Lecture 2

Einstein's Special Relativity

An Introduction

Common sense Gallilean Relations

$$\begin{aligned} x' &= (x - vt) \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

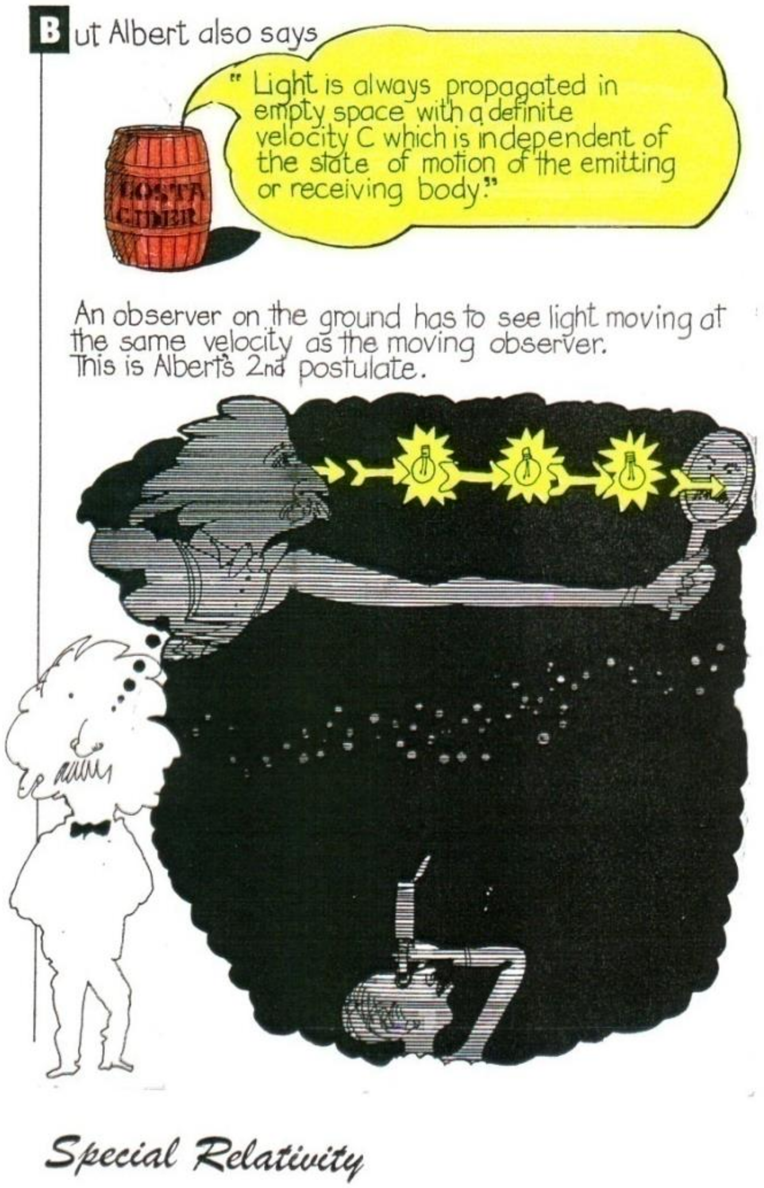
Einstein after changing the equations

$$\begin{aligned} x' &= (x - vt)\gamma \\ y' &= y \\ z' &= z \\ t' &= (t - \tfrac{vx}{c^2})\gamma \end{aligned}$$

https://mothership.sg/2016/12/albert-einstein-visited-spore-in-1922-met-a-14-year-old-boy-named-david-marshall/

Einstein's Mirror

1) Einstein also asked if he can see himself in the mirror when he rides on a beam of light ?

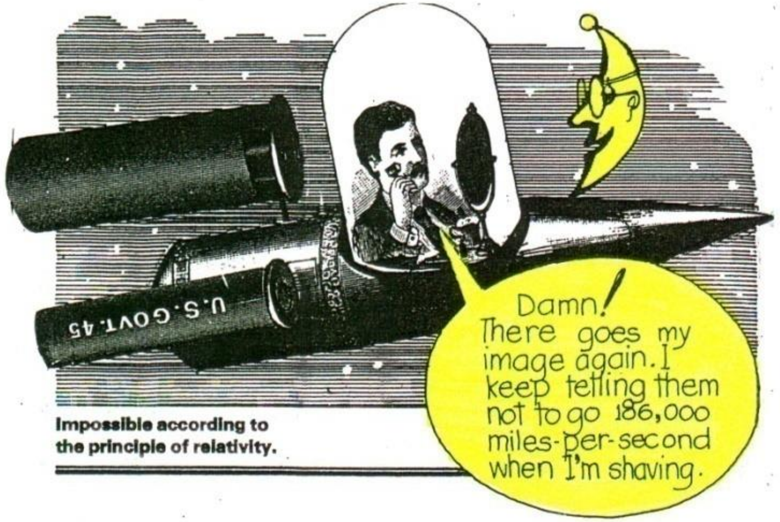


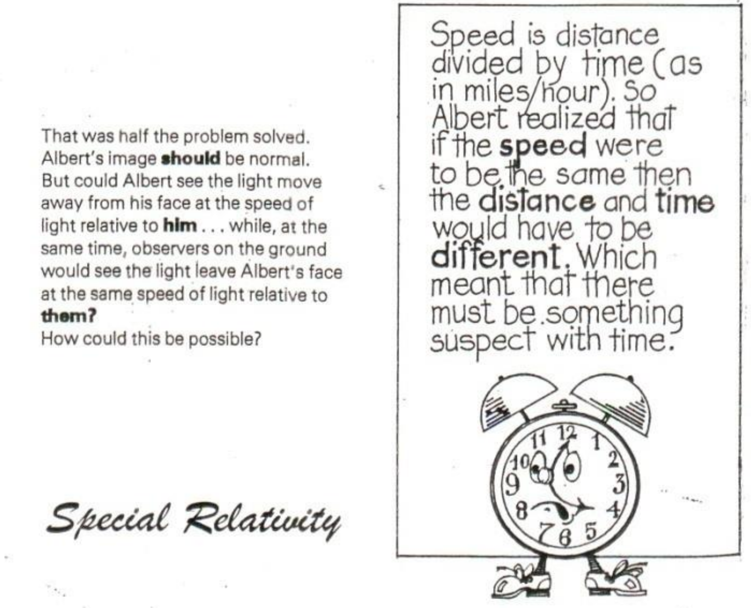
Special Relativity

2) At 16, he began to wonder whether clocks might behave differently while moving.

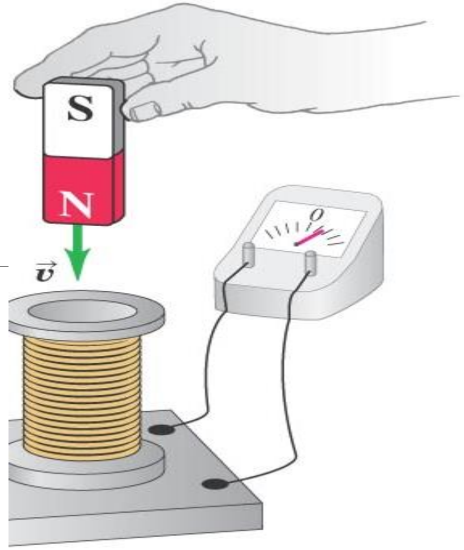
3) Einstein asked if there were some other transformations (relations) which may leave the famous equations of Maxwell unchanged in form.

(a covariant idea)

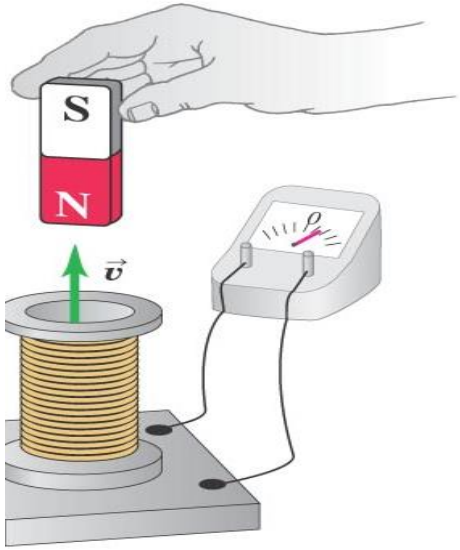




The Principle of Relativity



The laws of physics apply in the same fashion everywhere. Newton's Laws work no matter what your point of view.



A magnet moving in a coil of wire will induce a current, but … who's to say if the coil is moving over the magnet or if the magnet is moving through the coil.

There should be a new transformation/relations

Einstein believed that **there should be a new** transformation/relations (Lorentz) along the *x*-direction, where *E* (*B*) represents the electric field in the wave equation and behaves as follows :

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2}$$

$$\frac{\partial^2 E(x',t')}{\partial x'^2} = \frac{1}{c^2}\frac{\partial^2 E(x',t')}{\partial t'^2}$$

Notice no Extra term as in Galilean transformation $\Longrightarrow$ Implies covariant

There should be a new transformation/relations

Recall Galilean transformation along the *x*-direction, where *E* (*B*) represents the electric field in the wave equation :

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2}$$

$$\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2 E(x',t')}{\partial x'^2} + \frac{2v}{c^2}\frac{\partial^2 E(x',t')}{\partial x'\partial t'} = \frac{1}{c^2}\frac{\partial^2 E(x',t')}{\partial t'^2}$$

Extra terms … implies not covariant

University Physics by Young and Freedman, P1259

Hint : $\frac{\partial f(x,t)}{\partial t} = \frac{\partial f(x,t)}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f(x,t)}{\partial t}\frac{\partial t}{\partial t}$

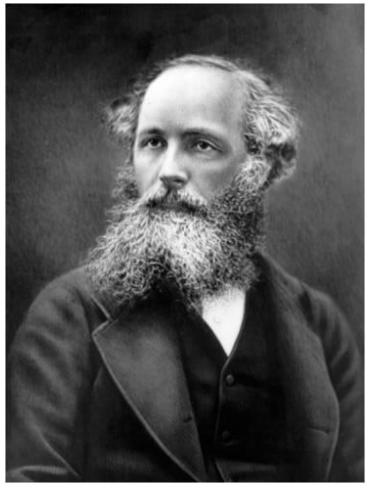
Einstein's Wave thoughts & Light

What will the lady surfer see?

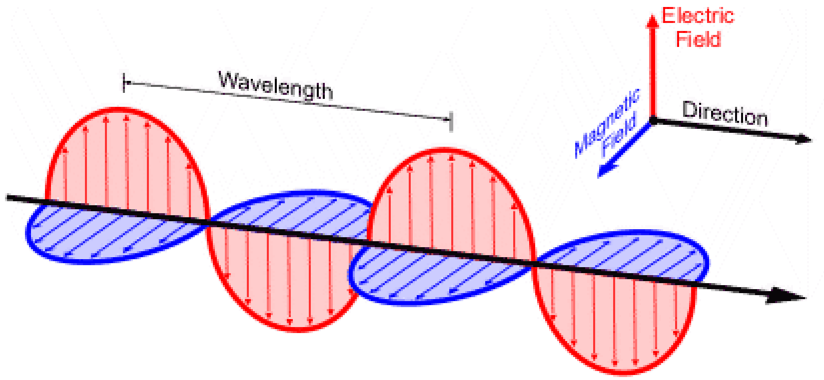
As seen from the lady going at the same speed, she and the wave are standing still.

