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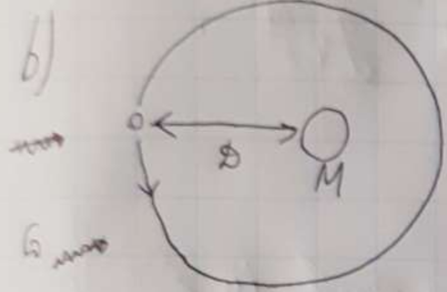
IPSP 3720433

Problem 2.1 Earth orbit around the Sun.

a) $c \approx 3\cdot10^8 m\cdot s^{-1}$, $t = 8min + 19s \approx 500s = 5\cdot10^2 s$ (because it is easier to calculate with powers of 10 and simple numbers)

$$\mathcal{D} = c\cdot t = 3\cdot10^8 m\cdot s^{-1}\cdot 5\cdot10^2 s = 1.5\cdot10^{11} m.$$

b)



$G = 6.7\cdot10^{-11} m^3 kg^{-1} s^{-2}$ $[G] = m^3 kg^{-1} s^{-2}$
$M = 2\cdot10^{30} kg$ $[M] = kg$
$\mathcal{D} = 1.5\cdot10^{11} m$ $[\mathcal{D}] = m$

$$[T] = s = \underline{m^0 kg^0 s^1} = [G^{p_G}]\cdot[M^{p_M}]\cdot[\mathcal{D}^{p_D}] = \underline{(m^3kg^{-1}s^{-2})^{p_G}\cdot kg^{p_M}\cdot m^{p_D}}$$

$$\begin{cases} 0 = 3p_G + p_D \\ 0 = -p_G + p_M \\ 1 = -2p_G \end{cases} \leftrightarrow \begin{cases} p_D = \frac{3}{2} \\ p_M = -\frac{1}{2} \\ p_G = -\frac{1}{2} \end{cases}$$

Therefore $T \approx \sqrt{\frac{\mathcal{D}^3}{GM}} = \sqrt{\frac{1.5^3\cdot10^{33}}{6.7\cdot10^{-11}\cdot2\cdot10^{30}}}\ s = \sqrt{\frac{1.5\cdot10^{14}\cdot(1.5)^2}{6.7\cdot2}}\ s \approx \frac{1}{2}\cdot10^7 s = 5\cdot10^6 s.$

c) $5\cdot10^6 s \simeq 5\cdot10^6\cdot s\cdot\frac{year}{365\,d}\cdot\frac{d}{24\,h}\cdot\frac{h}{3600\,s} = \frac{5\cdot10^6}{365\cdot\underbrace{24\cdot3600}_{\approx10^5}} year \approx \frac{50}{365} year \simeq \underline{\frac{1}{6} year.}$

It is $\approx 2\pi$ smaller than actual period.

d) $\vec{\xi}(t) = \frac{\vec{x}(t)}{L} = \left(\frac{x_1(t)}{L}, \frac{x_2(t)}{L}, \frac{x_3(t)}{L}\right)$

Most likely L is the average distance of planet to Sun.
But if we will consider dynamics with many planets at once, L can mean 1A.U. (Earth's distance) or average „radius" of solar system.

Problem 2.2 Oscillating period of particle attached to a spring.

$g_{Moon} = 1.6\ m\cdot s^{-2}$, $M = 10^{-1} kg$, $k = 1.6\ kg\cdot s^{-2}$.

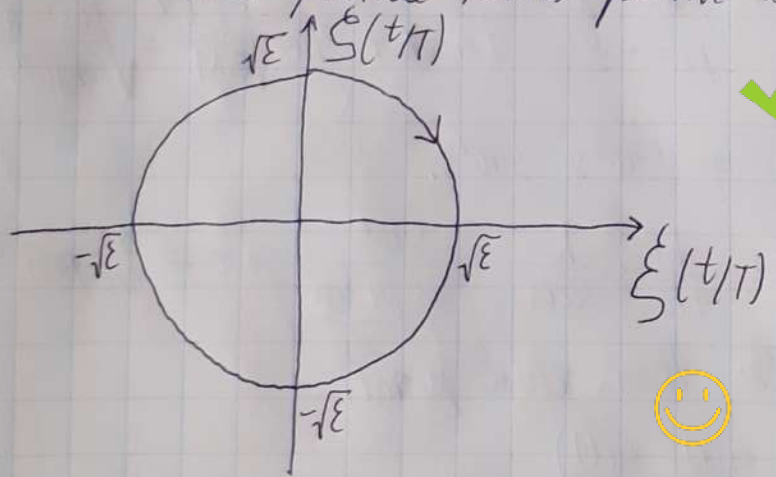
a) Construct length/time scale for non-dimensionalization of dynamics
Using B-π method:

$$m - [g_{Moon}^{1}]\cdot[k^{-1}]\cdot[M^{1}] = \left[\frac{gM}{k}\right]$$
$$kg = [M^{1}] = [M]$$
$$s = [M^{\frac{1}{2}}]\cdot[k^{-\frac{1}{2}}] = \left[\sqrt{\tfrac{M}{k}}\right]$$

