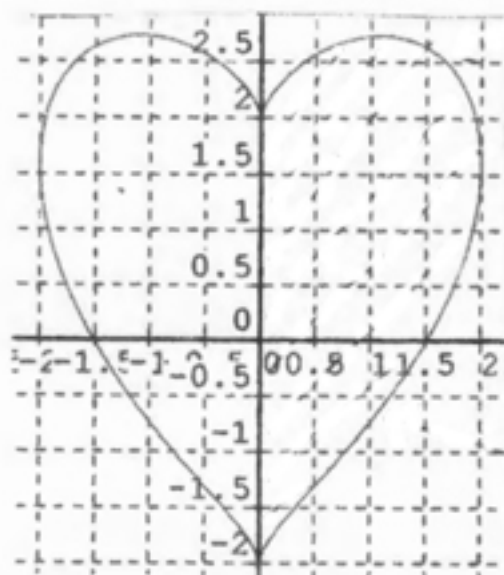


1. Let  $a$  denote a positive constant. Let  $L$  denote the normal line to the parabola  $y^2 = 4ax$  at the point  $(a, 2a)$ . If the distance from the origin to  $L$  is equal to 1521. Find the value of  $a$ . Give your answer correct to the nearest integer.
2. Let  $a$  denote a positive constant. If  $x = \sin t$  and  $y = \sin 5t$  and the equation  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + ay = 0$  holds for all values of  $t$ , find the **exact value** of  $a$ .
3. Let  $a$  denote a positive constant. Let  $R$  denote the finite region in the first quadrant bounded between the two curves  $x^2 - 2ax + y^2 = 0$  and  $y^2 - ax = 0$ . If the volume of revolution of  $R$  one round about the  $x$ -axis is equal to 1521, find the value of  $a$ . Give your answer correct to two decimal places.
4. Let  $a$  denote a positive constant. On Valentine's Day this year my wife sent me this gift:  $x^2 + (y - x^{2/3})^2 = a^2$ . When I plotted its graph, I got a curve that looks like the one in the picture below.



Being a maths guy I calculated the area bounded by this curve and found that it equals 2020. What is the value of  $a$ ? Give your answer correct to two decimal places.