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## 1 Настройка CLion

- В файле CMakeLists.txt дописать строку `add_compile_definitions(LCAL)`. Нажать появившуюся опцию в правом верхнем углу `enable auto-reload`.

- Вбить шаблон в main.cpp:

```
#ifndef LOCAL
#define _GLIBCXX_DEBUG
#endif
#include<bits/stdc++.h>

using namespace std;

#define int long long
#define app push_back
#define all(x) x.begin(), x.end()
#ifdef LOCAL
#define debug(...) [ ](auto...a){ ((cout << a <<
    ' '), ...) << endl; }(#__VA_ARGS__, ":",
    __VA_ARGS__)
#define debugv(v) do { cout << #v << ": "; for (
    auto x : v) cout << x << ' '; cout << endl;
    } while(0)
#else
#define debug(...)
#define debugv(v)
#endif

int32_t main() {
    cin.tie(0); ios_base::sync_with_stdio(0);
    int n = 2; vector<int> a(n, n);
    debug(n); debugv(a);
}
```

Скомпилировать, чтобы проверить отсутствие опечаток.

- Запустить терминал (ctrl + alt + T)

```
$ cd workspace/CLionProjects
$ for c in {A..Z}; do cp main.cpp $c.cpp && echo
    "add_executable($c $c.cpp)" >> CMakeLists.
    txt; done
```

Далее отключаем подсветку и форматирование в настройках (ctrl+alt+S)

- Editor → Code Style → Formatter → Do not format прописать \*
- Editor → Inspections → C/C++ → static analysis tools → CLang-Tidy отключить
- Editor → Inlay Hints → отключаем всё (достаточно первых трёх — code vision, parameter names, types).

Тёмная тема отключается в Appearance & Behavior → Appearance.

Чтобы добавить санитайзеры, надо дописать в CMakeLists.txt `set(CMAKE_CXX_FLAGS "-fsanitize=address -fsanitize=undefined")`

## 2 Теория чисел

### 2.1 КТО

```
int gcd(int a, int b, int &x, int &y) {
    if (b==0) { x = 1; y = 0; return a; }
    int d = gcd(b, a%b, y, x);
    y-=a/b*x;
    return d;
}

int inv(int r, int m) {
    int x, y;
    gcd(r, m, x, y);
    return (x+m)%m;
}

int crt(int r, int n, int c, int m) { return r + ((c
    - r) % m + m) * inv(n, m) % m * n; }

//8ed8ed
```

### 2.2 Алгоритм Миллера — Рабина

```

__int128 one=1;
int po(int a,int b,int p)
{
    int res=1;
    while(b) {if(b & 1) {res=(res*one*a)%p;--b;} else
        {a=(a*one*a)%p;b>>=1;}} return res;
}
bool chprime(int n) ///miller-rabin
{
    if(n==2) return true;
    if(n<=1 || n%2==0) return false;
    int h=n-1;int d=0;while(h%2==0) {h/=2;++d;}
    for(int a:{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
        37})
    {
        if(a==n) return true;
        int u=po(a,h,n);bool ok=0;
        if(u%n==1) continue;
        for(int c=0;c<d;++c)
        {
            if((u+1)%n==0) {ok=1;break;}
            u=(u*one*u)%n;
        }
        if(!ok) return false;
    }
    return true;
}
//86b2ed

```

## 2.3 Алгоритм Берлекэмп — Мессе

<https://mzhang2021.github.io/cp-blog/berlekamp-massey/>

```

template<typename T>
vector<T> berlekampMassey(const vector<T> &s) {
    int n = s.size(), l = 0, m = 1;
    vector<T> b(n), c(n);
    T ld = b[0] = c[0] = 1;
    for (int i=0; i<n; i++, m++) {
        T d = s[i];
        for (int j=1; j<=l; j++)
            d += c[j] * s[i-j];
        if (d == 0) continue;
        vector<T> temp = c;
        T coef = d / ld;
        for (int j=m; j<n; j++) c[j] -= coef * b[j-m];
        if (2 * l <= i) {
            l = i + 1 - l;
            b = temp;
            ld = d;
            m = 0;
        }
    }
    c.resize(l + 1);
    c.erase(c.begin());
    for (T &x : c)
        x = -x;
    return c;
}
//ff47ae

```

## 2.4 Линейное решето

```

const int C = 1e7+7;
vi pr, lp(C);
for (int i = 2; i < C; ++i) {
    if (lp[i] == 0) {
        lp[i] = i;
        pr.app(i);
    }
    for (int j = 0; j < (int)pr.size() && pr[j] <= lp[i]
        && pr[j] * i < C; ++j) {
        lp[pr[j] * i] = pr[j];
    }
}
//36b3d1

```

## 2.5 Алгоритм Шенкса

```

#define T int
int mod;
int gcd(int a, int b, int &x, int &y) {
    if (b==0) { x = 1; y = 0; return a; }
    int d = gcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}

```

```

int inv(int r, int m) {
    int x, y;
    gcd(r,m,x,y);
    return (x+m)%m;
}
int crt(int r, int n, int c, int m) { return r + ((c
    - r) % m + m) * inv(n, m) % m * n; }
T inv(T a)
{
    return inv(a,mod);
}
T mul(T a,T b)
{
    return (a*b)%mod;
}
vector<int> rasl(int x)
{
    vector<int> v;
    if(x==1) {return v;}
    for(int i=2;i*i<=x;++i)
    {
        if(x%i==0)
        {
            v=rasl(x/i);v.app(i);return v;
        }
    }
    v.app(x);return v;
}
T po(T a,int b) ///b>=1
{
    if(b==1) {return a;}
    if(b%2==0)
    {
        T u=po(a,b/2);
        return mul(u,u);
    }
    else
    {
        T u=po(a,b-1);
        return mul(a,u);
    }
}
T getper(T a,T one,int per,vector<int> v)
{
    for(int p:v)
    {
        if(po(a,per/p)==one)
        {
            per/=p;
        }
    }
    return per;
}
vector<pair<int,int> > shanks(T a,vector<T> b,T one,
    int per) ///a^per=1 and b[i]^per=1 /// all right
    numbers in output are equal
{
    if(b.empty()) {return {};}
    int n=b.size();
    vector<int> vp=rasl(per);
    int pera=getper(a,one,per,vp);per=pera;
    vp=rasl(pera);
    vector<int> have(n,0);
    int cur2=per;T cura=a;T invcura=inv(a);
    int curad=1;
    vector<pair<T,int> > v;
    vector<bool> ok(n,true);
    vector<T> poinvzx;
    for(int p:vp)
    {
        T ca=po(cura,cur2/p);
        if(ca==one) {continue;}
        T invca=po(invcura,cur2/p);
        int step=sqrt(b.size()*p)+2;
        int wee=p/step+2;
        v.clear();poinvzx.clear();
        T zx=one;T invzx=one;T buba=one;
        vector<T> zhe;
        T lu=one;
        for(int i=0;i<step;++i)
        {
            v.app({zx,i});zhe.app(lu);
            zx=mul(zx,ca);invzx=mul(invzx,invca);buba=mul(
                buba,cura);lu=mul(lu,invcura);
        }
        poinvzx.app(one);
        for(int j=0;j<wee;++j)
        {
            poinvzx.app(mul(poinvzx.back(),buba));
        }
    }
}

```

```

}
sort(all(v));
for(int i=0;i<n;++i)
{
    if(!ok[i]) {continue;}
    T uu=po(b[i],cur2/p);
    bool okkk=false;
    for(int j=0;j<wee;++j)
    {
        auto it=lower_bound(all(v),make_pair(uu,0LL)
    );
        if(it!=v.end() && (*it).first==uu)
        {
            okkk=true;
            have[i]--(curad*step*j);
            have[i]+=(curad*(it).second);
            have[i]%=pera;if(have[i]<0) {have[i]+=pera
        };
        b[i]=mul(b[i],poinvzx[j]);b[i]=mul(b[i],
zhe[(it).second]);
        assert(po(b[i],cur2/p)==one);
        break;
    }
    uu=mul(uu,zx);
    if(!okkk) {ok[i]=false;}
}
cur2/=p;cura=po(cura,p);invcura=po(invcura,p);
curad*=p;
}
vector<pair<int,int>> > res;
for(int i=0;i<n;++i)
{
    if(ok[i] && b[i]==one)
    {
        res.app({(have[i]%pera+pera)%pera,pera});
    }
    else
    {
        res.app({-1,pera});
    }
}
return res;
}
int shanks2(int x,int y,int mod1) ///only for T=long
long, 0^0 = 1 by default
{
    mod=mod1;
    vector<int> v=rasl(mod);sort(all(v));
    int per=1;for(int i=0;i<v.size();++i) {if(i==0 ||
v[i]!=v[i-1]) {per*=(v[i]-1);} else {per*=v[i
];}}
    if(y==1 || mod==1) {return 0;}
    int C=61;
    for(int i=1;i<C;++i)
    {
        if(po(x,i)==y) {return i;}
    }
    if(y==0) {return (-1);}
    T h=po(x,C);
    int lc1=gcd(h,mod);int lc2=gcd(y,mod);
    if(lc1!=lc2) {return (-1);}
    mod/=lc2;T h1=h/lc2;T y1=y/lc2;
    vector<pair<int,int>> > s=shanks(x%mod,{mul(y1,inv(
h1)),1,per);
    if(s[0].first!=(-1))
    {
        return s[0].first+C;
    }
    else
    {
        return (-1);
    }
}
}
//a75596

```

## 3 Графы

### 3.1 SCC и 2-SAT

Алгоритм ищет сильносвязные компоненты в графе  $g$ , если есть путь  $i \rightarrow j$ , то  $scc[i] \leq scc[j]$

```

vector<int> find_scc(vector<vector<int>> g) {
    int n = g.size();
    vector<vector<int>> r(n);
    for (int i = 0; i < n; ++i) {
        for (int j : g[i]) r[j].push_back(i);
    }
}

```

```

vector<int> used(n), tout(n);
int time = 0;
auto dfs = [&](auto dfs, int cur) -> void {
    used[cur] = 1;
    for (int nxt : g[cur]) {
        if (!used[nxt]) dfs(dfs, nxt);
    }
    tout[cur] = time++;
};
for (int i = 0; i < n; ++i) if (!used[i]) dfs(dfs,
i);
vector<int> ind(n);
iota(all(ind), 0);
sort(all(ind), [&](int i, int j){return tout[i] >
tout[j]});
vector<int> scc(n, -1);
auto go = [&](auto go, int cur, int color) -> void
{
    scc[cur] = color;
    for (int nxt : r[cur]) {
        if (scc[nxt] == -1) go(go, nxt, color);
    }
};
int color = 0;
for (int i : ind) {
    if (scc[i] == -1) go(go, i, color++);
}
return scc;
}
//4fd51f

```

Чтобы решать 2-SAT, надо создать граф на  $2n$  вершинах, рёбра  $i \Rightarrow j$  и  $(j \oplus 1) \Rightarrow (i \oplus 1)$  должны быть добавлены одновременно. После этого если  $scc[2 * i] = scc[2 * i + 1]$ , то решения нет; если  $scc[2 * i + 0] < scc[2 * i + 1]$ , то присутствует импликация  $\neg i \Rightarrow i$ , надо назначить  $i = \text{true}$ .

### 3.2 Эйлеров цикл

```

vector<int> euler(vector<vector<pair<int,int>>> g,
int m, int src) { // g[cur][i] = pair{nxt, idx}
    int n = g.size();
    vector<int> used(m), it(n), cycle;
    auto dfs = [&](auto dfs, int cur) -> void {
        while (true) {
            while (it[cur] < g[cur].size() && used[g[cur][
it[cur]].second]) it[cur]++;
            if (it[cur] == g[cur].size()) return;
            auto [nxt, idx] = g[cur][it[cur]];
            used[idx] = true;
            dfs(dfs, nxt);
            cycle.push_back(idx); // or {cur, nxt}
        }
    };
    dfs(dfs, src);
    reverse(cycle.begin(), cycle.end());
    if (cycle.size() != m) return {}; // check that
    all edges are present in the cycle, fail
    otherwise
    return cycle;
}
//f6b9d4

```

### 3.3 Компоненты рёберной двусвязности

```

//n - number of vertices, m - number of edges,
parallel edges -- ???, color of any edge is the
color of its lower end
vector<vector<int>> dfstree(n);
vector<int> used(n), cut(n), h(n), up(n);
auto findCutPoints = [&](auto self, int u) -> void
{
    used[u] = 1;
    up[u] = h[u];
    for (int v : g[u]) {
        if (!used[v]) {
            dfstree[u].push_back(v);
            h[v] = h[u] + 1;
            self(self, v);
            up[u] = min(up[u], up[v]);
            if (up[v] >= h[u]) {
                cut[v] = 1;
            }
        }
        else {
            up[u] = min(up[u], h[v]);
        }
    }
}

```

```

};
findCutPoints(findCutPoints, 0);
vector<vector<int>> tree(n + m);
vector<int> color(n); color[0] = 0; int ptr = n;
auto build = [&] (auto self, int u) -> void {
    for (int v : dfstree[u]) {
        if (cut[v]) {
            color[v] = ptr; ++ptr;
            self(self, v);
        }
        else {
            color[v] = color[u];
            self(self, v);
        }
    }
};
build(build, 0);
for (int i = 0; i < n; ++i) {
    set<int> to;
    for (int j : g[i]) {
        int x = i, y = j;
        if (h[x] < h[y]) swap(x, y);
        to.insert(color[x]);
    }
    for (int j : to) {
        tree[i].app(j); tree[j].app(i);
    }
}
//2ebfbb

```

### 3.4 DCP offline

```

struct Dsu {
    int n;
    vector<pair<int, int>> s;
    vector<int> p, sz;
    // other info

    Dsu(int n) : n(n), p(n), sz(n, 1) {
        iota(all(p), 0);
    }

    int get(int u) {
        while (u != p[u]) u = p[u];
        return u;
    }

    bool merge(int u, int v) {
        u = get(u), v = get(v);
        if (u == v) return false;
        if (sz[v] < sz[u]) swap(u, v);
        s.app({p[u], p[u]});
        s.app({sz[v], sz[v]});
        // app other info like s.app({comp, comp});
        p[u] = v;
        sz[v] += sz[u];
        return true;
    }

    void rollback(int sz) {
        while (s.size() != sz) {
            s.back().first = s.back().second;
            s.pop_back();
        }
    }
};

struct DcpOffline {
    int n;
    vector<vector<pair<int, int>>> d;

    void addEdgeOnSegment(int l, int r, int a, int b) {
        for (l += n, r += n; l < r; l /= 2, r /= 2) {
            if (l & 1) d[l++].app({a, b});
            if (r & 1) d[--r].app({a, b});
        }
    }

    template<typename T>
    void dfs(Dsu &dsu, T act) {
        dfs(1, 0, n, dsu, act);
    }

    template<typename T>
    void dfs(int v, int l, int r, Dsu &dsu, T act) {
        int sz = dsu.s.size();
        for (auto [u, v] : d[v]) {
            dsu.merge(u, v);
        }
    }
};

