

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1521
Tutorial 7

1. In an electric circuit, the voltage of V volts (V), current of I amperes (A), and resistance of R ohms (Ω) are governed by Ohm's Law $V = I \times R$.

- (i) If the resistance is fixed at 15Ω , how fast is the current increasing with respect to voltage?
- (ii) If the voltage is fixed at 120 V , how fast is the current increasing with respect to resistance at the instant when resistance is 20Ω ?
- (iii) If the resistance is slowly increasing as the resistor heats up, how is the current changing at the moment when $R = 400\Omega$, $I = 0.08\text{A}$, $dV/dt = -0.01 \text{ V/s}$ and $dR/dt = 0.03 \Omega/\text{s}$?

Ans. (i) $\approx 0.0667 \text{ A/V}$; (ii) *decreasing* at $0.3 \text{ A}/\Omega$; (iii) decreasing at $3.1 \times 10^{-5} \text{ A/s}$

2. Find the directional derivative of $f(x, y) = xe^{2y-x}$ at $P(-2, -1)$ in the direction

(i) $\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$; (ii) $3\mathbf{i} + 4\mathbf{j}$;

Find the direction that gives the *largest possible* directional derivative of f at P .

Ans. (i) $-\sqrt{2}/2$; (ii) $-7/5$; $f_x(-2, -1)\mathbf{i} + f_y(-2, -1)\mathbf{j} = 3\mathbf{i} - 4\mathbf{j}$

3. Let $f(x, y, z) = \sin(xyz)$ and $P = (\frac{1}{2}, \frac{1}{3}, \pi)$.

(i) Find the rate of change of f at P in the direction $\mathbf{u} = \frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}$.

- (ii) Suppose P moves 0.1 unit along \mathbf{u} in part (i). How much will the value of f have changed?

Ans. (i) $\frac{1}{12}(1 - \pi)$; (ii) decreases by ≈ 0.01785 .

4. Find the local maximum and minimum values and saddle points (if any) of each of the following functions.

(i) $f(x, y) = \ln(x^2y) - xy - 2x$, where $x > 0$, $y > 0$

(ii) $g(x, y) = xy(1 - x - y)$

(iii) $h(x, y) = x^2 + y^2 + x^{-2}y^{-2}$, where $x \neq 0$, $y \neq 0$

Ans. (i) $f(1/2, 2) = -\ln 2 - 2$ is a local maximum, (ii) $(0, 0)$, $(1, 0)$, $(0, 1)$ are saddle points, $g(1/3, 1/3) = 1/27$ is a local maximum, (iii) $h(\pm 1, \pm 1) = h(\pm 1, \mp 1) = 3$ are local minima.