

## NATIONAL UNIVERSITY OF SINGAPORE

## CS1231 - DISCRETE STRUCTURES

(Semester 1: AY2015/16)

Time Allowed: 90 minutes

INSTRUCTIONS TO CANDIDATES

1. Please write your Student Number (Matriculation Number) only. Do not write your name.
2. This assessment paper contains **TWO (2)** parts and comprises **TEN (10)** printed pages, including this page.
3. Answer **ALL** questions.
4. This is an **OPEN BOOK** assessment.
5. You are allowed to use **NUS APPROVED CALCULATORS**.
6. You may use pen or pencil, but please erase cleanly, and write legibly.
7. Please write your Student Number below.

STUDENT NUMBER: \_\_\_\_\_

EXAMINER'S USE ONLY			
Part	MaxScore	Mark	Remark
A	20	OCR	OCR
Q11	4		
Q12	6		
Q13	8		
Q14	4		
Q15	8		
TOTAL	50		

## Part A

**(20 marks) Multiple choice questions. Answer on the OCR form.**

For each multiple choice question, choose the best answer and **shade** the corresponding choice on the OCR form. Each multiple choice question is worth 2 marks. No mark is deducted for wrong answers. Shade your student number (check that it is correct!) on the OCR form as well. You should use a **2B pencil**.

Note that Appendix A on Page 10 may be useful. You may use the facts there, by citing, but without proving, them.

Q1. According to Prof. Sim, the CS1231 Message of the Day is:

- A. I may do it.
- B. I shall do it.
- C. I can't do it.
- D. I can do it.
- E. I think I can do it.

Q2. Which of the following are tautologies?

- (I)  $\sim(p \vee q) \vee [(\sim p) \wedge q] \vee p$ .
  - (II)  $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$ .
  - (III)  $(p \rightarrow r) \wedge (q \rightarrow r) \rightarrow [(p \vee q) \rightarrow r]$ .
- A. None of (I), (II) or (III).
  - B. (I) and (II) only.
  - C. (I) and (III) only.
  - D. (II) and (III) only.
  - E. All of (I), (II) and (III).

Q3. John was given the following statement:

“If the product of two integers  $a$  and  $b$  is even, then, either  $a$  is even or  $b$  is even.”

The following is John's proof:

1. Suppose  $a$  and  $b$  are both odd.
2. Therefore,  $a = 2m + 1$  and  $b = 2n + 1$  where  $m$  and  $n$  are integers.
3. Then,  $ab = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$ , which is odd.
4. Hence, the proof is complete.

What kind of proof did John use?

- A. Proof by contradiction.
- B. Proof by mathematical induction.
- C. Proof by contrapositive.
- D. Proof by converse.
- E. Proof by universal instantiation.

Q4. Which of the following is a negation for “Acer is inside and Beau is at the pool.”

- A. Acer is not inside or Beau is not at the pool.
- B. Acer is not inside or Beau is at the pool.
- C. Acer is not inside and Beau is not at the pool.
- D. Acer is inside or Beau is not at the pool.
- E. Acer is inside or Beau is at the pool.

Q5. Which of the following arguments are valid?

- (I) Every living thing is a plant or an animal.  
David’s dog is alive and it is not a plant.  
All animals have hearts.  
Hence, David’s dog has a heart.
  - (II) Some scientists are not engineers.  
Some astronauts are not engineers.  
Hence, some scientists are not astronauts.
  - (III) All astronauts are scientists.  
Some astronauts are engineers.  
Hence, some engineers are scientists.
  - (IV) Some females are not mothers.  
Some politicians are not females.  
Hence, some politicians are not mothers.
- A. (I) and (II) only.
  - B. (I) and (III) only.
  - C. (III) and (IV) only.
  - D. (I), (II) and (III) only.
  - E. (I), (III) and (IV) only.

Q6. Which of the following is the **negation** of this statement:

“There is a boy in the class such that all the girls in the class are younger than that boy.”

Let  $Boy(x)$  be “ $x$  is a boy”,  $Girl(x)$  be “ $x$  is a girl”, and  $Younger(x, y)$  be “ $x$  is younger than  $y$ ”.

- A.  $\exists x (Boy(x) \wedge \forall y (Girl(y) \rightarrow Younger(y, x)))$
- B.  $\forall x (\sim Boy(x) \vee \exists y (\sim Girl(y) \wedge Younger(y, x)))$
- C.  $\forall x (\sim Boy(x) \vee \exists y (Girl(y) \wedge \sim Younger(y, x)))$
- D.  $\forall x (Boy(x) \vee \exists y (Girl(y) \wedge \sim Younger(y, x)))$
- E. None of the above.

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The following definitions are for the next FOUR questions (Q7 — Q10).

