

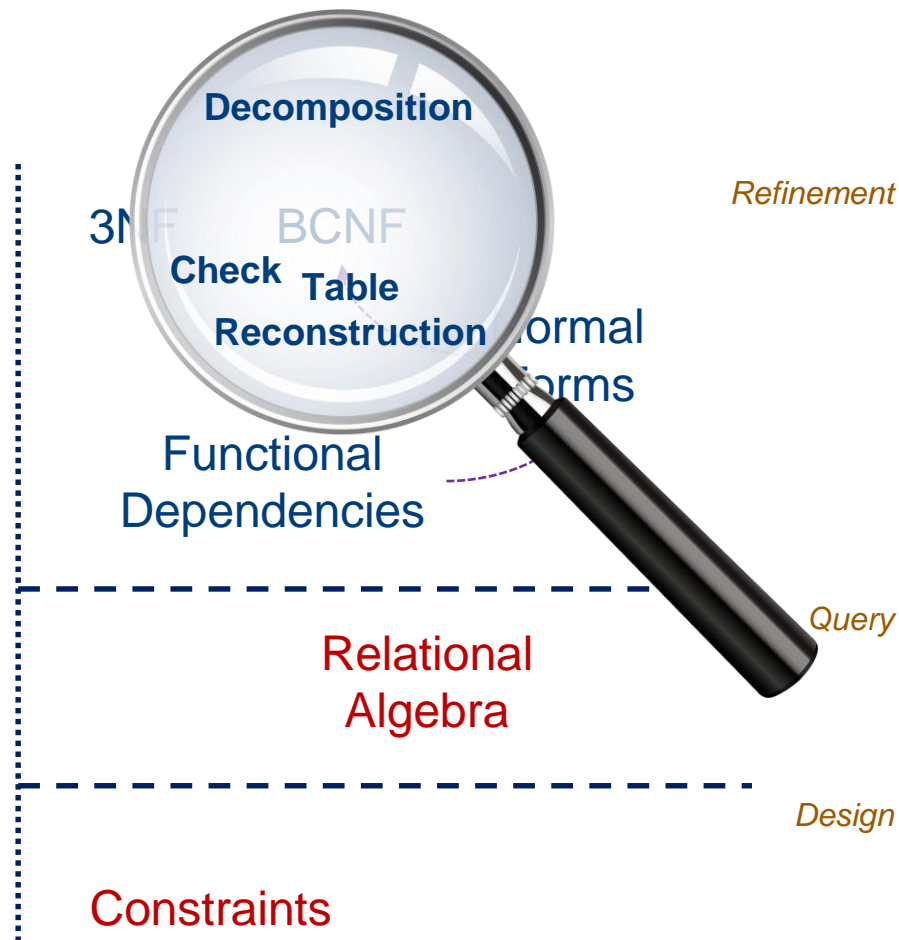
# **CS2102 Database Systems**

## Lecture 12 – Third Normal Form

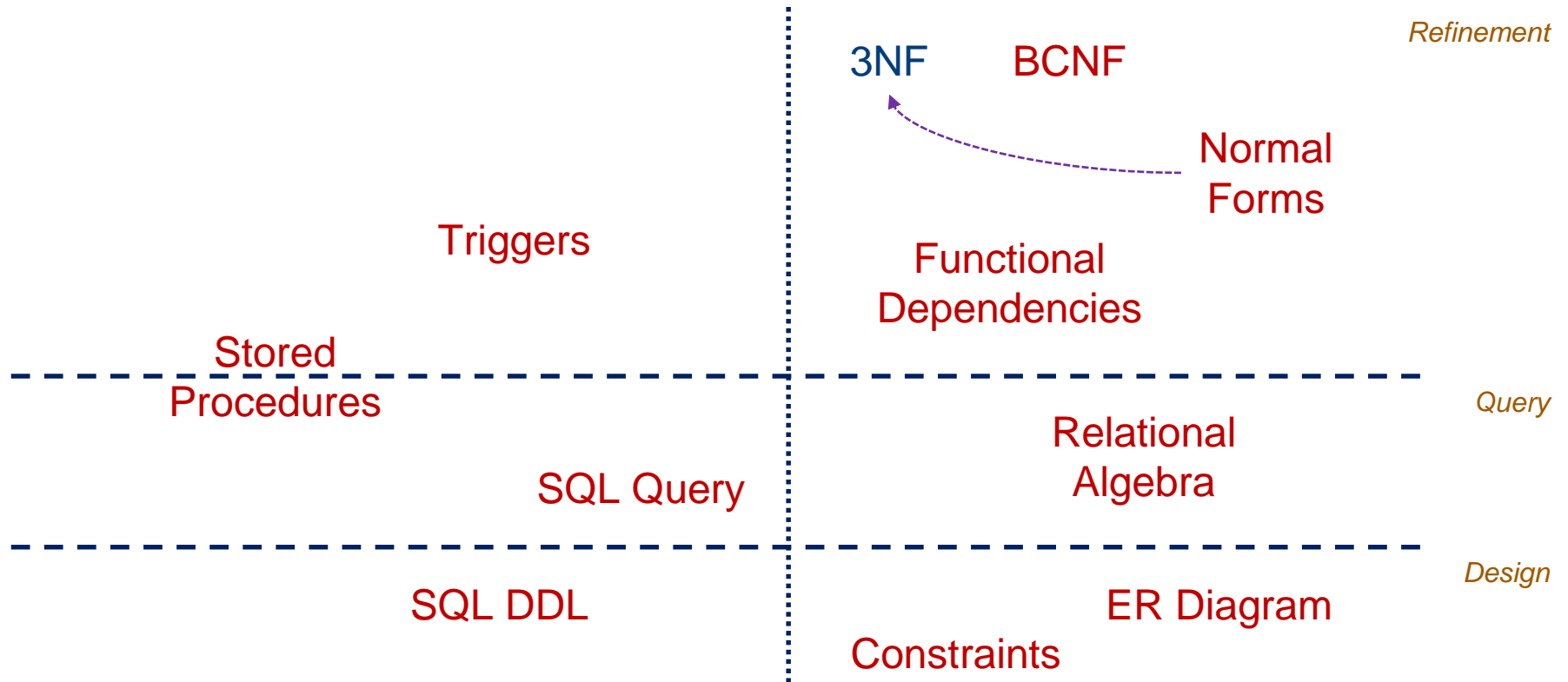
# Roadmap

## • Previously

- **BCNF Definition:**
  - Every *non-trivial & decomposed* FD has *superkey* as LHS
- **BCNF Check:**
  - Find violation: “*more but not all*”
- **BCNF Decomposition:**
  - For violation  $A \rightarrow A^+$ 
    - $R1(A^+)$
    - $R2(A \cup \{R - A^+\})$
  - *Repeat* until all tables are in BCNF
- **BCNF Properties:**
  - ✓ No update or deletion anomalies
  - ✓ Small redundancies
  - ✓ Original table can be reconstructed
  - ✗ *Dependencies may not be preserved in the decomposed table*



