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### 1 Теория чисел

### 1.1 KTO

```
2  if (b==0) { x = 1; y = 0; return a; }
3  int d = gcd(b,a%b,x,y);
4  swap(x,y);
5  y-=a/b*x;
6  return d;
7 }
8  int inv(int r, int m) {
9   int x, y;
10  gcd(r,m,x,y);
11  return (x+m)%m;
12 }
13  int crt(int r, int n, int c, int m) { return r + (( c - r) % m + m) * inv(n, m) % m * n; }
```

# 1.2 Алгоритм Миллера — Рабина

1 int gcd(int a, int b, int &x, int &y) {

```
__int128 one=1;
2 int po(int a, int b, int p)
3 {
4
     int res=1;
     while(b) {if(b & 1) {res=(res*one*a) %p; --b;} else
        {a=(a*one*a)%p;b>>=1;}} return res;
7 bool chprime(int n) ///miller-rabin
8 {
     if(n==2) return true;
     if(n<=1 || n%2==0) return false;</pre>
10
     int h=n-1; int d=0; while(h%2==0) {h/=2; ++d;}
     for(int a:{2, 3, 5, 7, 11, 13, 17, 19, 23, 29,
12
       31, 37})
13
      if(a==n) return true;
14
       int u = po(a,h,n); bool ok = 0;
       if(u%n==1) continue;
       for(int c=0;c<d;++c)
18
         if ((u+1) %n==0) {ok=1;break;}
20
         u=(u*one*u)%n;
21
22
       if(!ok) return false;
23
    }
24
     return true;
25 }
```

