Lecture "Experimental Physics I"
(Prof. Dr. R. Seidel)

Lecture 20

Hydrostatics & liquids at interfaces

- “Elasticity” of fluids
- Static pressure
- Principle of transmission of fluid-pressure (Pascal’s principle)
- Barometric height formula
- Buoyancy
- Surface tension & capillary effects

1) Elasticity of fluids

A) Ideal fluids

After studying solids and their deformation, we will now address the properties of fluids. To keep life simple, we will first define and study a model fluid - the so-called **ideal fluid**. It is defined by (**see slide**):

> *In an **ideal fluid** there are **no friction forces**, so called viscous drag forces. A friction-less fluid is also called **inviscid fluid**.*

There are two consequences arising from this definition for an ideal fluid:

- No mechanical energy is converted into heat.
- No force is required to reshape a given volume of fluid at zero flow (when moved slow enough, such that inertial effects can be neglected).
- The term ideal fluid is not precisely defined in literature. There are often additional idealized properties assumed for an ideal fluid, such as:
    - incompressibility
    - no surface tension
    - weightlessness

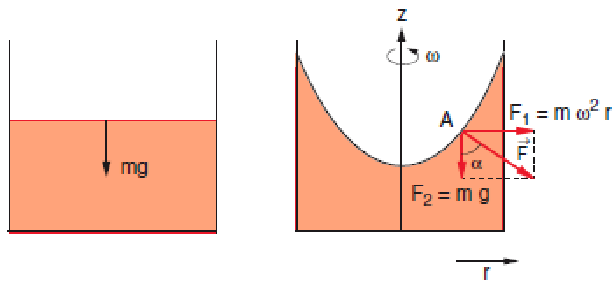
Depending on the problem, one has to decide which fluid properties need to be idealized and how many real properties have to be considered. We will first look at problems in the field of **hydrostatics** that investigates fluids at rest such that the ideal fluid picture can be applied, since due to the absence of motion we have in this case no friction forces.

B) Bulk compression of fluids

Let us first look at the elastic properties of an ideal fluid in analogy to the elasticity of solids. According to the definition above, there is no force needed to slowly move a fluid element in tangential direction along a surface. Thus, we can conclude consecutively for a **static fluid**:

- There are **no tangential forces** at the fluid surface
- **The shear modulus of fluids is zero**
- The **surface of a fluid is normal to the acting total force**

The normal orientation of the fluid surface to an acting force we experience in our everyday life. We have flat fluid surfaces in cups, lakes etc. in horizontal direction, normal to the gravity force. If there would be tangential forces along the surface, they would cause a redistribution of the fluid, i.e. a flow. We already applied the principle of the orientation of a fluid surface normal to an acting force when calculating the parabolic shape of a fluid surface in a spinning chamber:



Furthermore, if no force is needed to deform the shape of a fluid, there are no forces for tensile elongations, such that the concept of the **elastic modulus is not meaningful.** However, one can compress a fluid inside a vessel with an external pressure. From all the elastic deformation we discussed before, therefore only the **bulk compression** described by the **bulk modulus is meaningful.**

Last lecture we had for the bulk compression of an elastic solid:

$$\Delta p = -\underbrace{\frac{1}{\kappa}}_{K}\frac{\Delta V}{V}$$

where $\boldsymbol{K}$ **was the bulk modulus** and $\boldsymbol{\kappa}$ the **compressibility**. Transforming the equation gives:

$$\kappa = -\frac{\Delta V/V}{\Delta p}$$

i.e. κ is the relative volume change per pressure change of a medium:

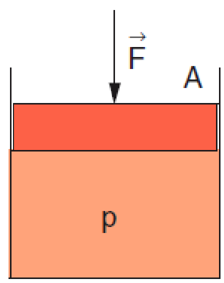
Experiment: Compression of air and water in a vessel, where we see that water is practically incompressible

Precise experimental measurements provide a compressibility of water of $\kappa_{H_2O} = 5 \times 10^{-10}\ Pa^{-1}$. The relative volume change for when applying an atmospheric pressure ($p_{atmos} \approx 10^5$ Pa) is $5 \times 10^{-5} = 0.005\%$. Therefore, volume changes of fluids can typically be neglected for pressure changes on the order of the atmospheric pressure. Thus, **fluids are often considered to be incompressible**. This provides that **their density $\boldsymbol{\rho} = \boldsymbol{const}$**.

2) Static pressure

A) Pascal's principle

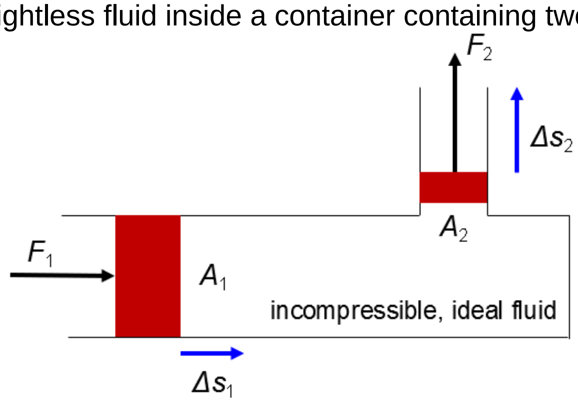
We saw before that in an ideal fluid, force can only act in orthogonal direction the fluid surface.



