GEH1027
Einstein's Universe and Quantum Weirdness

Tutorial 1

AY2020/21 Sem II

Contact

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(send email for appointment)

About tutorial sessions

- During the tutorial:
  - Go through problems in the tutorial (not all)
  - In-class assignments (at the last 10-15 minutes)
  - Q&A if time permits

- Remember to sign the attendance!

  *just turn on the camera so I know you are with me

Tutorial assessment (35%)

Attendance	5%
In-class assignments	20%
IVLE forum discussion	10%

Tutorial 1 – key questions

- Can you understand the relativistic velocity addition formula?
- How would a fast moving object appear to us (stationary)?
- Can you appreciate the second postulate of special relativity?

Special Relativity

Einstein’s 2 postulates:

1. The laws of Physics are the same in all inertial frames.

2. The speed of light in a vacuum has the same measured value regardless of the motion of the source or the motion of the observer.
(i.e. the speed of light is invariant)

Galilean transformation

$$x' = x - vt$$
$$t' = t$$

Inverse:

$$x = x' + vt'$$
$$t = t'$$

Lorentz transformation

$$x' = (x - vt)\gamma$$
$$t' = \left(t - \frac{vx}{c^2}\right)\gamma$$

Inverse:

$$x = (x' + vt')\gamma$$
$$t = \left(t' + \frac{vx'}{c^2}\right)\gamma$$

Lorentz factor: $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Remember: primed quantities belong to the moving frame

Velocity addition

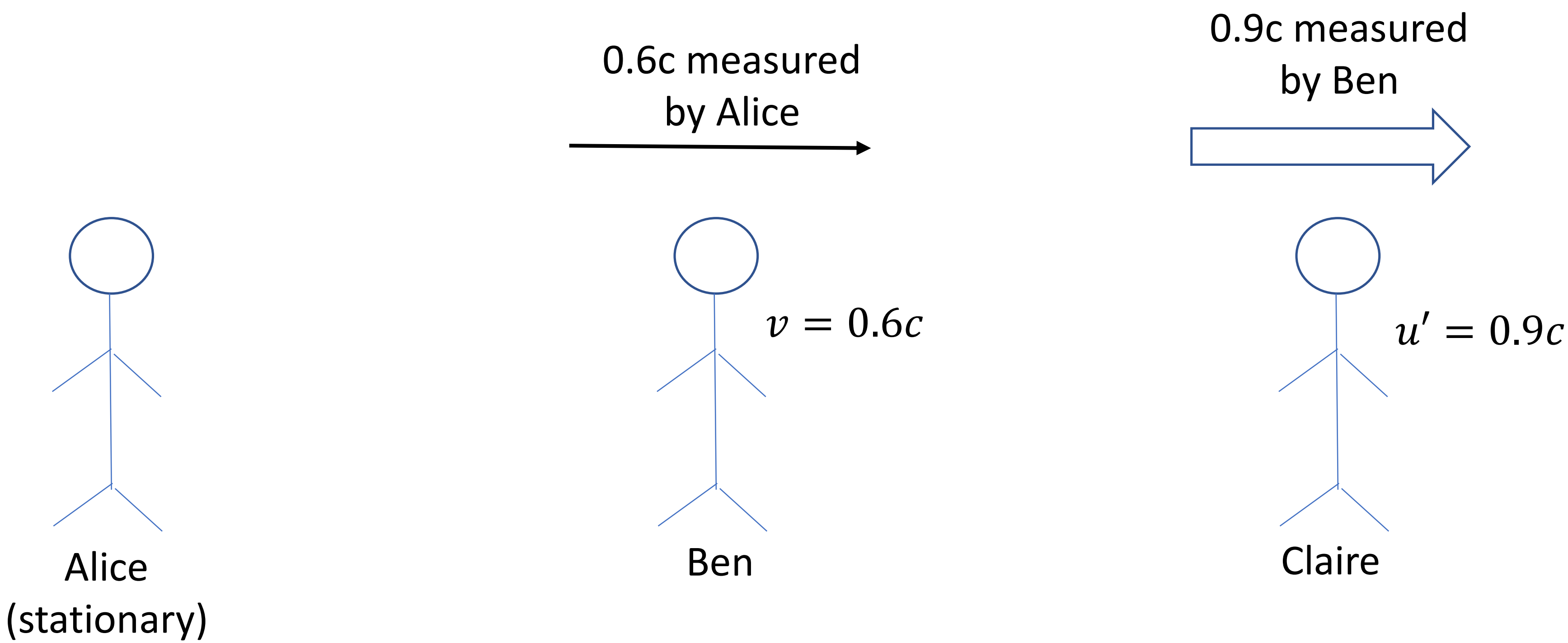
$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

