Study of the Accuracy of the Video Arbitration System in Tennis

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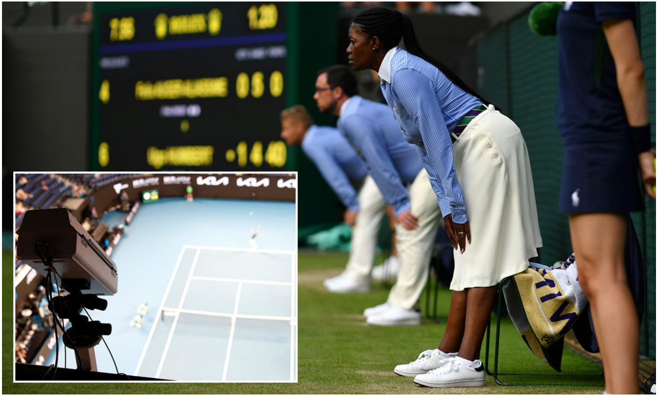
2024

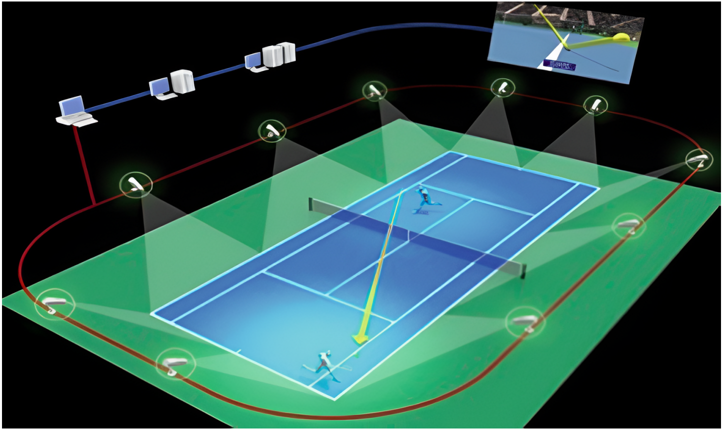
Introduction



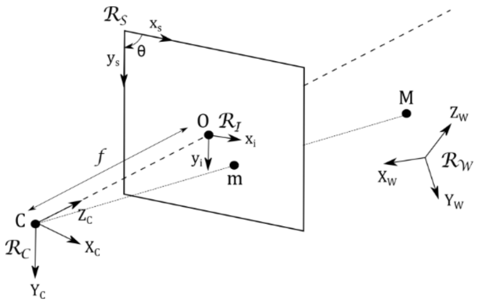
How can we develop a simplified model of the video arbitration system in tennis to determine its accuracy ?

Plan

Operation of the Video Arbitration System



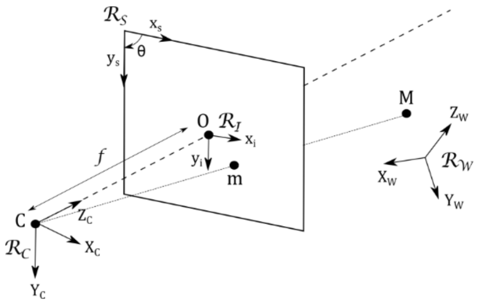
Camera : Calibration



- First transformation :

$$\left\{\begin{array}{c} X_C \\ Y_C \\ Z_C \\ 1 \end{array}\right\} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \left\{\begin{array}{c} X_W \\ Y_W \\ Z_W \\ 1 \end{array}\right\} = [T] \left\{\begin{array}{c} X_W \\ Y_W \\ Z_W \\ 1 \end{array}\right\}$$

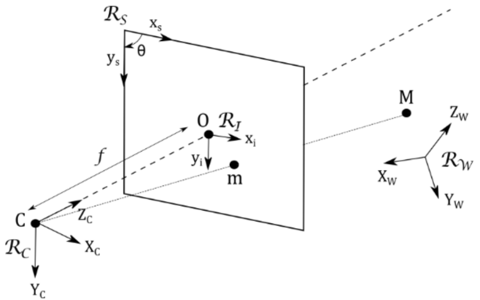
Camera : Calibration



- Second transformation

$$s \cdot \begin{Bmatrix} x_i \\ y_i \\ 1 \end{Bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{Bmatrix}$$

