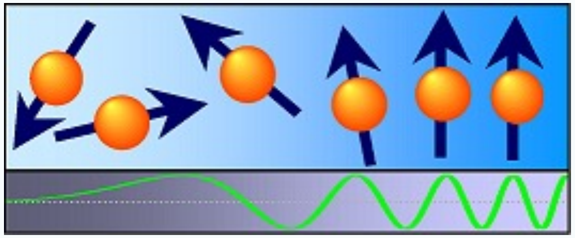
Experimental Physics
EP1 MECHANICS

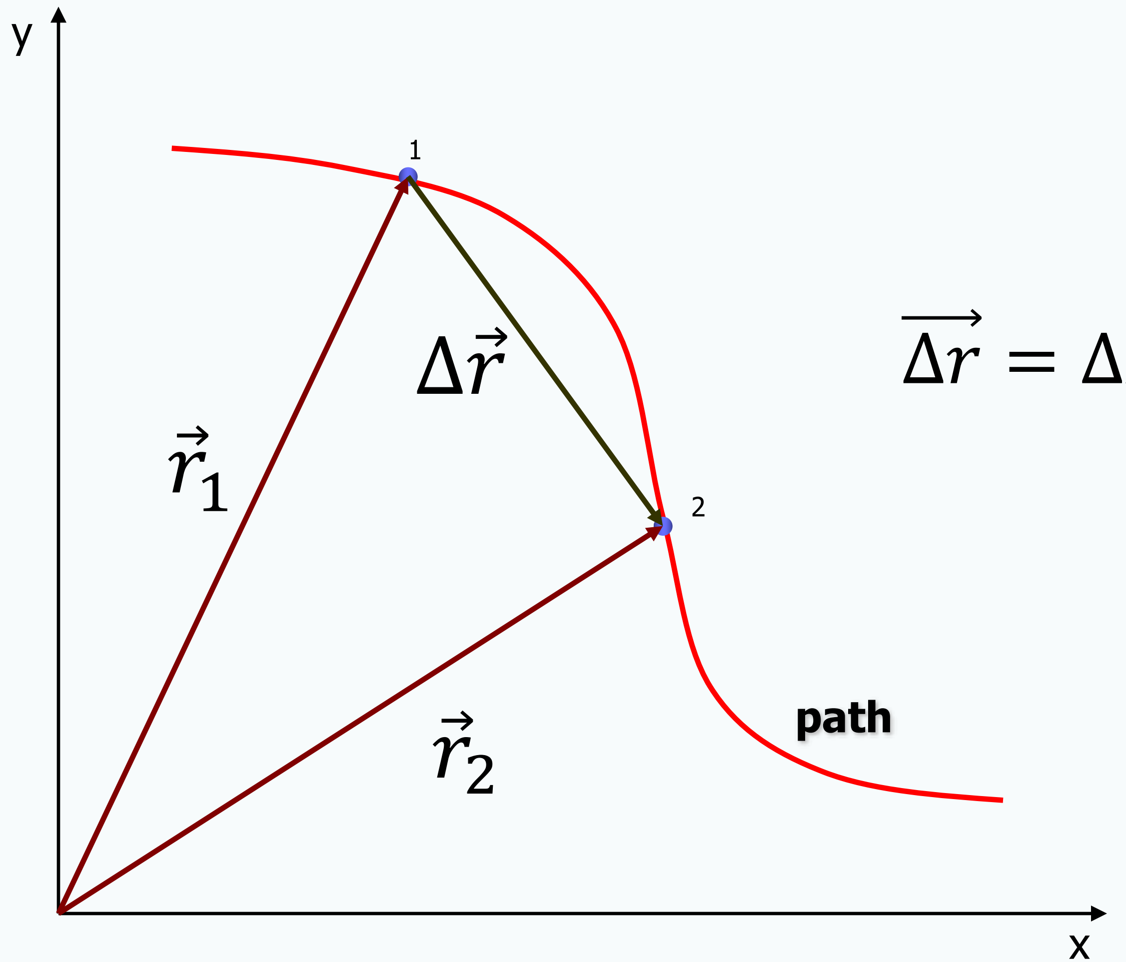
- Motion in 2D and 3D -



Rustem Valiullin

https://www.physgeo.uni-leipzig.de/en/fbi/applied-magnetic-resonance

Position and Displacement



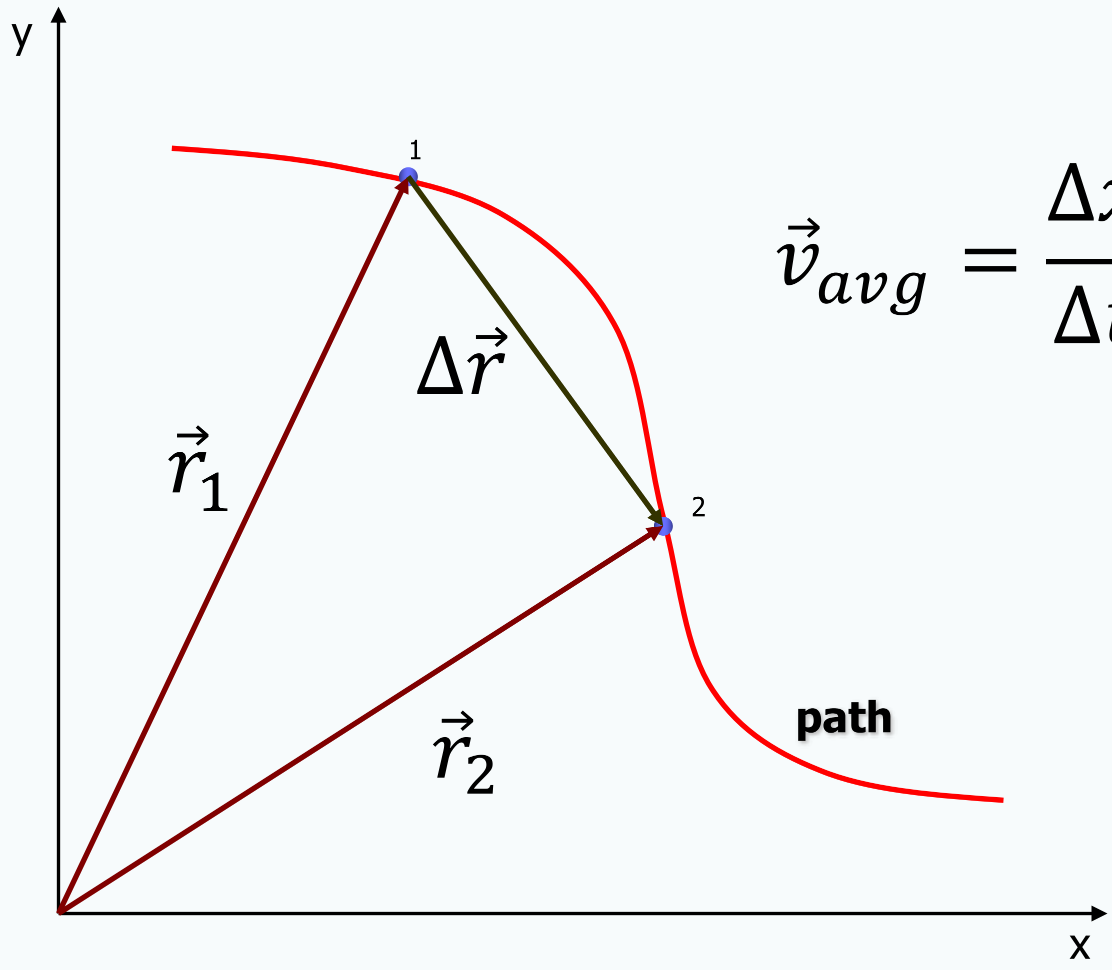
$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\overrightarrow{\Delta r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$$

