change of the state of the system determined by the measurement process is called wave packet reduction.
It is worth mentioning that the above conception of the measurement process is the most delicate aspect of the Copenhagen interpretation and it has raised a long debate that still continues. Here we only briefly mention some problematic points.
First, it is not explained why all or part of the experimental device, despite being made of atoms, must behave as a classical object with well-defined classical properties.
Moreover it is not clear where the borderline between the measurement apparatus (characterized by a classical behavior) and the system (characterized by a quantum behavior) should be put. The problem is usually solved pragmatically in each specific situation but, at a conceptual level, the ambiguity remains.
Finally, one has to renounce to describe the interaction of the system with the apparatus using the Schrödinger equation. This fact was formalized in 1932 by von Neumann ([vo]) who postulated two different kind of evolution for the system: a genuine quantum evolution governed by the Schrödinger equation when the system is not measured and a different (stochastic and non linear) evolution producing the wave packet reduction when the system is measured.
We only mention that many attempts have been made to clarify these conceptual problems within the framework of the standard interpretation but a reasonable and universally accepted solution has not been found.
From the point of view of our analysis, it is interesting to observe that the problematic aspects of the system-apparatus interaction are immediately evident when one approaches the description of a quantum particle in a cloud chamber.
In fact, for a quantum mechanical description of the process one must take into account that the initial state of the particle emitted by the source does not have the form of a semi-classical wave packet but rather a highly correlated continuous superposition of states with well localized position and momentum. In particular, according to the first theoretical analysis of the radioactive decay given by Gamow ([Ga]), the emitted α-particle must be described by a wave function having the form of a spherical wave, with center in the radioactive nucleus and isotropically propagating in space. Therefore, the non trivial problem arises as to how such a superposition state can produce the observed classical trajectories. As we shall see in the next sections, in such explanatory scheme a crucial role is played by the act of "observation", where the system to be measured can be the α-particle (in particular its position) as well as the atoms of the vapor (in particular their excitation).

3. Early contributions

In this section we shall briefly summarize the first theoretical attempts to explain the apparent contradiction between the highly correlated superposition state of the α-particle and the observed tracks in the chamber. In particular, we shall consider the contributions of Born (1927), Heisenberg (1929-30) and Darwin (1929).

3.1 Born (1927).
The theoretical explanation of the observed tracks in a cloud chamber was already approached by Born (for the first time to our knowledge) in 1927 during the general discussion at the Solvay Conference ([BV]). In his words: *"Mr. Einstein has considered the following problem: A radioactive sample emits α-particles in all directions; these are made visible by the method of the Wilson cloud chamber. Now, if one associates a spherical wave with each emission process, how can one understand that the track of each α-particle appears as a (very nearly) straight line? In other words: how can the corpuscular character of the phenomenon be reconciled here with the representation by waves?"*
According to Born, the answer can be given resorting to the notion of *"reduction of the probability packet"* discussed by Heisenberg in [He3]. According to this notion, the observation of the electron position by light of wavelength λ would produce the reduction of the probability packet for the position of the electron to a region of linear size λ. In the subsequent evolution the packet spreads in space until a new observation reduces it again to a packet of width λ. Then, in Heisenberg's words: *"Every determination of position reduces therefore the wave packet back to its original size λ"*. This mechanism of reduction would be responsible for the appearance of a (nearly) classical trajectory of the electron.
The same idea is used by Born in the case of the cloud chamber. Here the observation by means of light must be replaced by ionization of the atoms of the vapor in the chamber: *"As soon as such ionization is shown by the appearance of cloud droplets, in order to describe what happens afterwards one must reduce the wave packet in the immediate vicinity of the drops. One thus obtains a wave packet in the form of a ray, which corresponds to the corpuscular character of the phenomenon"*.
It is worth to emphasize that, according to this reasoning, the whole process is described in terms of the interaction of a quantum system (the α-particle) with a classical measurement apparatus (the atoms of the vapor). Such interaction, which is not described by Schrödinger equation, produces "reduction" of the spherical wave to a wave packet with definite position and momentum.
Following a suggestion by Pauli, Born continues discussing the possibility to describe both the α-particle and the atoms of the vapor as constituents of a unique quantum system, whose wave function depends on the coordinates of all the particles of the system.
In particular, Born proposes, as an example, a simplified one dimensional model consisting of the α-particle and only two atoms. The α-particle is initially in a superposition state of two wave packets with opposite momentum and position close to the origin. The two cases where atoms are placed on the same side or on opposite sides with respect to the origin are considered. Born discusses, at a purely qualitatively level, the probability that during the time evolution of the whole system the two atoms will be hit in the two different cases. We simply give the conclusions reached by Born without going into details (for a quantitative analysis of the same model see e.g. [DFT1]).
The first statement he makes, without invoking the reduction of the wave packet, is that the α-particle has negligible probability to hit both atoms unless they are on the same side with respect to the origin.
On the other hand, Born concludes his argument having an explicit recourse to the reduction postulate, saying: *"To the reduction of the wave packets corresponds the choice of one of the two directions*

