Tutorial 1

Exercise 1

6. Consider the system of linear equations

$$\begin{cases} 3x + 4y - 5z = -8 \\ x - 2y + z = 2. \end{cases}$$

(a) For any real number t, verify that $x = \frac{1}{5}(-4 + 3t)$, $y = \frac{1}{5}(-7 + 4t)$, $z = t$ is a solution to the linear system.

(b) Write down two particular solutions to the system.

9. (a) Does an inconsistent linear system with more unknowns than equations exist?

(b) Does a linear system which has only one solution, but more equations than unknowns, exist?

(c) Does a linear system which has only one solution, but more unknowns than equations, exist?

(d) Does a linear system which has infinitely many solutions, but more equations than unknowns, exist?

Justify your answers.

13. Determine whether the following augmented matrices are row equivalent to each other.

$$A = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 4 & 6 & 0 \end{array} \right), \quad B = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 6 & 9 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right), \quad C = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{array} \right).$$

23. Determine the values of a and b so that the linear system

$$\begin{cases} ax + bz = 2 \\ ax + ay + 4z = 4 \\ ay + 2z = b \end{cases}$$

(a) has no solution;

(b) has only one solution;

(c) has infinitely many solutions and a general solution has one arbitrary parameter;

(d) has infinitely many solutions and a general solution has two arbitrary parameters.

20. In the downtown section of a certain city, two sets of one-way streets intersect as shown in the following:
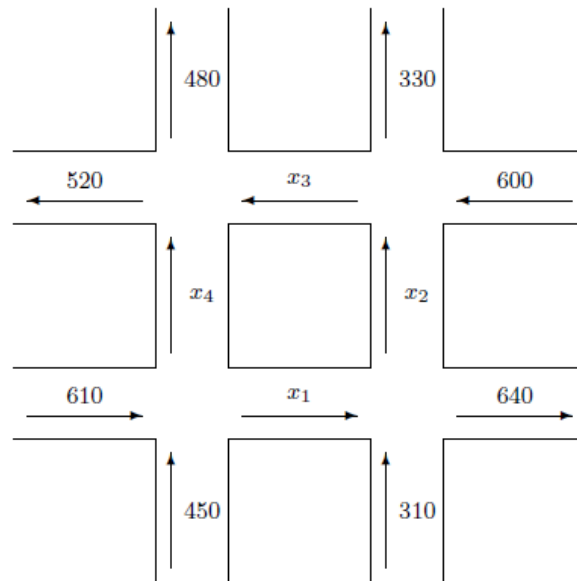
The average hourly volume of traffic entering and leaving this section during rush hour is given in the diagram.

(a) Do we have enough information to find the traffic volumes x_1, x_2, x_3, x_4? Explain your answer.

(b) Given that $x_4 = 500$, find x_1, x_2, x_3.

(The average hourly volume of traffic entering an intersection must be equal to the volume that leaving.)