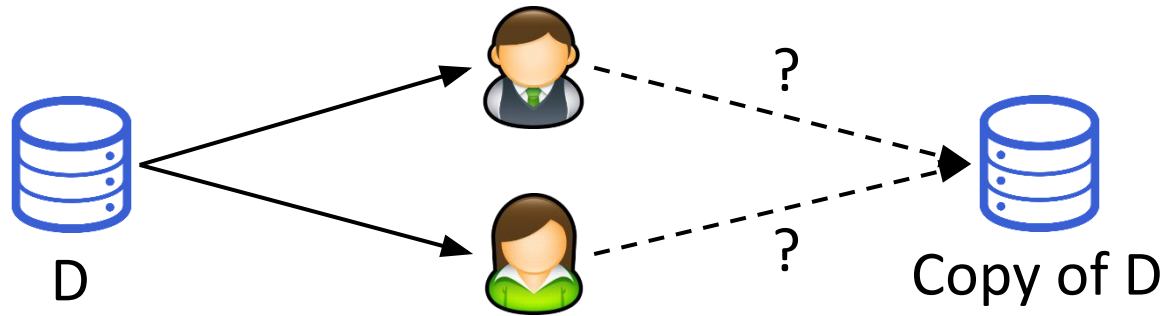
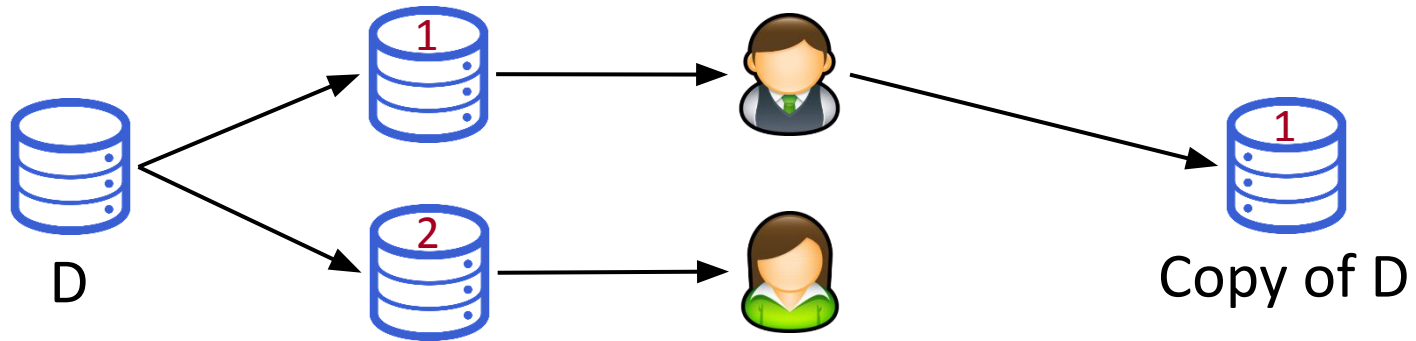

CS5322 Database Security

Watermarks: Motivation



- Suppose that we have a dataset D , and we share it with two users $U1$ and $U2$
- A while later, we find that a copy of D is leaked to the internet
- Both $U1$ and $U2$ deny being the culprit
- How can we find out whom we shall blame?

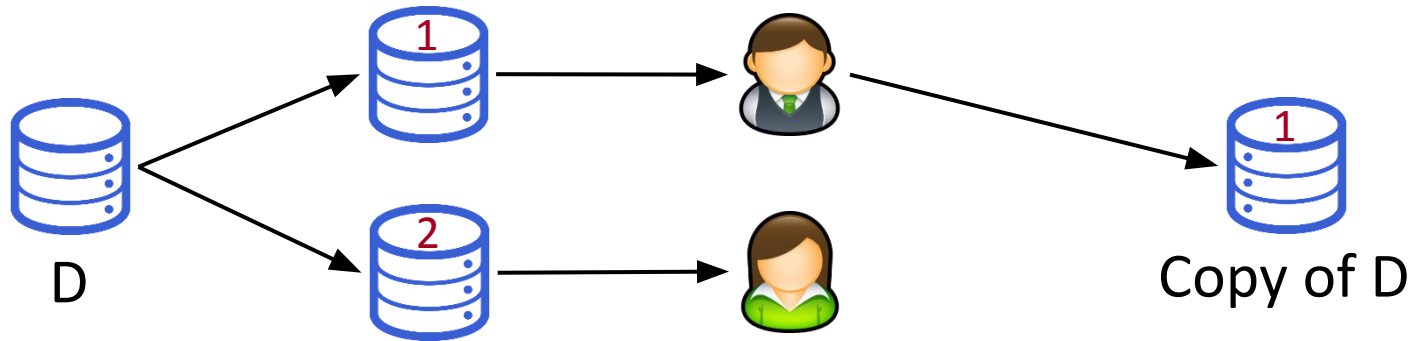
Watermarks: Motivation



- Idea:

- ❑ Secretly *watermark* the data shared with U1 and U2
- ❑ If a copy of the data is leaked, we just need to check the watermark of the leaked data to decide the culprit

