

10-PHY-BIMA1 Mathematics 1, EXAM
Leipzig University, WiSe 2023/24, February 21, 2024, Dr. Tim Shilkin
Start of the exam: 12:00, End of exam: 14:00

Last name, first name: _____

Matric. no.: _____

(Please, do not write in this box)

Problem	1	2	3	4	5	6	7	Σ
Grade	/5	/5	/5	/5	/5	/5	/8	/38

Pay attention to hints:

- Mark all pieces of paper with your name and your student number.
- Please, write the solution of each problem on a separate paper sheet. Please, do not forget to put your name on this sheet.
- Please, put your answers to each problem in your answer sheet. Please, do not forget to put your name on this sheet.
- Please, staple with a stapler all your papers and hand them in altogether. Please, verify that you submit
 - 1) the cover sheet
 - 2) the answer sheet with your answers to Problems 1–6
 - 3) sheets with your solutions of Problems 1–6
 - 4) sheets with your proof of the theoretical results (Problem 7).
- No supplementary materials and electronic devices are allowed during the exam. Use of any support (notes, communication) terminates your exam.
- The solutions to the exam problems will be available in the moodle next day after exam.

GOOD LUCK!

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1. **(5 points)** Find the oblique asymptotes of the following function as $x \rightarrow \pm\infty$

$$f(x) = x^2 \left(\sqrt{x^2 + 5} - \sqrt{x^2 + 1} \right)$$

2. **(5 points)** Find all inflection points of the following function

$$f(x) = x \ln(1 + x^2)$$

3. **(5 points)** Find the area of a plane figure bounded by the following three lines given in Cartesian coordinates ($0 < x < 1$):

$$y = x e^{x^2}, \quad y = 2x \ln x, \quad x = 1$$

4. **(5 points)** Find all values of the parameter $\alpha \in \mathbb{R}$ such that the following improper integral is convergent. Explain your answer. (Here $\operatorname{arctg} x := \arctan x$ is the arctangent of x).

$$\int_0^1 \frac{dx}{(\operatorname{arctg} x)^\alpha}$$

5. **(5 points)** Determine for what value of the parameter α the system is CONSISTENT and SOLVE the system for this value of α :

$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= 4 \\ x_1 + x_2 + 2x_3 + 3x_4 &= 8 \\ 2x_1 + 4x_2 + 5x_3 + 10x_4 &= 20 \\ 3x_1 + x_2 + 5x_3 + 5x_4 &= \alpha \end{aligned}$$

6. **(5 points)** Find the inverse of the following matrix A :

$$A = \begin{pmatrix} 1 & 4 & 1 \\ 3 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix}$$

7. **(8 points)** Write the rigorous proofs of the following two extreme value theorems:

THEOREM 1. If a function $f : [a, b] \rightarrow \mathbb{R}$ is continuous on the closed and bounded interval $[a, b]$ then f is bounded on this interval.

THEOREM 2. If a function $f : [a, b] \rightarrow \mathbb{R}$ is continuous on the closed and bounded interval $[a, b]$ then f must attain its maximum and a minimum on this interval.

10-PHY-BIMA1 Mathematics 1, Answer sheet
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1. Problem 1.

- Asymptote as $x \rightarrow +\infty$: _____
- Asymptote as $x \rightarrow -\infty$: _____

2. Problem 2.

- Inflection points: _____

3. Problem 3.

- The area of the figure is: _____

4. Problem 4.

- The integral is convergent if $\alpha \in$ _____

5. Problem 5.

- The system is consistent if $\alpha =$ _____
- In this case the solution is

$$x_1 = \underline{\hspace{2cm}}$$

$$x_2 = \underline{\hspace{2cm}}$$

$$x_3 = \underline{\hspace{2cm}}$$

$$x_4 = \underline{\hspace{2cm}}$$

6. Problem 6.

$$A^{-1} =$$