### Содержание

		•
1	Наст	ройка CLion
2	Теор	ия чисел
	2.1 2.2	КТО
	2.3	Алгоритм Берлекэмпа — Месси
	2.4	Линейное решето
	2.5	Алгоритм Шенкса
3	Граф	ОЫ
	3.1	SCC и 2-SAT
	3.2 3.3	Эйлеров цикл
	3.4	
	3.5	Взвешенное паросочетание
	3.6 3.7	Дерево доминаторов
	3.8	Алгоритм Чу-Лью
4	Свёр	тки
•	4.1	ХОR свёртка
	4.2	FFT & co
	4.3	Быстрое FFT
	1. 1	
5	Стру	ктуры данных
	5.1 5.2	Дерево Фенвика
	5.3	Массовое дерево отрезков
	5.4	Битовый бор
	5.5 5.6	Ordered set         1           Динамический битсет         1
	5.7	Convex hull trick
	5.8	Центроиды
	5.9 5.10	Дерево Ли Чао       1         Min-Kinetic Segment Tree       1
		Декартово дерево
		5.11.1 Декартово дерево по явному ключу. Multiset 1
6	Стро	оковые алгоритмы 1-
	6.1	Префикс-функция
	6.2 6.3	Z-функция       1         Алгоритм Манакера       1
		Суфмассив
	6.5	Алгоритм Ахо — Корасик
	6.6	Дерево палиндромов
7	Пото	
	7.1 7.2	Алгоритм Диница       1         Mincost k-flow       1
	1.2	Mincost k-flow
8		ритм Гаусса 1
	8.1 8.2	Решение $Av=b$
	0.2	Dashe 710 — 0
9	Гами	льтоновы путь и цикл
	9.1 9.2	Link-cut tree
		Directed case
10	Геом	ra 2
10		а
	10.2	Выпуклая оболочка
	10.3	Точка внутри многоугольника         2           Касательные         2
	10.5	Пересечение многоугольника и полуплоскости
	10.6	События для прямой
	10.7	Кривая Гильберта для алгоритма Мо         2           Симплекс         2
11		ные дроби
		Поиск нижней огибающей, сумма и минимум по модулю 2 Простая рекурсия
12	<b>Разн</b>	юе
	12.1	Трюки от Сергея Копелиовича
		12.2.1 Быстрый ввод
	12.3	12.2.2 Быстрый аллокатор       2         Редукция Барретта       2
	12.4	Флаги компияции
	12.5	Что сделать на пробном туре
	12.6	Хеш файла без комментариев

### 1 Настройка CLion

- 1. B файле CMakeLists.txt дописать строчку add\_compile\_definitions(LOCAL). Нажать появившуюся опцию в правом верхнем углу enable auto-reload.
- 2. Вбить шаблон в main.cpp:

```
#ifdef LOCAL
#define _GLIBCXX_DEBUG
#endif
#include<bits/stdc++.h>
using namespace std;
#define int long long
#define app push_back
#define all(x) x.begin(), x.end()
#ifdef LOCAL
#define debug(...) [](auto...a){ ((cout << a << '< a << '< table > ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' '< ta
         '), ...) << endl; }(#__VA_ARGS__, ":", __VA_ARGS__)
#define debugv(v) do { cout << #v << ": "; for (</pre>
       auto x : v) cout << x << ' '; cout << endl; }</pre>
        while(0)
#else
#define debug(...)
#define debugv(v)
#endif
int32 t main() {
  cin.tie(0);ios_base::sync_with_stdio(0);
    int n = 2; vector<int> a(n, n);
   debug(n); debugv(a);
//59124c
```

Скомпилировать, чтобы проверить отсутствие опечаток.

3. Запустить терминал (crtl + alt + T)

```
$ cd workspace/CLionProjects
$ for c in {A..Z}; do cp main.cpp $c.cpp && echo
    "add_executable($c $c.cpp)" >> CMakeLists.txt
; done
```

Далее отключаем подсветку и форматирование в настройках (ctrl+alt+S)

- Editor  $\to$  Code Style  $\to$  Formatter  $\to$  Do not format прописать
- Editor  $\to$  Inspections  $\to$  C/C++  $\to$  static analysis tools  $\to$  CLang-Tidy отключить
- Editor → Inlay Hints → отключаем всё (достаточно первых трёх code vision, parameter names, types).

Тёмная тема отключается в Appearance & Behavior  $\rightarrow$  Appearance. Чтобы добавить санитайзеры, надо дописать в CMakeLists.txt set(CMAKE\_CXX\_FLAGS "-fsanitize=address -fsanitize=undefined")

### 2 Теория чисел

### 2.1 KTO

```
int gcd(int a, int b, int &x, int &y) {
   if (b==0) { x = 1; y = 0; return a; }
   int d = gcd(b,a%b,y,x);
   y==a/b*x;
   return d;
}
int inv(int r, int m) {
   int x, y;
   gcd(r,m,x,y);
   return (x+m)%m;
}
int crt(int r, int n, int c, int m) { return r + ((c - r) % m + m) * inv(n, m) % m * n; }
//8ed8ed
```

### 2.2 Алгоритм Миллера — Рабина

```
int128 one=1;
int po(int a,int b,int p)
{
  int res=1;
  while(b) {if(b & 1) {res=(res*one*a)%p;--b;} else {
    a=(a*one*a)%p;b>>=1;}} return res;
bool chprime(int n) ///miller-rabin
  if(n==2) return true;
  if(n<=1 || n%2==0) return false;</pre>
  int h=n-1;int d=0;while(h%2==0) {h/=2;++d;}
  for(int a:{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
    37})
    if(a==n) return true;
    int u=po(a,h,n);bool ok=0;
    if(u%n==1) continue;
    for(int c=0;c<d;++c)</pre>
      if((u+1)%n==0) {ok=1;break;}
      u=(u*one*u)%n;
    if(!ok) return false;
 return true;
//86b2ed
```

### 2.3 Алгоритм Берлекэмпа — Месси

https://mzhang2021.github.io/cp-blog/berlekamp-massey/

```
template<typename T>
vector<T> berlekampMassey(const vector<T> &s) {
  int n = s.size(), l = 0, m = 1;
vector<T> b(n), c(n);
T ld = b[0] = c[0] = 1;
  for (int i=0; i<n; i++, m++) {</pre>
     T d = s[i];
     for (int j=1; j<=1; j++)
d += c[j] * s[i-j];
     if (d == 0) continue;
     vector<T> temp = c;
     T coef = d / ld;
     for (int j=m; j<n; j++) c[j] -= coef * b[j-m];
if (2 * 1 <= i) {</pre>
       1 = i + 1 - 1;
       b = temp;
       ld = d;
       m = 0;
     }
  c.resize(l + 1);
  c.erase(c.begin());
  for (T &x : c)
    x = -x;
  return c;
//ff47ae
```

### 2.4 Линейное решето

```
const int C = 1e7+7;
vi pr, lp(C);
for (int i = 2; i < C; ++i) {
   if (lp[i] == 0) {
      lp[i] = i;
      pr.app(i);
   }
   for (int j = 0; j < (int)pr.size() && pr[j] <= lp[i
      ] && pr[j] * i < C; ++j) {
      lp[pr[j] * i] = pr[j];
   }
}
//36b3d1</pre>
```

### 2.5 Алгоритм Шенкса

```
#define T int
int mod;
int gcd(int a, int b, int &x, int &y) {
   if (b==0) { x = 1; y = 0; return a; }
   int d = gcd(b,a%b,y,x);
   y-=a/b*x;
   return d;
```

```
int inv(int r, int m) {
  int x, y;
  gcd(r,m,x,y);
  return (x+m)%m;
T inv(T a)
{
  return inv(a, mod);
T mul(T a, T b)
  return (a*b)%mod;
vector<int> rasl(int x)
  vector<int> v;
  if(x==1) {return v;}
  for(int i=2;i*i<=x;++i)</pre>
    if(x\%i==0)
    {
      v=rasl(x/i); v.app(i); return v;
  v.app(x);return v;
T po(T a, int b) ///b>=1
  if(b==1) {return a;}
  if (b%2==0)
   T u=po(a,b/2);
    return mul(u,u);
  else
    T u=po(a,b-1);
    return mul(a,u);
Т
 getper(T a, T one, int per, vector<int> v)
  for(int p:v)
    if(po(a,per/p)==one)
     per/=p;
vector<pair<int, int> > shanks(T a, vector<T> b, T one,
    int per) ///a^per=1 and b[i]^per=1 /// all right
    numbers in output are equal
  if(b.empty()) {return {};}
  int n=b.size();
  vector<int> vp=rasl(per);
  int pera=getper(a, one, per, vp); per=pera;
  vp=rasl(pera);
  vector<int> have(n,0);
  int cur2=per;T cura=a;T invcura=inv(a);
  int curad=1;
  vector<pair<T, int> > v;
  vector<bool> ok(n,true);
  vector<T> poinvzx;
  for(int p:vp)
   T ca=po(cura,cur2/p);
    if(ca==one) {continue;}
    T invca=po(invcura, cur2/p);
    int step=sqrt(b.size()*p)+2;
    int wee=p/step+2;
    v.clear();poinvzx.clear();
    T zx=one; T invzx=one; T buba=one;
    vector<T> zhe;
    T lu=one:
    for(int i=0;i<step;++i)</pre>
      v.app({zx,i}); zhe.app(lu);
      zx=mul(zx,ca);invzx=mul(invzx,invca);buba=mul(
    buba, cura); lu=mul(lu,invcura);
    poinvzx.app(one);
    for(int j=0;j<wee;++j)</pre>
      poinvzx.app(mul(poinvzx.back(),buba));
```

```
sort(all(v));
    for(int i=0;i<n;++i)</pre>
      if(!ok[i]) {continue;}
      T uu=po(b[i],cur2/p);
      bool okkk=false;
      for(int j=0;j<wee;++j)</pre>
        auto it=lower_bound(all(v), make_pair(uu, 0LL))
        if(it!=v.end() && (*it).first==uu)
        {
           okkk=true;
           have[i]-=(curad*step*j);
           have[i]+=(curad*(*it).second);
           have[i]%=pera;if(have[i]<0) {have[i]+=pera</pre>
    ; }
          b[i]=mul(b[i],poinvzx[j]);b[i]=mul(b[i],zhe
    [(*it).secondl);
           assert(po(b[i],cur2/p)==one);
          break;
        uu=mul(uu,zx);
      if(!okkk) {ok[i]=false;}
    cur2/=p; cura=po(cura,p); invcura=po(invcura,p);
    curad*=p;
  vector<pair<int,int> > res;
  for(int i=0;i<n;++i)</pre>
    if(ok[i] && b[i]==one)
      res.app({(have[i]%pera+pera)%pera,pera});
    else
      res.app({-1,pera});
    }
  return res;
int shanks2(int x,int y,int mod1) ///only for T=long
    long, 0^0 = 1 by default
  mod=mod1;
  vector<int> v=rasl(mod);sort(all(v));
  int per=1;for(int i=0;i<v.size();++i) {if(i==0 || v</pre>
    [i]!=v[i-1]) {per*=(v[i]-1);} else {per*=v[i];}}
  if(y==1 || mod==1) {return 0;}
  int C=61;
  for(int i=1;i<C;++i)</pre>
  {
    if(po(x,i)==y) {return i;}
  if(y==0) {return (-1);}
  T h=po(x,C);
  int lc1=gcd(h, mod); int lc2=gcd(y, mod);
  if(lc1!=1c2) {return (-1);}
  mod/=lc2;T h1=h/lc2;T y1=y/lc2;
vector<pair<int,int> > s=shanks(x*mod, {mul(y1,inv())})
    h1))},1,per);
  if(s[0].first!=(-1))
  {
    return s[0].first+C;
  else
    return (-1);
  }
//a75596
```

## 3 Графы

### 3.1 SCC и 2-SAT

```
Алгоритм ищет сильносвязные компоненты в графе g, если есть путь i \to j, то scc[i] \le scc[j] vector<int> find_scc(vector<vector<int>> g) { int n = g.size(); vector<vector<int>> r(n); for (int i = 0; i < n; ++i) { for (int j : g[i]) r[j].push\_back(i); } vector<int>> used(n), tout(n);
```

```
int time = 0;
  auto dfs = [&](auto dfs, int cur) -> void {
    used[cur] = 1;
    for (int nxt : g[cur])
      if (!used[nxt]) dfs(dfs, nxt);
    tout[cur] = time++;
  for (int i = 0; i < n; ++i) if (!used[i]) dfs(dfs,</pre>
    i);
  vector<int> ind(n);
 iota(all(ind), 0);
sort(all(ind), [&](int i, int j){return tout[i] >
    tout[j];});
  vector<int> scc(n, -1);
  auto go = [&](auto go, int cur, int color) -> void
    scc[cur] = color;
for (int nxt : r[cur]) {
      if (scc[nxt] == -1) go(go, nxt, color);
    }
  };
 int color = 0;
for (int i : ind) {
    if (scc[i] == -1) go(go, i, color++);
  return scc;
//4fd51f
```

Чтобы решать  $2\text{-}\mathcal{SAT}$ , надо создать граф на 2n вершинах, рёбра  $i\Rightarrow j$  и  $(j\oplus 1)\Rightarrow (i\oplus 1)$  должны быть добавлены одновременно. После этого если  $\mathrm{scc}[2\ ^\star\ i]=\mathrm{scc}[2\ ^\star\ i+1]$ , то решения нет; если  $\mathrm{scc}[2\ ^\star\ i+0]<\mathrm{scc}[2\ ^\star\ i+1]$ , то присутствует импликация  $\neg i\Rightarrow i$ , надо назначить i=true.