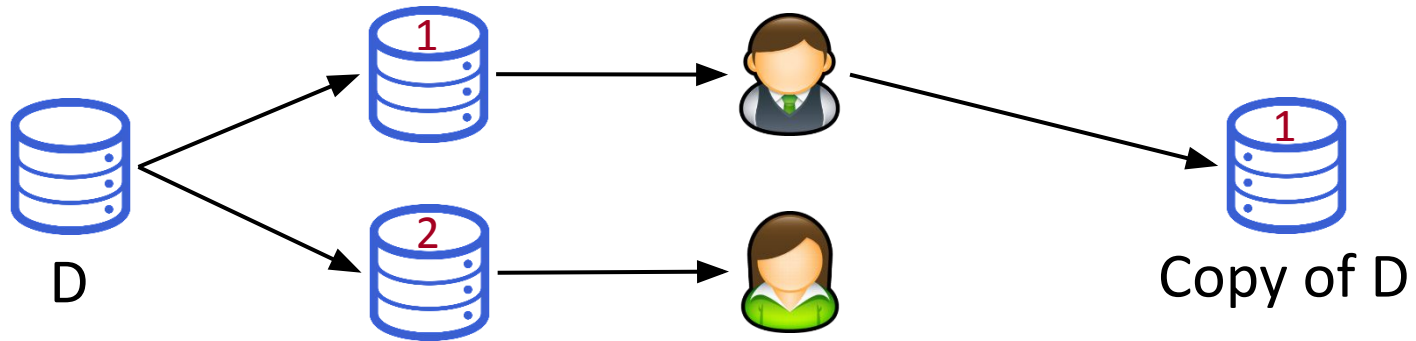
Watermarks: Challenges



■ Challenges:

- ❑ Users may not leak the complete copy of D
 - He may choose to leak only a subset of the records
 - Or he may choose to leak only a subset of the attributes
- ❑ Users may modify the records in D to try to destroy the watermarks

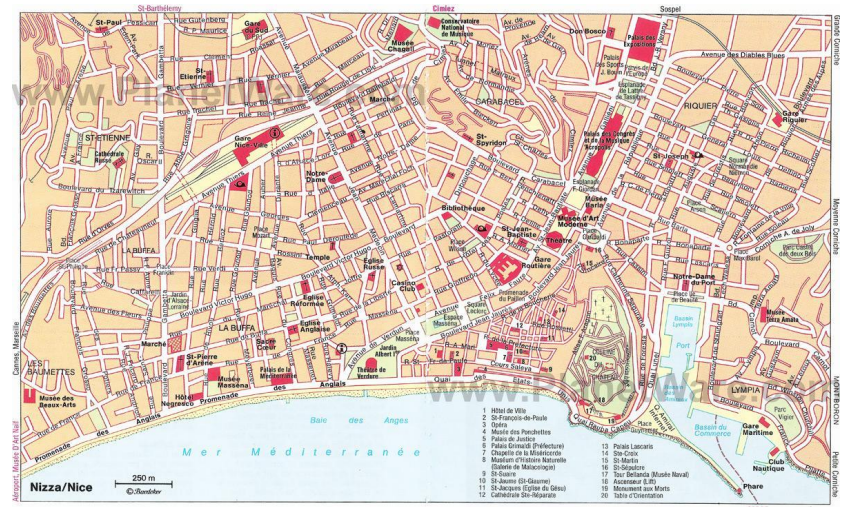
Watermarks: Challenges



- How do we address the above challenges?
- Let's revisit the watermarking techniques for other types of data, and borrow ideas from there

Traps in Maps

- Map making is costly
 - Especially in the old days
- To save cost, a map maker may just copy another's map
- To protect their copyrights, map makers often put *traps* in their maps
 - i.e., intentionally put incorrect information in maps
- Rationale: If the traps appear in someone else's map, then they can make a case against them

