

Wavelet-Kompression und Multiskalenanalyse

Vortrag zum Thema 'Mathematik in Computerspielen'

Simon Cordes & Niklas Budinger

22. Mai 2018

JGU Mainz

Inhalt...

1. Multiskalenanalyse
2. 2D Haar-Wavelets
3. Algorithmen
4. Anwendungen

Multiskalenanalyse

am Beispiel der eindimensionalen Haar-Wavelets

Inhalt

Inhalt 3

Inhalt 2

Inhalt

Inhalt 3

Inhalt 4

Inhalt 2

⇒ Inhalt 5

2D Haar-Wavelets

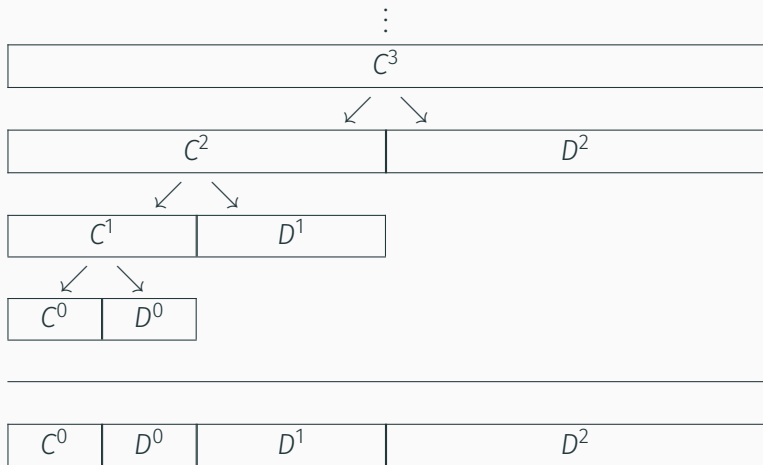
Bild-Kompression

Algorithmen

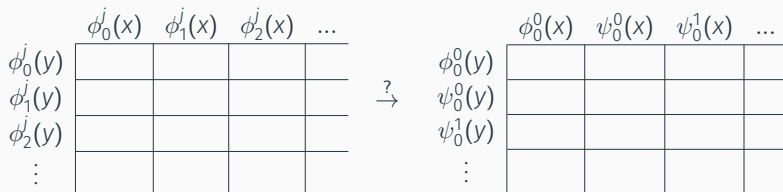
1D Haar-Wavelet-Transformation I

$$\begin{array}{c}
 \begin{array}{ccccccc}
 \phi_0^j & \phi_1^j & \dots & \phi_{2i}^j & \phi_{2i+1}^j & \dots & \phi_{n-2}^j & \phi_{n-1}^j
 \end{array} \\
 C : \begin{array}{|c|c|c|c|} \hline & & a & b \\ \hline \end{array} \\
 \begin{array}{ccc} \swarrow & & \searrow \end{array} \\
 \begin{array}{cccccccc}
 \phi_0^{j-1} & \dots & \phi_i^{j-1} & \dots & \phi_{\frac{n}{2}-1}^{j-1} & \psi_0^{j-1} & \dots & \psi_i^{j-1} & \dots & \phi_{\frac{n}{2}-1}^j
 \end{array} \\
 C' : \begin{array}{|c|c|c|c|c|c|} \hline & \frac{a+b}{\sqrt{2}} & & & \frac{a-b}{\sqrt{2}} & \\ \hline \end{array}
 \end{array}$$

1D Haar-Wavelet-Transformation II



2D Standard-Haar-Wavelet-Transformation I



2D Standard-Haar-Wavelet-Transformation II

$$\begin{array}{c} \phi_l^j(y) \end{array} \begin{array}{c} \phi_0^j(x) \quad \phi_1^j(x) \quad \phi_2^j(x) \quad \dots \\ \boxed{} \quad \boxed{} \quad \boxed{} \quad \boxed{} \end{array} \xleftrightarrow{1D} \begin{array}{c} \phi_l^j(y) \end{array} \begin{array}{c} \phi_0^0(x) \quad \psi_0^0(x) \quad \psi_0^1(x) \quad \dots \\ \boxed{} \quad \boxed{} \quad \boxed{} \quad \boxed{} \end{array}$$

$$\begin{array}{c} \phi_0^j(y) \\ \phi_1^j(y) \\ \phi_2^j(y) \\ \vdots \end{array} \begin{array}{c} \psi_k^i(x) \\ \boxed{} \\ \boxed{} \\ \boxed{} \\ \boxed{} \end{array} \xleftrightarrow{1D} \begin{array}{c} \phi_0^0(y) \\ \psi_0^1(y) \\ \psi_0^1(y) \\ \vdots \end{array} \begin{array}{c} \psi_k^i(x) \\ \boxed{} \\ \boxed{} \\ \boxed{} \\ \boxed{} \end{array}$$

