Meaning in Motion: A Category-Theoretic Framework for Conceptual Topology

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Introduction:

Natural language exhibits variability, ambiguity, and strong context-dependence, challenging formal semantic modeling. However, through the lens of category theory and Z-frame used in this theory, certain latent regularities emerge. We explore the idea that word meaning is not merely a function of proximity in embedding space, but a topological structure defined by morphic continuity over conceptual anchors, called Z-frames.

Content Note

This paper analyzes metaphorically charged vocabulary—including bodily and emotional imagery drawn from song lyrics—solely in terms of structural and semantic relationships.

In accordance with category theory, the focus is on morphic relationships and topological coherence, not object-level attributes or empirical interpretations.

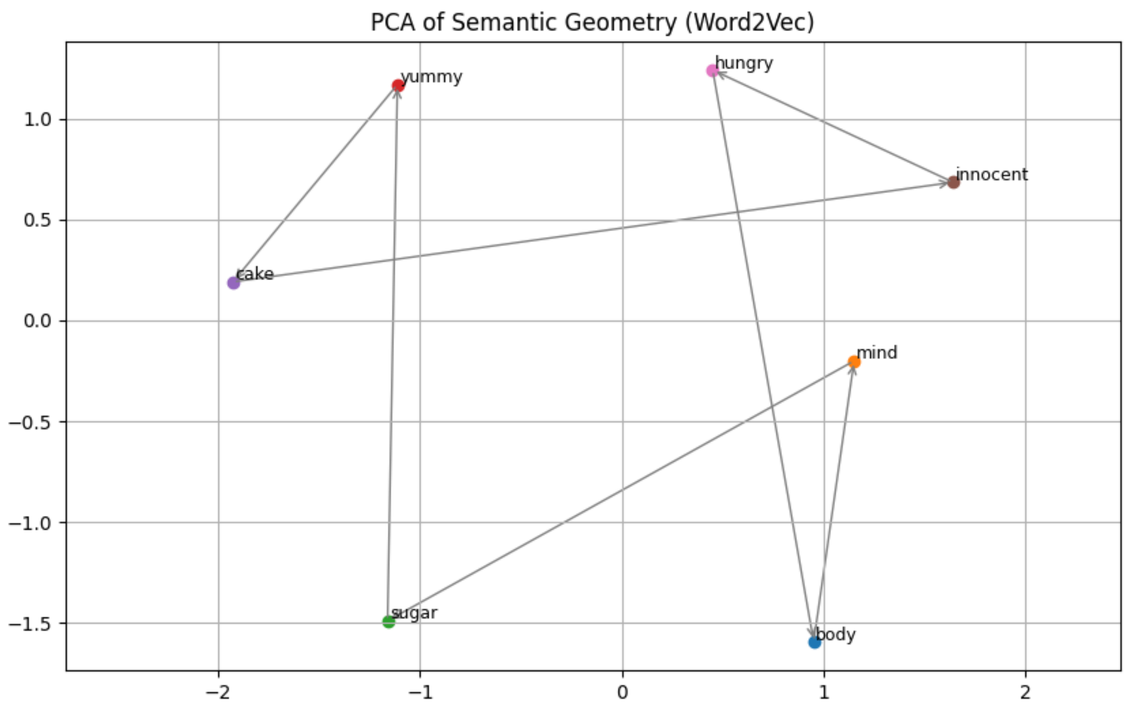
king - man + woman = queen

alt text

Semantic Projections and Topological Formalization

A principal component analysis (PCA) of embedding vectors corresponding to ["body", "mind", "sugar", "yummy", "cake", "innocent", "hungry"] reveals a triadic geometric structure anchored around the semantic attractor Z = sugar.

