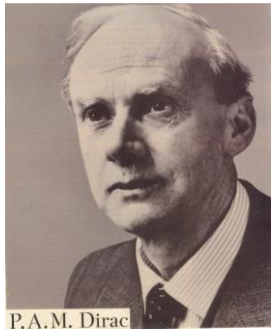
Lecture 12

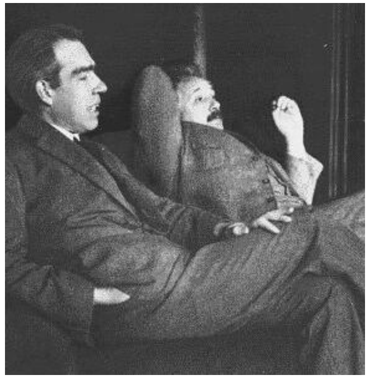
If you see one you see all !

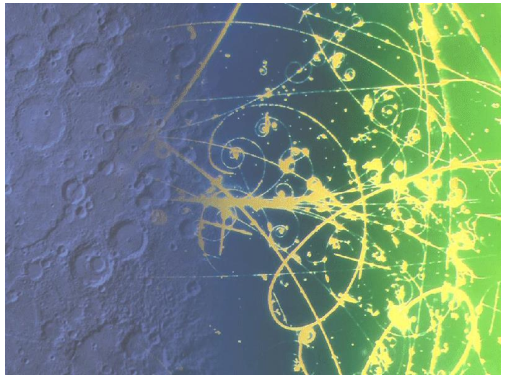
After the foundations of Quantum Mechanics were laid down …
It was a time when 2nd-rate men did 1st - rate work. Why ?

P. Dirac

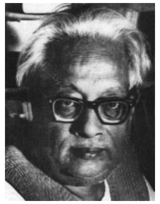
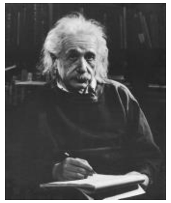
Nobel Laureate, 1902 – 1984



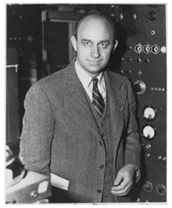
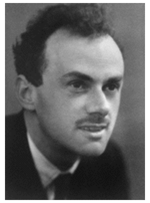




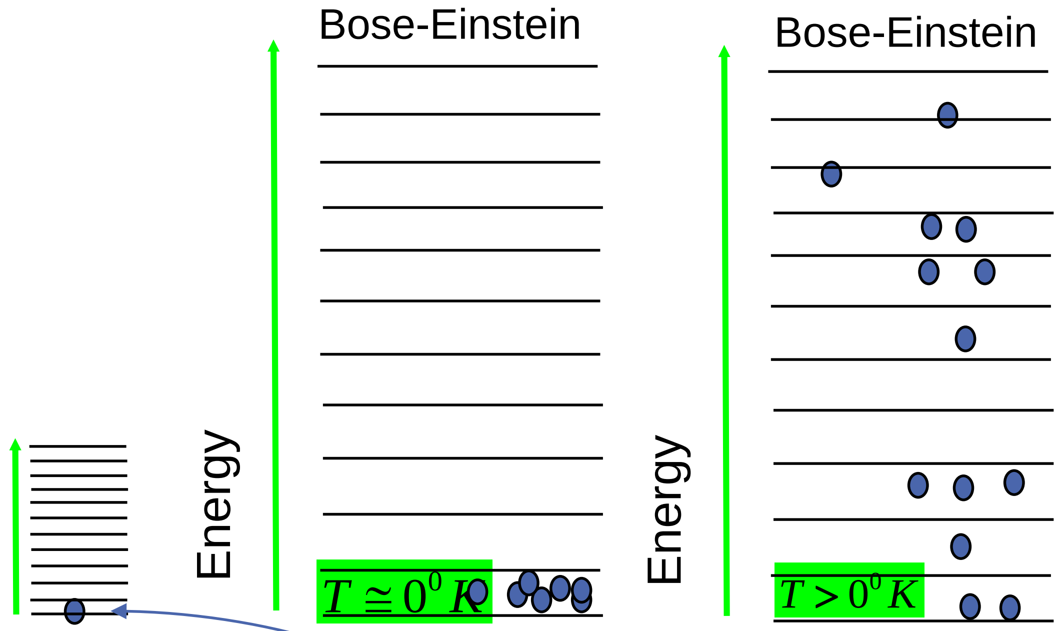
Some applications !

