

**Problem 1:            Keppler****1 + 2+1 Points**

A planet which is orbiting around the sun is discovered. It is observed that its orbital period is about 7 years.

- a. What is the planet's average distance to the sun?
- b. Assume that the maximum and minimum planet-sun-distances are  $r_{\max} = 4\text{AE}$  and  $r_{\min} = 3.2\text{AE}$ . Calculate the elliptic parameters  $\varepsilon$  and  $a$ .
- c. Another celestial body with mass  $m = 0.3 \cdot 10^{-10} M_{\text{sun}}$  is parabolically passing by the sun. How large is its angular momentum if its minimal distance to the sun is the same as that of the newly discovered planet?

**Problem 2:            Spinning earth****6 Points**

Calculate the kinetic energy of the earth due to its spinning about its axis. Compare your answer with the kinetic energy of the orbital motion of Earth's center of mass about the sun. Assume Earth to be a homogeneous sphere of mass  $6 \cdot 10^{24} \text{kg}$  and a radius  $6.4 \cdot 10^6 \text{m}$ . The radius of Earth's orbit is  $1.54 \cdot 10^{11} \text{m}$ .

### Problem 3: Momentum of Inertia

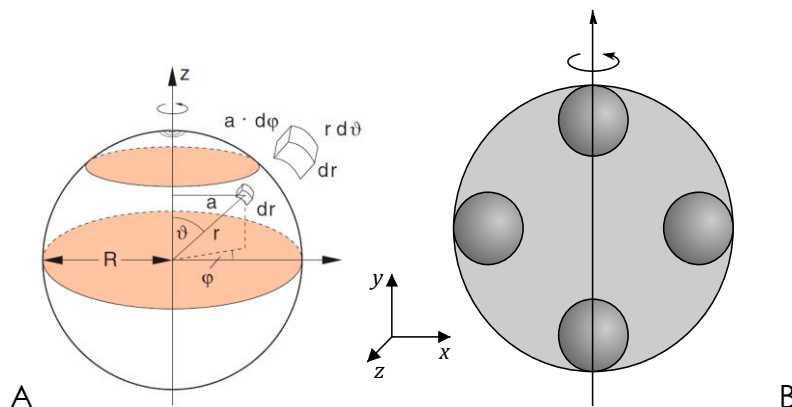
4+2 Points

The moment of inertia of the sphere depicted in **A** (mass  $M$ , radius  $R$ ) is:

$$I = \int a^2 \rho(\vec{r}) dV = \int a^2 dm$$

For a sphere with a constant density  $\rho$  this can be expressed in spherical coordinates:

$$I = \rho \int_0^R \int_0^\pi \int_0^{2\pi} a^3 r \cdot d\varphi \cdot d\theta \cdot dr$$



- a. Show that the Moment of Inertia  $I$  of an isotropic sphere is:

$$I = \frac{2}{5} MR^2$$

- b. In the image B, a sphere (Radius  $R$ , density  $\rho$ ) with 4 spherical holes (radius  $r = R/4$ , the holes are displaced from the centre of the sphere in  $x$  and  $y$  direction) is shown. Calculate the moment of inertia for a rotation around the  $y$  axis for this object.

*Hint: Assume the spherical holes to be spheres of negative density. The total inertia is the inertia of the large sphere minus the inertia of the hollowed spheres.*

**Problem 4: Rotational energy of molecule****4 Points**

Calculate the rotational energy of the  $\text{Na}_3$ -molecule composed of 3 Na atoms (mass = 23 AMU) which form an isosceles triangle with the apex angle  $\alpha = 79^\circ$  and a side length of  $d = 0.32 \text{ nm}$  when it rotates around the three principal axes with the angular momentum  $L = \sqrt{l(l+1)}\hbar$ . Determine at first the three axis and centre of mass. Assume quantum numbers  $n=m=1$ .

**Task to think about**

The gravitational force that the sun exerts on the moon is about twice as large as that which the earth exerts on it. Why does the moon nevertheless orbit the earth and not fly away?