Integrals Cheat Sheet

Common Integrals

$$\int x^{-1}dx = \ln(x)$$

$$\int \frac{1}{x}dx = \ln(x)$$

$$\int |x|dx = \frac{x\sqrt{x^2}}{2}$$

$$\int e^x dx = e^x$$

$$\int \sin(x)dx = -\cos(x)$$

$$\int \cos(x)dx = \sin(x)$$

$$\int x^a dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1$$

Trigonometric Integrals

$$\int \sec^2(x)dx = \tan(x)$$

$$\int \csc^2(x)dx = -\cot(x)$$

$$\int \frac{1}{\sin^2(x)}dx = -\cot(x)$$

$$\int \frac{1}{\cos^2(x)}dx = \tan(x)$$

Arc Trigonometric Integrals

$$\int \frac{1}{x^2+1}dx = \arctan(x)$$

$$\int \frac{-1}{x^2+1}dx = \operatorname{arccot}(x)$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x)$$

$$\int \frac{-1}{\sqrt{1-x^2}}dx = \arccos(x)$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}dx = \operatorname{arcsec}(x)$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}dx = \operatorname{arccsc}(x)$$

$$\int \frac{1}{\sqrt{x^2+1}}dx = \operatorname{arcsinh}(x)$$

$$\int \frac{1}{1-x^2}dx = \operatorname{arctanh}(x)$$

$$\int \frac{1}{|x|\sqrt{x^2+1}}dx = -\operatorname{arccsch}(x)$$

Hyperbolic Integrals

Hyperbolic Integrals

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

$$\int \operatorname{csch}^2(x)\,dx = \left(-\coth(x)\right)$$

$$\int \cosh(x)\,dx = \sinh(x)$$

$$\int \sinh(x)\,dx = \cosh(x)$$

$$\int \operatorname{csch}(x)\,dx = \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

$$\int \sec(x)\,dx = \ln\left(\tan(x) + \sec(x)\right)$$

Integrals of Special Functions

$$\int \cos\left(\frac{x^2\pi}{2}\right)dx = \mathbb{C}(x)$$

$$\int \frac{\sin(x)}{x}\,dx = \operatorname{Si}(x)$$

$$\int \frac{\cos(x)}{x}\,dx = \operatorname{Ci}(x)$$

$$\int \frac{\sinh(x)}{x}\,dx = \operatorname{Shi}(x)$$

$$\int \frac{\cosh(x)}{x}\,dx = \operatorname{Chi}(x)$$

$$\int \frac{\exp(x)}{x}\,dx = \operatorname{Ei}(x)$$

$$\int \exp\left(-x^2\right)dx = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x)$$

$$\int \exp x^2\,dx = \exp x^2 \operatorname{F}(x)$$

$$\int \sin\left(\frac{x^2\pi}{2}\right)dx = \operatorname{S}(x)$$

$$\int \sin\left(x^2\right)dx = \sqrt{\frac{\pi}{2}}\,\operatorname{S}\left(\sqrt{\frac{2}{\pi}}\,x\right)$$

$$\int \frac{1}{\ln(x)}\,dx = \operatorname{li}(x)$$

Indefinite Integrals Rules

Integration By Parts	$\int uv' = uv - \int u'v$
Integral of a constant	$\int f(a)\,dx = x \cdot f(a)$
Take the constant out	$\int a \cdot f(x)\,dx = a \cdot \int f(x)\,dx$
Sum Rule	$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
Add a constant to the solution	If $\frac{dF(x)}{dx} = f(x)$ then $\int f(x)\,dx = F(x) + C$
Power Rule	$\int x^a dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1$
Integral Substitution	$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du, \quad u = g(x)$

Definite Integrals Rules

Definite Integral Boundaries	$\int_a^b f(x)dx = F(b) - F(a)$ $= \lim_{x \to b-} (F(x)) - \lim_{x \to a+} (F(x))$
Odd function	If $f(x) = -f(-x) \Rightarrow \int_{-a}^{a} f(x)dx = 0$
Undefined points	If exist b, a < b < c, and $f(b) =$ undefined, Then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$
Same points defined	$\int_a^a f(x)dx = 0$