Lecture 3

Einstein's Special Relativity : Applications

Lecture 3

Einstein's Special Relativity : … a theory for fast and slow moving objects !

Revisited

Review results again

+ … due to Lorentz transformation

Review results again

+ … … another derivation for time dilation

Einstein’s Relativity

Time Dilation Revisited

Figure
A light clock. Light will bounce up and down between parallel mirrors and “tick off” equal intervals of time.

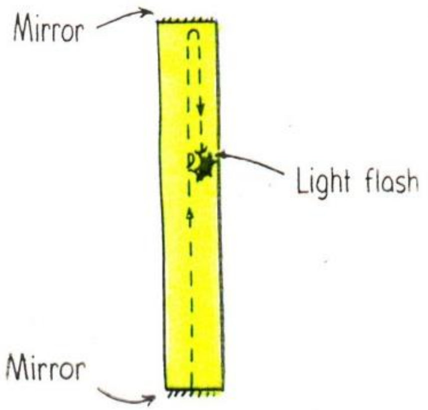
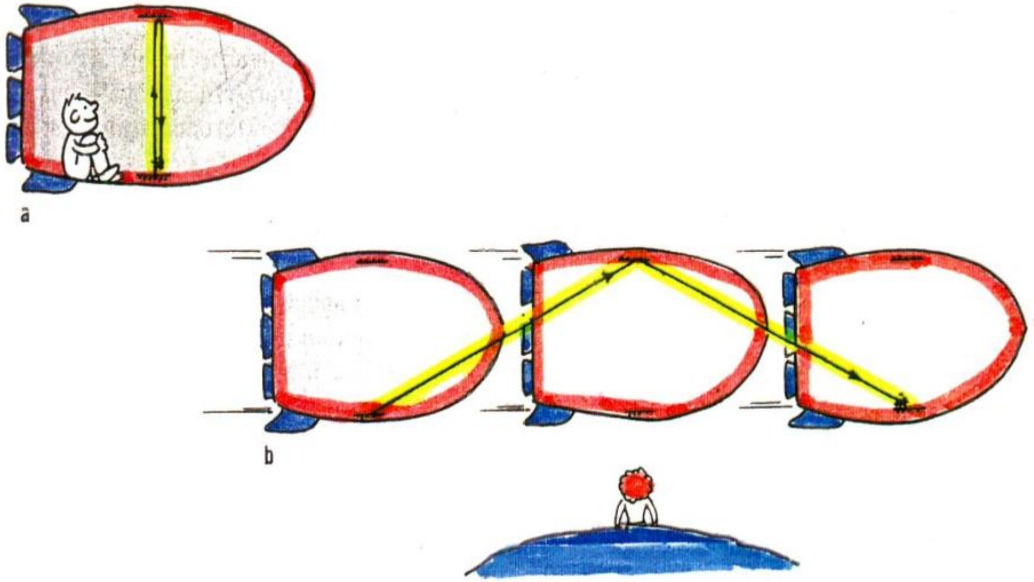


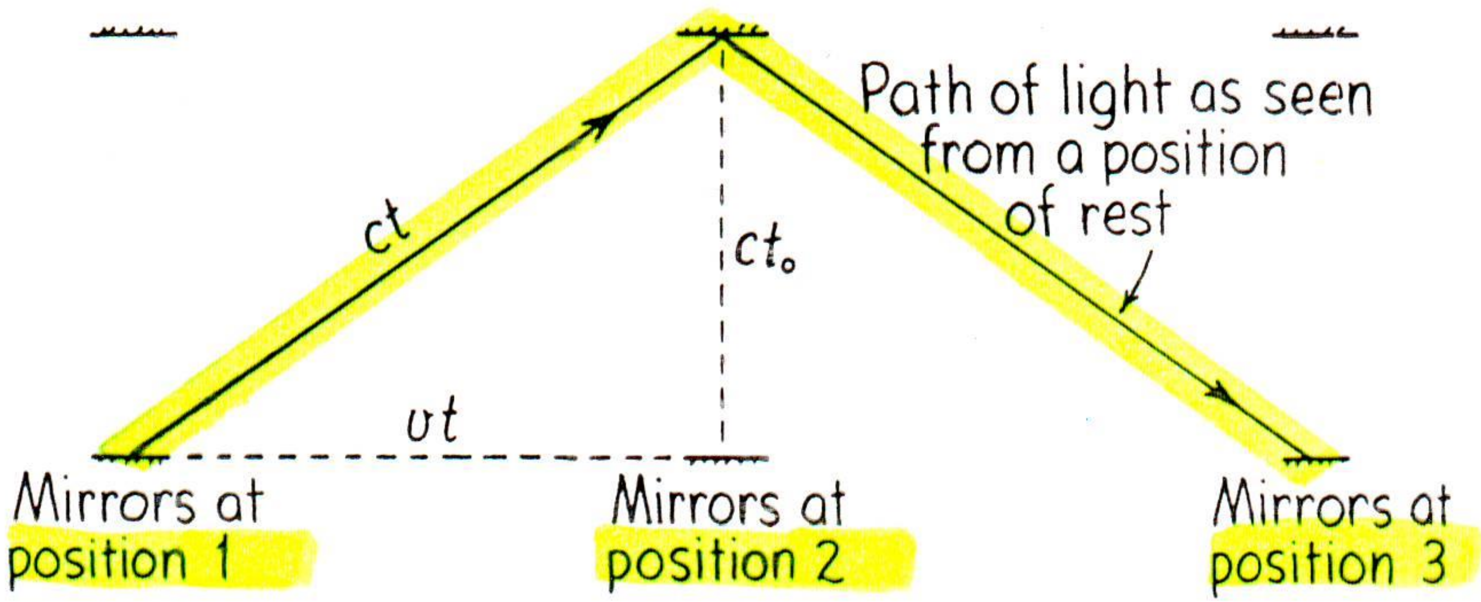
Figure
(*a*) An observer moving with the spaceship observes the light flash moving vertically between the mirrors of the light clock. (*b*) An observer who is passed by the moving ship observes the flash moving along a diagonal path.

