Lecture "Experimental Physics I"
(Prof. Dr. R. Seidel)

Lecture 27

Waves

- Longitudinal and transverse waves
- Wave equation(s)
- Sound waves
- Energy transport of a sound wave

1) Longitudinal and transverse waves

So far, we looked at stationary oscillations of coupled oscillators, such as a string, where we had a strict periodicity in time. Now we want to look at dynamic energy propagation, if only one of the terminal oscillators (e.g. the string end) is excited.

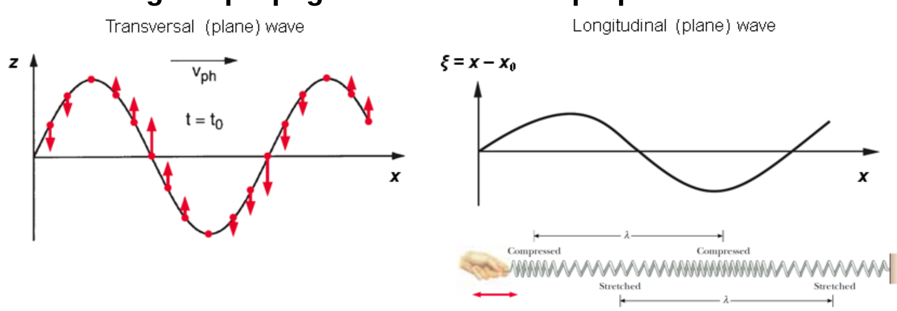
Experiment: We have a **coupled pendulum chain** where we displace one pendulum and get a dynamic propagation of the excitation energy from one pendulum to the other, which we call **wave.** (Note, that we can have for this system also stationary oscillation modes but it would be very difficult to excite a whole mode at once)

Observations:

- The excitation at the end provides a **moving amplitude across the system**, which **we call wave**
- In such a wave the **oscillations are propagating through space due to a coupling between neighboring oscillators.**
- **A wave transports oscillation energy, while the oscillators remain centered around fixed positions, such that there is no mass transport.**
- Later: The propagation velocity of the wave depends on the coupling and the mass of the oscillators.

Experiment: Stationary oscillations but a propagation of the oscillation can be illustrated by a rotating helix. At each position the circular motion of the helix represents the local oscillation. In the shadow projection, we see however a forward motion of the sinusoidal pattern.

We distinguish **longitudinal and transverse (plane) waves** depending on whether the **oscillations occur along the propagation direction or perpendicular** to it:



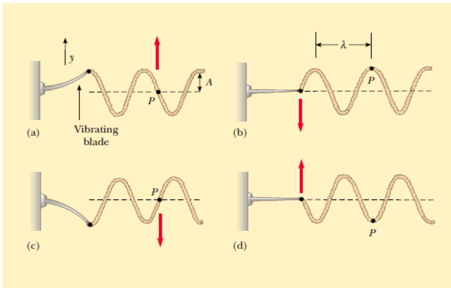
Experiments:

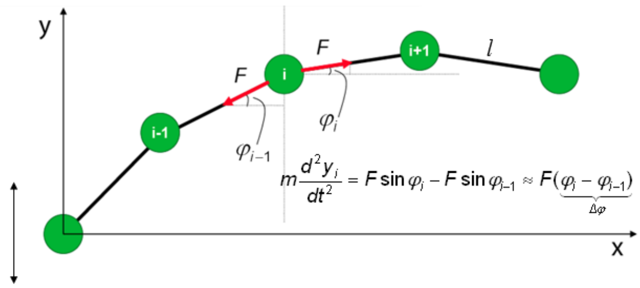
- Longitudinal waves we have seen for the pendulum chain, transverse waves are obtained in the “wave machine” which consists of coupled rotary oscillators.
- Helical elastic string: We can see nicely longitudinal and transverse pulse waves. For the latter we derive the wave equation in the following.

2) The wave equation

A) Derivation

Let us now derive an equation that describes the propagation of a wave. To this end we look again on the string under tension (since it can be seen as a chain of infinitesimally small coupled oscillators) and drive a wave by rapidly moving its end (a single time or periodically).