To simplify the description one defines a stress-like quantity, where the corresponding force direction is provided by the corresponding orientation of the surface. This quantity is the **pressure $\boldsymbol{p}$**, which is equivalent to the volume stress, as a scalar quantity:

$$p = \frac{F}{A}$$

with the SI unit of the pressure being $[p] = N\ m^{-2} = Pa$ (Pascal). Pressure is the crucial quantity in hydrostatics. To understand this, we carry out a thought experiment with a vessel filled with an ideal, incompressible, weightless fluid inside a container containing two pistons:



Pushing one piston inside will due to the incompressibility of the fluid push the other piston outside. Since no work is required to deform the fluid, the work done during pushing must equal the work done by the pushed piston:

$$F_1\, \Delta s_1 = F_2\, \Delta s_2$$

Incompressibility provides us furthermore volume conservation:

$$\Delta V_1 = A_1\, \Delta s_1 = A_2\, \Delta s_2 = \Delta V_2$$

Division of both equations yields:

$$p_1 = \frac{F_1}{A_1} = \frac{F_2}{A_2} = p_2$$

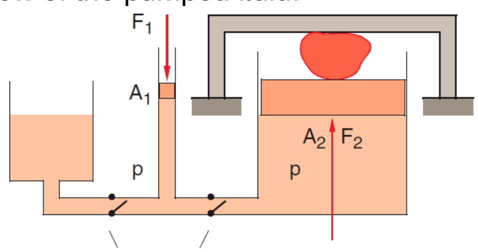
i.e. we have an equal pressure at both pistons non-depending on the orientation of the pistons. This pressure equality is generalized in the so-called **Pascal's principle**, which is also known as the **Principle of transmission of fluid-pressure**:

> A change in the **pressure** applied to a fluid is **transmitted undiminished to every point of the fluid** and to the walls of the container, i.e. the **pressure acts in all directions**.

Experiments: Pressure acts equally in all directions: A syringe filled with water drives the water out of a glass bulb with many holes. We see that the water leaves the bulb equally in all directions

Pressure transmission is basis for hydraulic devices

Pascal's principle is the basis for hydraulic devices, which are used in many applications to amplify forces (car breaks, power shovels, cranes, etc.). Here one has a similar system as the vessel with the two pistons. In addition, one has also an open storage vessel for the hydraulic fluid and two valves that prevent the backflow of the pumped fluid:



When pushing piston 1 with F_1 we obtain for the force F_2 at the actuator:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

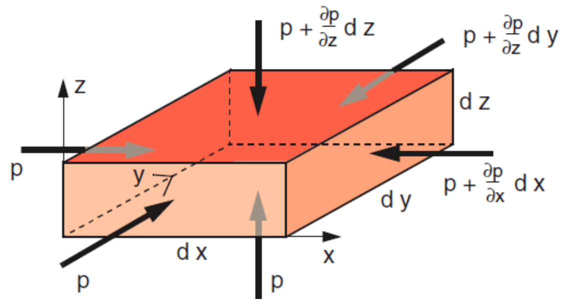
Thus,

$$F_2 = F_1 \frac{A_2}{A_1}$$

where we have a **force amplification** for appropriate area ratios. A "weak" pump can be employed to achieve very high forces. We nonetheless have energy conservation, since the pump has to move the piston over proportionally larger distances compared to the actuator. To allow larger movements, the fluid reservoir and the valves allow to pump in more and more fluid.

B) Force on a small fluid segment

To derive more general formulas for the pressure in a fluid or even gas, we look at the net force onto a small fluid element dV at which an external pressure acts. This pressure may not be constant throughout the solution, i.e. we may have a pressure gradient in the solution:



We first look at the effective force from the pressure onto the fluid element along the x-coordinate. If the **pressure on the left face** is $p_L = p$ then the **pressure on right face** can be approximated by a first order Taylor expansion:

$$p_R = p + \frac{\partial p}{\partial x} dx$$

for an infinitesimal edge length of the fluid element dx. The force difference along the x-direction is then:

$$dF_x = \underbrace{p\, dydz}_{p_L\, dA} - \underbrace{\left(p + \frac{\partial p}{\partial x} dx\right) dydz}_{p_R\, dA} = -\frac{\partial p}{\partial x} dx\, dy\, dz = -\frac{\partial p}{\partial x} dV$$

Analogously, we obtain for the other directions:

$$dF_y = -\frac{\partial p}{\partial y} dV \quad and \quad dF_z = -\frac{\partial p}{\partial z} dV$$

such that we get for the force acting on the fluid element the following vectorial form:

$$d\vec{F} = -\begin{pmatrix} \partial p/dx \\ \partial p/dy \\ \partial p/dz \end{pmatrix} dV = -\vec{\nabla} p\, dV$$

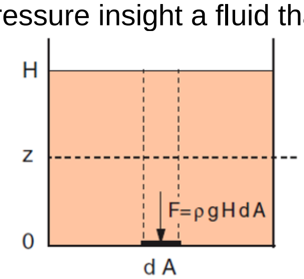
i.e. the force is given by the gradient of the pressure in the solution.
For a **static, weightless fluid,** the net force on every fluid element must be zero, since we would have otherwise an acceleration of the fluid element and thus a fluid flow. This provides:

$$0 = \vec{\nabla} p \quad \text{and consequently} \quad p = const.$$

In a **fluid without gravity we have everywhere the same pressure**, in analogy to Pascal's principle.