### 3.2 Эйлеров цикл

```
vector<int> euler(vector<vector<pair<int, int>>> g,
    int m, int src) { // g[cur][i] = pair{nxt, idx}
int n = g.size();
vector<int> used(m), it(n), cycle;
auto dfs = [&](auto dfs, int cur) -> void {
    while (true) {
        while (it[cur] < g[cur].size() && used[g[cur][it[cur]].second]) it[cur]++;
        if (it[cur] == g[cur].size()) return;
        auto [nxt, idx] = g[cur][it[cur]];
        used[idx] = true;
        dfs(dfs, nxt);
        cycle.push_back(idx); // or {cur, nxt}
    }
};
dfs(dfs, src);
reverse(cycle.begin(), cycle.end());
if (cycle.size() != m) return {}; // check that all
        edges are present in the cycle, fail otherwise
    return cycle;
}
//f6b9d4</pre>
```

#### 3.3 Компоненты рёберной двусвязности

```
//n - number of vertices, m - number of edges,
    parallel edges -- ???, color of any edge is the
    color of its lower end
vector <vector <int> > dfstree(n);
vector <int> used(n), cut(n), h(n), up(n);
auto findCutPoints = [&] (auto self, int u) -> void {
  used[u] = 1;
  up[u] = h[u];
  for (int v : g[u]) {
    if (!used[v]) {
      dfstree[u].push_back(v);
      h[v] = h[u] + 1;
      self(self, v);
      up[u] = min(up[u], up[v]);
      if (up[v] >= h[u]) {
        cut[v] = 1;
      }
    else {
      up[u] = min(up[u], h[v]);
  }
findCutPoints(findCutPoints, 0);
vector <vector <int> > tree(n + m);
```

```
vector<int> color(n);color[0]=0;int ptr=n;
auto build = [&] (auto self, int u) -> void {
  for (int v : dfstree[u]) {
     if (cut[v]) {
        color[v]=ptr;++ptr;
        self(self, v);
     else {
        color[v]=color[u];
        self(self, v);
  }
};
build(build, 0);
for(int i=0;i<n;++i) {</pre>
   set<int> to;
   for(int j:g[i]) {
     int x=i, y=j;
if(h[x]<h[y]) swap(x,y);</pre>
     to.insert(color[x]);
  for(int j:to) {
  tree[i].app(j);tree[j].app(i);
//2ebfbb
```

#### 3.4 DCP offline

```
struct Dsu {
  int n;
  vector<pair<int &, int>> s;
  vector<int> p, sz;
  // other info
  Dsu(int n) : n(n), p(n), sz(n, 1){
     iota(all(p), 0);
  int get(int u) {
     while (u != p[u]) u = p[u];
     return u:
  bool merge(int u, int v) {
     inerge(int u, int v) {
    u = get(u), v = get(v);
    if (u == v) return false;
    if (sz[v] < sz[u]) swap(u, v);</pre>
    s.app({p[u], p[u]});
s.app({sz[v], sz[v]});
// app other info like s.app({comp, comp});
p[u] = v;
     sz[v] += sz[u];
     return true;
  void rollback(int sz) {
  while (s.size() != sz) {
        s.back().first = s.back().second;
        s.pop_back();
struct DcpOffline {
  int n;
  vector<vector<pair<int, int>>> d;
  void addEdgeOnSegment(int 1, int r, int a, int b) {
  for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {
    if (1 & 1) d[1++].app({a, b});</pre>
        if (r & 1) d[--r].app({a, b});
     }
  template<typename T>
  void dfs(Dsu &dsu, T act) {
     dfs(1, 0, n, dsu, act);
  template<typename T>
  void dfs(int v, int l, int r, Dsu &dsu, T act) {
     int sz = dsu.s.size();
     for (auto [u, v]: d[v]) {
        dsu.merge(u, v);
     if (1 + 1 == r) {
       act(1, dsu);
     } else {
```

```
int m = (1 + r) / 2;
    dfs(v * 2, 1, m, dsu, act);
    dfs(v * 2 + 1, m, r, dsu, act);
}
dsu.rollback(sz);
}

DcpOffline(int maxt) : n(2 << __lg(maxt + 1)), d(2 * n) {}

//3c4e2d</pre>
```

```
3.5 Взвешенное паросочетание
https://judge.yosupo.jp/submission/201334
\#define\ d(x)\ (lab[x.u] + lab[x.v] - 2 * e[x.u][x.v].w
const int N = 403*2;
const int inf = 1e18;
struct Q{ int u, v, w; } e[N][N];
vector<int> p[N];
void ss(int v) {
  sl[v] = 0;
  for (int u = 1; u \le n; t+u) if (e[u][v].w > 0 &&
    st[u] != v && !s[st[u]]) upd(u, v);
void ins(int u){ if (u \le n) q[++t] = u; else for (
int v : p[u]) ins(v); }
void ch(int u, int w) { st[u] = w; if (u > n) for (
    int v : p[u]) ch(v, w); }
int gr(int u, int v)
  if ((v = find(all(p[u]), v) - p[u].begin()) & 1) {
    reverse(1 + all(p[u]));
    return (int)p[u].size() - v;
  return v;
}
void stm(int u, int v) {
 lk[u] = e[u][v].v;
  if (u <= n) return; Q w = e[u][v];</pre>
  int x = b[u][w.u], y = gr(u,x);
for (int i = 0; i < y; ++i) stm(p[u][i], p[u][i^1])</pre>
  stm(x, v); rotate(p[u].begin(), y+all(p[u]));
void aug(int u, int v) {
  int w = st[lk[u]];stm(u, v);if (!w) return;
stm(w, st[f[w]]);
  aug(st[f[w]], w);
int lca(int u, int v) {
  for (id++; u|v; swap(u, v)) {
  if (!u) continue;if(ed[u] == id) return u;
    ed[u] = id; if (u = st[lk[u]]) u = st[f[u]]; //
    =, not ==
  return 0;
//cf1d55
void add(int u, int a, int v) {
  int x = n + 1; while (x \le m \&\& st[x]) ++x;
  if (x > m) ++m;
  lab[x] = s[x] = st[x] = 0;

lk[x] = lk[a];
  p[x].clear();
  p[x].push_back(a);
\#define\ op(q)\ for\ (int\ i=q,\ j=0;\ i!=a;\ i=st[f[j]
    ]]) p[x].push_back(i), p[x].push_back(j=st[lk[i
  ]]), ins(j) // also not ==
op(u); reverse(1+all(p[x]));op(v);
  ch(x, x); for (int i = 1; i <= m; ++i) e[x][i].w =
    e[i][x].w = 0;
  fill(b[x]+1, b[x]+n+1, 0);
  for (int u : p[x]) {
  for (int v = 1; v <= m; ++v) if (!e[x][v].w || d(
    e[u][v]) < d(e[x][v])) e[x][v] = e[u][v], e[v][x]</pre>
      = e[v][u];
    for (int v = 1; v \le n; ++v) if (b[u][v]) b[x][v]
     = 11:
  ss(x);
```

```
void ex(int u) {
  for (int x : p[u]) ch(x, x);
  int a = b[u][e[u][f[u]].u], r = gr(u, a);
for (int i = 0; i < r; i += 2) {
  int x = p[u][i], y = p[u][i + 1];</pre>
     f[x] = e[y][x].u; s[x] = 1; s[y] = 0; sl[x] = 0;
     ss(y); ins(y);
  s[a] = 1; f[a] = f[u];
for (int i = r + 1; i < p[u].size(); ++i) s[p[u][i
     ]] = -1, ss(p[u][i]);
  st[u] = 0;
bool on(const Q &e) {
  int u = st[e.u], v = st[e.v], a;
       (s[v] == -1) {
    f[v] = e.u, s[v] = 1, a = st[lk[v]], sl[v] = sl[a]
  [v] - e.u, s[v] - 1, a - sc[lk[v]], sl[v]
] = s[a] = 0, ins(a);
} else if (!s[v]) {
  a = lca(u, v); if (!a) return aug(u, v), aug(v, u
  ), 1; else add(u, a, v);
  return 0;
//3f0f1d
bool bfs() {
  fill(s+1, s+m+1, -1); fill(sl+1, sl+m+1, 0); // s is
      filled with -1
  h = 1, t = 0; for (int i = 1; i \le m; ++i) if (st[i
     ] == i \&\& !lk[i]) f[i] = s[i] = 0, ins(i);
   if (h > t) return 0;
  while (1) {
     while (h \le t) {
        int u = q[h++];
     if (s[st[u]] != 1) {
   for (int v = 1; v <= n; ++v) if (e[u][v].w >
0 && st[u] != st[v]) {
             if (d(e[u][v])) upd(u, st[v]); else if (on(
     e[u][v])) return 1;
        }
     }
     int x = inf;
     for (int i = n+1; i <= m; ++i) if (st[i] == i &&</pre>
     s[i] == 1) x = min(x, lab[i]/2);
     for (int i = 1; i <= m; ++i) if (st[i] == i && sl
      [i] \&\& s[i] != 1) x = min(x, d(e[sl[i]][i])>>s[i]
     ]+1);
          (int i = 1; i \le n; ++i) if (\sim s[st[i]]) if ((
     lab[i] += (s[st[i]] * 2 - 1) * x) <=0) return 0;
for (int i = n + 1; i <= m; ++i) if (st[i] == i
&& ~s[st[i]]) lab[i] += (2 - 4 * s[st[i]]) * x;
     h = 1, t = 0;
     for (int i = 1; i <= m; ++i) if (st[i] == i && sl</pre>
     [i] && st[sl[i]] != i && !d(e[sl[i]][i]) && on(e[
     sl[i]][i])) return 1;
     for (int i = n+1; i <= m; ++i) if (st[i] == i &&
     s[i] == 1 \&\& !lab[i]) ex(i);
pair<int, vector<array<int, 2>>> run(int N, vector<</pre>
   array<int, 3>> edges) {
for (auto &[u, v, w] : edges) ++u, ++v;
  fill(ed+1, ed+m+1, 0);
fill(lk+1, lk+m+1, 0);
  n = m = N;
  id = 0;
  iota(st + 1, st + n + 1, 1);
  lota(st + 1, st + 11 + 1, 1),
int wm = 0, weight = 0;
for (int i = 1; i <= n; ++i) for (int j = 1; j <= n
   ; ++j) e[i][j] = {i,j,0};</pre>
  for (int i = 1; i <= n; ++i) p[i].clear();
for (int i = 1; i <= n; ++i) for (int j = 1; j <= n</pre>
      ; ++j) b[i][j] = i==j?i:0;
  fill_n(lab+1, n, wm); while (bfs());
vector<array<int, 2>> matching;
for (int i = 1; i <= n; ++i) if (i < lk[i]) weight</pre>
     += e[i][lk[i]].w, matching.push_back({i - 1, lk[i
     1 - 1});
  return {weight, matching};
//be682f
```

### 3.6 Дерево доминаторов

```
struct DominatorTree{
  struct DSU{
    struct Vert{
      int p;
      pair<int, int> val;
    };
    vector<Vert> t;
    vector<int> ord;
    DSU(vector<int> &ord): ord(ord) { t.resize(ord.
    size()); for (int i = 0; i < ord.size(); i++) t[i
    ].p = i; }
    int get(int v){
     if (t[v].p == v) return v;
      int new_p = get(t[v].p);
      if (ord[t[v].val.first] > ord[t[t[v].p].val.
    first]) t[v].val = t[t[v].p].val;
      t[v].p = new_p;
      return t[v].p;
    void merge(int a, int b){
      a = get(a); b = get(b);
if (a != b){
        t[b].p = a;
    void setVal(int v, pair<int, int> val){
     t[v].val = val;
    auto getVal(int v){
      get(v);
      return t[v].val;
  };
  vector<vector<int> > g, gr, lg;
  vector<int> idom, sdom, was, tin;
  int timer:
  void dfs(int v){
    tin[v] = timer++;
    was[v] = 1;
    for (int to : g[v]) if (!was[to]) dfs(to);
  vector<vector<int> > req;
  DominatorTree(int n, vector<pair<int, int> > &edges
    , int root){
    g.resize(n); gr.resize(n); lg.resize(n);
    idom.resize(n, -1); sdom.resize(n);
was.resize(n, 0), tin.resize(n);
    req.resize(n);
    for (auto &&e : edges){
      g[e.first].push_back(e.second);
      gr[e.second].push_back(e.first);
    timer = 0; dfs(root);
    vector<int> ord;
for (int i = 0; i < n; i++) ord.push_back(i);</pre>
    sort(ord.begin(), ord.end(), [this](int w1, int
    w2){ return tin[w1] > tin[w2]; });
    DSU dsu(tin);
    for (int v : ord) {
   sdom[v] = v;
      for (int to : gr[v]){
           (v == to) continue;
        int val = tin[to] < tin[v] ? to : dsu.getVal(</pre>
    to).first;
        if (tin[val] < tin[sdom[v]]) sdom[v] = val;</pre>
      req[sdom[v]].push_back(v);
      for (auto &&r : req[v]){
        auto val = dsu.getVal(r);
        if (tin[val.first] < tin[sdom[r]]){</pre>
          lg[val.second].push_back(r);
        } else {
          idom[r] = sdom[r];
```

}

```
dsu.setVal(v, make_pair(sdom[v], v));
      for (int to : g[v]){
   if (tin[to] > tin[v] && dsu.t[to].p == to){
          dsu.merge(v, to);
      }
    for (int i = 0; i < n; i++) was[i] = 0;</pre>
    for (int i = 0; i < n; i++) if (!was[i] && idom[i</pre>
    ] != -1) {
      vector<int> st;
      st.push_back(i);
      was[i] = 1;
      while(st.size()){
         int v = st.back(); st.pop_back();
         idom[v] = idom[i];
         for (int to : lg[v]) if (!was[to]) was[to] =
       st.push_back(to);
    }
  }
};
vector <pair <int, int> > e;
DominatorTree d(n,e,0);
auto par = d.idom;
//839464
```

# 3.7 Венгерский алгоритм решения задачи о назначениях

```
//choose one element in each row to minimize sum of
     the chosen elements, n <= m, INF>max(abs(a[i][j])
const int INF = 1e18;
vector<int> hungarian(int n, int m, vector<vector<int</pre>
     >> a, int &cost) { //1-indexed!, a.size()=n+1, a[
     i].size()=m+1
  vector<int> u(n+1), v(m+1), p(m+1), way(m+1);
for (int i=1; i<=n; ++i) {</pre>
    p[0] = i;
int j0 = 0;
     vector<int> minv(m+1, INF);
     vector<char> used(m+1, false);
       used[j0] = true;
       int i0 = p[j0], delta = INF, j1;
for (int j=1; j<=m; ++j)
          if (!used[j]) {
            int cur = a[i0][j]-u[i0]-v[j];
if (cur < minv[j])
  minv[j] = cur, way[j] = j0;
if (minv[j] < delta)</pre>
               delta = minv[j], j1 = j;
       for (int j=0; j<=m; ++j)</pre>
          if (used[j])
            u[p[j]] += delta, v[j] -= delta;
            minv[j] -= delta;
     j0 = j1;
} while (p[j0] != 0);
     do {
       int j1 = way[j0];
       p[j0] = p[j1];
    j0 = j1;
} while (j0);
  vector<int> ans(n+1);
  for (int j=1; j<=m; ++j) {
  if (p[j]!=0) {</pre>
       ans[p[j]] = j;
  cost = -v[0];
  return ans;
//6d564b
```