The initially determined equation of motion is still valid. For small angles, the force along the transverse direction and thus the acceleration of a single mass element is given by:

$$m\frac{d^2y_i}{dt^2} = F\,\sin\varphi_i - F\,\sin\varphi_{i-1} \approx F(\underbrace{\varphi_i - \varphi_{i-1}}_{\Delta\varphi}) = F\Delta\varphi$$

For smaller and smaller bead distances we get a continuous chain of mass segments $dm = \mu dx$ with μ being the linear mass density:

$$\underbrace{dm}_{\mu dx}\frac{d^2y}{dt^2} = \mu dx\frac{d^2y}{dt^2} = Fd\varphi$$

where $d\varphi$ denotes the angular difference to the next segment dx. Rewriting the differentials provides:

$$\frac{d^2y}{dt^2} = \frac{F}{\mu}\frac{d\varphi}{dx}$$

To get the derivative on the right side, for any curve we can write for small angles φ:

$$\varphi \approx \tan\varphi = \frac{dy}{dx}$$

Now we differentiate this equation again by x and arrive at:

$$\frac{\partial\varphi}{\partial x} = \frac{\partial^2 y}{\partial x^2}$$

Note, the correct expression for the differential of the tangent for larger angles is given by:

$$\frac{\partial\, tan\,\varphi\,(x)}{\partial x} = \frac{1}{\cos^2\varphi}\underbrace{[\cos\varphi\cos\varphi - (-\sin\varphi\sin\varphi)]}_{1}\frac{\partial\varphi}{\partial x} = \frac{1}{\cos^2\varphi}\frac{\partial\varphi}{\partial x} \approx \frac{\partial\varphi}{\partial x}$$

Inserting into the previous equation gives:

$$\frac{d^2y}{dt^2} = \underbrace{\frac{F}{\mu}}_{v^2}\frac{\partial^2 y}{\partial x^2}$$

Using the fact that the prefactor defines a squared velocity (as judged from the units), we derive the final form of the **wave equation**:

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad with \qquad v^2 = \frac{F}{\mu}$$

for a transverse wave along the x-direction. This general equation can be similarly derived for other transverse waves, such as electromagnetic waves or longitudinal waves such as linearly coupled oscillators (pendulums, mass-spring oscillator). Generally, we can write for a wave along x:

$$\frac{d^2\xi}{dx^2} = \frac{1}{v^2}\frac{d^2\xi}{dt^2}$$

with $\xi = x - x_0$ for a longitudinal wave and $\xi = y - y_0$ or $\xi = z - z_0$ (or any linear combination of y and z) for a transverse wave.

B) Plane wave solution for wave equation

So far, we looked only at "**stationary oscillations**" of a string where the **phase constant was equal over the whole string** and only the local amplitude was changing. The derived solutions for the string oscillation also fulfill the wave equation and are called standing waves as we will discuss later. An alternative and more general solution is the so-called **plane wave**, which is a wave that propagates in space. For propagation along x it has the following form:

$$y = A\cos\left(\frac{2\pi}{\lambda}x - \omega t\right) = A\cos(kx - \omega t)$$

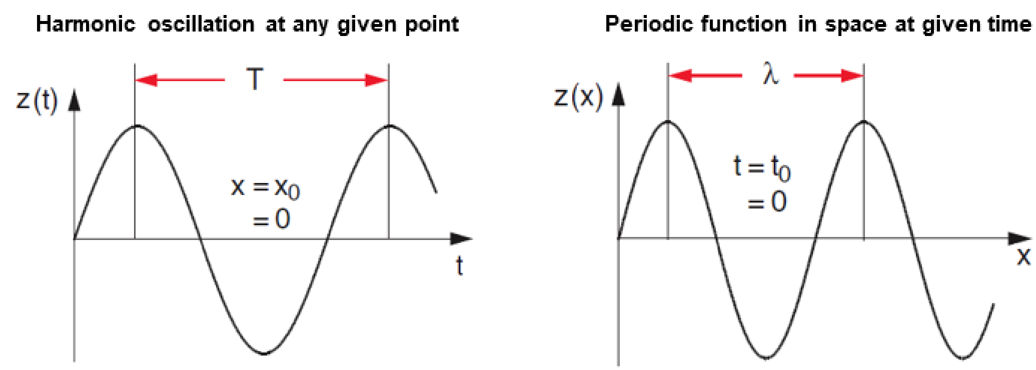
This equation can be seen from different views:

At a fixed position it describes an oscillation with **angular frequency $\boldsymbol{\omega}$** (see figure below). However, for each position we have a different initial phase of kx and thus also a total phase. The angular frequency can be expressed by the period of the oscillation at any given position x (as we knew from before):

$$\underset{\text{angular frequency}}{\omega} = \underset{\text{frequency}}{2\pi f} = \frac{2\pi}{\underset{\text{period}}{T}}$$

