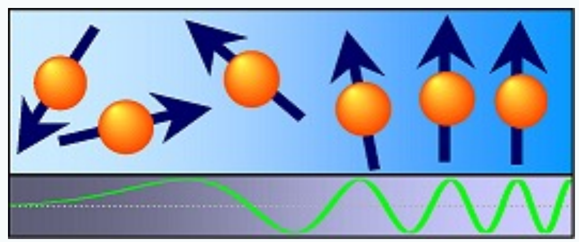
Experimental Physics
EP1 MECHANICS

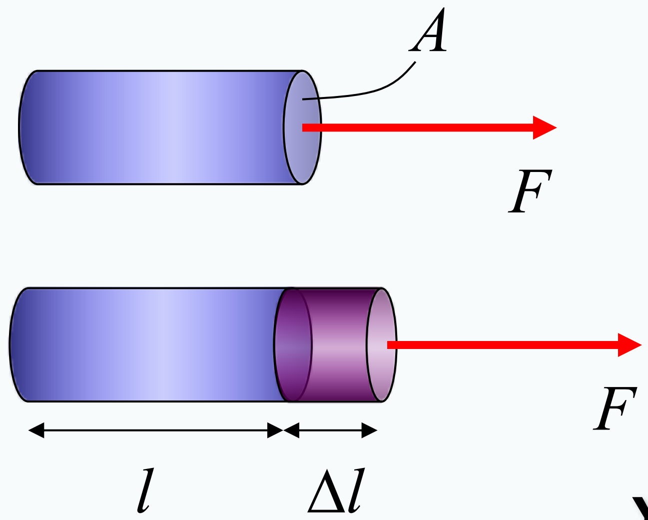
– Fluid mechanics –



Rustem Valiullin

https://www.physgeo.uni-leipzig.de/en/fbi/applied-magnetic-resonance

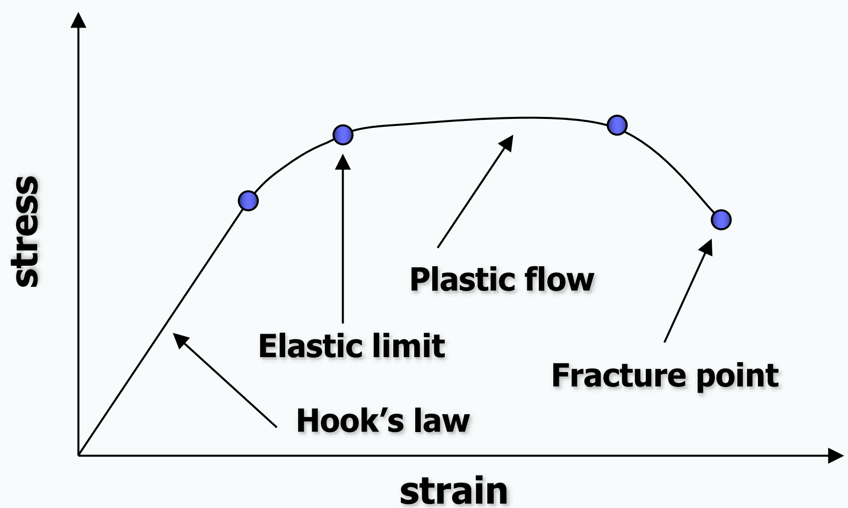
Elastic modulus



Tensile stress $= \dfrac{F}{A} \quad \left[\dfrac{N}{m^2}\right]$

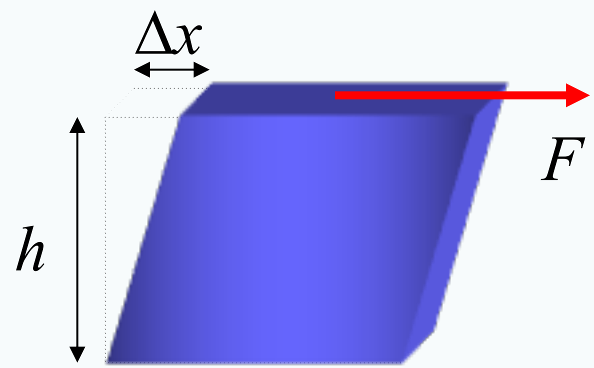
Strain $= \dfrac{\Delta l}{l}$ **degree of deformation**

Young's or elastic modulus $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{\Delta l / l} \quad \left[\dfrac{N}{m^2}\right]$



Material	Y, GPa (TS, MPa)
Rubber	0.01-0.1
Wood	~10 (50)
Aluminum	70 (90)
Glass	50-90 (50)
Silicon	185
Steel	200 (520)
Diamond	1220

Shear modulus



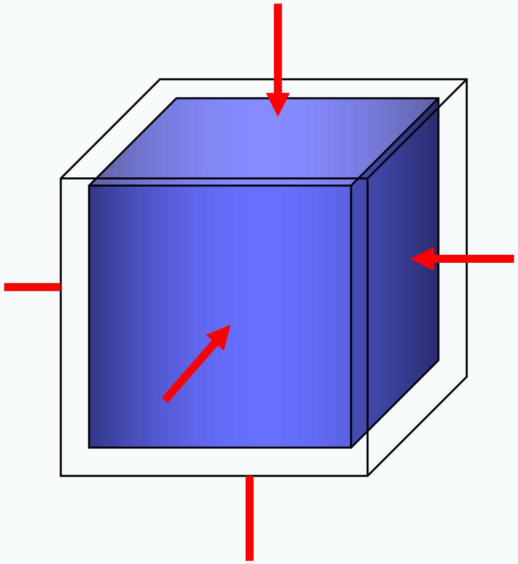
Shear stress $= \dfrac{F}{A} \quad \left[\dfrac{N}{m^2}\right]$

Shear strain $= \dfrac{\Delta x}{h} = \tan\theta$

Shear modulus $G = \dfrac{F/A}{\Delta x / h} \quad \left[\dfrac{N}{m^2}\right]$

Material	G, GPa
Rubber	0.0006
Aluminum	25.5
Glass	26
Steel	80
Diamond	478

Bulk modulus



Volume stress (pressure) $= \dfrac{F}{A} \quad \left[\dfrac{N}{m^2}\right]$

Volume strain $= \dfrac{\Delta V}{V}$

Bulk modulus $B = -\dfrac{F/A}{\Delta V/V} \quad \left[\dfrac{N}{m^2}\right]$

Water expands by about 9% upon freezing. What pressure would it then create in a bottle?

~ 200 MPa

1 km down a see the pressure of water rises by 10 MPa. What is the water density there?

