Formal Mathematical Structure of the Neuronova Cognitive Hyperspace with SU(2)–Bloch Spinorial Layer

Nizar El Imrani

Contents

1 Temporal Axis

Definition 1 (Temporal Axis)**.** *The temporal axis is defined as:*

$$T = \mathbb{R},$$

with elements $t \in T$.

We decompose:

$$T^+ = \{t > 0\}, \qquad T^- = \{t < 0\}, \qquad T^0 = \{0\}.$$

2 Spatial Axis with Negative Region

Definition 2 (Physical Space)**.** *The spatial component is*

$$S = \mathbb{R}^3.$$

We define:

$$S^+ = \{(x, y, z) \in \mathbb{R}^3 : x \geq 0, y \geq 0, z \geq 0\}, \qquad S^- = \{(x, y, z) \in \mathbb{R}^3 : \text{some coordinate negative}\}.$$

3 Scenario Axis E_sEs

Definition 3 (Scenario Axis)**.**

$$E = \mathbb{R}.$$

Definition 4 (Scenario Function)**.**

$$E_s : T \times S \to E, \qquad (t, x) \mapsto E_s(t, x).$$

Decomposition:

$$E^+ = \{e > 0\}, \quad E^- = \{e < 0\}, \quad E^0 = \{0\}.$$

4 Scenes as Intervals of E_sEs

Definition 5 (Scene)**.** *Given $I = [t_0, t_1]$ and $D \subset S$,*

$$\mathcal{S} = \{(t, x, E_s(t, x)) : t \in I, \ x \in D\} \subset T \times S \times E.$$

5 The Cognitive Hyperspace

Definition 6 (Cognitive Hyperspace)**.**

$$\mathcal{H} = T \times S \times E = \mathbb{R}_t \times \mathbb{R}^3 \times \mathbb{R}_{E_s}.$$

This induces the sign–decomposed regions $\mathcal{H}^{\pm\pm\pm}$.

6 Observer Regions

6.1 External Region (Hilbert Representation)

Definition 7 (External Region)**.** *Let $\mathcal{K}$ be a Hilbert space. Define:*

$$\mathcal{R}_{\text{ext}} = \mathcal{H} \times \mathcal{K}.$$

6.2 Internal Region (Minkowski Fiber)

Definition 8 (Minkowski Fiber Over a Scene)**.**

$$\mathcal{M}_{\mathcal{S}} = I \times \mathbb{R}^3,$$

with Minkowski metric

$$g = -dt^2 + dx^2 + dy^2 + dz^2.$$

Definition 9 (Internal Region)**.**

$$\mathcal{R}_{\text{int}}(\mathcal{S}) = \mathcal{S} \times \mathcal{M}_{\mathcal{S}}.$$

6.3 Transition Region

Definition 10 (Transition Region)**.** *A region* $\mathcal{R}_{\text{trans}}$ *such that:*

$$\Pi(\mathcal{R}_{\text{trans}}) = \mathcal{S}, \qquad \exists\ \iota : \mathcal{S} \hookrightarrow \mathcal{M}_{\mathcal{S}}.$$

7 The SU(2)–Bloch Spinorial Fiber over the Scenario Axis

We now extend the hyperspace by introducing the full spinorial layer (SU(2) fiber bundle) required for Neuronova's semantic dynamics.

7.1 Spinorial Fiber Attached to E_sEs

Definition 11 (Spinorial Fiber)**.** *To each point* $e \in E$ *we attach a two–dimensional complex Hilbert space*

$$\mathbb{C}^2_e,$$

with normalized states

$$|\psi\rangle_e = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \qquad |\alpha|^2 + |\beta|^2 = 1.$$

Thus the complete hyperspace becomes a fiber bundle:

$$\mathcal{H}_{\text{spin}} = \mathcal{H} \times \mathbb{C}^2.$$

Remark 1. *The coordinate* E_s *acts as the base variable of the fiber:*

$$(e, |\psi\rangle) \in E \times \mathbb{C}^2.$$

7.2 Pauli Matrices and Bloch Projection

The Pauli matrices are:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Definition 12 (Bloch Vector)**.** *For a spinor* $|\psi\rangle$ *define*

$$n_k = \langle\psi|\, \sigma_k \,|\psi\rangle\,, \qquad k \in \{x, y, z\}.$$

Then

$$\mathbf{n} = (n_x, n_y, n_z) \in S^2.$$

Thus each E_s–point carries an entire Bloch sphere.

