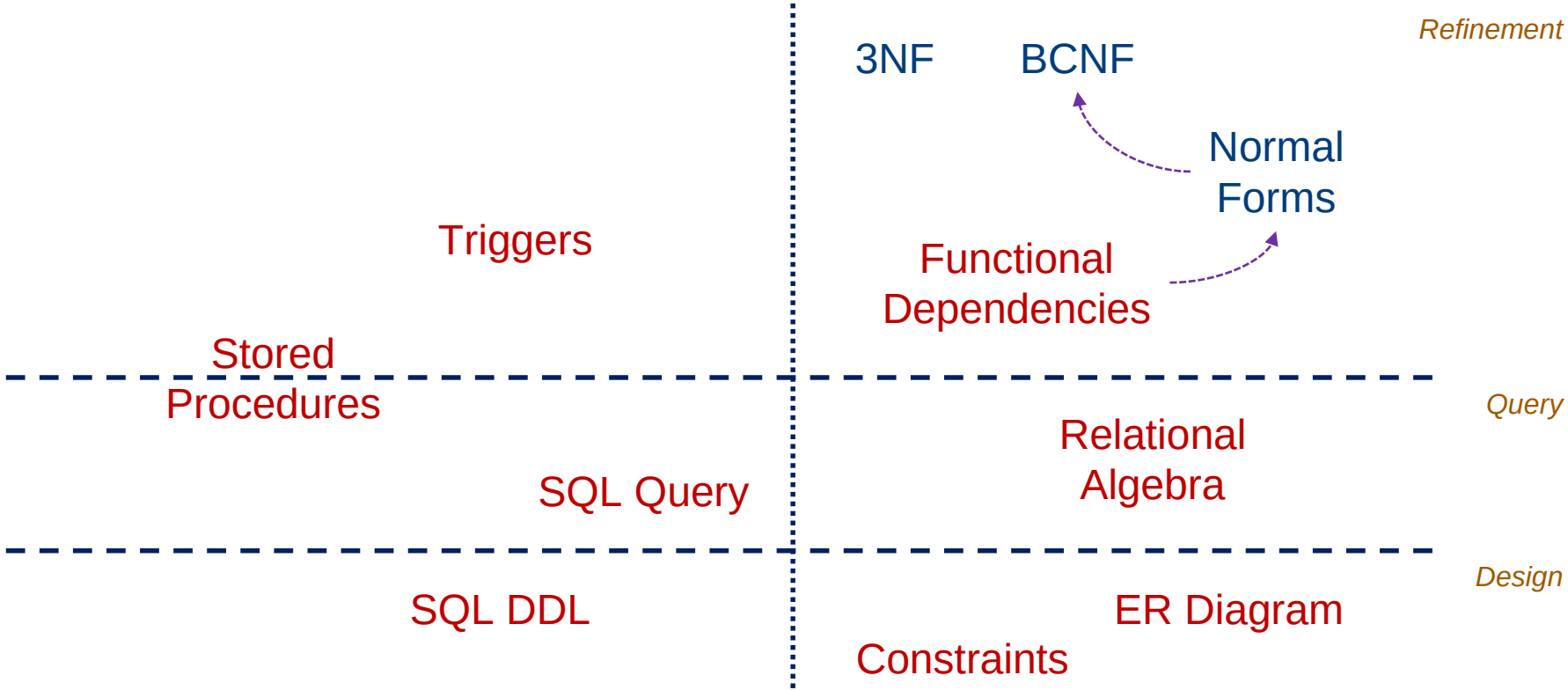
CS2102 Database Systems

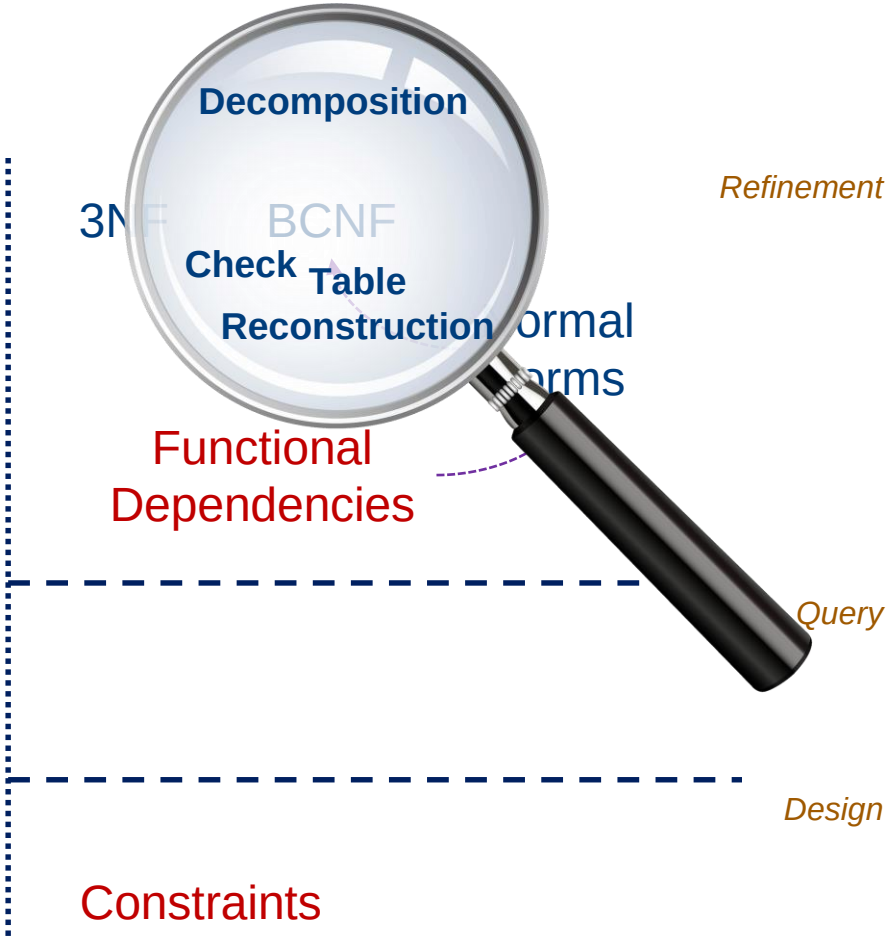
Lecture 11 – Boyce-Codd Normal Form

Roadmap

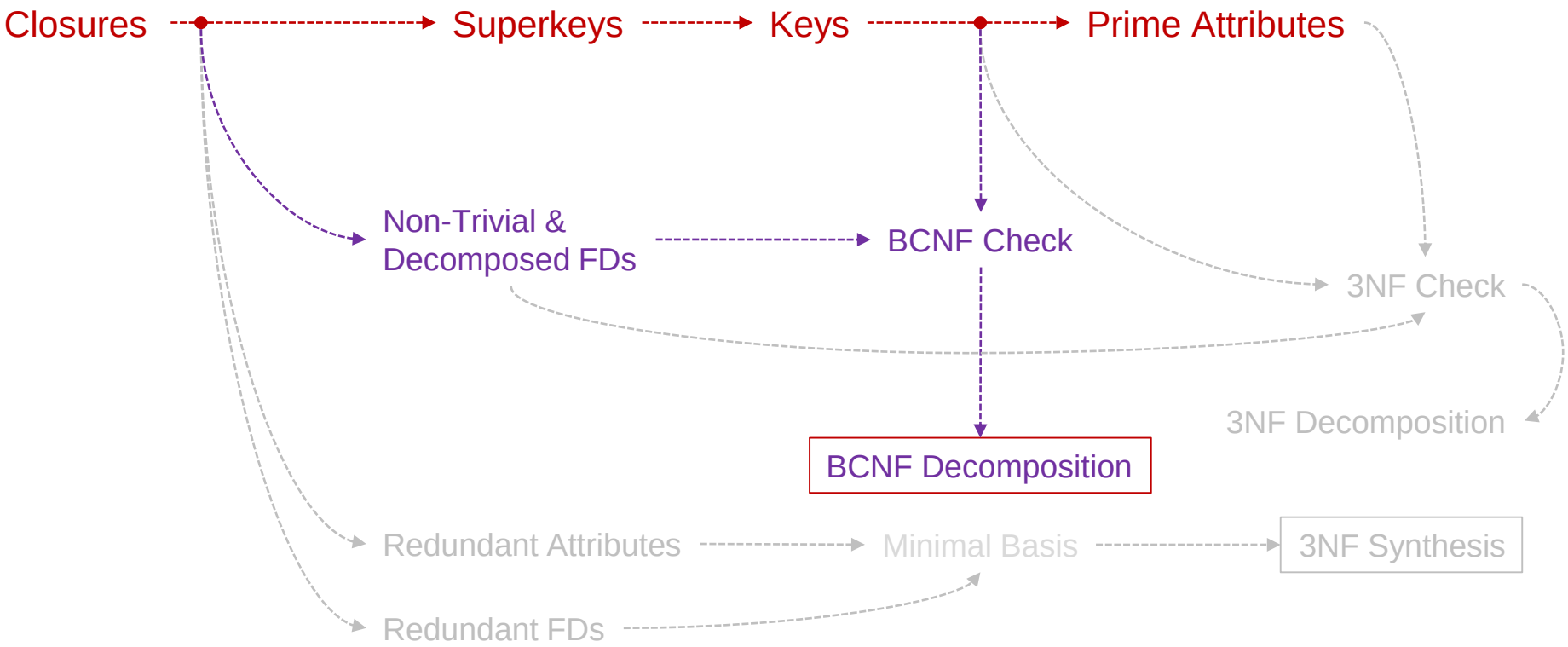

Refinement
3NF
BCNF
Normal Forms
Functional Dependencies
Triggers
Stored Procedures
Query
Relational Algebra
SQL Query
Design
SQL DDL
ER Diagram
Constraints

Roadmap

- We will do this step by step
  - Recap FD *(with non-trivial & decomposed FD)*
  - Define BCNF
  - Check BCNF
  - Normalize to BCNF
  - ❖ *Show that we can reconstruct the original table*



Algorithm Roadmap



Normal Form(s)

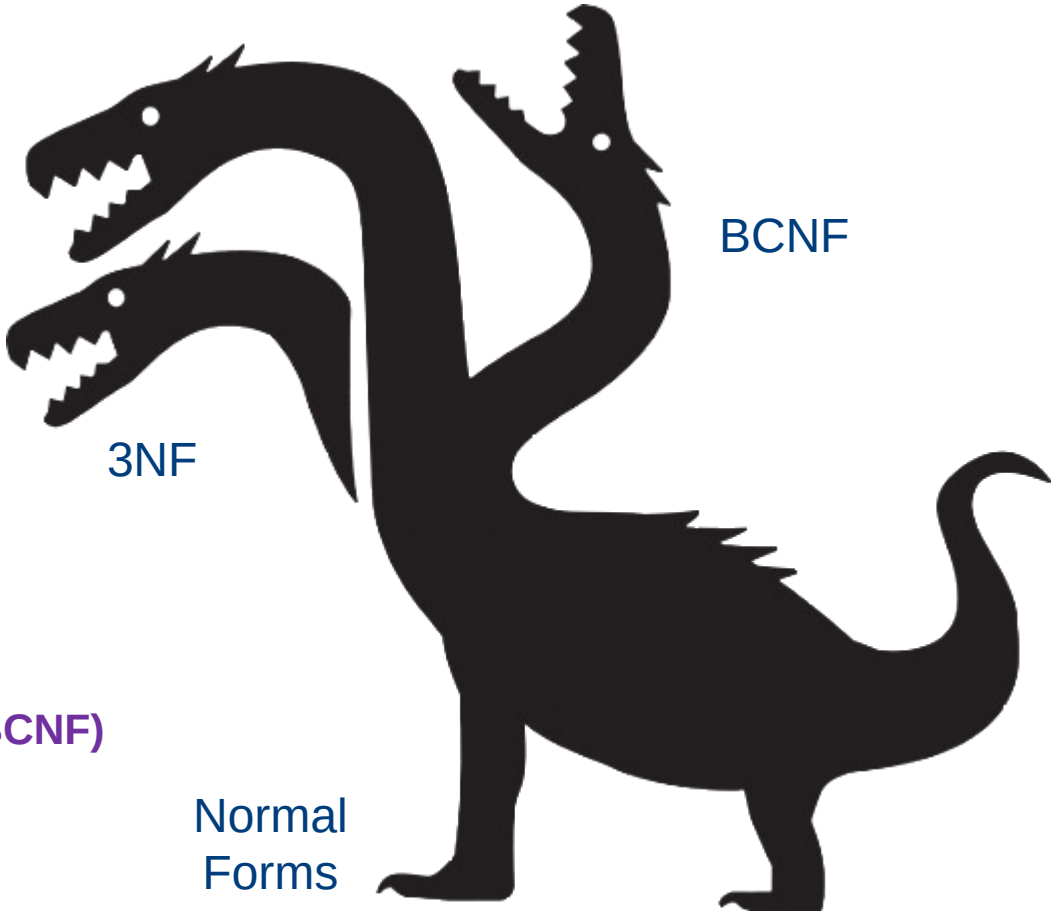
- What is it?
  - Conditions that a “good” table should satisfy
  - Various NFs in increasing order of strictness

Easy to satisfy. May have high redundancy.

1. 1st NF
2. 2nd NF
3. **3rd NF (3NF)**
4. **Boyce-Codd NF (BCNF)**

Very little redundancy. Not always possible to satisfy.

5. 4th NF
6. 5th NF
7. 6th NF



RECAP: FD

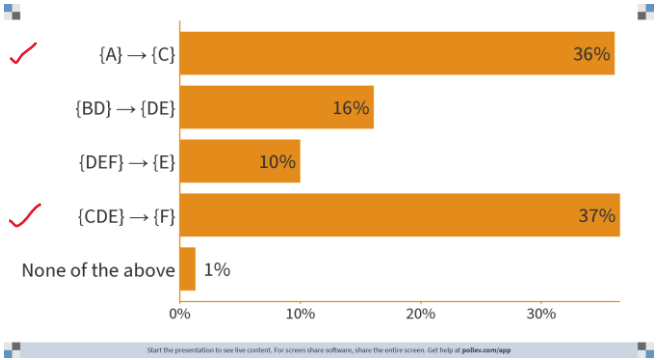
non-trivial and decomposed functional dependencies