At a fixed time, the plane wave solution describes a **sinusoidal displacement along the $\boldsymbol{x}$-coordinate**, with **$\boldsymbol{\lambda}$ being the spatial periodicity** of the sine function, i.e. it is the distance between 2 maxima or minima. We call $\boldsymbol{\lambda}$ the **wave length**. Analogous to the angular and (linear) frequencies we define the **angular wave number** as $\boldsymbol{k = 2\pi/\lambda}$ and a linear wave number ν:

$$\underset{\text{(angular) wave number}}{k} = \underset{\text{linear wave number}}{2\pi\nu} = \frac{2\pi}{\underset{\text{wave length}}{\lambda}}$$



In quantum mechanics we will see that the angular wave number is proportional to the linear momentum of a quantum state.

Looking at the spatial sine pattern as function of time we see that the **$\boldsymbol{\omega t}$-term shifts the sine function** with increasing time **to the right,** such that we get a sinusoidal **wave pattern that propagates with positive velocity** towards increasing x **(show animation)**

Vice versa an opposite sign in front of ω would shift the curve to the left:

$$y = A\cos(kx + \omega t) = A\cos(-kx - \omega t) = A\cos\left(-\frac{2\pi}{\lambda}x - \omega t\right)$$

We typically use the representation with a **negative $\boldsymbol{k}$ to describe waves that travel with negative velocity**, i.e. towards decreasing x, i.e. the sign of k indicates the propagation direction of the wave

Now let us test the proposed plane wave solution by inserting it into the wave equation. This provides:

$$-k^2 A\cos(kx-\omega t) = -\frac{1}{v^2}\omega^2 A\cos(kx-\omega t)$$

From which we get:

$$v = \frac{\omega}{k}$$

This can be transformed to

$$v = \frac{\omega}{2\pi}\frac{2\pi}{k} = \lambda f = v_{Ph}$$

The velocity constant v given by **the product of wave length and frequency defines the so-called phase velocity** v_{Ph}. It is **the velocity at which the wave pattern is traveling. This corresponds to the velocity of any given point of the wave with a given constant phase in the argument cosine of the plane wave solution** (e.g. 2π for a maximum or 0 for a zero amplitude, see constant phase position in animation**).** This can also be seen in our plane wave solution:

$$y = A\cos\overbrace{(kx-\omega t)}^{phase}$$

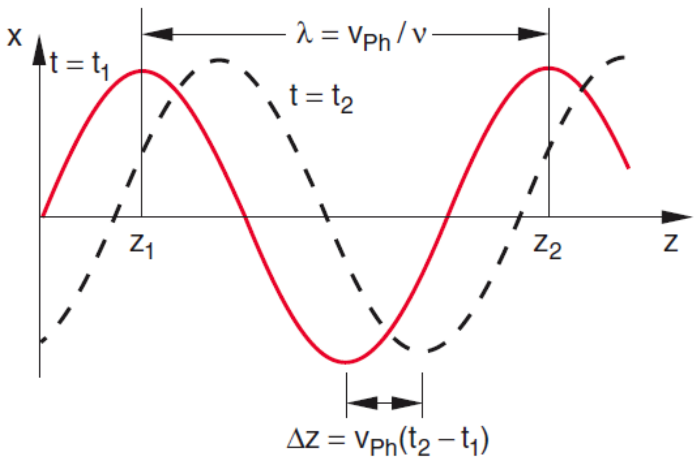
For a point of constant phase, the displacement y remains all the time the same. For a constant phase we can write:

$$const = kx - \omega t$$

Transforming provides:

$$x = \frac{const}{k} + \frac{\omega}{k}t = x_0 + v_{Ph}t$$

such that a point with constant phase travels with $v_{Ph} = \omega/k$ towards larger x with time. This makes intuitively sense, since after T the time component of the phase changed by -2π. The phase change of the spatial component must therefore be $+2\pi$, such that x must increase by λ. This means a **plane wave** that moves to the right **overlaps again with itself after period T, since in this time it moved by λ.** Thus:



For our **string wave** we get from the derived wave equation:

$$v_{Ph} = \sqrt{\frac{F}{\mu}}$$

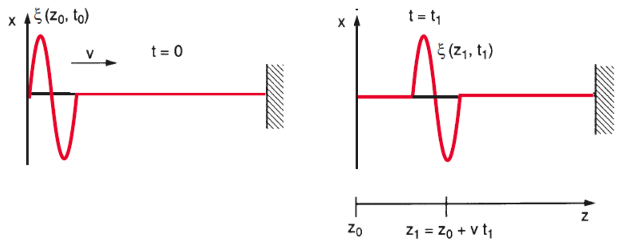
F defines hereby a kind of coupling strengths and μ the inertia. This means the relation represents a **balance between coupling and inertia** which together **define the phase velocity of a wave**.

