

$$1. \text{ (a) } \int_0^b \int_0^a (x^2 + y^2) dx dy = \int_0^b \left[\frac{1}{3}x^3 + xy^2 \right]_{x=0}^{x=a} dy = \int_0^b \left(\frac{1}{3}a^3 + ay^2 \right) dy \\ = \left[\frac{1}{3}a^3y + \frac{1}{3}ay^3 \right]_0^b = \frac{1}{3}a^3b + \frac{1}{3}ab^3.$$

$$\text{(b) } \int_1^2 \int_0^1 \frac{xy}{\sqrt{4-x^2}} dx dy = \int_1^2 \left[-\frac{1}{2}y \left(2(4-x^2)^{1/2} \right) \right]_{x=0}^{x=1} dy \\ = \int_1^2 -y(3^{1/2} - 4^{1/2}) dy \\ = (2 - \sqrt{3}) \left[\frac{1}{2}y^2 \right]_{y=1}^{y=2} = 3 - \frac{3}{2}\sqrt{3}.$$

2. (a) The region can be regarded as a Type A region

$$D : \quad 0 \leq y \leq x, \quad 0 \leq x \leq 1.$$

$$\int_0^1 \int_0^x e^{x^2} dy dx = \int_0^1 \left[ye^{x^2} \right]_{y=0}^{y=x} dx = \int_0^1 xe^{x^2} dx \\ = \frac{1}{2} \left[e^{x^2} \right]_0^1 = \frac{1}{2}(e - 1).$$

- (b) The region can be regarded as a type A region with bottom boundary $y = x^2$ and top boundary $y = \sqrt{x}$.

Since the two curves intersect at $x = 0$ and $x = 1$, the left and right are bounded by $x = 0$ and $x = 1$ respectively. So

$$D : \quad x^2 \leq y \leq \sqrt{x}, \quad 0 \leq x \leq 1.$$

$$\int_0^1 \int_{x^2}^{\sqrt{x}} (x + y) dy dx = \int_0^1 \left[xy + \frac{1}{2}y^2 \right]_{y=x^2}^{y=\sqrt{x}} dx = \int_0^1 \left(x^{3/2} + \frac{1}{2}x - x^3 - \frac{1}{2}x^4 \right) dx \\ = \left[\frac{1}{5}2x^{5/2} + \frac{1}{4}x^2 - \frac{1}{4}x^4 - \frac{1}{10}x^5 \right]_0^1 = \frac{3}{10}.$$

3. The line joining $(1, 0)$ and $(4, 2)$ has equation

$$\frac{y-0}{x-1} = \frac{2-0}{4-1} = \frac{2}{3} \iff y = \frac{2}{3}x - \frac{2}{3} \iff x = \frac{3}{2}y + 1.$$

The line joining $(1, 0)$ and $(9, -3)$ has equation

$$\frac{y-0}{x-1} = \frac{(-3)-0}{9-1} = -\frac{3}{8} \iff y = -\frac{3}{8}x + \frac{3}{8} \iff x = -\frac{8}{3}y + 1.$$

The region D is the union of D_1 and D_2 , where

$$D_1 : \quad y^2 \leq x \leq \frac{3}{2}y + 1, \quad 0 \leq y \leq 2,$$

$$D_2 : \quad y^2 \leq x \leq -\frac{8}{3}y + 1, \quad -3 \leq y \leq 0.$$

Hence the required answer is

$$\begin{aligned} \iint_D x \, dA &= \iint_{D_1} x \, dA + \iint_{D_2} x \, dA \\ &= \int_0^2 \int_{y^2}^{(3y/2)+1} x \, dx dy + \int_{-3}^0 \int_{y^2}^{-(8y/3)+1} x \, dx dy \\ &= \frac{19}{5} + \frac{106}{5} = 25, \end{aligned}$$

since

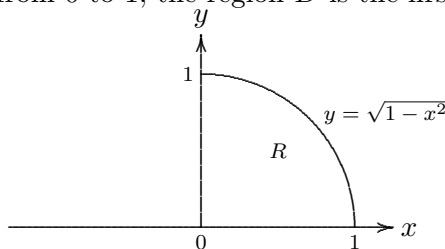
$$\begin{aligned} \int_0^2 \int_{y^2}^{(3y/2)+1} x \, dx dy &= \int_0^2 \frac{1}{8}(9y^2 + 12y + 4 - 4y^4) \, dy = \frac{19}{5}, \\ \int_{-3}^0 \int_{y^2}^{-(8y/3)+1} x \, dx dy &= \int_{-3}^0 \frac{1}{18}(64y^2 - 48y + 9 - 9y^4) \, dy = \frac{106}{5}. \end{aligned}$$

