GEH1027
Einstein's Universe and Quantum Weirdness

Tutorial 4

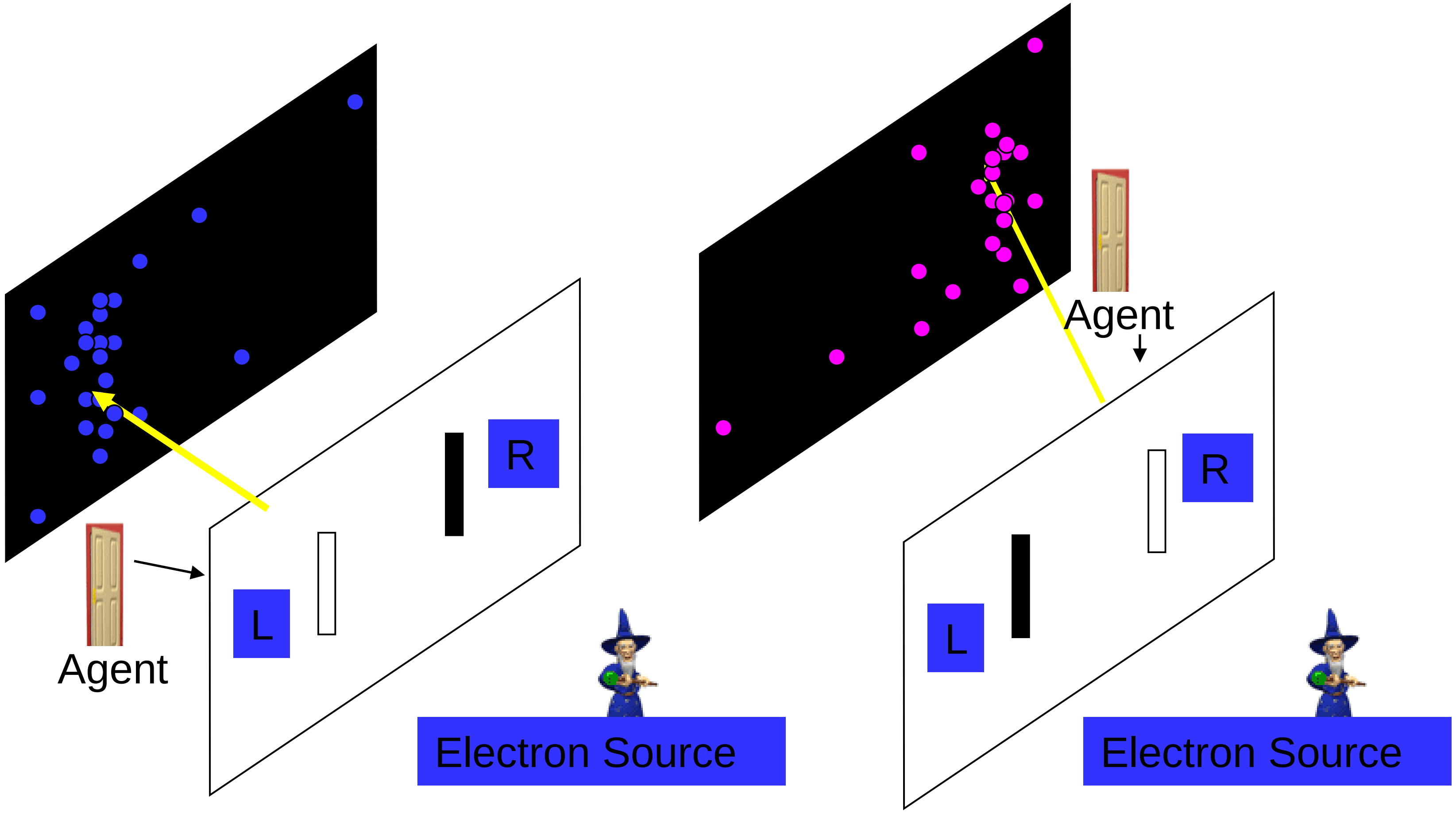
AY2020/21 Sem II

Tutorial 4 – menu

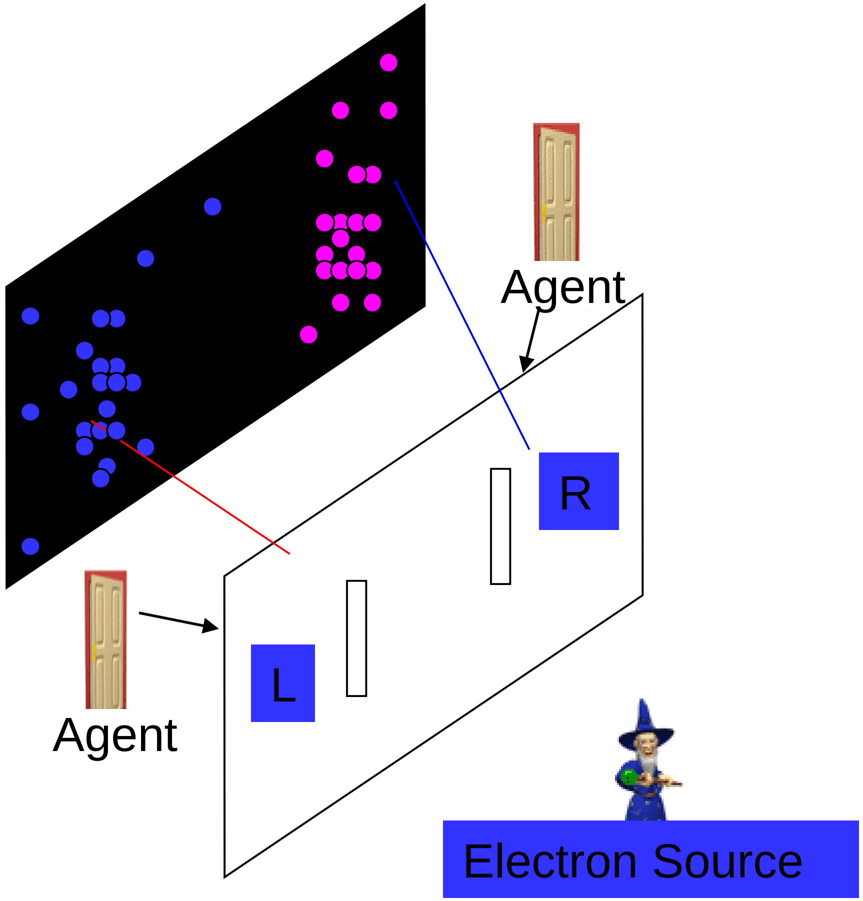
1. Two-slit experiment, electron microscope, Heisenberg’s Uncertainty Principle
2. Bohr’s atomic model (Epistemic questions)
3. Significance of selected phenomena/experiments
4. Why do quantum particles have variable mass?
5. Mach-Zehnder Interferometer
6. (Optional) Solvay Conference
7. (Optional) Solvay Conference - *gedanken*