Average velocity

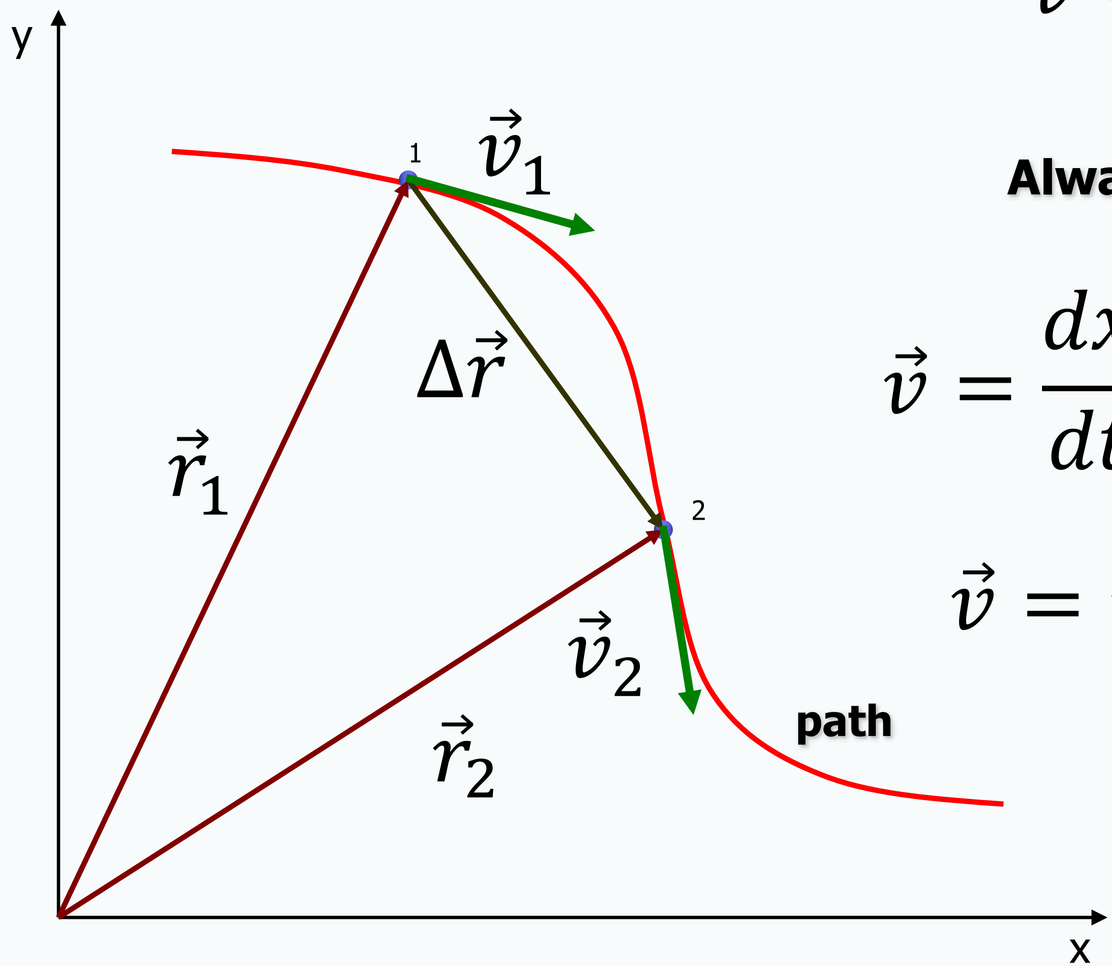
$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$



$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{k}$$

The average velocity is not a function of path connecting two points.

Instantaneous velocity



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Always tangent to the path.

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

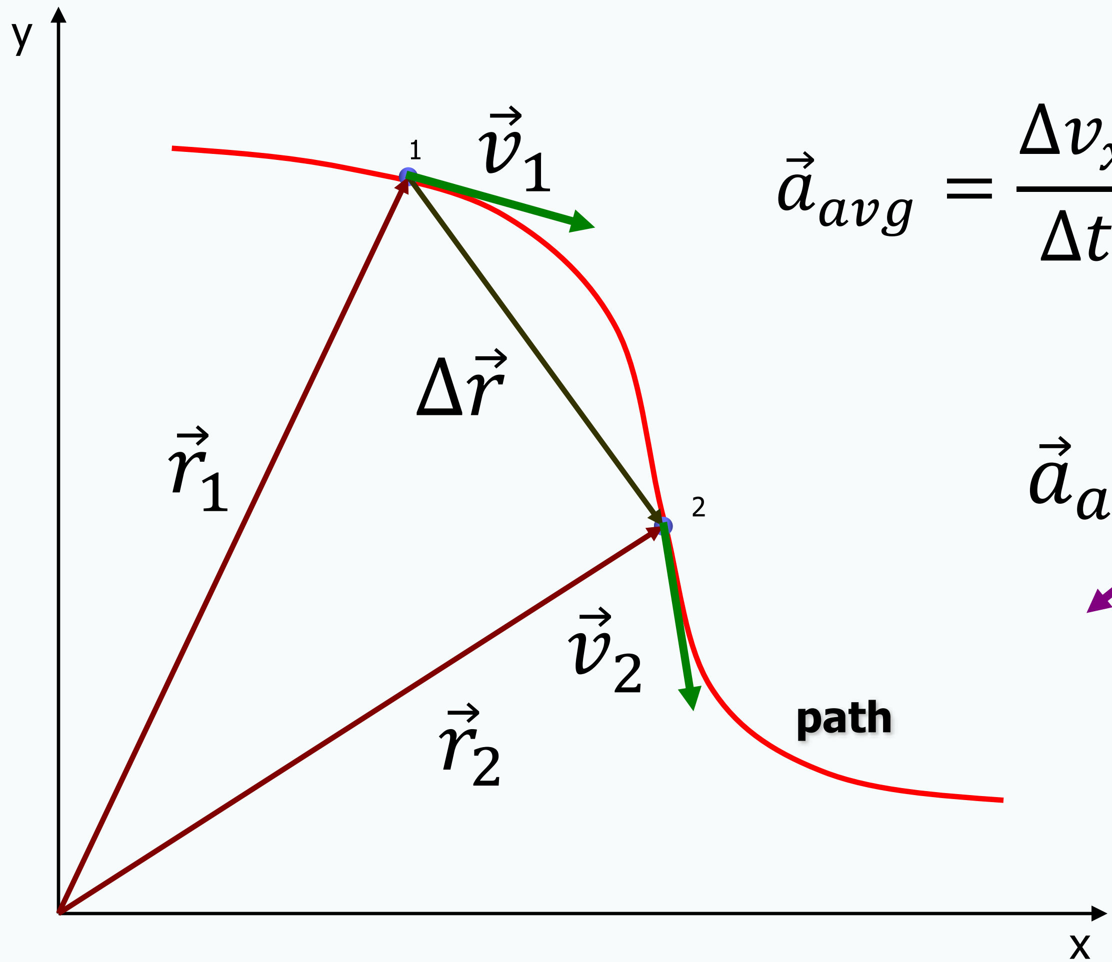
$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