2D Nicht-Standard-Haar-Wavelet-Transformation I

$$\begin{array}{cc} & \begin{array}{cc} \phi_{2k}^j(x) & \phi_{2k+1}^j(x) \end{array} \\ \begin{array}{c} \phi_{2l}^j(y) \\ \phi_{2l+1}^j(y) \end{array} & \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \end{array} \xrightarrow{?} \begin{array}{cc} & \begin{array}{cc} \phi_k^{j-1}(x) & \psi_k^{j-1}(x) \end{array} \\ \begin{array}{c} \phi_l^{j-1}(y) \\ \psi_l^{j-1}(y) \end{array} & \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \end{array}$$

2D Nicht-Standard-Haar-Wavelet-Transformation II

$$\begin{array}{c}
 \begin{array}{cc}
 & \phi_{2k}^j(x) & \phi_{2k+1}^j(x) \\
 \phi_{2l}^j(y) & \begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \\
 \phi_{2l+1}^j(y) & \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}
 \end{array}
 \xrightarrow[\text{Schritt}]{1D}
 \begin{array}{cc}
 & \phi_k^{j-1}(x) & \psi_k^{j-1}(x) \\
 \phi_{2l}^j(y) & \begin{array}{|c|c|} \hline \frac{a+b}{\sqrt{2}} & \frac{a-b}{\sqrt{2}} \\ \hline \end{array} \\
 \phi_{2l+1}^j(y) & \begin{array}{|c|c|} \hline \frac{c+d}{\sqrt{2}} & \frac{c-d}{\sqrt{2}} \\ \hline \end{array}
 \end{array}
 \end{array}$$

$$\begin{array}{cc}
 \downarrow 1D & \text{Schritt} \downarrow \\
 \begin{array}{cc}
 & \phi_k^{j-1}(x) & \psi_k^{j-1}(x) \\
 \phi_l^{j-1}(y) & \begin{array}{|c|c|} \hline \frac{a+b+c+d}{2} & \frac{a-b+c-d}{2} \\ \hline \end{array} \\
 \psi_l^{j-1}(y) & \begin{array}{|c|c|} \hline \frac{a+b-c+d}{2} & \frac{a-b-c+d}{2} \\ \hline \end{array}
 \end{array}
 \end{array}$$

2D Nicht-Standard-Haar-Wavelet-Transformation III

	$\phi_0^j(x)$	$\phi_1^j(x)$...	$\phi_{2k}^j(x)$	$\phi_{2k+1}^j(x)$...
$\phi_0^j(y)$						
$\phi_1^j(y)$						
\vdots						
$\phi_{2l}^j(y)$				a	b	
$\phi_{2l+1}^j(y)$				c	d	
\vdots						

1D Zeilen \downarrow 1D Spalten

	$\phi_0^{j-1}(x)$...	$\phi_k^{j-1}(x)$...	$\phi_{\frac{n}{2}-1}^{j-1}(x)$	$\psi_0^{j-1}(x)$...	$\psi_k^{j-1}(x)$...
$\phi_0^{j-1}(y)$									
\vdots									
$\phi_l^{j-1}(y)$			$\frac{a+b+c+d}{2}$					$\frac{a-b+c-d}{2}$	
\vdots									
$\phi_{\frac{n}{2}-1}^{j-1}(y)$									
$\psi_0^{j-1}(y)$									
\vdots									
$\psi_l^{j-1}(y)$			$\frac{a+b-c+d}{2}$					$\frac{a-b-c+d}{2}$	
\vdots									

Anwendungen

Inhalt

Inhalt 3

Inhalt 2

Inhalt

Inhalt 2

Inhalt 3

Inhalt 4

⇒ Inhalt 5

Vielen Dank!

