MA 1 – HW 11

(1)

$$(2A)^T-(3B)^T=\begin{pmatrix}6&2&8\\-4&0&2\\2&4&4\end{pmatrix}^T-\begin{pmatrix}3&0&6\\-9&3&3\\6&-12&3\end{pmatrix}^T=$$

$$=\begin{pmatrix}6&-4&2\\2&0&4\\8&2&4\end{pmatrix}-\begin{pmatrix}3&-9&6\\0&3&-12\\6&3&3\end{pmatrix}=\begin{pmatrix}3&5&-4\\2&-3&16\\2&-1&1\end{pmatrix}$$

(2)

a) $$\begin{pmatrix}3&5&1\\-2&0&2\end{pmatrix}\cdot\begin{pmatrix}2&1\\1&3\\4&1\end{pmatrix}=\begin{pmatrix}15&19\\4&0\end{pmatrix}$$

b) not possible $_m(\ )^n\cdot{}_k\underbrace{(\qquad)}_{l}$ $n=2,$ $k=1$

c) $$\begin{pmatrix}1&4&3\\0&1&4\\0&0&2\end{pmatrix}\cdot\begin{pmatrix}3&2\\1&1\\4&5\end{pmatrix}=\begin{pmatrix}19&21\\17&21\\8&10\end{pmatrix}$$

d) $$\begin{pmatrix}4&6\\2&1\end{pmatrix}\cdot\begin{pmatrix}3&1&5\\4&1&6\end{pmatrix}={}_2\Big\{\underbrace{\begin{pmatrix}36&10&56\\10&3&16\end{pmatrix}}_{3}$$

e) $\begin{pmatrix}4&6&1\\2&1&1\end{pmatrix}\cdot\begin{pmatrix}3&1&5\\4&1&6\end{pmatrix}$ not possible

f) $$\begin{pmatrix}2\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}3&2&4&5\end{pmatrix}=\begin{pmatrix}6&4&8&10\\-3&-2&-4&-5\\9&6&12&15\end{pmatrix}$$

(3)

$$AB=\begin{pmatrix}\alpha&1\\2&2\end{pmatrix}\cdot\begin{pmatrix}5&2\\\alpha&1\end{pmatrix}=\begin{pmatrix}6\alpha&2\alpha+1\\10+2\alpha&6\end{pmatrix}$$

$$BA=\begin{pmatrix}5&2\\\alpha&1\end{pmatrix}\cdot\begin{pmatrix}\alpha&1\\2&2\end{pmatrix}=\begin{pmatrix}5\alpha+4&9\\\alpha^2+2&\alpha+2\end{pmatrix}$$

$$\Longrightarrow\begin{cases}6\alpha=5\alpha+4\\10+2\alpha=\alpha^2+2\\6=\alpha+2\\2\alpha+1=9\end{cases}\Longrightarrow$$

$$\Longrightarrow \underline{\alpha=4}.$$

(4) $A=\begin{pmatrix}1 & 2024\\ 0 & 1\end{pmatrix}$ – elementary of type 3
(· 2nd row by 2024 & add to 1st)

$A^2=\begin{pmatrix}1 & 2024\cdot 2\\ 0 & 1\end{pmatrix}$ ...

$A^{10}=\begin{pmatrix}1 & 2024\cdot 10\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 20240\\ 0 & 1\end{pmatrix}$.

(5) $A\vec{x}=\vec{w}\times\vec{x}$ $\quad \vec{w}=\begin{pmatrix}1\\2\\3\end{pmatrix}$

1) $\forall \vec{a},\vec{b}\in\mathbb{R}^3$ $\vec{a}\times\vec{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$

$\Longrightarrow \vec{w}\times\vec{x}=\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2x_3-3x_2\\3x_1-1\cdot x_3\\1\cdot x_2-2\cdot x_1\end{pmatrix}$

2) By „reverse engineering" approach:

$\underbrace{\begin{pmatrix}0 & -3 & 2\\ 3 & 0 & -1\\ -2 & 1 & 0\end{pmatrix}}_{A}\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2x_3-3x_2\\3x_1-x_3\\x_2-2x_1\end{pmatrix}$

$A^T=\begin{pmatrix}0 & 3 & -2\\ -3 & 0 & 1\\ 2 & -1 & 0\end{pmatrix}=-A$, for such situations.

(6) a) Type 1
b) No
c) Type 3
d) Type 2

(7) a) $A=\begin{pmatrix}2 & -1\\ 5 & 3\end{pmatrix}$ $\quad B=\begin{pmatrix}-4 & 2\\ 5 & 3\end{pmatrix}$,

$\underline{E=\begin{pmatrix}-2 & 0\\ 0 & 1\end{pmatrix}}$

Check:
$\begin{pmatrix}-2 & 0\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}2 & -1\\ 5 & 3\end{pmatrix}=\begin{pmatrix}-4 & 2\\ 5 & 3\end{pmatrix}$ ✓

b) $E=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$

Check: $\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}2&1&3\\-2&4&5\\3&1&4\end{pmatrix}=\begin{pmatrix}2&1&3\\3&1&4\\-2&4&5\end{pmatrix}$

c) $E=\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}$

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(8)

$$\begin{array}{c}-6\ \ -3\ \ -3\\ A=\begin{pmatrix}2&1&1\\6&4&5\\4&1&3\end{pmatrix}\end{array}\xrightarrow{E_1=\begin{pmatrix}1&0&0\\-3&1&0\\0&0&1\end{pmatrix}}\begin{array}{c}-4\ \ -2\ \ -2\\ \begin{pmatrix}2&1&1\\0&1&2\\4&1&3\end{pmatrix}\end{array}\xrightarrow{E_2=\begin{pmatrix}1&0&0\\0&1&0\\-2&0&1\end{pmatrix}}\begin{pmatrix}2&1&1\\0&1&2\\0&-1&1\end{pmatrix}$$

$$\xrightarrow{E_3=\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}}\begin{pmatrix}2&1&1\\0&1&2\\0&0&3\end{pmatrix}=U$$

