

CS2100

COMPUTER ORGANISATION

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Lecture #16

Quine-McCluskey



NUS
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Computing

Lecture #16: Quine-McCluskey

This topic is only for your own reading only.

- A tabulation method **similar in concept** to K-map
- Applicable for functions with any number of variables
 - K-maps are useful for functions with at most 5 or 6 variables
- Tedious on paper, but can be automated (programmed)
- Non-examinable
 - But knowing it may enhance your understanding of K-maps

PIs and EPIs

- To find the simplest SOP expression from a K-map, you need to obtain:
 - Minimum number of literals per product term; and
 - Minimum number of product terms.
- Achieved through K-map using
 - *Biggest groupings* of minterms (**prime implicants**) where possible; and
 - *No redundant groupings* (look for **essential prime implicants**)

Eg: $F(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$

Step 1: List out all minterms in groups with same number of 1s in their binary codes.

1st column

2: 0010

4: 0100

8: 1000

Codes with one 1

3: 0011

5: 0101

10: 1010

Codes with two 1s

7: 0111

13: 1101

Codes with three 1s

15: 1111

Codes with four 1s

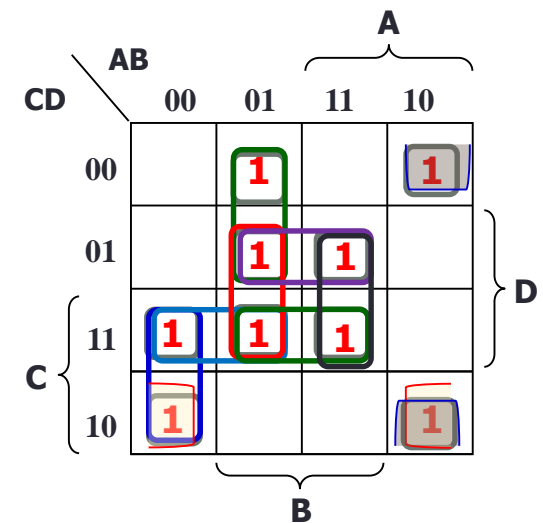
		A			
		AB		11	10
CD	00	01	11	10	D
	00	1		1	
01		1	1		
11	1	1	1		
10	1			1	
		B			

$$\text{Eg: } F(A,B,C,D) = \Sigma m(2,3,4,5,7,8,10,13,15)$$

Step 2: Combine codes that differ by 1 bit into bigger group, write the combined code in next column.

1 st column	2 nd column	
✓ 2: 0010	2,3: 001-	Codes with one 1
✓ 4: 0100	2,10: -010	
✓ 8: 1000	4,5: 010-	
-----	8,10: 10-0	
✓ 3: 0011	3,7: 0-11	Codes with two 1s
✓✓ 5: 0101	5,7: 01-1	
✓✓ 10: 1010	5,13: -101	
-----	7,15: -111	
✓✓ 7: 0111	7,15: -111	Codes with three 1s
✓✓ 13: 1101	13,15: 11-1	

✓✓ 15: 1111		



Eg: $F(A,B,C,D) = \Sigma m(2,3,4,5,7,8,10,13,15)$

Step 3: Repeat step 2 – Combine codes that differ by 1 bit into bigger group, write the combined code in next column.

