

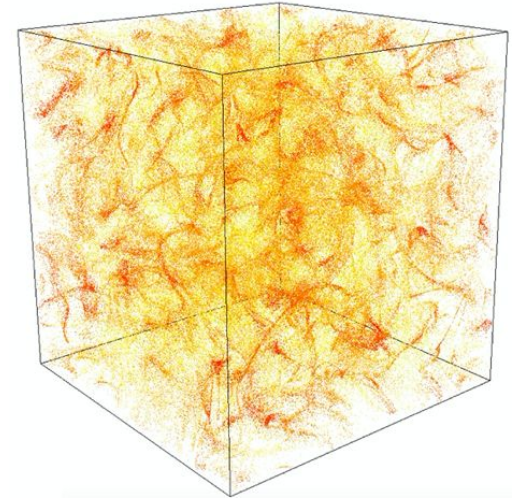
# Data Analysis of Particle Clustering in Turbulence

Mikhal Ben-Joseph, Rivca Chaver, Michael Ho, Nagaprasad  
Rudrapatna, Emma Runia, Qi Xuan Khoo, and Shine Wu



# Our Goal

- Our goal was to perform both prediction and inference on the effect of the Stokes number, Reynolds number, and Froude number, on turbulence
- We created two sets of models: one for prediction, one for inference
- Inference model judged alignment toward physical interpretation of the relationships between variables and potential for extrapolation
- Prediction model judged on minimization of cross-validation error





# Methodology

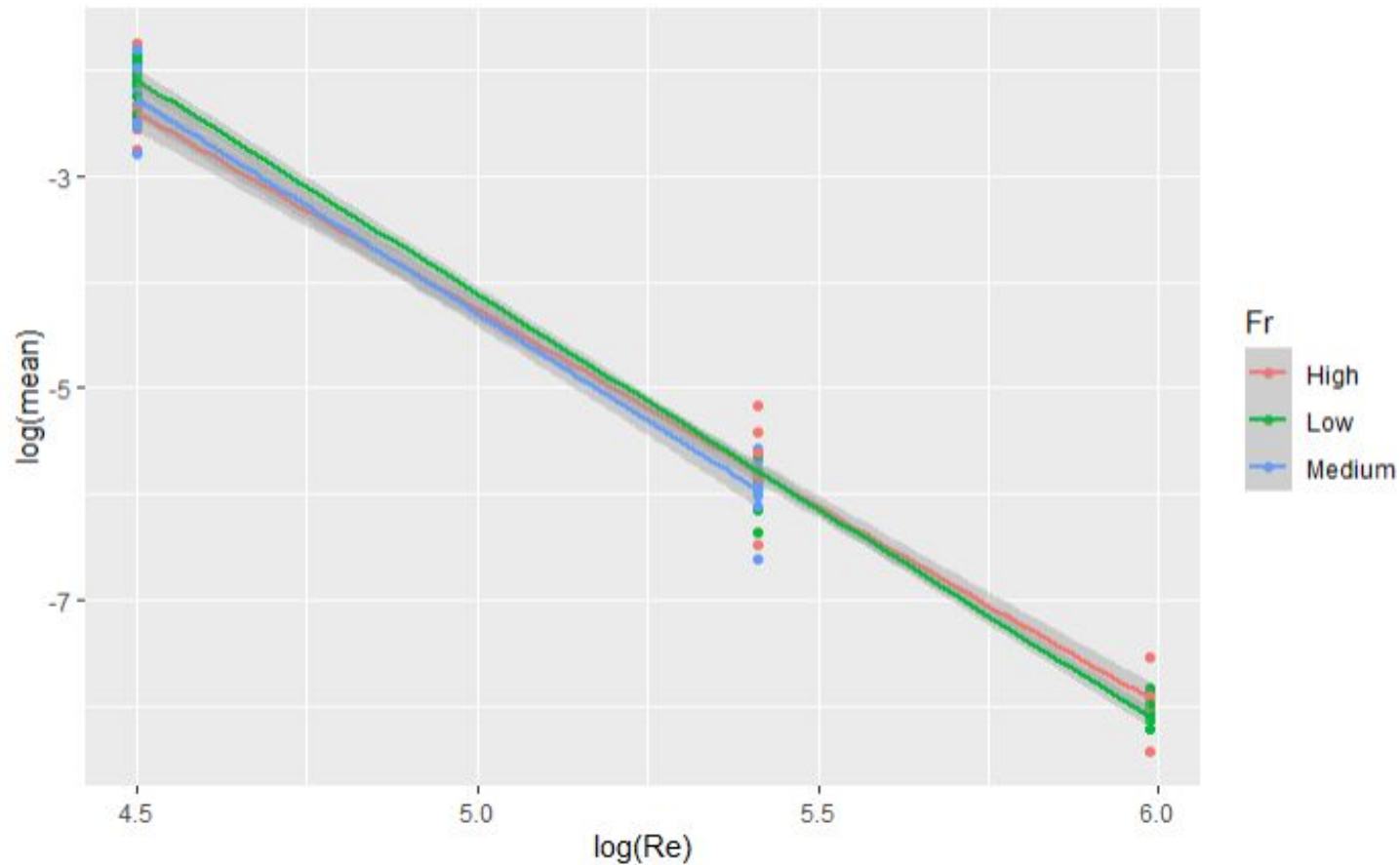
## Data Transformation

- Conversion of raw moments to mean, variance, skewness, and kurtosis
- Inverse logit transformation of Froude number and categorical binning

