# Содержание

1	Hac	тройка CLion
2	Teop	рия чисел
	2.1	KTO
	2.2	Алгоритм Миллера         Рабина         1           Алгоритм Берлекэмпа         Месси         2
	2.4	Линейное решето
	2.5	Алгоритм Шенкса
3	Грас	ры З
Ĭ	3.1	SCC и 2-SAT
	3.2	Эйлеров цикл
	3.3	Компоненты рёберной двусвязности
	3.5	Взвешенное паросочетание
	3.6	Дерево доминаторов
4	Свё	отки -
	4.1	ХОП свёртка
	4.2	FFT & co
	4.3 4.4	Быстрое FFT
_	_	
5	<b>Стр</b> у	уктуры данных
	5.2	Дерево отрезков в точке
	5.3	Массовое дерево отрезков
	5.4 5.5	Битовый бор       10         Ordered set       10
	5.6	Динамический битсет
	5.7	Convex hull trick
	5.8	Центроиды
	5.9 5.10	Дерево Ли Чао       1         Min-Kinetic Segment Tree       1
		Декартово дерево
		5.11.1 Декартово дерево по явному ключу. Multiset
6	Стр	оковые алгоритмы 13
	6.1	Префикс-функция
	6.2	Z-функция       15         Алгоритм Манакера       15
	6.4	Алгоритм Манакера         15           Суфмассив         15
	6.5	Алгоритм Ахо — Корасик
	6.6	Дерево палиндромов
7	Пот	оки 15
	7.1	Алгоритм Диница
	7.2	Mincost k-flow
8		рритм Гаусса
	8.1	Решение $Av = b$
	0.2	
9		<b>ильтоновы путь и цикл</b> Link-cut tree
	9.1	Undirected case
	9.3	Directed case
10	Геог	ла 19
		Примитивы
		Выпуклая оболочка
		Точка внутри многоугольника         15           Касательные         20
		Пересечение многоугольника и полуплоскости
		События для прямой
	10.7	Кривая Гильберта для алгоритма Мо
11		ные дроби 21
		Поиск нижней огибающей, сумма и минимум по модулю
	11.2	Простая рекурсия
12	Раз	
		Компараторы         2           Трюки от Сергея Копелиовича         2
	12.2	12.2.1 Быстрый ввод
		12.2.2 Быстрый аллокатор
	12.3	Редукция Барретта       22         Флаги компияции       22
	12.4	Флаги компияции       22         12.4.1 Сеточка в vim       25
		Что сделать на пробном туре
		Хеш файла без комментариев

# 1 Настройка CLion

```
1. B файле CMakeLists.txt дописать строчку add_compile_definitions(LOCAL). Нажать появившуюся опцию в правом верхнем углу enable auto-reload.
```

```
2. Вбить шаблон в main.cpp:
  #ifdef LOCAL
  #define _GLIBCXX_DEBUG
  #endif
  #include<bits/stdc++.h>
  using namespace std;
  #define int long long
  #define app push_back
  #define all(x) x.begin(), x.end()
  #ifdef LOCAL
  #define debug(...) [](auto...a){ ((cout << a << ' '),</pre>
  #else
  #define debug(...)
  #define debugv(v)
  #endif
  int32_t main() {
```

Скомпилировать, чтобы проверить отсутствие опечаток.

debug(n); debugv(a);

cin.tie(0);ios\_base::sync\_with\_stdio(0);
int n = 2; vector<int> a(n, n);

```
3. Запустить терминал (crtl + alt + T)
$ cd workspace/CLionProjects
$ for c in {A..Z}; do cp main.cpp $c.cpp && echo "
```

Далее отключаем подсветку и форматирование в настройках (ctrl+alt+S)

- ullet Editor o Code Style o Formatter o Do not format прописать  $^\star$
- ullet Editor o Inspections o C/C++ o static analysis tools o CLang-Tidy от-ключить

add\_executable(\$c \$c.cpp)" >> CMakeLists.txt; done

Тёмная тема отключается в Appearance & Behavior  $\rightarrow$  Appearance. Чтобы добавить санитайзеры, надо дописать в CMakeLists.txt set(CMAKE\_CXX\_FLAGS "-fsanitize=address -fsanitize=undefined")

# 2 Теория чисел

## 2.1 KTO

```
int gcd(int a, int b, int &x, int &y) {
    if (b==0) { x = 1; y = 0; return a; }
    int d = gcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
int inv(int r, int m) {
    int x, y;
    gcd(r,m,x,y);
    return (x+m)%m;
}
int crt(int r, int n, int c, int m) { return r + ((c - r) % m + m) * inv(n, m) % m * n; }
//8ed8ed
```

## 2.2 Алгоритм Миллера — Рабина

```
__int128 one=1;
int po(int a,int b,int p)
{
  int res=1;
  while(b) {if(b & 1) {res=(res*one*a)%p;--b;} else {a=(a*one*a)%p;b>>=1;}} return res;
}
bool chprime(int n) ///miller-rabin
```

# 2.3 Алгоритм Берлекэмпа — Месси

https://mzhang2021.github.io/cp-blog/berlekamp-massey/

```
template<tvpename T>
vector<T> berlekampMassey(const vector<T> &s) {
  int n = s.size(), 1 = 0, m = 1;
  vector<T> b(n), c(n);
T ld = b[0] = c[0] = 1;
  for (int i=0; i<n; i++, m++) {
    T d = s[i];
    for (int j=1; j<=1; j++)
  d += c[j] * s[i-j];</pre>
    if (d == 0) continue;
    vector<T> temp = c;
    T coef = d / ld;
    for (int j=m; j<n; j++) c[j] -= coef * b[j-m];</pre>
    if (2 * 1 <= i) {
      1 = i + 1 - 1;
      b = temp;
      1d = d;
      m = 0;
    }
  c.resize(1 + 1);
  c.erase(c.begin());
  for (T &x : c)
    x = -x;
  return c;
//ff47ae
```

#### 2.4 Линейное решето

```
const int C = 1e7+7;
vi pr, lp(C);
for (int i = 2; i < C; ++i) {
   if (lp[i] == 0) {
      lp[i] = i;
      pr.app(i);
   }
   for (int j = 0; j < (int)pr.size() && pr[j] <= lp[i] && pr[j]
      ] * i < C; ++j) {
      lp[pr[j] * i] = pr[j];
   }
}
//36b3d1</pre>
```

# 2.5 Алгоритм Шенкса

```
#define T int
int mod;
int gcd(int a, int b, int &x, int &y) {
    if (b==0) { x = 1; y = 0; return a; }
    int d = gcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
int inv(int r, int m) {
    int x, y;
    gcd(r,m,x,y);
    return (x+m)%m;
}
int crt(int r, int n, int c, int m) { return r + ((c - r) % m + m) * inv(n, m) % m * n; }
```

```
T inv(T a)
  return inv(a, mod);
T mul(T a, T b)
  return (a*b)%mod;
vector<int> rasl(int x)
  vector<int> v;
  if(x==1) {return v;}
  for(int i=2;i*i<=x;++i)</pre>
    if(x%i==0)
      v=rasl(x/i);v.app(i);return v;
  v.app(x);return v;
T po(T a, int b) ///b>=1
  if(b==1) {return a;}
  if(b%2==0)
    T u=po(a,b/2);
    return mul(u,u);
  else
    T u=po(a,b-1);
    return mul(a,u);
T getper(T a, T one, int per, vector <int> v)
  for(int p:v)
    if(po(a,per/p)==one)
    {
      per/=p;
    }
  return per;
vector<pair<int, int> > shanks(T a, vector<T> b, T one, int per)
    ///a^per=1 and b[i]^per=1 /// all right numbers in output
     are equal
  if(b.empty()) {return {};}
  int n=b.size();
  vector<int> vp=rasl(per);
  int pera=getper(a, one, per, vp); per=pera;
  vp=rasl(pera);
  vector<int> have(n,0);
  int cur2=per;T cura=a;T invcura=inv(a);
  int curad=1;
  vector<pair<T, int> > v;
  vector<bool> ok(n,true);
  vector<T> poinvzx;
  for(int p:vp)
    T ca=po(cura,cur2/p);
    if(ca==one) {continue;}
    T invca=po(invcura,cur2/p);
    int step=sqrt(b.size()*p)+2;
    int wee=p/step+2;
    v.clear();poinvzx.clear();
    T zx=one; T invzx=one; T buba=one;
    vector<T> zhe;
    T lu=one;
    for(int i=0;i<step;++i)</pre>
      v.app({zx,i}); zhe.app(lu);
      zx=mul(zx,ca);invzx=mul(invzx,invca);buba=mul(buba,cura)
     ;lu=mul(lu,invcura);
    poinvzx.app(one);
    for(int j=0;j<wee;++j)</pre>
    {
      poinvzx.app(mul(poinvzx.back(),buba));
    sort(all(v));
    for(int i=0;i<n;++i)</pre>
      if(!ok[i]) {continue;}
      T uu=po(b[i],cur2/p);
      bool okkk=false;
      for(int j=0;j<wee;++j)</pre>
```

```
auto it=lower_bound(all(v),make_pair(uu,OLL));
        if(it!=v.end() && (*it).first==uu)
          okkk=true;
          have[i]-=(curad*step*j);
          have[i]+=(curad*(*it).second);
          have[i]%=pera;if(have[i]<0) {have[i]+=pera;}</pre>
          \texttt{b[i]=mul(b[i],poinvzx[j]);b[i]=mul(b[i],zhe[(*it).}
     second]);
          assert(po(b[i],cur2/p)==one);
          break:
        uu=mul(uu,zx);
      if(!okkk) {ok[i]=false;}
    cur2/=p;cura=po(cura,p);invcura=po(invcura,p);curad*=p;
  vector<pair<int,int> > res;
  for(int i=0;i<n;++i)</pre>
  {
    if(ok[i] && b[i]==one)
      res.app({(have[i]%pera+pera)%pera,pera});
    else
      res.app({-1,pera});
 return res;
int shanks2(int x,int y,int mod1) //only for T=long long, 0^0
      = 1 by default
 mod=mod1;
 vector<int> v=rasl(mod);sort(all(v));
  int per=1; for (int i=0; i < v. size(); ++i) {if(i==0 || v[i]!=v[i</pre>
    -1]) {per*=(v[i]-1);} else {per*=v[i];}}
  if(y==1 || mod==1) {return 0;}
  int C=61;
  for(int i=1;i<C;++i)</pre>
    if(po(x,i)==y) {return i;}
  if(y==0) {return (-1);}
 T h=po(x,C);
  int lc1=gcd(h,mod);int lc2=gcd(y,mod);
  if(lc1!=lc2) {return (-1);}
 mod/=lc2;T h1=h/lc2;T y1=y/lc2;
  vector<pair<int, int> > s=shanks(x%mod, {mul(y1, inv(h1))},1,
    per);
  if(s[0].first!=(-1))
 {
   return s[0].first+C;
    return (-1);
//a75596
```

# 3 Графы

#### **3.1** *SCC* и **2-**SAT

```
Алгоритм ищет сильносвязные компоненты в графе g, если есть путь i 	o j, то
scc[i] \leq scc[j]
vector<int> find_scc(vector<vector<int>>> g) {
 int n = g.size();
  vector<vector<int>> r(n);
 for (int i = 0; i < n; ++i) {</pre>
    for (int j : g[i]) r[j].push_back(i);
 vector<int> used(n), tout(n);
 int time = 0;
 auto dfs = [&](auto dfs, int cur) -> void {
    used[curl = 1;
    for (int nxt : g[cur]) {
      if (!used[nxt]) dfs(dfs, nxt);
    tout[cur] = time++;
  };
 for (int i = 0; i < n; ++i) if (!used[i]) dfs(dfs, i);</pre>
  vector<int> ind(n);
  iota(all(ind), 0);
```

```
sort(all(ind), [&](int i, int j){return tout[i] > tout[j];})
;
vector<int> scc(n, -1);
auto go = [&](auto go, int cur, int color) -> void {
    scc[cur] = color;
    for (int nxt : r[cur]) {
        if (scc[nxt] == -1) go(go, nxt, color);
      }
};
int color = 0;
for (int i : ind) {
    if (scc[i] == -1) go(go, i, color++);
}
return scc;
}
//4fd51f
```

Чтобы решать 2- $\mathcal{SAT}$ , надо создать граф на 2n вершинах, рёбра  $i\Rightarrow j$  и  $(j\oplus 1)\Rightarrow (i\oplus 1)$  должны быть добавлены одновременно. После этого если  $\mathrm{scc}[2\ ^\star\ i]=\mathrm{scc}[2\ ^\star\ i+1]$ , то решения нет; если  $\mathrm{scc}[2\ ^\star\ i+0]<\mathrm{scc}[2\ ^\star\ i+1]$ , то присутствует импликация  $\neg i\Rightarrow i$ , надо назначить  $i=\mathrm{true}$ .