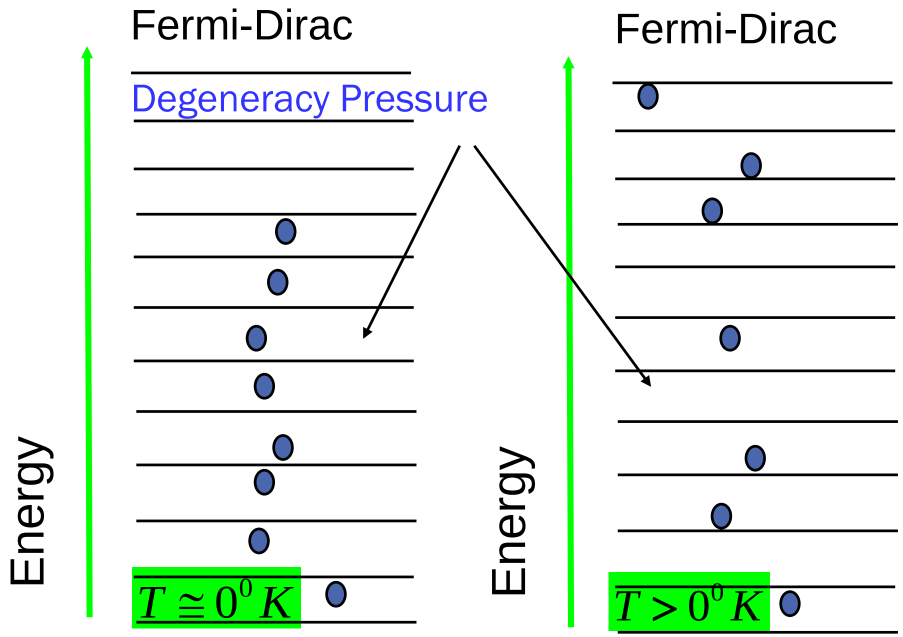



Bose Statistics

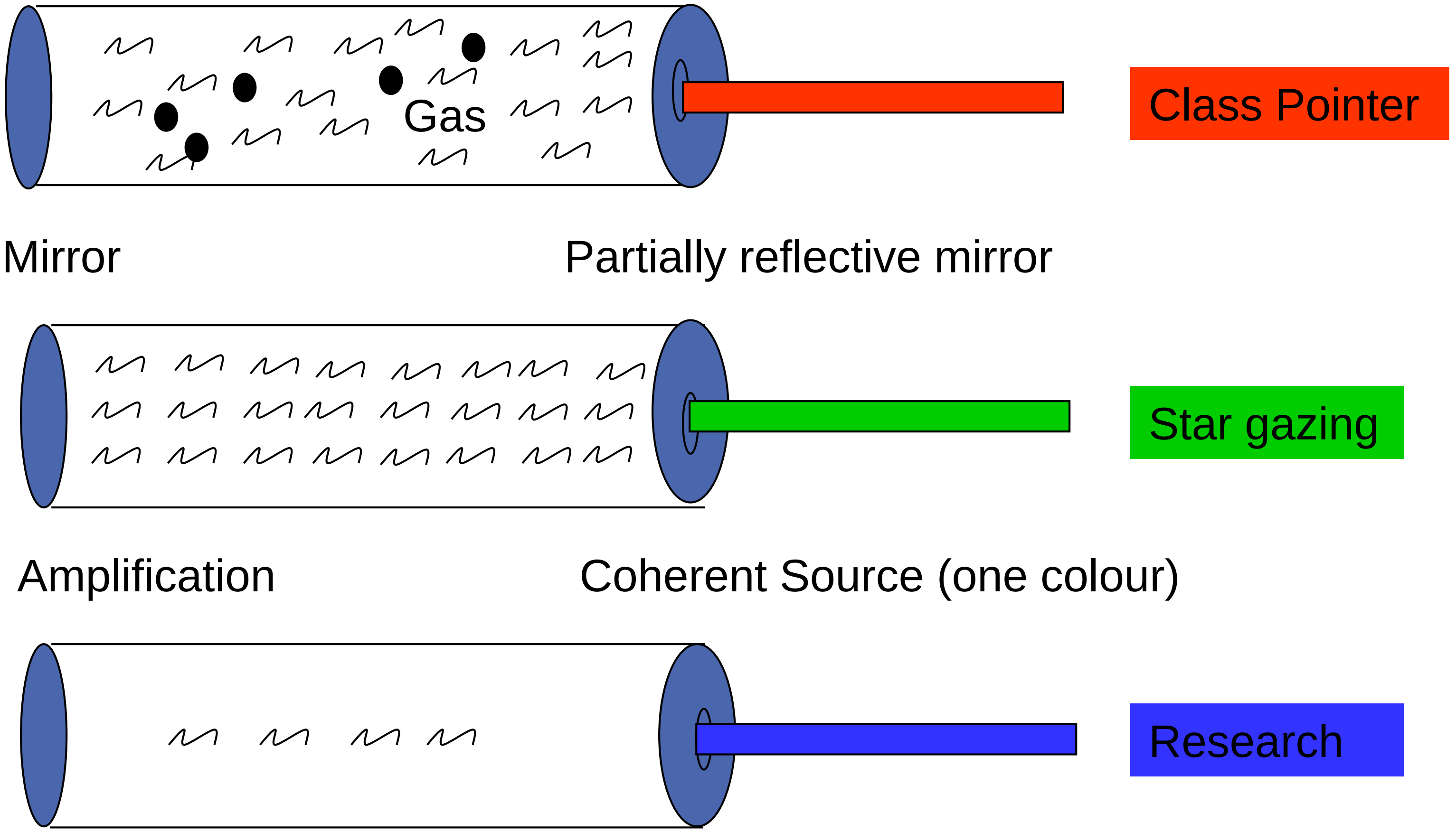
Fermion Statistics





Note : At high temperature Bosons (Integer Spin) & Fermions (Half fractional Spin) behave like Boltzmann (Maxwell) particles. The micro-particles become so cold that the uncertainty in their position is comparable to the separation between micro-particles. This means that that several micro-particles can occupy the same region of space at the same time … they lose their individuality completely and behave as a single quantum entity … Bose Einstein Condensate … classical tennis balls cannot do this right !

LASER (Maser)

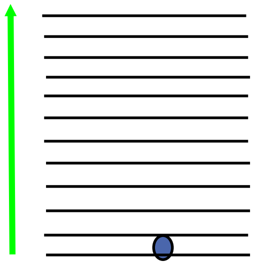


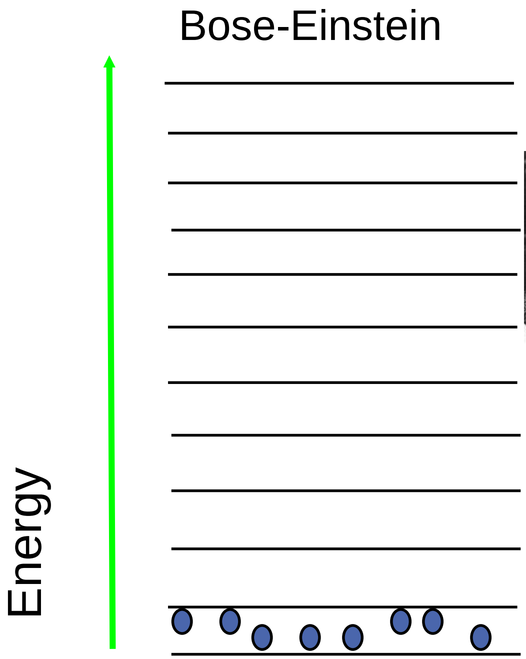
Lasing only occurs when more than half of the atoms are excited; also known as population inversion … it is opposite to the normal state when only a few atoms are excited.

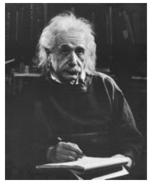
Bose-Einstein Condensation

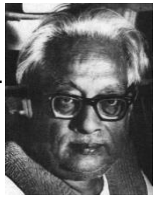
A dramatic increase in the occupancy of the lowest state at a critical temperature T_c is called Bose condensation.