Figure 1. Z = sugar

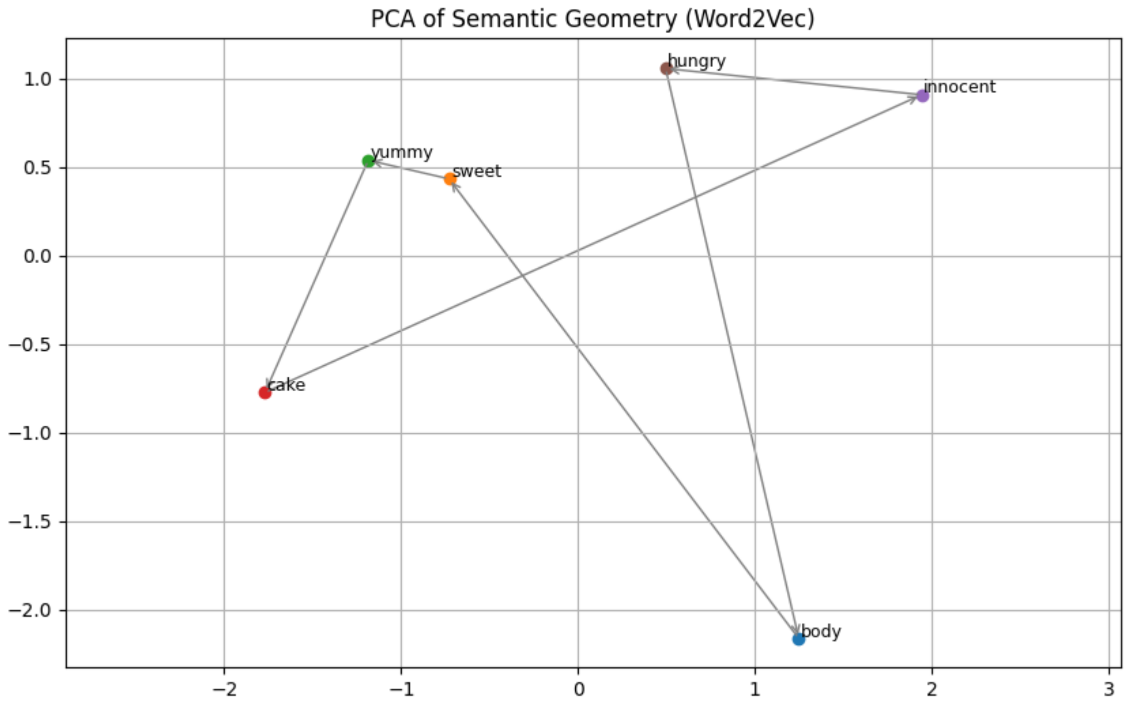


(*Lexical data sourced from the song "Yummy" perfomed by INNA, 2023.*)

This diagram reveals that clusters such as ["yummy", "cake"] and ["mind", "body"] emerge symmetrically around the semantic anchor sugar, forming a quasi-balanced configuration. Notably, "body" and "cake" align across opposing poles, indicating that the concept of sweetness (Z = sugar) functions as a mediating structure—bridging semantic crevasses while simultaneously inducing a communicative relation between otherwise distinct lexical domains.

Note: In colloquial English, "cake" often evokes connotations beyond dessert — frequently used as a symbol of bodily allure. Within our Z-frame (sugar), this metaphor is not accidental. Rather, "cake" and "body" form a morphic equivalence, mediated by sweetness, with semantic flows aligning across physical and cultural registers.

Figure 2. Z = body



(*Lexical data sourced from the song "Yummy" perfomed by INNA, 2023.*)

This reveals distinct lexical clusters symmetrically aligned over the semantic anchor *body*. The morphism Hungry → Sweet/Yummy | body (the symbol | indicates the Z-frame) can be interpreted as a desiring gaze projected onto a luscious body—an interpretation directly supported by the lyric line from INNA's "Yummy"(2023): *"Those hungry eyes / Tracing my body lines."*

