

Table 7.1 Common finite-difference formulas.

Type of approximation	Formula	Truncation error
Forward differences	$f'_i = (f_{i+1} - f_i)/(\Delta x)$	$O(\Delta x)$
	$f''_i = (f_{i+2} - 2f_{i+1} + f_i)/(\Delta x)^2$	
	$f'''_i = (f_{i+3} - 3f_{i+2} + 3f_{i+1} - f_i)/(\Delta x)^3$	
	$f^{(4)}_i = (f_{i+4} - 4f_{i+3} + 6f_{i+2} - 4f_{i+1} + f_i)/(\Delta x)^4$	
Backward differences	$f'_i = (f_i - f_{i-1})/(\Delta x)$	$O(\Delta x)$
	$f''_i = (f_i - 2f_{i-1} + f_{i-2})/(\Delta x)^2$	
	$f'''_i = (f_i - 3f_{i-1} + 3f_{i-2} - f_{i-3})/(\Delta x)^3$	
	$f^{(4)}_i = (f_i - 4f_{i-1} + 6f_{i-2} - 4f_{i-3} + f_{i-4})/(\Delta x)^4$	
Central differences	$f'_i = (f_{i+1} - f_{i-1})/(2\Delta x)$	$O(\Delta x^2)$
	$f''_i = (f_{i+1} - 2f_i + f_{i-1})/(\Delta x)^2$	
	$f'''_i = (f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2})/(2(\Delta x)^3)$	
	$f^{(4)}_i = (f_{i+2} - 4f_{i+1} + 6f_i - 4f_{i-1} + f_{i-2})/(\Delta x)^4$	

Table 7.2 Higher order finite-difference formulas.

Type of formula	Formula	Truncation error
Forward differences	$f'_i = (-f_{i+2} + 4f_{i+1} - 3f_i)/(2(\Delta x))$	$O(\Delta x)^2$
	$f''_i = (-f_{i+3} + 4f_{i+2} - 5f_{i+1} + 2f_i)/(\Delta x)^2$	
	$f'''_i = (-3f_{i+4} + 14f_{i+3} - 24f_{i+2} + 18f_{i+1} - 5f_i)/(2(\Delta x)^3)$	
	$f^{(4)}_i = (-2f_{i+5} + 11f_{i+4} - 24f_{i+3} + 26f_{i+2} - 14f_{i+1} + 3f_i)/(\Delta x)^4$	
Backward differences	$f'_i = (3f_i - 4f_{i-1} + f_{i-2})/(2(\Delta x))$	$O(\Delta x)^2$
	$f''_i = (2f_i - 5f_{i-1} + 4f_{i-2} - f_{i-3})/(\Delta x)^2$	
	$f'''_i = (5f_i - 18f_{i-1} + 24f_{i-2} - 14f_{i-3} + 3f_{i-4})/(2(\Delta x)^3)$	
	$f^{(4)}_i = (3f_i - 14f_{i-1} + 26f_{i-2} - 24f_{i-3} + 11f_{i-4} - 2f_{i-5})/(\Delta x)^4$	
Central differences	$f'_i = (-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2})/(12(\Delta x))$	$O(\Delta x^4)$
	$f''_i = (-f_{i+2} + 16f_{i+1} - 30f_i + 16f_{i-1} - f_{i-2})/(12(\Delta x)^2)$	
	$f'''_i = (-f_{i+3} + 8f_{i+2} - 13f_{i+1} + 13f_{i-1} - 8f_{i-2} + f_{i-3})/(8(\Delta x)^3)$	
	$f^{(4)}_i = (-f_{i+3} + 12f_{i+2} - 39f_{i+1} + 56f_i - 39f_{i-1} + 12f_{i-2} - f_{i-3})/(6(\Delta x)^4)$	