Posing Einstein's Question: Questioning Einstein's Pose

David Topper* and *Dwight E. Vincent

Einstein's face, as much as Einstein himself, has become an icon of science for our time. His picture has appeared on the covers of countless books and magazines and on numerous posters. It is found on postage stamps throughout the world. There are several annual "Einstein" calendars. In the latter part of his life he was hounded by photographers, like royalty or a media star—so much so that, when once asked his profession, he replied: "A photographer's model."

In the popular mind, an ideal image of Einstein pictures him standing in front of a blackboard covered with equations. And for the average layperson, most surely $E=mc^2$ would be on that blackboard—the equation being an icon itself. Of the several photographs of Einstein with a blackboard, Fig. 1 is probably reproduced the most, but this picture has the somewhat more obscure equation $R_{ik}= 0$, an equation that has great significance in Einstein's theory of gravitation: General Relativity. A closer look reveals a squiggle to the right of the zero, which at first appears to be a "2" but is, in fact, a question mark. Perhaps because of the equivocal nature of this squiggle, some reproductions of this photograph just crop it off![1] An even further look reveals an incompletely erased "ghost" equation, also $R_{ik}= 0$, but apparently without the question mark.

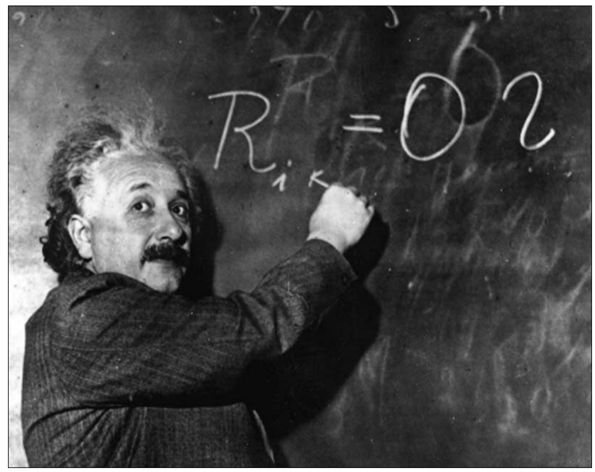

Fig. 1. Einstein at the blackboard of the Mount Wilson Observatory Library, January or early February 1931. In light of one of the themes of this paper, we must point out that our Fig. 1 is cut horizontally below Einstein's right arm. Compare with Fig. 5.
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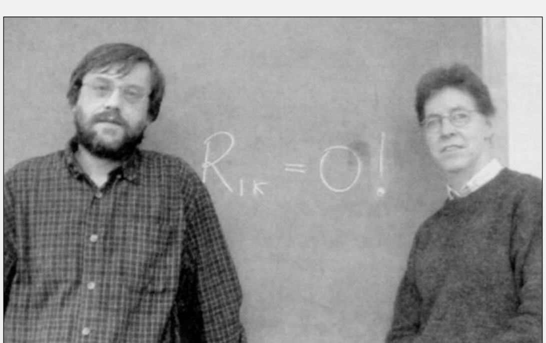
Cropping is not the only way this famous picture has been manipulated. On the cover of James Trefil's book (Fig. 2), the equation is completely eliminated and Einstein is supposedly writing the book's title on the blackboard.[2] A more recent book, one that discusses the possible theological implications of contemporary astronomy and cosmology,[3] places Einstein before a backdrop of stars and adds an index finger (!)—so that he seemingly is pointing to "God" (Fig. 3).

The Einstein question-mark picture has also been given a large degree of public exposure since Apple Computer decided to use it in its "Think Different" advertising campaign (Fig. 4). Initial inspection of this advertisement picture shows Einstein writing, not $R_{ik}= 0$, but another equation, $R = 0$. A comparison of the original photograph (Fig. 1) with Apple's billboard (Fig. 4) reveals a relative shift between Einstein's body and the blackboard: now his hand



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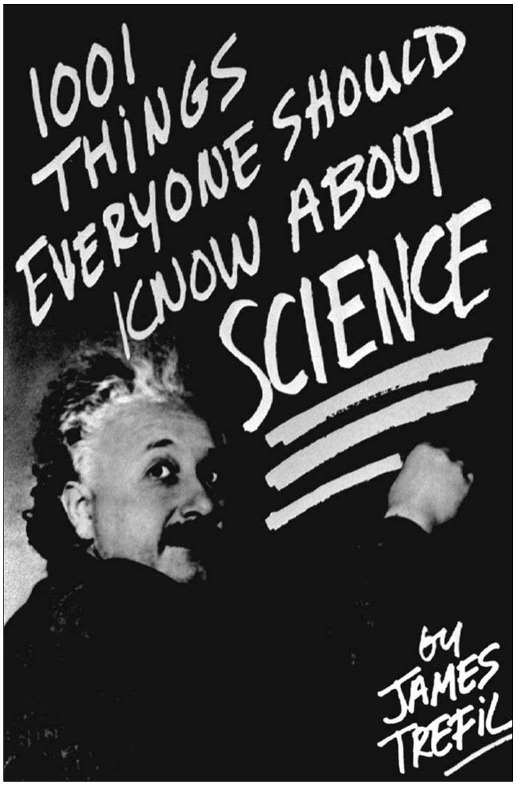

Fig. 2. Jacket cover of James Trefil's *1001 Things Everyone Should Know About Science* (Doubleday, New York 1992). *Reproduced with permission of Doubleday, a division of Bantam Doubleday Dell Publishing Group, Inc.*