## Model fitting

- Log-transformation of the response
- Interaction term between log-transformed Reynolds and Froude numbers
- GAM with natural spline on Stokes number to help resolve patterns in residuals
- Log-transformation of Reynolds number to correct fanning in residuals

# Interaction of Froude and Reynolds



# Final Model for Inference

$$\log(\text{response}) = f1(\text{St}) + f2(\text{St}) + f3(\text{St}) + \log(\text{Re}) + I(\text{Fr\_low}) + I(\text{Fr\_medium}) + I(\text{Fr\_high}) + I(\text{Fr\_low}) * \log(\text{Re}) + I(\text{Fr\_medium}) * \log(\text{Re}) + I(\text{Fr\_high}) * \log(\text{Re})$$

# Inference Results

01

## Stokes Number

A low Stokes number is associated with higher flow concentration and vice versa, and incremental changes in the Stokes number are most impactful when it is low

02

## Reynolds Number

The Reynolds number has a negative linear association with flow concentration

03

## Froude Number

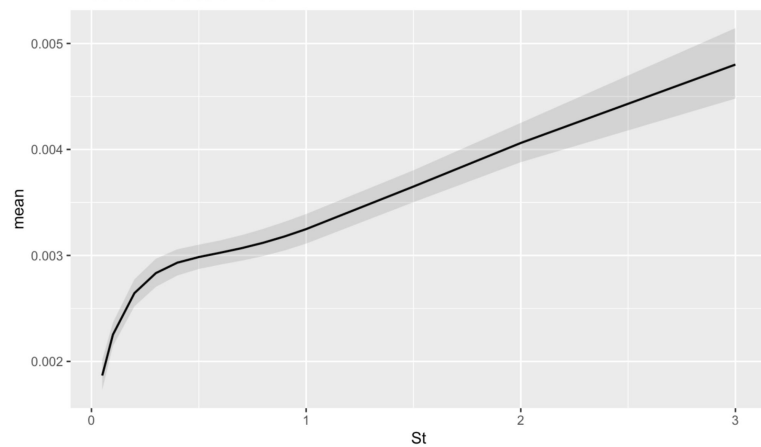
The Froude number indicates different types of flow where a low Froude number is subcritical and is associated with higher tessellation, and a high Froude number is supercritical and associated with lower tessellation

04

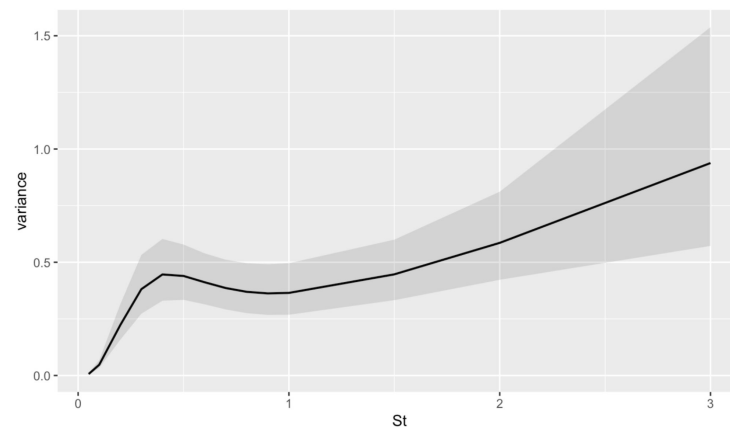
## Reynolds/Froude Interaction

When the Froude number is low, it increases the effect of the Reynolds number, suggesting that when flow is subcritical, a lower Reynolds number will result in more turbulence compared to when flow is supercritical

