CS2100 COMPUTER ORGANISATION

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Lecture #17

Combinational Circuits

NUS National University of Singapore | School of Computing

Lecture #17: Combinational Circuits

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2. Analysis Procedure
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4. Gate-Level (SSI) Design
5. Block-Level Design
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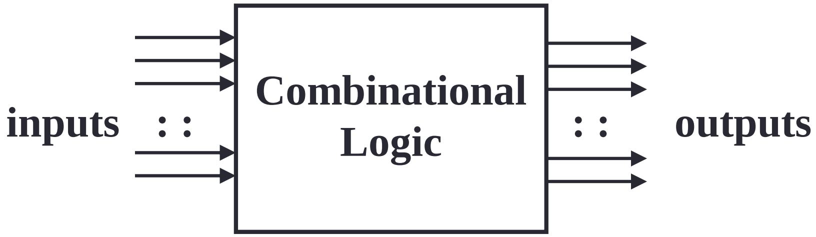
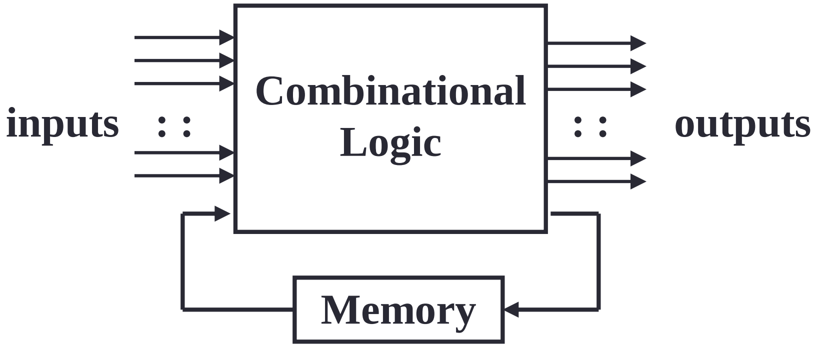
1. Introduction

- Two classes of logic circuits
  - Combinational
  - Sequential

- Combinational Circuit
  - Each output depends entirely on the immediate (present) inputs.



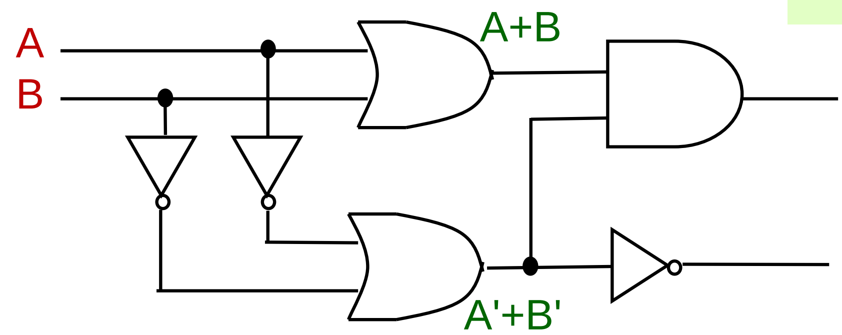
- Sequential Circuit
  - Each output depends on both present inputs and state.



2. Analysis Procedure

- Given a combinational circuit, how do you analyze its function?

What is this circuit?



- Steps:
  1. Label the inputs and outputs.
  2. Obtain the functions of intermediate points and the outputs.
  3. Draw the truth table.
  4. Deduce the functionality of the circuit ➲ ?

A	B	(A+B)	(A'+B')	F1 (S)	F2 (C)
0	0	0	1	0	0
0	1	1	1	1	0
1	0	1	1	1	0
1	1	1	0	0	1

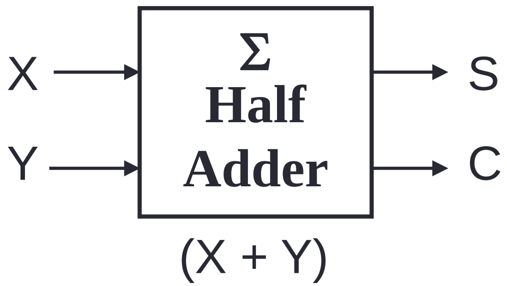
Half Adder
Carry
Sum

3. Design Methods

- Different combinational circuit design methods:
  - Gate-level design method (with logic gates)
  - Block-level design method (with functional blocks)
- Design methods make use of logic gates and useful function blocks
  - These are available as Integrated Circuit (IC) chips.
  - Types of IC chips (based on packing density): SSI, MSI, LSI, VLSI, ULSI.
- Main objectives of circuit design:
  - Reduce cost (number of gates for small circuits; number of IC packages for complex circuits)
  - Increase speed
  - Design simplicity (re-use blocks where possible)

4. Gate-Level (SSI) Design: Half Adder (1/2)

- Design procedure:
  1. State problem
     Example: Build a Half Adder.
  2. Determine and label the inputs and outputs of circuit.
     Example: Two inputs and two outputs labelled, as shown below.



X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

3. Draw the truth table.

4. Gate-Level (SSI) Design: Half Adder (2/2)

4. Obtain simplified Boolean functions.

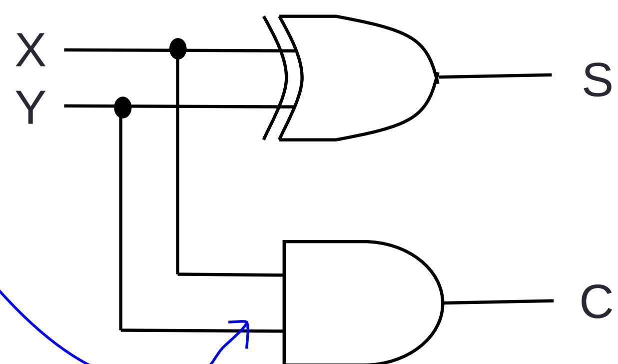
Example: $C = X \cdot Y$ | min term

$S = X' \cdot Y + X \cdot Y' = X \oplus Y$

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

5. Draw the logic diagram.

Half Adder



use AND gate

Block diagram of Half Adder

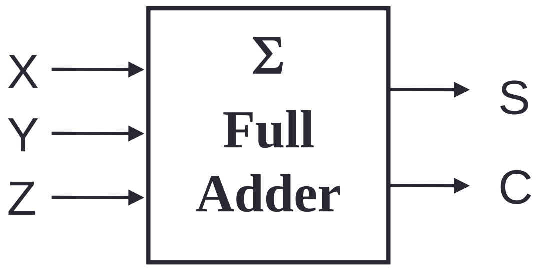
(X + Y)

4. Gate-Level (SSI) Design: Full Adder (1/5)

- Half adder adds up only two bits.
- To add two binary numbers, we need to add 3 bits (including the *carry*).
  - Example:

	1	1	1	0	carry
	0	0	1	1	X
+	0	1	1	1	Y
	1	0	1	0	S

- Need Full Adder (so called as it can be made from two half adders).