### 1.3 Алгоритм Берлекэмпа — Месси

https://mzhang2021.github.io/cp-blog/berlekamp-massey/

```
1 template < typename T>
 2 vector<T> berlekampMassey(const vector<T> &s) {
     int n = (int) s.size(), 1 = 0, m = 1;
      vector < T > b(n), c(n);
 5
      T ld = b[0] = c[0] = 1;
 6
      for (int i=0; i<n; i++, m++) {</pre>
        T d = s[i];
        for (int j=1; j<=1; j++)
d += c[j] * s[i-j];
 8
 9
 10
        if (d == 0)
          continue;
 11
 12
        vector<T> temp = c;
        T coef = d / ld;
 13
        for (int j=m; j<n; j++)</pre>
 14
        c[j] -= coef * b[j-m];
if (2 * 1 <= i) {
 16
          1 = i + 1 - 1;
 17
          b = temp;
          ld = d;
 19
          m = 0;
20
        }
22
      }
23
      c.resize(1 + 1);
24
      c.erase(c.begin());
25
      for (T &x : c)
26
       x = -x;
1
```

```
St. Petersburg State University (Ushakov, Turevskii, Danilevich)
2.7
    return c;
28 }
     Графы
   2.1 SCC и 2-SAT
   Алгоритм ищет сильносвязные компоненты в графе g, если есть
  путь i \to j, то scc[i] \le scc[j]
    В случае 2-\mathcal{SAT} рёбра i\Rightarrow j и (j\oplus 1)\Rightarrow (i\oplus 1) должны быть
   добавлены одновременно.
1 vector < vector < int >> g(2 * n);
2 vector < vector < int >> r(g.size());
3 for (int i = 0; i < g.size(); ++i) {
    for (int j : g[i]) r[j].push_back(i);
4
6 \text{ vector} < int > used(g.size()), tout(g.size());
7 int time = 0;
8 auto dfs = [&](auto dfs, int cur) -> void {
Q
   if (used[cur]) return;
10
     used[cur] = 1;
    for (int nxt : g[cur]) {
12
      dfs(dfs, nxt);
    // used[cur] = 2;
14
1.5
    tout[cur] = time++;
16 };
17 for (int i = 0; i < g.size(); ++i) if (!used[i])
       dfs(dfs, i);
18 vector<int> ind(g.size());
19 iota(ind.begin(), ind.end(), 0);
20 sort(all(ind), [&](int i, int j){return tout[i] >
       tout[j];});
21 vector < int > scc(g.size(), -1);
22 auto go = [&](auto go, int cur, int color) -> void
23
     if (scc[cur] != -1) return;
     scc[cur] = color;
25
    for (int nxt : r[cur]) {
      go(go, nxt, color);
27
    }
28 };
29 int color = 0;
30 for (int i : ind) {
31
    if (scc[i] == -1) go(go, i, color++);
33 for (int i = 0; i < g.size() / 2; ++i) {
     if (scc[2 * i] == scc[2 * i + 1]) "IMPOSSIBLE"
     if (scc[2 * i] < scc[2 * i + 1]) {</pre>
35
36
      // !i => i, assign i = true
37
     } else {
38
       // i => !i, assign i = false
39
     }
40 }
   2.2 Эйлеров цикл
 1 vector<vector<pair<int, int>>> g(n); // pair{nxt,
      idx}
2 vector<pair<int, int>> e(p.size());
3 // build graph
 4 vector < int > in(n), out(n);
5 for (auto [u, v] : e) in[v]++, out[u]++;
6 vector<int> used(m), it(n), cycle;
7 auto dfs = [&](auto dfs, int cur) -> void {
```

```
8
     while (true) {
       while (it[cur] < g[cur].size() && used[g[cur][it[cur]].second]) it[cur]++;</pre>
9
10
        if (it[cur] == g[cur].size()) return;
       auto [nxt, idx] = g[cur][it[cur]];
       used[idx] = true;
        dfs(dfs, nxt);
13
14
        cycle.push_back(idx);
   }
15
16 };
17 \text{ int } \text{cnt} = 0, \text{ odd} = -1;
18 for (int i = 0; i < n; ++i){
    if (out[i] && odd == -1) odd = i;
```

```
Page 2 of 9
20
     if (in[i] != out[i]) {
       if (in[i] + 1 == out[i]) odd = i;
22
       if (abs(in[i] - out[i]) > 1) return {}; // must
        hold
23
       cnt++;
24
     }
25 }
26 if (cnt != 0 && cnt != 2) return {}; // must hold
27 // for undirected find odd vertex (and count that #
        of odd is 0 or 2)
28 dfs(dfs, odd);
29 reverse(cycle.begin(), cycle.end());
30 if (cycle.size() != m) return {};
   2.3 Компоненты рёберной двусвязности
  int n, m;
 2 \text{ cin} >> \text{n} >> \text{m}:
 3 vector \langle vector \langle int \rangle \rangle g(n + 1);
4 map <pair <int, int>, int> comp, col;
5 for (int i = 0; i < m; ++i) {</pre>
   int u, v, c; cin >> u >> v >> c;c--;
     col[{u,v}]=col[{v,u}]=c;
     g[u].push_back(v);
    g[v].push_back(u);
10 }
11 vector <int> used(n + 1);
12 vector \langle int \rangle newCompWithoutParent(n + 1), h(n + 1),
        up(n + 1);
13 function <void(int,int)> findCutPoints = [&] (int u
       , int p) {
14
     used[u] = 1;
     up[u] = h[u];
16
     for (int v : g[u]) {
       if (!used[v]) {
         h[v] = h[u] + 1;
19
         findCutPoints(v, u);
20
          up[u] = min(up[u], up[v]);
21
         if (up[v] >= h[u]) {
22
           newCompWithoutParent[v] = 1;
23
24
       }
25
       else {
26
         up[u] = min(up[u], h[v]);
27
28
29 };
30 for (int u = 1; u <= n; ++u) {
     if (!used[u]) {
32
       findCutPoints(u, u):
33
     }
34 }
35 int ptr = 0;
36 vector <map <int, int> > colors(m);
37 function <void(int, int)> markComponents = [&] (int
       u, int cur) {
38
     used[u] = 1;