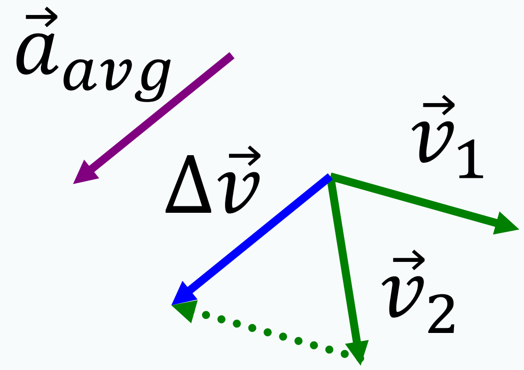
$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Average acceleration

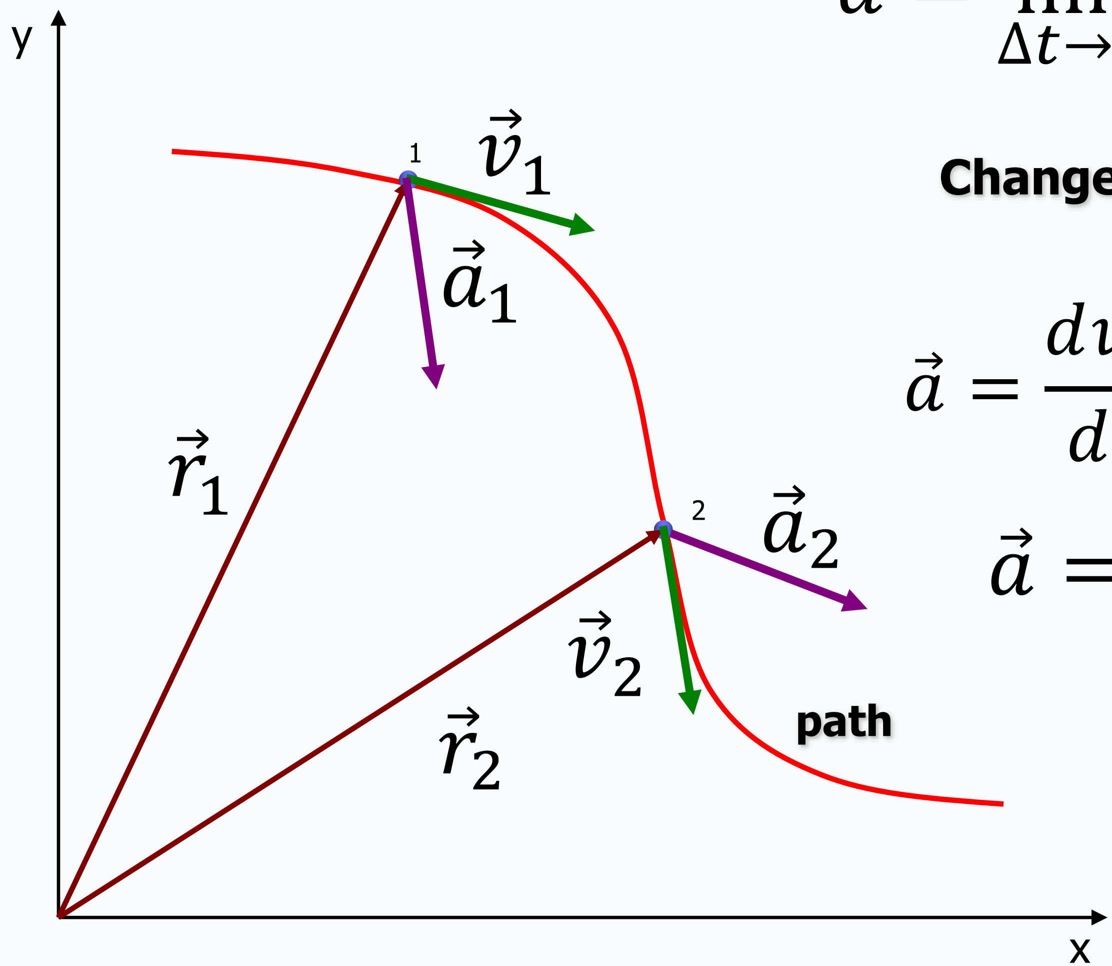
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{\imath} + \frac{\Delta v_y}{\Delta t}\hat{\jmath} + \frac{\Delta v_z}{\Delta t}\hat{k}$$





Instantaneous acceleration



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Change in either magnitude or in direction.

$$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Constant acceleration

$$\begin{cases} v_x = v_{x0} + a_x t \\ v_y = v_{y0} + a_y t \\ v_z = v_{z0} + a_z t \end{cases}$$

$$\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath} + v_z \hat{k}$$

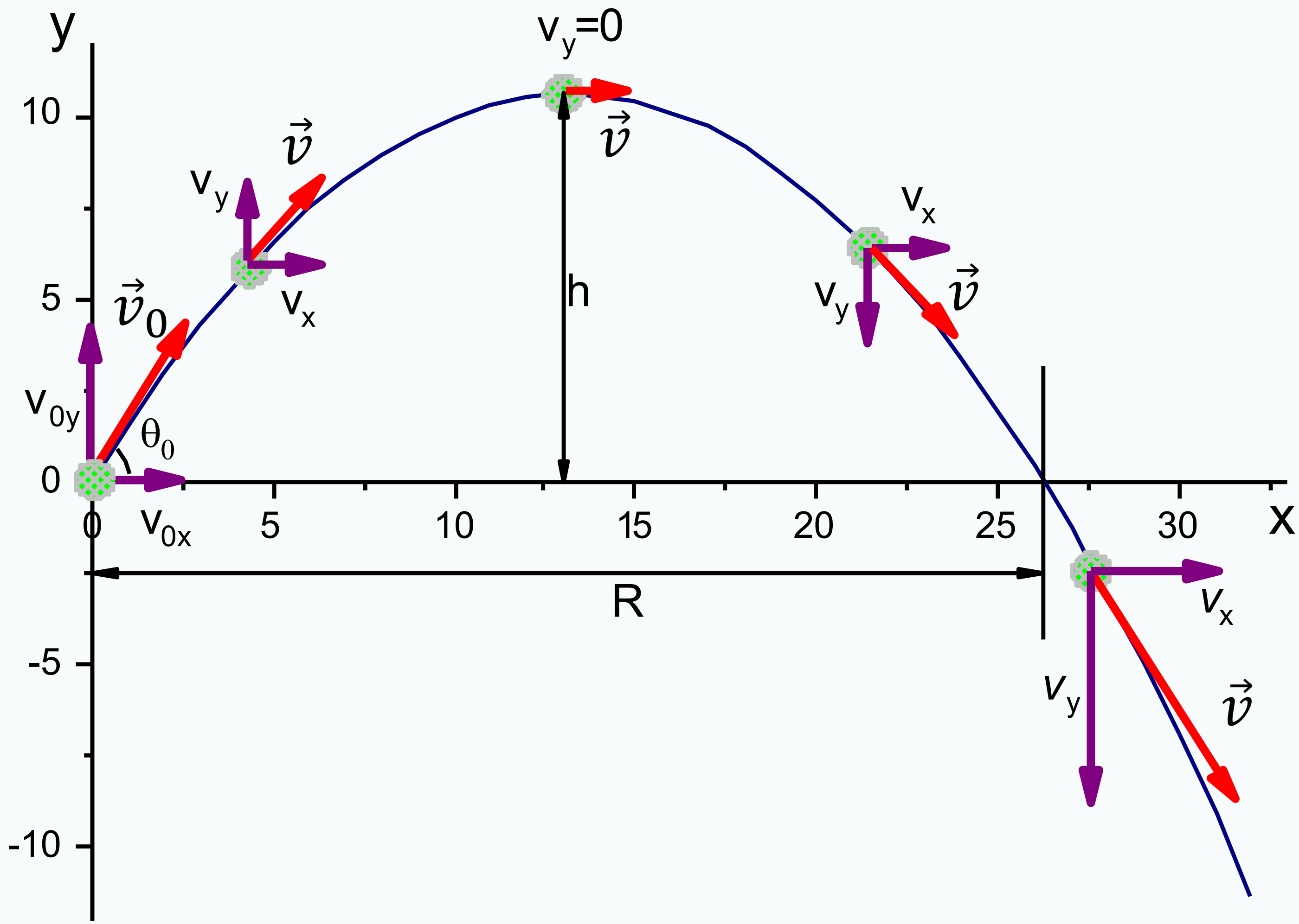
$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\begin{cases} x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \\ z = z_0 + v_{z0}t + \frac{1}{2}a_z t^2 \end{cases}$$