Special Relativity

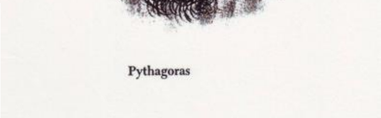


We will only derive the Time Dilation expression !
… sometimes found in books

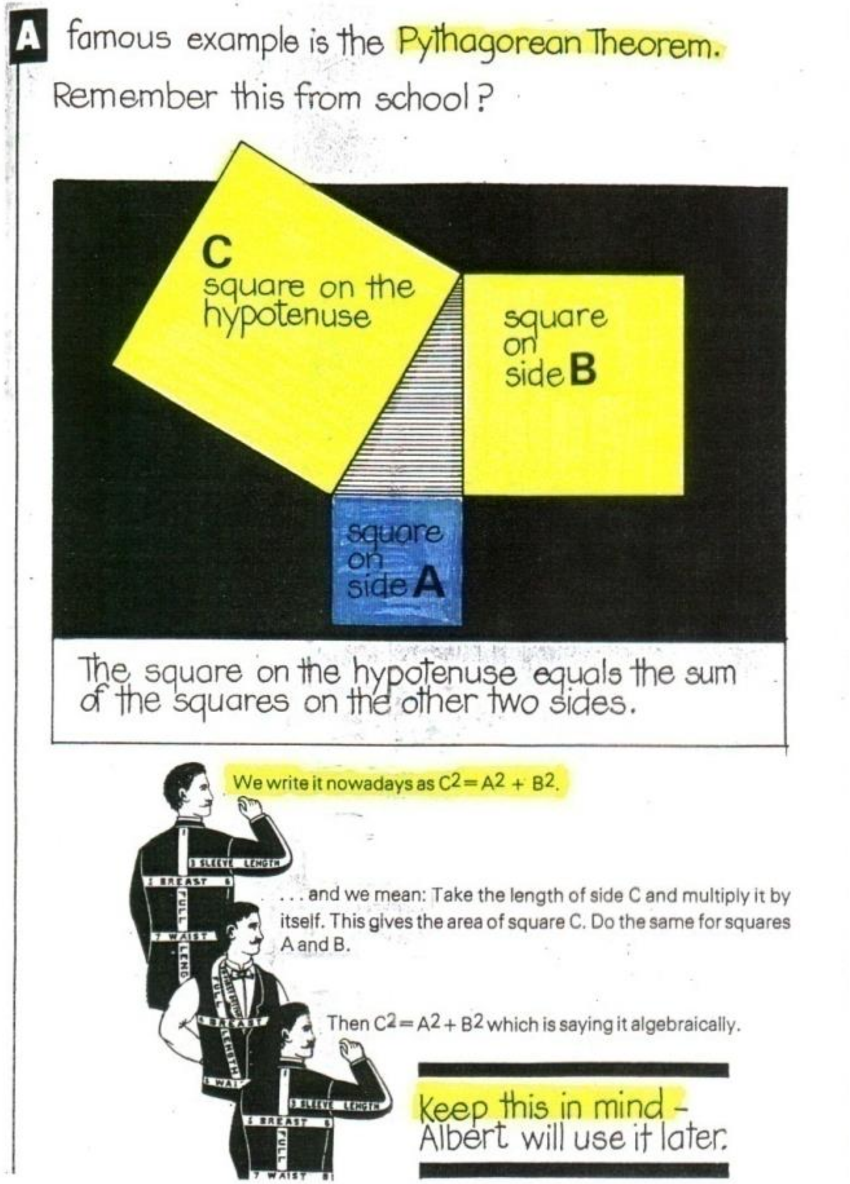
Time Dilation Detailed Analysis

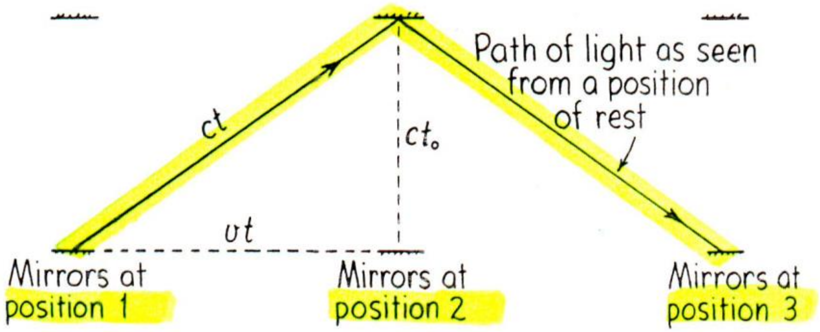


Einstein’s S. Relativity



Pythagoras





Time Dilation

Time is longer for moving frames.

By how much ?

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$c^2t^2 = c^2t_0^2 + v^2t^2$$

$$c^2t^2 - v^2t^2 = c^2t_0^2$$

$$t^2[1 - (v^2/c^2)] = t_0^2$$

$$t^2 = \frac{t_0^2}{1 - (v^2/c^2)}$$

$$t = \frac{t_0}{\sqrt{1 - (v^2/c^2)}}$$

$$t = \gamma\, t_0$$

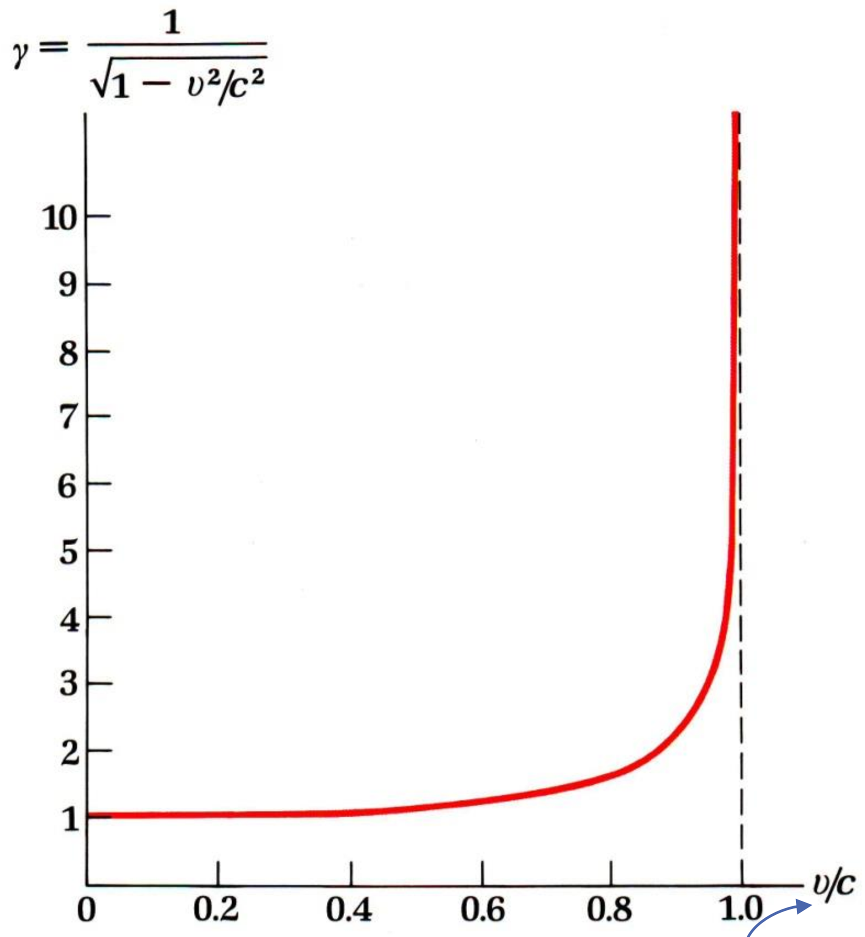
Try to check what happens when $v = 0$ and $v = c$!

What is Gamma again?

Any physical quantity multiplied by gamma will “blow up” eventually.