(X + Y + Z)

4. Gate-Level (SSI) Design: Full Adder (2/5)

- Truth table:

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Note:
Z - carry in (to the current position)
C - carry out (to the next position)

C

X \ YZ	00	01	11	10
0	0	0	1	0
1	0	1	1	1

- Using K-map, simplified SOP form:

C = ? X.Y + X.Z + Y.Z

S = ? X'.Y'.Z + X'.Y.Z' + X.Y'.Z' + X.Y.Z

S

X \ YZ	00	01	11	10
0	0	1	0	1
1	1	0	1	0

4. Gate-Level (SSI) Design: Full Adder (3/5)

- Alternative formulae using algebraic manipulation:

$$
\begin{aligned}
C &= X \cdot Y + X \cdot Z + Y \cdot Z \\
&= X \cdot Y + (X + Y) \cdot Z \\
&= X \cdot Y + (\,(X \oplus Y) + X \cdot Y\,) \cdot Z \\
&= X \cdot Y + (X \oplus Y) \cdot Z + X \cdot Y \cdot Z \\
&= X \cdot Y + (X \oplus Y) \cdot Z
\end{aligned}
$$

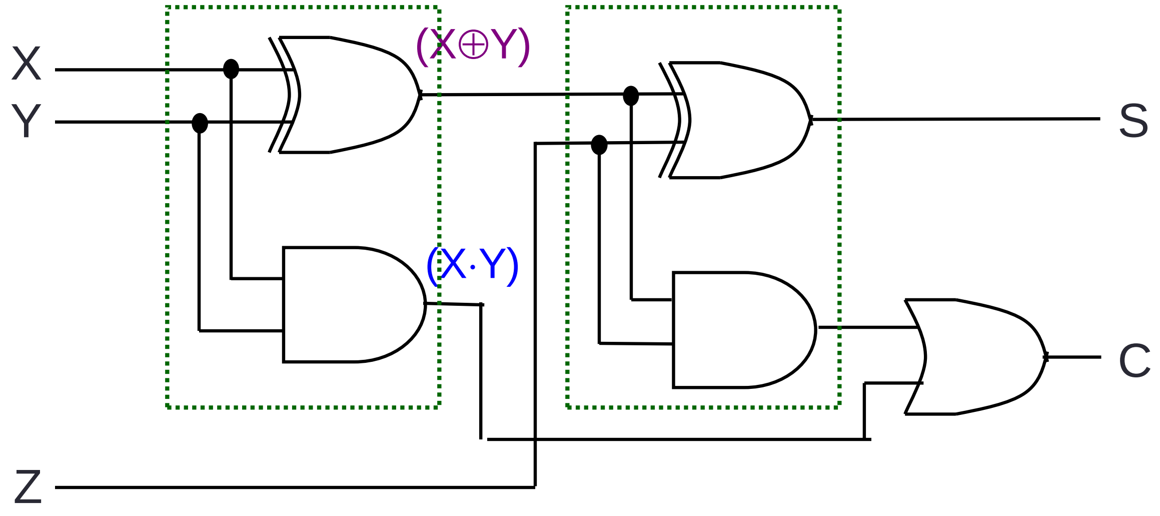
$$
\begin{aligned}
S &= X' \cdot Y' \cdot Z + X' \cdot Y \cdot Z' + X \cdot Y' \cdot Z' + X \cdot Y \cdot Z \\
&= X' \cdot (Y' \cdot Z + Y \cdot Z') + X \cdot (Y' \cdot Z' + Y \cdot Z) \\
&= X' \cdot (Y \oplus Z) + X \cdot (Y \oplus Z)' \\
&= X \oplus (Y \oplus Z)
\end{aligned}
$$

4. Gate-Level (SSI) Design: Full Adder (4/5)

- Circuit for above formulae:

  C = X·Y + (X⊕Y)·Z

  S = X⊕(Y⊕Z) = (X⊕Y)⊕Z (XOR is associative)



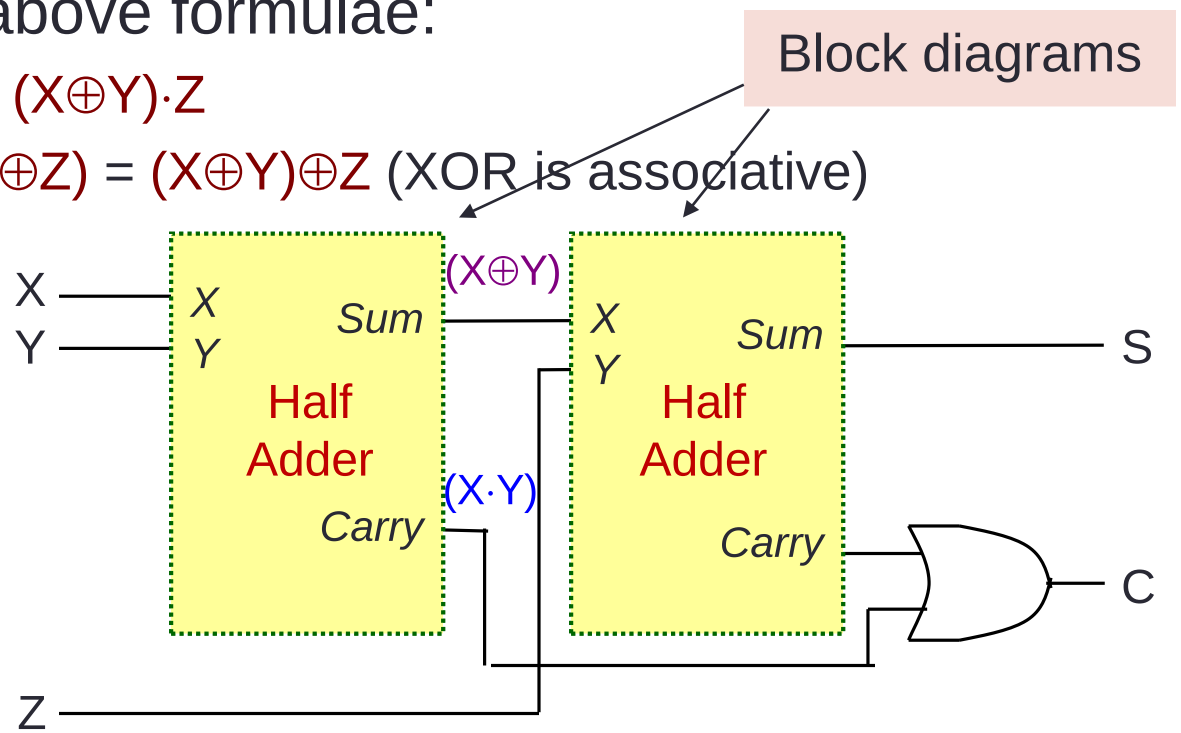
Full Adder made from two <u>Half-Adders</u> (+ an OR gate).

4. Gate-Level (SSI) Design: Full Adder (5/5)

- Circuit for above formulae:

  C = X·Y + (X⊕Y)·Z

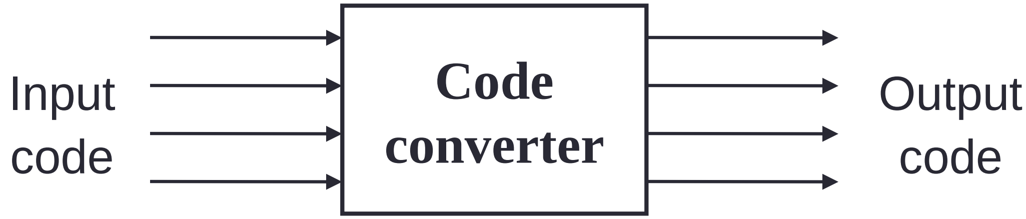
  S = X⊕(Y⊕Z) = (X⊕Y)⊕Z (XOR is associative)



Full Adder made from two <u>Half-Adders</u> (+ an OR gate).

4. Gate-Level (SSI) Design: Code Converters

- Code converter – takes an input code, translates to its equivalent output code.



