Lecture "Experimental Physics I"
(Prof. Dr. R. Seidel)

Lecture 5

Laws of motion

- Force
- Newton’s laws
- Atwood machine
- Inclined plane
- Friction forces

1. Forces

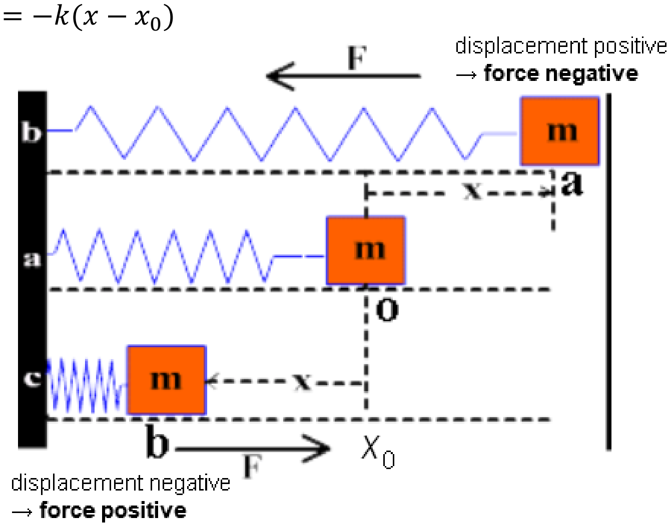
In kinematics we looked at the motion of objects without considering the origin of their motion. Now we turn to the **dynamics**, which tries to **understand the interactions between objects and how these relate to motion**, e.g. why is the earth moving around the sun and why is a stone falling down to the ground?

Newton: recognized that the origin for **any change of the motion** of an object are **interactions of the object with the environment**.
Such interactions are described by **forces**, a concept that we know from our everyday experience. For example, a light and a heavy object fall down with the same gravitational acceleration (Galileo), but to lift the heavy object one needs correspondingly much more force. Therefore, force and mass seem to be related.
Generally, we distinguish **"contact" forces and field forces** that do not require contact for the application of force. (see slides about different force types). Forces that act between disconnected objects was a problematic concept for early physicists. Michael Faraday (1791–1867) introduced the concept of a "field" that can be created by objects with certain properties (mass, charge etc.). The distinction between the force types is vanishing when looking at the atomic scale. In fact, there are **only four known fundamental forces in nature which are all field forces:**
(1) gravitational forces (infinite range)
(2) electromagnetic forces between electric charges (infinite range)
(3) strong nuclear forces between subatomic particles (range 10^{-15} m)
(4) weak nuclear forces that arise in certain radioactive decay processes (range 10^{-18} m)

A) Hooke's law

Springs are a very intuitive way to imagine, visualize and measure forces. In first approximation the **force that a spring exerts** when it is extended or compressed from its equilibrium position x_0 is given by Hooke's law:

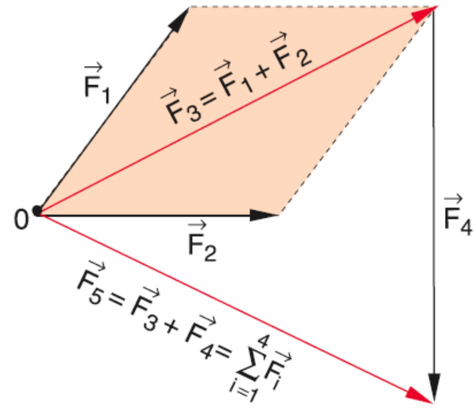
$$F = -k\Delta x = -k(x - x_0)$$



The force on a spring is a **backdriving force** since it counteracts its deformation. It exerts a retracting (negative force in the figure) for an extension (positive displacement) and an expansion force (positive force) for a compression. This is a very general concept that holds for the majority of (mechanical) displacements from an equilibrium position.

B) Forces as vectors

Forces are vectors as we saw from the experiment with the handcart. They can be split arbitrarily into components. The vector sum of the components must then always equal the original force vector:



For forces we can formulate the following **superposition rules**:

i) forces at one point add up to resulting force vector:

$$\vec{F} = \sum_i \vec{F}_i$$

ii) any force can be split into a desierde number of (orthogonal) components

Often a smart choice of the coordinate system or the component splitting helps to solve problems in mechanics more easily.

