Bayesian Analysis I, Fall 2023

Lab Exercise 0: on **Thursday Oct. 26, 14-16**

1. Suppose E and F are events (E_0 is the complement of E_1, F_0 complement of F_1) with joint probability (conditional on a common background knowledge S) given by

	E_0	E_1
F_0	0.10	0.05
F_1	0.50	0.35

Note that $P(E_0 \cap F_0) = 0.10 \equiv P(E_0 \cap F_0|S)$, etc. Find

a) $P(E_0|F_0)$ b) $P(E_0|F_1)$ c) $P(F_0|E_0)$ d) $P(F_1|E_0)$

e) $P(E_0)$ and $P(E_1)$: marginal probabilities

f) $P(F_0)$ and $P(F_1)$: marginal probabilities

g) Are the events E and F independent?

2. Suppose that X and Y are such that

$$P(X = 0, Y = 1) = P(X = 0, Y = -1) = P(X = 1, Y = 0) = P(X = -1, Y = 0) = 1/4$$

Show that X and Y are uncorrelated (i.e. Cov(X,Y)= 0) but that they are not independent.

3. It is known that about 30% of human twins are identical and the rest fraternal. Identical twins are always of the same gender ((M,M) or (F,F) with equal probability) while fraternal are male (M) or female (F) independently of each other (i.e. (M,M), (F,F), (M,F) or (F,M) with equal probability). Elsa and Ella are twins (and girls). Given this information, what is the posterior probability that they are identical twins?

4. You have four cubical dice, three of which are ordinary and one of which is defective with its faces showing 1,1,3,4,5,6 dots. You choose a die at random and throw it. Find

a) P(die is defective | die shows 3) b) P(die is defective | die shows 1)

c) P(die is defective | die shows 2)

Which outcome gives the most information about if the chosen die is defective?

5. Whether certain mice are black or brown depends on a pair of genes, each of which is either B or b. If both members of the pair are alike, the mouse is said to be homozygous, and if they are different, it is said to be heterozygous. The mouse is brown only if it is homozygous bb. The offspring of a pair of mice have two such genes, one from each parent, and if the parent is heterozygous, the inherited gene is equally likely to be B or b. Suppose that a black mouse results from a mating between two heterozygotes.

a) What are the probabilities that this mouse is homozygous and that it is heterozygous?

Now suppose that this mouse is mated with a brown mouse, resulting in seven offspring, all of which turn out to be black.

b) What is the probability that the black mouse was homozygous BB?