- Example: BCD to Excess-3 code converter.
  - Input: BCD code
  - Output: Excess-3 code

4. BCD to Excess-3 Code Converter (1/3)

Digit	BCD code	Excess-3 code
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

4. BCD to Excess-3 Code Converter (2/3)

- Truth table:

	BCD				Excess-3			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0
10	1	0	1	0	X	X	X	X
11	1	0	1	1	X	X	X	X
12	1	1	0	0	X	X	X	X
13	1	1	0	1	X	X	X	X
14	1	1	1	0	X	X	X	X
15	1	1	1	1	X	X	X	X

Just definition of BCD

- K-maps:

W

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

X

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	X	X	X	X
10	0	1	X	X

Y

AB \ CD	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

Z

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	X	X	X	X
10	1	0	X	X

4. BCD to Excess-3 Code Converter (3/3)

W

AB \ CD	00	01	11	10
00				
01		1	1	1
11	X	X	X	X
10	1	1	X	X

X

AB \ CD	00	01	11	10
00		1	1	1
01	1			
11	X	X	X	X
10		1	X	X

Y

AB \ CD	00	01	11	10
00	1		1	
01	1		1	
11	X	X	X	X
10	1		X	X

Z

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	X	X	X	X
10	1		X	X

W = ?

X = ?

Y = ?

Z = ?

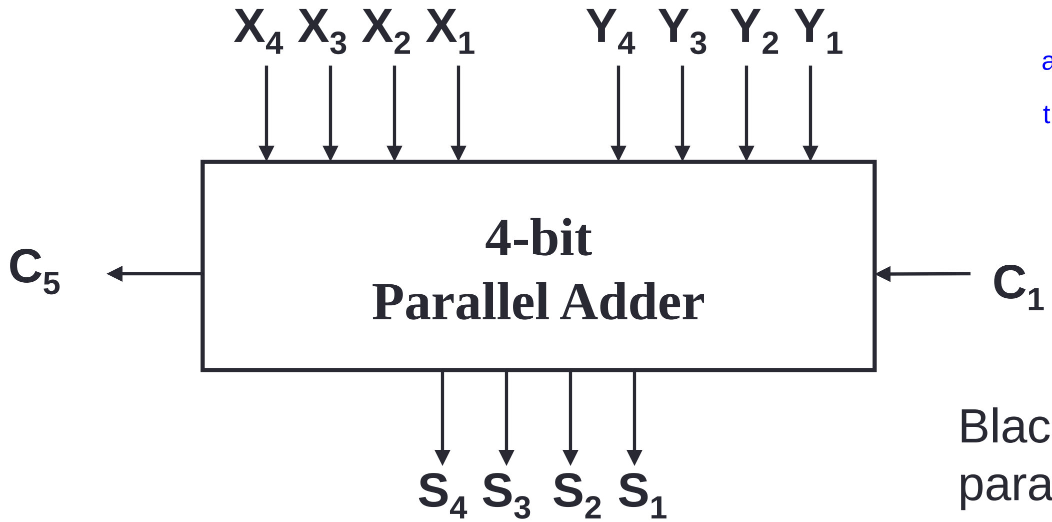
5. Block-Level Design

- More complex circuits can also be built using block-level method.
- In general, block-level design method (as opposed to gate-level design) relies on algorithms or formulae of the circuit, which are obtained by decomposing the main problem to sub-problems recursively (until small enough to be directly solved by blocks of circuits).
- First example shows how to create a 4-bit parallel adder using block-level design.
- Using 4-bit parallel adders as building blocks, we can create the following:
  1. BCD-to-Excess-3 Code Converter
  2. 16-bit Parallel Adder

Need creativity to think of how to solve block design

5. 4-bit Parallel Adder (1/4)

- Consider a circuit to add two 4-bit numbers together and a carry-in, to produce a 5-bit result.



all about using small blocks to create bigger things

Black-box view of 4-bit parallel adder

- 5-bit result is sufficient because the largest result is:

$$1111_2 + 1111_2 + 1_2 = 11111_2$$

5. 4-bit Parallel Adder (2/4)

- SSI design (gate-level design) technique should not be used here.
- Truth table for 9 inputs is too big: $2^9 = 512$ rows!

$X_4X_3X_2X_1$	$Y_4Y_3Y_2Y_1$	C_1	C_5	$S_4S_3S_2S_1$
0 0 0 0	0 0 0 0	0	0	0 0 0 0
0 0 0 0	0 0 0 0	1	0	0 0 0 1
0 0 0 0	0 0 0 1	0	0	0 0 0 1
…	…	…	…	…
0 1 0 1	1 1 0 1	1	1	0 0 1 1
…	…	…	…	…
1 1 1 1	1 1 1 1	1	1	1 1 1 1

- Simplification becomes too complicated!

5. 4-bit Parallel Adder (3/4)

- Alternative design possible.
- Addition formula for each pair of bits (with carry in),

$$C_{i+1}S_i = X_i + Y_i + C_i$$

has the same function as a full adder:

$$C_{i+1} = X_i \cdot Y_i + (X_i \oplus Y_i) \cdot C_i$$
$$S_i = X_i \oplus Y_i \oplus C_i$$

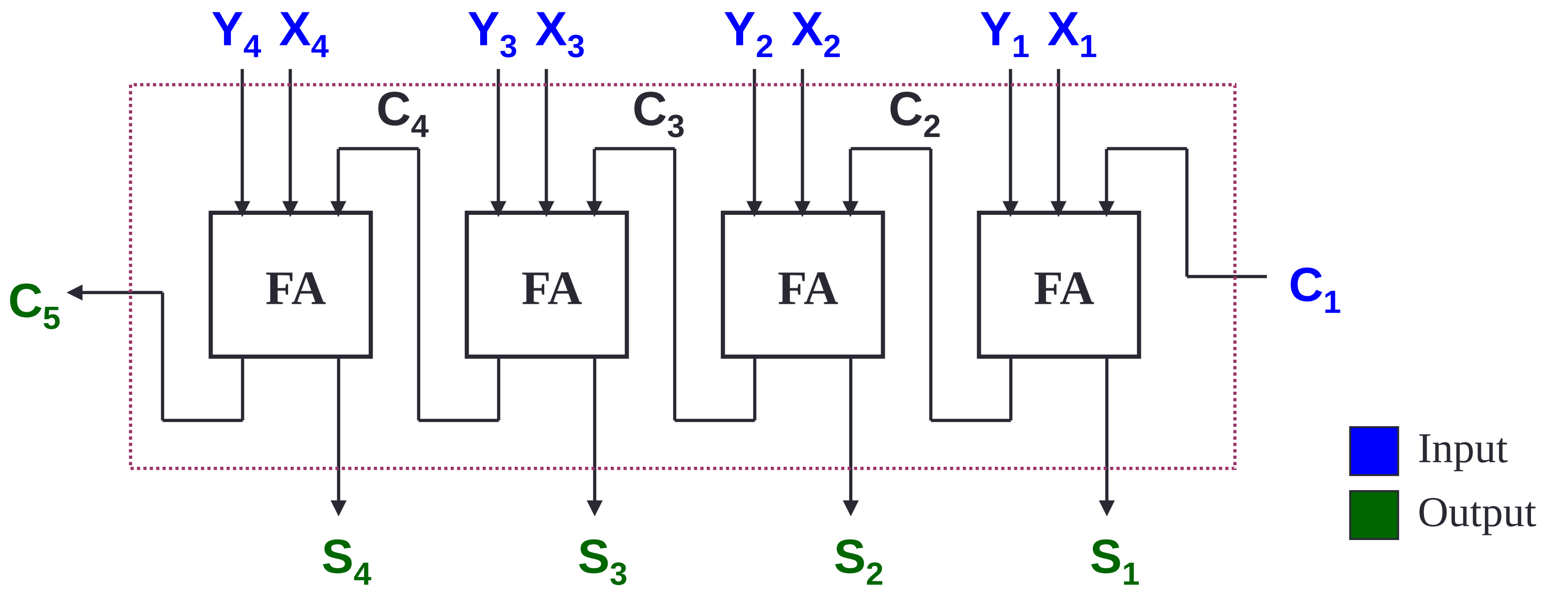
C =	1 1 0 0
X =	1 0 1 0
Y =	1 1 1 1
X + Y =	1 1 0 0 1

3 inputs: Carry in, X, Y

2 outputs: Carry Out , Sum

5. 4-bit Parallel Adder (4/4)

- Cascading 4 full adders via their carries, we get:



- Note that carry is propagated by cascading the carry from one full adder to the next.
- Called Parallel Adder because inputs are presented simultaneously (in parallel). Also called Ripple-Carry Adder.