How about riding on a Light wave ?

According to Maxwell this is impossible, why ?



E&M wave keeps itself going by its changing electric part moving forward and so instantly powering up the magnetic part … the cycles repeats.



4) Light wave must move to exist … Light cannot be made to stand still. If light beam is stationary, there will be no light.

Who was Maxwell ?

Born on 1831, Edinburgh, Scotland

Memorized all of Psalm 119. His mother taught him: God's scientific genius and compassionate hand in the beauties of nature.

He was also awarded a prestigious prize for original research in mathematically analyzing the stability of the rings around Saturn. Maxwell concluded that Saturn's rings could not be completely solid or fluid.

His Great Opus 1864 : Electricity and Magnetism

Others: Optics, color photography, kinetic theory, Thermodynamics,

Mathematics of control theory

Died in 1879

But who was Einstein ?

Born on March 15 (14), 1879, Ulm in Germany, π day

Uncle Jakob introduced mathematical problems. Max Talmud introduced popular science and discuss philosophy … both enjoyed very much.

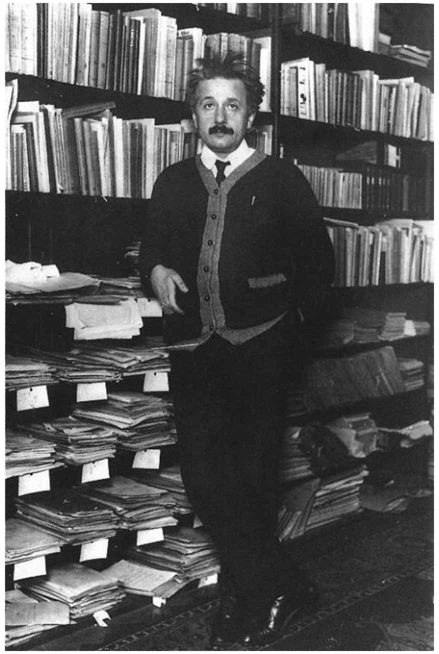
Grew interested in music and literature.

Polytechnic ETH, 1896-1900

School teacher till 1902 (Patent Clerk)

V17, *Annalen der Physik*, 1905

- p132 Photoelectric Effect
- p549 Brownian Motion
- p891 Special Theory of Relativity
- 1907 Doctorate, University of Zurich



Einstein's method of science, start with the postulate, then continue with the prove

Postulates of Einstein's Theory (Method)

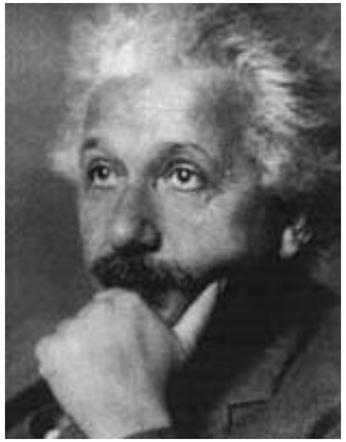
Recall from previous slides :

The laws of mechanics remain the same for observers in inertial frames that are in uniform motion with respect to each other.

1) The laws of **Physics** (mechanics & E&M) are the same in all inertial frames. i.e. no preferred frame exists.

Einstein extended the laws of mechanics to EM also

2) The speed of light in free space has the same value c in all inertial systems.



Notice : no ether was mentioned why ?

At 1905 … after 10 years since Einstein was 16 years old …

An even shorter writeup for the 2 axioms !

1) The laws of physics are the same.

2) The velocity of light in vacuo is the same.

Sander Bais

Some books :

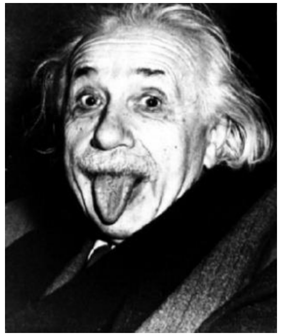
1) Principle of Relativity

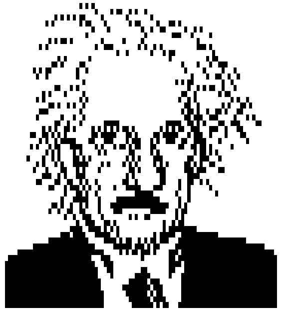
2) Principle of the constancy of the speed of light

Einstein's Theory

1905 paper:
Title: *On the Electrodynamics of Moving Bodies.*

In one sweeping stroke, he was not only able to preserve the principle of relativity, but also provide an unified description of mechanics and electrodynamics.





So what was Einstein's trick ?

Einsteinian Transformation

Got rid of Galilean transformation (commonsense ?).

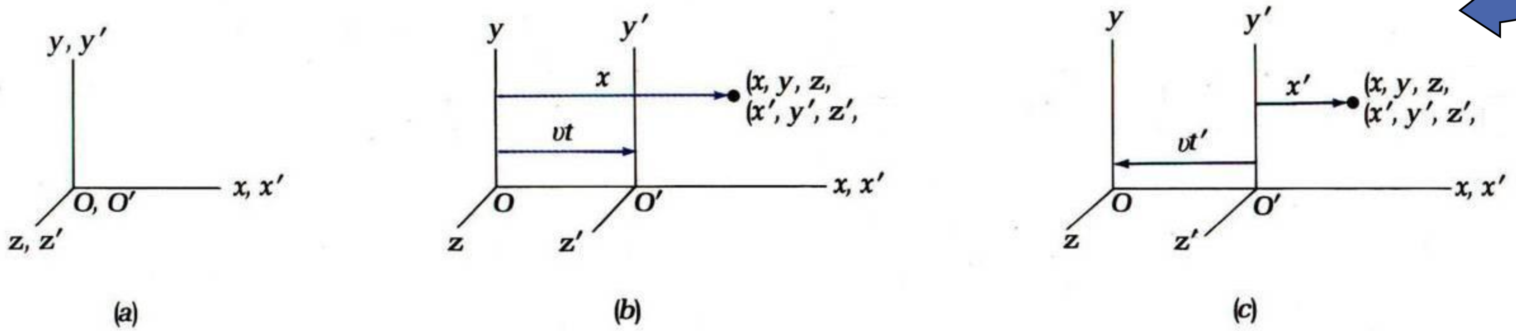
Lorentz transformations ensures that the speed of light c is the same in all the inertial frames while Galilean transformations do not.

Recall from last lecture: $u = u' + v$

But Note : An observer traveling with a light source would measure this light, a velocity greater than c in contradiction to the 2nd postulate of Einstein.

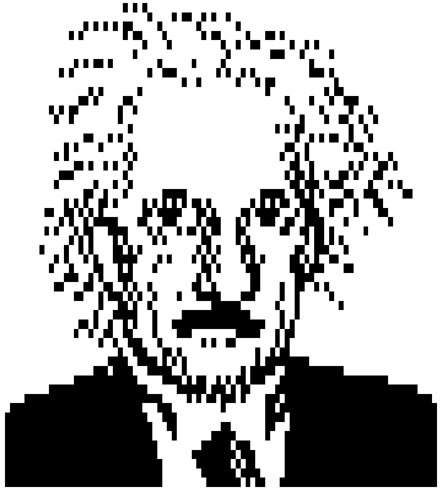
Recall Galilean Transformation

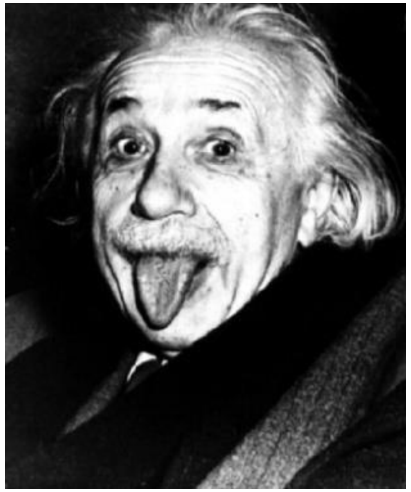
Wait a minute



$$(x' = x - vt, \quad y' = y, \quad z' = z) \quad and \quad t' = t$$

These rules are simple and follow from common sense. Notice the innocent looking statement ***t' = t***. It simply means that clocks behave in the same way in the two inertial frames.





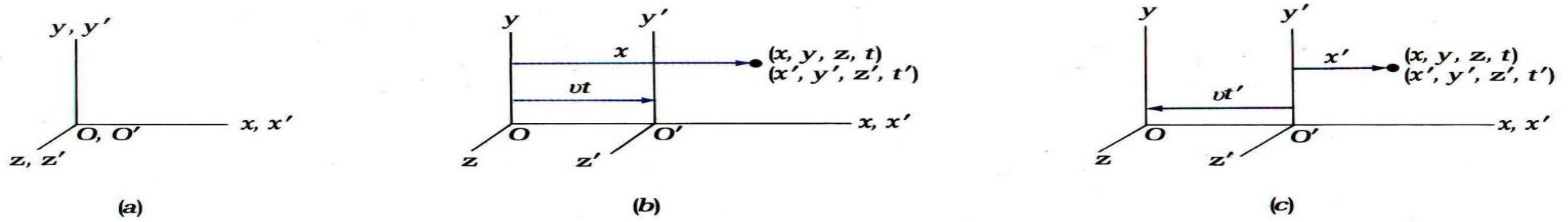
So what exactly did Einstein do ?

What is this Einsteinian/Lorentz Transformation?

Consider 2 frames *S* and *S'* moving uniformly wrt to each other with a velocity *v*.

Put time into the equation also, 4th dimension

$$S(x, y, z, t) \quad S'(x', y', z', t')$$



Space (x, y, t) and time (t) treated equal footing

What is this Einsteinian/Lorentz Transformation?

The co-ordinate x' depends on x, y, z and t, similarly for y', z', t'.

$$x' = ax + cy + dz + bt$$

Notice

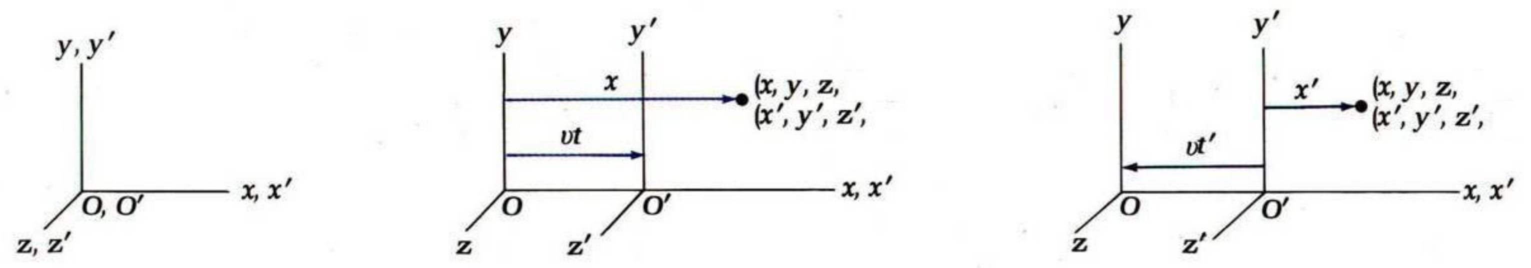
that it is **Linear**

due to **Homogeneous Isotropy**

… means that all points of space-time have the same properties … in all direction

I rarely think in words at all … A. Einstein

Where do Lorentz Transformations (Relations or "boost") come from ? Einstein's perspective !