of propagations, and the choice is made as soon as one observes the excitation of an atom, as a consequence of the collision. Only starting from such observation the evolution of the α-particle can be described as a real classical trajectory.
In conclusion, it seems that according to Born a more detailed description of the α-particle in a cloud chamber taking into account the presence of the environment is in fact possible *"but this does not lead us further as regards the fundamental questions."*

3.2 Heisenberg (1929-30).
As already mentioned, Born's analysis was explicitly inspired by the considerations of Heisenberg in [He2] on the applicability of the notion of classical trajectory of an electron. It is remarkable that Heisenberg himself explicitly reconsidered the cloud chamber problem in his lectures at the University of Chicago in 1929, published in [He3], following the same line of thought of Born but with considerable more details.
His approach, described in chapter 5 of the book, can be considered an exhaustive qualitative investigation of the problem according to the standard interpretation of Quantum Mechanics. This analysis had a deep influence on the physics community for many years.
Heisenberg's first (rather extreme) remark is that the whole experimental situation could be satisfactorily described using only Classical Mechanics, but it might also be of interest to discuss the problem from the point of view of Quantum Theory.
He stresses that, approaching a quantum theoretical description, one is immediately faced with the problem of separating the quantum system from the apparatus. In the case of the cloud chamber, one has two different reasonable choices: a) the quantum system consists of the α-particle alone (and then the molecules of the vapor play the role of the measurement apparatus); b) the quantum system consists of the α-particle and the molecules of the vapor.
It should be emphasized that the physical descriptions of the experimental situation obtained following the two different choices are explicitly considered equivalent.
Heisenberg's line of reasoning in examining the two cases proceeds as follows.
In case a) the single molecule of the vapor measures the position of the α-particle. Assume that the molecule (supposed at rest) occupies a volume Δq around the point q_1 and t_1 is the collision time between α-particle and molecule. As result of the measurement process, the state of the α-particle is suddenly reduced and therefore at time t_1 it has position q_1 with spread Δq. On the other hand one knows the position q_0 of the α-particle at time t_0, when it leaves the radioactive source. Since no external force is present, one infers that the momentum of the α-particle at time t_1 is $p_1 = M(q_1 - q_0)/(t_1 - t_0)$, where M denotes the mass.
It is then possible to conclude that for $t > t_1$ the α-particle is described by the free evolution of a wave packet starting from q_1, with initial spread Δq and momentum along the straight line γ joining q_0 and q_1. Therefore, the center of the wave packet moves along the same line γ.
During the time evolution the wave packet inevitably spreads out. On the other hand the α-particle collides with other molecules placed along γ and after each collision the same measurement process of the position takes place. In this way the spreading is repeatedly reduced and the wave packet remains focused around the straight line γ, corresponding to the observed "trajectory" of the α-particle.