u: velocity observed by a person in stationary inertial frame

v: velocity of moving inertial frame with respect to the stationary inertial frame

u': velocity observed by a person in the moving frame

So what is Claire's speed when measured by Alice?



$$u = \frac{0.6c + 0.9c}{1 + (0.6 * 0.9)}$$
$$= \frac{1.5c}{1.54} = 0.974c$$

Question 1

Could you explain to a classmate how *The Relativity Principle* is related to Galileo's Law of Inertia and Newton's First Law?

Newton's 1^{st} Law = Galileo's Law of Inertia

Subset of *Relativity Principle* which incorporates Electricity and Magnetism e.g. speed of light

Question 2

a) State the 2 Postulates of Einstein's Special Relativity.

The laws of physics applies all inertial reference frames or observers; no preferred frames.

c is constant in all inertial frames.

b) Which postulate was Einstein's real contribution?

The fact that c is constant (postulate 2), but also insight to the interconnectedness between the ideas.

Question 3

What does the Lorentz factor suggest to us?

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

There is an upper speed limit in the universe, regardless of reference frames. (Postulate 2).

What happens in the extremes when $v \to 0$ and $v \to c$?

“**Invariance**” vs “**covariance**”?

Invariance – a quantity that is invariant stays unchanged across frames e.g. your (rest) mass, space-time interval Δs^2

Covariance – takes the same mathematical form across different frames

Question 4

a) Alice's spaceship measures $100m$ long when it is at rest. How long does this spaceship appear to you when it is moving past you at $0.99c$?

