A								

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2018-2019

MA1521

CALCULUS FOR COMPUTING

December 2018 Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. **Write down your matriculation number neatly in the space provided above.** Do not write your name anywhere in this booklet. This booklet (and only this booklet) will be collected at the end of the examination. Do not insert any loose pages in the booklet.
2. This examination paper consists of **FOUR (4)** questions and comprises **NINE (9)** printed pages.
3. Answer **ALL** questions. For each question, write your answer in the box and your working in the space provided inside the booklet following that question. The marks for each question are indicated at the beginning of the question. The maximum possible total score for this examination paper is 40 marks.
4. This is a **closed book (with authorized material)** examination. Students are only allowed to bring into the examination hall **ONE** piece A4 size help-sheet which can be used on both sides.
5. Candidates may use any calculators that satisfy MOE A-Level examination guidelines. However, they should lay out systematically the various steps in the calculations.

For official use only. Do not write below this line.

Question	1	2	3	4
Marks				

Question 1 [10 marks]

(a) An open box (*without a top cover*) is to be made from a rectangular piece of metal sheet, 20 cm by 30 cm, by cutting out equal squares of side x cm from each of the four corners and folding up the sides (to be perpendicular to the base). Find the value of x to make the box with the largest possible volume. Give your answer correct to two decimal places.

(b) Let $\theta = \int_0^1 \frac{2e^x}{e^{2x}+2e^x+5}dx$. Find the EXACT VALUE of $(\tan\theta)$. Give your answer as a fraction in its simplest form which may involve the number e in the numerator and the denominator.

Answer 1(a)	3.92	**Answer 1(b)**	$\frac{e-1}{e+3}$

(Show your working below and on the next page.)

(a) $f(x) = x(20-2x)(30-2x)$

$= 4x^3 - 100x^2 + 600x = \text{max!}$

$0 \le x \le 10$

$f'(x) = 12x^2 - 200x + 600 = 0$

$3x^2 - 50x + 150 = 0$

$x = \frac{50 \pm \sqrt{2500-1800}}{6}$

$= 12.742\cdots$ or $3.923\cdots$

$\because 0 \le x \le 10$

$\therefore x = 3.923\cdots$

$\approx \underline{\underline{3.92}}$

(b) $\theta = \int_0^1 \frac{2e^x}{(e^x+1)^2+4}dx$

$= \frac{1}{2}\int_0^1 \frac{e^x}{1+\left(\frac{e^x+1}{2}\right)^2}dx$

$= \int_0^1 \frac{d\left(\frac{e^x+1}{2}\right)}{1+\left(\frac{e^x+1}{2}\right)^2}$

$= \tan^{-1}\left(\frac{e+1}{2}\right) - \tan^{-1}(1)$

$\tan\theta = \frac{\frac{e+1}{2} - 1}{1+\left(\frac{e+1}{2}\right)(1)}$

$= \underline{\underline{\frac{e-1}{e+3}}}$

Question 2 [10 marks]

(a) Let $f(x) = \int_0^x t^2 \left(\tan^{-1} t\right) dt$. Find the value of $f^{(8)}(0)$, where $f^{(8)}(0)$ denotes the eighth derivative of f at 0. Give your answer correct to the nearest integer.

(b) Let a and b denote two constants. It is known that $f(x,y)$ has continuous partial derivatives of all orders and that

$$\nabla f = \left(2x^3 + axy^2 + 3y + 1\right)\mathbf{i} + \left(5x^2 y + bx + 2\right)\mathbf{j}.$$

Find the directional derivative of f at the point $(2,3)$ in the direction of the vector joining $(2,3)$ to $(1,5)$. Give your answer correct to two decimal places.

Answer 2(a)	1008	Answer 2(b)	8.94

(Show your working below and on the next page.)

(a) $f(x) = \int_0^x t^2 \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{2n+1} dt$

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+1)(2n+4)}$

$\frac{f^{(8)}(0)}{8!} = \frac{1}{5 \times 8}$

$\therefore f^{(8)}(0) = \frac{8!}{5 \times 8}$

$= \underline{\underline{1008}}$

(b) $2axy + 3 = 10xy + b$

$\therefore a = 5,\ b = 3$

$\nabla f(2,3) = (16 + 90 + 9 + 1)\vec{i} + (60 + 6 + 2)\vec{j}$

$= 116\vec{i} + 68\vec{j}$

$\vec{u} = \frac{-\vec{i} + 2\vec{j}}{\sqrt{5}}$

$\therefore D_{\vec{u}} f(2,3) = \frac{-116 + 136}{\sqrt{5}}$

$= \frac{20}{\sqrt{5}}$

$= 8.944\ldots$

$\approx \underline{\underline{8.94}}$

Question 3 [10 marks]

(a) A stone was projected vertically upwards at a velocity of $20m/s$. It reached the top of its trajectory T seconds after it was projected. Find the value of T correct to two decimal places, based on the following assumptions: the stone's mass is $0.3kg$, the gravitational constant g equals to $9.8m/s^2$ and the value of the air resistance at any time equals to $0.6v$ Newtons where v is the value of the velocity of the stone at that time measured in m/s.