So $U=E_3E_2E_1A$, where

$E_1=\begin{pmatrix}1&0&0\\-3&1&0\\0&0&1\end{pmatrix}$, $E_2=\begin{pmatrix}1&0&0\\0&1&0\\-2&0&1\end{pmatrix}$, $E_3=\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}$.

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(9)

$$\begin{array}{c}-2\ \ -6\ \ -6\ \ 2\ \ 0\ \ 0\\ \left(\underbrace{\begin{matrix}-1&-3&-3\\2&6&1\\3&8&3\end{matrix}}_{A}\middle|\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\right)\end{array}\to\begin{array}{c}-3\ \ -9\ \ -9\ \ 3\ \ 0\ \ 0\\ \left(\begin{matrix}-1&-3&-3\\0&0&-5\\3&8&3\end{matrix}\middle|\begin{matrix}1&0&0\\2&1&0\\0&0&1\end{matrix}\right)\end{array}\to$$

$$\to\left(\begin{matrix}-1&-3&-3\\0&0&-5\\0&-1&-6\end{matrix}\middle|\begin{matrix}1&0&0\\2&1&0\\3&0&1\end{matrix}\right)\to\left(\begin{matrix}-1&-3&-3\\0&-1&-6\\0&0&-5\end{matrix}\middle|\begin{matrix}1&0&0\\3&0&1\\2&1&0\end{matrix}\right)\begin{array}{c}\\ \\ \\ 6\ \ -\frac{12}{5}\ \ -\frac{6}{5}\ \ 0\end{array}\to$$

$$\to\left(\begin{matrix}-1&-3&-3\\0&-1&0\\0&0&-5\end{matrix}\middle|\begin{matrix}1&0&0\\\frac{3}{5}&-\frac{6}{5}&1\\2&1&0\end{matrix}\right)\begin{array}{c}\\ \\ \\ 3\ \ \left(-\frac{6}{5}\right)\ \left(-\frac{3}{5}\right)\ 0\end{array}\to\left(\begin{matrix}-1&-3&0\\0&-1&0\\0&0&-5\end{matrix}\middle|\begin{matrix}-\frac{1}{5}&-\frac{3}{5}&0\\\frac{3}{5}&-\frac{6}{5}&1\\2&1&0\end{matrix}\right)$$

$$\to\left(\begin{matrix}-1&0&0\\0&-1&0\\0&0&-5\end{matrix}\middle|\begin{matrix}-2&3&-3\\\frac{3}{5}&-\frac{6}{5}&1\\2&1&0\end{matrix}\right)\to\left(\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\middle|\underbrace{\begin{matrix}2&-3&+3\\-\frac{3}{5}&\frac{6}{5}&-1\\-\frac{2}{5}&-\frac{1}{5}&0\end{matrix}}_{A^{-1}}\right)$$

Answer: Therefore $\begin{pmatrix} 2 & -3 & 3 \\ -3/5 & 6/5 & -1 \\ -2/5 & -1/5 & 0 \end{pmatrix} = A^{-1}$.

Check:

$$AA^{-1} = \begin{pmatrix} -1 & -3 & -3 \\ 2 & 6 & 1 \\ 3 & 8 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 & 3 \\ -3/5 & 6/5 & -1 \\ -2/5 & -1/5 & 0 \end{pmatrix} = \begin{pmatrix} -2+\frac{9}{5}+\frac{6}{5} & 3-\frac{18}{5}+\frac{3}{5} & -3+3 \\ 4-\frac{18}{5}-\frac{2}{5} & -6+\frac{36}{5}-\frac{1}{5} & 6-6 \\ 6-\frac{24}{5}-\frac{6}{5} & -9+\frac{48}{5}-\frac{3}{5} & 9-8 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

(10)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{pmatrix}$$

$$\begin{array}{ccc} -3 & -3 & -3 \end{array}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{pmatrix} \xrightarrow{E_1 = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \begin{array}{c} \begin{array}{ccc} 2 & 2 & 2 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ -2 & 2 & 7 \end{pmatrix} \end{array} \xrightarrow{E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 9 \end{pmatrix}$$

$$\begin{array}{cc} -4 & -6 \end{array}$$

$$\xrightarrow{E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix} = U$$

$$\Longrightarrow E_3 E_2 E_1 A = U \iff$$

$$A = \underbrace{E_1^{-1} E_2^{-1} E_3^{-1}}_{L} U$$

$$\Longrightarrow L = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{E_1^{-1}} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}}_{E_2^{-1}} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}}_{E_3^{-1}} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 2 & 1 \end{pmatrix}.$$

$$\Longrightarrow$$

$$A = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 2 & 1 \end{pmatrix}}_{L} \cdot \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix}}_{U}.$$