```

```

    }
    if (l + 1 == r) {
        act(l, dsu);
    }
    else {
        int m = (l + r) / 2;
        dfs(v * 2, l, m, dsu, act);
        dfs(v * 2 + 1, m, r, dsu, act);
    }
    dsu.rollback(sz);
}

DcpOffline(int maxt) : n(2 << __lg(maxt + 1)), d(2 * n) {}
};
//3c4e2d

```

### 3.5 Взвешенное паросочетание

<https://judge.yosupo.jp/submission/201334>

```

#define d(x) (lab[x.u] + lab[x.v] - 2 * e[x.u][x.v].w)
const int N = 403 * 2;
const int inf = 1e18;
struct Q { int u, v, w; } e[N][N];
vector<int> p[N];
int n, m = 0, id, h, t, lk[N], sl[N], st[N], f[N], b[N][N], s[N], ed[N], q[N], lab[N];
void upd(int u, int v) { if (!sl[v] || d(e[u][v]) < d(e[sl[v]][v])) sl[v] = u; }
void ss(int v) {
    sl[v] = 0;
    for (int u = 1; u <= n; ++u) if (e[u][v].w > 0 && st[u] != v && !s[st[u]]) upd(u, v);
}
void ins(int u) { if (u <= n) q[++t] = u; else for (int v : p[u]) ins(v); }
void ch(int u, int w) { st[u] = w; if (u > n) for (int v : p[u]) ch(v, w); }
int gr(int u, int v) {
    if ((v = find(all(p[u]), v) - p[u].begin()) & 1) {
        reverse(1 + all(p[u]));
        return (int)p[u].size() - v;
    }
    return v;
}
void stm(int u, int v) {
    lk[u] = e[u][v].v;
    if (u <= n) return; Q w = e[u][v];
    int x = b[u][w.u], y = gr(u, x);
    for (int i = 0; i < y; ++i) stm(p[u][i], p[u][i ^ 1]);
    stm(x, v); rotate(p[u].begin(), y + all(p[u]), p[u].end());
}
void aug(int u, int v) {
    int w = st[lk[u]]; stm(u, v); if (!w) return;
    stm(w, st[f[w]]);
    aug(st[f[w]], w);
}
int lca(int u, int v) {
    for (id++; u | v; swap(u, v)) {
        if (!u) continue; if (ed[u] == id) return u;
        ed[u] = id; if (u == st[lk[u]]) u = st[f[u]]; //
        =, not ==
    }
    return 0;
}
//cf1d55

```

```

void add(int u, int a, int v) {
    int x = n + 1; while (x <= m && st[x]) ++x;
    if (x > m) ++m;
    lab[x] = s[x] = st[x] = 0;
    lk[x] = lk[a];
    p[x].clear();
    p[x].push_back(a);
#define op(q) for (int i = q, j = 0; i != a; i = st[f[j]]) p[x].push_back(i), p[x].push_back(j = st[lk[i]]), ins(j) // also not ==
    op(u); reverse(1 + all(p[x])); op(v);
    ch(x, x); for (int i = 1; i <= m; ++i) e[x][i].w = e[i][x].w = 0;
    fill(b[x] + 1, b[x] + n + 1, 0);
    for (int u : p[x]) {
        for (int v = 1; v <= m; ++v) if (!e[x][v].w || d(e[u][v]) < d(e[x][v])) e[x][v] = e[u][v], e[v][x] = e[v][u];
        for (int v = 1; v <= n; ++v) if (b[u][v]) b[x][v] = u;
    }
}

```

```

    }
    ss(x);
}
void ex(int u) {
    for (int x : p[u]) ch(x, x);
    int a = b[u][e[u][f[u]].u], r = gr(u, a);
    for (int i = 0; i < r; i += 2) {
        int x = p[u][i], y = p[u][i + 1];
        f[x] = e[y][x].u; s[x] = 1; s[y] = 0; sl[x] = 0;
        ss(y); ins(y);
    }
    s[a] = 1; f[a] = f[u];
    for (int i = r + 1; i < p[u].size(); ++i) s[p[u][i]] = -1, ss(p[u][i]);
    st[u] = 0;
}
bool on(const Q &e) {
    int u = st[e.u], v = st[e.v], a;
    if (s[v] == -1) {
        f[v] = e.u, s[v] = 1, a = st[lk[v]], sl[v] = sl[a] = s[a] = 0, ins(a);
    } else if (!s[v]) {
        a = lca(u, v); if (!a) return aug(u, v), aug(v, u), 1; else add(u, a, v);
    }
    return 0;
}
//3f0fld

bool bfs() {
    fill(s+1, s+m+1, -1); fill(sl+1, sl+m+1, 0); // s is filled with -1
    h = 1, t = 0; for (int i = 1; i <= m; ++i) if (st[i] == i && !lk[i]) f[i] = s[i] = 0, ins(i);
    if (h > t) return 0;
    while (1) {
        while (h <= t) {
            int u = q[h++];
            if (s[st[u]] != 1) {
                for (int v = 1; v <= n; ++v) if (e[u][v].w > 0 && st[u] != st[v]) {
                    if (d(e[u][v])) upd(u, st[v]); else if (on(e[u][v])) return 1;
                }
            }
        }
        int x = inf;
        for (int i = n+1; i <= m; ++i) if (st[i] == i && s[i] == 1) x = min(x, lab[i]/2);
        for (int i = 1; i <= m; ++i) if (st[i] == i && sl[i] && s[i] != 1) x = min(x, d(e[sl[i]][i])>>s[i]+1);
        for (int i = 1; i <= n; ++i) if (~s[st[i]]) if ((lab[i] += (s[st[i]] * 2 - 1) * x) <= 0) return 0;
        for (int i = n + 1; i <= m; ++i) if (st[i] == i && ~s[st[i]]) lab[i] += (2 - 4 * s[st[i]]) * x;
        h = 1, t = 0;
        for (int i = 1; i <= m; ++i) if (st[i] == i && sl[i] && st[sl[i]] != i && !d(e[sl[i]][i]) && on(e[sl[i]][i])) return 1;
        for (int i = n+1; i <= m; ++i) if (st[i] == i && s[i] == 1 && !lab[i]) ex(i);
    }
}

pair<int, vector<array<int, 2>>> run(int N, vector<array<int, 3>> edges) {
    for (auto &[u, v, w] : edges) ++u, ++v;
    fill(ed+1, ed+m+1, 0);
    fill(lk+1, lk+m+1, 0);
    n = m = N;
    id = 0;
    iota(st + 1, st + n + 1, 1);
    int wm = 0, weight = 0;
    for (int i = 1; i <= n; ++i) for (int j = 1; j <= n; ++j) e[i][j] = {i, j, 0};
    for (auto [u, v, w] : edges) wm = max(wm, e[v][u].w = e[u][v].w = max(e[u][v].w, w));
    for (int i = 1; i <= n; ++i) p[i].clear();
    for (int i = 1; i <= n; ++i) for (int j = 1; j <= n; ++j) b[i][j] = i==j?i:0;
    fill_n(lab+1, n, wm); while (bfs());
    vector<array<int, 2>> matching;
    for (int i = 1; i <= n; ++i) if (i < lk[i]) weight += e[i][lk[i]].w, matching.push_back({i - 1, lk[i] - 1});
    return {weight, matching};
}

```

```

}
//be682f

```

### 3.6 Дерево доминаторов

```

struct DominatorTree{
    struct DSU{
        struct Vert{
            int p;
            pair<int, int> val;
        };

        vector<Vert> t;
        vector<int> ord;

        DSU(vector<int> &ord): ord(ord) { t.resize(ord.size()); for (int i = 0; i < ord.size(); i++) t[i].p = i; }

        int get(int v){
            if (t[v].p == v) return v;
            int new_p = get(t[v].p);
            if (ord[t[v].val.first] > ord[t[t[v].p].val.first]) t[v].val = t[t[v].p].val;
            t[v].p = new_p;
            return t[v].p;
        }

        void merge(int a, int b){
            a = get(a); b = get(b);
            if (a != b){
                t[b].p = a;
            }
        }

        void setVal(int v, pair<int, int> val){
            t[v].val = val;
        }

        auto getVal(int v){
            get(v);
            return t[v].val;
        }
    };

    vector<vector<int>> g, gr, lg;
    vector<int> idom, sdom, was, tin;

    int timer;
    void dfs(int v){
        tin[v] = timer++;
        was[v] = 1;
        for (int to : g[v]) if (!was[to]) dfs(to);
    }

    vector<vector<int>> req;

    DominatorTree(int n, vector<pair<int, int>> & edges, int root){
        g.resize(n); gr.resize(n); lg.resize(n);
        idom.resize(n, -1); sdom.resize(n);
        was.resize(n, 0); tin.resize(n);
        req.resize(n);
        for (auto &e : edges){
            g[e.first].push_back(e.second);
            gr[e.second].push_back(e.first);
        }
        timer = 0; dfs(root);
        vector<int> ord;
        for (int i = 0; i < n; i++) ord.push_back(i);
        sort(ord.begin(), ord.end(), [this](int w1, int w2){ return tin[w1] > tin[w2]; });
        DSU dsu(tin);
        for (int v : ord){
            sdom[v] = v;
            for (int to : gr[v]){
                if (v == to) continue;
                int val = tin[to] < tin[v] ? to : dsu.getVal(to).first;
                if (tin[val] < tin[sdom[v]]) sdom[v] = val;
            }

            req[sdom[v]].push_back(v);
            for (auto &r : req[v]){
                auto val = dsu.getVal(r);
                if (tin[val.first] < tin[sdom[r]]){
                    lg[val.second].push_back(r);
                }
            }
        }
    }
}

```

```

    } else {
        idom[r] = sdom[r];
    }
}

dsu.setVal(v, make_pair(sdom[v], v));
for (int to : g[v]){
    if (tin[to] > tin[v] && dsu.t[to].p == to){
        dsu.merge(v, to);
    }
}

for (int i = 0; i < n; i++) was[i] = 0;

for (int i = 0; i < n; i++) if (!was[i] && idom[i] != -1){
    vector<int> st;
    st.push_back(i);
    was[i] = 1;
    while(st.size()){
        int v = st.back(); st.pop_back();
        idom[v] = idom[i];
        for (int to : lg[v]) if (!was[to]) was[to] = 1, st.push_back(to);
    }
}

};

/*
vector <pair <int, int> > e;
DominatorTree d(n,e,0);
auto par = d.idom;
*/
//839464

```

### 3.7 Венгерский алгоритм решения задачи о назначениях

```

//choose one element in each row to minimize sum of
//the chosen elements, n <= m, INF>max(abs(a[i][j]
//))
const int INF = 1e18;
vector<int> hungarian(int n, int m, vector<vector<
int>> a, int &cost) { //1-indexed!, a.size()==n
+1, a[i].size()==m+1
vector<int> u(n+1), v(m+1), p(m+1), way(m+1);
for (int i=1; i<=n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m+1, INF);
    vector<char> used(m+1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j=1; j<=m; ++j)
            if (!used[j]) {
                int cur = a[i0][j]-u[i0]-v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j=0; j<=m; ++j)
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}
vector<int> ans(n+1);
for (int j=1; j<=m; ++j) {
    if (p[j]!=0) {
        ans[p[j]] = j;
    }
}
cost = -v[0];
return ans;
}
//6d564b

```

### 3.8 Алгоритм Чу-Лью

В ориентированном взвешенном графе ищет остовное дерево минимального веса (такое дерево, что все вершины достижимы из нуля, входящая степень любой ненулевой вершины равна 1, в нуль не входит ни одного ребра). Если `recover = true`, то восстанавливает ответ.

Предполагается, что все вершины достижимы из нуля.

```

using edge = array<int, 4>; // {from, to, w, i}

template<typename T, typename C>
using pq = priority_queue<T, vector<T>, C>;

pair<int, vector<int>> solve(int n, vector<edge> ed,
    bool recover) {
    auto cmp = [&](int i, int j) { return ed[i][2] >
        ed[j][2]; };
    vector r(n, pq<int, decltype(cmp)>(cmp));
    for (auto [u, v, w, i] : ed) r[v].push(i);
    vector<int> mod(n), p(n), color(n), take;
    iota(all(p), 0);
    auto get = [&](int u) {
        while (u != p[u]) u = p[u] = p[p[u]];
        return u;
    };
    auto unite = [&](int x, int y) {
        x = get(x), y = get(y);
        if (x == y) return;
        if (r[x].size() > r[y].size()) swap(x, y);
        p[x] = y;
        while (r[x].size()) {
            auto e = r[x].top();
            r[x].pop();
            ed[e][2] += mod[x] - mod[y];
            r[y].push(e);
        }
    };
    vector<vector<pair<int, int>>> g(n);
    int ans = 0;
    color[0] = 2;
    auto go = [&](int cur) {
        vector<pair<int, int>> stack;
        int time = 0;
        while (color[cur] < 2) {
            color[cur] = 1;
            edge e;
            do {
                e = ed[r[cur].top()];
                r[cur].pop();
            } while (get(e[0]) == cur);
            e[2] += mod[cur];
            ans += e[2];
            mod[cur] -= e[2];
            stack.push_back({cur, e[3]});
            int a = get(e[0]);
            if (color[a] == 1) {
                while (true) {
                    auto [nxt, i] = stack.back();
                    stack.pop_back();
                    g[ed[i][0]].push_back({time++, i});
                    unite(nxt, cur);
                    if (nxt == a) break;
                }
            }
            cur = get(e[0]);
        }
        for (auto [x, i] : stack) {
            color[x] = 2;
        }
    };
    if (recover) {
        pq<pair<int, int>, greater<>> dijkstra;
        for (auto [x, i] : stack) {
            dijkstra.emplace(x, i);
        }
        while (!dijkstra.empty()) {
            auto [t, i] = dijkstra.top();
            dijkstra.pop();
            if (color[ed[i][1]] == 3) {
                continue;
            }
            color[ed[i][1]] = 3;
            take.push_back(i);
            for (auto [t2, i2] : g[ed[i][1]]) {
                dijkstra.emplace(t2, i2);
            }
        }
    }
    for (int i = 1; i < n; ++i) go(get(i));
}

