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Section:- 6A

Assignment 3

Ex 3.2

Question 1:-

The population of certain community is known to increase at a rate propotional to number of people present at any time. If population has doubled in 5 years how long will it take to tripple? to Quadriple?

Solution:-

Let $P(t)$ be the amount of population in certain community

$$\left.\begin{aligned} \frac{dP}{dt} &= KP \\ P(0) &= P_0 \end{aligned}\right] \to \text{Initial Value Problem}$$

↳ Present any time so Initial time zero

$$P(5) = 2P_0 \quad] \to \text{Additional condition.}$$

When solve I.V.P it gives:-

$$P = P_0 e^{Kt} \quad - (1)$$

Now finding K:-

$\underbrace{P(5)}$ = $2P_0$

↓

replace it with eq (1)

$$\cancel{P_0} e^{Kt} = 2\cancel{P_0}$$

Taking ln on both sides

$$\cancel{\ln}|\cancel{e}^{Kt}| = \ln|2|$$

$$Kt = \ln|2|$$

$$K = \frac{\ln|2|}{t}$$ As time is 5 so

$$K = \frac{\ln|2|}{5}$$

$$K = 0.1386$$

Puting value of K in eq (1)

$$P = P_0 e^{0.1386t} \quad - (2)$$

i> How Long will it take to tripple?

$\underbrace{P(t)}$ = $3P_0$

↓

replace with eq (2)

$$\cancel{P_0} e^{0.1386t} = 3\cancel{P_0}$$

$$t = \frac{\ln|3|}{0.1386} \Rightarrow \boxed{t = 7.93 \text{ years}}$$

-> How Long will it take to Quadriple?

$$\underbrace{P(t)}_{\downarrow} = 4P_0$$

replace it with eq (2)

$$\cancel{P_0} e^{0.1386t} = 4\cancel{P_0}$$

$$t = \frac{\ln|4|}{0.1386}$$

$$t = 10.002$$

$$\boxed{t = 10 \text{ years.}}$$

Question 3:-

The population of town Grows at rate propotional to the population at any time. It's initial Population of 500 increases by 15% in 10 years. What will be population in 30 years?

Solution:-

Let P(t) be the population of town

$$\left.\begin{aligned} \frac{dP}{dt} &= KP \\ P(0) &= P_0 = 500 \end{aligned}\right] \to \text{Initial Value Problem}$$

Initial Population increases by 15% of 500

$$P(10) = 500 + 75 = 575$$

$$P(10) = 575 \,] \to \text{Additional condition.}$$

When solve I.V.P it gives

$$P = ce^{kt} \quad \text{— (1)}$$

Now Finding 'c'

$P = ce^{kt}$, $P(0) = 500$

$500 = ce^{k(0)}$

$500 = ce^{0}$

$c = 500$

Now Finding 'k'

$P = ce^{kt}$, $P(10) = 575$

$575 = 500\, e^{k(10)}$

$e^{10k} = \frac{575}{500} = 1.15$

$k = \frac{\ln|1.15|}{10}$

$k = 0.0139$

Now putting in eq ①

$P = 500\, e^{0.0139t}$ — ②

=> What will be population in 30 years?

$P(30) = 500\, e^{0.0139t}$ → As $t = 30$

$P(30) = 500\, e^{(0.0139)(30)}$

$P(30) = 760.43$

$\boxed{P(30) = 760}$

Question 5:-

The radioactive isotope of Lead 209 decays at a rate propotional to amount present at any time and has half Life of 3.3 hours. If 1 Gram of Lead present initially, how Long will it take for 90% of Lead to decay?