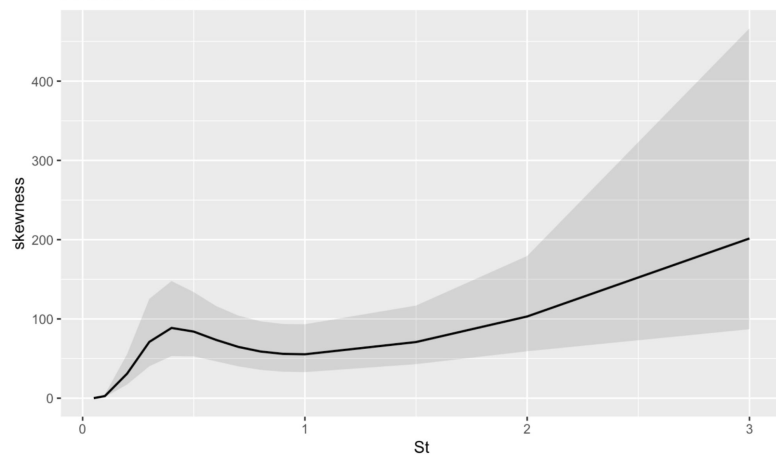
Predicted values of mean



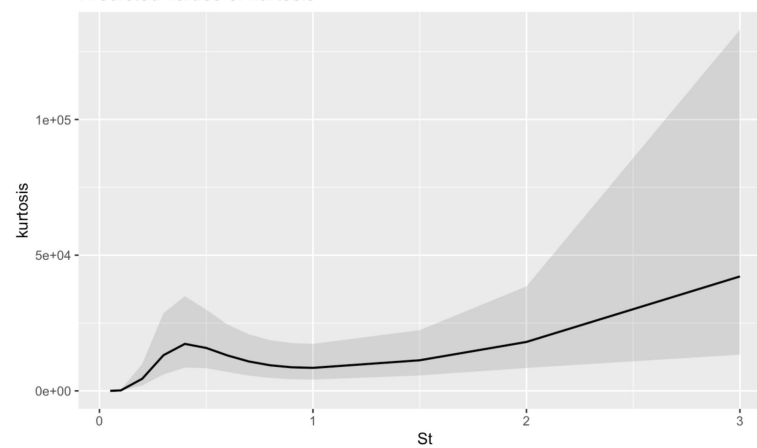
Predicted values of variance



Predicted values of skewness



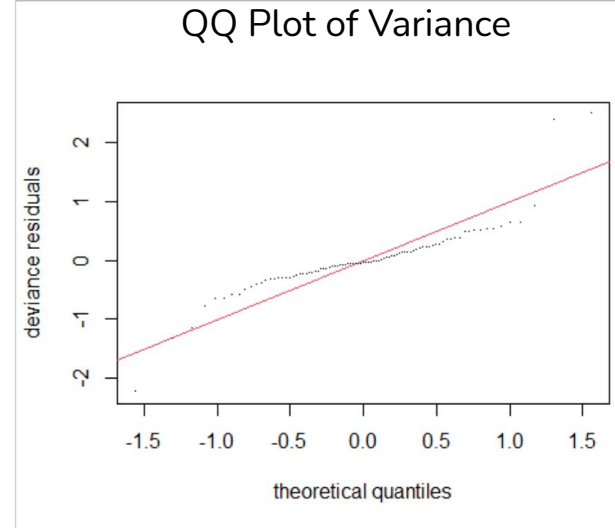
Predicted values of kurtosis





# Prediction Methodology

- Retains GAM framework from inferential model
- Violations of GAM assumptions
- Metric: Generalized Cross-Validation (GCV)
- Addition of a 4th-order polynomial in Stokes number improved fit for  $\log(\text{variance})$ ,  $\log(\text{skewness})$ , and  $\log(\text{kurtosis})$
- Low Froude number and log-transformed Reynolds number



	GCV	Mean
Mean	0.0114	0.0404
Variance	0.4805	92.486
Kurtosis	1.4571	6.19e9
Skewness	2.7995	7.53e5



# Final Models for Prediction

$$\begin{aligned} \log(\text{mean}) = & f1(\text{St}) + f2(\text{St}) + f3(\text{St}) + \text{St} + \text{St}^2 + \text{St}^3 + \text{St}^4 + \\ & \log(\text{Re}) + I(\text{Fr\_low}) + I(\text{Fr\_medium}) + I(\text{Fr\_high}) + I(\text{Fr\_low}) * \\ & \log(\text{Re}) + I(\text{Fr\_high}) * \log(\text{Re}) \end{aligned}$$

$$\begin{aligned} \log(\text{response}) = & f1(\text{St}) + f2(\text{St}) + f3(\text{St}) + \text{St} + \text{St}^2 + \text{St}^3 + \text{St}^4 \\ & + \log(\text{Re}) + I(\text{Fr\_low}) + I(\text{Fr\_medium}) + I(\text{Fr\_high}) + I(\text{Fr\_low}) \\ & * \log(\text{Re}) + I(\text{Fr\_high}) \end{aligned}$$



# Conclusion

Here are the key takeaways from our analysis:

- Influential predictors of the first four central moments are the Stokes number, log-transformed Reynolds number, Froude number, and the interaction between log-transformed Reynolds and Froude
- Stokes number is associated with **higher** flow concentration
- Log-transformed Reynolds number has a **negative linear** association with flow concentration
- Low Froude number is **subcritical** and is associated with **higher tessellation**, and a high Froude number is **supercritical** and associated with **lower tessellation**