We furthermore can define equilibrium using forces:

An object is in equilibrium when the forces along each (orthogonal) direction add up to zero, i.e.

$$\vec{F} = \sum_i \vec{F}_i = \begin{pmatrix} \sum_i F_{x,i} \\ \sum_i F_{y,i} \\ \sum_i F_{z,i} \end{pmatrix} = 0$$

We will see later that in this case an initially moving body keeps moving without a change.

Experiment: We illustrate the cancelation of forces in equilibrium by a mass hanging in the center of a rope that is counterbalanced by two objects that have the same mass as the original object. Using two roles their pulling direction is inclined by an angle α with respect to the vertical direction. For symmetry reasons, the lateral forces add up to zero. For the vertical forces, force cancelation demands:

$$2F_g \cos\alpha - F_g = 0 \;\Rightarrow \cos\alpha = 1/2 \Rightarrow \alpha = 60°$$

Thus, we have to choose an inclination of 60° for the two counterweights to achieve equilibrium.

2. Newton's laws

A mathematical description of motion under the influence of forces can be typically related to few basic equations that are based on the laws of motion. These laws have been derived from experimental observation, but are in fact just "eductated" assumptions.

They were formulated for the first time by **Newton in his "Philosophiae naturalis principia mathematica"** (1687–1726).

A) Newton's first law

Before 1600 the state at rest was considered to be the natural state. Galileo already recognized that it is not the nature of an object to stop once set in motion: rather, it is its nature to resist changes in its motion.
Newton formulated this in his 1st law more precisely:

> **An object continues to be in rest or continues its motion with a constant velocity unless it is acted on by a(n) (unbalanced) force (i.e. a non-zero total force).**

This is also called **law of inertia**, since each object resists a change of its movement. This **resistance is called inertia**

Experiment: The law of inertia can be nicely illustrated by a slider on a horizontal air track. The slider keeps on going with the same velocity without force. Only if we apply a force, e.g. by friction we see a change in velocity.

Newton's laws hold only in **inertial frames of reference** (inertial coordinate systems). An inertial frame of reference is **one that is not accelerating**. Imagine that one stands on roller blades in a train and the train accelerates. In the frame of the platform one stands still (at least for while ☺). Within the accelerating frame of reference of the train one accelerates without force, such that the law would not hold.
Newton used actually the first law to define inertial frames of reference as frames of reference were the first law holds. Any reference frame that is in uniform motion with respect to an inertial frame is also an inertial frame.

As an additional measure for the state of motion one introduces the **momentum**:

$$\vec{p} = m\vec{v}$$

> An alternative formulation of the first law is then:
> **The momentum of a force-free free particle is invariant in time.**

B) Newton's second law

Newton's 2nd law describes the change of the motion of an object:

> **A change in motion is proportional to the acting force and occurs along the direction of the acting (net) force according to:**
>
> $$\vec{F} = \frac{d\vec{p}}{dt}$$

Inserting $\vec{p} = m\vec{v}$, this transforms into:

$$\vec{F} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt} = \frac{dm}{dt}\vec{v} + m\vec{a}$$

Only if $m = const.$, one gets the more famous form of this formula:

$$\vec{F} = m\vec{a}$$

The term on the right side $m\vec{a}$ is hereby the **resistance against acceleration, i.e. the inertia.**

In this simplified form an alternative formulation of the 2nd law is:

> **The sum of the forces on an object is equal to the mass of that object multiplied by the acceleration of the object.**

The second law defines **force**: The force required to produce an acceleration of 1 m/s^2 of an object of a mass of 1 kg is defined to be 1 newton (N).