1 **Present** to a friend the following Quantum Mysteries **clearly.**

(a) The 2 slits Interference experiment for Electrons with spice agents
https://www.youtube.com/watch?v=Q1YqgPAtzh (watch it at home)



Electrons behave like bullets when only 1 slit is opened. When both slits are opened (no agents), we know there will be an interference pattern. What happens when there are agents???



When both slits are opened (with agents), there is no interference pattern!

When the agents are present, electrons behave like particles.

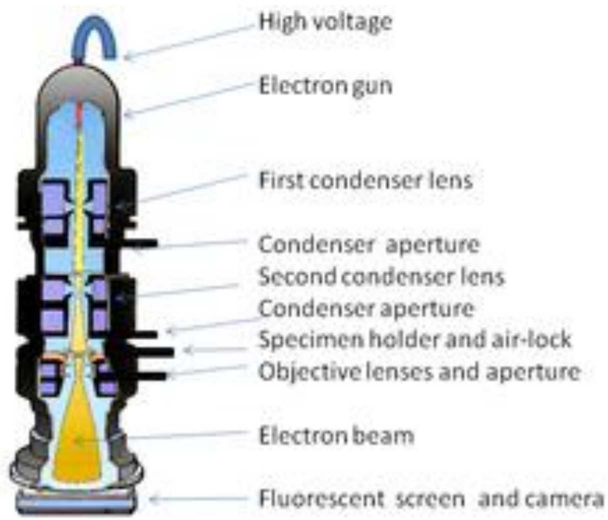
Agents interrupt the behaviour of the system!

Are electrons conscious? The act of measurement creates “Physical REALITY”.
The act of measurement collapses the wavefunction.

“Consciousness” of observer plays a crucial part in formulation of Q.M. (E.P Wigner)

1(b) Electron Microscope and Heisenberg Uncertainty Principle.

http://www.youtube.com/watch?v=KT7xJ0tjB4A (watch it before Tutorial class)



Transmission Electron Microscope

The resolving power of optical microscopes is limited by the wavelength of light: $\lambda \sim 10^{-7}$ m

Electrons can have a shorter wavelength: $\lambda \sim 10^{-12}$ m

de Broglie's formula: $\lambda = \frac{h}{p}$

Note: Δx and Δp are for measurements taken at the same time t.

$\Delta x(t_1)\Delta p(t_2) = 0$ if $t_1 \neq t_2$.

Discuss how $\Delta x \Delta p \approx \hbar$ is related to $\Delta E \Delta t \approx \hbar$.

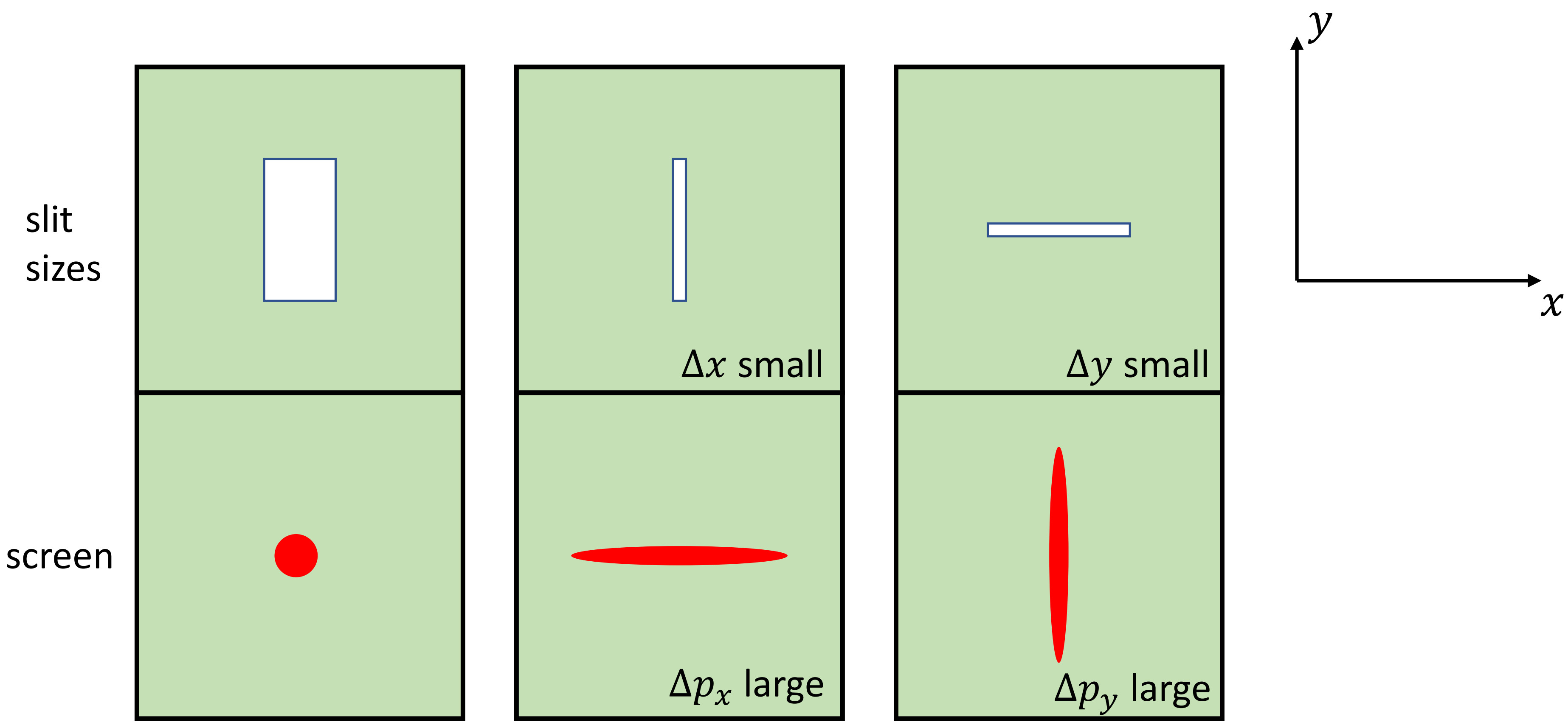
Recall $E = pc$.

