

If  $y = \frac{1}{3}x^3$  find the **exact value** of  $\frac{dy}{dx}$  when  $x = 39$ .

Let  $C$  denote the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ . Let  $P$  denote a point on the curve  $C$  in the first quadrant. Let  $L$  denote the normal line to the curve  $C$  at the point  $P$ . If  $L$  passes through the point  $(\frac{1}{2}, 0)$  find the gradient of the line  $L$ . Give your answer correct to two decimal places.

Let  $a$  denote a positive constant. Let  $R$  denote the part of the region in the first quadrant which is bounded above by the curve  $y = \sqrt{(a-x)(2a-x)(3a-x)}$  and bounded below by the  $x$ -axis from  $x = 2a$  to  $x = 3a$ . If the volume of the solid obtained by revolving  $R$  one complete round about the  $x$ -axis is equal to 2019, find the value of  $a$ . Give your answer correct to two decimal places.

Let  $a$  denote a positive constant. If the area under the curve  $y = x\sqrt{2ax - x^2}$  from  $x = 0$  to  $x = a$  is equal to 888, find the value of  $a$ . Give your answer correct to two decimal places.