Material	G, GPa
Air	$\sim 10^{-4}$
Water	2.2
Glass	35-50
Steel	160
Diamond	442

Fluids

- Elastic stress
- Shear stress
- **Bulk stress**

$$F_{\text{bot}} = F_{\text{top}} + mg = F_{\text{top}} + \rho A h g$$

$$P_{\text{bot}} - P_{\text{top}} = \rho h g$$

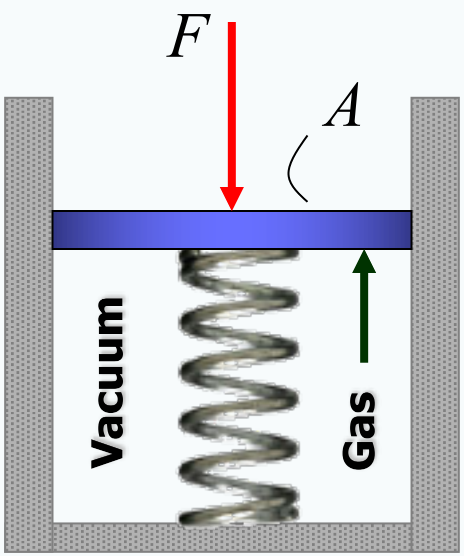
Compressibility:

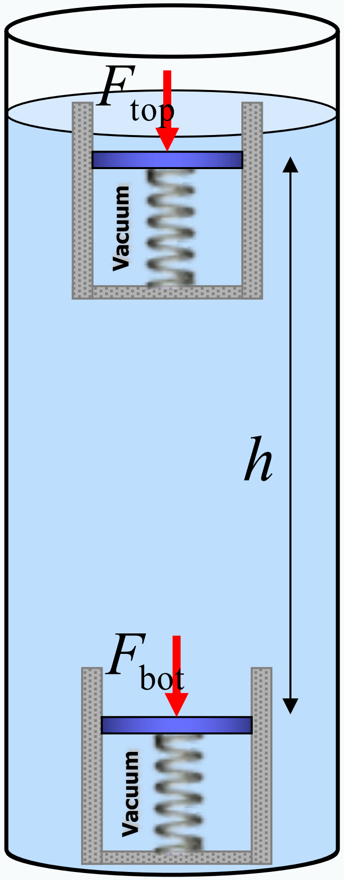
$$\kappa \equiv \frac{1}{B} = -\frac{1}{V}\frac{dV}{dP}$$

Pressure $P = \dfrac{F}{A}$

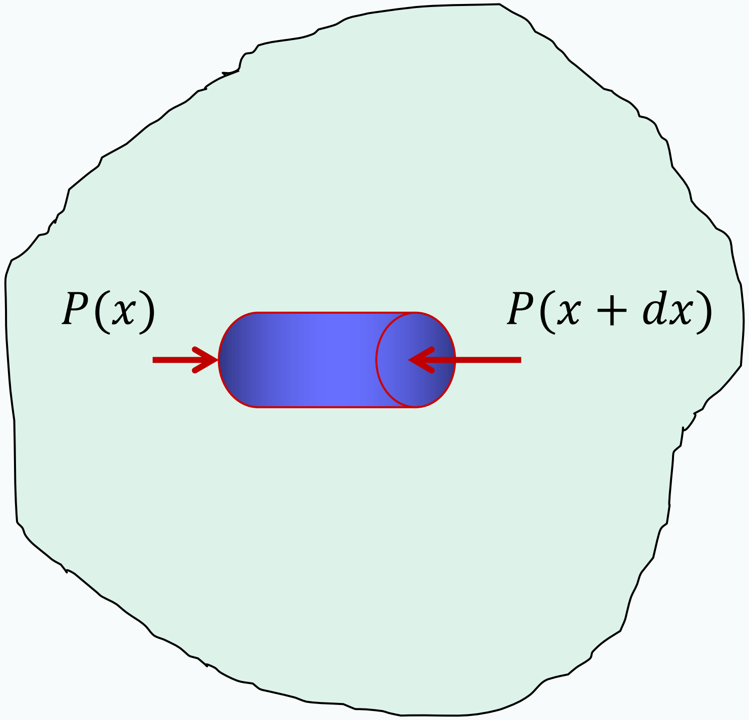
Pascal's principle:

Pressure applied to an enclosed fluid is transmitted to every point of the fluid and to the walls of the container.





Basic equation of hydrostatics



$$dF_{net,\,x} = P(x)dA - P(x+dx)dA$$

$$P(x+dx) - P(x) = dP = \frac{\partial P}{\partial x}dx$$

$$dF_{net,\,x} = -\frac{\partial P}{\partial x}dxdA = -\frac{\partial P}{\partial x}dV$$

$$dF_{net,\,y} = -\frac{\partial P}{\partial y}dydA = -\frac{\partial P}{\partial y}dV$$

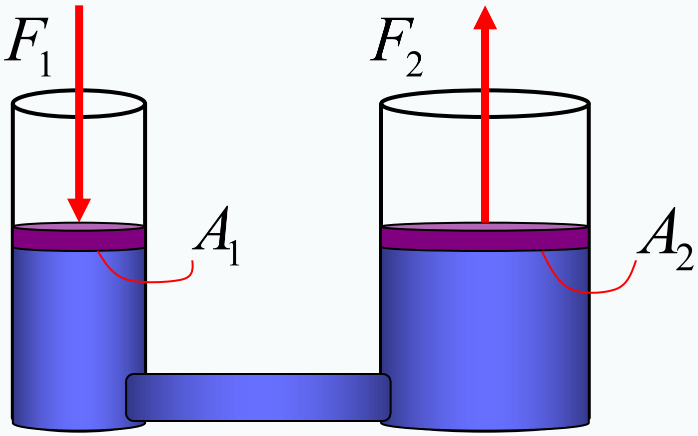
$$dF_{net,\,z} = -\frac{\partial P}{\partial z}dzdA = -\frac{\partial P}{\partial z}dV$$

$$grad(P) \equiv \frac{\partial P}{\partial x}\hat{\imath} + \frac{\partial P}{\partial y}\hat{\jmath} + \frac{\partial P}{\partial z}\hat{k}$$