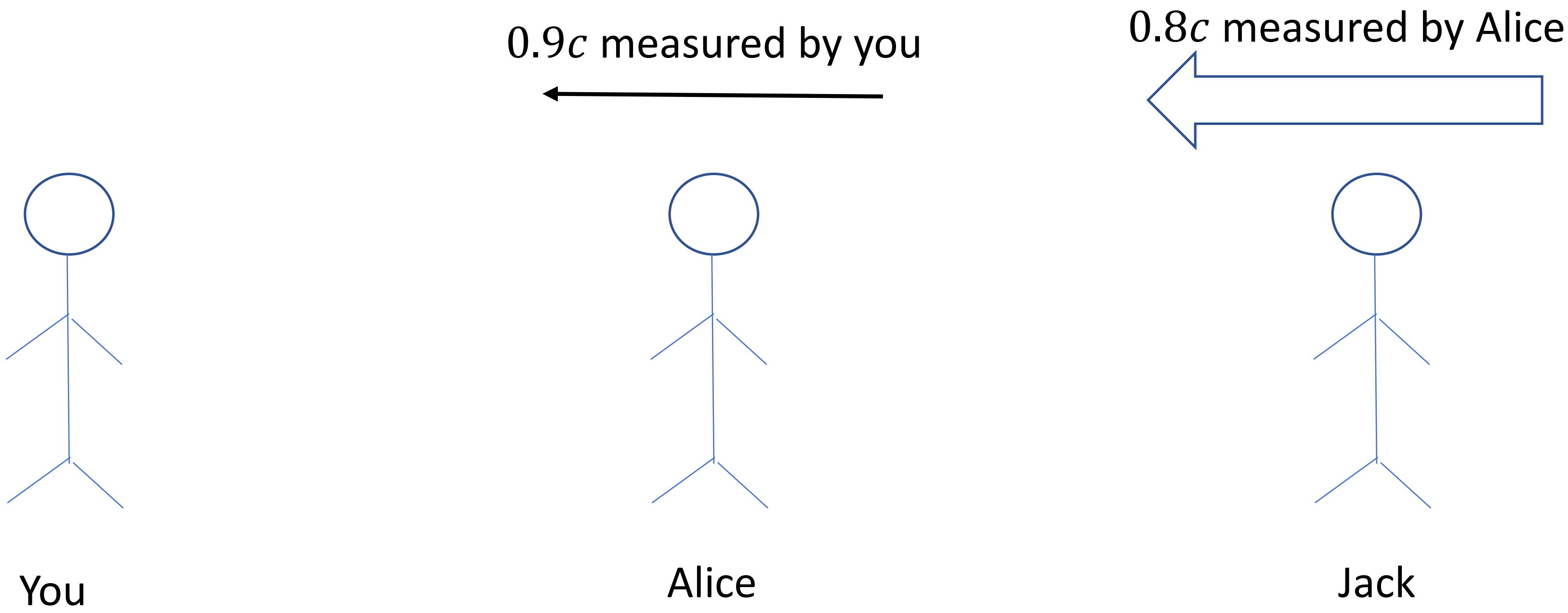
$$L = L_0 \times \sqrt{1 - \left(\frac{v}{c}\right)^2} = 100 \times \sqrt{1 - 0.99^2}$$

b) Alice's spaceship is now moving towards you at $0.9c$. Jack jumps onto a separate spaceship that Alice perceive to move towards her at $0.8c$. What is Jack's velocity perceived by you?

$$u = \frac{v + u'}{1 + \left(\frac{vu'}{c^2}\right)}$$

In this context, what are your v and u'?

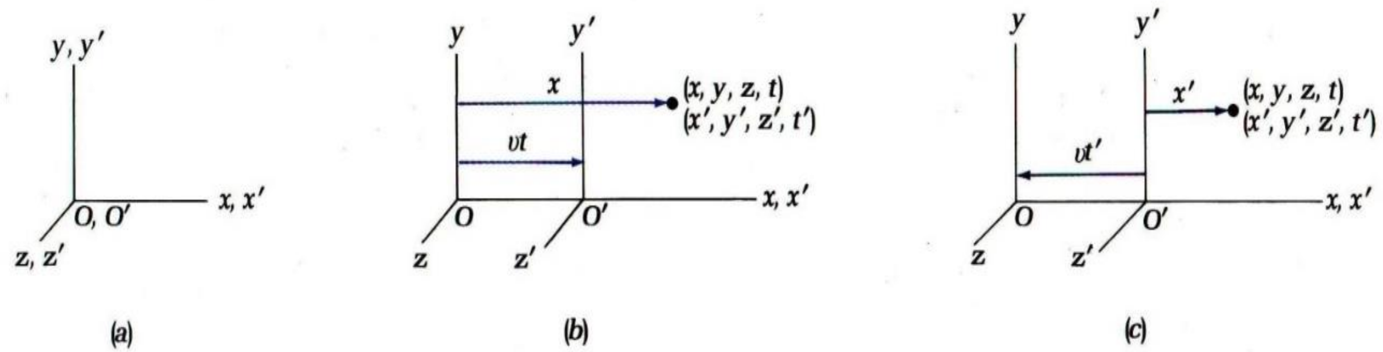
v = Alice's speed
u' = Jack's speed



$0.9c$ measured by you
$0.8c$ measured by Alice
You
Alice
Jack

Question 5

Recall that the peculiar velocity addition formula $u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$ where *u* is the velocity observed by a person in the stationary inertial frame and *v* is the velocity of the moving frame observed by the stationary inertial observer. Here *c* is the speed of light. Suppose a ball is thrown out of the moving frame at a velocity of *u'* (like a ball thrown out of a uniformly moving bus)



