1. Let a denote a positive constant. It is known that

$$a + \frac{5}{18}a + ...$$

is a geometric series and it converges to the sum 2106. Find the exact value of a.

Answer 1521

2. Use the Ratio Test to determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{8^n}{9^n - 5^n}$$

is convergent or divergent.

Answer Convergent

3. What is the coefficient of the term x^4 in the Taylor series of $3\cos^2 x$ at $x = 0$?
(Hint: you may want to use the formula $\cos^2 x = \frac{1+\cos 2x}{2}$ together with the standard Taylor series of $\cos t$.)

Answer: 1

4. It is known that one of the following four points lies on the plane $3x + 4y + 5z = 0$. Which one is it?
   $(3, 4, -5)$
   $(1.1.1)$
   $(2, -1, 0)$
   $(5, 4, -3)$

   Answer: $(3, 4, -5)$

5. Let O, A, B, C denote the four points $(0, 0, 0), (2, 0, 0), (0, 4, 0)$ and $(0, 0, 2)$ respectively. Let M denote the mid-point of AB and let N denote the mid-point of BC. Let L denote the line that passes through O and N. Find the perpendicular distance from M to the line L. Give your answer correct to two decimal places.

   Answer: 1.34

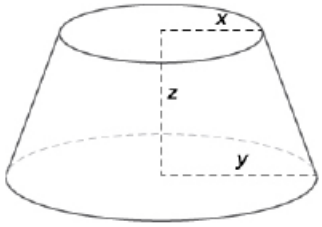
6. Let L_1, L_2 denote two lines in space. It is known that L_1 passes through the point $(-1, 0, 1)$ and L_1 is perpendicular to the plane $2x - y + 7z = 1521$. It is also known that L_2 passes through the two points $(2, -4, 18)$ and $(1, -6, 21)$. If L_1 intersects L_2 at the point (a, b, c), find the exact value of $a + b + c$.

   Answer: 16

7. Let $f(x, y, z) = \sqrt{\frac{yz^2}{x}}$. Find the directional derivative of f at the point $(2, 3, 8)$ in the direction of the vector joining $(2, 3, 8)$ to $(1, 5, 21)$. Give your answer correct to two decimal places.

   Answer: 1.64

8. The picture below shows the frustrum of a right circular cone where x is the top radius, y is the base radius and z is the height of the frustrum. If x is increasing at a rate of 1.521 m/s (i.e. metre per second), y is increasing at a rate of 2.02 m/s and z is decreasing at a rate of 5.99 m/s, estimate the rate of change of its volume in cubic metre per second when $x = 15$ metres, $y = 20$ metres and $z = 30$ metres. Give your answer correct to two decimal places.
(Hint: You can use the formula $V = \frac{1}{3}\pi z(x^2 + xy + y^2)$ for the volume of the frustrum.)



Answer: 77.23

9. Let a denote a positive constant. Let S denote the plane $x - 2y + 2z = a$. Let $f(x, y, z)$ denote a function of three variables defined in the following way: for each point (x, y, z), let d denote the perpendicular distance from this point to S; then we define $f(x, y, z) = d^2$. If the maximum rate of increasing of f at the point $(1, 2, 3)$ is equal to 2020, find the exact value of a.

   Answer: 3033

10. Let (r, θ) denote the polar coordinate system and (x, y) denote the Cartessian coordinate system of a two dimensional plane. Recall that they are related by the following two equations:

$$x = r\cos\theta, \quad y = r\sin\theta.$$

Find the value of

$$\frac{\partial^2\theta}{\partial y \partial x}$$

when $r = 0.7$ and $\theta = \frac{2\pi}{5}$. Give your answer correct to two decimal places.

Answer: 1.65