# 3.2 Эйлеров цикл

```
vector<int> euler(vector<vector<pair<int, int>>> g, int m, int
                                                 src) { // g[cur][i] = pair{nxt, idx}
                 int n = g.size();
                 vector<int> used(m), it(n), cycle;
                 auto dfs = [&](auto dfs, int cur) -> void {
                                   while (true) {
                                                   while (it[cur] < g[cur].size() && used[g[cur][it[cur]].</pre>
                                         second]) it[cur]++;
                                                   if (it[cur] == g[cur].size()) return;
                                                    auto [nxt, idx] = g[cur][it[cur]];
                                                    used[idx] = true;
                                                   dfs(dfs, nxt);
                                                   cycle.push_back(idx); // or {cur, nxt}
                                }
                 dfs(dfs, src);
                 reverse(cycle.begin(), cycle.end());
                if (cycle.size() != m) return {}; // check that all edges
                                        are present in the cycle, fail otherwise % \left\{ 1\right\} =\left\{ 1\right\} =\left
                 return cycle;
}
//f6b9d4
```

#### 3.3 Компоненты рёберной двусвязности

```
//n - number of vertices, m - number of edges, parallel edges
     -- ???, color of any edge is the color of its lower end
vector <vector <int> > dfstree(n);
vector <int> used(n), cut(n), h(n), up(n);
auto findCutPoints = [&] (auto self, int u) -> void {
    used[u] = 1;
    up[u] = h[u];
    for (int v : g[u]) {
        if (!used[v]) {
            dfstree[u].push_back(v);
            h[v] = h[u] + 1;
            self(self, v);
            up[u] = min(up[u], up[v]);
            if (up[v] >= h[u]) {
                cut[v] = 1;
        }
        else {
            up[u] = min(up[u], h[v]);
    }
findCutPoints(findCutPoints, 0);
vector <vector <int> > tree(n + m);
vector<int> color(n);color[0]=0;int ptr=n;
auto build = [&] (auto self, int u) -> void {
    for (int v : dfstree[u]) {
        if (cut[v]) {
            color[v]=ptr;++ptr;
            self(self, v);
        }
        else {
            color[v]=color[u];
            self(self, v);
        }
   }
build(build, 0);
```

```
for(int i=0;i<n;++i) {
    set<int> to;
    for(int j:g[i]) {
        int x=i,y=j;
        if(h[x]<h[y]) swap(x,y);
        to.insert(color[x]);
    }
    for(int j:to) {
        tree[i].app(j);tree[j].app(i);
    }
}
//2ebfbb</pre>
```

#### 3.4 DCP offline

```
struct Dsu {
 int n;
 vector<pair<int &, int>> s;
 vector<int> p, sz;
 // other info
  Dsu(int n) : n(n), p(n), sz(n, 1){
   iota(all(p), 0);
 int get(int u) {
   while (u != p[u]) u = p[u];
   return u:
 bool merge(int u, int v) {
   u = get(u), v = get(v);
    if (u == v) return false;
    if (sz[v] < sz[u]) swap(u, v);
   s.app({p[u], p[u]});
   s.app({sz[v], sz[v]});
    // app other info like s.app({comp, comp});
    sz[v] += sz[u];
   return true;
  void rollback(int sz) {
   while (s.size() != sz) {
     s.back().first = s.back().second;
      s.pop_back();
 }
struct DcpOffline {
 int n;
 vector<vector<pair<int, int>>> d;
  void addEdgeOnSegment(int 1, int r, int a, int b) {
   for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {
     if (1 & 1) d[1++].app({a, b});
      if (r & 1) d[--r].app({a, b});
   }
 }
 template<typename T>
 void dfs(Dsu &dsu, T act) {
   dfs(1, 0, n, dsu, act);
  template<typename T>
 void dfs(int v, int 1, int r, Dsu &dsu, T act) {
   int sz = dsu.s.size();
    for (auto [u, v]: d[v]) {
     dsu.merge(u, v);
   if (1 + 1 == r) {
     act(1, dsu);
    } else {
      int m = (1 + r) / 2;
      dfs(v * 2, 1, m, dsu, act);
     dfs(v * 2 + 1, m, r, dsu, act);
   dsu.rollback(sz);
 DcpOffline(int maxt) : n(2 \ll _lg(maxt + 1)), d(2 * n) {}
};
```

#### 3.5 Взвешенное паросочетание

https://judge.yosupo.jp/submission/201334

```
\#define\ d(x)\ (lab[x.u] + lab[x.v] - 2 * e[x.u][x.v].w)
const int N = 403*2;
const int inf = 1e18;
struct Q{ int u, v, w; } e[N][N];
vector<int> p[N];
int n, m = 0, id, h, t, lk[N], sl[N], st[N], f[N], b[N][N], s[
    N], ed[N], q[N], lab[N];
void upd(int u, int v) { if (!sl[v] \mid | d(e[u][v]) < d(e[sl[v]))
    ]][v])) sl[v] = u; }
void ss(int v) {
 sl[v] = 0;
  for (int u = 1; u \le n; ++u) if (e[u][v].w > 0 && st[u] != v
     && !s[st[u]]) upd(u, v);
void ins(int u){ if (u \le n) q[++t] = u; else for (int v : p[u]
     ]) ins(v); }
void ch(int u, int w) { st[u] = w; if (u > n) for (int v : p[u]
    ]) ch(v, w); }
int gr(int u, int v) {
   if ((v = find(all(p[u]), v) - p[u].begin()) & 1) {
    reverse(1 + all(p[u]));
    return (int)p[u].size() - v;
  return v;
void stm(int u, int v) {
  lk[u] = e[u][v].v;
  if (u \le n) return; Q w = e[u][v];
 int x = b[u][w.u], y = gr(u,x);
for (int i = 0; i < y; ++i) stm(p[u][i], p[u][i^1]);
  stm(x, v); rotate(p[u].begin(), y+all(p[u]));
void aug(int u, int v) {
 int w = st[lk[u]];stm(u, v);if (!w) return;
  stm(w, st[f[w]]);
  aug(st[f[w]], w);
int lca(int u, int v) {
  for (id++; u|v; swap(u, v)) {
    if (!u) continue;if(ed[u] == id) return u;
    ed[u] = id; if (u = st[lk[u]]) u = st[f[u]]; // =, not ==
  return 0;
//cf1d55
void add(int u, int a, int v) {
  int x = n + 1; while (x \le m \&\& st[x]) ++x;
  if (x > m) ++m;
  lab[x] = s[x] = st[x] = 0;
  lk[x] = lk[a];
  p[x].clear();
  p[x].push_back(a);
\#define op(q) for (int i = q, j = 0; i != a; i=st[f[j]]) p[x].
    push\_back(i), \; p[x].push\_back(j=st[lk[i]]), \; ins(j) \; // \; also
      not =
  op(u); reverse(1+all(p[x]));op(v);
  ch(x, x); for (int i = 1; i <= m; ++i) e[x][i].w = e[i][x].w
      = 0;
  fill(b[x]+1, b[x]+n+1, 0);
  for (int u : p[x]) {
    for (int v = 1; v \le m; ++v) if (!e[x][v].w || d(e[u][v])
     < d(e[x][v])) e[x][v] = e[u][v], e[v][x] = e[v][u];
    for (int v = 1; v \le n; ++v) if (b[u][v]) b[x][v] = u;
  ss(x);
void ex(int u) {
  for (int x : p[u]) ch(x, x);
  int a = b[u][e[u][f[u]].u], r = gr(u, a);
  for (int i = 0; i < r; i += 2) {
  int x = p[u][i], y = p[u][i + 1];</pre>
    f[x] = e[y][x].u; s[x] = 1; s[y] = 0; sl[x] = 0; ss(y);
     ins(y);
  s[a] = 1; f[a] = f[u];
  for (int i = r + 1; i < p[u].size(); ++i) s[p[u][i]] = -1,
     ss(p[u][i]);
  st[u] = 0;
bool on(const O &e) {
  int u = st[e.u], v = st[e.v], a;
  if (s[v] == -1) {
    f[v] = e.u, s[v] = 1, a = st[lk[v]], sl[v] = sl[a] = s[a]
     = 0, ins(a);
  } else if (!s[v]) {
    a = lca(u, v); if (!a) return aug(u, v), aug(v, u), 1;
```

else add(u, a, v);

```
return 0;
//3f0f1d
bool bfs() {
  fill(s+1, s+m+1, -1); fill(sl+1, sl+m+1, 0); // s is filled
    with -1
 if (h > t) return 0;
 while (1) {
   while (h <= t) {
  int u = q[h++];</pre>
      if (s[st[u]] != 1) {
        for (int v = 1; v \le n; ++v) if (e[u][v].w > 0 && st[u]
    ] != st[v]) {
         if (d(e[u][v])) upd(u, st[v]); else if (on(e[u][v]))
     return 1:
       }
     }
    int x = inf;
    for (int i = n+1; i <= m; ++i) if (st[i] == i && s[i] ==
    1) x = min(x, lab[i]/2);
    for (int i = 1; i <= m; ++i) if (st[i] == i && sl[i] && s[
    i] != 1) x = min(x, d(e[sl[i]][i])>>s[i]+1);
    for (int i = 1; i <= n; ++i) if (~s[st[i]]) if ((lab[i] +=</pre>
     (s[st[i]] * 2 - 1) * x) <=0) return 0;
    for (int i = n + 1; i <= m; ++i) if (st[i] == i && ~s[st[i]</pre>
    ]]) lab[i] += (2 - 4 * s[st[i]]) * x;
    h = 1, t = 0;
    for (int i = 1; i <= m; ++i) if (st[i] == i && sl[i] && st
    [sl[i]] != i && !d(e[sl[i]][i]) && on(e[sl[i]][i]))
    for (int i = n+1; i <= m; ++i) if (st[i] == i && s[i] == 1
     && !lab[i]) ex(i);
pair<int, vector<array<int, 2>>> run(int N, vector<array<int,</pre>
    3>> edges) {
  for (auto &[u, v, w] : edges) ++u, ++v;
  fill(ed+1, ed+m+1, 0);
  fill(lk+1, lk+m+1, 0);
 n = m = N;
  id = 0;
  iota(st + 1, st + n + 1, 1);
  int wm = 0, weight = 0;
  for (int i = 1; i <= n; ++i) for (int j = 1; j <= n; ++j) e[
    i][j] = \{i, j, 0\};
  for (auto [u, v, w]: edges) wm = max(wm, e[v][u].w = e[u][v
    ].w = max(e[u][v].w, w));
  for (int i = 1; i <= n; ++i) p[i].clear();</pre>
  for (int i = 1; i <= n; ++i) for (int j = 1; j <= n; ++j) b[
    i][j] = i==j?i:0;
  fill_n(lab+1, n, wm); while (bfs());
 vector<array<int, 2>> matching;
for (int i = 1; i <= n; ++i) if (i < lk[i]) weight += e[i][</pre>
    lk[i]].w, matching.push_back({i - 1, lk[i] - 1});
  return {weight, matching};
//be682f
```

#### 3.6 Дерево доминаторов

```
struct DominatorTree{
 struct DSU{
   struct Vert(
     int p;
     pair<int, int> val;
    };
    vector<Vert> t:
    vector<int> ord;
    DSU(vector<int> &ord): ord(ord) { t.resize(ord.size());
    for (int i = 0; i < ord.size(); i++) t[i].p = i; }</pre>
    int get(int v){
      if (t[v].p == v) return v;
      int new_p = get(t[v].p);
      if (ord[t[v].val.first] > ord[t[t[v].p].val.first]) t[v
    ].val = t[t[v].p].val;
      t[v].p = new_p;
```

```
return t[v].p;
  void merge(int a, int b){
    a = get(a); b = get(b);
    if (a != b) {
      t[b].p = a;
    }
  void setVal(int v, pair<int, int> val){
    t[v].val = val;
  auto getVal(int v){
    get(v);
    return t[v].val;
  }
};
vector<vector<int> > g, gr, lg;
vector<int> idom, sdom, was, tin;
int timer;
void dfs(int v){
 tin[v] = timer++;
was[v] = 1;
  for (int to : g[v]) if (!was[to]) dfs(to);
vector<vector<int> > req;
DominatorTree(int n, vector<pair<int, int> > &edges, int
  root){
  g.resize(n); gr.resize(n); lg.resize(n);
  idom.resize(n, -1); sdom.resize(n);
  was.resize(n, 0), tin.resize(n);
  req.resize(n);
  for (auto &&e : edges){
    g[e.first].push back(e.second);
    gr[e.second].push_back(e.first);
  timer = 0; dfs(root);
  vector<int> ord;
  for (int i = 0; i < n; i++) ord.push_back(i);</pre>
  sort(ord.begin(), ord.end(), [this](int w1, int w2){
   return tin[w1] > tin[w2]; });
  DSU dsu(tin);
  for (int v : ord){
    sdom[v] = v;
    for (int to : gr[v]){
      if (v == to) continue;
      int val = tin[to] < tin[v] ? to : dsu.getVal(to).first</pre>
      if (tin[val] < tin[sdom[v]]) sdom[v] = val;</pre>
    req[sdom[v]].push_back(v);
    for (auto &&r : req[v]){
      auto val = dsu.getVal(r);
      if (tin[val.first] < tin[sdom[r]]){</pre>
        lg[val.second].push_back(r);
        idom[r] = sdom[r];
    dsu.setVal(v, make_pair(sdom[v], v));
    for (int to : g[v]){
      if (tin[to] > tin[v] && dsu.t[to].p == to){
        dsu.merge(v, to);
    }
  }
  for (int i = 0; i < n; i++) was[i] = 0;</pre>
  for (int i = 0; i < n; i++) if (!was[i] && idom[i] != -1){
    vector<int> st;
    st.push back(i);
    was[i] = 1;
    while(st.size()){
      int v = st.back(); st.pop_back();
      idom[v] = idom[i];
      for (int to : lg[v]) if (!was[to]) was[to] = 1, st.
   push back(to);
    }
  }
}
```

};

```
/*
vector <pair <int, int> > e;
DominatorTree d(n,e,0);
auto par = d.idom;
*/
//839464
```