Solution:-

Let $A(t)$ be the radioactive isotope of Lead Pb-209

$$\left.\begin{aligned} \frac{dA}{dt} &= KA \\ A(0) &= A_0 = 1 \end{aligned}\right] \rightarrow \text{Initial Value Problem}$$

Half Life of 3.3 hours

$$A(t) = \frac{1}{2} A_0$$

$$A(3.3) = \frac{1}{2}(1) = 0.5$$

$$A(3.3) = 0.5 \;] \rightarrow \text{Additional condition.}$$

When solve initial value Problem it Gives:-

$$A = ce^{Kt} \quad \text{—— (1)}$$

Now finding 'c':-

$$1 = ce^{K(0)}$$

$$c = 1$$

Now finding 'K'

$$0.5 = 1e^{K(3.3)}$$

$$K = \frac{\ln|0.5|}{3.3}$$

$$K = -0.21$$

Put in eq ①

$$A = 1e^{-0.21t} \quad \text{— ②}$$

⇒ How long will it take to 90% decay

$$t = ? \quad , \quad A = 1 - \frac{90}{100} = 0.1$$

From eq ②

$$A = 1e^{-0.21t}$$

$$0.1 = e^{-0.21t}$$

$$t = \frac{\ln|0.1|}{-0.21}$$

$$t = 10.96$$

$$\boxed{t = 11 \text{ years}}$$

Question 6:-

Initially there were 100 miligram of radioactive substance present. After 6 years the mass decreased by 3%. If the rate of decay is propotional to amount of substance present at any time find the amount remaining after 24 hours?

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(4)

Solution:-

Let $A(t)$ be the amount of substance present at any time

$$\left.\begin{aligned} \frac{dA}{dt} &= KA \\ A(0) &= A_0 = 100mg \end{aligned}\right\} \rightarrow \text{Initial Value Problem}$$

$$A(6) = 100mg - \frac{100(3)}{100} = 97mg$$

$A(6) = 97mg$] → Additional condition.

When solves the I.V.P it gives:-

$$A = ce^{Kt} \quad - (1)$$

Finding 'c' :-

$$100 = ce^{K(0)}$$

$$c = 100$$

Finding 'K' :-

$$97 = ce^{K(6)}$$

$$K = \frac{\ln|97|}{6}$$

$$K = -5.076 \times 10^{-3}$$

Put 'c' and 'K' in eq (1)

$$A = 100e^{-5.076 \times 10^{-3} t} \quad - (2)$$

⇒ Now Finding time remain after 24 hours:-

$$A(24) = 100\, e^{-5.07\times10^{-3}(24)}$$

$$\boxed{A(24) = 88.52\, mg.}$$

Question 7:-

Determine the half Life of radioactive substa[nce] described in Problem 6?

Solution:-

As we know

$$A = A_0 = 100\, mg$$

Half life

$$\cancel{A(t)}\ A(t) = \frac{1}{2} A_0$$

$$A(t) = \frac{1}{2}(100) = 50$$

so,

$$A = ce^{kt}$$

where $c = 100$, $k = -5.076\times10^{-3}$

$$50 = 100\, e^{-5.076\times10^{-3}(t)}$$

$$t = \frac{\ln|0.5|}{-5.076\times10^{-3}}$$

$$\boxed{t = 136.5 \text{ hours.}}$$

Question 11:-

In a piece of wood burned or charcoal, it was found that 85.5% of the C-14 had decayed use the information in Example3 to determine age of wood?

Solution:-

Let A be the amount of wood so,

$$\frac{dA}{dt} = KA$$

and

$$A = ce^{Kt} \quad — (1)$$

Information from Example3:-

$$A(5600) = \frac{A_0}{2}$$

or

$$A_0 e^{5600K} = \frac{A_0}{2}$$

As we know that,

$$A = A_0 e^{Kt} \quad — (2)$$

$$\underbrace{A(5600)}_{\downarrow} = \frac{A_0}{2}$$

replace by eq (2)