i.e. when quantities moves near the speed of light … things behave strangely …

… there is a limit to speed



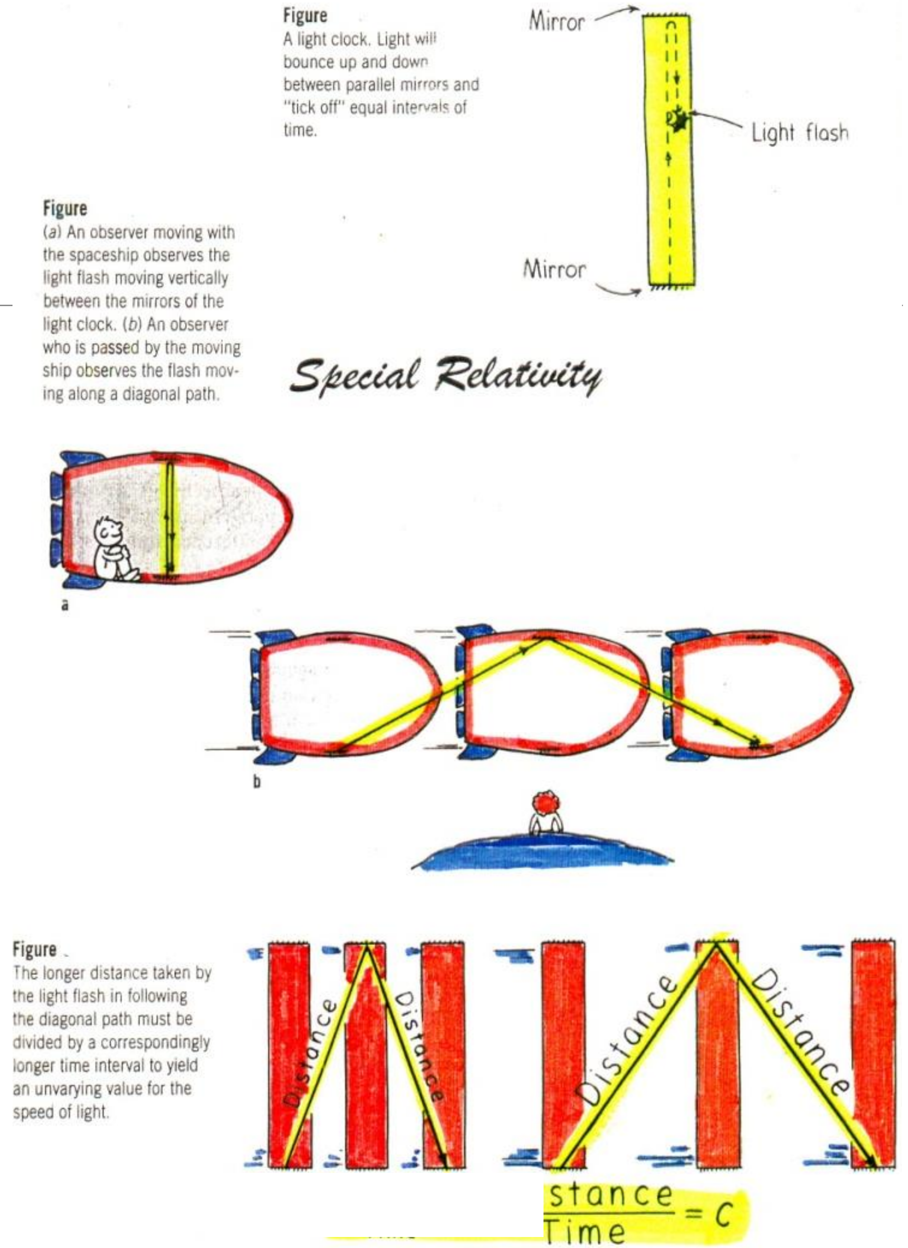
$$\beta = \frac{v}{c}$$

What happens to gamma, when $v = c$?

Einstein’s S. Relativity

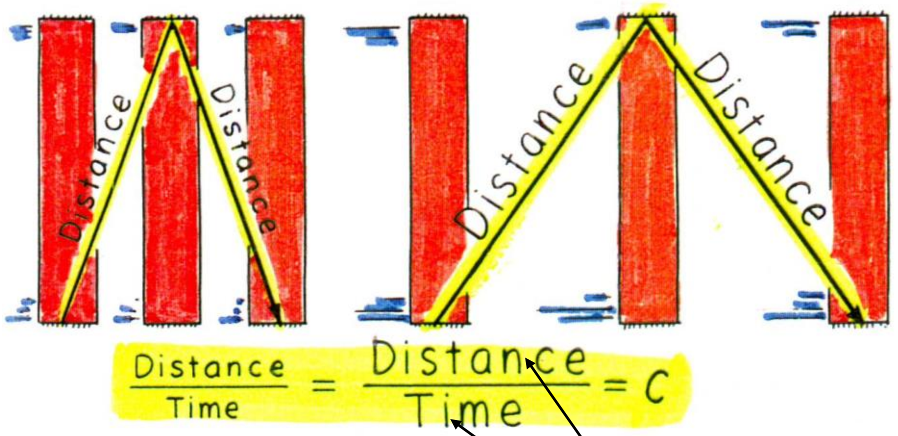
Time Dilation

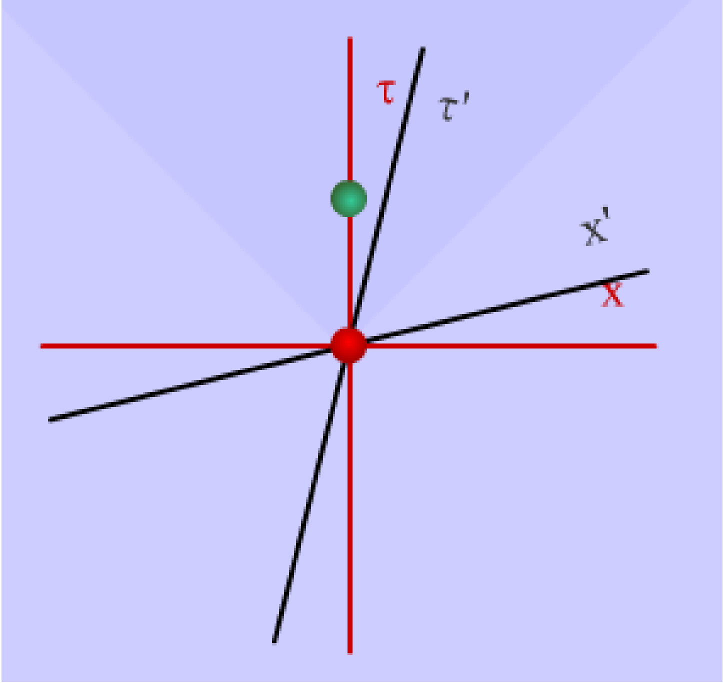
What else can we learn ?



Einstein's Special Relativity

Where is the culprit ?





Same Distortion

$$c = 1$$

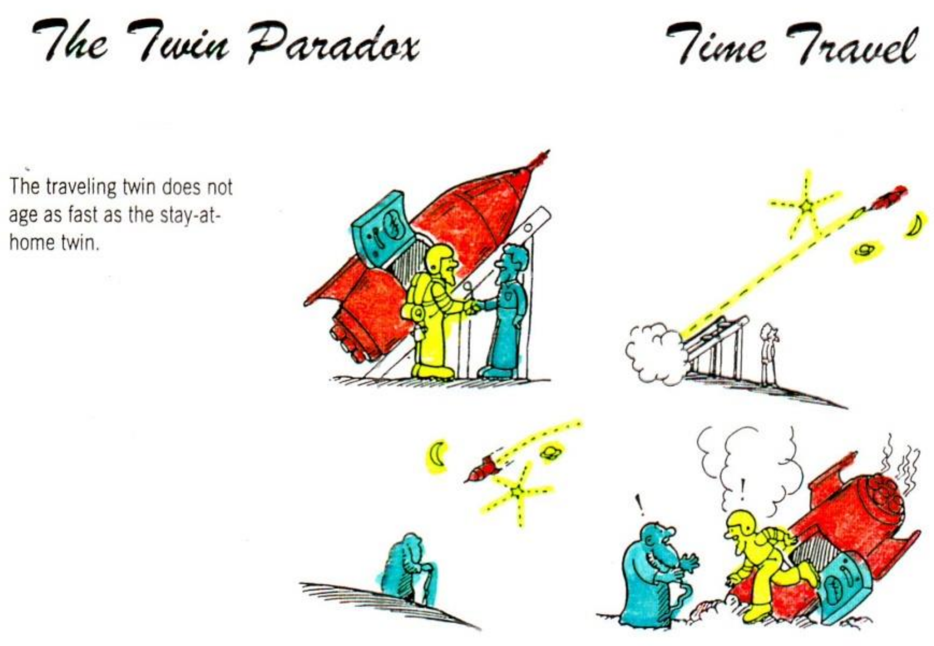
Like 2 sides of a coin … these distortions cannot be separated.

Time dilation & Twin Paradox

Seems Father & Son may "switch" roles as time flows.

Can we ask a question the other way round?

Where is the Paradox ?



It is quite easily resolved, but seems to possess some hidden emotional content that makes it the subject of interminable debate among dilettantes of relativity. W. Rindler

Space travel, Why don't we ?

Old Argument

Our life span is too short

Problems

Not enough propulsive energy

Shielding against radiation

Etc ...

Space Travel to think about.

We can see into the past, but we cannot trip into the past.

See if we can appreciate this poem ?

There was a young lady named Bright
Who traveled much faster than light
She departed one day, in a relative way
And returned on the previous night.

Paul Hewitt, Conceptual Physics

More consequences !

Revisited

Length contraction revisited

+ … !

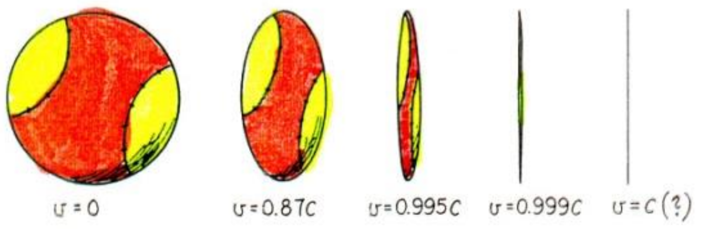
Figure
The Lorentz-FitzGerald contraction. As speed increases, length in the direction of motion decreases. Lengths in the perpendicular direction do not change.