### 3.8 Алгоритм Чу-Лью

В ориентированном взвешенном графе ищет остовное дерево минимального веса (такое дерево, что все вершины достижимы из нуля, входящая степень любой ненулевой вершины равна 1, в нуль не входит ни одного ребра). Если recover = true, то восстанавливает ответ.

```
Предполагается, что все вершины достижимы из нуля.
using edge = array<int, 4>; // {from, to, w, i}
template<typename T, typename C>
using pq = priority_queue<T, vector<T>, C>;
pair<int, vector<int>> solve(int n, vector<edge> ed,
    bool recover) {
  auto cmp = [&](int i, int j) { return ed[i][2] > ed
     [j][2]; };
  vector r(n, pq<int, decltype(cmp)>(cmp));
for (auto [u, v, w, i] : ed) r[v].push(i);
vector<int> mod(n), p(n), color(n), take;
  iota(all(p), 0);
  auto get = [&](int u) {
   while (u != p[u]) u = p[u] = p[p[u]];
     return u;
  auto unite = [&](int x, int y) {
    x = get(x), y = get(y);
if (x == y) return;
if (r[x].size() > r[y].size()) swap(x, y);
    p[x] = y;
     while (r[x].size()) {
       auto e = r[x].top();
       r[x].pop();
ed[e][2] += mod[x] - mod[y];
       r[y].push(e);
    }
  };
  vector<vector<pair<int, int>>> g(n);
  int ans = 0:
  color[0] = 2;
  auto go = [&](int cur) {
     vector<pair<int, int>> stack;
     int time = 0;
     while (color[cur] < 2) {</pre>
       color[cur] = 1;
       edge e;
       do {
         e = ed[r[cur].top()];
         r[cur].pop();
       } while (get(e[0]) == cur);
       e[2] += mod[cur];
       ans += e[2];
mod[cur] -= e[2];
       stack.push_back({cur, e[3]});
       int a = get(e[0]);
       if (color[a] == 1) {
          while (true) {
            auto [nxt, i] = stack.back();
stack.pop_back();
            g[ed[i][0]].push_back({time++, i});
            unite(nxt, cur);
if (nxt == a) break;
         }
       }
       cur = get(e[0]);
     for (auto [x, i] : stack) {
  color[x] = 2;
       pq<pair<int, int>, greater<>> dijkstra;
for (auto [x, i] : stack) {
          dijkstra.emplace(x, i);
       while (!dijkstra.empty()) {
          auto [t, i] = dijkstra.top();
          dijkstra.pop();
          if (color[ed[i][1]] == 3) {
            continue;
          color[ed[i][1]] = 3;
          take.push_back(i);
for (auto [t2, i2] : g[ed[i][1]]) {
  dijkstra.emplace(t2, i2);
       }
    }
  };
  for (int i = 1; i < n; ++i) qo(qet(i));
  return {ans, take};
//f245b7
```

### 4 Свёртки

#### XOR свёртка 4.1

```
vector<int> bxor(vector<int> a, vector<int> b)
  assert(p%2==1); int inv2=(p+1)/2;
  int n=0; while((1<<n)<a.size()) ++n;</pre>
  a.resize(1<<n);b.resize(1<<n);
  for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n);++
  mask) if(!(mask & (1<<i))) {int u=a[mask],v=a[</pre>
    mask+(1<<i)];a[mask+(1<<i)]=(u+v)%p;a[mask]=(u-v)
  for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n);++</pre>
    mask) if(!(mask & (1 << i))) {int u=b[mask], v=b[
    mask+(1<<i)];b[mask+(1<<i)]=(u+v)%p;b[mask]=(u-v)
  vector < int > c(1 << n, 0);
  for(int mask=0; mask<(1<< n); ++ mask) {c[mask]=a[mask]}
    ]*b[mask];c[mask]%=p;}
  for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n);++</pre>
    ]=((u+v)*inv2)%p;}
  return c;
//20cc50
```

```
4.2 FFT & co
typedef long long 11;
const int p=998244353;
int po(int a,int b) {if(b==0) return 1; if(b==1)
    return a; if(b%2==0) {int u=po(a,b/2);return (u*1
    LL*u)%p;} else {int u=po(a,b-1); return (a*1LL*u)%
    f { ; q
int inv(int x) {return po(x,p-2);}
the five first po(K,p 2);
template<int M, int K, int G> struct Fft {
    // 1, 1/4, 1/8, 3/8, 1/16, 5/16, 3/16, 7/16, ...
int g[1 << (K - 1)];
Fft() : g() { //if tl constexpr...</pre>
    // static_assert(K >= 2, "Fft: K >= 2 must hold")
    g[0] = 1;
    g[1 << (K - 2)] = G;
    for (int 1 = 1 \ll (K - 2); 1 >= 2; 1 >>= 1) {
g[1 >> 1] = (g[1] * 1LL* g[1]) % M;
    assert((g[1]*1LL * g[1]) % M == M - 1);
    }
  void fft(vector<int> &x) const {
    const int n = x.size();
    assert(n <= 1 << K);
    for (int j = i \ll (h+1); j \ll (((i \ll 1) + 1)
    << h); ++j) {
           const int t = (g[i] * 1LL* x[j | 1]) % M;
           x[j | 1] = x[j] - t;
           if(x[j|1] < 0) x[j | 1] += M;
          x[j]+=t;
           if (x[j] >= M) x[j] -= M;
        }
      }
    for (int i = 0, j = 0; i < n; ++i) {
      if (i < j) std::swap(x[i], x[j]);</pre>
      for (int l = n; (l >>= 1) && !((j ^= 1) & 1); )
     { }
    }
  vector<int> convolution(vector<int> a, vector<int>
    if(a.empty() || b.empty()) return {};
    for(int& x:a) \{x\%=p; if(x>=p) x-=p; if(x<0) x+=p;\}
     for(int& x:b) \{x\%=p; if(x>=p) \ x-=p; if(x<0) \ x+=p
    const int na = a.size(), nb = b.size();
    int n, invN = 1;
    for (n = 1; n < na + nb - 1; n <<= 1) invN = ((
    invN & 1) ? (invN + M) : invN) >> 1;
    vector<int> x(n, 0), y(n, 0);
```

```
std::copy(a.begin(), a.end(), x.begin());
std::copy(b.begin(), b.end(), y.begin());
    fft(x);
    fft(y);
    for (int i = 0; i < n; ++i) x[i] = (((static_cast
<long long>(x[i]) * y[i]) % M) * invN) % M;
    std::reverse(x.begin() + 1, x.end());
    fft(x):
    x.resize(na + nb - 1);
    return x;
 }
Fft<998244353,23,31> muls;
//a1b591
vector<int> form(vector<int> v,int n)
  while(v.size()<n) v.push_back(0);</pre>
  while(v.size()>n) v.pop_back();
  return v;
vector<int> operator *(vector<int> v1, vector<int> v2)
{
  return muls.convolution(v1, v2);
vector<int> operator +(vector<int> v1, vector<int> v2)
  while(v2.size()<v1.size()) v2.push_back(0); while(</pre>
    v1.size()<v2.size()) v1.push_back(0);
  for(int i=0;i<v1.size();++i) {v1[i]+=v2[i];if(v1[i])</pre>
    ]>=p) v1[i]-=p; else if(v1[i]<0) v1[i]+=p;}</pre>
  return v1:
vector<int> operator -(vector<int> v1, vector<int> v2)
  int sz=max(v1.size(),v2.size());while(v1.size()<sz)
  v1.push_back(0); while(v2.size()<sz) v2.</pre>
    push_back(0);
  for(int i=0;i<sz;++i) {v1[i]-=v2[i];if(v1[i]<0) v1[</pre>
    i]+=p; else if(v1[i]>=p) v1[i]-=p;} return v1;
vector<int> trmi(vector<int> v)
  for(int i=1;i<v.size();i+=2) {if(v[i]>0) v[i]=p-v[i
    ]; else v[i]=(-v[i]);}
  return v;
vector<int> deriv(vector<int> v)
  if(v.empty()) return{};
  vector<int> ans(v.size()-1);
  for(int i=1;i<v.size();++i) ans[i-1]=(v[i]*1LL*i)%p</pre>
  return ans:
vector<int> integ(vector<int> v)
  vector<int> ans(v.size()+1);ans[0]=0;
  for(int i=1;i<v.size();++i) ans[i-1]=(v[i]*1LL*i)%p</pre>
  return ans;
vector<int> mul(vector<vector<int> > v)
  if(v.size()==1) return v[0];
  vector<vector<int> > v1, v2; for(int i=0;i<v.size()</pre>
    /2;++i) v1.push_back(v[i]); for(int i=v.size()/2;
    i<v.size();++i) v2.push_back(v[i]);</pre>
  return muls.convolution(mul(v1), mul(v2));
vector<int> inv1(vector<int> v,int n)
  assert(v[0]!=0);
  int sz=1; v=form(v,n); vector<int> a={inv(v[0])};
  while(sz<n)
    vector<int> vsz;for(int i=0;i<min(n,2*sz);++i)</pre>
    vsz.push_back(v[i]);
    vector<int> b=((vector<int>) {1})-muls.
    convolution(a, vsz);
    for(int i=0;i<sz;++i) assert(b[i]==0);</pre>
    b.erase(b.begin(),b.begin()+sz);
    vector<int> c=muls.convolution(b,a);
    for(int i=0;i<sz;++i) a.push_back(c[i]);</pre>
    sz*=2:
  return form(a,n);
```

//12aa4e

### 4.3 Быстрое FFT

- Solution based on https://codeforces.com/blog/entry/117947
- Iterative and in-place version.
- Uses signed montgomery
- Optimized to minimize memory usage

```
const int MOD = 998244353;
const long long MOD2 = (long long) MOD * MOD;
const int root = 3;
const int alim = 64; // Bound for using O(n^2)
    polynomial mult
int modpow(int b, int e) {
  int ans = 1;
  for (; e; b = (long long) b * b % MOD, e /= 2)
     if (e & 1) ans = (long long) ans * b % MOD;
const int MODinv = 2 - MOD; // pow(-MOD, -1, 2**32)
inline int m_reduce(long long x) {
  int m = x * MODinv;
  return (x>>32) - (((long long) m * MOD) >> 32);
const int r2 = modpow(2, 64);
inline int m_transform(int x) {
  return m_reduce((long long)x * r2);
inline int m_add(int x, int y) {
  int z = x + y;
  return z < 0? z + MOD : z - MOD;
inline int m_sub(int x, int y) {
  int z = x - y;
return z < 0 ? z + MOD : z - MOD;</pre>
inline int m_mult(int x, int y) {
  return m_reduce((long long) x * y);
}
vector<int> rt = {1};
vector<int> transformed_rt;
vector<int> transformed_rt2;
template<int a>
void transform(vector<int> &P) {
  int m = P.size();
int n = m / a;
  int size = rt.size();
while (2 * size < n) {
  rt.resize(n / 2);</pre>
     int r = modpow(root, MOD / (4 * size));
for (int i = 0; i < size; ++i)
  rt[i + size] = (long long) r * rt[i] % MOD;</pre>
     size *= 2;
   // For montgomery
  for (int i = transformed_rt.size(); i < rt.size();</pre>
     ++i) {
     transformed_rt.resize(rt.size());
     transformed_rt[i] = m_transform(rt[i]);
     transformed_rt2.resize(rt.size());
     transformed_rt2[i] = (unsigned int) MODinv *
     transformed_rt[i];
  int k = n;
  while (k >= 4) k /= 4;
  if (k == 2) {
     int step = n * a;
     int half_step = step / 2;
for (int j1 = 0; j1 < half_step; ++j1) {
  int j2 = j1 + half_step;</pre>
       int diff = m_sub(P[j1], P[j2]);
       P[j1] = m_add(P[j1], P[j2]);
```

```
P[j2] = diff;
    k = n/2;
  } else {
  for (; k > 1; k /= 4) {
  for (int i = 0; i < n/k; ++i) {</pre>
       int step = k * a;
       int half_step = step / 2;
       int quarter_step = half_step / 2;
       int R20 = transformed_rt2[2 * i];
       int RR0 = transformed_rt[2 * i];
       int R21 = transformed rt2[2 * i + 1];
       int RR1 = transformed_rt[2 * i + 1];
       int R2 = transformed rt2[i];
       int RR = transformed_rt[i];
       int j1 = i * step;
       int j2 = j1 + quarter_step;
int j3 = j2 + quarter_step;
       int j4 = j3 + quarter_step;
    for (int j = 0; j < quarter_step; ++j, ++j1, ++
j2, ++j3, ++j4) {</pre>
         int z0;
           int z = P[j3];
int m = (unsigned int) R2 * z;
z0 = ((long long) z * RR - (long long) m *
    MOD) >> 32;
         }
         int z1;
           int z = P[j4];
           int m = (unsigned int) R2 * z;
           z1 = ((long long) z * RR - (long long) m *
    MOD) >> 32;
         int sum0 = m_add(P[j1], z0);
         int diff0 = m_sub(P[j1], z0);
int sum1 = P[j2] + z1;
         int diff1 = P[j2] - z1;
         // [sum0, sum1, diff0, diff1]
         int zz0;
           int z = sum1;
           int m = (unsigned int) R20 * z;
            zz0 = ((long long) z * RR0 - (long long) m
    * MOD) >> 32;
         }
         int zz1;
         {
            int z = diff1;
            int m = (unsigned int) R21 * z;
           zz1 = ((long long) z * RR1 - (long long) m
    * MOD) >> 32;
         }
         P[j1] = m_add(sum0, zz0);
         P[j2] = m_sub(sum0, zz0);
P[j3] = m_add(diff0, zz1);
         P[j4] = m_sub(diff0, zz1);
    }
  for (int i = 0; i < m; ++i)
if (P[i] < 0) P[i] += MOD;</pre>
template<int a>
void inverse_transform(vector<int> &P) {
  int m = P.size();
int n = m / a;
  int n_inv = m_transform(modpow(n, MOD - 2));
  vector<int> rev(n);
for (int i = 1; i < n; ++i) {</pre>
    rev[i] = rev[i / 2] / 2 + (i & 1) * n / 2;
```

```
}
   // P = [p * n_inv for p in P]
  for (int i = \overline{0}; i < m; ++i)
     P[i] = m_mult(n_inv, P[i]);
  // P = [P[a * rev[i // a] + (i % a)] for i in range
     (m)]
  for (int i = 1; i < n; ++i)</pre>
     if (i < rev[i])</pre>
       swap_ranges(P.begin() + a * i, P.begin() + a *
     i + a, P.begin() + a * rev[i]);
  // P = [P[-a * (i // a) + (i % a)] for i in range(m
    ) ]
  for (int i = 1; i < n/2; ++i)
  swap_ranges(P.begin() + a * i, P.begin() + a * i
  + a, P.begin() + a * (n - i));</pre>
  transform<a>(P);
  // P = [P[a * rev[i // a] + (i % a)] for i in range
     (m) 1
  for (int i = 1; i < n; ++i)</pre>
     if (i < rev[i])</pre>
     swap_ranges(P.begin() + a * i, P.begin() + a *
i + a, P.begin() + a * rev[i]);
template<int a>
void fast_polymult_mod(vector<int> &P, vector<int> &Q
  int m = P.size();
  int n = m / a;
  transform<a>(P);
  transform<a>(0);
  vector < int > &PO = P:
  for (int i = 0; i < n; ++i) {
  vector<unsigned long long> res(2 * a);
     for (int j = 0; j < a; ++j) {
    if (j >= 10 && j % 9 == 8)
    for (int k = j; k < j + a - 10; ++k)
        res[k] -= (res[k] >> 63) * 9 * MOD2;
       for (int k = 0; k < a; ++k)
         res[j + k] += (long long) P[i * a + j] * Q[i
     * a + k];
     int c = rt[i/2];
     if (i & 1) c = MOD - c;
for (int j = 0; j < a; ++j)
  PO[i * a + j] = (res[j] + c * (res[j + a] % MOD</pre>
     )) % MOD;
  inverse_transform<a>(PQ);
template <size_t... N>
void work(std::index_sequence<N...>, int x, std::
     vector<int>& a, std::vector<int>& b) {
  static void (*ptrs[])(std::vector<int>&, std::
     vector<int>&) = {&fast_polymult_mod<N+1>...};
  ptrs[x - 1](a, b);
void fast_polymult(vector<int> &P, vector<int> &Q) {
  int m1 = P.size();
  int m2 = Q.size();
  int res_len = m1 + m2 - 1;
  int b = 1;
  while ((alim << b) < res_len) ++b;</pre>
  int a = ((res_len - 1) >> b) + 1;
  int m = a \ll \overline{b};
  P.resize(m):
  Q.resize(m);
  // Call fast_polymult_mod<a>(P, Q);
  work(std::make_index_sequence<alim>{}, a, P, Q);
  P.resize(res len);
//239b3e
```