C) Pressure caused by the fluid weight

Now we will use the formula that connects the force on a small volume element with a pressure gradient to obtain a relation for the pressure insight a fluid that has a fluid weight:



In the static case (i.e. no flow) the total force on a given fluid element must be zero. In contrast to before, we have additionally to **take the gravity of the fluid element into account**. We can thus write for the force on a fluid element dV along the vertical direction:

$$dF_{z,tot} = 0 = -\underbrace{\rho\, dV}_{m}\, g - \frac{dp}{dz}\, dV$$

i.e. the gravity force must is compensated by a pressure difference along the z coordinate to ensure a zero net force. Transformation gives:

$$\frac{dp}{dz} = -\rho g$$

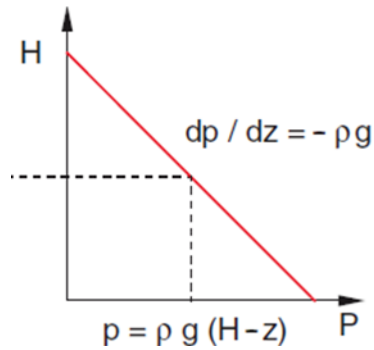
The pressure gradient is therefore negative, i.e. the pressure decreases with increasing position z. After separation of variables and integration we can calculate the absolute pressure at height z starting from a known pressure $p(H)$ at height H:

$$\int_{p(H)}^{p(z)} dp = -\int_{H}^{z} \rho g \, dz$$

For an incompressible fluid we have $\boldsymbol{\rho} = \boldsymbol{const}$ such that the integration provides:

$$p(z) - p(H) = -\rho \, g \left(\underbrace{z - H}_{\Delta H}\right) \quad or \quad \Delta p = -\rho \, g \, \Delta H$$

i.e. the pressure increases linearly with the depth in the fluid or decreases when one moves upwards insight the fluid:



With this we can formulate an **extended version of Pascal's principle:**

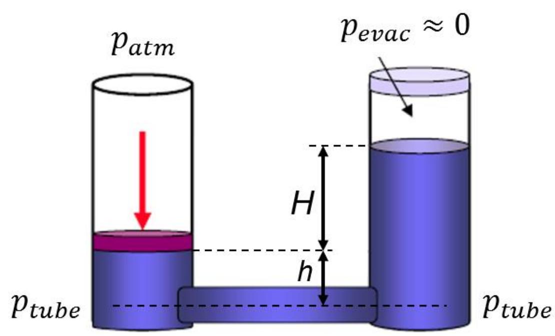
> The pressure in a static fluid depends only on the external pressure and the height difference (with respect to the surface).

Experiment: Despite the gravity force acting in just one direction on the fluid, the pressure from the fluid weight also acts equally in all directions as demonstrated by a rotatable pressure sensor at the bottom of a fluid column.

<u>Applications:</u>

The extended version of the fluid pressure has some interesting consequences (see **Experiments & slides**).

One can **measure the atmospheric** pressure using **two communicating vessels** filled with liquid that are connected via a horizontal connector tube. One of them is connected to the atmospheric pressure the other contains an evacuated reservoir on its top. Along the tube including its two outlets to the reservoirs, the pressure p_{tube} must be everywhere equal in the static case, due to the equal height.



Using the reformulated Pascal's principle, we can write down for the pressure in both vessel at the height of the tube:

$$p_{tube} = \underbrace{p_{atm} + \rho g h}_{p_1} = \underbrace{p_{evac} + \rho g (h + H)}_{p_1}$$

Transformation provides:

$$\Delta p = p_{atm} - \overbrace{p_{evac}}^{0} = \rho g H$$

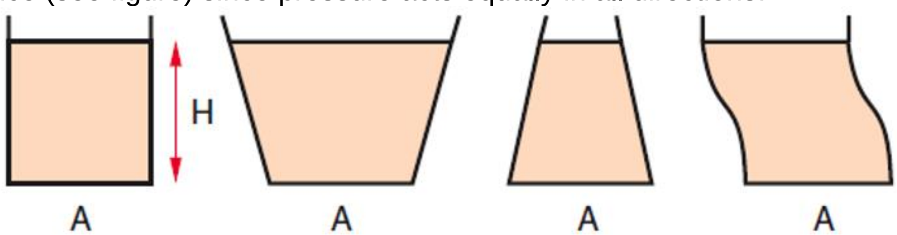
i.e. the atmospheric pressure pushes the liquid in the right vessel upwards until the hydrostatic pressure from the additional fluid height equals the pressure difference on the surface of both fluids. Assuming full evacuation ($p_{evac} = 0$) gives for the height difference H of the fluid levels:

$$H = \frac{p_{atm}}{\rho g}$$

If the vessel is filled with water, we get $H = 10.13\ \mathrm{m}$ for an atmospheric pressure of $p_{atm} \approx 101.3\ \mathrm{kPa} = 1\ \mathrm{atm}$. Thus, about 10 m of a water column generate our atmospheric pressure. One can therefore not use a pump to suck water into an apartment that is higher than 10 m. In order to lift the water higher one would need to have a pressure difference larger than the pressure of the water column, which is not possible. Larger height differences can thus only be created by pumps that push up the liquid.
If the vessel is filled with mercury as in some older barometers, we have a height difference of $H = 760\ \mathrm{mm}$. The height of such a mercury column defines the pressure in the unit Torr:

$$1\ \mathrm{Torr} = 1\ \mathrm{mm}\ Hg = \frac{101300}{760}\,\mathrm{Pa} \approx 133.3\ \mathrm{Pa}$$