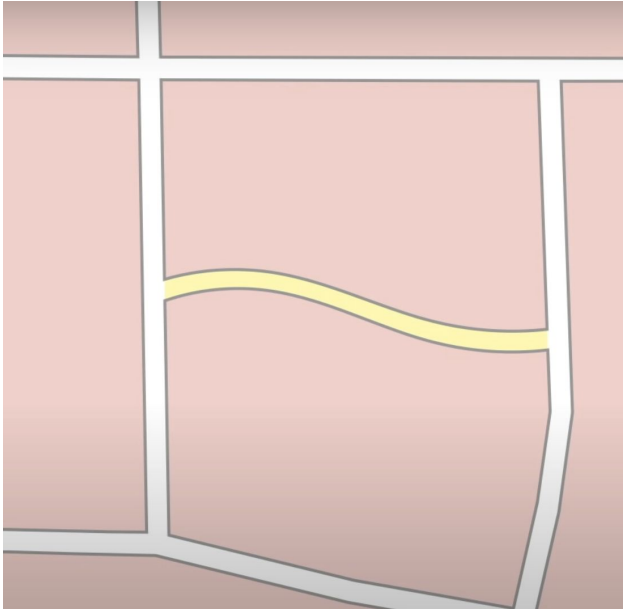


Traps in Maps



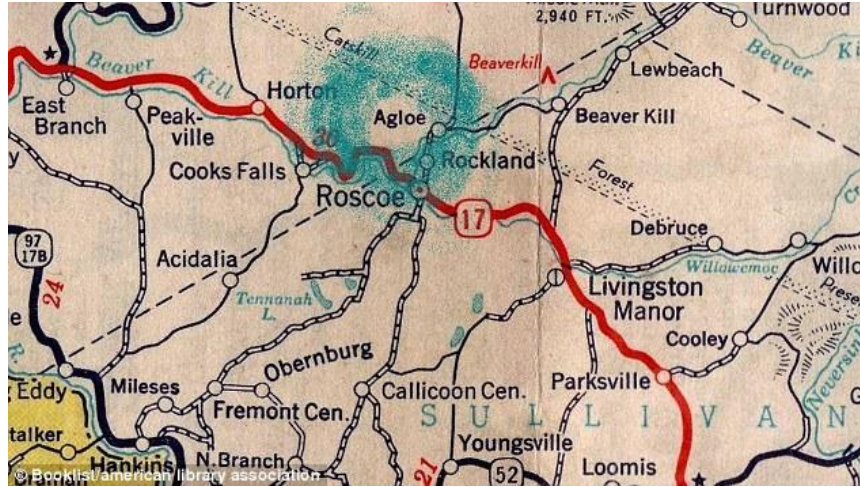
- Example:
 - Adding non-existing curves to roads

Traps in Maps



- Example:
 - Adding non-existing streets

Traps in Maps



- Example
 - Adding non-existing towns
- See
 - https://en.wikipedia.org/wiki/Agloe,_New_York
 - <https://en.wikipedia.org/wiki/Argleton>

Traps in Maps -> Watermarks in Data?

- Can we apply a similar approach to watermark relational data?
 - Idea: Introduce false information into the data
 - Just like traps in maps
 - Requirements:
 - The false information should be difficult to detect
 - And it should not affect the utility of the data
-

Watermarks for Relational Data

- We will discuss the approach in the following paper:
 - R. Agrawal, P. J. Hass, J. Kiernan, "Watermarking Relational Data: Framework, Algorithms and Analysis", VLDB Journal, 2003.
- We will refer to this approach as AHK
- Basic Idea:
 - Utilize the least significant bits (LSB) of the numeric values in the data

AHK: Assumptions

- The AHK approach makes the following assumptions:
 - The table T to be watermarked has a number of numeric attributes A_1, A_2, \dots, A_d , and a primary key P
 - The leaked data contains the primary key P and some numeric attributes
 - For each A_i , the ξ least significant bits (LSB) could be used for watermarking, i.e.,
 - Changes to a small number of them won't significantly affect the usefulness of the data
 - But completely removing those ξ bits would make the data much less useful
 - Detection of watermarks should not need to access T
 - Because T might be updated after being shared

AHK: Basic Idea

- For each tuple t , use its primary key $t.P$ to decide
 - Whether the tuple should be watermarked
 - Which attribute of the tuple should be watermarked
 - Which of the ξ least significant bits should be watermarked
- But can't let users figure out which bits are changed
 - Otherwise the watermarks can be removed
- Solution
 - Pick the watermarked tuples and bits using a cryptographic pseudo-random number generator (CPRNG)

Cryptographic Pseudo-Random Number Generator (CSPNG)

- A deterministic function G that given a *seed value*, generates a sequence of random numbers, such that
 - Without the seed, it is computationally infeasible to predict the next random number
- How it is used:
 - The data owner chooses a secret key K
 - For each tuple t , the data owner concatenates K and its primary key $t.P$, and use the concatenation result $K||t.P$ as the seed value for G
 - The data owner then uses the random numbers generated by G to decide whether and how to watermark t

AHK Watermarking Algorithm

- Input parameters:
 - G : a CPRNG
 - K : a secret key selected by the data owner
 - T : a table to be watermarked
 - d : the number of attributes in T
 - ξ : the number of least significant bits (LSB) suitable for watermarks
 - f : the fraction of tuples to be watermarked
- For each tuple t in T
 - Seed G with $K || t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$ <----- t has f probability to be watermarked
 - $i = r2 \bmod d$ <----- the i -th attribute $t.A_i$ will be marked
 - $j = r3 \bmod \xi$ <----- the j -th LSB of $t.A_i$ will be marked
 - if ($r4$ is even) then set the j -th LSB of $t.A_i$ to 0
 - else set the j -th LSB of $t.A_i$ to 1

AHK Watermarking Algorithm

- For each tuple t in T
 - Seed G with $K || t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$ <--- t has f probability to be watermarked
 - $i = r2 \bmod d$ <--- the i -th attribute $t.A_i$ will be marked
 - $j = r3 \bmod \xi$ <--- the j -th LSB of $t.A_i$ will be marked
 - if $(r4$ is even) then set the j -th LSB of $t.A_i$ to 0
 - else set the j -th LSB of $t.A_i$ to 1