Experiments: Inertia can be illustrated in ‘everyday’ experiments

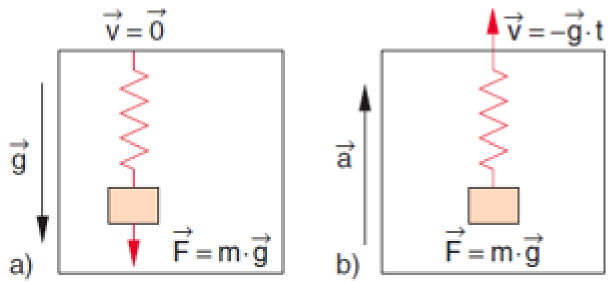
1. Imagine you are in a toilet in one hand the mobile phone, such that there is only one hand free to operate the toilet paper. If one pulls rapidly on the paper end, there is a large force acting to accelerate the paper role. Thus, the toilet paper eventually raptures.
2. If you want to take the table cloth from a table where a bottle of champagne is standing you can use the same trick by pulling rapidly on the cloth. The high force that accelerates the bottle overcomes the friction and we can pull the cloth away
3. Two masses connected in series with a small rope: When slowly pulling the upper rope ruptures, since it bears the additional weight of the upper mass. When rapidly pulling the lower rope ruptures, since the additional inertia for accelerating the upper mass lowers the force in the rope and adds force to the lower rope.

C) Mass and Weight

(see slides)

Netwon’s 2nd law brings mass into the description of the object dynamics. **Mass** is a scalar quantity and an inherent property of an object, independent of the object’s surroundings and of the method used to measure it. Inertial **mass is a measure of the resistance to acceleration.**

Mass is a tricky concept and it is not really clear what it is and where it comes from. General relativity theory states, however, in the **equivalence principle** that **inertial mass and gravitational mass are the same**. This can be illustrated by a mass on a spring in a box:



For the static box under an external gravitational acceleration of $-g$ the same force is seen on the spring as for an accelerated system with $-g$ and no gravity. An observer within the reference system of the box cannot distinguish from looking at the spring in which reference system he/she actually is.

With Newton’s 2nd law we can also define **weight**:

The weight of a body is a net force required to prevent the body from falling. It has the same magnitude as the gravitational force F_g that acts on the body.

$$weight = -F_g = mg$$

It is taken as a positive quantity. m is here the gravitational mass

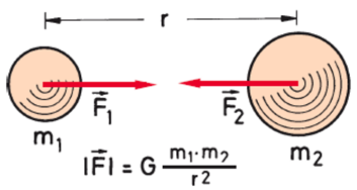
D) Newton's third law

The 3rd Newton's law states that:

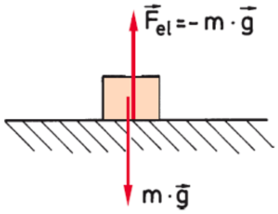
> **Forces always occur in pairs. That means that if an object A exerts a force on another object B, then B will exert an equal but opposite force on A.**

> Alternatively, one can write:
> **Actio equals Reactio**
> $$\vec{F}_{AB} = -\vec{F}_{BA}$$

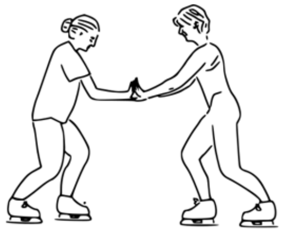
Examples:



gravitational force between objects including Newton's apple



object on a plane



ice skater

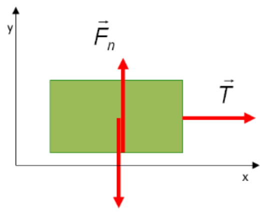
Experiments:

1. Weight of a person is not balanced on a card board box. The box does not provide enough counter force that is normally necessary
2. Two persons on a skate board with a rope in between. Nondependent on who is pulling, both persons move towards each other. (unequal body masses just cause different accelerations)
3. Recoil: When a bullet is shot from a wagon, the wagon starts to move in the opposite direction.
4. Recoil: Rockets filled with pressured air and water. The water rocket flies faster since it ejects more mass such that there is a higher momentum transfer onto the rocket.