… for simplicity concentrate on one direction … x direction

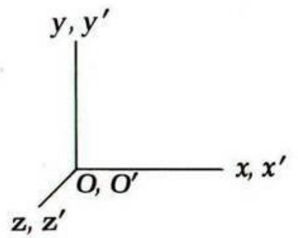
1) The transformation between the 2 frames must be linear, in order to ensure that Newton's 1st law holds in all inertial frames.

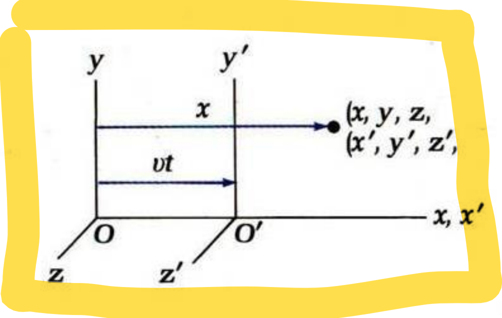
2) We can write the transformations such that the co-ordinates of an event in one frame are **linear combinations** of the co-ordinates in the other frame.

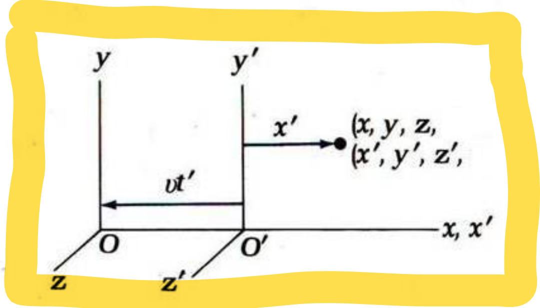
$(x,\ y,\ z,\ t) \longleftrightarrow (x',\ y',\ z',\ t')$

$$x' = ax + bt \qquad\qquad x = a'x' + b't'$$

$a', b', a \text{ and } b$ are coefficients to be determined







If the origin of the frame O' is at $x' = 0$

$$x' = ax + bt$$

$$0 = ax + bt$$

$$-\frac{b}{a} = \frac{x}{t} = \boxed{v}$$

If the origin of the frame O is at $x = 0$

$$x = a'x' + b't'$$

$$0 = a'x' + b't'$$

$$-\frac{b'}{a'} = \frac{x'}{t'} = \boxed{-v}$$

Why – ve ?

Substitute

$$-\frac{b}{a} = \frac{x}{t} = v \qquad\qquad -\frac{b'}{a'} = \frac{x'}{t'} = -v$$

Back into the original equation

$$x' = ax + bt \qquad\qquad x = a'x' + b't'$$

We get one equation in Lorentz Transformation … wait a minute !

$$x' = a(x - vt) \qquad\qquad x = a'(x' + vt')$$

Argument : One should note that going from O to O' frame should be similar to going from O' to O i.e. $a = a'$

So how do we find ? $a \text{ or } a'$ The Principle of Relativity (Postulate 1)

In order to determine the co-efficient *a* (and hence *a'*), we now make use of the second axiom of Einstein (speed of light is the same, Postulate 2)

Let us suppose that we send out a light signal at $t = t' = 0$ when $x = x' = 0$. The signal propagates in both O and O' satisfying

$$x' = ct' \qquad\qquad x = ct$$

Substitute into previous equations

$$x' = a(x - vt) \qquad\qquad x = a'(x' + vt')$$

$$ct' = a(ct - vt) \qquad\qquad ct = a'(ct' + vt')$$

Multiplying both sides

$$c^2 tt' = a^2 tt'(c^2 - v^2)$$

$$\Rightarrow a^2 = \frac{c^2}{c^2 - v^2} \qquad \Rightarrow a^2 = \frac{1}{1 - \frac{v^2}{c^2}} \qquad \Rightarrow a = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma$$

Finally, to get Lorentz Transformation for x direction

$$x' = a(x - vt) \qquad x = a'(x' + vt')$$

$$x' = \gamma(x - vt)$$
$$x' = (x - vt)\gamma$$

$$x = \gamma(x' + vt')$$
$$x = (x' + vt')\gamma$$

What about the other equations, namely y and z direction ?

What about last equation for the time variable, namely t direction ?
Recall t is equal footing with x, y, and z.

How to get Lorentz Transformation for t variable ?

Again Recall

$$x' = a(x - vt) \qquad x = a'(x' + vt')$$

$$but\ recall\ a = a' = \gamma$$

Rearrange

$$t' = \frac{x}{\gamma v} - \frac{\gamma}{v}(x - vt)$$

$$t' = \gamma\, t + \frac{x(1-\gamma^2)}{\gamma v}$$

$$h\mathrm{int} : (1-\gamma^2) = \left(-\tfrac{v^2}{c^2}\right)\gamma^2$$

Notice : no mention of ether

$$t' = \gamma\left(t - \tfrac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \tfrac{vx'}{c^2}\right)$$

The Special Relativity Theory (Summary)

$$
\begin{aligned}
x' &= (x - vt) \\
y' &= y \\
z' &= z \\
t' &= t
\end{aligned}
$$

$$
\begin{aligned}
x' &= (x - vt)\gamma \\
y' &= y \\
z' &= z \\
t' &= \left(t - \frac{vx}{c^2}\right)\gamma
\end{aligned}
$$

Galileo

$$\gamma^2 = \frac{1}{1 - \left(\frac{v}{c}\right)^2}$$

$$(1 - \gamma^2) = \left(-\frac{v^2}{c^2}\right)\gamma^2$$

Lorentz

Recall Galilean transformation along the x-direction, where E (B) represents the electric field in the wave equation :

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2}$$

$$\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2 E(x',t')}{\partial x'^2} + \frac{2v}{c^2}\frac{\partial^2 E(x',t')}{\partial x'\partial t'} = \frac{1}{c^2}\frac{\partial^2 E(x',t')}{\partial t'^2}$$

Extra terms … implies not covariant

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Hint : $\frac{\partial f(x,t)}{\partial t} = \frac{\partial f(x,t)}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f(x,t)}{\partial t}\frac{\partial t}{\partial t}$

Einstein believed that **there should be a new** transformation (Lorentz) along the *x*-direction, where *E* (*B*) represents the electric field in the wave equation and behaves as follows :

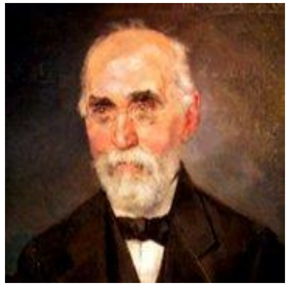
$$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2}$$

$$\frac{\partial^2 E(x',t')}{\partial x'^2} = \frac{1}{c^2}\frac{\partial^2 E(x',t')}{\partial t'^2}$$

Notice no Extra term as in Galilean transformation

$\Longrightarrow$ Implies covariant idea

Einstein's (Lorentz) Transformation Equations in Full Glory



Lorentz

$$
\begin{aligned}
x' &= (x - vt)\gamma \\
y' &= y \\
z' &= z \\
t' &= \boxed{\left(t - \frac{vx}{c^2}\right)\gamma}
\end{aligned}
$$

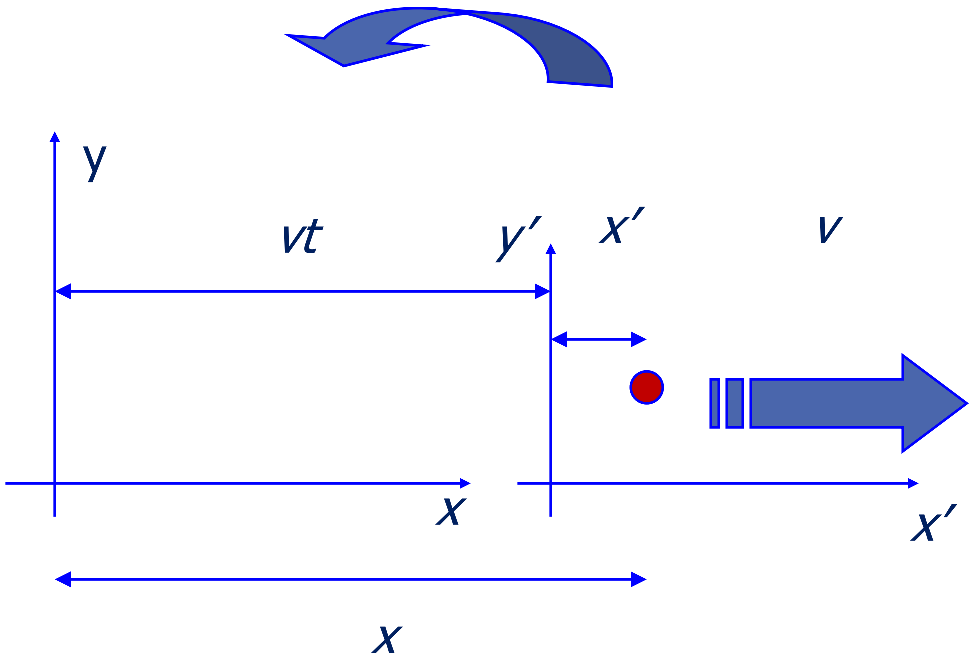
One Striking Feature $t' \neq t$

Moving clocks run differently compared to stationary clocks.

Note: Moving clocks run differently as compared to stationary clocks. It seems that t' depends on x, y and z which means that the way a moving clock runs also depends on where it is (*strange indeed* !)