- Summary:
 - Around f fraction of the tuples are watermarked
 - Each watermarked tuple has one watermarked bit
 - So if there are n tuples in T , there are around $n * f$ watermarked bits in total

AHK Watermarking Algorithm

- For each tuple t in T
 - Seed G with $K || t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$ <--- t has f probability to be watermarked
 - $i = r2 \bmod d$ <--- the i -th attribute $t.A_i$ will be marked
 - $j = r3 \bmod \xi$ <--- the j -th LSB of $t.A_i$ will be marked
 - if ($r4$ is even) then set the j -th LSB of $t.A_i$ to 0
 - else set the j -th LSB of $t.A_i$ to 1
- Observation:
 - If the j -th LSB of $t.A_i$ is originally 0, and $r4$ happens to be even, then the watermark does not change anything
 - If the j -th LSB of $t.A_i$ is originally 1, and $r4$ happens to be odd, then the watermark does not change anything
- So how do we know check whether a bit is really watermarked?

AHK Watermark Detection Algorithm

- Input Parameters:
 - ...
 - S : a set of tuples leaked
 - T : a threshold
- $\text{total_count} = \text{match_count} = 0$
- For each tuple t in S
 - Seed G with $K_u \parallel t.P$ <----- K_u is the key used to mark the data given to u
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $\text{total_count} = \text{total_count} + 1$
 - if $(r4 \text{ is even and the } j\text{-th LSB of } t.A_i \text{ is } 0) \text{ or } (r4 \text{ is odd and the } j\text{-th LSB of } t.A_i \text{ is } 1)$
 - $\text{match_count} = \text{match_count} + 1$
- If $(\text{match_count} > \text{total_count} - T)$ then suspect data leak from User u
- Else if $(\text{match_count} < T)$ then suspect data leak and watermark manipulation from User u
 - innocent data will have match_count half of total_count
 - because 50% chance of match_count matching

AHK Watermark Detection Algorithm

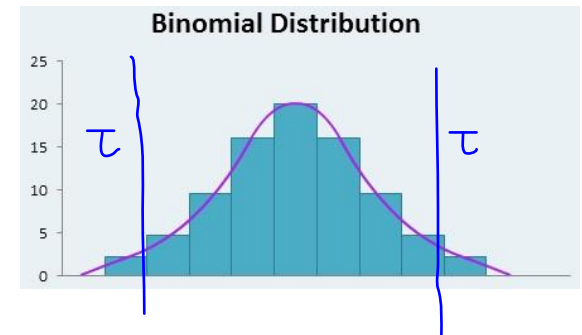
- $\text{total_count} = \text{match_count} = 0$
- For each tuple t in S
 - Seed G with $K_u \parallel t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $\text{total_count} = \text{total_count} + 1$
 - if ($r4$ is even and the j -th LSB of $t.Ai$ is 0) or ($r4$ is odd and the j -th LSB of $t.Ai$ is 1)
 - $\text{match_count} = \text{match_count} + 1$
- If ($\text{match_count} > \text{total_count} - \tau$) then suspect data leak from User u
- Else if ($\text{match_count} < \tau$) then suspect data leak and watermark manipulation from User u

■ Intuition:

- If the data is leaked by User u , then there should be a lot of match bits
- If the data is not leaked by User u , then there should be around 50% match bits
- If the fraction of matched bits is much smaller than 50%, then it is likely that User u has figured out which bits are watermarked, and has deliberately modified those bits

AHK Watermark Detection Algorithm

- $total_count = match_count = 0$
- For each tuple t in S
 - Seed G with $K_u || t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $total_count = total_count + 1$
 - if ($r4$ is even and the j -th LSB of $t.Ai$ is 0) or ($r4$ is odd and the j -th LSB of $t.Ai$ is 1)
 - $match_count = match_count + 1$
- If ($match_count > total_count - \tau$) then suspect data leak from User u
- Else if ($match_count < \tau$) then suspect data leak and watermark manipulation from User u



- How should we decide τ ?
- If the data is not leaked by User u , then each "marked" bit has $1/2$ probability to produce a match
- When we have $total_count$ marked tuples, the number of matched bits should follow a binomial distribution $B(total_count, 1/2)$
- We can set τ such that the probability of getting a binomial variable larger than $total_count - \tau$ or smaller than τ is small (e.g., 5%)

AHK Watermark Detection Algorithm

- $\text{total_count} = \text{match_count} = 0$
 - For each tuple t in S
 - Seed G with $K_u \parallel t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $\text{total_count} = \text{total_count} + 1$
 - if ($r4$ is even and the j -th LSB of $t.Ai$ is 0) or ($r4$ is odd and the j -th LSB of $t.Ai$ is 1)
 - $\text{match_count} = \text{match_count} + 1$
 - If $(\text{match_count} > \text{total_count} - \tau)$ then suspect data leak from User u
 - Else if $(\text{match_count} < \tau)$ then suspect data leak and watermark manipulation from User u
-
- Note that the above algorithm can be easily extended to handle the case when S contains only a subset of the attributes
 - How?
 - After choosing i , we only increase total_count when Ai exists in S