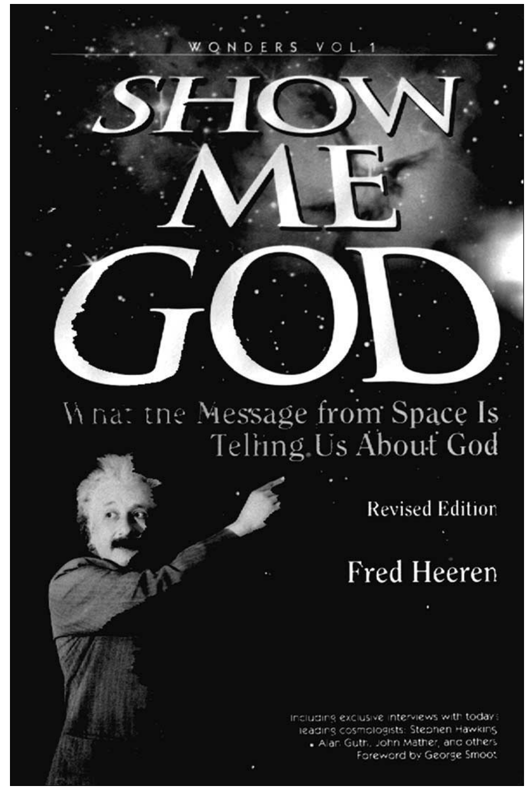

Fig. 3. Jacket cover of Fred Heeren's *Show Me God: What the Message from Space Is Telling Us About God*, rev. ed. (Wheeling, Ill., Day Star, 1997). *Reproduced with permission of Fred Heeren.*

Fig. 4. Billboard advertising Apple Computers. Photo, taken by *Sylvia Topper* in Toronto, August 1998 includes author D.T.

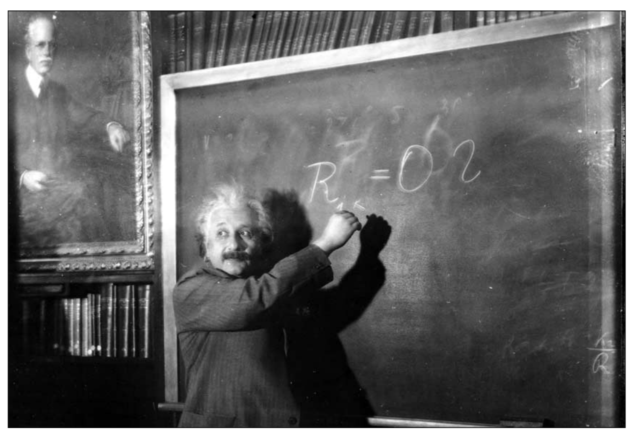

Fig. 5. Second picture of Einstein at the blackboard of the Mount Wilson Observatory Library. *This item is reproduced by permission of The Huntington Library, San Marino, California.*

covers the *ik* subscripts. The shift was probably made to create a horizontal image that fits into the long billboard. However, by taking such liberty with the equation, with little or no concern about content, Apple has transformed the physical meaning of Einstein's presumed discussion. The apparent indifference with regard to the impact of what such a change conveys is a subtle example of the lack of mathematical and scientific common sense among even well-educated members of the general public.

Figure 1 is not the only photograph taken of Einstein in front of this blackboard. Figure 5 shows the identical pose, but from an angle shifted slightly to the right and further back.[4] Indeed, this one was used by Heeren; see Fig. 3. There is yet another photograph, but that shot is so close that even the zero is cut off. So the word "pose" is purposely used: if Einstein were actually writing the equation, his chalk would be near the question mark, not the subscripts. Obviously the photographer or photographers (the number of poses and the diverse institutional sources of the photographs seem to imply that there was more than one photographer) asked Einstein to pose for the shots, presumably sometime after his lecture. All were taken about the same time, as may be inferred from the identical wrinkles in his sleeve. In light of the "ghost" equation, we may also reasonably assume that Einstein was asked to rewrite the equation that he had discussed in his lecture, and hence he added the question mark because in the talk he had queried something about the validity of $R_{ik}= 0$.

Einstein at CalTech

Before delving into the meaning of this equation, let us set these pictures in their historical context. They were taken at Mount Wilson Observatory near Pasadena, California, sometime in January or early February, 1931. At the time, Einstein was a Visiting Professor at the California Institute of Technology (CalTech), having been invited by Robert A. Millikan, Director of Physics. It was to be an annual arrangement (this being the first) whereby Einstein would spend the winter term at CalTech, returning to Berlin in the spring. This was not Einstein's first visit to the United States; he had been on a fundraising tour in 1921 for the planned Hebrew University in Jerusalem, a tour that included several scientific lectures, too. But it was his first trip to California, and many people, including Einstein himself, have written about specific aspects of his stay there.[5-9]

This second visit to the United States began rather controversially, for upon initially arriving in New York he delivered a speech on the rise of militarism in which he proposed that governments would be powerless to wage war if 2% of those called up refused military service. This became known as "the 2% speech" and it did not sit well among some very patriotic Americans. In particular, Millikan, a political conservative, was rather embarrassed by the event.

This second visit to the United States began rather controversially, for upon initially arriving in New York, Einstein sailed to California, stopping in Cuba, proceeding through the Panama Canal, and landing in San Diego on Dec. 30, 1930, to much fanfare. The entourage included his wife, Elsa, his secretary, Helen Dukas, and his assistant (often called "the calculator"), Walther Mayer. The following day they were driven to Pasadena. It was during this visit that Einstein met, among other celebrities, Charlie Chaplin and Upton Sinclair. Since Sinclair was a radical socialist, the meeting caused Millikan further grief.