```



```
    return {ans, take};
}
//f245b7
```

## 4 Свёртки

### 4.1 XOR свёртка

```
vector<int> bxor(vector<int> a, vector<int> b)
{
    assert(p%2==1); int inv2=(p+1)/2;
    int n=0; while((1<<n)<a.size()) ++n;
    a.resize(1<<n); b.resize(1<<n);
    for(int i=0; i<n; ++i) for(int mask=0; mask<(1<<n); ++mask)
        if(!(mask & (1<<i))) {int u=a[mask], v=a[mask+(1<<i)];
            a[mask+(1<<i)]=(u+v)%p; a[mask]=(u-v)%p;}
    for(int i=0; i<n; ++i) for(int mask=0; mask<(1<<n); ++mask)
        if(!(mask & (1<<i))) {int u=b[mask], v=b[mask+(1<<i)];
            b[mask+(1<<i)]=(u+v)%p; b[mask]=(u-v)%p;}
    vector<int> c(1<<n, 0);
    for(int mask=0; mask<(1<<n); ++mask) {c[mask]=a[mask]*b[mask]; c[mask]%=p;}
    for(int i=0; i<n; ++i) for(int mask=0; mask<(1<<n); ++mask)
        if(!(mask & (1<<i))) {int u=c[mask], v=c[mask+(1<<i)];
            c[mask+(1<<i)]=(v-u)*inv2%p; c[mask]=(u+v)*inv2%p;}
    return c;
}
//20cc50
```

### 4.2 FFT & co

```
typedef long long ll;
const int p=998244353;
int po(int a, int b) {if(b==0) return 1; if(b==1) return a;
    if(b%2==0) {int u=po(a, b/2); return (u*1LL*u)%p;}
    else {int u=po(a, b-1); return (a*1LL*u)%p;}}
int inv(int x) {return po(x, p-2);}
template<int M, int K, int G> struct Fft {
    // 1, 1/4, 1/8, 3/8, 1/16, 5/16, 3/16, 7/16, ...
    int g[1<<(K-1)];
    Fft() : g() { //if t1 constexpr...
        // static_assert(K >= 2, "Fft: K >= 2 must hold");
        g[0] = 1;
        g[1<<(K-2)] = G;
        for (int l = 1<<(K-2); l >= 2; l >>= 1) {
            g[l>>1] = (g[l] * 1LL * g[l]) % M;
        }
        assert((g[1]*1LL*g[1]) % M == M - 1);
        for (int l = 2; l <= 1<<(K-2); l <<= 1) {
            for (int i = 1; i < l; ++i) {
                g[l+i] = (g[l] * 1LL * g[i]) % M;
            }
        }
    }
    void fft(vector<int> &x) const {
        const int n = x.size();
        assert(n <= 1<<K);
        for (int h = __builtin_ctz(n); h--;) {
            const int l = (1<<h);
            for (int i = 0; i < n; ++i) {
                for (int j = i<<(h+1); j < (((i<<1)+1)<<h); ++j) {
                    const int t = (g[i] * 1LL * x[j+l]) % M;
                    x[j+l] = x[j] - t;
                    if (x[j+l] < 0) x[j+l] += M;
                    x[j] += t;
                    if (x[j] >= M) x[j] -= M;
                }
            }
        }
        for (int i = 0, j = 0; i < n; ++i) {
            if (i < j) std::swap(x[i], x[j]);
            for (int l = n; (l>>=1) && !((j^=1) & 1);) {}
        }
    }
    vector<int> convolution(vector<int> a, vector<int> b) const {
        if(a.empty() || b.empty()) return {};
```

```
        for(int& x:a) {x%=p; if(x>=p) x-=p; if(x<0) x+=p;}
        for(int& x:b) {x%=p; if(x>=p) x-=p; if(x<0) x+=p;}
        const int na = a.size(), nb = b.size();
        int n, invN = 1;
        for (n = 1; n < na + nb - 1; n <= 1) invN = ((invN & 1) ? (invN + M) : invN) >> 1;
        vector<int> x(n, 0), y(n, 0);
        std::copy(a.begin(), a.end(), x.begin());
        std::copy(b.begin(), b.end(), y.begin());
        fft(x);
        fft(y);
        for (int i = 0; i < n; ++i) x[i] = (((static_cast<long long>(x[i]) * y[i]) % M) * invN) % M);
        std::reverse(x.begin() + 1, x.end());
        fft(x);
        x.resize(na + nb - 1);
        return x;
    }
};
Fft<998244353, 23, 31> muls;
//a1b591

vector<int> form(vector<int> v, int n)
{
    while(v.size()<n) v.push_back(0);
    while(v.size()>n) v.pop_back();
    return v;
}
vector<int> operator *(vector<int> v1, vector<int> v2)
{
    return muls.convolution(v1, v2);
}
vector<int> operator +(vector<int> v1, vector<int> v2)
{
    while(v2.size()<v1.size()) v2.push_back(0); while(v1.size()<v2.size()) v1.push_back(0);
    for(int i=0; i<v1.size(); ++i) {v1[i]+=v2[i]; if(v1[i]>=p) v1[i]-=p; else if(v1[i]<0) v1[i]+=p;}
    return v1;
}
vector<int> operator -(vector<int> v1, vector<int> v2)
{
    int sz=max(v1.size(), v2.size()); while(v1.size()<sz) v1.push_back(0); while(v2.size()<sz) v2.push_back(0);
    for(int i=0; i<sz; ++i) {v1[i]-=v2[i]; if(v1[i]<0) v1[i]+=p; else if(v1[i]>=p) v1[i]-=p;}
    return v1;
}
vector<int> trmi(vector<int> v)
{
    for(int i=1; i<v.size(); i+=2) {if(v[i]>0) v[i]=p-v[i]; else v[i]=-v[i];}
    return v;
}
vector<int> deriv(vector<int> v)
{
    if(v.empty()) return {};
    vector<int> ans(v.size()-1);
    for(int i=1; i<v.size(); ++i) ans[i-1]=(v[i]*1LL*i)%p;
    return ans;
}
vector<int> integ(vector<int> v)
{
    vector<int> ans(v.size()+1); ans[0]=0;
    for(int i=1; i<v.size(); ++i) ans[i-1]=(v[i]*1LL*i)%p;
    return ans;
}
vector<int> mul(vector<vector<int>> v)
{
    if(v.size()==1) return v[0];
    vector<vector<int>> > v1, v2; for(int i=0; i<v.size()/2; ++i) v1.push_back(v[i]); for(int i=v.size()/2; i<v.size(); ++i) v2.push_back(v[i]);
    return muls.convolution(mul(v1), mul(v2));
}
vector<int> invl(vector<int> v, int n)
{
    assert(v[0]!=0);
    int sz=1; v=form(v, n); vector<int> a={inv(v[0])};
    while(sz<n)
    {
```

```

vector<int> vsz;for(int i=0;i<min(n,2*sz);++i)
vsz.push_back(v[i]);
vector<int> b=((vector<int>) {1})-muls.
convolution(a,vsz);
for(int i=0;i<sz;++i) assert(b[i]==0);
b.erase(b.begin(),b.begin()+sz);
vector<int> c=muls.convolution(b,a);
for(int i=0;i<sz;++i) a.push_back(c[i]);
sz*=2;
}
return form(a,n);
}
//12aa4e

```

### 4.3 Быстрое FFT

- Solution based on <https://codeforces.com/blog/entry/117947>
- Iterative and in-place version.
- Uses signed montgomery
- Optimized to minimize memory usage

```

const int MOD = 998244353;
const long long MOD2 = (long long) MOD * MOD;
const int root = 3;
const int alim = 64; // Bound for using O(n^2)
                        polynomial mult

int modpow(int b, int e) {
    int ans = 1;
    for (; e; b = (long long) b * b % MOD, e /= 2)
        if (e & 1) ans = (long long) ans * b % MOD;
    return ans;
}

const int MODinv = 2 - MOD; // pow(-MOD, -1, 2**32)
inline int m_reduce(long long x) {
    int m = x * MODinv;
    return (x >> 32) - (((long long) m * MOD) >> 32);
}

const int r2 = modpow(2, 64);
inline int m_transform(int x) {
    return m_reduce((long long)x * r2);
}

inline int m_add(int x, int y) {
    int z = x + y;
    return z < 0 ? z + MOD : z - MOD;
}

inline int m_sub(int x, int y) {
    int z = x - y;
    return z < 0 ? z + MOD : z - MOD;
}

inline int m_mult(int x, int y) {
    return m_reduce((long long) x * y);
}

vector<int> rt = {1};
vector<int> transformed_rt;
vector<int> transformed_rt2;

template<int a>
void transform(vector<int> &P) {
    int m = P.size();
    int n = m / a;

    int size = rt.size();
    while (2 * size < n) {
        rt.resize(n / 2);
        int r = modpow(root, MOD / (4 * size));
        for (int i = 0; i < size; ++i)
            rt[i + size] = (long long) r * rt[i] % MOD;
        size *= 2;
    }

    // For montgomery
    for (int i = transformed_rt.size(); i < rt.size();
        ++i) {
        transformed_rt.resize(rt.size());
        transformed_rt[i] = m_transform(rt[i]);
        transformed_rt2.resize(rt.size());
        transformed_rt2[i] = (unsigned int) MODinv *
            transformed_rt[i];
    }
}

```

```

int k = n;
while (k >= 4) k /= 4;

if (k == 2) {
    int step = n * a;
    int half_step = step / 2;
    for (int j1 = 0; j1 < half_step; ++j1) {
        int j2 = j1 + half_step;

        int diff = m_sub(P[j1], P[j2]);
        P[j1] = m_add(P[j1], P[j2]);
        P[j2] = diff;
    }
    k = n/2;
} else {
    k = n;

    for (; k > 1; k /= 4) {
        for (int i = 0; i < n/k; ++i) {
            int step = k * a;
            int half_step = step / 2;
            int quarter_step = half_step / 2;

            int R20 = transformed_rt2[2 * i];
            int RR0 = transformed_rt[2 * i];

            int R21 = transformed_rt2[2 * i + 1];
            int RR1 = transformed_rt[2 * i + 1];

            int R2 = transformed_rt2[i];
            int RR = transformed_rt[i];

            int j1 = i * step;
            int j2 = j1 + quarter_step;
            int j3 = j2 + quarter_step;
            int j4 = j3 + quarter_step;

            for (int j = 0; j < quarter_step; ++j, ++j1,
                ++j2, ++j3, ++j4) {
                int z0;
                {
                    int z = P[j3];
                    int m = (unsigned int) R2 * z;
                    z0 = ((long long) z * RR - (long long) m *
                        MOD) >> 32;
                }

                int z1;
                {
                    int z = P[j4];
                    int m = (unsigned int) R2 * z;
                    z1 = ((long long) z * RR - (long long) m *
                        MOD) >> 32;
                }

                int sum0 = m_add(P[j1], z0);
                int diff0 = m_sub(P[j1], z0);
                int sum1 = P[j2] + z1;
                int diff1 = P[j2] - z1;

                // [sum0, sum1, diff0, diff1]

                int zz0;
                {
                    int z = sum1;
                    int m = (unsigned int) R20 * z;
                    zz0 = ((long long) z * RR0 - (long long) m *
                        MOD) >> 32;
                }

                int zz1;
                {
                    int z = diff1;
                    int m = (unsigned int) R21 * z;
                    zz1 = ((long long) z * RR1 - (long long) m *
                        MOD) >> 32;
                }

                P[j1] = m_add(sum0, zz0);
                P[j2] = m_sub(sum0, zz0);
                P[j3] = m_add(diff0, zz1);
                P[j4] = m_sub(diff0, zz1);
            }
        }
    }

    for (int i = 0; i < m; ++i)

```



```

    if (P[i] < 0) P[i] += MOD;
}

template<int a>
void inverse_transform(vector<int> &P) {
    int m = P.size();
    int n = m / a;
    int n_inv = m_transform(modpow(n, MOD - 2));

    vector<int> rev(n);
    for (int i = 1; i < n; ++i) {
        rev[i] = rev[i / 2] / 2 + (i & 1) * n / 2;
    }

    // P = [p * n_inv for p in P]
    for (int i = 0; i < m; ++i)
        P[i] = m_mult(n_inv, P[i]);

    // P = [P[a * rev[i] // a] + (i % a)] for i in range(m)
    for (int i = 1; i < n; ++i)
        if (i < rev[i])
            swap_ranges(P.begin() + a * i, P.begin() + a *
                i + a, P.begin() + a * rev[i]);

    // P = [P[-a * (i // a) + (i % a)] for i in range(m)]
    for (int i = 1; i < n/2; ++i)
        swap_ranges(P.begin() + a * i, P.begin() + a * i
            + a, P.begin() + a * (n - i));

    transform<a>(P);

    // P = [P[a * rev[i] // a] + (i % a)] for i in range(m)
    for (int i = 1; i < n; ++i)
        if (i < rev[i])
            swap_ranges(P.begin() + a * i, P.begin() + a *
                i + a, P.begin() + a * rev[i]);
}

template<int a>
void fast_polymult_mod(vector<int> &P, vector<int> &
    Q) {
    int m = P.size();
    int n = m / a;

    transform<a>(P);
    transform<a>(Q);

    vector<int> &PQ = P;
    for (int i = 0; i < n; ++i) {
        vector<unsigned long long> res(2 * a);
        for (int j = 0; j < a; ++j) {
            if (j >= 10 && j % 9 == 8)
                for (int k = j; k < j + a - 10; ++k)
                    res[k] -= (res[k] >> 63) * 9 * MOD2;
            for (int k = 0; k < a; ++k)
                res[j + k] += (long long) P[i * a + j] * Q[i
                    * a + k];
        }

        int c = rt[i/2];
        if (i & 1) c = MOD - c;
        for (int j = 0; j < a; ++j)
            PQ[i * a + j] = (res[j] + c * (res[j + a] %
                MOD)) % MOD;
    }

    inverse_transform<a>(PQ);
}

template <size_t... N>
void work(std::index_sequence<N...>, int x, std:::
    vector<int>& a, std:::vector<int>& b) {
    static void (*ptrs[])(std:::vector<int>&, std:::
        vector<int>&) = {&fast_polymult_mod<N+1>...};
    ptrs[x - 1](a, b);
}

void fast_polymult(vector<int> &P, vector<int> &Q) {
    int m1 = P.size();
    int m2 = Q.size();
    int res_len = m1 + m2 - 1;

    int b = 1;
    while ((alim << b) < res_len) ++b;
    int a = ((res_len - 1) >> b) + 1;
    int m = a << b;

```

```

P.resize(m);
Q.resize(m);

// Call fast_polymult_mod<a>(P, Q);
work(std::make_index_sequence<alim>{}, a, P, Q);

P.resize(res_len);
}
//239b3e

```

## 4.4 FFT в double'ax

```

using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> &a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j < bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (cd &x : a)
            x /= n;
    }
}

vector<int> multiply(vector<int> const& a, vector<
    int> const& b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b
        .end());
    int n = 1;
    while (n < a.size() + b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);

    vector<int> result(n);
    for (int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    while(!result.empty() && !result.back()) result.
        pop_back();
    return result;
}
//35d9d0

