EXPERIMENTAL PHYSICS 1234

EXERCISE 4321

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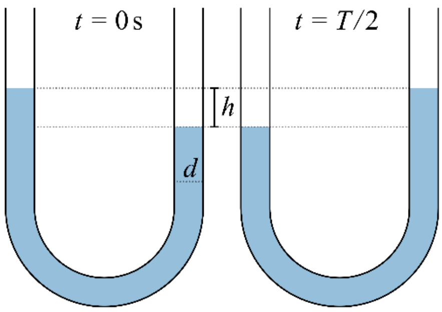
Lecturer: Prof. ABC
Tutor: Dr. DEF
E-Mail: asd.qwe@uni-leipzig.de

Problem 1: Oscillating Fluids **$2+2+2$ Points**

A **U**-shaped hollow glass tube with circular cross-section and diameter of $d = 1\,\mathrm{cm}$ is filled with 30 grams of water, $m_{\text{water}} = 0.03\,\mathrm{kg}$. You gently shake it and the water level starts to oscillate quickly up and down, see figure aside. The amplitude between highest and lowest water level is $h = 4\,\mathrm{cm}$. Friction can be neglected.



a. Find the equation of motion of the water level.
b. Calculate the oscillation frequency f of the water level.
c. Calculate the maximum speed of the water level.

Solution

Two steps are needed for this task. First, recovering the harmonic oscillator equation and second, identifying the "spring constant" and oscillation frequency ω.
The restoring force is the additional weight of the water column height difference. The amount of moved water is the whole water inside the U-tube. So,

$$F_{\text{restore}} = -\Delta m g = -\rho A \Delta h g = \boldsymbol{-2g\rho A x(t)} \stackrel{!}{=} m_{\text{water}} a = \boldsymbol{m_{\text{water}} \ddot{x}(t)}\,.$$

Note, that $x(t)$ is measured from the unperturbed column height and therefore $h = 2x$.

$$\rightarrow 0 = \ddot{x} + \frac{2g\rho A}{m_{\text{water}}} x$$

This is the harmonic oscillator differential equation. One can now identify D_{spring}, ω and f:

$$\omega^2 = \frac{D_{\text{spring}}}{m_{\text{water}}} = \frac{2g\rho A}{m_{\text{water}}}\,, \qquad f = \frac{\omega}{2\pi} = 1.14\,Hz\,.$$

Furthermore, the solution of the harmonic oscillator is known:

$$x(t) = A\sin(\omega t) = \frac{h}{2}\sin\left(\sqrt{\frac{2g\rho A}{m_{\text{water}}}}\,t\right),$$

$$v(t) = \dot{x}(t) = \frac{h}{2}\sqrt{\frac{2g\rho A}{m_{\text{water}}}}\cos\left(\sqrt{\frac{2g\rho A}{m_{\text{water}}}}\, t\right) = 0.0357\frac{m}{s}.$$

Problem 2: Trigonometric properties 2 + 2 Points

38 ••• (*a*) Show that $A_0 \cos(\omega t + \delta)$ can be written as $A_s \sin(\omega t) + A_c \cos(\omega t)$, and determine A_s and A_c in terms of A_0 and δ. (*b*) Relate A_c and A_s to the initial position and velocity of a particle undergoing simple harmonic motion.

Picture the Problem We can use the formula for the cosine of the sum of two angles to write $x = A_0 \cos(\omega t + \delta)$ in the desired form. We can then evaluate x and dx/dt at $t = 0$ to relate A_c and A_s to the initial position and velocity of a particle undergoing simple harmonic motion.

(*a*) Apply the trigonometric identity $\cos(\omega t + \delta) = \cos\omega t\cos\delta - \sin\omega t\sin\delta$ to obtain:

$$\begin{aligned} x &= A_0\cos(\omega t + \delta) \\ &= A_0[\cos\omega t\cos\delta - \sin\omega t\sin\delta] \\ &= -A_0\sin\delta\sin\omega t + A_0\cos\delta\cos\omega t \\ &= \boxed{A_s\sin\omega t + A_c\cos\omega t} \end{aligned}$$

provided

$A_s = -A_0\sin\delta$ and $A_c = A_0\cos\delta$

(*b*) When $t = 0$:

$$x(0) = \boxed{A_0\cos\delta = A_c}$$

Evaluate dx/dt:

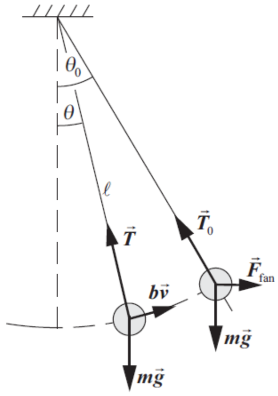
$$\begin{aligned} v &= \frac{dx}{dt} = \frac{d}{dt}[A_s\sin\omega t + A_c\cos\omega t] \\ &= A_s\omega\cos\omega t - A_c\omega\sin\omega t \end{aligned}$$

Evaluate $v(0)$ to obtain:

$$v(0) = \omega A_s = \boxed{-\omega A_0\sin\delta}$$

Problem 3: Pendulum 4 + 3 Points

Picture the Problem The diagram shows 1) the pendulum bob displaced through an angle θ_0 and held in equilibrium by the force exerted on it by the air from the fan and 2) the bob accelerating, under the influence of gravity, tension force, and drag force, toward its equilibrium position. We can apply Newton's second law to the bob to obtain the equation of motion of the damped pendulum and then use its solution to find the decay time constant and the time required for the amplitude of oscillation to decay to 1°.



(*a*) Apply $\sum \tau = I\alpha$ to the pendulum to obtain:

$$-mg\ell\sin\theta + \ell F_{\text{d}} = I\frac{d^2\theta}{dt^2}$$

Express the moment of inertia of the pendulum about an axis through its point of support:

$$I = m\ell^2$$

Substitute for I and F_{d} to obtain:

$$m\ell^2\frac{d^2\theta}{dt^2} + \ell bv + mg\ell\sin\theta = 0$$

Because $\theta << 1$ and $v = \ell\omega = \ell d\theta/dt$:

$$m\ell^2\frac{d^2\theta}{dt^2} + \ell^2 b\frac{d\theta}{dt} + mg\ell\theta \approx 0$$

or

$$m\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + \frac{mg}{\ell}\theta \approx 0$$

The solution to this second-order homogeneous differential equation with constant coefficients is:

$$\theta = \theta_0 e^{-t/2\tau}\cos(\omega' t + \delta) \qquad (1)$$

where θ_0 is the maximum amplitude, $\tau = m/b$ is the time constant, and the frequency $\omega' = \omega_0\sqrt{1-(b/2m\omega_0)^2}$.

Apply $\sum \vec{F} = m\vec{a}$ to the bob when it is at its maximum angular displacement to obtain:

$$\sum F_x = F_{\text{fan}} - T\sin\theta_0 = 0$$

and

$$\sum F_y = T\cos\theta_0 - mg = 0$$

Divide the x-direction equation by the y-direction equation to obtain:

$$\frac{F_{\text{fan}}}{mg} = \frac{T\sin\theta_0}{T\cos\theta_0} = \tan\theta_0$$

or

$$F_{\text{fan}} = mg\tan\theta_0$$

When the bob is in equilibrium, the drag force on it equals F_{fan}:

$$bv = mg\tan\theta_0 \Rightarrow \tau = \frac{m}{b} = \frac{v}{g\tan\theta_0}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{7.0\,\text{m/s}}{(9.81\,\text{m/s}^2)\tan 5.0°} = 8.16\,\text{s} = \boxed{8.2\,\text{s}}$$

(*b*) From equation (1), the angular amplitude of the motion is given by:

$$\theta = \theta_0 e^{-t/2\tau}$$

When the amplitude has decreased to 1.0°:

$$1.0° = 5.0° e^{-t/2\tau} \text{ or } e^{-t/2\tau} = 0.20$$

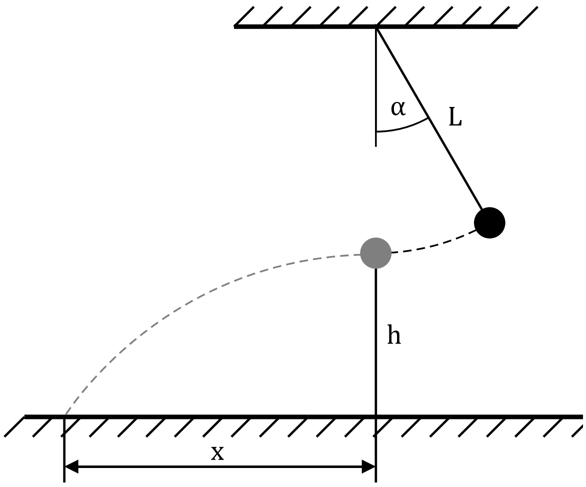
Take the natural logarithm of both sides of the equation to obtain:

$$-\frac{t}{2\tau} = \ln(0.20) \Rightarrow t = -2\tau\ln(0.20)$$

Substitute the numerical value of τ and evaluate t:

$$t = -2(8.16\,\text{s})\ln(0.20) = \boxed{26\,\text{s}}$$

Problem 1: Pendulum Collision **$1 + 3 + 1$ Points**



A pendulum with a length L and mass m starts at a time $t = 0$ with a velocity $v = 0$, displaced by an angle α. At the lowest point it hits another mass which sits on a pole, and an elastic collision happens.

a. How much time passes from the beginning of the movement unto the collision?
b. What are the speeds of the pendulum and the mass on the pole before and after the collision?
c. How far does the mass on the pole fly until it hits the ground?

Solution

(a)

It is quarter of the full pendulum period:

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}}$$

They should be able to take the formula from the lecture.

(b)

Speeds before the collision are 0 for the pole mass and $v_1 = \sqrt{2 \cdot g \cdot L \cdot (1 - \cos\alpha)}$ for the pendulum mass (calculated form energy conservation).

Energy conservation law for the collision: $\frac{m_1 \cdot v_1^2}{2} = \frac{m_1 \cdot v'^2_1}{2} + \frac{m_2 \cdot v'^2_2}{2}$, where v' is speed after collision for each of the bodies.

Momentum conservation law for the collision:

$$m_1 v_1 = m_1 v_1' + m_2 v_2', \qquad \frac{m_2}{m_1} = p$$

Then:

$$\begin{cases} v_1^2 = v'^2_1 + p \cdot v'^2_2 \\ v_1 = v_1' + p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases} v_1^2 - v'^2_1 = p \cdot v'^2_2 \\ v_1 - v_1' = p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases} (v_1 - v_1') \cdot (v_1 + v_1') = p \cdot v'^2_2 \\ v_1 - v_1' = p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases} p \cdot v_2' \cdot (v_1 + v_1') = p \cdot {v'}_2^2 \\ v_1 - v_1' = p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases} v_1 + v_1' = v_2' \\ v_1 - v_1' = p \cdot v_2' \end{cases}$$

$$\downarrow$$

$$\begin{cases} v_1' = v_1 \cdot \dfrac{1-p}{1+p} = v_1 \cdot \dfrac{m_1 - m_2}{m_1 + m_2} \\ v_2' = v_1 \cdot \dfrac{2}{1+p} = v_1 \cdot \dfrac{2 \cdot m_1}{m_1 + m_2} \end{cases}$$

That means that if $m_1 > m_2$ then the pendulum will continue motion in the same direction; if $m_1 < m_2$ then the pendulum will bounce back to ($v_1' < 0$) case; if $m_1 = m_2$ then all motion will be transferred to the mass on the pole and pendulum will stop.

It does not matter how students derive the equations, if they use energy and momentum conservation laws. I think it's reasonable to give them 1 point if they put these laws into the right equations (see above). And another point if they come up with the correct final formulas for v_1' *and* v_2'.

(c)

$$t_{flight} = \sqrt{\frac{2h}{g}} \rightarrow x = \sqrt{\frac{2h}{g}} \cdot v_1 \cdot \frac{2 \cdot m_1}{m_1 + m_2} = \sqrt{\frac{2h}{g}} \cdot \sqrt{2 \cdot g \cdot L \cdot (1 - \cos \alpha)} \cdot \frac{2 \cdot m_1}{m_1 + m_2}$$
$$= \sqrt{h \cdot L \cdot (1 - \cos \alpha)} \cdot \frac{4 \cdot m_1}{m_1 + m_2}$$

That's cool. It means that this experiment on Earth and Mars would provide the same value x for the mass flight distance.

Any form of these answers should count for the score. Students don't have to simplify, if they mentioned before what t_{flight} *or* v_1 *are.*