There is an ironic turn to Millikan's role as the major host of Einstein at CalTech. It is true that Millikan's experimental work on the photoelectric effect confirmed Einstein's prediction in 1905 of the particle nature of light; for this work, plus his experiments on the electron's charge, Millikan received the Nobel Prize in 1923. Yet not only did he initially perform the photoelectric experiment to disprove the photon, but even after his own experimental confirmation he did not believe in the theory. He wrote this extraordinary sentence on the photon theory: "Experiment has outrun theory, or, better, guided by erroneous theory, it has discovered relationships which seem to be of the greatest interest and importance, but the reasons for them are as yet not at all understood."[10] Moreover, he was skeptical about relativity too. Even after the famous 1919 British solar eclipse experiments, which seemed to verify General Relativity's prediction that the path of light should bend when moving close to the Sun, Millikan went on record as postulating a different explanation of the deflection of the Sun's rays; perhaps they were deflected by the refraction of gases in the solar atmosphere, an explanation that was "plausible," and which he said he hoped to be true.[11] But Millikan was not the only skeptic Einstein would meet during this sojourn.

Einstein's first stay in California lasted about two months. In a letter to Max Born in February, he referred to the visit as "loafing in...paradise."[12] By Feb. 28 Einstein was on his way east, stopping at the Grand Canyon before proceeding to Chicago and New York, and then setting sail back to Germany. Einstein made another winter trip to CalTech the following year (1931-32). Then in October 1932 he accepted an appointment to the newly founded Institute for Advanced Studies in Princeton, New Jersey. When he left Germany in December 1932, on his fourth trip to America, he intended to divide his time between Berlin and Princeton, but after the Nazis seized power he never again set foot on German soil.

While at CalTech (in early 1931) Einstein made numerous visits to the Mount Wilson Observatory. On one of those visits Einstein gave the "$R_{ik}= 0$?" lecture in the library. We see some of those who attended the lecture in Fig. 6. (Note the equation on the blackboard.) All, except Einstein, were experimentalists and, importantly, all were involved with experiments that had some bearing on Relativity Theory.

Adams (far right), who succeeded Hale as director of Mount Wilson, had worked on the gravitational redshift of the burnt-out star Sirius B, the companion star of Sirius (the brightest naked-eye star). Einstein had used General Relativity to make the prediction that large gravitational fields should extract energy from escaping light to the extent that the wavelength of the light would be lengthened (i.e., redshifted) by well-defined values. In 1915 Adams began working on Sirius B, which he found, incredibly, to be about 50,000 times as dense as water, hence a good candidate for testing Einstein's prediction. By 1925 his measurements of the redshift were interpreted as confirming Einstein's prediction, although recent measurements have revised, if not discredited, Adams's results.[13]

Campbell, president emeritus of the University of California and former director of Lick Observatory, meas-

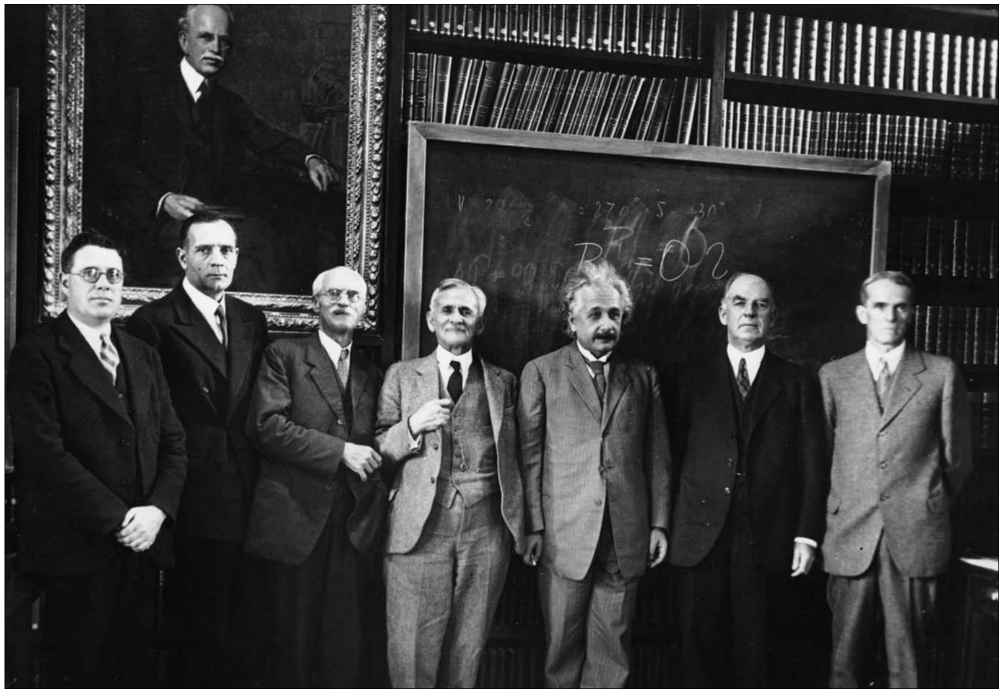

Fig. 6. Group picture at the Mount Wilson Observatory Library, January or early February 1931. From the left: Milton L. Humason, Edwin P. Hubble, Charles E. St. John, Albert A. Michelson, Einstein, William W. Campbell, and Walter S. Adams. The portrait is of George Ellery Hale, founder of the Mount Wilson Observatory. *Reproduced with permission of the Observatories of the Carnegie Institution of Washington.*

ured the gravitational bending of light in the 1922 eclipse and found full agreement with Einstein's prediction. Campbell was originally skeptical about General Relativity; in fact his earlier experiments on the 1918 eclipse seemed to indicate no deflection of light.[14]

St. John, one of the early collaborators with Hale at Mount Wilson, performed a series of experiments to detect the gravitational redshift in the solar light that bombards Earth. When he began these experiments in 1917 he was, like Campbell and unlike Adams, a skeptic about General Relativity, and indeed his first results showed no solar redshift. But by 1923 he had confirmed Einstein's prediction.[15] There are several photographs of Einstein and St. John at Mount Wilson, with St. John showing him—proudly, we suspect—the solar telescope.

