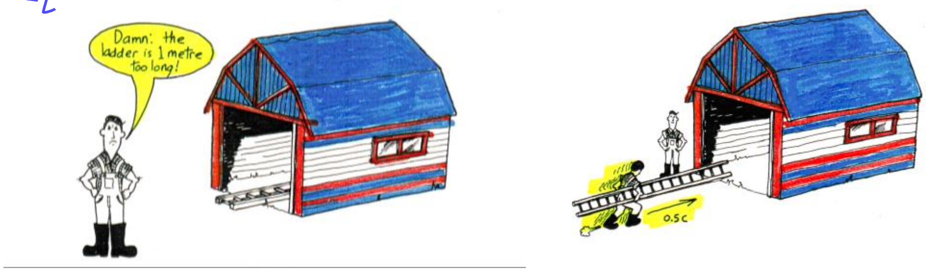
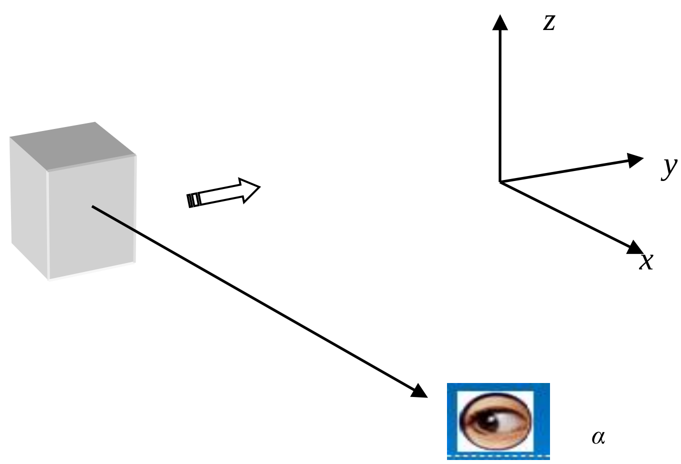
**1)** A farmer has a ladder which is 10 meters long and a barn which is only 8 meters long and so, much to the farmer's consternation, the ladder will not fit in the barn. How can he fit the ladder into the barn using the length contraction theory?

$\frac{L_0}{\gamma} = L$ $10\sqrt{1-\frac{v^2}{c^2}} = 8$




**2)** Consider a cube whose edge has a length $L_0$ in its own rest frame. See figure below. The cube is moving at a speed close to that of light in the $y$ direction. How would it appear to an observer far away at $\alpha$? See figures below and discuss with your classmates.

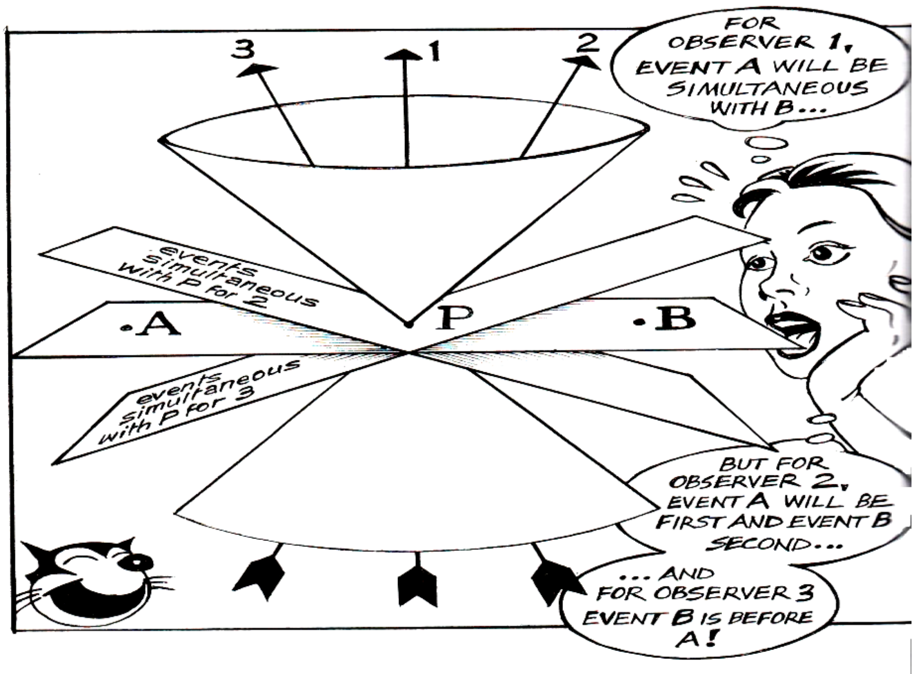


**3)** Recall this slide 67 in Lecture 2. Discuss with your classmates what is wrong with the slide.

## Time and Observer Dependency

One important fact about lightcones is that they represent the limits of which events can affect one another. Nothing goes faster than light, so anything that will influence you must be travelling either on the lightcone itself (if it's light) or within the lightcone (if it's going slower than light). The same goes for anywhere you hope to go or influence.

Now, we've drawn our picture with one time, but that was merely for the sake of simplicity. The march of time is observer-dependent, to a certain extent. Within one particular observer's lightcone, the order of events is definite. But another observer, moving relative to our first observer, will disagree with the first as to what events are simultaneous with event P.



$$(m_0c^2)^2 + (pc)^2 = m_0^2c^4 + \gamma^2 m_0^2 v^2 c^2$$
$$= m_0^2 c^4\left(1+\gamma^2 \frac{v^2}{c^2}\right) = \gamma^2 m_0^2 c^4$$
$$= E^2 = (\gamma m_0 c^2)^2$$

$$1 + \frac{\gamma^2 v^2}{c^2} = 1 + \frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \frac{1\left(1-\frac{v^2}{c^2}\right)+\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \frac{1}{1-\frac{v^2}{c^2}}$$

**4a)** If modified relativistic quantities are given as $p = \gamma m_0 v$ and $E = \gamma m_0 c^2$.

Show that $E^2 = (m_0c^2)^2 + (pc)^2$**,** where $p$ is the momentum, $v$ is the speed and $m_0$ is the rest mass of moving object under consideration. Note that this formula may be used for all particles whether they are massive or massless.

$$m_0c^2 + \frac{p^2}{2m_0} = m_0c^2 + \frac{1}{2}\gamma^2 m_0 v^2$$
$$= m_0c^2\left(1 + \frac{1}{2}\gamma^2 \frac{v^2}{c^2}\right)$$

Can you comment on this equation and what can you learn? (Tutor will assist).

**4b)** From $E^2 = (m_0c^2)^2 + (pc)^2$ **above** show that if $v << c$, we will get $E \approx m_0c^2 + \dfrac{p^2}{2m_0}$.

Can you comment on what is new here as compared to Newtonian energy?

**5)** Recall Lorentz Transformations: $t' = \left(t - \frac{vx}{c^2}\right)\gamma$, $x' = (x - vt)\gamma$, $y' = y$, $z' = z$. The 'prime' co-ordinates represent the moving frame.

**a)** Show that $t' = \gamma(t - \beta x)$, $x' = \gamma(x - \beta t)$, $y' = y$, $z' = z$, if $c$ is set to unity (for convenience)

**b)** Check this **Invariant** quantity: $-t^2 + x^2 + y^2 + z^2 = -t'^2 + x'^2 + y'^2 + z'^2$.
Can you think of another invariant quantity where 2 different observers will agree on?

**c)** Discuss (with the Tutor) the significance of **2b)** above.

Can you comment on this equation and what can you learn? (Tutor will assist)