### 4.4 FFT в double'ax

```
using cd = complex<double>;
const double PI = acos(-1);
void fft(vector<cd> & a, bool invert) {
  int n = a.size();
  for (int i = 1, j = 0; i < n; i++) {
  int bit = n >> 1;
     for (; j & bit; bit >>= 1)
  j ^= bit;
     j ^= bit;
    if (i < j)</pre>
       swap(a[i], a[j]);
  for (int len = 2; len <= n; len <<= 1) {
  double ang = 2 * PI / len * (invert ? -1 : 1);</pre>
     cd wlen(cos(ang), sin(ang));
     for (int i = 0; i < n; i += len) {</pre>
        cd w(1);
        for (int j = 0; j < len / 2; j++) {</pre>
          cd u = a[i+j], v = a[i+j+len/2] * w;
          a[i+j] = u + v;
          a[i+j+len/2] = u - v;
          w *= wlen;
       }
     }
  if (invert) {
     for (cd & x : a)
       x /= n;
  }
vector<int> multiply(vector<int> const& a, vector<int</pre>
    > const& b) {
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.
    end());
  int n = 1;
  while (n < a.size() + b.size())</pre>
    n <<= 1;
  fa.resize(n);
  fb.resize(n);
  fft(fa, false);
fft(fb, false);
  for (int i = 0; i < n; i++)
fa[i] *= fb[i];
  fft(fa, true);
  vector<int> result(n);
  for (int i = 0; i < n; i++)</pre>
  result[i] = round(fa[i].real());
while(!result.empty() && !result.back()) result.
    pop_back();
  return result;
//35d9d0
```

### 5 Структуры данных

### 5.1 Дерево Фенвика

```
int fe[maxn];
void pl(int pos,int val) {while(pos<maxn) {fe[pos]+=
    val;pos|=(pos+1);}}
int get(int pos) {int ans=0;while(pos>=0) {ans+=fe[
    pos];pos&=(pos+1);--pos;} return ans;} /// [0,pos
    ] - vkluchitelno!!!
int get(int l,int r) {return get(r-1)-get(l-1);} ///
    sum of [l,r)
//2991a1
```

### 5.2 Дерево отрезков в точке

```
template<typename T, typename U>
struct SegmentTree {
  int h, n;
  T neutral;
  U unite;
  vector<T> data;

template<typename I>
```

```
SegmentTree(int sz, T neutral, U unite, I init) : h
  (__lg(sz) + 1), n(1 << h), neutral(neutral),
  unite(unite), data(2 * n) {</pre>
   for (int i = 0; i < sz; ++i) data[i + n] = init(i</pre>
  for (int i = n - 1; i > 0; --i) data[i] = unite(
data[2 * i], data[2 * i + 1]);
SegmentTree(int sz, T neutral, U unite) : h(__lg(sz
) + 1), n(1 << h), neutral(neutral), unite(unite)
, data(2 * n, neutral) {}</pre>
void set(int i, T x) {
  data[i += n] = x;
for (i /= 2; i > 0; i /= 2) data[i] = unite(data
[2 * i], data[2 * i + 1]);
T get(int 1, int r) {
  T leftRes = neutral, rightRes = neutral;
  for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {
    if (1 & 1) leftRes = unite(leftRes, data[1++]);</pre>
      if (r & 1) rightRes = unite(data[--r], rightRes
   ) ;
   return unite(leftRes, rightRes);
int left(int i) {
  int lvl = __lg(i);
return (i & ((1 << lvl) - 1)) * (1 << (h - lvl));</pre>
int right(int i) {
  int lvl = __lg(i);
  return ((i & ((1 << lvl) - 1)) + 1) * (1 << (h -</pre>
   lv1));
// l \in [0; n) && ok(get(l, l), l);
// returns last r: ok(get(1, r), r)
template<typename C>
int lastTrue(int 1, C ok) {
   T cur = neutral;
   1 += n;
   do {
      1 >>= __builtin_ctz(l);
T with1 = unite(cur, data[1]);
      if (ok(with1, right(1))) {
         cur = with1;
         ++1;
      } else {
         while (1 < n) {
            T with2 = unite(cur, data[2 * 1]);
            if (ok(with2, right(2 * 1))) {
              cur = with2;
1 = 2 * 1 + 1;
            } else {
 1 = 2 * 1;
           }
        return 1 - n;
   } while (1 & (1 - 1));
  return n;
// r \in [0; n) && ok(get(r, r), r);
// returns first 1: ok(get(1, r), 1)
template<typename C>
int firstTrue(int r, C ok) {
  T cur = neutral;
   r += n;
   while (r & (r - 1)) {
   r >>= __builtin_ctz(r);
   T with1 = unite(data[--r], cur);
      if (ok(with1, left(r))) {
        cur = with1;
      } else {
         while (r < n) {
           T with2 = unite(data[2 * r + 1], cur);
if (ok(with2, left(2 * r + 1))) {
              cur = with2;
r = 2 * r;
            } else {
   r = 2 * r + 1;
        return r - n + 1;
```

```
}
  return 0;
}
};
//64190d
```

### 5.3 Массовое дерево отрезков

```
template<typename T, typename M, typename Ud,
    typename Um, typename A>
struct MassSegmentTree {
  int h, n;
  T zd;
 M zm;
  vector<T> data;
  vector<M> mod;
 Ud ud; // T (T, T)
Um um; // M (M, M);
  A a; // T (T, M, int); last argument is the length
    of current segment (could be used for range += and sum counting, for instance)
  template<typename I>
 MassSegmentTree(int sz, T zd, M zm, Ud ud, Um um, A
     a, \tilde{I} init): h(\underline{I}g(sz) + 1), n(1 << h), zm(zm), zd(zd), data(2 * n, zd), mod(n, zm), ud(ud), um(
    um), a(a) {
    for (int i = 0; i < sz; ++i) data[i + n] = init(i</pre>
    for (int i = n - 1; i > 0; --i) data[i] = ud(data
    [2 * i], data[2 * i + 1]);
 void push(int i) {
    if (mod[i] == zm) return;
    apply(2 * i, mod[i]);
apply(2 * i + 1, mod[i]);
   mod[\bar{i}] = zm;
 // is used only for apply
int length(int i) { return 1 << (h - __lg(i)); }</pre>
  // used only for descent
 int left(int i) {
  int lvl = __lg(i);
    return (i \frac{1}{6} (1 << lvl) - 1)) * (1 << (h - lvl));
  // used only for descent
 int right(int i) {
    return left(i) + length(i);
 template<typename S>
 void apply(int i, S x) {
    data[i] = a(data[i], x, length(i));
    if (i < n) mod[i] = um(mod[i], x);
  void update(int i) {
    if (mod[i] != zm) return;
    data[i] = ud(data[2 * i], data[2 * i + 1]);
 template<typename S>
  void update(int 1, int r, S x) { // [1; r)
    1 += n, r += n;
    for (int shift = h; shift > 0; --shift) {
      push(1 >> shift);
      push((r - 1) >> shift);
    for (int lf = 1, rg = r; lf < rg; lf /= 2, rg /=
    2) {
      if (lf & 1) apply(lf++, x);
      if (rg & 1) apply(--rg, x);
    for (int shift = 1; shift <= h; ++shift) {</pre>
      update(1 >> shift);
      update((r - 1) >> shift);
  T get(int 1, int r) { // [1; r)
```

```
1 += n, r += n;
    for (int shift = h; shift > 0; --shift) {
      push(1 >> shift);
      push((r - 1) >> shift);
    T leftRes = zd, rightRes = zd;
for (; 1 < r; 1 /= 2, r /= 2) {
   if (1 & 1) leftRes = ud(leftRes, data[1++]);</pre>
       if (r & 1) rightRes = ud(data[--r], rightRes);
    return ud(leftRes, rightRes);
  // l \in [0; n) && ok(get(1, 1), 1);
  // returns last r: ok(get(l, r), r)
  template<typename C>
  int lastTrue(int 1, C ok) {
    1 += n;
    for (int shift = h; shift > 0; --shift) push(1 >>
     shift);
    T cur = zd;
    do {
      1 >>= __builtin_ctz(l);
T with = ud(cur, data[l]);
       if (ok(with, right(1))) {
         cur = with;
         ++1;
       } else {
         while (1 < n) {
           push(1);
1 = 2 * 1;
           with = ud(cur, data[1]);
           if (ok(with, left(1 + 1))) {
              cur = with;
              ++1;
           }
         return 1 - n;
    } while (1 & (1 - 1));
    return n;
  }
  // r \in [0; n) && ok(get(r, r), r);
  // returns first 1: ok(get(1, r), 1)
  template<typename C>
  int firstTrue(int r, C ok) {
    r += n;
    for (int shift = h; shift > 0; --shift) push((r -
     1) >> shift);
    auto cur = zd;
    while (r & (r - 1)) {
   r >>= __builtin_ctz(r);
   T with = ud(data[--r], cur);
       if (ok(with, left(r))) {
         cur = with;
       } else {
         while (r < n) {
           push(r);
           r = 2 * r;
with = ud(data[r + 1], cur);
           if (ok(with, left(r + 1))) {
              cur = with;
            } else {
              ++r;
           }
         return r - n + 1;
    return 0;
//fc0cde
```

### 5.4 Битовый бор

```
template<unsigned int sz,typename T=int>
struct binarytrie{
  using Bit=typename conditional<sz<=32,unsigned int,
    unsigned long long>::type;
  struct node{
    T cnt;
    array<int,2>nxt;
    node():cnt(0),nxt({-1,-1}){}
};
  vector<node>v;
binarytrie(){v.emplace_back();}
  void insert(Bit x){add(x,1);}
```

```
void erase(Bit x){add(x,-1);}
void add(Bit x,T k)
  assert(0 <= x&&(x>> sz) == 0);
  int p=0;
  v[p].cnt+=k;
  for(int i=sz;i--;)
  {
    int j=x>>i&1;
    if(v[p].nxt[j]==-1)
    {
      v[p].nxt[j]=v.size();
      v.emplace back();
    p=v[p].nxt[i];
    v[p].cnt+=k;
T count(Bit x,Bit xor val=0)const//[0,x)
  assert(0<=xor_val&&(xor_val>>sz)==0);
if(x<0)return 0;</pre>
  else if(x>>sz)return v[0].cnt;
  T ret=0;
  int p=0;
  for(int i=sz;i--;)
    int j=x>>i&1,k=xor_val>>i&1;
    if(j==0)p=v[p].nxt[k];
    else
    {
      if(v[p].nxt[k] \ge 0) ret += v[v[p].nxt[k]].cnt;
      p=v[p].nxt[!k];
    if(p==-1)break;
  return ret;
Bit max(Bit xor_val=0)const
  assert(0 \le xor_val \& (xor_val >> sz) == 0);
  int p=0;
  Bit ret=0;
  if(v[p].cnt==0)return ret;
  for(int i=sz;i--;)
    ret<<=1;
    int k=xor_val>>i&1;
    if(v[p].nxt[!k] \ge 0\&v[v[p].nxt[!k]].cnt > 0)
      p=v[p].nxt[!k];
      ret|=1;
    else p=v[p].nxt[k];
  return ret;
Bit min(Bit xor_val=0)const
  assert(0<=xor_val&&(xor_val>>sz)==0);
  int p=0;
  Bit ret=0;
  for(int i=sz;i--;)
    ret<<=1;
    int k=xor_val>>i&1;
    if(v[p].nxt[k] >= 0 & v[v[p].nxt[k]].cnt > 0)p=v[p].
  nxt[k];
    else
    {
      p=v[p].nxt[!k];
      ret|=1;
  return ret;
Bit find_by_order(T ord,Bit xor_val=0)const
  assert(0<=xor_val&&(xor_val>>sz)==0);
  assert(0<=ord&&ord<v[0].cnt);
  int p=0;
  Bit ret=0;
  for(int i=sz;i--;)
  {
    ret<<=1;
    int k=xor_val>>i&1;
    if(v[p].nxt[k]>=0)
```

if(ord>=v[v[p].nxt[k]].cnt)

#### 5.5 Ordered set

### 5.6 Динамический битсет

```
#include <tr2/dynamic_bitset>
using namespace tr2;
using bs=dynamic_bitset<>;
//26f8b6
```

### 5.7 Convex hull trick

```
int div_up(int a, int b) { return a/b+((a^b)>0&&a%b);
     } // divide a by b rounded up
const int LQ = ..., RQ = ...; //leftmost query,
    rightmost query
int in(ii L, int x) {
  return L.x * x + L.y;
struct Hull {
vector <pair <int, int> > lines;
vector <int> borders;
void push(ii L) {
  while (lines.size() && in(L,borders.back()) < in(</pre>
    lines.back(),borders.back())) {
    lines.pop_back();
    borders.pop_back();
  if (lines.empty()) {
    lines = \{L\};
    borders = {LQ};
  else if (lines.back().x > L.x) {
    int x = div_up(L.y - lines.back().y, lines.back()
     .x - L.x);
    if (x \le RQ) {
      lines.app(L);
      borders.app(x);
  }
Hull (){}
Hull (vector <ii> a) {
  auto comp = [\&] (ii u, ii v) {
    return u.x > v.x \mid \mid (u.x == v.x \&\& u.y < v.y);
  sort(all(a), comp);
  for (auto L : a) {
   push(L);
int get(int x) {
  int pos = upper_bound(all(borders), x) - borders.
   begin();
  assert(pos>0);
  pos--;
  return in(lines[pos],x);
```

```
}
};
//04555a
```