Michelson, of course, had performed what became known as the Michelson-Morley experiment (with chemist Edward W. Morley) at Case Institute of Technology in Cleveland, Ohio, in 1887. This experiment precisely verified the independence of the speed of light on motion of source or receiver through an "ether," a conjectured background medium that light would propagate through. By the mid-1920s the experiment was credited as providing an empirical foundation to Einstein's Special Theory of Relativity, which concerns itself with the ramifications arising from the universal constancy of the speed of light. However, the historical evidence indicates that the Michelson-Morley experiment probably had little influence on the genesis of Special Relativity, namely Einstein's 1905 paper, "On the Electrodynamics of Moving Bodies."[16,17] In the one formal gathering during Einstein's visit to CalTech (a dinner on Jan. 15 at the Athenaeum, the faculty club), Einstein, in his speech to the more than 200 invited guests, acknowledged the role of the Michelson-Morley experiment in supporting Relativity Theory. Nonetheless, he subtly implied in the speech that it had only a negligible influence on him.[18,19] Furthermore, it seems that Michelson was not enamored with the Theory of Relativity, for he continued to believe to the end of his life in the existence of the ether. Einstein had demonstrated that Relativity made the concept of ether superfluous. One poignant feature of the picture: Michelson is looking rather frail, having had a stroke about two years earlier. He died not long after, on May 9th.

Humason was Hubble's key assistant, and of course their work on the redshift of the galaxies ultimately led to the expanding theory of the universe in the 1930s. Indeed, Einstein arrived at Mount Wilson at an auspicious time, for in 1929 Hubble had published the first of his now-classic papers revealing that the receding velocity of galaxies and the increasing distances to these same galaxies were directly related. This relation later became known as Hubble's Law.[20] This result implied but did not confirm a non-static, expanding model of the universe. Hubble, a staunch empiricist, was not entirely convinced that the redshifts were necessarily Doppler shifts due to recessional motion.[21]

In his 1917 paper[22] on the "Cosmological Considerations of the General Theory of Relativity," Einstein had assumed a static universe and consequently added the "cosmological constant" to his equations in order to preserve this state. Einstein essentially dismissed the theoretical work of Aleksandr A. Friedmann and Georges Lemaître in the 1920s on non-static models of the universe. It seems, however, that it was during this first stay at CalTech when Einstein abandoned the static hypothesis. The front page of the *New York Times* in early January quoted Einstein as saying: "New observations by Hubble and Humason...concerning the redshift of light in distant nebulae make the presumptions near [i.e., make it appear likely] that the general structure of the universe is not static."[23] After meeting Hubble and seeing first-hand the experimental evidence, Einstein was further convinced. In early February the front page of the *Times* stated that he announced at a lecture on Mount Wilson that he dropped the idea of a closed universe.[24] A week later they reported that during another lecture there he said, "The redshift of distant nebulae has smashed my old construction like a hammer blow," and they noted further that he said this while "swinging down his hand to illustrate."[25] In time he would call the cosmological constant the "biggest blunder" of his life.

Einstein's Equation

But what about the equation $R_{ik}= 0$ in the picture? Of the many and varied reproductions of the famous photographs, the meaning of the equation is almost never broached. In Banesh Hoffmann's splendid book, *Albert Einstein: Creator and Rebel*, the picture appears with the question mark cropped, and with the caption, "Einstein lecturing in Pasadena, ca. 1930 [1931]. On the blackboard is $R_{ik}= 0$, a tensor form of his 10 field equations for pure gravity."[26] By seemingly ignoring the question mark, we believe that Hoffmann may have missed the significance of what Einstein was talking about, since it is not obvious that Einstein was strictly presenting just 10 field equations for pure gravity at the time. Allan Sandage, famed astronomer at Mount Wilson, has tried to explain the significance of the equation and the relevance of the question mark. He writes:

> *During an extended visit to Southern California in 1931 Einstein lectured on General Relativity at the offices of the Mount Wilson Observatory of the Carnegie Institute of Washington. The famous photograph taken during this lecture shows his query as to whether the contracted Riemann curvature tensor for the universe as whole, R_{ik}, can equal zero.*[27]

Sandage then notes that Hubble, upon hearing this lecture, may have been spurred to commence his massive observational program of galaxy counts. Since Sandage is interested in experimental astronomy, this possible link to Hubble's work is his focus of attention, not Einstein's theory itself. Hence he goes on to discuss this experimental work, having mentioned Einstein's equation only in passing. Sandage made a rather ironic, even slightly humorous, point about the equation in a letter to us. He wrote: "I expect all this was over the astronomer's heads [since] they were all observers, not theoreticians."[28] At least one author has taken a purely observational interpretation to Einstein's blackboard question mark. Robert Jastrow states that Einstein is asking a question about whether the universe is spatially flat or not, a question having to do with whether the universe might continue to expand forever or eventually collapse back on itself.[29] Although Einstein may well have been talking to the gathered astronomers about the observational criteria for concluding whether the universe is expanding forever or eventually contracting, there is nothing about the assumed four-dimensional $R_{ik}= 0$ equation that would *formally* lead to this conclusion. We will come back to this point later.