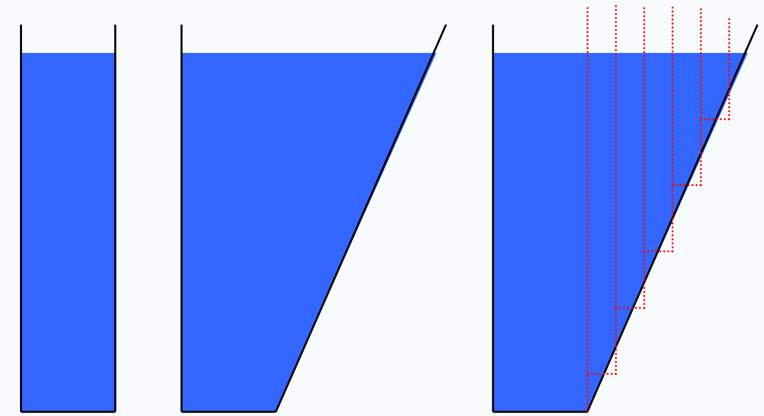
Under equilibrium:

$$\boxed{grad\;(P) = \nabla P = \vec{f}}$$

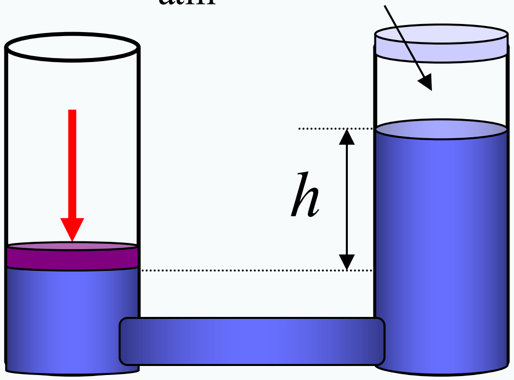
Hydraulic pressure



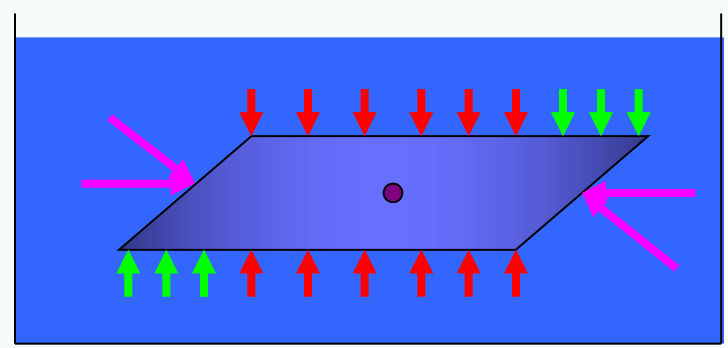
$$P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{A_2}$$



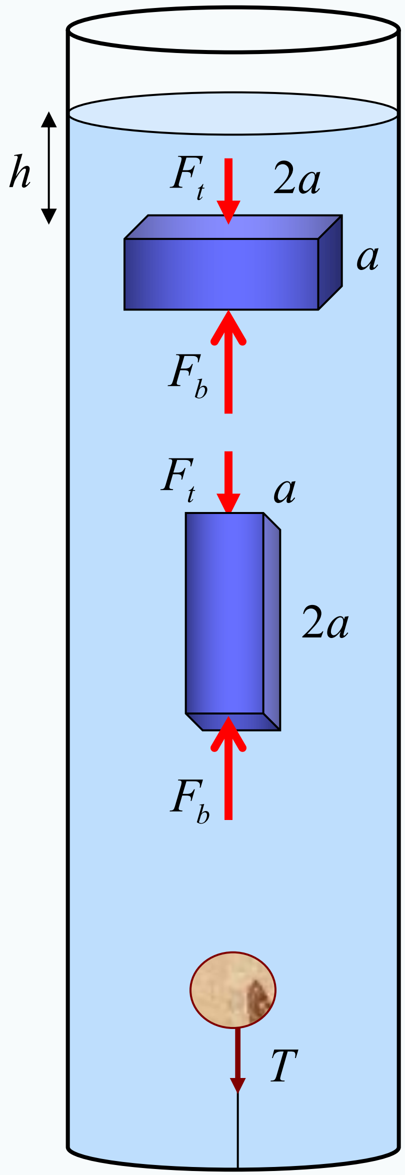
$$F = AP_{atm} \qquad P = 0$$



$$P_{atm} = \rho g h$$



Buoyancy



$$F_t = -\left(P_0 \cdot 2a^2 + \rho g h \cdot 2a^2\right)$$

$$F_b = P_0 \cdot 2a^2 + \rho g (h + a) \cdot 2a^2$$

$$F_{\text{net}} = \rho g 2a^3 = \rho V g$$

buoyant force

<u>Archimed's principle</u>:

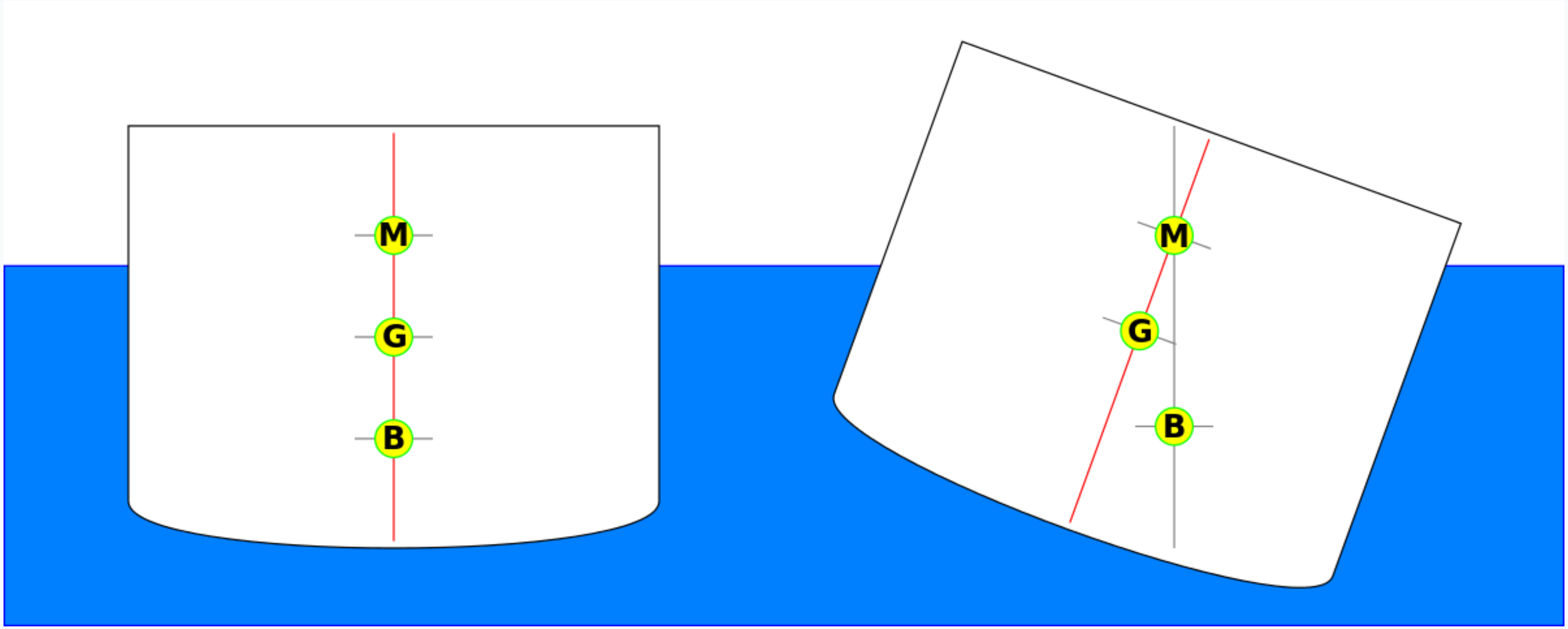
A body submerged in a fluid is acted (buoyed up) by a force equal to the weight of the displaced fluid.

