Tutorial 4

Exercise 3

5. Let $A = \{\, (1+t, 1+2t, 1+3t) \mid t \in \mathbb{R} \,\}$ be a subset of $\mathbb{R}^3$.

   (a) Describe A geometrically.

   (b) Show that $A = \{\, (x, y, z) \mid x + y - z = 1 \text{ and } x - 2y + z = 0 \,\}$.

   (c) Write down a matrix equation $M\boldsymbol{x} = \boldsymbol{b}$ where M is a 3×3 matrix and $\boldsymbol{b}$ is a 3×1 matrix such that its solution set is A.

7. Let P represent a plane in $\mathbb{R}^3$ with equation $x - y + z = 1$ and A, B, C represent three different lines given by the following set notation:

$$A = \{\, (a, a, 1) \mid a \in \mathbb{R} \,\}, \quad B = \{\, (b, 0, 0) \mid b \in \mathbb{R} \,\}, \quad C = \{\, (c, 0, -c) \mid c \in \mathbb{R} \,\}.$$

   (a) Express the plane P in explicit set notation.

   (b) Does any of the three lines above lie completely on the plane P? Briefly explain your answer.

   (c) Find all the points of intersection of the line B with the plane P.

   (d) Find the equation of another plane that is parallel to (but not overlapping) the plane P, and contains exactly one of the three lines above.

   (e) Can you find a nonzero linear system whose solution set contains all the three lines? Justify your answer.

8. Let $\boldsymbol{u_1} = (2, 1, 0, 3)$, $\boldsymbol{u_2} = (3, -1, 5, 2)$, and $\boldsymbol{u_3} = (-1, 0, 2, 1)$. Which of the following vectors are linear combinations of $\boldsymbol{u_1}$, $\boldsymbol{u_2}$, $\boldsymbol{u_3}$?

   (a) $(2, 3, -7, 3)$, (b) $(0, 0, 0, 0)$, (c) $(1, 1, 1, 1)$, (d) $(-4, 6, -13, 4)$.

10. Let $V = \{\, (x, y, z) \mid x - y - z = 0 \,\}$ be a subset of $\mathbb{R}^3$.

   (a) Let $S = \{(1, 1, 0), (5, 2, 3)\}$. Show that $\text{span}(S) = V$.

   (b) Let $S' = \{(1, 1, 0), (5, 2, 3), (0, 0, 1)\}$. Show that $\text{span}(S') = \mathbb{R}^3$.

12. Let $\boldsymbol{u_1} = (2, 0, 2, -4)$, $\boldsymbol{u_2} = (1, 0, 2, 5)$, $\boldsymbol{u_3} = (0, 3, 6, 9)$, $\boldsymbol{u_4} = (1, 1, 2, -1)$, $\boldsymbol{v_1} = (-1, 2, 1, 0)$, $\boldsymbol{v_2} = (3, 1, 4, 0)$, $\boldsymbol{v_3} = (0, 1, 1, 3)$, $\boldsymbol{v_4} = (-4, 3, -1, 6)$. Determine if the following are true.

   (a) $\text{span}\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}\} \subseteq \text{span}\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}, \boldsymbol{v_4}\}$.

   (d) $\text{span}\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}, \boldsymbol{v_4}\} = \mathbb{R}^4$.

16. Determine which of the following are subspaces of $\mathbb{R}^4$. Justify your answers.

   (b) $\{\, (w, x, y, z) \mid wx = yz \,\}$.

   (e) $\{\, (w, x, y, z) \mid w = 0 \text{ or } y = 0 \,\}$.

   (g) $\{\, (w, x, y, z) \mid w + z = 0 \text{ and } x + y - 4z = 0 \text{ and } 4w + y - z = 0 \,\}$.