3. Applications of Newton's laws

Newton's laws provide a powerful toolbox to address a wide range of problems for the motion of a point mass under external force. A **general efficient way to solve such problems is** (see slides):

1. Isolate the object or the objects of interest.
2. Draw detailed diagram of all external forces acting on the object.
3. Choose a convenient coordinate system.
4. Write down the second Newton's law in the component form (forces on one side, inertia on the other side)
5. Solve the resulting system of equations for the unknown quantity.
6. Check yourselves (by considering extreme cases).



$$\sum F_x = T = ma_x$$

$$\sum F_y = -mg + F_n = ma_y = 0$$

We will apply this scheme now in a couple of simple examples.

A) Forces and acceleration on inclined plane

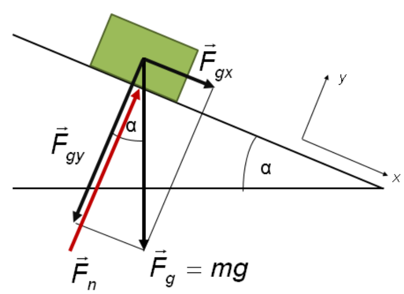
We first look at the forces and accelerations at an inclined plane. We choose a tilted coordinate system with the x-axis along the plane surface along which the motion occurs and the y-axis perpendicular to the plane.
For each component we write down the net force on the left side and the inertia on the right side. For the given coordinate system, the gravitational force $\vec{F}_g$ splits into two components ($\vec{F}_{gx}$ and $\vec{F}_{gy}$). We furthermore have a force $\vec{F}_n$ normal to the surface of the inclined plane that stops the object to sink into the plane. The object is accelerated only parallel to the plane surface along the x-direction. This gives for the two coordinates:

x-direction:

$$\sum_i F_x = ma_x$$

$$mg\sin\alpha = ma_x$$

$$a_x = g\sin\alpha$$



y-direction:

$$\sum_i F_y = ma_y$$

$$F_n - mg\cos\theta = 0$$

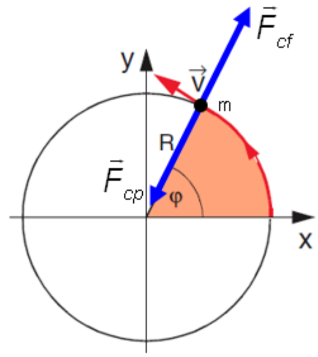
Experiment: We measure the acceleration of a slider parallel to the inclined plane to show the scaling with $\sin\alpha$ and determine the gravitational acceleration:

$$F_{||} = mg\sin\alpha$$

$$g = \frac{a_x}{\sin\alpha}$$

B) Centripetal (and centrifugal) force

(see slides)



We introduced before the centripetal acceleration. According to Newton's 2nd law, we have to generate a corresponding centripetal force in order to force the object on the circular trajectory:

$$\vec{F}_{cp} = m\frac{v^2}{R}(-\hat{e}_r) = m\omega^2 R(-\hat{e}_r)$$

We now measure this force and show its proportionality to the mass:

Experiment: Measure the centripetal force $\vec{F}_{cp}$ as function of mass (2x) **and angular velocity**. For this a force sensor on the rotation axis is connected via a cord to a rotating mass. The force sensor is illustrated with a simple model with a spring scale before the experiment.

The concept of the centripetal force does not fit our everyday experience, e.g. when sitting in a car that follows a curve. Here we feel that we are pushed out of the curve.

The reason is that in the car we are here in a rotating frame of reference, where we stand still. The **centripetal force must within this system thus be counterbalanced by a second force of equal magnitude**. For the force balance in the radial direction we thus get:

$$\sum F_r = 0 \rightarrow \vec{F}_{cf} = -\vec{F}_{cp}$$

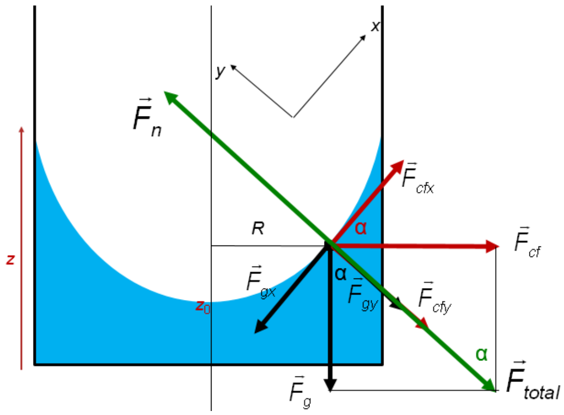
The counteracting force $\vec{F}_{cf}$ away from the center of rotation **we call centrifugal force**. This force is the one that we actually feel. It substitutes for the actual inertial force in the rotating frame of reference. It is directed away from the center of rotation:

$$\vec{F}_{cf} = m\frac{v^2}{R}\hat{e}_r = m\omega^2 R\hat{e}_r$$

The **centrifugal force is a pseudo force** since it exists only in the rotating reference system but not in the non-rotating one. We have to introduce such a pseudo force, since the **rotating frame of reference is not on inertial reference system**, since it has a constant acceleration.

To further practice the concept of the centrifugal force, we carry out the following experiment:

Experiment: We look at the curved surface arising when spinning a fluid. The **centrifugal force creates in fact a parabola-shaped water surface.**



To quantitatively describe the shape of the liquid surface, we look at a small mass element Δm at the fluid surface. We are now in the rotating reference system and have thus to use the centrifugal force that points laterally outwards. We also have the acting gravity force. In equilibrium we have a static (non-moving) fluid, i.e. the forces in each direction must be balanced. For convenience we chose a coordinate system with the x-axis parallel to the surface tangent and the y axis perpendicular to it. We then consider the components of both forces along these coordinate axes. For the y-direction we can write:

$$\sum_i F_y = 0$$

$$-\Delta mg\cos\alpha - \Delta m\omega^2 R\sin\alpha + F_n = 0$$

where the first term is the *y*-component of the gravitational force, the second term the *y*-component of the centrifugal force and $\vec{F}_n$ the force normal to the surface that counterbalances these forces. $\vec{F}_n$ is generated by the fluid pressure.

For the *x*-direction we can write:

$$\sum_i F_x = 0$$

$$-\Delta mg\sin\alpha + \Delta m\omega^2 R\cos\alpha = 0$$

where the two terms are the x components of gravitational and centrifugal force. Transforming the latter equation provides the tangent of the local tilt angle, which is nothing else than the slope or the derivative at that position:

$$\frac{dz}{dR} = \tan\alpha = \frac{\omega^2 R}{g}$$

Separation of the variables provides:

$$dz = \frac{\omega^2 R}{g} dR$$

Subsequent integration strting from $R = 0$ at initial height z_0 gives:

$$\int_{z_0}^{z} dz = \int_0^R \frac{\omega^2 R}{g} dR = z(R=0) + \frac{\omega^2 R^2}{2g}$$

This yields:

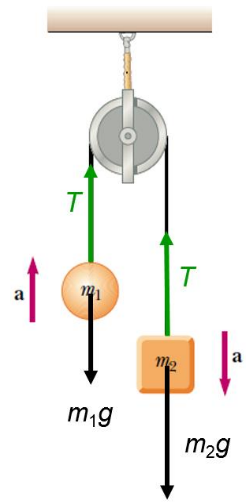
$$z - z_0 = \frac{\omega^2 R^2}{2g}$$

We thus get:

$$z = z_0 + \frac{\omega^2 R^2}{2g}$$

which provides a parabola, which gets steeper for increasing ω. One can show that due to volume conservation all the parabolas obtained at different angular velocities intersect at the same two points (see slides).

B) Atwood's machine



Atwood's machine (see figure) is a device with which any acceleration smaller than g can be obtained by adjusting the weight difference between two masses that are connected via a rope and a wheel. It is a convenient way to measure g with simple tools.