Exercise

- For each tuple t in T
 - Seed G with $K || t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$ <--- t has $1/f$ probability to be watermarked
 - $i = r2 \bmod d$ <--- the i -th attribute $t.A_i$ will be marked
 - $j = r3 \bmod \xi$ <--- the j -th LSB of $t.A_i$ will be marked
 - if $(r4 \bmod 3 = 1)$ then set the j -th LSB of $t.A_i$ to 0
 - else set the j -th LSB of $t.A_i$ to 1
- Suppose that we apply the above watermarking algorithm on a table T
- Explain whether and/or how we can detect watermarks from the modified table without accessing T

Exercise

- Input Parameters:
 - ...
 - S : a set of tuples leaked
 - T : a threshold
- $\text{total_count} = \text{match_count} = 0$
- For each tuple t in S
 - Seed G with $K_u \parallel t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $\text{total_count} = \text{total_count} + 1$
 - if $(r4 \bmod 3 = 1 \text{ and the } j\text{-th LSB of } t.A_i \text{ is } 0)$ or $(r4 \bmod 3 \neq 1 \text{ and the } j\text{-th LSB of } t.A_i \text{ is } 1)$
 - $\text{match_count} = \text{match_count} + 1$
- If $(\text{match_count} > \text{total_count} - T1)$ then suspect data leak from User u
- Else if $(\text{match_count} < T2)$ then suspect data leak and watermark manipulation from User u

Exercise

- $total_count = match_count = 0$
 - For each tuple t in S
 - Seed G with $K_u || t.P$
 - Get random numbers $r1, r2, r3, r4$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $total_count = total_count + 1$
 - if $(r4 \bmod 3 = 1$ and the j -th LSB of $t.Ai$ is 0) or $(r4 \bmod 3 \neq 1$ and the j -th LSB of $t.Ai$ is 1)
 - $match_count = match_count + 1$
 - If $(match_count > total_count - \tau1)$ then suspect data leak from User u
 - Else if $(match_count < \tau2)$ then suspect data leak and watermark manipulation from User u
-
- How should we decide $\tau1$ and $\tau2$?
 - If the data is not leaked by User u , then each "marked" bit has $1/3 \sim 2/3$ probability to produce a match
 - When we have $total_count$ marked tuples, the number of matched bits should follow a binomial distribution $B(total_count, p)$, where p is in $[1/3, 2/3]$
 - We can set $\tau1$ and $\tau2$ such that the probability of getting a binomial variable larger than $total_count - \tau1$ or smaller than $\tau2$ is small (e.g., 5%)

Exercise

- For each tuple t in T
 - Seed G with $K || t.P$
 - Get random numbers $r1, r2, r3$ from G
 - If $(r1 \bmod 1/f) = 0$ <--- t has $1/f$ probability to be watermarked
 - $i = r2 \bmod d$ <--- the i -th attribute $t.A_i$ will be marked
 - $j = r3 \bmod \xi$ <--- the j -th LSB of $t.A_i$ will be marked
 - Flip the j -th LSB of $t.A_i$
- Suppose that we apply the above watermarking algorithm on a table T
- Explain whether and/or how we can detect watermarks from the modified table **without accessing T**

Exercise

- For each tuple t in T
 - Seed G with $K || t.P$
 - Get random numbers $r1, r2, r3$ from G
 - If $(r1 \bmod 1/f) = 0$ <--- t has $1/f$ probability to be watermarked
 - $i = r2 \bmod d$ <--- the i -th attribute $t.A_i$ will be marked
 - $j = r3 \bmod \xi$ <--- the j -th LSB of $t.A_i$ will be marked
 - Flip the j -th LSB of $t.A_i$
- Suppose that we apply the above watermarking algorithm on a table T
- Explain whether and/or how we can detect watermarks from the modified table ~~without accessing T~~

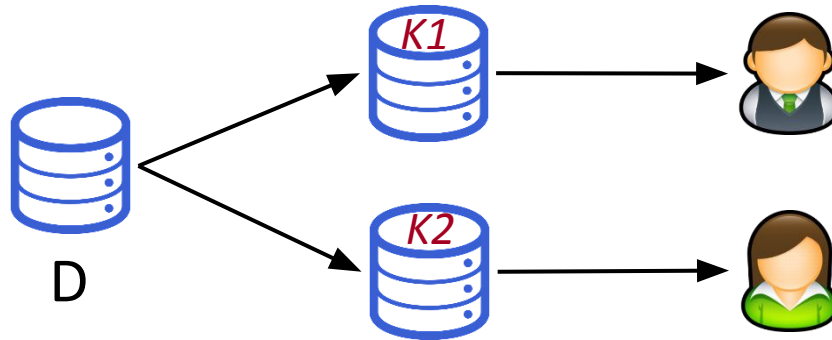
Exercise

- Input Parameters:
 - ...
 - S : a set of tuples leaked
 - T : a threshold
- $\text{total_count} = \text{match_count} = 0$
- For each tuple t in S
 - Seed G with $K_u \parallel t.P$
 - Get random numbers $r1, r2, r3$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $\text{total_count} = \text{total_count} + 1$
 - if (the j -th LSB of $t.A_i$ is different from the original tuple)
 - $\text{match_count} = \text{match_count} + 1$
- If $(\text{match_count} > T)$ then suspect data leak from User u

