1. How many critical points does the function

$$f(x) = (x - 0.1)\, x^{\frac{1521}{2019}}$$

have in the interval $[-1, 1]$?

$$f'(x) = x^{\frac{1521}{2019}} + \frac{1521}{2019}(x - 0.1)\, x^{-\frac{498}{2019}}$$

$$= \frac{1}{6730}\left(\frac{11800x - 507}{x^{\frac{166}{673}}}\right)$$

$$f'(x) = 0 \Rightarrow x = \frac{507}{11800} \in (-1, 1)$$

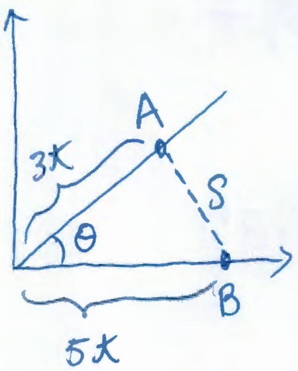
$\therefore \frac{507}{11800}$ is a critical point.

$f'(x)$ does not exist at $x = 0 \in (-1, 1)$

$\therefore 0$ is also a critical point

Answer: $\underline{\underline{2}}$

2. Let θ denote a positive constant which represents the radian measurement of an angle with $0 < \theta < \frac{\pi}{2}$. At time $t = 0$ minute, a point A starts at the origin and moving away from the origin into the first quadrant along the line $y = (\tan\theta)\,x$ at a uniform speed of 3 metre per minute. At the same time $t = 0$ minute, a point B starts at the origin and moving away from the origin towards the right along the x-axis at a uniform speed of 5 metre per minute. It is observed that at time $t = 1$ minute, the distance between A and B is increasing at a rate of 4.7 metre per minute. Find the value of θ. Give your answer correct to two decimal places.



$$s^2 = 9t^2 + 25t^2 - 30t^2\cos\theta$$

$$2s\frac{ds}{dt} = 68t - 60t\cos\theta$$

$\therefore$ at $t = 1$, we have

$$s^2 = 34 - 30\cos\theta$$

$$s\frac{ds}{dt} = 34 - 30\cos\theta$$

$$\therefore 4.7 = \frac{ds}{dt} = s = \sqrt{34 - 30\cos\theta}$$

$$\therefore \cos\theta = \frac{11.91}{30}$$

$$\theta = \cos^{-1}\left(\frac{11.91}{30}\right) = 1.162\ldots$$

$$\approx \underline{\underline{1.16}}$$

3. The region bounded by the graphs of $y = \frac{1}{\sqrt{1+x^2}}$, $y = \frac{1}{\sqrt{4+x^2}}$, $x = 0$ and $x = b$ where b denotes a positive constant is rotated about the x-axis to generate a solid of revolution. Let $V(b)$ denote the volume of this solid of revolution. By taking the value of π to be equal to $\frac{22}{7}$ you find that the value of $\lim_{b\to\infty} V(b)$ is equal to $\frac{m}{n}$ where m and n are two positive integers with no common factors. What is the value of $m+n$?

$$V(b) = \int_0^b \pi\left(\frac{1}{\sqrt{1+x^2}}\right)^2 dx - \int_0^b \pi\left(\frac{1}{\sqrt{4+x^2}}\right)^2 dx$$

$$= \pi \tan^{-1}(x)\Big|_0^b - \frac{\pi}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big|_0^b$$

$$= \pi \tan^{-1}(b) - \frac{\pi}{2}\tan^{-1}\left(\frac{b}{2}\right)$$

$$\lim_{b\to\infty} V(b) = \pi\left(\frac{\pi}{2}\right) - \frac{\pi}{2}\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi^2}{4} = \frac{121}{49} \quad (\text{using } \pi = \tfrac{22}{7})$$

$$\therefore m+n = 121+49 = \underline{\underline{170}}$$

4. It is known that f is a differentiable function which satisfies

$$\int_1^x f(t)\,dt = \sin 1 - \frac{\sin x}{x}$$

for all $x > 0$. Find the value of $\int_1^2 x f'(x)\,dx$. Give your answer correct to two decimal places.

$$\frac{d}{dx}\int_1^x f(t)\,dt = \frac{d}{dx}\left(\sin 1 - \frac{\sin x}{x}\right)$$

$$\therefore f(x) = -\frac{x\cos x - \sin x}{x^2}$$

$$\int_1^2 x f'(x)\,dx = \int_1^2 x\,d(f(x))$$

$$= \left[x f(x)\right]_1^2 - \int_1^2 f(x)\,dx$$

$$= \left[-\frac{x\cos x - \sin x}{x}\right]_1^2 - \int_1^2 f(t)\,dt$$

$$= -\frac{2\cos 2 - \sin 2}{2} + \frac{\cos 1 - \sin 1}{1} - \left(\sin 1 - \frac{\sin 2}{2}\right)$$

$$= -\cos 2 + \sin 2 + \cos 1 - 2\sin 1$$

$$= 0.182\ldots$$

$$\approx \underline{\underline{0.18}}$$