MATH-411 Numerical Analysis—Homework 4
Rochester Institute of Technology, Fall 2022

Due: Friday October 7, 2022 at 11.59pm EST.
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Remark:

- All assignments are uploaded on MyCourses as pdf.
- For this assignment your can handwrite your solution **or** you can type your solution in Microsoft word and then convert to pdf **or** you can use Latex and automatically generate your pdf.
- Figure out how to upload your files on MyCourses before the due dates. Late homeworks are **not** accepted.
- You can discuss ideas on how to tackle the problems on **Piazza** but do not post solutions. Thanks.

Please show all your work clearly. If the assignment involves MATLAB, please turn in your code and figures as well.

1. Given the data points $(1,0),(2,\ln 2),(4,\ln 4)$

   (a) Determine the degree 2 interpolating polynomial.

   (b) Use the result of (a) to approximate $\ln 3$.

   (c) Give an error bound for the approximation in part (b).

   (d) Compare the actual error (assuming the $\ln$ function) to your error bound.

2. In addition to using Lagrange polynomials to estimate values between points, they could also be used to estimate derivatives.

   (a) Consider a function, f, and three points x_0, x_1, and x_2. Estimate the derivative of f at x_0 by differentialing the Lagrange polynomial through the three points. That is, find $p_3'(x_0)$ along with the error estimate.

   (b) Now assume that $x_i = x + ih$, for $i = 0,1,2$. Plug this in to your result from (a) to derive a three-point formula for the first derivative (along with the error). How does the error here compare to the error you see from the typical two-point estimate of the derivative you learned in Calc I? in particular, discuss how accuracy is affected in each case by changes in h.

   (c) In general, are the derivatives from the Lagrange polynomials going to be more or less accurate than the function values? Give some explanation for your answer, though formal proof is not required.

3. Determine k_1, k_2, k_3 in the following cubic spline. Which of the three end conditions—natural, parabolically terminated, or not-a-knot—if any, are satisfied?

$$S(x) = \begin{cases} 4 + k_1 x + 2x^2 - \frac{1}{6}x^3 & \text{on}[0,1] \\ 1 - \frac{4}{3}(x-1) + k_2(x-1)^2 - \frac{1}{6}(x-1)^3 & \text{on}[1,2] \\ 1 + k_3(x-2) + (x-2)^2 - \frac{1}{6}(x-2)^3 & \text{on}[2,3] \end{cases}$$