# Roadmap



# Roadmap

- We will do this step by step

- Dependency Preservation

*(why we need 3NF)*



- Define 3NF



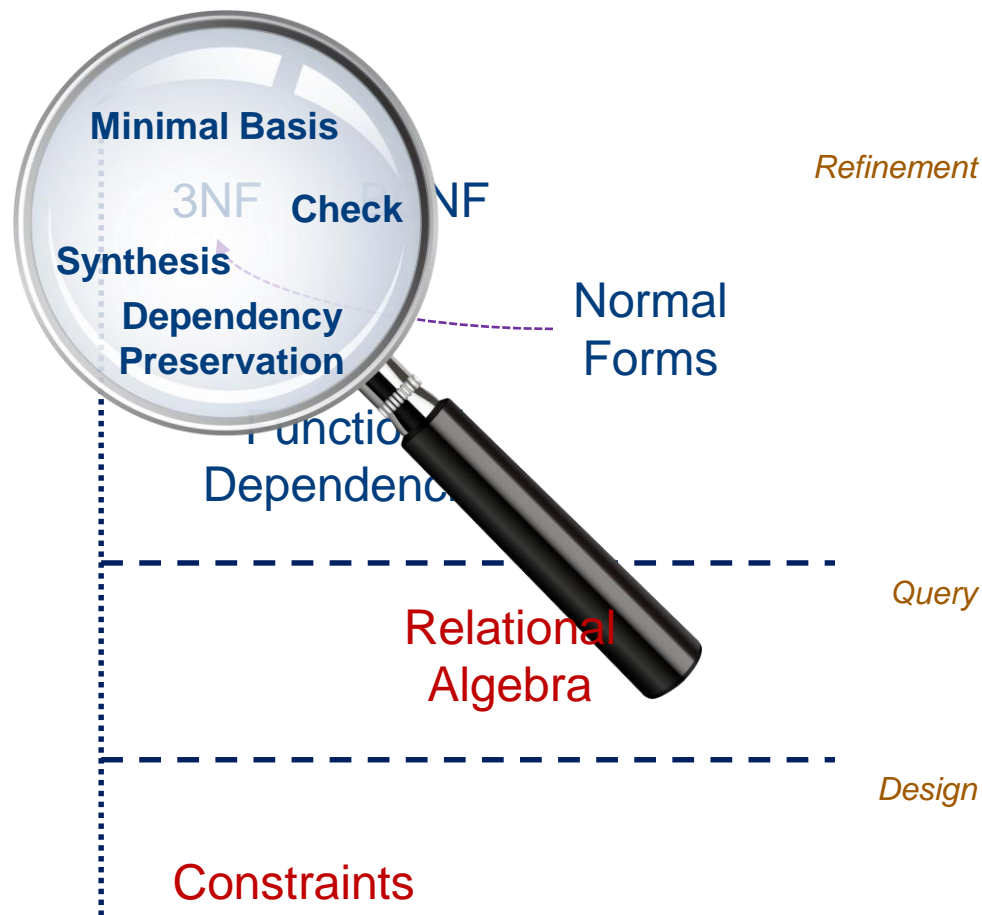
- Check 3NF



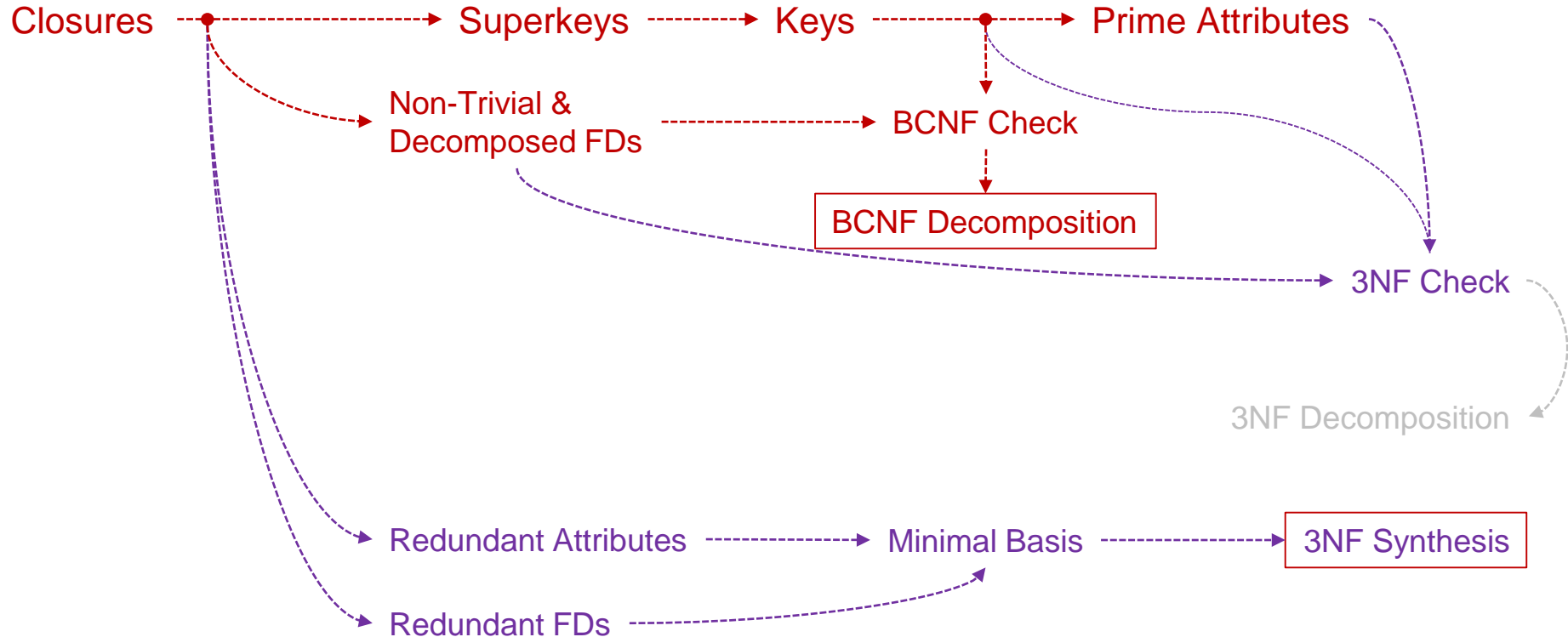
- Minimal Basis



- 3NF Synthesis



# Algorithm Roadmap



# Dependency Preservation

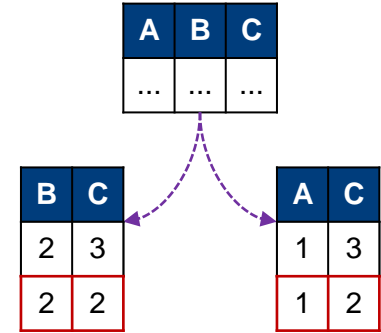
or why we need 3NF



# Dependency Preservation

## ● Motivating Example

- Table  $R(A, B, C)$  with  $AB \rightarrow C$  and  $C \rightarrow B$
- $\{C\} \rightarrow \{B, C\}$  violates BCNF of  $R(A, B, C)$ 
  - $R1(B, C)$  *(non-trivial & decomposed FD on  $R1$ :  $C \rightarrow B$ )*
  - $R2(A, C)$  *(non-trivial & decomposed FD on  $R2$ : nothing)*
- Where is  $AB \rightarrow C$ ??
  - Cannot even be derived from FDs on  $R1$  and  $R2$
  - Totally “lost” *when a functional dependency is lost, then it is possible to insert values that violates the functional dependency*
- ❖ This is why we say that a BCNF decomposition may not always preserve all FDs *(non dependency preserving decomposition)*
  - ❖ **Dilemma!**
    - ❖ *either the table has anomalies OR does not preserve constraints!*



# Dependency Preservation

- “What We Want” Example

- Table  $R(A, B, C)$  with  $A \rightarrow B$ ,  $B \rightarrow C$  and  $A \rightarrow C$

- $\{B\} \rightarrow \{B, C\}$  violates BCNF of  $R(A, B, C)$

- $R1(B, C)$  *(non-trivial & decomposed FD on  $R1: B \rightarrow C$ )*

- $R2(A, B)$  *(non-trivial & decomposed FD on  $R2: A \rightarrow B$ )*

- Where is  $A \rightarrow C$ ??

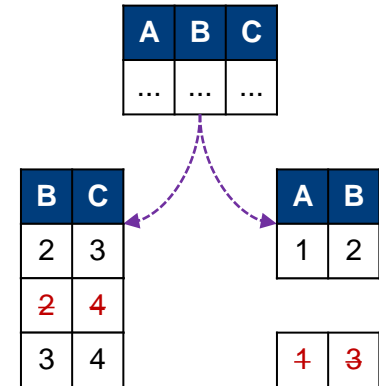
- Can be derived from FDs on  $R1$  and  $R2$

- Compute  $\{A\}^+$  w.r.t.  $\{B \rightarrow C, A \rightarrow B\} = \{A, B, C\}$

- So this FD is *preserved* even when it spans across multiple tables

- ❖ No valid insertion into  $R1$  or  $R2$  can violate this functional dependencies

- ❖ Gives rise to the notion of *FD equivalence*





# Functional Dependency Equivalence

## ● Definition

essentially just using closure to ensure can derive one from the other

- Let  $F1$  and  $F2$  be sets of FDs
- We say that  $F1$  is *equivalent* to  $F2$  (i.e.,  $F1 \equiv F2$ ) if and only if
  - Every FD in  $F1$  can be derived from  $F2$  (i.e.,  $F2 \vdash F1$ )
  - Every FD in  $F2$  can be derived from  $F1$  (i.e.,  $F1 \vdash F2$ )
  - ❖ *They can look different, but they carry the same information*
- ❖ In the context of decomposition, we can let  $F2$  be the FD from the decomposed table
  - ❖ How do we get this FD from decomposed table?
  - ❖ *Union of projection*