Lorentz Equations in Full Glory

$$
\begin{aligned}
x' &= (x - vt)\gamma \\
y' &= y \\
z' &= z \\
t' &= \left(t - \frac{vx}{c^2}\right)\gamma
\end{aligned}
$$



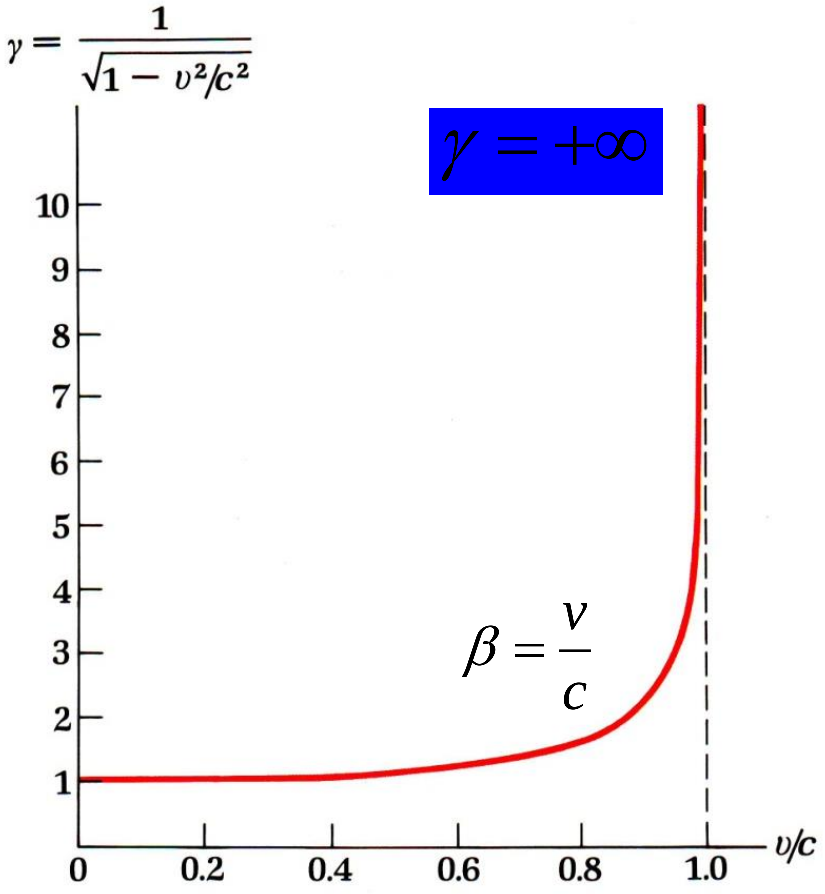
The Lorentz transformation mixed up space and time in an unusual way and this was the source of all confusion in the early days of special relativity.

What is Gamma (speed limit) ?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Called Lorentz factor

What can we learn ?



This gamma (called **Lorentz factor**) term imposes the speed of light limit !

How so ?

Reverse Equations … who uses them ?

$$
\begin{aligned}
x &= (x' + vt') \\
y &= y' \\
z &= z' \\
t &= t'
\end{aligned}
$$

Inverse Galilean Transformation

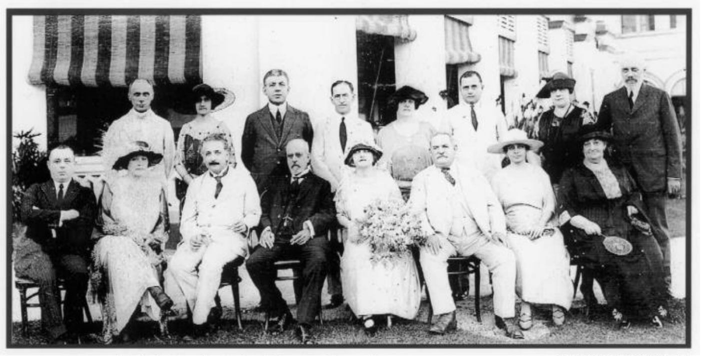


photo courtesy of Alan Frankel

Albert Einstein with prominent members of Singapore's Jewish community.

Photo includes, seated from left to right: the Einsteins' hosts, Alfred Montor and his wife; Albert Einstein; Sir Menasseh Meyer, whom Einstein refers to as 'Croesus'; and Einstein's wife Elsa. Meyer's daughter Moselle stands behind Einstein.

Divide by *time* and recall $\gamma = \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$$
\begin{aligned}
x &= (x' + vt')\gamma \\
y &= y' \\
z &= z' \\
t &= \left(t' + \frac{vx'}{c^2}\right)\gamma
\end{aligned}
$$

Inverse Lorentz Transform

Useful expression $\gamma^2 = \dfrac{1}{1-\left(\frac{v}{c}\right)^2}$

Some strange & unexpected results & consequences !

Result 1: Peculiar Velocity Addition

From Galilean Transformation (Secondary school)

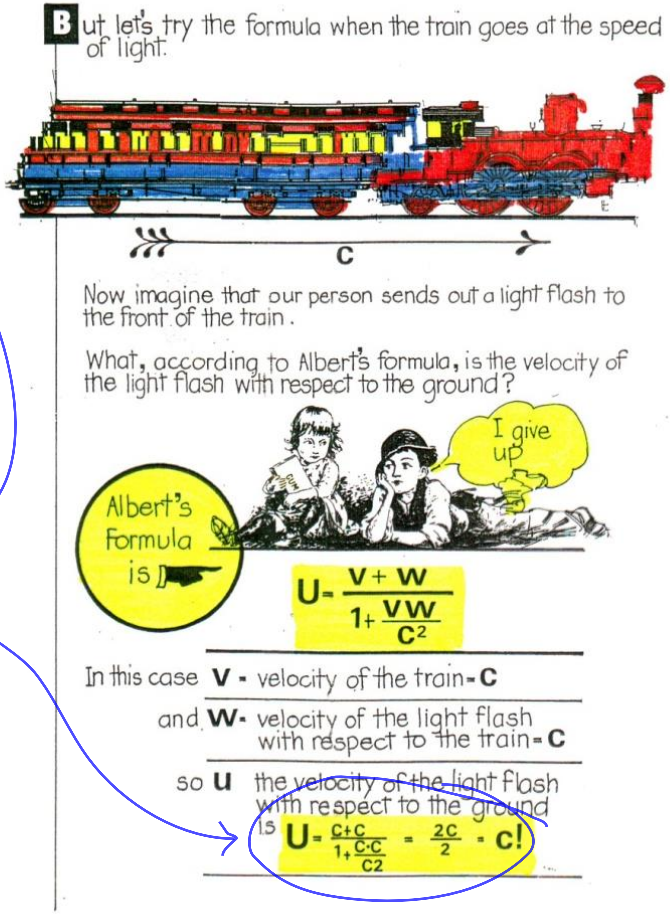
$$u = u' + v \qquad u = \frac{u' + v}{1}$$

From Lorentz Transformation

$$\boxed{u = \frac{u' + v}{1 + \frac{u'v}{c^2}}}$$

- Note

The numerator of this formula makes common sense. But this simple sum of 2 velocities is altered by the second term in the denominator.



Perculiar Velocity Addition

$$\begin{aligned} x &= (x'+vt') \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}$$

Inverse Galilean Transformation

$$\boxed{u = u'+v}$$

Albert Einstein with prominent members of Singapore's Jewish community.

photo courtesy of Alan Frankel

Photo includes, seated from left to right: the Einsteins' hosts, Alfred Montor and his wife; Albert Einstein; Sir Menasseh Meyer, whom Einstein refers to as 'Croesus'; and Einstein's wife Elsa. Meyer's daughter Moselle stands behind Einstein.

Divide by *time* and recall

$$\gamma^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2} \qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\begin{aligned} x &= (x'+vt')\gamma \\ y &= y' \\ z &= z' \\ t &= \left(t'+\frac{vx'}{c^2}\right)\gamma \end{aligned}$$

Inverse Lorentz Transform

$$\boxed{u = \frac{u'+v}{1+\frac{u'v}{c^2}}}$$

So what can you conclude ?

Try yourself now !

$$x' = (x - vt)\gamma$$
$$y' = y$$
$$z' = z$$
$$t' = (t - \tfrac{vx}{c^2})\gamma$$

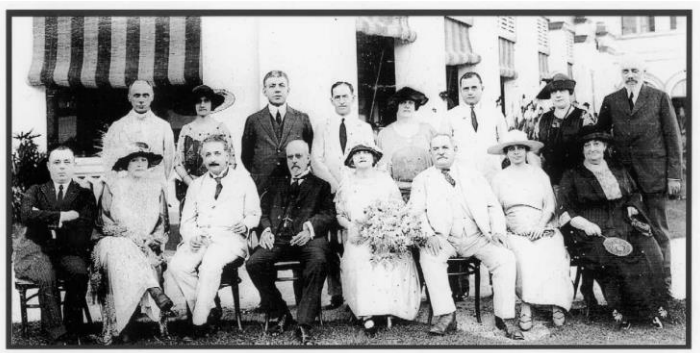
$$\frac{x'}{t'} = \frac{(x - vt)}{(t - \frac{vx}{c^2})} = \frac{\left(\frac{x}{t} - v\right)}{\left(1 - \frac{xv}{tc^2}\right)}$$

$$\boxed{u = \frac{u' + v}{1 + \frac{u'v}{c^2}}}$$

Reduce the number of Equations

$$x' = (x - vt)$$

$$t' = t$$



Albert Einstein with prominent members of Singapore's Jewish community.

photo courtesy of Alan Frankel

Photo includes, seated from left to right: the Einsteins' hosts, Alfred Montor and his wife; Albert Einstein; Sir Menasseh Meyer, whom Einstein refers to as 'Croesus'; and Einstein's wife Elsa. Meyer's daughter Moselle stands behind Einstein.

$$\gamma^2 = \frac{1}{1 - \left(\frac{v}{c}\right)^2}$$

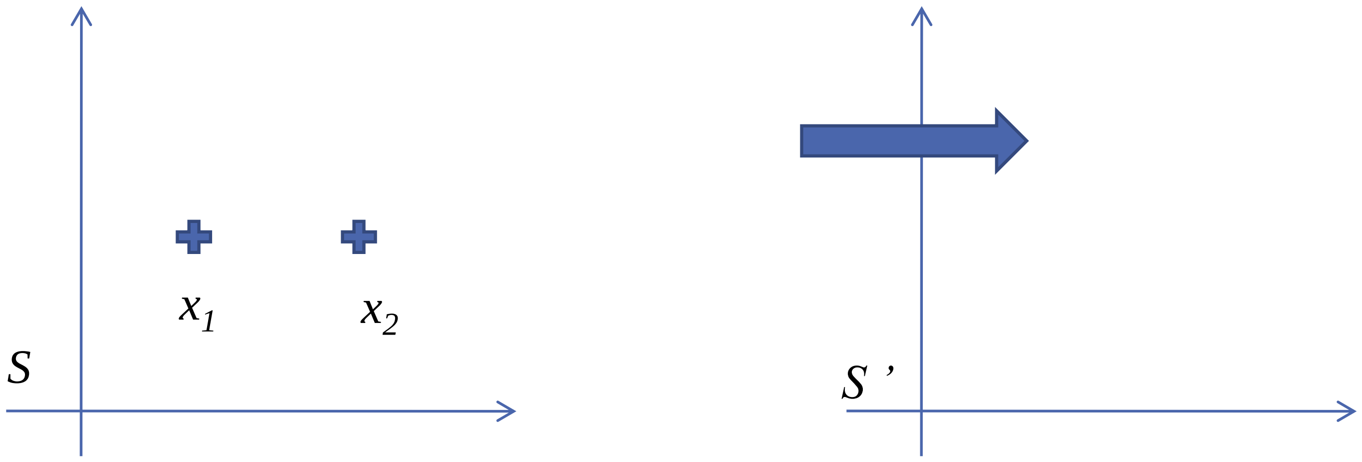
$$x' = (x - vt)\gamma$$

$$t' = \left(t - \frac{vx}{c^2}\right)\gamma$$

We really need only 2 equations

Result 2 : Simultaneity

Consider events at the $(x_1\,, t_1)$ and $(x_2\,, t_2)$



$$t'_1 = \left(t_1 - \frac{vx_1}{c^2} \right)\gamma$$

$$t'_2 = \left(t_2 - \frac{vx_2}{c^2} \right)\gamma$$

If $t_1 = t_2$, which means that they are simultaneous events in the S reference frame, we have :

$$t'_2 - t'_1 = \overset{0}{(t_2 - t_1)\gamma} - \frac{v}{c^2}(x_2 - x_1)\gamma$$