Non-Trivial and Decomposed FD

- **Simplifying Discussion**
  - **Non-trivial** $\{A\} \rightarrow \{B\}$ where $\{B\} \not\subseteq \{A\}$ *(literally, not trivial)*
  - **Decomposed** $\{A\} \rightarrow \{B\}$ where B is a single attribute *(singular)*
  - ❖ A non-decomposed FD can always be transformed into the equivalent set of decomposed FD by *decomposition rule*
    - $\{BC\} \rightarrow \{DE\} \Leftrightarrow \{BC\} \rightarrow \{D\}$ and $\{BC\} \rightarrow \{E\}$

Non-Trivial and Decomposed FD

- **Simplifying Discussion**
  - **Non-trivial** $\{A\} \rightarrow \{B\}$ where $\{B\} \not\subseteq \{A\}$ *(literally, not trivial)*
  - **Decomposed** $\{A\} \rightarrow \{B\}$ where B is a single attribute *(singular)*
  - ❖ A non-decomposed FD can always be transformed into the equivalent set of decomposed FD by *decomposition rule*
    - $\{BC\} \rightarrow \{DE\} \Leftrightarrow \{BC\} \rightarrow \{D\}$ and $\{BC\} \rightarrow \{E\}$
  - **Exercise:** Which of these are non-trivial decomposed FDs?