Another interesting conclusion/observation of the extended Pascal's principle is the **hydrostatic paradox**. The pressure insight a fluid is not dependent on the shape of the vessel, only on the height difference (see figure) since pressure acts equally in all directions:



A nice demonstration of this effect is Pascal's barrel where on attaches a thin, 10 m long tube filled with water vertically to a barrel. The atmospheric pressure that acts in addition to the pressure of the water column can then make such a barrel to burst (**see slide**).

Experiments:

- Communicating vessels with a reduced pressure in one of the vessels
- Hydrostatic paradox using differently shaped but connected glass vessels

D) Barometric formula

For an incompressible fluid we have a linear pressure increase with decreasing height inside the fluid. This changes inside a compressible medium such as a gas (e.g. earth atmosphere). Here we have to **consider the compressibility** of the medium. At larger heights with lower pressure the gas is less compressed and thus has a lower density compared to small heights. Correspondingly, the pressure gradient that is proportional to the density is thus larger at lower compared to larger height levels.

Experiment: Modell for the pressure inside a compressible medium made by repelling magnets put on a vertically oriented rod. We have a larger compression in the lower compared to the upper layers of the magnets.

We will learn in the thermodynamic lectures that for an ideal gas at constant temperature there is a proportionality between density and pressure:

$$\frac{p}{p_0} = \frac{\rho}{\rho_0}$$

Our previous formula thus becomes.

$$\frac{dp}{dz} = -\rho\, g = -\frac{p}{p_0}\rho_0\, g$$

Separation of variables and integration then gives:

$$\int_{p_0}^{p(h)} \frac{dp}{p} = -g\frac{\rho_0}{p_0}\int_0^h dz$$

where we start with pressure p_0 at zero height (e.g. the earth surface). With this we get:

$$\underbrace{\ln p(h) - \ln p_0}_{\ln(p(h)/p_0)} = -g\frac{\rho_0}{p_0}h$$

Transformation provides the **Barometric height formula for an isothermal atmosphere** (i.e. at constant temperature), which predicts an exponential decay of the pressure with height:

$$p(h) = p_0 e^{-\rho_0 g h/p_0}$$

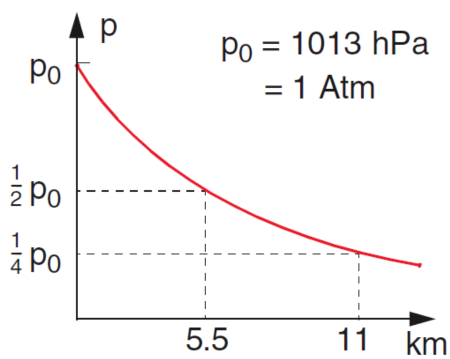
Due to the proportionality between pressure and density we also get an exponential decay for the density:

$$\rho(h) = \rho_0 e^{-\rho_0 g h/p_0}$$

At the surface of the earth we have $\rho_0 \approx 1.24\ \mathrm{kg\,m^{-3}}$ and $p_0 \approx 101.3\ \mathrm{kPa}$. Inserting provides for our height formula:

$$p(h) = p_0 e^{-h/8.33\ \mathrm{km}}$$

i.e. at 8.33 km the pressure drops by a factor of $1/e \approx 1/2.7$:



Experiment: We measure the pressure reduction between ground and sealing of the lecture hall using a pressure sensor. For small height differences $h \ll 8.33\ \mathrm{km}$ we can use a Taylor expansion of the exponential

$$e^{-x} \approx 1 - \frac{x}{1!}\left(+\frac{x^2}{2!} - \cdots\right)$$

for small x. Using only the first order term we get for the barometric height formula:

$$p(h) \approx p_0 - \underbrace{p_0\frac{h}{8.33\ \mathrm{km}} + \cdots}_{\Delta p}$$

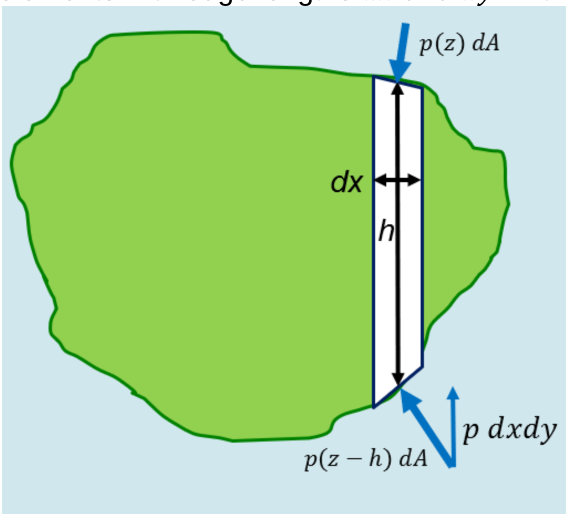
The expected pressure difference is thus

$$\Delta p(h) \approx -101.30\ kPa\ \frac{6.2\ m}{8330\ m} = -0.08\ kPa$$

In 1648, Blaise Pascal and his brother-in-law Florin Périer supported Torricelli's theory that the barometric pressure was caused by the weight of air in the atmosphere. The measured the height of a column of mercury at three elevations on the mountain Puy de Dôme, an extinct volcano in the Auvergne region in France.

