Introduction to Bayesian Analysis 1

Lab Exercise 0

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	E_0	E_1
F_0	0.10	0.05
F_1	0.50	0.35

e) Marginal probabilities

$$P(E_0) = P(E_0, F_0) + P(E_0, F_1) = 0.10 + 0.50 = 0.60$$
$$P(E_1) = P(E_1, F_0) + P(E_1, F_1) = 0.15 + 0.35 = 0.40$$

f) Marginal probability

$$P(F_0) = P(F_0, E_0) + P(F_0, E_1) = 0.10 + 0.05 = 0.15$$
$$P(F_1) = P(F_1; E_0) + P(F_1, E_1) = 0.50 + 0.35 = 0.85$$

g) For independent event,

$$P(E_0 \cap F_0) = P(E_0) \times P(F_0)$$

In this case $0.10 \neq 0.60 \times 0.15$, therefore, they are NOT

a) $P(E_0 | F_0) = \dfrac{P(E_0 \cap F_0)}{P(F_0)} = \dfrac{0.10}{0.15} = 2/3$

b) $P(E_0 | F_1) = \dfrac{0.50}{0.85} = \dfrac{10}{17}$

c) $P(F_0 | E_0) = \dfrac{0.10}{0.60} = \dfrac{1}{6}$

d) $P(F_1 | E_0) = \dfrac{0.50}{0.60} = 5/6$

(2)

$X \backslash Y$	0	1	-1
0	0	1/4	1/4
1	1/4	0	0
-1	1/4	0	0

$P(x=0) = 0 + 1/4 + 1/4 = 1/2$ $\quad$ $P(y=0) = 0 + 1/4 + 1/4 = 1/2$

i) For independent events

$$P(x=0, y=0) = P(X=0) \times P(Y=0)$$

However, in this case,

$0 \neq 1/2 \times 1/2$ Therefore, X and Y are not independent

ii) $\mathrm{Cov}(x,y) = E(XY) - E(X)E(Y)$

if $\mathrm{cov}(x,y) = 0 \iff E(XY) = E(X)E(Y)$

$E(X) = \sum x p(x) = -1 \times 1/4 + 0 \times 1/2 + 1 \times 1/4 = 0$

$E(Y) = \sum y p(y) = -1 \times 1/4 + 0 \times 1/2 + 1 \times 1/4 = 0$

$E(XY) = \sum (xy \times p(x,y)) = 0 \times 0 + \cdots + 1 \times 0 = 0$

therefore, x and y are uncorrelated.

(3) $P(\text{Identical twins}) = 0.3$

$P(\text{Fraternal twins}) = 0.7$

$$P(\text{Identical twins} \mid FF) = \frac{P(FF \mid \text{Identical twins})\,P(\text{Identical twins})}{P(FF)}$$

$$= \frac{1/2 \times 3/10}{13/40} = 6/13 \approx \underline{0.46154}$$

$P(FF) = P(FF, Id) + P(FF, Fr) = P(FF|Id)P(Id) + P(FF|Fr)P(Fr)$

$= 1/2 \times 3/10 + 1/4 \times 7/10 = 13/40$ //

(4) a) $P(\text{die is defective} \mid \text{die shows } 3) \equiv P(D \mid '3') = \dfrac{P('3' \mid D)\,P(D)}{P('3' \mid D)P(D) + P('3' \mid \sim D)P(\sim D)}$

$= \dfrac{1/6 \times 1/4}{1/6 \times 1/4 + 1/6 \times 3/4} = \dfrac{1}{4}$

b) $P(D \mid '1') = \dfrac{P('1' \mid D)\,P(D)}{P('1' \mid D)P(D) + P('1' \mid \sim D)P(\sim D)} = \dfrac{2/6 \times 1/4}{2/6 \times 1/4 + 1/6 \times 3/4} = 2/5$

c) $P(D \mid '2') = \dfrac{P('2' \mid D)\,P(D)}{P('2' \mid D)P(D) + P('2' \mid \sim D)P(\sim D)} = \dfrac{0 \times \frac{1}{4}}{0 \times \frac{1}{4} + \frac{1}{6} \times \frac{3}{4}} = 0$

(5) a) $P(BB) = P(BB \text{ out of } (BB, Bb, bB)) = 1/3$

$\Rightarrow P(Bb \text{ or } bB) = 2/3$

b)

	Pups	P(a pup is black)
BB × bb	Bb or Bb	1
Bb × bb	Bb, Bb, bb, bb	1/2

$P(BB \mid \text{mouse mated and 7 pups born}) = \dfrac{1/3 \times 1^7}{1/3 \times 1^7 + 2/3 \times (1/2)^7} = \dfrac{64}{65}$