```

## 5 Структуры данных

### 5.1 Дерево Фенвика

```

int fe[maxn];
void pl(int pos, int val) {while(pos<maxn) {fe[pos]+=
    val;pos|=(pos+1);}}
int get(int pos) {int ans=0;while(pos>=0) {ans+=fe[
    pos];pos&=(pos+1);--pos;} return ans;} /// [0,
    pos] - vkluchitelno!!!
int get(int l, int r) {return get(r-1)-get(l-1);} ///
    sum of [l,r)
//2991a1

```

## 5.2 Дерево отрезков в точке

```
template<typename T, typename U>
struct SegmentTree {
    int h, n;
    T neutral;
    U unite;
    vector<T> data;

    template<typename I>
    SegmentTree(int sz, T neutral, U unite, I init) :
        h(__lg(sz) + 1), n(1 << h), neutral(neutral),
        unite(unite), data(2 * n) {
        for (int i = 0; i < sz; ++i) data[i + n] = init(i);
        for (int i = n - 1; i > 0; --i) data[i] = unite(
            data[2 * i], data[2 * i + 1]);
    }

    SegmentTree(int sz, T neutral, U unite) : h(__lg(
        sz) + 1), n(1 << h), neutral(neutral), unite(
        unite), data(2 * n, neutral) {}

    void set(int i, T x) {
        data[i + n] = x;
        for (i /= 2; i > 0; i /= 2) data[i] = unite(data
            [2 * i], data[2 * i + 1]);
    }

    T get(int l, int r) {
        T leftRes = neutral, rightRes = neutral;
        for (l += n, r += n; l < r; l /= 2, r /= 2) {
            if (l & 1) leftRes = unite(leftRes, data[l++]);
            if (r & 1) rightRes = unite(data[--r],
                rightRes);
        }
        return unite(leftRes, rightRes);
    }

    int left(int i) {
        int lvl = __lg(i);
        return (i & ((1 << lvl) - 1)) * (1 << (h - lvl))
            ;
    }

    int right(int i) {
        int lvl = __lg(i);
        return ((i & ((1 << lvl) - 1)) + 1) * (1 << (h -
            lvl));
    }

    // l \in [0; n) && ok(get(l, l), l);
    // returns last r: ok(get(l, r), r)
    template<typename C>
    int lastTrue(int l, C ok) {
        T cur = neutral;
        l += n;
        do {
            l >>= __builtin_ctz(l);
            T with1 = unite(cur, data[l]);
            if (ok(with1, right(l))) {
                cur = with1;
                ++l;
            } else {
                while (l < n) {
                    T with2 = unite(cur, data[2 * l]);
                    if (ok(with2, right(2 * l))) {
                        cur = with2;
                        l = 2 * l + 1;
                    } else {
                        l = 2 * l;
                    }
                }
                return l - n;
            }
        } while (l & (l - 1));
        return n;
    }

    // r \in [0; n) && ok(get(r, r), r);
    // returns first l: ok(get(l, r), l)
    template<typename C>
    int firstTrue(int r, C ok) {
        T cur = neutral;
        r += n;
        while (r & (r - 1)) {
            r >>= __builtin_ctz(r);
            T with1 = unite(data[--r], cur);
            if (ok(with1, left(r))) {
                cur = with1;
            }
        }
    }
};
```

```
    } else {
        while (r < n) {
            T with2 = unite(data[2 * r + 1], cur);
            if (ok(with2, left(2 * r + 1))) {
                cur = with2;
                r = 2 * r;
            } else {
                r = 2 * r + 1;
            }
        }
        return r - n + 1;
    }
}

return 0;
};
//64190d
```

## 5.3 Массовое дерево отрезков

```
template<typename T, typename M, typename Ud,
    typename Um, typename A>
struct MassSegmentTree {
    int h, n;
    T zd;
    M zm;
    vector<T> data;
    vector<M> mod;

    Ud ud; // T (T, T)
    Um um; // M (M, M);
    A a; // T (T, M, int); last argument is the
    length of current segment (could be used for
    range += and sum counting, for instance)

    template<typename I>
    MassSegmentTree(int sz, T zd, M zm, Ud ud, Um um,
        A a, I init) : h(__lg(sz) + 1), n(1 << h), zm(
        zm), zd(zd), data(2 * n, zd), mod(n, zm), ud(ud)
        , um(um), a(a) {
        for (int i = 0; i < sz; ++i) data[i + n] =
            init(i);
        for (int i = n - 1; i > 0; --i) data[i] = ud
            (data[2 * i], data[2 * i + 1]);
    }

    MassSegmentTree(int sz, T zd, M zm, Ud ud, Um um,
        A a) : h(__lg(sz) + 1), n(1 << h), zm(zm), zd(
        zd), data(2 * n, zd), mod(n, zm), ud(ud), um(um)
        , a(a) {}

    void push(int i) {
        if (mod[i] == zm) return;
        apply(2 * i, mod[i]);
        apply(2 * i + 1, mod[i]);
        mod[i] = zm;
    }

    // is used only for apply
    int length(int i) { return 1 << (h - __lg(i)); }

    // used only for descent
    int left(int i) {
        int lvl = __lg(i);
        return (i & ((1 << lvl) - 1)) * (1 << (h -
            lvl));
    }

    // used only for descent
    int right(int i) {
        return left(i) + length(i);
    }

    template<typename S>
    void apply(int i, S x) {
        data[i] = a(data[i], x, length(i));
        if (i < n) mod[i] = um(mod[i], x);
    }

    void update(int i) {
        if (mod[i] != zm) return;
        data[i] = ud(data[2 * i], data[2 * i + 1]);
    }

    template<typename S>
    void update(int l, int r, S x) { // [l; r)
        l += n, r += n;
        for (int shift = h; shift > 0; --shift) {
            push(l >> shift);
            push((r - 1) >> shift);
        }
    }
};
```

```

    }
    for (int lf = 1, rg = r; lf < rg; lf /= 2,
    rg /= 2) {
        if (lf & 1) apply(lf++, x);
        if (rg & 1) apply(--rg, x);
    }
    for (int shift = 1; shift <= h; ++shift) {
        update(l >> shift);
        update((r - 1) >> shift);
    }
}

T get(int l, int r) { // [l; r)
    l += n, r += n;
    for (int shift = h; shift > 0; --shift) {
        push(l >> shift);
        push((r - 1) >> shift);
    }
    T leftRes = zd, rightRes = zd;
    for (; l < r; l /= 2, r /= 2) {
        if (l & 1) leftRes = ud(leftRes, data[l
++]);
        if (r & 1) rightRes = ud(data[--r],
rightRes);
    }
    return ud(leftRes, rightRes);
}

// l \in [0; n) && ok(get(l, l), l);
// returns last r: ok(get(l, r), r)
template<typename C>
int lastTrue(int l, C ok) {
    l += n;
    for (int shift = h; shift > 0; --shift) push
(l >> shift);
    T cur = zd;
    do {
        l >>= __builtin_ctz(l);
        T with = ud(cur, data[l]);
        if (ok(with, right(l))) {
            cur = with;
            ++l;
        } else {
            while (l < n) {
                push(l);
                l = 2 * l;
                with = ud(cur, data[l]);
                if (ok(with, left(l + 1))) {
                    cur = with;
                    ++l;
                }
            }
            return l - n;
        }
    } while (l & (l - 1));
    return n;
}

// r \in [0; n) && ok(get(r, r), r);
// returns first l: ok(get(l, r), l)
template<typename C>
int firstTrue(int r, C ok) {
    r += n;
    for (int shift = h; shift > 0; --shift) push
((r - 1) >> shift);
    auto cur = zd;
    while (r & (r - 1)) {
        r >>= __builtin_ctz(r);
        T with = ud(data[--r], cur);
        if (ok(with, left(r))) {
            cur = with;
        } else {
            while (r < n) {
                push(r);
                r = 2 * r;
                with = ud(data[r + 1], cur);
                if (ok(with, left(r + 1))) {
                    cur = with;
                } else {
                    ++r;
                }
            }
            return r - n + 1;
        }
    }
    return 0;
}
};
//fc0cde

```

## 5.4 Битовый бор

```

template<unsigned int sz, typename T=int>
struct binarytrie{
    using Bit=typename conditional<sz<=32, unsigned int
, unsigned long long>::type;
    struct node{
        T cnt;
        array<int, 2>nxt;
        node():cnt(0),nxt({-1,-1}){}
    };
    vector<node>v;
    binarytrie(){v.emplace_back();}
    void insert(Bit x){add(x, 1);}
    void erase(Bit x){add(x, -1);}
    void add(Bit x, T k)
    {
        assert(0<=x&&(x>>sz)==0);
        int p=0;
        v[p].cnt+=k;
        for(int i=sz; i--;)
        {
            int j=x>>i&1;
            if(v[p].nxt[j]==-1)
            {
                v[p].nxt[j]=v.size();
                v.emplace_back();
            }
            p=v[p].nxt[j];
            v[p].cnt+=k;
        }
    }
    T count(Bit x, Bit xor_val=0) const//[0, x)
    {
        assert(0<=xor_val&&(xor_val>>sz)==0);
        if(x<0) return 0;
        else if(x>>sz) return v[0].cnt;
        T ret=0;
        int p=0;
        for(int i=sz; i--;)
        {
            int j=x>>i&1, k=xor_val>>i&1;
            if(j==0)p=v[p].nxt[k];
            else
            {
                if(v[p].nxt[k]>=0) ret+=v[v[p].nxt[k]].cnt;
                p=v[p].nxt[!k];
            }
            if(p==-1) break;
        }
        return ret;
    }
    Bit max(Bit xor_val=0) const
    {
        assert(0<=xor_val&&(xor_val>>sz)==0);
        int p=0;
        Bit ret=0;
        if(v[p].cnt==0) return ret;
        for(int i=sz; i--;)
        {
            ret<=1;
            int k=xor_val>>i&1;
            if(v[p].nxt[!k]>=0&&v[v[p].nxt[!k]].cnt>0)
            {
                p=v[p].nxt[!k];
                ret|=1;
            }
            else p=v[p].nxt[k];
        }
        return ret;
    }
    Bit min(Bit xor_val=0) const
    {
        assert(0<=xor_val&&(xor_val>>sz)==0);
        int p=0;
        Bit ret=0;
        for(int i=sz; i--;)
        {
            ret<=1;
            int k=xor_val>>i&1;
            if(v[p].nxt[k]>=0&&v[v[p].nxt[k]].cnt>0)p=v[p
].nxt[k];
            else
            {
                p=v[p].nxt[!k];
                ret|=1;
            }
        }
        return ret;
    }
};

```

```

}
Bit find_by_order(T ord, Bit xor_val=0) const
{
    assert(0<=xor_val&&(xor_val>>sz)==0);
    assert(0<=ord&&ord<v[0].cnt);
    int p=0;
    Bit ret=0;
    for(int i=sz; i-->0)
    {
        ret<<=1;
        int k=xor_val>>i&1;
        if(v[p].nxt[k]>=0)
        {
            if(ord>=v[v[p].nxt[k]].cnt)
            {
                ord-=v[v[p].nxt[k]].cnt;
                p=v[p].nxt[!k];
                ret|=1;
            }
            else p=v[p].nxt[k];
        }
        else
        {
            p=v[p].nxt[!k];
            ret|=1;
        }
    }
    return ret;
}
T order_of_key(Bit x, Bit xor_val=0) const {return
    count(x, xor_val);}
};
binarytrie<32>bt;
//0b3855

```

## 5.5 Ordered set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;
using namespace std;

using ordered_set = tree<int, null_type, less<>,
    rb_tree_tag, tree_order_statistics_node_update>;
//f589b9

```

## 5.6 Динамический битсет

```

#include <tr2/dynamic_bitset>
using namespace tr2;
using bs=dynamic_bitset<>;
//26f8b6

```

## 5.7 Convex hull trick

```

int div_up(int a, int b) { return a/b+((a^b)>0&&a%b); }
// divide a by b rounded up
const int LQ = ..., RQ = ...; //leftmost query,
    rightmost query
int in(ii L, int x) {
    return L.x * x + L.y;
}
struct Hull {
    vector<pair<int, int>> lines;
    vector<int> borders;
    void push(ii L) {
        while (lines.size() && in(L, borders.back()) < in(
            lines.back(), borders.back())) {
            lines.pop_back();
            borders.pop_back();
        }
        if (lines.empty()) {
            lines = {L};
            borders = {LQ};
        }
        else if (lines.back().x > L.x) {
            int x = div_up(L.y - lines.back().y, lines.back().x - L.x);
            if (x <= RQ) {
                lines.append(L);
                borders.append(x);
            }
        }
    }
};
Hull (){}
Hull (vector<ii> a) {

```

```

    auto comp = [&] (ii u, ii v) {
        return u.x > v.x || (u.x == v.x && u.y < v.y);
    };
    sort(all(a), comp);
    for (auto L : a) {
        push(L);
    }
}
int get(int x) {
    int pos = upper_bound(all(borders), x) - borders.begin();
    assert(pos>0);
    pos--;
    return in(lines[pos], x);
}
};
//04555a

```

## 5.8 Центроиды

```

vector<int> sz(n), lvl(n, -1);
auto dfs = [&] (auto dfs, int cur, int prev) -> int {
    if (lvl[cur] != -1) return 0;
    sz[cur] = 1;
    for (auto [nxt, w] : g[cur]) {
        if (nxt != prev) sz[cur] += dfs(dfs, nxt, cur);
    }
    return sz[cur];
};
auto find = [&] (auto find, int cur, int prev, int tot) -> int {
    int bch = -1, bsz = 0;
    for (auto [nxt, w] : g[cur]) {
        if (nxt == prev || lvl[nxt] != -1) continue;
        if (sz[nxt] > bsz) {
            bch = nxt;
            bsz = sz[nxt];
        }
    }
    if (bsz + bsz <= tot) return cur;
    return find(find, bch, cur, tot);
};
dfs(dfs, 0, 0);
auto c = find(find, 0, 0, sz[0]);
vector<pair<int, int>> stack{{c, 0}};
while (!stack.empty()) {
    auto [centroid, l] = stack.back();
    stack.pop_back();
    lvl[centroid] = l;
    for (auto [nxt, w] : g[centroid]) {
        if (lvl[nxt] != -1) continue;
        dfs(dfs, nxt, centroid);
        int new_centroid = find(find, nxt, centroid, sz[nxt]);
        stack.push_back({new_centroid, lvl[centroid] + 1});
    }
}
//0e1e52

```

## 5.9 Дерево Ли Чао