3) Buoyancy

A very important phenomenon of static fluids (gases) is the phenomenon of buoyancy, which makes ships floating on the water surface and balloons filled with hot air or helium to fly. The formulas derived above provide a simple way to derive the central principle for buoyant forces. Let us consider a rigid body that is fully submerged in a fluid of density ρ. We now slice the body into small vertical volume elements with edge lengths dx and dy in the horizontal direction:



With h being the thickness of the body in the vertical direction at the particular position, we get for the volume of the element:

$$dV = h\,dx\,dy$$

The force acting on the small surface elements on the top and the bottom of the volume element is given by

$$d\vec{F} = -p\,\vec{n}\,dA$$

where $\vec{n}$ is the normal outward pointing vector of the surface element. The force along the vertical direction is given by the projection of the area element into the xy-plane for which $dA_z = dxdy$:

$$dF_z = \pm p(z)\,dA_z = \pm p(z)dx\,dy$$

The force sign is positive on the bottom of the element and negative on the top. The total vertical force on both surfaces of the volume element is then as:

$$dF_z^{net} = dF_z^{bot} + dF_z^{top} = (p_{bot} - p_{top})dxdy = \rho_{fl}g\,\underbrace{h\,dx\,dy}_{dV}$$

i.e. it is given by the pressure difference. Expressing the pressure difference using the height of the corresponding fluid column $\Delta p = \rho_{fl}gh$ gives:

$$dF_z^{net} = \rho_{fl}g\,\underbrace{h\,dx\,dy}_{dV}$$

which can be simplified using the volume of the element. The total buoyant force is then provided by integration over the total volume of the body:

$$F_z^{net} = \int dF_z^{net} = \rho_{fl}g\underbrace{\int dV}_{V}$$

such that we arrive at

$$F_z^{net} = m_{fl}g$$

where m_{fl} is the mass of the fluid that is displaced by the body. This provides the famous **Archimedes' principle** stating that :

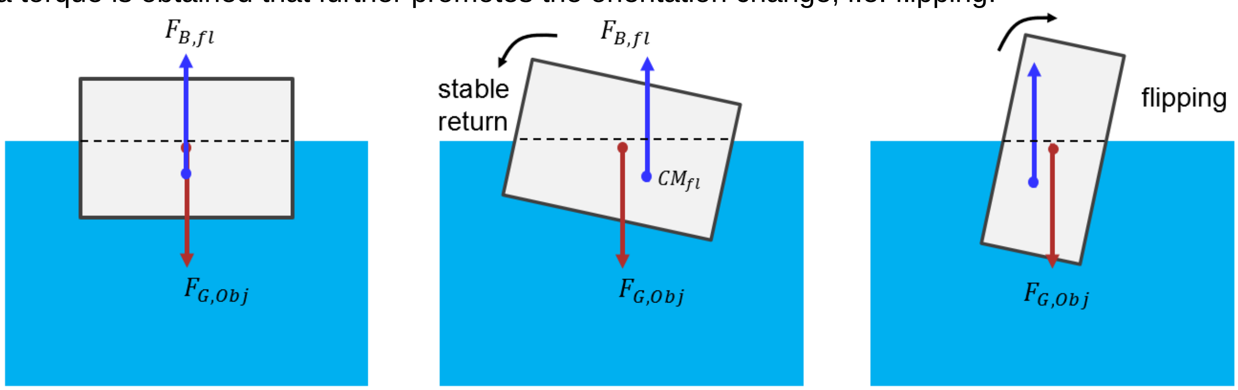
> A body submerged in a fluid is lifted (buoyed up) by a force equal to the weight of the displaced fluid.

Archimedes' principle and the buoyant forces can be demonstrated in a number of experiments:

Experiments:

- Buoyancy balance: Measurement of the buoyant force of a mass in a trough as well as the weight of the displaced fluid (that left the trough) using a balance
- Buoyant force, Archimedes cylinder, comparison of buoyancy and weight of the displaced fluid
- Dasymeter - a device to demonstrate the buoyant effect of gases. It can be built from beam balance connected to two objects of equal weight but different density and thus different volume. Evacuating the chamber around the balance provides imbalance due to the different buoyancy in the air atmosphere before