Equilibrium (under water): $\rho_f g V = \rho_w g V \quad \Rightarrow \quad \rho_f = \rho_w$

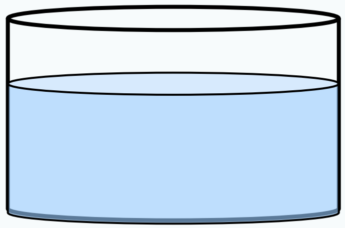
Equilibrium (floating): $\rho_f g V' = \rho_w g V \quad \Rightarrow \quad \dfrac{V'}{V} = \dfrac{\rho_w}{\rho_f}$

$$mg = 0.285\ \text{N} \qquad T = 0.855\ \text{N} \qquad \rho - ?$$

$$\rho = \rho_w \left(1 + \frac{T}{mg}\right)^{-1} \approx 0.25 \times 10^3\ \text{kg/m}^3$$

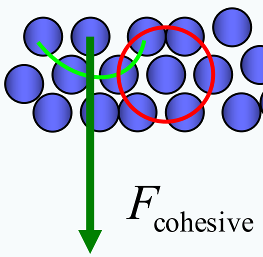

M
G
B
M
G
B

Surface tension



$$\sigma = \frac{dE}{dA}$$ **- specific surface energy**

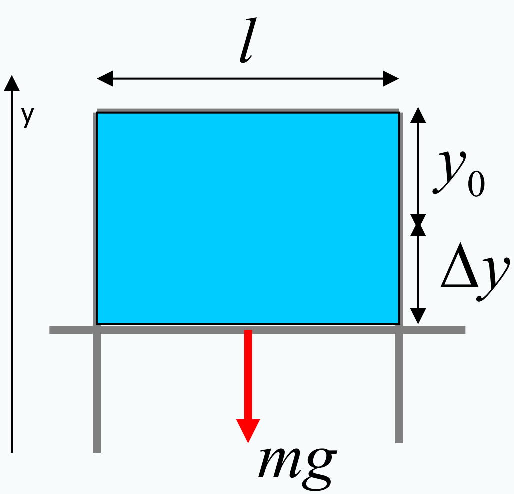
$$[\sigma] = \frac{J}{m^2} = \frac{N}{m}$$ **- surface tension**

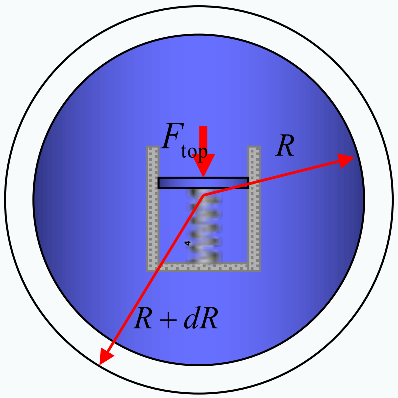


$$\Delta U_g = -mg\Delta y$$

$$\Delta U_s = -\sigma\left(A_f - A_i\right) = -\sigma \cdot l\Delta y \cdot 2$$

$$\sigma = \frac{mg}{2l}$$



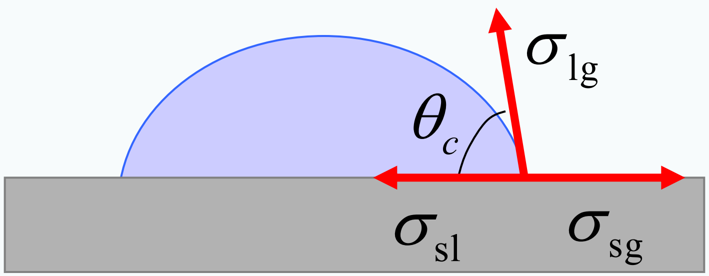


$$dU_s = \sigma\left(4\pi(R+dR)^2 - 4\pi R^2\right)$$

$$F = -\frac{dU_s}{dR} \qquad F = \frac{P}{A}$$

$$P_{\text{cohesive}} = \frac{2\sigma}{R}$$

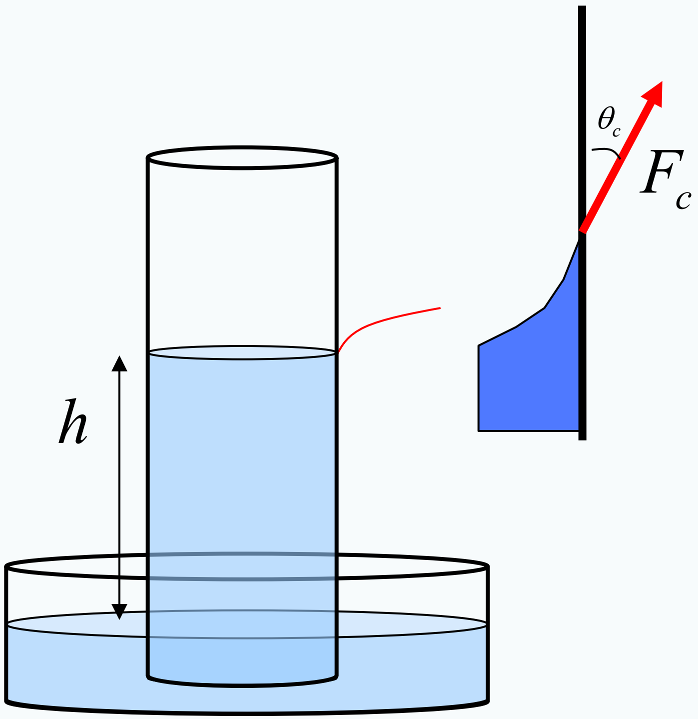
Capillary effects



The Young equation:

$$\sigma_{\mathrm{lg}} \cos\theta_c + \sigma_{\mathrm{sl}} = \sigma_{\mathrm{sg}}$$

Contact angle	Degree of wetting	Strength of interaction:	
		Sol.-Liq.	Liq.-Liq.
θ = 0	Perfect wetting	strong	weak
0 < θ < 90°	high wettability	strong	strong
		weak	weak
90°≤θ<180°	low wettability	weak	strong
θ = 180°	perfectly non-wetting	weak	strong