     for (int v : g[u]) {
40
       if (!used[v]) {
          if (newCompWithoutParent[v]) {
           ptr++;
            markComponents(v, ptr - 1);
         }
          else {
46
           markComponents(v, cur);
47
       }
48
       else if (h[v] < h[u]) {</pre>
         comp [{u,v}] = comp [{v,u}] = cur;
          int c = col[{u,v}];
          colors[cur][u] |= 1 << c;
          colors[cur][v] |= 1 << c;
54
55
     }
56 };
57 used.assign(n + 1, 0);
58 for (int u = 1; u <= n; ++u) {
59
     if (!used[u]) {
       markComponents(u, -1);
```

```
61
    }
62 }
                                                                9
63 for (int comp = 0; comp < m; ++comp) {
     vector <int> cnt(4);
64
     int tot = 0;
66
     for (auto [u, mask] : colors[comp]) {
67
       tot |= mask;
       cnt[bp(mask)]++;
68
                                                                    return c;
69
70
    if (bp(tot)<3) {
71
       continue:
72
     if (cnt[2] || cnt[3]>2) {
74
       cout << "Yes" << endl;</pre>
75
       return;
77 }
78 cout << "No" << endl;
   3 хог, and, or-cвёртки
                                                                       / summa na [1,r)
  3.1 and-свёртка
1 vector<int> band(vector<int> a, vector<int> b)
                                                                  4.2 Ordered set
2 {
3
     int n=0; while((1<<n)<a.size()) ++n;</pre>
     a.resize(1<<n);b.resize(1<<n);
4
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
       ; ++ mask) if (mask & (1<<i)) {a[mask-(1<<i)]+=a[
       mask];a[mask-(1<<ii)]%=p;}
                                                                5 using namespace std;
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
       ; ++ mask) if (mask & (1 << i)) {b[mask - (1 << i)] += b[}
       mask];b[mask-(1<<ii)]%=p;}
     vector < int > c(1 << n,0);
     for(int mask=0; mask<(1<<n); ++ mask) {c[mask]=a[</pre>
       mask] * b [mask]; c [mask] % = p;}
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
       ; ++ mask) if(!(mask & (1<<i))) {c[mask]-=c[mask]}
       +(1<<i)];c[mask]%=p;}
10
     return c;
11 }
                                                                    int h, n;
                                                                3
  3.2 от-свёртка
                                                                    Data zd;
                                                                    Mod zm:
                                                                    vector < Data > data:
1 vector<int> bor(vector<int> a, vector<int> b)
                                                                    vector < Mod > mod;
2 -
3
     int n=0; while((1<<n)<a.size()) ++n;</pre>
                                                                9
     a.resize(1<<n);b.resize(1<<n);
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
5
                                                               11
       ; ++ mask) if (!(mask & (1<<i))) {a[mask+(1<<i)]+=
       a[mask];a[mask+(1<<i)]%=p;}
6
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
                                                               12
       ;++mask) if(!(mask & (1<<i))) {b[mask+(1<<i)]+=
       b[mask]; b[mask+(1<<ii)]%=p;}
     vector < int > c(1 << n,0);
     for(int mask=0; mask<(1<<n); ++ mask) {c[mask]=a[</pre>
       mask] * b [mask]; c [mask] %=p;}
9
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
       ; ++ mask) if (mask & (1<<i)) {c[mask] -= c[mask]
       -(1<<i)];c[mask]%=p;}
                                                                       (i);
     return c;
                                                               16
11 }
                                                               17
  3.3 хот-свёртка
1 vector<int> bxor(vector<int> a, vector<int> b)
     assert(p%2==1); int inv2=(p+1)/2;
3
     int n=0; while((1<<n)<a.size()) ++n;</pre>
                                                                       ), a(a) {}
     a.resize(1<<n);b.resize(1<<n);
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
                                                                    void push(int i) {
       ;++mask) if(!(mask & (1<<i))) {int u=a[mask],v=
                                                               22
       a[mask+(1<<i)]; a[mask+(1<<i)]=(u+v)%p; a[mask]=(
                                                               23
                                                               24
       u-v)%p;}
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
                                                                       mod[i] = zm;
       ;++mask) if(!(mask & (1<<i))) {int u=b[mask], v=
```

