

Mathematics 1, Homework 03
Leipzig University, WiSe 2023/24, Tim Shilkin
Due Date: 12.11.23 until 23:59 on-line
or 13.11.23 until 9:15 am in person

This homework will be estimated from 10 points (each problem is one point).

In problems 1 — 6 it is necessary to compute the limits. Explain your answers.

1. $\lim_{x \rightarrow 5} \frac{x}{x^2+5} =$

2. $\lim_{x \rightarrow 0} \frac{\sin 6x}{\operatorname{tg} 2x} =$

3. $\lim_{x \rightarrow +\infty} \left(\sin \frac{1}{x} + \cos \frac{1}{x} \right)^x =$

4. $\lim_{x \rightarrow -\infty} \frac{\ln(1+3^x)}{\ln(1+2^x)} =$

5. $\lim_{x \rightarrow 0} \frac{2^{3 \sin x} - 1}{x} =$

6. $\lim_{x \rightarrow 0} \frac{\sqrt{1+\operatorname{tg} x} - \sqrt{1+\sin x}}{x^3} =$

7. Investigate the behavior of the function $f(x)$ at infinity and find horizontal or oblique asymptotes of this function (if exist)

$$f(x) = \frac{x^2(x-2)}{(x+1)^2}$$

8. For a function

$$f(x) = \frac{1}{1 - e^{\frac{x}{1-x}}}$$

(a) find points of discontinuity of $f(x)$

(b) for each point of discontinuity x_0 compute one-sided limits $\lim_{x \rightarrow x_0-0} f(x)$ and $\lim_{x \rightarrow x_0+0} f(x)$

(c) indicate jump discontinuities and vertical asymptotes of $f(x)$ (if exist).

9. The function $f(x)$ is meaningless at $x = 0$. Define the value $f(0)$ so that the extended function $f(x)$ is continuous at $x = 0$. Prove the continuity of the extended function.

$$f(x) = \sin x \cdot \sin \frac{1}{x}$$

10. Determine if the function

$$f(x) = x \sin(2\pi x)$$

is uniformly continuous on $(0, +\infty)$. Explain your answer.