

NATIONAL UNIVERSITY OF SINGAPORE
CS1231/CS1231S – Discrete Structures
 (AY2019/20 Semester 1)

ANSWER SHEET

Time Allowed: 1 hour 20 minutes

FOR EXAMINERS' USE ONLY		
Questions	Marks	Obtained
Q11	12	
Q12	9	
Q13	9	
Total	30	

INSTRUCTIONS TO CANDIDATES

1. This Answer Sheet consists of **TWO (2)** printed pages.
2. Fill in your Tutorial Group Number (eg: T12) and Student Number below with a pen. Do not write your name.
3. You may write your answers in pencil.
4. Write within the boxes provided. Illegible handwriting and unnecessarily long answers will be penalized.

TUTORIAL GROUP:

STUDENT NUMBER:

A								
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(Write your Student Number legibly with a pen to prevent accidental erasure.)

MCQ answers:

1. **A** 2. **D** 3. **D** 4. **C** 5. **A** 6. **D** 7. **D** 8. **B** 9. **B** 10. **E**

Q11. Logic [12 marks]

- (a) $\forall x (Bird(x) \wedge Fly(x))$ [3 marks]

All animals are birds and (all animals) can fly.

- (b) "There is no free lunch." [3 marks]

$\forall x (\sim Free(x) \vee \sim Lunch(x))$ or $\forall x (Lunch(x) \rightarrow \sim Free(x))$

- (c) "Every child irritates his or her parent." [3 marks]

or $\forall x (Child(x) \rightarrow \exists y (Parent(y, x) \wedge Irritate(x, y)))$
 $\forall x (Child(x) \rightarrow \forall y (Parent(y, x) \rightarrow Irritate(x, y)))$

- (d) "For every odd integer there is a different integer such that the sum of these two numbers is even." [3 marks]

$\forall x (\sim Even(x) \rightarrow \exists y (x \neq y \wedge Even(x + y)))$

Q12. $A - (B - C) = (A - B) \cup (A \cap C)$.

[9 marks]

$$\begin{aligned}
 A - (B - C) &= A - (B \cap \bar{C}) && \text{by the Set Difference Law;} \\
 &= A \cap \overline{(B \cap \bar{C})} && \text{by the Set Difference Law;} \\
 &= A \cap (\bar{B} \cup \bar{\bar{C}}) && \text{by the De Morgan's Law;} \\
 &= A \cap (\bar{B} \cup C) && \text{by the Double Complement Law;} \\
 &= (A \cap \bar{B}) \cup (A \cap C) && \text{by the Distributive Law;} \\
 &= (A - B) \cup (A \cap C) && \text{by the Set Difference Law;}
 \end{aligned}$$

Q13. $\sum_{i=1}^n i(i+1)(i+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$.

[9 marks]

1. For each $n \in \mathbb{Z}^+$,
let $P(n) = \left(\sum_{i=1}^n i(i+1)(i+2) = \frac{1}{4}n(n+1)(n+2)(n+3) \right)$.
2. **Base step:** $P(1) = \left((1)(2)(3) = 6 = \frac{1}{4}(1)(2)(3)(4) \right)$, which is true.
3. **Inductive step:**
Assume $P(k)$, i.e. $\sum_{i=1}^k i(i+1)(i+2) = \frac{1}{4}k(k+1)(k+2)(k+3)$.
4. Then $P(k+1)$:

$$\begin{aligned}
 &\sum_{i=1}^{k+1} i(i+1)(i+2) \\
 &= \sum_{i=1}^k i(i+1)(i+2) + (k+1)(k+2)(k+3) \\
 &= \frac{1}{4}k(k+1)(k+2)(k+3) + (k+1)(k+2)(k+3) \quad (\text{applying } P(k)) \\
 &= \left(\frac{1}{4}k+1\right)(k+1)(k+2)(k+3) \quad (\text{basic algebra}) \\
 &= \frac{1}{4}(k+4)(k+1)(k+2)(k+3) \quad (\text{basic algebra})
 \end{aligned}$$
5. Therefore, $P(k+1)$ is true.
6. By MI, $P(n)$ is true for all $n \in \mathbb{Z}^+$.

=== END OF PAPER ===