2. Basic Notions and Dimensional Analysis

Chapter 1 of my lecture notes provides the background to solve the following exercises. Your solution to the problems 2.1–2.4 should uploaded

to your Moodle account

as a PDF-file

by Tuesday, Oct 25, 12:00 (with a grace time till the start of the seminars)

The parts marked by $\star$ are suggestions for further exploration that will be followed up in the seminars. These are bonus problems that need not be submitted and that will not be graded. Problems marked by might take some extra effort to solve. They should always be considered as Bonus problems.

Problems

Problem 2.1. Earth orbit around the Sun

a) Light travels with a speed of $c \approx 3 \times 10^8\,\mathrm{m/s}$. It takes 8 minutes and 19 seconds to travel from Sun to Earth. What is the distance D of Earth and Sun in meters? Why is it admissible for this estimate to assume that the light takes 500 seconds for the trip?

b) The period, T, of the trajectory of the Earth around the Sun depends on D, on the mass $M = 2 \times 10^{30}\,\mathrm{kg}$ of the Sun, and on the gravitational constant $G = 6.7 \times 10^{-11}\,\mathrm{m^3/kg\,s^2}$. Estimate, based on this information, how long it takes for the Earth to travel once around the Sun.
Hint: Which combination of D, M, and G has a unit of seconds?

c) Express your estimate in terms of years. The estimate of (b) is of order one, but still off by a considerable factor. Do you recognize the numerical value of this factor?

d) Upon discussing the trajectory $\mathbf{x}(t)$ of planets around the Sun later on in this course, we will introduce dimensionless positions of the planets $\xi(t) = \mathbf{x}(t)/L = (x_1(t)/L, x_2(t)/L, x_3(t)/L)$. What would be L in this definition?

Problem 2.2. Oscillation period of a particle attached to a spring

In a gravitational field with acceleration $g_{\text{Moon}} = 1.6\,\text{m/s}^2$ a particle of mass $M = 100\,\text{g}$ is hanging at a spring with spring constant $k = 1.6\,\text{kg/s}^2$. It oscillates with a frequency ω when it is slightly pulled downwards and released. We describe the oscillation by the distance $x(t)$ from its rest position.

a) Construct a length scale L and a time scale T based on the parameters g_{Moon}, M, and k of the problem. This provides a dimensionless distance $\xi(t) = x(t)/L$, and the associated dimensionless velocity $\zeta(t) = \dot{x}(t)\,T/L$.

b) Provide an order-of-magnitude guess of the oscillation frequency ω.

c) For a Hookian spring the quantity $\mathcal{E} = \xi^2(t/T) + \zeta^2(t/T)$ is a constant number for the motion of the particle: $\mathcal{E}$ does *not* depend on the dimensionless time t/T even though $\xi(t/T)$ and $\zeta(t/T)$ are both time dependent.
What does this tell about the shape of trajectories in phase space?
Sketch a phase-space portrait of the motion.

$\star$ d) How does the phase-space portrait of the particle attached to a string differ from the one of a pendulum that we discussed in class?

In experiments one observes that $\mathcal{E}$ is only constant for small amplitude oscillations.
What does this mean in terms of the trajectories in phase space?
What happens for large amplitudes?

Problem 2.3. Conversion from Joule to Calories

A Calorie (cal) is defined as the heat energy needed to increase the temperature of $1\,\text{g}$ of water by $1\,\text{K}$, and an electric kettle typically is heating water with a power of $P = 1\,\text{kW}$.

Hint: Power refers to the energy released per unit time, i.e., $\text{J}\,\text{s}^{-1}$ in SI units.

a) Check out the power of your kettle and how long it takes to boil $1\,\text{l}$ of water. Estimate based on this information the conversion factor between Joule (J) and calories.

$\star$ b) Look up the literature value for the factor and compare it to your estimate. What might be the reasons for the discrepancy?

Problem 2.4. Water waves

The speed of waves on the ocean depends only on their wave length L and the gravitational acceleration $g \simeq 10\,\mathrm{m/s^2}$.

a) How does the speed v_w of the waves depend on L and g?

b) Unless it is surfing, the speed of a yacht is limited by its hull speed, i.e. the speed of a wave with wave length identical to the length of the yacht. Estimate the top speed of a 30 ft yacht.
Hint: $1\,\mathrm{ft} = 304.8 \times 10^{-3}\,\mathrm{m}$.

c) Often the conversion of feet into meters is approximated by a factor of 7/23, i.e. by assuming that $7\,\mathrm{m} = 23\,\mathrm{ft}$. Is that a reasonable approximation?