MA1521 TUTORIAL 11

1. Solve the following differential equations:

(a) $xy' + (1+x)y = e^{-x}, \qquad x > 0$

(b) $y' - (1 + \frac{3}{x})y = x + 2, \qquad y(1) = e - 1, \qquad x > 0$

(c) $y' + y + \frac{x}{y} = 0$ (d) $2xyy' + (x-1)y^2 = x^2 e^x, \qquad x > 0$

2. Psychologists talk about something called a **Performance Curve**. Suppose an MA1521 student is solving mathematics problems. She starts with differential equations. Let P(t) be a non-negative function that measures her performance, that is, her success rate at solving DEs. Her performance increases rapidly at first, but then the rate of increase slows down as she becomes more expert. Let M, a positive constant, be the best possible performance; then one can suppose that P satisfies

$$\frac{dP}{dt} = C[M - P],$$

where C is a constant. Solve this equation assuming that she is completely incompetent at t = 0 [that is, P(0) = 0].

3. A certain person starts a rumour in a small town. The number of people who have heard the rumour, R(t), is given by

$$\frac{dR}{dt} = KR[1300 - R],$$

where K is a positive constant, and 1300 is the number of residents in that small town. What is the meaning of K? Is this equation reasonable? [Hint: surely the rumour will spread slowly both when hardly anyone has heard it yet, **but also** when nearly everyone has already heard it!] By regarding this equation as a Bernoulli equation, find R(t).

4. The half-life of Thorium 230 is about 75000 years, while that of Uranium 234 is about 245000 years. A certain sample of ancient coral has a Thorium/Uranium ratio of 10 percent. How old is the coral?

5. The bacteria in a certain culture number 10000 initially. Two and a half hours later there are 11000 of them. Assuming a Malthus model, how many bacteria will there be 10 hours after the start of the experiment? How long will it take for the number to reach 20000? [Answers: About 14600; about 18.18 hours.]

6. You have 200 bugs in a bottle. Every day you supply them with food and count them. After two days you have 360 bugs. It is known that the birth rate for this kind of bug is 150% per day. [Is this a sensible way of stating a birth rate per capita? Why?] Assuming that the population is given by a logistic model, find the number of bugs after 3 days. Predict how many bugs you will have eventually. [Answers: about 372; about 376.]