# Functional Dependency Equivalence

best way is to solve with a hyper graph

## • Example

### ■ Example: $R(A, B, C, D, E)$

■  $F1 = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$

■  $F2 = \{A \rightarrow BC, D \rightarrow AE\}$

❖ Show  $F1 \equiv F2$

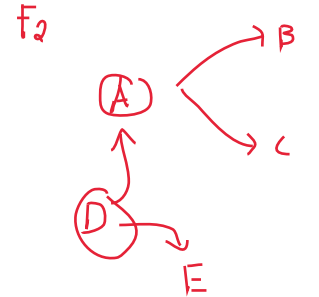
1. Prove that  $F1$  can be derived from  $F2$

■  $A \rightarrow B$  and  $D \rightarrow E$  can be derived easily using *decomposition rule*

■  $\{AB\}^+ = \{ABC\}$  so  $AB \rightarrow C$  is implied by  $F2$

■  $\{D\}^+ = \{ABCDE\}$  so  $D \rightarrow AC$  is implied by  $F2$

$\therefore F1$  can be derived from  $F2$



# Functional Dependency Equivalence

to prove equivalence, need to show both directions

- **Example**

- **Example:**  $R(A, B, C, D, E)$

- $F1 = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$

- $F2 = \{A \rightarrow BC, D \rightarrow AE\}$

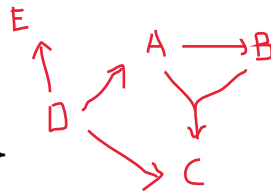
- ❖ Show  $F1 \equiv F2$

2. Prove that  $F2$  can be derived from  $F1$

- $\{A\}^+ = \{ABC\}$  so  $A \rightarrow BC$  is implied by  $F1$

- $\{D\}^+ = \{ABCDE\}$  so  $D \rightarrow AE$  is implied by  $F1$

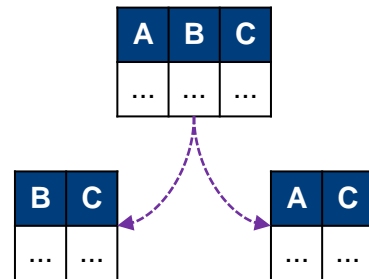
$\therefore F2$  can be derived from  $F1$



# Dependency Preservation

## • Motivating Example

- Table  $R(A, B, C)$  with  $AB \rightarrow C$  and  $C \rightarrow B$
- $\{C\} \rightarrow \{B, C\}$  violates BCNF of  $R(A, B, C)$ 
  - $R1(B, C)$  *(non-trivial & decomposed FD on  $R1$ :  $C \rightarrow B$ )*
  - $R2(A, C)$  *(non-trivial & decomposed FD on  $R2$ : nothing)*



- $F1 = \{AB \rightarrow C, C \rightarrow B\}$
- $FR1 = \{C \rightarrow B\}, FR2 = \{\}$ 
  - **F2** =  $\{C \rightarrow B\}$

❖ We can show that  $F1 \not\equiv F2$

- Can we still enforce  $AB \rightarrow C$  on the decomposed table?
- YES! But not easy, need to use trigger!

# 3NF

## Third Normal Form



# 3NF

if not decomposed FD then RHS is a subset of prime attributes

## • Definition

- A table R is in 3NF if *every non-trivial and decomposed FD* **either**:
    - *its left hand side is a superkey* BCNF condition only
    - *the right hand side is a prime attributes* (appears in a key)
  - **Example:** R(A, B, C) with  $C \rightarrow B$ ,  $AC \rightarrow B$  and  $AB \rightarrow C$ 
    - Key: {AB} and {AC}
    - Prime Attributes: {ABC}
    - Non-trivial and decomposed FDs on R:
      - $C \rightarrow B$ 
        - B is PA
      - $AB \rightarrow C$ 
        - AB is SK
      - $AC \rightarrow B$ 
        - AC is SK
- ∴ R satisfies 3NF

# 3NF

another definition: all attribute does not transitively depend on a key



## • Definition

- A table R is in 3NF if *every non-trivial and decomposed FD either:*
  - *its left hand side is a superkey*
  - *the right hand side is a prime attributes* (appears in a key)
- **Example:** R(A, B, C) with  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $AC \rightarrow B$  and  $AB \rightarrow C$ 
  - Key: {A}
  - Prime Attributes: {A}
  - Non-trivial and decomposed FDs on R:
    - $A \rightarrow B$ 
      - A is SK
    - $B \rightarrow C$ 
      - B is **NOT** SK
      - C is **NOT** PA

$\therefore$  R violates 3NF

# 3NF vs BCNF

- **BCNF:** for any non-trivial and decomposed FD
  - The left hand side is a superkey

*“Every attribute must depend ONLY on superkeys!”*

**NO EXCEPTION**



- **3NF:** for any non-trivial and decomposed FD
  - The left hand side is a superkey
  - OR, the right hand side is a prime attribute

*“Exceptions can be made for prime attributes”*





# 3NF vs BCNF

- **BCNF**: for any non-trivial and decomposed FD
  - The left hand side is a superkey
- **3NF**: for any non-trivial and decomposed FD
  - The left hand side is a superkey
  - OR, the right hand side is a prime attribute
- **3NF** is more “*lenient*” than **BCNF**
  - Therefore
    - Satisfying BCNF  $\Rightarrow$  satisfying 3NF but not necessarily vice versa
    - Violating 3NF  $\Rightarrow$  violating BCNF but not necessarily vice versa

# 3NF

- **Checking 3NF**

- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*
- **By Counterexample:**
  1. Consider all non-trivial and decomposed FDs of R
  2. For each non-trivial and decomposed FDs of R, check that
    - a. the left hand side is superkey
    - b. the right hand side is in prime attributes
      - If not one of them, then we have a counterexample
  3. If no counterexample found, then R satisfies 3NF

**More but NOT All**  
 $\{S\} \subset \{S\}^+ \subset R$

# 3NF

- **Checking 3NF**

- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*
- **By Counterexample:**
  1. Consider all subset of attributes of R
  2. Compute the closure of each subset
  3. Consider only the subset that are not superkey *(closure  $\neq R$ )*
  4. Remove the “trivial” attributes
  5. We have a counterexample, if
    - a. The resulting set is non-empty, **and**
    - b. There is an attribute in the right hand side that is not in the left hand side and not a prime attribute

# 3NF

## • Checking 3NF

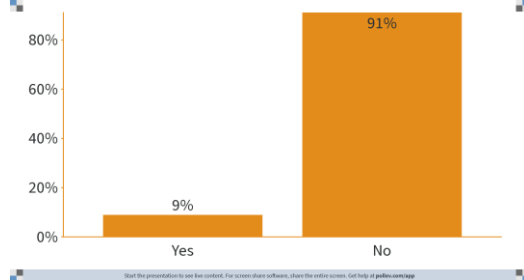
- **Example:**  $R(A, B, C, D)$  with  $AB \rightarrow C$ ,  $C \rightarrow D$  and  $D \rightarrow A$

1. Consider all subset of attributes of  $R$
2. Compute the closure of each subset
3. Consider only the subset that are not superkey *(closure  $\neq R$ )*
4. Remove the “trivial” attributes
5. We have a counterexample, if
  - a. The resulting set is non-empty, **and**
  - b. There is an attribute in the right hand side that is not in the left hand side and not a prime attribute

- Keys =  $\{AB\}$ ,  $\{BC\}$ ,  $\{BD\}$  Prime attributes =  $\{ABCD\}$

$\{A\}^+ = \{A\}$	$\{B\}^+ = \{B\}$	$\{C\}^+ = \{ACD\}$	$\{D\}^+ = \{AD\}$
$\{AB\}^+ = \{ABCD\}$	$\{AC\}^+ = \{ACD\}$	$\{AD\}^+ = \{AD\}$	
$\{BC\}^+ = \{ABCD\}$	$\{BD\}^+ = \{ABCD\}$	$\{CD\}^+ = \{ACD\}$	
$\{ABC\}^+ = \{ABCD\}$	$\{ABD\}^+ = \{ABCD\}$	$\{ACD\}^+ = \{ACD\}$	$\{BCD\}^+ = \{ABCD\}$