Time Dilation

Length Contraction

Compare

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$



with

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\, t_0$$

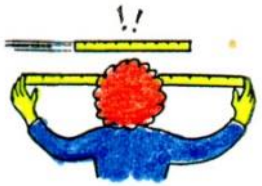
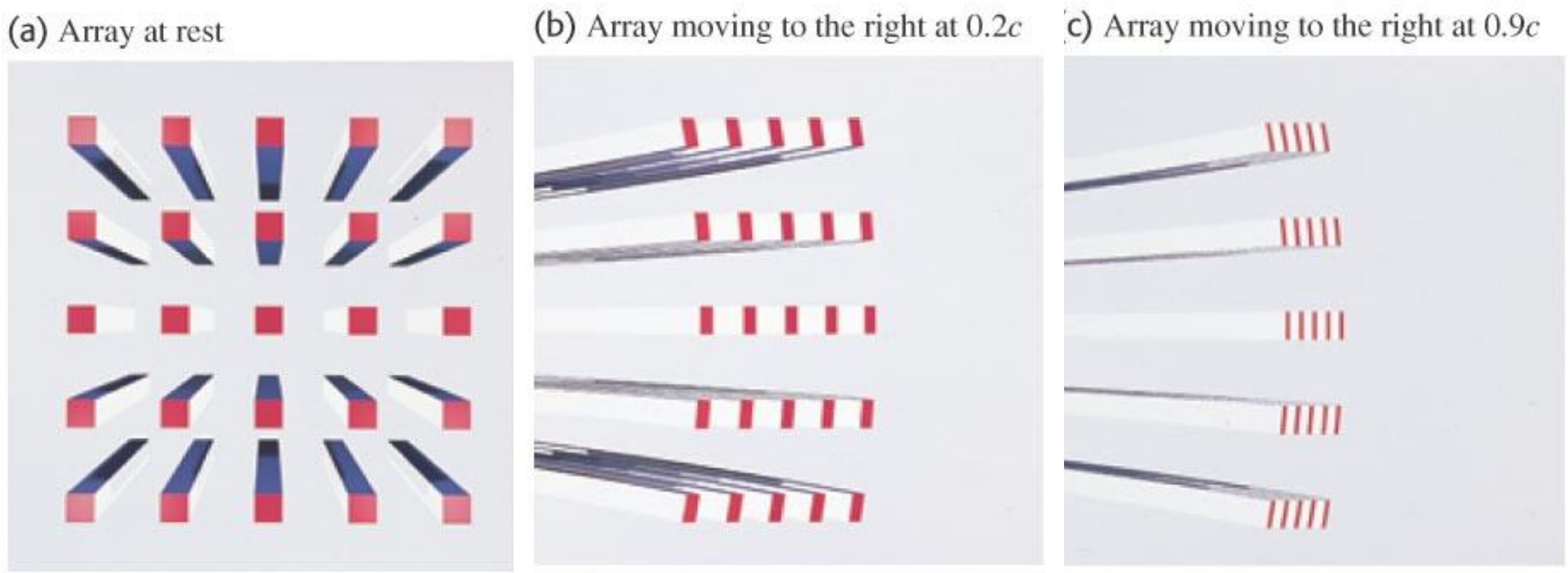


Figure
The meter stick is measured to be half as long when traveling at 87 percent the speed of light relative to the observer.

Are the appearance of 3D objects in relativistic motion really shorten ?

How does an object moving at velocities appear to an observer ?



It will appear rotated !

We do not see all the points simultaneously; light from points farther from us take longer to reach us does light from points near to us, so we see the farther points at the positions they had at earlier times.

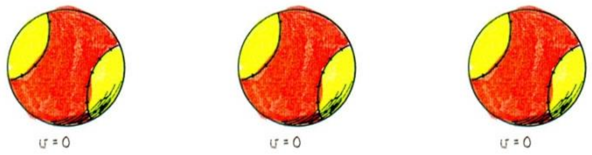
We can see some points that we couldn't see when the rod was at rest because the rod moves out of the way of the light rays from those points to us. Conversely, some light that can get to us when the rod is at rest just blocked by the moving rod. Hence it appears rotated and distorted.

University Physics by Young & Freedman

(c) Array moving to the right at $0.9c$

Length Contraction

But in reality (3D) …
what will you see ?



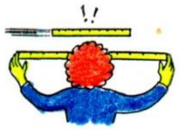
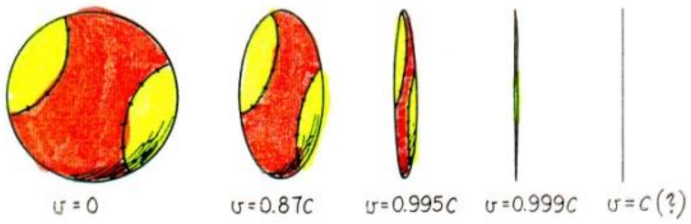


Figure
The meter stick is measured to be half as long when traveling at 87 percent the speed of light relative to the observer.

Figure
The Lorentz-FitzGerald contraction. As speed increases, length in the direction of motion decreases. Lengths in the perpendicular direction do not change.

Time Dilation



Ball is 2D

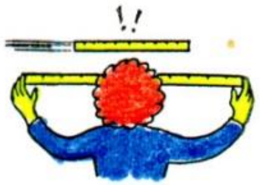
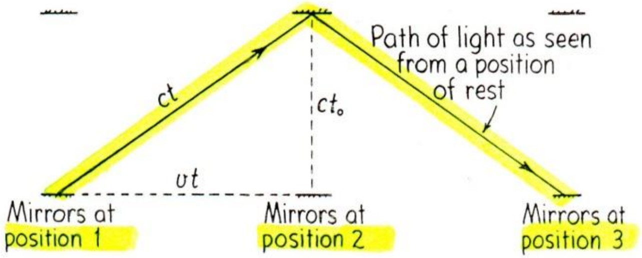


Figure
The meter stick is measured to be half as long when traveling at 87 percent the speed of light relative to the observer.

Dilation & Contraction

Time dilation (longer) is related to Length contraction (shorter)

The Muon, μ Puzzle



Muon Decay Paradox

$$In\ \ Lab: 2.2\times10^{-6}\,\mathrm{sec}\ \ or\ \ 2.2\ micro\,\mathrm{sec}$$

But the time for the muon to come down by 2000m is ~ 6.4 micro sec.

$$c^2t^2 = c^2t_0^2 + v^2t^2$$

$$c^2t^2 - v^2t^2 = c^2t_0^2$$

$$t^2[1-(v^2/c^2)] = t_0^2$$

$$t^2 = \frac{t_0^2}{1-(v^2/c^2)}$$

$$t = \frac{t_0}{\sqrt{1-(v^2/c^2)}}$$

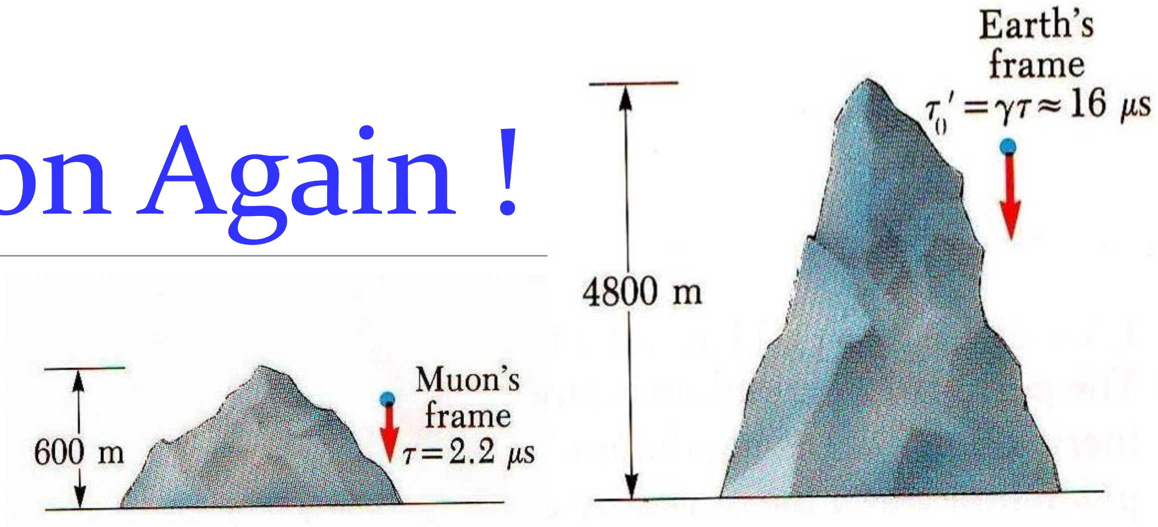
Cosmic.mov

$$t = \gamma\, t_0$$

Time Dilation

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

An Example: Muon Again !



