National University of Singapore

Semester 1, 2020/2021   MA1101R   Practice Assignment 1

(a) Use A4 size paper and pen (blue or black ink) to write your answers.

(b) Write down your student number and full name clearly on the top left of every page of the answer scripts.

(c) Write the page number on the top right corner of each page of answer scripts.

(d) There are four questions in this worksheet (see next page) with a total of 20 marks.

(e) To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (arrange them in order of the page. Name the pdf file by **<u>MatricNo P1</u>** (e.g. **A123456R P1**). Upload your pdf into the LumiNUS folder <u>Practice 1 submission</u>.

(f) Hand in your answers by the end of this session. **Late submission will not be accepted.**

1. A certain linear system has the augmented matrix

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \\ 0 & 2 & 2 & c \end{array}\right)$$

for some real numbers a, b, c.

   (i) [3 marks] Reduce the augmented matrix to a row echelon form using <u>two</u> elementary row operations (show the two e.r.o. in your working.)

   (ii) [3 marks] Write down the condition in terms of a, b, c (if possible) for the system to have (a) no solution; (b) only one solution; (c) infinitely many solutions.

   (iii) [2 marks] If the above linear system is a homogeneous system in variables x, y, z (in that order), write down a general solution of this system.

2. [4 marks] Let

   $\boldsymbol{A} = (a_{ij})_{2\times 3}$ with $a_{ij} = 2i - j$ and $\boldsymbol{B} = (b_{ij})_{3\times 2}$ with $b_{ij} = \begin{cases} 1 & \text{if } j = 1 \\ 2 & \text{if } j = 2 \end{cases}$.

   Write down $\boldsymbol{A}$ and $\boldsymbol{B}$ explicitly.

3. Given that the following linear system is consistent:

$$\begin{cases} x & + & y & = & 1 \\ x & - & y & = & 1 \\ x & - & 3y & = & 1 \\ 3x & + & y & = & 3 \end{cases}$$

   (i) [1 mark] Write the linear system in matrix equation form $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$.

   (ii) [2 marks] Compute $\boldsymbol{A}^T \boldsymbol{A}$ and $\boldsymbol{A}^T \boldsymbol{b}$ for $\boldsymbol{A}$ and $\boldsymbol{b}$ in part (i).

   (iii) [2 marks] Pre-multiply $\boldsymbol{A}^T$ on both sides of the matrix equation in (i), derive the solution of the linear system <u>without using Gaussian Elimination</u>. Show your working.

4. [3 marks] Consider the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & a & b & a \\ 0 & a & b & a \\ 0 & 0 & b & a \end{array}\right)$$

for some real numbers a and b.
Suppose the linear system has only one solution. Find the <u>reduced row echelon form</u> of the above augmented matrix. Show how you derive your answer.