National University of Singapore

Semester 1, 2020/2021 **MA1101R** **Practice Assignment 3**

(a) Use A4 size paper and pen (blue or black ink) or electronic device to write your answers.

(b) Write down your student number and full name clearly on the top left of every page of the answer scripts.

(c) Write the page number on the top right corner of each page of answer scripts.

(d) There are four questions in this worksheet (see next page) with a total of 20 marks.

(e) To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (arrange them in order of the page. Name the pdf file by **StudentNo P3** (e.g. **A123456R P3**). Upload your pdf into the LumiNUS folder Practice 3 submission.

(f) Hand in your answers by the end of this session. **Late submission will not be accepted.**

1. Let $V = \{(w, x, y, z) \mid y = w - x, z = 2w + x\}$ be a subset of $\mathbb{R}^4$.

   (i) [2 marks] Show that V is s subspace of $\mathbb{R}^4$ by expression V as a linear span.

   (ii) [2 marks] Write down a basis for V and $\dim V$.

   (iii) [1 mark] If $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is a subset of V, can we tell whether it is a linearly independent set? Why?

2. Let $S = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ be a basis for $\mathbb{R}^3$.

   (i) [3 marks] Show that $T = \{\boldsymbol{u}_1 + \boldsymbol{u}_2, \boldsymbol{u}_1 - \boldsymbol{u}_2, \boldsymbol{u}_3\}$ is a basis for $\mathbb{R}^3$.

   (ii) [2 marks] Find the transition matrix from T to S. Briefly explain how you get the answer.

3. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$.

   (i) [2 marks] Find a basis for the column space of $\boldsymbol{A}$.

   (ii) [2 marks] Find a basis for the nullspace of $\boldsymbol{A}$. (Show your working.)

   (iii) [2 mark] Extend the basis in (i) to a basis for $\mathbb{R}^4$. (Show your working.)

4. [4 marks] Let $\boldsymbol{C} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & x-2 & 0 & 0 \\ 0 & 0 & x^2 - x - 2 & x+1 \end{pmatrix}$.

   Find all the values of x such that
   (i) $\text{rank}(\boldsymbol{C}) = 1$; (ii) $\text{rank}(\boldsymbol{C}) = 2$; (iii) $\text{rank}(\boldsymbol{C}) = 3$.