# 4 Свёртки

# 4.1 XOR свёртка

```
vector<int> bxor(vector<int> a, vector<int> b)
 assert(p%2==1); int inv2=(p+1)/2;
  int n=0; while((1<<n)<a.size()) ++n;</pre>
  a.resize(1<<n);b.resize(1<<n);
  for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n);++mask) if</pre>
     (!(mask \& (1<<i))) \{int u=a[mask], v=a[mask+(1<<i)]; a[mask]\}
     +(1<<i)]=(u+v)p;a[mask]=(u-v)p;}
  for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n);++mask) if</pre>
     (!(mask \& (1<<i))) \{int u=b[mask], v=b[mask+(1<<i)]; b[mask]\}
     +(1<<i)]=(u+v)%p;b[mask]=(u-v)%p;}
  vector<int> c(1<<n,0);</pre>
  for(int mask=0; mask<(1<<n); ++ mask) {c[mask]=a[mask]*b[mask];</pre>
     c[mask]%=p;}
  for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n);++mask) if</pre>
      (!(mask \& (1<<i))) \ \{ \\ int \ u=c[mask], v=c[mask+(1<<i)]; c[mask \\
     +(1<<i)]=((v-u)*inv2)*p;c[mask]=((u+v)*inv2)*p;}
//20cc50
```

#### 4.2 FFT & co

```
typedef long long 11;
const int p=998244353;
int po(int a,int b) {if(b==0) return 1; if(b==1) return a; if(
     b%2==0) {int u=po(a,b/2);return (u*1LL*u)%p;} else {int u
     =po(a,b-1); return (a*1LL*u)%p; } }
int inv(int x) {return po(x,p-2);}
template<int M, int K, int G> struct Fft {
  // 1, 1/4, 1/8, 3/8, 1/16, 5/16, 3/16, 7/16, ...
  int g[1 << (K - 1)];</pre>
  Fft() : g() { //if tl constexpr...
    // static_assert(K >= 2, "Fft: K >= 2 must hold");
    g[0] = 1;
    q[1 << (K - 2)] = G;
    for (int 1 = 1 << (K - 2); 1 >= 2; 1 >>= 1) {
      g[1 >> 1] = (g[1] * 1LL* g[1]) % M;
    assert((g[1]*1LL * g[1]) % M == M - 1);
    for (int 1 = 2; 1 <= 1 << (K - 2); 1 <<= 1) {
  for (int i = 1; i < 1; ++i) {
        g[1 + i] = (g[1] * 1LL * g[i]) % M;
   }
 void fft(vector<int> &x) const {
    const int n = x.size();
    assert(n <= 1 << K);
    for (int h = __builtin_ctz(n); h--; ) {
  const int l = (1 << h);</pre>
      for (int i = 0; i < n >> (h+1); ++i) {
        for (int j = i \ll (h+1); j < (((i \ll 1) + 1) \ll h); ++
     j) {
          const int t = (g[i] * 1LL* x[j | 1]) % M;
          x[j | 1] = x[j] - t;
          if (x[j|1] < 0) x[j | 1] += M;
          x[j]+=t;
          if (x[j] >= M) x[j] -= M;
        }
     }
    for (int i = 0, i = 0; i < n; ++i) {
      if (i < j) std::swap(x[i], x[j]);
      for (int 1 = n; (1 >>= 1) && !((j ^= 1) & 1); ) {}
    }
 vector<int> convolution(vector<int> a, vector<int> b) const
    if(a.empty() || b.empty()) return {};
    for(int& x:a) \{x\%=p; if(x>=p) x-=p; if(x<0) x+=p;\} for(int&
     x:b) \{x\%=p; if(x>=p) x-=p; if(x<0) x+=p;\}
    const int na = a.size(), nb = b.size();
    int n, invN = 1;
```

```
for (n = 1; n < na + nb - 1; n <<= 1) invN = ((invN & 1) ?
      (invN + M) : invN) >> 1;
    vector < int > x(n, 0), y(n, 0);
    std::copy(a.begin(), a.end(), x.begin());
    std::copy(b.begin(), b.end(), y.begin());
    fft(x);
    fft(y);
    for (int i = 0; i < n; ++i) x[i] = (((static_cast<long</pre>
     long>(x[i]) * y[i]) % M) * invN) % M;
    std::reverse(x.begin() + 1, x.end());
    fft(x);
    x.resize(na + nb - 1);
    return x;
 }
Fft<998244353,23,31> muls;
//a1b591
vector<int> form(vector<int> v.int n)
  while(v.size()<n) v.push back(0);</pre>
  while(v.size()>n) v.pop_back();
  return v;
vector<int> operator *(vector<int> v1, vector<int> v2)
  return muls.convolution(v1, v2);
vector<int> operator +(vector<int> v1, vector<int> v2)
  while(v2.size()<v1.size()) v2.push_back(0); while(v1.size()<</pre>
    v2.size()) v1.push back(0);
  for(int i=0;i<v1.size();++i) {v1[i]+=v2[i];if(v1[i]>=p) v1[i
     ]-=p; else if(v1[i]<0) v1[i]+=p;}</pre>
  return v1;
vector<int> operator -(vector<int> v1, vector<int> v2)
  int sz=max(v1.size(),v2.size());while(v1.size()<sz) v1.</pre>
    push_back(0); while(v2.size()<sz) v2.push_back(0);</pre>
  for(int i=0;i<sz;++i) {v1[i]-=v2[i];if(v1[i]<0) v1[i]+=p;</pre>
    else if(v1[i]>=p) v1[i]-=p;} return v1;
vector<int> trmi(vector<int> v)
  for(int i=1;i<v.size();i+=2) {if(v[i]>0) v[i]=p-v[i]; else v
     [i]=(-v[i]);}
  return v;
vector<int> deriv(vector<int> v)
  if(v.empty()) return{};
  vector<int> ans(v.size()-1);
  for(int i=1;i<v.size();++i) ans[i-1]=(v[i]*1LL*i)%p;</pre>
  return ans;
vector<int> integ(vector<int> v)
  vector<int> ans(v.size()+1);ans[0]=0;
  for(int i=1;i<v.size();++i) ans[i-1]=(v[i]*1LL*i)%p;</pre>
  return ans;
vector<int> mul(vector<vector<int> > v)
  if(v.size()==1) return v[0];
  vector<vector<int> > v1, v2; for(int i=0; i<v.size()/2; ++i) v1.</pre>
     push_back(v[i]); for(int i=v.size()/2;i<v.size();++i) v2.</pre>
     push back(v[i]);
  return muls.convolution(mul(v1), mul(v2));
vector<int> inv1(vector<int> v, int n)
  assert(v[0]!=0);
  int sz=1;v=form(v,n);vector<int> a={inv(v[0])};
  while(sz<n)
    vector<int> vsz; for(int i=0;i<min(n,2*sz);++i) vsz.</pre>
    push back(v[i]);
    vector<int> b=((vector<int>) {1})-muls.convolution(a,vsz);
    for(int i=0;i<sz;++i) assert(b[i]==0);</pre>
    b.erase(b.begin(),b.begin()+sz);
    vector<int> c=muls.convolution(b,a);
    for(int i=0;i<sz;++i) a.push_back(c[i]);</pre>
    sz*=2;
  return form(a,n);
```

//12aa4e

# 4.3 Быстрое FFT

- Solution based on https://codeforces.com/blog/entry/117947
- · Iterative and in-place version.
- · Uses signed montgomery
- Optimized to minimize memory usage

```
const int MOD = 998244353;
const long long MOD2 = (long long) MOD * MOD;
const int root = 3:
const int alim = 64; // Bound for using O(n^2) polynomial mult
int modpow(int b, int e) {
  int ans = 1:
  for (; e; b = (long long) b * b % MOD, e /= 2)
    if (e & 1) ans = (long long) ans * b % MOD;
  return ans:
const int MODinv = 2 - MOD; // pow(-MOD, -1, 2**32)
inline int m_reduce(long long x) {
  int m = x * MODinv;
  return (x>>32) - (((long long) m * MOD) >> 32);
const int r2 = modpow(2, 64);
inline int m_transform(int x) {
 return m_reduce((long long)x * r2);
inline int m_add(int x, int y) {
 int z = x + y;
  return z < 0 ? z + MOD : z - MOD;</pre>
inline int m_sub(int x, int y) {
  int z = x - y;
  return z < 0 ? z + MOD : z - MOD;</pre>
inline int m_mult(int x, int y) {
  return m_reduce((long long) x * y);
vector<int> rt = {1};
vector<int> transformed_rt;
vector<int> transformed_rt2;
template<int a>
void transform(vector<int> &P) {
  int m = P.size();
  int n = m / a;
  int size = rt.size();
while (2 * size < n) {</pre>
    rt.resize(n / 2);
    int r = modpow(root, MOD / (4 * size));
    for (int i = 0; i < size; ++i)</pre>
      rt[i + size] = (long long) r * rt[i] % MOD;
    size *= 2;
  // For montgomery
  for (int i = transformed_rt.size(); i < rt.size(); ++i) {</pre>
    transformed rt.resize(rt.size());
    transformed_rt[i] = m_transform(rt[i]);
    transformed_rt2.resize(rt.size());
    transformed_rt2[i] = (unsigned int) MODinv *
     transformed_rt[i];
  int k = n:
  while (k >= 4) k /= 4;
  if (k == 2) {
    int step = n * a;
    int half_step = step / 2;
for (int j1 = 0; j1 < half_step; ++j1) {
  int j2 = j1 + half_step;</pre>
      int diff = m_sub(P[j1], P[j2]);
      P[j1] = m_add(P[j1], P[j2]);
      P[j2] = diff;
    k = n/2;
```

```
} else {
 for (; k > 1; k \neq 4) {
    for (int i = 0; i < n/k; ++i) {
     int step = k * a;
      int half_step = step / 2;
      int quarter_step = half_step / 2;
      int R20 = transformed_rt2[2 * i];
      int RR0 = transformed_rt[2 * i];
      int R21 = transformed_rt2[2 * i + 1];
      int RR1 = transformed_rt[2 * i + 1];
      int R2 = transformed_rt2[i];
      int RR = transformed_rt[i];
      int j1 = i * step;
      int j2 = j1 + quarter_step;
      int j3 = j2 + quarter_step;
      int j4 = j3 + quarter_step;
      for (int j = 0; j < quarter_step; ++j, ++j1, ++j2, ++j3,</pre>
      ++j4) {
       int z0;
        {
          int z = P[j3];
int m = (unsigned int) R2 * z;
          z0 = ((long long) z * RR - (long long) m * MOD) >>
    32:
        int z1:
          int z = P[i4];
          int m = (unsigned int) R2 * z;
          z1 = ((long long) z * RR - (long long) m * MOD) >>
    32;
        int sum0 = m_add(P[j1], z0);
        int diff0 = m_sub(P[j1], z0);
        int sum1 = P[j2] + z1;
        int diff1 = P[j2] - z1;
        // [sum0, sum1, diff0, diff1]
        int zz0;
        {
          int z = sum1;
          int m = (unsigned int) R20 * z;
          zz0 = ((long long) z * RR0 - (long long) m * MOD) >>
       }
        int zz1;
        {
          int z = diff1;
          int m = (unsigned int) R21 * z;
          zz1 = ((long long) z * RR1 - (long long) m * MOD) >>
        P[j1] = m_add(sum0, zz0);
       P[j2] = m_sub(sum0, zz0);
        P[j3] = m_add(diff0, zz1);
        P[j4] = m_sub(diff0, zz1);
  for (int i = 0; i < m; ++i)
   if (P[i] < 0) P[i] += MOD;</pre>
template<int a>
void inverse_transform(vector<int> &P) {
 int m = P.size();
 int n = m / a;
 int n_inv = m_transform(modpow(n, MOD - 2));
 vector<int> rev(n);
for (int i = 1; i < n; ++i) {</pre>
   rev[i] = rev[i / 2] / 2 + (i & 1) * n / 2;
  // P = [p * n_inv for p in P]
  for (int i = 0; i < m; ++i)
```

```
P[i] = m_mult(n_inv, P[i]);
  // P = [P[a * rev[i // a] + (i % a)] for i in range(m)]
  for (int i = 1; i < n; ++i)</pre>
    if (i < rev[i])</pre>
      swap\_ranges(P.begin() + a * i, P.begin() + a * i + a, P.
     begin() + a * rev[i]);
  // P = [P[-a * (i // a) + (i % a)] for i in range(m)]
  for (int i = 1; i < n/2; ++i)
    swap_ranges(P.begin() + a * i, P.begin() + a * i + a, P.
     begin() + a * (n - i));
  transform<a>(P);
  // P = [P[a * rev[i // a] + (i % a)] for i in range(m)]
  for (int i = 1; i < n; ++i)
    if (i < rev[i])</pre>
      swap_ranges(P.begin() + a * i, P.begin() + a * i + a, P.
     begin() + a * rev[i]);
template<int a>
void fast_polymult_mod(vector<int> &P, vector<int> &Q) {
 int m = P.size();
 int n = m / a;
  transform<a>(P):
 transform < a > (0):
 vector<int> &PQ = P;
  for (int i = 0; i < n; ++i) {
    vector<unsigned long long> res(2 * a);
    for (int j = 0; j < a; ++j) {
      if (j >= 10 && j & 9 == 8)
for (int k = j; k < j + a - 10; ++k)
res[k] -= (res[k] >> 63) * 9 * MOD2;
      for (int k = 0; k < a; ++k)
        res[j + k] += (long long) P[i * a + j] * Q[i * a + k];
    int c = rt[i/2];
    if (i & 1) c = MOD - c;
    for (int j = 0; j < a; ++j)
      PQ[i * a + j] = (res[j] + c * (res[j + a] % MOD)) % MOD;
 inverse transform<a>(PQ);
template <size_t... N>
void work(std::index_sequence<N...>, int x, std::vector<int>&
     a, std::vector<int>& b) {
  static void (*ptrs[])(std::vector<int>&, std::vector<int>&)
     = {&fast_polymult_mod<N+1>...};
 ptrs[x - 1](a, b);
void fast_polymult(vector<int> &P, vector<int> &Q) {
 int m1 = P.size();
  int m2 = Q.size();
 int res_len = m1 + m2 - 1;
  int b = 1;
  while ((alim << b) < res_len) ++b;</pre>
  int a = ((res_len - 1) >> b) + 1;
  int m = a << b;</pre>
 P.resize(m);
 Q.resize(m);
  // Call fast_polymult_mod<a>(P, Q);
 work(std::make_index_sequence<alim>{}, a, P, Q);
 P.resize(res_len);
}
//239b3e
```

