National University of Singapore

Semester 1, 2020/2021 MA1101R Practice Assignment 2

(a) Use A4 size paper and pen (blue or black ink) to write your answers.

(b) Write down your student number and full name clearly on the top left of every page of the answer scripts.

(c) Write the page number on the top right corner of each page of answer scripts.

(d) There are three questions in this worksheet (see next page) with a total of 20 marks.

(e) To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (arrange them in order of the page. Name the pdf file by **StudentNo P2** (e.g. **A123456R P2**). Upload your pdf into the LumiNUS folder Practice 2 submission.

(f) Hand in your answers by the end of this session. **Late submission will not be accepted.**

1. $\boldsymbol{A}$ and $\boldsymbol{B}$ are 3×3 row equivalent matrices related by the following diagram:

$$\boldsymbol{A} \xrightarrow{R_2-2R_1} \xrightarrow{R_1 \leftrightarrow R_3} \xrightarrow{3R_2} \xrightarrow{R_1+R_3} \boldsymbol{B}$$

   (i) [4 marks] Write down four elementary matrices $\boldsymbol{E}_1, \boldsymbol{E}_2, \boldsymbol{E}_3, \boldsymbol{E}_4$ such that

$$\boldsymbol{E}_4\boldsymbol{E}_3\boldsymbol{E}_2\boldsymbol{E}_1\boldsymbol{A} = \boldsymbol{B}.$$

   (ii) [2 marks] Find an invertible matrix $\boldsymbol{C}$ such that $\boldsymbol{A} = \boldsymbol{C}\boldsymbol{B}$. (You may use MATLAB, but you need to show how $\boldsymbol{C}$ is obtained.)

   (iii) [2 marks] If $\det(\boldsymbol{B}) = 12$, find $\det(\boldsymbol{A})$. You need to show how you obtain the answer.

   (iv) [2 marks] If $\boldsymbol{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ is the inverse of $\boldsymbol{B}$, find $\boldsymbol{A}^{-1}$. (You may use MATLAB, but you need to show how $\boldsymbol{A}^{-1}$ is obtained.)

2. Let $\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ a & b & c & d \end{pmatrix}$ for some real numbers a, b, c, d.

   (i) [2 marks] Find $\det(\boldsymbol{A})$. (Show your working)

   (ii) [2 marks] Write down the condition among a, b, c, d such that the homogeneous system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions. (Briefly explain your answer.)

3. Let $U = \{(x, y, z) \mid x + y + z = 0\}$ and $V = \{(x, y, z) \mid 2x - y - z = 0\}$ be the implicit set notations representing two planes in the xyz-space.

   (i) [2 marks] Write down the explicit set notation of U.

   (ii) [2 marks] Write down the explicit set notation of $U \cap V$.

   (iii) [1 mark] Write down a vector that is parallel to the line of intersection of U and V.

   (iv) [1 mark] Is $W = \{(a, a, a) \mid a \in \mathbb{R}\}$ a subset of V? (Briefly explain your answer.)