

基础部分 (共 80 分)**一、填空题 (每空 2 分, 共 20 分)**

1、(1) $a_t = \frac{d^2 s}{dt^2} = 2b_2$; (2) $a_{\text{总}} = \sqrt{\frac{(b_1 + 2b_2 t)^4}{R^2} + 4b_2^2}$; (3) $\alpha = \frac{a_t}{R} = \frac{b_1 + 2b_2 t}{R}$;

2、(4) $F = ma = 6t$; (5) $A = \left(3t^2 + \frac{9}{2}t^4\right)_0^1 = 7.5\text{J}$;

3、(6) $\alpha = \frac{M}{I} = \frac{3g}{2L}$; (7) $\omega = \sqrt{\omega_0^2 + \frac{3g}{L}}$; (8) $v_A = \sqrt{L^2 \omega_0^2 + 3gL}$;

4、(9) $a_{\text{max}} = \omega^2 A = (4\pi)^2 \times 0.02 = 16\pi^2 \times 0.02 = 0.32\pi^2 = 3.16\text{m/s}^2$;

(10) $x = 0.02 \cos\left(4\pi t + \frac{\pi}{3}\right)\text{m}$;

二、填空题 (每空 2 分, 共 20 分)

1、(1) 不守恒; (2) 相等;

2、(3) $1.12 \times 10^4\text{J}$; (4) $6.21 \times 10^{-21}\text{J}$; (5) $444 \sim 448\text{ m/s}$;

3、(6) $RT \ln 2$; (7) $\Delta E = C_{V,m} \Delta T = \frac{i}{2} R \Delta T = \frac{6}{2} \times R \times 2 = 6R$; (8) $Q = C_{p,m} \Delta T = \frac{i+2}{2} R \Delta T = 8R$;

4、(9) $\frac{5}{4}m_0$; (10) $\frac{4}{5}\pi R^2$;

三、计算题 (10 分) 解: (1) 角动量守恒: $m_2 v_0 l = (m_2 l^2 + \frac{1}{3} m_1 l^2) \omega_0$, $\omega = \frac{3m_2 v_0}{(3m_2 + m_1)l}$; (5 分)

(2) (一) 由转动定律 $M = I\beta$ 可得: $\beta = \frac{-M_f}{I} = \frac{M_f}{m_2 l^2 + \frac{1}{3} m_1 l^2} = \frac{3M_f}{(3m_2 + m_1)l^2}$

由匀角加速度转动公式有: $\omega = 0 = \omega_0 + \beta t$, $t = -\frac{\omega_0}{\beta} = \frac{m_2 v_0 l}{M_f}$ (5 分)

(二) 用动量矩定理: $Mt = I\omega - I\omega_0 = I\omega_0$, $t = -\frac{I\omega_0}{-M_f} = \frac{m_2 v_0 l}{M_f}$

四、计算题 (10 分) 解: (1) 设入射波的波动方程为: $y_1(x, t) = A \cos\left[\omega\left(t + \frac{x}{u}\right) + \varphi_0\right]$

由图知 $A = 5\text{ (m)}$, $\lambda = 4 \times 4 = 16\text{ (m)}$, 因此, $\omega = 2\pi \frac{u}{\lambda} = \frac{5\pi}{4}\text{ (rad/s)}$,

$$t=0 \text{ 时, } \begin{cases} y_0 = A \cos \varphi_0 = -\frac{A}{2}, \\ v_0 = -\omega A \sin \varphi_0 < 0 \end{cases} \quad \text{即} \quad \begin{cases} \cos \varphi_0 = -\frac{1}{2}, \\ \sin \varphi_0 > 0 \end{cases} \quad \text{共同定出} \quad \varphi_0 = \frac{2\pi}{3},$$