Suppose that we use a "light" ball and it is moving in the same direction as the moving frame (to the right)

Question 5

a) Find u if $v = u' = c$

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

Using the velocity addition formula,

$$u = \frac{c + c}{1 + \frac{c^2}{c^2}} = c$$

d) Find u if $u' = -c$

Mathematically indeterminate

$$u = \frac{c-c}{1+\frac{c(-c)}{c^2}} = \frac{0}{0}$$

In this case how should we determine *u*? Taking limits by letting $v = c - \Delta v$!

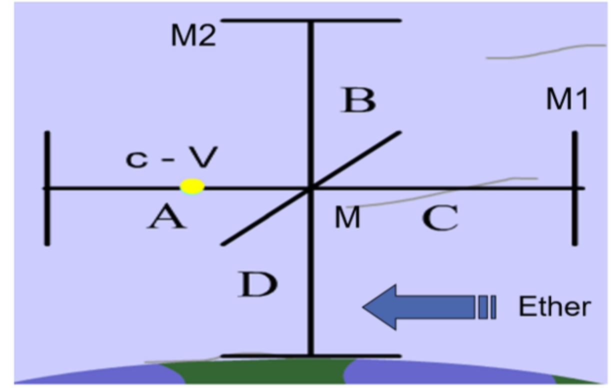
Δv here is to represent a small velocity such that v is *slightly* below c. At the end, we will take the limit where $\Delta v \to 0$

$$u = \frac{(c - \Delta v) - c}{1 + \frac{(c - \Delta v)(-c)}{c^2}} = \frac{-\Delta v}{\Delta v/_c}$$

$$= -c$$

Question 6

a) Find the time t_A which is the time it takes for light to travel from M to M1, and back again, if the length of M – M1 arm is l_1.



An ether wind is blowing from M1 towards M i.e. to the left.

Time from M to M1

$$t_1 = \frac{l_1}{c - v}$$

Time from M1 back to M

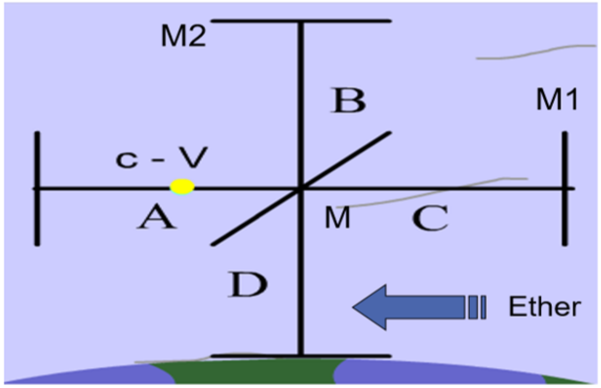
$$t_2 = \frac{l_1}{c + v}$$

$$t_A = t_1 + t_2$$

$$= \frac{2l_1}{c} \frac{1}{\left(1 - \frac{v^2}{c^2}\right)}$$

Question 6

b) Find the time t_B which is the time it takes for light to travel from M to M2, and back again, if the length of M – M2 arm is l_2.



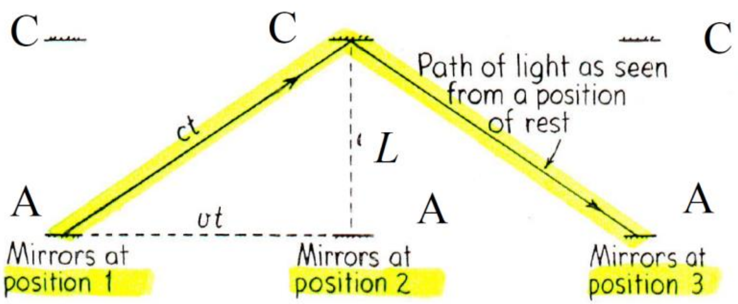
An ether wind is blowing from M1 towards M i.e. to the left.