So lengths will be scaled by $\frac{gM}{k}$, time by $\sqrt{\frac{M}{k}}$.
Therefore $\xi(t) = \frac{x(t)}{(gM/k)}$, $\zeta(t) = \frac{\dot{x}(t)}{(gM/k)\cdot\sqrt{\frac{k}{M}}} = \frac{\dot{x}(t)}{g\sqrt{\frac{M}{k}}}$.

b) $\omega \approx \frac{1}{T} = \sqrt{\frac{k}{M}} = \sqrt{\frac{1.6}{10^{-1}}}\, s^{-1} = 4 s^{-1}$.

c) $\mathcal{E} = \xi^2(t/T) + \zeta^2(t/T)$ is const.
That means phase space visualization is a circle in $\xi(t/T) 0 \zeta(t/T)$ plane.
Radius is $\sqrt{\mathcal{E}}$.
It is traversed clockwise (velocity is positive and goes to 0 when particle reaches positive coordinate amplitude, etc.).



d) [for seminars)

Problem 2.3 Conversion Joule - Calorie.

a) • $P = 2000\,W$, $\tau \approx 1\,min\ 40s = 10^2 s$.
So $Q_1 = 2\cdot10^3\cdot10^2\,J = 2\cdot10^5\,J$, on the one hand.
• On the other, $\Delta T \approx 80\,K$, $V = 10^{-3} m^3$, $m = 10^3 \frac{kg}{m^3}\cdot 10^{-3} m^3 = 1 kg$.
So $Q_2 = c m \Delta t = 1 kg \cdot 80 K \cdot \frac{1\,cal}{kg\cdot K} = 10^3 g \cdot 80 K \cdot \frac{1\,cal}{g\cdot K} = 8\cdot10^4 cal$.
• $Q_1 \overset{?}{=} Q_2$. So $8\cdot10^4\,cal = 20\cdot10^4\,J \longrightarrow 1\,cal = 2.5\,J$.

b) From literature cal = 4 J.
Possibilities for discrepancy:
- actual $P_{act} > P$ (it was written „2000-2400W", I took 2000 for simplicity) — will give higher Q_1
- actual $\Delta T_{act} < \Delta T$ — will give less Q_2
(I took 20°C as t_0, while in my house it is more)
- actual ρ of water can be smaller — will give less Q_2
(due to dissolved gases)
- (volume is not the reason, it is marked enough accurately for such error)
Combination of factors can work even if some go in other direction.

Problem 1.4 Water waves

$\vartheta = f(L, g)$

a) $[L] = m$
$[g] = m \cdot s^{-2}$
$[\vartheta] = m \cdot s^{-1}$, so $[\vartheta] = [L^{\frac{1}{2}}] \cdot [g^{\frac{1}{2}}]$, $\vartheta \simeq \sqrt{Lg}$

b) $\vartheta \simeq \sqrt{30 ft \cdot 10 m \cdot s^{-2}} = \sqrt{300 \cdot 3 \cdot 10^{-1} m^2 s^{-2}} \simeq 10 \, ms^{-1}$

c) $1 ft = 304{,}8 \cdot 10^{-3} m$
$\frac{7}{23} \simeq 0.3043$

Feasibility depends only on goal. If we have very accurate ft value and need to maintain accuracy — it is not enough. If we do not need, it is all right.
Also, if we do simple calculations, it is easier to use 0.3m, then $\frac{7}{23}$ m.

Index der Kommentare

1.1 2.1: 18/19

1.2 2.1 was done very clear and concise :)

1.3 6/7

1.4 please go more into an argument concerning the margin of error when reasoning for approximations

1.5 7/7

1.6 3/3

1.7 keen eye :)

1.8 great explanation :)

1.9 2/2

2.1 2.3: 3/4
For one minute you would need a more powerful kettle