# 3NF



## • Checking 3NF

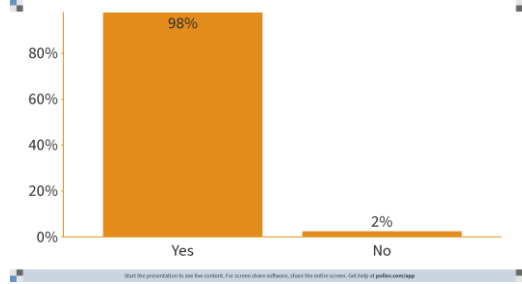
- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*

### ■ Exercise: R(A, B, C, D) with $B \rightarrow C$ and $B \rightarrow D$

- Keys = {AB}      Prime attributes = {AB}

$\{A\}^+ = \{A\}$	$\{B\}^+ = \{B, C, D\}$	$\{C\}^+ = \{C\}$	$\{D\}^+ = \{D\}$
$\{AB\}^+ = \{AB, C, D\}$	$\{AC\}^+ = \{A, C\}$	$\{AD\}^+ = \{A, D\}$	
$\{BC\}^+ = \{B, C, D\}$	$\{BD\}^+ = \{B, C, D\}$	$\{CD\}^+ = \{C, D\}$	
$\{ABC\}^+ = \{AB, C, D\}$	$\{ABD\}^+ = \{AB, C, D\}$	$\{ACD\}^+ = \{A, C, D\}$	$\{BCD\}^+ = \{B, C, D\}$

# 3NF



- **Checking 3NF**

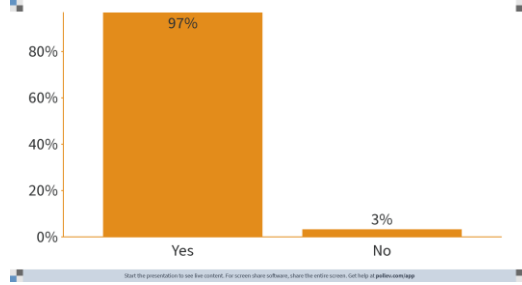
- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*

- **Exercise:** R(A, B, C, D) with  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $C \rightarrow D$  and  $D \rightarrow A$

- Keys = {A}, {B}, {C}, {D}      Prime attributes = {ABCD}

$\{A\}^+ = \{ABCD\}$	$\{B\}^+ = \{ABCD\}$	$\{C\}^+ = \{ABCD\}$	$\{D\}^+ = \{ABCD\}$
$\{AB\}^+ = \{ABCD\}$	$\{AC\}^+ = \{ABCD\}$	$\{AD\}^+ = \{ABCD\}$	
$\{BC\}^+ = \{ABCD\}$	$\{BD\}^+ = \{ABCD\}$	$\{CD\}^+ = \{ABCD\}$	
$\{ABC\}^+ = \{ABCD\}$	$\{ABD\}^+ = \{ABCD\}$	$\{ACD\}^+ = \{ABCD\}$	$\{BCD\}^+ = \{ABCD\}$

# 3NF

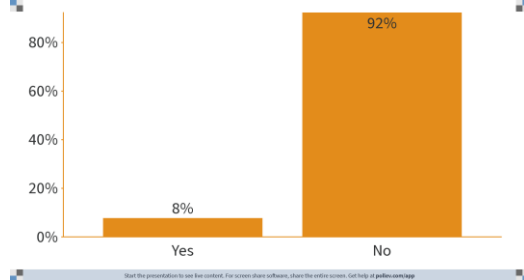


## • Checking 3NF

- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*
- **Exercise:** R(A, B, C, D) with  $AB \rightarrow D$ ,  $BD \rightarrow C$ ,  $CD \rightarrow A$  and  $AC \rightarrow B$ 
  - Keys = {AB}, {AC}, {BD}, {CD} Prime attributes = {ABCD}

$\{A\}^+ = \{A\}$	$\{B\}^+ = \{B\}$	$\{C\}^+ = \{C\}$	$\{D\}^+ = \{D\}$
$\{AB\}^+ = \{ABCD\}$	$\{AC\}^+ = \{ABCD\}$	$\{AD\}^+ = \{AD\}$	
$\{BC\}^+ = \{BC\}$	$\{BD\}^+ = \{ABCD\}$	$\{CD\}^+ = \{ABCD\}$	
$\{ABC\}^+ = \{ABCD\}$	$\{ABD\}^+ = \{ABCD\}$	$\{ACD\}^+ = \{ABCD\}$	$\{BCD\}^+ = \{ABCD\}$

# 3NF



## • Checking 3NF

- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*
- **Exercise:** R(A, B, C, D, E) with  $AB \rightarrow C$ ,  $C \rightarrow E$ ,  $E \rightarrow A$  and  $E \rightarrow D$ 
  - Keys = {AB} , {BC} , {BE}      Prime attributes = {ABCE}

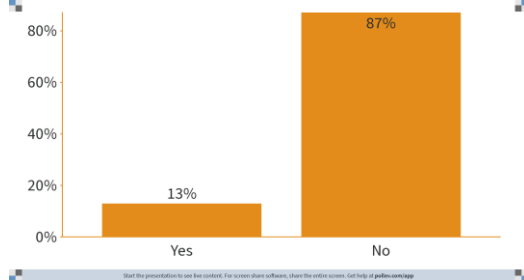
$$\{A\}^+ = \{A\} \quad \bigg| \quad \{B\}^+ = \{B\} \quad \bigg| \quad \{C\}^+ = \{A, C, D, E\}$$

Sometimes the violation is obvious like this  $E \rightarrow D$

Or you can always start from smallest subset of attributes and try to find violation



# 3NF



- **Checking 3NF**

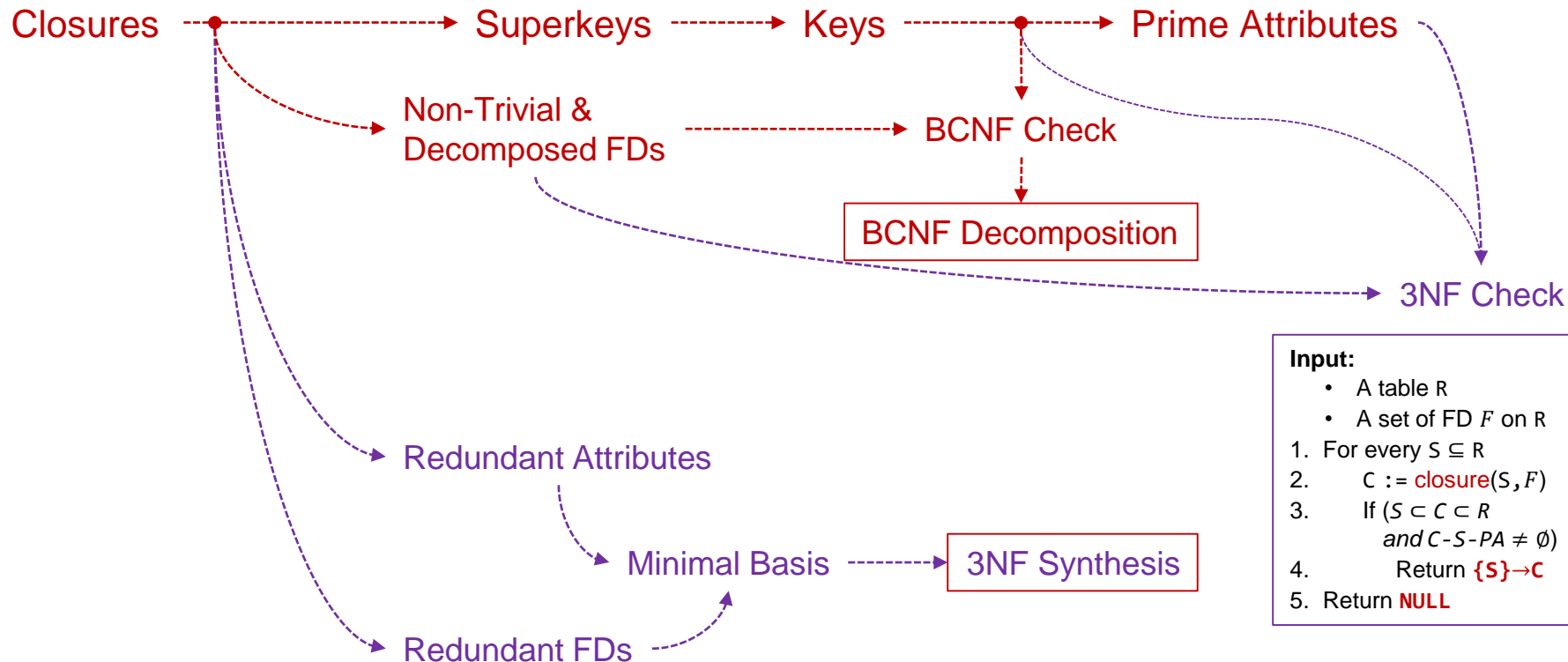
- A table R is in **NOT** 3NF if *every non-trivial and decomposed FD both:*
  - *its left hand side is NOT a superkey*
  - *the right hand side is NOT a prime attributes (does not appear in any key)*