Non-Trivial and Decomposed FD

- **Why?**
  - We will check normal forms based on the non-trivial and decomposed FDs on a table
    - Trivial FDs are not interesting *(i.e., trivial)* because they convey no information
    - Decomposed FDs are easier to reason and they are still valid
- **Deriving Non-Trivial and Decomposed FD from a Table**
  - Closure! *... non-trivialize, then decompose ...*
    1. Consider all subset of attributes in R *(do we need to consider R?)*
    2. Compute the closure of each subset
    3. Remove the “trivial” attributes
    4. Derive the decomposed FD from each closure

Non-Trivial and Decomposed FD

- **Deriving Non-Trivial and Decomposed FD from a Table**
  - **Example:** R(A, B, C) with {A} → {B}, {B} → {A} and {B} → {C}
    1. Consider all subset of attributes in R

       {A} {B} {C} {AB} {AC} {BC} {ABC}

    2. Compute the closure of each subset

       $\{A\}^+ = \{ABC\}$ $\{B\}^+ = \{ABC\}$ $\{C\}^+ = \{C\}$ $\{AB\}^+ = \{ABC\}$ $\{AC\}^+ = \{ABC\}$ $\{BC\}^+ = \{ABC\}$

    3. Remove the “trivial” attributes

       $\{A\}^+ = \{ABC\}$ $\{B\}^+ = \{ABC\}$ $\{C\}^+ = \{C\}$ $\{AB\}^+ = \{ABC\}$ $\{AC\}^+ = \{ABC\}$ $\{BC\}^+ = \{ABC\}$

    4. Derive the decomposed FD from each closure

       {A} → {B} {A} → {C} {B} → {A} {B} → {C} {AB} → {C} {AC} → {B} {BC} → {A}

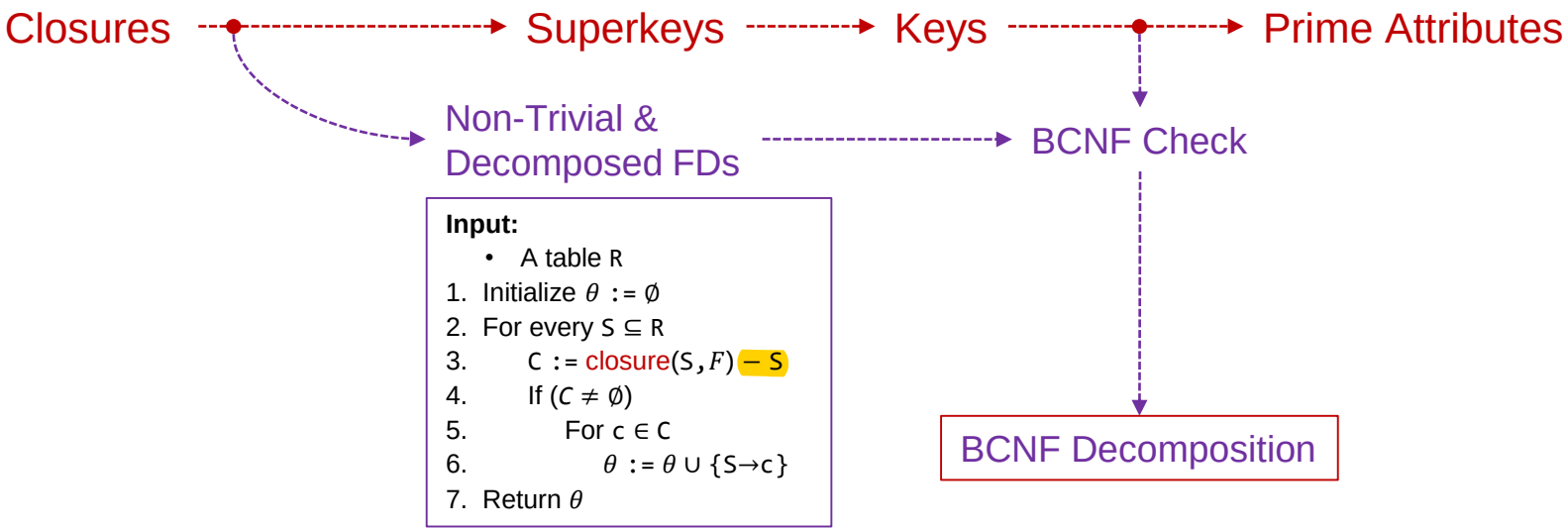
Non-Trivial and Decomposed FD

- **Deriving Non-Trivial and Decomposed FD from a Table**
  - **Exercise:** R(A, B, C, D) with {AB} → {C}, {C} → {D} and {D} → {A}

$\{A\}^+$ =	{A}	$\{B\}^+$ =	{B}	$\{C\}^+$ =	{ACD}	$\{D\}^+$ =	{AD}
$\{AB\}^+$ =	{ABCD}	$\{AC\}^+$ =	{ACD}	$\{AD\}^+$ =	{AD}		
$\{BC\}^+$ =	{ABCD}	$\{BD\}^+$ =	{ABCD}	$\{CD\}^+$ =	{ACD}		
$\{ABC\}^+$ =	{ABCD}	$\{ABD\}^+$ =	{ABCD}	$\{ACD\}^+$ =	{ACD}	$\{BCD\}^+$ =	{ABCD}

{C} → {A}	{AB} → {C}	{AC} → {D}	{ABC} → {D}
{C} → {D}	{AB} → {D}	{BD} → {A}	{ABD} → {C}
{D} → {A}	{BC} → {A}	{BD} → {C}	{BCD} → {A}
	{BC} → {D}	{CD} → {A}	

Algorithm Roadmap



BCNF

Boyce-Codd Normal Form

BCNF

- **Definition**
  - A table R is in BCNF if *every non-trivial and decomposed FD has a superkey as its left hand side*

  - **Example:** R(A, B, C) with {A} → {B}, {B} → {A} and {B} → {C}
    - Key: {A} and {B}
    - Non-trivial and decomposed FDs on R:
      - {A} → {B} - {A} → {C} - {B} → {A} - {B} → {C}
      - {AB} → {C} - {AC} → {B} - {BC} → {A}
    - For each of the above FD, the left hand side is a superkey

      ∴ R satisfies BCNF

BCNF

- **Definition**
  - A table `R` is in BCNF if *every non-trivial and decomposed FD has a superkey as its left hand side*

  - **Example:** `R(A, B, C)` with `{A}` → `{B}` and `{B}` → `{C}`
    - Key: `{A}`
    - Non-trivial and decomposed FDs on `R`:
      - `{A}` → `{B}`
      - `{A}` → `{C}`
      - `{B}` → `{C}`
      - `{AB}` → `{C}`
      - `{AC}` → `{B}`

      this is not a superkey as it does not contain a key
    - B is **NOT** superkey in `{B}` → `{C}`

      ∴ `R` does **NOT** satisfy BCNF

BCNF

- **Intuition**
  - A table R is in BCNF if *every non-trivial and decomposed FD has a superkey as its left hand side*

  - **In other words,** any attribute B can depend ***only*** on superkeys
    - "In superkeys we trust"
    - Any dependency on non-superkeys is prohibited
  - **Why?**
    - Suppose B depends on non-superkey $C_1 C_2 \dots C_n$
    - Since $C_1 C_2 \dots C_n$ is not a superkey, the same $C_1 C_2 \dots C_n$ may appear multiple times in the table *(why?)*
    - Whenever this happens, the same B would appear multiple times in the table

Table "Example"

C_1	C_2	$\cdots$	C_n	B
c1	c2	...	cn	b
c1	c2	...	cn	b

BCNF

- **Intuition**
  - A table `R` is in BCNF if *every non-trivial and decomposed FD has a superkey as its left hand side*

Table "Student_Data"

Name	NRIC	Phone	Address
Alice	1234	67899876	Jurong East
Alice	1234	83838484	Jurong East
Bob	5678	98765432	Pasir Ris

  - **Example:** The table shown on the right
    - Key: `{NRIC, Phone}`
    - FD that violates BCNF
      - `{NRIC}` → `{Name}`
      - `{NRIC}` → `{Address}`
    - Since `{NRIC}` is not superkey, the same `NRIC` can appear multiple times in the table
      - Every time `NRIC` is repeated, the corresponding `Name` and `Address` are also repeated

BCNF

proving that it is a BCNF means that need to list down everything to prove

but to show violation just need 1 example

- **Checking BCNF**
  - A table R is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*

  - **By Counterexample:**

Can be done at the same time. Generate + check!

