**Mathematics 1, Homework 04**
**Leipzig University, WiSe 2023/24, Tim Shilkin**
**Due Date: 19.11.23 until 23:59 on-line**
**or 20.11.23 until 9:15 am in person**

Each problem is estimated by one point. Explain your answers.

1. Find the following limit:

$$\lim_{n\to\infty} \frac{n^2}{2n+1} \cdot \left( \text{arctg}\left(1 + \frac{2n+1}{n^2}\right) - \frac{\pi}{4} \right)$$

2. Determine wether the following function is differentiable at $x = 0$

$$f(x) = \begin{cases} \dfrac{x}{\ln|\ln|x||}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

In problems 3—9 it is necessary to compute the first derivatives of functions:

3. $f(x) = 2^x \cos x + \ln x \cdot \text{arctg}\, x$

4. $f(x) = 2^{\text{tg}\frac{1}{x}}$

5. $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

6. $f(x) = \ln(\ln^2(\ln^3 x))$ $\quad \left( \text{here we denote } \ln^m x := (\ln x)^m \right)$

7. $f(x) = \arcsin \frac{1-x^2}{1+x^2}, \quad x > 0$

8. $f(x) = \left(1 + \frac{1}{x}\right)^x$

9. $f(x) = \sin(\cos^2 x) \cdot \cos(\sin^2 x)$

10. Compute the second derivative of the following function:

$$f(x) = \frac{x^2(x+2)}{(x-1)^2}$$