Let us consider case b), where the molecules of the vapor are considered part of the quantum system. Heisenberg starts with an interesting claim that would have probably deserved further analysis: in case b) the procedure to account for the the observed trajectories is more complicated but, on the other hand, it allows to hide the role of the reduction of the wave packet.
Then he goes on to describe a simplified model made of the α-particle plus only two molecules. The molecules are non interacting, their centers of mass are fixed in the positions a_I, a_{II} and the internal coordinates are denoted by q_I, q_{II}. It is assumed that the Hamiltonians of the two molecules have a complete set of eigenfunctions $\varphi_{n_I}(q_I)$, $\varphi_{n_{II}}(q_{II})$, corresponding to a discrete set of eigenvalues labeled by integers n_I, n_{II}.
The initial state of the system is chosen in the form of a product of the ground states of the molecules (labeled by n_I^0, n_{II}^0) times a plane wave with momentum p for the α-particle.
The interesting object to compute is the probability that both molecules are excited and the result of the computation is that such a probability is significantly different from zero only if the momentum p is parallel to the line joining a_I and a_{II}. Since the passage of the α-particle is indirectly observed through the excitations of the molecules, the result explains why one can only see straight trajectories in a cloud chamber.
The solution of the Schrödinger equation for the three-particle system is approached treating the interaction between the α-particle and the molecules as small perturbation of the free dynamics and assuming that the momentum p is large with respect to the spacing of energy levels of the molecules. The perturbative computation is not developed in all details by Heisenberg. Here we only summarize the main steps of his procedure in order to clarify the line of reasoning.
The Schrödinger equation is solved by iteration and at the first order the wave function can be written in the form

$$\psi^{(1)} = e^{-i\frac{t}{\hbar}E_0} \sum_{n_I, n_{II}} w^{(1)}_{n_I n_{II}}(x)\varphi_{n_I}(q_I)\varphi_{n_{II}}(q_{II}) \tag{3.1}$$

where E_0 denotes the total energy of the system, x the position coordinate of the α-particle and the coefficients $w^{(1)}_{n_I n_{II}}$ satisfy an equation which is easily derived from the original Schrödinger equation. From Born's rule it follows that $|w^{(1)}_{n_I n_{II}}(x)|^2$ represents the probability density (at first order) to find the α-particle in x when the molecules are in the states labeled by n_I, n_{II}.
From the equation for $w^{(1)}_{n_I n_{II}}$ it is immediately evident the first result: the probability that both molecules are excited is zero at first order.
The second result claimed by Heisenberg is definitively less evident and nevertheless it is stated without a detailed proof. It says that $w^{(1)}_{n_I n^0_{II}}$ is significantly different from zero only in a strip parallel to p located behind the molecule I, whose thickness (close to the molecule) is of the same order of the dimension of the molecule. The same kind of result is obviously true for $w^{(1)}_{n^0_I n_{II}}$.
Then he continues with the analysis of the wave function at the second order

$$\psi^{(2)} = e^{-i\frac{t}{\hbar}E_0} \sum_{n_I, n_{II}} w^{(2)}_{n_I n_{II}}(x)\varphi_{n_I}(q_I)\varphi_{n_{II}}(q_{II}) \tag{3.2}$$

Writing the equation for $w^{(2)}_{n_I n_{II}}$ and exploiting the results obtained at the first order, he derives the desired final result. The probability density at second order $|w^{(2)}_{n_I n_{II}}(x)|^2$, with $n_I \neq n_I^0$ and $n_{II} \neq n_{II}^0$, is significantly different from zero only if the following two situations occurs: the molecule II is in the strip of $w^{(1)}_{n_I n_{II}^0}$ or the molecule I is in the strip of $w^{(1)}_{n_I^0 n_{II}}$.
The procedure can be iterated with an arbitrary number of molecules and therefore the linearity of the trajectories is proved.
At the end of the analysis, Heisenberg makes a second claim on the problem of the wave packet reduction. In particular, he explains that in case b) the reduction takes place when one arranges a measurement process "to observe" the excitation of the molecules. This simply means that the unavoidable line of separation between the system and the apparatus has been moved to include the molecules in the system. In this sense one should probably understand the previous claim that the reduction in case b) is hidden.
Summarizing, Heisenberg's analysis of the cloud chamber, like Born's analysis, insists to consider the treatments in case a) and b) as conceptually equivalent. This belief is founded on the fact that in any case the recourse to the reduction of the wave packet cannot be avoided.