b[mask+(1<<i)];b[mask+(1<<i)]=(u+v)%p;b[mask]=(

u-v)%p;}

```
vector < int > c(1 << n, 0);
     for(int mask=0; mask<(1<<n); ++ mask) {c[mask]=a[</pre>
      mask]*b[mask];c[mask]%=p;
     for(int i=0;i<n;++i) for(int mask=0;mask<(1<<n)</pre>
       ; ++mask) if (!(mask & (1<<i))) {int u=c[mask], v=
       c[mask+(1<<i)];c[mask+(1<<i)]=((v-u)*inv2)%p;c[
       mask]=((u+v)*inv2)%p;}
   4 Структуры данных
   4.1 Дерево Фенвика
 1 int fe[maxn]; /// fenwick tree
 2 void pl(int pos,int val) {while(pos<maxn) {fe[pos</pre>
       ]+=val;pos|=(pos+1);}}
 3 int get(int pos) {int ans=0; while(pos>=0) {ans+=fe[
       pos];pos&=(pos+1);--pos;} return ans;} /// [0,
       pos] - vkluchitelno!!!
 4 int get(int 1, int r) {return get(r-1)-get(1-1);} //
 1 #include <ext/pb_ds/assoc_container.hpp>
 2 #include <ext/pb_ds/tree_policy.hpp>
 4 using namespace __gnu_pbds;
 7 using ordered_set = tree<int, null_type, less<>,
       \verb"rb_tree_tag", | | | tree_order_statistics_node_update"|
   4.3 Дерево отрезков
 UniteData, typename UniteMod, typename Apply>
  struct MassSegmentTree {
     UniteData ud; // Data (Data, Data)
     UniteMod um; // Mod (Mod, Mod);
     Apply a; // Data (Data, Mod, int); last argument
       is the length of current segment (could be used
       for range += and sum counting, for instance)
     template < typename I>
     MassSegmentTree(int sz, Data zd, Mod zm,
       UniteData ud, UniteMod um, Apply a, I init) : h
       (_{-1}g(sz > 1 ? sz - 1 : 1) + 1), n(1 << h), zm(
       zm), zd(zd), data(2 * n, zd), mod(n, zm), ud(ud
       ), um(um), a(a) {
       for (int i = 0; i < sz; ++i) data[i + n] = init</pre>
       for (int i = n - 1; i > 0; --i) data[i] = ud(
       data[2 * i], data[2 * i + 1]);
     MassSegmentTree(int sz, Data zd, Mod zm,
       UniteData ud, UniteMod um, Apply a) : h(__lg(sz
       > 1 ? sz - 1 : 1) + 1), n(1 << h), zm(zm), zd(
       zd), data(2 * n, zd), mod(n, zm), ud(ud), um(um)
       if (mod[i] == zm) return;
       apply(2 * i, mod[i]);
       apply(2 * i + 1, mod[i]);
26
28
     // is used only for apply
```

```
29
     int length(int i) { return 1 << (h - __lg(i)); }</pre>
30
31
     // is used only for descent
39
      int left(int i) {
33
       int lvl = __lg(i);
       return (i & ((1 << lvl) - 1)) * (1 << (h - lvl)
34
35
     }
36
     // is used only for descent
37
38
     int right(int i) {
39
       int lvl = __lg(i);
       return ((i & ((1 << lvl) - 1)) + 1) * (1 << (h
        - lvl)):
41
     }
42
43
      template < typename S>
      void apply(int i, S x) {
44
       data[i] = a(data[i], x, length(i));
45
46
       if (i < n) mod[i] = um(mod[i], x);</pre>
 47
48
 49
      void update(int i) {
       if (mod[i] != zm) return;
50
        data[i] = ud(data[2 * i], data[2 * i + 1]);
51
52
53
54
      template < typename S>
      void update(int 1, int r, S x) { // [1; r)
56
       1 += n, r += n;
57
        for (int shift = h; shift > 0; --shift) {
         push(1 >> shift);
59
          push((r - 1) >> shift);
60
61
       for (int lf = 1, rg = r; lf < rg; lf /= 2, rg</pre>
        /= 2) {
         if (lf & 1) apply(lf++, x);
62
         if (rg & 1) apply(--rg, x);
63
64
       }
65
        for (int shift = 1; shift <= h; ++shift) {</pre>
          update(1 >> shift);
66
67
          update((r - 1) >> shift);
68
     }
69
70
71
     Data get(int 1, int r) { // [1; r)
72
       1 += n, r += n;
73
        for (int shift = h; shift > 0; --shift) {
74
          push(1 >> shift);
          push((r - 1) >> shift);
75
76
       Data leftRes = zd, rightRes = zd;
 78
        for (; 1 < r; 1 /= 2, r /= 2) {</pre>
         if (1 & 1) leftRes = ud(leftRes, data[1++]);
79
80
          if (r & 1) rightRes = ud(data[--r], rightRes)
81
82
       return ud(leftRes, rightRes);
83
     }
84
     // l \in [0; n) && ok(get(1, 1), 1);
85
86
     // returns last r: ok(get(1, r), r)
87
      template < typename C>
      int lastTrue(int 1, C ok) {
89
       1 += n;
90
        for (int shift = h; shift > 0; --shift) push(1
       >> shift);
91
       Data cur = zd;
92
        do {
          1 >>= __builtin_ctz(1);
93
94
          Data with1;
95
          with1 = ud(cur, data[1]);
96
          if (ok(with1, right(1))) {
97
            cur = with1;
            ++1;
98
          } else {
99
100
            while (1 < n) {
              push(1);
101
102
              Data with2;
              with2 = ud(cur, data[2 * 1]);
```

```
104
              if (ok(with2, right(2 * 1))) {
                cur = with2;
06
                1 = 2 * 1 + 1;
              } else {
08
                1 = 2 * 1;
09
10
            }
            return 1 - n;
13
       } while (1 & (1 - 1));
14
       return n;
15
16
     // r \in [0; n) && ok(get(r, r), r);
17
     // returns first 1: ok(get(1, r), 1)
18
19
     template < typename C>
     int firstTrue(int r, C ok) {
21
       r += n;
       for (int shift = h; shift > 0; --shift) push((r
        - 1) >> shift);
23
       Data cur = zd;
       while (r & (r - 1)) {
24
         r >>= __builtin_ctz(r);
         Data with1;
27
         with1 = ud(data[--r], cur);
28
         if (ok(with1, left(r))) {
29
           cur = with1:
         } else {
           while (r < n) {
31
32
             push(r);
33
              Data with2;
34
              with2 = ud(data[2 * r + 1], cur);
35
              if (ok(with2, right(2 * r))) {
36
                cur = with2;
                r = 2 * r;
37
38
              } else {
39
                r = 2 * r + 1;
40
41
            }
42
            return r - n + 1;
43
44
       }
45
       return 0;
     }
46
47 };
   4.3.1 Примеры использования
     • Взятие максимума и прибавление константы
    1 MassSegmentTree segtree(n, OLL, OLL,
    2 [](int x, int y) { return max(x, y); },
    3 [](int x, int y) { return x + y; },
    4 [](int x, int y, int len) { return x + y; });
     • Взятие суммы и прибавление константы
    1 MassSegmentTree segtree(n, OLL, OLL,
    2 [](int x, int y) { return x + y; },
3 [](int x, int y) { return x + y; },
    4 [](int x, int y, int len) { return x + y * len;
           }):
     • Взятие суммы и присовение
    1 MassSegmentTree segtree(n, OLL, -1LL,
    2 [](int x, int y) { return x + y; },
3 [](int x, int y) { return y; },
    4 [](int x, int y, int len) { return y * len; });
```

