National University of Singapore

Semester 1, 2020/2021 **MA1101R** **Practice Assignment 4**

(a) Use A4 size paper and pen (blue or black ink).

(b) Write down your student number and full name clearly on the top left of every page of the answer scripts.

(c) Write the page number on the top right corner of each page of answer scripts.

(d) There are two questions in this worksheet (see next page) with a total of 20 marks.

(e) To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (arrange them in order of the page. Name the pdf file by **StudentNo P4** (e.g. **A123456R P4**). Upload your pdf into the LumiNUS folder Practice 4 submission.

(f) Hand in your answers by the end of this session. **Late submission will not be accepted.**

1. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$.

   (i) [2 marks] Show that $S = \{(1,1,1,1)^T, (1,0,1,0)^T, (1,0,0,1)^T\}$ is a basis for the column space V of $\boldsymbol{A}$.

   (ii) [2 marks] Use Gram Schmidt process (Theorem 5.2.19) to convert S to an orthogonal basis T for V <u>without</u> normalising the resulting vectors. (Show your working. You may use MATLAB command to check your answer.)

   (iii) [2 mark] Find the coordinate vector of $\boldsymbol{w} = (8,4,12,0)^T$ with respect to the orthogonal basis T in (ii) using Theorem 5.2.8.

   (iv) [2 mark] Find the projection $\boldsymbol{p}$ of $\boldsymbol{b} = (1,2,3,4)^T$ onto V using Theorem 5.2.15.

   (v) [2 marks] Find the least squares solutions of $\boldsymbol{Ax} = \boldsymbol{b}$ using the linear system $\boldsymbol{A}^T\boldsymbol{Ax} = \boldsymbol{A}^T\boldsymbol{b}$, where $\boldsymbol{b}$ is the vector in (iv).

   (vi) [2 marks] Find the least squares solutions of $\boldsymbol{Ax} = \boldsymbol{b}$ by using $\boldsymbol{p}$ in part (iv) directly. (Refer to Example 5.3.9)

2. (Do not simply use MATLAB to obtain the answers for this question. Show your working clearly. MATLAB command can be used to check your answer.)

   Let $\boldsymbol{B} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$.

   (i) [3 marks] Find the characteristic polynomial and all the eigenvalues of $\boldsymbol{B}$.

   (ii) [4 marks] Find a basis for each eigenspace of $\boldsymbol{B}$.

   (iii) [1 mark] Write down the nullspace of $\boldsymbol{B}$ without any further working. Justify your answer.