EXPERIMENTAL PHYSICS 1

SOLUTION EXERCISE 10 (*TASK)

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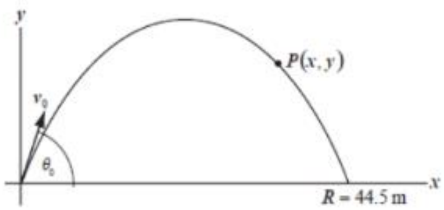
Solution Exercice 10 (*Tasks)

Problem 1:

37 •• In 1978, Geoff Capes of Great Britain threw a heavy brick a horizontal distance of 44.5 m. Find the approximate speed of the brick at the highest point of its flight, neglecting any effects due to air resistance. Assume the brick landed at the same height it was launched.

Picture the Problem We'll ignore the height of Geoff's release point above the ground and assume that he launched the brick at an angle of 45°. Because the velocity of the brick at the highest point of its flight is equal to the horizontal component of its initial velocity, we can use constant-acceleration equations to relate this velocity to the brick's x and y coordinates at impact. The diagram shows an appropriate coordinate system and the brick when it is at point P with coordinates (x, y).



Using a constant-acceleration equation, express the x and y coordinates of the brick as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
and
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Because $x_0 = 0$, $y_0 = 0$, $a_y = -g$, and $a_x = 0$:

$$x = v_{0x}t$$
and
$$y = v_{0y}t - \tfrac{1}{2}gt^2$$

Eliminate t between these equations to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_{0x}^2}x^2$$

where we have used $\tan\theta_0 = \dfrac{v_{0y}}{v_{0x}}$.

When the brick strikes the ground $y = 0$ and $x = R$:

$$0 = (\tan\theta_0)R - \frac{g}{2v_{0x}^2}R^2$$

where R is the range of the brick.

Solve for v_{0x} to obtain:

$$v_{0x} = \sqrt{\frac{gR}{2\tan\theta_0}}$$

Substitute numerical values and evaluate v_{0x}:

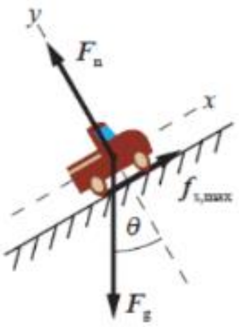
$$v_{0x} = \sqrt{\frac{(9.81\,\text{m/s}^2)(44.5\,\text{m})}{2\tan 45°}} \approx \boxed{15\,\text{m/s}}$$

Note that, at the brick's highest point, $v_y = 0$.

Problem 2:

58 •• The coefficient of static friction between a rubber tire and the road surface is 0.85. What is the maximum acceleration of a 1000-kg four-wheel-drive truck if the road makes an angle of 12° with the horizontal and the truck is (*a*) climbing and (*b*) descending?

Picture the Problem Choose a coordinate system in which the $+x$ direction is up the incline and apply Newton's second law of motion. The free-body diagram shows the truck climbing the incline with maximum acceleration.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the truck when it is climbing the incline:

$$\sum F_x = f_{s,max} - mg\sin\theta = ma_x \quad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

Solve equation (2) for F_n and use the definition $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s mg\cos\theta \quad (3)$$

Substitute $f_{s,max}$ in equation (1) and solve for a_x:

$$a_x = g(\mu_s \cos\theta - \sin\theta)$$

Substitute numerical values and evaluate a_x:

$$a_x = (9.81\,\text{m/s}^2)[(0.85)\cos 12° - \sin 12°] = \boxed{6.1\,\text{m/s}^2}$$

(*b*) When the truck is descending the incline with maximum acceleration, the static friction force points down the incline; that is, its direction is reversed Apply $\sum F_x = ma_x$ to the truck under these conditions to obtain:

$$-f_{s,max} - mg\sin\theta = ma_x \quad (4)$$

Substituting for $f_{s,max}$ in equation (4) and solving for a_x gives:

$$a_x = -g(\mu_s \cos\theta + \sin\theta)$$

Substitute numerical values and evaluate a_x:

$$a_x = (-9.81\,\text{m/s}^2)[(0.85)\cos 12° + \sin 12°] = \boxed{-10\,\text{m/s}^2}$$

Problem 3:

45 •• A homogeneous solid object floats on water, with 80.0 percent of its volume below the surface. When placed in a second liquid, the same object floats on that liquid with 72.0 percent of its volume below the surface. Determine the density of the object and the specific gravity of the liquid.

Picture the Problem Let V be the volume of the object and V' be the volume that is submerged when it floats. The weight of the object is $\rho V g$ and the buoyant force due to the water is $\rho_w V' g$. Because the floating object is in translational equilibrium, we can use $\sum F_y = 0$ to relate the buoyant forces acting on the object in the two liquids to its weight.