### 5.8 Центроиды

```
vector < int > sz(n), lvl(n, -1);
auto dfs = [&](auto dfs, int cur, int prev) -> int {
  if (lvl[cur] != -1) return 0;
  sz[cur] = 1;
  for (auto [nxt, w] : g[cur]) {
  if (nxt != prev) sz[cur] += dfs(dfs, nxt, cur);
  return sz[cur];
auto find = [&](auto find, int cur, int prev, int tot
    ) -> int {
  int bch = -1, bsz = 0;
  for (auto [nxt, w] : g[cur]) {
  if (nxt == prev || lvl[nxt] != -1) continue;
    if (sz[nxt] > bsz) {
      bch = nxt;
      bsz = sz[nxt];
    }
  if (bsz + bsz <= tot) return cur;
  return find(find, bch, cur, tot);
dfs(dfs, 0, 0);
auto c = find(find, 0, 0, sz[0]);
vector<pair<int, int>> stack{{c, 0}};
while (!stack.empty()) {
  auto [centroid, 1] = stack.back();
  stack.pop_back();
  lvl[centroid] = 1;
  for (auto [nxt, w] : g[centroid]) {
  if (lvl[nxt] != -1) continue;
    dfs(dfs, nxt, centroid);
    int new_centroid = find(find, nxt, centroid, sz[
    nxtl):
    stack.push_back({new_centroid, lvl[centroid] +
    1});
  }
//0e1e52
```

### 5.9 Дерево Ли Чао

```
struct Line{
  int a, b;
Line(){}
  Line (int a, int b) : a(a), b(b) {}
int get(int x) { return a + b * x;}
struct Lichao {
  int n;
  vector <int> x;
vector <Line> t;
  Lichao(){}
  Lichao (int n, vector\langle int \rangle x) : n(n), t(n << 2,
     Line(inf, 0)), x(x) {}
  void put(int v, int l, int r, Line L) {
     if (1 + 1 == r)
       if (L.get(x[1]) < t[v].get(x[1])) {
          t[v] = L;
       }
       return;
     int m = (1 + r) / 2;
     if (L.get(x[m]) < t[v].get(x[m])) {
       swap(L, t[v]);
     if (L.b > t[v].b) {
  put(2 * v + 1, 1, m, L);
     else {
       put(2 * v + 2, m, r, L);
  int get(int v, int 1, int r, int i) {
  if (1 + 1 == r) {
       return t[v].get(x[1]);
     int m = (1 + r) / 2;
     int ans = t[v].get(x[i]);
```

```
if (i < m) {
    ans = min(ans, get(2 * v + 1, 1, m, i));
} else {
    ans = min(ans, get(2 * v + 2, m, r, i));}
    return ans;
}

void put(Line L) {
    put(0, 0, n, L);
}

int get(int i) {
    return get(0, 0, n, i);
}
};
//99f5fa</pre>
```

```
5.10 Min-Kinetic Segment Tree
I guess the source is https://koosaga.com/307
using lint = long long;
const lint inf = 4e18;
const int MAXT = 4100000;
using pi = array<lint, 2>;
struct line {
  lint A, B;
  int idx;
  lint eval(lint x) { return A * x + B; }
  // returns the x-intercept of intersection "
     strictly" larger than T
   lint cross_after(line &x, lint T) {
     if (x.A == A) {
       return inf;
     lint up = x.B - B;
     lint dn = A - x.A;
     if (dn < 0) {
       dn *= -1;
       up *= -1;
     lint incep = (up \le 0 ? -((-up) / dn) : (up + dn)
     - 1) / dn);
     if (incep > T)
       return incep;
     return inf;
struct kst { // min kinetic segment tree
  line tree[MAXT];
  lint melt[MAXT], T;
  pi lazy[MAXT];
  int n;
  bool cmp(line &a, line &b) {
     lint 1 = a.eval(T), r = b.eval(T);
if (1 != r)
     return 1 > r;
return a.A > b.A;
  }
  void pull(int p) {
     tree[p] = cmp(tree[2 * p], tree[2 * p + 1]) ?
tree[2 * p + 1] : tree[2 * p];
melt[p] = min({melt[2 * p], melt[2 * p + 1], tree
[2 * p].cross_after(tree[2 * p + 1], 0)});
  void init(int s, int e, int p, vector<line> &1) {
     if (s == e) {
       tree[p] = l[s];
        melt[p] = inf;
        lazy[p] = {0, 0};
     lazy[p] = {0, 0};
     int m = (s + e) / 2;
init(s, m, 2 * p, 1);
init(m + 1, e, 2 * p + 1, 1);
     pull(p);
  void lazydown(int p) {
  for (int i = 2 * p; i < 2 * p + 2; i++) {
    lazy[i][0] += lazy[p][0];</pre>
```

lazy[i][1] += lazy[p][1];

```
tree[i].B += lazy[p][0] * tree[i].A + lazy[p
    ][1];
      melt[i] -= lazy[p][0];
    lazy[p][0] = lazy[p][1] = 0;
 void propagate(int p) {
  if (melt[p] > 0)
      return;
    lazydown(p);
    propagate(2 * p);
    propagate(2 * p + 1);
    pull(p);
 lint query(int s, int e, int ps, int pe, int p = 1)
    if (e < ps || pe < s)</pre>
      return inf;
    if (s <= ps && pe <= e)
      return tree[p].eval(0);
    int pm = (ps + pe) / 2;
    lazydown(p);
    return min(query(s, e, ps, pm, 2 * p), query(s, e
, pm + 1, pe, 2 * p + 1));
 void heaten(int s, int e, int ps, int pe, int p,
    lint v) {
if (e < ps || pe < s)
      return;
    if (s <= ps && pe <= e) {</pre>
      lazy[p][0] += v;
       tree[p].B += v * tree[p].A;
      melt[p] -= v;
      propagate(p);
      return;
    lazydown(p);
    int pm = (ps + pe) / 2;
heaten(s, e, ps, pm, 2 * p, v);
heaten(s, e, pm + 1, pe, 2 * p + 1, v);
    pull(p);
  void add(int s, int e, int ps, int pe, int p, lint
    if (e < ps || pe < s)</pre>
      return;
    if (s <= ps && pe <= e) {</pre>
      lazy[p][1] += v;
      tree[p].B += v;
    lazydown(p);
    int pm = (ps + pe) / 2;
add(s, e, ps, pm, 2 * p, v);
add(s, e, pm + 1, pe, 2 * p + 1, v);
    pull(p);
  void init(vector<line> &1, lint _T) {
    n = l.size();
    T = _T;
    init(0, n - 1, 1, 1);
 }
//66f9a9
```

#### 5.11 Декартово дерево

#### 5.11.1 Декартово дерево по явному ключу. Multiset

```
mt19937 rng(0);
using ptr = int32_t;

struct vertex {
   ptr lf = 0, rg = 0;
   int32_t heap = rng(), rnd = rng(), sz = 1;
   int val, sum = 0;

   vertex(int x = 0) : val(x), sum(x) {}
};

vector <vertex> mem;
```

```
ptr new_vertex(int x) {
  mem.app(vertex(x));
  return (int)mem.size()-1;
ptr update(ptr v) {
  mem[v].sz = mem[mem[v].lf].sz + 1 + mem[mem[v].rg].
  mem[v].sum = mem[mem[v].lf].sum + mem[v].val + mem[
    mem[v].rg].sum;
  return v;
ptr merge(ptr 1, ptr r) {
   if (!1 || !r) return 1 ^ r;
   if (mem[1].heap > mem[r].heap) {
    mem[1].rg = merge(mem[1].rg, r);
     return update(1);
  } else {
    mem[r].lf = merge(1, mem[r].lf);
     return update(r);
}
pair<ptr, ptr> splitkey(ptr v, int x, int32_t rnd) {
   if (!v) return {v, v};
  if (pair{mem[v].val, mem[v].rnd} < pair{x, rnd}) {
  auto [lf, rg] = splitkey(mem[v].rg, x, rnd);
  mem[v].rg = lf;
  return (red);</pre>
     return {update(v), rg};
  } else {
    auto [lf, rg] = splitkey(mem[v].lf, x, rnd);
mem[v].lf = rg;
return {lf, update(v)};
}
void insert(ptr &a, ptr b) {
  if (!a) {
    a = b;
     return;
  if (mem[a].heap > mem[b].heap) {
     if (pair{mem[a].val, mem[a].rnd} < pair{mem[b].</pre>
     val, mem[b].rnd}) {
       insert(mem[a].rg, b);
       else ·
       insert(mem[a].lf, b);
     update(a);
  } else {
     auto [lf, rg] = splitkey(a, mem[b].val, mem[b].
     mem[b].lf = lf;
     mem[b].rg = rg;
     a = update(b);
void join(ptr &a, ptr b) {
  auto dfs = [&](auto dfs, ptr b) -> void {
     if (!b) return;
    ptr lf = mem[b].lf;
    ptr rg = mem[b].rg;
    mem[b].lf = mem[b].rg = 0;
     insert(a, update(b));
dfs(dfs, lf);
     dfs(dfs, rg);
  dfs(dfs, b);
pair <ptr, ptr> splitsz(ptr v, int k) {
  if (!v) return {v, v};
if (k <= mem[mem[v].lf].sz) {</pre>
    auto [1, r] = splitsz(mem[v].lf, k);
mem[v].lf = r;
     return {1, update(v)};
    else {
     auto [1, r] = splitsz(mem[v].rg, k - mem[mem[v].
lf].sz - 1);
    mem[v].rg = 1;
     return {update(v), r};
  }
}
int32_t main() {
  mem = {vertex()};
```

```
mem[0].sz = 0;
}
//54a637
```

### 6 Строковые алгоритмы

### 6.1 Префикс-функция

```
vector<int> prefix_function(string s) {
   int n = s.size();
   vector<int> p(n);
   for (int i = 1; i < n; ++i) {
      p[i] = p[i - 1];
      while (p[i] && s[p[i]] != s[i]) p[i] = p[p[i] -
      1];
      p[i] += s[i] == s[p[i]];
   }
   return p;
}
//91103c</pre>
```

### **6.2** Z-функция

```
vector<int> z_function (string s) { // z[i] - lcp of
    s and s[i:]
int n = s.size();
vector<int> z(n);
for (int i=1, l=0, r=0; i<n; ++i) {
    if (i <= r) z[i] = min(r-i+1, z[i-l]);
    while (i+z[i] < n && s[z[i]] == s[i+z[i]]) ++z[i
    ];
    if (i+z[i]-1 > r) {
        l = i, r = i+z[i]-1;
     }
}
return z;
}
//48cccd
```

### 6.3 Алгоритм Манакера

```
vector<int> manacher(const string &s, int even) {
  int 1 = 0, r = 0, n = s.size();
  vector<int> man(n, 0);
  for (int i = 1; i < n; i++) {
  int j = i - even;</pre>
    if (j <= r) {</pre>
      man[i] = min(r - j, man[l + r - j]);
    while (j + man[i] + 1 < n && i - man[i] > 0 && s[
j + man[i] + 1] == s[i - man[i] - 1]) {
      man[i]++;
    if (j + man[i] > r) {
      l = i - man[i];
       r = j + man[i];
    }
  return man;
// abacaba : odd : (0 1 0 3 0 1 0); even : (0 0 0 0 0
      0 0)
// abbaa : odd : (0 0 0 0 0); even : (0 0 2 0 1)
bool pal(int from, int len) {
   if (len == 0) {
         return true;
    int m = len/2;
    if (len & 1) {
         return odd[from + m] >= m;
    else {
         return even[from + m] >= m;
//8a64d6
```

### 6.4 Суфмассив

```
Переработанный китайский суффмассив
```

```
const int inf = 1e9;
struct rmq {
  int n;
  vector<int> a;
```

```
void build(const vector<int> &x) {
     assert(x.size() == n);
     for (int i = 0; i < n; ++i) a[n + i] = x[i];
for (int i = n - 1; i > 0; --i) a[i] = min(a[2 *
     i], a[2 * i + 1]);
  rmq(int n) : n(n), a(2 * n, inf) {}
  void put(int i, int x) {
     a[i + n] = min(a[i + n], x);

for (i = (i + n) / 2; i > 0; i /= 2) {

a[i] = min(a[i * 2], a[i * 2 + 1]);
  int getMin(int 1, int r) { //[1;r)
     assert(1 < r);
     int res = inf;
     for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {
       if (1 & 1) res = min(res, a[1++]);
if (r & 1) res = min(res, a[--r]);
     return res:
  }
};
template <typename T>
vector <int> SA(const T &a) {
  int m = *max_element(all(a)) + 1, n = a.size();
vector <int> sa(n), nsa(n), pre(max(n, m)), x(a.
     begin(), a.end()), y(n);
  for (int e : x) pre[e]++;
for (int i = 1; i < m; ++i) pre[i] += pre[i - 1];
for (int i = 0; i < n; ++i) sa[--pre[x[i]]]=i;</pre>
  int dif = 1;
  y[sa.front()]=0;
for (int i = 1; i < n; ++i) {
  dif += x[sa[i]]!=x[sa[i-1]];</pre>
     y[sa[i]] = dif - 1;
  x = y;
for (int h = 1; dif < n; h *= 2) {
     fill(all(pre), 0);
     for (int e : x) pre[e]++;
     for (int i = 1; i < dif; ++i) pre[i] += pre[i -</pre>
     for (int t = n; t--; ) {
        int i = sa[t];
        if (i>=h) {
          nsa[--pre[x[i-h]]]=i-h;
        else if (i + 1 != h) {
          nsa[--pre[x[i-h+n+1]]]=i-h+n+1;
     nsa[--pre[x[n - h]]]=n-h;
     sa = nsa;
     auto getr = [&] (int i) {
       if (i + h < n) {
         return x[i + h];
       else {
         return x[i + h - n - 1];
     dif = 1;
     y[sa.front()]=0;
     for (int i = 1; i < n; ++i) {
        if (x[sa[i]]!=x[sa[i-1]] || sa[i-1]+h==n) {
        else {
          dif += getr(sa[i]) != getr(sa[i-1]);
       y[sa[i]]=dif-1;
     x = y;
  return sa;
template <typename T>
struct suar {
  vector <int> sa, lcp, pos; rmq t;
  suar (const T \&a) : t((int)a.size() - 1) {
     sa = SA(a);
     int n = (int)a.size(), k = 0;
     lcp.resize(n - 1);
     pos.resize(n);
     for (int i = 0; i < n; ++i) pos[sa[i]] = i;
for (int i = 0; i < n; ++i) {
        if (pos[i]+1<n) {</pre>
```

```
int j = sa[pos[i]+1];
   while (i+k<n&&j+k<n&&a[i+k]==a[j+k])k++;
   lcp[pos[i]]=k;
}
if (k) {
   k--;
}
t.build(lcp);
}
int getLcp(int i, int j) {
   i = pos[i]; j = pos[j];
   if (j < i) {
      swap(i, j);
   }
   if (i == j) {
      return inf;
   }
   else {
      return t.getMin(i, j);
   }
};
//6327c9</pre>
```