## 5 Строковые алгоритмы

### 5.1 Префикс-функция

```
1 vector<int> prefix_function(string s) {
2   vector<int> p(s.size());
3   for (int i = 1; i < s.size(); ++i) {
4    p[i] = p[i - 1];</pre>
```

```
while (p[i] && s[p[i]] != s[i]) p[i] = p[p[i] -
5
      p[i] += s[i] == s[p[i]];
    1
     return p;
9 }
  5.2 Z-функция
1 vector<int> z_function (string s) { // z[i] - lcp
      of s and s[i:]
   int n = (int) s.length();
   vector <int> z (n);
3
    for (int i=1, 1=0, r=0; i<n; ++i) {
    if (i <= r)</pre>
     z[i] = min (r-i+1, z[i-1]);
    while (i+z[i] < n && s[z[i]] == s[i+z[i]])
     ++z[i];
9
    if (i+z[i]-1 > r)
10
    l = i, r = i+z[i]-1;
11 }
   return z;
12
13 }
  5.3
      Алгоритм Манакера
1 vector<int> manacher_odd(const string &s) {
    vector<int> man(s.size(), 0);
3
    int 1 = 0, r = 0;
    int n = s.size();
    for (int i = 1; i < n; i++) {</pre>
      if (i <= r) {
6
        man[i] = min(r - i, man[l + r - i]);
9
      while (i + man[i] + 1 < n && i - man[i] - 1 >=
       0 && s[i + man[i] + 1] == s[i - man[i] - 1]) {
10
        man[i]++;
11
      if (i + man[i] > r) {
        1 = i - man[i];
13
14
         r = i + man[i];
15
16
    }
17
    return man;
18 }
19 // abacaba : (0 1 0 3 0 1 0)
20 // abbaa : (0 0 0 0 0)
21
22 vector <int> manacher_even(const string &s) {
    assert(s.size());
24
    string t;
25
    for (int i = 0; i + 1 < s.size(); ++i) {</pre>
26
      t += s[i];
27
      t += '#';
29
    t += s.back();
30
    auto odd = manacher_odd(t);
    vector <int> ans;
32
    for (int i = 1; i < odd.size(); i += 2) {</pre>
33
      ans.push_back((odd[i]+1)/2);
34
35
    return ans;
36 }
37 // abacaba : (0 0 0 0 0 0)
38 // abbaa : (0 2 0 1)
  5.4 Суфмассив
  Китайский суффмассив
 1 struct SuffixArray {
    vector <int> sa, lcp;
    SuffixArray (string &s, int lim=256) {
3
      int n = (int)s.size() + 1, k = 0, a, b;
```

```
5
      vector \langle int \rangle x(s.begin(), s.end() + 1), y(n),
      ws(max(n, lim)), rank(n);
      sa = lcp = y, iota(sa.begin(), sa.end(), 0);
```

```
for (int j = 0, p = 0; p < n; j = max(111, j *</pre>
       2), lim = p) {
8
        p = j, iota(y.begin(), y.end(), n - j);
Q
         for (int i = 0; i < n; i++) if (sa[i] >= j) y
       [p++] = sa[i] - j;
10
         fill(ws.begin(), ws.end(), 0);
         for (int i = 0; i < n; i++) ws[x[i]]++;</pre>
12
         for (int i = 1; i < lim; i++) ws[i] += ws[i -</pre>
        17:
         for (int i = n; i--; ) sa[--ws[x[y[i]]]] = y[
       il:
14
         swap(x, y), p = 1, x[sa[0]] = 0;
         for (int i = 1; i < n; i++) a = sa[i - 1], b
       = sa[i], x[b] = (y[a] == y[b] && y[a + j] == y[
       b + j]) ? p - 1 : p++;
16
       for (int i = 1; i < n; i++) rank[sa[i]] = i;
17
18
       for (int i = 0, j; i < n - 1; lcp[rank[i++]]=k)</pre>
         for (k && k--, j = sa[rank[i] - 1];
s[i + k] == s[j + k]; k++);
19
21
22 };
23 struct Rmq {
24
    const int INF = 1e9;
25
    int n;
     vector<int> rmq;
26
27
    Rmq() {}
28
     void build(const vector<int> &x) {
29
       assert(x.size() == n);
       for (int i = 0; i < n; ++i) rmq[n + i] = x[i];</pre>
30
       for (int i = n - 1; i > 0; --i) rmq[i] = min(
      rmq[2 * i], rmq[2 * i + 1]);
32
     Rmq(int n) : n(n), rmq(2 * n, INF) {}
34
     void put(int i, int x) {
       rmq[i + n] = min(rmq[i + n], x);
       for (i = (i + n) / 2; i > 0; i /= 2) {
38
         rmq[i] = min(rmq[i * 2], rmq[i * 2 + 1]);
     }
40
41
     int getMin(int 1, int r) { //[1;r)
42
      assert(1 < r);
       int res = INF;
43
       for (1 += n, r += n; 1 < r; 1 /= 2, r /= 2) {</pre>
         if (1 & 1) res = min(res, rmq[1++]);
46
         if (r & 1) res = min(res, rmq[--r]);
48
       return res:
49
    }
50 };
52 struct Lc {
    vector<int> pos;
53
54
     Rmq rmq;
     Lc(string s) : rmq(s.size()) {
       SuffixArrav sa(s):
57
       auto ss = sa.sa;
58
       ss.erase(ss.begin());
       auto lcp = sa.lcp;
       lcp.erase(lcp.begin());
       lcp.erase(lcp.begin());
64
       pos.resize(s.size());
       assert(s.size() == ss.size());
       for (int i = 0; i < ss.size(); ++i) {</pre>
         pos[ss[i]] = i;
68
       int n = s.size();
70
       assert(lcp.size() == n - 1);
       rmq.build(lcp);
72
     int getLcp(int i, int j) {
74
       i = pos[i]; j = pos[j];
75
       if (j < i) {</pre>
76
         swap(i, j);
77
78
       if (i == j) {
79
         return 1e18;
```

10 vector<int> g[N + 7];

11 int dp[N + 7];

