2022级模拟卷答案（下）

一：填空选择$(3'\times 10=30')$

⑴ 2；　⑵ $x^2-3y^2=1$；　⑶ A；　⑷ $2-2\ln 2$；

⑸ $\ln|x|+\dfrac{x}{y}=C$ 或 $y(\ln|x|+C)+x=0$；

⑹ $\int_0^1 dx\int_0^{x^2} f(x,y)\,dy+\int_1^{\sqrt{2}} dx\int_0^{\sqrt{2-x^2}} f(x,y)\,dy$；

⑺ $\sqrt{2}\pi$；　⑻ $2\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n}\sin nx$；　⑼ A；　⑽ $0<a<\dfrac{2}{3}$。

二：$\bar{n}_1=(1,0,2)$，$\bar{n}_2=(0,1,-3)$，

$\bar{s}=\bar{n}_1\times\bar{n}_2=(-2,3,1)$，

直线方程为：$\dfrac{x}{-2}=\dfrac{y-1}{3}=\dfrac{z-2}{1}$。

三：$\dfrac{\partial z}{\partial x}=yf_1+\dfrac{1}{y}f_2$，

$$\frac{\partial^2 z}{\partial x\partial y}=f_1+y(xf_{11}-\frac{x}{y^2}f_{12})-\frac{1}{y^2}f_2+\frac{1}{y}(xf_{21}-\frac{x}{y^2}f_{22})$$

$$=f_1+xyf_{11}-\frac{1}{y^2}f_2-\frac{x}{y^3}f_{22}$$

四：特征方程$r^2+3r+2=0$，特征值$r_1=-2;r_2=-1$，

齐次的通解　$Y=Ce^{-2x}+De^{-x}$，

$y''+3y'+2y=2$的特解$y_1=1$；

$y''+3y'+2y=xe^{-x}$的特解$y_2=x(Ax+B)e^{-x}$

代入方程得$A=\dfrac{1}{2},B=-1$，

通解为$y=Ce^{-2x}+De^{-x}+1+x(\dfrac{x}{2}-1)e^{-x}$。

五：(1) $L:\begin{cases} x=1+\cos t \\ y=\sin t \end{cases}(0\le t\le 2\pi)$

$$\int_L \sqrt{x^2+y^2}ds=\int_0^{2\pi}\sqrt{2(1+\cos t)}\sqrt{\cos^2 t+\sin^2 t}dt=8$$。

(2) $$\iiint\limits_{\Omega}(x^2+y^2)\,\mathrm{d}v=\iiint\limits_{\Omega_1}\rho^3\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z-\iiint\limits_{\Omega_2}\rho^3\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$=\int_0^{2\pi}d\varphi\int_0^4\rho^3d\rho\int_{\frac{\rho^2}{2}}^8 dz-\int_0^{2\pi}d\varphi\int_0^2\rho^3d\rho\int_{\frac{\rho^2}{2}}^2 dz=336\pi$$。

(3) 作辅助面 $z=1\quad(x^2+y^2\le 1)$ 取上侧，

$$\oiint\limits_{\Sigma+\Sigma_1}xy^2dydz+x^2ydzdx+zdxdy=\iiint\limits_{\Omega}(y^2+x^2+1)dv$$

$$=\int_0^{2\pi}d\varphi\int_0^1\rho d\rho\int_{\rho^2}^1(\rho^2+1)dz=\frac{2}{3}\pi,$$

$$\iint\limits_{\Sigma_1}xy^2dydz+x^2ydzdx+zdxdy=\iint\limits_{D}dxdy=\pi,$$

$$I=\iint\limits_{\Sigma+\Sigma_1}-\iint\limits_{\Sigma_{21}}=-\frac{\pi}{3}$$。

六：$f(x)=\dfrac{1}{1+x}+\dfrac{1}{1+2x}=\sum\limits_{n=0}^{\infty}(-1)^n(1+2^n)x^n;|x|<\dfrac{1}{2}$

七：$\because$ 平面 $\dfrac{x}{3}+\dfrac{y}{4}+\dfrac{z}{5}=1,\quad\therefore z=5(1-\dfrac{x}{3}-\dfrac{y}{4})$ 设 $F(x,y)=5(1-\dfrac{x}{3}-\dfrac{y}{4})+\lambda\left(x^2+y^2-1\right)$

由 $\begin{cases} F'_x=2\lambda x-\dfrac{5}{3}=0 \\ F'_y=2\lambda y-\dfrac{5}{4}=0 \end{cases}$ 得 $x=\dfrac{5}{6\lambda},\quad y=\dfrac{5}{8\lambda}$

上式代入 $x^2+y^2-1=0,\quad\dfrac{1}{\lambda}=\pm\dfrac{24}{25}$, 从而 $x=\pm\dfrac{4}{5},\quad y=\pm\dfrac{3}{5},\quad\therefore z_1=\dfrac{35}{12},\quad z_2=\dfrac{85}{12}$

所求点为 $\left(\dfrac{4}{5},\ \dfrac{3}{5},\ \dfrac{35}{12}\right)$。

八：由 $\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi \end{cases}$，得：$\dfrac{\partial z}{\partial\rho}=\dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial\rho}+\dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial\rho}=\dfrac{\partial f}{\partial x}\cos\varphi+\dfrac{\partial f}{\partial y}\sin\varphi$,

将上式两端同乘ρ，得到 $\rho\dfrac{\partial f}{\partial \rho}=\dfrac{\partial f}{\partial x}\rho\cos\varphi+\dfrac{\partial f}{\partial y}\rho\sin\varphi=xf'_x+yf'_y$。

于是有

$$I=\iint\limits_{D_\varepsilon}\frac{xf'_x+yf'_y}{x^2+y^2}dxdy=\iint\limits_{D_\varepsilon}\frac{1}{\rho^2}\rho\frac{\partial f}{\partial \rho}\rho d\rho d\varphi=\int_0^{2\pi}d\varphi\int_\varepsilon^1\frac{\partial f}{\partial \rho}d\rho=\int_0^{2\pi}f(\rho\cos\varphi,\rho\sin\varphi)\Big|_\varepsilon^1 d\varphi$$
$$=\int_0^{2\pi}f(\cos\varphi,\sin\varphi)d\varphi-\int_0^{2\pi}f(\varepsilon\cos\varphi,\varepsilon\sin\varphi)d\varphi=0-\int_0^{2\pi}f(\varepsilon\cos\varphi,\varepsilon\sin\varphi)d\varphi$$
$$=-\int_0^{2\pi}f(\varepsilon\cos\varphi,\varepsilon\sin\varphi)d\varphi$$

由积分中值定理，有

$I=-2\pi\cdot f(\varepsilon\cos\varphi_1,\varepsilon\sin\varphi_1)$，其中$0\le\varphi_1\le 2\pi$，

故$\lim\limits_{\varepsilon\to 0^+}\dfrac{-1}{2\pi}\iint\limits_{D}\dfrac{xf'_x+yf'_y}{x^2+y^2}dxdy=\lim\limits_{\varepsilon\to 0^+}f(\varepsilon\cos\varphi_1,\varepsilon\sin\varphi_1)=f(0,0)=-1$。