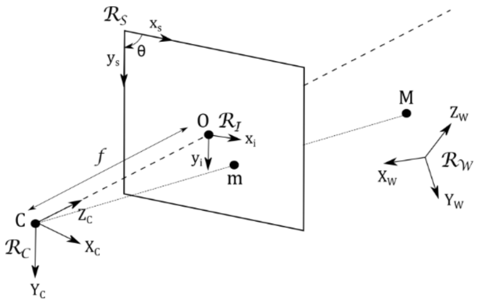
Camera : Calibration



- Third transformation

$$s \cdot \begin{Bmatrix} x_s \\ y_s \\ 1 \end{Bmatrix} = \begin{bmatrix} k_x & 0 & c_x \\ 0 & k_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_i \\ y_i \\ 1 \end{Bmatrix} = K \begin{Bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{Bmatrix}$$

Camera : Calibration



$$\mathrm{s} \cdot \{x_s\} = [K][T]\{X_W\} = [P]\{X_W\}$$

$$[\mathrm{P}] = \begin{bmatrix} r_{11}f_x + r_{31}c_x & r_{12}f_x + r_{32}c_x & r_{13}f_x + r_{33}c_x & t_xf_x + t_zc_x \\ r_{21}f_y + r_{31}c_y & r_{22}f_y + r_{32}c_y & r_{23}f_y + r_{33}c_y & t_yf_y + t_zc_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}$$

Camera : Calculation of the Ball's 2D Position

Detection of Moving Objects by Background Subtraction

Image Extracted from the Video

Camera : Calculation of the Ball's 2D Position

Detection of Moving Objects by Background Subtraction

Reference Background

Camera : Calculation of the Ball's 2D Position

Detection of Moving Objects by Background Subtraction

Smoothed Image

Camera : Calculation of the Ball's 2D Position

Detection of Moving Objects by Background Subtraction

Resulting Image from the Subtraction of the Smoothed Image and the Reference Background

Camera : Calculation of the Ball's 2D Position

Removal of the Player and the Racket

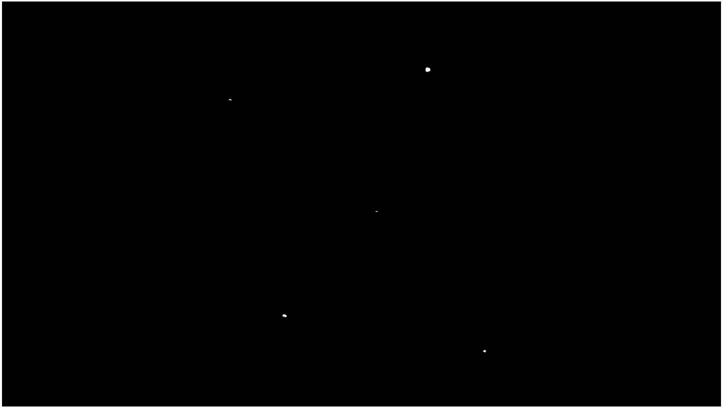
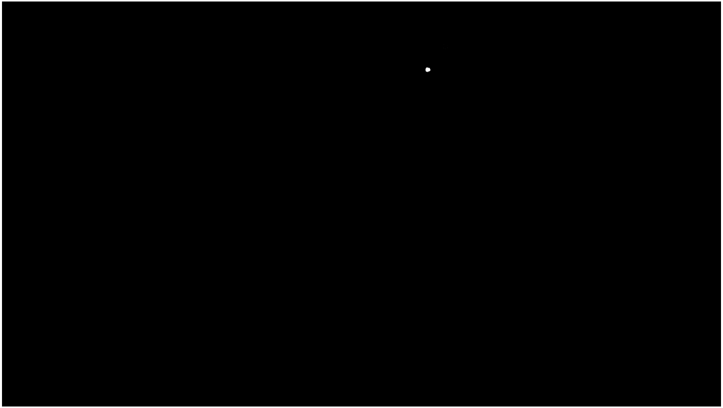


