MATH-411 Numerical Analysis—Homework 5
Rochester Institute of Technology, Fall 2022

Due: Friday October 28, 2022 at 11.59pm EST.
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Remark:

- All assignments are uploaded on MyCourses as pdf.
- You can discuss ideas on how to tackle the problems on **Piazza** but do not post solutions. Thanks.
- Please show all your work clearly. If the assignment involves MATLAB, please turn in your code and figures as well.
- Do you put your answers on your code comments, I strongly ask that you write down your answers for any assignment (homework and projects) on a different sheet (handwritten or typeset), include your figures and tables on this sheet and all your supporting calculations and narratives.

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1. **Fourier series:** Calculate the Fourier series for the following functions using $n = 8, 16, 32$ and 64 interpolation points on the interval $0 \leq x \leq 2\pi$, and for the $n = 64$ case, plot the resulting sine and cosine mode coefficients for $\sin(kx)$ and $\cos(kx)$ vs. K:

   (a) $f(x) = \ln(2 + \sin x)$

   (b) $f(x) = \begin{cases} x & ; \ 0 \leq x < \pi \\ 2\pi - x & ; \ \pi < x \leq 2\pi \end{cases}$

   (c) $f(x) = x^2(x - 2\pi)^2$

2. **Numerical Differentiation:**

   (a) Use the three-point centered difference formula for the second derivative to approximate $f''(1)$, where $f(x) = \frac{1}{x}$, for $h = 0.1, 0.01$, and 0.001. Find the approximation error and verify the error estimate predicted by theory (what does the theory say?).

   (b) Develop a first-order method for approximating $f''(x)$ that uses the data $f(x-h), f(x)$, and $f(x+3h)$ only. Find the error term.

3. **Riemann Sums:** In each of the following problems, a function and interval of definition are given. Also a partition of the interval is specified, as well as a point in each of the sub-intervals that the partition determines. In each case, sketch the graph of f and the rectangles that this information provides and compute the Riemann sum.

   (a) $f(x) = 25 - x^2$, $[0,5]$, $\{0,2,3,4,5\}$, use right endpoint

   (b) $f(x) = 16 - x^2$, $[0,4]$, $\{0,1,2,3,4\}$, use midpoint

   (c) $f(x) = x^3 + 2$, $[-1,2]$, $\{-1,-0.5,0,0.5,1,1.5,2\}$, use left endpoint

   (d) $f(x) = \frac{1}{x+1}$, $[0,2]$, $\{0,0.5,1,1.5,2\}$, use midpoint