7.3 Semantic Interpretation of Bloch Axes

Mathematically:

- n_z measures projection onto σ_z eigenstates ("truth–axis").
- n_x measures projection onto σ_x (polarity).
- n_y measures projection onto σ_y (semantic phase).

This yields:

$$\text{Factuality} \leftrightarrow n_z, \qquad \text{Polarity} \leftrightarrow n_x, \qquad \text{Phase / nuance} \leftrightarrow n_y.$$

7.4 Spinorial Dynamics: Geometric Hamiltonian

We define a smooth time–dependent Hamiltonian:

$$H_{\text{geom}}(\tau) = \Omega_x(\tau)\sigma_x + \Omega_y(\tau)\sigma_y + \Omega_z(\tau)\sigma_z.$$

Definition 13 (Intrinsic Evolution)**.**

$$i\frac{d}{d\tau}|\psi(\tau)\rangle = H_{\text{geom}}(\tau)|\psi(\tau)\rangle.$$

This induces SU(2) motion on spinors and continuous trajectories on the Bloch sphere.

7.5 TSIM Synchronization Term

TSIM introduces a gradient flow:

$$\frac{d}{d\tau}|\psi\rangle = -iH_{\text{geom}}|\psi\rangle - \eta\frac{\partial E_{\text{TSIM}}}{\partial\langle\psi|}, \quad \eta > 0.$$

When the synchronization energy reaches threshold:

$$E_{\text{TSIM}}(\tau) < \varepsilon,$$

we get a **reversible collapse**:

$$|\psi(\tau)\rangle \longrightarrow |\psi_{\text{aligned}}\rangle,$$

where $|\psi_{\text{aligned}}\rangle$ is aligned with the observer's cone in Minkowski.

7.6 Lissajous and Polar Curves from Spinorial Motion

- Under H_{geom}, $|\psi(\tau)\rangle$ moves in SU(2) generating Lissajous-type curves.
- Under Bloch projection, $\mathbf{n}(\tau)$ traces a smooth curve on S^2.
- Under TSIM collapse, this curve aligns with a polar direction determined by observer metrics.

7.7 Bloch–Spinor Bundle Structure

Definition 14 (Spinorial–Bloch Bundle)**.** *Define*

$$\pi : \mathcal{H}_{\text{spin}} \to \mathcal{H}, \qquad (t, x, e, |\psi\rangle) \mapsto (t, x, e).$$

The fiber is:

$$\pi^{-1}(t, x, e) \simeq \mathbb{C}^2.$$

The Bloch sphere is the projectivization:

$$\mathcal{B}_e := \mathbb{C}^2/\mathrm{U}(1) \simeq S^2.$$

Remark 2. *The full geometry is now:*

$$\mathcal{H}_{\text{full}} = \mathcal{H} \oplus \mathcal{B} \oplus \mathcal{K},$$

with:

- *Minkowski fiber: internal observer dynamics,*
- *Hilbert fiber: external representation of all possibilities,*
- *Bloch fiber: semantic micro–state.*

Summary of All Regions with Spinorial Additions

- Base axes:
$$T,\ S,\ E_s.$$
- Regions of sign:
$$T^{\pm},\ S^{\pm},\ E^{\pm}.$$
- Scenes:
$$\mathcal{S} \subset \mathcal{H}.$$
- Observer structures:
$$\mathcal{R}_{\text{ext}},\ \mathcal{R}_{\text{int}},\ \mathcal{R}_{\text{trans}}.$$
- Spinorial–Bloch layer:
  - Spinors $|\psi\rangle \in \mathbb{C}^2$,
  - Pauli matrices, Bloch projection S^2,
  - Geometric Hamiltonian H_{geom},
  - TSIM synchronization term,
  - Lissajous $\to$ collapse $\to$ polar curve.