(b) Let $y(x)$ denote the solution to the differential equation $x\frac{dy}{dx} - y = 2x^2 \cos 2x$, with $x > 0$ and $y\left(\frac{\pi}{2}\right) = \pi$. Find the value of $y\left(\frac{5\pi}{4}\right)$. Give your answer correct to two decimal places.

Answer 3(a)	0.81	**Answer 3(b)**	11.78

(Show your working below and on the next page.)

(a)

$$0.3\frac{dV}{dt} = -0.3g - 0.6V$$

$$\frac{dV}{9.8+2V} = -dt$$

$$\int_{20}^{0} \frac{dV}{9.8+2V} = \int_0^T -dt$$

$$T = \int_0^{20} \frac{dV}{2V+9.8}$$

$$= \frac{1}{2}\ln\frac{40+9.8}{9.8}$$

$$= 0.812\cdots$$

$$\approx \underline{0.81}$$

(b)

$$\frac{dy}{dx} - \frac{1}{x}y = 2x\cos 2x$$

$$R = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

$$y = x\int 2\cos 2x\,dx$$

$$= x(\sin 2x + C)$$

$$\pi = \frac{\pi}{2}C \Rightarrow C = 2$$

$$y = x(\sin 2x + 2)$$

$$y\left(\frac{5\pi}{4}\right) = \frac{5\pi}{4}\left(\sin\frac{5\pi}{2} + 2\right)$$

$$= 11.780\cdots$$

$$\approx \underline{11.78}$$

Question 4 [10 marks]

(a) The monkey population at the Bukit Timah Nature Reserve follows a logistic model with a birth rate per capita of 12% per year. After a very long time, the population settled down to the carrying capacity of 2200 monkeys. It is known that at time $t = 10$ year, there were 1521 monkeys. How many monkeys were there initially at time $t = 0$ year? Give your answer correct to the nearest integer.

~~(b) Let $w = w(x, y)$ denote a function of two variables x and y. If $w(x, y)$ is the answer that you get by applying the method of separation of variables to solve the partial differential equation $x(\frac{\partial w}{\partial x}) = \frac{\partial w}{\partial y}$, with $x > 0$, $w(2, 0) = 10$ and $w(10, 0) = 2$, find the value of $w(2, 2)$. Give your answer correct to two decimal places.~~

Answer 4(a)	886	Answer 4(b)	1.35

(Show your working below and on the next page.)

(a)
$$N = \frac{2200}{1 + \left(\frac{2200}{\hat{N}} - 1\right)e^{-0.12t}}$$

$N(10) = 1521$

$$\Rightarrow 1521 = \frac{2200}{1 + \left(\frac{2200}{\hat{N}} - 1\right)e^{-1.2}}$$

$$\therefore \hat{N} = \frac{2200}{\left(\frac{2200}{1521} - 1\right)e^{1.2} + 1}$$

$$= 886.3\ldots$$

$$\approx \underline{\underline{886}}$$

(b) Let $w = XY$

$$xX'Y = XY'$$

$$\frac{xX'}{X} = \frac{Y'}{Y} = k$$

$$x\frac{X'}{X} = k \Rightarrow X = Ax^k$$

$$\frac{Y'}{Y} = k \Rightarrow Y = Be^{ky}$$

$$\therefore w = Cx^k e^{ky}$$

$$w(2,0) = 10 \Rightarrow 10 = C(2^k)$$

$$w(10,0) = 2 \Rightarrow 2 = C(10^k)$$

$$\therefore \frac{10}{2} = \left(\frac{2}{10}\right)^k \Rightarrow k = -1, \quad C = 20$$

$$\therefore w = 20x^{-1}e^{-y}$$

$$w(2,2) = 20\left(\tfrac{1}{2}\right)e^{-2} = \frac{10}{e^2} = 1.353\ldots$$

$$\approx \underline{\underline{1.35}}$$