1. Consider all non-trivial and decomposed FDs of R
2. For each non-trivial and decomposed FDs of R, check that the left hand side is superkey
   - If not, then we have a counterexample → $LHS^+ \neq R$
3. If no counterexample found, then R satisfies BCNF

BCNF

- **Checking BCNF**
  - A table R is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*

  - **By Counterexample:**
    1. Consider all subset of attributes of R
    2. Compute the closure of each subset
    3. Consider only the subset that are not superkey *(closure ≠ R)*
    4. Remove the "trivial" attributes
    5. If the resulting set is non-empty, we have a counterexample

Non superkey: 3

Non-trivial FD: 4, 5

More but NOT All

Alternatively, $\{S\} \subset \{S\}^+ \subset R$

The FD violating BCNF are $\{S\} \rightarrow (\{S\}^+ - \{S\})$
and can be decomposed if needed

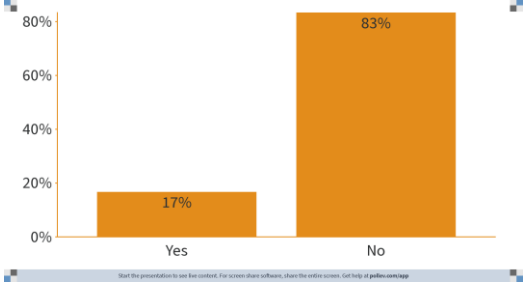
BCNF

- **Checking BCNF**
  - A table `R` is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*

  - **Example:** `R(A, B, C, D)` with $\{AB\} \rightarrow \{C\}$, $\{C\} \rightarrow \{D\}$ and $\{D\} \rightarrow \{A\}$
    1. Consider all subset of attributes of `R`
    2. Compute the closure of each subset
    3. Consider only the subset that are not superkey *(closure ≠ R)*
    4. Remove the “trivial” attributes
    5. If the resulting set is non-empty, we have a counterexample

$\{A\}^+$	=	$\{A\}$	$\{B\}^+$	=	$\{B\}$	$\{C\}^+$	=	$\{ACD\}$	$\{D\}^+$	=	$\{AD\}$
$\{AB\}^+$	=	$\{ABCD\}$	$\{AC\}^+$	=	$\{ACD\}$	$\{AD\}^+$	=	$\{AD\}$			
$\{BC\}^+$	=	$\{ABCD\}$	$\{BD\}^+$	=	$\{ABCD\}$	$\{CD\}^+$	=	$\{ACD\}$			
$\{ABC\}^+$	=	$\{ABCD\}$	$\{ABD\}^+$	=	$\{ABCD\}$	$\{ACD\}^+$	=	$\{ACD\}$	$\{BCD\}^+$	=	$\{ABCD\}$

BCNF

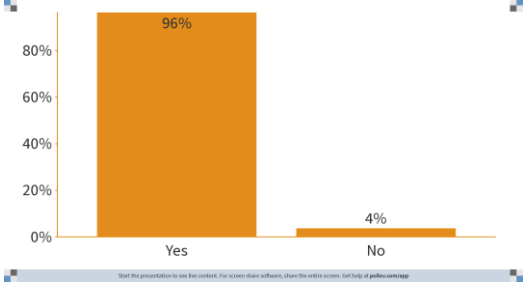


- **Checking BCNF**
  - A table R is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*

  - **Exercise:** R(A, B, C, D) with {B} → {C} and {B} → {D}

$\{A\}^+$ = {A}	$\{B\}^+$ = {BCD}	$\{C\}^+$ = {C}	$\{D\}^+$ = {D}
$\{AB\}^+$ = {ABCD}	$\{AC\}^+$ = {AC}	$\{AD\}^+$ = {AD}	
$\{BC\}^+$ = {BCD}	$\{BD\}^+$ = {BCD}	$\{CD\}^+$ = {CD}	
$\{ABC\}^+$ = {ABCD}	$\{ABD\}^+$ = {ABCD}	$\{ACD\}^+$ = {ACD}	$\{BCD\}^+$ = {BCD}

BCNF



- **Checking BCNF**
  - A table R is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*

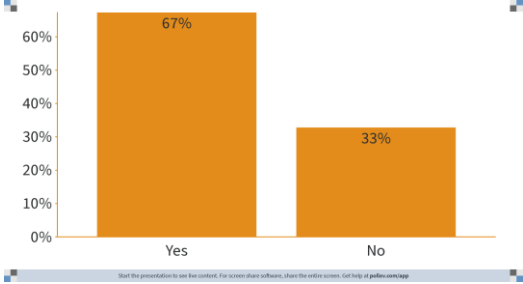
  - **Exercise:** R(A, B, C, D) with {A} → {B}, {B} → {C}, {C} → {D} and {D} → {A}

{A}+ = {ABCD}	{B}+ = {ABCD}	{C}+ = {ABCD}	{D}+ = {ABCD}
{AB}+ = {ABCD}	{AC}+ = {ABCD}	{AD}+ = {ABCD}	
{BC}+ = {ABCD}	{BD}+ = {ABCD}	{CD}+ = {ABCD}	
{ABC}+ = {ABCD}	{ABD}+ = {ABCD}	{ACD}+ = {ABCD}	{BCD}+ = {ABCD}

Note

Whenever we have {A} → {B} and {B} → {A}, we say that the attributes A and B are *functionally equivalent*. In such cases, we may actually remove either attribute A or attribute B (not the FD, but the attribute) from the database. For instance, eid and email in project. email can always be derived! Maybe as something like eid@company.name

BCNF

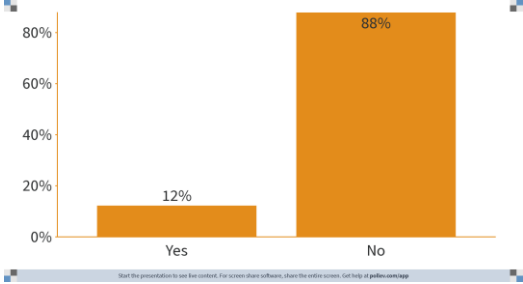


- **Checking BCNF**
  - A table R is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*

  - **Exercise:** R(A, B, C, D) with {AB} → {D}, {BD} → {C}, {CD} → {A} and {AC} → {B}

{A}$^+$	=	{A}	{B}$^+$	=	{B}	{C}$^+$	=	{C}	{D}$^+$	=	{D}
{AB}$^+$	=	{ABCD}	{AC}$^+$	=	{ABCD}	{AD}$^+$	=	{AD}			
{BC}$^+$	=	{BC}	{BD}$^+$	=	{ABCD}	{CD}$^+$	=	{ABCD}			
{ABC}$^+$	=	{ABCD}	{ABD}$^+$	=	{ABCD}	{ACD}$^+$	=	{ABCD}	{BCD}$^+$	=	{ABCD}

