Mathematics 1, Homework 09
Leipzig University, WiSe 2023/24, Tim Shilkin
Due Date: 14.01.24 until 23:59 on-line
or 15.01.24 until 9:15 am in person

Each problem is estimated by one point. Explain your answers.

Compute the value of the following improper integral:

1. $\int\limits_0^1 \dfrac{\ln x}{\sqrt{x}}\, dx$

Investigate (non-absolute) convergence of the following improper integrals:

1. $\int\limits_0^1 \dfrac{dx}{(2-x)\sqrt{1-x}}$
2. $\int\limits_1^{+\infty} \dfrac{dx}{x^2+4x-5}$
3. $\int\limits_1^{+\infty} \dfrac{x\cos(x^2)}{1+x}\, dx$

Compute the values of the following infinite sums:

5. $\dfrac{1}{1\cdot 5} + \dfrac{1}{5\cdot 9} + \dfrac{1}{9\cdot 13} + \ldots + \dfrac{1}{(4n+1)\cdot(4n+5)} + \ldots$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n}$

Investigate (non-absolute) convergence of the following infinite series:

7. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-\sqrt{n}}$
8. $\dfrac{2\cdot 1!}{1} + \dfrac{2^2\cdot 2!}{2^2} + \dfrac{2^3\cdot 3!}{3^3} + \dfrac{2^4\cdot 4!}{4^4} + \ldots + \dfrac{2^n\cdot n!}{n^n} + \ldots$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n\sqrt{n}}$
10. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+100}$