It is as if a gas has condensed into a liquid.









Ideal case $T \approx 0^0 K$

Discovering the coolest spot on the Equator

QUANTUM RESEARCH: Dr Tony Tan (centre) with Dr Murray Barrett (second from right) and Kyle Arnold (far left) at CQT lab.

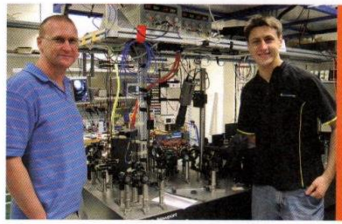
THE CENTRE for Quantum Technologies (CQT) – Singapore's first Research Centre of Excellence – might well be considered the coolest place on the Equator. Investigators at the CQT laboratories have successfully accomplished the Bose-Einstein condensation, in which a collection of atoms reach a state of matter that is as cold as the fundamental laws of physics will allow.

The Bose-Einstein condensate is named after Satyendra Nath Bose and Albert Einstein who predicted such a state in 1924. However, it was not until 1995 that a group of researchers managed to produce the Bose-Einstein condensate in the laboratory. In recognition of their work, they were awarded the Nobel Prize in Physics in 2001. In Singapore, CQT's Principal Investigator Dr Murray Barrett and PhD student Kyle Arnold have successfully produced the state near the Equator.

Dr Murray Barrett likened a gas of atoms or molecules to a collection of billiard balls bouncing off one another. He noted that temperature provides a measure of how energetic or fast the molecules move. "However, in the peculiar world of quantum physics, it is impossible to obtain precise knowledge of both the position and speed of an atom at the same time," he said.

He explained that the "billiard balls" description of atoms break down when they reach a critical temperature less than a millionth of a degree above absolute zero, which is 0 Kelvin or -273.15 degrees Celsius. "The atoms become so cold that the uncertainty in their position is comparable to the separation between atoms. This means that several atoms can occupy the same region of space at the same time and this is something that billiard balls simply cannot do! At this critical temperature, all the atoms condense into the lowest energy state available. They lose their individuality completely and behave as a single quantum entity – the Bose-Einstein condensate.

The condensates are produced only in the world's most advanced laboratories. The achievement of the CQT is testament to Singapore being in the premier league of experimental atomic physics and provides local researchers with the most sophisticated tools in which to investigate the quantum properties of matter. While to-date there is yet applications for the condensates, there are potential applications in fields such as nanotechnology and holography on the horizon. KE



BOSE-EINSTEIN CONDENSATE RESEARCHERS: Dr Murray Barrett (left) with Kyle Arnold.

Building research excellence

SINGAPORE is able to attract the brightest and most committed research talent to carry out breakthrough science.

Speaking at the NUS Centre for Quantum Technologies (CQT) where two of the Centre's researchers achieved the "Bose-Einstein condensate" on 22 April 2009, Chairman of the National Research Foundation Dr Tony Tan said: "I commend the CQT for achieving such a difficult undertaking in a relatively short time. The remarkable achievement of Dr Murray Barrett and Kyle Arnold makes Singapore one of a small number of countries in the world to achieve this complex atomic physics experiment." To-date, only a select number of countries in the region including Japan and Australia are known to have been able to produce the condensate.

Dr Tan highlighted that despite the current economic downturn, Singapore will still continue to invest in research and development. It is only in doing so that the country will not stagnate and be overtaken by other developed nations.

The CQT is the first Research Centre of Excellence (RCE) established in Singapore to help local universities become research-intensive through world-class, investigator-led research that has a global impact. Besides helping first-rate academic investigators perform high quality, high-impact research, RCEs aim to enhance graduate education and train quality research manpower. It also aims to create new knowledge in areas of strategic relevance to Singapore. KE

⟨ℒ|Â|ℋ⟩ quantumlah

One of the coldest (coolest) place on earth is Singapore !

Bose-Einstein Condensation

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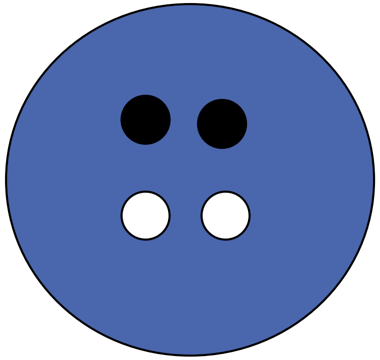
Superfluid Helium

A collection of helium atoms could therefore be expected to show Bose condensation. http://www.youtube.com/watch?v=2Z6UJbwxBZI



Helium, He^4 Nucleus

Superfluids flow without experiencing any viscosity (friction). 2.2^0 K

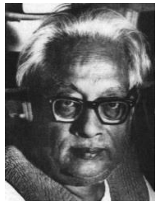
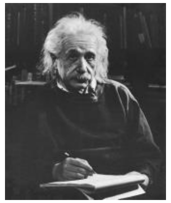
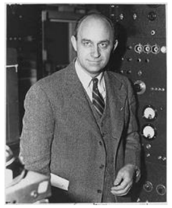
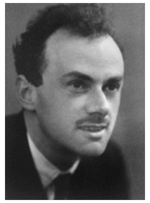
Total Spin is zero implies it is a Boson

- Proton
- Neutrons

Note: It seems that liquid helium is the only known liquid that cannot be frozen into a solid by merely lowering the temperature.