### 6.5 Алгоритм Ахо — Корасик

```
const int alpha = 26;
const char a = 'a';
struct node{
 int next[alpha] = {}, link[alpha] = {};
  int suf = 0;
  int visited = 0, ans = 0;
  int bad = 0; // any term is reachable by suf links
  vector<int> term;
  node() {
    fill(next, next + alpha, -1);
}:
vector<node> mem;
int get_next_or_create(int nd, char c) {
  if (mem[nd].next[c - a] == -1) { mem[nd].next[c - a
    ] = mem.size(); mem.emplace_back(); }
  return mem[nd].next[c - a];
void build(vector<string> t) {
  mem.reserve(1e6 + 100); mem.clear();
  mem.emplace_back();
  // Oth element is nullptr, 1st is the root
  for (int j = 0; j < t.size(); ++j) {</pre>
    int cur = 0;
    for (char c : t[j]) cur = get_next_or_create(cur,
     c);
    mem[cur].term.push_back(j);
  vector<int> bfs order;
  queue<int> bfs;
    node &root = mem[0];
    root.suf = 0;
    for (char c = a; c < a + alpha; ++c) {
    root.link[c - a] = (root.next[c - a] == -1 ? 0
: root.next[c - a]);
    bfs.push(0);
  while (!bfs.empty()) {
    int cur_idx = bfs.front();
    bfs.pop();
    node &cur = mem[cur_idx];
    cur.bad = cur.term.size() > 0 || mem[cur.suf].bad
    bfs_order.push_back(cur_idx);
    for (char c = a; c < a + alpha; ++c) {
      int nxt_idx = cur.next[c - a];
if (nxt_idx == -1) continue;
      node &nxt = mem[nxt_idx];
      nxt.suf = (cur_idx ? mem[cur.suf].link[c - a] :
      for (char c = a; c < a + alpha; ++c) {
        nxt.link[c - a] = (nxt.next[c - a] == -1 ?
    mem[nxt.suf].link[c - a] : nxt.next[c - a]);
```

```
bfs.push(nxt_idx);
    }
} // do something
} //be16ed
```

### 6.6 Дерево палиндромов

```
const int alpha = 26;
const char a = 'a';
struct palindromic{
  int n;
  vector<int> p, suf{0, 0}, len{-1, 0};
//d[u] is a difference of lengths of u and suf[u],
    go is jump by chain constant d
  vector<array<int, alpha>> to{{}, {}};
  int sz = 2;
  palindromic(const string &s) : n(s.size()), p(n +
    1, 0) {
    suf.reserve(n);
    len.reserve(n);
     for (int i = 0; i < n; ++i) {</pre>
       auto check = [&] (int 1) {
        return i > 1 && s[i] == s[i - 1 - 1];
       int par = p[i];
       while (!check(len[par])) {
        par = suf[par];
       if (to[par][s[i]-a] == 0) {
         p[i+1]=to[par][s[i]-a]=sz++;
         to.emplace_back();
         len.emplace_back(len[par]+2);
         if (par == \overline{0}) {
           suf.emplace_back(1);
         else {
           do {
             par = suf[par];
            } while (!check(len[par]));
           suf.emplace_back(to[par][s[i]-a]);
         }
       else {
        p[i+1]=to[par][s[i]-a];
       }
    }
  int partition() {
    vector <int> d(sz), up(sz, 1); //d[1] = 0 sic
for (int i = 2; i < sz; ++i) {
  d[i] = len[i] - len[suf[i]];</pre>
       if (d[i] == d[suf[i]]) {
  up[i] = up[suf[i]];
       else {
         up[i] = suf[i];
    vector <int> dp(n + 1, n), last(sz);
dp[0] = 0;
     for (int i = 1; i <= n; ++i) {
       int u = p[i];
       while (u != 1) {
         if (suf[u] == up[u]) {
           last[u] = dp[i - len[u]];
         else {
           last[u] = min(last[suf[u]], dp[i - len[up[u
     ]] - d[u]]);
         dp[i] = min(dp[i], last[u] + 1);
         u = up[u];
    return dp.back();
  }
//acac02
```

### 7 Потоки

### 7.1 Алгоритм Диница

```
#define pb push_back
struct Dinic{
```

```
struct edge{
  int to, flow, cap;
const static int N = 555; //count of vertices
vector<edge> e;
vector<int> g[N + 7];
int dp[N + 7];
int ptr[N + 7];
void clear(){
  for (int i = 0; i < N + 7; i++) g[i].clear();
  e.clear();
void addEdge(int a, int b, int cap){
  g[a].pb(e.size());
  e.pb({b, 0, cap});
  g[b].pb(e.size());
  e.pb({a, 0, 0});
int minFlow, start, finish;
bool bfs(){
  for (int i = 0; i < N; i++) dp[i] = -1;</pre>
  dp[start] = 0;
  vector<int> st;
  int uk = 0;
  st.pb(start);
  while(uk < st.size()){
  int v = st[uk++];
  for (int to : g[v]){
    auto ed = e[to];</pre>
       if (ed.cap - ed.flow >= minFlow && dp[ed.to] ==
     -1) {
         dp[ed.to] = dp[v] + 1;
         st.pb(ed.to);
       }
    }
  return dp[finish] != -1;
int dfs(int v, int flow){
  if (v == finish) return flow;
  for (; ptr[v] < g[v].size(); ptr[v]++){</pre>
    int to = g[v][ptr[v]];
    edge ed = e[to];
    if (ed.cap - ed.flow >= minFlow && dp[ed.to] ==
    dp[v] + 1){
       int add = dfs(ed.to, min(flow, ed.cap - ed.flow
      if (add) {
        e[to].flow += add;
e[to ^ 1].flow -= add;
         return add;
    }
  return 0;
int dinic(int start, int finish){
  Dinic::start = start;
  Dinic::finish = finish;
  int flow = 0;
  for (minFlow = (1 << 30); minFlow; minFlow >>= 1){
    while(bfs()){
       for (int i = 0; i < N; i++) ptr[i] = 0;</pre>
       while(int now = dfs(start, (int)2e9 + 7)) flow
    += now;
    }
  }
  return flow;
} dinic;
//15c079
7.2 Mincost k-flow
```

```
struct edge {
  int next, capacity, cost, flow = 0;
  edge() = default;
  edge(int next, int capacity, int cost) : next(next)
  , capacity(capacity), cost(cost) {}
```

```
int rem() const { return capacity - flow; }
  int operator+=(int f) { return flow += f; }
  int operator-=(int f) { return flow -= f; }
g[from].push_back(e.size());
  e.emplace_back(next, capacity, cost);
  g[next].push_back(e.size());
  e.emplace back(from, 0, -cost);
};
/* in case of undirected graph use this:
addEdge(u, v, capacity, cost);
addEdge(v, u, capacity, cost);
vector<1l> phi(n, 0);
auto fordBellman = [&](int s, int t) {
  phi.assign(n, 0);
for (int iter = 0; iter < n; ++iter) {</pre>
    bool changed = false;
for (int u = 0; u < n; ++u) {</pre>
      for (auto index : g[u]) {
   auto edge = e[index];
         if (edge.rem() > 0 && phi[edge.next] > phi[u]
     + edge.cost) {
          phi[edge.next] = phi[u] + edge.cost;
           changed = true;
      }
    if (!changed) break;
  }
};
fordBellman(s, t);
// now shortest path using dijkstra with potentials
vector<ll> dist:
vector<int> from;
vector<bool> cnt;
auto dijkstra = [&](int s, int t) {
  dist.assign(n, 1e18);
  from.assign(n, -1);
  cnt.assign(n, false);
  dist[s] = 0;
set <pair <int,</pre>
                   int> > se;
  se.insert({0, s});
  while ((int)(se.size())) {
    int cur = se.begin()->y;
    se.erase(se.begin());
    cnt[cur] = true;
for (int index : g[cur]) {
      auto &edge = e[index];
       if (edge.rem() == 0) continue;
      ll weight = edge.cost + phi[cur] - phi[edge.
    nextl:
      if (dist[edge.next] > dist[cur] + weight) {
         se.erase({dist[edge.next], edge.next});
         dist[edge.next] = dist[cur] + weight;
         se.insert({dist[edge.next], edge.next});
         from[edge.next] = cur;
    }
  if (dist[t] == (11) 1e18) return -1LL;
  11 cost = 0;
  for (int p = t; p != s; p = from[p]) {
    for (auto index : g[from[p]]) {
      auto &edge = e[index];
      11 weight = edge.cost + phi[from[p]] - phi[edge
    .nextl;
      if (edge.rem() > 0 && edge.next == p && dist[
    edge.next] == dist[from[p]] + weight) {
         edge += 1;
         e[index ^ 1] -= 1;
        cost += edge.cost;
        break:
      }
    }
  for (int i = 0; i < n; ++i) {
    phi[i] += dist[i];
  return cost;
11 cost = 0;
for (int flow = 0; flow < k; ++flow) {</pre>
  ll a = dijkstra(s, t);
```

```
if (a == -1) {
    cout << "-1\n";
    return;
  cost += a;
// now recover answer
auto findPath = [&](int s, int t) {
  vector<int> ans;
  int cur = s;
  while (cur != t) {
    for (auto index : g[cur]) {
      auto &edge = e[index];
      if (edge.flow <= 0) continue;</pre>
      edge -= 1;
e[index ^ 1] += 1;
      ans.push back(index / 4);
// index / 4 because each edge has 4 copies
    cur = edge.next;
      break;
    }
  }
  return ans;
};
for (int flow = 0; flow < k; ++flow) {</pre>
  auto p = findPath(s, t);
cout << p.size() << ' ';</pre>
  for (int x : p) cout << x + 1 << ' ';
cout << '\n';</pre>
.
//94b9cb
template <typename T, typename C>
class mcmf {
  public:
  static constexpr T eps = (T) 1e-9;
  struct edge {
    int from;
    int to;
    T c;
T f;
    C cost;
  };
  vector< vector<int> > q:
  vector<edge> edges;
  vector<C> d;
  vector<int> q;
vector<bool> in_queue;
  vector<int> pe;
  int n;
  int st, fin;
  T flow:
  C cost;
  mcmf(int _n, int _st, int _fin) : n(_n), st(_st),
    fin(_fin) {
    assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
    g.resize(n);
    d.resize(n);
    in_queue.resize(n);
    pe.resize(n);
    flow = 0;
    cost = 0;
  void clear_flow() {
    for (const edge &e : edges) {
      e.f = 0;
    flow = 0;
  void add(int from, int to, T forward_cap, T
    backward_cap, C cost) {
    assert(0 \le from \&\& from < n \&\& 0 \le to \&\& to < n
    g[from].push_back((int) edges.size());
    edges.push_back({from, to, forward_cap, 0, cost})
    g[to].push_back((int) edges.size());
    edges.push_back({to, from, backward_cap, 0, -cost
    });
  }
  bool expath() {
    fill(d.begin(), d.end(), numeric_limits<C>::max()
```

```
);
    q.clear();
    q.push_back(st);
    d[st] = 0;
    in_queue[st] = true;
    int beg = 0;
    bool found = false;
    while (beg < (int) q.size()) {</pre>
      int i = q[beg++];
if (i == fin) {
        found = true;
      in_queue[i] = false;
      for (int id : g[i]) {
        const edge &e = edges[id];
        if (e.c - e.f > eps && d[i] + e.cost < d[e.to
    1) {
          d[e.to] = d[i] + e.cost;
          pe[e.to] = id;
           if (!in_queue[e.to]) {
             q.push_back(e.to);
             in_queue[e.to] = true;
        }
      }
    if (found) {
      T push = numeric_limits<T>::max();
int v = fin;
      while (v != st) {
        const edge &e = edges[pe[v]];
        push = min(push, e.c - e.f);
        v = e.from;
      v = fin;
      while (v != st) {
  edge &e = edges[pe[v]];
        e.f += push;
        edge &back = edges[pe[v] ^ 1];
        back.f -= push;
        v = e.from;
      flow += push;
      cost += push * d[fin];
    return found;
 pair<T, C> max_flow_min_cost() {
    while (expath()) {
    return make_pair(flow, cost);
//b7bbb2
```

## 8 Алгоритм Гаусса

#### **8.1** Решение Av = b

```
optional<vector<int> > gauss(vector<vector<int> > A,
    vector<int> b) ///returns v such that Av=b
{
    int n=A.size();assert(b.size()==n);int m=A[0].
    size();
    for(int &x:b) {x%=p;x+=p;x%=p;}
    for(int i=0;i<n;++i) {for(int &x:A[i]) {x%=p;x+=p</pre>
    ;x%=p;}}
    int bi=0;
    for(int i=0;i<n;++i)</pre>
        if(bi==m) break;
        for(int j=i;j<n;++j)</pre>
             if(A[j][bi])
                 if(j!=i) {swap(A[i],A[j]);swap(b[i],b
    [j]);}
                 break:
        if(A[i][bi])
             int o=inv(A[i][bi]);
             for(int j=i+1;j<n;++j)</pre>
                 int we=(A[j][bi]*o)%p;
```

```
b[j]=we*b[i];b[j]%=p;if(b[j]<0) b[j]
    ]+=p;
                 for(int k=bi; k<m;++k)</pre>
                 {
                     A[j][k]=we*A[i][k];A[j][k]%=p;if
    (A[j][k]<0) A[j][k]+=p;
             }
        }
        else
        {
             ++bi; --i; continue;
    vector<int> v(m);
    for(int i=n-1;i>=0;--i)
        int bi=0;
        while(bi<m && !A[i][bi]) {++bi;}</pre>
        if(bi==m)
        {
             if(b[i]) {return nullopt;}
            else {continue;}
        }
        int cur=b[i];
        for(int j=bi+1;j<m;++j)</pre>
            cur-=A[i][j]*v[j];cur%=p;
        v[bi]=cur*inv(A[i][bi]);v[bi]%=p;if(v[bi]<0)
    v[bi]+=p;
    return v;
//bcc622
8.2 Базис Av = 0
vector<vector<int> > gaussbasis(vector<vector<int> >
    A, int m) ///returns basis of Av=0
    int n=A.size();if(n) assert(m==A[0].size());
    for(int i=0;i<n;++i) {for(int &x:A[i]) {x%=p;x+=p</pre>
    ;x%=p;}}
    int bi=0;
    for(int i=0;i<n;++i)</pre>
        if(bi==m) break:
        for(int j=i;j<n;++j)</pre>
             if(A[j][bi])
                 if(j!=i) {swap(A[i],A[j]);}
                 break;
        if(A[i][bi])
             int o=inv(A[i][bi]);
             for(int j=i+1;j<n;++j)</pre>
                 int we=(A[j][bi]*o)%p;
                 for(int k=bi; k<m;++k)</pre>
                     A[j][k]=we*A[i][k];A[j][k]%=p;if
    (A[j][k]<0) A[j][k]+=p;
        else
             ++bi;--i;continue;
    vector<int> indices(m);iota(all(indices),0);
    for(int i=n-1;i>=0;--i)
        int bi=0;
        while(bi<m && !A[i][bi]) {++bi;}</pre>
        if(bi<m)
        {
             indices.erase(find(all(indices),bi));
    vector<vector<int> > v(indices.size(), vector<int</pre>
    >(m,0));
    for(int i=0;i<indices.size();++i)</pre>
```

```
{
    v[i][indices[i]]=1;
}
for(int i=n-1;i>=0;--i)
{
    int bi=0;
    while(bi<m && !A[i][bi]) {++bi;}
    if(bi=m) continue;
    for(int k=0;k<indices.size();++k) {
        int cur=0;
        for(int j=bi+1;j<m;++j)
        {
            cur-=A[i][j]*v[k][j];cur%=p;
        }
        v[k][bi]=cur*inv(A[i][bi]);v[k][bi]%=p;if(v[k][bi]<0) v[k][bi]+=p;
        }
    return v;
}
//ef40f3</pre>
```

### 9 Гамильтоновы путь и цикл

https://codeforces.com/blog/entry/90513, https://codeforces.com/blog/entry/90743.