- **Exercise:** R(A, B, C, D, E) with  $AB \rightarrow C$ ,  $DE \rightarrow C$  and  $B \rightarrow E$

- Keys = {ABD}      Prime attributes = {ABD}

$$\{A\}^+ = \{A\} \quad \bigg| \quad \{B\}^+ = \{B, E\}$$

# Algorithm Roadmap



## Input:

- A table  $R$
- A set of FD  $F$  on  $R$

1. For every  $S \subseteq R$
2.  $C := \text{closure}(S, F)$
3. If  $(S \subset C \subset R$   
and  $C - S - \text{PA} \neq \emptyset)$
4. Return  $\{S\} \rightarrow C$
5. Return **NULL**

# 3NF Decomposition



# 3NF Decomposition

- **Normalization**

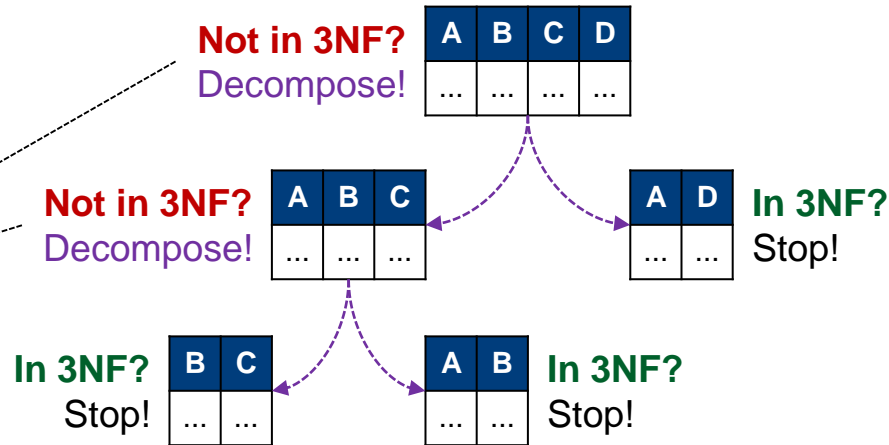
- **Same Idea as BCNF Decomposition:**

- Same potential problem of non dependency preserving decomposition
    - Is there a better idea?

**Input:**

- A table  $R$
- A set of FD  $F$  on  $R$

1. For every  $S \subseteq R$
2.  $C := \text{closure}(S, F)$
3. If  $(S \subset C \subset R$   
and  $C-S-PA \neq \emptyset)$
4. Return  $\{S\} \rightarrow C$
5. Return **NULL**



# Minimal Basis

on our path to 3NF synthesis



# Minimal Basis

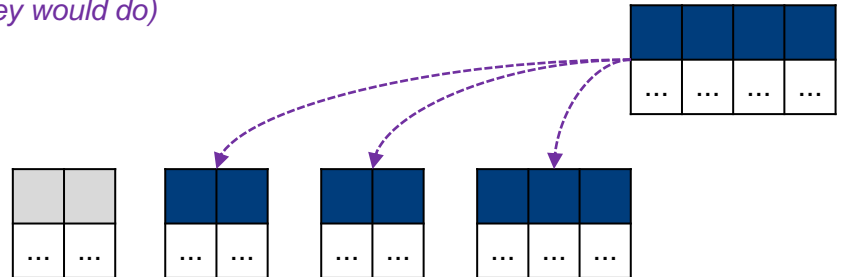
$\{AB \rightarrow C, C \rightarrow DE\}$   $\rightarrow$   $R1(A, B, C)$   
 $R2(C, D, E)$

## • A Sneak Peek

### ■ 3NF Synthesis

■ **Input:** Table R and a set of FDs F

1. **Derive minimal basis of F**
2. From the minimal basis, combine the FDs for which the left hand sides are the same  
*(union rule, producing what's called as canonical cover)*
3. Create a table for each FDs remained in the minimal basis after union
4. If none of the tables contains a key of the original table R, create a table that contains a key of R  
*(any key would do)*



# Minimal Basis

- Definition

- Given a set  $F$  of FDs, the minimal basis of  $F$  is a *simplified* version of  $F$ 
  - Also called minimal cover *(let's call this  $F_b$ )*
- How simplified?
  - Four conditions
    1. Every FD in  $F_b$  can be derived from  $F$  and vice versa
    2. Every FD in  $F_b$  are non-trivial and decomposed FD
    3. For each FD in  $F_b$ , none of the attributes on the left hand side is redundant *(no redundant attributes)*
    4. No FD in  $F_b$  are redundant *(no redundant FD)*
- ❖ Redundant means that we can remove them *(attributes or FDs)* without affecting the original FD *(i.e., still equivalent)*

# Minimal Basis

## • Definition

- Given a set  $F$  of FDs, the minimal basis of  $F$  is a *simplified* version of  $F$ 
  - Also called minimal cover *(let's call this  $F_b$ )*
- How simplified?
  - Four conditions
    1. Every FD in  $F_b$  can be derived from  $F$  and vice versa
      - $F = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
      - ✓  $F1 = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$
      - ✗  $F2 = \{A \rightarrow D, A \rightarrow C, C \rightarrow D\}$
      - ✗  $F3 = \{A \rightarrow B, A \rightarrow C, C \rightarrow D, D \rightarrow C\}$

### NOTE

$F$  cannot be derived from  $F2$

$F3$  cannot be derived from  $F$



# Minimal Basis

- **Definition**


- Given a set  $F$  of FDs, the minimal basis of  $F$  is a *simplified* version of  $F$ 
  - Also called minimal cover *(let's call this  $F_b$ )*
- How simplified?
  - Four conditions
  - 2. Every FD in  $F_b$  are non-trivial and decomposed FD
    - $F = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
    - ✓  $F1 = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$
    - ✗  $F2 = \{A \rightarrow BD, A \rightarrow C, C \rightarrow D\}$
    - ✓  $F3 = \{A \rightarrow B, A \rightarrow C, C \rightarrow D, BC \rightarrow D\}$

**NOTE**

$A \rightarrow BD$  is non-decomposed

# Minimal Basis

- Definition

- Given a set  $F$  of FDs, the minimal basis of  $F$  is a *simplified* version of  $F$ 
  - Also called minimal cover *(let's call this  $F_b$ )*
- How simplified?
  - Four conditions
- 3. For each FD in  $F_b$ , none of the attributes on the left hand side is redundant *(no redundant attributes)*
  - $F = \{A \rightarrow BD, \text{   $B \rightarrow C, C \rightarrow D, BC \rightarrow D\}$$
  - Consider  $AB \rightarrow C$ , if we remove  $B$  from left hand side, we have  $A \rightarrow C$
  - We can derive  $A \rightarrow C$  from  $F$  since  $\{A\}^+ = \{ABCD\}$  *(i.e.,  $A \rightarrow C$  is "hidden" in  $F$ )*
  - So we can add  $A \rightarrow C$  without adding extraneous constraints
  - But once we add  $A \rightarrow C$  then  $AB \rightarrow C$  is redundant!
    - Effectively, we found that  $B$  is redundant in  $AB \rightarrow C$
  - ∴  $AB \rightarrow C$  should not be in minimal basis

# Minimal Basis

- **Definition**

- Given a set  $F$  of FDs, the minimal basis of  $F$  is a *simplified* version of  $F$ 
  - Also called minimal cover *(let's call this  $F_b$ )*
- How simplified?
  - Four conditions
  - 4. No FD in  $F_b$  are redundant *(no redundant FD)*
    - $F = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
    - Consider  $BC \rightarrow D$
    - We can derive it from  $C \rightarrow D$ 
      - So we can remove it without removing any important information
    - $\therefore BC \rightarrow D$  should not be in minimal basis

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

- $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$        $F_b = \{A \rightarrow B, B \rightarrow C\}$
- Is  $F_b$  a minimal basis for  $F$ ?
  1.  $A \rightarrow C$  in  $F$  can be derived from  $F_b$ 
    - $F_b$  is  $F$  by removal of  $A \rightarrow C$
  2. All FDs in  $F_b$  are non-trivial and decomposed
  3. For any FD in  $F_b$ , if we remove an attribute from left hand side, then the FD cannot be derived from  $F$  *(in fact, they have no left hand side!)*
  4. If any FD in  $F_b$  is removed, then some FD in  $F$  cannot be derived

$\therefore F_b$  is a minimal basis for  $F$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$        $F_b = \{A \rightarrow B, AB \rightarrow C\}$

■ Is  $F_b$  a minimal basis for  $F$ ?

