Math-411 Numerical Analysis HW 3

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1. Let $f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$. Does Newton's Method converge quadratically to the root $r = 2$?. Find $\lim_{x\to\infty} \frac{e_{i+1}}{e_i}$, where ei denotes the error at step i.

$f(x)$ is defined to be $f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$

```
f=@(x) x^4 -7*x^3+18*x^2-20*x+8;
f(2)
```

```
ans =
     0
```

so $f'(x) = 4x^3 - 21x^2 + 36x - 20$

```
fp=@(x) 4*x^3 -21*x^2+36*x-20;
fp(2)
```

```
ans =
     0
```

so $f''(x) = 12x^2 - 42x + 36$

```
fpp=@(x) 12*x^2 -42*x^1+36;
fpp(2)
```

```
ans =
     0
```

so $f'''(x) = 24x - 42$

```
fppp=@(x) 24*x -42;
fppp(2)
```

```
ans =
     6
```

$r(x)$ is define to be newton method of $f(x)$

$\Rightarrow r(x) = x - \frac{f(x)}{f'(x)}$

Test for covergence using fixed point thereom

$\Rightarrow r'(x) = 1 - \frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2}$ since $f'(2) = 0$ we have to take the limit x goes to 2

$$r'(2) \to \lim_{x\to 2} 1 - \frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2} \to \lim_{x\to 2} 1 - \lim_{x\to 2}\frac{(f'(x))^2}{(f'(x))^2} + \lim_{x\to 2}\frac{f(x)f''(x)}{(f'(x))^2}$$

$\to 1 - 1 + \lim_{x\to 2}\frac{f(x)f''(x)}{(f'(x))^2}$ since $\frac{0}{0}$ use L'Hopital

$$\to \lim_{x\to 2}\frac{f'(x)f''(x) + f(x)f'''(x)}{2f'(x)f''(x)}$$

$$\to \lim_{x\to 2}\frac{f'(x)f''(x)}{2f'(x)f''(x)} + \lim_{x\to 2}\frac{f(x)f'''(x)}{2f'(x)f''(x)}$$

$\to 1/2 + \lim_{x\to 2}\frac{f(x)f'''(x)}{2f'(x)f''(x)}$ since $\frac{0}{0}$ use L'Hopital

$\to 1/2 + 1/2\lim_{x\to 2}\frac{f(x)f^{(4)}(x) + f'(x)f'''(x)}{f''(x)f''(x) + f'(x)f'''(x)}$ since $\frac{0}{0}$ use L'Hopital

$$\to 1/2 + 1/2\lim_{x\to 2}\frac{f(x)f^{(5)}(x) + f'(x)f^{(4)}(x) + f''(x)f'''(x) + f'(x)f^{(4)}(x)}{f'''(x)f''(x) + f''(x)f'''(x) + f''(x)f'''(x) + f'(x)f^{(4)}(x)}$$

$\to 1/2 + 1/2\lim_{x\to 2}\frac{f(x)f^{(5)}(x) + f''(x)f'''(x) + 2f'(x)f^{(4)}(x)}{3f'''(x)f''(x) + f'(x)f^{(4)}(x)}$ since $\frac{0}{0}$ use L'Hopital

$$\to 1/2 + 1/2\lim_{x\to 2}\frac{f(x)f^{(6)}(x) + f'(x)f^{(5)}(x) + f''(x)f^{(4)}(x) + f'''(x)f'''(x) + 2(f''(x)f^{(4)}(x) + f'(x)f^{(5)}(x))}{3(f'''(x)f'''(x) + f^{(4)}(x)f''(x)) + f''(x)f^{(4)}(x) + f'(x)f^{(5)}(x)}$$

since there now term that don't equal zero on the top and bottom we plug in and reduce

$$\to 1/2 + 1/2\lim_{x\to 2}\frac{f'''(x)f'''(x)}{3f'''(x)f'''(x)}$$

$$\to 1/2 + 1/2\lim_{x\to 2}\frac{1}{3}$$

$$\to 1/2 + 1/2 * 1/3$$

$$r'(2) \to \frac{4}{6}$$

since $r'(2) < 1$ is less the one $\lim_{x\to\infty}\frac{e_{i+1}}{e_i}$ tends towards 0

but since $r'(2) \neq 0$ it doesn't have quadratic convergence

2.Each equation has one root. Use Newton’s method to approximate the root to eight correct decimal places.