#### 4.4 FFT B double'ax

```
using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> & a, bool invert) {
  int n = a.size();

for (int i = 1, j = 0; i < n; i++) {
  int bit = n >> 1;
```

```
for (; j & bit; bit >>= 1)
      j ^= bit;
    j ^= bit;
   if (i < j)</pre>
      swap(a[i], a[j]);
 for (int len = 2; len <= n; len <<= 1) {
  double ang = 2 * PI / len * (invert ? -1 : 1);</pre>
    cd wlen(cos(ang), sin(ang));
    for (int i = 0; i < n; i += len) {</pre>
      cd w(1);
      for (int j = 0; j < len / 2; j++) {</pre>
        cd u = a[i+j], v = a[i+j+len/2] * w;
        a[i+j] = u + v;
        a[i+j+len/2] = u - v;
        w *= wlen;
     }
   }
 if (invert) {
   for (cd & x : a)
      x /= n;
 }
vector<int> multiply(vector<int> const& a, vector<int> const&
    b) {
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
  int n = 1;
  while (n < a.size() + b.size())</pre>
   n <<= 1:
  fa.resize(n);
 fb.resize(n);
  fft(fa, false);
  fft(fb, false);
  for (int i = 0; i < n; i++)
   fa[i] *= fb[i];
  fft(fa, true);
  vector<int> result(n);
 for (int i = 0; i < n; i++)
   result[i] = round(fa[i].real());
  while(!result.empty() && !result.back()) result.pop_back();
 return result;
//35d9d0
```

# 5 Структуры данных

#### 5.1 Дерево Фенвика

```
int fe[maxn];
void pl(int pos,int val) {while(pos<maxn) {fe[pos]+=val;pos|=(
    pos+1);}}
int get(int pos) {int ans=0;while(pos>=0) {ans+=fe[pos];pos&=(
    pos+1);--pos;} return ans;} /// [0,pos] - vkluchitelno!!!
int get(int l,int r) {return get(r-1)-get(l-1);} /// sum of [l
    ,r)
//2991a1
```

#### 5.2 Дерево отрезков в точке

```
template<typename T, typename U>
struct SegmentTree {
 int h, n;
 T neutral:
 U unite:
 vector<T> data;
  template<tvpename I>
  SegmentTree(int sz, T neutral, U unite, I init) : h(__lg(sz)
     + 1), n(1 << h), neutral(neutral), unite(unite), data(2
    * n) {
   for (int i = 0; i < sz; ++i) data[i + n] = init(i);</pre>
   for (int i = n - 1; i > 0; --i) data[i] = unite(data[2 * i
    ], data[2 * i + 1]);
  SegmentTree(int sz, T neutral, U unite) : h(__lg(sz) + 1), n
    (1 << h), neutral(neutral), unite(unite), data(2 * n,
    neutral) {}
  void set(int i, T x) {
```

```
data[i += n] = x;
    for (i /= 2; i > 0; i /= 2) data[i] = unite(data[2 * i],
    data[2 * i + 1]);
 T get(int 1, int r) {
    T leftRes = neutral, rightRes = neutral;
    for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {
      if (1 & 1) leftRes = unite(leftRes, data[1++]);
      if (r & 1) rightRes = unite(data[--r], rightRes);
    return unite(leftRes, rightRes);
  int left(int i) {
    int lvl = __lg(i);
    return (i & ((1 << lvl) - 1)) * (1 << (h - lvl));
  int right(int i) {
    int lvl = __lg(i);
    return ((i & ((1 << lvl) - 1)) + 1) * (1 << (h - lvl));
  // l \in [0; n) \&\& ok(get(1, 1), 1);
  // returns last r: ok(get(l, r), r)
  template<typename C>
  int lastTrue(int 1, C ok) {
   T cur = neutral;
   1 += n;
    do (
             _builtin_ctz(1);
     1 >>=
      T with1 = unite(cur, data[1]);
      if (ok(with1, right(1))) {
        cur = with1:
        ++1;
      } else {
        while (1 < n) {
          T with2 = unite(cur, data[2 * 1]);
          if (ok(with2, right(2 * 1))) {
            cur = with2;
           1 = 2 * 1 + 1;
          } else {
           1 = 2 * 1;
         }
       return 1 - n;
    } while (1 & (1 - 1));
   return n;
  // r \in [0; n) && ok(get(r, r), r);
  // returns first 1: ok(get(1, r), 1)
  template<typename C>
  int firstTrue(int r, C ok) {
   T cur = neutral;
    r += n;
    while (r & (r - 1)) {
      r >>= __builtin_ctz(r);
      T with1 = unite(data[--r], cur);
      if (ok(with1, left(r))) {
        cur = with1;
      } else {
        while (r < n) {
          T with2 = unite(data[2 * r + 1], cur);
          if (ok(with2, left(2 * r + 1))) {
            cur = with2;
            r = 2 * r;
          } else {
   r = 2 * r + 1;
         }
        return r - n + 1;
     }
    return 0;
 }
};
//64190d
```

#### 5.3 Массовое дерево отрезков

```
Um um; // M (M, M);
A a; // T (T, M, int); last argument is the length of
current segment (could be used for range += and sum
counting, for instance)
template<typename I>
MassSegmentTree(int sz, T zd, M zm, Ud ud, Um um, A a, I
init) : h(_lg(sz) + 1), n(1 << h), zm(zm), zd(zd), data
 (2 * n, zd), mod(n, zm), ud(ud), um(um), a(a) {
    for (int i = 0; i < sz; ++i) data[i + n] = init(i);</pre>
    for (int i = n - 1; i > 0; --i) data[i] = ud(data[2 *
i], data[2 * i + 1]);
{\tt MassSegmentTree(int\ sz,\ T\ zd,\ M\ zm,\ Ud\ ud,\ Um\ um,\ A\ a)\ :\ h}
(\underline{\ \ }lg(sz) + 1), n(1 << h), zm(zm), zd(zd), data(2 * n, zd
), mod(n, zm), ud(ud), um(um), a(a) {}
void push(int i) {
    if (mod[i] == zm) return;
    apply(2 * i, mod[i]);
apply(2 * i + 1, mod[i]);
    mod[i] = zm;
// is used only for apply
int length(int i) { return 1 << (h - __lg(i)); }</pre>
// used only for descent
int left(int i) {
   int lvl = __lg(i);
   return (i & ((1 << lvl) - 1)) * (1 << (h - lvl));</pre>
// used only for descent
int right(int i) {
    return left(i) + length(i);
template<tvpename S>
void apply(int i, S x) {
    data[i] = a(data[i], x, length(i));
    if (i < n) mod[i] = um(mod[i], x);
void update(int i) {
    if (mod[i] != zm) return;
    data[i] = ud(data[2 * i], data[2 * i + 1]);
template<typename S>
void update(int 1, int r, S x) { // [1; r)
   1 += n, r += n;
    for (int shift = h; shift > 0; --shift) {
        push(l >> shift);
        push((r - 1) >> shift);
    for (int lf = 1, rg = r; lf < rg; lf /= 2, rg /= 2) {
        if (lf & 1) apply(lf++, x);
        if (rg & 1) apply(--rg, x);
    for (int shift = 1; shift <= h; ++shift) {</pre>
        update(1 >> shift);
        update((r - 1) >> shift);
}
T get(int 1, int r) { // [1; r)
    1 += n, r += n;
    for (int shift = h; shift > 0; --shift) {
        push(1 >> shift);
push((r - 1) >> shift);
    T leftRes = zd, rightRes = zd;
for (; 1 < r; 1 /= 2, r /= 2) {
        if (1 & 1) leftRes = ud(leftRes, data[l++]);
        if (r & 1) rightRes = ud(data[--r], rightRes);
    return ud(leftRes, rightRes);
// l \in [0; n) && ok(get(1, 1), 1);
// returns last r: ok(get(1, r), r)
template<typename C>
int lastTrue(int 1, C ok) {
    1 += n:
    for (int shift = h; shift > 0; --shift) push(1 >>
shift);
```

T cur = zd;

```
1 >>= __builtin_ctz(l);
T with = ud(cur, data[l]);
             if (ok(with, right(l))) {
                 cur = with;
                 ++1;
             } else {
                 while (1 < n) {
                     push(1);
                      1 = 2 * 1;
                      with = ud(cur, data[1]);
                      if (ok(with, left(1 + 1))) {
                          cur = with;
                          ++1;
                      }
                 return 1 - n;
            }
        } while (1 & (1 - 1));
        return n;
    }
    // r \in [0; n) && ok(get(r, r), r);
    // returns first 1: ok(get(1, r), 1)
    template<typename C>
    int firstTrue(int r, C ok) {
        r += n;
        for (int shift = h; shift > 0; --shift) push((r - 1)
     >> shift);
        auto cur = zd;
        while (r & (r - 1)) {
            r >>= __builtin_ctz(r);
T with = ud(data[--r], cur);
             if (ok(with, left(r))) {
                 cur = with;
             } else {
                 while (r < n) {
                     push(r);
                      r = 2 * r;
                      with = ud(data[r + 1], cur);
                      if (ok(with, left(r + 1))) {
                          cur = with;
                      } else {
                          ++r;
                      }
                 }
                 return r - n + 1;
            }
        return 0;
    }
//fc0cde
```

### 5.4 Битовый бор

```
template<unsigned int sz,typename T=int>
struct binarytrie{
 using Bit=typename conditional<sz<=32,unsigned int,unsigned
    long long>::type;
  struct node{
   T cnt:
   array<int,2>nxt;
   node():cnt(0),nxt({-1,-1}){}
 };
 vector<node>v;
 binarytrie(){v.emplace_back();}
 void insert(Bit x){add(x,1);}
 void erase(Bit x) {add(x,-1);}
 void add(Bit x,T k)
  {
   assert(0 <= x & (x>> sz) == 0);
   int p=0;
    v[p].cnt+=k;
    for(int i=sz;i--;)
    {
      int i=x>>i&1;
      if(v[p].nxt[j]==-1)
        v[p].nxt[j]=v.size();
        v.emplace_back();
      p=v[p].nxt[i];
      v[p].cnt+=k;
    }
 T count(Bit x,Bit xor_val=0)const//[0,x)
    assert(0<=xor_val&&(xor_val>>sz)==0);
    if(x<0)return 0;</pre>
```

```
else if(x>>sz)return v[0].cnt;
    T ret=0;
    int p=0;
    for(int i=sz;i--;)
      int j=x>>i&1,k=xor_val>>i&1;
      if(j==0)p=v[p].nxt[k];
      else
        if(v[p].nxt[k]>=0)ret+=v[v[p].nxt[k]].cnt;
        p=v[p].nxt[!k];
      if(p==-1)break;
    return ret;
  Bit max(Bit xor_val=0)const
    assert(0 \le xor_val \le (xor_val >> sz) == 0);
    int p=0;
    Bit ret=0;
    if(v[p].cnt==0)return ret;
    for(int i=sz;i--;)
    {
      ret<<=1;
      int k=xor_val>>i&1;
      if(v[p].nxt[!k] \ge 0 & v[v[p].nxt[!k]].cnt > 0)
        p=v[p].nxt[!k];
        ret|=1;
      else p=v[p].nxt[k];
    }
    return ret;
  Bit min(Bit xor val=0)const
    assert(0<=xor val&&(xor val>>sz)==0);
   int p=0;
Bit ret=0;
    for(int i=sz;i--;)
    {
      ret<<=1;
      int k=xor val>>i&1;
      if(v[p].nxt[k]>=0&&v[v[p].nxt[k]].cnt>0)p=v[p].nxt[k];
      else
      {
        p=v[p].nxt[!k];
        ret|=1;
    return ret;
  Bit find_by_order(T ord,Bit xor_val=0)const
    assert(0<=xor_val&&(xor_val>>sz)==0);
    assert(0<=ord&&ord<v[0].cnt);
    int p=0;
    Bit ret=0;
    for(int i=sz;i--;)
    {
      int k=xor_val>>i&1;
      if(v[p].nxt[k]>=0)
      {
        if(ord>=v[v[p].nxt[k]].cnt)
          ord-=v[v[p].nxt[k]].cnt;
          p=v[p].nxt[!k];
          ret|=1;
        else p=v[p].nxt[k];
      }
      else
        p=v[p].nxt[!k];
        ret|=1;
      }
    return ret;
  T order_of_key(Bit x,Bit xor_val=0)const{return count(x,
     xor_val);}
binarytrie<32>bt:
//0b3855
```

### 5.5 Ordered set

## 5.6 Динамический битсет

```
#include <tr2/dynamic_bitset>
using namespace tr2;
using bs=dynamic_bitset<>;
//26f8b6
```

#### 5.7 Convex hull trick

```
int div up(int a, int b) { return a/b+((a^b)>0&a^b); } //
    divide a by b rounded up
const int LQ = ..., RQ = ...; //leftmost query, rightmost
    query
int in(ii L, int x) {
 return L.x * x + L.y;
struct Hull {
vector <pair <int, int> > lines;
vector <int> borders;
void push(ii L) {
 while (lines.size() && in(L,borders.back()) < in(lines.back</pre>
     (),borders.back())) {
    lines.pop_back();
    borders.pop_back();
  if (lines.empty()) {
   lines = \{L\};
   borders = {LQ};
 else if (lines.back().x > L.x) {
   int x = div_up(L.y - lines.back().y, lines.back().x - L.x)
    if (x \le RO) {
     lines.app(L);
     borders.app(x);
 }
Hull (){}
Hull (vector <ii> a) {
 auto comp = [&] (ii u, ii v) {
   return u.x > v.x || (u.x == v.x && u.y < v.y);</pre>
 sort(all(a), comp);
 for (auto L : a) {
   push(L);
 }
int get(int x) {
 int pos = upper_bound(all(borders), x) - borders.begin();
 assert(pos>0);
 pos--;
 return in(lines[pos],x);
};
//04555a
```