1.  $B \rightarrow C$  in  $F$  can **NOT** be derived from  $F_b$

$\therefore F_b$  is **NOT** a minimal basis for  $F$

## ERRATA

There was a typo error:  
Originally it was written as  
 $A \rightarrow C$  in  $F$  can NOT be  
derived from  $F_b$   
It should be  $B \rightarrow C$  in  $F$  can  
NOT be derived from  $F_b$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

- $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$        $F_b = \{A \rightarrow BC, A \rightarrow C\}$
- Is  $F_b$  a minimal basis for  $F$ ?
  1.  $B \rightarrow C$  in  $F$  can **NOT** be derived from  $F_b$
  2. Also...
    - $A \rightarrow BC$  is **NOT** a decomposed FD

$\therefore F_b$  is **NOT** a minimal basis for  $F$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{A \rightarrow B, A \rightarrow C, C \rightarrow B\}$        $F_b = \{A \rightarrow B, AB \rightarrow C, C \rightarrow B\}$

■ Is  $F_b$  a minimal basis for  $F$ ?

1. ✓
2. ✓

3. B in  $AB \rightarrow C$  can be removed in  $F_b$   
-  $\{A\}^+ = \{ABC\}$

*(redundant attribute)*

$\therefore F_b$  is **NOT** a minimal basis for  $F$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

$F_b = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

■ Is  $F_b$  a minimal basis for  $F$ ?

1. ✓
2. ✓
3. ✓
4.  $A \rightarrow C$  can be removed!

(*redundant FD*)

since it is just transitive

$\therefore F_b$  is **NOT** a minimal basis for  $F$



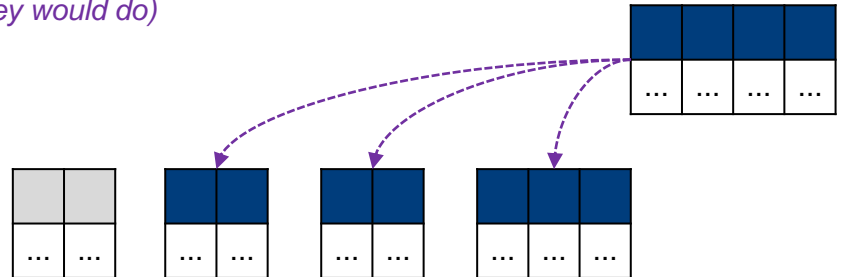
# Minimal Basis

- A Reminder on Why We Need Minimal Basis

- 3NF Synthesis

- **Input:** Table R and a set of FDs F

1. Derive *minimal basis* of F
2. From the *minimal basis*, combine the FDs for which the left hand sides are the same *(union rule, producing what's called as canonical cover)*
3. Create a table for each FDs remained in the *minimal basis* after union
4. If none of the tables contains a key of the original table R, create a table that contains a key of R *(any key would do)*



# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Algorithm

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
  3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
  4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*
- ❖ *Clearly, at the end, we have satisfied all conditions* *(as long as we maintain  $F_b \equiv F$ )*
- But how do we perform step 3?

### IDEA

Start with  $F = F1 \cup \{AB \rightarrow C\}$ .

- If A is redundant, then  $F1 \cup \{A \rightarrow C\}$  is equivalent to F.
- F can definitely be derived from F1 because  $\{AB \rightarrow C\}$  can be derived from  $\{A \rightarrow C\}$  using Augmentation + Decomposition.
- So we only have to check that F1 can be derived from F.
- This is done by deriving  $\{AB \rightarrow C\}$  from  $F1 \cup \{A \rightarrow C\}$ 
  - How? Compute  $\{A\}^+$  and check if C is inside

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant Attribute

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

- Consider an FD  $\{A\} \rightarrow B$  in  $F$  for any  $A$  and  $B$  *(in here,  $\{A\}$  is a set of attributes)*
  1. Consider an attribute  $C$  in  $\{A\}$ 
    - Compute  $\{A-C\}^+$  using  $F$  *(in here,  $\{A-C\}$  means we remove  $C$  from  $\{A\}$ )*
    - If  $B \in \{A-C\}^+$ , then we can remove  $C$  *(because  $\{A-C\} \rightarrow B$ )*
      - Repeat step 1 but with  $\{A\} \rightarrow B$  changed with  $\{A-C\} \rightarrow B$
  2. Do this for all FDs

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant Attribute

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow D, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$ 
  - Consider  $AB \rightarrow C$ 
    - $\{A\}^+ = \{A, B, C, D\}$ 
      - B can be removed from  $AB \rightarrow C$
  - Repeat with
    - $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D, BC \rightarrow D\}$

## Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Consider an attribute  $C$  in  $\{A\}$ 
    - Compute  $\{A-C\}^+$  using  $F$
    - If  $B$  in  $\{A-C\}^+$ , replace  $\{A\} \rightarrow B$  with  $\{A-C\} \rightarrow B$  then repeat
  2. Do this for all FDs

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant Attribute

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D, BC \rightarrow D\}$ 
  - Consider  $BC \rightarrow D$ 
    - $\{B\}^+ = \{B\}$ 
      - C can **NOT** be removed from  $BC \rightarrow D$
    - $\{C\}^+ = \{CD\}$ 
      - B can be removed from  $BC \rightarrow D$
  - Repeat with
    - $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D, C \rightarrow D\}$

## Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Consider an attribute  $C$  in  $\{A\}$ 
    - Compute  $\{A-C\}^+$  using  $F$
    - If  $B$  in  $\{A-C\}^+$ , replace  $\{A\} \rightarrow B$  with  $\{A-C\} \rightarrow B$  then repeat
  2. Do this for all FDs

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## Algorithm: Remove Redundant Attribute

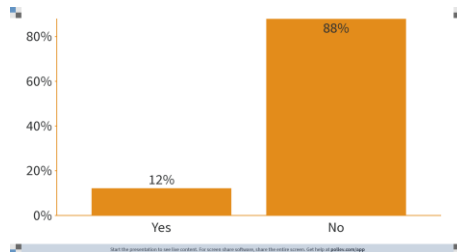
2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### Example:

- $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D, C \rightarrow D\}$ 
  - Nothing else to remove

### Question:

- Is this always gives us a unique solution?



## Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Consider an attribute  $C$  in  $\{A\}$ 
    - Compute  $\{A-C\}^+$  using  $F$
    - If  $B$  in  $\{A-C\}^+$ , replace  $\{A\} \rightarrow B$  with  $\{A-C\} \rightarrow B$  then repeat
  2. Do this for all FDs

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant Attribute

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, B \rightarrow A, AB \rightarrow C\}$ 
  - Consider  $AB \rightarrow C$ 
    - $\{A\}^+ = \{A, B, C\}$ 
      - B can be removed from  $AB \rightarrow C$
  - We could have started with
    - $\{B\}^+ = \{A, B, C\}$ 
      - A can be removed from  $AB \rightarrow C$
  - BUT, we cannot remove both!
    - So two possible solutions

## Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Consider an attribute  $C$  in  $\{A\}$ 
    - Compute  $\{A-C\}^+$  using  $F$
    - If  $B$  in  $\{A-C\}^+$ , replace  $\{A\} \rightarrow B$  with  $\{A-C\} \rightarrow B$  then repeat
  2. Do this for all FDs

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Algorithm

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

❖ *Clearly, at the end, we have satisfied all conditions* *(as long as we maintain  $F_b \equiv F$ )*

➤ But how do we perform **step 4**?

### IDEA

Start with  $F = F1 \cup \{A \rightarrow B\}$ .