$$\cancel{A_0} e^{5600K} = \frac{\cancel{A_0}}{2}$$

$$\ln|e^{5600K}| = \ln\left|\frac{1}{2}\right|$$

$$K = \frac{\ln|0.5|}{5600}$$

$$K = -1.23776 \times 10^{-4}$$

$$A = A_0 e^{-1.23776 \times 10^{-4}(t)} \quad \text{—— (3)}$$

Now we know that 85.5% of Decay

$$A = 100\% A_0 - 85.5\% A_0 = 14.5\% A_0$$

$$A = \frac{14.5}{100} A_0$$

$$A = 0.145 A_0$$

Put in eq (3)

$$0.145 \cancel{A_0} = \cancel{A_0} e^{-1.23776 \times 10^{-4}(t)}$$

$$t = \frac{\ln|0.145|}{-1.23776 \times 10^{-4}}$$

$$t = 15600.936$$

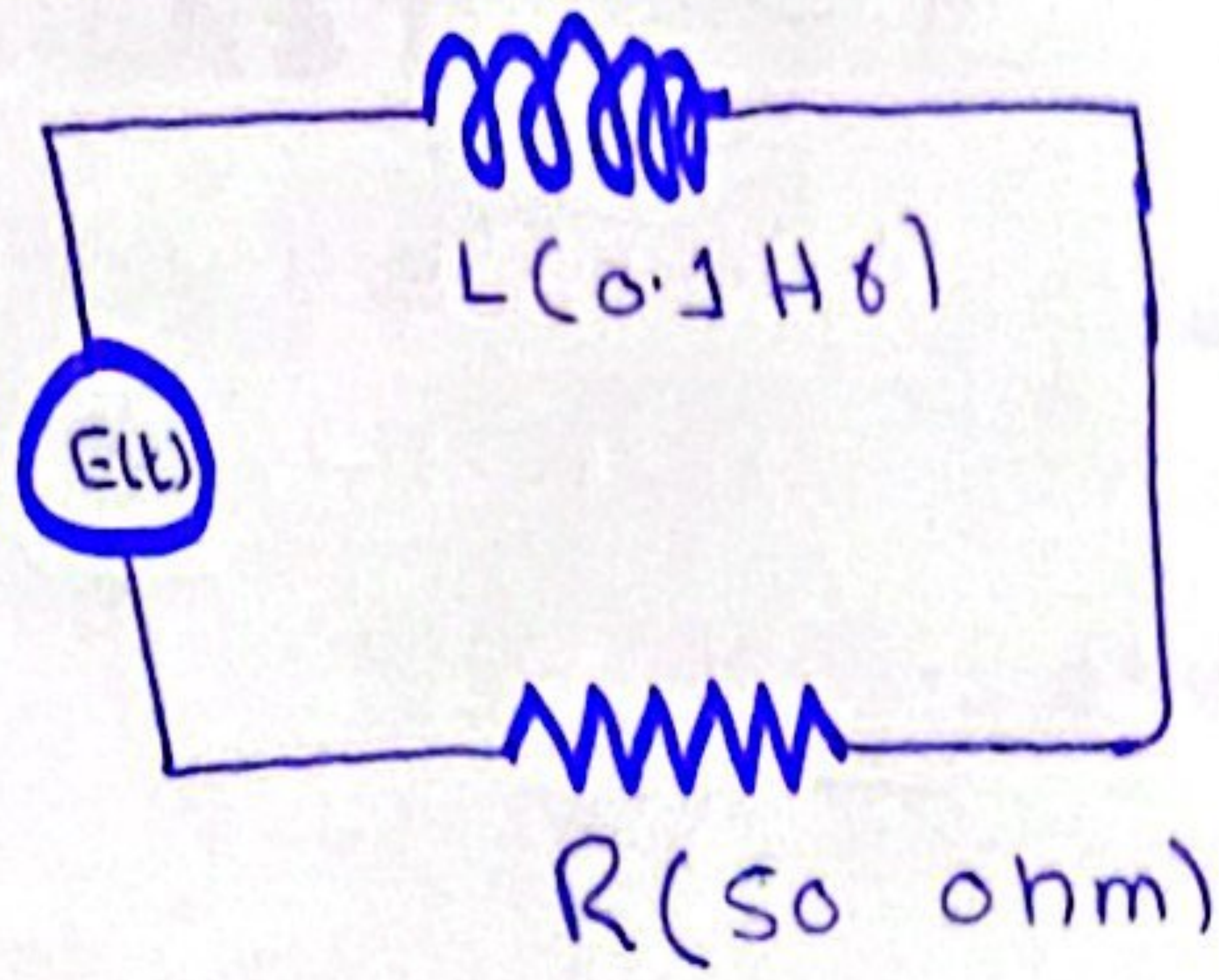
$$\boxed{t = 15600 \text{ years.}}$$

Question 15:-

A 30 volt electromotive force is applied to LR-Circuit in which the inductance is 0.1 Henry and resistance is 50 ohms. Find current $i(t)$ if $i(0) = 0$? Determine current as $t \to \infty$?

Solution:-

30 volt



Formula for LR circuit:-