$$\Delta x \Delta p = \Delta x \frac{\Delta E}{c} = \frac{\Delta x}{c} \Delta E = \Delta t \Delta E \approx \hbar$$

Energy-time uncertainty and position-momentum uncertainty are actually quite different…

Give an example of $\Delta E \Delta t \approx \hbar$ at work in a micro quantum system: Transition in atomic systems

1(b) http://www.youtube.com/watch?v=KT7xJ0tjB4A (watch it before Tutorial class)



Heisenberg Uncertainty principle: We cannot measure position and momentum **simultaneously**.

The more we know about the position, the less we know about the momentum.

$$\boxed{\Delta \mathrm{x} \Delta p_x \geq \hbar; \;\; \Delta y \Delta p_y \geq \hbar; \;\; \Delta y \Delta p_x = 0; \;\; \Delta x \Delta p_y = 0}$$

2 Recall the 3 Epistemic Problems posted in Lecture 11 and see if you could **use Bohr's atomic model** to throw light into the following questions.

(a) Why does the electron stay in the excited energy level for an unspecified period before it descends to a lower energy level? What is the physical mechanism that makes it wait?

(b) Why the law of conservation of energy does not seem to hold true at these interim times of transition between states?

(c) What is the physical mechanism that causes the creation of a photon, when the electron arrives at the lower energy level?

Discuss on forum…

Hint: Bohr's atomic model is not accurate! So here, we're really using a not-so-accurate model to come up with an explanation for the above questions.

How then, do we properly explain them?

3 <u>To "Think" about it only</u>

Write **very briefly** (2 lines only) the significance for each of the following phenomena.

(a) Planck's Blackbody Radiation **(b)** Rutherford Scattering

(c) Einstein's Photoelectric effect **(d)** Franck-Hertz Experiment

(e) Compton's experiment

a) Energy is quantized

b) Discovery of the nucleus

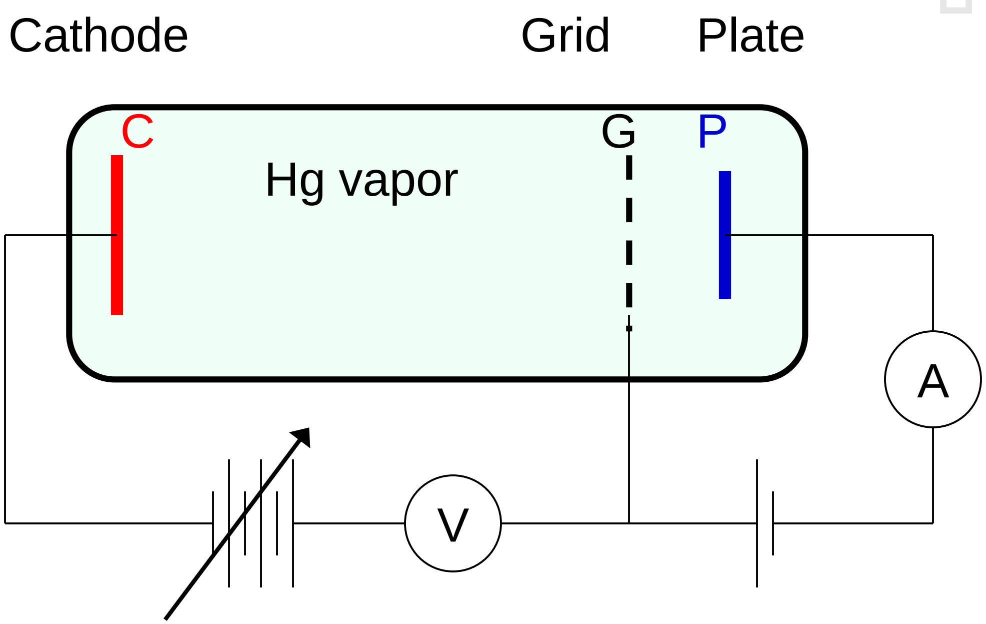
c) Light behaves like a particle (with energy $E = h\nu$)

d) Experimental proof for the existence of the discrete energy levels

e) Light behaves like a particle and bounces off electron

Franck-Hertz Experiment

- The first experiment to provide support for the discrete energy levels was performed by James Frank and Gustav Hertz (nephew of Heinrich Hertz) in 1914. The setup is similar to the one below.