(a) $x^3 = 2x + 2$

```
f= @(x) x^3-2*x-2;
fp= @(x) 3*x^2-2;
```

```
r= @(x) x-f(x)/fp(x);

guess = 1;
for i = 1:8
    guess = r(guess);
end
```

(b) $e^x + x = 7$

```
f= @(x) exp(x)+x-7;
fp= @(x) exp(x)+1;
r= @(x) x-f(x)/fp(x);

guess = 1;
for i = 1:6
    guess = r(guess);
end
```

(c) $e^x + \sin(x) = 4$

```
f= @(x) exp(x)+sin(x)-4;
fp= @(x) exp(x)+cos(x);
r= @(x) x-f(x)/fp(x);

guess = 1;
for i = 1:4
    guess = r(guess);
end
```

3.Apply Newton's Method to find the only root to as much accuracy as possible and find the root's multiplicity. Then use Modified Newton's Method to converge to the root quadratically. Report the forward and backward error of the best approximation obtained from each method.

(a) $f(x) = 27x^3 + 54x^2 + 36x + 8$

```
f= @(x) 27*x^3+54*x^2+36*x+8;
fp= @(x) 81*x^2+108*x+36;
r= @(x) x-f(x)/fp(x);

guess = 1;
for i = 1:40
    guess = r(guess);
end
guess
```

```
guess =
  -0.666664832347452
```

```
fp(guess)
```

```
ans =
     2.725428771555016e-10
```

```
fpp=@(x) 162*x+108;
fpp(guess)
```

```
ans =
     2.971597127015002e-04
```

```
fppp=@(x) 108;
fppp(guess)
```

```
ans =
   108
```

Since $f'''(x) \neq 0$ at the root and it is the first non zero dervative we have a mulitplciaty of 3

```
m = @(x) x - (f(x)*fp(x))/((fp(x))^2 -f(x)*fpp(x));

guess = -.5;
for i = 1:1
    guess = m(guess);
end
guess
```

```
guess =
  -0.666666666666667
```

(b) $f(x) = 36x^4 - 12x^3 + 37x^2 - 12x + 1$

```
f= @(x) 36*x^4 -12*x^3 +37*x^2 -12*x+1;
fp= @(x) 144*x^3 -36*x^2 +74*x -12;
r= @(x) x-f(x)/fp(x);

guess = 1;
for i = 1:40
    guess = r(guess);
end
guess
```

```
guess =
   0.166666669054496
```

```
fp(guess)
```

```
ans =
     1.766993413809814e-07
```

```
fpp=@(x) 432*x^2 -72*x +74;
fpp(guess)
```

```
ans =
  74.000000171923688
```

Since $f''(x) \neq 0$ at the root and it is the first non zero dervative we have a mulitplciaty of 2