Image without the Player and the Racket

Camera : Calculation of the Ball's 2D Position

Removal of False Balls



Final Image with the Tennis Ball

Camera : Calculation of the Ball's 2D Position

Recognition of the Ball and Extraction of its Coordinates

Ball Detected and Coordinates Extracted

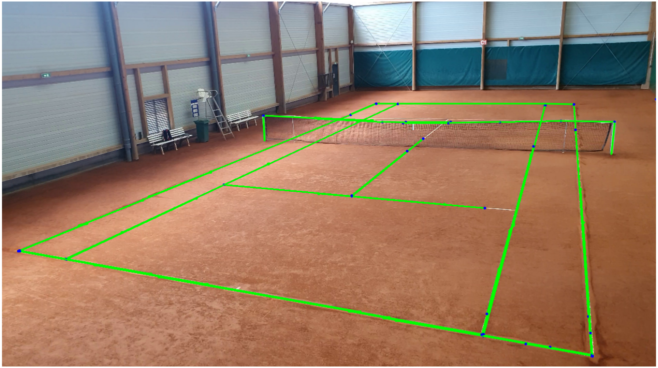
Computer System : Detection of Key Point Coordinates on the Court

Line Identification

Reference Background in Grayscale

Computer System : Detection of Key Point Coordinates on the Court

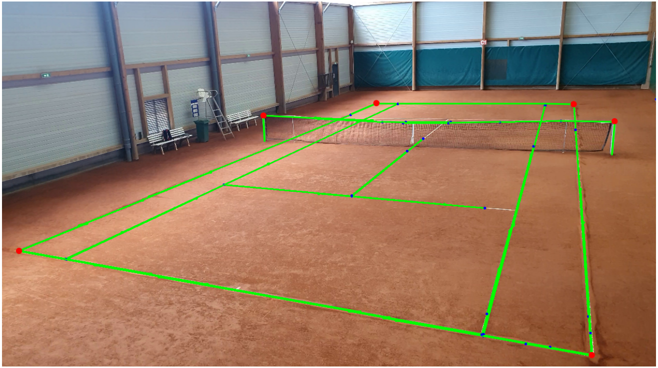
Line Identification



Line Detection by Hough Transform

Computer System : Detection of Key Point Coordinates on the Court

Extraction of Corner Coordinates



Computer System : Detection of 3D Coordinates of the Ball

Method

- Determine the Projection Matrix P for Each Camera P=K T
- Triangulate All Points Provided by the Two Cameras

Computer System : Detection of 3D Coordinates of the Ball

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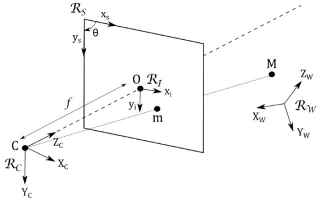
Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points



Input : correspondences $\mathrm{x}_i \leftrightarrow \mathbf{X}_i$

We are looking for : $\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

Method :

For each $\mathrm{x}_i \leftrightarrow \mathbf{X}_i$, we have :

$$\mathbf{x}_i = \mathrm{P}\mathbf{X}_i \quad \Rightarrow \quad \mathbf{x}_i \wedge \mathrm{P}\mathbf{X}_i = \mathbf{0}$$

So

$$\begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \wedge \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

$$\begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \wedge \begin{pmatrix} X_i p_{11} + Y_i p_{12} + Z_i p_{13} + p_{14} \\ X_i p_{21} + Y_i p_{22} + Z_i p_{23} + p_{24} \\ X_i p_{31} + Y_i p_{32} + Z_i p_{33} + p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We expand :

$$\begin{pmatrix} y_i X_i p_{31} + y_i Y_i p_{32} + y_i Z_i p_{33} + y_i p_{34} - X_i p_{21} - \cdots \\ X_i p_{11} + Y_i p_{12} + Z_i p_{13} + p_{14} - x_i X_i p_{31} - \cdots \\ x_i X_i p_{21} + x_i Y_i p_{22} + x_i Z_i p_{23} + x_i p_{24} - y_i X_i p_{11} - \cdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

$$\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \rightarrow \mathbf{p} = \begin{pmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{pmatrix}$$

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -X_1 & -Y_1 & -Z_1 & -1 & x_1X_1 & y_1Y_1 & y_1Z_1 & y_1 \\ X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & 0 & -X_i & -Y_i & -Z_i & -1 & y_iX_i & y_iY_i & y_iZ_i & y_i \\ X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -x_iX_i & -x_iY_i & -x_iZ_i & -x_i \end{bmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{33} \\ p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \\ \vdots \end{pmatrix}$$

- Each correspondence generates 2 equations
- 12 unknowns (up to a scale factor), thus 11 degrees of freedom
- 5.5 correspondences to calibrate