5. BCD to Excess-3 Converter: Revisit (1/2)

- Excess-3 code can be converted from BCD code using truth table:
- Gate-level design can be used since only 4 inputs.
- However, alternative design is possible.
- Use problem-specific formula:

  Excess-3 code
  $= \text{BCD Code} + 0011_2$

	BCD				*Excess-3*			
	A	**B**	**C**	**D**	**W**	**X**	**Y**	**Z**
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0
10	1	0	1	0	X	X	X	X
11	1	0	1	1	X	X	X	X
12	1	1	0	0	X	X	X	X
13	1	1	0	1	X	X	X	X
14	1	1	1	0	X	X	X	X
15	1	1	1	1	X	X	X	X

5. BCD to Excess-3 Converter: Revisit (2/2)

- Block-level circuit:

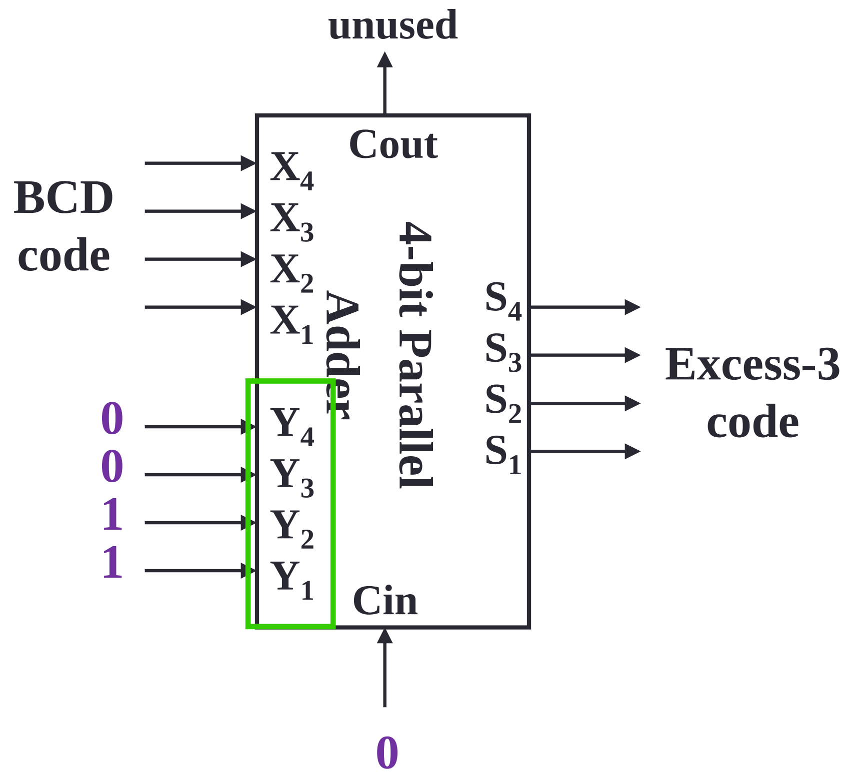
4 bit parallel adder has 9 inputs 5 outputs

A BCD to Excess-3 Code Converter

aka connecting

0 to ground

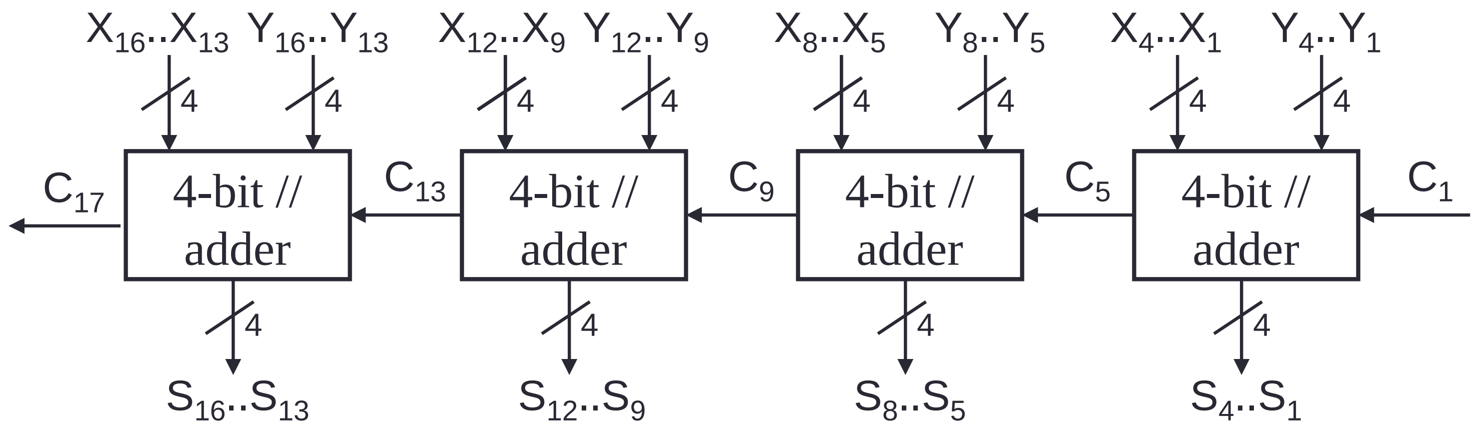
1 to VCC



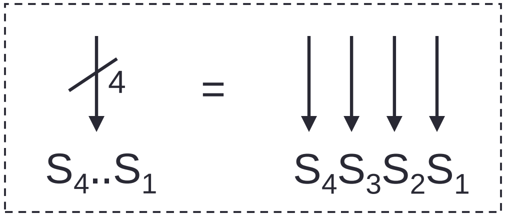
Note: In the lab, input 0 (low) is connected to GND, 1 (high) to Vcc.

5. 16-bit Parallel Adder

- Larger parallel adders can be built from smaller ones.
- Example: A 16-bit parallel adder can be constructed from four 4-bit parallel adders:



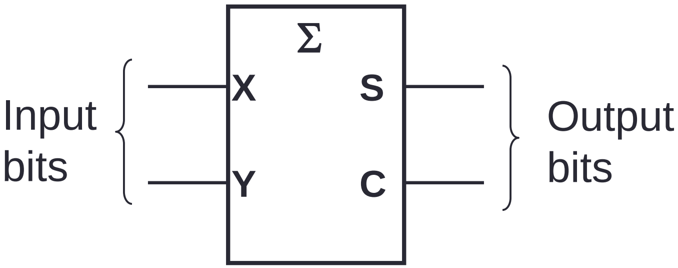
A 16-bit parallel adder

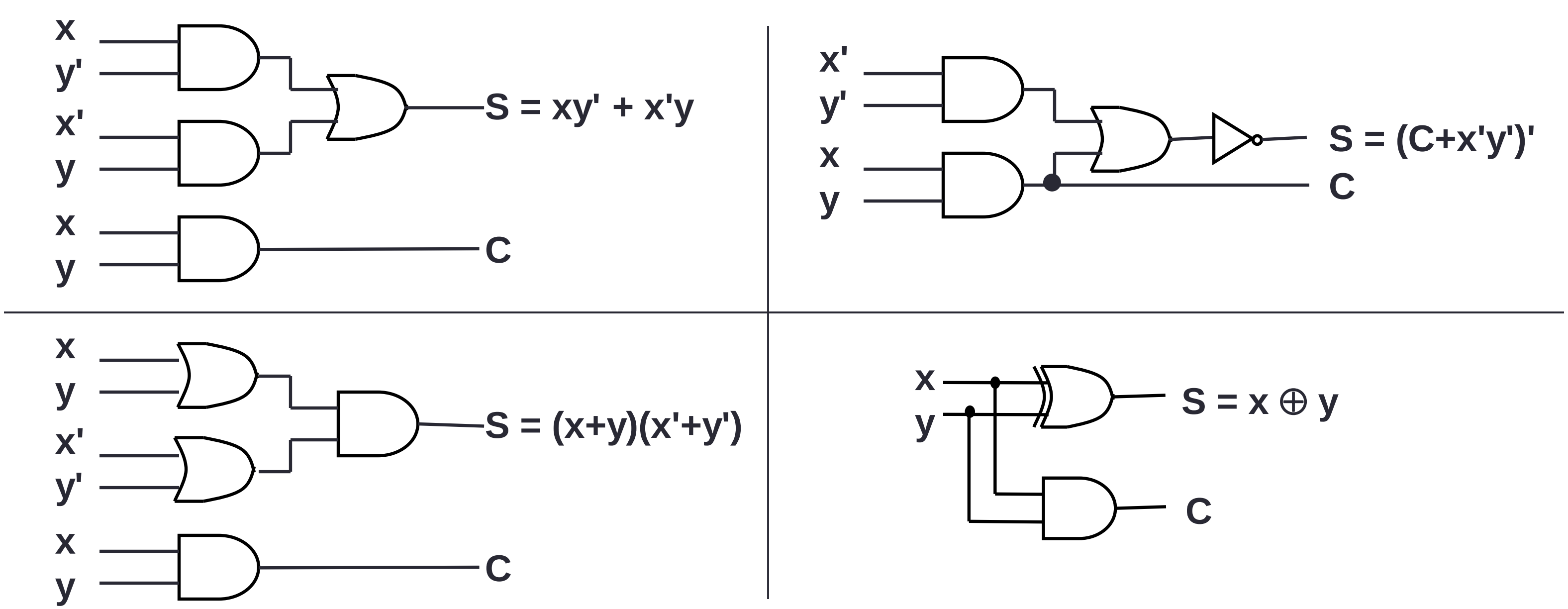


6. Summary of Arithmetic Circuits (1/4)

- Half adder

x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

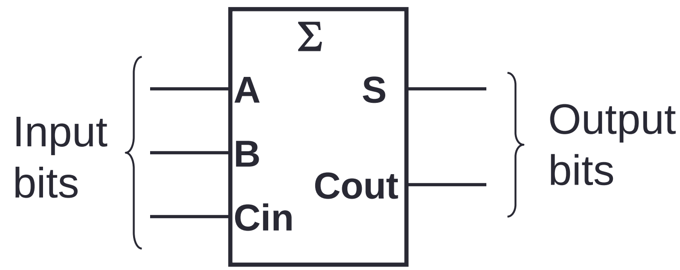




6. Summary of Arithmetic Circuits (2/4)

- Full adder

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

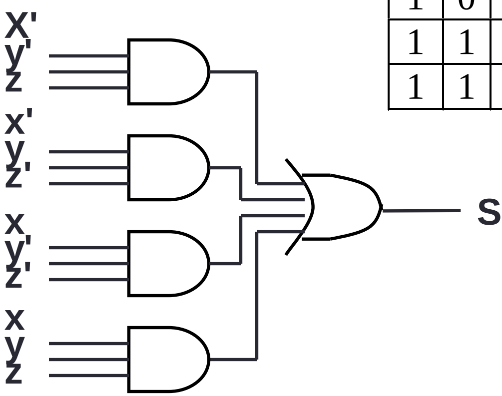


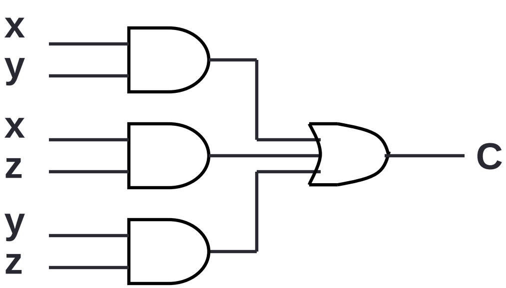
x \ yz	00	01	11	10
0			1	
1		1	1	1

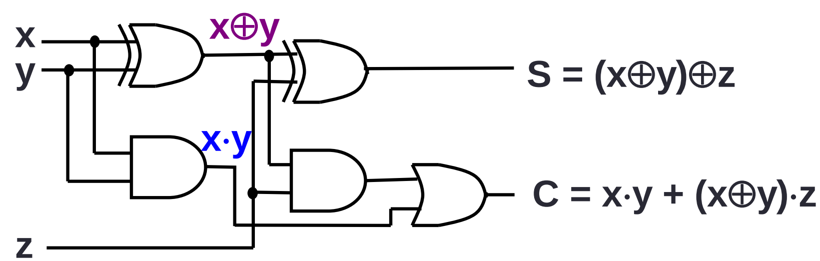
C = xy + xz + yz

x \ yz	00	01	11	10
0		1		1
1	1		1	

S = x'y'z + x'yz' + xy'z' + xyz







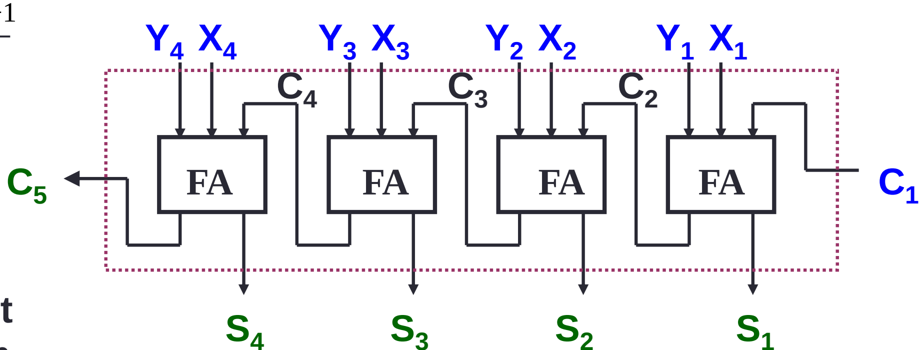
6. Summary of Arithmetic Circuits (3/4)

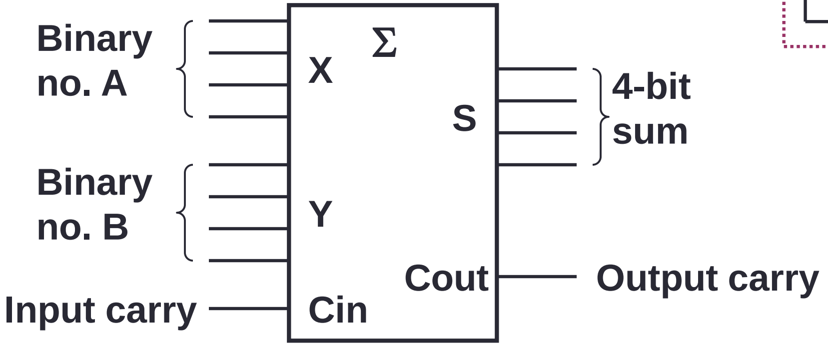
- 4-bit parallel adder

Subscript i	4	3	2	1	
Input carry	0	1	1	0	C_i
Augend	1	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output carry	0	0	1	1	C_{i+1}

2 ways:

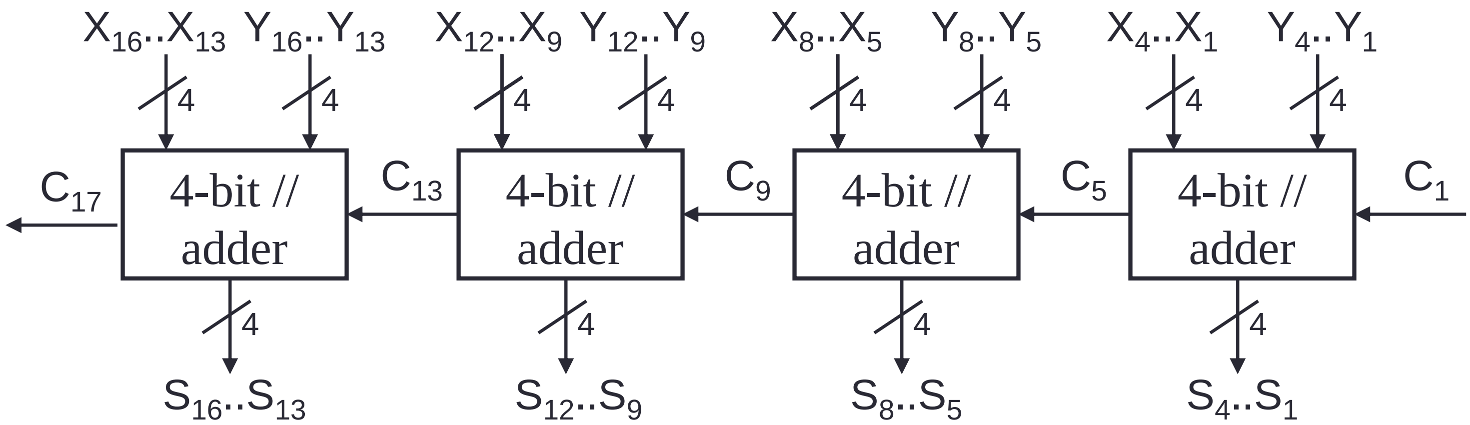
- Serial (one FA)
- Parallel (*n* FAs for *n* bits)





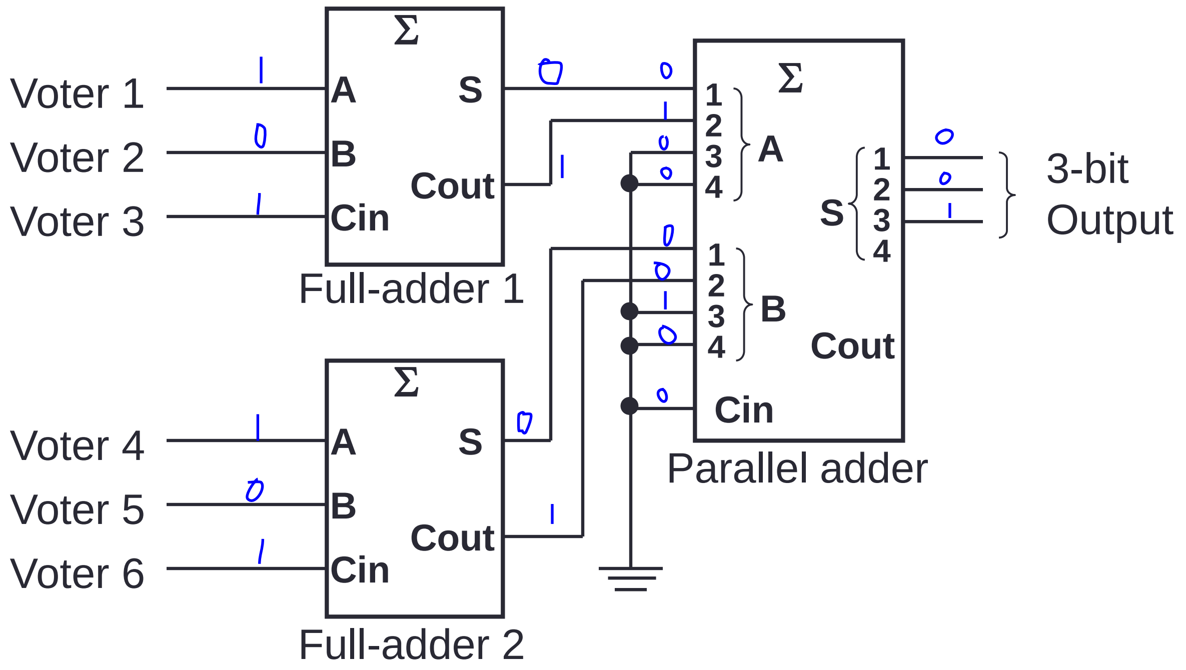
6. Summary of Arithmetic Circuits (4/4)

- Cascading 4 full adders (FAs) gives a 4-bit parallel adder.
  - Classical method: 9 input variables → $2^9 = 512$ rows in truth table!
- Cascading method can be extended to larger adders.
  - Example: 16-bit parallel adder.



7. Example: 6-Person Voting System

- Application: 6-person voting system.
  - Use FAs and a 4-bit parallel adder.
  - Each FA can sum up to 3 votes.



8. Magnitude Comparator (1/4)

- Magnitude comparator: compares 2 unsigned values A and B, to check if A>B, A=B, or A<B.
- To design an *n*-bit magnitude comparator using classical method, it would require 2^{2n} rows in truth table!
- We shall exploit regularity in our design.
- Question: How do we compare two 4-bit unsigned values A ($a_3a_2a_1a_0$) and B ($b_3b_2b_1b_0$)?