1st column

✓ 2: 0010
 ✓ 4: 0100
 ✓ 8: 1000

 ✓ 3: 0011
 ✓ 5: 0101
 ✓ 10: 1010

 ✓ 7: 0111
 ✓ 13: 1101

 ✓ 15: 1111

2nd column

2,3: 001-
 2,10: -010
 4,5: 010-
 8,10: 10-0

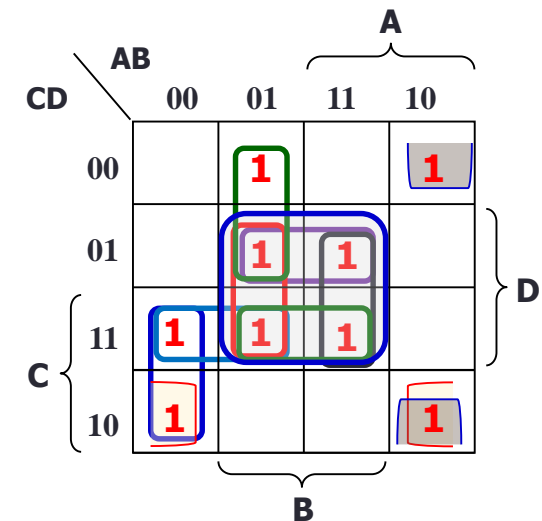
 3,7: 0-11

 ✓ 5,7: 01-1
 ✓ 5,13: -101

 ✓ 7,15: -111
 ✓ 13,15: 11-1

3rd column

5,7,13,15: -1-1
~~5,7,13,15: -1-1~~



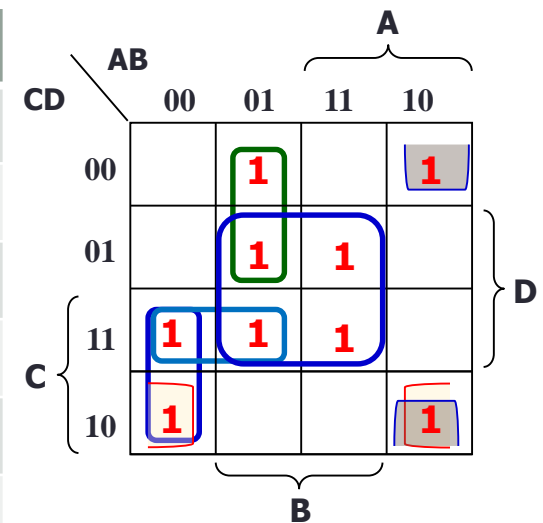
We have completed
Phase 1: Identifying all the
 Prime Implicants (PIs)!

$$\text{Eg: } F(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

Phase 2: Identify the **Essential Prime Implicants (EPIs)**

- Draw the **PI chart**

	2	3	4	5	7	8	10	13	15
2,3: 001- ($A'.B'.C$)	✓	✓							
2,10: -101 ($B.C'.D$)	✓						✓		
EPI 4,5: 010- ($A'.B.C'$)			✓	✓					
EPI 8,10: 10-0 ($A.B'.D'$)						✓	✓		
3,7: 0-11 ($A'.C.D$)		✓			✓				
EPI 5,7,13,15: -1-1 ($B.D$)				✓	✓			✓	✓



Where are the EPIs? Look for columns containing a single tick.

EPIs are: $A'.B.C'$, $A.B'.D'$, and $B.D$

But we are not done yet. There are still minterms not covered by the EPIs!

Eg: $F(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$

Phase 2: After identifying the EPIs

- Draw the **reduced PI chart** if there are minterms not covered

	2	3	4	5	7	8	10	13	15
2,3: 001- ($A'.B'.C$)	✓	✓							
2,10: -101 ($B.C'.D$)	✓						✓		
EPI → 4,5: 010- ($A'.B.C'$)			✓	✓					
EPI → 8,10: 10-0 ($A.B'.D'$)						✓	✓		
3,7: 0-11 ($A'.C.D$)		✓			✓				
EPI → 5,7,13,15: -1-1 ($B.D$)				✓				✓	✓

AB		A			
		00	01	11	10
CD	00		1		1
	01		1	1	
	11	1	1	1	
	10	1			1

Diagram labels: A (columns 01, 11), B (columns 00, 10), C (rows 11, 10), D (rows 01, 11).

- Find out what are the minterms covered by the EPIs.
- Remove the EPIs and minterms they cover from the chart → **reduced PI chart**.
- Find the minimum number of remaining PIs to cover the remaining minterms.

Answer:

$$B.D + A'.B.C' + A.B'.D' + A'.B'.C$$

End of File