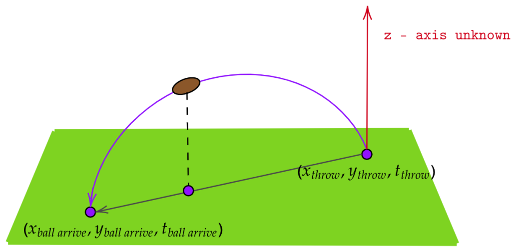
Physics of Adding Z axis

Figure 1: Problem construction



What we know

- x, y coordinates of the ball at any given time t
- Speed of the ball parallel to the field
- We know the forces acting on the ball (for now we just assume this is gravity)

Adding air resistance will be done eventually, but the assumption is that air resistance does not change too much about the arc that we are generating. Furthermore, we will assume the launch height z_0 is 2 yards (this is a proxy for the height of the quarterback)

Consider the forces on the ball:

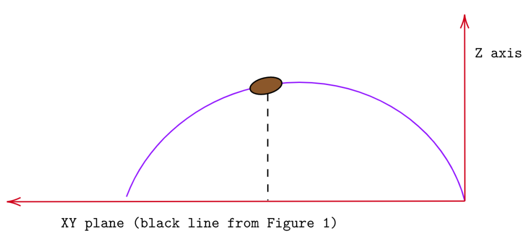
- x-dir : None
- y-dir : None
- z-dir : Gravity (g) in the direction down

Since the components behave independently:

- $V_x = \frac{dx}{dt}$ (we know this)
- $V_y = \frac{dy}{dt}$ (we know this)
- $V_z = \frac{dz}{dt}$ (we don't know this since we only have positions in the xy plane)

Now we slice along the black line in Figure 1 to generate a 2 dimensional parabola:

Figure 2: 3D to 2D

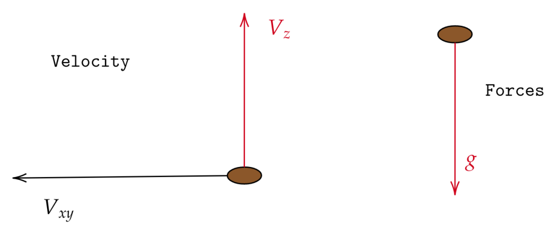


The velocity of the ball in the xy direction of this figure is just the distance the ball travels in that plane by the time it takes. From Figure 1 this tells us that:

$$V_{xy} = \frac{\sqrt{(x_{ballarrive} - x_{throw})^2 + (y_{ballarrive} - y_{throw})^2}}{t_{ballarrive} - t_{throw}}$$

So upon release, the ball has the following velocities and forces being applied:

Figure 3: Velocity and Force



We need to find the value of z at any given time stamp so we must first find the initial velocity V_z. Since the only force being applied in the z axis is g, we will use the basic kinematics equation to find how far above the throwing point the ball is at time t. In this case Δz represents the difference in height between the throw point and the arrival point (we assume they are the same for the sake of simplicity):

$$\Delta z = V_{0z}t + \frac{1}{2}gt^2$$

$$\Delta z - \frac{1}{2}gt^2 = V_{0z}t$$

$$\frac{\Delta z - \frac{1}{2}gt^2}{t} = V_{0z}$$

Now with the initial velocity in the z direction, we can use the following kinematics equation to find the position z, let t_s be the number of seconds since the ball is thrown and let z_0 be the launch height which we defined above:

$$z = z_0 + V_{0z} * t_s - \frac{1}{2}gt_s^2$$

Thus for a given time we are able to calculate the position the ball is in the air.