```

struct Line {
    int a, b;
    Line(){}
    Line (int a, int b) : a(a), b(b) {}
    int get(int x) { return a + b * x; }
};

struct Lichao {
    int n;
    vector<int> x;
    vector<Line> t;
    Lichao(){}
    Lichao (int n, vector<int> x) : n(n), t(n << 2,
        Line(0, 0)), x(x) {}

    void put(int v, int l, int r, Line L) {
        if (l + 1 == r) {
            if (L.get(x[l]) < t[v].get(x[l])) {
                t[v] = L;
            }
            return;
        }
        int m = (l + r) / 2;
        if (L.get(x[m]) < t[v].get(x[m])) {
            swap(L, t[v]);
        }
    }
};

```

```

    if (L.b > t[v].b) {
        put(2 * v + 1, l, m, L);
    }
    else {
        put(2 * v + 2, m, r, L);
    }
}

int get(int v, int l, int r, int i) {
    if (l + 1 == r) {
        return t[v].get(x[l]);
    }
    int m = (l + r) / 2;
    int ans = t[v].get(x[i]);
    if (i < m) {
        ans = min(ans, get(2 * v + 1, l, m, i));
    }
    else {
        ans = min(ans, get(2 * v + 2, m, r, i));
    }
    return ans;
}

void put(Line L) {
    put(0, 0, n, L);
}

int get(int i) {
    return get(0, 0, n, i);
}
};
//99f5fa

```

## 5.10 Min-Kinetic Segment Tree

I guess the source is <https://koosaga.com/307>

```

using lint = long long;
const lint inf = 4e18;
const int MAXT = 4100000;
using pi = array<lint, 2>;

struct line {
    lint A, B;
    int idx;

    lint eval(lint x) { return A * x + B; }

    // returns the x-intercept of intersection "
    // strictly" larger than T
    lint cross_after(line &x, lint T) {
        if (x.A == A) {
            return inf;
        }
        lint up = x.B - B;
        lint dn = A - x.A;
        if (dn < 0) {
            dn *= -1;
            up *= -1;
        }
        lint incep = (up <= 0 ? -((-up) / dn) : (up + dn
            - 1) / dn);
        if (incep > T)
            return incep;
        return inf;
    }
};

struct kst { // min kinetic segment tree
    line tree[MAXT];
    lint melt[MAXT], T;
    pi lazy[MAXT];
    int n;

    bool cmp(line &a, line &b) {
        lint l = a.eval(T), r = b.eval(T);
        if (l != r)
            return l > r;
        return a.A > b.A;
    }

    void pull(int p) {
        tree[p] = cmp(tree[2 * p], tree[2 * p + 1]) ?
            tree[2 * p + 1] : tree[2 * p];
        melt[p] = min({melt[2 * p], melt[2 * p + 1],
            tree[2 * p].cross_after(tree[2 * p + 1], 0)});
    }

    void init(int s, int e, int p, vector<line> &l) {
        if (s == e) {
            tree[p] = l[s];
            melt[p] = inf;

```

```

            lazy[p] = {0, 0};
            return;
        }
        lazy[p] = {0, 0};
        int m = (s + e) / 2;
        init(s, m, 2 * p, l);
        init(m + 1, e, 2 * p + 1, l);
        pull(p);
    }

    void lazydown(int p) {
        for (int i = 2 * p; i < 2 * p + 2; i++) {
            lazy[i][0] += lazy[p][0];
            lazy[i][1] += lazy[p][1];
            tree[i].B += lazy[p][0] * tree[i].A + lazy[p]
                ][1];
            melt[i] -= lazy[p][0];
        }
        lazy[p][0] = lazy[p][1] = 0;
    }

    void propagate(int p) {
        if (melt[p] > 0)
            return;
        lazydown(p);
        propagate(2 * p);
        propagate(2 * p + 1);
        pull(p);
    }

    lint query(int s, int e, int ps, int pe, int p =
        1) {
        if (e < ps || pe < s)
            return inf;
        if (s <= ps && pe <= e)
            return tree[p].eval(0);
        int pm = (ps + pe) / 2;
        lazydown(p);
        return min(query(s, e, ps, pm, 2 * p), query(s,
            e, pm + 1, pe, 2 * p + 1));
    }

    void heaten(int s, int e, int ps, int pe, int p,
        lint v) {
        if (e < ps || pe < s)
            return;
        if (s <= ps && pe <= e) {
            lazy[p][0] += v;
            tree[p].B += v * tree[p].A;
            melt[p] -= v;
            propagate(p);
            return;
        }
        lazydown(p);
        int pm = (ps + pe) / 2;
        heaten(s, e, ps, pm, 2 * p, v);
        heaten(s, e, pm + 1, pe, 2 * p + 1, v);
        pull(p);
    }

    void add(int s, int e, int ps, int pe, int p, lint
        v) {
        if (e < ps || pe < s)
            return;
        if (s <= ps && pe <= e) {
            lazy[p][1] += v;
            tree[p].B += v;
            return;
        }
        lazydown(p);
        int pm = (ps + pe) / 2;
        add(s, e, ps, pm, 2 * p, v);
        add(s, e, pm + 1, pe, 2 * p + 1, v);
        pull(p);
    }

    void init(vector<line> &l, lint _T) {
        n = l.size();
        T = _T;
        init(0, n - 1, 1, l);
    }
};
//66f9a9

```

## 5.11 Декартово дерево

### 5.11.1 Декартово дерево по явному ключу. Multiset

```

mt19937 rng(0);

using ptr = int32_t;

struct vertex {
    ptr lf = 0, rg = 0;
    int32_t heap = rng(), rnd = rng(), sz = 1;
    int val, sum = 0;

    vertex(int x = 0) : val(x), sum(x) {}
};

vector<vertex> mem;

ptr new_vertex(int x) {
    mem.app(vertex(x));
    return (int)mem.size()-1;
}

ptr update(ptr v) {
    mem[v].sz = mem[mem[v].lf].sz + 1 + mem[mem[v].rg].sz;
    mem[v].sum = mem[mem[v].lf].sum + mem[v].val + mem[mem[v].rg].sum;
    return v;
}

ptr merge(ptr l, ptr r) {
    if (!l || !r) return l ^ r;
    if (mem[l].heap > mem[r].heap) {
        mem[l].rg = merge(mem[l].rg, r);
        return update(l);
    } else {
        mem[r].lf = merge(l, mem[r].lf);
        return update(r);
    }
}

pair<ptr, ptr> splitkey(ptr v, int x, int32_t rnd) {
    if (!v) return {v, v};
    if (pair{mem[v].val, mem[v].rnd} < pair{x, rnd}) {
        auto [lf, rg] = splitkey(mem[v].rg, x, rnd);
        mem[v].rg = lf;
        return {update(v), rg};
    } else {
        auto [lf, rg] = splitkey(mem[v].lf, x, rnd);
        mem[v].lf = rg;
        return {lf, update(v)};
    }
}

void insert(ptr &a, ptr b) {
    if (!a) {
        a = b;
        return;
    }
    if (mem[a].heap > mem[b].heap) {
        if (pair{mem[a].val, mem[a].rnd} < pair{mem[b].val, mem[b].rnd}) {
            insert(mem[a].rg, b);
        } else {
            insert(mem[a].lf, b);
        }
        update(a);
    } else {
        auto [lf, rg] = splitkey(a, mem[b].val, mem[b].rnd);
        mem[b].lf = lf;
        mem[b].rg = rg;
        a = update(b);
    }
}

void join(ptr &a, ptr b) {
    auto dfs = [&](auto dfs, ptr b) -> void {
        if (!b) return;
        ptr lf = mem[b].lf;
        ptr rg = mem[b].rg;
        mem[b].lf = mem[b].rg = 0;
        insert(a, update(b));
        dfs(dfs, lf);
        dfs(dfs, rg);
    };
    dfs(dfs, b);
}

pair<ptr, ptr> splitsz(ptr v, int k) {
    if (!v) return {v, v};

```

```

    if (k <= mem[mem[v].lf].sz) {
        auto [l, r] = splitsz(mem[v].lf, k);
        mem[v].lf = r;
        return {l, update(v)};
    } else {
        auto [l, r] = splitsz(mem[v].rg, k - mem[mem[v].lf].sz - 1);
        mem[v].rg = l;
        return {update(v), r};
    }
}

int32_t main() {
    mem = {vertex()};
    mem[0].sz = 0;
}
//54a637

```

## 6 Строковые алгоритмы

### 6.1 Префикс-функция

```

vector<int> prefix_function(string s) {
    int n = s.size();
    vector<int> p(n);
    for (int i = 1; i < n; ++i) {
        p[i] = p[i - 1];
        while (p[i] && s[p[i]] != s[i]) p[i] = p[p[i] - 1];
        p[i] += s[i] == s[p[i]];
    }
    return p;
}
//91103c

```

### 6.2 Z-функция

```

vector<int> z_function(string s) { // z[i] - lcp of s and s[i:]
    int n = s.size();
    vector<int> z(n);
    for (int i=1, l=0, r=0; i<n; ++i) {
        if (i <= r) z[i] = min(r-i+1, z[i-l]);
        while (i+z[i] < n && s[z[i]] == s[i+z[i]]) ++z[i];
        if (i+z[i]-1 > r) {
            l = i, r = i+z[i]-1;
        }
    }
    return z;
}
//48cccd

```

### 6.3 Алгоритм Манакера

```

vector<int> manacher(const string &s, int even) {
    int l = 0, r = 0, n = s.size();
    vector<int> man(n, 0);
    for (int i = 1; i < n; i++) {
        int j = i - even;
        if (j <= r) {
            man[i] = min(r - j, man[l + r - j]);
        }
        while (j + man[i] + 1 < n && i - man[i] > 0 && s[j + man[i] + 1] == s[i - man[i] - 1]) {
            man[i]++;
        }
        if (j + man[i] > r) {
            l = i - man[i];
            r = j + man[i];
        }
    }
    return man;
}

// abacaba : odd : (0 1 0 3 0 1 0); even : (0 0 0 0 0 0 0)
// abbaa : odd : (0 0 0 0 0); even : (0 0 2 0 1)
bool pal(int from, int len) {
    if (len == 0) {
        return true;
    }
    int m = len/2;
    if (len & 1) {
        return odd[from + m] >= m;
    }
}

```



```

    else {
        return even[from + m] >= m;
    }
}
//8a64d6

```

## 6.4 Суфмассив

Переработанный китайский суфмассив

```

const int inf = 1e9;
struct rmq {
    int n;
    vector<int> a;
    void build(const vector<int> &x) {
        assert(x.size() == n);
        for (int i = 0; i < n; ++i) a[n + i] = x[i];
        for (int i = n - 1; i > 0; --i) a[i] = min(a[2 * i], a[2 * i + 1]);
    }
    rmq(int n) : n(n), a(2 * n, inf) {}
    void put(int i, int x) {
        a[i + n] = min(a[i + n], x);
        for (i = (i + n) / 2; i > 0; i /= 2) {
            a[i] = min(a[i * 2], a[i * 2 + 1]);
        }
    }
    int getMin(int l, int r) { //[l;r)
        assert(l < r);
        int res = inf;
        for (l += n, r += n; l < r; l /= 2, r /= 2) {
            if (l & 1) res = min(res, a[l++]);
            if (r & 1) res = min(res, a[--r]);
        }
        return res;
    }
};

template <typename T>
vector<int> SA(const T &a) {
    int m = *max_element(all(a)) + 1, n = a.size();
    vector<int> sa(n), nsa(n), pre(max(n, m)), x(a.begin(), a.end()), y(n);
    for (int e : x) pre[e]++;
    for (int i = 1; i < m; ++i) pre[i] += pre[i - 1];
    for (int i = 0; i < n; ++i) sa[--pre[x[i]]] = i;
    int dif = 1;
    y[sa.front()] = 0;
    for (int i = 1; i < n; ++i) {
        dif += x[sa[i]] != x[sa[i - 1]];
        y[sa[i]] = dif - 1;
    }
    x = y;
    for (int h = 1; dif < n; h *= 2) {
        fill(all(pre), 0);
        for (int e : x) pre[e]++;
        for (int i = 1; i < dif; ++i) pre[i] += pre[i - 1];
        for (int t = n; t--; ) {
            int i = sa[t];
            if (i >= h) {
                nsa[--pre[x[i - h]]] = i - h;
            }
            else if (i + 1 != h) {
                nsa[--pre[x[i - h + n + 1]]] = i - h + n + 1;
            }
        }
        nsa[--pre[x[n - h]]] = n - h;
        sa = nsa;
        auto getr = [&](int i) {
            if (i + h < n) {
                return x[i + h];
            }
            else {
                return x[i + h - n - 1];
            }
        };
        dif = 1;
        y[sa.front()] = 0;
        for (int i = 1; i < n; ++i) {
            if (x[sa[i]] != x[sa[i - 1]] || sa[i - 1] + h == n) {
                dif++;
            }
            else {
                dif += getr(sa[i]) != getr(sa[i - 1]);
            }
            y[sa[i]] = dif - 1;
        }
        x = y;
    }
    return sa;
}

```

```

}

template <typename T>
struct suar {
    vector<int> sa, lcp, pos; rmq t;
    suar(const T &a) : t((int)a.size() - 1) {
        sa = SA(a);
        int n = (int)a.size(), k = 0;
        lcp.resize(n - 1);
        pos.resize(n);
        for (int i = 0; i < n; ++i) pos[sa[i]] = i;
        for (int i = 0; i < n; ++i) {
            if (pos[i] + 1 < n) {
                int j = sa[pos[i] + 1];
                while (i + k < n && j + k < n && a[i + k] == a[j + k]) k++;
                lcp[pos[i]] = k;
            }
            if (k) {
                k--;
            }
        }
        t.build(lcp);
    }
    int getLcp(int i, int j) {
        i = pos[i]; j = pos[j];
        if (j < i) {
            swap(i, j);
        }
        if (i == j) {
            return inf;
        }
        else {
            return t.getMin(i, j);
        }
    }
};
//6327c9

```

## 6.5 Алгоритм Ахо — Корасик

```

const int alpha = 26;
const char a = 'a';

struct node {
    int next[alpha] = {}, link[alpha] = {};
    int suf = 0;
    int visited = 0, ans = 0;
    int bad = 0; // any term is reachable by suf
    links
    vector<int> term;
    node() {
        fill(next, next + alpha, -1);
    }
};

vector<node> mem;

int get_next_or_create(int nd, char c) {
    if (mem[nd].next[c - a] == -1) { mem[nd].next[c - a] = mem.size(); mem.emplace_back(); }
    return mem[nd].next[c - a];
}

void build(vector<string> t) {
    mem.reserve(1e6 + 100); mem.clear();
    mem.emplace_back();
    // 0th element is nullptr, 1st is the root
    for (int j = 0; j < t.size(); ++j) {
        int cur = 0;
        for (char c : t[j]) cur = get_next_or_create(cur, c);
        mem[cur].term.push_back(j);
    }
    vector<int> bfs_order;
    queue<int> bfs;
    {
        node &root = mem[0];
        root.suf = 0;
        for (char c = a; c < a + alpha; ++c) {
            root.link[c - a] = (root.next[c - a] == -1 ? 0 : root.next[c - a]);
        }
        bfs.push(0);
    }
    while (!bfs.empty()) {
        int cur_idx = bfs.front();
        bfs.pop();
    }
}

```