In the example above, an observer in the **muon's frame** would measure the t_0 = 2.2μs lifetime, while an earth-based observer measures the L_0 = 4800m height of the mountain.

In the muon's frame, there is no time dilation, but the distance of travel, L is observed to be shorter when measured in this frame.

Likewise, on the **earth observer's frame;** observes that there is time dilation on the muon, but the distance of travel is measured to be actual height, L_0 of the mountain.

There is a kind of "offsetting" effect but **the outcome should be the same !**

Simultaneity (is relative !)

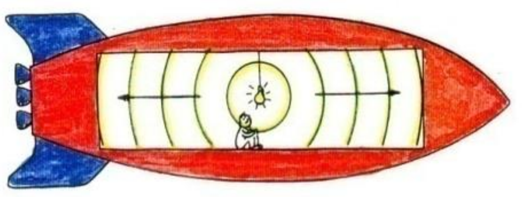
Note:

Simultaneous events in the ordinary sense means that they occur at the same time.

But consequence of Einstein's second postulate:

2 events that are simultaneous in one frame of reference need not be simultaneous in a frame moving relative to the first frame.

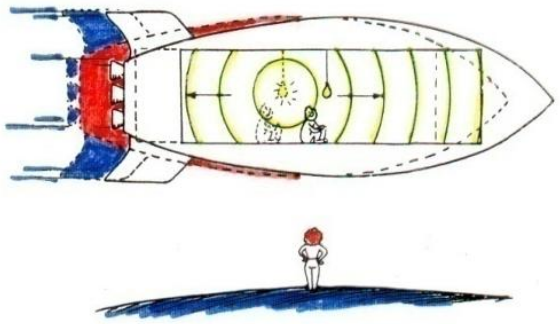
Figure
From the point of view of the observer who travels with the compartment, light from the source travels equal distances to both ends of the compartment and therefore strikes both ends simultaneously.



Relativity

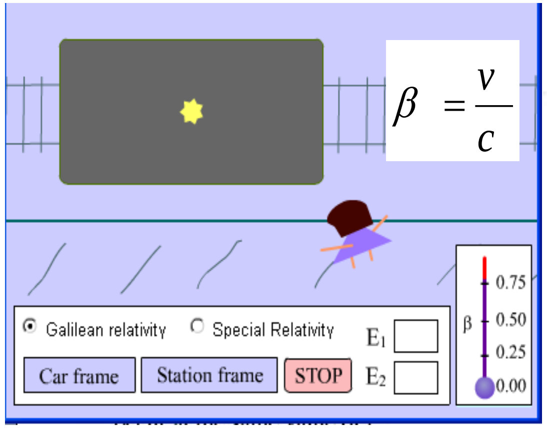
Figure
The same events of light striking the front and back of the compartment are not simultaneous from the point of view of an observer in a different frame of reference. Because of the ship's motion, light that strikes the back of the compartment doesn't have as far to go and strikes sooner than light that strikes the front of the compartment.

Special Relativity

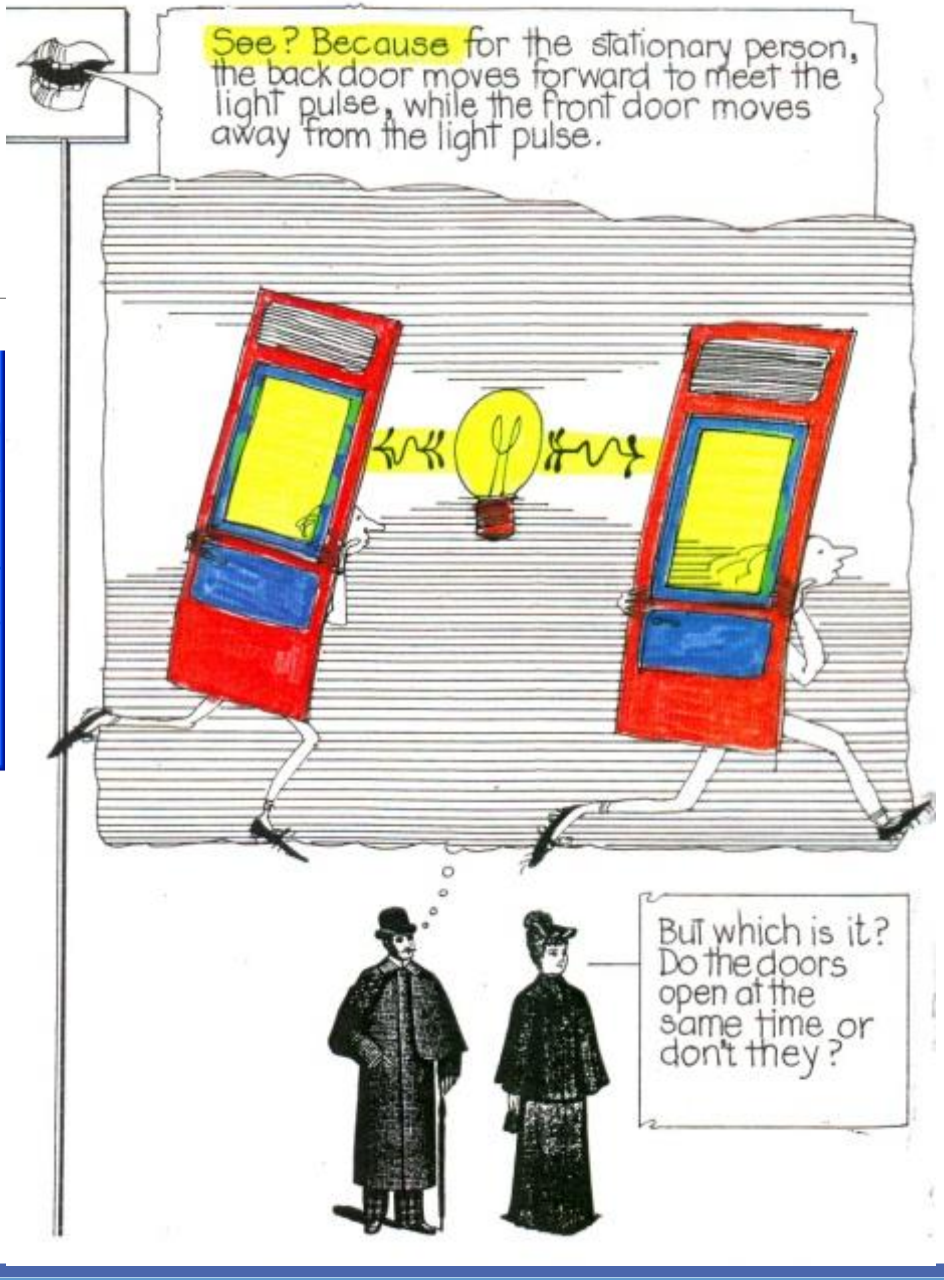


Simultaneity

Non-simultaneity of events in one frame may be simultaneous in another co-ordinate system is a relativistic result. Speed of light is always the same for all observers.