When an element in the coupled system starts to experience a neighboring displacement and thus a force it starts to accelerate. Its inertia provides however a retarded response (since it needs to get accelerated) compared to the incoming displacement. We thus get a progressively increasing phase shift along the chain.

Experiment with helical elastic string: One can qualitatively test the equation for the phase velocity of the string wave by pulling the string with different forces while keeping the length constant. One can clearly see that the velocity of the pulse waves increases with force.

Experiment with the students in the lecture hall: The retardation due to the inertia can be intuitively illustrated by a "la Ola" wave, where the participants only see what the neighbor is doing. When the neighbor is rising his/her hands the limited speed at which a given person can react (like "inertia") leads to a slightly later rising of the hands compared to the neighbor and thus to a wave.

Note: Any solution to the wave equation is a wave including non-harmonic and non-periodic waves. For example, a short pulse/distortion that propagates along the coupled system is also a wave:



C) Complex number representation of a plane wave

So far, we have not considered any initial phase at $t = 0$ for the wave. This can be done by introducing an initial phase constant using the sum of a cosine and sine function:

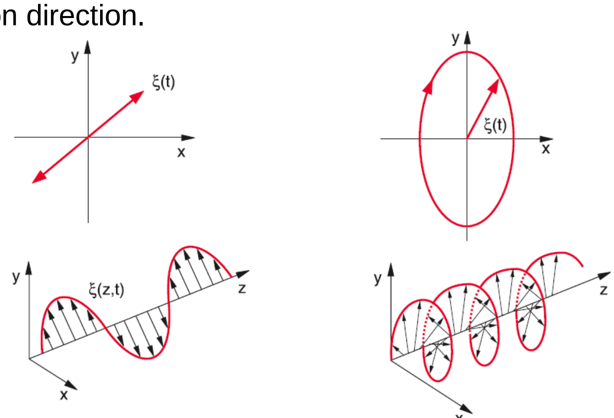
$$y = A\cos(kx - \omega t + \varphi_0) = C_1 \cos(kx - \omega t) + C_2 \sin(kx - \omega t)$$

The latter motivates a complex notation of the plane wave solution with complex amplitude C that comprises the initial phase:

$$y = C\, e^{i(kx-\omega t)} = \underbrace{|C|e^{i\varphi}}_{\text{polar rep. of C}}\; e^{i(kx-\omega t)}$$

D) Polarization of a transverse plane wave

A transverse wave can be polarized depending on how its amplitude vector behaves in time. We call the wave **linearly polarized if the amplitude vector $\vec{\xi}_0$ is constant**, i.e. it keeps pointing in the same direction. Thus, the displacement of a transverse wave occurs in a fixed plane that is parallel to the propagation direction.



In general, polarized transverse waves can be described by a superposition of two transverse waves with identical frequency whose oscillation directions are perpendicular to each other (e.g. along y and along z):

$$y = y_0 \cos(kx - \omega t + \varphi_1)$$
$$z = z_0 \cos(kx - \omega t + \varphi_2)$$

For a fixed position x this corresponds to the same superposition as seen for the Lissajous curves. Thus, we get:

- **Linear polarization** if $\varphi_2 = \varphi_1$ such that:
$$\vec{\xi} = \begin{pmatrix} y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} y_0 \\ z_0 \end{pmatrix}}_{\vec{\xi}_0} \cos(kx - \omega t + \varphi)$$
- **Circular polarization** if $\varphi_2 = \varphi_1 \pm \pi/2$ AND $y_0 = z_0$ such that
$$\vec{\xi} = y_0 \begin{pmatrix} \cos(kx - \omega t + \varphi) \\ \mp \sin(kx - \omega t + \varphi) \end{pmatrix}$$
- **Elliptic polarization** for all other cases of phases and amplitudes

For circular/elliptic polarization the resulting amplitude vector describes a helical path that is squeezed along one direction for elliptical polarization.