BCNF



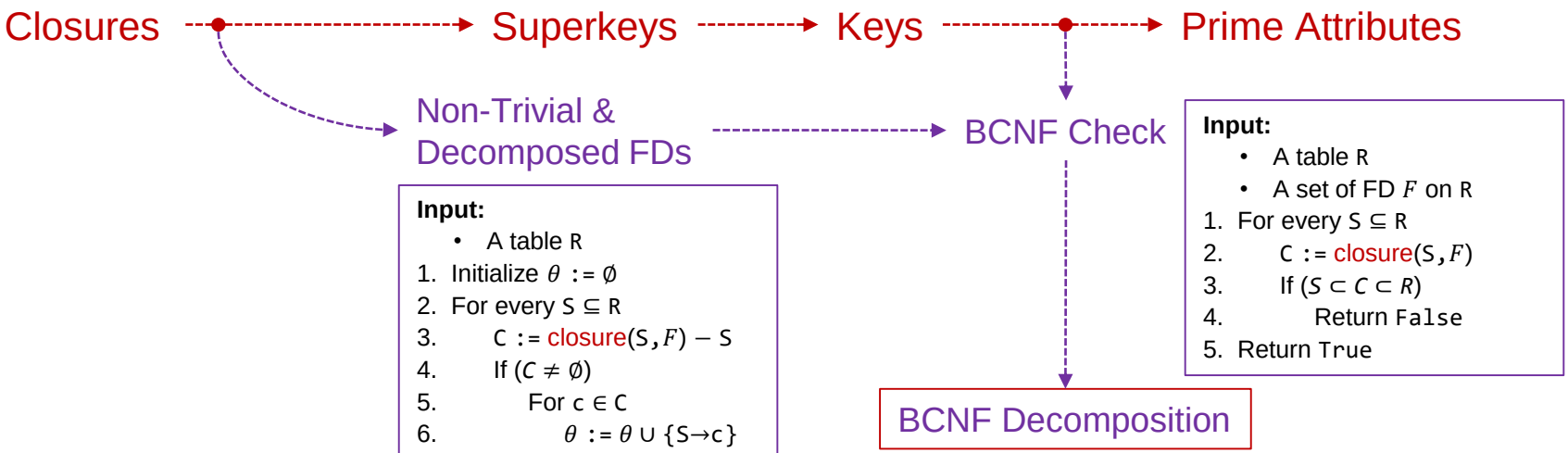
- **Checking BCNF**
  - A table R is **NOT** in BCNF if *there exists at least one non-trivial and decomposed FD such that its left hand side is NOT superkey*
  - **Exercise:** R(A, B, C, D, E) with {AB} → {C}, {C} → {E}, {E} → {A} and {E} → {D}

$\{A\}^+ = \{A\}$ | $\{B\}^+ = \{B\}$ | $\{C\}^+ = \{ACDE\}$

Note

Remember the trick from before, we can always start from smallest subset. This way, the *chance* of the closure being non-trivial is greater.

Algorithm Roadmap



BCNF Decomposition

How to normalize a table to satisfy BCNF property

BCNF Decomposition

projection is just decomposition

- **Normalization**

Table "Student_Data"

Name	NRIC	Phone	Address
Alice	1234	67899876	Jurong East
Alice	1234	83838484	Jurong East
Bob	5678	98765432	Pasir Ris

Table "Student_Info"

Name	NRIC	Address
Alice	1234	Jurong East
Bob	5678	Pasir Ris

Table "Student_Contact"

NRIC	Phone
1234	67899876
1234	83838484
5678	98765432

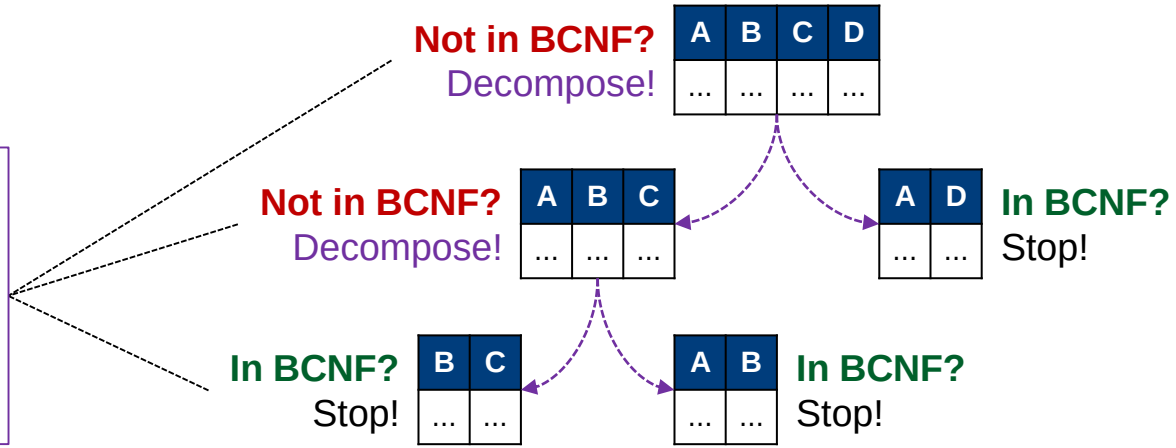
  - **Idea:**
    - Decompose until all are in BCNF
      - Split the attributes into another table and remove any duplicates
      - The splitting need to ensure that BCNF violations are removed

BCNF Decomposition

- **Normalization**
  - **Idea:**
    - Decompose until all are in BCNF
      - Split the attributes into another table and remove any duplicates
      - The splitting need to ensure that BCNF violations are removed

Input:
- A table R
- A set of FD F on R

1. For every S ⊆ R
2. C := closure(S)
3. If ($S \subset C \subset R$)
4. Return False
5. Return True



BCNF Decomposition

- **Normalization**
  - **Problem:**
    - How to decompose such that we remove at least one BCNF violation?
  - **Idea:**
    - Let's look at the violation again
      - Say {B} → {CD} violates BCNF property of R(A, B, C, D)
    - In what table does {B} → {CD} not violate BCNF property?
      - Make B a superkey!
      - R1(B, C, D)
    - We need to return the ***violation*** and not just True/False

Input:
- A table R
- A set of FD F on R

1. For every S ⊆ R
2. C := closure(S)
3. If ($S \subset C \subset R$)
4. Return **False**
5. Return **True**

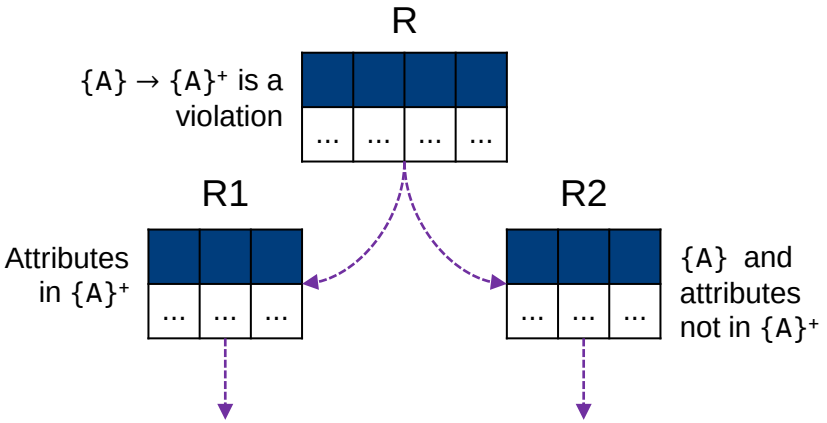
This is True/False, no information about violations

More but NOT All

Alternatively, $\{S\} \subset \{S\}^+ \subset R$

The FD violating BCNF are $\{S\} \rightarrow (\{S\}^+ - \{S\})$
but $\{S\} \rightarrow \{S\}^+$ is sufficient

BCNF Decomposition



- **Normalization Algorithm**
  - **Input:**
    - A table R
    - A set of FD F on R