The puzzle of what Einstein was actually talking about that day at Mount Wilson would easily be solved if there were a transcript of Einstein's talk. Unfortunately, in the myriad reports of Einstein's visit to California and to the Mount Wilson Observatory in early 1931, we have been unable to find any record of the talk that Einstein gave just before the photograph was taken. Einstein's personal diary[9] is a particular disappointment in this respect since it barely mentions the Mount Wilson visits and is uncharacteristically concerned with nonscientific events. Since no one (at least to our satisfaction) has discussed Einstein's true meaning in writing this equation, we will delve into the theoretical background of the equation. By doing this we hope to uncover at least a probable explanation.

To get an understanding of what Einstein is trying to convey with

$$R_{ik}= 0 \qquad (1)$$

we must understand how the gravitational field is defined in General Relativity. In Newtonian theory the gravitational field can be represented by one function, the gravitational potential

$$V(r) = -\frac{GM}{r} \qquad (2)$$

where M is the mass of the source and G is the gravitational constant. The variable r is the distance between the location of the source of the gravitational field and the point where the field is being evaluated. Usually we do not include any time dependence in the potential function since the nearby gravitational fields we deal with in Newtonian physics are static. In terms of three-dimensional Cartesian coordinates, x, y, and z, we can write the separation distance r as

$$r = \sqrt{x^2 + y^2 + z^2} \qquad (3)$$

To make a connection with what follows, we note that this relation can be squared on both sides and rewritten for infinitesimal changes in the coordinates ($\mathrm{d}x$, $\mathrm{d}y$, $\mathrm{d}z$) and distance ($\mathrm{d}r$) as

$$\mathrm{d}r^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \qquad (4)$$

In General Relativity we must always bring in the time t as an extra dimension on a similar footing to the other dimensions; hence, the number of coordinates increases to four: x, y, z, and t. We then say we are dealing with spacetime. Instead of the one gravitational potential in Newtonian theory, General Relativity requires an array of such potential functions given by

$$\begin{pmatrix} V_{tt} & V_{tx} & V_{ty} & V_{tz} \\ V_{xt} & V_{xx} & V_{xy} & V_{xz} \\ V_{yt} & V_{yx} & V_{yy} & V_{yz} \\ V_{zt} & V_{zx} & V_{zy} & V_{zz} \end{pmatrix} \qquad (5)$$

Each of the 16 elements in this array is a function of x, y, z, and t. The rationale for the subscript labels will become obvious later. The conventional symbol to use here is the small letter g instead of V. We are implementing this switch to V so as to make the connection with Newtonian theory a little more obvious.

Why then is Newtonian gravitational theory so good, you might ask, if Einstein's theory requires so much more information to be known about the gravitational field? The answer is that except in very exotic locations (near neutron stars, black holes, and rapidly collapsing or exploding astronomical objects) Einstein's gravity theory closely approximates Newton's gravity theory. Earth, and even our Sun, are not strong sources of gravity by universe standards and their gravity fields certainly do not change quickly. In describing the motion of any body not traveling at speeds close to the speed of light, in the vicinity of these static low intensity gravitational fields, the array of gravitational potentials (5) effectively reduces to just the V_{tt} component:

$$V_{tt} = 1 + \frac{2V(r)}{c^2} \qquad (6)$$

Here, c stands for the velocity of light in free space. Thus, from a General Relativistic perspective, in Newtonian theory we can make much simpler computations, needing only one of the gravitational components for our gravity calculations, because we are living in a boring (but hospitable!) gravitational environment.

The array (5), the embodiment of gravity information in General Relativity, is called the metric tensor. The word metric is used to convey the idea that these potentials are linked to how distance is measured in the four-dimensional spacetime. Let s be the four-dimensional distance parameter, similar to the three-dimensional r separation parameter we defined earlier. Then the distance relationship that Einstein put forward for a four-dimensional spacetime, where gravity has a nontrivial influence, was the following:

$$\begin{aligned} \mathrm{d}s^2 = {} & V_{tt}\mathrm{d}(ct) \bullet \mathrm{d}(ct) + V_{tx}\mathrm{d}(ct) \bullet \mathrm{d}x + V_{ty}\mathrm{d}(ct) \bullet dy + \\ & V_{tz}\mathrm{d}(ct) \bullet \mathrm{d}z + V_{xt}\,\mathrm{d}x \bullet \mathrm{d}(ct) + V_{xx}\mathrm{d}x \bullet \mathrm{d}x + \\ & V_{xy}\mathrm{d}x \bullet \mathrm{d}y + V_{xz}\mathrm{d}x \bullet \mathrm{d}z + V_{yt}\mathrm{d}y \bullet \mathrm{d}(ct) + \\ & V_{yx}\mathrm{d}y \bullet \mathrm{d}x + V_{yy}\mathrm{d}y \bullet \mathrm{d}y + V_{yz}\mathrm{d}y \bullet \mathrm{d}z + \\ & V_{zt}\mathrm{d}z \bullet \mathrm{d}(ct) + V_{zx}\mathrm{d}z \bullet \mathrm{d}x + V_{zy}\mathrm{d}z \bullet \mathrm{d}y + \\ & V_{zz}\mathrm{d}z \bullet \mathrm{d}z \end{aligned} \qquad (7)$$

This somewhat complicated looking expression is a generalization of the three-dimensional distance relation (4) to four dimensions. All combinations of the coordinate differentials dx, dy, dz, and d(ct) are being allowed. For all the differentials to have length units, the time variable t is multiplied by the speed of light c and enters as ct. Note that each differential product is matched with the appropriately labeled array functions V_{tt}, V_{xx}, V_{yz}, etc. The presence of the array functions as coefficients of the differential products allows gravity to influence how lengths get measured in the various dimensional directions. Any link with the way that distance is measured in a space is of necessity a link with the geometry of that space. A comparison of the distance function (4) with the distance function (7) shows that General Relativity is set up to allow for much more complicated geometries than the Newtonian three-dimensional background space requires. Since Einstein postulated that matter distributions curved the space that surrounded them, and since matter distributions in the real world come in many varieties, his theory had to be capable of handling many types of geometrical arrangements.