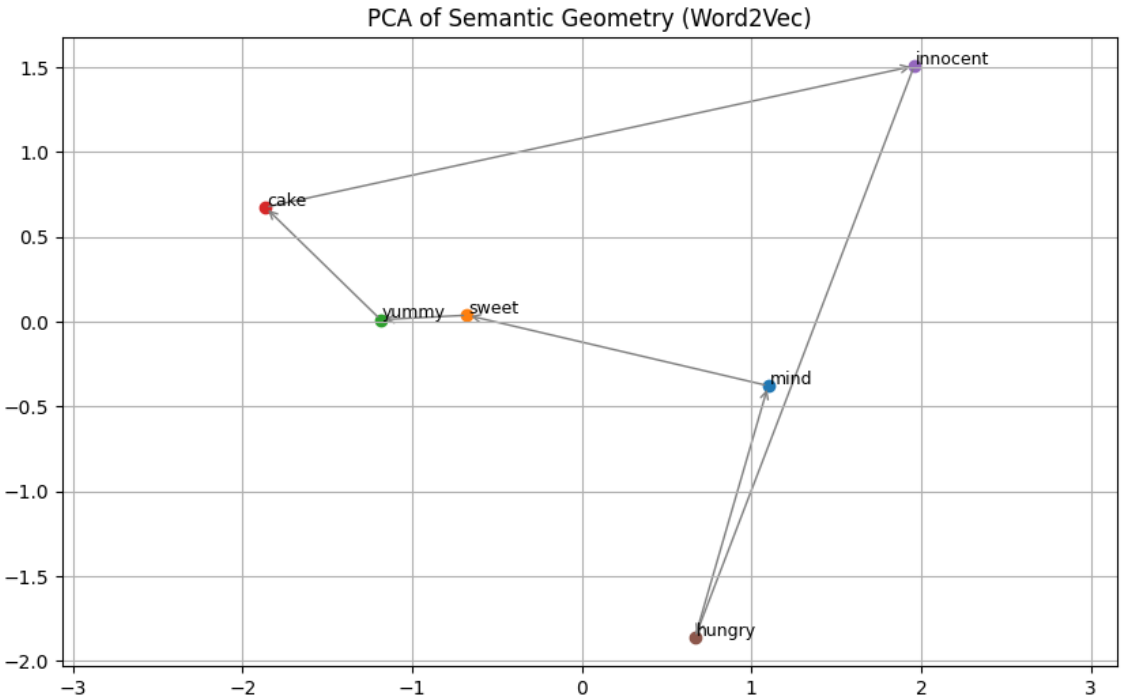
Conversely, the inversion of the morphism can also be read causally: sweetness or yumminess evokes hunger, completing a morphic loop.

Another morphism, cake → innocent | body encodes a metaphor of virginity or naive allure. Innocence, connoting sweetness, is metaphorically grounded in the figure of cake — suggesting a playful or untouched body that invites desire through charm rather than overt sensuality.

Note: While the lyrics include the phrase *"Ain't no innocent, innocent mind, in this club tonight"* (INNA, "Yummy", 2023), we do not evaluate the actual properties of objects in category theory. Instead, we focus exclusively on the relational structure among the words used in the lyrics. Our interpretation is grounded in how semantic morphisms operate within the Z-frame, independent of referential truth.

Figure3. Z = mind

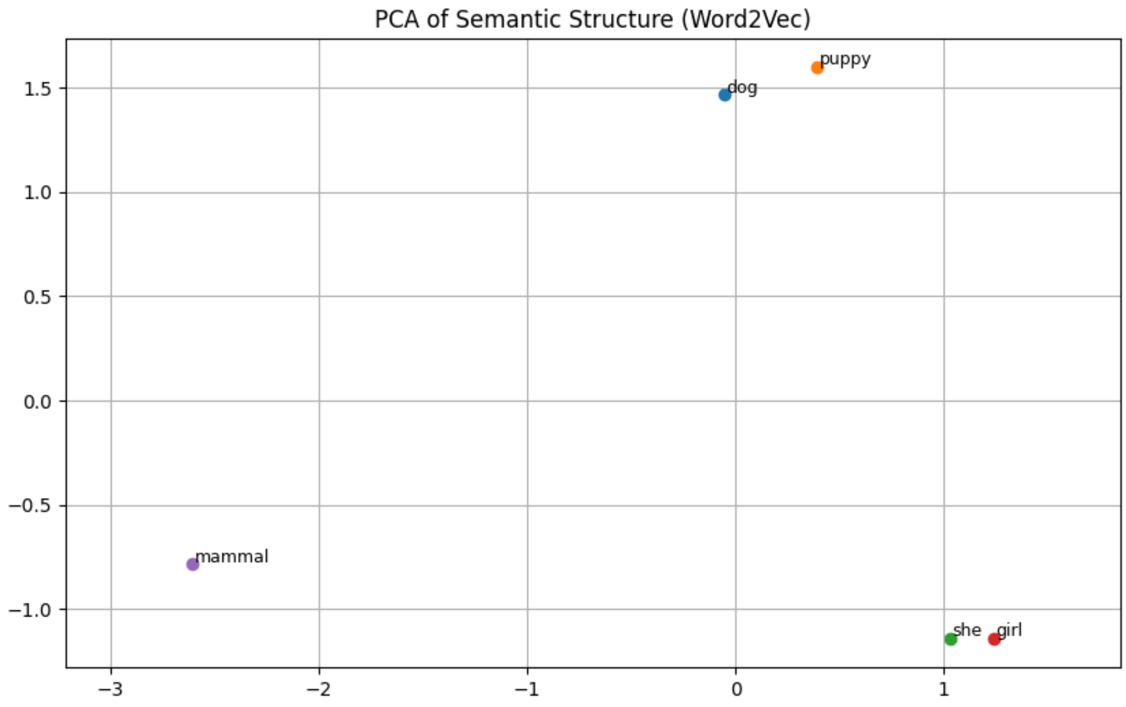
The lexical structure must be carefully configured to preserve morphic coherence. While Z = body functions as a stable semantic anchor—maintaining the commutative integrity of conceptual flows — Z = mind fails to support a coherent morphic path. As shown in this PCA projection, morphic stability breaks down, collapsing into asymmetric disruption and losing structural coherence.