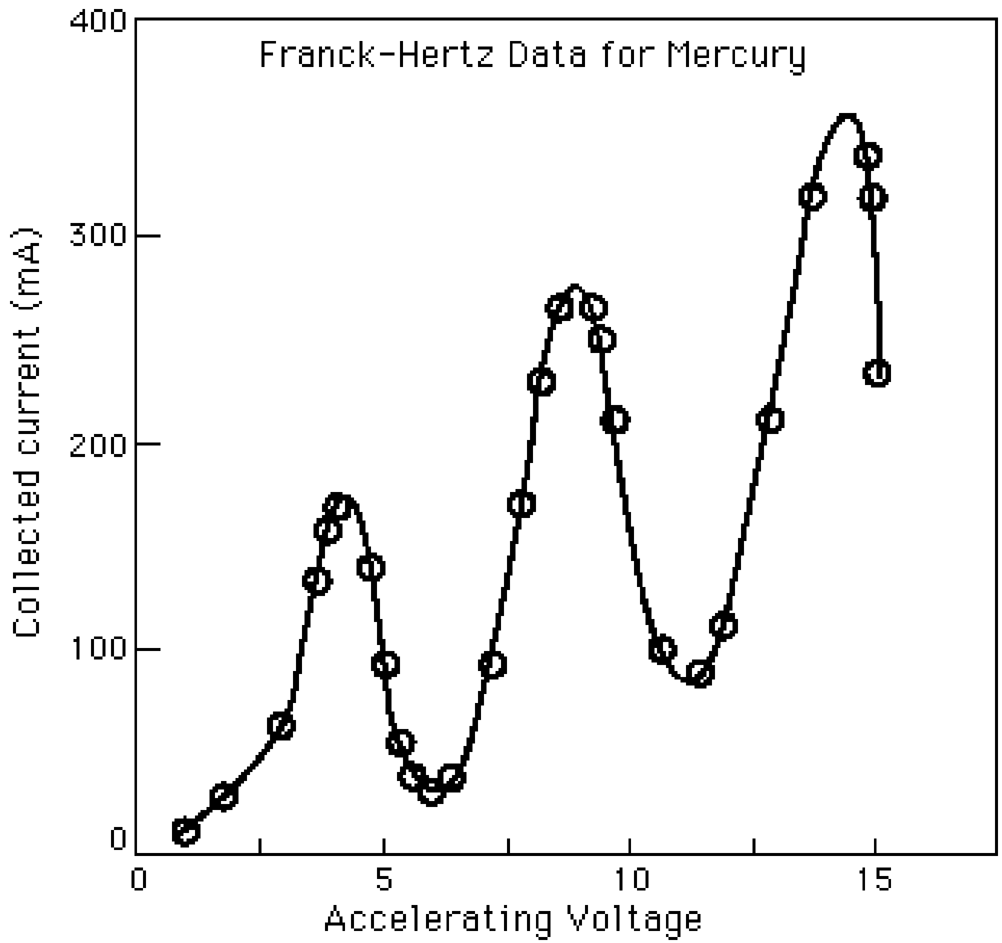
- The cathode C is heated (not shown) and emitted electrons are accelerated toward grid G. They gained KE = eV. Some passed through the grid and collected by plate P, showing up as current I. Plate P is at slightly lower potential than G.

- The tube is filled with mercury vapor. Electrons collided with Hg atoms. If the collision is elastic, the current I is not affected by the vapor as the electrons lose very little KE ($m_{atom} >> m_{electrons}$.)

Franck-Hertz Experiment

- When $V < 4.9$ V, I increases with increasing V as expected, as the collisions are elastic and the electrons have enough KE to overcome the slight reversed bias between G and P.

- When $V = 4.9$ V, current drops drastically as the electrons suffered inelastic collisions and transferred most of its KE to the atoms (raising them to their first excited state) and the remaining KE is not enough for the electrons to P.



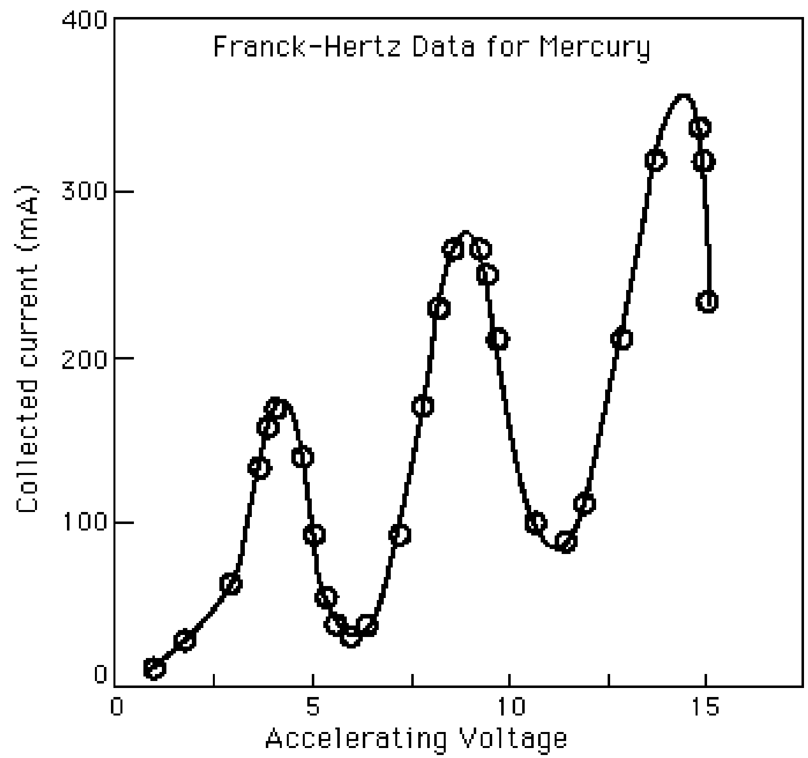
Franck-Hertz Experiment

- Frank and Hertz also noted that at this voltage the 253.6 nm spectral line of mercury appeared in the emission spectrum. This corresponds to photons of energy:

$$E = \frac{hc}{253.6 \times 10^{-9}} = 7.838 \times 10^{-19}\ \mathrm{J} = 4.89\ \mathrm{eV}$$

which was emitted when the atoms drop from the first excited state back to their ground state.

- The behavior was repeated at multiples of 4.9 V, indicating the electrons exciting two or more atoms to their first excited states.