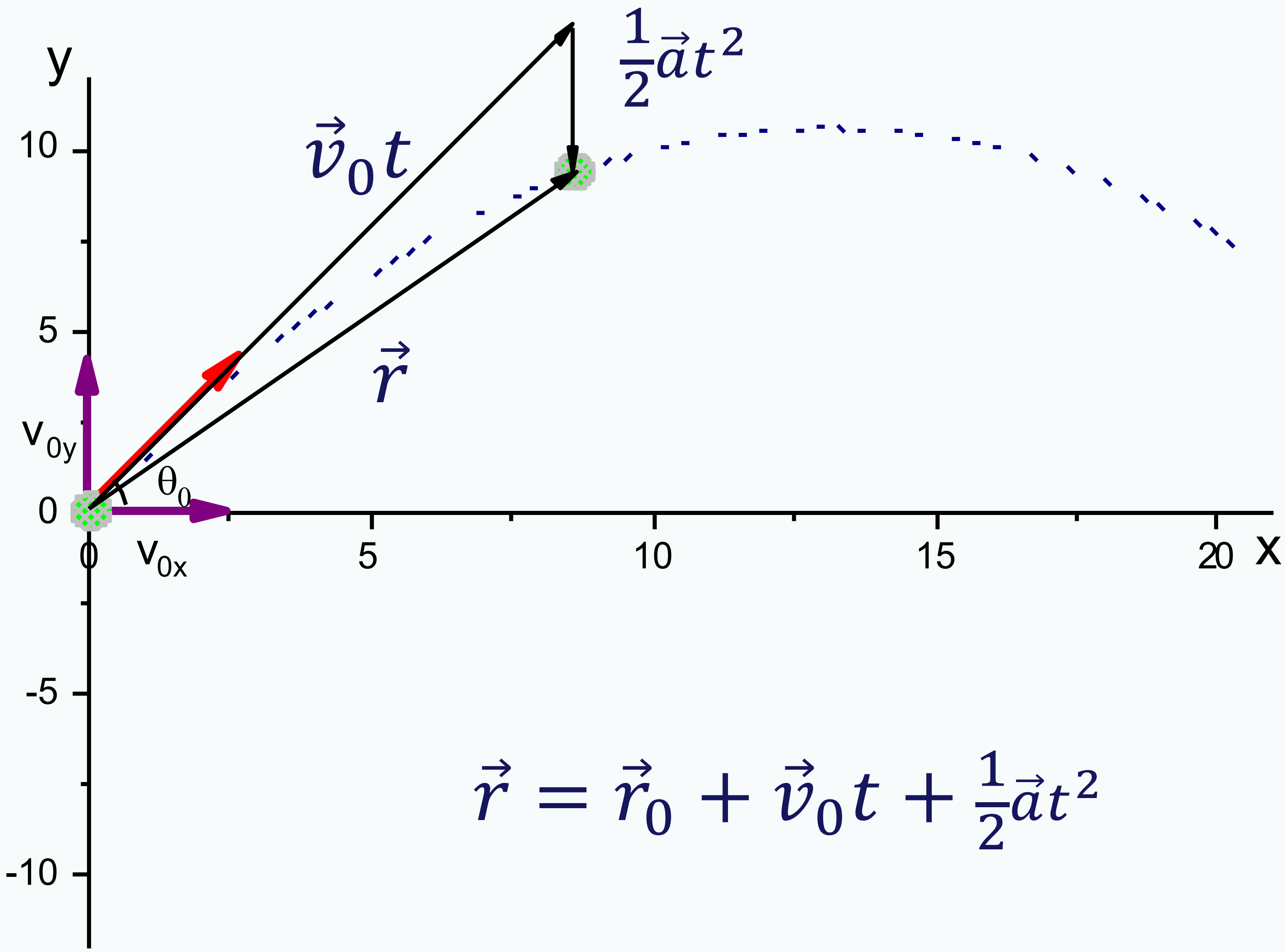
$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

