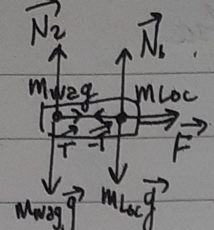
Ex. 4 Stanislav 3720433



1 - Constant friction

Static friction with locom. = drive for whole system.

$$F = (m_{wag} + m_{Loc})a$$

For locom. $F = \mu N_1 = \mu m_{Loc} \cdot g$, hence

$$\mu m_{Loc} g = (m_{wag} + m_{Loc}) \cdot \frac{v_1^2 - v_0^2}{2d} \longrightarrow$$

$$\mu g = \left(1 + \frac{m_{wag}}{m_{Loc}}\right) \cdot \frac{v_1^2 - v_0^2}{2d} \longrightarrow$$

$$\boxed{m_{Loc} = \frac{m_{wag}}{-1 + \frac{2\mu g d}{v_1^2 - v_0^2}}}$$

$$m_{Loc} = \frac{5 \cdot 10^5 \text{ kg}}{-1 + \frac{2 \cdot 0.12 \cdot 9.8 \cdot 12 \cdot 10^3}{(80/3.6)^2}} = \left[\text{units dropped because clear that denom. is dim.-less}\right] \approx 8.9 \text{ tonn.}$$

2 - Dynamic friction

EOM: $ma = -kv$

$$m\dot{v} = -kv \longrightarrow \int \frac{dv}{v} = -\frac{k}{m}\int dt$$

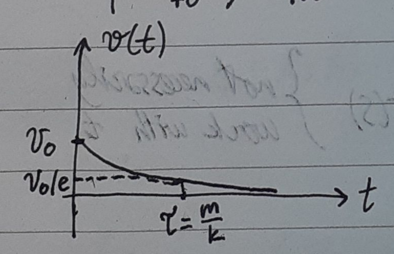
$$\ln v = -\frac{k}{m}t + C \quad |e^{\wedge}$$

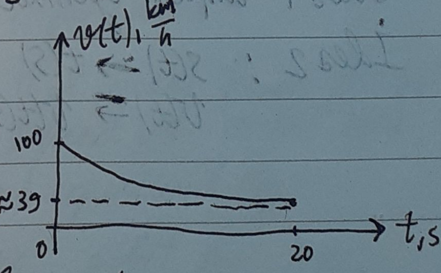
$$v = v_0 e^{-\frac{k}{m}t}$$

a) $v_1 = v_0 e^{-\frac{k}{m}t} \longrightarrow t = \left(-\frac{m}{k}\right) \ln \frac{v_1}{v_0}$,

$$t = \left(\frac{-1.5 \cdot 10^3 \text{ s}}{70}\right) \ln \frac{50}{100} \approx 14.85 \text{ s} \approx 15 \text{ s}.$$

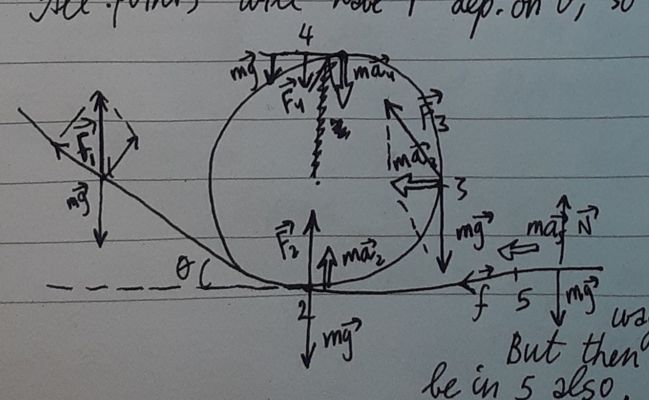
b)





3 - Circular motion

All points will have F dep. on v, so I write generically. Also, $v = \text{const} \rightarrow a_{tot} = a_c = \frac{v^2}{r} = |\vec{a_i}|, i \neq 5, 1.$



For points 2, 3, 4 forces from track adjust so that $m\vec{a_i}$ point to center.

There is friction on 1, otherwise wagon would accelerate there. But then I considered friction to be in 5 also.

In 1: $F(t_1) = mg$, $F_{net} = 0$

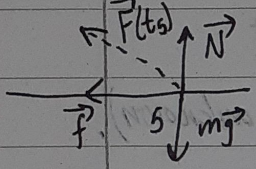
In 2: $F(t_2) = ma + mg = m\left(g + \frac{v^2}{r}\right)$, $F_{net} = m\frac{v^2}{r}$

In 3: $F_{(t_3)} = \sqrt{(mg)^2 + (ma)^2} = m\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$, $F_{net} = \frac{mv^2}{r}$
(see picture)

In 4: $F_{(t_4)} = ma - mg = m\left(\frac{v^2}{r} - g\right)$, $F_{net} = \frac{mv^2}{r}$

In 5: $F(t_5) = \sqrt{N^2 + f^2} = \sqrt{\mu^2 + 1}\, mg$ (because I consider „track force" = all friction + normal impact)
(see picture)

μ can be found from 1:

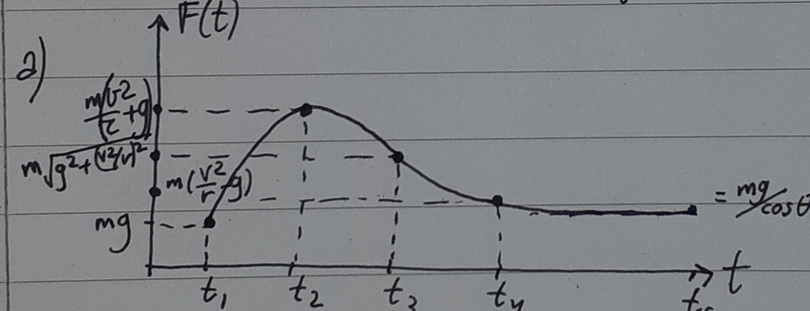


$mg\sin\theta = \mu mg\cos\theta \rightarrow \mu = \tan\theta$, so

$F(t_5) = \sqrt{1 + \tan^2}\, mg = \frac{mg}{\cos\theta}$, $F_{net} = \frac{mg}{\cos\theta}$.

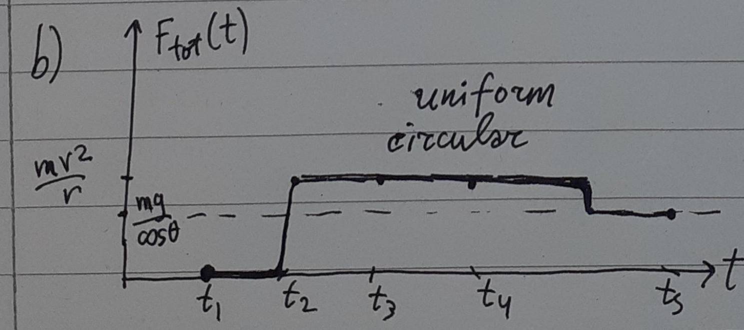
Hence $F_2 > F_3 > F_1$, and F_4/F_5 can relate differently to them, because θ and v can vary. Below is ~~approxi~~ some possibility for F_4, F_5

a)



(if consider $F_5 = 0$ – must admit there is no friction in 5, and there is in 1 → track differs in positions 1 and 5. I assumed it is same)

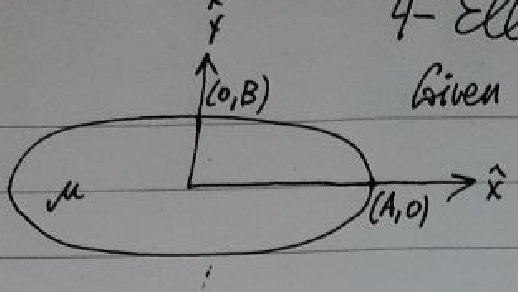
b)



c) $a_{max} = 6g = \frac{v_{max}^2}{r} \rightarrow v_{max} = \sqrt{6gr} \approx \sqrt{6 \cdot 9.8 \cdot 30}\ m \cdot s^{-1} = 42\ m \cdot s^{-1}$.

d) $a_{min} = g$ (when $F_4 = 0$, barely touching) $\rightarrow v_{min} = \sqrt{gR} \approx 17\ m \cdot s^{-1}$.

4- Elliptical Paths



Given $v = \text{const}$

a) Point A has smallest rad. of curvature $\rightarrow a_{max} = \frac{v^2}{r_{min}}$ is in A.

Point B ... largest ... $\rightarrow$
$a_{min} = \frac{v^2}{r_{max}}$ is in B.

b) Curvature for a curve $(x(t), y(t))$ can be found

$$K = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{3/2}}, \quad R = K^{-1}, \quad [K] = m^{-1}, [R] = m.$$

(deriving this is great exercise, but it is pure math, and I will derive for elliptic case)

Ellipse parametrization

$$\begin{cases} x = A\cos t \rightarrow \dot{x} = -A\sin t \rightarrow \ddot{x} = -A\cos t \\ y = B\sin t \rightarrow \dot{y} = B\cos t \rightarrow \ddot{y} = -B\sin t \end{cases}$$

Then $K = \dfrac{-A\sin t(-B\sin t) - B\cos t(-A\cos t)}{(A^2\sin^2 t + B^2\cos^2 t)^{3/2}} = \dfrac{AB}{(A^2\sin^2 t + B^2\cos^2 t)^{3/2}}$

In point $(A,0)$ $t = 0 \rightarrow K = \dfrac{AB}{B^3} = \dfrac{A}{B^2} \rightarrow R = \dfrac{B^2}{A}.$

The accel. is max at A, assume it is also the max. possible from all others at A: μg (from $F_{frict,\,max}$). Then

$$\boxed{v_{max} = \sqrt{\mu g \frac{B^2}{A}}}$$

Note: I can reason that $v = \sqrt{a R_{curv}}$ can be bigger in other points even when $a < \mu g$, but $R_{curv} > \frac{B^2}{A}$ (for example at B $R_{curv} = \frac{A^2}{B} > \frac{B^2}{A}$), but then task says v is same everywhere, so v_A will be bigger than $\sqrt{\mu g \frac{B^2}{A}}$, which is impossible!