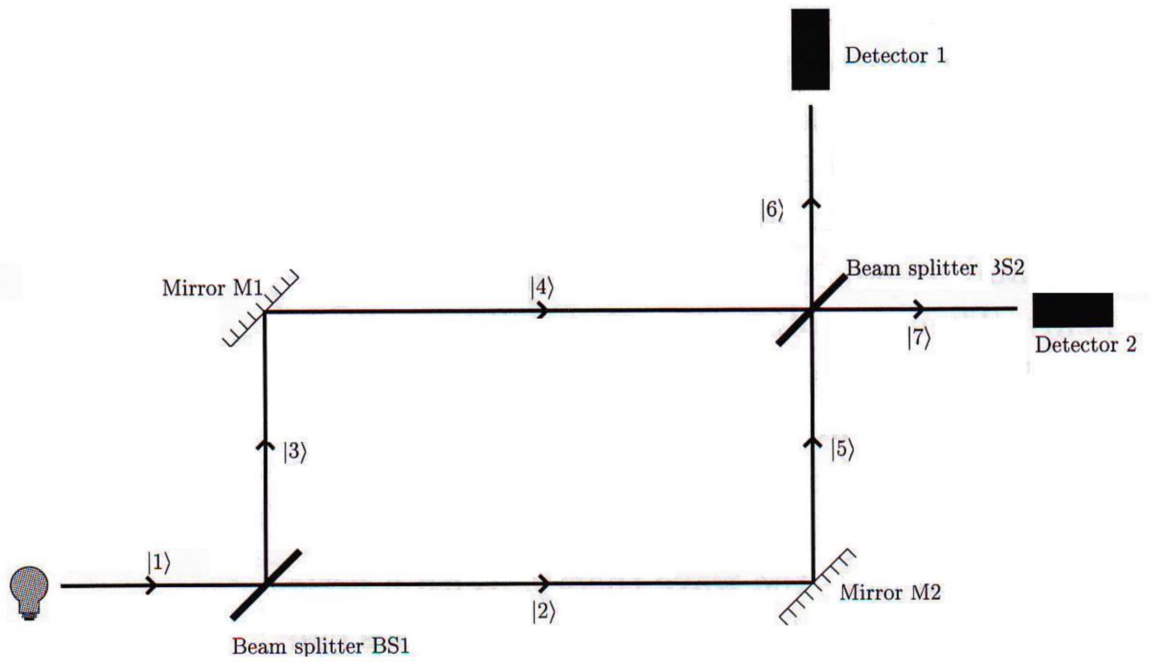


4 It is often said that a quantum particle has variable mass (i.e. not fixed mass). Does this surprise you as opposed to the classical particle? Can you give a reason? Hint: Use the appropriate Heisenberg Uncertainty Principle.

- Use the uncertainty relation for energy and time: $\Delta E \Delta t \approx \hbar$

- Uncertainty in energy is non-zero. Since mass and energy are related ($E = mc^2$), the uncertainty in mass is also non-zero ($\Delta E = (\Delta m)c^2$). Thus a quantum particle has variable mass.

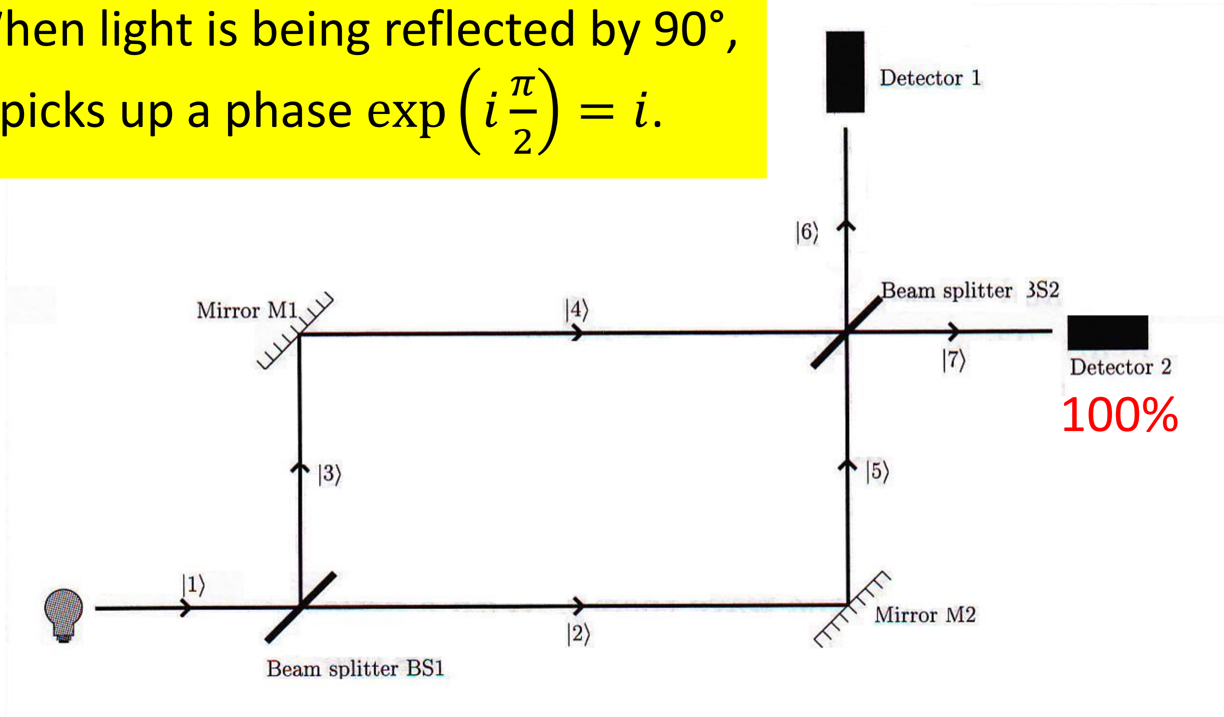
5 Consider the Mach-Zehnder Interferometer with 2 beam splitters (BS) below. When light is detected, detector 1 or detector 2 will click. We want to investigate what will happen when a **single photon** (weak light source) enters this interferometer.



(a) Reason which detector will click more often and why?

Intuitively, we may conclude that both detector 1 and detector 2 will click with the same rate (i.e. 50% - 50% each). Is that true…?

Quantum theoretical point of view:
When light is being reflected by 90°, it picks up a phase $\exp\left(i\frac{\pi}{2}\right) = i$.



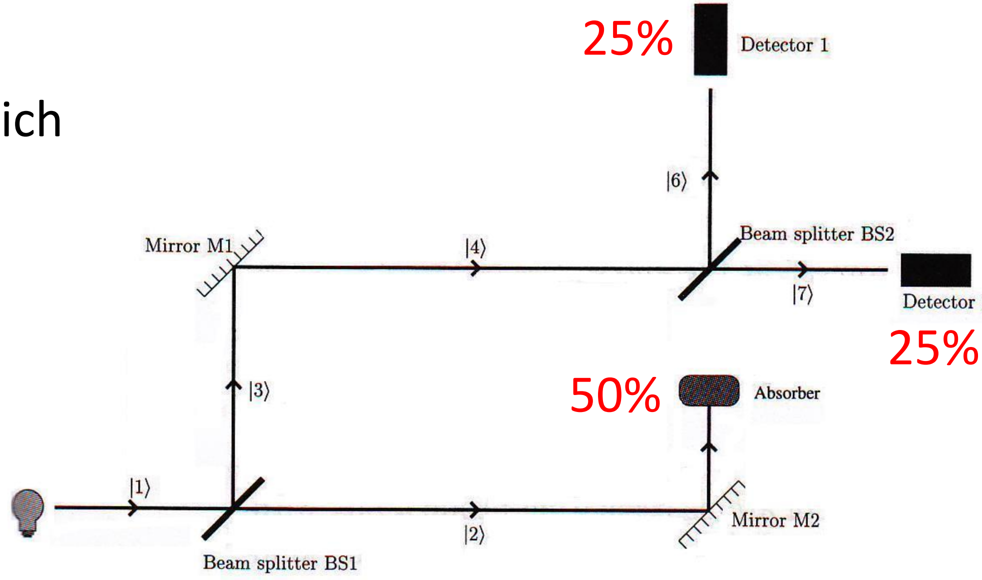
- This means that detector 2 will <u>always</u> click. (Different from intuitive prediction!)
- This experiment is realisable. We are forced to conclude that Quantum Theory is richer than classical probability.