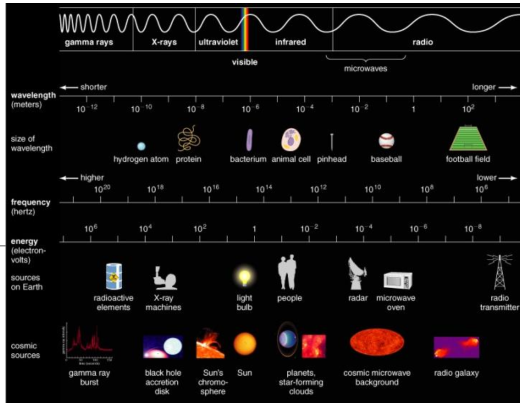
Simult.swf

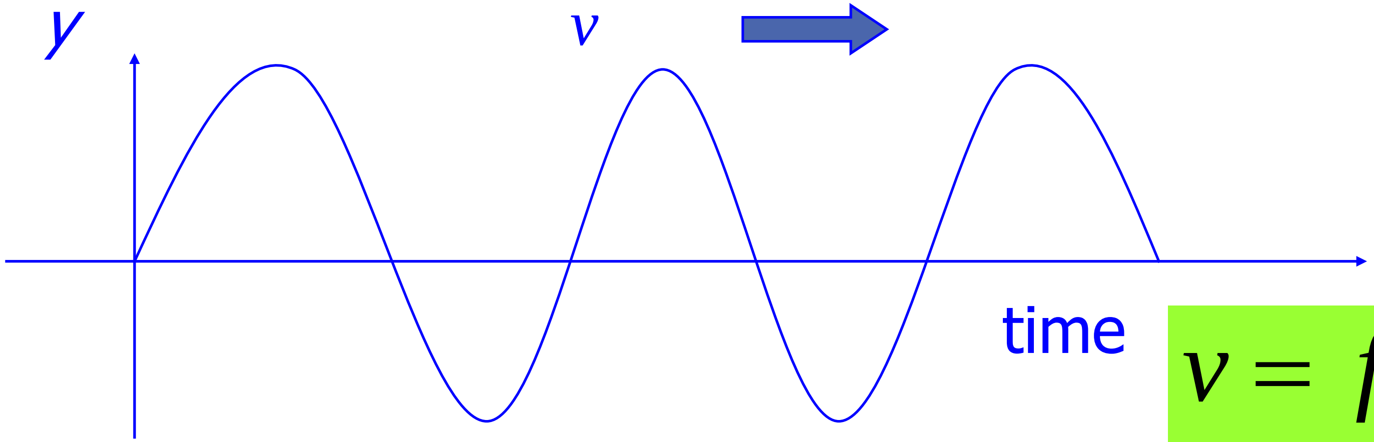


So why don't we see this effect easily in our MRT train ?

Relativistic effects of light !

Wave Ideas (Colours)





$$c = f\lambda$$

$$v = f\lambda$$

$$f = \frac{1}{T}$$

v = velocity or speed

f = frequency

Lamda = Wavelength

T = period

Shorter wavelength means **bluish**.

Longer wavelength means reddish

Recall Sound Doppler Effect

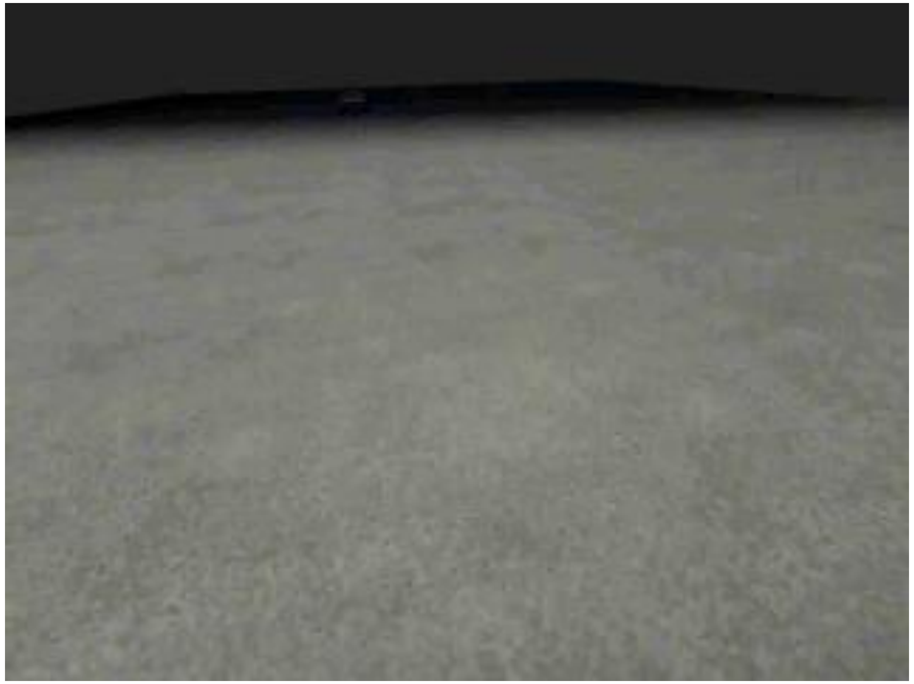
Source

v

330 to 340 m/s

Listener

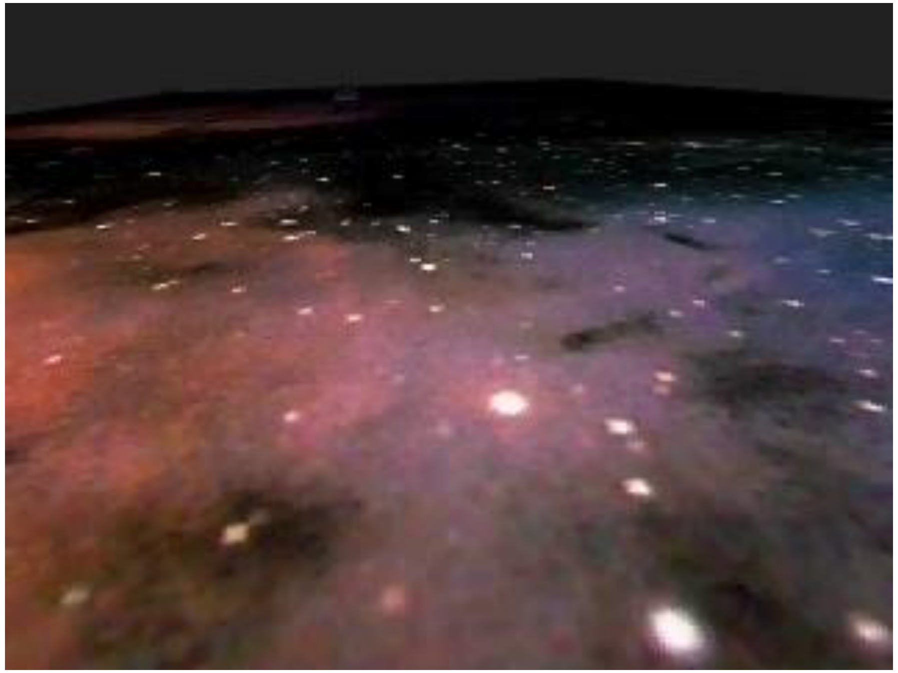
Sound seems to change pitch (frequency) !



Relativistic Doppler Effect

[Electromagnetic Waves]()

[c = speed of light]()



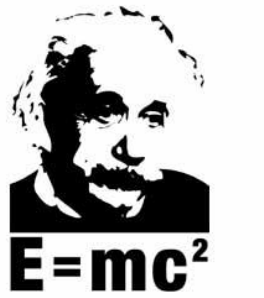
Light seems to change colour !

Other Optical Effects

http://www.youtube.com/watch?v=JQnHTKZBTI4

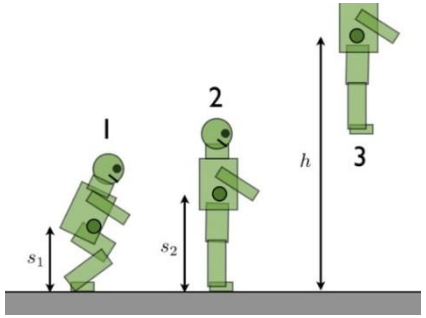
Other physical quantities may need modifications !

