1. Additional Notes

Note that "logic of quantified statements" (chapter 3) is commonly known as "predicate logic", as opposed to "propositional logic" in chapter 2.

We picked up some frequently asked questions and created this *Additional Notes* section to include some materials not covered in lecture that might be of interest to you.

Equivalent expressions: The following quantified statements are equivalent. We use the shorter notation on the left.

- $\forall x \in D\ P(x) \equiv \forall x\big((x \in D) \rightarrow P(x)\big)$
- $\exists x \in D\ P(x) \equiv\ \exists x\big((x \in D) \land P(x)\big)$

Scope of quantifiers

- When a quantifier is used on a variable x in a predicate statement, we say that x is *bound*. If no quantifier is used on a variable, we say that the variable is *free*.

  Examples: In the statement $\forall x \in X\ \exists y \in Y\ P(x,y)$, both x and y are bound. In the statement $\forall x \in X\ P(x,y)$, x is bound but y is free.

- A statement is called a *well-formed formula* (*wff*) when all variables are properly quantified. You should write *wff* statements.

- The set of all variables bound by a common quantifier is the *scope* of that quantifier. If there are no parentheses, then the scope is the smallest *wff* following the quantification.

  Example: For this statement

  $$\forall x \in X\ \big(\exists y \in Y\ P(x,y) \lor Q(x,y)\big),$$

  the variables x and y in $P(x,y)$ are bound, while the variable y in $Q(x,y)$ is free, because the scope of $\exists y$ covers only $P(x,y)$, whereas the scope of $\forall x$ covers $\big(\exists y \in Y\ P(x,y) \lor Q(x,y)\big)$.

  If we write $\forall x \in X\ \exists y \in Y\ \big(P(x,y) \lor Q(x,y)\big)$, then the variables x and y in both $P(x,y)$ and $Q(x,y)$ are bound.

2. Common Mistakes

I have gone over some common mistakes in lecture. Here are some more:

- Using commas (,) in place of appropriate connectives, for example, writing $\forall x\ P(x), Q(x)$ when you actually mean $\forall x\ (P(x) \rightarrow Q(x))$ or $\forall x\ (P(x) \wedge Q(x))$ or something else. Commas don't mean anything in logic statements, they are mainly there for readability.
- Treating predicates as if they are functions returning some value. Example: given the following predicates
    - $Loves(x, y)$: x loves y
    - $Reindeer(x)$: x is a reindeer

  Some students wrote "statements" like $Loves(x, Reindeer(y))$. Since $Reindeer(y)$ is a predicate, its value is either true or false. So, the above is akin to writing $Loves(x, true)$ or $Loves(x, false)$ which does not make sense! The correct way is to use the appropriate connectives.

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