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -X_1 & -Y_1 & -Z_1 & -1 & x_1X_1 & y_1Y_1 & y_1Z_1 & y_1 \\ X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & 0 & -X_i & -Y_i & -Z_i & -1 & y_iX_i & y_iY_i & y_iZ_i & y_i \\ X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -x_iX_i & -x_iY_i & -x_iZ_i & -x_i \end{bmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{33} \\ p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \\ \vdots \end{pmatrix}$$

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Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -X_1 & -Y_1 & -Z_1 & -1 & x_1X_1 & y_1Y_1 & y_1Z_1 & y_1 \\ X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & 0 & -X_i & -Y_i & -Z_i & -1 & y_iX_i & y_iY_i & y_iZ_i & y_i \\ X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -x_iX_i & -x_iY_i & -x_iZ_i & -x_i \end{bmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{33} \\ p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \\ \vdots \end{pmatrix}$$

- Each correspondence generates 2 equations
- 12 unknowns (up to a scale factor), thus 11 degrees of freedom
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Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

$$\mathrm{M}\begin{pmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{33} \\ p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \\ \vdots \end{pmatrix}$$

With :

$$\mathrm{M}=\begin{bmatrix} 0 & 0 & 0 & 0 & -X_1 & -Y_1 & -Z_1 & -1 & x_1X_1 & y_1Y_1 & y_1Z_1 & y_1 \\ X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & 0 & -X_6 & -Y_6 & -Z_6 & -1 & y_6X_6 & y_6Y_6 & y_6Z_6 & y_6 \\ X_6 & Y_6 & Z_6 & 1 & 0 & 0 & 0 & 0 & -x_6X_6 & -x_6Y_6 & -x_6Z_6 & -x_6 \end{bmatrix}$$

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

Least Squares Problem :

$$\text{Minimize } \|M \cdot p\|$$

Find the Vector p that minimizes this norm

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

Singular Value Decomposition and Solution :

$$M = U\Sigma V^T$$

- M Initial Matrix
- U Orthogonal Matrix of Left Eigenvectors.
- Σ Diagonal Matrix of the Square Roots of Eigenvalues of M^TM
- V Orthogonal Matrix of Right Eigenvectors

Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

Singular Value Decomposition and Solution :

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Computer System : Detection of the Ball's 3D Coordinates

P from Correspondence Points

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Computer System : Detection of the Ball's 3D Coordinates

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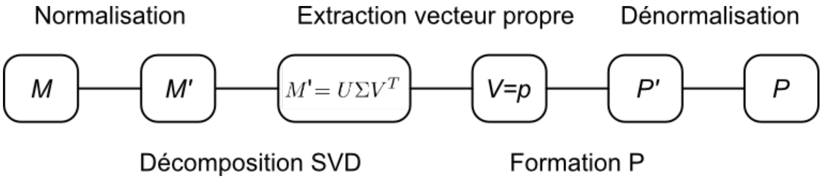
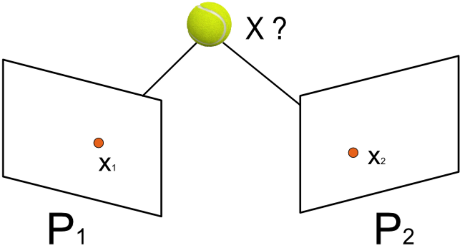
Computer System : Detection of the Ball's 3D Coordinate

P from Correspondence Points



Computer System : Detection of the Ball's 3D Coordinate

Triangulation



$$\left.\begin{aligned} x_1 &= P_1 X \\ x_2 &= P_2 X \end{aligned}\right\} X?$$

Computer System : Detection of the Ball's 3D Coordinate

Triangulation

2D Points in the Images :

$x_1 = (x_1, y_1, w_1) \quad x_2 = (x_2, y_2, w_2)$

Camera Projection Matrices :

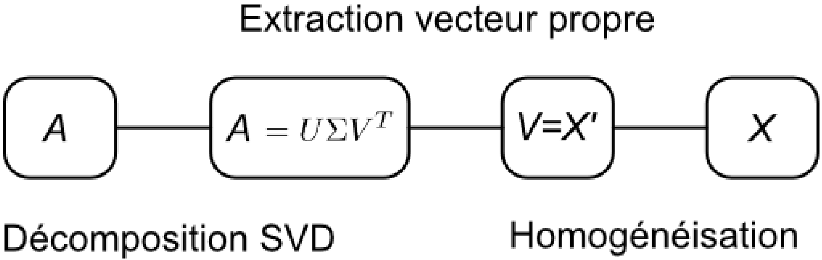
$$P1 = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{pmatrix} \quad P2 = \begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \end{pmatrix}$$