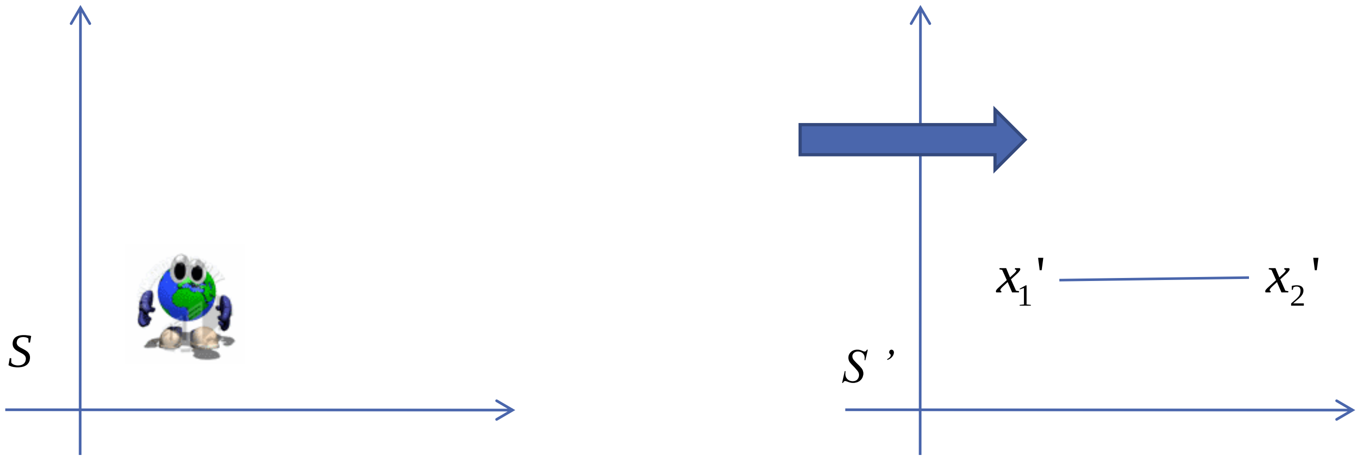
$$\boxed{t'_2 - t'_1 = \frac{v}{c^2}(x_1 - x_2)\gamma}$$

What can you conclude ?

Simultaneous events in one frame are not simultaneous in moving frames.

Result 3 : Lorentz Contraction

Consider a rod at rest in S' lying parallel to x–axis at x_1' *and* x_2'



$$x'_1 = (x_1 - vt_1)\gamma$$
$$x'_2 = (x_2 - vt_2)\gamma$$

$$x'_2 - x'_1 = (x_2 - x_1)\gamma - v(t_2 - t_1)\gamma$$

$$\boxed{L_0 = L\gamma}$$

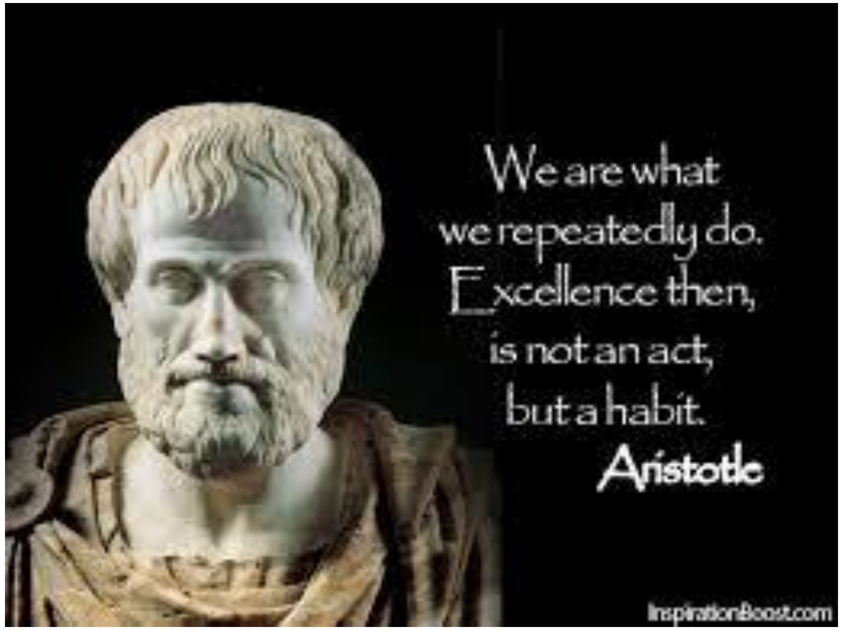
We **must measure** x_1 and x_2 **simultaneously** in S, so as before we take $t_1 = t_2$, we have :
What can you conclude ?

$$\boxed{L = \frac{L_0}{\gamma}}$$

About Excellence & Comprehension

We are what we repeatedly do. Excellence then, is not an act but a habit … teaching is the highest form of understanding.

Aristotle



Quotes in Thomas Friedman,... *The World is Flat...*

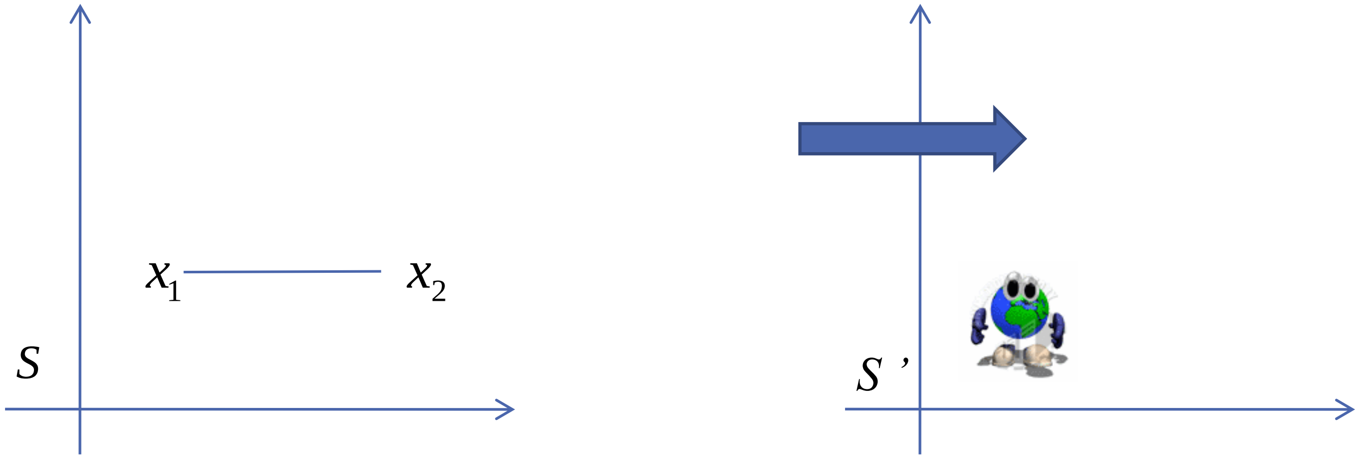
When I asked Bill Gates about the supposed American education advantage -- an education that stresses creativity, not rote learning -- he was utterly dismissive. In his view, those who think that the more rote learning systems of China and Japan can't turn out innovators who can compete with Americans are sadly mistaken.

Said Gates, "I have never met the guy who doesn't know how to multiply who created software ... Who has the most creative video games in the world? Japan! I never met these 'rote people' ... Some of my best software developers are Japanese. You need to understand things in order to invent beyond them."

http://philosophy.hku.hk/think/creative/quotes.ph

Self-Study (Optional)

Consider a rod at rest in S lying parallel to x–axis at x_1 *and* x_2



Question:

Consider the usual scenario above and use Lorentz transformations in your analysis. The length of a straight rod of length L_0 is measured in S.

What is the length this straight rod according to the observer in the moving S'. ***i.e. find an algebraic expression for this rod.***

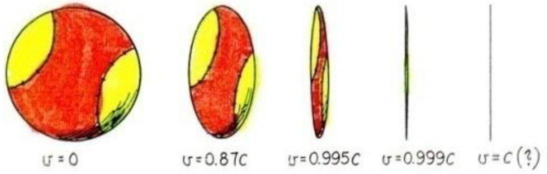
Result 3: Length Contraction (Example)

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\boxed{L = \frac{L_0}{\gamma}}$$

Length observed by moving person at rest with ball

Assuming a person moving with the ball, will not notice that the ball is flattened



Length observed by person at rest outside (stationary frame)

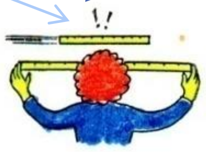


Figure The meter stick is measured to be half as long when traveling at 87 percent the speed of light relative to the observer.

Recall, this was obtained by Lorentz … now called Lorentz contraction

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma$$

In the Literature, L_0 is called the **proper** length It is measured by the person at rest.

A Thought Experiment ?

A Farmer and his son, would like to fit the ladder in the barn.

The son had learned some relativity in college and he is back at the farm for summer vacation.

Discuss how can he apply Einstein's Relativity Theory to overcome this problem.

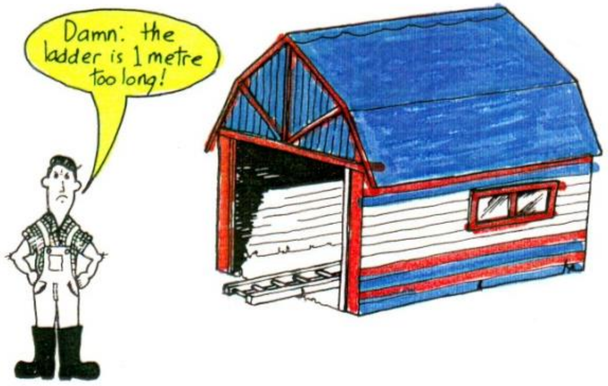


Figure 1

One day, however, the farmer hits upon a method for getting the ladder into the barn. He gets his son to hold the ladder in the middle and run towards the barn at 50% of the speed of light, while he stands by the barn's open door (see Figure 2).