$$|1\rangle \xrightarrow{BS1} \underbrace{\left(\frac{1}{\sqrt{2}}\right)}_{\text{Normalization}} \left(|2\rangle + i|3\rangle\right) \xrightarrow{M1,M2} \left(\frac{1}{\sqrt{2}}\right)\left(i|5\rangle + i\left(i|4\rangle\right)\right)$$

$$\left(\frac{1}{\sqrt{2}}\right)\left(i|5\rangle - |4\rangle\right) \xrightarrow{BS2} \left(\frac{1}{\sqrt{2}}\right)\left[\underbrace{i\frac{1}{\sqrt{2}}\left(|6\rangle + i|7\rangle\right)}_{=|5\rangle} - \underbrace{\frac{1}{\sqrt{2}}\left(i|6\rangle + |7\rangle\right)}_{=|4\rangle}\right] \boxed{= -|7\rangle}$$

5(b) We now block path 5 with a light absorber. Reason carefully again which detector will click more often and why?

Detector 1 and detector 2 click with equal probabilities. Oh dear, something strange is happening… Classical approach cannot explain the single photon weirdness….



$$|1\rangle \xrightarrow{\text{BS1}} \left(\frac{1}{\sqrt{2}}\right)\left(|2\rangle + i|3\rangle\right) \xrightarrow{\text{M1,M2}} \left(\frac{1}{\sqrt{2}}\right)\Big(\underbrace{i|5\rangle}_{\text{"blocked"}\to 0} + i\left(i|4\rangle\right)\Big)$$

$$\overset{\text{Normalization}}{\left(\frac{-1}{\sqrt{2}}\right)}|4\rangle \xrightarrow{\text{BS2}} \left(\frac{-1}{\sqrt{2}}\right)\left(\underbrace{\frac{1}{\sqrt{2}}\left(i|6\rangle + |7\rangle\right)}_{=|4\rangle}\right) = \left(\frac{-1}{2}\right)\left(i|6\rangle + |7\rangle\right)$$

Overall final state = $\left(\frac{-1}{2}\right)\left(i|6\rangle + |7\rangle\right) \boxed{+\frac{i}{\sqrt{2}}|5\rangle}$

photons blocked at absorber

The normalisation factor (for those interested)

Let the input *ket* state be: $|1\rangle = a(|2\rangle + i|3\rangle)$

where a is an arbitrary complex number in general.

Bra is the adjoint or the complex conjugate of *ket* and vice versa.

The *bra* state would be $\langle 1| = a^*(\langle 2| - i\langle 3|)$.

Total probability is $|\psi|^2 = \langle 1|1\rangle = 1$

$$1 = \langle 1|1\rangle = |a|^2[\langle 2|2\rangle + i\langle 2|3\rangle - i\langle 3|2\rangle + \langle 3|3\rangle]$$

Using $\langle 2|3\rangle = 0$, $\langle 3|2\rangle = 0$ and $\langle 2|2\rangle = \langle 3|3\rangle = 1$,

$$\therefore |a|^2 = \frac{1}{2} \Rightarrow a = \frac{1}{\sqrt{2}}$$

In other words, the sum of “squares” of numbers in front of the states (wavefunctions) must be 1.

For instance, for an arbitrary state (expressed in different notations below)

$$|\psi\rangle = a|1\rangle + b|2\rangle$$

$$\psi = a\psi_1 + b\psi_2$$

we must have $a^*a + b^*b = 1$.

The probability to find $|1\rangle$ or ψ_1 is a^*a.

The probability to find $|2\rangle$ or ψ_2 is b^*b.

Question 5 (cont'd): some comments about the Mach-Zehnder Interferometer

- If we do not know the path taken by the photon like in case (a), we know for sure where the photon will go (detector 2).