Relativistic Momentum



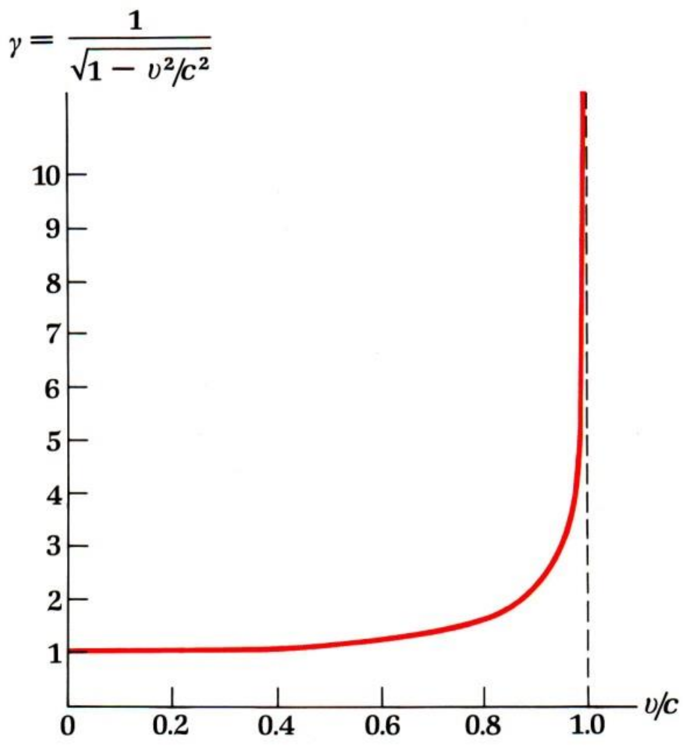
Recall Impulse in Newton's Laws

$$F = m_0 a$$
$$= m_0 \left(\frac{\Delta v}{\Delta t}\right)$$
$$F\Delta t = \Delta m_0 v = \Delta(m_0 v)$$
$$= \Delta p$$



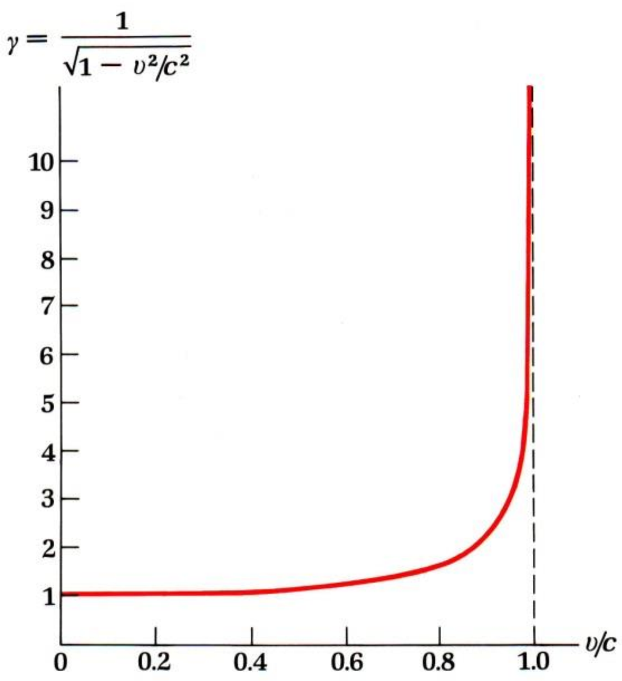
Relativistic Momentum is now modified

$$\boxed{p = \gamma m_0 v}$$



Relativistic Momentum

Does this mean that momentum can increase without any limit?
Does this mean that speed can also increase without any limit?



Caveat Emptor

In Newtonian ideas, the particle behave as if their masses increase with speed. Einstein initially favoured this interpretation but later changed his mind to keep mass a constant, an intrinsic property of matter that is the same in all frames of reference. So it is gamma that changes with speed, not mass.

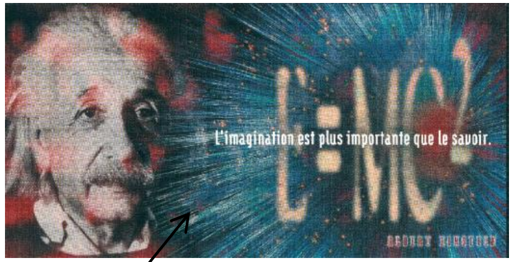
Valid Equation but …

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\, m_0$$

Mathematics is not Physics

Physics is also not Mathematics

Kinetic Energy, K & ($E = mc^2$)



(Newtonian) $K = \int_{x_1}^{x_2} F\,dx$

$$K = \int_{x_1}^{x_2} F\,dx = \int_{x_2}^{x_1} m_0 \frac{dv}{dt}\,dx = \int_0^v m_0 v\,dv = \frac{1}{2} m_0 v^2$$

(Einsteinian)

$$K = \int_{x_1}^{x_2} F\,dx = \int_0^v \frac{d}{dt}(mv)\,dx = \int_0^v (mv\,dv + v^2\,dm)$$

$$K = \int_{m_0}^{m} c^2\,dm = mc^2 - m_0 c^2$$

Hint :

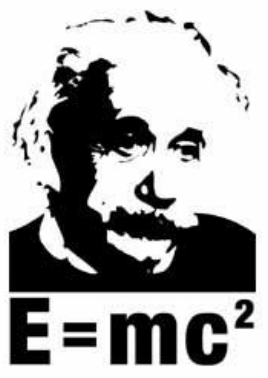
$$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \gamma\, m_0$$

Note: $mvdv + v^2 dm = c^2 dm$

Rest Mass, $E = m_o c^2$

Compare with (Classical Newtonian)

$$K = \frac{1}{2} m_0 v^2$$



Einstein (Relativistic)

$$K = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2 = (\gamma - 1) m_0 c^2$$

recall $m = \gamma m_0$

$$K = mc^2 - m_0 c^2$$

If $\frac{v^2}{c^2}$ is very small, $v << c$

Recall : $(1+x)^n = 1 + nx + n(n-1)\frac{x^2}{2!} + \ldots$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots$$

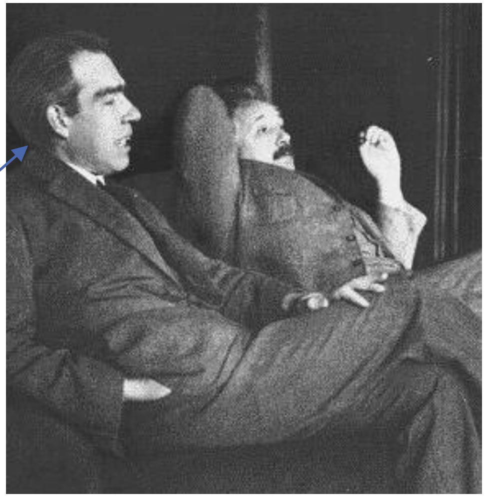
$$K = \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots - 1\right) m_0 c^2$$

$$K = \frac{1}{2} m_0 v^2 + \frac{3}{8}\frac{m_0 v^4}{c^2} + \ldots$$

Sec. school …

Binomial expansion

$$(1+x)^2 = 1 + 2x + x^2$$



Correspondence Principle … due to Bohr

Are there such real and physical Evidences in nature ?

Atomic (Nuclear) Bombs

$$\text{Re}\,call \quad : \; K = mc^2 - m_0c^2$$

Rearrange

$$mc^2 = m_0c^2 + K$$

$$E_{total} = mc^2$$

A Comment !

It tells us that even if Kinetic energy, K is zero (mass is not moving), it still has tremendous energy lockup as rest mass, m_0.

In short, it also tells us that mass can be converted to energy and vice versa. Unified Mass and Energy ...

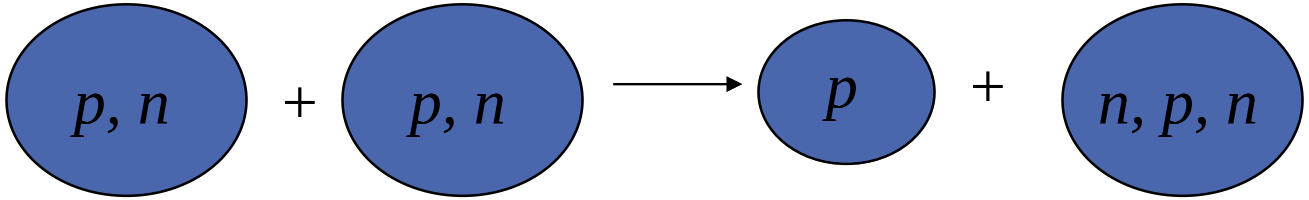
Mass is not equivalent (*only loosely*) to energy !

Einstein on $E = mc^2$

http://www.youtube.com/watch?v=CC7Sg41Bp-U&feature=related

Are there more (look deeper) such real and physical Evidences in nature ?

