

1. Find the exact value of

$$\lim_{x \rightarrow (-1)} \frac{1 + x^{1521}}{1 + x}.$$

Answer 1521

2. Let a denote a positive constant. Let L denote the tangent line to the curve

$$y = \frac{a - \sqrt{x}}{a + \sqrt{x}}$$

at the point $(a^2, 0)$. If L passes through the point $(-1, \frac{2020}{1521})$, find the value of a . Give your answer correct to two decimal places.

Answer 0.48

3. Let a denote a positive constant. The torch on the Statue of Liberty is 93 metres above the ground. At time $t = 0$ a ball is dropped from the same height as the torch at a distance a metres from the torch. It is known that the ball falls a distance of $4.9t^2$ metres at time t seconds. If the speed (i.e. the absolute value of the velocity) of the shadow of the ball on the ground is $\sqrt{1521}$ metre per second at the moment just before the ball hits the ground, find the value of a . Give your answer correct to two decimal places.

Answer 84.95

4. Let a denote a positive constant. Let C denote the Cissoid which has equation $r = \frac{2a \sin^2 \theta}{\cos \theta}$ in polar coordinates. Let L denote the tangent line to C at the point when $\theta = \frac{\pi}{3}$. If L passes through the point $(0, -2020)$ in Cartesian coordinates, find the value of a . Give your answer correct to two decimal places.

Answer 388.75

5. Let P denote the point $(1521, 2020)$. Let L denote a straight line that passes through P . It is known that L intersects the positive x -axis at A and L intersects the positive y -axis at B . Find the smallest possible length of the line segment AB . Give your answer correct to the nearest integer.

Answer 4991

6. Let m and n denote two positive integers with $m+n=3$. Find the smallest possible value of the integral

$$\int_0^{\frac{\pi}{3}} \cos^m x \sin^n x dx.$$

Give your answer correct to two decimal places.

Answer 0.22

7. Let $g(t)$ denote a continuous function which satisfies

$$g(2) = 2, \quad g(4) = 3, \quad g(6) = 4, \quad \int_0^2 g(t)dt = 4, \quad \int_0^6 g(t)dt = 18$$

and $\int_4^6 g(t)dt = 6$. Let

$$f(x) = \int_0^{x^2} x^3 g(t)dt.$$

Find the exact value of $f'(2)$.

Answer 240

8. Let a denote a positive constant with $a > 1$. If

$$\tan \left(\int_{\ln a}^{\ln(8a)} \frac{e^x}{e^{2x} + 1} dx \right) = 0.1521$$

find the value of a . Give your answer correct to two decimal places.

Answer 5.73

9. Let a denote a positive constants. Let R denote the finite region in the first quadrant bounded between the x -axis, the y -axis, the line $x = a$ and the curve $y = \frac{1}{4a^2 - x^2}$. If the area of R is equal to 1.521, find the value of a . Give your answer correct to two decimal places.

Answer 0.18

10. Let a denote a positive constant. Let C denote the curve

$$y = a \ln x.$$

Let L denote the tangent line at a point on C in the first quadrant such that L passes through the origin. Let R denote the finite region in the first quadrant bounded by the x -axis, the curve C and the line L . If the volume of the solid of revolution obtained by rotating R one complete round about the y -axis is equal to 1521, find the value of a . Give your answer correct to two decimal places.

Answer 661.85