(*Lexical data sourced from the song "Yummy" perfomed by INNA, 2023.*)

This illustrates that Z-frame selection is not arbitrary. A well-chosen Z-frame acts as a semantic attractor, stabilizing morphic trajectories across the lexical topology. In contrast, inadequate anchors—such as mind in this instance—induce morphic rupture, resulting in the disintegration of structural stability.

Figure4. Z = mammal
This reveals different clusters are symmetrically placed through mammal. These lexical elements semantically converge through the conceptual anchor mammal, forming a coherent fiber structure.

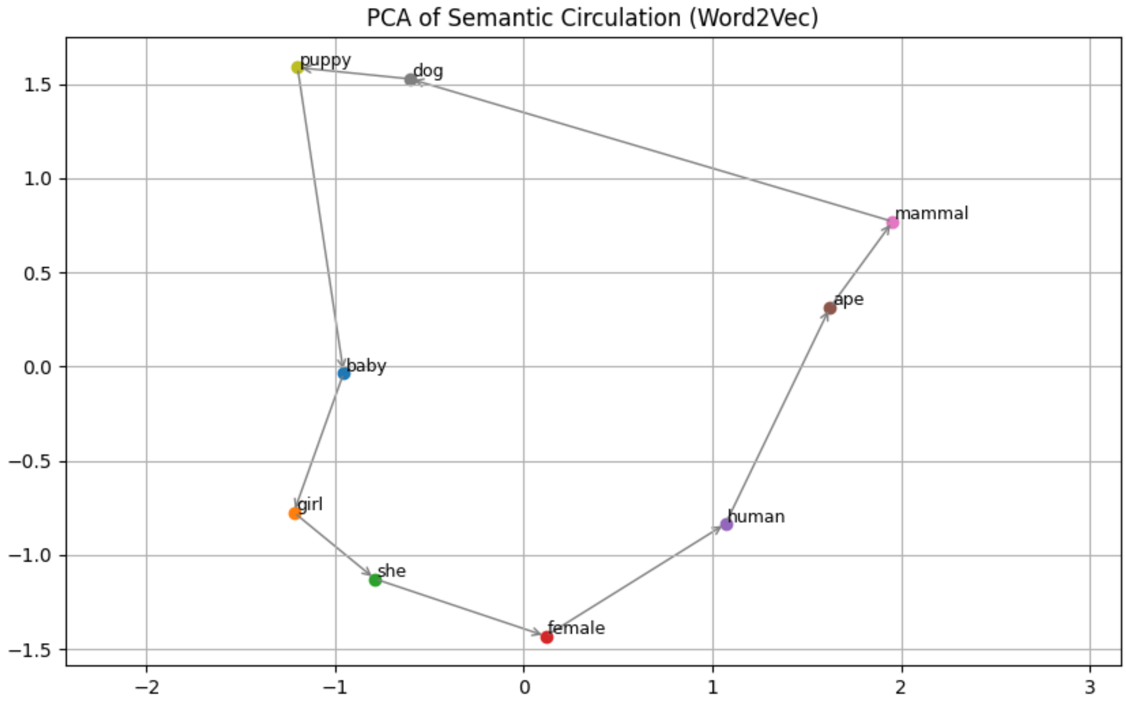


Quasi Communicative-Diagram

```
puppy      girl    //specified: size + young
 |          |
 |          |
dog        she     //abstraction
  \        /
    Mammal
```

