Answers to mid term test

1. A
2. A
3. D
4. B
5. C
6. B
7. C
8. A
9. B
10. D

1). A

2). A

$$r = \sec\theta + \cos\theta$$

$$\frac{dr}{d\theta} = \sec\theta\tan\theta - \sin\theta$$

$x = r\cos\theta$, $y = r\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$\theta = -\frac{\pi}{3} \Rightarrow \frac{dr}{d\theta} = -2\sqrt{3} + \frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2}$$

$$\frac{dy}{dx} = \frac{\frac{-3\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{5}{2}\left(\frac{1}{2}\right)}{\frac{-3\sqrt{3}}{2}\left(\frac{1}{2}\right) - \frac{5}{2}\left(-\frac{\sqrt{3}}{2}\right)} = 4.041\ldots$$

$$\approx \underline{\underline{4.04}}$$

3). D

$$4x^3 + 4y^3y' = \frac{17}{2}y + \frac{17}{2}xy'$$

$x=1$, $y=2 \Rightarrow$ $4 + 32y' = 17 + \frac{17}{2}y'$

$$y' = \frac{26}{47} = 0.553\ldots$$

$$\approx \underline{\underline{0.55}}$$

4). B

$$\ln y = \cos x \ln x \Rightarrow \frac{1}{y}y' = -\sin x\ln x + \frac{1}{x}\cos x$$

$$x = \frac{\pi}{6} \Rightarrow y' = \left\{\left(\frac{\pi}{6}\right)^{\cos\frac{\pi}{6}}\right\}\left\{-\sin\frac{\pi}{6}\ln\frac{\pi}{6} + \frac{6}{\pi}\cos\frac{\pi}{6}\right\}$$

$$= 1.129\ldots \approx \underline{\underline{1.13}}$$

5). C

$$y' = \frac{1}{-(x+1521)(x+2017)} \left\{ -(x+2017) - (x+1521) \right\}$$

$$= \frac{2(x+1769)}{(x+1521)(x+2017)}$$

$$y' = 0 \Rightarrow x = \underline{\underline{-1769}}$$

6). B

$$9c^5 = \int_0^c (ax^2+bx)^4 \, d(ax^2+bx)$$

$$= \frac{1}{5}(ax^2+bx)^5 \Big|_0^c = \frac{1}{5}(ac^2+bc)^5$$

$$ac+b = (45)^{1/5} = 2.141\ldots$$

$$\approx \underline{\underline{2.14}}$$

7). C

$$\int_0^{\pi/2} \sin^{225}x \cos^3 x \, dx = \int_0^{\pi/2} (\sin^{225}x)(1-\sin^2 x) \, d(\sin x)$$

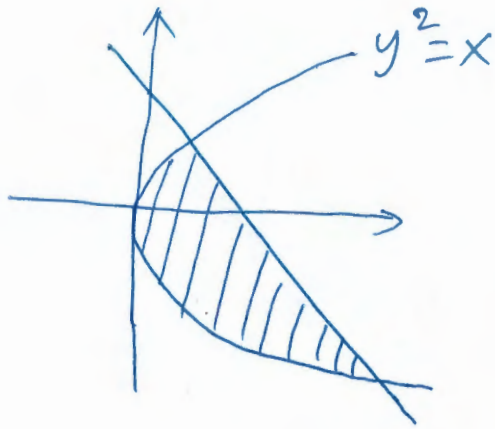
$$= \left[\frac{1}{226}\sin^{226}x - \frac{1}{228}\sin^{228}x \right]_0^{\pi/2} = \frac{1}{226} - \frac{1}{228}$$

$$\int_0^{\pi/2} (\sin^3 x)(\cos^{223}x) \, dx = -\int_0^{\pi/2} (1-\cos^2 x)(\cos^{223}x) \, d(\cos x)$$

$$= \left[-\frac{1}{224}\cos^{224}x + \frac{1}{226}\cos^{226}x \right]_0^{\pi/2} = \frac{1}{224} - \frac{1}{226}$$

$$\frac{\frac{1}{226} - \frac{1}{228}}{\frac{1}{224} - \frac{1}{226}} = \underline{\underline{\frac{56}{57}}}$$

8). A



$y^2=x$ and $2y=x-15$

$\Rightarrow y^2=2y+15$

$\Rightarrow y^2-2y-15=0$

$\Rightarrow (y-5)(y+3)=0$

$\Rightarrow y=-3,\ 5$

$$\text{Area} = \int_{-3}^{5} [(2y+15)-y^2]\,dy = \frac{256}{3}$$

9). B

$$\text{Vol} = \int_0^2 \pi(x^2+x+1)\,dx = 20.943\ldots$$

$$\approx 20.94$$

10). D

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} |\tan x|\,dx$$

$$= \int_{-\frac{\pi}{6}}^{0} -\tan x\,dx + \int_0^{\frac{\pi}{4}} \tan x\,dx$$

$$= 0.490\ldots$$

$$\approx 0.49$$