1. Check if R is in BCNF

a. If not , there is an FD {A} → {A}⁺ that violates it then R decompose into
   i. R1(*everything in* {A}⁺)
   ii. R2(*everything in* {A} *as well as attributes in R but not in* {A}⁺)
   iii. Recursively check R1 and R2
   iv. Return the union of the result from recursive check

b. Otherwise, stop
   i. Return {R}

output will be schemas/relations {R1, R2, ...}

Input:
- A table R
- A set of FD F on R

1. For every S ⊆ R
2. C := closure(S)
3. If ($S \subset C \subset R$)
4. Return **{S} → C**
5. Return **NULL**

BCNF Decomposition

- **Normalization Algorithm**
  - **Example:**
    - Known FD:
      - `{NRIC} → {Name, Address}`
  - **Steps:**
    1. Check BCNF property
       - `{NRIC} → {Name, NRIC, Address}` violates BCNF property
       a. Decompose `Student_Data` into two tables
          i. `Student_Info(Name, NRIC, Address)` *(`{NRIC}`⁺)*
          ii. `Student_Contact(NRIC, Phone)` *(`{NRIC}` and attributes not in `{NRIC}`⁺)*
          iii. Check if `Student_Info(Name, NRIC, Address)` and `Student_Contact(NRIC, Phone)` are in BCNF *(they are...)*

Table "Student_Data"

Name	NRIC	Phone	Address
Alice	1234	67899876	Jurong East
Alice	1234	83838484	Jurong East
Bob	5678	98765432	Pasir Ris

BCNF Decomposition

And that is how we got this decomposition

Table "Student_Data"

Name	NRIC	Phone	Address
Alice	1234	67899876	Jurong East
Alice	1234	83838484	Jurong East
Bob	5678	98765432	Pasir Ris

Table "Student_Info"

Name	NRIC	Address
Alice	1234	Jurong East
Bob	5678	Pasir Ris

Table "Student_Contact"

NRIC	Phone
1234	67899876
1234	83838484
5678	98765432

BCNF Decomposition

Collected Answers

1. `R1(A, B, C)` ✗
2. `R2(A, D)` ✓

- **Normalization Algorithm**
  - **Example:** `R(A, B, C, D)` with $\{A\} \rightarrow \{B\}$ and $\{B\} \rightarrow \{C\}$
  - **Steps:**
    1. Check BCNF property
       - $\{A\} \rightarrow \{A, B, C\}$ violates BCNF property
       a. Decompose `R` into two tables
          i. `R1(A, B, C)` *($\{A\}^+$)*
          ii. `R2(A, D)` *($\{A\} \cup (R - \{A\}^+)$)*
          iii. Check if `R1(A, B, C)` and `R2(A, D)` are in BCNF

NO! **YES!**

BCNF Decomposition

Collected Answers

~~1. R1(A, B, C)~~ ✗
- a. R3(B, C) ✓
- b. R4(A, B) ✓

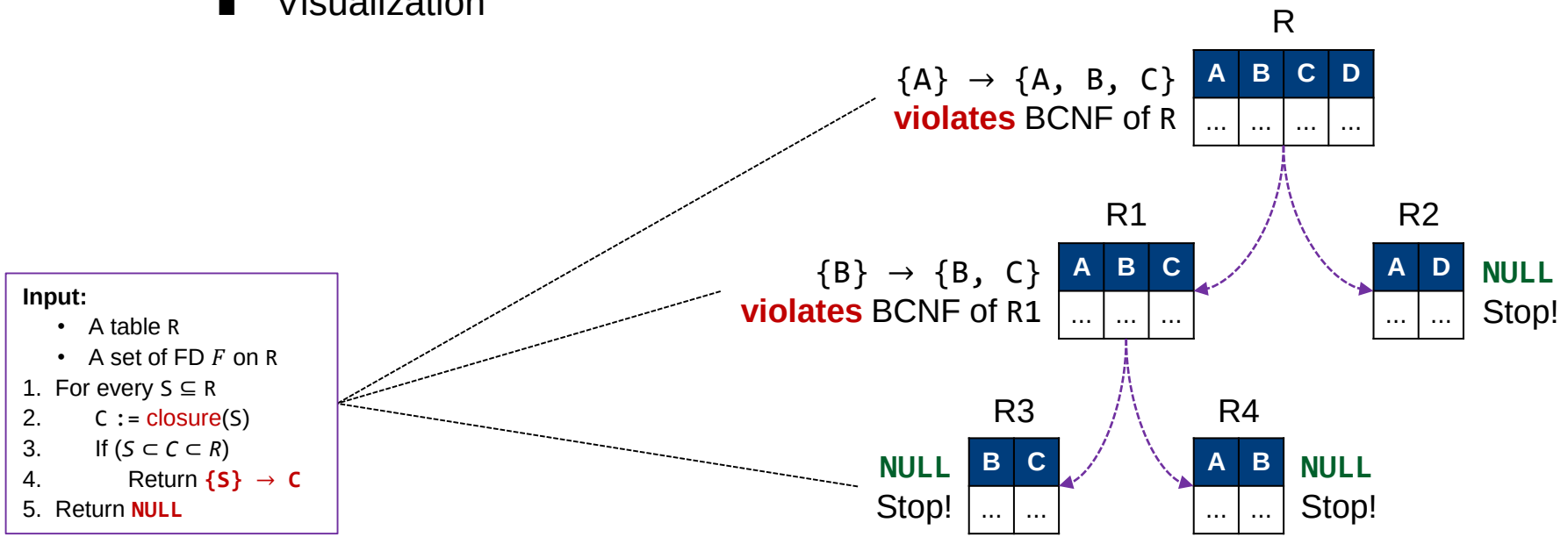
2. R2(A, D) ✓

- **Normalization Algorithm**
  - **Example:** R(A, B, C, D) with {A} → {B} and {B} → {C}
  - **Steps:**
    1. Check BCNF property of R1(A, B, C) with {A} → {B} and {B} → {C}
       - {B} → {B, C} violates BCNF property
       a. Decompose R1 into two tables
          i. R3(B, C) *($\{B\}^+$)*
          ii. R4(A, B) *($\{B\} \cup (R1 - \{B\}^+)$)*
          iii. Check if R3(B, C) and R4(A, B) are in BCNF

YES! **YES!**

BCNF Decomposition

- **Normalization Algorithm**
  - **Example:** `R(A, B, C, D)` with `{A}` → `{B}` and `{B}` → `{C}`
    - Visualization



BCNF Decomposition

- **Normalization Algorithm**
  - **Notes:**
    - The BCNF decomposition of a table may NOT be *unique*
      - Because any FD violating BCNF can be used for decomposition
    - If a table only has two attributes, then it is in BCNF
      - Why?
        - No need to check tables with only two attributes
  - **Issues:**
    - What happen if an FD has attributes *across* multiple tables?
    - Compute closure!
      - Then remove attributes that are irrelevant to the current table
    - Let's illustrate this with an example

BCNF Decomposition

Collected Answers

1. R1(A, B) ✓
2. R2(A, C, D, E) ?

- **Normalization Algorithm**
  - **Example:** R(A,B,C,D,E) with {A} → {B} and {BC} → {D}
  - **Steps:**
    1. Check BCNF property
       - {A} → {A, B} violates BCNF property
       a. Decompose R into two tables
          i. R1(A, B) ($\{A\}^+$)
          ii. R2(A, C, D, E) ($\{A\} \cup (R - \{A\}^+)$)
          iii. Check if R1(A, B) and R2(A, C, D, E) are in BCNF

thus there is a need to compute the closure

$A \rightarrow B$

$Ac^+ \rightarrow ABCD$

$\therefore AC \rightarrow D$

YES! **???**

Do we even know what are the FDs that holds on R2?