Exercise

- $total_count = match_count = 0$
- For each tuple t in S
 - Seed G with $K_u || t.P$
 - Get random numbers $r1, r2, r3$ from G
 - If $(r1 \bmod 1/f) = 0$
 - $i = r2 \bmod d$
 - $j = r3 \bmod \xi$
 - $total_count = total_count + 1$
 - if (the j -th LSB of $t.A_i$ is different from the original tuple)
 - $match_count = match_count + 1$
- If $(match_count > \tau1)$ then suspect data leak from User u
- How should we decide $\tau1$?
- If the data is not leaked by User u but another User v , then each "marked" bit has $f/d/\xi$ probability to produce a match
- When we have $total_count$ marked tuples, the number of matched bits should follow a binomial distribution $B(total_count, f/d/\xi)$
- We can set $\tau1$ such that the probability of getting a binomial variable larger than $\tau1$ is small (e.g., 5%)

Attacking AHK Watermarks



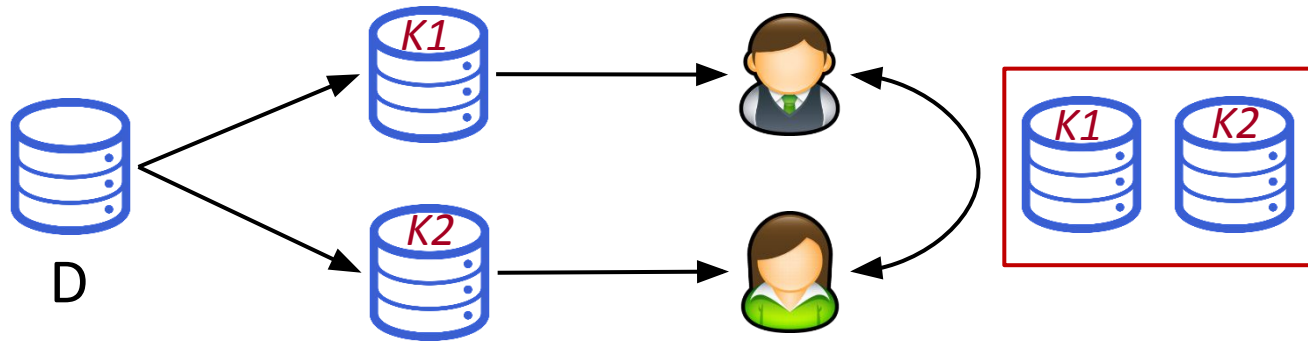
- Summary:

- Choose a secret key K_u for each user u
- Use K_u to decide which bit of which tuple should be watermarked

- Question:

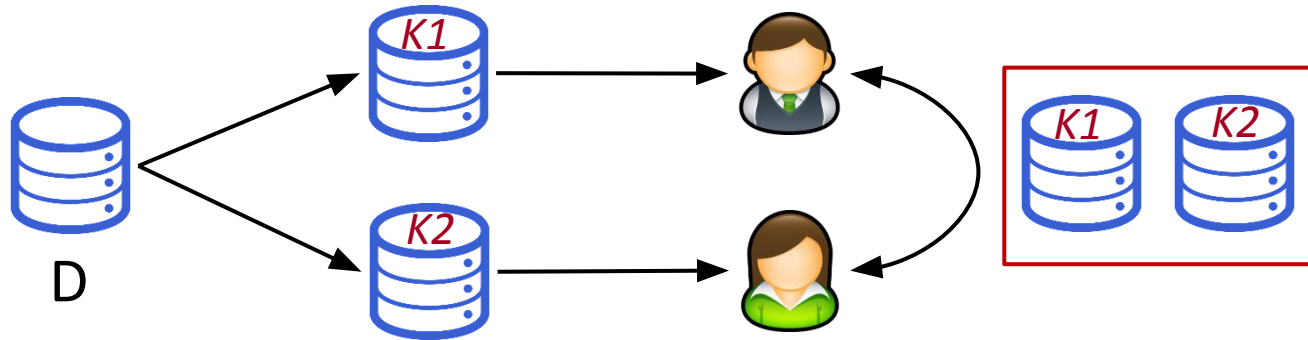
- If you are one of the users, what would you do to attack the watermarks?