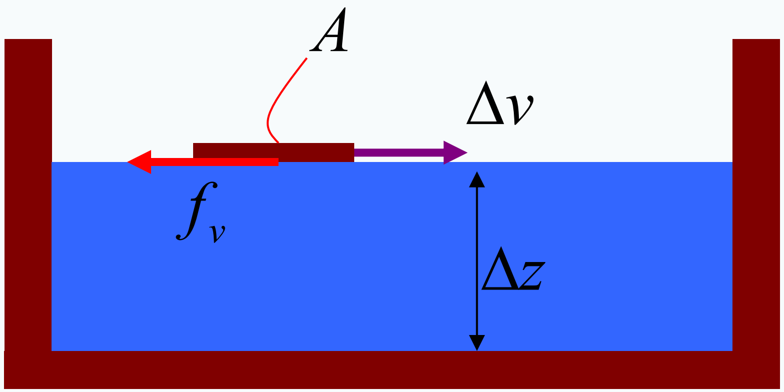


$$\sigma \cos\theta_c \cdot 2\pi R = \pi R^2 h \rho_l g$$

$$h = \frac{2\sigma\cos\theta_c}{\rho_l g R}$$

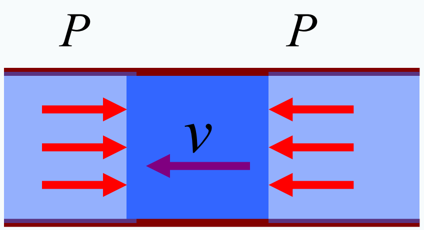
capillary action

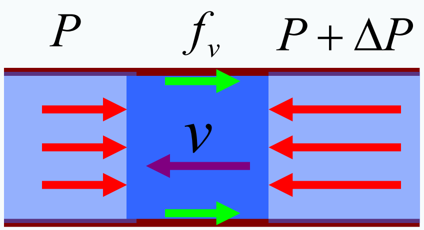
Viscous flow



$$f_v = -\eta A \frac{\Delta v}{\Delta z} = -\eta A \frac{dv}{dz}$$

viscosity or dynamic viscosity





$$F_{\text{net,p}} + f_v = 0$$

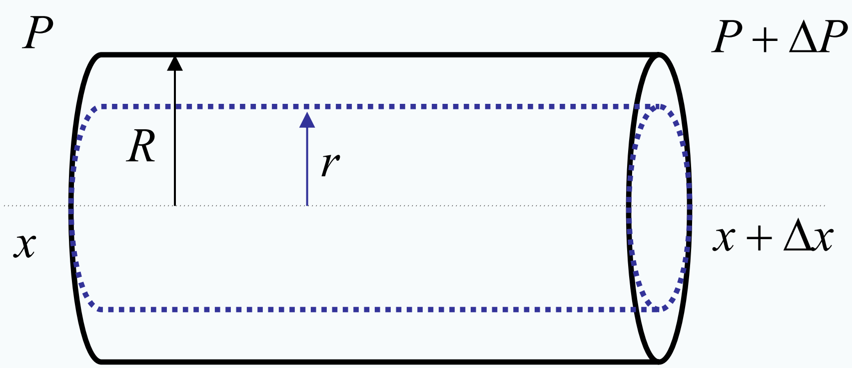
$$F_{\text{net,p}} = -\Delta P \cdot \pi r_a^2$$