{A} → {B}? But no B

{BC} → {D}? Also no B

BCNF Decomposition

Collected Answers

1. R1(A, B) ✓
2. R2(A, C, D, E) ✗

- **Normalization Algorithm**
  - **Example:** R(A,B,C,D,E) with {A} → {B} and {BC} → {D}
  - **Steps:**
    1. Check BCNF property of R2(A, C, D, E) with {A} → {B} and {BC} → {D}
       - But all FD contains B and R2 has no attribute B
       a. Compute closure of each subset of attributes
       b. Remove attributes not in the current table *(projection)*
       - Now we can find the violation

$\{A\}^+ = \{AB\}$	$\{C\}^+ = \{C\}$	$\{D\}^+ = \{D\}$	$\{E\}^+ = \{E\}$
$\{AC\}^+ = \{ABCD\}$	$\{AD\}^+ = \{ABD\}$	$\{AE\}^+ = \{AE\}$	
$\{CD\}^+ = \{CD\}$	$\{CE\}^+ = \{CE\}$	$\{DE\}^+ = \{DE\}$	
$\{ACD\}^+ = \{ABCD\}$	$\{ACE\}^+ = \{ABCDE\}$	$\{ADE\}^+ = \{ABDE\}$	$\{CDE\}^+ = \{CDE\}$

BCNF Decomposition

Collected Answers

1. R1(A, B) ✓
2. ~~R2(A, C, D, E)~~ *
   a. R3(A, D, E) ✓
   b. R4(A, C, E) ✓

- **Normalization Algorithm**
  - **Example:** R(A,B,C,D,E) with {A} → {B} and {BC} → {D}
  - **Steps:**
    1. Check BCNF property of R2(A, C, D, E) with {A} → {B} and {BC} → {D}
       - {A, C} → {A, C, D} violates BCNF property
       a. Decompose R2 into two tables
          i. R3(A, C, D) *($\{AC\}^+$)*
          ii. R4(A, C, E) *($\{AC\} \cup (R2 - \{AC\}^+)$)*
          iii. Check if R3(A, C, D) and R4(A, C, E) are in BCNF

YES! **YES!**

We used {A} → {A, B} *as our FD violation.*
But we can also use {B, C} → {B, C, D} *as our FD violation!*
Let's redo our normalization algorithm

BCNF Decomposition

Collected Answers

1. R1(B, C, D) ✓
2. R2(A, B, C, E) ✗

- **Normalization Algorithm**
  - **Example:** R(A,B,C,D,E) with {A} → {B} and {BC} → {D}
  - **Steps:**
    1. Check BCNF property
       - {B, C} → {B, C, D} violates BCNF property
       a. Decompose R into two tables
          i. R1(B, C, D) *({BC}⁺)*
          ii. R2(A, B, C, E) *({BC} ∪ (R – {BC}⁺))*
          iii. Check if R1(B, C, D) and R2(~~A, C, D, E~~) are in BCNF

YES! **NO!**

BCNF Decomposition

Collected Answers

1. R1(B, C, D) ✓
2. ~~R2(A, B, C, E)~~ ✱
   a. R3(A, B) ✓
   b. R4(A, C, E) ✓

- **Normalization Algorithm**
  - **Example:** R(A,B,C,D,E) with {A} → {B} and {BC} → {D}
  - **Steps:**
    1. Check BCNF property of R2(A, B, C, E) with {A} → {B} and {BC} → {D}
       - {A} → {A, B} violates BCNF property
       a. Decompose R2 into two tables
          i. R3(A, B) *({A}⁺)*
          ii. R4(A, C, E) *({A} ∪ (R2 − {A}⁺))*
          iii. Check if R3(A, B) and R4(A, C, E) are in BCNF

YES! **YES!**

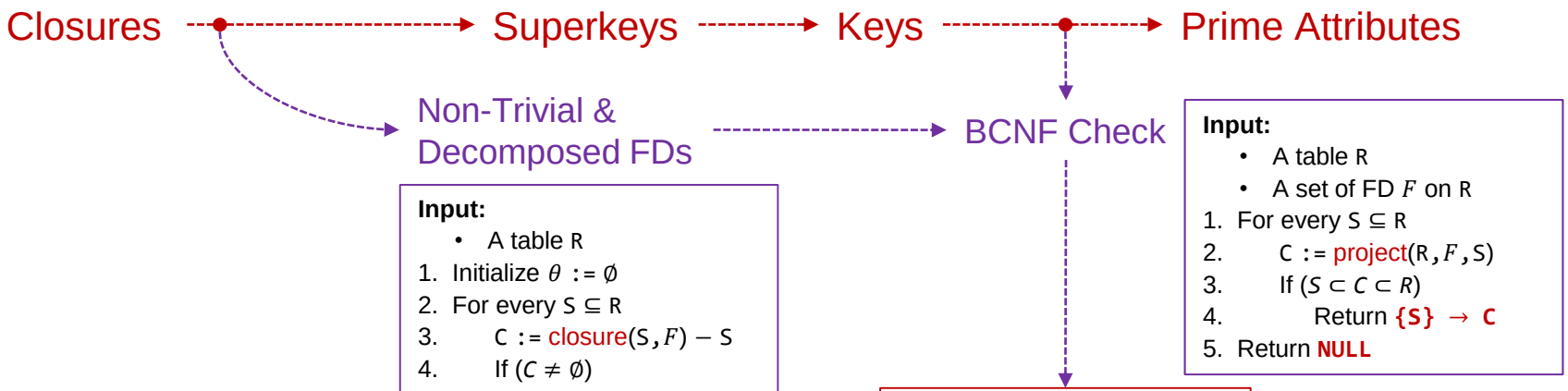
Starting with {B, C} → {B, C, D}

1. R1(B, C, D)
2. R3(A, B)
3. R4(A, C, E)

Starting with {A} → {A, B}

1. R1(A, B)
2. R3(A, D, E)
3. R4(A, C, E)

Algorithm Roadmap



Input:
- A table R
- A set of FD F on R

1. For every S ⊆ R
2. C := project(R, F, S)
3. If ($S \subset C \subset R$)
4. Return {S} → C
5. Return NULL

Input:
- A table R

1. Initialize $\theta := \emptyset$
2. For every S ⊆ R
3. C := closure(S, F) − S
4. If ($C \neq \emptyset$)
5. For c ∈ C
6. $\theta := \theta \cup \{S \rightarrow c\}$
7. Return θ

Input:
- A table R
- A set of FD F on R
- A set of attributes {A} ⊆ R

1. Initialize $\theta := \emptyset$
2. For every S ⊆ R
3. C := closure(S, F)
4. $\theta := \theta \cup \{\{S\} \rightarrow (C \cap A)\}$
5. Return θ

Input:
- A table R
- A set of FD F on R

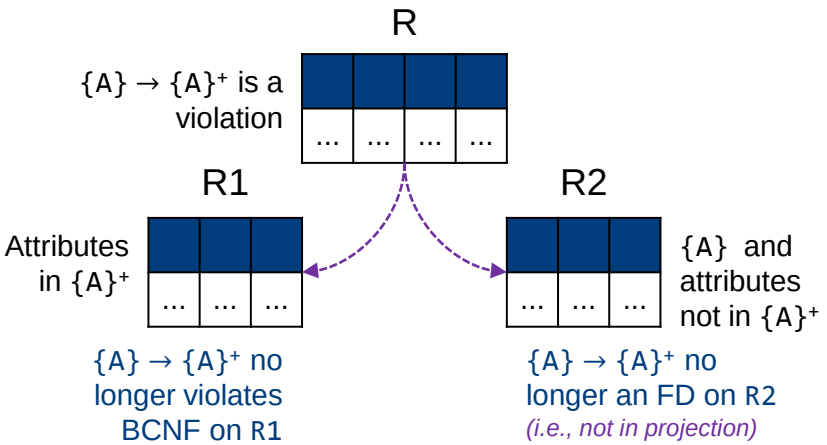
1. f := BCNF Check(R, F)
2. If (f = NULL)
3. Return {R}
4. $\{A\} \rightarrow \{A\}^+$:= f
5. Return BCNF(R1({A}+)) ∪ BCNF(R2({A}+(R−{A}+)))

BCNF Decomposition

- **Normalization Algorithm**
  - Why does the BCNF decomposition algorithm work?
    - In other words, how can it eliminate violations of BCNF?

❖ In general, each decomposition step removes at least one BCNF violation
  - Then we recursively removes all violation
  - How do we know it will not add new violations?

