

NATIONAL UNIVERSITY OF SINGAPORE  
Department of Mathematics

MA 1521  
Tutorial 2

1. For each of the following functions:

(a)  $y = \frac{x+1}{x^2+1}, x \in [-3, 3]$       (b)  $y = (x-1)\sqrt[3]{x^2}, x \in (-\infty, \infty)$

determine

- (i) the critical points;
- (ii) the intervals where it is increasing and decreasing;
- (iii) the local and absolute extreme values.

**Ans.** (a) local min.  $-\frac{1}{2(\sqrt{2}+1)}$  at  $x = -1 - \sqrt{2}$  and  $\frac{2}{5}$  at  $x = 3$ ;

local max.  $\frac{1}{2(\sqrt{2}-1)}$  at  $x = -1 + \sqrt{2}$  and  $-\frac{1}{5}$  at  $x = -3$ .

(b) local min.  $-\frac{3}{5}\left(\frac{2}{5}\right)^{2/3}$  at  $x = \frac{2}{5}$ ; local max. 0 at  $x = 0$ .

2. Ornithologists have determined that some species of birds tend to avoid flights over large bodies of water during daylight hours. It is believed that more energy is required to fly over water than land because air generally rises over land and falls over water during the day. A bird with these tendencies is released from an island that is 5 km from the nearest point B on a straight shoreline, flies to a point C on the shoreline, and then flies along the shoreline to its nesting area D. Assume that the bird instinctively chooses a path that will minimize its energy expenditure. Points B and D are 13 km apart. If it takes 1.4 times as much energy to fly over water as land, find the distance between B and C.

**Ans.** 5.1 km.

3. A light shines from the top of a lamp post 15 m high. A ball is dropped from the same height from a point 9 m away from the light. It is known that the ball falls a distance  $s = 4.9t^2$  m in  $t$  seconds. Find the speed of the shadow of the ball on the ground 0.5 second later.

**Ans.** 440.8 m/sec.

4. Use L'Hopital's rule to find the following limits.

(a)  $\lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{1 + \cos 2x}$       (b)  $\lim_{x \rightarrow 0} \frac{\ln(\cos ax)}{\ln(\cos bx)}, a, b > 0$

(c)  $\lim_{x \rightarrow \infty} x \tan \frac{1}{x}$       (d)  $\lim_{x \rightarrow 0^+} x^a \ln x, a > 0$

(e)  $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$       (f)  $\lim_{x \rightarrow 0^+} x^{\sin x}$

(g)  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{1}{x^2}}$

**Ans.** (a)  $\frac{1}{4}$       (b)  $\frac{a^2}{b^2}$       (c) 1      (d) 0      (e)  $e^{-1}$       (f) 1      (g)  $e^{-1/6}$