$$L\frac{di}{dt} + R(i) = E(t) \quad \text{—— ①}$$

$$i(t_0) = i_0$$

Putting values in eq ①

$$0.1\frac{di}{dt} + 50\,i = 30$$

Multiply with $\frac{1}{0.1}$

$$\frac{di}{dt} + 500\,i = 300 \quad \text{— ②}$$

$\hookrightarrow \mu(t) = 500$

$$\text{I.Factor} \Rightarrow e^{\int \mu(t)dt} \Rightarrow e^{\int 500 dt} \Rightarrow e^{500t}$$

Multiply it with eq ②

$$e^{500t}\frac{di}{dt} + e^{500t} \cdot 500\,i = 300e^{500t}$$

$$\int \frac{d}{dt}(e^{500t} \cdot i)\cancel{d}t = 300\int e^{500t}\,dt$$

$$e^{500t} \cdot i = \frac{300}{500} e^{500t} + c$$

$$e^{500t} \cdot i = \frac{3}{5} e^{500t} + c$$

$$i = \frac{3}{5} + c e^{-500t} \quad \text{— (3)}$$

i) $i(0) = 0$

Put in eq (3)

$$0 = \frac{3}{5} + c e^{-500(0)}$$

$$c = -\frac{3}{5}$$

Put in eq (3)

$$\boxed{i(t) = \frac{3}{5} - \frac{3}{5} e^{-500t}} \quad \text{— (4)}$$

ii) $t \to \infty$

$t = \infty$

Put in eq (4)