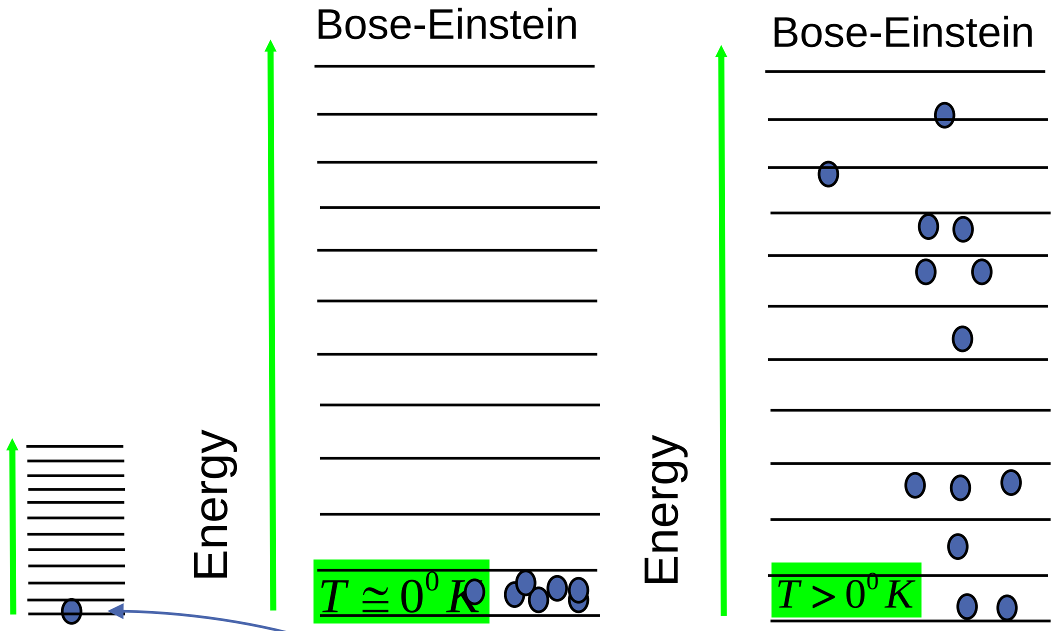
Comments about Statistics

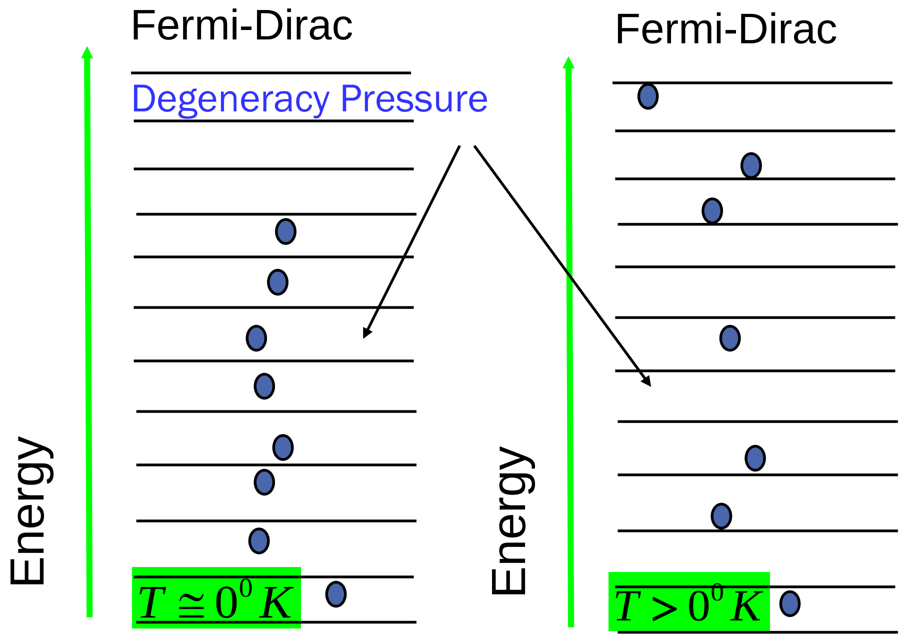
in quantum mechanics ?

Bose Statistics

Fermion Statistics





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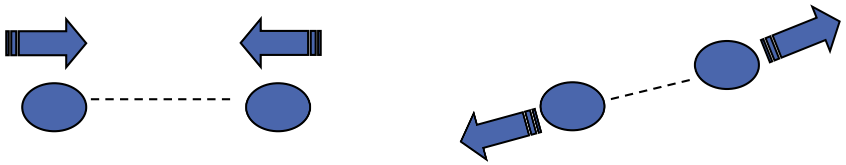
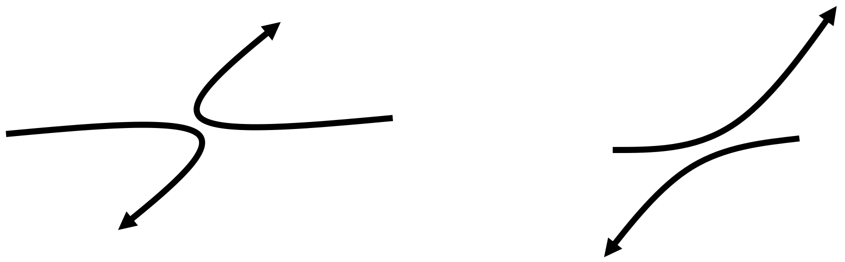
Indistinguishable micro-Particles

If you see one you see all !

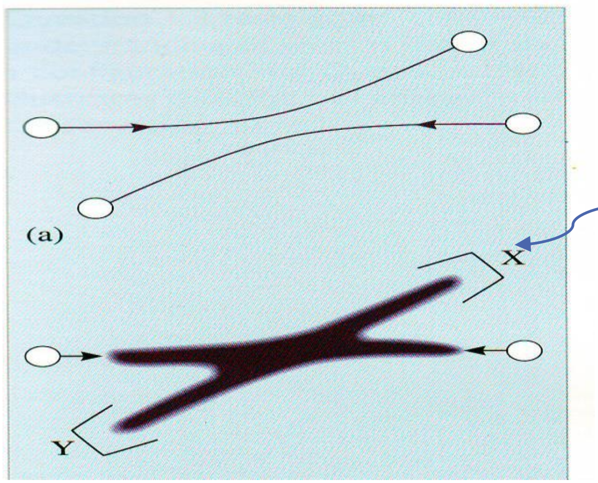
About Statistics (Probability) ?