Attacking AHK Watermarks



- Idea:
 - Collude with another user to obtain two differently watermarked datasets
 - Compare the two datasets to figure out which bits are different
- Rationale:
 - In each dataset, the watermarked tuples and watermarked bits are all randomly selected
 - The chance that the same bit is watermarked in both dataset is fairly low
 - So a watermarked bit in one dataset is likely to be different in the other dataset

Attacking AHK Watermarks



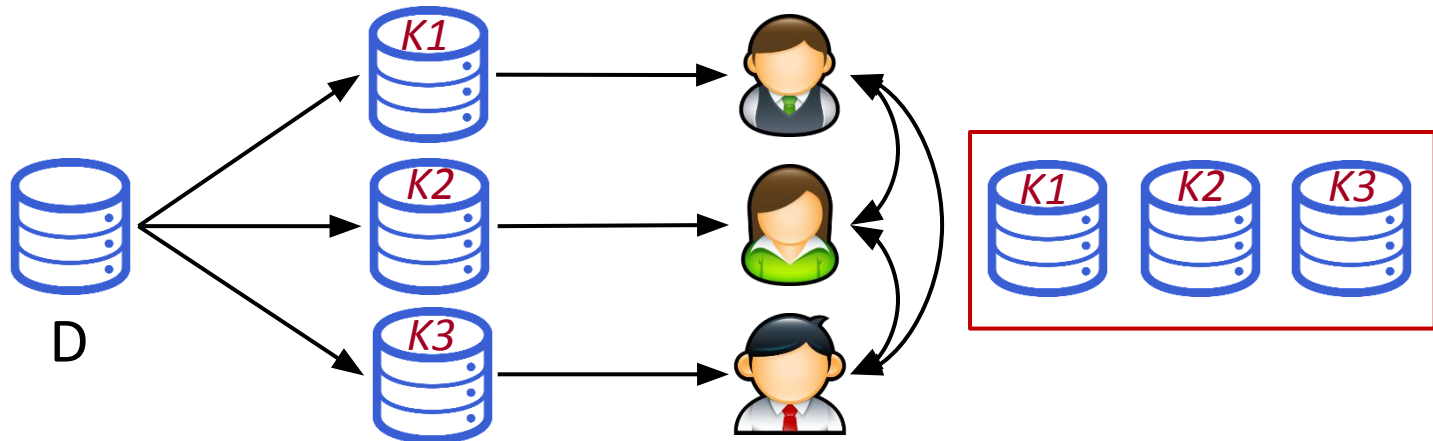
- Problem:

- We only know which bits are different in the two datasets
- But we don't know whether a "different" bit is watermarked in Dataset 1 or Dataset 2

- Solution 1:





- We don't care; just flip half of those bits
- This could fool the watermark detection algorithm described previously

Attacking AHK Watermarks







- Solution 2:
 - Find a third user to collude with
 - Now whenever there is a bit that the datasets do not agree on, we can use a majority vote to infer the original value
- Question:
 - How do we guard against such collusion attacks?
- Idea:
 - Make the watermark bits *correlated* among the datasets

Guarding Against Collusion Attacks

$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$





- Suppose that we are to give a dataset D to four users $u1$, $u2$, $u3$, and $u4$
- We randomly divide D into three parts: $D1$, $D2$, $D3$
- We choose a secret key K , and produce two watermarked versions of D_i
 - The first version D_i+ is generated by the AHK algorithm
 - The second version D_i- is generated by flipping the watermarked bits in D_i+
- We then assign watermarked parts to users as shown above

Guarding Against Collusion Attacks

$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$





- Suppose that $u1$ and $u3$ collude
- They could compromise the watermarks in $D1$ and $D2$
- But the watermarks in $D3$ still exists
- When the data is leaked, if we detect watermarks in $D3$ (but not in the other parts),
- then we know that $u1$ and $u3$ must be among the culprits

Guarding Against Collusion Attacks

$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$





- Suppose that $u1$, $u2$, and $u3$ collude
- They could compromise the watermarks in $D1$ and $D2$
- But the watermarks in $D3$ still exist
- When the data is leaked, if we find that the watermarks exist in $D3$ (but not in the other parts)
- then we know that $u1$ and $u3$ be among the culprits
- Why can't we be sure about $u2$?
 - Because $u1$ and $u3$ can already compromise $D1$ and $D2$

Guarding Against Collusion Attacks

$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$





- Suppose that $u1$ and $u4$ collude
- They could compromise the watermarks in $D1$ and $D2$ and $D3$
- When the data is leaked, if we cannot detect any watermarks
- then we know that $u1$ and $u4$ must be among the culprits
 - Otherwise, the watermarks in $D1$, $D2$, and $D3$ cannot be compromised at the same time

Guarding Against Collusion Attacks

$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$





- Suppose that $u1$, $u2$, and $u4$ collude
- They could compromise the watermarks in $D1$ and $D2$ and $D3$
- When the data is leaked, if we cannot detect any watermarks
- then we know that $u1$ and $u4$ must be among the culprits
 - Otherwise, the watermarks in $D1$, $D2$, and $D3$ cannot be compromised at the same time
- But we cannot be sure about $u2$

Guarding Against Collusion Attacks

$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$

- Suppose that $u1$, $u2$, and $u4$ collude
- They could compromise the watermarks in $D1$ and $D2$ and $D3$
- But they could be more strategic
- Can they leak $D1-$, $D2-$, $D3+$, and pretend that the leak is caused by $u3$?
- No.
- Because they don't know which tuples are from $D2-$ and $D3+$

Guarding Against Collusion Attacks





$u1$ 	$D1+$,	$D2+$,	$D3+$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1-$,	$D2-$,	$D3-$

- In general, if
 - there are n users, and
 - we assume that at most c of them collude
- Then
 - there are ways to distribute watermarked parts to users to ensure successful detection under our assumption
- See
 - D. Boneh and J. Shaw. "Collusion-Secure Fingerprinting for Digital Data". IEEE Transaction on Information Theory, 1998.