Floating objects and their stability:
Static objects swim, i.e. float, if their average density is smaller than the density of water, since in this case the buoyancy of the submerged objected is larger than the weight. Such objects immerse in the fluid until the displaced fluid weight balances the objects weight, i.e. $m_{fl} = m_{Obj}$.
For floating objects not only the immersion depth is important but also the stability of the floating object against flips etc. One can see whether an object is floating in a stable orientation by considering that the buoyancy acts in the center of mass of the displaced fluid CM_{fl}. When displacing the object from the equilibrium position, the force vectors of buoyancy and weight do not act along the same line anymore and generate a torque on the object. For a stably swimming object the force couple must generate a back-driving torque while for an unstably swimming object a torque is obtained that further promotes the orientation change, i.e. flipping:



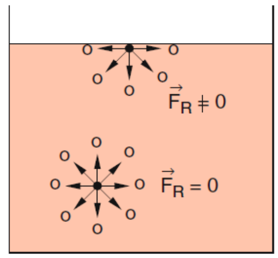
Stable conditions can be achieved by appropriate design of the shape of the floating object and a location of the center of mass, which is as low as possible. The latter can be seen by moving the center of mass of the object downwards along the object center.

4) Liquid surfaces

At the surfaces of liquids, we have a number of interesting phenomena that will be discussed in the following.

A) Surface tension

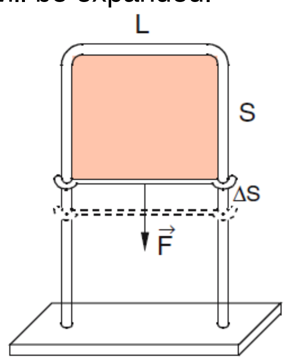
Inspecting a the molecules at a fluid surface provides that there is actually a net force onto each surface molecule that is non-zero and directed inwards. It arises from the fact that all molecular interaction forces are directed into the fluid but none out of the fluid. This requires a **counter force to keep a fluid molecule at the surface.** We will see later that this counter force is provided by an internal pressure. We also see from the cartoon that increasing the fluid surface is energetically not beneficial, since one has to break additional molecular bonds when bringing additional molecule to the fluid surface.



Thus, an **increase of the fluid surface requires work/energy.** This work is in first approximation proportional to the increase in the number of surface molecules and thus proportional to the increase in surface area. The required work per surface area is given by the **specific surface energy:**

$$\varepsilon = \frac{\Delta W}{\Delta A}$$

with $[\varepsilon] = J/m^2$. The specific surface area is a material property, i.e. it is specific to the particular liquid-gas interface. **ε is as larger as lower the interaction between gas and liquid is, i.e. as more bond breaking in the liquid matters.** Now we develop this concept further by looking at a trapped fluid lamella, which surface will be expanded.



According to the figure, the required work to extend the lamella downwards by length Δs is given by:

$$\Delta W = \varepsilon\, \Delta A = \varepsilon\, 2L\, \Delta s = F\, \Delta s$$

The factor of 2 arises since we increase the area of the front and the backside. The work is proportional to Δs such that we have to generate a force along Δs, which thus acts tangential to the surface:

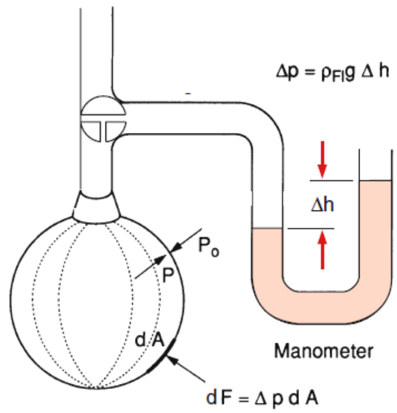
$$F = \varepsilon\, 2L$$

This force acts normal to the line at which the surface increase occurs (i.e. the air/water interface). We call the normalized force per line length the **surface tension**:

$$\sigma = \frac{F}{2L} = \varepsilon$$

with $[\sigma] = N/m$. **Specific surface energy and surface tension represent thus the same quantity**.

To better understand the concept of surface tension we now look at a soap bubble with a thin fluid layer between the inner and outer soap surface. The bubble tries (due to the discussed energetic reasons) to minimize its surface due to the surface tension. Bubble collapse is impeded by an increased internal pressure inside the bubble.



The total soap surface area of outer and inner surface is given by:

$$A = 2\,4\pi r^2$$

When increasing the bubble radius by dr, the increase in surface area is given by:

$$dA = \left(\frac{\partial A}{\partial r}\right) dr = 16\pi r\, dr$$

The required work to increase the surface area is then

$$dW = \varepsilon\, dA = \varepsilon\, 16\pi r\, dr$$

The work done by the internal pressure acting on the bubble surface is given by:

$$dW = F dr = p\, 4\pi r^2 dr$$

The internal pressure can only expand the bubble until the point where the work done equals the work needed to increase the surface area. Thus,

$$p\, 4\pi r^2 dr = \varepsilon\, 16\pi r\, dr$$

From which we get:

$$p = \frac{4\varepsilon}{r} = \frac{4\sigma}{r}$$

i.e. as **smaller the bubble, as larger the internal pressure** that keeps the bubble from collapsing.

An **experiment** where this effect can be demonstrated are two differently sized bubbles connected by a valve. When connecting the bubbles by the valve the larger one grows at the expense of the smaller one, since in the smaller one exists a larger pressure that drives the air into the larger bubble. Rationally one can explain this also by the fact that the surface area of a sphere is growing slower than the volume, such that after the transfer we have a smaller total surface area.