Mass-Energy Equivalence



$$\text{Deuterium} + \text{Deuterium} \longrightarrow \text{Hydrogen} + \text{Tritium}$$

$$m_0 \ of \ Deuterium = 2.0141019 \ amu$$

$$m_0 \ of \ Hydrogen = 1.0078252 \ amu$$

$$m_0 \ of \ Tritium = 3.0160494 \ amu$$

$$1 \ amu = 1.66 x 10^{-24} \ gm$$

Notice something strange ?

Mass Energy Conversion

$$LHS \neq RHS$$

$2D = 4.0282038\, amu \qquad H + T = 4.0238746\, amu$

Where did the extra 0.0043292 *amu* mass go to?

The mass destroyed appears as energy in this reaction.

Other examples:

A hot cup of coffee has more mass than the same cup of coffee when it is cold.
A wound up clock has more mass than same clock when it is unwound.

Note: The whole is slightly less than it's parts.

About this cartoon

$$E = m_0 c^2 \quad \text{but} \quad E^2 \neq \left(m_0 c^2\right)^2$$

You will know more at Tutorial ?

But a more “**correct**” Quantity: E^2

$$E^2 = (m_0 c^2)^2 + (pc)^2$$

Relates:

Total energy equals rest energy and momentum, *pc* of **any** moving particle.

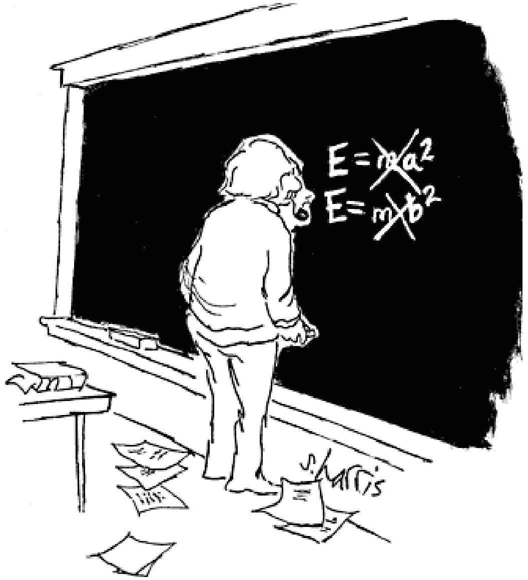
Valid Equation but …

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0$$

Also suggests that a particle may have energy & momentum even when it has no mass.

Example: A photon’s mass is zero but it has got momentum. … *p = mv, how so ?* But what is a photon ?

About this cartoon



$$E = m_0 c^2 \quad \text{but} \quad E^2 \neq \left(m_0 c^2\right)^2$$

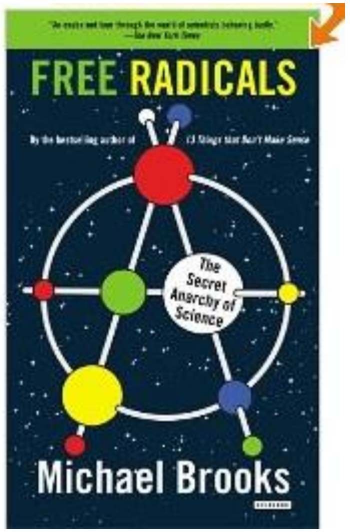
You will know more at Tutorial ?

Surprise : Einstein did not manage to prove $E = mc^2$!

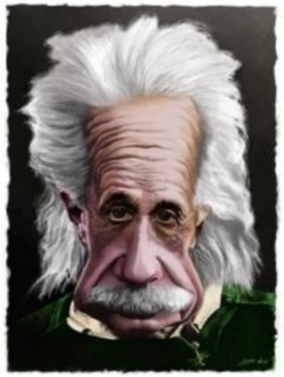
"Einstein was entirely cavalier about the sacred process of science …
Einstein once advised that if you want to know how theoretical physics gets done, the last person *you should ask is a theorist …*

The final attempt to prove $E = mc^2$ came in 1934 … four hundred scientists were given the treat of watching him remodel his universe … But the proof was still wrong … The error had been pointed out years by no less an authority than Max Planck."

Michael Brooks



Why should I learn some Relativity ?



Nothing puzzles me more than time and space; and yet nothing troubles me less, as I never think about them.

C. Lamb

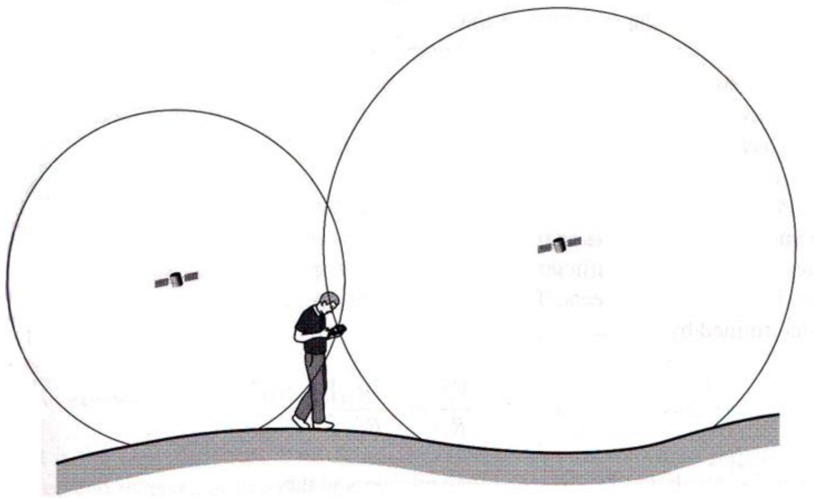
While I know what it is to be treated something like a lion, I would rather like to become something of a Lamb.

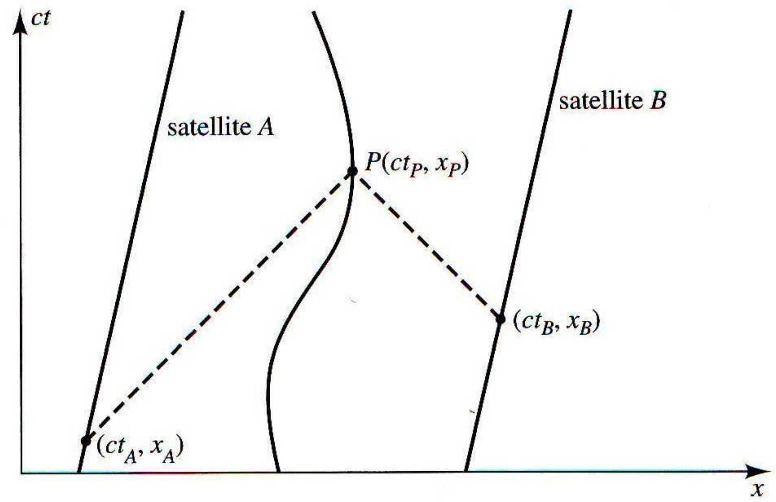
Eddington

Global Positioning System (GPS)

You will be thankful that there is relativistic corrections.

Lost in a jungle or out at sea … and waiting for rescue … ~ 10 meter error.





Radar Gun

The gun emits a radio beam of frequency f_0 which in the frame of reference of an approaching car has a higher frequency, f. The reflected beam also has a frequency in the car's frame, but has an higher $f\,'$ in the police' officer's frame. The radar gun calculates the car's speed by comparing the frequencies of the emitted beam and the doubly Doppler–shifted reflected beam.

$$v = f\lambda \qquad c = f\lambda$$

Young and Freedman, *University Physics*

Original Video of Einstein

http://www.youtube.com/watch?v=-GhN3_4kOdo

A Summary

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

No Ether (Aether) Lorentz/Einstein transformation

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Speed of Light is always *c* for all observers (Invariant quantity)

Simultaneous Events (not always)

$$t = \gamma t_0 \qquad L = \frac{L_0}{\gamma}$$

Distortion in Space and Time (mixed)

Massless particles can have momentum ?

$$E^2 = (m_0 c^2)^2 + (pc)^2$$

Non-moving mass can have energy too

$$E_{Total} = mc^2 = m_0 c^2 + K$$

Time Dilation is Real

Mass-Energy conversion (Unification)

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0$$

You may add one more equation ??

Mathematics is not Physics

Physics is also not Mathematics …

… physics is closer to philosophy