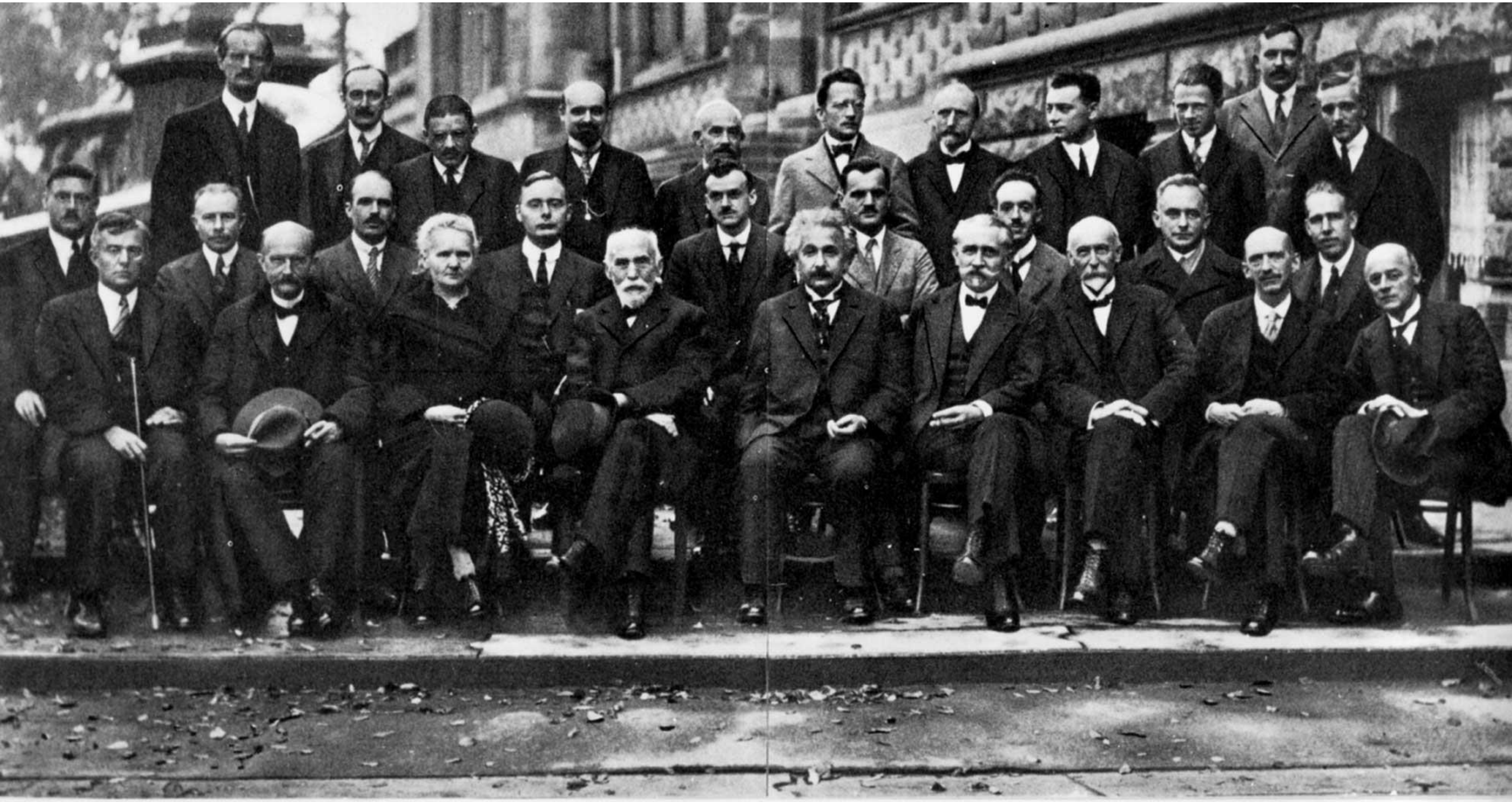
- If we know the path taken by the photon like in case (b), we lose the knowledge of the destination of the photon.

- How does the photon know the existence of the absorber?

- It seems that single photon has non-local property? It has some knowledge of the presence of the absorber?

- If the two arms of the interferometer are separated very far away, perhaps faster than speed of light scenario would seem possible in quantum mechanical description?

- Note: the same story can be repeated with electrons!
  (We describe electrons with a wavefunction – a quantum mechanical concept)



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R.H. FOWLER

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P. EHRENFEST   E. VERSCHAFFELT

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I. LANGMEIR   M. PLANCK   MADAME CURIE   H. A. LORENTZ   A. EINSTEIN   P. LANGEVIN   CH. E. GUYE   C.T.R. WILSON

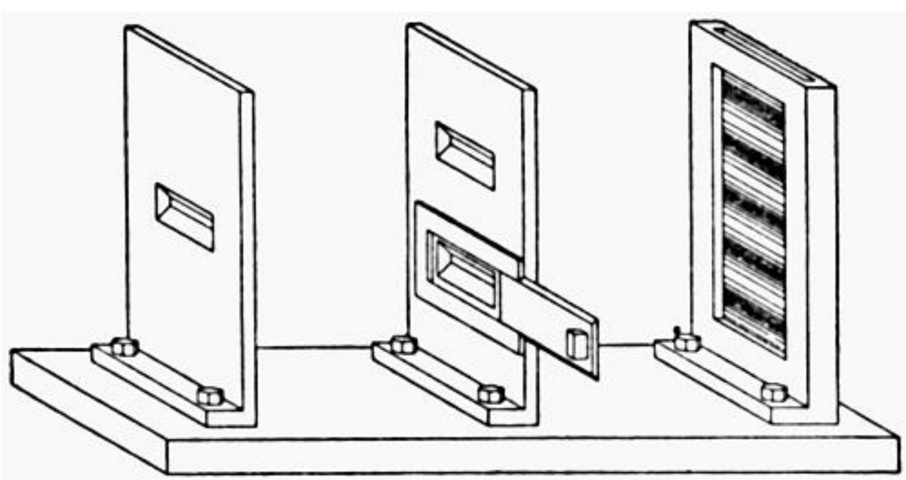
O.W. RICHARDSON

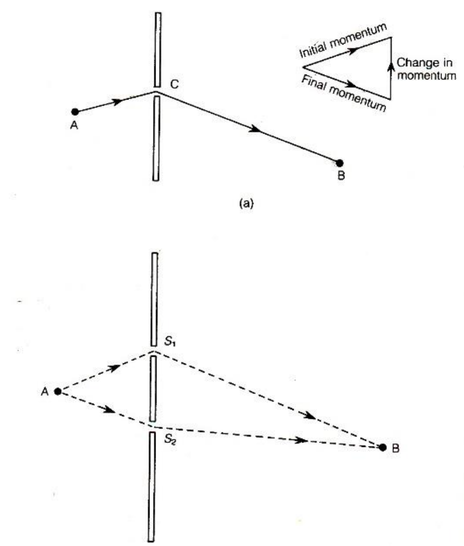
(Optional questions)

6 Recall the 2 slits experiment discussed in class.
At the 5th Solvay conference in 1927 (see picture below), Einstein raised doubts regarding Bohr's Quantum interpretation of the 2 slits experiment.

Einstein's doubt:

"When it (the photon) passes through the slit, it would impart a momentum to the slit arrangement. When arriving at a particular point on the target screen, the momentum transferred to the screen would be different for the 2 slits. By measuring the momentum, I know which slit the photon went through. So you cannot tell me that interference is produced only when we don't know anything about how photons go through the slits because here we have interference and I also know which slit the photon came through."





Can you comment or rebut Einstein's objection with the help of the figure below supplied by Bohr?