### 9.1 Link-cut tree

```
namespace LCT {
  vector<vi> ch;
  vi fa, rev;
  void init(int n)
    ch.resize(n + 1);
    fa.resize(n + 1);
    rev.resize(n + 1);
    for (int i = 0; i <= n; i++)</pre>
      ch[i].resize(2),
      ch[i][0] = ch[i][1] = fa[i] = rev[i] = 0;
  bool isr(int a)
    return !(ch[fa[a]][0] == a || ch[fa[a]][1] == a);
  void pushdown(int a)
    if(rev[a])
      rev[ch[a][0]] ^= 1, rev[ch[a][1]] ^= 1;
      swap(ch[a][0], ch[a][1]);
      rev[a] = 0;
  void push(int a)
    if(!isr(a)) push(fa[a]);
   pushdown(a);
  void rotate(int a)
    int f = fa[a], gf = fa[f];
int tp = ch[f][1] == a;
    int son = ch[a][tp ^ 1];
    if(!isr(f))
      ch[gf][ch[gf][1] == f] = a;
    fa[a] = gf;
    ch[f][tp] = son;
    if(son) fa[son] = f;
    ch[a][tp ^ 1] = f, fa[f] = a;
  void splay(int a)
  {
    push(a);
    while(!isr(a))
      int f = fa[a], gf = fa[f];
      if(isr(f)) rotate(a);
        int t1 = ch[gf][1] == f, t2 = ch[f][1] == a;
        if(t1 == t2) rotate(f), rotate(a);
        else rotate(a), rotate(a);
    }
```

```
void access(int a)
    int pr = a;
    splay(a);
    ch[a][1] = 0;
    while(1)
      if(!fa[a]) break;
      int u = fa[a];
      splay(u);
      ch[u][1] = a;
     a = u;
    splay(pr);
 void makeroot(int a)
    access(a);
   rev[a] ^= 1;
 void link(int a, int b)
   makeroot(a);
   fa[a] = b;
 void cut(int a, int b)
   makeroot(a);
    access(b);
   fa[a] = 0, ch[b][0] = 0;
 int fdr(int a)
    access(a);
    while(1)
      pushdown(a);
      if (ch[a][0]) a = ch[a][0];
      else {
        splay(a);
        return a;
    }
 }
//647cca
```

#### 9.2 Undirected case

```
#include <bits/stdc++.h>
using namespace std;
namespace hamil {
  template <typename T> bool chkmax(T &x,T y){return
    x<y?x=y,true:false;}
  template <typename T> bool chkmin(T &x,T y){return
    x>y?x=y,true:false;}
  #define vi vector<int>
  #define pb push_back
  #define mp make_pair
  #define pi pair<int, int>
  #define fi first
  #define se second
  #define 11 long long
  using namespace LCT;
  vector<vi> used;
  unordered_set<int> caneg;
  void cut(int a, int b) {
    LCT::cut(a, b);
    for (int s = 0; s < 2; s++) {
  for (int i = 0; i < used[a].size(); i++)</pre>
         if (used[a][i] == b) {
           used[a].erase(used[a].begin() + i);
       if (used[a].size() == 1) caneg.insert(a);
      swap(a, b);
    }
  void link(int a, int b) {
    LCT::link(a, b);
for (int s = 0; s < 2; s++) {
      used[a].pb(b);
       if (used[a].size() == 2) caneg.erase(a);
      swap(a, b);
  vi work(int n, vector<pi> eg, ll mx_ch = -1) {
    // mx_ch : max number of adding/replacing default is (n + 100) * (n + 50)
```

```
// n : number of vertices. 1-indexed.
    // eg: vector<pair<int, int> > storing all the
    // return a vector<int> consists of all indices
    of vertices on the path. return empty list if
    failed to find one.
    LCT::init(n);
       (mx_ch == -1) mx_ch = 111 * (n + 100) * (n +
    50); /\overline{/}default
    used.resize(n + 1);
    caneg.clear();
    for (int i = 1; i <= n; i++) used[i].clear();</pre>
    vector<vi> edges(n + 1);
    for (auto v : eg)
  edges[v.fi].pb(v.se),
       edges[v.se].pb(v.fi);
    for (int i = 1; i <= n; i++)</pre>
      caneg.insert(i);
    mt19937 x(chrono::steady_clock::now().
    time_since_epoch().count());
    int tot = \overline{0};
    while (mx_ch >= 0) {
// cout << tot << ' ' << mx_ch << endl;</pre>
      vector<pi> eg;
      for (auto v : caneg)
  for (auto s : edges[v])
           eg.pb(mp(v, s));
      shuffle(eg.begin(), eg.end(), x);
if (eg.size() == 0) break;
for (auto v : eg) {
         mx_ch--;
         int a = v.fi, b = v.se;
         if (used[a].size() < used[b].size()) swap(a,</pre>
         if (used[b].size() >= 2) continue;
         if (x() & 1) continue;
         if (LCT::fdr(a) == LCT::fdr(b)) continue;
         if (used[a].size() < 2 && used[b].size() < 2)</pre>
           tot++;
         if (used[a].size() == 2)
           int p = used[a][x() % 2];
           cut(a, p);
         link(a, b);
       if (tot == n - 1) {
         vi cur;
         for (int i = 1; i <= n; i++)</pre>
           if (used[i].size() <= 1) {</pre>
              int pl = i, ls = 0;
while (pl) {
                cur.pb(pl);
                int flag = 0;
                for (auto v : used[pl])
                  if (v != ls) {
                    ls = pl;
pl = v;
                     flag = 1;
                    break;
                if (!flag) break;
             break;
         return cur;
    //failed to find a path
    return vi();
  }
//c35638
```

#### 9.3 Directed case

```
namespace hamil {
  template <typename T> bool chkmax(T &x,T y) {return
  x<y?x=y,true:false;}
template <typename T> bool chkmin(T &x,T y){return
  x>y?x=y,true:false;}
#define vi vector<int>
  #define pb push_back
  #define mp make_pair
  #define pi pair<int, int>
```

```
#define fi first
#define se second
#define ll long long
using namespace LCT;
vi out, in;
vi work(int n, vector<pi> eg, ll mx_ch = -1) {
  // mx_ch : max number of adding/replacing
default is (n + 100) * (n + 50)
  // n : number of vertices. 1-indexed.
  // eg: vector<pair<int, int> > storing all the
  edges.
  // return a vector<int> consists of all indices
  of vertices on the path. return empty list if
  failed to find one.
  out.resize(n + 1), in.resize(n + 1);
  LCT::init(n);
  for (int i = 0; i \le n; i++) in[i] = out[i] = 0;
  if (mx_ch == -1) mx_ch = 111 * (n + 100) * (n +
  50); /\overline{/}default
  vector\langle vi \rangle from(n + 1), to(n + 1);
  for (auto v : eg)
  from[v.fi].pb(v.se),
    to[v.se].pb(v.fi);
  unordered_set<int> canin, canout;
for (int i = 1; i <= n; i++)</pre>
    canin.insert(i)
    canout.insert(i);
  mt19937 x(chrono::steady_clock::now().
  time_since_epoch().count());
int tot = 0;
  while (mx_ch >= 0) {
// cout << tot << ' ' << mx_ch << endl;
vector<pi> eg;
for (auto v : canout)
      for (auto s : from[v])
  if (in[s] == 0) {
           assert(canin.count(s));
           continue;
         else eg.pb(mp(v, s));
    for (auto v : canin)
       for (auto s : to[v])
         eg.pb(mp(s, v));
    shuffle(eg.begin(), eg.end(), x);
    if (eg.size() == 0) break;
    for (auto v : eg) {
      mx_ch--;
       if (in[v.se] && out[v.fi]) continue;
       if (LCT::fdr(v.fi) == LCT::fdr(v.se))
      if (in[v.se] || out[v.fi])
         if (x() & 1) continue;
       if (!in[v.se] && !out[v.fi])
         tot++;
       if (in[v.se])
         LCT::cut(in[v.se], v.se);
         canin.insert(v.se);
         canout.insert(in[v.se]);
         out[in[v.se]] = 0;
         in[v.se] = 0;
       if (out[v.fi]) {
         LCT::cut(v.fi, out[v.fi]);
         canin.insert(out[v.fi]);
         canout.insert(v.fi);
         in[out[v.fi]] = 0;
         out[v.fi] = 0;
       LCT::link(v.fi, v.se);
       canin.erase(v.se);
       canout.erase(v.fi);
       in[v.se] = v.fi;
       out[v.fi] = v.se;
    if (tot == n - 1) {
       vi cur;
       for (int i = 1; i <= n; i++)
  if (!in[i]) {</pre>
           int pl = i;
while (pl) {
             cur.pb(pl)
             pl = out[pl];
           break;
         }
      return cur;
    }
  //failed to find a path
```

```
return vi();
}
}
//43ae60
```

### 10 Геома

### 10.1 Примитивы

```
struct Point {
  int x, y;
  Point(){}
  Point (int x_, int y_) {
    x = x_; y = y_;
  Point operator + (Point p)
    return Point(x+p.x,y+p.y);
  Point operator - (Point p) {
    return Point(x - p.x, y - p.y);
  int operator * (Point p) {
    return x * p.y - y * p.x;
  int operator % (Point p) {
    return x * p.x + y * p.y;
  bool operator < (Point v) {</pre>
    return (*this) * v > 0;
  bool operator > (Point v) {
    return v < (*this);
  bool operator <= (Point v) {
  return (*this) * v >= 0;
  }
};
bool line(Point a, Point b, Point c) {
  return (b - a) * (c - b)==0;
bool ord(Point a, Point p, Point b) {
  return (p - a)%(p - b)<0;</pre>
int hp(Point a) {
  if (a.y == 0) return a.x >= 0;
  return a.y > 0;
bool comp(Point a, Point b) {
  if (hp(a) != hp(b)) return hp(a) < hp(b);
return a.x * b.y - a.y * b.x > 0;
//a48b68
```

### 10.2 Выпуклая оболочка

```
using pt = pair<int, int>;
#define x first
#define y second
int cross(pt p, pt q) { return p.x * q.y - p.y * q.x;
pt operator-(pt a, pt b) { return {a.x - b.x, a.y - b
    .y}; }
vector<point> convex(vector<point> a) {
 sort(all(a));
  a.erase(unique(all(a)), a.end());
  vector<point> h;
  for (int t = 0; t < 2; ++t) {
    int sz = h.size() - t;
    for (auto p: a) {
      while (h.size() >= sz + 2 \&\& cross(p - h.end()
    [-1], h.end()[-2] - h.end()[-1]) <= 0 h.pop_back
    ();
     h.push_back(p);
   reverse(all(a));
  return h; // h is circular: h.empty() || h.front()
    == h.back()
//ef9132
```

### 10.3 Точка внутри многоугольника

```
auto inT = [&] (Point a, Point b, Point c, Point p) {
  a = a-p; b = b-p; c = c-p;
int ab = a * b, bc = b * c, ca = c * a;
  return abs(ab)+abs(bc)+abs(ca) == abs(ab+bc+ca);
auto inP = [&] (Point p) {
//a must be in counterclockwise order!
//assuming no three points of a are collinear
 if (n == 1) return p == a[0];
  if (n == 2) return (p-a[0]) * (a[1]-a[0]) == 0 && (
  p-a[0]) % (p-a[1]) <= 0;
int 1 = 1, r = n - 1;
while (1 < r - 1) {</pre>
    int \dot{m} = (1 + r) / 2;
    if ((a[m] - a[0]) < (p - a[0])) {
      1 = m;
    } else {
      r = m;
    }
  return inT(a[1], a[0], a[r], p);
//9e04bc
10.4 Касательные
auto max = [&] (auto cmp) {
  int k = 0;
  for (int lg = 18; lg >= 0; --lg) {
    int i = k + (1 \ll lg), j = k - (1 \ll lg);
    i = (i % n + n) % n;
    j = (j % n + n) % n;
    array<int, 3> ind{i, j, k};
    sort(all(ind), cmp);
    k = ind[2];
  }
```

```
return k:
};
auto uppert = [&](Point p) { //last vertex in
    counterclockwise order about p
  auto cmp = [\&] (int i, int j) {return (a[i] - p) <
    (a[j] - p); };
  return max(cmp);
};
auto lowert = [&](Point p) { //first vertex in
    counterclockwise order about p
  auto cmp = [\&] (int i, int j) {return (a[i] - p) >
    (a[j] - p); };
  return max(cmp);
}:
auto uppertinf = [&](Point p) { //upper tangent line
   parallel to vector p
  swap(p.x, p.y);
 p.x = -p.x;
  auto cmp = [&] (int i, int j) { return a[i] % p < a</pre>
   [j] % p; };
  return max(cmp);
auto lowertinf = [&](Point p) { //lower tangent line
   parallel to vector p
  swap(p.x, p.y);
  p.x = -p.x;
  auto cmp = [&] (int i, int j) { return a[i] % p > a
    [j] % p; };
  return max(cmp);
//90f89d
```

