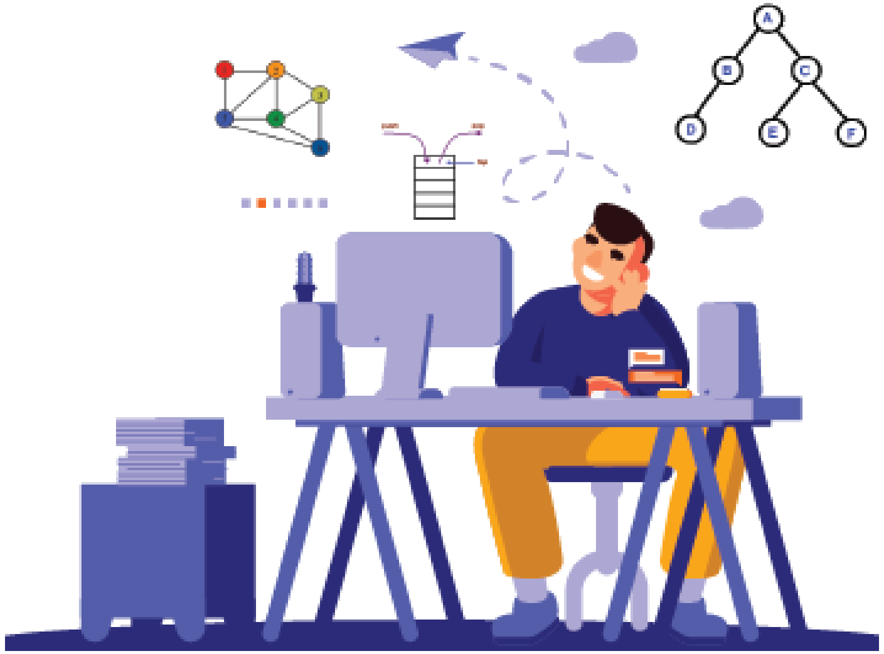
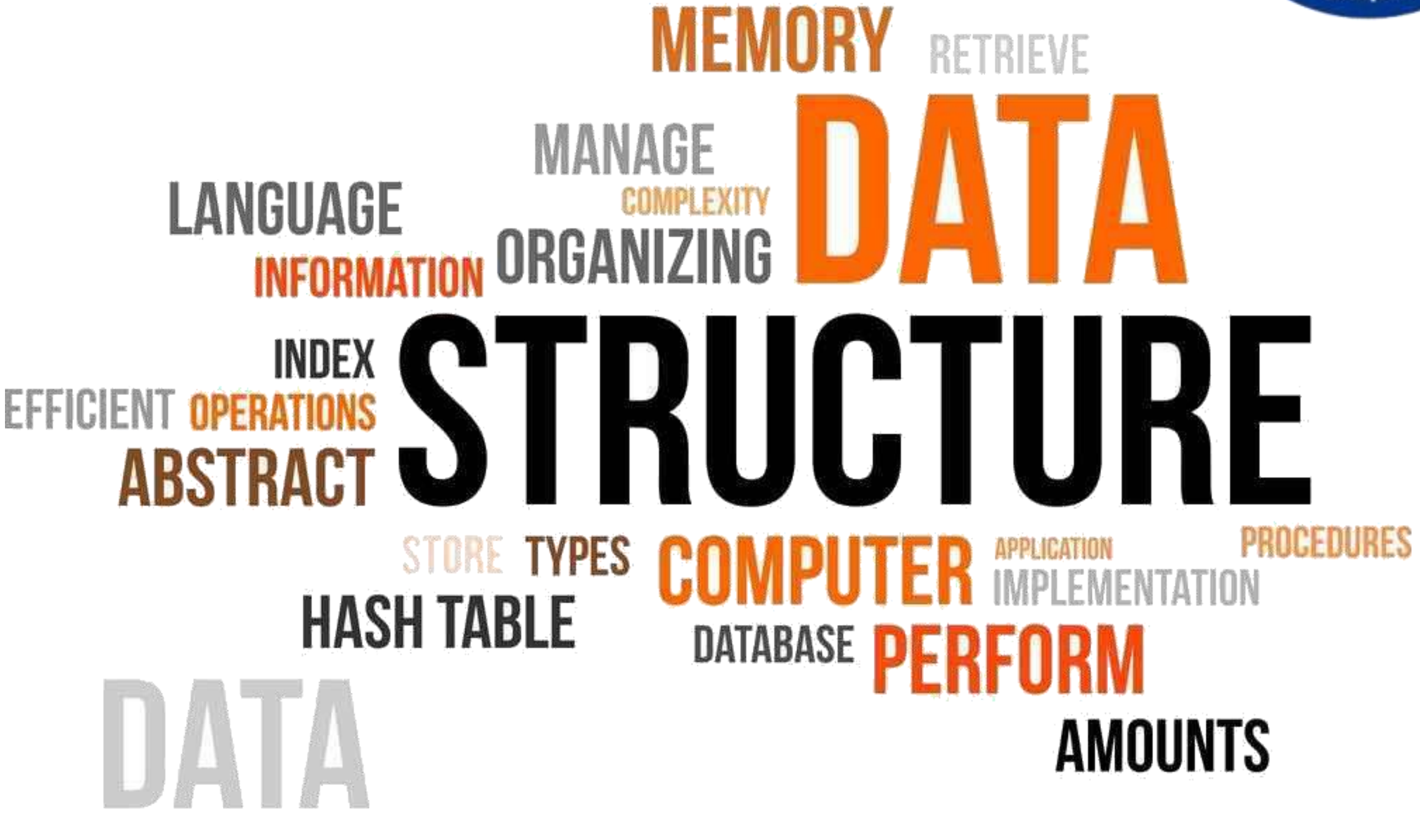
Data Structures