- If  $\{A \rightarrow B\}$  is redundant, then  $F1$  is equivalent to  $F$ .
- $F1$  can definitely be derived from  $F$  because  $F1$  contains everything that  $F$  has but  $F$  has an additional  $\{A \rightarrow B\}$ .
- So we only have to check that  $F$  can be derived from  $F1$ .
- But the difference is only  $\{A \rightarrow B\}$ .
- This is done by deriving  $\{A \rightarrow B\}$  from  $F1$ 
  - How? Compute  $\{A\}^+$  and check if  $B$  is inside



# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

- Consider an FD  $\{A\} \rightarrow B$  in  $F$  for any  $A$  and  $B$  *(in here,  $\{A\}$  is a set of attributes)*

### ■ Observation:

- If we can remove  $\{A\} \rightarrow B$ , that means  $F - \{\{A\} \rightarrow B\} \equiv F$
- The only difference is the removal of  $\{A\} \rightarrow B$ 
  - Clearly, all FD in  $F - \{\{A\} \rightarrow B\}$  can be derived from  $F$
  - So what we need to show is only that  $\{A\} \rightarrow B$  can be derived from  $F - \{\{A\} \rightarrow B\}$
- Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
  - Then check if  $B$  is in  $\{A\}^+$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

- Consider an FD  $\{A\} \rightarrow B$  in  $F$  for any  $A$  and  $B$  *(in here,  $\{A\}$  is a set of attributes)*
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B \in \{A\}^+$ , then we can remove  $\{A\} \rightarrow B$  *(because  $\{A\} \rightarrow B$  even in its absence)*
      - Repeat with next FD but with  $\{A\} \rightarrow B$  removed
  2. Do this for all FDs

# Minimal Basis

## • Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D\}$ 
  - Consider  $A \rightarrow B$ 
    - $\{A\}^+$  w.r.t  $\{A \rightarrow D, A \rightarrow C, C \rightarrow D\}$   
 $= \{A, C, D\}$ 
      - $A \rightarrow B$  can **NOT** be remove

### Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

### Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$  then repeat with next FD
  2. Do this for all FDs

# Minimal Basis

## ● Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D\}$ 
  - Consider  $A \rightarrow D$ 
    - $\{A\}^+$  w.r.t  $\{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$   
 $= \{A, B, C, D\}$ 
      - $A \rightarrow D$  can be removed
  - Repeat with
    - $F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$

### Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

### Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$   
then repeat with next FD
  2. Do this for all FDs

# Minimal Basis

## ● Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$ 
  - Consider  $A \rightarrow C$ 
    - $\{A\}^+$  w.r.t  $\{A \rightarrow B, C \rightarrow D\}$   
 $= \{A, B\}$ 
      - $A \rightarrow C$  can **NOT** be removed

### Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

### Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$   
then repeat with next FD
  2. Do this for all FDs

# Minimal Basis

## ● Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$ 
  - Consider  $C \rightarrow D$ 
    - $\{C\}^+$  w.r.t  $\{A \rightarrow B, A \rightarrow C\}$   
 $= \{C\}$ 
      - $C \rightarrow D$  can **NOT** be removed

### Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

### Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$   
then repeat with next FD
  2. Do this for all FDs

# Minimal Basis

- **Algorithm: Remove Redundant FD**

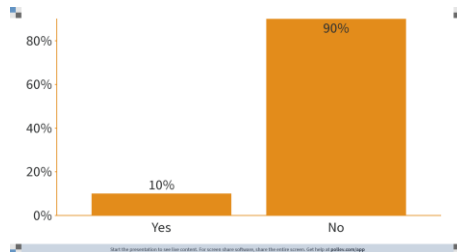
2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

- **Example:**

- $F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$ 
  - No other FDs to consider

- **Question:**

- Is this always gives us a unique solution?



## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$  then repeat with next FD
  2. Do this for all FDs

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## ● Algorithm: Remove Redundant FD

2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*

### ■ Example:

- $F = \{A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C, C \rightarrow A, C \rightarrow B\}$ 
  - Can remove either one but not both
    - $A \rightarrow B$ 
      - $\{A\}^+$  w.r.t.  $\{A \rightarrow C, B \rightarrow A, B \rightarrow C, C \rightarrow A, C \rightarrow B\}$   
 $= \{A, B, C\}$
    - $A \rightarrow C$ 
      - $\{A\}^+$  w.r.t.  $\{A \rightarrow B, B \rightarrow A, B \rightarrow C, C \rightarrow A, C \rightarrow B\}$   
 $= \{A, B, C\}$

## Algorithm

- For  $\{A\} \rightarrow B$  in  $F$ 
  1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$ 
    - If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$  then repeat with next FD
  2. Do this for all FDs



# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Algorithm

2. Transform the FDs so that it is non-trivial and decomposed

*(maintain  $F_b \equiv F$ )*

3. Remove redundant attributes on the left hand sides of each FDs

*(maintain  $F_b \equiv F$ )*

▪ For  $\{A\} \rightarrow B$  in  $F$

1. Consider an attribute  $C$  in  $\{A\}$

- Compute  $\{A-C\}^+$  using  $F$

- If  $B$  in  $\{A-C\}^+$ , replace  $\{A\} \rightarrow B$  with  $\{A-C\} \rightarrow B$  then repeat

2. Do this for all FDs

4. Remove redundant FDs

*(maintain  $F_b \equiv F$ )*

▪ For  $\{A\} \rightarrow B$  in  $F$

1. Compute  $\{A\}^+$  using  $F - \{\{A\} \rightarrow B\}$

- If  $B$  in  $\{A\}^+$ , remove  $\{A\} \rightarrow B$  then repeat with next FD

2. Do this for all FDs

❖ *Clearly, at the end, we have satisfied all conditions*

*(as long as we maintain  $F_b \equiv F$ )*

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow DE, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

2. Transform the FDs so that it is non-trivial and decomposed

*(maintain  $F_b \equiv F$ )*

■  $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

3. Remove redundant attributes on the left hand sides of each FDs

*(maintain  $F_b \equiv F$ )*

■ Two candidates

1.  $BC \rightarrow D$

2.  $BC \rightarrow E$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

3. Remove redundant attributes on the left hand sides of each FDs

*(maintain  $F_b \equiv F$ )*

■ Consider  $BC \rightarrow D$

- $\{B\}^+ = \{B\}$ 
  - So C can **NOT** be removed
- $\{C\}^+ = \{C\}$ 
  - So B can **NOT** be removed

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

3. Remove redundant attributes on the left hand sides of each FDs

*(maintain  $F_b \equiv F$ )*

■ Consider  $BC \rightarrow E$

- $\{B\}^+ = \{B\}$ 
  - So C can **NOT** be removed
- $\{C\}^+ = \{C\}$ 
  - So B can **NOT** be removed

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

- $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

2. Transform the FDs so that it is non-trivial and decomposed

*(maintain  $F_b \equiv F$ )*

- $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

3. Remove redundant attributes on the left hand sides of each FDs

*(maintain  $F_b \equiv F$ )*

- No redundant attributes

4. Remove redundant FDs

*(maintain  $F_b \equiv F$ )*

- Need to check everything

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

- $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

### 4. Remove redundant FDs

*(maintain  $F_b \equiv F$ )*

- Consider  $BC \rightarrow D$

- $\{BC\}^+$  w.r.t.  $\{BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$   
=  $\{B, C\}$ 
  - So  $BC \rightarrow D$  is **NOT** redundant

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

4. Remove redundant FDs

*(maintain  $F_b \equiv F$ )*

■ Consider  $BC \rightarrow E$

- $\{BC\}^+$  w.r.t.  $\{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$   
=  $\{A, B, C, D, E\}$ 
  - So  $BC \rightarrow E$  is **redundant**

❖ Continue with  $F = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

4. Remove redundant FDs

■ Consider  $A \rightarrow E$

- $\{A\}^+$  w.r.t.  $\{BC \rightarrow D, D \rightarrow A, E \rightarrow B\}$   
=  $\{A\}$ 
  - So  $A \rightarrow E$  is **NOT** redundant

*(maintain  $F_b \equiv F$ )*



# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

4. Remove redundant FDs

■ Consider  $D \rightarrow A$

- $\{D\}^+$  w.r.t.  $\{BC \rightarrow D, A \rightarrow E, E \rightarrow B\}$   
=  $\{D\}$ 
  - So  $D \rightarrow A$  is **NOT** redundant