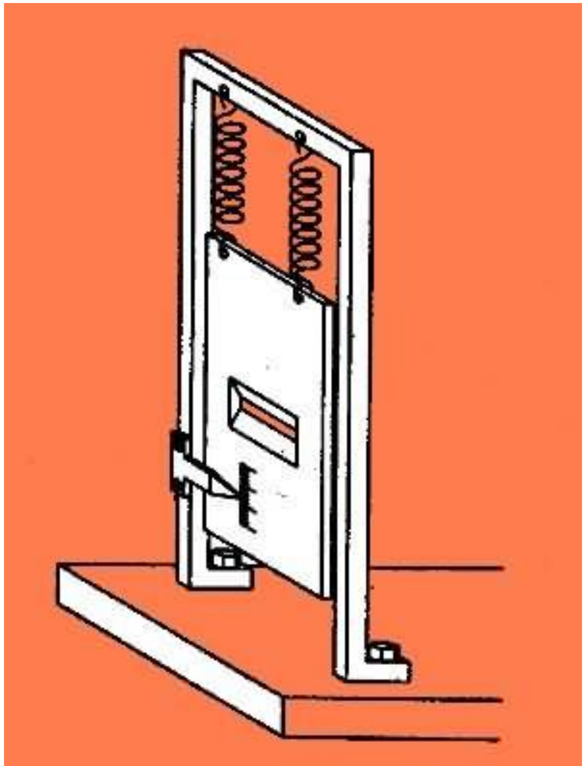


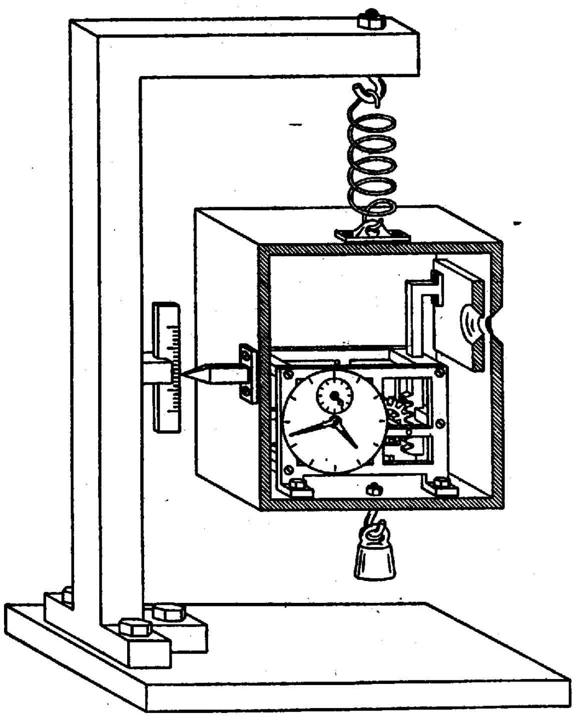
FIG. 5

Bohr attached a scale and a pointer to the slit (figure above) so that one can measure the momentum of the slit by observing how much the pointer moves.

- One can measure the momentum of the slit by observing how much the pointer moves. <u>This observation is a measurement of the position</u> of the pointer. However from Heisenberg's uncertainty principle ($\Delta x \Delta p \approx \hbar$), this measurement will <u>upset the momentum of the slit</u> (Δx small, Δp large).

- The **very act of taking the pointer reading disturbs the slit and alters whatever momentum it might have acquired** when the particle went through it. Thus, we cannot (in 2 slit experiment) ever say what the deflection of the particle is and through which slit it arrived at the screen.

7 At the 6th Solvay conference in 1930, Einstein again raised doubts but this time regarding the Uncertainty principle, $\Delta E \Delta t \approx \hbar$.
He considered the *gedanken* (a thought experiment) given in the figure below, where we have a box installed with a shutter and a clock.



According to Einstein:

"At t_1 the clock opens the shutter releasing one photon out. The shutter is closed at t_2. By making $t = t_2 - t_1$ very small, we will know with high accuracy, the time when the photon escaped. In summary, the error Δt in t can be made as small as possible as we please.

How much energy went out with the photon? All we have to do is to compare the weight of the box before and after the photon has escaped. One would know accurately the energy of the photon.

So you see in the above both ΔE and Δt can be independently be made practically zero, violating the sacred uncertainty law, $\Delta E \Delta t \sim \hbar$"

Is there a problem in the above argument? Bohr had a splendid reply to Einstein.

- Bohr's argument made use of Einstein's theory of general relativity to refute Einstein!

- Suppose the pointer is at the zeroth position before the photon escapes. After the photon escapes, the box will be lighter by $E = mc^2$.

- Approach: try to "squeeze" $\Delta x \Delta p$ out of $\Delta E \Delta t$, then conclude that $\Delta x \Delta p \approx \hbar$

To find ΔE:

Uncertainty in force, $\Delta F = \frac{\Delta p}{t}$

Uncertainty in mass, $\Delta m = \frac{\Delta F}{g} = \frac{\Delta p}{tg}$

Uncertainty in energy, $\Delta E = (\Delta m)c^2 = \frac{c^2 \Delta p}{tg}$

Recall:

$$F = ma \rightarrow m = \frac{F}{a}$$

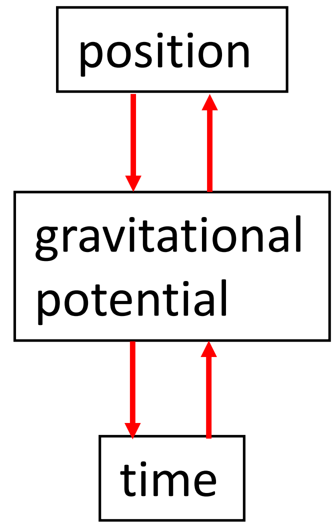
To find $\boldsymbol{\Delta t}$:

$$\frac{GMm}{r^2} = mg \Rightarrow \frac{GM}{r} = gr \Rightarrow V = gr$$

- According to general relativity, the time measured by the clock is affected by the gravitational potential acting on it.

- Uncertainty in position of clock
  - Uncertainty in gravitational potential, $\Delta V = g\Delta r$
  - (Fractional) uncertainty in the ticking of clock = $g\Delta r/c^2$
  - Uncertainty in time $\Delta t = t\left[\frac{g\Delta r}{c^2}\right]$

Time dilation due to GR $\approx V/c^2$



- Combining we find $\Delta E\Delta t$ to be

$$\Delta E\Delta t = \frac{c^2\Delta p}{tg}\cdot\frac{tg\Delta r}{c^2} = \Delta p\Delta r \geq \hbar$$

- Einstein's thought experiment did not refute the uncertainty principle, but rather <u>confirmed</u> it!