Figure5. Z = baby and mammal

This projection reveals a closed morphic cycle, in which semantic transitions form a circular structure across intersecting Z-frames—baby and mammal. Initially, replacing she with female in Fig.4 disrupted the geometric coherence of the semantic projection. To restore structural continuity, the conceptual chain was extended with the additional elements she, human, ape and baby, thereby completing the circulation. The resulting configuration formed a closed morphic loop across intersecting Z-frames — baby and mammal — as originally intended with baby serving as a bridge between distinct morphic chains and thereby closing the loop.



Depending on the granularity of interpretation, the circulation can be decomposed into two or three morphic chains:

- **Two-chain view**:
  puppy → *mammal*
  girl → *mammal*
- **Three-chain view**:
  puppy → *mammal*
  girl → *female*
  female → *mammal*

This morphic loop demonstrates how semantic trajectories from seemingly disparate domains converge onto a shared conceptual attractor (*mammal*), while originating from distinct frames such as *gender*, *human*, and *canine*. In the three-chain interpretation, these domains are connected through intermediate nodes like *baby* or *female*, forming a continuous morphic cycle. The resulting closed circulation reflects topological stability within a morphic semantic system.

Concrete Communicative Model: 3-chain view

```
          b
     G ←────── P
     |         ↑
   g |         | c
     ↓         |
     F ──────→ M
         h
P: Puppy
G: Girl
F: Female
M: Mammal

b: baby (Z Frame)
g: gender (gender morphic chain)
h: human (human morphic chain)
c: canine (canine morphic chain)
```

Simplified Quasi Communicative-Diagram:

```
         Z = baby
     puppy  ←   girl    //specified: size + young
      |          |
      |          |
     dog        she     //abstraction
       \        /
         Mammal
```

These lexical arrangements are unlikely to be coincidental. Rather, they appear to instantiate a deeper semantic topology—one governed by morphic continuity structured over a Z-frame. The underlying theory formalizing this behavior is defined as follows:

```
CT := (C, B, π: E → B, Fb := π⁻¹(b), A ≅ b ⋃ Nat(Hom(−, A), Fb))

Where:
  - C is the category of concepts (objects = words or concepts)
  - B is the base space of Z-frames (semantic continuity anchors)
  - E is the total semantic space (word vector embedding space)
  - π projects each concept to its semantic base (Z-frame)
  - Fb is the fiber (semantic morphic chain) over a base b
  - A ≅ b ⋃ Nat(Hom(−, A), Fb) interprets each concept A via its morphisms
relative to its   Z-frame b (Yoneda perspective defined in appendix)
```

Morphic Chain Complexes and Semantic Dynamism

We formalize the observed phenomena in Fig.4 or Fig.5 as a *Morphic Chain Complex*, representing the continuity of meaning across lexical transitions. These morphic chains capture structured semantic flows that operate within and across shared Z-frames.

Quasi Communicative-Diagram (Fig.4)

```
puppy  ←   girl    //specified: size + young
 |          |
 |          |
dog        she     //abstraction
  \        /
    Mammal
```

Morphic Chain Complex

We define morphic chains such as:

- *girl → she → mammal*
- *puppy → dog → mammal*

as instances of a **Morphic Chain Complex**, representing layered conceptual transitions.

Formally we define:

```
Let **D(Cₙ₋₁ | Z)** := Category of Morphic Chains over **Ob(Cₙ₋₁)** within a given Z-frame.
```

These chains represent **meaning-preserving semantic flows** under the continuity constraints of a shared Z-frame **Z**.

Objects in this category are chain complexes:

```
C₀ → C₁ → C₂ → ...
```

which encode stepwise conceptual transitions (e.g., *she → she → mammal*), typically reflecting trajectories such as *human-like life abstraction*.