Classical (macro-particles)



Quantum (micro-particles); we do not know ?



$$\Delta x \Delta p \sim \hbar$$

$$\boxed{QP - PQ = \frac{ih}{2\pi} = i\hbar}$$

Quantum (micro-particles)
A better picture description

In quantum mechanics one could have only fuzzy "trajectories" if one insist on trajectories

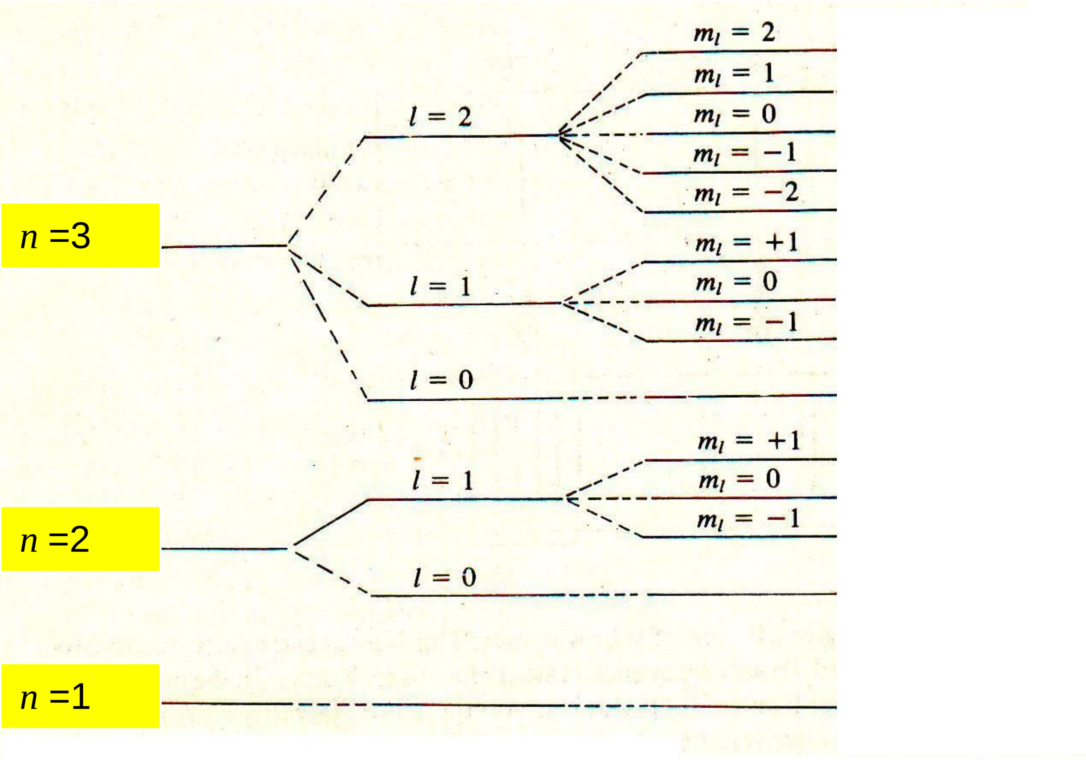
When doing QM calculations (of the collisions that are performed) we must allow for 2 possibilities … each of which has its own amplitude.

Closely related to the Heisenberg Uncertainty Principle … indistinguishable particles

What about other Particles ?

A Socratic question

Recall : “understand”? More labels $\left|\psi_{n,l,m_l,m_s}\right|^2$



Spin up(1/2) or Spin down(-1/2)

Electron State ψ_{n,l,m_l,m_s}

Example : $\psi_{3,2,-1,\frac{1}{2}}$

How to know how many energy levels? $(2x + 1)$ number of levels,

$x = l$ or s

After the discovery of the electron spin, we now need 3 + 1 = 4 labels (or quantum numbers) to describe the electron. This Spin did not come from solving the Schrodinger equation. It is a special kind of Spin (*intrinsic*).

Let us Spin !

What toys were Pauli and Bohr playing ?

Reportedly

Sir Winston Churchill (former UK Prime Minister)

also enjoyed this top !

This is a Mechanical Spin : NOT Quantum Spin

http://www.youtube.com/watch?v=AhmUEIH6cTY

http://www.youtube.com/watch?v=AyAgeUneFds

What about other Particles ?

Do they also have this so called Intrinsic Spin (new quantum number or we called label) ?

Examine the good old photons again !

Is there such a spin for photon ?

Photon is a Spin One particle

According to our rule, it should have 3 states. Why ?

Experts have established a connection between photon spin and polarization familiar to us in optics.

Expert opinion : only 2 spin substates are necessary to describe the photon particle. So the zero state is not needed.

Recall :

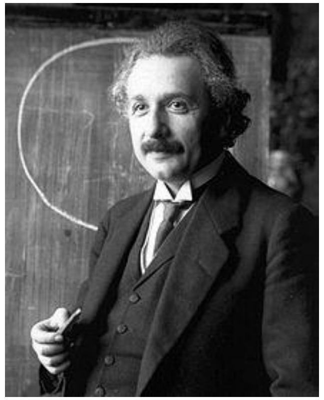
How to know how many energy levels? $(2x + 1)$ number of levels,

$x = l$ or s

One more Surprise !

"These days, *every Tom*, *Dick and Harry* thinks he *knows* what a photon is, but he is wrong",

Albert *Einstein*

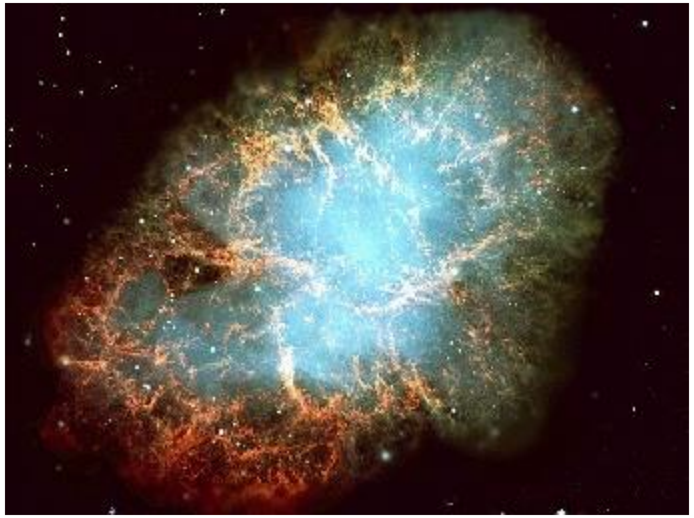


A demonstration !

