NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1521
Tutorial 1

1. Let $f(x) = \frac{6}{x}$ and $g(x) = \sqrt{|3-x|}$. Find an expression for $(g \circ f)(x) - (f \circ g)(x)$.
Ans. $\sqrt{|3 - \frac{6}{x}|} - \frac{6}{\sqrt{|3-x|}}$.

2. Find the first derivatives of the following functions.

(a) $y = \dfrac{ax+b}{cx+d}$ (b) $y = \sin^n x \cos(mx)$

(c) $y = e^{x^2+x^3}$ (d) $y = x^3 - 4(x^2 + e^2 + \ln 2)$

(e) $y = \left(\dfrac{\sin\theta}{\cos\theta - 1}\right)^2$ (f) $y = t\tan(2\sqrt{t}) + 7$

(g) $r = \sin(\theta + \sqrt{\theta+1})$ (h) $s = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$

Ans. (a) $y' = \dfrac{ad-bc}{(cx+d)^2}$ (b) $y' = n\sin^{n-1} x \cos x \cos mx - m\sin^n x \sin mx$

(c) $y' = e^{x^2+x^3}(2x + 3x^2)$ (d) $y' = 3x^2 - 8x$

(e) $y' = -2\sin\theta(\cos\theta - 1)^{-2}$ (f) $y' = \sqrt{t}\sec^2(2\sqrt{t}) + \tan(2\sqrt{t})$

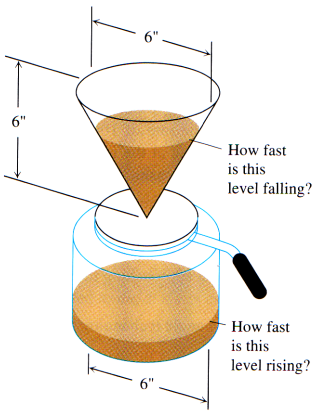
(g) $r' = \frac{2\sqrt{\theta+1}+1}{2\sqrt{\theta+1}}\cos(\theta + \sqrt{\theta+1})$ (h) $s' = 4\tan x \sec x - \csc^2 x$

3. Coffee is drained from a conical filter into a cylindrical coffeepot at the rate of 10 in^3/min.

(a) How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?

(b) How fast is the level in the cone falling then?

(Volume of cone: $\frac{1}{3}\times$ base area $\times$ height)



Ans. (a) $\frac{10}{9\pi}$ in/min; (b) $\frac{8}{5\pi}$ in/min.

4. For the following functions, find y' and y''.

   (a) $x^{2/3} + y^{2/3} = a^{2/3},\ \ 0 < x < a,\ 0 < y$

   (b) $y = (\sin x)^{\sin x},\ \ 0 < x < \frac{\pi}{2}$

   (c) $x = a\cos t,\ y = a\sin t$

Ans. (a) $y' = -\sqrt{(\frac{a}{x})^{2/3} - 1}, \quad y'' = \dfrac{a^{2/3}}{3x^{4/3}\sqrt{a^{2/3} - x^{2/3}}}$.

(b) $y' = (\sin x)^{\sin x}(1 + \ln\sin x)\cos x,$

$y'' = (\sin x)^{\sin x}[(1 + \ln\sin x)^2\cos^2 x + \dfrac{\cos^2 x}{\sin x} - (1 + \ln\sin x)\sin x]$.

(c) $y' = -\cot t\,,\ y'' = -\dfrac{1}{a\sin^3 t}$.