Math-411 Numerical Analysis Homework 2

Nate Valla

1. Suppose that you were to run the bisection method on the function $f(x) = \frac{1}{x}$ with starting interval [-1, 2].

Will the method converge to a real number? If so, is this a root? Give some explanation of why you will see this behavior.

$$c_0 = midpoint[-1, 2] = \frac{-1+2}{2} = 0.5 \Rightarrow f(c_0) = 2$$

$$c_1 = midpoint[-1, 0.5] = \frac{-1+0.5}{2} = -0.25 \Rightarrow f(c_1) = -4$$

$$c_2 = midpoint[-0.25, 0.5] = \frac{-0.25+0.5}{2} = 0.25 \Rightarrow f(c_2) = 4$$

$$c_4 = midpoint[-0.25, 0.25] = \frac{-0.25+0.25}{2} = -0 \Rightarrow lim_{x\to 0} f(c_4) = \infty$$

Yes this method does converge to a real number, however it is not a root. It is pole of odd degree so oneside is negative and the other one positive so it micks a root of odd degree with the bisection Method.

2. Apply two steps of Newton's method with initial guess $x_0 = 1$ to find the roots of $f(x)$ where (a) $f(x) = x^3 + x^2 - 1$ and (b) $f(x) = x^2 + (x-1)^{-1} - 3x$

Newtons mathtod : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

a) $f(x) = x^3 + x^2 - 1 : f'(x) = 3x^2 + 2x : x_0 = 1$

$$x_{n+1} = x_n - \frac{x^3 + x^2 - 1}{3x^2 + 2x}$$

$$x_1 = 1 - \frac{(1)^3 + (1)^2 - (1)}{3(1)^2 + 2(1)} = 0.8$$

$$x_2 = 0.8 - \frac{(0.8)^3 + (0.8)^2 - (0.8)}{3(0.8)^2 + 2(0.8)} = 0.7$$

b) $f(x) = x^2 + (x-1)^{-1} - 3x : f'(x) = 2x - (x-1)^{-2} - 3 : x_o = 1$

$$x_{n+1} = x_n - \frac{x^2 + (x-1)^{-1} - 3x}{2x - (x-1)^{-2} - 3} \to x_{n-1} = x_n - \frac{(x^2 + (x-1)^{-1} - 3x)(x-1)^2}{(2x - (x-1)^{-2} - 3)(x-1)^2}$$

$$\to x_{n-1} = x_n - \frac{(x^2 - 3x)(x-1)^2 + (x-1)}{(2x-3)(x-1)^2 + 1}$$

$$x_1 = 1 - \frac{((1)^2 - 3(1))((1) - 1)^2 + ((1) - 1)}{(2(1) - 3)(((1) - 1)^2 + 1} = 1$$

$$x_2 = 1 - \frac{((1)^2 - 3(1))((1) - 1)^2 + ((1) - 1)}{(2(1) - 3)(((1) - 1)^2 + 1} = 1$$

3. Consider the following 5 methods for calculating $2^{\frac{1}{4}}$.

(a) Bisection methods applied to $f(x) = x^4 - 2$.

(b) Secant Method applied to $f(x) = x^4 - 2$.

(c) Fixed point iteration applied to $g(x) = \frac{x}{2} + \frac{1}{x^3}$.

(d) Fixed point iteration applied to $g(x) = \frac{2x}{3} + \frac{2}{3x^3}$.

(e) Fixed point iteration applied to $g(x) = x - \frac{2}{5}(x^4 - 2)$.

(f) Newton's Method applied to $f(x) = x^4 - 2$

Rank them in order of speed of convergence from fastest to slowest. Give the reasons

for your ranking.

1. F Newton method has been stated to be the fastest
2. B This is a variation of newtons method
3. D its a convergent Fix point method
4. A it converges with a set persion
5. E doesnt converge
6. doesn't converges

```
f = @(x) 1/2-3/x^4;
f(2^(1/4))
```

```
ans = -1.0000
```

```
f = @(x) 2/3-2/(x^4);
f(2^(1/4))
```

```
ans = -0.3333
```

```
f = @(x) 1-2/5*4*x^3;
f(2^(1/4))
```

```
ans = -1.6909
ans = 0.1554
```

4. Let $f(x) = x^2 - 6$ With $p_0 = 3$ and $p_1 = 2$ find p_3 using the Secant method.

$$x_{n+1} = x_n \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})},$$

$$x_{n+1} = x_n \frac{(x_n^2 - 6)(x_n - x_{n-1})}{x_n^2 - x_{n-1}^2}, \; x_0 = 3, x_1 = 2$$

$$x_2 = 2 \frac{((2)^2 - 6)((2) - (3))}{(2)^2 - (3)^2} = -0.8$$

$$x_3 = -0.8 \frac{((0.8)^2 - 6)((-0.8) - (2))}{(0.8)^2 - (2)^2} = 3.573$$