Mathematics 1, Homework 08
Leipzig University, WiSe 2023/24, Tim Shilkin
Due Date: 07.01.24 until 23:59 on-line
or 08.01.24 until 9:15 am in person

Each problem is estimated by one point. Explain your answers.

Find the following limits using L'Hôpital's rule:

1. $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x - x}{x - \sin x} =$

2. $\lim\limits_{x\to 6} \dfrac{6^x - x^6}{x - 6} =$

Using Taylor's formula, expand the function in powers x

3. $\sqrt[3]{\sin(x^3)}$ up to $o(x^{13})$ as $x \to 0$

4. $\ln\left(\dfrac{\sin x}{x}\right)$ up to $o(x^6)$ as $x \to 0$

Using Taylor's expansion, find the following limits:

5. $\lim\limits_{x\to 0} \dfrac{e^{-\frac{x^2}{2}} - \cos x}{x^4} =$

6. $\lim\limits_{x\to 0} \dfrac{e^x \sin x - x(1+x)}{x^3} =$

Find the following limits using an appropriate method to your choice:

7. $\lim\limits_{x\to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right) =$

8. $\lim\limits_{x\to 0} \dfrac{\arcsin(2x) - 2\arcsin x}{x^3} =$

9. $\lim\limits_{x\to 0} \dfrac{x\sqrt[3]{\sin(x^3)} + \ln\left(\frac{\sin x}{x}\right)}{x^2} =$

10. $\lim\limits_{x\to +\infty} \left(x - x^2 \ln\left(1 + \dfrac{1}{x}\right)\right) =$