With Polarizers

From Junior College and Polytechnic

Polarization & M1 Celestial Object



Polarized

Light

Recall the Crab Nebula … Super Nova Remnant

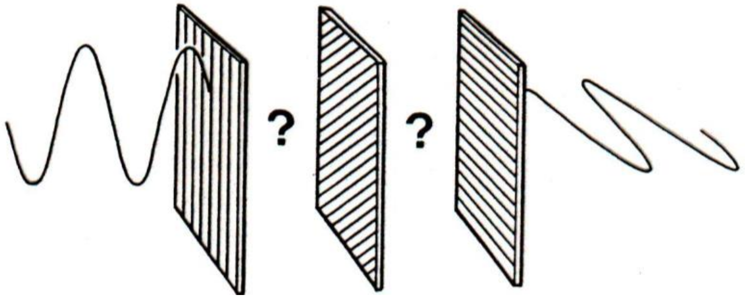
Something interesting here !

Think about this "bull crab" !

It is not bull ….

From Junior College

Light is a wave.



Consider 3 fences … Analogy

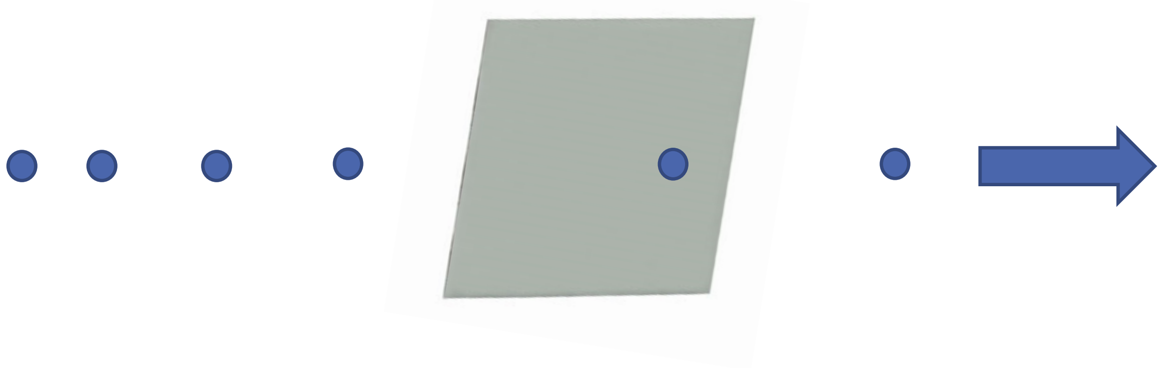
I am free !

How can I
get out ?

Questions

Consider when a light particle encounters the polarizer, what will happen to the particle ?

What will happen to the next light particle (photon) ?

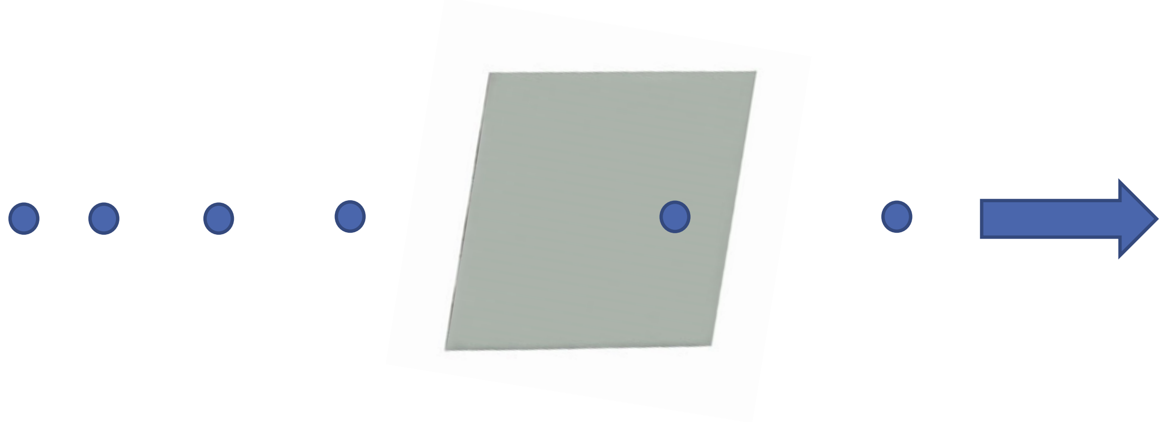


Amazing Answer : We do not know !

We cannot predict the behavior of individual photon, only statistical behavior of many photons.

More Question

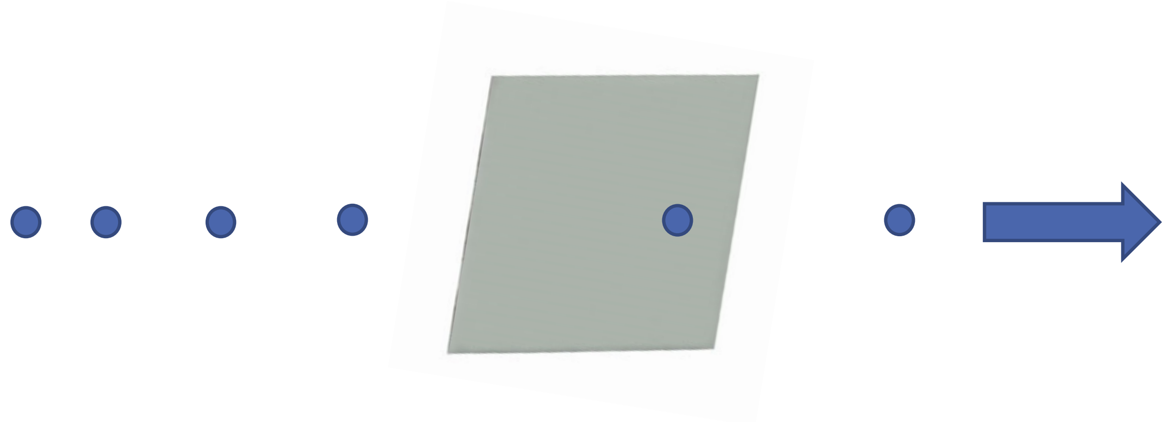
Could it be that there is some hidden mechanism within each light particle that could determine whether or not the light particle would pass through ?



Surprising Answer : Not at all no hidden variables ! This not like tossing a coin randomly. Because the information is really not even there in the first place.

