Mathematics 1, Homework 07
Leipzig University, WiSe 2023/24, Tim Shilkin
Due Date: 17.12.23 until 23:59 on-line
or 18.12.23 until 9:15 am in person

Each problem is estimated by one point. Explain your answers.

In problems 1–4 it is necessary to compute the definite integrals.

1. $\displaystyle\int\limits_{-1}^{8} \sqrt[3]{x}\, dx =$

2. $\displaystyle\int\limits_{0}^{\ln 2} x\, e^{-x}\, dx =$

3. $\displaystyle\int\limits_{-1}^{1} \frac{dx}{x^2 - x + 1}\, dx =$

4. $\displaystyle\int\limits_{-3}^{3} \Big| \big| |x| - 1 \big| - 1 \Big|\, dx =$

5. Find the derivative of an integral with the variable limits: $\displaystyle\frac{d}{dx}\int\limits_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^4}}$

6. Find the area of the part of the plane bounded by curves $y = 2x^2 - 2$ and $y = x^2 - x$.

7. The set Ω on the plane is bounded by the curve given in the polar coordinates $x = r\cos\varphi$, $y = r\sin\varphi$ by the relation

$$r = 3 + 2\cos\varphi, \qquad \varphi \in [0, 2\pi]$$

   Shade the set Ω on the plane and compute its area.

8. A material point moves on the plane so that its coordinates are

$$x(t) = \tfrac{1}{2}t^2, \qquad y(t) = \tfrac{1}{3}t^3$$

   Find the traveled distance of this point for the interval of time from $t = 0$ till $t = 1$.

9. Find the volume of a body $\Omega \subset \mathbb{R}^3$ bounded by the planes $z = -1$, $z = 1$ and the surface

$$\Sigma = \Big\{ (x, y, z) \in \mathbb{R}^3 : \ x^2 + y^2 - z^2 = 1, \ -1 \le z \le 1 \Big\}$$

10. Find the area of a surface Σ given by the relation

$$\Sigma = \Big\{ (x, y, z) \in \mathbb{R}^3 : \ z = \ln\sqrt{x^2 + y^2}, \ 0 \le z \le 1 \Big\}$$