NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1521
Tutorial 4

1. Find the area of the following region.

   (a) The region bounded between $y = \frac{1}{2}\sec^2 x$, $y = -4\sin^2 x$, $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$.

   (b) The region in the first quadrant bounded by $y = x$, $y = \frac{1}{4}x^2$ and below $y = 1$.

   (c) The region between the graphs of $y = 4 - x^2$ and $y = 2 - x$ from $x = -2$ to $x = 3$.

   Ans. (a) $\frac{4}{3}\pi$ (b) $\frac{5}{6}$ (c) $\frac{49}{6}$

2. (a) Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

   (b) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

   Ans. (a) $\frac{64}{15}\sqrt{2}\pi$ (b) $\frac{8}{3}\pi$

3. Find the exact value of $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$, where a is a positive constant.

4. Let a be a positive constant. Find the area of the finite region bounded by the curves $y^2 = x + 4a^2$ and $x - ay + 2a^2 = 0$.

5. A finite region R is bounded by the curve $y = \sqrt{\tan x}$, and the lines $x = \frac{\pi}{4}$ and $y = 0$. Find the volume of the solid formed by revolving R one complete round about the x-axis.