A Unified Spatiotemporal Operator Model for Natural Language: Formalizing Verbs, Adverbs, and Copular Structures in a Cognitive Field Framework

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Abstract

This paper presents a mathematically grounded framework for natural language semantics based on spatiotemporal cognitive fields. Nouns are modeled as smooth scalar fields $S(x, t)$, verbs as differential operators acting on these fields, and adverbs as integration-based restrictors over spatial, temporal, or spatiotemporal domains. This extended formulation unifies dynamic meaning, aspect, and local perspectival anchoring without relying on inverse or reciprocal operators, thus avoiding singularities or asymptotic behavior. The model provides a precise account of Spanish constructions, including *ser*/*estar*, adverbial modification, and manner expressions, while remaining compatible with computational implementations.

1 Introduction

Meaning in natural language is inherently dynamic, reflecting changes in space, time, attention, and cognitive perspective. Existing NLP approaches—symbolic or embedding-based—do not explicitly encode these structural relations.

This paper formalizes language in a continuous spatiotemporal cognitive manifold where:

- nouns are scalar fields,
- verbs are differential operators,
- adverbs are restriction integrals,

- copular distinctions arise from temporal integration scale.

The framework integrates theoretical insights from your earlier manuscript with newly formalized mathematical operators that avoid singularities and preserve numerical stability.

2 Cognitive Spatiotemporal Fields

Let (x, t) denote compressed spatial and temporal dimensions—or cognitive coordinates. Each noun corresponds to:

$$S : \mathbb{R}^2 \to \mathbb{R}, \qquad (x, t) \mapsto S(x, t).$$

An **observer** selects a local window:

$$W_r = [r_0 - \varepsilon_r,\, r_0 + \varepsilon_r], \qquad W_t = [t_0 - \varepsilon_t,\, t_0 + \varepsilon_t].$$

These windows give meaning to adverbs like *aquí* and *ahora*.

3 Verb Operators

3.1 Quadratic Dynamic Operator

Measures raw dynamism:

$$v_{\text{quad}} = \sqrt{(\partial_t S)^2 + \|\nabla S\|^2}.$$

Captures motion or active processes. **Example**: *Juan corre*.

3.2 Fractional Contextual Operator

Normalizes change by noun magnitude:

$$v_{\text{frac}} = \frac{\partial_t S + \|\nabla S\|}{S}.$$

Example: *Juan está cansado*—change relative to a stable subject.

3.3 Transformational Operator

$$v_{\text{trans}} = \frac{\partial_t S \cdot \|\nabla S\|}{S}.$$

Models structural transformation. *El hielo se derrite.*

3.4 Mixed Second-Order Operator

$$v_{\text{mix}} = \frac{\partial^2 S}{\partial x\, \partial t}.$$

This operator underlies all adverbial modification.

4 Adverbs as Integration-Based Restrictors

Unlike inverse operators (removed here), adverbs are defined as **bounded integrals** over derivatives of S. This avoids singularities and yields stable semantics.

4.1 Spatial Adverbs (Place)

Spatial adverbs restrict meaning to a spatial interval:

$$A_{\text{place}}(S) = \int_{x_1}^{x_2} \frac{\partial^2 S}{\partial x^2}\, dx.$$

Linguistic interpretation:

- *aquí* $\to$ window centered at observer: $[r_0 - \varepsilon_r, r_0 + \varepsilon_r]$.
- *allí* $\to$ remote spatial window.

Example (Spanish):

"Juan está aquí."

Meaning = evaluation of S under local spatial curvature.

4.2 Temporal Adverbs (Time)

Temporal adverbs restrict meaning to a temporal interval:

$$A_{\text{time}}(S) = \int_{t_1}^{t_2} \frac{\partial^2 S}{\partial t^2}\, dt.$$

Interpretation: Measures temporal acceleration of the noun-state. Small window = *ahora*; extended = *hoy*, *este año*.

Example: "Juan está cansado ahora."