12 int ptr[N + 7];

```
}
                                                            13
80
81
      else {
                                                            14 void clear(){
                                                               for (int i = 0; i < N + 7; i++) g[i].clear();</pre>
82
        return rmq.getMin(i, j);
83
                                                            16
                                                                 e.clear();
84
                                                            17 }
    }
85 };
                                                            18
                                                            19 void addEdge(int a, int b, int cap){
                                                                g[a].pb(e.size());
                                                            20
       Алгоритм Ахо — Корасик
                                                            21
  5.5
                                                                 e.pb({b, 0, cap});
                                                                 g[b].pb(e.size());
                                                            23
                                                                 e.pb({a, 0, 0});
1 struct node{
                                                            24 }
    node *next[26] = {}, *link[26] = {};
3
    node *suf = nullptr;
                                                            26 int minFlow, start, finish;
    vector<int> term;
                                                            27
    int visited = 0;
                                                            28 bool bfs(){
    node() {}
                                                                 for (int i = 0; i < N; i++) dp[i] = -1;</pre>
                                                            29
    node *get_next(char c) {
                                                                 dp[start] = 0;
      if (next[c - 'a'] == nullptr) next[c - 'a'] =
                                                            31
                                                                 vector < int > st:
      new node();
      return next[c - 'a'];
                                                            32
                                                                 int uk = 0;
                                                                 st.pb(start);
    }
10
                                                            34
                                                                 while(uk < st.size()){</pre>
11 };
                                                                   int v = st[uk++];
12 node *root = new node();
                                                                   for (int to : g[v]){
13 for (int i = 0; i < s.size(); ++i) {
                                                                    auto ed = e[to];
                                                            37
    node *cur = root;
                                                                     if (ed.cap - ed.flow >= minFlow && dp[ed.to]
                                                            38
    for (char c : s[i]) cur = cur->get_next(c);
15
                                                                   == -1){
    cur -> term.push_back(i);
                                                                       dp[ed.to] = dp[v] + 1;
17 }
                                                                       st.pb(ed.to);
18 vector<node *> bfs_order;
                                                            41
19 queue < node *> bfs;
20 root->suf = root;
                                                                   }
                                                            43
21 for (char c = 'a'; c <= 'z'; ++c) root->link[c - 'a
                                                            44
                                                                 return dp[finish] != -1;
       '] = (root->next[c - 'a'] ? root->next[c - 'a']
                                                            45 }
       : root);
                                                            46
22 bfs.push(root);
                                                            47 int dfs(int v, int flow){
23 while (!bfs.empty()) {
                                                                 if (v == finish) return flow;
2.4
    node *cur = bfs.front();
                                                                 for (; ptr[v] < g[v].size(); ptr[v]++){</pre>
    bfs_order.push_back(cur);
                                                                   int to = g[v][ptr[v]];
26
    bfs.pop();
                                                                   edge ed = e[to];
    for (char c = 'a'; c <= 'z'; ++c) {
                                                                   if (ed.cap - ed.flow >= minFlow && dp[ed.to] ==
      node *nxt = cur->next[c - 'a'];
                                                                    dp[v] + 1){
      if (!nxt) continue;
29
                                                                     int add = dfs(ed.to, min(flow, ed.cap - ed.
      nxt->suf = (cur == root ? cur : cur->suf->link[
30
      c - 'a']);
                                                                   flow));
                                                                     if (add){
      for (char c = 'a'; c <= 'z'; ++c) nxt->link[c -
                                                                       e[to].flow += add;
       'a'] = (nxt->next[c - 'a'] ? nxt->next[c - 'a'
                                                                       e[to ^ 1].flow -= add;
      ] : nxt->suf->link[c - 'a']);
                                                            56
                                                                       return add;
32
      bfs.push(nxt);
                                                            58
33
                                                            59
                                                                   }
34 }
                                                                 }
35 node *cur = root;
                                                                 return 0;
36 for (char c : t) {
                                                            62 }
    cur = cur->link[c - 'a'];
    cur->visited++;
                                                            64 int dinic(int start, int finish){
39 }
                                                                 Dinic::start = start;
40 vector < int > count(n);
                                                                 Dinic::finish = finish:
41 for (int i = bfs_order.size() - 1; i >= 0; --i) {
                                                                 int flow = 0;
    node *cur = bfs_order[i];
42
                                                            68
                                                                 for (minFlow = (1 << 30); minFlow; minFlow >>= 1)
    for (int idx : cur->term) count[idx] = cur->
      visited;
                                                            69
44
     cur->suf->visited += cur->visited;
                                                                     for (int i = 0; i < N; i++) ptr[i] = 0;</pre>
45 }
                                                            71
                                                                     while(int now = dfs(start, (int)2e9 + 7))
                                                                   flow += now;
                                                            72
     Потоки
                                                            73
                                                                 }
                                                            74
                                                                 return flow;
  6.1 Алгоритм Диница
                                                            75 }
                                                            76 } dinic;
 1 #define pb push_back
2 struct Dinic{
3 struct edge{
                                                               6.2 Mincost k-flow
    int to, flow, cap;
5 1:
                                                               6.2.1 Строим граф
7 const static int N = 555; //count of vertices
                                                             1 struct edge {
9 vector<edge> e;
                                                                int next, capacity, cost, flow = 0;
```