得入射波的波动方程: $y_1(x, t) = A \cos \left[\omega \left(t + \frac{x}{u} \right) + \varphi_0 \right] = 5 \cos \left[\frac{5\pi}{4} \left(t + \frac{x}{10} \right) + \frac{2\pi}{3} \right] \text{ (m)}$ (2 分)

$$(2) \quad t=0 \text{ 时, } \begin{cases} y_p = A \cos \left[\omega \left(0 + \frac{x_p}{u} \right) + \varphi_0 \right] = 0 \\ v_p = -\omega A \sin \left[\omega \left(0 + \frac{x_p}{u} \right) + \varphi_0 \right] > 0 \end{cases}, \quad \text{即} \quad \begin{cases} \cos \left[\frac{5\pi}{4} \left(0 + \frac{x_p}{10} \right) + \frac{2\pi}{3} \right] = 0 \\ \sin \left[\frac{5\pi}{4} \left(0 + \frac{x_p}{10} \right) + \frac{2\pi}{3} \right] < 0 \end{cases}$$

共同定出 $\varphi_p = \frac{5\pi}{4} \left(0 + \frac{x_p}{10} \right) + \frac{2\pi}{3} = \frac{3\pi}{2}$, 得 $x_p = \frac{20}{3} = 6.67 \text{ (m)}$ (2 分)

(3) 入射波在反射点 O 的振动方程: $y_{10}(t) = 5 \cos \left[\frac{5\pi}{4} t + \frac{2\pi}{3} \right],$

计入半波损失, 得反射波在反射点 O 的振动方程: $y_{20}(t) = 5 \cos \left[\frac{5\pi}{4} t + \frac{2\pi}{3} + \pi \right]$

由时间推迟法, 得反射波的波动方程:

$$y_2(x, t) = 5 \cos \left[\frac{5\pi}{4} \left(t - \frac{x}{10} \right) + \frac{2\pi}{3} + \pi \right] = -5 \cos \left[\frac{5\pi}{4} \left(t - \frac{x}{10} \right) + \frac{2\pi}{3} \right] \quad (2 \text{ 分})$$

$$(4) \quad y(x, t) = y_1(x, t) + y_2(x, t) = 5 \cos \left[\frac{5\pi}{4} \left(t + \frac{x}{10} \right) + \frac{2\pi}{3} \right] - 5 \cos \left[\frac{5\pi}{4} \left(t - \frac{x}{10} \right) + \frac{2\pi}{3} \right]$$

$$= -10 \sin \frac{\pi}{8} x \cdot \sin \left(\frac{5\pi}{4} t + \frac{2\pi}{3} \right)$$

波节点位置: $|\sin \frac{\pi}{8} x| = 0$, 则 $\frac{\pi}{8} x = k\pi$, ($k=0, 1, 2, \dots$),

波节点: $x_k = 8k = 0, 8, 16, \dots \text{ (m)}$ (2 分)

(5) 驻波的平均能流密度: $\bar{I} = \bar{I}_1 + \bar{I}_2 = (\bar{w}_1 - \bar{w}_2)u = 0$ (2 分)

五、计算题 (10 分) 解: (1) $Q_{ca} = \nu R T_a \ln \frac{V_a}{V_c} = 8.31 \times 600 \times \ln 2 = 3456 \text{ (J)}$, 吸热

$$Q_{bc} = \nu C_{V,m} (T_c - T_b) = \frac{5}{2} \times 8.31 \times (600 - 300) = 6232.5 \text{ (J)}, \text{ 吸热}$$

$$Q_{ab} = \nu C_{p,m} (T_b - T_a) = \frac{7}{2} R (T_b - T_a) = \frac{7}{2} \times 8.31 \times (300 - 600) = -8725.5 \text{ (J)}, \text{ 放热} \quad (3 \text{ 分})$$

(2) $A = Q_{ab} + Q_{bc} + Q_{ca} = -8725.5 + 6232.5 + 3456 = 963 \text{ (J)}$ (3 分)

$$(3) \eta = \frac{A}{Q_1} = \frac{963}{6232.5 + 3456} = 9.94\% \quad (4 \text{ 分})$$

