Answers and Hints to Problems for Exam Preparation

1) Yes. Yes.
2) Mathematical induction.
3) $\binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!} = 210$.
4) Write a binomial theorem for $(1+1)^n$.
5) -1
6) $1/3$
7) $1/2$. Hint: divide and mutiply by $\sqrt{n^4+n^2}+n^2$. Alternatively, you can use Taylor theorem.
8) $e^{-21/4}$
9) No. Example: $(1/2)^{1/n}$.
10) Use definition of a limit.
11) Converges.
12) 5
13) Converges. Hint: Use ratio test.
14) Converges. Hint: use comparison test with $\sum_{n=1}^{\infty} \frac{1}{2n^{3/2}}$.
15) $e^3 - 4$
16) Yes. (and it is equal to $1/2$)
17) 2
18) 0
19) e^{10}
20)

$$\sin(x)^{\cos(x)} \left(-\sin(x)\ln(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right)$$

21) Hint: check points $x = 1$ and $x = -1$, and also $-\infty$ and $+\infty$.
22) Local and global maximum at $x = 1$, $f(1) = e^{-1}$, global minimum at $x = -2$, $f(-2) = -2e^2$.
23) Local and global maximum at $x = e$, $f(e) = 1/e$, global minimum at $x = 1$, $f(1) = 0$. Note that one needs to prove $\lim_{x \to +\infty} f(x) = 0$ and that $f(x)$ is monotonically decreasing at $(1; +\infty)$.
24) $-1/12$. Hint: use Taylor series (or L'Hospital, but L'Hospital involves a bit more calculations).
25) $9\ln(3) - 26/9$. Hint: Integrate by parts.
26) 4.
27) $3/\ln(2) - 8/3$.
28) Any vectors (x, y, z) satisfying $3x - y + 2z = 0$. For example, $(0, 2, 1)$, $(1, 0, -3/2)$, $(1, 3, 0)$.
29) Only a). Hint: Solve the system of linear equations in order to see that in a) the vectors are linearly independent, while in b) and c) the vectors are linearly dependent.
30) $x = \frac{7}{3}t - 1$, $y = \frac{4}{3}t$, $z = t$. (there are various equivalent equations for the same line).