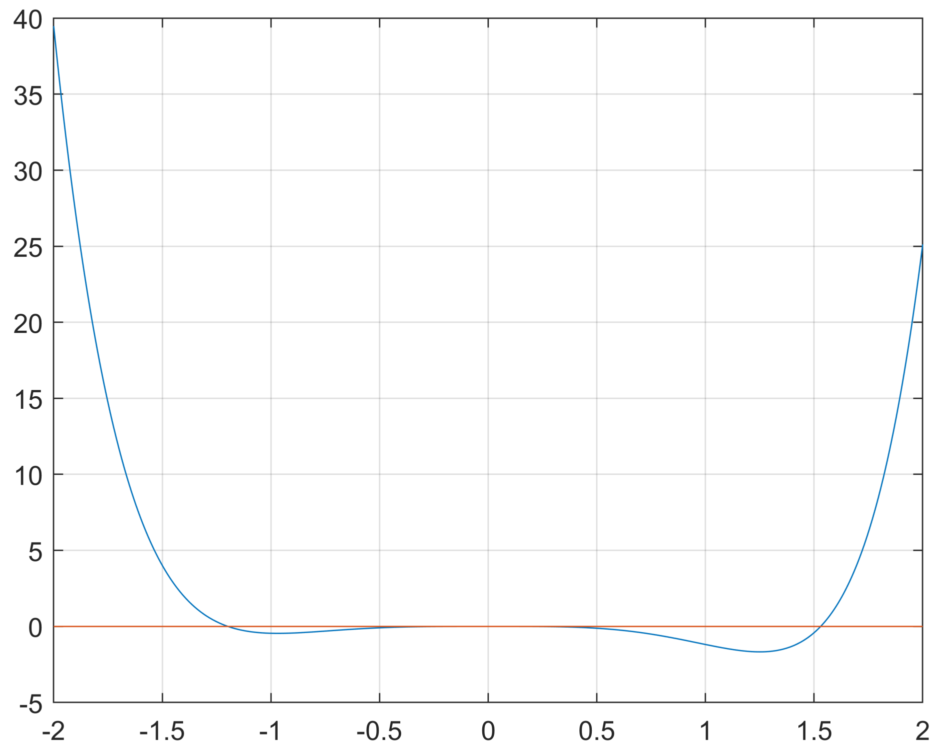
```
m = @(x) x - (f(x)*fp(x))/((fp(x))^2 -f(x)*fpp(x));

guess = 1;
for i = 1:8
    guess = m(guess);
end
guess
```

```
guess =
   0.166666666400369
```

4. Consider the function $f(x) = e^{\sin^3(x)} + x^6 - 2x^4 - x^3 - 1$ on the interval [−2,2]. Plot the function on the interval, and find all three roots to six correct decimal places. Determine which roots converge quadratically, and find the multiplicity of the roots that converge linearly.

```
f= @(x) exp((sin(x)).^3) + x.^6 -2*x.^4- x.^3 -1;
z=@(x) 0*x;
t = -2:0.001:2;
plot(t,f(t))
hold on
grid on
plot(t,z(t))
```




```
fp =@(x) x^2*(-3 -8*x + 6*x^3)+ 3*exp(sin(x)^3)*cos(x)*sin(x)^2;
fpp=@(x) 6*x*(-1-4*x+5*x^3)-3*exp(sin(x)^3)*sin(x)^3+3*exp(sin(x)^3)*cos(x)^2*sin(x)*(2+3*sin(x
```

```
fppp=@(x) 6*(-1-8*x+20*x^3)+3*exp(sin(x)^3)*cos(x)*(-sin(x)^2*(1 + 3*sin(x)^3) + cos(x)^2*(2 +
fp4=@(x) 6*(-8+60*x^2)+3*exp(sin(x)^3)*sin(x)*(3*sin(x)^5+sin(x)^2+3*(9*sin(x)^6+36*sin(x^3)+20
```

```
fp4 = function_handle with value:
    @(x)6*(-8+60*x^2)+3*exp(sin(x)^3)*sin(x)*(3*sin(x)^5+sin(x)^2+3*(9*sin(x)^6+36*sin(x^3)+20)*sin(x)*cos(x)^4-4*(9
```

```
r= @(x) x-f(x)/fp(x);
guess = .0001;
for i = 1:1
    guess = r(guess);
end
guess
```

```
guess =
     8.612264570337472e-05
```

The root is at zero

```
fp(guess)
```

```
ans =
    -5.110386655480147e-12
```

```
fpp(guess)
```

```
ans =
    -1.780170278066658e-07
```

```
fppp(guess)
```

```
ans =
  -0.004133975884102
```

```
fp4(guess)
```

```
ans =
 -48.002062938237664
```

The multiplicity is 4

```
guess = 1.53;
for i = 1:5
    guess = r(guess);
end
guess
```

```
guess =
   1.530133508166615
```

The root is at 1.530133508

```
fp(guess)
```

```
ans =
  14.972731159968262
```

The multiplicity is 1

```
guess = -1.19;
for i = 1:5
    guess = r(guess);
end
guess
```

```
guess =
  -1.197623722133570
```

The root is at -1.1976237221

```
fp(guess)
```

```
ans =
  -4.920576858818949
```

The multiplicity is 1