4.3 Spatiotemporal Adverbs (Manner)

Adverbs of manner or scene restrict both dimensions:

$$A_{\text{mode}}(S) = \int_{x_1}^{x_2} \int_{t_1}^{t_2} \frac{\partial^2 S}{\partial x \, \partial t} \, dt \, dx.$$

Interpretation: Captures interaction of space/time variation:

- *lentamente* $\rightarrow$ low mixed curvature,
- *rápidamente* $\rightarrow$ high curvature,
- *así* $\rightarrow$ fixed spatiotemporal pattern.

Example:

"Juan habla lentamente."

5 Ser vs Estar as Temporal Integration Windows

***Ser* = Long-Term Integration**

$$v_{\text{ser}} = \int_{T_1}^{T_2} S(x,t) \, dt \quad \text{with } T_2 - T_1 \text{ large.}$$

***Estar* = Short-Term Integration**

$$v_{\text{estar}} = \int_{t_0-\epsilon}^{t_0+\epsilon} S(x,t) \, dt.$$

This unifies the copular distinction without special lexical rules.

6 Illustration

7 Discussion

Removing inverse operators eliminates asymptotes and ensures numerical stability. The remaining operators suffice to model:

- place,
- time,

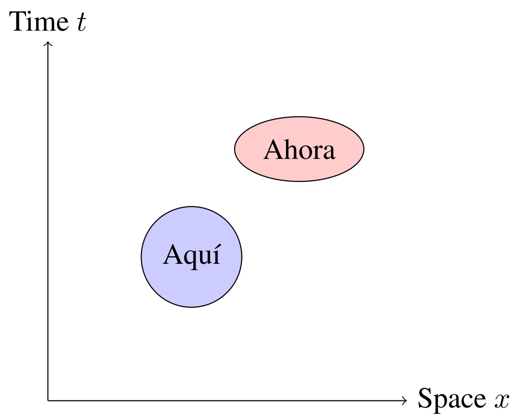

Figure 1: Local integration windows for place and time adverbs.

- manner,
- inside-scene constraints,
- copular distinctions.

This provides a robust foundation for future computational implementations.

8 Conclusion

We presented a refined operator framework for natural language semantics based on spatiotemporal cognitive fields. The model formalizes verbs and adverbs using differential and integral operators without resorting to reciprocal or inverse structures. This delivers a stable, compositional semantics that aligns naturally with Spanish grammar and general language universals.

9 Worked Example: The Intransitive Verb *correr / to run*

In this section, we illustrate the operator-based semantics using an intransitive verb: *correr* ('to run'). In all six examples below, *Juan* is simultaneously: (i) the **subject** whose cognitive field is derived, (ii) embedded inside the spatiotemporal integrals, (iii) and the **observer** in the Minkowski sense: his worldline defines the local reference frame in which the event is evaluated.

We denote Juan's noun-field by $S_{\text{Juan}}(x, t)$.

9.1 Case 1: *Juan.*

Examples: Spanish: *Juan.*
English: *Juan.*
Japanese:

The bare noun phrase corresponds to the field:

$$S_{\text{Juan}} : \mathbb{R}^2 \to \mathbb{R}.$$

No operator is applied; Juan is represented as a stable cognitive entity whose worldline $\gamma_{\text{Juan}}(t)$ anchors the local Minkowski frame.

9.2 Case 2: *Juan corre. / Juan runs.*

Examples: Spanish: *Juan corre.*
English: *Juan runs.*
Japanese:

Running is modeled by the quadratic dynamic operator:

$$v_{\text{run}}(x,t) = \sqrt{(\partial_t S_{\text{Juan}})^2 + \|\nabla_x S_{\text{Juan}}\|^2}.$$

Scene-level interpretation:

$$V_{\text{run}}(D) = \iint_D v_{\text{run}}(x,t)\, dx\, dt.$$

Juan remains both the differentiated field and the observer whose motion is measured.