3

4

5

6

edge() = default;

```
edge(int next, int capacity, int cost) : next(
  next), capacity(capacity), cost(cost) {}
                                                                       if (edge.rem() > 0 && edge.next == p && dist[
6
                                                                     edge.next] == dist[from[p]] + weight) {
                                                              32
                                                                         edge += 1;
e[index ^ 1] -= 1;
8
     int rem() const { return capacity - flow; }
                                                              33
9
                                                                          cost += edge.cost;
10
     int operator+=(int f) { return flow += f; }
                                                              35
                                                                          break;
                                                              36
11
12
    int operator -=(int f) { return flow -= f; }
                                                                     }
13 };
                                                                   }
                                                              38
                                                                   for (int i = 0; i < n; ++i) {</pre>
14 auto addEdge = [&](auto from, auto next, auto
                                                              40
                                                                     phi[i] += dist[i];
       capacity, int cost) {
     g[from].push_back(e.size());
                                                              41
     e.emplace_back(next, capacity, cost);
                                                                   return cost;
                                                              43 ]:
17
     g[next].push_back(e.size());
18
     e.emplace_back(from, 0, -cost);
                                                              44 \ 11 \ cost = 0;
                                                              45 for (int flow = 0; flow < k; ++flow) {
                                                                   ll a = dijkstra(s, t);
  Если граф ориентированный, то addEdge вызываем один раз. Если
                                                                   if (a == -1) {
   неориентированный, то два, вот так:
                                                                     cout << " -1\n";
                                                              49
 1 addEdge(u, v, capacity, cost);
                                                                     return;
                                                                   }
2 addEdge(v, u, capacity, cost);
                                                                   cost += a;
                                                              52 }
  6.2.2 Запускаем Форда — Беллмана
1 vector<11> phi(n, 0);
                                                                 6.2.4 Восстанавливаем ответ
2 auto fordBellman = [&](int s, int t) {
    phi.assign(n, 0);
                                                               1 auto findPath = [&](int s, int t) {
     for (int iter = 0; iter < n; ++iter) {</pre>
                                                                  vector < int > ans;
5
       bool changed = false;
                                                                   int cur = s;
       for (int u = 0; u < n; ++u) {</pre>
                                                               4
                                                                   while (cur != t) {
         for (auto index : g[u]) {
                                                                     for (auto index : g[cur]) {
           auto edge = e[index];
                                                                       auto &edge = e[index];
9
           if (edge.rem() > 0 && phi[edge.next] > phi[
                                                                       if (edge.flow <= 0) continue;</pre>
       u] + edge.cost) {
                                                                        edge -= 1;
             phi[edge.next] = phi[u] + edge.cost;
10
                                                                       e[index ^ 1] += 1;
11
              changed = true;
                                                                       ans.push_back(index / 4);
                                                              10
           }
                                                              11 // index / 4 because each edge has 4 copies
13
         }
                                                                       cur = edge.next;
       7.
14
                                                              13
                                                                       break;
15
       if (!changed) break;
                                                              14
16
    }
                                                              15
                                                                   }
17 };
                                                              16
                                                                   return ans;
18 fordBellman(s, t);
                                                              17 };
                                                              18 for (int flow = 0; flow < k; ++flow) {
                                                              19
                                                                   auto p = findPath(s, t);
  6.2.3 Ищем кратчайший путь Дейкстрой с потенциалами
                                                                   cout << p.size() << ' ';</pre>
                                                                   for (int x : p) cout << x + 1 << ', ';</pre>
                                                              2.1
1 vector<11> dist;
                                                              22
                                                                   cout << '\n';
2 vector < int > from;
                                                              23 }
3 vector < bool > cnt;
4 auto dijkstra = [&](int s, int t) {
     dist.assign(n, 1e18);
                                                                 7 FFT & co
     from.assign(n, -1);
     cnt.assign(n, false);
                                                                 7.1 NTT & co
     dist[s] = 0;
9
     for (int i = 1; i < n; ++i) {</pre>
       int cur = find(cnt.begin(), cnt.end(), false) -
10
                                                               1 #define int long long
        cnt.begin();
                                                               2 using namespace std;
                                                               3 typedef long long 11;
11
       for (int j = 0; j < n; ++j) {</pre>
12
         if (!cnt[j] && dist[j] < dist[cur]) cur = j;</pre>
                                                               4 const int p=998244353;
                                                               5 int po(int a, int b) {if(b==0) return 1; if(b==1)
13
                                                                     return a; if(b%2==0) {int u=po(a,b/2); return (u
14
       cnt[cur] = true;
       for (int index : g[cur]) {
                                                                     *1LL*u)%p;} else {int u=po(a,b-1);return (a*1LL
        auto &edge = e[index];
                                                                     *u)%p;}}
                                                               6 int inv(int x) {return po(x,p-2);}
         if (edge.rem() == 0) continue;
                                                                 template < int M, int K, int G> struct Fft {
18
         ll weight = edge.cost + phi[cur] - phi[edge.
                                                                   // 1, 1/4, 1/8, 3/8, 1/16, 5/16, 3/16, 7/16, ...
19
         if (dist[edge.next] > dist[cur] + weight) {
                                                                   int g[1 << (K - 1)];</pre>
                                                                   Fft() : g() { //if tl constexpr...
20
           dist[edge.next] = dist[cur] + weight;
                                                                     static_assert(K >= 2, "Fft: K >= 2 must hold");
21
           from[edge.next] = cur;
                                                              11
22
         }
                                                              12
                                                                     g[0] = 1;
                                                                     g[1 << (K - 2)] = G;
                                                                     for (int 1 = 1 << (K - 2); 1 >= 2; 1 >>= 1) {
24
    }
                                                              14
25
     if (dist[t] == (11) 1e18) return -1LL;
                                                                       g[1 >> 1] = (g[1] * 1LL* g[1]) % M;
26
     11 cost = 0:
                                                              16
     for (int p = t; p != s; p = from[p]) {
                                                              17
                                                                     assert((g[1]*1LL * g[1]) % M == M - 1);
       for (auto index : g[from[p]]) {
                                                                     for (int 1 = 2; 1 <= 1 << (K - 2); 1 <<= 1) {
                                                                       for (int i = 1; i < 1; ++i) {</pre>
29
                                                              19
         auto &edge = e[index];
30
         11 weight = edge.cost + phi[from[p]] - phi[
                                                                         g[1 + i] = (g[1] * 1LL * g[i]) % M;
                                                              21
       edge.next];
```

