t_1 t - \frac{gt^2}{2}.$

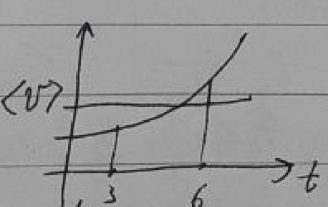
Hits the ground at $h(t_3) = 0 = \frac{a_1 t_1^2}{2} + a_1 t_1 t_2 + a_1 t_1 t_3 - \frac{g t_3^2}{2}$

a) Then $a_1 = \frac{g t_3^3}{2\left[t_1 t_2 + t_1 t_3 + \frac{t_1^2}{2}\right]} = \frac{9.8 \cdot 2.25^3}{2\left[4\cdot 3.5 + 4\cdot 2.25 + \frac{4^2}{2}\right]}\frac{m}{s^2} =$

$= 2.47\ \frac{m}{s^2}$

b) $h(t_2) = \frac{a_1 t_1^2}{2} + a_1 t_1 t_2 = a_1 t_1\left[\frac{t_1}{2} + t_2\right] \approx 54.3\,m$

2. $v(t) = a_1 + a_2 \cdot t + a_3 t^2$



3, 5, 2

$\langle v \rangle = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)\,dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}\left[a_1 + a_2 t + a_3 t^2\right]dt = \frac{1}{t_2 - t_1}\left[a_1 t + a_2\frac{t^2}{2} + a_3\frac{t^3}{3}\right]_{t_1}^{t_2} =$

$= \frac{1}{t_2 - t_1}\left[a_1(t_2 - t_1) + \frac{a_2}{2}\left[t_2^2 - t_1^2\right] + \frac{a_3}{3}\left[t_2^3 - t_1^3\right]\right] = a_1 + \frac{a_2}{2}(t_1 + t_2) +$

$+ \frac{a_3}{3}\left[t_2^2 + t_1^2 + t_1 t_2\right] = \left[3 + \frac{5}{2}[3+6] + \frac{2}{3}\left[3^2 + 6^2 + 3\cdot 6\right]\right]\frac{m}{s} =$

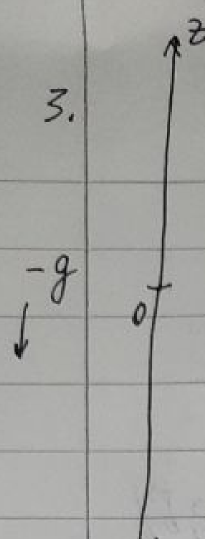
$= 67.5\ \frac{m}{s}.$

Compare with $v\left(\frac{t_1 + t_2}{2}\right) = \left[3 + 5\left(\frac{3+6}{2}\right) + 2\left(\frac{3+6}{2}\right)^2\right]\frac{m}{s} =$

$= 66\ \frac{m}{s}$, $\langle v \rangle$ is higher, which should be since

$v(t)$ is $\cup$ concave.

3.



1st stone		2nd stone
$z_0 = 0$		$z'_0 = 0$
$v_{z0} = 0 = v_0$	$\xrightarrow{\Delta t}$	$v'_{z0} = v_z$ – signed velocity
$a = -g$		$a = -g$

Therefore EOM for the first stone:

$$z(t) = -\frac{gt^2}{2}.$$

EOM for the 2nd: $z'(t) = v_z(t-\Delta t) - \frac{g(t-\Delta t)^2}{2}$.

Condition to pass: $z(t_1) = z'(t_1)$, $t_1 > \Delta t$.

$$-\frac{gt_1^2}{2} = v_z(t_1 - \Delta t) - \frac{g}{2}(t_1 - \Delta t)^2$$

$$\cancel{-\frac{gt_1^2}{2}} = v_z t_1 - v_z \Delta t \cancel{-\frac{gt_1^2}{2}} - g\frac{\Delta t^2}{2} + g t_1 \Delta t$$

$$t_1 = \frac{v_z \Delta t + g\frac{\Delta t^2}{2}}{v_z + g\Delta t} = \frac{(2v_z + g\Delta t)\Delta t}{2v_z + 2g\Delta t} > \Delta t$$

So $2v_z + g\Delta t > 2v_z + 2g\Delta t$ if $v_z + g\Delta t > 0$, but $0 > g\Delta t$ is impossible.