$$\boxed{\left.\begin{array}{l} x_1 = P_1 X \\ x_2 = P_2 X \end{array}\right\} AX = 0}$$

$$\text{A}=\begin{pmatrix} x_1p_{31}-p_{11} & x_1p_{32}-p_{12} & x_1p_{33}-p_{13} & x_1p_{34}-p_{14} \\ y_1p_{31}-p_{21} & y_1p_{32}-p_{22} & y_1p_{33}-p_{23} & y_1p_{34}-p_{24} \\ x_2q_{31}-q_{11} & x_2q_{32}-q_{12} & x_2q_{33}-q_{13} & x_2q_{34}-q_{14} \\ y_2q_{31}-q_{21} & y_2q_{32}-q_{22} & y_2q_{33}-q_{23} & y_2q_{34}-q_{24} \end{pmatrix}$$

Computer System : Detection of the Ball's 3D Coordinate

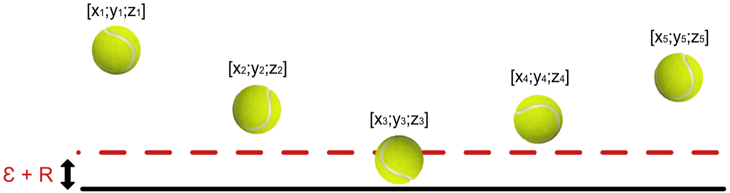
Triangulation



Computer System : Coordinates of the First Bounce

Hypothesis :

- Non-deformable Ball : Point Impact
- Radius : 0.033 m
- Perfectly Flat Court



Model Verification on a Tennis Court

Camera Setup

((a)) Phone 1

((b)) Phone 2

Figure – Camera Arrangement

Model Verification on a Tennis Court

Recording and Service

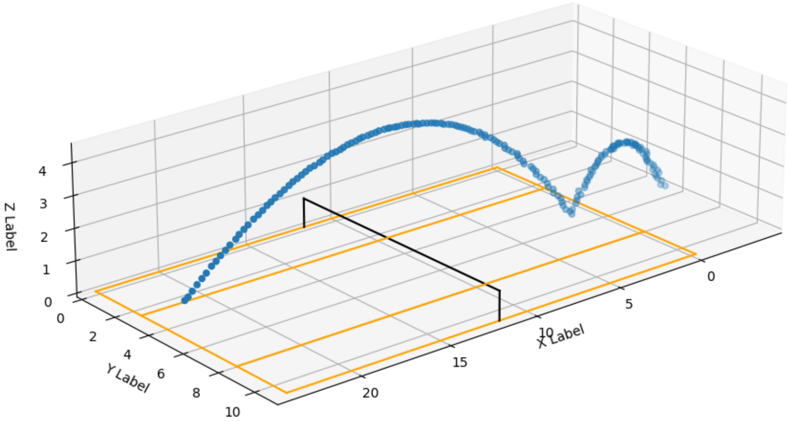
((a)) Phone 1

((b)) Phone 2

Figure – Recordings

Model Verification on a Tennis Court

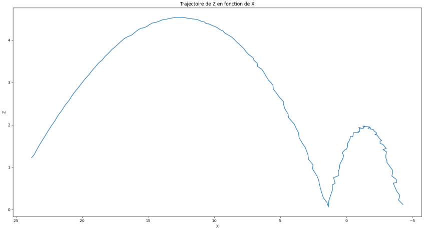
Trajectory Obtained for a Serve



Model Verification on a Tennis Court

Comparison with the Actual Distance

((a)) Measured Distance



((b)) Video Arbitration Distance

Model Validity

Criteria

$$\text{True Positive Rate} = \frac{\sum A}{\sum A + \sum B + \sum C}$$

$$\text{False Alarm Rate} = \frac{\sum D}{\sum D + \sum E}$$

Detection Results	Ball Detected		Ball Not Detected
	Localization detected correct	Localization detected incorrect	
Ball Present in field	A (Correct)	B (Incorrect localization)	C (Error)
Ball not present	D (False alarm)		**E** (Correct)

Model Validity

Implementation

Indoor/Outdoor Choice

- Repeat the protocol 20 times
- Optimal Court : Best Success Rate

Camera Position Selection

- Repeat the protocol 20 times with the optimal court.
- Best Position : Highest Success Rate