86

87 }

return v;

```
}
22
23
     }
24
     void fft(vector<int> &x) const {
25
       const int n = x.size();
       assert(n <= 1 << K);
26
       for (int h = __builtin_ctz(n); h--; ) {
27
         const int 1 = (1 << h);</pre>
2.8
29
         for (int i = 0; i < n >> (h+1); ++i) {
           for (int j = i \iff (h+1); j \iff (((i \iff 1) +
30
       1) << h); ++j) {
             const int t = (g[i] * 1LL* x[j | 1]) % M;
31
             x[j | 1] = x[j] - t;
32
             if (x[j|1] < 0) x[j | 1] += M;
34
             x[j]+=t;
35
              if (x[j] >= M) x[j] -= M;
37
         }
38
       }
39
       for (int i = 0, j = 0; i < n; ++i) {</pre>
40
         if (i < j) std::swap(x[i], x[j]);</pre>
41
         for (int 1 = n; (1 >>= 1) && !((j ^= 1) & 1);
        ) {}
42
       }
43
     }
     vector <int> convolution(const vector <int> &a.
44
       const vector<int> &b) const {
45
       if(a.empty() || b.empty()) return {};
46
       const int na = a.size(), nb = b.size();
       int n, invN = 1;
47
       for (n = 1; n < na + nb - 1; n <<= 1) invN = ((
48
       invN & 1) ? (invN + M) : invN) >> 1;
49
       vector < int > x(n, 0), y(n, 0);
       \mathtt{std}: \mathtt{copy(a.begin(), a.end(), x.begin());}
50
51
       std::copy(b.begin(), b.end(), y.begin());
52
       fft(x);
53
       fft(y);
54
       for (int i = 0; i < n; ++i) x[i] = (((</pre>
       static_cast<long long>(x[i]) * y[i]) % M) *
       invN) % M;
       std::reverse(x.begin() + 1, x.end());
56
       fft(x):
57
       x.resize(na + nb - 1);
58
       return x;
   }
59
60 };
61 Fft < 998244353,23,31 > muls;
62 vector<int> form(vector<int> v,int n)
64
     while(v.size()<n) v.push_back(0);</pre>
65
     while(v.size()>n) v.pop_back();
66
     return v;
67 }
68 vector<int> operator *(vector<int> v1, vector<int>
      v2)
69 {
70
     return muls.convolution(v1, v2);
71 1
72 vector <int > operator +(vector <int > v1, vector <int >
      v2)
73 {
     while(v2.size()<v1.size()) v2.push_back(0); while</pre>
       (v1.size() < v2.size()) v1.push_back(0);
     for(int i=0;i<v1.size();++i) {v1[i]+=v2[i];if(v1[</pre>
       i]>=p) v1[i]-=p; else if(v1[i]<0) v1[i]+=p;}
76
     return v1;
77 }
78 vector <int> operator - (vector <int> v1, vector <int>
79 {
     int sz=max(v1.size(), v2.size()); while(v1.size()
80
       sz) v1.push_back(0); while(v2.size()<sz) v2.
       push_back(0);
     for(int i=0;i<sz;++i) {v1[i]-=v2[i];if(v1[i]<0)</pre>
81
       v1[i]+=p; else if(v1[i]>=p) v1[i]-=p;} return
82. }
83 vector<int> trmi(vector<int> v)
84 €
     for(int i=1;i<v.size();i+=2) {if(v[i]>0) v[i]=p-v
       [i]; else v[i]=(-v[i]);}
```

```
88 vector<int> deriv(vector<int> v)
89 {
90
     if(v.empty()) return{};
     vector < int > ans(v.size()-1):
92
     for(int i=1;i<v.size();++i) ans[i-1]=(v[i]*1LL*i)</pre>
      %p;
93
    return ans;
94 }
95 vector<int> integ(vector<int> v)
96 -
97
     vector < int > ans(v.size()+1); ans[0]=0;
98
     for(int i=1;i<v.size();++i) ans[i-1]=(v[i]*1LL*i)</pre>
      %p;
99
     return ans;
00 }
01 vector<int> mul(vector<vector<int> > v)
02 {
0.3
     if(v.size() == 1) return v[0];
04
     vector < vector < int> > v1, v2; for (int i=0; i < v. size()</pre>
      /2;++i) v1.push_back(v[i]); for(int i=v.size()
       /2;i<v.size();++i) v2.push_back(v[i]);
     return muls.convolution(mul(v1), mul(v2));
06 }
07 vector<int> inv1(vector<int> v,int n)
08 {
0.9
     assert(v[0]!=0);
     int sz=1; v=form(v,n); vector < int > a = { inv(v[0]) };
11
     while (sz<n)
12
       vector<int> vsz;for(int i=0;i<min(n,2*sz);++i)</pre>
       vsz.push_back(v[i]);
14
       vector<int> b=((vector<int>) {1})-muls.
       convolution(a, vsz);
       for(int i=0;i<sz;++i) assert(b[i]==0);</pre>
       b.erase(b.begin(),b.begin()+sz);
17
       vector<int> c=muls.convolution(b.a):
18
       for(int i=0;i<sz;++i) a.push_back(c[i]);</pre>
19
       sz*=2:
     }
     return form(a,n);
22 }
  7.2 старое доброе FFT
1 using cd = complex < double >;
2 const double PI = acos(-1);
4 void fft(vector < cd > & a, bool invert) {
5
    int n = a.size();
     if (n == 1)
       return:
     vector < cd > a0(n / 2), a1(n / 2);
     for (int i = 0; 2 * i < n; i++) {
10
       a0[i] = a[2*i];
12
       a1[i] = a[2*i+1];
14
     fft(a0, invert);
15
     fft(a1, invert);
16
     double ang = 2 * PI / n * (invert ? -1 : 1);
18
     cd w(1), wn(cos(ang), sin(ang));
19
     for (int i = 0; 2 * i < n; i++) {</pre>
      a[i] = a0[i] + w * a1[i];
21
       a[i + n/2] = a0[i] - w * a1[i];
       if (invert) {
23
         a[i] /= 2:
24
         a[i + n/2] /= 2;
26
       w *= wn:
27
    }
28 1
29 vector<int> multiply(vector<int> const& a, vector<
      int> const& b) {
30
     vector < cd > fa(a.begin(), a.end()), fb(b.begin(),
      b.end());
     int n = 1;
```

```
32
     while (n < a.size() + b.size())</pre>
33
       n <<= 1;
34
     fa.resize(n);
35
     fb.resize(n);
36
37
     fft(fa, false);
     fft(fb, false);
38
     for (int i = 0; i < n; i++)
fa[i] *= fb[i];</pre>
39
40
     fft(fa, true);
41
42
     vector<int> result(n);
43
     for (int i = 0; i < n; i++)
  result[i] = round(fa[i].real());</pre>
44
45
      while(!result.empty() && !result.back()) result.
46
       pop_back();
47
      return result;
48 }
```