六、计算题 (10 分) 解: (1) 由题意 $\gamma = \frac{m}{m_0} = 2 = \frac{E}{E_0} \Rightarrow E = 2E_0$;

由狭义相对论的能量公式, 加速过程电子获得的动能: $E_k = E - E_0 = E_0$;

电子的动能来源于加速电压所作的功: $eU = E_k = E_0 = 0.51 \text{ MeV} \Rightarrow U = 0.51 \text{ MV} = 5.1 \times 10^5 \text{ V} \quad (5 \text{ 分})$

(2) 方法一: 由 $\gamma = 2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow v = \frac{\sqrt{3}}{2}c$;

$$p = mv = \gamma m_0 v = 2 \cdot 9.0 \times 10^{-31} \frac{\sqrt{3}}{2} 3 \times 10^8 \text{ kg} \cdot \text{m/s} = 4.68 \times 10^{-22} \text{ kg} \cdot \text{m/s}; \quad (5 \text{ 分})$$

方法二: 由狭义相对论的能量动量关系: $E^2 = p^2 c^2 + E_0^2$;

$$p = \frac{1}{c} \sqrt{E^2 - E_0^2} = \frac{1}{c} \sqrt{3} E_0 = \sqrt{3} m_0 c = 4.68 \times 10^{-22} \text{ kg} \cdot \text{m/s};$$

加强部分 (力学加强, 热学加强和电学加强各 20 分)

七、(10 分) 请选做你所学的对应模块题, 选错模块不给分

大学物理 L: 解: 取车厢为参照系, 坐标系 oxy 固定于车厢, 小球受到重力 P、悬线拉

力 T 和惯性力 F 惯的作用: $P + T + F_{\text{惯}} = 0 \quad (3 \text{ 分})$

$$T \sin \theta - ma = 0 \quad (1) \quad (3 \text{ 分})$$

$$-mg + T \cos \theta = 0 \quad (2)$$

解之得: $\tan \theta = \frac{a}{g} = \frac{\sqrt{3}}{3} \Rightarrow \theta = 30^\circ \quad (4 \text{ 分})$

大学物理 R: 解: (1) $p = nkT \Rightarrow n = \frac{p}{kT} = \frac{1.00 \times 10^{-4} \times 1.013 \times 10^5}{1.38 \times 10^{-23} \times 300} = 2.45 \times 10^{21} (\text{个}/\text{m}^3); \quad (3 \text{ 分})$

$$(2) \bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi \mu}} = \sqrt{\frac{8 \times 8.31 \times 300}{3.14 \times 28 \times 10^{-3}}} = 476.3 (\text{m/s}); \quad (3 \text{ 分})$$

$$(3) \bar{\lambda} = \frac{kT}{\sqrt{2} \pi d^2 p} = \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2} \times 3.14 \times 3.1^2 \times 10^{-20} \times 10^{-4} \times 1.013 \times 10^5} = 9.57 \times 10^{-4} (\text{m}); \quad (2 \text{ 分})$$

$$(4) \bar{Z} = \frac{\bar{v}}{\bar{\lambda}} = \frac{476.3}{9.57 \times 10^{-4}} = 5.0 \times 10^5 (\text{s}^{-1}); \quad (2 \text{ 分})$$

大学物理 D: 解: (1) 由高斯定理得: $E = \begin{cases} 0, & r < R \\ \frac{Q}{4\pi\epsilon_0 r^2}, & r > R \end{cases} \quad (3 \text{ 分})$

