MA 1521
Tutorial 3

1. Evaluate the following definite integrals.

   (a) $\displaystyle\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$.

   (b) $\displaystyle\int_{-4}^4 |x|\, dx$.

   (c) $\displaystyle\int_0^\pi \frac{1}{2}(\cos x + |\cos x|)\, dx$.

   (d) $\displaystyle\int_0^\pi \sin^2\left(1 + \frac{\theta}{2}\right) d\theta$.

   Ans. (a) $1 + \sqrt{2} - 2^{3/4}$ (b) 16   (c) 1   (d) $\frac{1}{2}\pi + \sin 2$

2. Using the fundamental theorem of Calculus, find the derivative dy/dx for the following functions.

   (a) $y = \displaystyle\int_0^{\sqrt{x}} \cos t\, dt$.

   (b) $y = \displaystyle\int_0^{x^2} \cos \sqrt{t}\, dt$.

   (c) $y = \displaystyle\int_0^{\sin x} \frac{dt}{\sqrt{1 - t^2}}, \qquad |x| < \frac{\pi}{2}$.

   Ans. (a) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (b) $2x\cos x$ (c) 1

3. Using the *substitution* method, or otherwise, find the following integrals.

   (a) $\displaystyle\int x^{1/2} \sin(x^{3/2} + 1)\, dx$.

   (b) $\displaystyle\int \csc^2 2t \, \cot 2t \, dt$.

   (c) $\displaystyle\int \frac{1}{\theta^2} \sin\frac{1}{\theta} \cos\frac{1}{\theta}\, d\theta$.

   (d) $\displaystyle\int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)}\, dx$.

   (e) $\displaystyle\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta} \cos^3 \sqrt{\theta}}\, d\theta$.

   Ans.
   (a) $-\frac{2}{3} \cos(x^{3/2} + 1) + C$
   (b) $-\frac{1}{4} \cot^2 2t + C$
   (c) $-\frac{1}{2} \sin^2 \frac{1}{\theta} + C$
   (d) $6 \ln |\tan^3 x + 2| + C$
   (e) $\sec^2 \sqrt{\theta} + C$

4. Applying the method of *integration by parts*, or otherwise, find the following integrals.

   (a) $\displaystyle\int x \sin\left(\frac{x}{2}\right) dx$.

   (b) $\displaystyle\int t^2 \, e^{4t}\, dt$.

   (c) $\displaystyle\int e^{-y} \cos y \, dy$.

   (d) $\displaystyle\int \theta^2 \sin(2\theta)\, d\theta$.

   (e) $\displaystyle\int z \, (\ln z)^2 \, dz$.

   Ans.
   (a) $-2 \left[x \cos\left(\frac{x}{2}\right) - 2 \sin\left(\frac{x}{2}\right) \right] + C$
   (b) $(\frac{t^2}{4} - \frac{t}{8} + \frac{1}{32}) e^{4t} + C$
   (c) $\frac{e^{-y}}{2}(\sin y - \cos y) + C$
   (d) $-\frac{1}{2} \left[\theta^2 \cos(2\theta) - \theta \sin(2\theta) - \frac{1}{2} \cos(2\theta) \right] + C$
   (e) $\frac{1}{2} \left[z^2 (\ln z)^2 - z^2 (\ln z) + \frac{z^2}{2} \right] + C$