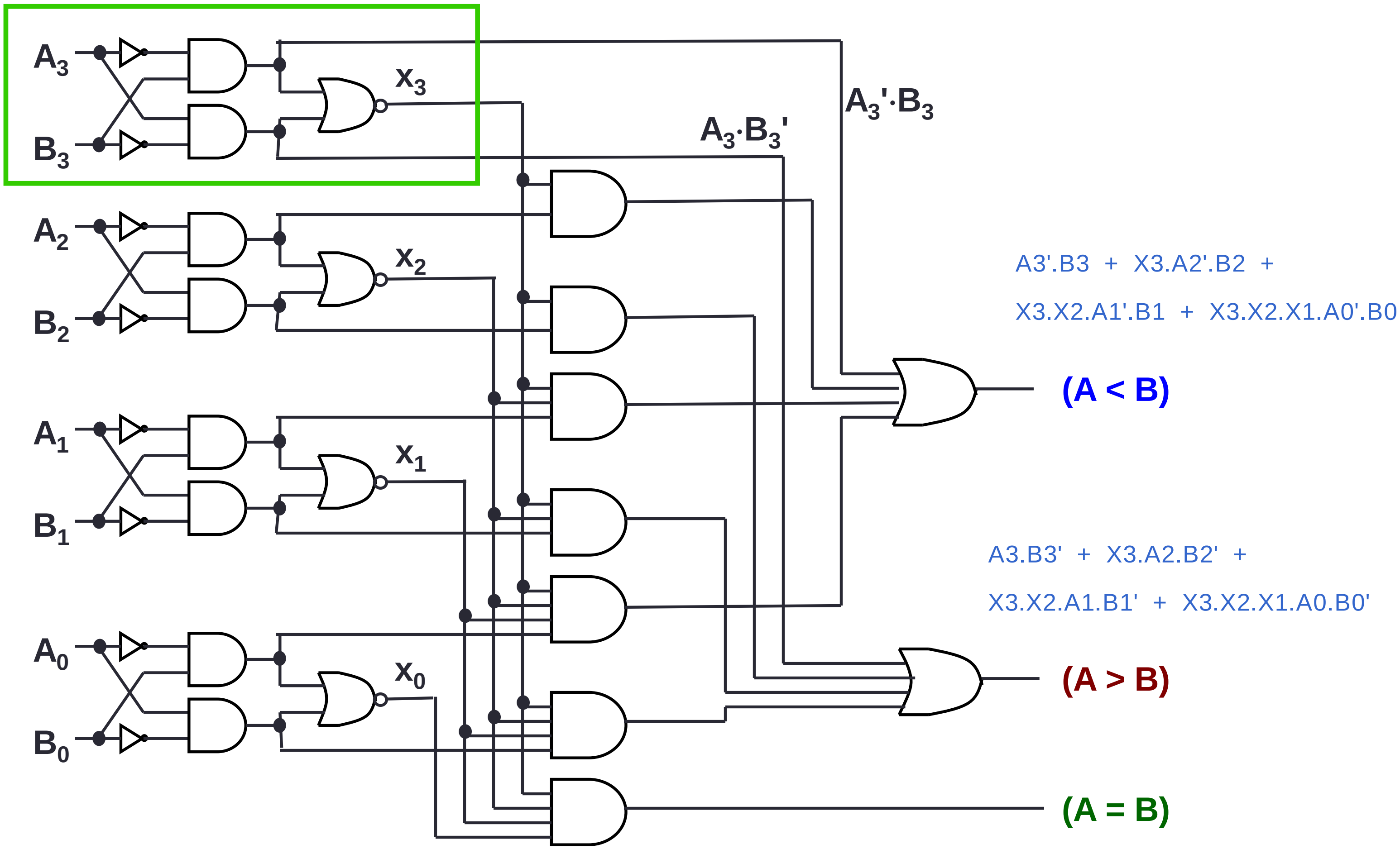
  If ($a_3 > b_3$) then A > B

  If ($a_3 < b_3$) then A < B

  If ($a_3 = b_3$) then if ($a_2 > b_2$) …

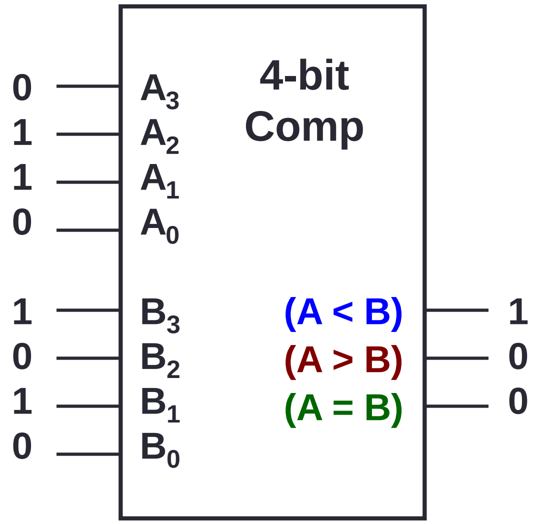
8. Magnitude Comparator (2/4)

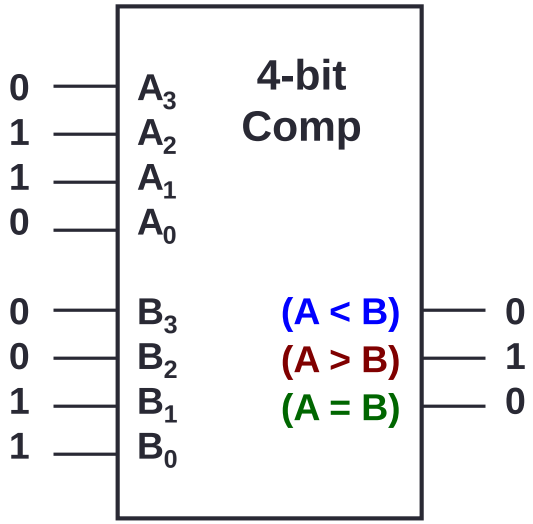
Let $A = A_3A_2A_1A_0$, $B = B_3B_2B_1B_0$; $x_i = A_i \cdot B_i + A_i' \cdot B_i'$



8. Magnitude Comparator (3/4)

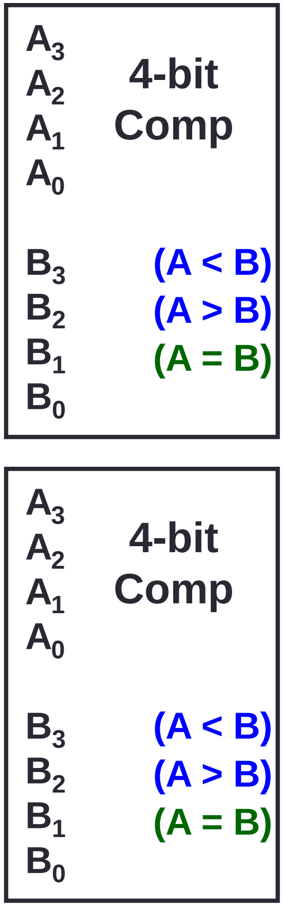
- Block diagram of a 4-bit magnitude comparator





8. Magnitude Comparator (4/4)

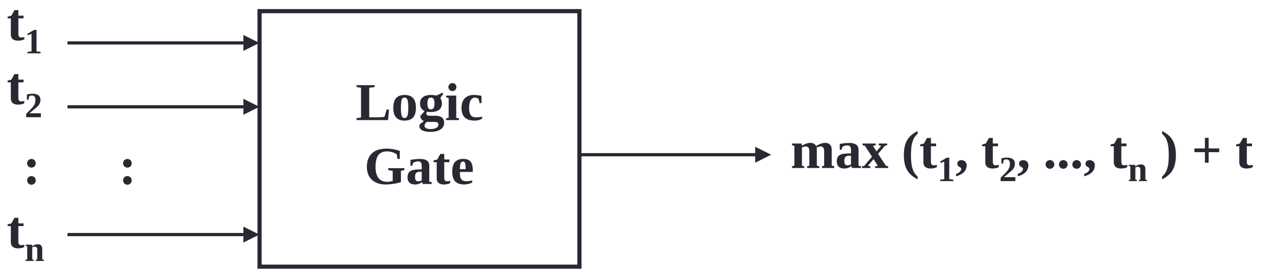
- A function F accepts a 4-bit binary value ABCD, and returns 1 if $3 \leq ABCD \leq 12$, or 0 otherwise. How would you implement F using magnitude comparators and a suitable logic gate?



9. Circuit Delays (1/5)

- Given a logic gate with delay t. If inputs are stable at times $t_1, t_2, \ldots, t_n$, then the earliest time in which the output will be stable is:

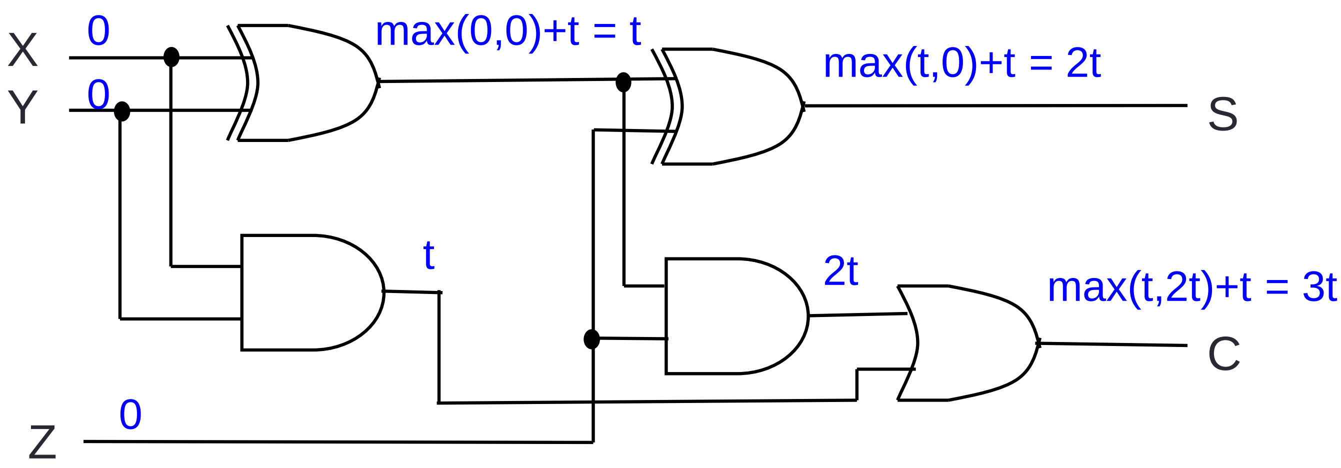
$$\max(t_1, t_2, \ldots, t_n) + t$$



- To calculate the delays of all outputs of a combinational circuit, repeat above rule for all gates.