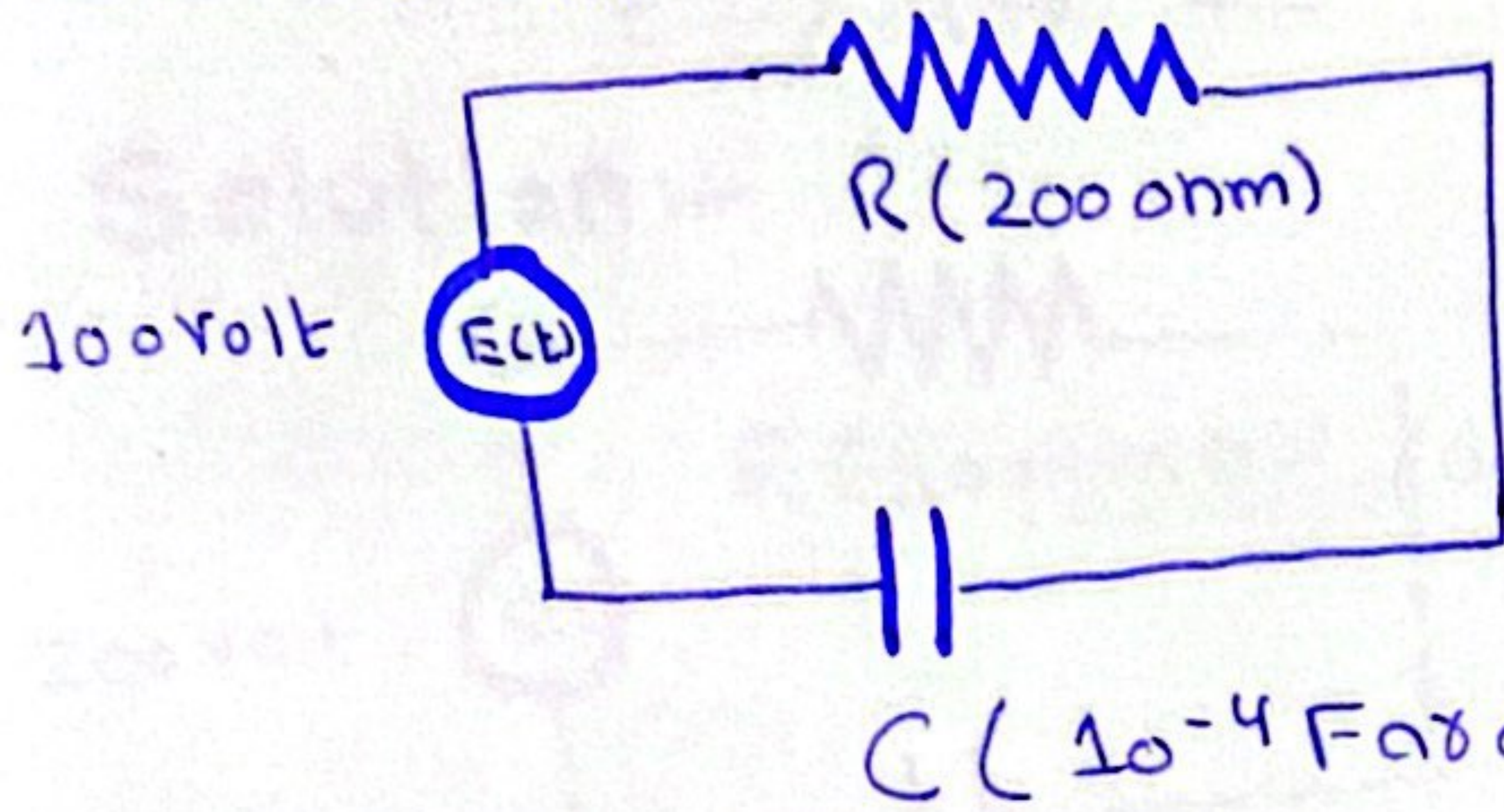
$$i(t) = \frac{3}{5} - 0$$

$$\boxed{i = \frac{3}{5}}$$

Question 17:-

A 100 volt electromotive force is applied to an RC series circuit in which the resistance is 200 ohms and capacitance is 10^{-4} farad. Find the charge $q(t)$ on capacitor if $q(0)=0$. Find current $i(t)$?

Solution:-



∴ RC circuit

Formula:-

$$R\frac{dq}{dt} + \frac{1}{C}q = E(t) \quad -(1)$$

$$q(t_0) = q_0$$

put values in eq (1)

$$200\frac{dq}{dt} + 10^4 q = 100$$

$$\frac{dq}{dt} + 50q = \frac{1}{2} \quad -(2)$$

$\hookrightarrow u(t) = 50$

$$\text{I. Factor} \Rightarrow e^{\int u(t)dt} \Rightarrow e^{\int 50 dt} \Rightarrow e^{50t}$$

Multiply to eq (2)

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$$e^{50t}\frac{dq}{dt} + e^{50t}50q = \frac{1}{2}e^{50t}$$

$$\int \frac{d}{dt}\left(e^{50t}\cdot q\right)dt = \frac{1}{2}\int e^{50t}$$

$$e^{50t}\cdot q = \frac{1}{100}e^{50t} + C$$

$$q = \frac{1}{100} + Ce^{-50t} \quad — (3)$$

i) $q(0) = 0$

$$0 = \frac{1}{100} + Ce^{-50(0)}$$

$$C = -\frac{1}{100}$$

Put in eq (3)

$$\boxed{q = \frac{1}{100} - \frac{1}{100}e^{-50t}}$$

ii) $i(t)$?

