NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 2 EXAMINATION 2018-2019

MA1521 CALCULUS FOR COMPUTING

May 2019 Time allowed: 2 hours

Question 1 [10 marks]

(a) (Multiple Choice Question)

Let $f(x) = 1384(1-2x)e^{3x}$, $-\infty < x < \infty$. Find the absolute maximum value of f. Give your answer correct to the nearest integer.

(A) 1521 (B) 1001 (C) 2001

(b) Let m and n denote two positive even integers with $m < n$. It is known that the area of the region between the graphs of $y = 2\cos x$ and $y = \sin 2x$ from $x = m\pi$ to $x = (n+1)\pi$ is equal to 8554. Find the **exact value** of $n - m$.

Question 2 [10 marks]

(a) Let $P(x)$ denote the degree two Taylor polynomial of the function $\ln(2+\tan x)$ at $x=0$. Find the value of $P\left(\frac{9}{10}\right)$. Give your answer correct to two decimal places.

(b) Find the directional derivative of the function $f(x,y,z) = 4xyz - 2x^2 + y^2 + z^2 + 321$ at the point $(1,1,2)$ in the direction of the vector which joins $(2,3,1)$ to $(1,2,3)$. Give your answer correct to two decimal places.

Question 3 [10 marks]

(a) It is known that the function $f(x, y) = 3xy - x^2 - y^3 - 5$ has exactly one local maximum point at (a, b). If $a + b = \frac{m}{n}$ where m and n are two positive integers without any common factors, find the **exact value** of $m + n$.

(b) The region R lies above the paraboloid $z = 4 - x^2 - y^2$ and below the paraboloid $z = 8 - 3x^2 - 3y^2$. Find the volume of R. Give your answer correct to two decimal places.

Question 4 [10 marks]

(a) Evaluate $\int_{-2}^{0} \left(\int_{0}^{x^2} e^{\left(y - \frac{1}{3}y^{\frac{3}{2}}\right)} dy \right) dx$. Give your answer correct to two decimal places.

(b)At time $t = 0$ a tank contains 20 pounds of salt dissolved in 120 gallons of water. Assume that water containing 0.5 pound of salt per gallon is entering the tank at a rate of 4 gallons per minute and the well stirred solution is leaving the tank at the same rate. Find the amount of salt in the tank at time $t = 16$ minutes. Give your answer in pounds correct to two decimal places.

Question 5 [10 marks]

(a) Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^2}, \quad \text{with} \quad x > 0, y > 0 \quad \text{and} \quad y(1) = \sqrt{\frac{5}{7}}.$$

Find the value of $y(\frac{3}{2})$. Give your answer correct to two decimal places.

(b) The growth of the sandhill crane population follows a logistic model with a birth rate per capita of 10% per year. Initially at time $t = 0$ there were 1521 sandhill cranes. It is known that at time $t = 10$ years there were 2019 sandhill cranes. How many sandhill cranes will there be after a very long time? Give your answer correct to the nearest integer.