Projectile motion

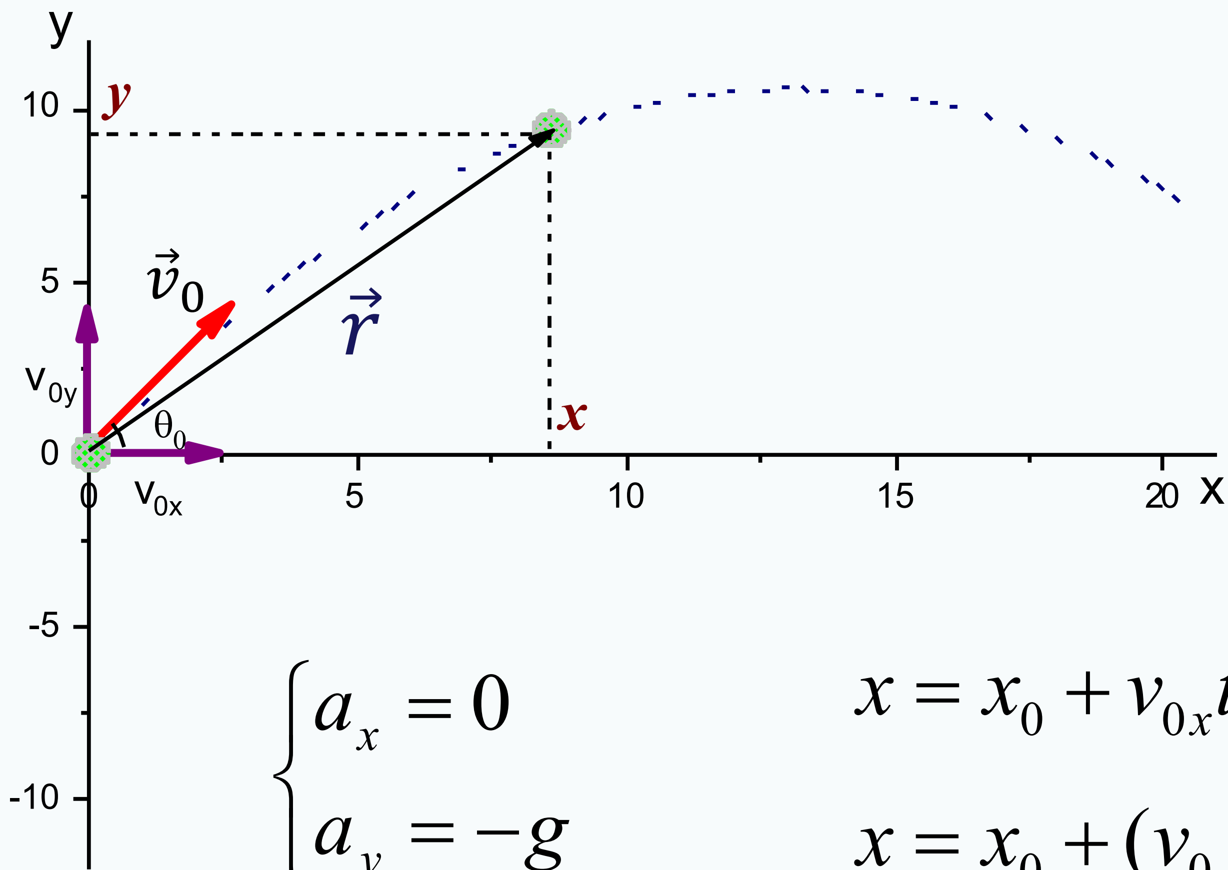


Projectile motion



$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

Projectile – horizontal motion

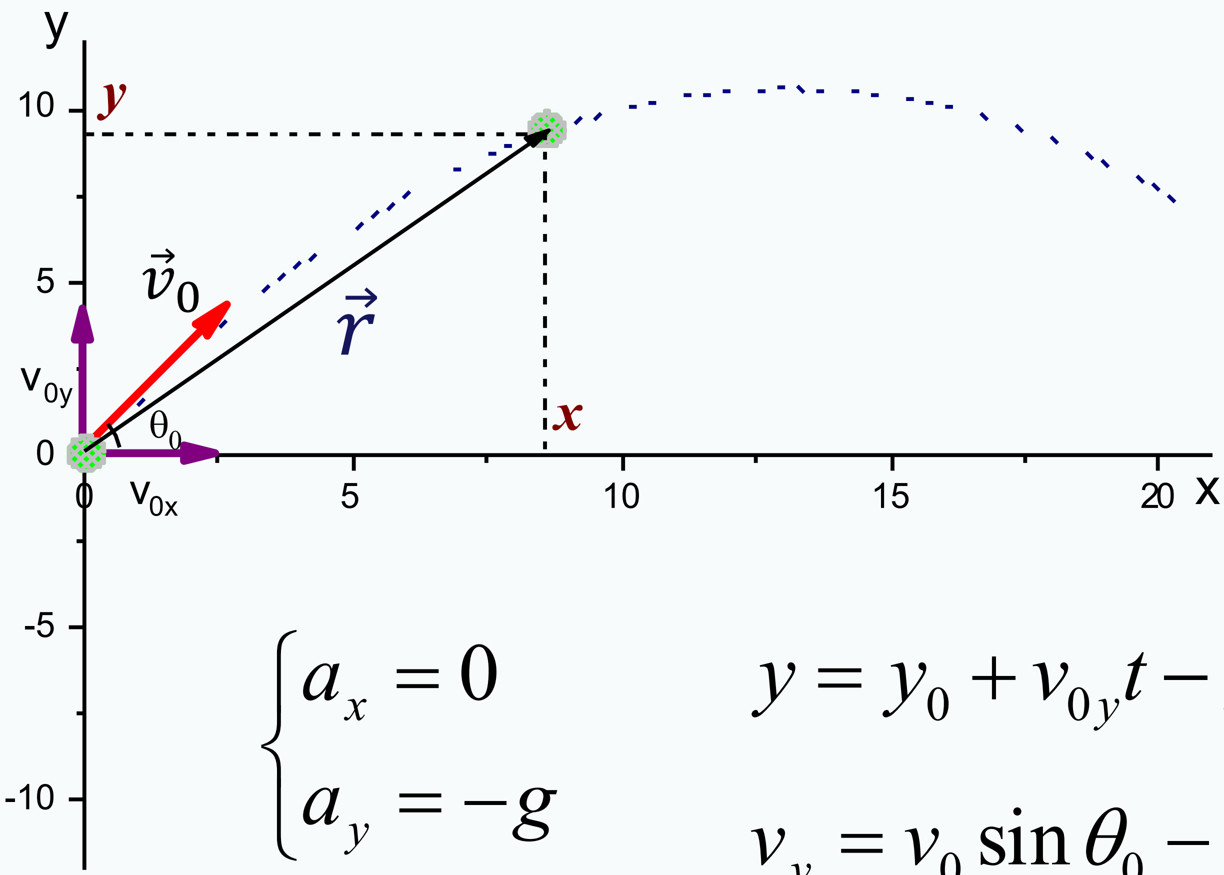


$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$x = x_0 + v_{0x}t$$

$$x = x_0 + (v_0 \cos\theta_0)t$$

Projectile – vertical motion



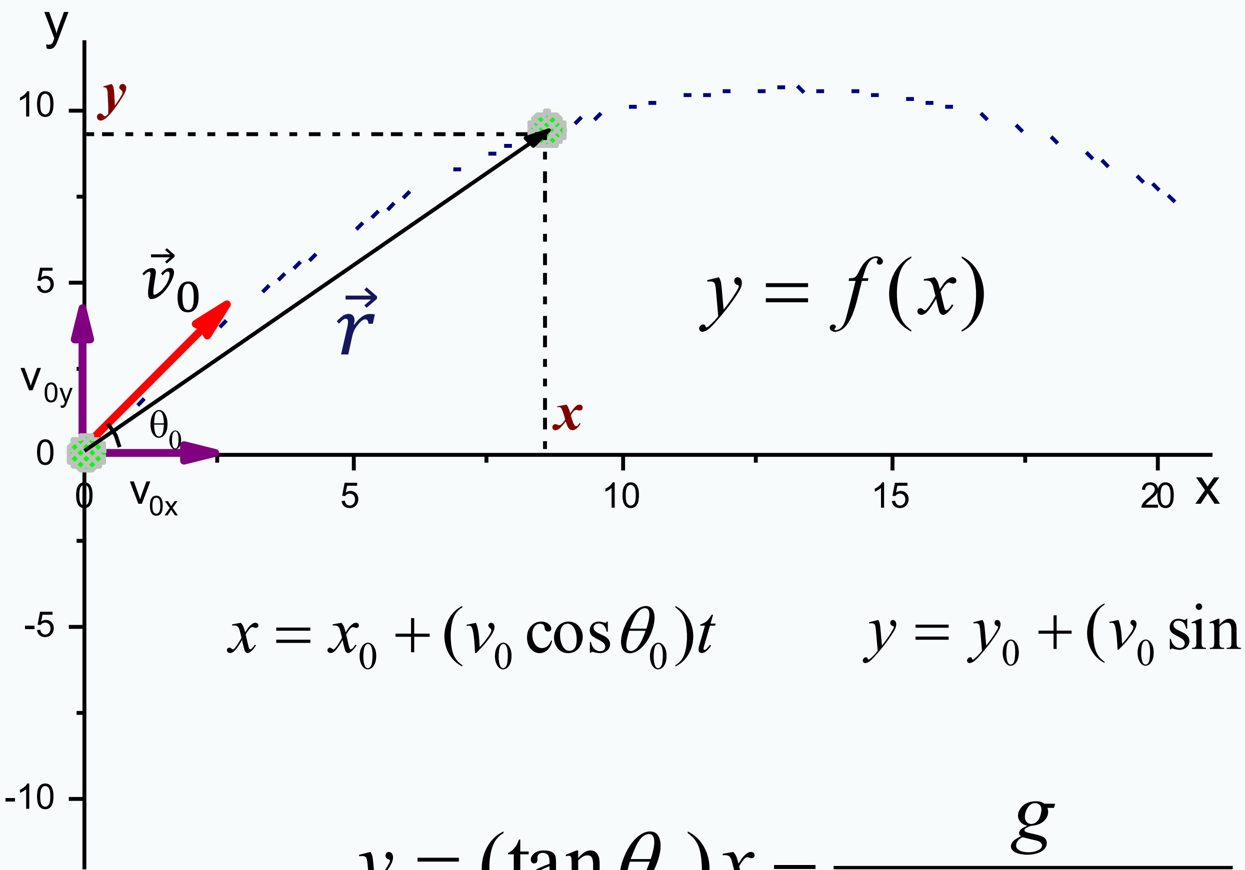
$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$y = y_0 + v_{0y}t - gt^2/2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