3.3 Darwin (1929).
A further relevant contribution to the explanation of the trajectories in a cloud chamber was given by Darwin ([Da]) in 1929. In such paper one does not find any analysis of specific models. Nevertheless there is a detailed discussion of the problem and it is clearly stated a possible strategy for an approach entirely based on the use of Schrödinger equation.
Darwin approaches a collision problem in the framework of Wave Mechanics with the aim to *"take a problem which would be regarded at first sight as irreconcilable with a pure wave theory, but thoroughly typical of the behavior of particles, and show how in fact the correct result arises naturally from the consideration of waves alone."*
He emphasizes that in order to obtain the correct predictions on the behavior of a given system $\mathcal{S}$ one has to take into account its interaction with (part of) the environment $\mathcal{E}$. Therefore the wave function ψ is not a wave in ordinary three dimensional space but rather it is a function of the coordinates of $\mathcal{S}$ and of $\mathcal{E}$. Only when such ψ has been computed, the probabilistic predictions on $\mathcal{S}$ are obtained by taking an average over all possible final configurations of $\mathcal{E}$.
Such procedure, even if *"discouragingly complicated"*, can account for the particle-like behavior working only on ψ and without invoking any act of observation. It is worth noticing that the program outlined by Darwin essentially coincides with the basic strategy of modern decoherence theory.
After these considerations, Darwin discusses a concrete example where the intuitive particle behavior can be derived from the analysis of the wave function. That part of the analysis is not directly connected with the cloud chamber and therefore it is not relevant for our purposes. However, in the final part of the paper, one finds some other interesting considerations.
He examines the case of the ray tracks of α-particles in a cloud chamber, *"one of the most striking manifestations of particle characters"*, in connection with Gamow's theory of radioactive decay ([Ga]), distinguishing two different points of view.

According to the first one *"we must regard Gamow's calculations as determining only the probability of disintegration, and that when this has taken place, we start the next stage by assigning a definite direction for the motion of the α-particle; after which we reconvert it into a wave, but now on a narrow front, so as to find its subsequent history"*.
As an alternative to this point of view, he first notices that α-rays can in principle exhibit diffraction and therefore it is reasonable to assign a real existence to the spherical wave outside the nucleus. Then he discusses a possible wave description of the experiment. The wave function ψ is a function of the coordinates of the α-particle and of the coordinates of the atoms in the chamber and, before the first collision, it is a product of the spherical wave for the α-particle times a set of stationary (in general ground) states for the atoms. *"But the first collision changes this product into a function in which the two types of coordinates are inextricably mixed, and every subsequent collision makes it worse."* Such complicated function contains a phase factor and *"without in the least seeing the details, it looks quite natural to expect that this phase factor will have some special character, such as vanishing, when the various co-ordinates satisfy a condition of collinearity."*
It is interesting to notice that Darwin clearly identifies stationary phase arguments as the crucial technical tools required to predict the particle-like behavior.
Then he continues: *"So without pretending to have mastered the details, we can understand how it is possible that the ψ function, so to speak, not to know in what direction the track is to be, but yet to insist that it should be a straight line. The decision as to actual track can be postponed until the wave reaches the uncovered part, where the observations are made."*
This approach seems to have a general validity. In Darwin's view the wave-particle duality proposed by Bohr can be avoided. The whole quantum theory can be based on the wave function ψ, considered as the central object from which all the particle or wave properties can be accurately described, at least until a real measurement is performed. In his words *"it thus seems legitimate to suppose that it is always admissible to postpone the stage, at which we are forced to think of particles, right up to the point at which they are actually observed."*

4. Mott's paper

The program enunciated by Darwin was concretely realized by Mott in his seminal paper of 1929.
In the introduction Mott recognizes to have been inspired by Darwin's paper in his attempt to explain the typical particle-like properties of an α-particle in a cloud chamber using only Wave Mechanics. He admits that such point of view seems at first sight counterintuitive , since *"it is a little difficult to picture how it is that an outgoing spherical wave can produce a straight track; we think intuitively that it should ionise atoms at random throughout space"*. Like Heisenberg, Mott points out that the crucial point is to establish the borderline between the system under consideration and the measuring device. He recalls the two possible approaches: in the first the α-particle is the quantum system under consideration (and the gas of the chamber is part of the measuring device) while in the second approach the quantum system consists of the α-particle and of the atoms of the gas.