Predicates  $P(x)$ ,  $Q(x, y)$  and  $R(x, y, z)$  are defined as follows:

$$\begin{aligned}
 P(x) &= (x > 1 \wedge \forall y \in \mathbb{N} (y \mid x \rightarrow (y = 1 \vee y = x))), \forall x \in \mathbb{N}. \\
 Q(x, y) &= (x \mid y), \forall x, y \in \mathbb{N}. \\
 R(x, y, z) &= (z \mid x \wedge z \mid y \wedge \forall u \in \mathbb{N} ((u \mid x \wedge u \mid y) \rightarrow u \leq z)), \forall x, y, z \in \mathbb{N}.
 \end{aligned}$$

Q7. Which of the following statements is true?

- (I)  $Q(0, 5)$       (II)  $Q(3, 0)$       (III)  $Q(21, 7)$       (IV)  $Q(7, 9)$

- A. (II) only.  
 B. (I) and (II) only.  
 C. (II) and (III) only.  
 D. (I) and (IV) only.  
 E. None of (I), (II), (III) or (IV).

Q8. Which integer  $x$  makes  $P(x)$  true?

- (I)  $x = 1$       (II)  $x = 2$       (III)  $x = 9$       (IV)  $x = 97$

- A. (I) only.  
 B. (II) only.  
 C. (II) and (IV) only.  
 D. (III) and (IV) only.  
 E. All of (I), (II), (III) and (IV).

Q9. Which pair of  $x, y$  makes true the statement:  $\sim Q(x, y) \wedge R(x, y)$  ?

- (I)  $x = 2, y = 2$       (II)  $x = 2, y = 6$       (III)  $x = 0, y = 4$       (IV)  $x = 4, y = 0$

- A. (I) only.  
 B. (III) only.  
 C. (II) and (III) only.  
 D. (I) and (IV) only.  
 E. None of (I), (II), (III) or (IV).

Q10. Which pair of  $x, y$  makes **false** the statement:  $R(x, y, 1) \rightarrow (P(x) \wedge P(y) \wedge x \neq y)$  ?

- (I)  $x = 3, y = 5$       (II)  $x = 2, y = 6$       (III)  $x = 7, y = 7$       (IV)  $x = 5, y = 6$

- A. (I) only.  
 B. (IV) only.  
 C. (II) and (III) only.  
 D. (II) and (III) and (IV) only.  
 E. None of (I), (II), (III), or (IV).

**Part B**

**(30 marks) Structured questions. Write your answer in the space provided.**

Q11. (4 marks) For each of the following statements, indicate whether the statement is true or false and justify your answer.

(a) (2 marks)  $\forall$  integers  $a$ ,  $\exists$  an integer  $b$  such that  $a + b = 0$ .

(b) (2 marks)  $\exists$  an integer  $a$  such that  $\forall$  integers  $b$ ,  $a + b = 0$ .

Q12. (6 marks) You are given the following English statements:

1. All swimmers are able to swim across the river.
2. No archers are short-sighted.
3. Patrick wears glasses.
4. Everybody is either an archer or a swimmer.

(a) (2 marks) Rewrite each of the above sentences into formal statements, using quantifiers wherever appropriate, and well-named predicates. You may assume that the domain is the set of people, which may be omitted in your statements. You may use the logically equivalent form in your statements.

(b) (2 marks) There is a missing statement above. Adding that missing statement would allow you to answer this question “Is Patrick able to swim across the river?” Write down the missing statement (as a formal quantified statement) and the conclusion (in English) about Patrick.

(c) (2 marks) Show your proof to derive the conclusion about Patrick in part (b) above.

Q13. (8 marks) On the island of knights (who always tell the truth) and knaves (who always lie), you meet three natives  $A$ ,  $B$ , and  $C$ , who address you as follows:

$A$ : At least one of us is a knave.

$B$ : At most two of us are knaves.

What are  $A$ ,  $B$  and  $C$ ? In your derivation, assume that  $A$  is a knave first. Later in your derivation, again assume that  $B$  is a knave first.

- Q14. (4 marks) In Q9 of Tutorial 1, you proved a shortcut to test for divisibility by 9. Here, you will prove a shortcut to test for divisibility by 5. That is, prove that any non-negative integer  $n$  is divisible by 5 if, and only if, its rightmost decimal digit is 0 or 5.



Q15. (8 marks) Find a positive integer  $n$  such that: (i) its prime factorization contains no repeated prime factors; and (ii) for any prime  $p$ ,  $p \mid n \iff (p-1) \mid n$ .

Be sure to clearly explain and justify how you obtain  $n$ .  
(Note: the list of primes in Appendix A may be useful.)

## Appendix A

### List of primes less than 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29,  
31, 37, 41, 43, 47, 53, 59, 61, 67, 71,  
73, 79, 83, 89, 97.

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END OF PAPER

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