Course code: IT623





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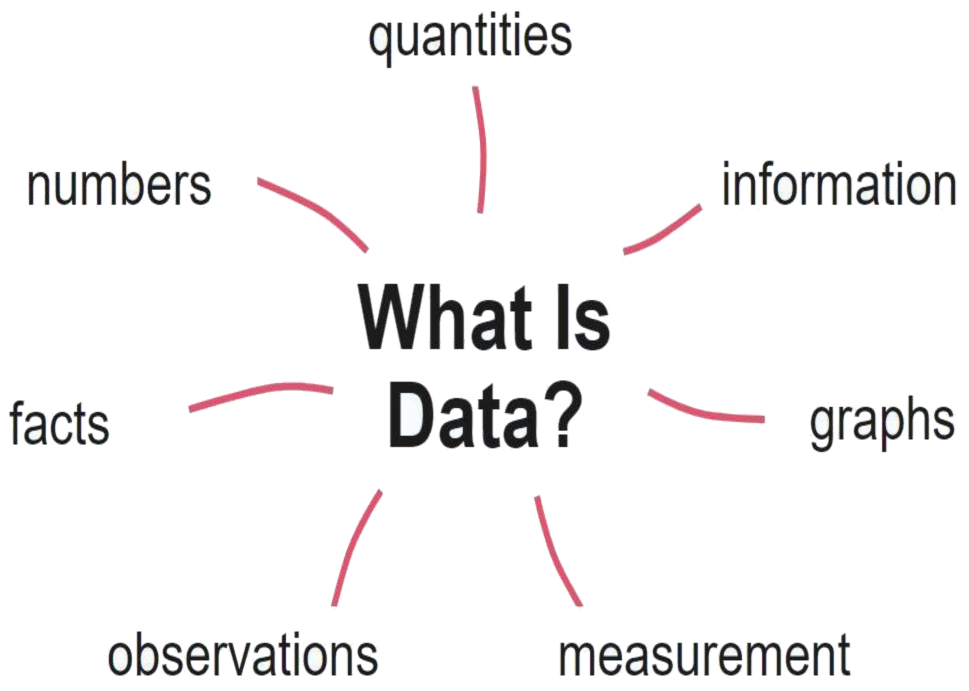
Lectures 2

Data

- In order to be useful, data must be organized, analyzed, and interpreted.
- Data analysis involves using statistical and computational tools to identify patterns, trends, and relationships within the data.
- This analysis can be used to draw conclusions, make predictions, and inform decision-making in a wide variety of fields, including business, healthcare, science, and more.



Data Structure

- Data may be organized in many different ways; the logical or mathematical model of a particular organization of data is called a ***data structure.***

- The choice of a particular data structure depends on two considerations:
  - It must be rich enough in structure to mirror the actual relationships of the data in the real world.
  - The structure should be simple enough that one can effectively process the data when necessary.

Data Structure Operations

- ***Traversing:*** Accessing each record exactly once.
- ***Searching:*** Particular record finding
- ***Inserting:*** Adding a new record
- ***Deleting:*** Removing the record

- **Difference between linear and non-linear data structures**

Standard Notations and common functions

1. Monotonicity

* A function $f(n)$ is monotonically increasing if $m \le n$ implies $f(m) \le f(n)$.

* Similarly, it is monotonically decreasing if $m \le n$ implies $f(m) \ge f(n)$.

* A function $f(n)$ is strictly increasing if $m < n$ implies $f(m) < f(n)$

* Strictly decreasing if $m < n$ implies $f(m) > f(n)$.

