

1. How many critical points does the function

$$f(x) = (x - 0.1)x^{\frac{1521}{2019}}$$

have in the interval $[-1, 1]$?

2. Let θ denote a positive constant which represents the radian measurement of an angle with $0 < \theta < \frac{\pi}{2}$. At time $t = 0$ minute, a point A starts at the origin and moving away from the origin into the first quadrant along the line $y = (\tan \theta)x$ at a uniform speed of 3 metre per minute. At the same time $t = 0$ minute, a point B starts at the origin and moving away from the origin towards the right along the x -axis at a uniform speed of 5 metre per minute. It is observed that at time $t = 1$ minute, the distance between A and B is increasing at a rate of 4.7 metre per minute. Find the value of θ . Give your answer correct to two decimal places.

3. The region bounded by the graphs of $y = \frac{1}{\sqrt{1+x^2}}$, $y = \frac{1}{\sqrt{4+x^2}}$, $x = 0$ and $x = b$ where b denotes a positive constant is rotated about the x -axis to generate a solid of revolution. Let $V(b)$ denote the volume of this solid of revolution. By taking the value of π to be equal to $\frac{22}{7}$ you find that the value of $\lim_{b \rightarrow \infty} V(b)$ is equal to $\frac{m}{n}$ where m and n are two positive integers with no common factors. What is the value of $m + n$?

4. It is known that f is a differentiable function which satisfies

$$\int_1^x f(t) dt = \sin 1 - \frac{\sin x}{x}$$

for all $x > 0$. Find the value of $\int_1^2 x f'(x) dx$. Give your answer correct to two decimal places.