#### 5.8 Центроиды

```
vector<int> sz(n), lvl(n, -1);
auto dfs = [&](auto dfs, int cur, int prev) -> int {
    if (lvl[cur] != -1) return 0;
    sz[cur] = 1;
    for (auto [nxt, w] : g[cur]) {
        if (nxt != prev) sz[cur] += dfs(dfs, nxt, cur);
    }
    return sz[cur];
};
auto find = [&](auto find, int cur, int prev, int tot) -> int
    {
    int bch = -1, bsz = 0;
    for (auto [nxt, w] : g[cur]) {
        if (nxt == prev || lvl[nxt] != -1) continue;
        if (sz[nxt] > bsz) {
            bch = nxt;
        }
}
```

```
bsz = sz[nxt];
   }
  if (bsz + bsz <= tot) return cur;</pre>
 return find(find, bch, cur, tot);
dfs(dfs, 0, 0);
auto c = find(find, 0, 0, sz[0]);
vector<pair<int, int>> stack{{c, 0}};
while (!stack.empty()) {
 auto [centroid, 1] = stack.back();
  stack.pop_back();
  lvl[centroid] = 1;
 for (auto [nxt, w] : g[centroid]) {
  if (lvl[nxt] != -1) continue;
    dfs(dfs, nxt, centroid);
    int new_centroid = find(find, nxt, centroid, sz[nxt]);
    stack.push_back({new_centroid, lvl[centroid] + 1});
//0e1e52
```

## 5.9 Дерево Ли Чао

```
struct Line{
  int a, b;
  Line(){}
  Line (int a, int b) : a(a), b(b) {}
  int get(int x) { return a + b * x;}
struct Lichao {
  int n;
  vector <int> x;
  vector <Line> t;
  Lichao(){}
  Lichao (int n, vector\langle int \rangle x) : n(n), t(n \langle \langle 2, Line(inf, 0) \rangle
     ), x(x) {}
  void put(int v, int l, int r, Line L) {
    if (1 + 1 == r) {
      if (L.get(x[1]) < t[v].get(x[1])) {</pre>
        t[v] = L;
      return;
    int m = (1 + r) / 2;
    if (L.get(x[m]) < t[v].get(x[m])) {
      swap(L, t[v]);
    if (L.b > t[v].b) {
     put(2 * v + 1, 1, m, L);
      put(2 * v + 2, m, r, L);
  int get(int v, int 1, int r, int i) {
   if (1 + 1 == r) {
      return t[v].get(x[1]);
    int m = (1 + r) / 2;
    int ans = t[v].get(x[i]);
    if (i < m) {
      ans = min(ans, get(2 * v + 1, 1, m, i));
    } else {
      ans = min(ans, get(2 * v + 2, m, r, i));}
    return ans;
  void put(Line L) {
    put(0, 0, n, L);
 int get(int i) {
    return get(0, 0, n, i);
};
//99f5fa
```

## 5.10 Min-Kinetic Segment Tree

```
I guess the source is https://koosaga.com/307
using lint = long long;
const lint inf = 4e18;
const int MAXT = 4100000;
```

```
using pi = array<lint, 2>;
struct line {
  lint A, B;
  int idx;
  lint eval(lint x) { return A * x + B; }
  \//\ returns the x-intercept of intersection "strictly" larger
  lint cross_after(line &x, lint T) {
    if (x.A == A) {
      return inf;
    lint up = x.B - B;
    lint dn = A - x.A;
    if (dn < 0) {
      dn *= -1;
      up *= -1;
    lint incep = (up \le 0 ? -((-up) / dn) : (up + dn - 1) / dn
    if (incep > T)
      return incep;
    return inf;
1:
struct kst { // min kinetic segment tree
  line tree[MAXT];
  lint melt[MAXT], T;
  pi lazy[MAXT];
  int n:
  bool cmp(line &a, line &b) {
   lint l = a.eval(T), r = b.eval(T);
    if (1 != r)
      return 1 > r;
   return a.A > b.A;
  void pull(int p) {
    tree[p] = cmp(tree[2 * p], tree[2 * p + 1]) ? tree[2 * p +
     1] : tree[2 * p];
    melt[p] = min(\{melt[2 * p], melt[2 * p + 1], tree[2 * p].
    cross_after(tree[2 * p + 1], 0)});
  void init(int s, int e, int p, vector<line> &l) {
   if (s == e) {
      tree[p] = l[s];
      melt[p] = inf;
      lazy[p] = {0, 0};
      return;
   lazy[p] = \{0, 0\};
    int m = (s + e) / 2;
init(s, m, 2 * p, 1);
    init(m + 1, e, 2 * p + 1, 1);
   pull(p);
  void lazydown(int p) {
  for (int i = 2 * p; i < 2 * p + 2; i++) {</pre>
      lazy[i][0] += lazy[p][0];
      lazy[i][1] += lazy[p][1];
      tree[i].B += lazy[p][0] * tree[i].A + lazy[p][1];
      melt[i] -= lazy[p][0];
    lazy[p][0] = lazy[p][1] = 0;
  void propagate(int p) {
   if (melt[p] > 0)
      return;
   lazydown(p);
    propagate(2 * p);
    propagate(2 * p + 1);
   pull(p);
  }
  lint query(int s, int e, int ps, int pe, int p = 1) {
    if (e < ps || pe < s)</pre>
      return inf;
    if (s <= ps && pe <= e)
     return tree[p].eval(0);
    int pm = (ps + pe) / 2;
    lazydown(p);
    return min(query(s, e, ps, pm, 2 * p), query(s, e, pm + 1,
     pe, 2 * p + 1));
```

```
}
  void heaten(int s, int e, int ps, int pe, int p, lint v) {
    if (e < ps || pe < s)</pre>
      return;
    if (s <= ps && pe <= e) {
     lazy[p][0] += v;
      tree[p].B += v * tree[p].A;
      melt[p] -= v;
      propagate(p);
   lazydown(p);
    int pm = (ps + pe) / 2;
    heaten(s, e, ps, pm, 2 * p, v);
    heaten(s, e, pm + 1, pe, 2 * p + 1, v);
   pull(p);
  void add(int s, int e, int ps, int pe, int p, lint v) {
   if (e < ps || pe < s)</pre>
      return;
    if (s <= ps && pe <= e) {</pre>
     lazy[p][1] += v;
      tree[p].B += v;
      return;
   lazydown(p);
    int pm = (ps + pe) / 2;
    add(s, e, ps, pm, 2 * p, v);
    add(s, e, pm + 1, pe, 2 * p + 1, v);
    pull(p);
 void init(vector<line> &1, lint T) {
   n = 1.size();
    T = _T;
   init(0, n - 1, 1, 1);
 }
};
//66f9a9
```

### 5.11 Декартово дерево

#### 5.11.1 Декартово дерево по явному ключу. Multiset

```
mt19937 rng(0);
struct vertex {
  int heap = rng(), val;
int sz = 1, cnt = 1;
  vertex *lf = nullptr, *rg = nullptr;
  vertex(int x, int cnt) : val(x), cnt(cnt), sz(cnt) {}
  friend int get_sz(vertex *v) {
   return v ? v->sz : 0;
  vertex *update() {
   sz = get_sz(lf) + cnt + get_sz(rg);
    return this;
};
vertex *merge(vertex *1, vertex *r) {
  if (!1) return r;
  if (!r) return 1;
  if (1->heap < r->heap) {
   r->lf = merge(l, r->lf);
    return r->update();
  } else {
   1->rg = merge(1->rg, r);
    return 1->update();
 }
pair<vertex *, vertex *> split(vertex *v, int x) {
  if (!v) return {v, v};
  if (v->val < x) {
   auto [lf, rg] = split(v->rg, x);
    v->rg = lf;
   return {v->update(), rg};
  } else {
   auto [lf, rg] = split(v->lf, x);
    v->lf = rq;
    return {lf, v->update()};
```

```
}
vertex *add(vertex *v, int x, int cnt) {
    auto [1, mr] = split(v, x);
    auto [m, r] = split(mr, x + 1);
    if (m == nullptr) {
        m = new vertex(x, cnt);
    } else {
        m->cnt += cnt;
        if (m->cnt == 0) m = nullptr; else m->update();
    }
    return merge(1, merge(m, r));
}
//91cc3a
```

# 6 Строковые алгоритмы

# 6.1 Префикс-функция

```
vector<int> prefix_function(string s) {
  int n = s.size();
  vector<int> p(n);
  for (int i = 1; i < n; ++i) {
    p[i] = p[i - 1];
    while (p[i] && s[p[i]] != s[i]) p[i] = p[p[i] - 1];
    p[i] += s[i] == s[p[i]];
  }
  return p;
}
//91103c</pre>
```

# **6.2** Z-функция

# 6.3 Алгоритм Манакера

```
vector<int> manacher(const string &s, int even) {
  int 1 = 0, r = 0, n = s.size();
 vector<int> man(n, 0);
 for (int i = 1; i < n; i++) {
  int j = i - even;
  if (j <= r) {</pre>
      man[i] = min(r - j, man[l + r - j]);
    while (j + man[i] + 1 < n \&\& i - man[i] > 0 \&\& s[j + man[i])
     ] + 1] == s[i - man[i] - 1]) {
      man[i]++;
    if (j + man[i] > r) {
      l = i - man[i];
      r = j + man[i];
    }
 return man:
// abacaba : odd : (0 1 0 3 0 1 0); even : (0 0 0 0 0 0)
// abbaa : odd : (0 0 0 0 0); even : (0 0 2 0 1)
bool pal(int from, int len) {
   if (len == 0) {
        return true;
    int m = len/2;
    if (len & 1) {
        return odd[from + m] >= m;
    else {
        return even[from + m] >= m;
}
```

//8a64d6

```
6.4 Суфмассив
Переработанный китайский суффмассив
const int inf = 1e9;
struct rmq {
 int n;
  vector<int> a;
  void build(const vector<int> &x) {
    assert(x.size() == n);
    for (int i = 0; i < n; ++i) a[n + i] = x[i];
    for (int i = n - 1; i > 0; --i) a[i] = min(a[2 * i], a[2 *
      i + 11):
 }
  rmq(int n) : n(n), a(2 * n, inf) {}
 void put(int i, int x) {
   a[i + n] = min(a[i + n], x);
    for (i = (i + n) / 2; i > 0; i /= 2) {
    a[i] = min(a[i * 2], a[i * 2 + 1]);
  int getMin(int 1, int r) { //[1;r)
    assert(1 < r);</pre>
    int res = inf;
    for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {
      if (1 & 1) res = min(res, a[1++]);
      if (r & 1) res = min(res, a[--r]);
    return res;
 }
};
template <typename T>
vector <int> SA(const T &a) {
  int m = *max_element(all(a)) + 1, n = a.size();
  vector \leq int > sa(n), nsa(n), pre(max(n, m)), x(a.begin(), a.
    end()), y(n);
  for (int e : x) pre[e]++;
for (int i = 1; i < m; ++i) pre[i] += pre[i - 1];</pre>
  for (int i = 0; i < n; ++i) sa[--pre[x[i]]]=i;</pre>
  int dif = 1;
  y[sa.front()]=0;
  for (int i = 1; i < n; ++i) {</pre>
    dif += x[sa[i]]!=x[sa[i-1]];
    y[sa[i]] = dif - 1;
  for (int h = 1; dif < n; h *= 2) {</pre>
    fill(all(pre), 0);
    for (int e : x) pre[e]++;
    for (int i = 1; i < dif; ++i) pre[i] += pre[i - 1];</pre>
    for (int t = n; t--; ) {
      int i = sa[t];
      if (i>=h) {
        nsa[--pre[x[i-h]]]=i-h;
      else if (i + 1 != h) {
        nsa[--pre[x[i-h+n+1]]]=i-h+n+1;
      }
    nsa[--pre[x[n - h]]]=n-h;
    sa = nsa;
    auto getr = [&] (int i) {
      if (i + h < n) {
        return x[i + h];
      else {
        return x[i + h - n - 1];
      }
    };
    dif = 1:
    y[sa.front()]=0;
    for (int i = 1; i < n; ++i) {</pre>
      if (x[sa[i]]!=x[sa[i-1]] || sa[i-1]+h==n) {
        dif++;
      }
      else {
        dif += getr(sa[i]) != getr(sa[i-1]);
      y[sa[i]]=dif-1;
    x = v;
  return sa;
template <typename T>
```

struct suar {

vector <int> sa, lcp, pos; rmq t;

```
suar (const T &a) : t((int)a.size() - 1) {
    sa = SA(a);
    int n = (int)a.size(), k = 0;
    lcp.resize(n - 1);
    pos.resize(n);
    for (int i = 0; i < n; ++i) pos[sa[i]] = i;</pre>
    for (int i = 0; i < n; ++i) {</pre>
      if (pos[i]+1<n) {</pre>
        int j = sa[pos[i]+1];
        while (i+k<n&&j+k<n&&a[i+k]==a[j+k])k++;
        lcp[pos[i]]=k;
      if (k) {
        k--;
      }
    t.build(lcp);
  int getLcp(int i, int j) {
    i = pos[i]; j = pos[j];
    if (j < i) {</pre>
      swap(i, j);
    if (i == j) {
      return inf;
    else {
      return t.getMin(i, j);
};
//6327c9
```