3) Sound waves

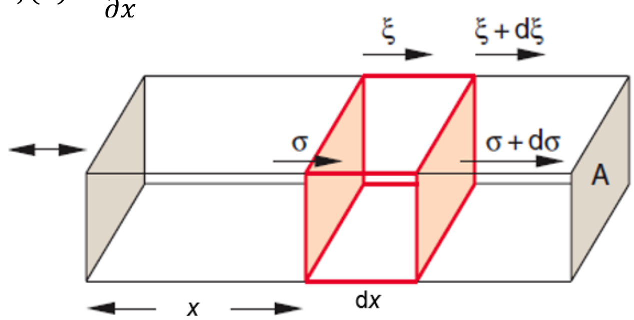
A very important example of mechanical waves are sound waves. They arise due to the interplay of the elasticity of the medium (body) defining the coupling between neighboring medium elements and its inertia (mass density). In the following we want to derive expressions for the phase velocity of sound waves, i.e. the speed of sound in rigid bodies as well as gases/ideal fluids. **Sound waves locally stretch or compress the material and thus cause density oscillations**.

A) Longitudinal sound waves in solid states

We first want to look at longitudinal sounds waves in a solid state. They can e.g. be excited using a loudspeaker that causes periodic pressure oscillations at the body surface, which in turn causes density oscillations within the material. The **relevant material properties** are the **Young's modulus E** describing the elasticity and the **density ρ.**

Let us consider a **wave along x with longitudinal displacements ξ of material elements perpendicular to a plane** $x = const$. In a wave the displacement ξ of the material from the equilibrium position is position dependent. Compared to position x with displacement $\xi(x)$, the displacement $\xi(x + dx)$ of volume elements at position $x + dx$ can be described by a Taylor expansion:

$$\xi(x + dx) = \xi(x) + \frac{\partial \xi}{\partial x} dx$$



The produced local strain, i.e. the relative extension $\varepsilon = \Delta L / L$ at a given spot is then given by:

$$\varepsilon = \frac{\xi(x+dx)-\xi(x)}{dx} = \frac{\partial \xi}{\partial x}$$

The local stress at a given point is given according to Hooke's law for a rigid body:

$$\sigma = E\varepsilon = E\frac{\partial \xi}{\partial x}$$

Thus, the stress is proportional to the change of the displacement (per length) across a given volume otherwise no compression/stretching would be obtained (see orange element volume in figure).

Now let us look at the net force/stress that acts on the volume element $dV = A\,dx$. It is provided by the stress difference on both sides of the volume element. The stress at position $x + dx$ can again be obtained from the stress at *x* using a Taylor expansion:

$$\sigma(x+dx) = \sigma(x) + \frac{\partial \sigma}{\partial x}dx = \sigma(x) + E\frac{\partial^2 \xi}{\partial x^2}dx$$

Where we inserted in the right side our derived expression for the stress. The net force on the volume element dV from the stress difference is then given by:

$$dF = A[\sigma(x+dx) - \sigma(x)] = EA\frac{\partial^2 \xi}{\partial x^2}dx$$

i.e. we have only a net force when the stress is changing across the volume element. The right side of this equation was obtained by inserting the expression for $\sigma(x+dx)$.

According to Newton's 2nd law, the inertia of the volume element with mass $dm = \rho\, dV$ must be equal to the effective force such that we can write:

$$\underbrace{\rho\, dV}_{dm}\frac{\partial^2 \xi}{\partial t^2} = E\frac{\partial^2 \xi}{\partial x^2}\underbrace{dxA}_{dV}$$

which can be transformed to a wave equation:

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 \xi}{\partial x^2}$$

In analogy to before, the prefactor defines the **phase velocity of an acoustic wave**, which is in first approximation only **defined by the ration of Young's modulus and the mass density** as initially hypothesized:

$$v_{Ph} = \sqrt{E/\rho}$$

E defines hereby the coupling strength and ρ the magnitude of the inertia. By measuring the speed of sound we can determine the Young's modulus of an material. When considering **Poisson's effect, i.e. the transverse contraction of a solid upon elongation** one can derive the following slightly different expression (**see slides**):

$$v_{Ph} = \sqrt{\frac{E(1-\mu)}{\rho(1+\mu)(1-2\mu)}}$$

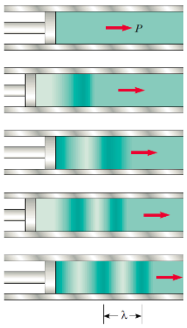
In addition to longitudinal sound waves we can also have transverse material displacements in solids and thus **transverse sound waves** in a solid. Transverse displacements are nothing else than shear deformations, such that one can derive in full analogy to the longitudinal waves the relation:

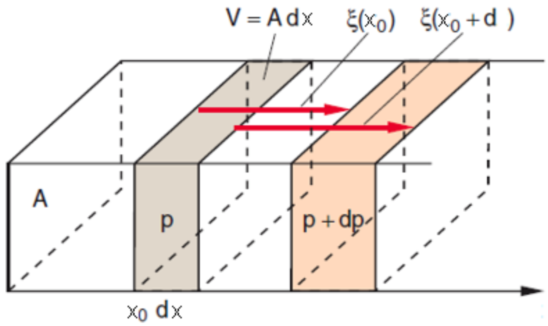
$$v_{Ph} = \sqrt{G/\rho}$$

for the **speed of sound of transverse sound waves**.

B) Sound waves in fluids and gases

In **gases and inviscid fluids** the shear module is zero. Therefore, we have **only longitudinal waves**. These are local density oscillations and thus pressure oscillations due to Pascals's principle. If air gets compressed at one position by an oscillating piston, then the pressure gradient from the increased pressure at one end starts to accelerate the neighboring gas layer. This gas layer gets compressed and maintains due its inertia compression even when the piston is decompressing the initial gas layer. Only after a while it experiences decompression but at this point the piston starts to compress again. This finally causes a propagating wave (Don't show derivation but say that bulk modulus replaces the Youngs modulus for a gas.)





We previously discussed the mechanics of bulk compression and found that a pressure change and volume strain are related by the bulk modulus K according to:

$$\Delta p = -K\frac{\Delta V}{V} = -K\frac{\partial \xi}{\partial x}$$

Considering as before a thin slice of thickness dx in perpendicular direction to the longitudinal displacement x (see figure), its volume can be expressed as $V = A\,dx$. Replacing volume changes using the linear displacement $\xi(x)$ allows in analogy to before deriving a similar relationship between pressure (being the bulk stress) and strain (right side in equation above).

Interlude: Formal derivation of wave equation inside compressible liquid/gas (not shown):

Let us consider a volume element V between x_0 and $x_0 + dx$ (gray in figure above) that experiences a displacement due to a wave propagating along x. For the displacement of the right side of V at $x_0 + dx$ we can write using the Taylor expansion:

$$\xi(x_0 + dx) = \xi(x_0) + \frac{\partial \xi}{\partial x}dx$$

The volume strain after the displacement is then given as:

$$\frac{\Delta V}{V} = \frac{[\xi(x_0 + dx) - \xi(x_0)]A}{A\,dx} = \frac{\partial \xi}{\partial x}$$

The pressure change at position $x_0 + dx$ can be obtained from a Taylor expansion around x:

$$\Delta p(x_0 + dx) = \Delta p(x_0) + \frac{\partial \Delta p}{\partial x}dx = \Delta p(x_0) - K\frac{\partial^2 \xi}{\partial x^2}dx$$

The force on V along x is given by the negative pressure difference:

$$dF = -A\big(\Delta p(x_0 + dx) - \Delta p(x_0)\big) = A\,K\frac{\partial^2 \xi}{\partial x^2}dx$$

This is an analogous expression to the force in a solid except that the bulk modulus replaces the Young's modulus. This force accelerates the mass comprised in $V = Adx$, such that we can write according to Newton's 2nd law:

$$\underbrace{\rho\,Adx}_{dm}\frac{\partial^2 \xi}{\partial t^2} = A\,K\frac{\partial^2 \xi}{\partial x^2}dx$$

Simplification provides then the analogous wave equation shown below.
and we arrive at the wave equation:

Due to the analogy between the stress strain relations in solids and liquids/gases we can derive an analogous wave equation:

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{K}{\rho}\frac{\partial^2 \xi}{\partial x^2}$$

The speed of sound in the gas or the inviscid fluid is given by:

$$v_{Ph} = \sqrt{K/\rho}$$

This equation is approximately valid for many liquids if shear forces play a minor role. For an **ideal gas** at a constant temperature $T = const$ (which we will see in thermodynamics), the pressure changes proportionally with the volume:

$$\frac{\Delta p}{p} = -\frac{\Delta V}{V}$$

Thus, the pressure p provides directly the bulk modulus:

$$\Delta p = -\underbrace{p}_{K}\frac{\Delta V}{V}$$

The speed of sound in an ideal gas is thus given as:

$$v_{Ph} = \sqrt{p/\rho}$$

For air at standard pressure and temperature we get $v_{air} = \sqrt{101{,}300Pa/1.24kg/m^3} = 286m/s$, which is only slightly lower than the real value of 340 m/s.