As we know $i(t) = \frac{dq}{dt}$

Put values of q

$$i(t) = \frac{d}{dt}\left(\frac{1}{100} - \frac{1}{100}e^{-50t}\right)$$

$$i(t) = \frac{d}{dt}(1/100) - \frac{d}{dt}\left(\frac{1}{100}e^{-50t}\right)$$

$$i(t) = 0 - \left(\frac{1}{100}e^{-50t}(-50)\right)$$

$$\boxed{i(t) = \frac{1}{2}e^{-50t}}$$

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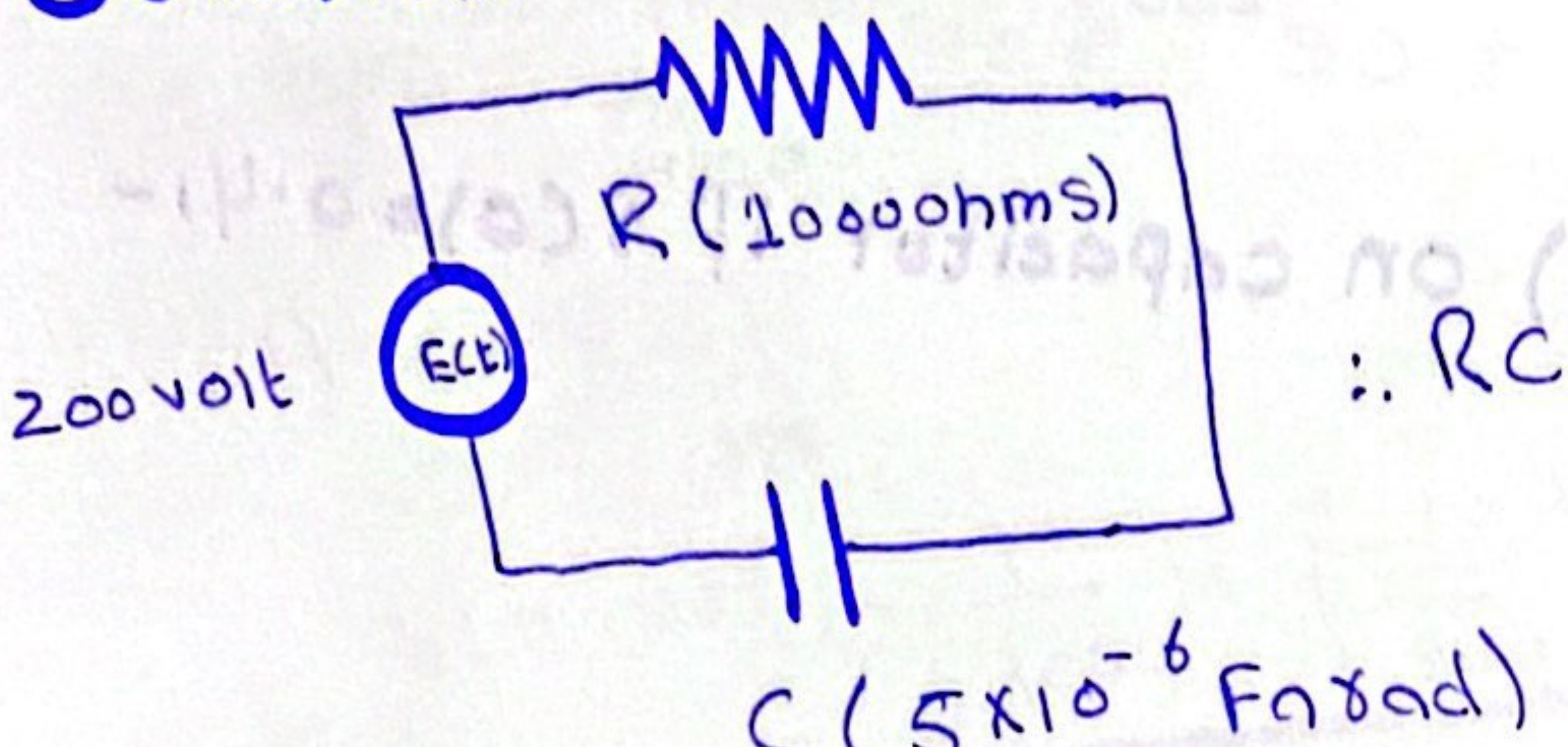
Question 18:-