Morphisms between chains are **chain maps** that preserve *morphic identity*—either fully (preserving semantic structure) or partially (up to rupture, i.e., structural non-invertibility).
In other words, they are mappings between semantic flows that maintain structural coherence even under conceptual deformation.

Mirror Morphism Definition:

Each mirror maps conceptual transitions across vocabularies while preserving morphic identity up to rupture—that is, it allows for semantic divergence that still respects underlying structural continuity, even if exact invertibility is not preserved.

```
f : X  → Y  | Z ∈ Dᵢ
f': X' → Y' | Z ∈ Dᵢ₊₁
  ⇒X' ≠ X, but cod(f) = cod(f') | CD (CD = codomain)

We define f' as a mirror-correspondent morphism of f under a given Z-frame,
if and only if:

∃Z: rupture(f, f' | Z) ≠ ∅
∧ cod(f) = cod(f') | CD
```

Note: Z: rupture(f, f' | Z) ≠ ∅ means that there exists a Z-frame under which *f* and *f'* exhibit structural divergence—i.e., they are not fully invertible but still converge at the codomain level.

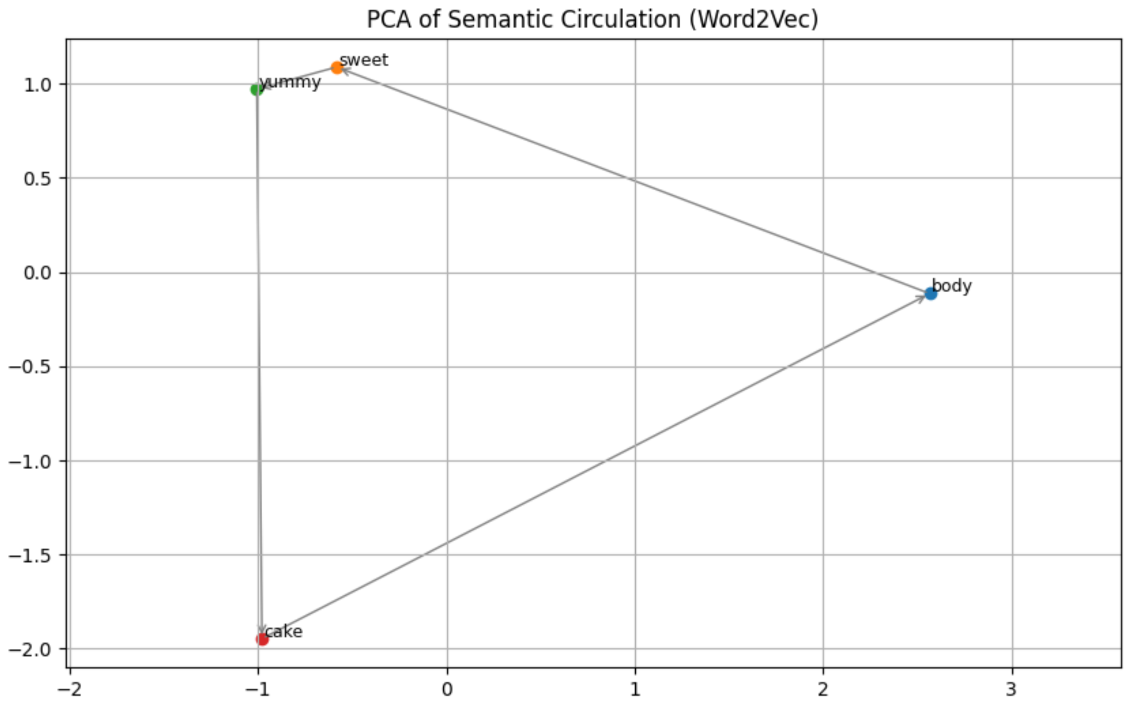
For example, let *Z = abstraction*. This allows a semantic transition from *girl* → *she* and *puppy* → *dog*, treating them as mirror morphisms under a shared conceptual frame.
However, if we take *Z = agency*, a rupture emerges: *puppy* → *dog* lacks agency, while *girl* → *she* retains it.
Hence, **rupture(f, f' | agency) ≠ ∅**, yet *f* and *f'* still align toward the same codomain (e.g., *mammal*).