Experiment: Measurement of speed of sound. We use a loud speaker with 4 kHz excitation and a detector at various distances. Putting the input and the output signals on an oscilloscope reveals the delay of the detection due to the speed of sound. Moving 5 periods, i.e. 5λ requires a displacement of 435 mm

$$v_{Ph} = \lambda f = \frac{0.435\,m}{5} 4000\frac{1}{s} = 348\,\frac{m}{s}$$

Gases have generally the lowest speed of sound, followed by liquids and solids, which have much higher elastic moduli compared to their densities:

TABLE 17.1
Speeds of Sound in Various Media

Medium	v (m/s)	Medium	v (m/s)
Gases		**Solids**	
Hydrogen (0°C)	1 286	Diamond	12 000
Helium (0°C)	972	Pyrex glass	5 640
Air (20°C)	343	Iron	5 130
Air (0°C)	331	Aluminum	5 100
Oxygen (0°C)	317	Brass	4 700
Liquids at 25°C		Copper	3 560
Glycerol	1 904	Gold	3 240
Sea water	1 533	Lucite	2 680
Water	1 493	Lead	1 322
Mercury	1 450	Rubber	1 600
Kerosene	1 324		
Methyl alcohol	1 143		
Carbon tetrachloride	926		

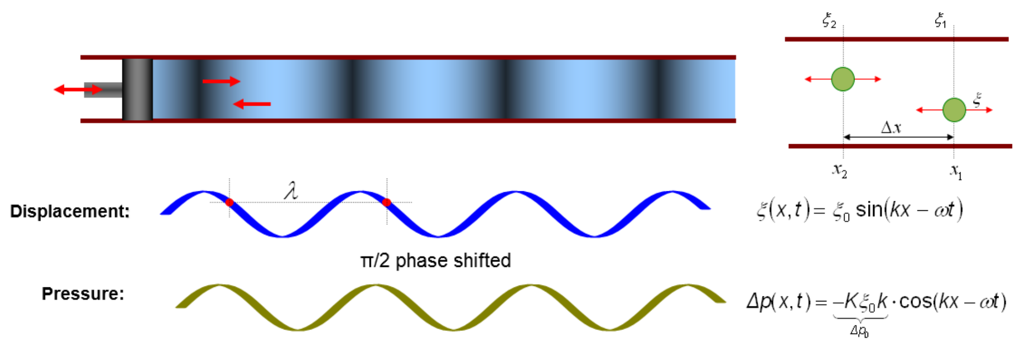
Since we found a wave equation for sound waves in gases, our plane wave solution will also solve this equation describing a continuous periodic wave with a single frequency. The plane wave solution describes the displacement of the gas at a given point according to:

$$\xi(x,t) = \xi_0 \cos(kx - \omega t)$$

Using the equation derived before, this translates into pressure oscillations of the following magnitude:

$$\Delta p(x,t) = -K\frac{\partial \xi}{\partial x} = \underbrace{K\xi_0 k}_{\Delta p_0} \sin(kx - \omega t)$$

These pressure oscillations are the ones felt in a concert with loud basses. Importantly **the pressure is phase shifted in time but most importantly in space by $\pi/2$ compared to the displacement amplitude**. Thus, we have zero pressure where we have displacement maxima and minima and maximum/minimum pressure where we have displacement.
The $\pi/2$ phase shift can be intuitively be understood considering that adjacent displacement maxima and minima shift the medium towards each other such that a pressure change in between arises (see figure).



The pressure amplitude is called **sound pressure** whose magnitude defines the "loudness", i.e. of the sound (see below). With $|K| = v_{Ph}^2\,\rho$ and $\omega = k\,v_{Ph}$ we get:

$$\Delta p_0 = K\xi_0 k = v_{Ph}\omega\rho\xi_0$$

C) Energy transport by a mechanical wave

We have seen in the experiment with the coupled pendulum chain that a (pulse) wave transports energy. The transmitted power equals the introduced oscillator energy per time required for its propagation. Similarly, one can also calculate the energy transported by a continuous wave in a continuous medium. Let us consider the displacement of a volume element with mass $\Delta m = \rho\,\Delta V$ in a medium through which a plane wave with local displacement ξ propagates:

$$\xi = \xi_0\cos(kx - \omega t)$$

Its kinetic energy is given by:

$$E_{kin} = \frac{1}{2}\Delta m\dot{\xi}^2 = \frac{1}{2}\rho\Delta V\,\xi_0^2\omega^2\sin^2(kx - \omega t)$$