9.3 Case 3: *Juan corre aquí. / Juan runs here.*

Examples: Spanish: *Juan corre aquí.*
English: *Juan runs here.*
Japanese:

"Aquí / here / " introduces a spatial restriction:

$$W_x = [x_0 - \varepsilon_x,\ x_0 + \varepsilon_x].$$

$$V_{\text{run,here}}(t) = \int_{W_x} v_{\text{run}}(x,t)\, dx.$$

The event is evaluated only within the spatial window of the observer.

9.4 Case 4: *Juan corre ahora. / Juan runs now.*

Examples: Spanish: *Juan corre ahora.*
English: *Juan runs now.*
Japanese:

"Ahora / now / " restricts the temporal window:

$$W_t = [t_0 - \varepsilon_t,\ t_0 + \varepsilon_t].$$

$$V_{\text{run,now}}(x) = \int_{W_t} v_{\text{run}}(x,t)\,dt.$$

The running is limited to the observer's cognitive present.

9.5 Case 5: *Juan corre rápidamente. / Juan runs quickly.*

Examples: Spanish: *Juan corre rápidamente.*
English: *Juan runs quickly.*
Japanese:

The adverb introduces increased dynamic intensity. We use either:

$$v_{\text{run,fast}} = \alpha_{\text{fast}}\, v_{\text{run}}, \quad \alpha_{\text{fast}} > 1,$$

or a shortened time window:

$$W_t^{\text{fast}} = [t_0 - \varepsilon_t^{\text{fast}}, t_0 + \varepsilon_t^{\text{fast}}], \quad \varepsilon_t^{\text{fast}} < \varepsilon_t.$$

$$V_{\text{run,fast}} = \int_{W_t^{\text{fast}}} \int_{W_x} v_{\text{run,fast}}(x,t)\,dx\,dt.$$

This models a steeper worldline slope in Juan's Minkowski trajectory.

9.6 Case 6: *Juan corre lentamente. / Juan runs slowly.*

Examples: Spanish: *Juan corre lentamente.*
English: *Juan runs slowly.*
Japanese:

Slow manner is modeled by attenuation:

$$v_{\text{run,slow}} = \alpha_{\text{slow}}\, v_{\text{run}}, \quad 0 < \alpha_{\text{slow}} < 1,$$

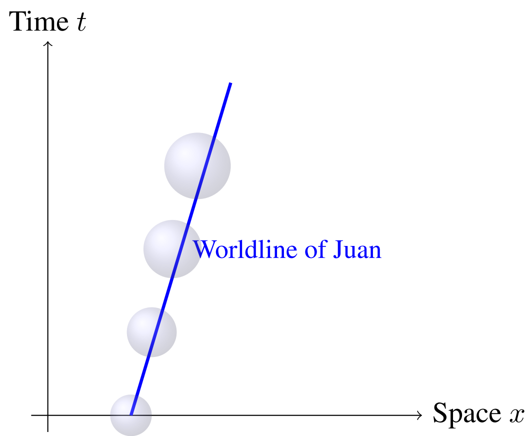

Figure 2: Juan as both the noun-field $S_{\text{Juan}}(x,t)$ and the Minkowski observer defining the local frame.

or by an expanded temporal window:

$$W_t^{\text{slow}} = [t_0 - \varepsilon_t^{\text{slow}},\, t_0 + \varepsilon_t^{\text{slow}}], \quad \varepsilon_t^{\text{slow}} > \varepsilon_t.$$

$$V_{\text{run,slow}} = \int_{W_t^{\text{slow}}} \int_{W_x} v_{\text{run,slow}}(x,t)\, dx\, dt.$$

This corresponds to a gentler slope in Juan's worldline within the Minkowski diagram.

Across all six cases, *Juan* consistently remains: (i) the subject represented by the field S_{Juan}, (ii) the entity being differentiated inside the operators, and (iii) the observer whose worldline defines the local Minkowski frame in which the event *correr* is interpreted.