Similarly, as for a soap bubble also a water droplet has an internal pressure sine it has an outer surface subjected to surface tension. Compared to the derivation above we must take half of the surface area (single vs. double surface) and arrive for the pressure inside a water droplet at:

$$p = \frac{2\sigma}{r}$$

B) Interfaces and adhesion tension

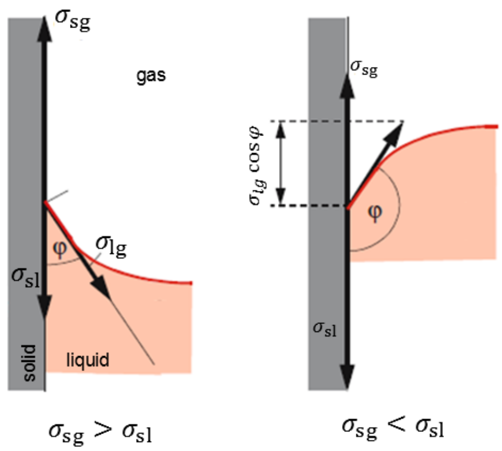
Now we consider a more complex scenario by looking at a **liquid boundary on a surface**, as for example occurring for a droplet on a surface. Here we have to consider three different interfaces: the **liquid-gas** as well as the **solid-liquid** and the **gas-solid interface**.

In analogy to before we can define **specific surface energies** and **surface tensions $\boldsymbol{\sigma_{ik}}$**. The specific surface energies represent the energy penalties per area for an interface between phase *i* and *k*. The penalties cause corresponding surface tension forces in tangential direction along each interface. Each surface tension of the particular interface is trying to shrink its interface size. At the intersection point of all 3 interfaces, we have thus 3 forces that try to pull the intersection point along the different interface direction. The droplet is held onto the surface by adhesion forces normal to the surface. Along the surface the droplet edge is however mobile. The droplet edge will be in equilibrium when the surface tension components along the surface cancel each other. The net force per edge length along the surface is given by:

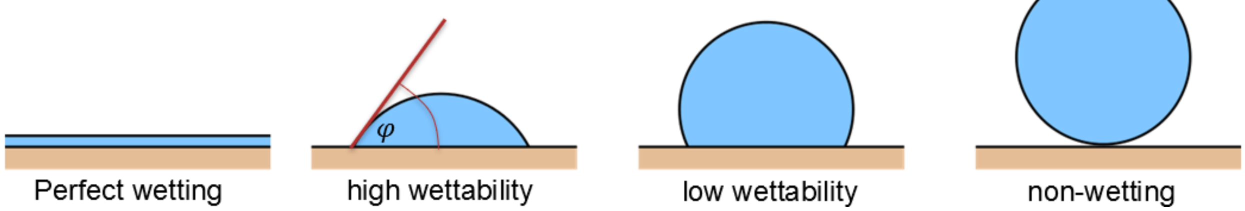
$$\sigma_{sg} - \sigma_{sl} - \sigma_{lg} \cos\varphi = 0$$

which is called the Young's equation for triple interfaces. Thus, the angle of the droplet surface, called the **contact angle** at the interface, is given by:

$$\cos\varphi = \frac{\sigma_{sg} - \sigma_{sl}}{\sigma_{lg}}$$



It is 90° ($\cos\varphi = 0$) for equal solid-liquid and solid-gas interface energies. The contact angle is smaller than 90° ($\cos\varphi > 0$) if the liquid-solid interaction is stronger than the gas-solid interaction ($\sigma_{sg} > \sigma_{sl}$ in this case since the surface tensions represent penalties). The contact angle is larger than 90° ($\cos\varphi < 0$) if the liquid-solid interaction is weaker than the gas-solid interaction ($\sigma_{sg} < \sigma_{sl}$). The contact angle determines whether a liquid is wetting, i.e. spreading on a surface, or whether it wants to remain a rather contracted droplet that avoids surface contact. We distinguish hereby:



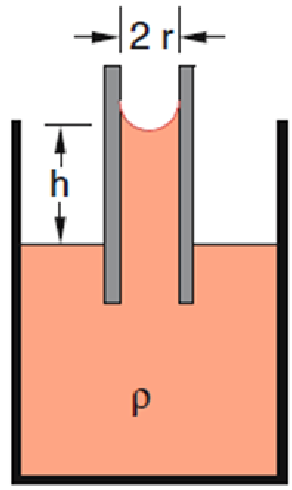
Contact angle φ	$\cos\varphi$	Degree of wetting	Strength of interaction Solid-liquid vs sol-gas	
$\varphi = 0°$		Perfect wetting	>>	$\sigma_{sg} \gg \sigma_{sl}$
$0 < \varphi < 90°$	> 0	high wettability	>	$\sigma_{sg} > \sigma_{sl}$
$90° < \varphi < 180°$	< 0	low wettability	<	$\sigma_{sg} < \sigma_{sl}$
$\varphi = 180°$		non-wetting	<<	$\sigma_{sg} \ll \sigma_{sl}$

An extreme example of a non-wetting surface is the Lotus effect, where the droplet is practically not at all interacting with the surface due to the unique nanoscopic structure of the surface.