$$f_v = \eta \cdot 2\pi r_a \Delta x \frac{dv}{dr_a}$$

$$\int_R^r r_a dr_a = \frac{2\eta\Delta x}{\Delta P} \int_0^{v_r} dv$$

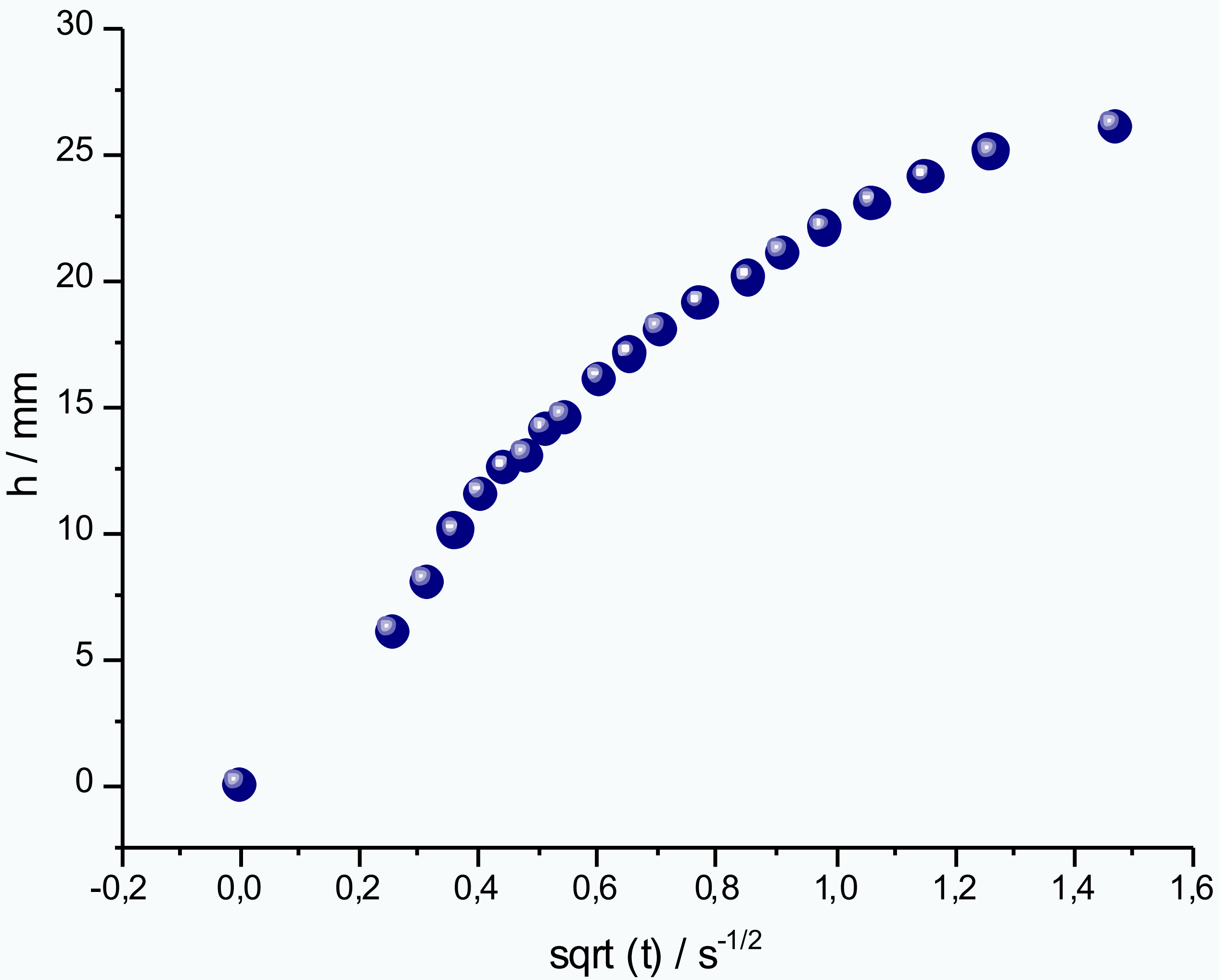
laminar flow

$$v_r = -\frac{\left(R^2 - r^2\right)}{4\eta} \frac{\Delta P}{\Delta x}$$

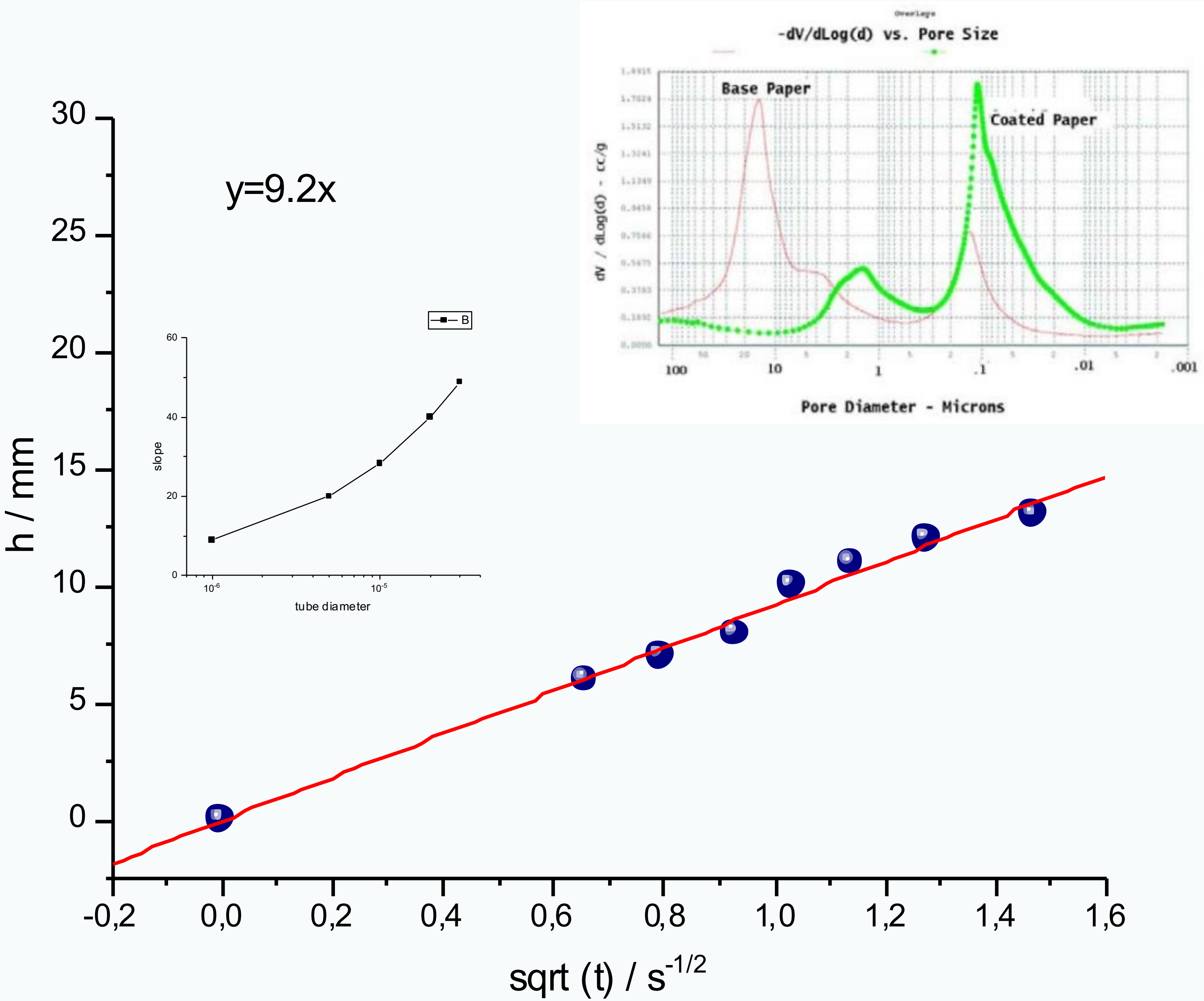


Hagen-Poiseuille law:

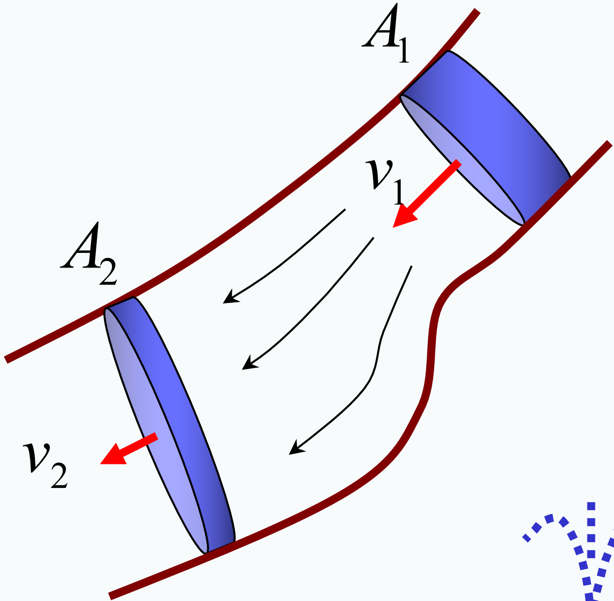
$$I_v = -\frac{\pi}{8\eta} \frac{\Delta P}{\Delta x} R^4$$



30
25
20
15
10
5
0
h / mm
-0,2
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
sqrt (t) / s-1/2


y=9.2x
h / mm
sqrt (t) / s-1/2
30
25
20
15
10
5
0
-0,2
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
B
slope
tube diameter
-dV/dLog(d) vs. Pore Size
Base Paper
Coated Paper
dV / dLog(d) - cc/g
Pore Diameter - Microns
100
10
1
.1
.01
.001

