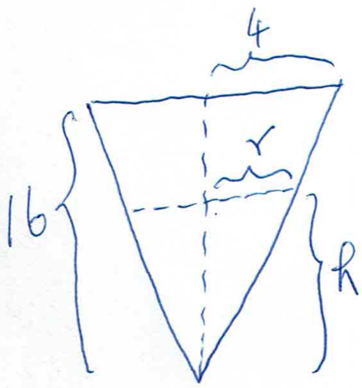
1. Let a denote a positive constant. A water tank in the shape of an inverted right circular cone is being filled at a rate of 2 cubic meters per minute. The height of the tank is 16 meters and the radius at the top is 4 meters. If the water level is rising at a rate of a meters per minute when the water is 5 meters deep, find the value of a. Give your answer correct to two decimal places.

Answer 0.41



$$\frac{h}{r} = \frac{16}{4} = 4$$

$$h = 4r, \text{ i.e. } r = \frac{1}{4}h$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{48}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{16}\pi h^2 \frac{dh}{dt}$$

$$\therefore \quad 2 = \frac{1}{16}\pi (5)^2 a$$

$$a = \frac{32}{25\pi} = 0.4074\ldots$$

$$\approx \underline{\underline{0.41}}$$

2. Find the value of

$$\lim_{x\to 0}\frac{\int_0^x \sqrt[3]{1+t^2}dt - x}{x-\sin x}.$$

Give your answer correct to two decimal places.

Answer 0.67

$$\lim_{x\to 0}\frac{\int_0^x \sqrt[3]{1+t^2}\,dt - x}{x-\sin x} \quad \left(\tfrac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 0}\frac{(1+x^2)^{1/3}-1}{1-\cos x} \quad \left(\tfrac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 0}\frac{\frac{1}{3(1+x^2)^{2/3}}(2x)}{\sin x}$$

$$= \left(\lim_{x\to 0}\frac{2}{3(1+x^2)^{2/3}}\right)\left(\lim_{x\to 0}\frac{x}{\sin x}\right)$$

$$= \frac{2}{3} = 0.666\ldots$$

$$\approx \underline{\underline{0.67}}$$