*(maintain  $F_b \equiv F$ )*

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

4. Remove redundant FDs

■ Consider  $E \rightarrow B$

- $\{E\}^+$  w.r.t.  $\{BC \rightarrow D, A \rightarrow E, D \rightarrow A\}$   
=  $\{E\}$ 
  - So  $E \rightarrow B$  is **NOT** redundant

*(maintain  $F_b \equiv F$ )*

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Example

■  $F = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

2. Transform the FDs so that it is non-trivial and decomposed

*(maintain  $F_b \equiv F$ )*

■  $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

3. Remove redundant attributes on the left hand sides of each FDs

*(maintain  $F_b \equiv F$ )*

■ No redundant attributes

4. Remove redundant FDs

*(maintain  $F_b \equiv F$ )*

■  $BC \rightarrow E$  is redundant

❖  $F_b = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

# Minimal Basis

## Conditions

1.  $F_b \equiv F$
2. Non-trivial and decomposed
3. No redundant attributes
4. No redundant FD

## • Exercise

- $F = \{A \rightarrow C, AC \rightarrow D, AD \rightarrow B\}$
- 2. Transform the FDs so that it is non-trivial and decomposed *(maintain  $F_b \equiv F$ )*
  - $F = \{A \rightarrow C, AC \rightarrow D, AD \rightarrow B\}$
- 3. Remove redundant attributes on the left hand sides of each FDs *(maintain  $F_b \equiv F$ )*
  - Two candidates
    1.  $AC \rightarrow D$  *(C is redundant)*
    2.  $AD \rightarrow B$  *(D is redundant)*
  - $F = \{A \rightarrow C, A \rightarrow D, A \rightarrow B\}$
- 4. Remove redundant FDs *(maintain  $F_b \equiv F$ )*
  - No redundant FD
  - $F_b = \{A \rightarrow C, A \rightarrow D, A \rightarrow B\}$

# 3NF Synthesis

our final product



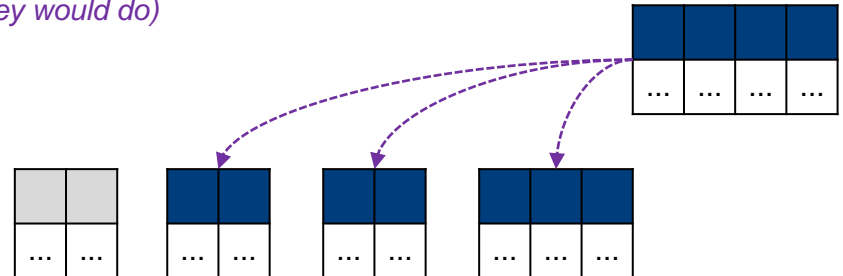
# 3NF Synthesis

- Algorithm

- 3NF Synthesis

- **Input:** Table R and a set of FDs F

1. Derive minimal basis of F
2. From the minimal basis, combine the FDs for which the left hand sides are the same *(union rule, producing what's called as canonical cover)*
3. **Create a table for each FDs remained in the minimal basis after union**
4. If none of the tables contains a key of the original table R, create a table that contains a key of R *(any key would do)*



# 3NF Synthesis

## 3NF Synthesis

1. Derive *minimal basis* of F
2. Produce *canonical cover*
3. Create a table for each FD
4. Add the *key* if missing

## • Algorithm

■ **Example:**  $R(A, B, C, D, E)$  with  $BC \rightarrow DE, A \rightarrow E, D \rightarrow A$  and  $E \rightarrow B$

■  $F = \{BC \rightarrow DE, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

1. Derive minimal basis of F

-  $F_b = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

2. Produce **canonical cover**

-  $F_c = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

3. Create a table for each FD

-  $R_1(B, C, D), R_2(A, E), R_3(A, D)$  and  $R_4(B, E)$

4. Add the key if missing

- Keys =  $\{AC\}, \{BC\}, \{CD\}, \{CE\}$

- R1 contains a key

### Why add Key?

Ensure *lossless join* decomposition

# 3NF Synthesis

## 3NF Synthesis

1. Derive *minimal basis* of F
2. Produce *canonical cover*
3. Create a table for each FD
4. Add the *key* if missing

## • Lossless Join Decomposition

■ **Example:**  $R(A, B, C, D)$  with  $A \rightarrow B$  and  $C \rightarrow D$

■  $F = \{A \rightarrow B, C \rightarrow D\}$

1. Derive minimal basis of F

-  $F_b = \{A \rightarrow B, C \rightarrow D\}$

2. Produce canonical cover

-  $F_c = \{A \rightarrow B, C \rightarrow D\}$

3. Create a table for each FD

-  $R_1(A, B)$  and  $R_2(C, D)$

4. Add the key if missing

- Keys =  $\{AC\}$

- Add  $R_3(A, C)$

### Why add Key?

$R_1(A, B)$  and  $R_2(C, D)$  cannot be used to reconstruct  $R(A, B, C, D)$



# 3NF Synthesis

## 3NF Synthesis

1. Derive *minimal basis* of F
2. Produce *canonical cover*
3. Create a table for each FD
4. Add the *key* if missing

## • Algorithm

- **Exercise:**  $R(A, B, C, D, E)$  with  $A \rightarrow B, A \rightarrow C, B \rightarrow C, E \rightarrow C$  and  $E \rightarrow D$ 
  - $F = \{A \rightarrow B, A \rightarrow C, B \rightarrow C, E \rightarrow C, E \rightarrow D\}$ 
    1. Derive minimal basis of F
      - $F_b = \{A \rightarrow B, B \rightarrow C, E \rightarrow C, E \rightarrow D\}$
    2. Produce canonical cover
      - $F_c = \{A \rightarrow B, B \rightarrow C, E \rightarrow CD\}$
    3. Create a table for each FD
      - $R1(A, B), R2(B, C)$  and  $R3(C, D, E)$
    4. Add the key if missing
      - Keys =  $\{AE\}$
      - Add  $R4(A, E)$

# 3NF Synthesis

## 3NF Synthesis

1. Derive *minimal basis* of F
2. Produce *canonical cover*
3. Create a table for each FD
4. Add the *key* if missing

## • Algorithm

- **Exercise:**  $R(A, B, C, D, E)$  with  $A \rightarrow B, AB \rightarrow C, C \rightarrow DE, E \rightarrow C$  and  $E \rightarrow D$ 
  - $F = \{A \rightarrow B, AB \rightarrow C, C \rightarrow DE, E \rightarrow C, E \rightarrow D\}$ 
    1. Derive minimal basis of F
      - $F_b = \{A \rightarrow B, A \rightarrow C, C \rightarrow D, C \rightarrow E, E \rightarrow C\}$
    2. Produce canonical cover
      - $F_c = \{A \rightarrow BC, C \rightarrow DE, E \rightarrow C\}$
    3. Create a table for each FD
      - $R1(A, B, C), R2(C, D, E)$  and  $R3(C, E)$
    4. Add the key if missing
      - Keys =  $\{A\}$
      - R1 contains a key

# Final Thought

closing statement

# Summary

- Poorly designed tables give rise to redundancy, update anomalies and deletion anomalies
- **BCNF** eliminates these problems
  - BCNF: for any non-trivial and decomposed FD on a table R, its left hand side is a superkey for R
  - But BCNF does not always preserve all FDs *(non dependency preserving)*
  - We may need to perform a join of multiple tables to check whether an FD holds
- **3NF** is slightly weaker than BCNF *(has more redundancies, has update and deletion anomalies in some rare cases)* but preserves all FDs
  - 3NF: for any non-trivial and decomposed FD on a table R, either its left hand side is a superkey for R, *or its right hand side is a prime attribute*

# BCNF or 3NF or *Lower*?

- BCNF is only *inferior* to 3NF in the sense that sometimes it does not preserve all FDs
- **Idea:**
  - Go for BCNF if we can find a BCNF decomposition that preserves all FDs
  - If such decomposition cannot be found, then
    - Still go for BCNF if preserving all FDs is not important
    - Or go for 3NF otherwise
      - If we are lucky, 3NF synthesis may actually find a BCNF decomposition!
- Should we go **lower** than 3NF?
  - Notice how there can be many tables produced by 3NF
  - What will happen to queries?
    - We may have to perform lots of joins
    - This can slow down queries by a lot since joins are expensive
  - Even 3NF may not be suitable in that case

# QUESTION?