# 6.5 Алгоритм Ахо — Корасик

```
const int alpha = 26;
const char a = 'a';
struct node{
    int next[alpha] = {}, link[alpha] = {};
    int suf = 0;
    int visited = 0, ans = 0;
    int bad = 0; // any term is reachable by suf links
    vector<int> term;
    node() {
        fill(next, next + alpha, -1);
vector<node> mem;
int get_next_or_create(int nd, char c) {
   if (mem[nd].next[c - a] == -1) { mem[nd].next[c - a] = mem
     .size(); mem.emplace_back(); }
    return mem[nd].next[c - a];
void build(vector<string> t) {
    mem.reserve(1e6 + 100); mem.clear();
    mem.emplace_back();
    // Oth element is nullptr, 1st is the root
    for (int j = 0; j < t.size(); ++j) {</pre>
        int cur = 0;
        for (char c : t[j]) cur = get_next_or_create(cur, c);
        mem[cur].term.push_back(j);
    vector<int> bfs order;
    queue<int> bfs;
        node &root = mem[0];
        root.suf = 0:
        for (char c = a; c < a + alpha; ++c) {
            root.link[c - a] = (root.next[c - a] == -1 ? 0 :
     root.next[c - a]);
        bfs.push(0);
    while (!bfs.empty()) {
        int cur idx = bfs.front();
        bfs.pop();
        node &cur = mem[cur_idx];
        cur.bad = cur.term.size() > 0 || mem[cur.suf].bad;
        bfs_order.push_back(cur_idx);
        for (char c = a; c < a + alpha; ++c) {
            int nxt idx = cur.next[c - a];
            if (nxt_idx == -1) continue;
            node &nxt = mem[nxt_idx];
```

## 6.6 Дерево палиндромов

```
const int alpha = 26;
const char a = 'a';
struct palindromic{
 int n:
  vector<int> p, suf{0, 0}, len{-1, 0};
//d[u] is a difference of lengths of u and suf[u], go is
     jump by chain constant d
  vector<array<int, alpha>> to{{}, {}};
  int sz = 2;
  palindromic(const string &s) : n(s.size()), p(n + 1, 0) {
    suf.reserve(n);
    len.reserve(n);
    for (int i = 0; i < n; ++i) {</pre>
      auto check = [&] (int 1) {
         return i > 1 && s[i] == s[i - 1 - 1];
       int par = p[i];
      while (!check(len[par])) {
         par = suf[par];
       if (to[par][s[i]-a] == 0) {
         p[i+1]=to[par][s[i]-a]=sz++;
         to.emplace_back();
         len.emplace_back(len[par]+2);
         if (par == \overline{0}) {
           suf.emplace_back(1);
             par = suf[par];
            } while (!check(len[par]));
            suf.emplace_back(to[par][s[i]-a]);
        p[i+1]=to[par][s[i]-a];
      }
    }
  int partition() {
    vector <int> d(sz), up(sz, 1); //d[1] = 0 sic
for (int i = 2; i < sz; ++i) {
  d[i] = len[i] - len[suf[i]];</pre>
      if (d[i] == d[suf[i]]) {
  up[i] = up[suf[i]];
      else {
         up[i] = suf[i];
      }
    vector \langle int \rangle dp(n + 1, n), last(sz);
    dp[0] = 0;
    for (int i = 1; i <= n; ++i) {</pre>
      int u = p[i];
      while (u != 1) {
        if (suf[u] == up[u]) {
  last[u] = dp[i - len[u]];
         else {
           last[u] = min(last[suf[u]], dp[i - len[up[u]] - d[u]
     11);
         dp[i] = min(dp[i], last[u] + 1);
         u = up[u];
    return dp.back();
};
//acac02
```

#### 7 Потоки

## 7.1 Алгоритм Диница

```
#define pb push_back
struct Dinic{
struct edge{
 int to, flow, cap;
const static int N = 555; //count of vertices
vector<edge> e;
vector<int> g[N + 7];
int dp[N + 7];
int ptr[N + 7];
void clear(){
 for (int i = 0; i < N + 7; i++) g[i].clear();</pre>
void addEdge(int a, int b, int cap){
 g[a].pb(e.size());
  e.pb({b, 0, cap});
 g[b].pb(e.size());
 e.pb({a, 0, 0});
int minFlow, start, finish;
bool bfs(){
  for (int i = 0; i < N; i++) dp[i] = -1;
  dp[start] = 0;
  vector<int> st;
  int uk = 0;
  st.pb(start);
  while(uk < st.size()){</pre>
    int v = st[uk++];
    for (int to : g[v]){
      auto ed = e[to];
      if (ed.cap - ed.flow >= minFlow && dp[ed.to] == -1){
   dp[ed.to] = dp[v] + 1;
        st.pb(ed.to);
   }
 return dp[finish] != -1;
int dfs(int v, int flow){
  if (v == finish) return flow;
  for (; ptr[v] < g[v].size(); ptr[v]++){</pre>
    int to = g[v][ptr[v]];
edge ed = e[to];
    if (ed.cap - ed.flow >= minFlow && dp[ed.to] == dp[v] + 1)
      int add = dfs(ed.to, min(flow, ed.cap - ed.flow));
      if (add) {
        e[to].flow += add;
        e[to ^ 1].flow -= add;
        return add;
      }
   }
 return 0;
int dinic(int start, int finish){
  Dinic::start = start;
  Dinic::finish = finish;
  int flow = 0;
 for (minFlow = (1 << 30); minFlow; minFlow >>= 1){
    while(bfs()){
      for (int i = 0; i < N; i++) ptr[i] = 0;</pre>
      while(int now = dfs(start, (int)2e9 + 7)) flow += now;
 return flow;
} dinic;
//15c079
```

#### 7.2 Mincost k-flow

```
struct edge {
  int next, capacity, cost, flow = 0;
```

```
edge() = default;
  edge(int next, int capacity, int cost) : next(next),
    capacity(capacity), cost(cost) {}
  int rem() const { return capacity - flow; }
  int operator+=(int f) { return flow += f; }
 int operator-=(int f) { return flow -= f; }
auto addEdge = [&](auto from, auto next, auto capacity, int
    cost) {
  g[from].push_back(e.size());
  e.emplace_back(next, capacity, cost);
  g[next].push_back(e.size());
  e.emplace_back(from, 0, -cost);
/* in case of undirected graph use this:
addEdge(u, v, capacity, cost);
addEdge(v, u, capacity, cost);
vector<ll> phi(n, 0);
auto fordBellman = [&](int s, int t) {
 phi.assign(n, 0);
for (int iter = 0; iter < n; ++iter) {</pre>
   bool changed = false;
    for (int u = 0; u < n; ++u) {
      for (auto index : g[u]) {
       auto edge = e[index];
        if (edge.rem() > 0 && phi[edge.next] > phi[u] + edge.
    cost) {
          phi[edge.next] = phi[u] + edge.cost;
          changed = true;
        }
     }
   if (!changed) break;
 }
fordBellman(s, t);
// now shortest path using dijkstra with potentials
vector<ll> dist;
vector<int> from;
vector<bool> cnt;
auto dijkstra = [&](int s, int t) {
 dist.assign(n, 1e18);
  from.assign(n, -1);
  cnt.assign(n, false);
  dist[s] = 0;
  set <pair <int, int> > se;
  se.insert({0, s});
  while ((int)(se.size())) {
   int cur = se.begin()->y;
    se.erase(se.begin());
   cnt[cur] = true;
    for (int index : g[cur]) {
     auto &edge = e[index];
      if (edge.rem() == 0) continue;
      11 weight = edge.cost + phi[cur] - phi[edge.next];
      if (dist[edge.next] > dist[cur] + weight) {
        se.erase({dist[edge.next], edge.next});
        dist[edge.next] = dist[cur] + weight;
        se.insert({dist[edge.next], edge.next});
        from[edge.next] = cur;
   }
  if (dist[t] == (11) 1e18) return -1LL;
  11 cost = 0;
  for (int p = t; p != s; p = from[p]) {
   for (auto index : g[from[p]]) {
      auto &edge = e[index];
      ll weight = edge.cost + phi[from[p]] - phi[edge.next];
      if (edge.rem() > 0 && edge.next == p && dist[edge.next]
    == dist[from[p]] + weight) {
        edge += 1;
        e[index ^ 1] -= 1;
        cost += edge.cost;
        break:
 for (int i = 0; i < n; ++i) {
  phi[i] += dist[i];</pre>
 return cost;
11 cost = 0;
for (int flow = 0; flow < k; ++flow) {</pre>
```

```
11 a = dijkstra(s, t);
  if (a == -1) {
  cout << "-1\n";</pre>
    return;
  cost += a;
// now recover answer
auto findPath = [&](int s, int t) {
  vector<int> ans;
  int cur = s;
  while (cur != t) {
    for (auto index : g[cur]) {
       auto &edge = e[index];
       if (edge.flow <= 0) continue;</pre>
       edge -= 1;
       e[index ^ 1] += 1;
       ans.push_back(index / 4);
// index / 4 because each edge has 4 copies
    cur = edge.next;
      break;
    }
  return ans;
for (int flow = 0; flow < k; ++flow) {</pre>
  auto p = findPath(s, t);
cout << p.size() << ' ';</pre>
  for (int x : p) cout << x + 1 << ' ';
cout << '\n';</pre>
//94b9cb
template <typename T, typename C>
class mcmf {
  public:
  static constexpr T eps = (T) 1e-9;
  struct edge {
   int from;
    int to;
    T c;
    Tf;
    C cost;
  vector< vector<int> > g;
  vector<edge> edges;
  vector<C> d;
  vector<int> q;
  vector<bool> in_queue;
  vector<int> pe;
  int n;
  int st, fin;
  T flow;
  mcmf(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin)
    \texttt{assert}(\texttt{0} \mathrel{<=} \texttt{st \&\& st < n \&\& 0} \mathrel{<=} \texttt{fin \&\& fin < n \&\& st !=}
     fin):
    g.resize(n);
    d.resize(n);
    in_queue.resize(n);
    pe.resize(n);
    flow = 0;
    cost = 0;
  void clear_flow() {
    for (const edge &e : edges) {
      e.f = 0;
    flow = 0;
  void add(int from, int to, T forward_cap, T backward_cap, C
     cost.) {
    assert(0 \le from && from < n && 0 \le to && to < n);
    g[from].push_back((int) edges.size());
    edges.push_back({from, to, forward_cap, 0, cost});
    g[to].push back((int) edges.size());
    edges.push_back({to, from, backward_cap, 0, -cost});
  bool expath() {
    fill(d.begin(), d.end(), numeric limits<C>::max());
    q.clear();
    q.push back(st);
```

```
d[st] = 0;
    in_queue[st] = true;
    int beg = 0;
    bool found = false;
    while (beg < (int) q.size()) {</pre>
      int i = q[beg++];
if (i == fin) {
        found = true;
      in_queue[i] = false;
      for (int id : g[i]) {
        const edge &e = edges[id];
        if (e.c - e.f > eps && d[i] + e.cost < d[e.to]) {</pre>
          d[e.to] = d[i] + e.cost;
          pe[e.to] = id;
          if (!in_queue[e.to]) {
            q.push_back(e.to);
            in_queue[e.to] = true;
        }
     }
    if (found) {
      T push = numeric_limits<T>::max();
      int v = fin;
      while (v != st) {
        const edge &e = edges[pe[v]];
        push = min(push, e.c - e.f);
        v = e.from;
      }
      v = fin;
      while (v != st) {
        edge &e = edges[pe[v]];
        e.f += push;
        edge &back = edges[pe[v] ^ 1];
        back.f -= push;
        v = e.from;
      flow += push;
      cost += push * d[fin];
    return found;
  pair<T, C> max_flow_min_cost() {
    while (expath()) {
    return make pair(flow, cost);
};
```

# 8 Алгоритм Гаусса

#### **8.1** Решение Av = b

```
optional<vector<int> > gauss(vector<vector<int> > A, vector<int
     > b) ///returns v such that Av=b
    int n=A.size();assert(b.size()==n);int m=A[0].size();
    for(int &x:b) {x%=p;x+=p;x%=p;}
    for(int i=0;i<n;++i) {for(int &x:A[i]) {x%=p;x+=p;x%=p;}}</pre>
    int bi=0;
    for(int i=0;i<n;++i)</pre>
        if(bi==m) break;
        for(int j=i;j<n;++j)</pre>
            if(A[j][bi])
                 if(j!=i) {swap(A[i],A[j]);swap(b[i],b[j]);}
                break;
        if(A[i][bi])
            int o=inv(A[i][bi]);
            for(int j=i+1;j<n;++j)</pre>
                 int we=(A[j][bi]*o)%p;
                 b[j]-=we*b[i];b[j]%=p;if(b[j]<0) b[j]+=p;
                 for(int k=bi; k<m; ++k)</pre>
                     A[j][k]=we*A[i][k];A[j][k]%=p;if(A[j][k]
     ]<0) A[j][k]+=p;
                }
            }
        }
```

```
else
         {
             ++bi;--i;continue;
    vector<int> v(m);
    for(int i=n-1;i>=0;--i)
         int bi=0;
         while(bi<m && !A[i][bi]) {++bi;}</pre>
         if(bi==m)
         {
             if(b[i]) {return nullopt;}
             else {continue;}
         int cur=b[i];
         for(int j=bi+1;j<m;++j)</pre>
             cur-=A[i][j]*v[j];cur%=p;
         \label{eq:vbi} $$v[bi]=cur*inv(A[i][bi]);v[bi]=p;$$if(v[bi]<0)$$ $v[bi]+=p;$
    return v;
//bcc622
```

# **8.2** Базис Av = 0

```
vector<vector<int> > gaussbasis(vector<vector<int> > A,int m)
     ///returns basis of Av=0
    int n=A.size();if(n) assert(m==A[0].size());
    for(int i=0;i<n;++i) {for(int &x:A[i]) {x%=p;x+=p;x%=p;}}</pre>
    int bi=0;
    for(int i=0;i<n;++i)</pre>
    {
         if(bi==m) break;
        for(int j=i;j<n;++j)</pre>
        {
             if(A[j][bi])
                 if(j!=i) {swap(A[i],A[j]);}
                 break;
        if(A[i][bi])
             int o=inv(A[i][bi]);
             for(int j=i+1;j<n;++j)</pre>
                 int we=(A[j][bi]*o)%p;
                 for(int k=bi; k<m;++k)</pre>
                     A[j][k]=we*A[i][k];A[j][k]%=p;if(A[j][k]
     ]<0) A[j][k]+=p;
             }
        else
        {
             ++bi; --i; continue;
    vector<int> indices(m);iota(all(indices),0);
    for(int i=n-1;i>=0;--i)
        int bi=0:
        while(bi<m && !A[i][bi]) {++bi;}</pre>
        if(bi<m)</pre>
        {
             indices.erase(find(all(indices),bi));
    vector<vector<int> > v(indices.size(), vector<int>(m,0));
    for(int i=0;i<indices.size();++i)</pre>
    {
        v[i][indices[i]]=1;
    for(int i=n-1;i>=0;--i)
        int bi=0;
        while(bi<m && !A[i][bi]) {++bi;}</pre>
        if(bi==m) continue;
        for(int k=0;k<indices.size();++k) {</pre>
        int cur=0;
        for(int j=bi+1;j<m;++j)</pre>
```

```
cur=A[i][j]*v[k][j];cur%=p;
}
v[k][bi]=cur*inv(A[i][bi]);v[k][bi]%=p;if(v[k][bi]<0)
v[k][bi]+=p;
}
return v;
}
//ef40f3</pre>
```

# 9 Гамильтоновы путь и цикл

https://codeforces.com/blog/entry/90513, https://codeforces.com/blog/entry/90743.