To calculate the acceleration of the machine we first look at the force balance of each weight separately. For each weight we must have:

$$\sum F_z = ma_z$$

with ***T*** being the tension force in the cord that according to Newton must be equal (for a mass-free rope) at each weight. We have to carefully consider the directions of the forces and accelerations such that we get for each mass:

$$T - m_1 g = m_1 a \qquad (mass\ m_1)$$
$$T - m_2 g = -m_2 a \qquad (mass\ m_2)$$

Subtracting both equations provides:

$$g(m_2 - m_1) = a(m_2 + m_1)$$

This equation can also be read as net effective gravity force on the system (left side) which accelerates both masses with the same effective acceleration (right side).

Transformation of this equation provides:

$$a = g\frac{(m_2 - m_1)}{(m_2 + m_1)}$$

Multiplication of the 1st equation from above with m_2 and the 2nd equation. with m_1 followed by addition of the two equation provides:

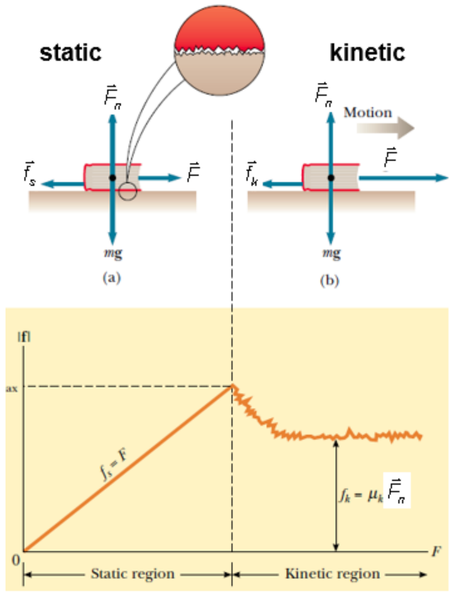
$$T(m_1 + m_2) - 2m_1 m_2 g = 0$$

From which the tension force is obtained:

$$T = g\frac{2m_1 m_2}{m_1 + m_2}$$

Experiment: Determination of g with an Atwood machine.

C) Forces of friction



Experiment: In an experiment we investigate the friction of a wood cuboid on the table. We can show that the friction is higher for a static cuboid compared to the moving cuboid. We also can demonstrate that the friction depends on the material but not on the contact area of the object and that it is proportional to the mass.

From the experiment we saw that we have different values for the friction force depending on whether the object is moving. Initially the static object is held on its position by the static friction $\boldsymbol{f_s}$ that balances the applied force. After overcoming the **maximum static friction $\boldsymbol{f_{s,max}}$**, the object moves against a lower friction, which is called **kinetic friction $\boldsymbol{f_k}$**

Thus, we have static friction and kinetic friction forces with:

$$f_{s,max} > f_k$$

For the different friction forces we can note the following general dependencies:

$$f_{s,max}, f_k \neq f(area)$$
$$f_{s,max}, f_k \propto F_N$$
$$f_{s,max}, f_k = f(material)$$

Thus, for a given material, friction depends only on the normal force F_n that acts on the object. We thus define a dimensionless material-dpendent friction coefficient to describe the friction force:

$$\mu = \frac{f}{F_n}$$

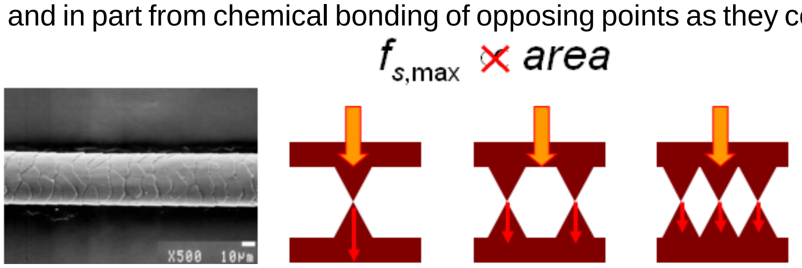
Consquently, we have two different friction coefficients one for static and one kinetic friction (see slides):

Static friction force

For the static friction we can write:

$$f_s \leq \mu_s F_n, f_{s,\boldsymbol{max}} = \mu_s F_n$$

with μ_s being the coefficient of static friction. Friction forces arise from contacting points that protrude beyond the general level of the surfaces in contact, even for surfaces that are apparently very smooth, as shown in the magnified view of a hair in the figure below. (If the surfaces are clean and smooth at the atomic level, they are likely to weld together when contact is made.) The frictional force arises in part from one peak physically blocking the motion of a peak of the opposing surface, and in part from chemical bonding of opposing points as they come into contact.



