

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1521
Tutorial 5

1. Find the radius of convergence of the following series.

$$\begin{array}{lll} \text{(a)} \sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n} & \text{(b)} \sum_{n=1}^{\infty} \frac{(3x-2)^n}{n} & \text{(c)} \sum_{n=1}^{\infty} (-1)^n (4x+1)^n \\ \text{(d)} \sum_{n=0}^{\infty} \frac{3^n x^n}{n!} & \text{(e)} \sum_{n=1}^{\infty} n^n x^n & \text{(f)} \sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}} \end{array}$$

Ans. (a) 1 (b) 1/3 (c) 1/4 (d) ∞ (e) 0 (f) 1/4

2. Find the sum of the geometric series inside the interval of convergence

$$1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 - + \cdots + \left(-\frac{x-3}{2}\right)^n + \cdots .$$

Ans. $\frac{2}{x-1}$

3. Find the Taylor series for the following functions:

$$\begin{array}{ll} \text{(a)} \frac{x}{1-x} \text{ at } x=0; & \\ \text{(b)} \frac{1}{x^2} \text{ at } x=1; & \\ \text{(c)} \frac{x}{1+x} \text{ at } x=-2; & \end{array}$$

Ans. (a) $\sum_{n=0}^{\infty} x^{n+1}$ (b) $\sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$ (c) $2 + \sum_{n=1}^{\infty} (x+2)^n$

4. Use the Taylor polynomial $P_3(x)$ of $f(x) = \sin x$ at $x = 0$ of order 3 to approximate $\sin(0.1)$.

Ans. 0.09983

5. Let

$$S = \sum_{n=0}^{\infty} \frac{1}{n!(n+2)}.$$

In this question, we will introduce two different ways to find the value of S , one by integration and the other by differentiation.

(i) Integrate the Taylor series of xe^x to show that $S = 1$.

(ii) Differentiate the Taylor series of $\frac{e^x-1}{x}$ to show that $S = 1$.