Extensions to Categorical Attributes





- For categorical attributes, there is no least significant bits that we can exploit
- Solution:
 - When we decide to watermark a categorical value, we change it to another value (instead of just changing a bit)
 - Example: changing the education of a person from "Bachelor" to "High School"
- Challenge:
 - Could be tricky to maintain the validity of the data
 - E.g., change a male patient's disease from dyspepsia to breast cancer

Exercise 1

$u1$ 	$D1-$,	$D2-$,	$D3-$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1+$,	$D2-$,	$D3+$





- Suppose that we are to give a dataset D to four users $u1$, $u2$, $u3$, and $u4$
- We randomly divide D into three parts: $D1$, $D2$, $D3$
- We choose a secret key K , and watermark each part D_i **twice**
 - The first version D_i+ is generated the AHK algorithm
 - The second version D_i- is generated by flipping the watermarked bits in D_i+
- We then assign watermarked parts to users as shown above
- Analyze the security guarantee of this watermarking scheme

Exercise 1: Answer

$u1$ 	$D1-$,	$D2-$,	$D3-$
$u2$ 	$D1-$,	$D2+$,	$D3+$
$u3$ 	$D1-$,	$D2-$,	$D3+$
$u4$ 	$D1+$,	$D2-$,	$D3-$





- This watermarking scheme is not good because
 - If $u1$, $u2$, and $u4$ collude and use a majority vote, they can obtain a copy of the data that corresponds to $\{D1-, D2-, D3+\}$, which is the same as $u3$'s copy
 - By leaking this copy, $u1$, $u2$, $u4$ can frame $u3$ as the data leaker

Exercise 2

$u1$ 	$D1+$,	$D2-$,	$D3+$,	$D4-$
$u2$ 	$D1-$,	$D2+$,	$D3-$,	$D4-$
$u3$ 	$D1-$,	$D2-$,	$D3+$,	$D4+$
$u4$ 	$D1-$,	$D2+$,	$D3-$,	$D4+$





- Suppose that we are to give a dataset D to four users $u1$, $u2$, $u3$, and $u4$
- We randomly divide D into four parts: $D1$, $D2$, $D3$, $D4$
- We choose a secret key K , and watermark each part D_i **twice**
 - The first version D_i+ is generated the AHK algorithm
 - The second version D_i- is generated by flipping the watermarked bits in D_i+
- We then assign watermarked parts to users as shown above
- Analyze the security guarantee of this watermarking scheme

Exercise 2: Answer

$u1$ 	$D1+$,	$D2-$,	$D3+$,	$D4-$
$u2$ 	$D1-$,	$D2+$,	$D3-$,	$D4-$
$u3$ 	$D1-$,	$D2-$,	$D3+$,	$D4+$
$u4$ 	$D1-$,	$D2+$,	$D3-$,	$D4+$





- This watermarking scheme is OK, because:
 - No 3-user-collusion can frame the remaining user, regardless of whether they follow the majority vote or the minority vote
 - If the users collude to destroy watermarks and they involve $u1$, then the watermarks in $D1$ will be destroyed; in that case, $u1$ will get caught, since he is the only user who has $D1+$
 - Every user collusion that does not involve $u1$ would destroy a different set of watermarks, and hence, we can identify the colluding users by observing which watermarks are destroyed

Exercise 3

$u1$ 	$D1+$,	$D2-$,	$D3-, D4-$
$u2$ 	$D1-$,	$D2+$,	$D3-, D4-$
$u3$ 	$D1-$,	$D2-$,	$D3+, D4-$
$u4$ 	$D1-$,	$D2-$,	$D3-, D4+$

- Suppose that we are to give a dataset D to four users $u1$, $u2$, $u3$, and $u4$
- We randomly divide D into four parts: $D1$, $D2$, $D3$, $D4$
- We choose a secret key K , and watermark each part D_i **twice**
 - The first version D_i+ is generated the AHK algorithm
 - The second version D_i- is generated by flipping the watermarked bits in D_i+
- We then assign watermarked parts to users as shown above
- Analyze the security guarantee of this watermarking scheme

Exercise 3: Answer

$u1$ 	$D1+$,	$D2-$,	$D3-, D4-$
$u2$ 	$D1-$,	$D2+$,	$D3-, D4-$
$u3$ 	$D1-$,	$D2-$,	$D3+, D4-$
$u4$ 	$D1-$,	$D2-$,	$D3-, D4+$

- This watermarking scheme is not OK, because
 - If any 3 users collude and follow the majority rule, they can leak $\{D1-, D2-, D3-, D4-\}$
 - When the data owner sees $\{D1-, D2-, D3-, D4-\}$, she would be unable to pinpoint any of the culprits, because every user can claim that it was the other 3 users who leaked the data