

MATH.APP.210 Spring 2022

Exercise 1

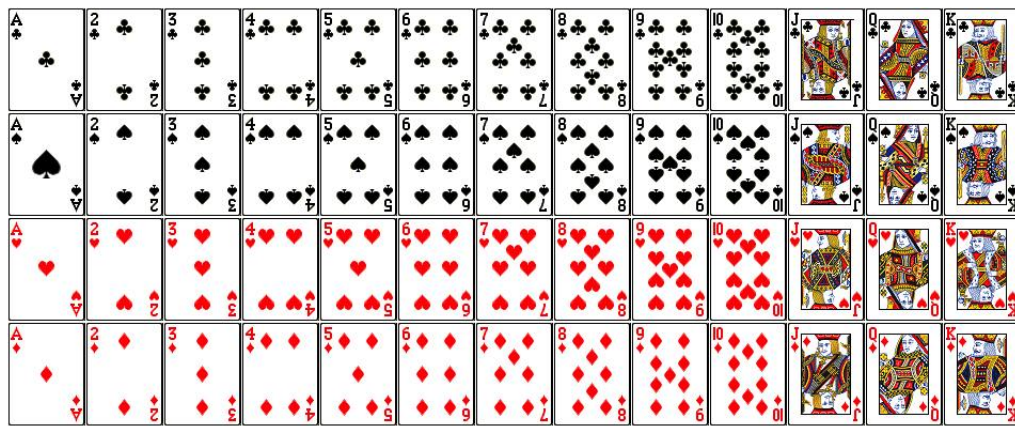
1. Assume that $P(A)$ and $P(B)$ are known. Express the following probabilities by using $P(A)$, $P(B)$ and $P(A \cap B)$.

(a) $P(\overline{A \cap B})$

(b) $P((\overline{A} \cap B) \cup A)$

(2 points)

2. Consider the following random experiment: A card is drawn from a standard deck of playing cards with 4 suites, 13 cards each, numbered 1 – 13. A card with number 1 is known as an ace. The four suites are *hearts*, *diamonds*, *spades*, and *clubs*. Ω is the set of all cards. (i.e. 52 cards in total).



Answer the questions by using only the following events:

- Event A : "the card is either spades or clubs", i.e., the card is *black*
- Event B : "the card is hearts"
- Event C : "the card is 11-13 or an ace", i.e., the card is a *high card* (a card that is not a high card is a "low card")

Express the following events by using set operations and give their probabilities, assuming one card is drawn at random.

(a) Event: The card is a black high card.

(b) Event: The card is a diamond

(c) Event: The card is not a diamond, but it is a high card.

(d) Event: The card is either red or it is a low card, but not both.

(4 points)

3. Consider a coin toss and assign a H for head and T for tail, each equally probable. The purpose of this exercise is to formally describe the sample space and events.

- (a) The coin is tossed three times in a row. This sequence of three tosses is the experiment for the random variable x . What is the sample space in this case?

(Write the whole set!) Express the following events as sets:

Event A : "the first and third toss are the same" and

Event B : "both the first and third, and the second and the third toss are different"

- (b) Three coins are tossed simultaneously and the coins are not identifiable in any way. Also their order does not matter. What is the sample space in this case?

Express the following events as sets:

Event C : "There are more heads than tails"

Event D : "There is an equal number of heads and tails"

- (c) A coin is tossed until we get the result "head". The sample points are now sequences that end in "head". What is the sample space in this case? Express the following events as sets:

Event E : "There are at most three tails before a heads"

Event F : "There are at least four tosses"

(6 points)

4. This exercise is about combinatorics.

- (a) Recall that if we pick k elements from an n -element set, there are $\binom{n}{k}$ possible outcomes. Assuming all numbers are equal likely. Calculate the probability that one gets all 7 numbers correct in the Finnish lottery, where 7 numbers are drawn from 1 to 40.

- (b) A *straight flush* is a set of 5 cards that consists of consecutive numbers, all of which are of the same suite. Try to assess the probability of getting a straight flush if you are dealt five cards at random.

(2 points)

5. Apply the rules of De Morgan to evaluate the following expressions. The superscript c indicates the complement of a set.

(a) $A \cup (B^c \cup A)^c$

(b) $A \cap (B \cup A^c)$

(c) $A \cap (B^c \cap A)^c$

(d) $A \cup (A \cap B)^c$

(4 points)

6. A patient arrives sick at a doctor's office. After an examination the doctor decides the patient has disease A with probability 0.7 or disease B with probability of 0.4. What is the probability that the patient has both diseases (Assuming it can only be A or B)? (2 points)