```

    node &cur = mem[cur_idx];
    cur.bad = cur.term.size() > 0 || mem[cur.suf
].bad;
    bfs_order.push_back(cur_idx);
    for (char c = a; c < a + alpha; ++c) {
        int nxt_idx = cur.next[c - a];
        if (nxt_idx == -1) continue;
        node &nxt = mem[nxt_idx];
        nxt.suf = (cur_idx ? mem[cur.suf].link[c
- a] : 0);
        for (char c = a; c < a + alpha; ++c) {
            nxt.link[c - a] = (nxt.next[c - a]
== -1 ? mem[nxt.suf].link[c - a] : nxt.next[c -
a]);
        }
        bfs.push(nxt_idx);
    }
    // do something
}
//bel6ed

```

## 6.6 Дерево палиндромов

```

const int alpha = 26;
const char a = 'a';
struct palindromic{
    int n;
    vector<int> p, suf{0, 0}, len{-1, 0};
    //d[u] is a difference of lengths of u and suf[u],
    go is jump by chain constant d
    vector<array<int, alpha>> to{{}, {}};
    int sz = 2;
    palindromic(const string &s) : n(s.size()), p(n +
1, 0) {
        suf.reserve(n);
        len.reserve(n);
        for (int i = 0; i < n; ++i) {
            auto check = [&] (int l) {
                return i > l && s[i] == s[i - l - 1];
            };
            int par = p[i];
            while (!check(len[par])) {
                par = suf[par];
            }
            if (to[par][s[i]-a] == 0) {
                p[i+1]=to[par][s[i]-a]=sz++;
                to.emplace_back();
                len.emplace_back(len[par]+2);
                if (par == 0) {
                    suf.emplace_back(1);
                }
                else {
                    do {
                        par = suf[par];
                    } while (!check(len[par]));
                    suf.emplace_back(to[par][s[i]-a]);
                }
            }
            else {
                p[i+1]=to[par][s[i]-a];
            }
        }
    }
    int partition() {
        vector<int> d(sz), up(sz, 1); //d[1] = 0 sic
        for (int i = 2; i < sz; ++i) {
            d[i] = len[i] - len[suf[i]];
            if (d[i] == d[suf[i]]) {
                up[i] = up[suf[i]];
            }
            else {
                up[i] = suf[i];
            }
        }
        vector<int> dp(n + 1, n), last(sz);
        dp[0] = 0;
        for (int i = 1; i <= n; ++i) {
            int u = p[i];
            while (u != 1) {
                if (suf[u] == up[u]) {
                    last[u] = dp[i - len[u]];
                }
                else {
                    last[u] = min(last[suf[u]], dp[i - len[up[
u]]] - d[u]);
                }
                dp[i] = min(dp[i], last[u] + 1);
                u = up[u];
            }
        }
    }
}

```

```

    }
    }
    return dp.back();
}
//acac02

```

## 7 Потоки

### 7.1 Алгоритм Диница

```

#define pb push_back
struct Dinic{
    struct edge{
        int to, flow, cap;
    };

    const static int N = 555; //count of vertices

    vector<edge> e;
    vector<int> g[N + 7];
    int dp[N + 7];
    int ptr[N + 7];

    void clear(){
        for (int i = 0; i < N + 7; i++) g[i].clear();
        e.clear();
    }

    void addEdge(int a, int b, int cap){
        g[a].pb(e.size());
        e.pb({b, 0, cap});
        g[b].pb(e.size());
        e.pb({a, 0, 0});
    }

    int minFlow, start, finish;

    bool bfs(){
        for (int i = 0; i < N; i++) dp[i] = -1;
        dp[start] = 0;
        vector<int> st;
        int uk = 0;
        st.pb(start);
        while(uk < st.size()){
            int v = st[uk++];
            for (int to : g[v]){
                auto ed = e[to];
                if (ed.cap - ed.flow >= minFlow && dp[ed.to]
== -1){
                    dp[ed.to] = dp[v] + 1;
                    st.pb(ed.to);
                }
            }
        }
        return dp[finish] != -1;
    }

    int dfs(int v, int flow){
        if (v == finish) return flow;
        for (; ptr[v] < g[v].size(); ptr[v]++){
            int to = g[v][ptr[v]];
            edge ed = e[to];
            if (ed.cap - ed.flow >= minFlow && dp[ed.to] ==
dp[v] + 1){
                int add = dfs(ed.to, min(flow, ed.cap - ed.
flow));
                if (add){
                    e[to].flow += add;
                    e[to ^ 1].flow -= add;
                    return add;
                }
            }
        }
        return 0;
    }

    int dinic(int start, int finish){
        Dinic::start = start;
        Dinic::finish = finish;
        int flow = 0;
        for (minFlow = (1 << 30); minFlow; minFlow >>= 1){
            while(bfs()){
                for (int i = 0; i < N; i++) ptr[i] = 0;
                while(int now = dfs(start, (int)2e9 + 7)) flow
+= now;
            }
        }
    }
}

```

```

    return flow;
}
} dinic;
//15c079

```

## 7.2 Mincost k-flow

```

struct edge {
    int next, capacity, cost, flow = 0;

    edge() = default;

    edge(int next, int capacity, int cost) : next(next),
        capacity(capacity), cost(cost) {}

    int rem() const { return capacity - flow; }

    int operator+=(int f) { return flow += f; }

    int operator-=(int f) { return flow -= f; }
};

auto addEdge = [&](auto from, auto next, auto capacity, int cost) {
    g[from].push_back(e.size());
    e.emplace_back(next, capacity, cost);
    g[next].push_back(e.size());
    e.emplace_back(from, 0, -cost);
};

/* in case of undirected graph use this:
addEdge(u, v, capacity, cost);
addEdge(v, u, capacity, cost);
*/

vector<ll> phi(n, 0);
auto fordBellman = [&](int s, int t) {
    phi.assign(n, 0);
    for (int iter = 0; iter < n; ++iter) {
        bool changed = false;
        for (int u = 0; u < n; ++u) {
            for (auto index : g[u]) {
                auto edge = e[index];
                if (edge.rem() > 0 && phi[edge.next] > phi[u]
                    + edge.cost) {
                    phi[edge.next] = phi[u] + edge.cost;
                    changed = true;
                }
            }
        }
        if (!changed) break;
    }
};

fordBellman(s, t);
// now shortest path using dijkstra with potentials
vector<ll> dist;
vector<int> from;
vector<bool> cnt;
auto dijkstra = [&](int s, int t) {
    dist.assign(n, 1e18);
    from.assign(n, -1);
    cnt.assign(n, false);
    dist[s] = 0;
    set<pair<int, int>> se;
    se.insert({0, s});
    while ((int)(se.size())) {
        int cur = se.begin()->y;
        se.erase(se.begin());
        cnt[cur] = true;
        for (int index : g[cur]) {
            auto &edge = e[index];
            if (edge.rem() == 0) continue;
            ll weight = edge.cost + phi[cur] - phi[edge.next];
            if (dist[edge.next] > dist[cur] + weight) {
                se.erase({dist[edge.next], edge.next});
                dist[edge.next] = dist[cur] + weight;
                se.insert({dist[edge.next], edge.next});
                from[edge.next] = cur;
            }
        }
    }
    if (dist[t] == (ll) 1e18) return -1LL;
    ll cost = 0;
    for (int p = t; p != s; p = from[p]) {
        for (auto index : g[from[p]]) {
            auto &edge = e[index];
            ll weight = edge.cost + phi[from[p]] - phi[edge.next];
            if (edge.rem() > 0 && edge.next == p && dist[edge.next] == dist[from[p]] + weight) {
                edge += 1;
            }
        }
    }
}

```

```

        e[index ^ 1] -= 1;
        cost += edge.cost;
        break;
    }
}
}
for (int i = 0; i < n; ++i) {
    phi[i] += dist[i];
}
return cost;
};

ll cost = 0;
for (int flow = 0; flow < k; ++flow) {
    ll a = dijkstra(s, t);
    if (a == -1) {
        cout << "-1\n";
        return;
    }
    cost += a;
}
// now recover answer
auto findPath = [&](int s, int t) {
    vector<int> ans;
    int cur = s;
    while (cur != t) {
        for (auto index : g[cur]) {
            auto &edge = e[index];
            if (edge.flow <= 0) continue;
            edge -= 1;
            e[index ^ 1] += 1;
            ans.push_back(index / 4);
            // index / 4 because each edge has 4 copies
            cur = edge.next;
            break;
        }
    }
    return ans;
};

for (int flow = 0; flow < k; ++flow) {
    auto p = findPath(s, t);
    cout << p.size() << ' ';
    for (int x : p) cout << x + 1 << ' ';
    cout << '\n';
}
//94b9cb

```

```

template <typename T, typename C>
class mcmf {
public:
    static constexpr T eps = (T) 1e-9;

    struct edge {
        int from;
        int to;
        T c;
        T f;
        C cost;
    };

    vector< vector<int>> > g;
    vector<edge> edges;
    vector<C> d;
    vector<int> q;
    vector<bool> in_queue;
    vector<int> pe;
    int n;
    int st, fin;
    T flow;
    C cost;

    mcmf(int _n, int _st, int _fin) : n(_n), st(_st),
        fin(_fin) {
        assert(0 <= st && st < n && 0 <= fin && fin < n
            && st != fin);
        g.resize(n);
        d.resize(n);
        in_queue.resize(n);
        pe.resize(n);
        flow = 0;
        cost = 0;
    }

    void clear_flow() {
        for (const edge &e : edges) {
            e.f = 0;
        }
        flow = 0;
    }
}

```

```

void add(int from, int to, T forward_cap, T
backward_cap, C cost) {
    assert(0 <= from && from < n && 0 <= to && to <
n);
    g[from].push_back((int) edges.size());
    edges.push_back({from, to, forward_cap, 0, cost
});
    g[to].push_back((int) edges.size());
    edges.push_back({to, from, backward_cap, 0, -
cost});
}

bool expath() {
    fill(d.begin(), d.end(), numeric_limits<C>::max
());
    q.clear();
    q.push_back(st);
    d[st] = 0;
    in_queue[st] = true;
    int beg = 0;
    bool found = false;
    while (beg < (int) q.size()) {
        int i = q[beg++];
        if (i == fin) {
            found = true;
        }
        in_queue[i] = false;
        for (int id : g[i]) {
            const edge &e = edges[id];
            if (e.c - e.f > eps && d[i] + e.cost < d[e.
to]) {
                d[e.to] = d[i] + e.cost;
                pe[e.to] = id;
                if (!in_queue[e.to]) {
                    q.push_back(e.to);
                    in_queue[e.to] = true;
                }
            }
        }
    }
    if (found) {
        T push = numeric_limits<T>::max();
        int v = fin;
        while (v != st) {
            const edge &e = edges[pe[v]];
            push = min(push, e.c - e.f);
            v = e.from;
        }
        v = fin;
        while (v != st) {
            edge &e = edges[pe[v]];
            e.f += push;
            edge &back = edges[pe[v] ^ 1];
            back.f -= push;
            v = e.from;
        }
        flow += push;
        cost += push * d[fin];
    }
    return found;
}

pair<T, C> max_flow_min_cost() {
    while (expath()) {
    }
    return make_pair(flow, cost);
}
};
//b7bbb2

```

## 8 Алгоритм Гаусса

### 8.1 Решение $Av = b$

```

optional<vector<int>> > gauss(vector<vector<int>> > A,
vector<int> > b) ///returns v such that Av=b
{
    int n=A.size();assert(b.size()==n);int m=A[0].
size();
    for(int &x:b) {x%=p;x+=p;x%=p;}
    for(int i=0;i<n;++i) {for(int &x:A[i]) {x%=p;x+=
p;x%=p;}}
    int bi=0;
    for(int i=0;i<n;++i)
    {
        if(bi==m) break;
        for(int j=i;j<n;++j)
        {

```

```

            if(A[j][bi])
            {
                if(j!=i) {swap(A[i],A[j]);swap(b[i],
b[j]);}
                break;
            }
        }
        if(A[i][bi])
        {
            int o=inv(A[i][bi]);
            for(int j=i+1;j<n;++j)
            {
                int we=(A[j][bi]*o)%p;
                b[j]-=we*b[i];b[j]%=p;if(b[j]<0) b[j
]+=p;
                for(int k=bi;k<m;++k)
                {
                    A[j][k]-=we*A[i][k];A[j][k]%=p;
                }
                if(A[j][k]<0) A[j][k]+=p;
            }
        }
        else
        {
            ++bi;--i;continue;
        }
    }
    vector<int> v(m);
    for(int i=n-1;i>=0;--i)
    {
        int bi=0;
        while(bi<m && !A[i][bi]) {++bi;}
        if(bi==m)
        {
            if(b[i]) {return nullopt;}
            else {continue;}
        }
        {
            int cur=b[i];
            for(int j=bi+1;j<m;++j)
            {
                cur-=A[i][j]*v[j];cur%=p;
            }
            v[bi]=cur*inv(A[i][bi]);v[bi]%=p;if(v[bi]<0)
v[bi]+=p;
        }
    }
    return v;
}
//bcc622

```

### 8.2 Базис $Av = 0$

```

vector<vector<int>> > gaussbasis(vector<vector<int>> >
A,int m) ///returns basis of Av=0
{
    int n=A.size();if(n) assert(m==A[0].size());
    for(int i=0;i<n;++i) {for(int &x:A[i]) {x%=p;x+=
p;x%=p;}}
    int bi=0;
    for(int i=0;i<n;++i)
    {
        if(bi==m) break;
        for(int j=i;j<n;++j)
        {
            if(A[j][bi])
            {
                if(j!=i) {swap(A[i],A[j]);}
                break;
            }
        }
        if(A[i][bi])
        {
            int o=inv(A[i][bi]);
            for(int j=i+1;j<n;++j)
            {
                int we=(A[j][bi]*o)%p;
                for(int k=bi;k<m;++k)
                {
                    A[j][k]-=we*A[i][k];A[j][k]%=p;
                }
                if(A[j][k]<0) A[j][k]+=p;
            }
        }
        else
        {
            ++bi;--i;continue;
        }
    }
}

```