In pure Riemannian geometry, the associated cross terms in (7) are always equal, and thus

$$V_{tx} = V_{xt}\,,\ V_{ty} = V_{yt}\,,\ V_{tz} = V_{zt}\,,\ V_{xy} = V_{yx}\,,\ V_{xz} = V_{zx}$$
$$\text{and } V_{yz} = V_{zy} \tag{8}$$

With these conditions in effect, we see that the gravitational array (5) contains only 10 independent functions.

The word tensor, in reference to the metric tensor array and to other arrays of functions that pop up all over the place in General Relativity discussions, signifies that the array has a special mathematical property. Whenever we need to change coordinates so as to more suitably model some region of spacetime curvature, the array, being a tensor quantity, will transform along with the coordinates in such a way that the physics remains the same. Equations involving tensors are said to be "covariant," which means that the form of the equations remain invariant; that is, the equations have the same form after the transformation as before. In General Relativity, distributions of matter can deform spacetime in odd ways, so the utilization of customized coordinates is crucial for making simple models. Realizing this, Einstein formulated General Relativity in terms of tensors so as to write the fundamental gravity equations in a coordinate independent manner.

We are now in a position to see what Einstein's blackboard equation really means. The array R_{ik} that Einstein has written in the picture's equation is called the Ricci curvature tensor. It represents an array of spacetime curvature functions, whose elements are labeled in one-to-one correspondence with the way that the elements of the metric tensor are labeled. The "*ik*" is a shorthand way of cycling through the various dimension labels without explicitly writing them all down. For instance, we can have ($i = x$, y, z, or t) and ($k = x$, y, z, or t). Any combination of the members of the i list or the k list is acceptable. According to taste, people will use numbers like 0, 1, 2, 3 or 1, 2, 3, 4 instead of the t, x, y, z labels that we have used. Each element of R_{ik} is in fact a sum of terms involving derivatives of the 10 gravitational potentials appearing in (5). When Eq. (1) is in effect in four dimensions, we have 10 nonlinear second-order differential equations that are known as the vacuum Einstein field equations. These equations are used for figuring out how the gravitational field will behave *outside* of matter/energy distributions (hence, the usage of the adjective *vacuum*). For example, in the space surrounding the Sun or the space surrounding a black hole we assume that there is a classical vacuum. The actual matter/energy distribution for these gravitational fields is treated as if it were a point source of gravity. When the matter/energy distribution is important for many points in space, as it is for cosmological solutions, the more general nonvacuum equations are necessary. In these nonvacuum equations something nonzero is written on the right-hand side of Eq. (1) representing matter/energy. For an example illustrating these two variations of the Einstein equations, consider the simplified form of Einstein's Ricci tensor equation that arises when we describe the physical situation associated with Eq. (6). To the level of approximation consistent with (6) we only need to consider one component of the array R_{ik} and that component is R_{tt}. For comparison we write the two forms of Einstein's equations on the same line, one for the vacuum case and one for the nonvacuum case, respectively:

$$R_{tt} = \frac{1}{r}\frac{\partial V_{tt}}{\partial r} + \frac{1}{2}\frac{\partial^2 V_{tt}}{\partial r^2} = 0 \qquad \text{versus}$$

$$R_{tt} = \frac{1}{r}\frac{\partial V_{tt}}{\partial r} + \frac{1}{2}\frac{\partial^2 V_{tt}}{\partial r^2} = \frac{4\pi G\rho}{c^2} \tag{9}$$

where ρ is the mass density. When all coordinates and all of the elements of the metric tensor are involved, these expressions are significantly more complicated but the overall interpretation is the same.

The Question Mark

If you know a little about General Relativity and you see Einstein with Eq. (1), it is natural to think that Einstein is just writing down the vacuum gravitational field equations. However, the presence of the question mark gives a good clue that what was on Einstein's mind may not have been the standard solutions of the General Relativity equations.[30]

A simple interpretation of the question mark might be that Einstein is questioning the validity of his gravitation theory. Although most of those shown in the group picture had been attempting to test Einstein's General Relativity predictions, some of them were very skeptical. Perhaps Einstein sensed that he was in front of a hard-nosed group of empiricists and tailored his talk to portray General Relativity as a tentative theory. He may have mentioned the successful tests that had already been carried out and he may have given the astronomers present some ideas about implications that General Relativity might have for the curvature of the universe. But it is difficult to fully accept that this is all Einstein meant with his question-mark equation. Einstein had been working hard on extending General Relativity. It seems inconsistent that he would question the underpinnings of the entire theory in 1931 while attempting to generalize the same theory with the same underpinnings.