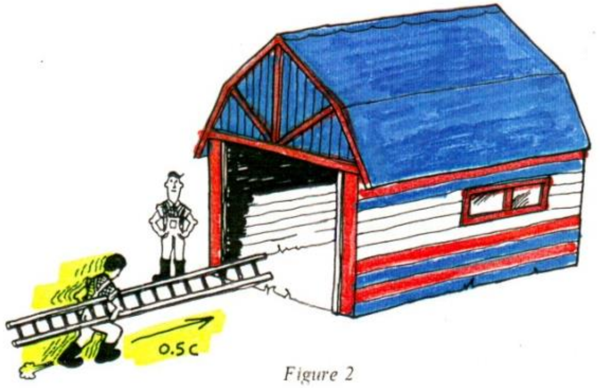


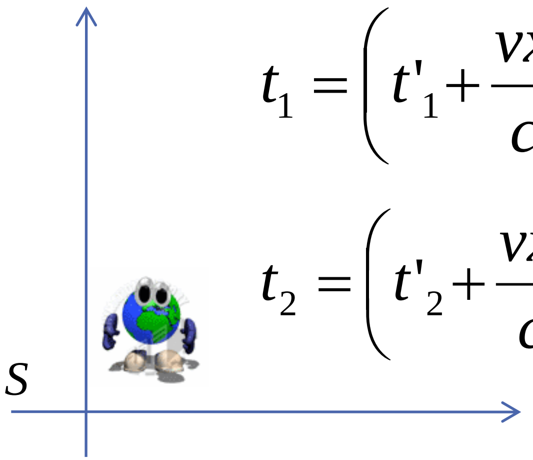
Figure 2

The Price to pay …for new ideas

It is not until we attempt to bring the theoretical part of our training into contact with the practical that we begin to experience the full effect of what Faraday has called "mental inertia" … not only the difficulty of recognizing among the objects before us, the abstract relations which we have learned from books, but ***the distracting pain of wrenching the mind away from the symbols to the objects, and from the objects back to symbols***. This, however, is the price we have to pay for new ideas.

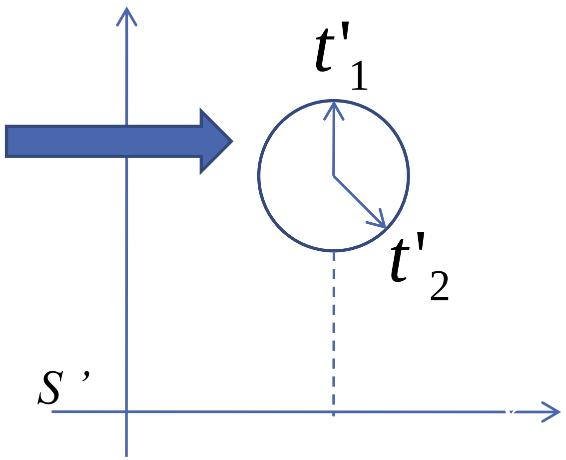
J.C. Maxwell, Introductory lecture at Cavendish Lab. 1871

Result 4 : Time Dilation



$$t_1 = \left(t'_1 + \frac{vx'_1}{c^2}\right)\gamma$$

$$t_2 = \left(t'_2 + \frac{vx'_2}{c^2}\right)\gamma$$



Consider a clock at rest in S '

$$t_2 - t_1 = (t'_2 - t'_1)\gamma + \frac{v}{c^2}(x'_2 - x'_1)\gamma$$

0

$$\boxed{t_2 - t_1 = (t'_2 - t'_1)\gamma}$$

Here the clock is fixed at one place $x'_1 = x'_2 = x'$

An observer in S measures at instants t_1 *and* t_2

What can we conclude ?

Time ticks more slowly in moving frames.

we have : $\boxed{T = T_0\,\gamma}$

as compared to

$L_0 = L\gamma$

Time Dilation : Example
Famous Twin Paradox

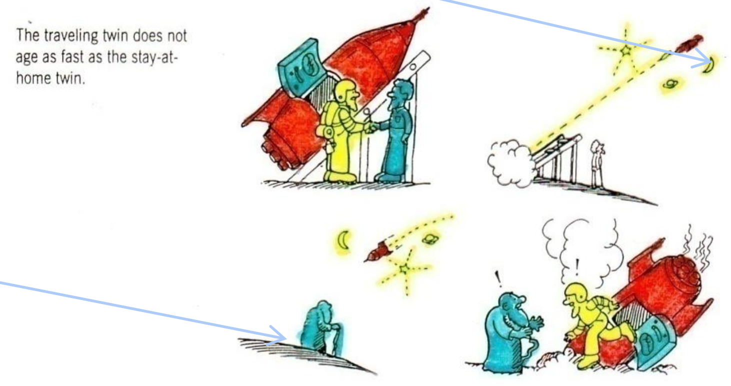
$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time observed by moving person

$$\boxed{T = \gamma \, T_0}$$

Time of his twin brother observed by person at rest

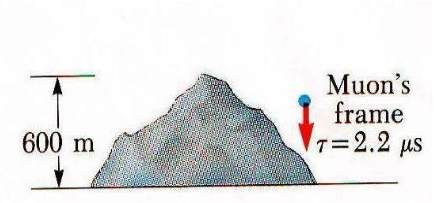
The moving observer's time appears *"to live longer"* (time slows down) as compared to the stationary observer.

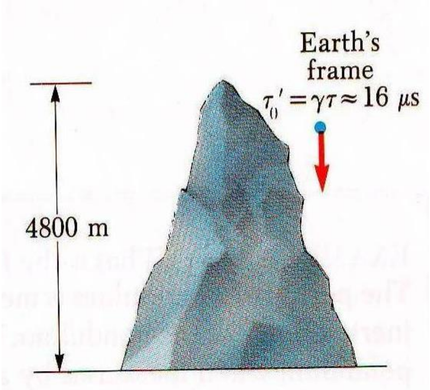


In the Literature, T_0 is called the proper time. It is measured by the moving person at rest with the moving frame.

A nice way of looking at Time dilation and Length contraction !

An Example: Muon particle !





In the example above, an observer in the muon's frame would measure the t_0 = 2.2μs lifetime, while an earth-based observer measures the L_0 = 4800m height of the mountain.

In the muon's frame, there is no time dilation, but the distance of travel, L is observed to be shorter when measured in this frame.

Likewise, on the earth observer's frame; observes that there is time dilation on the muon, but the distance of travel is measured to be actual height, L_0 of the mountain.

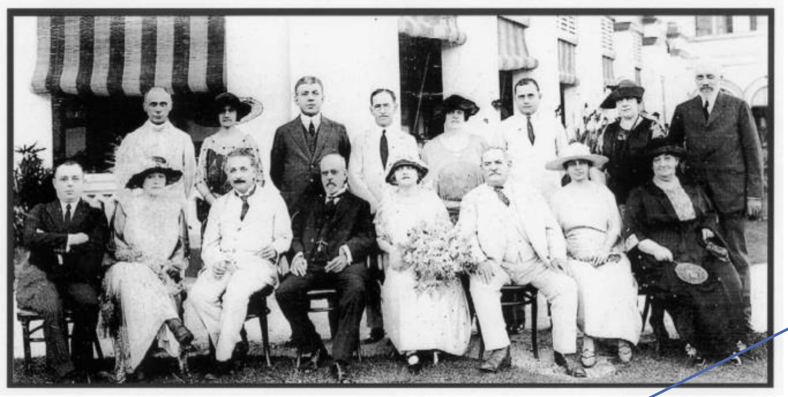
There is a kind of "offsetting" effect but the outcome should be the same !

Let us recap at this point !

2 *really Simple* steps to … … Special Relativity Theory

$$\begin{aligned} x' &= (x - vt) \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

Galileo



$$\gamma^2 = \frac{1}{1 - \left(\frac{v}{c}\right)^2}$$

$$\begin{aligned} x' &= (x - vt)\gamma \\ y' &= y \\ z' &= z \\ t' &= (t - \tfrac{vx}{c^2})\gamma \end{aligned}$$

Lorentz

Step 1 : Understand the 2 types of relations & postulates. History and philosophy surrounding them … ether

Step 2 : Repeat Step 1. Einstein: Space and Time must be treated as equal footing

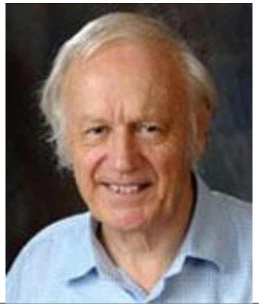
Keep on reminding yourselves : 2 axioms !

1) The laws of physics are the same.
2) The velocity of light in vacuo is the same.

Sander Bais

Some books :
1) Principle of Relativity
2) Principle of the constancy of the speed of light

Physics & Philosophy



Sir Anthony Leggett, Nobel Laureates (2003)

"In the next 20 to 30 years, there are likely to be major revolutions in science, particularly in physics." he said.
At the same time, he stressed the importance of a varied approach to learning. Sir Anthony himself read Latin, Greek and philosophy as a first degree in Oxford before later studying physics. Philosophy proved particularly useful later in his career as a physicist. "It taught me never to take anything for granted and constantly question many of the scientific ideas which, at the time, were well established ideals.

Report by A. Gunasingham
Straits Times Jan 20 2009

International Science Youth Forum at Hwa Chong Institution, 90th HCI Anniversary, Jan 2009

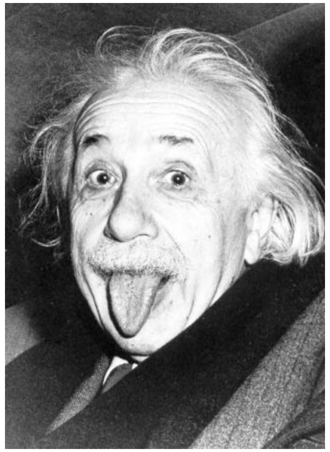
About doing Science (Physics) …

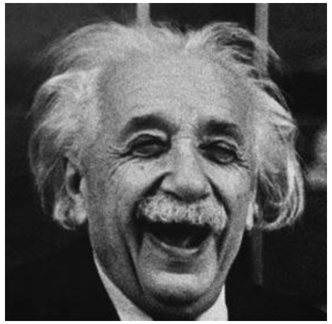
"One should have a knowledge of *mathematics* in order to become a physicist, but not too much,

one must be critical of earlier theories, but not too much,

one must have the capacity for original thinking, but not too much."

A. Einstein





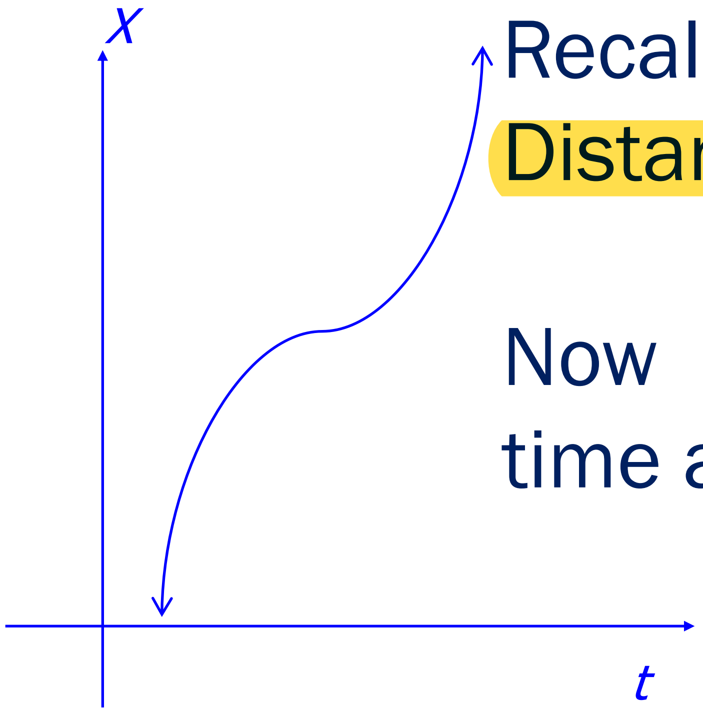
(profited from discussions with a few of his colleagues (mainly from Besso) who were unknown to the world of physics)

How can we represent x, y, z, t on a graph ?