#### 9.1 Link-cut tree

```
namespace LCT {
 vector<vi> ch;
  vi fa, rev;
  void init(int n) {
   ch.resize(n + 1);
   fa.resize(n + 1);
   rev.resize(n + 1);
   for (int i = 0; i <= n; i++)</pre>
     ch[i].resize(2),
     ch[i][0] = ch[i][1] = fa[i] = rev[i] = 0;
  bool isr(int a)
   return !(ch[fa[a]][0] == a || ch[fa[a]][1] == a);
  void pushdown(int a)
  {
    if(rev[a])
   {
     rev[ch[a][0]] ^= 1, rev[ch[a][1]] ^= 1;
     swap(ch[a][0], ch[a][1]);
     rev[a] = 0;
  void push(int a)
    if(!isr(a)) push(fa[a]);
   pushdown(a);
  void rotate(int a)
    int f = fa[a], gf = fa[f];
    int tp = ch[f][1] == a;
    int son = ch[a][tp ^ 1];
    if(!isr(f))
     ch[gf][ch[gf][1] == f] = a;
    fa[a] = gf;
   ch[f][tp] = son;
   if(son) fa[son] = f;
    ch[a][tp ^ 1] = f, fa[f] = a;
  void splay(int a)
   push(a):
    while(!isr(a))
     int f = fa[a], gf = fa[f];
     if(isr(f)) rotate(a);
     else
     {
        int t1 = ch[gf][1] == f, t2 = ch[f][1] == a;
        if(t1 == t2) rotate(f), rotate(a);
        else rotate(a), rotate(a);
     }
   }
  void access(int a)
   int pr = a;
   splay(a);
   ch[a][1] = 0;
    while(1)
    {
      if(!fa[a]) break;
     int u = fa[a];
     splay(u);
     ch[u][1] = a;
     a = u;
```

```
splay(pr);
  void makeroot(int a)
  {
   rev[a] ^= 1;
  void link(int a, int b)
    fa[a] = b;
 void cut(int a, int b)
   makeroot(a);
    access(b);
   fa[a] = 0, ch[b][0] = 0;
  int fdr(int a)
    access(a);
    while(1)
      pushdown(a);
      if (ch[a][0]) a = ch[a][0];
      else {
        splay(a);
        return a;
     }
   }
 }
//647cca
```

## 9.2 Undirected case

```
#include <bits/stdc++.h>
using namespace std;
namespace hamil {
  template <typename T> bool chkmax(T &x,T y){return x<y?x=y,</pre>
     true:false;}
  template <typename T> bool chkmin(T &x,T y){return x>y?x=y,
     true:false;}
  #define vi vector<int>
  #define pb push_back
  #define mp make_pair
  #define pi pair<int, int>
  #define fi first
  #define se second
  #define ll long long
  using namespace LCT;
  vector<vi> used;
 unordered_set<int> caneg;
  void cut(int a, int b) {
    LCT::cut(a, b);
    for (int s = 0; s < 2; s++) {
      for (int i = 0; i < used[a].size(); i++)</pre>
        if (used[a][i] == b) {
          used[a].erase(used[a].begin() + i);
      if (used[a].size() == 1) caneg.insert(a);
      swap(a, b);
    }
 void link(int a, int b) {
    LCT::link(a, b);
    for (int s = 0; s < 2; s++) {
      used[a].pb(b);
      if (used[a].size() == 2) caneg.erase(a);
      swap(a, b);
    }
 vi work(int n, vector<pi> eg, ll mx_ch = -1) {
    // mx_ch : max number of adding/replacing default is (n +
      100) * (n + 50)
    // n : number of vertices. 1-indexed.
    // eg: vector<pair<int, int> > storing all the edges.
// return a vector<int> consists of all indices of
     vertices on the path. return empty list if failed to find
      one.
    LCT::init(n);
    if (mx_ch == -1) mx_ch = 111 * (n + 100) * (n + 50); //
     default
    used.resize(n + 1);
    caneq.clear();
    for (int i = 1; i <= n; i++) used[i].clear();</pre>
```

```
vector<vi> edges(n + 1);
    for (auto v : eg)
      edges[v.fi].pb(v.se),
      edges[v.se].pb(v.fi);
    for (int i = 1; i <= n; i++)</pre>
      caneg.insert(i);
    \verb|mt19937 x(chrono::steady_clock::now().time_since_epoch().|\\
     count());
    int tot = 0;
    while (mx_ch >= 0) {
    // cout << tot << ' ' << mx_ch << endl;
      vector<pi> eg;
      \quad \text{for (auto } v \ : \ \text{caneg)} \\
        for (auto s : edges[v])
          eg.pb(mp(v, s));
      shuffle(eg.begin(), eg.end(), x);
      if (eg.size() == 0) break;
      for (auto v : eg) {
        mx_ch--;
        int a = v.fi, b = v.se;
        if (used[a].size() < used[b].size()) swap(a, b);</pre>
        if (used[b].size() >= 2) continue;
        if (x() & 1) continue;
        if (LCT::fdr(a) == LCT::fdr(b)) continue;
        if (used[a].size() < 2 && used[b].size() < 2)</pre>
          tot++:
        if (used[a].size() == 2) {
          int p = used[a][x() % 2];
          cut(a, p);
        link(a, b);
      if (tot == n - 1) {
        vi cur;
        for (int i = 1; i <= n; i++)</pre>
          if (used[i].size() <= 1) {</pre>
            int pl = i, ls = 0;
            while (pl) {
              cur.pb(pl);
               int flag = 0;
               for (auto v : used[pl])
                 if (v != ls) {
                  ls = pl;
                   pl = v;
                   flag = 1;
                   break;
               if (!flag) break;
            break;
        return cur;
     }
    //failed to find a path
    return vi();
//c35638
```

#### 9.3 Directed case

```
namespace hamil {
  template <typename T> bool chkmax(T &x,T y){return x<y?x=y,</pre>
    true:false: }
  template <typename T> bool chkmin(T &x,T y){return x>y?x=y,
    true:false;}
  #define vi vector<int>
  #define pb push_back
  #define mp make pair
  #define pi pair<int, int>
  #define fi first
  #define se second
  #define ll long long
  using namespace LCT;
  vi out, in;
  vi work(int n, vector<pi> eq, ll mx ch = -1) {
   // mx_ch : max number of adding/replacing default is (n +
     100) * (n + 50)
    // n : number of vertices. 1-indexed.
    // eg: vector<pair<int, int> > storing all the edges.
    // return a vector<int> consists of all indices of
    vertices on the path. return empty list if failed to find
     one.
    out.resize(n + 1), in.resize(n + 1);
   LCT::init(n);
```

```
for (int i = 0; i <= n; i++) in[i] = out[i] = 0;</pre>
    if (mx_ch == -1) mx_ch = 111 * (n + 100) * (n + 50); //
    vector\langle vi \rangle from(n + 1), to(n + 1);
    for (auto v : eg)
      from[v.fi].pb(v.se),
      to[v.se].pb(v.fi);
    unordered_set<int> canin, canout;
    for (int i = 1; i <= n; i++)</pre>
      canin.insert(i),
      canout.insert(i);
    mt19937 x(chrono::steady_clock::now().time_since_epoch().
    count());
    int tot = 0;
    while (mx_ch >= 0) {
// cout << tot << ' ' << mx_ch << endl;</pre>
      vector<pi> eg;
      for (auto v : canout)
        for (auto s : from[v])
          if (in[s] == 0) {
            assert(canin.count(s));
            continue;
          else eg.pb(mp(v, s));
      for (auto v : canin)
        for (auto s : to[v])
          eg.pb(mp(s, v));
      shuffle(eg.begin(), eg.end(), x);
      if (eg.size() == 0) break;
      for (auto v : eg) {
        mx_ch--;
        if (in[v.se] && out[v.fi]) continue;
        if (LCT::fdr(v.fi) == LCT::fdr(v.se)) continue;
        if (in[v.se] || out[v.fi])
          if (x() & 1) continue;
        if (!in[v.se] && !out[v.fi])
          tot++;
        if (in[v.se]) {
          LCT::cut(in[v.se], v.se);
          canin.insert(v.se);
          canout.insert(in[v.se]);
          out[in[v.sell = 0;
          in[v.se] = 0;
        if (out[v.fi]) {
          LCT::cut(v.fi, out[v.fi]);
          canin.insert(out[v.fi]);
          canout.insert(v.fi);
          in[out[v.fi]] = 0;
          out[v.fi] = 0;
        LCT::link(v.fi, v.se);
        canin.erase(v.se);
        canout.erase(v.fi);
        in[v.se] = v.fi;
        out[v.fi] = v.se;
      if (tot == n - 1) {
        for (int i = 1; i <= n; i++)</pre>
          if (!in[i]) {
            int pl = i;
            while (pl) {
              cur.pb(pl),
              pl = out[pl];
            break;
        return cur;
     }
    //failed to find a path
    return vi();
//43ae60
```

# 10 Геома

#### 10.1 Примитивы

```
struct Point {
  int x, y;
  Point(){}
  Point (int x_, int y_) {
    x = x_; y = y_;
  }
  Point operator + (Point p) {
```

```
return Point(x+p.x,y+p.y);
  Point operator - (Point p) {
    return Point(x - p.x, y - p.y);
  int operator * (Point p) {
   return x * p.y - y * p.x;
  int operator % (Point p) {
    return x * p.x + y * p.y;
  bool operator < (Point v) {</pre>
   return (*this) * v > 0;
  bool operator > (Point v) {
    return v < (*this);</pre>
  bool operator <= (Point v) {</pre>
    return (*this) * v >= 0;
}:
bool line(Point a, Point b, Point c) {
  return (b - a) * (c - b)==0;
bool ord(Point a, Point p, Point b) {
 return (p - a)%(p - b)<0;</pre>
int hp(Point a) {
 if (a.y == 0) return a.x >= 0;
 return a.y > 0;
bool comp(Point a, Point b) {
 if (hp(a) != hp(b)) return hp(a) < hp(b);</pre>
  return a.x * b.y - a.y * b.x > 0;
//a48b68
```

# 10.2 Выпуклая оболочка

```
using pt = pair<int, int>;
#define x first
#define y second
int cross(pt p, pt q) { return p.x * q.y - p.y * q.x; }
pt operator-(pt a, pt b) { return {a.x - b.x, a.y - b.y}; }
vector<point> convex(vector<point> a) {
  sort(all(a));
  a.erase(unique(all(a)), a.end());
  vector<point> h;
  for (int t = 0; t < 2; ++t) {
   int sz = h.size() - t;
    for (auto p: a) {
     while (h.size() \ge sz + 2 \&\& cross(p - h.end()[-1], h.
    end()[-2] - h.end()[-1]) <= 0) h.pop_back();
      h.push_back(p);
    reverse(all(a));
  return h; // h is circular: h.empty() || h.front() == h.back
//ef9132
```

### 10.3 Точка внутри многоугольника

```
auto inT = [&] (Point a, Point b, Point c, Point p) {
 a = a-p; b = b-p; c = c-p;
int ab = a * b, bc = b * c, ca = c * a;
 return abs(ab)+abs(bc)+abs(ca) == abs(ab+bc+ca);
auto inP = [&] (Point p) {
//a must be in counterclockwise order!
//assuming no three points of a are collinear
 if (n == 1) return p == a[0];
 if (n == 2) return (p-a[0]) * (a[1]-a[0]) == 0 && (p-a[0]) %
     (p-a[1]) <= 0;
  int 1 = 1, r = n - 1;
  while (1 < r - 1) {
   int m = (1 + r) / 2;
   if ((a[m] - a[0]) < (p - a[0])) {
     1 = m;
    } else {
     r = m;
```

```
}
return inT(a[1], a[0], a[r], p);
};
//9e04bc
```