Model Validity

Implementation

Indoor/Outdoor Choice

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Model Validity

Implementation

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- Repeat the protocol 20 times
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Model Validity

Results : Courts

((a)) Outdoor Court

((b)) Indoor Court

Model Validity

Results : Courts

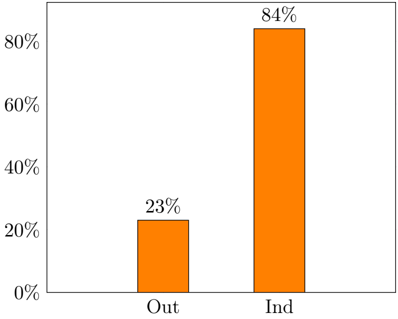


Figure – True Positives

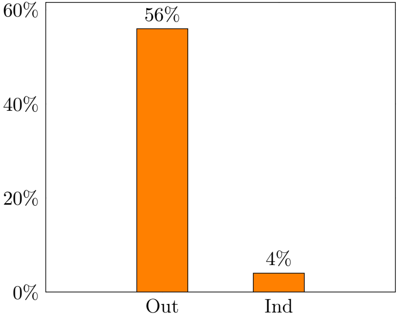
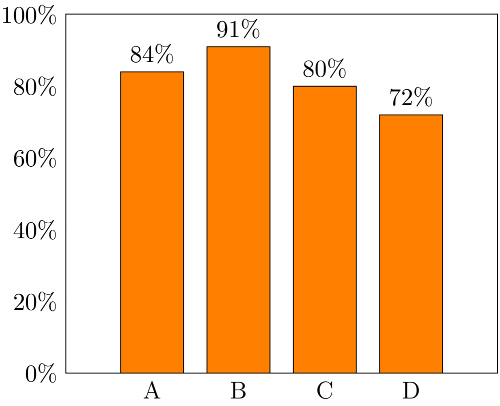


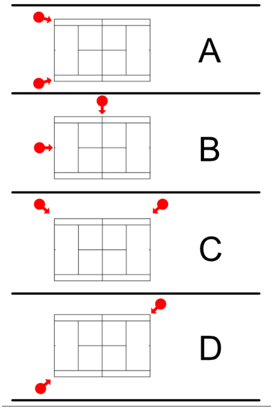
Figure – False Alarms

Model Validity

Results : Camera Positions



True positives



Camera Positions

Model Accuracy

Implementation

- Best Configuration : Court and Camera Positions
- Repeat the protocol 20 times.
- Compare Real Value and Computed Value.

Model Accuracy

Implementation

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Model Accuracy

Implementation

- Best Configuration : Court and Camera Positions
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Model Accuracy

Results

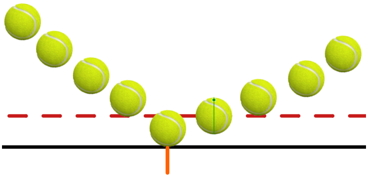
Service Number	Measured Distance (in mm)	Program Distance (in mm)	Delta (in mm)	Out ?
1	1402	1378	24	IN
2	1770	1797	27	IN
3	659	678	19	IN
4	121	109	12	IN
5	304	325	21	IN
6	164	182	18	IN
7	193	210	17	IN
8	281	299	18	IN
9	742	761	19	IN
10	811	828	17	IN
11	-65	-85	20	OUT
12	-182	-214	32	OUT
13	-356	-334	22	OUT
14	-678	-661	17	OUT
15	-1282	-1313	31	OUT
16	-1102	-1126	24	OUT
17	-834	-812	22	OUT
18	-943	-959	16	OUT
19	-232	-208	24	OUT
20	-571	-591	20	OUT

$$\boxed{\Delta_{\text{moy}} = 19mm}$$

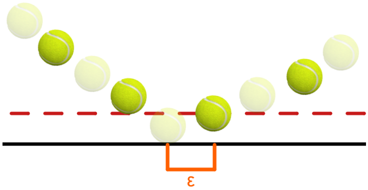
$$\Delta_{\text{HawkEye}} = 3.6mm$$

Model Accuracy

Discussion



((a)) Camera 60 fps



((b)) Camera 30 fps

Number of fps	30	60
Δ	42mm	19mm

Conclusion

- A simplified model...
- ...providing an estimate of the accuracy of video arbitration.

Conclusion

- A simplified model...
- ...providing an estimate of the accuracy of video arbitration.