(2) 选 ∞ 处电势为 0, 并沿径向为积分路径, 由 $V_p = \int_p^\infty \vec{E} \cdot d\vec{l} = \int_r^\infty E \cdot dr$ 得:

$$\text{球内 } r < R: V_p = \int_p^\infty \vec{E} \cdot d\vec{l} = \int_r^R E_{\text{内}} \cdot dr + \int_R^\infty E_{\text{外}} \cdot dr = \int_r^R 0 \cdot dr + \int_R^\infty \frac{Q}{4\pi\epsilon_0 r^2} \cdot dr = \frac{Q}{4\pi\epsilon_0 R}; \quad (2 \text{ 分})$$

$$\text{球内 } r > R: V_p = \int_p^\infty E_{\text{外}} \cdot dr = \int_R^\infty \frac{Q}{4\pi\epsilon_0 r^2} \cdot dr = \frac{Q}{4\pi\epsilon_0 R}; \quad (2 \text{ 分})$$

$$(3) U = \frac{Q}{4\pi\epsilon_0 R}; C = \frac{Q}{U} = 4\pi\epsilon_0 R; \quad (3 \text{ 分})$$

八、(10 分) 请选做你所学的对应模块题, 选错模块不给分

大学物理 L: 解: 无外力, 选惯性系是行星尘埃在其中静止的参考系, 则有

$$\vec{F} = m \frac{d\vec{v}}{dt} - \vec{u} \frac{dm}{dt}, \text{ 其中: } \vec{u} = -\vec{v}, \text{ 所以有: } m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt} = 0; \quad (5 \text{ 分})$$

以卫星速度方向为正方向, 即: $m \frac{dv}{dt} + v \frac{dm}{dt} = 0$

$$\text{所以有: } a = \frac{dv}{dt} = -\frac{v}{m} \frac{dm}{dt} = -\frac{cv^2}{m}; \quad (5 \text{ 分})$$

大学物理 R: 解: (1) 实际过程为有限温差热传导, 故不可逆, 因此, 需假想可逆过程连结始末态, 先考

$$\text{虑铜块有 } \Delta S_{\text{Cu}} = \int_{T_1}^{T_2} mc \frac{dT}{T} = mc \ln \frac{T_2}{T_1} < 0; \quad (4 \text{ 分})$$

(2) 假想水跟一温差无限小的热源接触并从其吸热, 所吸收热量跟实际过程铜块传递给水的热量一样多:

$$\Delta S_{\text{水}} = \frac{Q_{\text{吸}}}{T_2} = \frac{mc(T_1 - T_2)}{T_2} > 0; \quad (3 \text{ 分})$$

$$(3) \Delta S_{\text{总}} = \Delta S_{\text{Cu}} + \Delta S_{\text{水}} = mc \left(\frac{T_1}{T_2} - 1 - \ln \frac{T_1}{T_2} \right) > 0; \quad (3 \text{ 分})$$

大学物理 D: 解: (1) 此时电容器的电容: $C = \frac{\epsilon_0 S}{2d} \quad (3 \text{ 分})$

$$(2) \text{极板间场强: } E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 S}, \quad \text{极板间的电压: } U = 2Ed = \frac{2Qd}{\epsilon_0 S} \quad (3 \text{ 分})$$

$$(3) \text{电场能量密度: } w_e = \frac{1}{2} \epsilon_0 E^2 = \frac{Q^2}{2\epsilon_0 S^2}$$

$$\text{由间距 } d \text{ 拉开到 } 2d, \text{ 电场能量增加: } \Delta W_e = w_e (2V_{\text{体}} - V_{\text{体}}) = \frac{Q^2}{2\epsilon_0 S^2} \cdot Sd = \frac{Q^2 d}{2\epsilon_0 S} \quad (2 \text{ 分})$$

(4) 两极板带等量异号电荷, 外力 \vec{F} 将其缓缓拉开时, 应有 $\vec{F} = -\vec{F}_e$, 则外力所作功为

$$A = -\vec{F}_e \cdot \Delta \vec{r} = \Delta W_e = \frac{Q^2 d}{2\epsilon_0 S} \text{——外力克服静电引力所作的功等于静电场能量的增加。} \quad (2 \text{ 分})$$