Apply $\sum F_y = 0$ to the object floating in water:

$$\rho_w V' g - mg = \rho_w V' g - \rho V g = 0 \quad (1)$$

Solving for ρ yields:

$$\rho = \rho_w \frac{V'}{V}$$

Substitute numerical values and evaluate ρ:

$$\rho = \left(1.00\times10^3\ \text{kg/m}^3\right)\frac{0.800V}{V} = \boxed{800\ \text{kg/m}^3}$$

Apply $\sum F_y = 0$ to the object floating in the second liquid and solve for mg:

$$mg = 0.720 V \rho_L g$$

Solve equation (1) for mg:

$$mg = 0.800 \rho_w V g$$

Equate these two expressions to obtain:

$$0.720 \rho_L = 0.800 \rho_w$$

Substitute in the definition of specific gravity to obtain:

$$\text{specific gravity} = \frac{\rho_L}{\rho_w} = \frac{0.800}{0.720} = \boxed{1.11}$$

Problem 7:

The Tunnel through the Earth **$3 + 2 + 1$ Points**

Assume that there is a straight tunnel through the earth connecting the North Pole with the South Pole. An object with a mass of $m = 10^3$ kg is released at the North Pole and falls towards the earth's centre.

a. What is the time needed for the object to reach the South Pole?
a. What is its velocity at the earths centre?
b. Assume that at the centre of the earth the object receives an impulse of 10^4 Ns in the direction of its velocity (i.e. the linear momentum of the object is instantly increased by 10^4 Ns). What is the velocity of the object at the South Pole?

Hint: Assume that the earth is a sphere with a homogenous density. The mass is $m_E = 5.97 \cdot 10^{24}\, kg$, the radius is $6378\ km$.

The gravitational constant is $G = 6.67 \cdot 10^{-11}\, m^3 kg^{-1} s^{-2}$.

Hint 2: It might help to do the following consideration: How does the force depend on R proportionally? What other system works like this?

Solution

(a)

Force is depending on R as:

$$|\vec{F}| = \frac{4}{3}\pi \cdot Gm\rho R$$

The equation of the motion to the center will be (1P):

$$m\ddot{R}(t) = -\frac{4}{3}\pi \cdot Gm\rho R(t)$$

Solution (1P):

$$R(t) = \mathrm{R_{earth}} \cdot \cos\left(t \cdot \sqrt{\frac{4}{3}\pi G\rho}\right) = \mathrm{R_{earth}} \cdot \cos\left(t \cdot \sqrt{\frac{4}{3}\pi G \frac{m_E}{\frac{4}{3}\pi R_{earth}^3}}\right) =$$

$$= \mathrm{R_{earth}} \cdot \cos\left(t \cdot \sqrt{G\frac{m_E}{R_{earth}^3}}\right)$$

Note: the solution can be derived either by solving the differential equation or by comparing the system to a hookian spring.

What leads to the time till the other side of the Earth (1P):

$$T_{North \to South\ Pole} = \pi \cdot \frac{1}{\sqrt{G\frac{m_E}{R_{earth}^3}}} \cong 2535{,}9\ \mathrm{s} \cong 42{,}3\ \mathrm{min} \cong 0{,}7\ \mathrm{h}\ (1P)$$

(b)

Can be done in two ways:

$$v = \left|\dot{R}(t)\right| = \mathrm{R_{earth}} \cdot \sqrt{G\frac{m_E}{R_{earth}^3}} \cdot \sin\left(t \cdot \sqrt{G\frac{m_E}{R_{earth}^3}}\right)$$

At the center: $R(t) = 0$, so $\cos\left(t \cdot \sqrt{G\frac{m_E}{R_{earth}^3}}\right) = 0$, thus $\sin\left(t \cdot \sqrt{G\frac{m_E}{R_{earth}^3}}\right) = 1$

$$V_{center} = \mathrm{R_{earth}} \cdot \sqrt{G\frac{m_E}{R_{earth}^3}} = 7901.5\frac{m}{s}$$

Or from the energy point of view should be the same answer.

Potential gravitational energy: $E(r) = \frac{2}{3}\pi \cdot Gm\rho R^2$, thus at the surface of the Earth energy can be set as

$E(R_{earth}) = \frac{2}{3}\pi \cdot Gm\rho R_{earth}^2$, and at the center of Earth it will be 0.

Difference of the potential energy will transform into kinetic, so (1P):

$$m\frac{v^2}{2} = \frac{2}{3}\pi \cdot Gm\rho R_{earth}^2$$

$$\downarrow (1P)$$

$$v_{max} = \mathrm{R_{earth}} \cdot \sqrt{\frac{4}{3}\pi \cdot G\rho} = \mathrm{R_{earth}} \cdot \sqrt{G\frac{m_E}{R_{earth}^3}} = 7901.5\frac{m}{s} = 7.9\ \frac{km}{s}$$

(c)

The most reasonable way is to solve it from the energy considerations:

$$v_{at\ the\ center\ after\ additional\ impulse} = v_{max} + \frac{P_a}{m}$$

Where P_a is $P_{additional\ impulse}$, then:

$$E_{at\ the\ center}(R = 0) = m\frac{\left(v_{max} + \frac{P_a}{m}\right)^2}{2}$$

$$E_{at\ the\ surface}(R_{earth}) = m\frac{\left(v_{final}\right)^2}{2} + \frac{2}{3}\pi \cdot Gm\rho R_{earth}^2$$

$$\downarrow$$

$$v_{final}^2 = \left(v_{max} + \frac{P_a}{m}\right)^2 - \frac{4}{3}\pi \cdot Gm\rho R_{earth}^2 = \left(\frac{P_a}{m}\right)^2 + 2 \cdot v_{max} \cdot \frac{P_a}{m}$$

$$\downarrow (1\mathrm{P})$$

$$v_{final} = \sqrt{\left(\frac{P_a}{m}\right)^2 + 2 \cdot v_{max} \cdot \frac{P_a}{m}} = \sqrt{100 + 2 \cdot 7901 \cdot 10} = 397.6\ \frac{m}{s}$$