Projectile – equation of path

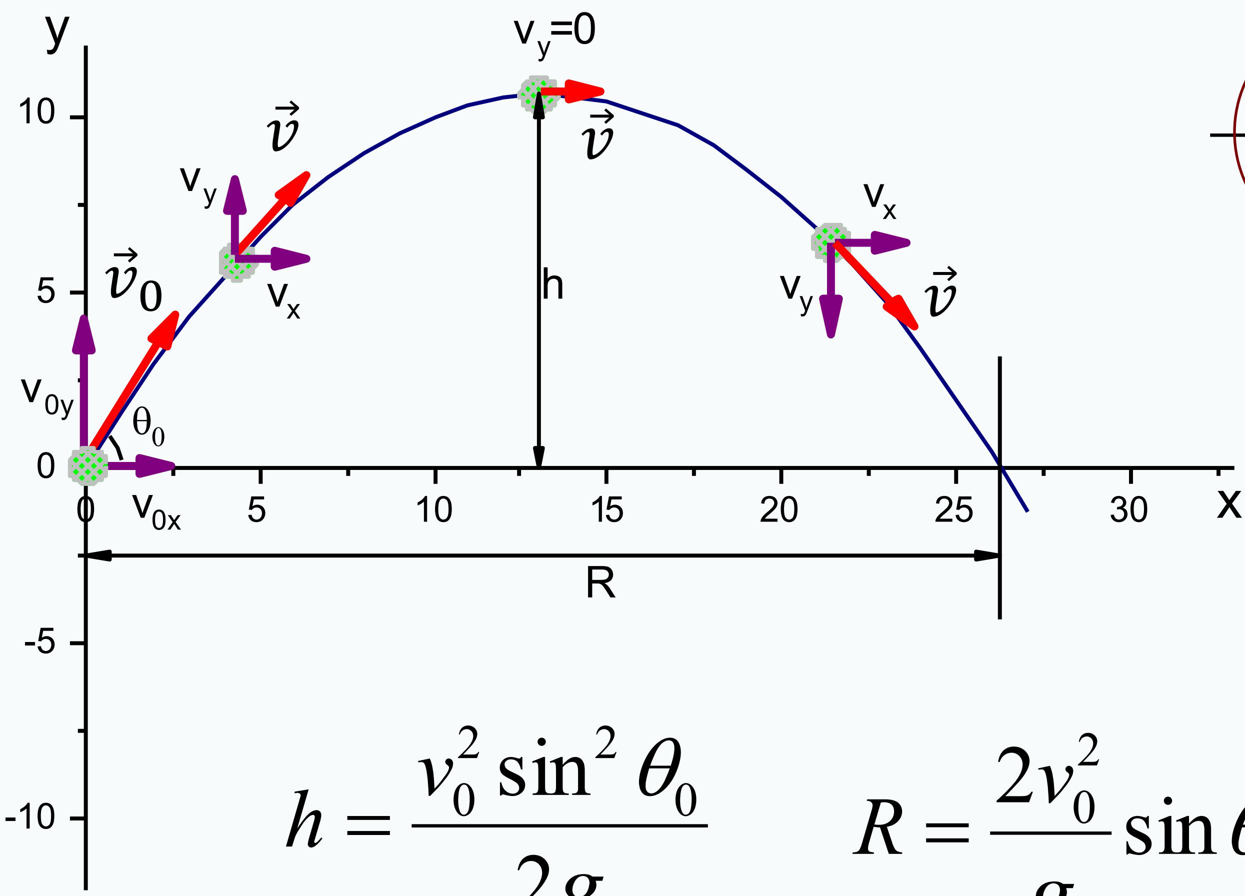


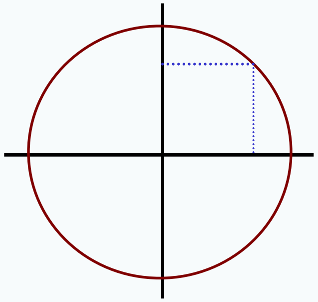
$$y = f(x)$$

$$x = x_0 + (v_0 \cos\theta_0)t \qquad y = y_0 + (v_0 \sin\theta_0)t - gt^2/2$$