9. Circuit Delays (2/5)

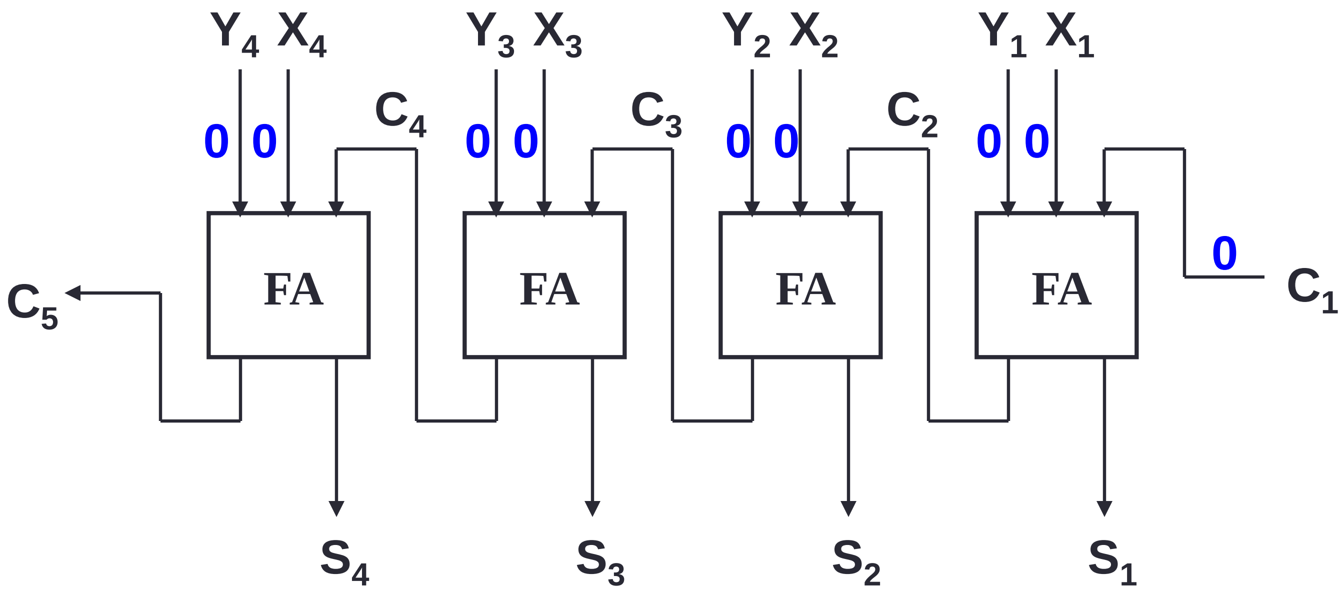
- As a simple example, consider the full adder circuit where all inputs are available at time 0. Assume each gate has delay t.



- Outputs **S** and **C** experience delays of **2t** and **3t** respectively.

9. Circuit Delays (3/5)

- More complex example: 4-bit parallel adder.

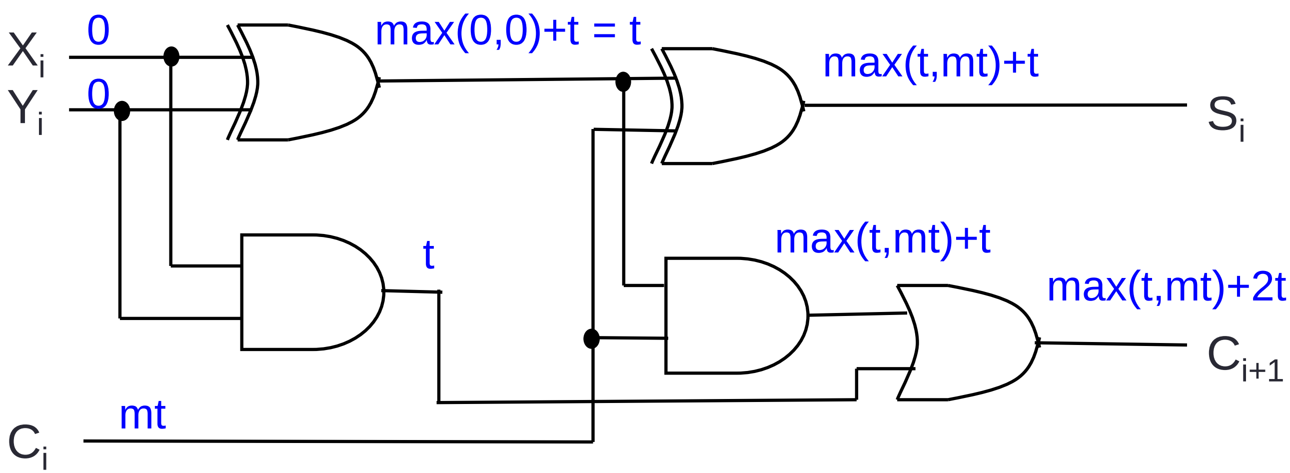


9. Circuit Delays (4/5)

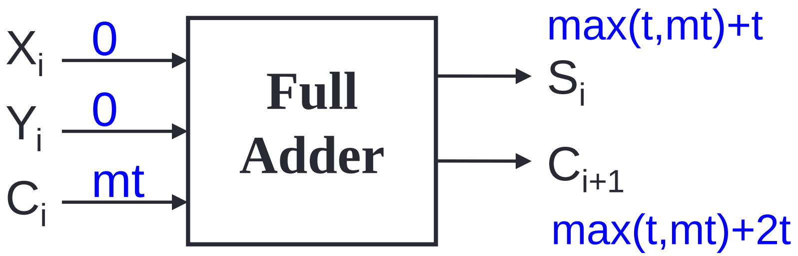
- Analyse the delay for the repeated block.

where X_i, Y_i are stable at 0t, while C_i is assumed to be stable at mt.

- Performing the delay calculation:



9. Circuit Delays (5/5)



- Calculating:

  When i=1, m=0; $S_1 = 2t$ and $C_2 = 3t$
  When i=2, m=3; $S_2 = 4t$ and $C_3 = 5t$
  When i=3, m=5; $S_3 = 6t$ and $C_4 = 7t$
  When i=4, m=7; $S_4 = 8t$ and $C_5 = 9t$

- In general, an *n*-bit ripple-carry parallel adder will experience the following delay times:

  $S_n = ((n - 1)2 + 2)\ t$
  $C_{n+1} = ((n - 1)2 + 3)\ t$

- Propagation delay of ripple-carry parallel adders is proportional to the number of bits it handles.

- Maximum delay: $((n - 1)2 + 3)\ t$

Quick Review Questions

- DLD pages 128 – 129
  Questions 6-1 to 6-4.

Block level has the problem of dependency, easy to implement but timing might take longer, since inputs might depend on previous output

End of File