Quasi-Natural Transformation of Meaning Systems

A **Morphic Chain Mirror** is a contextual correspondence between two morphic chains drawn from distinct but meaning-aligned vocabularies. This correspondence is realized through a **quasi-natural transformation** under a shared intermidiating Z-frame.

```
η: Dᵢ ⇒ Dᵢ₊₁  | CD (CD = codomain)
η_X ∘ Dᵢ({f₁ | Z₁, ..., fₙ | Zₙ}) ≈ Dᵢ₊₁({f'₁ | Z₁, ..., f'ₙ | Zₙ}) ∘ η_Y |
CD
for all fⱼ: Xⱼ → Yⱼ | Zⱼ ∈ Dᵢ ,
where f'ⱼ: η_X(Xⱼ) → η_Y(Yⱼ) | Zⱼ

Then, η is said to be a quasi-natural transformation under the Z-frame
i.e. η ∈ Mor(C) where C is the contextual meaning category
Example: η: girl → puppy | Z = baby
```

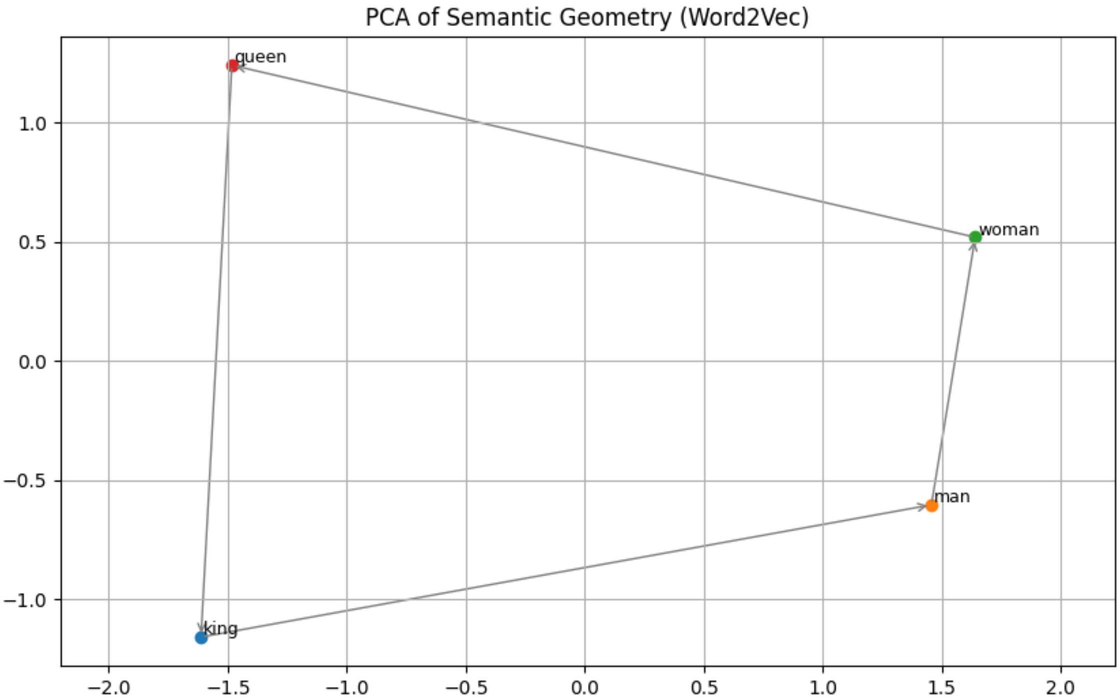
Communicative Diagram of Semantics




```
Under Z-frame: sugar

         g
 Yummy <---- Mind
   |          ↑
 h |          | f
   ↓          |
 Cake  ----> Body

f: Body → Mind | sugar // Bodily sweetness gives rise to a mental
perception of sweetness
g: Mind → Yummy | sugar //Mental sweetness leads to the subjective
pleasure: "this is yummy."
h: Yummy → Cake | sugar //The abstract sense of yumminess gets grounded in
a concrete symbol: cake.

if:
i = h∘g∘f | sugar
Thus:
Cake ≅ Body | sugar // metaphor
```

king - man + woman = queen

This relationship can be expressed as the following semantic morphism diagram.




```
        r
    Q ←──── W
    |       ↑
  f |       |h
    ↓       |
    K ────→ M
        g

K = king
Q = queen
M = man
W = woman

g: Differential Morphism(King → Man)
h: Differential Morphism(Man → Woman)
r: Differential Morphism(Woman → Queen)
f: Composed Semantic Morphism r ∘ h ∘ g
```