A 200 volt electromotive force is applied to an RC-series circuit in which the resistance is 1000 ohms and the capacitance is 5×10^{-6} farad. Find charge $q(t)$ on capacitor if $i(0) = 0.4$. Determine charge and current at $t = 0.005$ second. Determine charge as $t \to \infty$?

Solution:-



∴ RC-circuit

Formula:-

$$R\frac{dq}{dt} + \frac{1}{C}q = E(t) \quad —① $$

$$q(t_0) = q_0$$

put values in eq ①

$$1000\frac{dq}{dt} + \frac{1}{(5\times10^{-6})}q = 200$$

$$\frac{dq}{dt} + 200q = \frac{1}{5} \quad —②$$

$\hookrightarrow u(t) = 200$

Integration Factor:- e^{200t}

Multiply with eq (2)

$$e^{200t}\frac{dq}{dt} + e^{200t}\,200q = \frac{1}{5}e^{200t}$$

$$\int \frac{d}{dt}\left(e^{200t}\cdot q\right)dt = \frac{1}{5}\int e^{200t}$$

$$e^{200t}\cdot q = \frac{1}{1000}e^{200t} + c$$

$$q = \frac{1}{1000} + ce^{-200t}$$

i) Find charge q(t) on capacitor if i(0) = 0.4 :-

As we know

$$i(t) = \frac{dq}{dt}$$

Put values of q

$$i(t) = \frac{d}{dt}\left(\frac{1}{1000} + ce^{-200t}\right)$$

$$i(t) = \frac{d}{dt}\left(\frac{1}{1000}\right) + \frac{d}{dt}\left(ce^{-200t}\right)$$

$$i(t) = 0 + \left(-200ce^{-200t}\right)$$

$$i(t) = -200ce^{-200t} \quad \text{--- (3)}$$

As $i(0) = 0.4$ put in above eq

$$0.4 = -200ce^{-200(0)}$$

$$c = \frac{0.4}{-200}$$

$$c = \frac{-1}{500}$$

$$\boxed{c = -2\times10^{-3}}$$

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ut value of 'c' in eq 3

$$i(t) = -200\left(\frac{-1}{500}\right)e^{-200t}$$

$$i(t) = \frac{2}{5}e^{-200t} \quad — 4$$

ii) Determine charge and current at t = 0.005 :-

Put t = 0.005 in eq

$$q(t) = \frac{1}{1000} + ce^{-200t}$$

As $c = \frac{-1}{500}$

$$q(0.005) = \frac{1}{1000} + \left(\frac{-1}{500}\right)e^{-200(0.005)}$$

$$\boxed{q(0.005) = 0.003 \text{ columbs.}}$$

put t = 0.005 in eq 4 we get

$$i(0.005) = \frac{2}{5}e^{-200(0.005)}$$

$$\boxed{i(0.005) = 0.1472 \text{ amperes.}}$$

iii) Determine charge as $t \to \infty$:-

$t = \infty$

put in eq

$$q(t) = \frac{1}{1000} + \left(\frac{-1}{500}\right)e^{-200(t)}$$

$$q(\infty) = \frac{1}{1000} + \left(\frac{-1}{500}\right)e^{-200(\infty)}$$

$$\boxed{q(\infty) = \frac{1}{1000}}$$