… *recall equal footing*

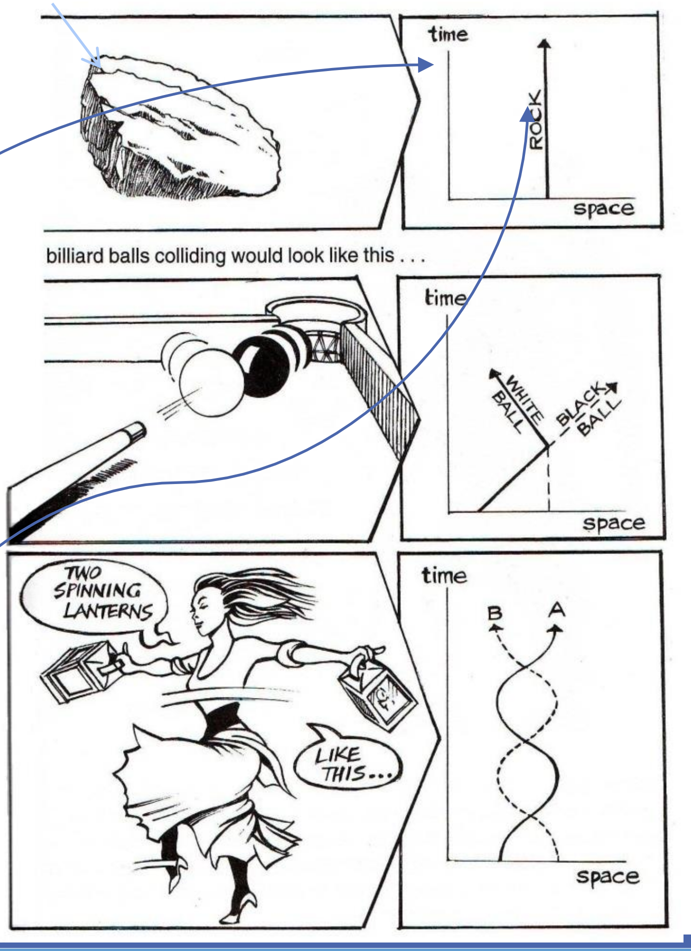
How to visualize (x, y, z, t) ?

Space-time Diagram



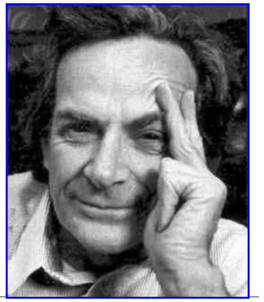
Recall in secondary school :
Distance –Time graph

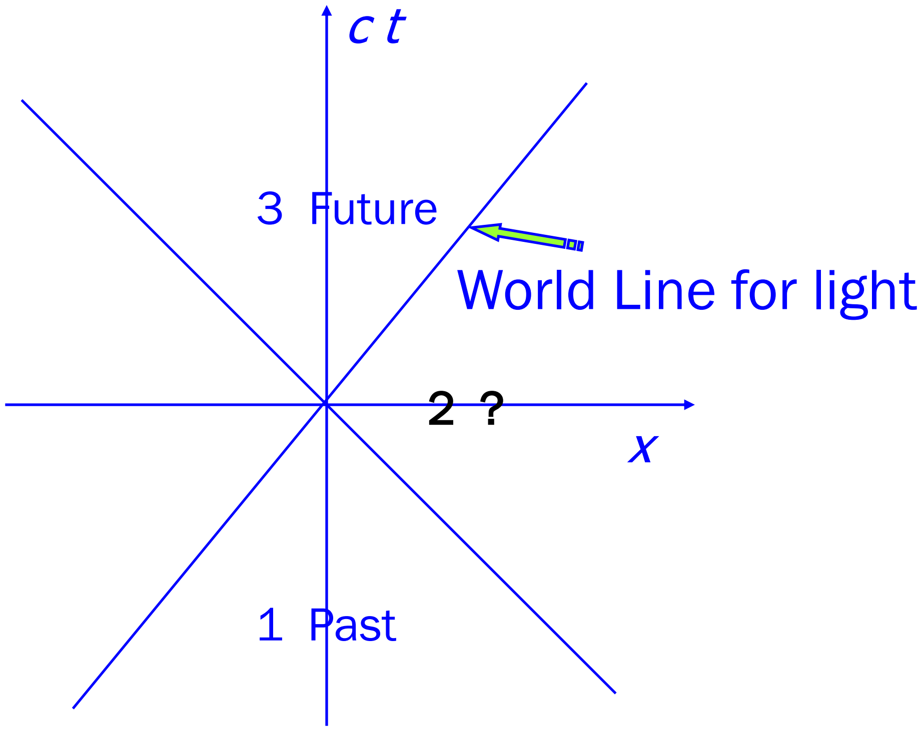
Now we want to exchange time axis !



The Universe is made of events ! ... World Lines (Loci)

Light … in Space-Time Diagram





1 Past

2 ?

3 Future

R. Feynman once said:

"No one can tell the future … as they

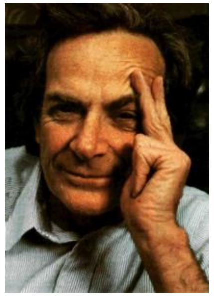
cannot even tell us the present."

For a very nice discussion

Feynman Lectures Volume 1 P17-4

Note : c is set to 1. The world line for light is a 45 degrees line, Why ?

No one can tell the future …



sure we will avoid it by doing the right thing at the right time, and so on. But actually there is no fortune teller who can even tell us the *present*! There is no one who can tell us what is really happening right now, at any reasonable distance, because that is unobservable. We might ask ourselves this question, which we leave to the student to try to answer: Would any paradox be produced if it were suddenly to become possible to know things that are in the space-like intervals of region 1?

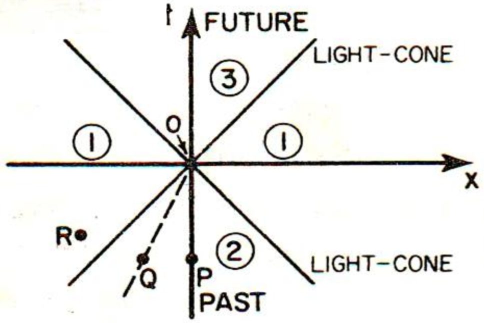
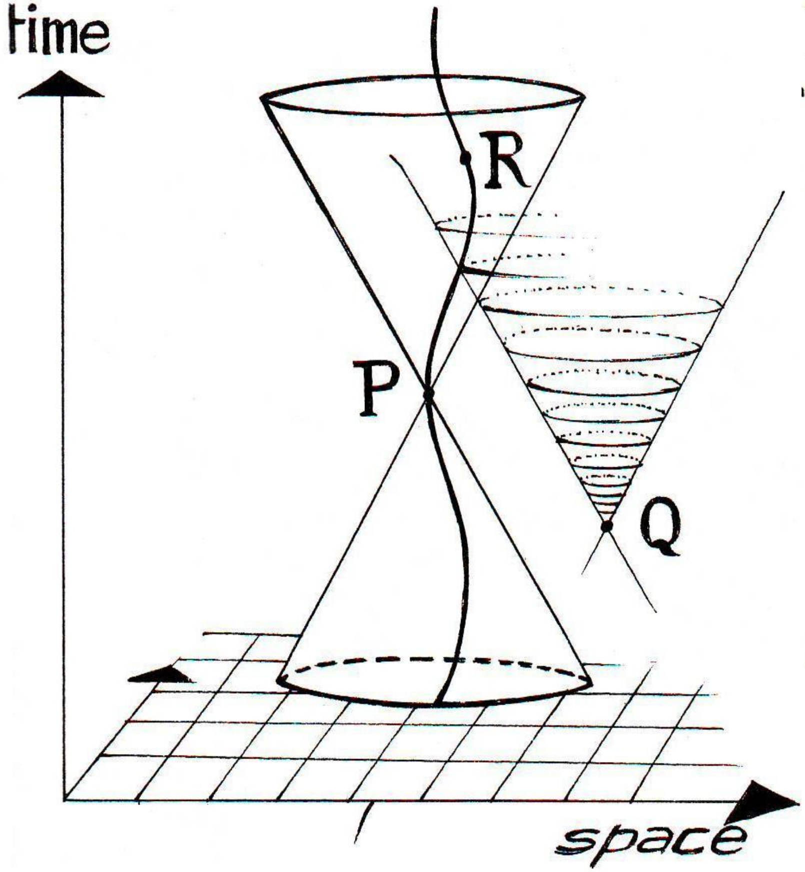
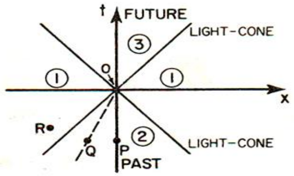


Fig. 17–3. The space-time region surrounding a point at the origin.

Readings





17–3. The space-time region
ding a point at the origin.

17–3 Past, present, and future

The space-time region surrounding a given space-time point can be separated into three regions, as shown in Fig. 17–3. In one region we have space-like intervals, and in two regions, time-like intervals. Physically, these three regions into which space-time around a given point is divided have an interesting physical relationship to that point: a physical object or a signal can get from a point in region 2 to the event O by moving along at a speed less than the speed of light. Therefore events in this region can affect the point O, can have an influence on it from the past. In fact, of course, an object at P on the negative t-axis is precisely in the "past" with respect to O; it is the same space-point as O, only earlier. What happened there then, affects O now. (Unfortunately, that is the way life is.) Another object at Q can get to O by moving with a certain speed less than c, so if this object were in a space ship and moving, it would be, again, the past of the same space-point. That is, in another coordinate system, the axis of time might go through both O and Q. So all points of region 2 are in the "past" of O, and anything that happens in this region *can* affect O. Therefore region 2 is sometimes called the *affective past*, or affecting past; it is the locus of all events which can affect point O in any way.

Region 3, on the other hand, is a region which we can affect *from* O, we can "hit" things by shooting "bullets" out at speeds less than c. So this is the world whose future can be affected by us, and we may call that the *affective future*. Now the interesting thing about all the rest of space-time, i.e., region 1, is that we can neither affect it now *from* O, nor can it affect us now *at* O, because nothing can go faster than the speed of light. Of course, what happens at R *can* affect us *later*; that is, if the sun is exploding "right now," it takes eight minutes before we know about it, and it cannot possibly affect us before then.