Conclusion

This geometric coherence reveals an underlying semantic structure in natural language, formally characterized as *semantic geometry through conceptual topology*. The proposed framework integrates category-theoretic notions such as morphisms, commutativity, and fiber structures into the modeling of linguistic meaning, enabling a rigorous yet visually tractable formalization of metaphor, analogy, and poetic logic.

Appendix:

The Definition Of Word
A word that includes itself or other words is subsumed by a higher-order concept. That concept, in turn, is structurally embedded within the word. Therefore, a word is a self-referential structure.

```
Let C be a concept (e.g., black)

//A concept is a structure of oppositional differences
Let OC be the opposite concept or not relative concept (e.g., white, not pink)
C = {-OC₁, −OC₂, ..., −OCₙ} (Yoneda × Structuralism)

// A word is a concept, and belongs to the category of words
word ≅ C and Ob(Word) ∋ word

// A concept is a bundle of semantic morphisms (in the word2vec sense)
// Interpretable as a more concrete model via Grothendieck fibration
∀ fᵢ ∈ Mor(C), ∃∞_f s.t. fᵢ ⊆ ∞_f

// The structure of semantic morphisms is abstracted into a word
∀ ∞_f, ∃ Word s.t. ∞_f ⊆ Word

and then: Word≅Nat(h∞f,Language)
// The ∞-structure of morphisms is deployed into the linguistic category as a natural transformation

// Self-identity of the word: word is word
id_word ≈word
```

Appendix 2

Meaning can be formalized through semantic monoid computation, and alternatively expressed via set-theoretic composition as follows:

```
Let king = {Royalty□, Male□, Human□}
Let queen = {Royalty□, Female□, Human□}
Let man = {Male□} Let woman = {Female□}

king - man + woman = queen
{Royalty□, Male□, Human□} ⊖ {Male□} ⊕ {Female□}
= {Royalty□, Female□, Human□}
```

In this framework, lexical items are treated as elements of a semantic monoid, where addition ⊕ and subtraction ⊖ correspond to meaning-preserving operations in vector space semantics.

Used Code

```python
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
from gensim.models import KeyedVectors
import numpy as np

model_path = ''
model = KeyedVectors.load_word2vec_format(model_path, binary=True)

words = ["body","mind","sugar","yummy","cake","innocent","hungry"]

vectors = [model[word] for word in words]
labels = words

pca = PCA(n_components=2)
reduced = pca.fit_transform(vectors)

plt.figure(figsize=(10, 6))
for i, label in enumerate(labels):
    x, y = reduced[i]
    plt.scatter(x, y)
    plt.text(x + 0.01, y + 0.01, label, fontsize=9)

#Add Arrows
from matplotlib.patches import FancyArrowPatch
for i in range(len(reduced) - 1):
    start = reduced[i]
    end = reduced[i + 1]
    arrow = FancyArrowPatch(start, end, arrowstyle='->', mutation_scale=10,
color='gray')
    plt.gca().add_patch(arrow)
```

```
    start = reduced[len(reduced)-1]
    end = reduced[0]
    arrow = FancyArrowPatch(start, end, arrowstyle='->', mutation_scale=10,
    color='gray')
    plt.gca().add_patch(arrow)

    plt.title("PCA of Semantic Structure (Word2Vec)")
    plt.grid(True)
    plt.axis("equal")
    plt.savefig("image.png")
    plt.show()
```

Word2Vec Data Set

https://code.google.com/archive/p/word2vec/

Reference

INNA, "Yummy", Global Records, 2023

概念位相論 / Conceptual Topology

This theory, named 概念位相論 or Conceptual Topoloy, was proposed by **No Name Yet Exist**.

GitHub: https://github.com/No-Name-Yet-Exist/Conceptual-Topology
Note: https://note.com/xoreaxeax/n/n3711c1318d0b

Author Verification(sha-256 generated 2025-05-06):
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