Standard Notations and common functions

$\boxed{\text{2. Floor and Ceiling Functions}}$

* For any real number x, we denote the greatest integer less than or equal x by $\boxed{\lfloor x \rfloor}$ (read "the floor of x") $\Rightarrow$ "greatest integer that does not exceed x" $\lfloor \ \rfloor$ floor value

* The least integer greater than or equal to x by $\lceil x \rceil$ (read "the ceiling of x") $\lceil \ \rceil$
$\Rightarrow$ "least integer that is not less than x"
Ceiling value

* For any real x decimal

i) $x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$ e.g.; $3.14-1 < \lfloor 3.14 \rfloor \le 3.14 \le \lceil 3.14 \rceil < 3.14+1$
$\Rightarrow 2.14 < 3 \le 3.14 \le 4 < 4.14$

* For any integer n

ii) $\lceil n/2 \rceil + \lfloor n/2 \rfloor = n$ e.g.; $n = 5$
$\lceil 2.5 \rceil + \lfloor 2.5 \rfloor = 3 + 2 = 5$

Standard Notations and common functions

Other properties of $\lfloor\ \rfloor$ & $\lceil\ \rceil$

For any real number $x \geq 0$ and integers $a, b > 0$,

ⓐ $\left\lceil \frac{\lceil x/a \rceil}{b} \right\rceil = \left\lceil \frac{x}{ab} \right\rceil$ e.g; $x = 7.32$, $a = 3$, $b = 2$

$\left\lceil \frac{\lceil 7.32/3 \rceil}{2} \right\rceil = \left\lceil \frac{\lceil 2.\ldots \rceil}{2} \right\rceil = \left\lceil \frac{3}{2} \right\rceil = 2$ L.H.S

$\left\lceil \frac{7.32}{6} \right\rceil = \lceil 1.\ldots \rceil = 2$ R.H.S Proved

ⓑ $\left\lfloor \frac{\lfloor x/a \rfloor}{b} \right\rfloor = \left\lfloor \frac{x}{ab} \right\rfloor$; Do by your self

ⓒ $\left\lceil \frac{a}{b} \right\rceil \leq \frac{a + (b-1)}{b}$; —"—

ⓓ $\left\lfloor \frac{a}{b} \right\rfloor \geq \frac{a - (b-1)}{b}$, —"—

Consider an example & prove it.

Standard Notations and common functions

* The floor function $f(x) = \lfloor x \rfloor$ is monotonically increasing, as is the ceiling function $f(x) = \lceil x \rceil$.

$\lfloor 3.14 \rfloor =$ $\lceil 3.14 \rceil =$

$\lfloor \sqrt{5} \rfloor =$ $\lceil \sqrt{5} \rceil =$

$\lfloor -8.5 \rfloor =$ $\lceil -8.5 \rceil =$

$\lfloor 7 \rfloor =$ $\lceil 7 \rceil =$

Standard Notations and common functions

3. Modular function & Arithmetic

* For any integer a and any positive integer n, the value $a \bmod n$ is the remainder (or residue) of the quotient a/n.

read as: "a modulo n"

* More exactly $K \pmod M$ is the unique integer r such that

$$\boxed{K = Mq + r \quad \text{where} \quad 0 \le r < M.}$$

* When K is positive, simple divide K by M to obtain remainder r. Thus,

$25 \pmod 7 = 4$; $25 \pmod 5 = 0$, $35 \pmod{11} = 2$, $3 \pmod 8 = 3$

* If $(a \bmod n) = (b \bmod n)$, we write $\boxed{a \equiv b \pmod n}$ and say that a is equivalent to b, modulo n.

* $\Rightarrow$ $\boxed{\text{Congruent}}$ $\rightarrow$

The mathematical Congruence relation is defined as follows:

$a \equiv b \pmod M$ if and only if M divides $b - a$.

Standard Notations and common functions

* In other words, $a \equiv b \pmod{n}$ if a and b have the same remainder when divided by n.

* Equivalently, $a \equiv b \pmod{n}$, if and only if n is a divisor of $b - a$.

* $a \not\equiv b \pmod{n}$ if a is not equivalent to b, modulo n.

$\boxed{\text{4. Integer and Absolute value Functions}}$

* Let x be any real number. The integer value of x, written $INT(x)$, converts x into an integer by deleting (truncating) the fractional part of the number.

$$INT(3.14) = 3, \quad INT(\sqrt{5}) = 2, \quad INT(-8.5) = -8, \quad INT(7) = 7$$

$INT(x) = \lfloor x \rfloor$ when x is positive

$INT(x) = \lceil x \rceil$ when x is negative.

* Similarly, absolute value gives a positive integer.