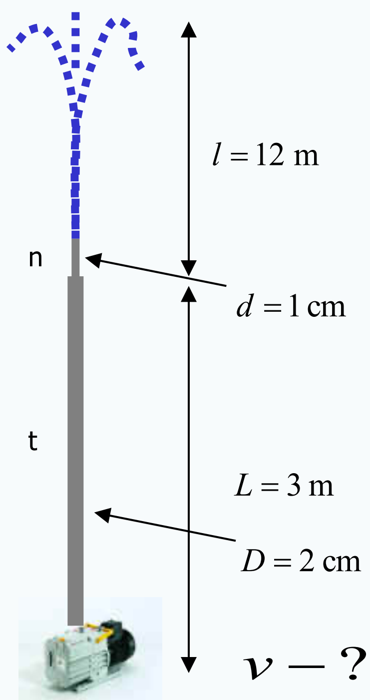
Continuity



$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$\boxed{I_V = vA}$ - volume flow rate **= constant**

- continuity equation



$$v_n A_n = v_t A_t$$

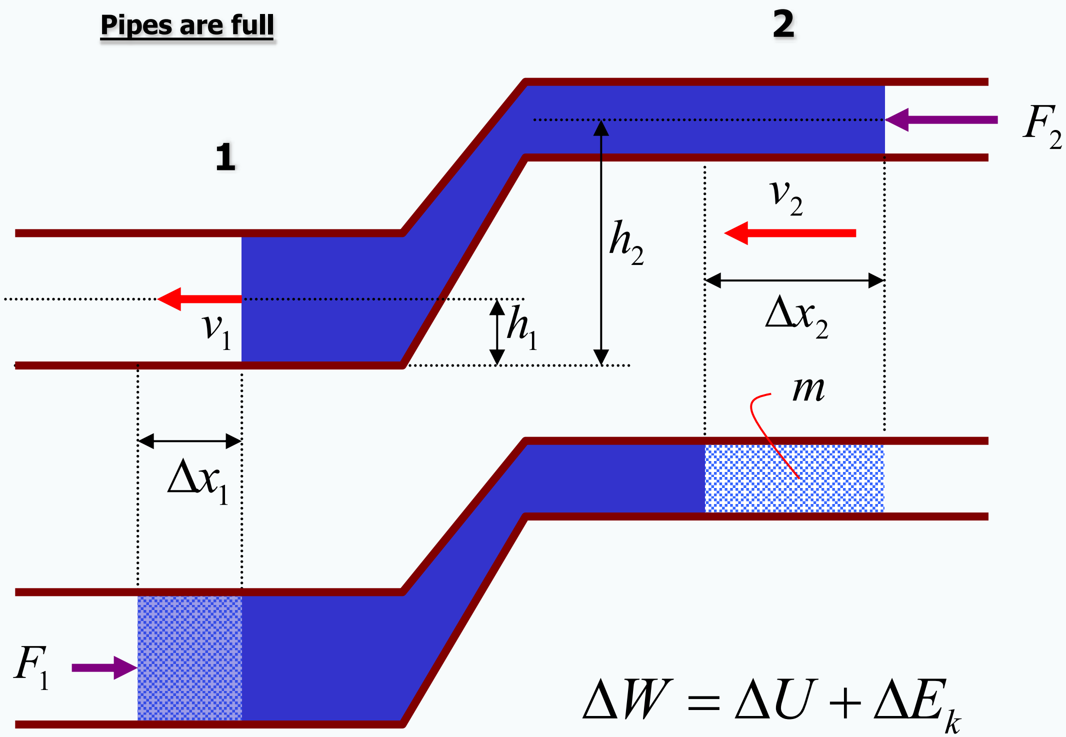
$$\tfrac{1}{2} m v_n^2 = mgl$$

$$v_t = \left(\frac{d}{D}\right)^2 \sqrt{2gl}$$

$$v_t = 3\ \text{m/s}$$

What pressure should the pump provide?

Bernoulli's equation



$$\Delta W = \Delta U + \Delta E_k$$

$$\Delta U = mgh_1 - mgh_2$$

$$\Delta E_k = \frac{m}{2}\left(v_1^2 - v_2^2\right)$$

$$W_2 = \int -F_2 dx = -P_2 A_2\left(-\Delta x_2\right)$$

$$W_1 = \int F_1 dx = P_1 A_1\left(-\Delta x_1\right)$$

$$\Delta W = P_2 V_2 - P_1 V_1 = P_2 V - P_1 V$$

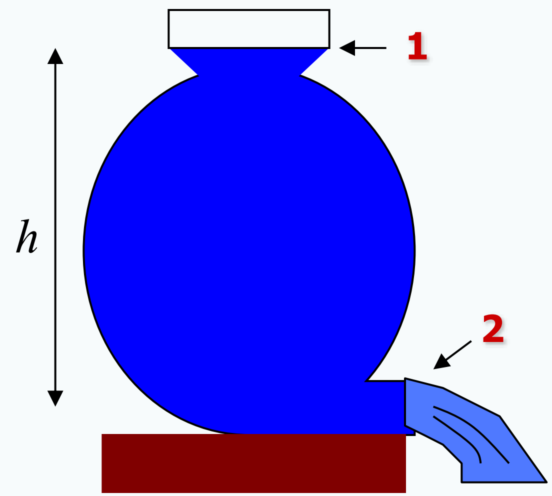
$$P_2 V - P_1 V = \rho V g h_1 - \rho V g h_2 + \tfrac{1}{2}\rho V v_1^2 - \tfrac{1}{2}\rho V v_2^2$$

$$P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 = P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2$$