```

vector<int> indices(m);iota(all(indices),0);
for(int i=n-1;i>=0;--i)
{
    int bi=0;
    while(bi<m && !A[i][bi]) {++bi;}
    if(bi<m)
    {
        indices.erase(find(all(indices),bi));
    }
}
vector<vector<int> > v(indices.size(),vector<int>
>(m,0));
for(int i=0;i<indices.size();++i)
{
    v[i][indices[i]]=1;
}
for(int i=n-1;i>=0;--i)
{
    int bi=0;
    while(bi<m && !A[i][bi]) {++bi;}
    if(bi==m) continue;
    for(int k=0;k<indices.size();++k) {
        int cur=0;
        for(int j=bi+1;j<m;++j)
        {
            cur=A[i][j]*v[k][j];cur%=p;
        }
        v[k][bi]=cur*inv(A[i][bi]);v[k][bi]%=p;if(v[
k][bi]<0) v[k][bi]+=p;
    }
}
return v;
}
//ef40f3

```

## 9 Гамильтоновы путь и цикл

<https://codeforces.com/blog/entry/90513>,  
<https://codeforces.com/blog/entry/90743>.

### 9.1 Link-cut tree

```

namespace LCT {
vector<vi> ch;
vi fa, rev;
void init(int n) {
    ch.resize(n + 1);
    fa.resize(n + 1);
    rev.resize(n + 1);
    for (int i = 0; i <= n; i++)
        ch[i].resize(2),
        ch[i][0] = ch[i][1] = fa[i] = rev[i] = 0;
}
bool isr(int a)
{
    return !(ch[fa[a]][0] == a || ch[fa[a]][1] == a)
;
}
void pushdown(int a)
{
    if(rev[a])
    {
        rev[ch[a][0]] ^= 1, rev[ch[a][1]] ^= 1;
        swap(ch[a][0], ch[a][1]);
        rev[a] = 0;
    }
}
void push(int a)
{
    if(!isr(a)) push(fa[a]);
    pushdown(a);
}
void rotate(int a)
{
    int f = fa[a], gf = fa[f];
    int tp = ch[f][1] == a;
    int son = ch[a][tp ^ 1];
    if(!isr(f))
        ch[gf][ch[gf][1] == f] = a;
    fa[a] = gf;

    ch[f][tp] = son;
    if(son) fa[son] = f;

    ch[a][tp ^ 1] = f, fa[f] = a;
}
void splay(int a)

```

```

{
    push(a);
    while(!isr(a))
    {
        int f = fa[a], gf = fa[f];
        if(isr(f)) rotate(a);
        else
        {
            int t1 = ch[gf][1] == f, t2 = ch[f][1] == a;
            if(t1 == t2) rotate(f), rotate(a);
            else rotate(a), rotate(a);
        }
    }
}
void access(int a)
{
    int pr = a;
    splay(a);
    ch[a][1] = 0;
    while(1)
    {
        if(!fa[a]) break;
        int u = fa[a];
        splay(u);
        ch[u][1] = a;
        a = u;
    }
    splay(pr);
}
void makeroot(int a)
{
    access(a);
    rev[a] ^= 1;
}
void link(int a, int b)
{
    makeroot(a);
    fa[a] = b;
}
void cut(int a, int b)
{
    makeroot(a);
    access(b);
    fa[a] = 0, ch[b][0] = 0;
}
int fdr(int a)
{
    access(a);
    while(1)
    {
        pushdown(a);
        if (ch[a][0]) a = ch[a][0];
        else {
            splay(a);
            return a;
        }
    }
}
}
//647cca

```

### 9.2 Undirected case

```

#include <bits/stdc++.h>
using namespace std;
namespace hamil {
    template <typename T> bool chkmax(T &x,T y){return
        x<y?x=y,true:false;}
    template <typename T> bool chkmin(T &x,T y){return
        x>y?x=y,true:false;}
    #define vi vector<int>
    #define pb push_back
    #define mp make_pair
    #define pi pair<int, int>
    #define fi first
    #define se second
    #define ll long long
    using namespace LCT;
    vector<vi> used;
    unordered_set<int> caneg;
    void cut(int a, int b) {
        LCT::cut(a, b);
        for (int s = 0; s < 2; s++) {
            for (int i = 0; i < used[a].size(); i++)
                if (used[a][i] == b) {
                    used[a].erase(used[a].begin() + i);
                    break;
                }
            if (used[a].size() == 1) caneg.insert(a);
        }
    }
}

```

```

    swap(a, b);
}
}
void link(int a, int b) {
    LCT::link(a, b);
    for (int s = 0; s < 2; s++) {
        used[a].pb(b);
        if (used[a].size() == 2) caneg.erase(a);
        swap(a, b);
    }
}
vi work(int n, vector<pi> eg, ll mx_ch = -1) {
    // mx_ch : max number of adding/replacing
    // default is (n + 100) * (n + 50)
    // n : number of vertices. 1-indexed.
    // eg: vector<pair<int, int> > storing all the
    // edges.
    // return a vector<int> consists of all indices
    // of vertices on the path. return empty list if
    // failed to find one.

    LCT::init(n);
    if (mx_ch == -1) mx_ch = 1ll * (n + 100) * (n +
50); //default
    used.resize(n + 1);
    caneg.clear();
    for (int i = 1; i <= n; i++) used[i].clear();

    vector<vi> edges(n + 1);
    for (auto v : eg)
        edges[v.fi].pb(v.se),
        edges[v.se].pb(v.fi);

    for (int i = 1; i <= n; i++)
        caneg.insert(i);

    mt19937 x(chrono::steady_clock::now().
time_since_epoch().count());
    int tot = 0;
    while (mx_ch >= 0) {
        // cout << tot << ' ' << mx_ch << endl;
        vector<pi> eg;
        for (auto v : caneg)
            for (auto s : edges[v])
                eg.pb(mp(v, s));

        shuffle(eg.begin(), eg.end(), x);
        if (eg.size() == 0) break;
        for (auto v : eg) {
            mx_ch--;
            int a = v.fi, b = v.se;
            if (used[a].size() < used[b].size()) swap(a,
b);
            if (used[b].size() >= 2) continue;
            if (x() & 1) continue;
            if (LCT::fdr(a) == LCT::fdr(b)) continue;
            if (used[a].size() < 2 && used[b].size() <
2)
                tot++;
            if (used[a].size() == 2) {
                int p = used[a][x() % 2];
                cut(a, p);
            }
            link(a, b);
        }
        if (tot == n - 1) {
            vi cur;
            for (int i = 1; i <= n; i++)
                if (used[i].size() <= 1) {
                    int pl = i, ls = 0;
                    while (pl) {
                        cur.pb(pl);
                        int flag = 0;
                        for (auto v : used[pl])
                            if (v != ls) {
                                ls = pl;
                                pl = v;
                                flag = 1;
                                break;
                            }
                        if (!flag) break;
                    }
                    break;
                }
            return cur;
        }
    }
    //failed to find a path
    return vi();
}

```

```

    }
}
//c35638

```

### 9.3 Directed case

```

namespace hamil {
    template <typename T> bool chkmax(T &x, T y){return
        x<y?x=y,true:false;}
    template <typename T> bool chkmin(T &x, T y){return
        x>y?x=y,true:false;}
    #define vi vector<int>
    #define pb push_back
    #define mp make_pair
    #define pi pair<int, int>
    #define fi first
    #define se second
    #define ll long long
    using namespace LCT;
    vi out, in;
    vi work(int n, vector<pi> eg, ll mx_ch = -1) {
        // mx_ch : max number of adding/replacing
        // default is (n + 100) * (n + 50)
        // n : number of vertices. 1-indexed.
        // eg: vector<pair<int, int> > storing all the
        // edges.
        // return a vector<int> consists of all indices
        // of vertices on the path. return empty list if
        // failed to find one.
        out.resize(n + 1), in.resize(n + 1);
        LCT::init(n);
        for (int i = 0; i <= n; i++) in[i] = out[i] = 0;
        if (mx_ch == -1) mx_ch = 1ll * (n + 100) * (n +
50); //default
        vector<vi> from(n + 1), to(n + 1);
        for (auto v : eg)
            from[v.fi].pb(v.se),
            to[v.se].pb(v.fi);
        unordered_set<int> canin, canout;
        for (int i = 1; i <= n; i++)
            canin.insert(i),
            canout.insert(i);
        mt19937 x(chrono::steady_clock::now().
time_since_epoch().count());
        int tot = 0;
        while (mx_ch >= 0) {
            // cout << tot << ' ' << mx_ch << endl;
            vector<pi> eg;
            for (auto v : canout)
                for (auto s : from[v])
                    if (in[s] == 0) {
                        assert(canin.count(s));
                        continue;
                    }
                    else eg.pb(mp(v, s));
            for (auto v : canin)
                for (auto s : to[v])
                    eg.pb(mp(s, v));
            shuffle(eg.begin(), eg.end(), x);
            if (eg.size() == 0) break;
            for (auto v : eg) {
                mx_ch--;
                if (in[v.se] && out[v.fi]) continue;
                if (LCT::fdr(v.fi) == LCT::fdr(v.se))
                    continue;
                if (in[v.se] || out[v.fi])
                    if (x() & 1) continue;
                if (!in[v.se] && !out[v.fi])
                    tot++;
                if (in[v.se]) {
                    LCT::cut(in[v.se], v.se);
                    canin.insert(v.se);
                    canout.insert(in[v.se]);
                    out[in[v.se]] = 0;
                    in[v.se] = 0;
                }
                if (out[v.fi]) {
                    LCT::cut(v.fi, out[v.fi]);
                    canin.insert(out[v.fi]);
                    canout.erase(v.fi);
                    in[out[v.fi]] = 0;
                    out[v.fi] = 0;
                }
                LCT::link(v.fi, v.se);
                canin.erase(v.se);
                canout.erase(v.fi);
                in[v.se] = v.fi;
                out[v.fi] = v.se;
            }
        }
    }
}

```



```

    if (tot == n - 1) {
        vi cur;
        for (int i = 1; i <= n; i++)
            if (!in[i]) {
                int pl = i;
                while (pl) {
                    cur.pb(pl);
                    pl = out[pl];
                }
                break;
            }
        return cur;
    }
    //failed to find a path
    return vi();
}
//43ae60

```

## 10 Геометрия

### 10.1 Примитивы

```

struct Point {
    int x, y;
    Point(){}
    Point (int x_, int y_) {
        x = x_; y = y_;
    }
    Point operator + (Point p) {
        return Point(x+p.x, y+p.y);
    }
    Point operator - (Point p) {
        return Point(x - p.x, y - p.y);
    }
    int operator * (Point p) {
        return x * p.y - y * p.x;
    }
    int operator % (Point p) {
        return x * p.x + y * p.y;
    }
    bool operator < (Point v) {
        return (*this) * v > 0;
    }
    bool operator > (Point v) {
        return v < (*this);
    };
    bool operator <= (Point v) {
        return (*this) * v >= 0;
    }
};
bool line(Point a, Point b, Point c) {
    return (b - a) * (c - b) == 0;
}
bool ord(Point a, Point p, Point b) {
    return (p - a) % (p - b) < 0;
}

int hp(Point a) {
    if (a.y == 0) return a.x >= 0;
    return a.y > 0;
}

bool comp(Point a, Point b) {
    if (hp(a) != hp(b)) return hp(a) < hp(b);
    return a.x * b.y - a.y * b.x > 0;
}
//a48b68

```

### 10.2 Выпуклая оболочка

```

using pt = pair<int, int>;
#define x first
#define y second

int cross(pt p, pt q) { return p.x * q.y - p.y * q.x; }
pt operator-(pt a, pt b) { return {a.x - b.x, a.y - b.y}; }

vector<point> convex(vector<point> a) {
    sort(all(a));
    a.erase(unique(all(a)), a.end());
    vector<point> h;
    for (int t = 0; t < 2; ++t) {
        int sz = h.size() - t;

```

```

        for (auto p: a) {
            while (h.size() >= sz + 2 && cross(p - h.end()
                [-1], h.end()[-2] - h.end()[-1]) <= 0) h.
                pop_back();
            h.push_back(p);
        }
        reverse(all(a));
    }
    return h; // h is circular: h.empty() || h.front()
        == h.back()
}
//ef9132

```

### 10.3 Точка внутри многоугольника

```

auto inT = [&] (Point a, Point b, Point c, Point p)
{
    a = a-p; b = b-p; c = c-p;
    int ab = a * b, bc = b * c, ca = c * a;
    return abs(ab)+abs(bc)+abs(ca) == abs(ab+bc+ca);
};
auto inP = [&] (Point p) {
    //a must be in counterclockwise order!
    //assuming no three points of a are collinear
    if (n == 1) return p == a[0];
    if (n == 2) return (p-a[0]) * (a[1]-a[0]) == 0 &&
        (p-a[0]) % (p-a[1]) <= 0;
    int l = 1, r = n - 1;
    while (l < r - 1) {
        int m = (l + r) / 2;
        if ((a[m] - a[0]) < (p - a[0])) {
            l = m;
        } else {
            r = m;
        }
    }
    return inT(a[l], a[0], a[r], p);
};
//9e04bc

```

### 10.4 Касательные

```

auto max = [&] (auto cmp) {
    int k = 0;
    for (int lg = 18; lg >= 0; --lg) {
        int i = k + (1 << lg), j = k - (1 << lg);
        i = (i % n + n) % n;
        j = (j % n + n) % n;
        array<int, 3> ind{i, j, k};
        sort(all(ind), cmp);
        k = ind[2];
    }
    return k;
};
auto uppert = [&] (Point p) { //last vertex in
    counterclockwise order about p
    auto cmp = [&] (int i, int j) { return (a[i] - p) <
        (a[j] - p); };
    return max(cmp);
};
auto lowert = [&] (Point p) { //first vertex in
    counterclockwise order about p
    auto cmp = [&] (int i, int j) { return (a[i] - p) >
        (a[j] - p); };
    return max(cmp);
};
auto uppertinf = [&] (Point p) { //upper tangent line
    parallel to vector p
    swap(p.x, p.y);
    p.x = -p.x;
    auto cmp = [&] (int i, int j) { return a[i] % p <
        a[j] % p; };
    return max(cmp);
};
auto lowertinf = [&] (Point p) { //lower tangent line
    parallel to vector p
    swap(p.x, p.y);
    p.x = -p.x;
    auto cmp = [&] (int i, int j) { return a[i] % p >
        a[j] % p; };
    return max(cmp);
};
//90f89d

```

### 10.5 Пересечение многоугольника и полу-плоскости

```

template<typename Check>
vector<point> intersect(vector<point> h, Check value
) {
    int n = h.size();
    vector<int> pos(n);
    for (int i = 0; i < n; ++i) {
        pos[i] = value(h[i]) >= 0;
    }
    if (count(all(pos), 0) == 0) return h;
    if (count(all(pos), 1) == 0) return {};
    auto intersect = [&](point p, point q) {
        auto s = value(p);
        auto t = value(q);
        assert(s < 0 && t >= 0);
        return q + (p - q) * (t / (t - s));
    };
    int t01 = -1, t10 = -1;

    for (int i = 0; i < n; ++i) {
        h.push_back(h[i]);
        pos.push_back(pos[i]);
        if (pos[i] == 0 && pos[i + 1] == 1) t01 = i;
        if (pos[i] == 1 && pos[i + 1] == 0) t10 = i;
    }
    if (t10 < t01) t10 += n;
    //for (int i = t01 + 1; i <= t10; ++i) assert(pos[i]
    == 1);
    //for (int i = t10 + 1; i <= t01 + n; ++i) assert(
    pos[i] == 0);
    vector<point> res(intersect(h[t01], h[t01 + 1]));
    for (int i = t01 + 1; i <= t10; ++i) {
        res.push_back(h[i]);
    }
    res.push_back(intersect(h[t10 + 1], h[t10]));
    return res;
}