The mass element has in addition potential energy due to the elastic deformations of the medium which is for linear elasticity proportional to the square of the displacement amplitude. The potential energy has the same magnitude as the kinetic energy, since for a plane wave the local oscillator has a constant mechanical energy (analogous to a simple mechanical oscillator). We thus get:

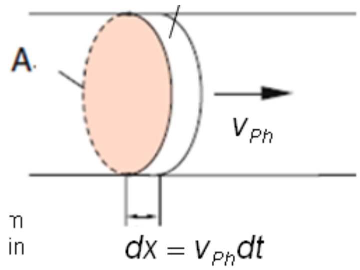
$$E_{pot} = \underbrace{\frac{1}{2}\rho\Delta V\omega^2\,\xi_0^2}_{\text{same as for } \mathrm{E_{kin}}}\cos^2(kx - \omega t)$$

The total mechanical energy per volume, i.e. the energy density, of the wave is thus

$$\rho_E = E_{kin} + E_{pot} = \frac{1}{2}\rho\xi_0^2\omega^2$$

Within the time dt the wave travels by $dx = v_{Ph}dt$. The energy that is transported per time dt by the wave across an area A is then given by the energy within the volume Adx:

$$dE = \rho_E A\underbrace{v_{Ph}dt}_{dx}$$



The transported power (or energy current) is then

$$P = \frac{dE}{dt} = \rho_E v_{Ph} A$$

We now define the **intensity** or **energy flux** of a wave (similar to mass flux) as the **energy transported per time and area perpendicular to the propagation direction.** Since the wave is moving with velocity v_{Ph} this is simply given by the relation:

$$I = \rho_E v_{Ph} = \frac{1}{2}\rho {\xi_0}^2 \omega^2 v_{Ph}$$

Thus, the intensity of a wave is proportional to the squares of amplitude and frequency. We can use this expression to calculate the **intensity of a sound wave** using the sound pressure $\Delta p_0 = \rho v_{Ph} \omega \xi_0$ derived before. Replacing $\omega \xi_0$ with the sound pressure yields:

$$I = \frac{1}{2} \frac{{\Delta p_0}^2}{v_{Ph}\rho}$$

Thus, the sound intensity is proportional to the square of the sound pressure. Sound levels extend over many orders of magnitude (see figure). One therefore uses a logarithmic scale for quantifying the intensity. It is called the **sound level a**nd it is **given in dB (decibel)**:

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-12}\ \mathrm{W/m^2}}\right)$$

where the normalization constant in the denominator of the base-10 logarithm is the minimal **threshold for sound sensation by the human ear.** The table below shows the sound levels of everyday noises from which we see that the **ear has a huge dynamic range of sound sensation over many orders of magnitude** (see right table column that gives you the actual intensity increase compared to the minimal threshold)**!**

	Sound pressure p	Sound level	X-fold increase
Pain limit	100 Pa	134 dB	$10^{13.4}$
Jet airplane from 100 m	6,3–200 Pa	110–140 dB	10^{11} – 10^{14}
Jack hammer from 1 m / Disco	2 Pa	~ 100 dB	10^{10}
Hearing damage at long term exposure	0,36 Pa	ab 85 dB	$10^{8.5}$
Busy traffic (10 m)	0,2–0,63 Pa	80–90 dB	10^8 - 10^9
Normal conversation (1m)	$2\cdot10^{-3}$–$6{,}3\cdot10^{-3}$ Pa	40–50 dB	10^4 - 10^5
Rustling leaves, quiet breathing	$6{,}3\cdot10^{-5}$ Pa	10 dB	10
Threshold of hearing at 2 kHz	$2 \cdot 10^{-5}$ Pa	0 dB	1

Lecture 27: Experiments

1) Wave pulse in a coupled pendulum chain where we displace one pendulum and get a longitudinal wave that propagates through the chain
2) Helical elastic string to demonstrate longitudinal and transverse pulse waves.
3) Stationary oscillations but propagation illustrated by a rotating helix model
4) Transverse waves in the wave machine
5) Measurement of speed of sound. Loud speacker with 4 kHz excitation and detector at various differences. Detection of phase shift between excitation and detection due to finite propagation speed of the wave