4. The region in Cartesian coordinates is given by

$$D : \quad 0 \leq y \leq \sqrt{1-x^2}, \quad 0 \leq x \leq 1$$

This is a type A region with x -axis as the bottom boundary and upper half of the unit circle as the upper boundary.

Since the range of x is from 0 to 1, the region D is the first quadrant of the unit disk.



In polar coordinates, this is given by

$$D : \quad 0 \leq r \leq 1, \quad 0 \leq \theta \leq \pi/2.$$

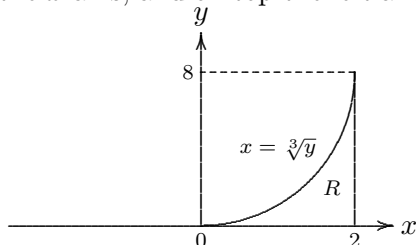
$$\begin{aligned} \int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy dx &= \int_0^{\pi/2} \int_0^1 e^{r^2} r \, dr d\theta = \int_0^{\pi/2} d\theta \int_0^1 e^{r^2} r \, dr \\ &= \frac{\pi}{2} \left[\frac{1}{2} e^{r^2} \right]_0^1 = \frac{1}{4} \pi (e - 1). \end{aligned}$$

5. (a) The type B region R is given by

$$\sqrt[3]{y} \leq x \leq 2, \quad 0 \leq y \leq 8.$$

It is bounded on the left by the cubic curve $\sqrt[3]{y} = x$ and on the right by the vertical line $x = 2$.

Below it is bounded by the x -axis, and on top the left and right boundaries intersect at $y = 8$.



Converting to type A region, the lower boundary is $y = 0$, the top boundary is the cubic curve $y = x^3$.

On the left, these two boundaries intersect at $x = 0$ and on the right, it is bounded by $x = 2$. So the region is given by

$$0 \leq y \leq x^3, \quad 0 \leq x \leq 2.$$

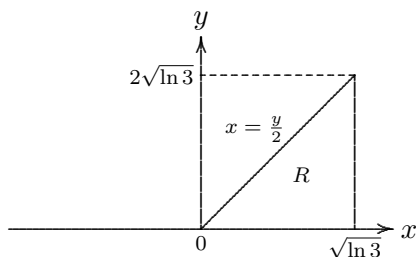
$$\begin{aligned} \int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy &= \int_0^2 \int_0^{x^3} e^{x^4} dy dx = \int_0^2 e^{x^4} [y]_{y=0}^{y=x^3} dx = \int_0^2 x^3 e^{x^4} dx \\ &= \left[\frac{1}{4} e^{x^4} \right]_0^2 = \frac{1}{4} (e^{16} - 1). \end{aligned}$$

- (b) The type B region R is given by

$$y/2 \leq x \leq \sqrt{\ln 3}, \quad 0 \leq y \leq 2\sqrt{\ln 3}.$$

It is bounded on the left by the straight line $x = y/2$ and on the right by the vertical line $x = \sqrt{\ln 3}$.

Below it is bounded by the x -axis, and on top the left and right boundaries intersect at $y = 2\sqrt{\ln 3}$.



Converting to type A region, the lower boundary is $y = 0$, the top boundary is the line $y = 2x$.

On the left, these two boundaries intersect at $x = 0$ and on the right, it is bounded by $x = \sqrt{\ln 3}$.

So the region is given by

$$0 \leq y \leq 2x, \quad 0 \leq x \leq \sqrt{\ln 3}.$$

$$\begin{aligned}\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy &= \int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2} dy dx = \int_0^{\sqrt{\ln 3}} e^{x^2} [y]_{y=0}^{y=2x} dx = \int_0^{\sqrt{\ln 3}} 2xe^{x^2} dx \\ &= \left[e^{x^2} \right]_0^{\sqrt{\ln 3}} = e^{\ln 3} - 1 = 2.\end{aligned}$$