More Questions Still

Why we cannot predict ? Does this mean nature play Dice ?



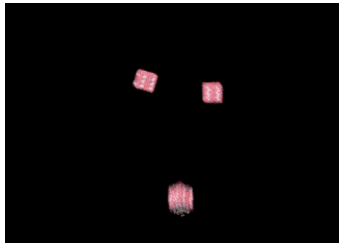
Shocking Answer : Yes. Nature is random. In fact this randomness is intrinsic (& completely). The Quantum randomness is very different from thrown a random coin. The information in QM is really not there. Why ?

Comments about probability

From Junior College and Polytechnic

What about tossing of a coin ?

Recall : Classical Probability

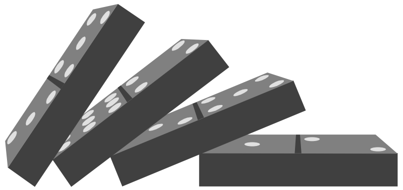


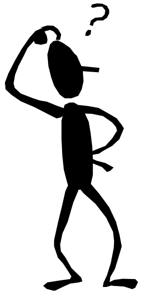
Why do we use probability theory in the prediction of an outcome in tossing a coin.

Since tossing a coin is a mechanical act, so why one cannot use ordinary (classical) mechanics and calculate precisely the outcome if it is a head and a tail instead of relying on a probabilistic forecast ?

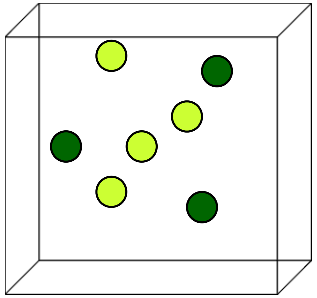
Ignorance: There is almost no hope … we are forced to make a probabilistic prediction of the outcome of the toss rather than deterministic one.

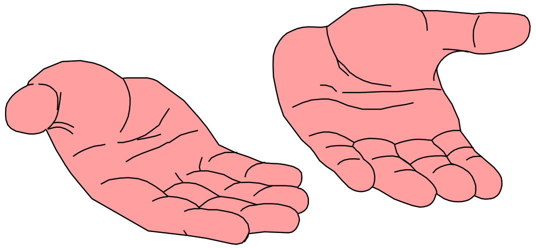
Recall : what is Probability?



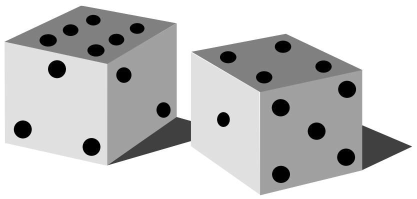


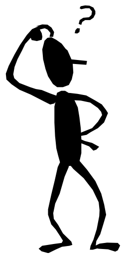
In probability, the student already knows the number of black and white balls in the container. He or she is interested to find the probability of drawing a given proportion of black and white balls.



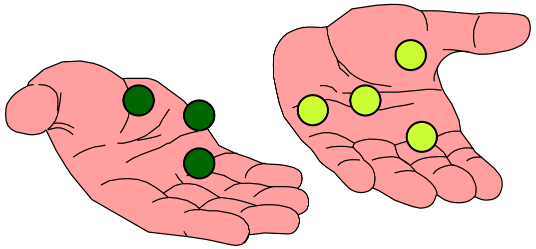


Recall : what are Statistics?





In Statistics, *we solve the opposite problem*. The student observes the proportion of black and white balls drawn from the container and attempts to deduce the number of black and white balls already in the covered container.



A Summary

All Particles in the Universe

Classical Case

Non identical
Distinguishable

Boltzmann (Maxwell) particles
(real Physical *Spin*)
"Tennis balls" if you like

Examples : really "no such" classical micro particles in nature. Atoms ? ? Molecules ? ?

Newtonian Mechanics
(Deterministic)

Quantum Case

Identical
Indistinguishable

Bosons (Bose-Einstein) (integer *Spins*)
Fermions (Fermi-Dirac) (half integer *Spins*)

Examples :
Photons, Electrons, Protons, Nuclei

Quantum Mechanics
(Random)

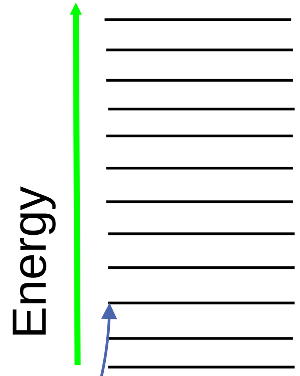
For the Curious !

$\langle \mathcal{L} | \hat{A} | \mathcal{H} \rangle$
quantumlah
http://www.quantumlah.org/

How do we do Quantum Mechanics ?

Recall Schrodinger Equation

Suppose we are studying the electron (hydrogen atom)
... we want to measure the energy of the system ?



$$\left[\left(-\frac{\hbar^2}{2m}\right)\frac{d^2}{dx^2}+V\right]\psi(x)=E\psi(x) \quad \Rightarrow \quad H\psi = E\psi$$

$$E_1, \quad E_2, \quad E_3, \quad E_4, \quad E_5,\ldots$$

In general, we have energy labels

$$\left|\psi\right\rangle = C_1\left|1\right\rangle + C_2\left|2\right\rangle + C_3\left|3\right\rangle \ldots + C_k\left|k\right\rangle + \ldots$$

$$|\psi\rangle = C_1|1\rangle + C_2|2\rangle + C_3|3\rangle ... + C_n|n\rangle + ...$$

1) Suppose we do an experiment and measure the energy of the system.

What answer would we get ? May be $E3$

2) We repeat the experiment under the same initial conditions i.e. $|\psi\rangle$

May be $E7$ ... (why not the same result as **1)** above ?