$$y = (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2$$

Projectile – h and R



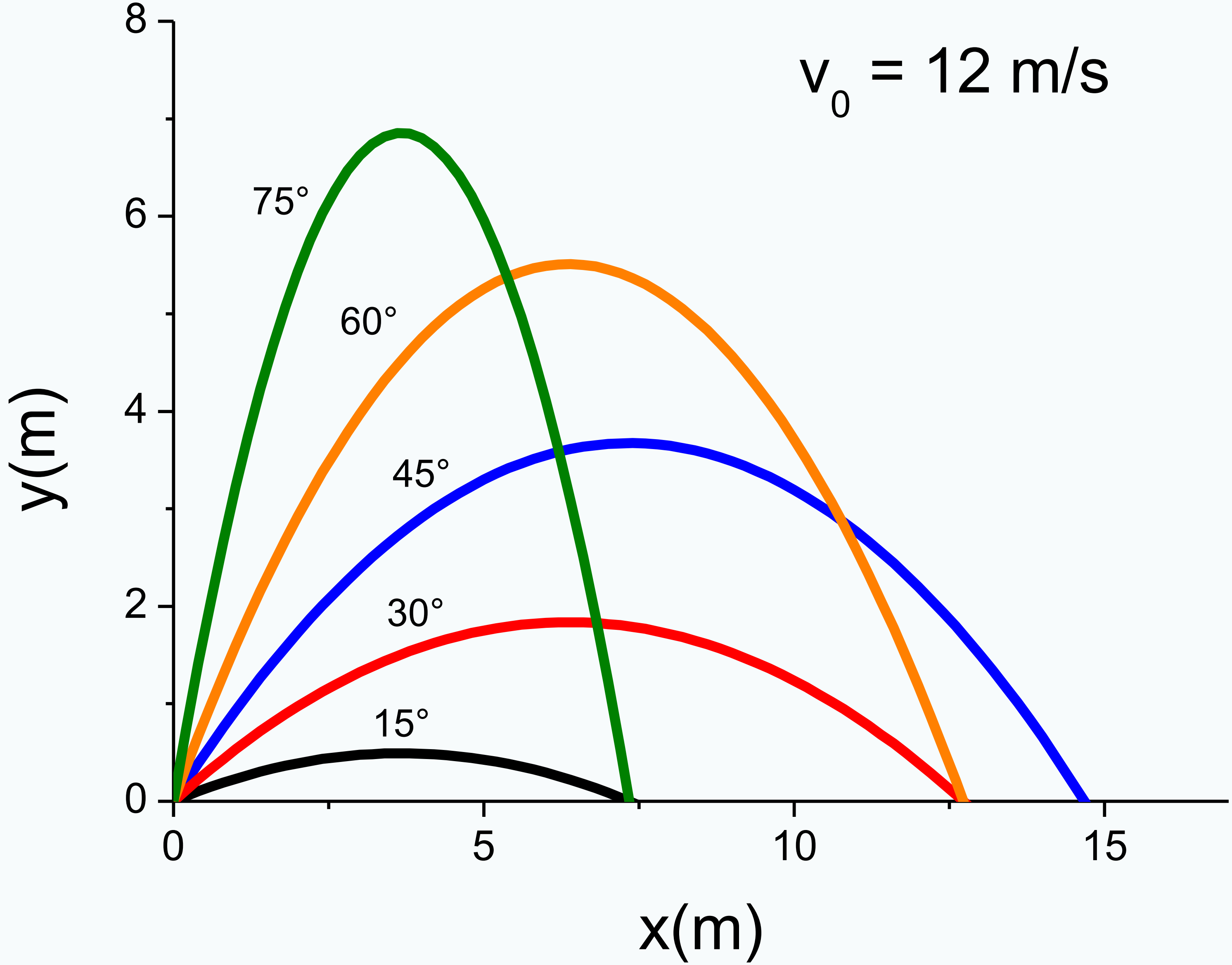


$$h = \frac{v_0^2 \sin^2 \theta_0}{2g}$$

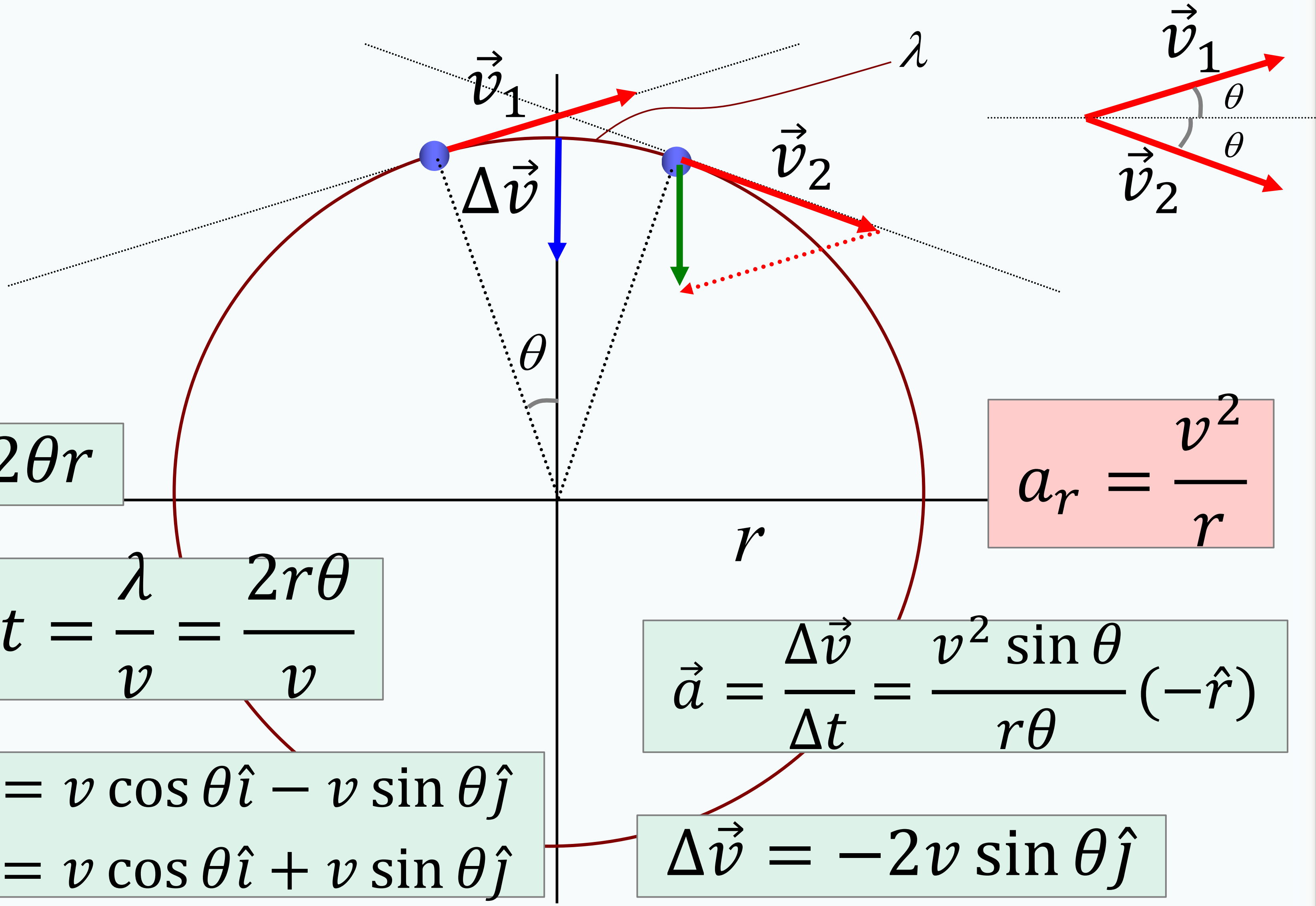
$$R = \frac{2v_0^2}{g} \sin\theta_0 \cos\theta_0$$