Bernoulli's equation

$$\boxed{P + \rho g h + \tfrac{1}{2}\rho v^2 = \text{const}}$$

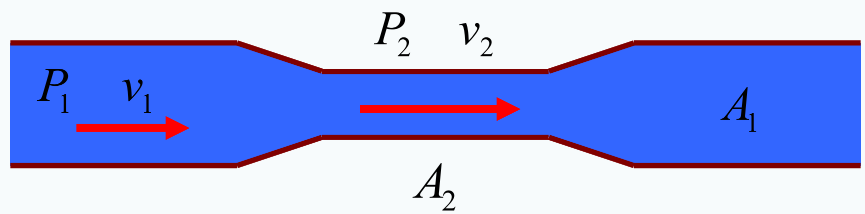
Applications of Bernoulli's equation



$$P_0 + \rho g h \approx P_0 + \tfrac{1}{2}\rho v_2^2$$

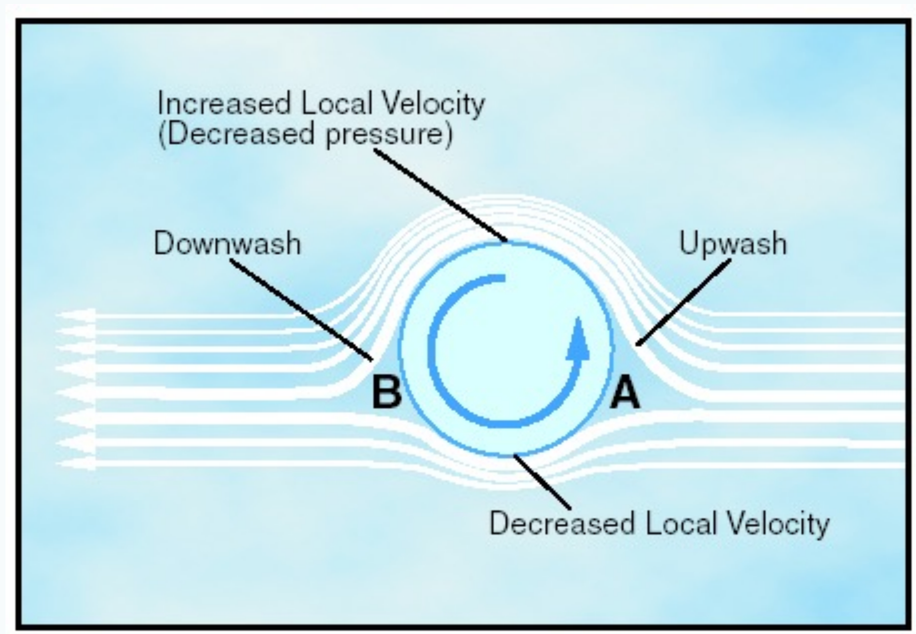
$$v_2 = \sqrt{2gh}$$

Torichelli's law

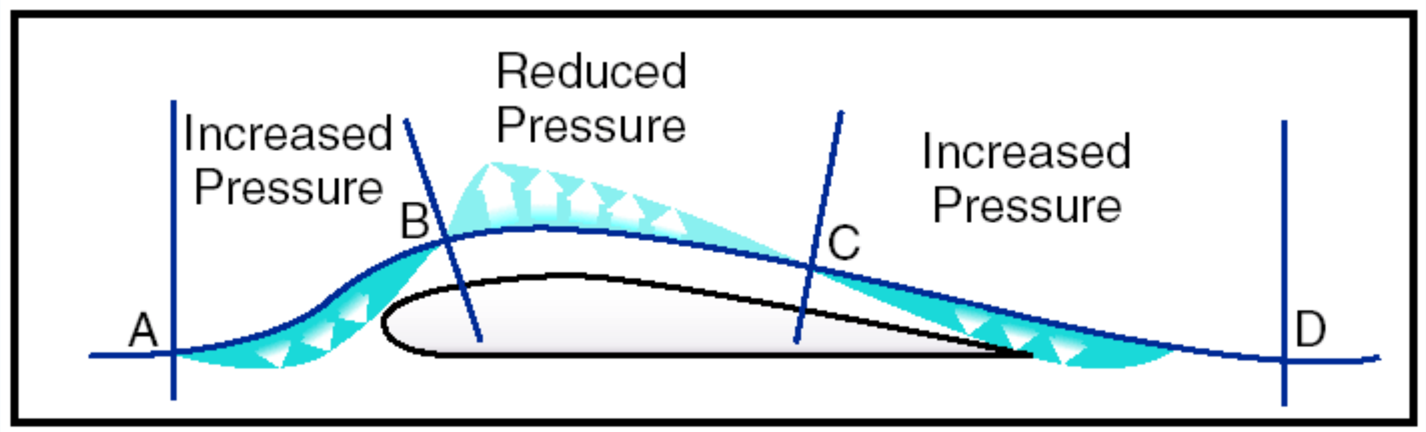


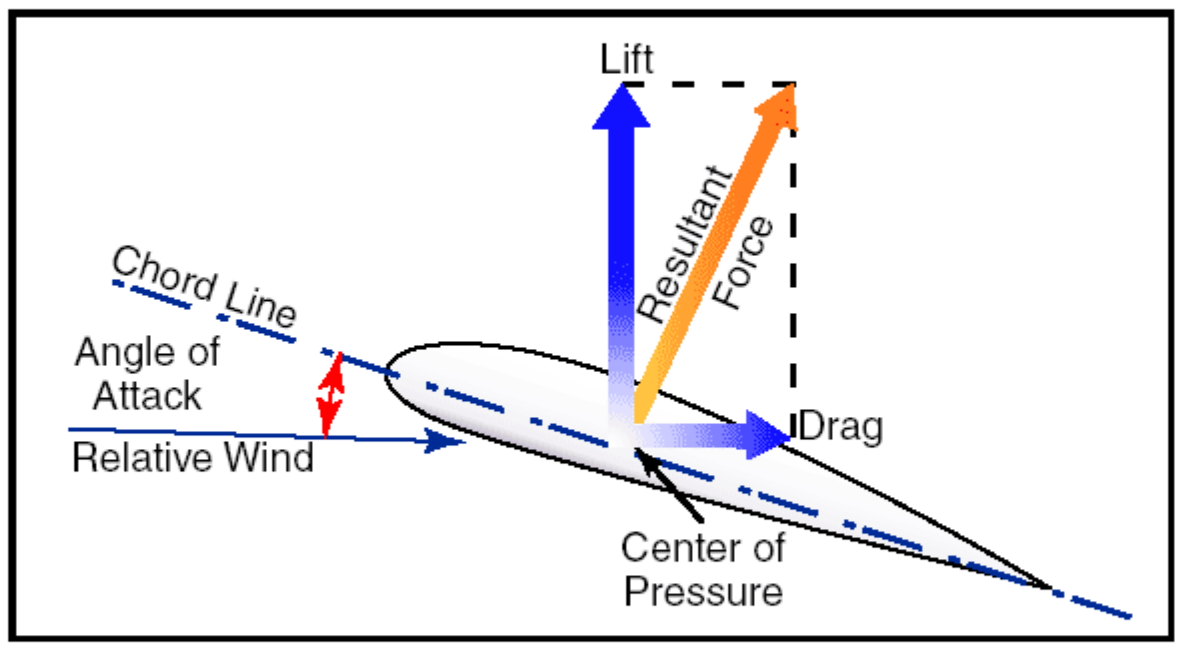
$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \qquad P + \tfrac{1}{2}\rho v^2 = \text{const}$$

Magnus effect



Physics of an airfoil





To remember!

- **<u>Pascal's principle</u>: pressure applied to an enclosed liquid is transmitted to any point.**

- **<u>Archimed's principle</u>: a body submerged in fluid is buoyed up with a force equal to the weight of the displaced fluid.**

- **Intermolecular forces: <u>surface tension</u> and <u>capillarity</u>.**

- **For steady-state flow of an incompressible fluid the volume flow rate is constant (<u>continuity</u>).**

- **<u>Bernoulli's equation</u> - conservation of total mechanical energy applied to fluids.**

- **<u>Pouseuille's law:</u> fluid velocity is inversely proportional to the distance from the wall.**