➢ At the worst case, we have only two attributes ⇒ always BCNF



BCNF Decomposition

Simplified Algorithm

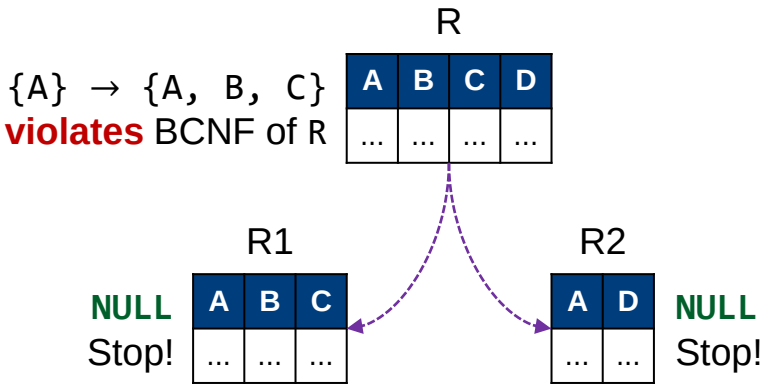
Find violation $\{A\} \rightarrow \{A\}^+$

a. $R1(\{A\}^+)$

b. $R2(\{A\} \cup (R-\{A\}+))$

Repeat with R1 and R2

- **Normalization Algorithm**
  - **Exercise:** R(A, B, C, D) with $\{A\} \rightarrow \{B\}$ and $\{A\} \rightarrow \{C\}$
  - **Steps:**



BCNF Decomposition

Simplified Algorithm

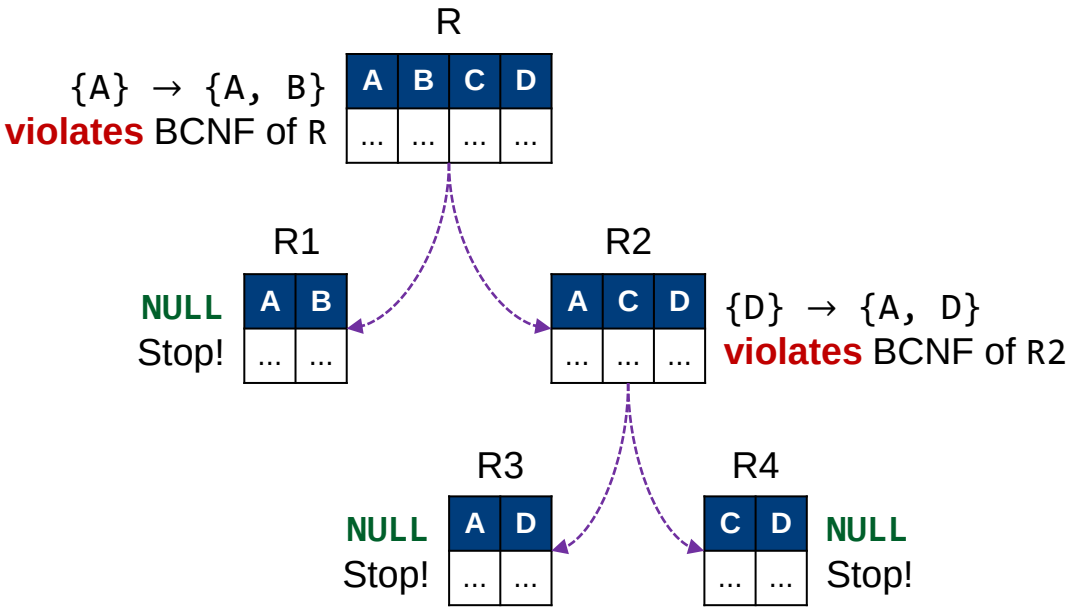
Find violation $\{A\} \rightarrow \{A\}^+$

a. $R1(\{A\}^+)$

b. $R2(\{A\} \cup (R-\{A\}+))$

Repeat with R1 and R2

- **Normalization Algorithm**
  - **Exercise:** $R(A, B, C, D)$ with $\{BC\} \rightarrow \{D\}$, $\{D\} \rightarrow \{A\}$ and $\{A\} \rightarrow \{B\}$
  - **Steps:**



BCNF Decomposition

- **Properties of BCNF**
  - **Good** properties
    - No update or deletion or insertion anomalies
    - Small redundancies *(very hard to completely remove redundancies)*
    - The original table can always be reconstructed from the decomposed tables
      - How?
      - `SELECT * FROM Student_Info, Student_Contact`
        `WHERE  Student_Info.NRIC = Student_Contact.NRIC`

This is called **"Lossless Join"**

Table "Student_Data"

Name	NRIC	Phone	Address
Alice	1234	67899876	Jurong East
Alice	1234	83838484	Jurong East
Bob	5678	98765432	Pasir Ris

Table "Student_Info"

Name	NRIC	Address
Alice	1234	Jurong East
Bob	5678	Pasir Ris

Table "Student_Contact"

NRIC	Phone
1234	67899876
1234	83838484
5678	98765432

BCNF Decomposition

$R_1 = \pi_{R_1}(R)$ $R_2 = \pi_{R_2}(R)$

$R = R_1 \bowtie R_2$

$= \pi(R) \bowtie \pi(R_1)$

- **Lossless Join Decomposition**
  - Say we decompose a table `R` into two tables `R1` and `R2`
  - The decomposition guarantees ***lossless join***, whenever the common attributes in `R1` and `R2` constitute a superkey of `R1` or `R2` $(R_1 \cap R_2) \longrightarrow R_1$ OR $(R_1 \cap R_2) \longrightarrow R_2$
    - **Example:**
      - `R(A, B, C)` decomposed into `R1(A, B)` and `R2(B, C)` with `B` being a superkey of `R2`
      - `R(A, B, C, D)` decomposed into `R1(A, B, C)` and `R2(B, C, D)` with `BC` being a superkey of `R1`

BCNF Decomposition

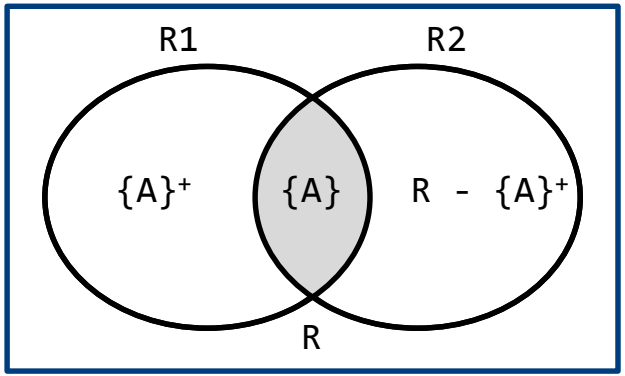
Lossless Join

The decomposition guarantees lossless join, whenever the *common attributes* in R1 and R2 constitute a superkey of R1 or R2

- **Lossless Join Decomposition of BCNF**
  - BCNF decomposition ***guarantee*** lossless join decomposition
  - **Why?**
    - Consider a violation $\{A\} \rightarrow \{A\}^+$
    - Decomposed into R1({A}+) and R2($\{A\} \cup (R - \{A\}^+)$)
    - **Common attributes?**
      - $\{A\}^+ \cap (\{A\} + (R - \{A\}^+))$

        $\Rightarrow \{A\}$
    - But $\{A\} \rightarrow \{A\}^+$ and R1({A}+)
      - So {A} is a superkey of R1

        $\therefore$ This decomposition guarantees lossless join decomposition



BCNF Decomposition

- **Properties of BCNF**
  - **Good** properties
    - No update or deletion or insertion anomalies
    - Small redundancies *(very hard to completely remove redundancies)*
    - The original table can always be reconstructed from the decomposed tables
  - **Bad** properties
    - Dependencies may not be preserved *(non dependency-preserving decomposition)*
      - *Topic for next week lecture*

QUESTION?