GEH1027 **2020/21 S2**

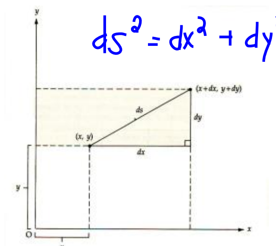
Einstein's Universe and Quantum Weirdness **Tutorial 3**

1) From the discussion given in the Lectures, the Einstein's famous Field Equations of General Relativity, are given as $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$ and the coupling constant is $\kappa = \frac{8\pi G}{c^4}$.

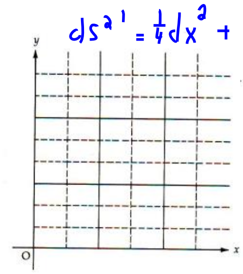
a) Discuss **qualitatively** the meaning of LHS and RHS.
b) How many field equations are there if ***time*** is now treated as equal footing as ***space***?
c) What do we by solve the above field equation? What are we looking for?
d) What is the dual role of $g_{\mu\nu}$?
e) Recall the meaning of $R_{\mu\nu} = 0$?

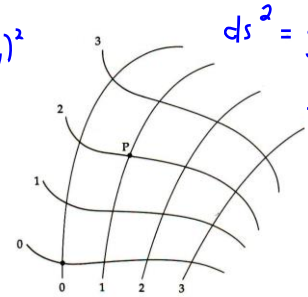
2) Consider the 2 dimensional Euclidean figures below.

$ds^2 = dx^2 + dy^2$



$ds^{2\,\prime} = \frac{1}{4}dx^2 + (\frac{1}{2}dy)^2$





$ds^2 = g_{xx}dx^2 + g_{yy}dy^2 + g_{xy}dxdy + g_{yx}dydx$

crossterm is significant

a) Write an expression for ds^2 in terms of dx and dy for extreme left figure.
b) If the scales of the figure in the center is altered such as both the x and y axes have twice as many co-ordinate lines as before, write an expression for ds^2 in terms of dx and dy.
c) Write the most general expression for ds^2 in terms of dx and dy for the extreme right figure.

3a) Given a metric tensor where components: $g_{00} = 1, g_{11} = g_{22} = g_{33} = -1,$ also that $g_{\mu\nu} = 0,\ when\ \mu \neq \nu$. Show that $g_{\mu\nu} g^{\mu\nu} = 4$.

3b) Recall the Standard Einstein's Field Equation, $R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta}$.

As mentioned in the lecture, note that $R = g^{\alpha\beta} R_{\alpha\beta}$. Show that if $T_{\alpha\beta} = 0$ then $R_{\alpha\beta} = 0$.

Hint: **i)** multiply both sides of the Field Equation by $g^{\alpha\beta}$ after setting $T_{\alpha\beta} = 0$.
ii) use result 3a)
iii) The Tutors will assist you.

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$R_{\alpha\beta} - \underbrace{\frac{1}{2} g_{\alpha\beta} R}_{0} = 0$

$\therefore R_{\alpha\beta} = 0$