The invariance on the contact area is due to a higher normal force on the protrusions when shrinking the contact area.

Kinetic friction force

For the kinetic friction we can write:

$$f_k = \mu_k F_n$$

with μ_k being the **coefficient of kinetic friction**. The **kinetic friction points always hinders the motion of an object**, i.e. it points into the opposite direction of the motion. Experimentally (in line with the explanations above) we find that:

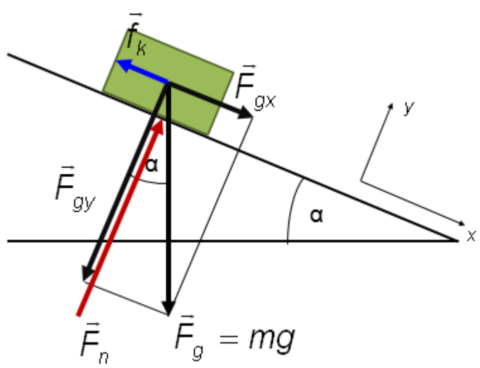
$$\mu_k < \mu_s$$

Since the friction force is nearly independent of the pulling speed between 1 cm/s and 10 m/s, μ_k is nearly constant. As one knows from real life experience lubrication helps to reduce kinetic friction.

Experiment: Asymmetrically driven Stick-slip motion for linear forward propulsion

Movie: Stick-slip motion for nanopositioning of a scanning tunneling microscope (STM)

D) Inclined plane with friction (see slides)



We can now build a more realistic model of an object on the inclined plane by considering the kinetic friction in opposite direction of the motion. The force/inertia balance along the surface plane (*x* direction) then changes into:

$$-\mu_k F_n + mg \sin\alpha = ma_x$$

Inserting $F_n = mg \cos\alpha$ and transformation provides then for the acceleration along x:

$$a_x = g(\sin\alpha - \mu_k \cos\alpha)$$

By setting the acceleration to zero this provides a way to determine the static friction coefficient from the maximum angle before sliding starts, which we will explore in a later lecture.

Lecture 5: Experiments

1. Vector addition of forces: mass hanging in the center of a rope that is counterbalanced by two identical masses. Rope is inclined by an angle of 60°.
2. 1st law: slider on a horizontal air track. The slider keeps on going with the same velocity without force. Only if we apply a force, e.g. by friction we see a change in velocity
3. 2nd law: toilet paper rupture with one hand, champagne bottle on table cloth
4. 2nd law: Two masses connected in series with a small cord: Slow and fast pulling changes the cord that ruptures
5. 3rd law: Two persons on a skate board with a rope in between. independent of who is pulling, both persons move towards each other.
6. 3rd law - recoil: When a bullet is shot from a wagon the wagon starts to move in teh opposite direction
7. 3rd law - recoil: Rockets filled with pressured air and water. The water rocket flies faster since it ejects more mass such that there is a higher momentum transfer onto the rocket).
8. We measure the acceleration of a slider parallel to the inclined plane to show the scaling with $\sin\alpha$ and determine the gravitational acceleration. This proves that one can decompose the total gravity force into a component normal and a component horizontally to the plane.
9. Measure the centripetal force F_{cp} as function of mass (2x) and angular velocity (continuous). For this a force sensor on the rotation axis is connected via a cord to a rotating mass. The force sensor is illustrated with a simple model with a spring scale beforehand
10. Parabola of a rotating water surface
11. Atwood's machine
12. Determination of the maximum static frictional force as well as the kinetic frictional force (Wooden block on wood and a spring); Independence of the friction coefficient (f_H = 4N/20N ≈ 0.2) from the normal force and surface.
13. Application stick-slip friction: Model for motion based on stick-slip friction + movie STM-coarse approach with asymmetrically driven piezo elements.