C) Capillary effect

An important manifestation of interface tension is the capillary effect, where liquid is either pulled inside a small capillary or pushed out of a small capillary.

The capillary effect is driven by the energy gain (or penalty) upon the wetting of the capillary surface by the liquid. If wetting is beneficial, the liquid rises within the capillary until other forces like the weight of the fluid counterbalances the effective surface tension. Let us better understand this by calculating the height at which fluid from a reservoir rises within a vertical capillary against the gravity.



For a given contact angle, the contact areas that are changing upon capillary wetting are the areas of the capillary surface covered by the fluid and gas phase.

For an increase dh of the height h of the fluid column in the capillary of radius r the change in surface energy is thus given as:

$$dE_{surf} = \underbrace{2\pi r dh}_{dA} \, (\sigma_{sl} - \sigma_{sg})$$

The corresponding surface tension force that pulls the liquid up (or down) at the circular contact line of the fluid with the capillary is then given as:

$$F_{surf} = -\frac{dE_{surf}}{dh} = 2\pi r\left(\sigma_{sg} - \sigma_{sl}\right) = 2\pi r \, \sigma_{lg} \cos\varphi$$

The surface tension is counteracted by the weight of the extruded fluid column given by:

$$F_g = -mg = -\rho \underbrace{h\pi r^2}_{V} g$$

The liquid stops rising in the capillary when the weight fully balances the surface tension force:

$$0 = F_{surf} + F_g$$

Inserting the force terms provides:

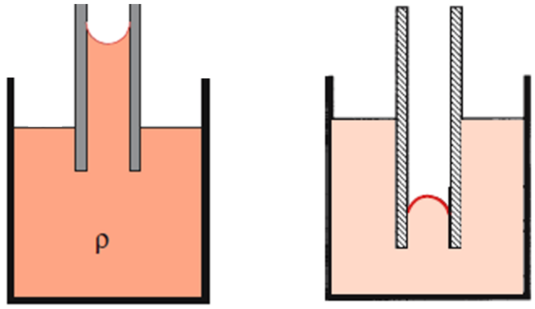
$$2\pi r \, \sigma_{lg} \cos\varphi = \rho h \, \pi r^2 g$$

and we get the following formula for the height of the fluid in the capillary:

$$h = \frac{2\sigma_{lg} \cos\varphi}{\rho r g}$$

We can again distinguish:

- **High wettability** with $\varphi < 90°$ and $\cos\varphi > 0$, such that $h > 0$. The **liquid is pulled up** inside the capillary
- **Low wettability** with $\varphi > 90°$ and $\cos\varphi < 0$, such that $h < 0$. The **liquid is pushed out** of the capillary



High wettability
$\varphi < 90°$

Low wettability
$\varphi > 90°$

In both cases the height change increases for decreasing capillary radius, which can be shown experimentally.

Experiments:

- Picture of receding liquid in a capillary using mercury (Hg) within acrylic capillaries, which provides low wettability
- Capillary effect of in glass capillaries connected to water. The high wettability provides a pull-up of the liquid in the capillaries.
- Capillary effect between two glass plates that form a wedge-like cleft with a small opening angle connected to a water reservoir. The liquid rises more in the narrow sections compared to the wider sections oft he cleft. The resulting curve shape is a hyperbola, since the height of the liquid level is inversely proportional to the plate distance.

Lecture 20: Experiments

1) Compressibility of water and air
2) Pressure acts equally in all directions: A syringe filled with water drives the water out of a glass bulb with many holes. We see that the water leaves the bulb equally in all directions
3) Pressure acts equally in all directions: Despite the gravity force acting in just one direction on the fluid, the pressure from the fluid weight also acts equally in all directions as demonstrated by a rotatable pressure sensor at the bottom of a fluid column.
4) Communicating vessels with a reduced pressure in one of the vessels
5) Hydrostatic paradox using differently shaped but connected glass vessels
6) Modell for the pressure inside a compressible medium made by repelling magnets put on a vertically oriented rod. We have a larger compression in the lower compared to the upper layers of the magnets.
7) Barometric formula: Pressure reduction between ground and sealing of the lecture hall using a pressure sensor.
8) Buoyancy balance: Measurement of the buoyant force as well as the weight of the displaced fluid (that left the trough) using a balance von
9) Buoyant force, Archimedes cylinder, comparison of buoyancy and weight of the displaced water
10) Dasymeter is a device to demonstrate the buoyant effect of gases. It can be build from beam balance connected to two objects of equal weight but different density and thus different volume. Evacuating the chamber around the balance provides imbalance due to the different buoyancy in the air atmosphere before
11) Picture of receding liquid in a capillary using mercury (Hg) within acrylic capillaries, which provides low wettability
12) Capillary effect of in glass capillaries connected to water. The high wettability provides a pull-up of the liquid in the capillaries.
13) Capillary effect between two glass plates that form a wedge-like cleft with a small opening angle connected to a water reservoir. The liquid rises more in the narrow sections compared to the wider sections of the cleft. The resulting curve shape is a hyperbola, since the height of the liquid level is inversely proportional to the plate distance.