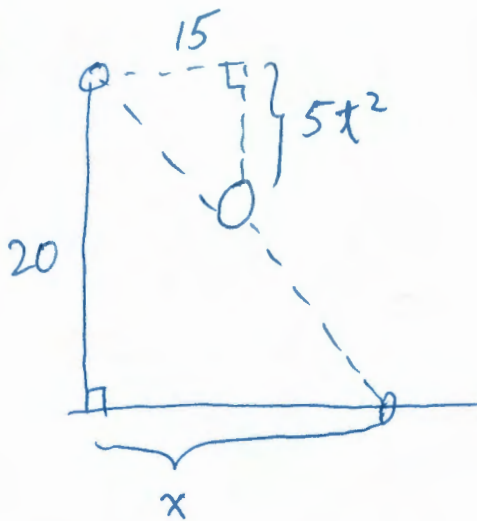Answers to mid term test

1. A
2. D
3. A
4. C
5. D
6. A
7. B
8. B
9. D
10. C

1). A

$$\frac{dy}{dx} = \frac{2x}{1+x^2}$$

2). D

$$\frac{x}{15} = \frac{20}{5t^2} \implies x = \frac{60}{t^2}$$

$$\frac{dx}{dt} = -\frac{120}{t^3}$$

$$t = 1.3 \implies \frac{dx}{dt} = -\frac{120}{(1.3)^3}$$

$$\approx -54.6$$

$$\therefore \text{speed} = 54.6$$

3). A $x^3 + y^3 = 6xy \implies 3x^2 + 3y^2 y' = 6y + 6xy' \cdots ①$

$6x + 6y(y')^2 + 3y^2 y'' = 6y' + 6y' + 6xy'' \cdots ②$

$x = y = 3$ and ① $\implies 27 + 27y' = 18 + 18y' \implies y' = -1$

$x = y = 3$, $y' = -1$ and ②

$\implies 18 + 18 + 27y'' = -6 - 6 + 18y''$

$\implies 9y'' = -48$

$\implies y'' = -\frac{16}{3}$

4). C

$$\frac{dy}{dx} = - \frac{2x+2}{(x^2+2x+1521)^2}$$

$$\frac{d^2y}{dx^2} = - \frac{2(x^2+2x+1521)^2 - (2x+2)2(x^2+2x+1521)(2x+2)}{(x^2+2x+1521)^4}$$

$$= \frac{-2(x^2+2x+1521)}{(x^2+2x+1521)^4} \left\{ x^2+2x+1521 - (2x+2)^2 \right\}$$

$$= \frac{-2}{(x^2+2x+1521)^3} \left\{ -3x^2 - 6x + 1517 \right\}$$

$$= \frac{2(3x^2+6x-1517)}{(x^2+2x+1521)^3}$$

$$\frac{d^2y}{dx^2} = 0 \implies x = \frac{-6 \pm \sqrt{36+12(1517)}}{6}$$

Reject -ve root ∵ a is +ve

∴ $a = \frac{-6 + \sqrt{36+12(1517)}}{6} \approx \underline{\underline{21.5}}$

5). D

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 + 4t - 1}{2t + 1}$$

$$t = 2 \implies \frac{dy}{dx} = \frac{12 + 8 - 1}{4 + 1} = \frac{19}{5}$$

6). A

$$\int \sec(1-2x)\tan(1-2x)\,dx$$

$$= -\frac{1}{2}\int \sec(1-2x)\tan(1-2x)\,d(1-2x)$$

$$= -\frac{1}{2}\sec(1-2x) + C$$

7). B

$$\int_{\sqrt{3^{1521}-1}}^{\sqrt{3^{2016}-1}} \left(\frac{x}{1+x^2}\right) dx = \frac{1}{2}\int_{\sqrt{3^{1521}-1}}^{\sqrt{3^{2016}-1}} \frac{d(1+x^2)}{1+x^2}$$

$$= \frac{1}{2}\ln(1+x^2)\Big|_{\sqrt{3^{1521}-1}}^{\sqrt{3^{2016}-1}}$$

$$= \frac{1}{2}\left(\ln 3^{2016} - \ln 3^{1521}\right)$$

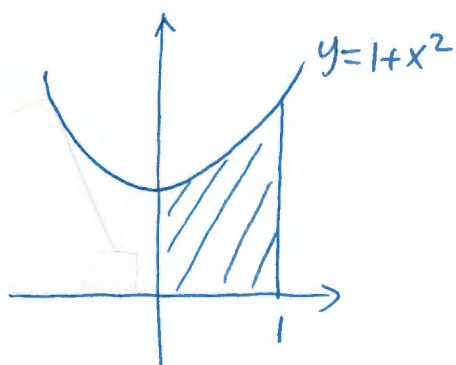$$= \frac{1}{2}(2016 - 1521)\ln 3 \approx 271.9$$

8). B

$$\int_1^{2^x} \frac{\sqrt{\ln t}}{t}\, dt = \int_1^{2^x} \sqrt{\ln t}\; d(\ln t) = \frac{2}{3}(\ln t)^{3/2}\Big|_1^{2^x}$$

$$= \frac{2}{3}(x \ln 2)^{3/2}$$

$$\frac{2}{3}(x \ln 2)^{3/2} = 1521 \implies x = \frac{\left(\frac{4563}{2}\right)^{2/3}}{\ln 2} \approx \underline{\underline{250}}$$
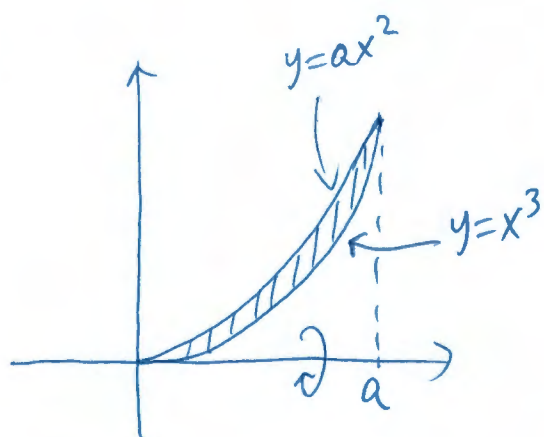
9). D

$$\text{area} = \int_0^1 (1+x^2)\, dx$$

$$= \left[x + \frac{1}{3}x^3\right]\Big|_0^1$$

$$= 1 + \frac{1}{3} = \underline{\underline{\frac{4}{3}}}$$

10). C

$$ax^2 = x^3 \implies x = 0,\ a$$

$$\text{vol} = \int_0^a \pi(a^2 x^4 - x^6)\, dx$$

$$= \pi\left[\frac{1}{5}a^2 x^5 - \frac{1}{7}x^7\right]\Big|_0^a$$

$$= \pi\left(\frac{1}{5}a^7 - \frac{1}{7}a^7\right) = \frac{2\pi}{35}a^7$$

$$\frac{2\pi}{35}a^7 = 20 \implies a = \left(\frac{350}{\pi}\right)^{1/7} \approx \underline{\underline{1.96}}$$