3) So patiently we accumulate a large number N of such readings and enumerate how many times we have $E3$, how many times we get $E7$, etc

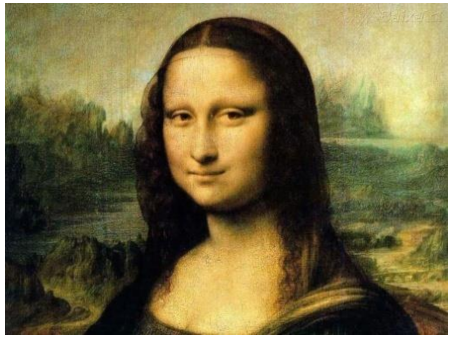
4) In fact, if we keep on trying, our log book would end up with a sequence of entries as in $E3$, $E7$, $E1$, … $E10$, $E3$ … i.e. completely random

$$Note: \quad N = n_3 + n_7 + n_2 + n_4 + ...$$

I cannot believe that God plays Dice. A. Einstein

$$|\psi\rangle = C_1|1\rangle + C_2|2\rangle + C_3|3\rangle \ldots + C_k|k\rangle + \ldots$$

Recall how do we find averages in sec. school



$$E_{average} = \frac{n_1E_1 + n_2E_2 + n_3E_3 + \ldots + n_7E_7 + \ldots}{n_1 + n_2 + n_3 + \ldots + n_7 + \ldots} = \sum_{k=1} \frac{n_kE_k}{N}$$

$$E_{average} = \sum_k |C_k|^2 E_k$$

$$E_{average} = \langle \psi^* | H | \psi \rangle$$

conjugate

Sometimes called expectation value (average) in Sec. Sch. or JC

Notice Something ?

It is procedural, like a Car Mechanics.

Deepen our “understanding”

Quantum Weirdness

ψ Revisited ? Why Probabilistic ψ ?

Classically, when we take a photograph (same condition) of a ordinary pendulum, we are sure to find it in some position or the other.

In the quantum pendulum, every time we try to observe (same condition) its position we would find it somewhere or other.

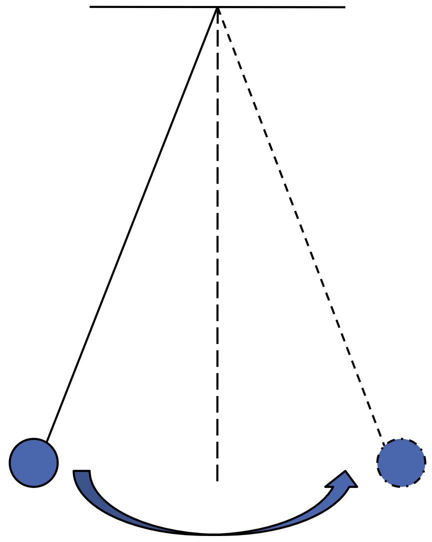
So what is the difference ?

Need to remember: every time when we talk about probability it implies that the experiment is repeated several times, only then is there any meaning in referring to probabilities.   Weekend : Think about Psi?

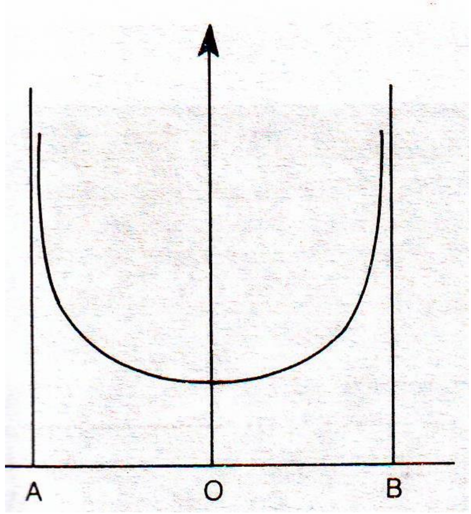
So ψ is sometimes called probability amplitude (or wave function).

$\psi * \psi$ *is called the probability of finding … to be verified by experiment*

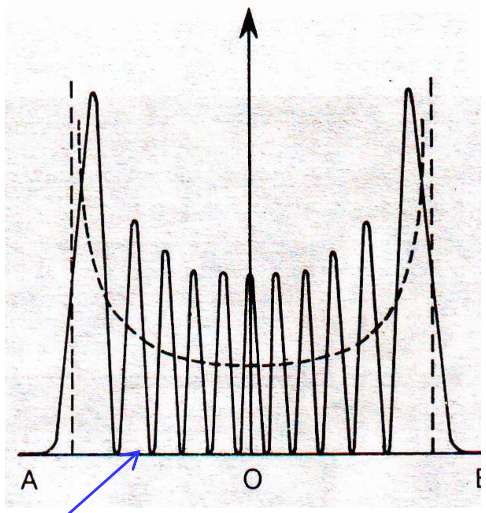
Quantum Pendulum (Oscillator)



Classical



Quantum



Another note :

However, in contrast to the classical pendulum, there are positions between A and B where the probability of finding the pendulum dips down to practically zero.

So “no” trajectory ?

We cannot discuss the trajectory of micro-particles

The professors' teacher

The career of a young theoretical physicist consists of treating the harmonic **oscillator** in ever-increasing levels of abstraction.

Sidney Coleman

Persuade you to think differently !

Vain Desire to see Something Familiar

The behaviour of things on a very small scale is simply different. An atom does not behave like a weight hanging on a spring and oscillating. Nor does it behave like a miniature representation of a solar system with little planets going around in orbits. Nor does it appear to be like a cloud or fog of some sort surrounding the nucleus. *It behaves like nothing you have ever seen before.*

There is one simplification at least. Electrons behave in this respect in exactly the same way as photons; they are both screwy, but in exactly the same way.

The difficulty really is psychological and exists in the perpetual torment that results from your saying to yourself "but how can it really be like that?" which is a reflection of an uncontrolled but ***vain desire*** to see it in terms of ***something familiar***. I will not describe it in terms of an analogy with something familiar. I will simply describe it ...

R. Feynman

Admit it Please !

I am going to tell you what nature behaves like. If you will simply admit that maybe she does behave like this, you will find her a delightful; and entrancing thing.

Do not keep saying to yourself, if you can possibly avoid it, ‘but how can it be like that ?’ because you will get ‘down the drain’, into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.’

R. Feynman

Is there a real world out there?

What we observe **is not nature itself**, but nature exposed to our method of questioning."

Heisenberg

Perhaps this is what we mean by "understanding" nature at its most fundamental level !

Are you upset ?

… tarassei tous anthropous ou ta pragmata alla ta peri ton pragmaton dogmata …

… what upsets people is not things themselves, but their theories about things …

Epictetus,
Greek & Stoic Philosopher,
55-135 AD