Time from M to M2 (Pythagoras' Theorem)

$$(ct_{M2})^2 = l_2^2 + (vt_{M2})^2$$

$$t_{M2}^2(c^2 - v^2) = l_2^2$$

$$t_{M2}^2 = \frac{l_2^2}{c^2 - v^2}$$



Note that $t_B = 2t_{M2}$

$$t_B = \frac{2l_2}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

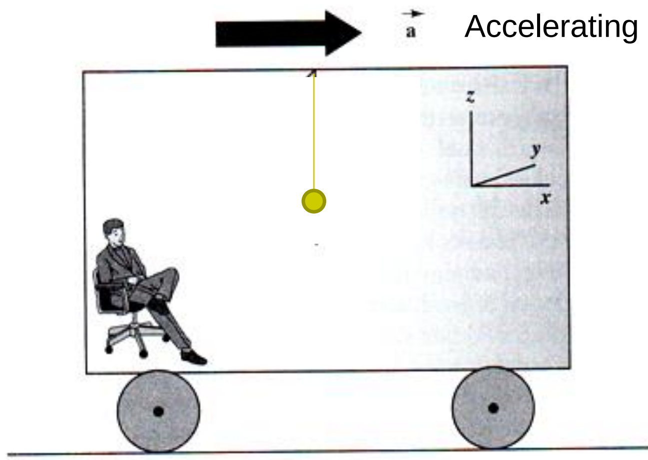
$$t_B = t_A \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} < 1, \qquad \therefore t_B < t_A$$

Question 7

Recall your experience in a train…

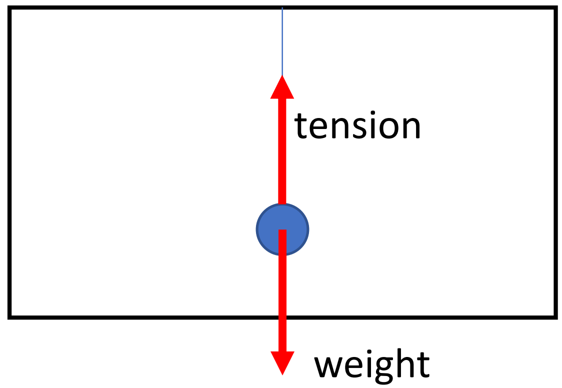
What will happen to the pendulum? Discuss a) when the box is in uniform motion and b) in accelerated motion.

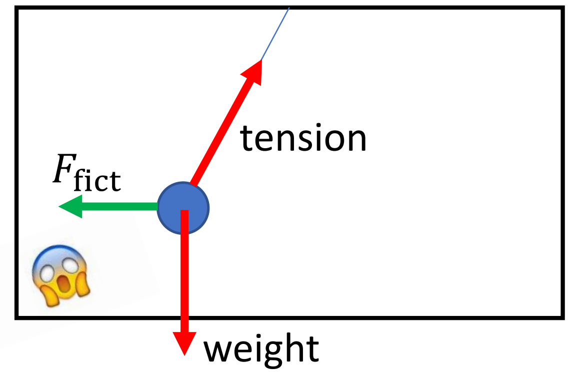


Accelerating box

i.e. how would the person in the box describe the both motions of the pendulum ? which motion has an extra force and why ?

Uniform motion





Person in the box observes:

- The pendulum hangs vertically
- There are only 2 forces acting on the pendulum: tension and weight
- If box has no windows, person wouldn't know if the box is moving or not!
- INERTIAL FRAME

Person in the box observes:

- The pendulum hangs at an angle
- There is "some force" pushing it towards the back!?
- **Fictitious force** not physical in nature, but due to the frame accelerating (recall: passenger in car doing a turn)
- NON-INERTIAL FRAME
- Note: if there are no windows, the person might even conclude that gravity is acting at an angle!