The purely observational interpretation put forward by Jastrow and others presumably arises from the following intuitive cosmological argument. If there were little overall matter/energy in the universe, then the universe could not have sufficient attractive gravity to collapse back on itself. The argument then continues with respect to Einstein's blackboard equation, that Einstein has the right-hand side of his field equation (which in a cosmological context represents the matter/energy of the universe) set equal to zero. Furthermore, Einstein has placed a question mark after the zero. So Einstein must be questioning whether the quantity of matter/energy of the universe is sufficiently small (represented in some approximate way as zero) to allow the universe to expand forever. The problem with arguing in this manner is that the field equations represented so succinctly by the one tensor equation $R_{ik} = 0$ are actually a system of 10 coupled nonlinear differential equations with many distinct solutions. For instance, there is a class of solutions of this equation in General Relativity called Kasner solutions that have space expanding in two directions while contracting in a third. There are any number of such seemingly wild solutions of the vacuum Einstein equation $R_{ik} = 0$, many of which were known to Einstein at the time of the photograph. We cannot make a deduction that a specific nonvacuum solution is being discussed by observing Einstein with such a general vacuum equation having so many diverse inherent solutions of its own. When a physicist such as Einstein writes down an equation of this form he is referring to the general formalism not to particular solutions. As documented by the *New York Times* record of Einstein's trip to California in 1931[31] and by the written reports on this trip by Walter Adams,[8,19] Einstein was not hesitant to introduce observationally based audiences to formal theoretical ideas. Discussions of his search for sourceless singularity-free solutions and his Unified Field Theory quest were mixed in with his more empirically based insights. In 1931 Einstein's cachet was such that his mathematically unprepared but psychologically set audiences eagerly appreciated any degree of obscure musings concerning his theoretical endeavors.

At the time of his Mount Wilson visit, Einstein had been doing research on extending General Relativity to include any other known force within a pure geometrical context.[32] This basically meant that the metric tensor (5) had to be changed in some way so as to encompass electromagnetism, since at the time, electromagnetism was the only other force recognized by physicists as being clearly fundamental. Einstein, from the early 1920s onward, had become more and more preoccupied with the idea of constructing a Unified Field Theory that would include electromagnetism. As already mentioned, nonvacuum field equations of General Relativity have the following form:

$$R_{ik} = [\textit{stuff involving a nongravitational energy distribution}] \tag{10}$$

We are deliberately suppressing the explicit form of the usual terms on the right-hand side of this equation involving what is called the energy-momentum tensor since its actual form is not important for this discussion. Solution of Eq. (10) usually involves making some assumption about the energy distribution (or equivalently, the mass distribution, since energy and mass are related by that other famous Einstein equation, $E=mc^2$). This nongravitational energy distribution is responsible for the gravitational curvature on the left-hand side of the equation. Thus, for example, if we were using some electromagnetic field energy as the energy source, the electromagnetic energy-momentum tensor would be constructed, it would be substituted into the right-hand side of the equations, and these 10 equations would be solved to see what the generated gravitational field would be. Physicists have carried out this process thousands of times since Einstein's day. All kinds of nongravitational energy sources have been proposed, from dense nuclear incompressible fluids for neutron stars to the energy tensors of exotic quantum fields for early-universe cosmology. Many solutions have direct astrophysical application while many others are found solely for the educational experience.

There is a similarity here with what we do with the solution of the Newton's second-law equation. We seek solutions of

$$\text{(Acceleration)} = \text{(External Force)/(Mass)} \tag{11}$$

whenever we require a practical model of some real-world physical phenomenon. But there are also times when a solution is sought primarily for the educational experience of seeing what type of motion ensues in the presence of some exotic force. In the case of Newton's second law, however, few people actively seek out ways of extending the concept of acceleration such that, for all external forces, Eq. (11) becomes

$$\text{(Acceleration)} = 0 \tag{12}$$

Yet this was exactly the sort of thing that Einstein was trying to do with Eq. (10). He wanted to extend the concept of the geometrical interpretation of General Relativity to incorporate all forces external to gravity. Hence, what Einstein wanted to convey with his blackboard equation

$$R_{ik} = 0? \tag{13}$$

is whether it is possible to attain a unified theory of nature (including the fundamental forces other than gravity) that has all of the beauty of the geometrical framework of standard gravity.

After writing this equation on the board for the esteemed scientists who were in his audience, and then probably rewriting it for the gathered photographers that day in January 1931 (Fig. 7), Einstein went on to spend the rest of his life fiddling with this equation. He hoped

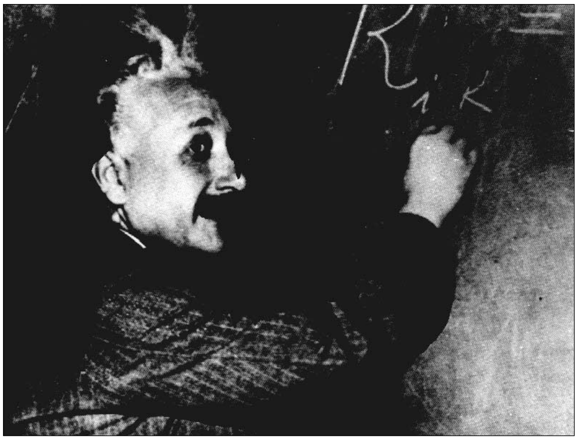

Fig. 7. Third picture of Einstein posed at the blackboard of the Mount Wilson Observatory Library; here he is more in profile and both zero and question mark are cut off. ***Courtesy of Dover Publications.***

that this formula would be the archetype for a theory that extended the General Relativity structure. In an essay he wrote in 1936, "Physics and Reality," he metaphorically expressed the source of his motivation for reconstructing his General Relativity Theory. He stated it as an aesthetic dissatisfaction—a dissatisfaction comparable to that of a person who had constructed a building, one part of which was the purest marble while the other part was low-grade wood.[33] Within the context of this metaphor, Eq. (10) can be rewritten as

$$MARBLE = WOOD \qquad (14)$$

The discontent with the "wooden" part of his construction led Einstein to a quest that is expressed in the question-mark equation. Can the matter/energy distribution be reformulated geometrically such that only a pure "marble" structure remains? Thus (13) becomes

$$\text{MARBLE} \equiv R_{ik} = 0\ ? \qquad (15)$$

where ≡ means exactly equivalent to. To get an idea of Einstein's methods in eliminating the "wood" while increasing the "marble" we will give a few examples illustrating his Unified Field Theory strategies. Among the things he tried were the following: He changed the metric tensor array (5) by making the potentials that are located off the main diagonal of the array, antisymmetric. The relations (8) then became

$$V_{tx} = -V_{xt}\,,\ V_{ty} = -V_{yt}\,,\ V_{tz} = -V_{zt}\,,\ V_{xy} = -V_{yx}\,,\ V_{xz} = -V_{zx}$$
$$\text{and } V_{yz} = -V_{zy} \qquad (16)$$