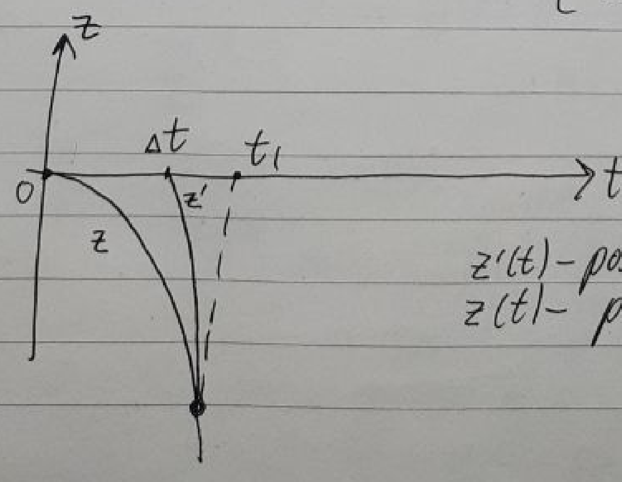
or $2v_z + g\Delta t < 2v_z + 2g\Delta t$ if $v_z + g\Delta t < 0$, which can be.

a) $\boxed{\text{So } v_z < -g\Delta t,\ |v_z| > g\Delta t \text{ – minimum speed to pass.}}$
Intuitively means that second stone must have more initial velocity then first will have at Δt (otherwise no chance to overcome since they also fall with same a).

b) $t_1 = \frac{(2v_z + g\Delta t)\Delta t}{2(v_z + g\Delta t)}$, see above ($v_z = -|v_z|$).

c) $z_1 = z(t_1) = -\frac{g}{2}t_1^2 = -\frac{g}{2}\cdot\left[\frac{(2v_z + g\Delta t)\Delta t}{2(v_z + g\Delta t)}\right]^2$.

d)



$z'(t)$ – position of 2nd
$z(t)$ – position of 1st

Vectors

1. a) $a = 4x + x - 1\cdot 0 = 5x$

b)
$$\vec{a} = \det\begin{bmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 2x & 1 & -1 \\ 2 & x & 0 \end{bmatrix} = \begin{pmatrix} x \\ 2 \\ 2x^2-2 \end{pmatrix}$$

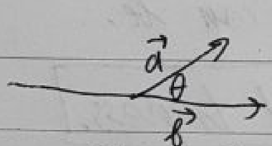
c) Use the
$$\left.\begin{aligned} \vec{a}\cdot\vec{b} &= \sum_{i=1}^{3} a_i b_i = |\vec{a}||\vec{b}|\cos\theta, \\ |\vec{a}\times\vec{b}| &= |\vec{a}||\vec{b}|\sin\theta, \end{aligned}\right\} \theta = \angle(\vec{a},\vec{b})$$

$$\theta = \operatorname{acos}\frac{\sum a_i b_i}{|\vec{a}|\,|\vec{b}|} = \operatorname{acos}\frac{3-10-14}{\sqrt{9+4+49}\sqrt{1+25+4}} = \operatorname{acos}\frac{-21}{\sqrt{62\cdot 30}} \approx 2.$$

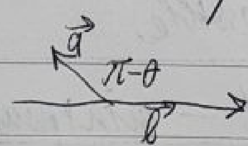
$$\theta = \operatorname{asin}\left(\left|\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}\right| \cdot \frac{1}{\sqrt{62}\cdot\sqrt{30}}\right) = \operatorname{asin}\left(\left|\begin{pmatrix} 4-35 \\ -7-6 \\ -15-2 \end{pmatrix}\right| \cdot \frac{1}{\sqrt{62\cdot 30}}\right) =$$

$$= \operatorname{asin}\left(\left|\begin{matrix} 31 \\ 13 \\ 17 \end{matrix}\right| \cdot \frac{1}{\sqrt{62\cdot 30}}\right) = \operatorname{asin}\sqrt{\frac{31^2+13^2+17^2}{62\cdot 30}} = \begin{bmatrix} 1\text{ rad} \\ \pi - 1\text{ rad} \approx 2 \end{bmatrix}.$$

! In 2nd case there are 2 possible solutions because



and



give same $|\vec{a}\times\vec{b}|$.

Then choose $\theta = 2$.