Exercise 1

(1) a) $P(\overline{A \cap B})$

$P(\bar{A}) = 1 - P(A)$ $\therefore P(\overline{A \cap B}) = 1 - P(A \cap B)$

b) $P((\bar{A} \cap B) \cup A)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$$\begin{aligned}
\therefore P((\bar{A} \cap B) \cup A) &= P(\bar{A} \cup A) \cap (A \cup B) \\
&= P(\Omega \cap (A \cup B)) \\
&= P(A \cup B) \\
&= P(A) + P(B) - P(A \cap B)
\end{aligned}$$

(2) a) Let the event be O. Therefore, O is $(A \cap C)$.

$P(A \cap C) = \frac{8}{52} = 2/13$

b) let the event be P. Therefore, P is $(\bar{A} \cap \bar{B})$

$P(\bar{A} \cap \bar{B}) = 13/52 = 1/4$

c) Let the event be Q. Therefore, Q is $\overline{\bar{A} \cap \bar{B}} \cap C$ which could be written as $(A \cup B) \cap C$

$P((A \cup B) \cap C) = 12/52 = 3/13$

d) let the event be R. Therefore, $R = (\bar{A} \cap C) \cup (A \cap \bar{C})$

$$p((\bar{A} \cap C) \cup (A \cap \bar{C})) = p(\bar{A} \cap C) + p(A \cap \bar{C}) = \frac{8}{52} + \frac{18}{52} = \frac{26}{52} = \frac{1}{2}$$

(3) For the random variable X, let the sample space be Ω_X.

$$\Omega_X = \{(H\,H\,H), (T\,H\,H), (H\,T\,H), (H\,H\,T), (T\,T\,H), (T\,H\,T), (H\,T\,T), (T\,T\,T)\}$$

a) $A = \{(H,H,H), (H,T,H), (T,H,T), (T,T,T)\}$

$B = \{(T,T,H), (H,H,T)\}$

b) The sample space here is

$$\Omega = \{0, 1, 2, 3\}$$

$C = \{2, 3\}$

$D = \emptyset$

c) In this case the sample spa would be

$\Omega = \{1, 2, 3, \cdots\}$ which represent the number of tosses to get a head.

$E = \{1, 2, 3, 4\}$

$F = \{4, 5, 6, \cdots\}$

(4) a) $k = 7 \quad n = 40$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{40!}{7!\,33!} = 18643560$$

$\therefore$ probability of 7 numbers $= \dfrac{1}{18643560}$

b) $k = 5 \;,\; n = 52$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{52!}{5!\,47!} = 2598960$$

There are 9 or 10 flushes per suite. Therefore, we have 36 or 40 straight flushes for the 4 suites.

$$P_1 = \frac{36}{2598960} \quad \text{or} \quad P_2 = \frac{40}{2598960}$$

(5) a) $A \cup (B^c \cup A)^c$

$$\begin{aligned}
&= A \cup (B \cap A^c) \\
&= (A \cup B) \cap (A \cup A^c) \\
&= (A \cup B) \cap \Omega \\
&= A \cup B
\end{aligned}$$

b) $A \cap (B \cup A^c)$

$$\begin{aligned}
&= (A \cap B) \cup (A \cap A^c) \\
&= (A \cap B) \cup \phi \\
&= A \cap B
\end{aligned}$$

c) $A \cap (B^c \cap A)^c$

$$\begin{aligned}
&= A \cap (B \cup A^c) \\
&= (A \cap B) \cup (A \cap A^c) \\
&= (A \cap B) \cup \phi \\
&= A \cap B
\end{aligned}$$

d) $A \cup (A \cap B)^c = A \cup (A^c \cup B^c)$

$= (A \cup A^c) \cup B^c$

$= \Omega \cup B^c$

$= \Omega$

⑥ $P(A) = 0.7$ $\quad P(B) = 0.4$ $\quad$ let $P(A \cup B) = \Omega = 1$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$1 = 0.7 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.1$ //