#### 10.4 Касательные

```
auto max = [&] (auto cmp) {
  int k = 0;
  for (int lg = 18; lg >= 0; --lg) {
   int i = k + (1 \ll lg), j = k - (1 \ll lg);
   i = (i % n + n) % n;
    j = (j % n + n) % n;
   array<int, 3> ind{i, j, k};
    sort(all(ind), cmp);
   k = ind[2];
 return k;
auto uppert = [&](Point p) { //last vertex in counterclockwise
     order about p
  auto cmp = [\&] (int i, int j) {return (a[i] - p) < (a[j] - p)
    ); };
 return max(cmp);
auto lowert = [&](Point p) { //first vertex in
    counterclockwise order about p
  auto cmp = [&] (int i, int j) {return (a[i] - p) > (a[j] - p)
    ); };
 return max(cmp);
auto uppertinf = [&](Point p) { //upper tangent line parallel
    to vector p
 swap(p.x, p.y);
 p.x = -p.x;
 auto cmp = [&] (int i, int j) { return a[i] % p < a[j] % p;</pre>
    };
 return max(cmp);
};
auto lowertinf = [&](Point p) { //lower tangent line parallel
    to vector p
  swap(p.x, p.y);
 p.x = -p.x;
  auto cmp = [&] (int i, int j) { return a[i] % p > a[j] % p;
 return max(cmp);
//90f89d
```

# 10.5 Пересечение многоугольника и полуплоскости

```
template<typename Check>
vector<point> intersect(vector<point> h, Check value) {
  int n = h.size();
 vector<int> pos(n);
  for (int i = 0; i < n; ++i) {
   pos[i] = value(h[i]) >= 0;
 if (count(all(pos), 0) == 0) return h;
 if (count(all(pos), 1) == 0) return {};
 auto intersect = [&](point p, point q) {
   auto s = value(p);
   auto t = value(q);
   assert(s < 0 \&\& t >= 0);
   return q + (p - q) * (t / (t - s));
 int t01 = -1, t10 = -1;
  for (int i = 0; i < n; ++i) {
   h.push back(h[i]);
   pos.push back(pos[i]);
    if (pos[i] == 0 && pos[i + 1] == 1) t01 = i;
    if (pos[i] == 1 \&\& pos[i + 1] == 0) t10 = i;
  if (t10 < t01) t10 += n:
//for (int i = t01 + 1; i \le t10; ++i) assert(pos[i] == 1);
//for (int i = t10 + 1; i <= t01 + n; ++i) assert(pos[i] == 0)
  vector<point> res{intersect(h[t01], h[t01 + 1])};
 for (int i = t01 + 1; i <= t10; ++i) {</pre>
   res.push_back(h[i]);
 res.push_back(intersect(h[t10 + 1], h[t10]));
 return res;
```

## 10.6 События для прямой

```
int cross(point p, point q) { return p.x * q.y - q.x * p.y; }
point operator-(point p, point q) { return {p.x - q.x, p.y - q
     .y}; }
int sgn(int x) \{ return x < 0 ? -1 : (x > 0); \}
double dist(point p) { return sqrt(p.x * p.x + p.y * p.y); }
const __int128 one = 1;
double solve(vector<point> a, point p, point q) {
  int n = a.size();
  a.push_back(a[0]);
  point pq = q - p;
vector<array<int, 3>> ev;
  for (int i = 0; i < n; ++i) {
  point u = a[i], v = a[i + 1];</pre>
     int s = sgn(cross(u - p, pq)), t = sgn(cross(v - p, pq));
    if (s == t) continue;
    int top = cross(u - p, v - u), bot = cross(pq, v - u);
ev.push_back({sgn(bot) * top, abs(bot), t - s});
  sort(all(ev), [](auto e, auto f) {
  return e[0] * one * f[1] < f[0] * one * e[1];</pre>
  int bal = 0:
  for (int i = 0; i + 1 < ev.size(); ++i) {</pre>
    if (bal + ev[i][2] < 0 || bal + ev[i][2] > 2) {
       assert(ev[i][0] * ev[i+1][1] == ev[i+1][0] * ev[i][1]);
       swap(ev[i], ev[i + 1]);
    bal += ev[i][2];
  // example usage: now calculating length of longest segment
     inside
  bal = 0;
  double from = 0, ans = 0;
  for (auto [t, b, w] : ev) {
  double x = t * 1.0 / b;
     if (bal == 0) from = x;
     assert(0 <= bal && bal <= 2);
    if (bal == 0) ans = max(ans, x - from);
  return ans * dist(pq);
//fe0649
```

### 10.7 Кривая Гильберта для алгоритма Мо

```
const int logn = 30; // any number, such that maxn is greater
     than coordinates. 20 is ok.
const int maxn = 1 << logn;</pre>
int hilbertorder(int x, int y) { // returns long long
    int d = 0; // long long
    for (int s = 1 << (logn - 1); s; s >>= 1)
        bool rx = x & s, ry = y & s;
d = (4 * d) | ((3 * rx) ^ ry);
        if (!ry) {
            if (rx) {
                 x = maxn - x:
                 y = maxn - y;
             swap(x, y);
        }
    }
    return d:
// Usage example:
vector<int> sort_indices(int q, vector<pair<int, int>> &qs) {
    vector<int> ind(q), ord(q);
    iota(all(ind), 0);
    for (int i = 0; i < q; ++i) ord[i] = hilbertorder(qs[i].</pre>
     first, qs[i].second);
    sort(all(ind), [&](int i, int j) { return ord[i] < ord[j];</pre>
     });
    return ind;
}
```

//134578

# 11 Цепные дроби

 $\verb|https://cp-algorithms.com/algebra/continued-fractions.html|\\$ 

# 11.1 Поиск нижней огибающей, сумма и минимум по модулю

```
int floor(int a, int b) {
  return a / b - ((a ^ b) < 0 && a % b);
vector<int> decompose(int p, int q) {
  vector<int> f;
  while (q != 0) {
    f.push_back(floor(p, q));
    p -= q * f.back();
    swap(p, q);
  return f;
using matrix = array<int, 4>;
matrix operator*(matrix a, matrix b) {
  matrix c{0,0,0,0};
  for (int i = 0; i < 2; ++i) {</pre>
    for (int j = 0; j < 2; ++j) {
      for (int k = 0; k < 2; ++k) {
        c[2 * i + k] += a[2 * i + j] * b[2 * j + k];
      }
   }
  return c;
#define x first
#define y second
// computes lower convex hull for 0 <= x <= N, 0 <= y <= (ax +  
vector<pair<int, int>> lower_convex_hull(int a, int b, int c,
     int n) {
  matrix m = \{1, 0, 0, 1\};
  auto f = decompose(a, c);
  vector<pair<int, int>> conv{{1, 0}, {0, 1}};
  for (int x : f) {
    m = m * matrix{x, 1, 1, 0};
    conv.emplace_back(m[2], m[0]);
    if (m[2] > n) break; // there should be one (if any) with
  auto diff = [&](int x, int y) {
    return c * y - a * x;
  int x = 0, y = b / c;
  vector<pair<int, int>> res{{x, y}};
  for (i = 2; i + 1 < conv.size(); i += 2) {</pre>
    while (diff(x + conv[i + 1].x, y + conv[i + 1].y) <= b) {
  int t = 1 + (diff(x + conv[i - 1].x, y + conv[i - 1].y)</pre>
     - b - 1) / abs(diff(conv[i].x, conv[i].y));
      auto [dx, dy] = tuple{conv[i - 1].x + t * conv[i].x,
     conv[i - 1].y + t * conv[i].y);
int k = (n - x) / dx;
      if (k == 0) break;
      if (diff(dx, dy)) k = min(k, (b - diff(x, y)) / diff(dx,
      dy));
      x += k * dx, y += k * dy;
      res.push_back({x, y});
  if (i >= conv.size()) i -= 2;
  for (; i > 0; i -= 2) {
    auto [dx1, dy1] = conv[i];
    if (x + dx1 > n) continue;
    x += dx1, y += dy1;
    if (i + 1 < conv.size()) {</pre>
      auto [dx2, dy2] = conv[i + 1];
      int k = (n - x) / dx2;

x += k * dx2;
      y += k * dy2;
    res.emplace back(x, y);
    int k = (n - x) / dx1;
    if (k == 0) continue;
    x += k * dx1;
```

```
y += k * dy1;
    res.emplace_back(x, y);
  return res;
// number of (x, y) under pq line such that p.x \le x < q.x &&
     0 < y
int area(auto p, auto q) {
  int integers = gcd(q.x - p.x, q.y - p.y);
return ((p.y + q.y - 1) * (q.x - p.x + 1) + integers + 1) /
// sum of (ax + b) / c for 0 \le x \le n
int get_area(int a, int b, int c, int n) { // SUM (ax + b) / c
     for 0 <= x <= n
  auto ch = lower_convex_hull(a, b, c, n + 1);
  int sum = 0;
  for (int i = 0; i + 1 < ch.size(); ++i) {</pre>
    sum += area(ch[i], ch[i + 1]);
  return sum;
// min of (ax + b) % c for 0 <= x <= n
int get_min(int a, int b, int c, int n) {
  auto ch = lower_convex_hull(a, b, c, n);
  \ensuremath{//} in fact, here we need only the last point of the first
    half of the algo (that is going up)
  int mn = c;
  for (auto [x, y]: ch) mn = min(mn, (a * x + b) % c);
  return mn;
//87941e
```

## 11.2 Простая рекурсия

```
Число точек (x,y):0\leqslant x< n,0< y\leqslant (kx+b)/d. To ects \sum_{x=0}^{n-1} \lfloor \frac{kx+b}{d} \rfloor. int cnt (int n, int k, int b, int d) { if (k=0) return (b/d) * n; if (k>=d\mid\mid b>=d) { return (k/d) * n * (n-1)/2 + (b/d) * n + cnt(n,k) % d, b % d, d); } return cnt((k*n+b)/d,d,(k*n+b) % d, k); } //11a6a0
```

# 12 Разное

#### 12.1 Компараторы

```
bool cmp1(int x, int y) { return x > y; }

struct cmp2{
    bool operator()(int x, int y) const { return x > y; }
};

int32_t main() {
    set<int, decltype(cmp1)*> s1({1, 2, 3}, cmp1);
    for (int x : s1) cout << x << ' '; cout << '\n';
    set<int, cmp2> s2({4, 5, 6});
    for (int x : s2) cout << x << ' '; cout << '\n';
    auto cmp3 = [&](int x, int y) { return x > y; };
    set<int, decltype(cmp3)> s3({7, 8, 9}, cmp3); // second cmp3 could be omitted if cmp3 = [](...) { ... }
    for (int x : s3) cout << x << ' '; cout << '\n';

    vector<int> v{3, 2, 1};
    cout << lower_bound(all(v), 2, cmp1) - v.begin();
    cout << lower_bound(all(v), 2, cmp2()) - v.begin();
    cout << lower_bound(all(v), 2, cmp3) - v.begin();
}</pre>
```

#### 12.2 Трюки от Сергея Копелиовича

#### 12.2.1 Быстрый ввод

//adea08

https://acm.math.spbu.ru/~sk1/algo/input-output

```
const int buf_size = 4096;
int getChar() {
 static char buf[buf_size];
  static int len = 0, pos = 0;
  if (pos == len)
   pos = 0, len = fread(buf, 1, buf_size, stdin);
  if (pos == len)
    return -1;
  return buf[pos++];
int readChar() {
 while (1) {
   int c = getChar();
    if (c > 32) return c;
int readInt() {
 int s = 1, c = readChar(), x = 0;
if (c == '-')
    s = -1, c = getChar();
 while (isdigit(c))
 x = x * 10 + c - '0', c = getChar();
return s * x;
//dc0a77
double read double() {
    string s;
    cin >> s;
    double sgn = 1, p10 = 0, num = 0;
    for (char c : s) {
        if (c == '-') {
            sgn = -1;
        } else if (c == '.') {
            p10 = 1;
        } else {
            p10 *= 10;
            num = (num * 10 + c - '0');
    if (p10 < 0.5) p10 = 1;
    return sgn * num / p10;
//b77b67
```

https://acm.math.spbu.ru/~sk1/algo/memory.cpp.html

#### 12.2.2 Быстрый аллокатор

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void * operator new (size_t n) {
   assert((mpos += n) <= MAX_MEM);
   return (void *)(mem + mpos - n);
}
void operator delete (void *) noexcept { }
void operator delete (void *, size_t) noexcept { }
//8726b1
```

#### 12.3 Редукция Барретта

```
using u64 = unsigned long long;
using u128 = __uint128_t;
struct barrett{
  u64 p, m;
  barrett() {}
  \texttt{barrett(u64 p) : p(p), m(-1ULL / p) } \ \{ \}
  int reduce(u64 x) {
  u64 q = (u128(m) * x) >> 64, r = x - q * p;
    return r - p * (r >= p);
} ba;
// Usage example:
void solve() {
  int p = ...;
  ba = barrett(p);
  int x = ..., y = ...;
  int prod = ba.reduce(x * y);
//a8b4c7
```

#### 12.4 Флаги компияции

```
-DLOCAL -Wall -Wextra -pedantic -Wshadow -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -Wshift- overflow=2 -Wduplicated-cond -Wcast-qual -Wcast-align -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -D_FORTIFY_SOURCE=2 -fsanitize=address -fsanitize=undefined -fno-sanitize-recover -fstack-protector -std=c++2a
```

#### **12.4.1** Сеточка в vim

https://codeforces.com/blog/entry/122540

```
i|<esc>25A |<esc>
o+<esc>25A---+<esc>
Vky35Pdd
```

# 12.5 Что сделать на пробном туре

- Послать клар
- Распечатать что-то
- Получить ML (stack & heap)
- Максимальный размер отправляемого файла?
- Убедиться, что чекер регистронезависимый (yes/YES)
- Позапускать Флойда Варшалла
- Посмотреть, насколько быстр быстрый ввод
- Перебить что-то, проверить хеш
- Проверить санитайзеры

#### 12.6 Хеш файла без комментариев

```
Хеш файла, игнорирующий переводы строк и комментарии:
```

```
$ cpp -dD -P -fpreprocessed "$filename" | tr -d '[:space:]' |
    md5sum | cut -c-6
```