// Usage example:
void solve() {
    vector<point> p, q;
    // ... q must be in counterclockwise order; q[0]
    == q[m]
    for (int i = 0; i < m; ++i) {
        p = intersect(p, [&](point pt) { return
        cross(q[i + 1] - q[i], pt - q[i]); });
    }
}
//bd15f0

```

## 10.6 События для прямой

```

int cross(point p, point q) { return p.x * q.y - q.x
    * p.y; }
point operator-(point p, point q) { return {p.x - q.
    x, p.y - q.y}; }
int sgn(int x) { return x < 0 ? -1 : (x > 0); }
double dist(point p) { return sqrt(p.x * p.x + p.y *
    p.y); }
const __int128 one = 1;

double solve(vector<point> a, point p, point q) {
    int n = a.size();
    a.push_back(a[0]);
    point pq = q - p;
    vector<array<int, 3>> ev;
    for (int i = 0; i < n; ++i) {
        point u = a[i], v = a[i + 1];
        int s = sgn(cross(u - p, pq)), t = sgn(cross(v -
        p, pq));
        if (s == t) continue;
        int top = cross(u - p, v - u), bot = cross(pq, v
        - u);
        ev.push_back({sgn(bot) * top, abs(bot), t - s});
    }
    sort(all(ev), [](auto e, auto f) {
        return e[0] * one * f[1] < f[0] * one * e[1];
    });
    int bal = 0;
    for (int i = 0; i + 1 < ev.size(); ++i) {
        if (bal + ev[i][2] < 0 || bal + ev[i][2] > 2) {
            assert(ev[i][0] * ev[i + 1][1] == ev[i + 1][0] *
            ev[i][1]);
            swap(ev[i], ev[i + 1]);
        }
        bal += ev[i][2];
    }
    // example usage: now calculating length of
    longest segment inside
    bal = 0;
}

```

```

double from = 0, ans = 0;
for (auto [t, b, w] : ev) {
    double x = t * 1.0 / b;
    if (bal == 0) from = x;
    bal += w;
    assert(0 <= bal && bal <= 2);
    if (bal == 0) ans = max(ans, x - from);
}
return ans * dist(pq);
}
//fe0649

```

## 10.7 Кривая Гильберта для алгоритма Мо

```

const int logn = 30; // any number, such that maxn
is greater than coordinates. 20 is ok.
const int maxn = 1 << logn;
int hilbertorder(int x, int y) { // returns long
long
    int d = 0; // long long
    for (int s = 1 << (logn - 1); s; s >>= 1)
    {
        bool rx = x & s, ry = y & s;
        d = (4 * d) | ((3 * rx) ^ ry);
        if (!ry) {
            if (rx) {
                x = maxn - x;
                y = maxn - y;
            }
            swap(x, y);
        }
        return d;
    }
}
// Usage example:
vector<int> sort_indices(int q, vector<pair<int, int
>> &qs) {
    vector<int> ind(q), ord(q);
    iota(all(ind), 0);
    for (int i = 0; i < q; ++i) ord[i] =
    hilbertorder(qs[i].first, qs[i].second);
    sort(all(ind), [&](int i, int j) { return ord[i]
    < ord[j]; });
    return ind;
}
//134578

```

## 10.8 Симплекс

```

#define int long long
using namespace std;

typedef double ld;

const ld EPS = 1e-9;

struct LPSolver {
    int m, n;
    vector<int> B, N;
    vector<vector<ld>> D;

    LPSolver(const vector<vector<ld>> &A, const
    vector<ld> &b, const vector<ld> &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m +
    2, vector<ld>(n + 2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j <
        n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) { B[i] = n + i; D[i
        ][n] = -1; D[i][n + 1] = b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j]
        = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }

    void pivot(int r, int s) {
        double inv = 1.0 / D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] * inv;
        for (int j = 0; j < n + 2; j++) if (j != s) D[r
        ][j] *= inv;
        for (int i = 0; i < m + 2; i++) if (i != r) D[i
        ][s] -= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
}

```

```

int x = phase == 1 ? m + 1 : m;
while (true) {
    int s = -1;
    for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j]
            == D[x][s] && N[j] < N[s]) s = j;
    }
    if (D[x][s] > -EPS) return true;
    int r = -1;
    for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
            (D[i][n + 1] / D[i][s] == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
    }
    if (r == -1) return false;
    pivot(r, s);
}

ld solve(vector<ld> &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
        pivot(r, n);
        if (!simplex(1) || D[m + 1][n + 1] < -EPS)
            return -numeric_limits<ld>::infinity();
        for (int i = 0; i < m; i++) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++)
                if (s == -1 || D[i][j] < D[i][s] || D[i][j]
                    == D[i][s] && N[j] < N[s]) s = j;
            pivot(i, s);
        }
    }
    if (!simplex(2)) return numeric_limits<ld>::infinity();
    x = vector<ld>(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
}

};
/*
//      maximize      c^T x
//      subject to    Ax <= b
//                      x >= 0
vector<vector<ld>> A(m, vector<ld> (n))
vector<ld> b(m), c(n)
LPSolver solver(A, b, c);
vector<ld> x;
ld value = solver.solve(x);
// OUTPUT: value of the optimal solution (infinity
//          if unbounded above, -infinity if infeasible)
*/
//31e155

```

## 11 Цепные дроби

<https://cp-algorithms.com/algebra/continued-fractions.html>

### 11.1 Поиск нижней огибающей, сумма и минимум по модулю

```

int floor(int a, int b) {
    return a / b - ((a ^ b) < 0 && a % b);
}

vector<int> decompose(int p, int q) {
    vector<int> f;
    while (q != 0) {
        f.push_back(floor(p, q));
        p -= q * f.back();
        swap(p, q);
    }
    return f;
}

using matrix = array<int, 4>;

matrix operator*(matrix a, matrix b) {
    matrix c{0,0,0,0};
    for (int i = 0; i < 2; ++i) {
        for (int j = 0; j < 2; ++j) {

```

```

            for (int k = 0; k < 2; ++k) {
                c[2 * i + k] += a[2 * i + j] * b[2 * j + k];
            }
        }
    }
    return c;
}

#define x first
#define y second

// computes lower convex hull for 0 <= x <= N, 0 <=
// y <= (ax + b) / c
vector<pair<int, int>> lower_convex_hull(int a, int
    b, int c, int n) {
    matrix m = {1, 0, 0, 1};
    auto f = decompose(a, c);
    vector<pair<int, int>> conv{{1, 0}, {0, 1}};
    for (int x : f) {
        m = m * matrix{x, 1, 1, 0};
        conv.emplace_back(m[2], m[0]);
        if (m[2] > n) break; // there should be one (if
            any) with .x > n
    }
    auto diff = [&](int x, int y) {
        return c * y - a * x;
    };
    int x = 0, y = b / c;
    vector<pair<int, int>> res{{x, y}};
    int i;
    for (i = 2; i + 1 < conv.size(); i += 2) {
        while (diff(x + conv[i + 1].x, y + conv[i + 1].y)
            <= b) {
            int t = 1 + (diff(x + conv[i - 1].x, y + conv[i - 1].y) - b - 1) / abs(diff(conv[i].x, conv[i].y));
            auto [dx, dy] = tuple{conv[i - 1].x + t * conv[i].x, conv[i - 1].y + t * conv[i].y};
            int k = (n - x) / dx;
            if (k == 0) break;
            if (diff(dx, dy)) k = min(k, (b - diff(x, y)) / diff(dx, dy));
            x += k * dx, y += k * dy;
            res.push_back({x, y});
        }
    }
    if (i >= conv.size()) i -= 2;
    for (; i > 0; i -= 2) {
        auto [dx1, dy1] = conv[i];
        if (x + dx1 > n) continue;
        x += dx1, y += dy1;
        if (i + 1 < conv.size()) {
            auto [dx2, dy2] = conv[i + 1];
            int k = (n - x) / dx2;
            x += k * dx2;
            y += k * dy2;
        }
        res.emplace_back(x, y);
        int k = (n - x) / dx1;
        if (k == 0) continue;
        x += k * dx1;
        y += k * dy1;
        res.emplace_back(x, y);
    }
    return res;
}

// number of (x, y) under pq line such that p.x <= x
// < q.x && 0 < y
int area(auto p, auto q) {
    int integers = gcd(q.x - p.x, q.y - p.y);
    return ((p.y + q.y - 1) * (q.x - p.x + 1) +
        integers + 1) / 2 - q.y;
}

// sum of (ax + b) / c for 0 <= x < n
int get_area(int a, int b, int c, int n) { // SUM (
    ax + b) / c for 0 <= x <= n
    auto ch = lower_convex_hull(a, b, c, n + 1);
    int sum = 0;
    for (int i = 0; i + 1 < ch.size(); ++i) {
        sum += area(ch[i], ch[i + 1]);
    }
    return sum;
}

// min of (ax + b) % c for 0 <= x <= n
int get_min(int a, int b, int c, int n) {
    auto ch = lower_convex_hull(a, b, c, n);

```

```
// in fact, here we need only the last point of
// the first half of the algo (that is going up)
int mn = c;
for (auto [x, y] : ch) mn = min(mn, (a * x + b) % c);
return mn;
}
//87941e
```

## 11.2 Простая рекурсия

Число точек  $(x, y) : 0 \leq x < n, 0 < y \leq (kx+b)/d$ . То есть  $\sum_{x=0}^{n-1} \lfloor \frac{kx+b}{d} \rfloor$ .

```
int cnt (int n, int k, int b, int d)
{
    if (k == 0) return (b / d) * n;
    if (k >= d || b >= d) {
        return (k / d) * n * (n - 1) / 2 + (b / d) * n +
            cnt(n, k % d, b % d, d);
    }
    return cnt((k * n + b) / d, d, (k * n + b) % d, k)
        ;
}
//11a6a0
```

## 12 Разное

### 12.1 Компараторы

```
bool cmp1(int x, int y) { return x > y; }

struct cmp2{
    bool operator()(int x, int y) const { return x > y; }
};

int32_t main() {
    set<int, decltype(cmp1)> s1({1, 2, 3}, cmp1);
    for (int x : s1) cout << x << ' '; cout << '\n';
    set<int, cmp2> s2({4, 5, 6});
    for (int x : s2) cout << x << ' '; cout << '\n';
    auto cmp3 = [&](int x, int y) { return x > y; };
    set<int, decltype(cmp3)> s3({7, 8, 9}, cmp3); //
    second cmp3 could be omitted if cmp3 = [](...)
    { ... }
    for (int x : s3) cout << x << ' '; cout << '\n';

    vector<int> v{3, 2, 1};
    cout << lower_bound(all(v), 2, cmp1) - v.begin()
        ;
    cout << lower_bound(all(v), 2, cmp2()) - v.begin()
        ;
    cout << lower_bound(all(v), 2, cmp3) - v.begin()
        ;
}
//adea08
```

## 12.2 Трюки от Сергея Копелиовича

### 12.2.1 Быстрый ввод

<https://acm.math.spbu.ru/~sk1/algo/input-output>

```
const int buf_size = 4096;

int getChar() {
    static char buf[buf_size];
    static int len = 0, pos = 0;
    if (pos == len)
        pos = 0, len = fread(buf, 1, buf_size, stdin);
    if (pos == len)
        return -1;
    return buf[pos++];
}

int readChar() {
    while (1) {
        int c = getChar();
        if (c > 32) return c;
    }
}

int readInt() {
    int s = 1, c = readChar(), x = 0;
```

```
if (c == '-')
    s = -1, c = getChar();
while (isdigit(c))
    x = x * 10 + c - '0', c = getChar();
return s * x;
}
//dc0a77
```

```
double read_double() {
    string s;
    cin >> s;
    double sgn = 1, p10 = 0, num = 0;
    for (char c : s) {
        if (c == '-') {
            sgn = -1;
        } else if (c == '.') {
            p10 = 1;
        } else {
            p10 *= 10;
            num = (num * 10 + c - '0');
        }
    }
    if (p10 < 0.5) p10 = 1;
    return sgn * num / p10;
}
//b77b67
```

<https://acm.math.spbu.ru/~sk1/algo/memory.cpp.html>

### 12.2.2 Быстрый аллокатор

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void * operator new (size_t n) {
    assert((mpos += n) <= MAX_MEM);
    return (void *) (mem + mpos - n);
}
void operator delete (void *) noexcept {}
void operator delete (void *, size_t) noexcept {}
//8726b1
```

## 12.3 Редукция Барретта

```
using u64 = unsigned long long;
using u128 = __uint128_t;
struct barrett{
    u64 p, m;
    barrett() {}
    barrett(u64 p) : p(p), m(-1ULL / p) {}
    int reduce(u64 x) {
        u64 q = (u128(m) * x) >> 64, r = x - q * p;
        return r - p * (r >= p);
    }
} ba;

// Usage example:
void solve() {
    int p = ...;
    ba = barrett(p);
    int x = ..., y = ...;
    int prod = ba.reduce(x * y);
}
//a8b4c7
```

## 12.4 Флаги компиляции

```
-DLOCAL -Wall -Wextra -pedantic -Wshadow -Wformat=2
-Wfloat-equal -Wconversion -Wlogical-op -Wshift-overflow=2
-Wduplicated-cond -Wcast-qual -Wcast-align -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
-D_FORTIFY_SOURCE=2 -fsanitize=address -fsanitize=undefined
-fno-sanitize-recover -fstack-protector -std=c++2a
```

### 12.4.1 Сеточка в vim

<https://codeforces.com/blog/entry/122540>

```
i|<esc>25A |<esc>
o+<esc>25A---+<esc>
Vky35Pdd
```

## 12.5 Что сделать на пробном туре

- Послать клар
- Распечатать что-то
- Получить ML (stack & heap)
- Максимальный размер отправляемого файла?
- Убедиться, что чекер регистронезависимый (yes/YES)
- Позапустить Флойда — Варшалла
- Посмотреть, насколько быстр быстрый ввод
- Перебить что-то, проверить хеш
- Проверить санитайзеры

## 12.6 Хеш файла без комментариев

Хеш файла, игнорирующий переводы строк и комментарии:

```
$ cpp -dD -P -fpreprocessed "$filename" | tr -d '[:space:]' | md5sum | cut -c-6
```