Later he also tried using complex numbers to extend this equation as follows:

$$\mathrm{Re}(V_{tx}) = \mathrm{Re}(V_{xt}),\ \mathrm{Re}(V_{ty}) = \mathrm{Re}(V_{yt}),\ \mathrm{Re}(V_{tz}) = \mathrm{Re}(V_{zt}),$$
$$\mathrm{Re}(V_{xy}) = \mathrm{Re}(V_{yx}),\ \mathrm{Re}(V_{xz}) = \mathrm{Re}(V_{zx})$$
$$\text{and } \mathrm{Re}(V_{yz}) = \mathrm{Re}(V_{zy})$$

and

$$\mathrm{Im}(V_{tx}) = -\mathrm{Im}(V_{xt}),\ \mathrm{Im}(V_{ty}) = -\mathrm{Im}(V_{yt}),\ \mathrm{Im}(V_{tz}) = -\mathrm{Im}(V_{zt}),$$
$$\mathrm{Im}(V_{xy}) = -\mathrm{Im}(V_{yx}),\ \mathrm{Im}(V_{xz}) = -\mathrm{Im}(V_{zx}) \text{ and}$$
$$\mathrm{Im}(V_{yz}) = -\mathrm{Im}(V_{zy}) \qquad (17)$$

In these relations Re means to take the real part of what follows and Im means to take the imaginary part of what follows. These non-Riemannian changes in the metric tensor array do give rise to equations that mimic electromagnetism in some ways, but they require modifications to the usual General Relativity field equations. Extra field equations are necessary because of the added structure. In these extended cases, however, there is always an equation like (1) without the unwanted right-hand side.

Another extension (that was very much on Einstein's mind in 1931)[32] was the idea of enlarging the number of dimensions from four to five. Einstein had tried this earlier in the 1920s and was revisiting the idea while he stayed at CalTech that winter. With a straightforward generalization to five dimensions, the geometry stays purely Riemannian and no extra field equations are necessary besides (1). The metric tensor in this case has 25 components, but only 15 independent functions [using similar logic to that used after the relations (8) to arrive at 10 independent functions for the four dimensional case]. The $R_{ik} = 0$ equation then contains extra parts that give rise to terms that are highly reminiscent of electromagnetic terms.

Einstein also experimented with the idea of leaving the number of spacetime dimensions at four, but allowing an internal space, associated with each spacetime point, to be higher-dimensional. He wanted a theory that would allow the extra dimension being introduced to live freely as the other usual dimensions did, without the curling-up assumptions that were previously tried (and that are still used today). Einstein had hopes of an extra dimension not only helping with the unification of electromagnetism and gravitation, but also somehow providing the hidden variable that might fix his disillusionment with the accepted ideas of quantum mechanics.

Whatever Unified Field Theory Einstein tried, he discarded not long afterwards. He always found some flaw that made his interest wane. By the time he died in 1955, he had left a long wake of abandoned theories, every one of which had been examined for their "marblelike" consistency and their lack of points where relations might go uncontrollably to infinity. The effort that Einstein put into his lonely pursuit of a Unified Field Theory was a precursor to the effort that is now being carried out in the modern search for The Theory of Everything. Such popular unification theories as Superstrings require their creators to make use of higher dimensions and to use extended symmetry tactics to accomplish their all-encompassing enterprise. Einstein made primitive stabs in the dark, by modern standards, with his Unified Field Theory ideas. Nevertheless, he was undoubtedly one of the first to think that such unification problems were important enough to devote a significant part of a career to them, and the techniques he employed have serious theoretical analogs today.

It seems that $R_{ik} = 0$ was, to put it simply, Einstein's favorite equation—at least for the latter half of his life. It encompassed the essence of the Unified Field Theory as he conceived it—and the forging of such a theory was the quest of his scientific life from the time of his first trip to America. As late as 1950, in an article commissioned by the editors of *Scientific American* who had asked him to write about his recent work, we find the equation, $R_{ik} = 0$, front and center.[34] In his continuing search for the "generalization" of General Relativity he was still pondering the equation, asking himself if it contained all the necessary "marble" for the universe. Whether the question mark added to the blackboard in early 1931 was for the audience or the photographer we do not know. We surmise, however, in light of Einstein's later work that the "?" attached to the equation $R_{ik} = 0$ was a portent of a problem that would define the later research of one of the greatest scientists of all time.

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30. The Apple version of the Einstein picture shows Einstein having just written an equation looking like $R = 0$?. The right-hand side of this equation involves what is called the Ricci scalar, a quantity obtained from multiplying each element of the array (5) with each corresponding element of the Ricci tensor, R_{ik}, and then adding up all of the products. If the normal General Relativity equation (1) is in effect, each one of the products in the Ricci scalar summation is zero since each element of R_{ik} is zero. Then Apple's Einstein picture would be asking the "profound" question: Is zero equal to zero? Most physicists would no doubt agree that Einstein would really be "thinking differently" with such a question! Perhaps in this counterfactual advertising universe Einstein is presenting some exciting new variation of General Relativity as a rhetorical question. It is clear, though, that in this universe he will have severe intractability problems with a theory where he had to solve one equation, while the gravitational field is described by the 10 (or more) independent functions present in the gravitational metric tensor (5).
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