$$R_{max} = R(\theta_0 = \pi / 4) = v_0^2 / g$$

Projectile – the longest range R



Circular uniform motion



$$\lambda = 2\theta r$$

$$a_r = \frac{v^2}{r}$$

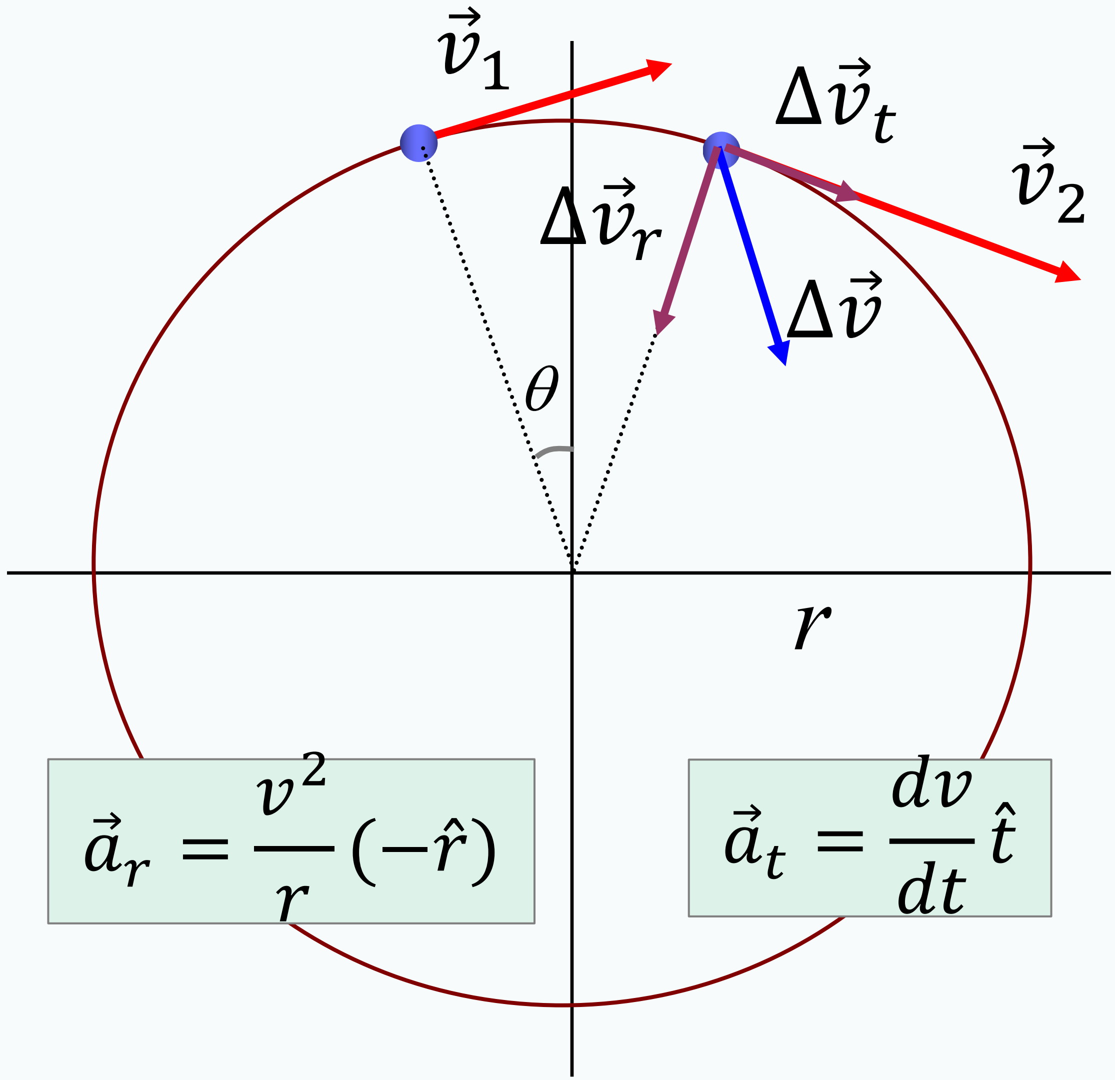
$$\Delta t = \frac{\lambda}{v} = \frac{2r\theta}{v}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v^2 \sin\theta}{r\theta}(-\hat{r})$$

$$\begin{cases} \vec{v}_2 = v\cos\theta\,\hat{\imath} - v\sin\theta\,\hat{\jmath} \\ \vec{v}_1 = v\cos\theta\,\hat{\imath} + v\sin\theta\,\hat{\jmath} \end{cases}$$

$$\Delta\vec{v} = -2v\sin\theta\,\hat{\jmath}$$

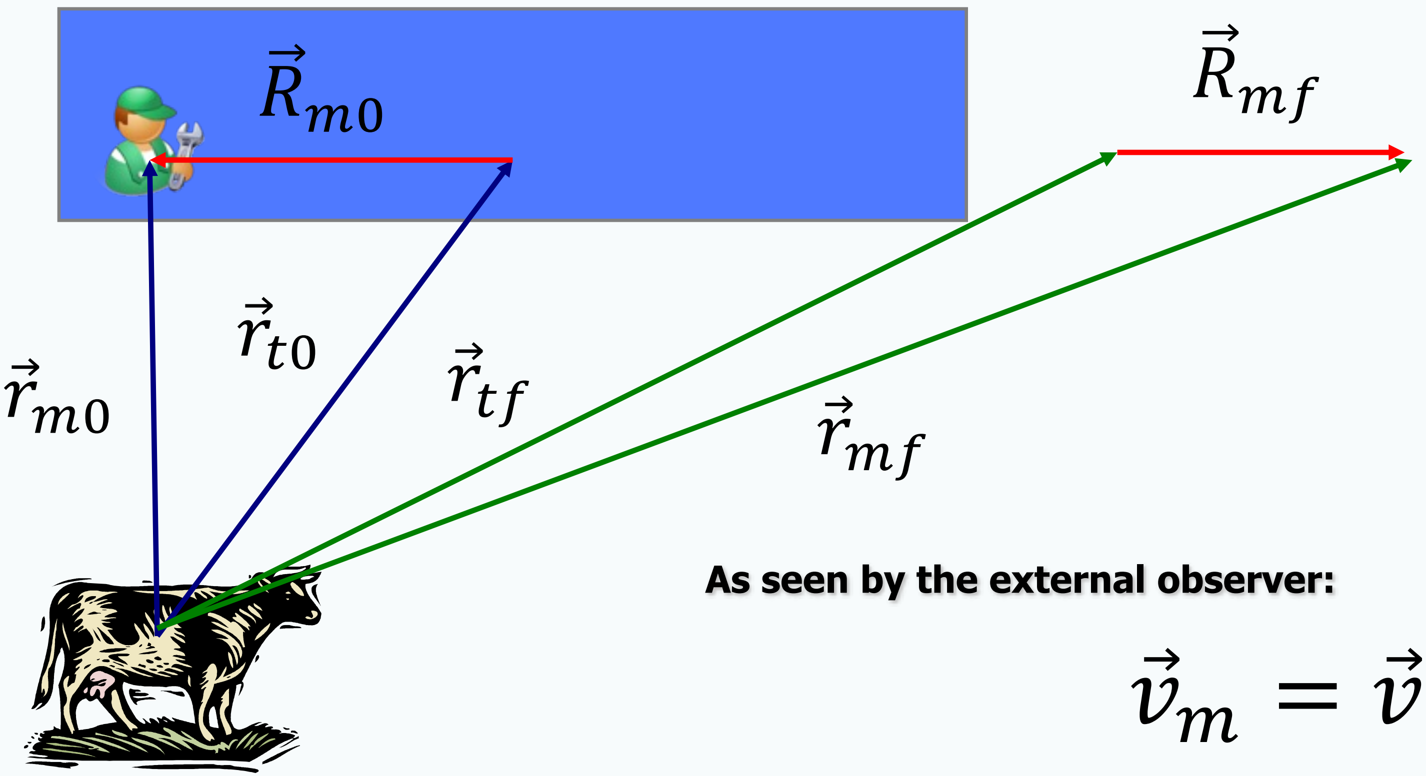
Tangential and radial accelerations



$$\vec{a}_r = \frac{v^2}{r}(-\hat{r})$$

$$\vec{a}_t = \frac{dv}{dt}\hat{t}$$

Relative motion



As seen by the external observer:

$$\vec{v}_m = \vec{v}_t + \vec{V}_m$$

If the train is moving with a constant velocity,
but the man is moving with an acceleration in the train

$$\vec{a}_m = \vec{A}_m$$

To remember!

- **<u>Equations of motion</u> in the vector form.**
- **<u>Projectile motion</u> – motion in a plane with the free-fall acceleration.**
- **Uniform circular motion leads to <u>centripetal</u> acceleration directed towards the center.**
- **There might be a tangential acceleration.**
- **There are simple rules relating velocities and accelerations in <u>two reference systems</u> moving with respect to each other.**