### 10.5 Пересечение многоугольника и полуплоскости

```
for (int i = 0; i < n; ++i) {
   h.push_back(h[i]);
   pos.push back(pos[i]);
    if (pos[i] == 0 && pos[i + 1] == 1) t01 = i;
    if (pos[i] == 1 && pos[i + 1] == 0) t10 = i;
  if (t10 < t01) t10 += n;
//for (int i = t01 + 1; i <= t10; ++i) assert(pos[i]
    == 1);
//for (int i = t10 + 1; i <= t01 + n; ++i) assert(pos
   [i] == 0);
  vector<point> res{intersect(h[t01], h[t01 + 1])};
  for (int i = t01 + 1; i <= t10; ++i) {
   res.push_back(h[i]);
 res.push back(intersect(h[t10 + 1], h[t10]));
  return res;
// Usage example:
void solve() {
   vector<point> p, q;
    // ... q must be in counterclockwise order; q[0] == q[m]
    for (int i = 0; i < m; ++i) {</pre>
       p = intersect(p, [&](point pt) { return cross
    (q[i + 1] - q[i], pt - q[i]); ));
//bd15f0
```

### 10.6 События для прямой

```
int cross(point p, point q) { return p.x * q.y - q.x
     * p.y; }
point operator-(point p, point q) { return {p.x - q.x
, p.y - q.y}; } int sgn(int x) \{ return x < 0 ? -1 : (x > 0); \}
double dist(point p) { return sqrt(p.x * p.x + p.y *
    p.y); }
const __int128 one = 1;
double solve(vector<point> a, point p, point q) {
  int n = a.size();
  a.push_back(a[0]);
  point pq = q - p;
  vector<array<int, 3>> ev;
  for (int i = 0; i < n; ++i) {
    point u = a[i], v = a[i + 1];
    int s = sgn(cross(u - p, pq)), t = sgn(cross(v - p, pq))
    p, pq));
     if (s == t) continue;
    int top = cross(u - p, v - u), bot = cross(pq, v)
    - u);
    ev.push_back({sgn(bot) * top, abs(bot), t - s});
  sort(all(ev), [](auto e, auto f) {
  return e[0] * one * f[1] < f[0] * one * e[1];</pre>
  int bal = 0;
  for (int i = 0; i + 1 < ev.size(); ++i)</pre>
    if (bal + ev[i][2] < 0 || bal + ev[i][2] > 2) {
   assert(ev[i][0] * ev[i+1][1] == ev[i+1][0] * ev
     [i][1]);
       swap(ev[i], ev[i + 1]);
    bal += ev[i][2];
  // example usage: now calculating length of longest
     segment inside
  bal = 0;
  double from = 0, ans = 0;
for (auto [t, b, w] : ev) {
   double x = t * 1.0 / b;
     if (bal == 0) from = x;
    assert(0 <= bal && bal <= 2);
    if (bal == 0) ans = max(ans, x - from);
  return ans * dist(pq);
//fe0649
```

#### 10.7 Кривая Гильберта для алгоритма Мо

```
const int logn = 30; // any number, such that maxn is
   greater than coordinates. 20 is ok.
```

```
const int maxn = 1 << logn;</pre>
int hilbertorder(int x, int y) { // returns long long
  int d = 0; // long long
     for (int s = 1 << (logn - 1); s; s >>= 1)
         bool rx = x & s, ry = y & s;
d = (4 * d) | ((3 * rx) ^ ry);
          if (!ry) {
               <u>if</u> (rx) {
                   x = maxn - x;
                   y = maxn - y;
              swap(x, y);
          }
     }
    return d;
// Usage example:
vector<int> sort_indices(int q, vector<pair<int, int</pre>
     >> &as) {
     vector<int> ind(q), ord(q);
     iota(all(ind), 0);
     for (int i = 0; i < q; ++i) ord[i] = hilbertorder</pre>
    (qs[i].first, qs[i].second);
sort(all(ind), [&](int i, int j) { return ord[i]
     < ord[j]; });
    return ind;
//134578
```

### 10.8 Симплекс

```
#define int long long
using namespace std;
typedef double ld;
const ld EPS = 1e-9;
struct LPSolver {
  int m, n;
vector <int> B, N;
  vector <vector <ld> > D;
  LPSolver(const vector <vector <ld> > &A, const
     vector <ld> &b, const vector <ld> &c) :
     m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 1)
     2, vector <ld>(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n
; j++) D[i][j] = A[i][j];</pre>
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][
n] = -1; D[i][n + 1] = b[i]; }
     for (int j = 0; j < n; j++) { N[j] = j; D[m][j] =
    -c[j]; } N[n] = -1; D[m + 1][n] = 1;
  void pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
for (int i = 0; i < m + 2; i++) if (i != r)
  for (int j = 0; j < n + 2; j++) if (j != s)
    D[i][j] -= D[r][j] * D[i][s] * inv;</pre>
     for (int j = 0; j < n + 2; j++) if (j != s) D[r][
     j] *= inv;
     for (int i = 0; i < m + 2; i++) if (i != r) D[i][
     s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  bool simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
while (true) {
       int s = -1;
       for (int j = 0; j <= n; j++) {
  if (phase == 2 && N[j] == -1) continue;
  if (s == -1 || D[x][j] < D[x][s] || D[x][j]</pre>
     == D[x][s] && N[j] < N[s]) s = j;
       if (D[x][s] > -EPS) return true;
       int r = -1;
       for (int i = 0; i < m; i++) {</pre>
         if (D[i][s] < EPS) continue;</pre>
          if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n
      + 1] / D[r][s] ||
             (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D
     [r][s]) && B[i] < B[r]) r = i;
       if (r == -1) return false;
```

```
pivot(r, s);
  ld solve(vector <ld> &x) {
     int r = 0;
     for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r]
     ][n + 1]) r = i;
     if (D[r][n + 1] < -EPS) {
       pivot(r, n);
       if (!simplex(1) || D[m + 1][n + 1] < -EPS)
     return -numeric_limits<ld>::infinity();
for (int i = 0; i < m; i++) if (B[i] == -1) {
          int s = -1;
      for (int j = 0; j <= n; j++)
   if (s == -1 || D[i][j] < D[i][s] || D[i][j]
== D[i][s] && N[j] < N[s]) s = j;</pre>
         pivot(i, s);
     if (!simplex(2)) return numeric_limits<ld>::
     infinity();
     x = vector < ld > (n);
     for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]]
      = D[i][n + 1];
    return D[m][n + 1];
  }
};
/*
,
//
                         c^T x
        maximize
                       Ax \le b
        subject to
                        x >= 0
//
vector <vector <ld> > A(m, vector <ld> (n))
vector <ld> b(m), c(n)
LPSolver solver(A, b, c);
vector <ld> x;
ld value = solver.solve(x);
// OUTPUT: value of the optimal solution (infinity if
    unbounded above, -infinity if infeasible)
//31e155
```

#### 11 Цепные дроби

https://cp-algorithms.com/algebra/continued-fractions.html

### Поиск нижней огибающей, сумма и минимум по модулю

```
int floor(int a, int b) {
  return a / b - ((a ^ b) < 0 && a % b);</pre>
vector<int> decompose(int p, int q) {
  vector<int> f;
while (q != 0) {
     f.push_back(floor(p, q));
p -= q * f.back();
     swap(p, q);
  return f;
using matrix = array<int, 4>;
matrix operator*(matrix a, matrix b) {
  matrix c{0,0,0,0};
  for (int i = 0; i < 2; ++i) {
     for (int j = 0; j < 2; ++j) {
  for (int k = 0; k < 2; ++k) {
    c[2 * i + k] += a[2 * i + j] * b[2 * j + k];</pre>
    }
  return c;
#define x first
#define y second
// computes lower convex hull for 0 <= x <= N, 0 <= y
         (ax + b) / c
vector<pair<int, int>> lower_convex_hull(int a, int b
  , int c, int n) {
matrix m = {1, 0, 0, 1};
  auto f = decompose(a, c);
  vector<pair<int, int>> conv{{1, 0}, {0, 1}};
```

```
for (int x : f) {
  m = m * matrix{x, 1, 1, 0};
     conv.emplace_back(m[2], m[0]);
if (m[2] > n) break; // there should be one (if
     any) with .x > n
  auto diff = [&](int x, int y) {
     return c * y - a * x;
  int x = 0, y = b / c;
  vector<pair<int, int>> res{{x, y}};
  int i;
  for (i = 2; i + 1 < conv.size(); i += 2) {</pre>
     while (diff(x + conv[i + 1].x, y + conv[i + 1].y)
      <= b) {
       int t = 1 + (diff(x + conv[i - 1].x, y + conv[i
       - 1].y) - b - 1) / abs(diff(conv[i].x, conv[i].y
     ));
     auto [dx, dy] = tuple{conv[i - 1].x + t * conv[i].x, conv[i - 1].y + t * conv[i].y};
int k = (n - x) / dx;
if (k == 0) break;
        if (diff(dx, dy)) k = min(k, (b - diff(x, y)) /
      diff(dx, dy));
x += k * dx, y += k * dy;
res.push_back({x, y});
     }
  if (i >= conv.size()) i -= 2;
  for (; i > 0; i -= 2) {
  auto [dx1, dy1] = conv[i];
     if (x + dx1 > n) continue;
     x += dx1, y += dy1;
if (i + 1 < conv.size()) {
       auto [dx2, dy2] = conv[i + 1];
int k = (n - x) / dx2;
x += k * dx2;
       y += k * dy2;
     res.emplace_back(x, y);
     int k = (n - x) / dx1;
     if (k == 0) continue;
x += k * dx1;
y += k * dy1;
     res.emplace_back(x, y);
  return res;
}
// number of (x, y) under pq line such that p.x <= x < q.x && 0 < y
int area(auto p, auto q) {
  int integers = gcd(q.x - p.x, q.y - p.y);
  return ((p.y + q.y - 1) * (q.x - p.x + 1) +
   integers + 1) / 2 - q.y;
// sum of (ax + b) / c for 0 \le x \le n
int sum = 0;
  for (int i = 0; i + 1 < ch.size(); ++i) {</pre>
    sum += area(ch[i], ch[i + 1]);
  return sum;
// min of (ax + b) % c for 0 <= x <= n
int get_min(int a, int b, int c, int n) {
  auto ch = lower_convex_hull(a, b, c, n);
  // in fact, here we need only the last point of the
      first half of the algo (that is going up)
  int mn = c;
  for (auto [x, y] : ch) mn = min(mn, (a * x + b) % c
     );
  return mn;
.
//87941e
11.2
        Простая рекурсия
```

```
Число точек (x,y): 0 \leqslant x < n, 0 < y \leqslant (kx+b)/d. То есть \sum \lfloor \frac{kx+b}{d} \rfloor.
int cnt (int n, int k, int b, int d)
  if (k == 0) return (b / d) * n;
  if (k >= d || b >= d) {
```

```
return (k / d) * n * (n - 1) / 2 + (b / d) * n +
cnt(n, k % d, b % d, d);
}
return cnt((k * n + b) / d, d, (k * n + b) % d, k);
}
//11a6a0
```

### 12 Разное

### 12.1 Компараторы

```
bool cmp1(int x, int y) { return x > y; }

struct cmp2{
  bool operator()(int x, int y) const { return x > y;
  }
};

int32_t main() {
  set<int, decltype(cmp1)*> s1({1, 2, 3}, cmp1);
  for (int x : s1) cout << x << ' '; cout << '\n';
  set<int, cmp2> s2({4, 5, 6});
  for (int x : s2) cout << x << ' '; cout << '\n';
  auto cmp3 = [&](int x, int y) { return x > y; };
  set<int, decltype(cmp3)> s3({7, 8, 9}, cmp3); //
    second cmp3 could be omitted if cmp3 = [](...) {
        ... }
  for (int x : s3) cout << x << ' '; cout << '\n';

  vector<int> v{3, 2, 1};
  cout << lower_bound(all(v), 2, cmp1) - v.begin();
  cout << lower_bound(all(v), 2, cmp3) - v.begin();
  cout << lower_bound(all(v), 2, cmp3) - v.begin();
}
//adea08</pre>
```

### 12.2 Трюки от Сергея Копелиовича

#### 12.2.1 Быстрый ввод

https://acm.math.spbu.ru/~sk1/algo/input-output

```
const int buf_size = 4096;
int getChar() {
  static char buf[buf size];
  static int len = 0, pos = 0;
  if (pos == len)
  pos = 0, len = fread(buf, 1, buf_size, stdin);
if (pos == len)
    return -1;
  return buf[pos++];
int readChar() {
  while (1) {
    int c = getChar();
if (c > 32) return c;
int readInt() {
  int s = 1, c = readChar(), x = 0;
if (c == '-')
    s = -1, c = getChar();
  while (isdigit(c))
  x = x * 10 + c - '0', c = getChar();
return s * x;
//dc0a77
double read_double() {
  string s;
  cin >> s;
  double sgn = 1, p10 = 0, num = 0;
  for (char c : s)
if (c == '-') {
                     {
      sgn = -1;
    } else if (c == '.') {
      p10 = 1;
    } else {
      p10 *= 10;
      num = (num * 10 + c - '0');
  if (p10 < 0.5) p10 = 1;
```

```
return sgn * num / p10;
}
//b77b67
https://acm.math.spbu.ru/~sk1/algo/memory.cpp.html
```

#### 12.2.2 Быстрый аллокатор

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void * operator new (size_t n) {
   assert((mpos += n) <= MAX_MEM);
   return (void *) (mem + mpos - n);
}
void operator delete (void *) noexcept { }
void operator delete (void *, size_t) noexcept { }
//8726b1</pre>
```

### 12.3 Редукция Барретта

```
using u64 = unsigned long long;
using u128 = __uint128_t;
struct barrett{
  u64 p, m;
  barrett() {}
  barrett(u64 p) : p(p), m(-lULL / p) {}
  int reduce(u64 x) {
    u64 q = (u128(m) * x) >> 64, r = x - q * p;
    return r - p * (r >= p);
  }
} ba;

// Usage example:
void solve() {
  int p = ...;
  ba = barrett(p);
  int x = ..., y = ...;
  int prod = ba.reduce(x * y);
}
//a8b4c7
```

#### 12.4 Флаги компияции

-DLOCAL -Wall -Wextra -pedantic -Wshadow -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -Wshift-overflow=2 -Wduplicated-cond -Wcast-qual -Wcast-align -D\_GLIBCXX\_DEBUG -D\_GLIBCXX\_DEBUG\_PEDANTIC -D\_FORTIFY\_SOURCE=2 -fsanitize=address - fsanitize=undefined -fno-sanitize-recover -fstack-protector -std=c++2a

#### 12.5 Что сделать на пробном туре

- Послать клар
- Распечатать что-то
- Получить ML (stack & heap)
- Максимальный размер отправляемого файла?
- Убедиться, что чекер регистронезависимый (yes/YES)
- Позапускать Флойда Варшалла
- Посмотреть, насколько быстр быстрый ввод
- Перебить что-то, проверить хеш
- Проверить санитайзеры

### 12.6 Хеш файла без комментариев

```
Хеш файла, игнорирующий переводы строк и комментарии:
```

```
$ cpp -dD -P -fpreprocessed "$filename" | tr -d '[:
    space:]' | md5sum | cut -c-6
```