What we mean by "right now" is a mysterious thing which we cannot define and we cannot affect, but it can affect us later, or we could have affected it if we had done something far enough in the past. When we look at the star Alpha Centauri, we see it as it was four years ago; we might wonder what it is like "now." yes "Now" means at the same time from our special coordinate system. We can only see Alpha Centauri by the light that has come from our past, up to four years ago, but we do not know what it is doing "now"; it will take four years before what it is doing "now" can affect us. Alpha Centauri "now" is an idea or concept of our mind; it is not something that is really definable physically at the moment, because we have to wait to observe it; we cannot even define it right "now." Furthermore, the "now" depends on the coordinate system. If, for example, Alpha Centauri were moving, an observer there would not agree with us because he would put his axes at an angle, and his "now" would be a *different* time. We have already talked about the fact that simultaneity is not a unique thing.

There are fortune tellers, or people who tell us they can know the future, and there are many wonderful stories about the man who suddenly discovers that he has knowledge about the affective future. Well, there are lots of paradoxes produced by that because if we know something is going to happen, then we can make

17–4

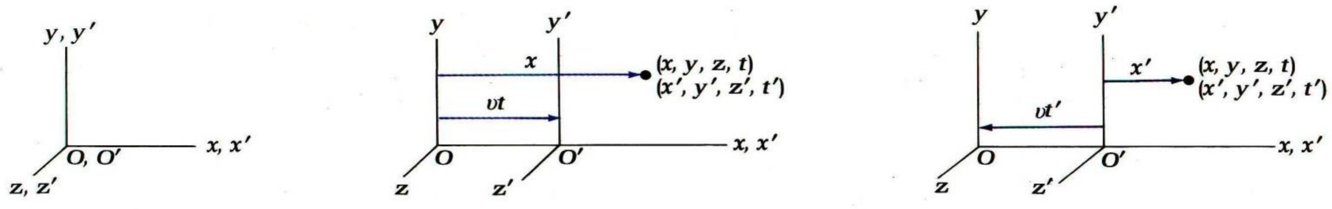
How can we superimpose 2 co-ordinate frames in …

the same space-time diagram for our discussion ?

Recall how we superimpose 2 co-ordinate frames

Consider 2 frames *S* and *S′* moving uniformly wrt to each other with a velocity *v*.

$$S(x, y, z, t) \qquad S'(x', y', z', t')$$



(a) (b) (c)

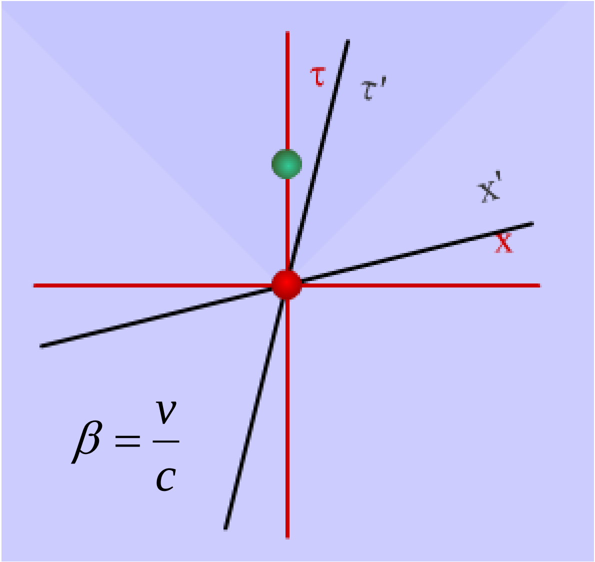
Space-Time diagram: 3 + 1 dimensions

The x - τ axes look normal but the x'- τ' axes are ***somewhat skewed.***

The angle of tilt depends on the velocity *v*.

This means that the Lorentz transformations are "***not orthogonal***"

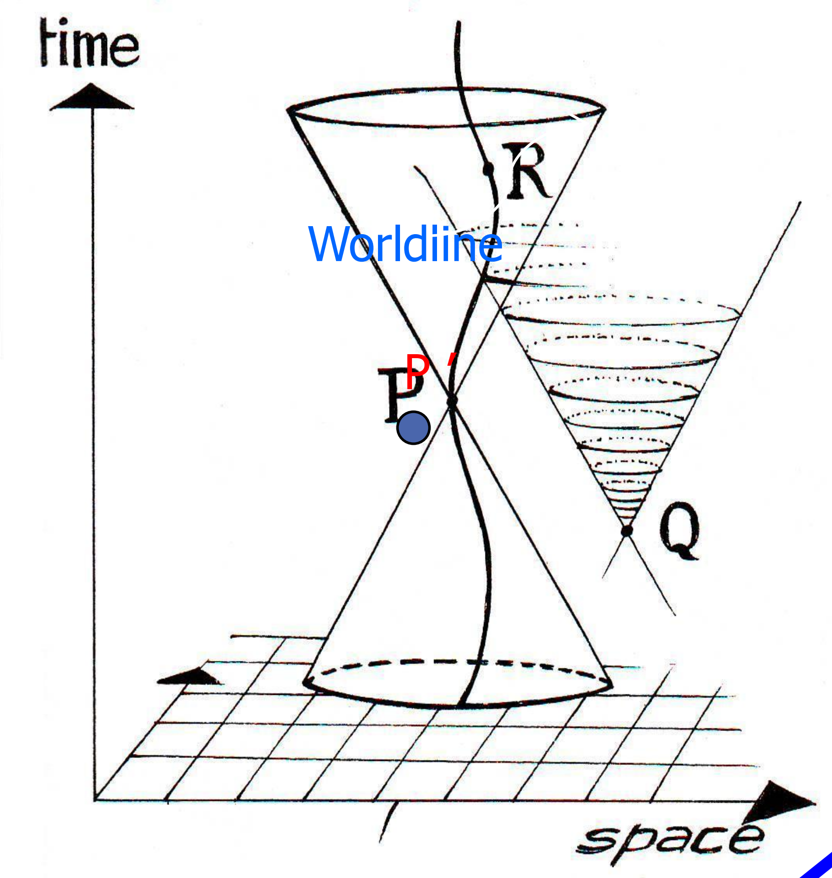
Take some time to ponder !



Give a reason as to why the space-time axes …

may be skewed towards each other ?

Space-time Cones

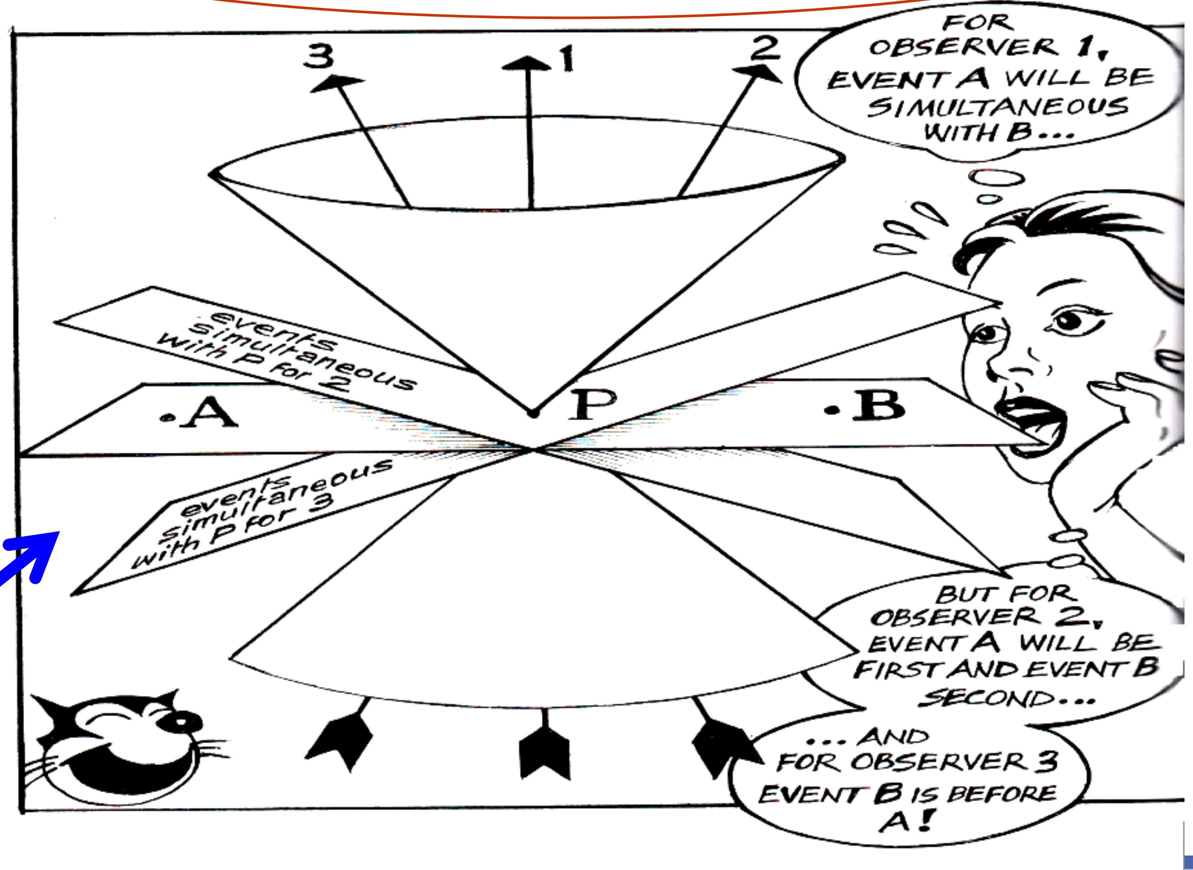


Can you spot the printing mistake !

Time and Observer Dependency

One important fact about lightcones is that they represent the limits of which events can affect one another. Nothing goes faster than light, so anything that will influence you must be travelling either on the lightcone itself (if it's light) or within the lightcone (if it's going slower than light). The same goes for anywhere you hope to go or influence.

Now, we've drawn our picture with one time, but that was merely for the sake of simplicity. The march of time is observer-dependent, to a certain extent. Within one particular observer's lightcone, the order of events is definite. But another observer, moving relative to our first observer, will disagree with the first as to what events are simultaneous with event P.



Conclusion

Reminders again !

2 *really* *Simple* steps to … … Special Relativity Theory

$$\begin{aligned} x' &= (x - vt) \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

$$\begin{aligned} x' &= (x - vt)\gamma \\ y' &= y \\ z' &= z \\ t' &= (t - \tfrac{vx}{c^2})\gamma \end{aligned}$$

Galileo

$$\gamma^2 = \frac{1}{1 - \left(\frac{v}{c}\right)^2}$$

Lorentz

Step 1 : Understand the 2 types of relations & postulates. History and philosophy surrounding them … ether

Step 2 : Repeat Step 1. Einstein: Space and Time must be treated as equal footing