Conceptual Topos As Semantic Cage: An Algebraic Topology of Meaning based on Conceptual Topology

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Introduction

Conceptual Topos v1.0 is an initial formalization of the algebraic topology of meaning based on Conceptual Topology.

This version sketches core axioms for Topos including:

- Initial Object
- Subobject Classifier Ω
- Terminal Object
- Fibered Topos structure
- Semantic Exponential via σ operator

Finite Limits (Pullbacks, Products, Equalizers) are under ongoing formalization.

Future versions (v1.x) will refine the formalization and extend it.

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1. Monoid Structure of Semantic Flow (M|Z):

In Conceptual Topology, Z is defined as a mediating point/semantic anchor.

```
Let C and D, Z be categories,
with semantic projection π: C ∪ D → Z, such that for each X ∈ Ob(C ∪ D):

π(X) ∈ Ob(Z)

For each X ∈ Ob(C ∪ D), there exists morphism:

f_X: X → π(X)
f_X⁻¹: π(X) → X

such that:

f_X⁻¹ ∘ f_X ≅ id_X

For morphism f: X → Y | Z,
this corresponds to:

f_Z: π(X) → π(Y) in Z

For any X, Y ∈ Ob(C ∪ D):

Let [X]_Z := semantic representation of X under frame Z (i.e., π(X))

Then:

[ X ]_Z1 ≅ [ Y ]_Z2 | Z1, Z2 ∈ Z //or Z1, Z2 >> Z
which means:
["Dog"]_Pet = [Retriever, Dachshund, Poodle, Bulldog, ...]
["girl"]_Human = [girl, woman, person, ...]
["Dog"]_Pet ≅ ["girl"]_Human | Life

Then the set of semantic flow morphisms under Z forms a monoid:

M|Z = { fₙ ∘ ... ∘ f₁ | all fᵢ: Xᵢ → Xᵢ₊₁ | Z ∧ ∀ i, j: fᵢ ≅ fⱼ | Z }

This is also defined as Morphic Chain:

Let **D(Cₙ₋₁ | Z)** := Category of Morphic Chains over **Ob(Cₙ₋₁)** within a
given Z-frame.
where:D(Cₙ₋₁ | Z) ={ C₀ → C₁ → C₂ → ... | Z }

or as a set
D(Cₙ₋₁ | Z) ={ C₀, C₁, C₂, ... | Z }
```

2. Identity Element of M|Z

```
Let: M|Z = { fₙ ∘ ... ∘ f₁ | all fᵢ: Xᵢ → Xᵢ₊₁ | Z ∧ ∀ i, j: fᵢ ≅ fⱼ | Z }

Define the identity element of M|Z as a family of identity morphisms over
the shared Z frame:

For each X ∈ Ob(C ∪ D), there exists a unique identity morphism under a Z
frame:

e|Z_X := id_X | Z

Then, for any f: X → Y | Z ∈ M|Z:

e|Z_X ∘ f = f
f ∘ e|Z_Y = f

Therefore, the identity structure of M|Z is given by the family:

{ id_X | Z | X ∈ Ob(C ∪ D) }

which forms a pointwise identity across the objects under the common Z
frame.
This ensures that M|Z satisfies the identity axiom of a monoid.
```

3. Associativity of M | Z

```
Let: M|Z = { fₙ ∘ ... ∘ f₁ | all fᵢ: Xᵢ → Xᵢ₊₁ | Z ∧ ∀ i, j: fᵢ ≅ fⱼ | Z }

Then for all f, g, h ∈ M|Z:

(f ∘ g) ∘ h = f ∘ (g ∘ h)

Thus, the composition ∘ in M|Z is associative.
```

4. Axioms

4.1. Identity Element

Unit Axiom 1: Identity Element Z

```
id_Z:= Z → Z | Z
∀f ∈ M|Z: id_Z ∘ f = f  and f∘id_Z =
```

Definition:

```
Statement:
Z-frame itself is the unit of M | Z.

Formal Definition:
Z:= Z → Z|Z

Justification:
Since any morphism in M | Z is defined as:

f: X→Y|Z

and Z itself is defined as its own identity morphism:

Z:= Z→Z|Z

then:
id_Z = Z

Conclusion:

Therefore:
id_Z is the unit element of M|Z.

∀f ∈ M|Z: (id_Z ∘ f | Z) = f and (f ∘ id_Z | Z)= f
(with frame-preserving composition)

∴id_Z is the unit of M|Z.
```

Note:

```
idZ :Z→Z | Z
f:X→Y | Z
(idZ|Z)∘(X→Y|Z)
```

Unit Axiom2: Void Concept

```
f ∈ M|Z
"" ∘ f = f  and f ∘ "" = f
id_Z ∘ f = f and f ∘ id_Z = f
```

Definition:

```


The empty concept is a theoretically assumed concept, denoted as "",
which acts as the unit element at the conceptual / lexical level.

Formal Definition:

"" ∘ f = f and f ∘ "" = f

Justification:

The empty concept "" represents no lexical or semantic content.
Composing any morphism f with the empty concept does not alter the flow of
meaning.
Conclusion:

"" is the unit element at the conceptual level of Conceptual Topology.

```

4.2. Zero Morphism: Negation Morphism

We define semantic zero morphism, negation morphism: n_f In CT as the result of applying Not() to a morphism

```
g:σ(Z). Not(g){ A ↛ B | Z} = A ↛ B|Z = n_f
where: g: A → B

Formal Properties (Axiom):

∀g:X→Y|Z where composition with n_f is defined:

∀g: g ∘ n_f = n_f and n_f ∘ g = n_f

Left Side:
g: A→B
g∘(A↛B|Z) = A↛ B|Z

Right Side
g: A→B(A↛B|Z)∘g=A↛B|Z
```

```
Interpretation:
Applying Not() to any morphism produces a semantic zero morphism, which
collapses any further semantic flow.

Natural Language:
Left Side: g∘(A↛B|Z)
"A is not B"
The apple is not a fruit

Right Side: (A↛B|Z)∘g
"B is not A"
This is a fruit, but this is not an apple which is a fruit.


In CT, this was called rutpure().
Now defined:
rupture(A,B,Z)= σ(Z).Not(g) = n_f = A↛B|Z
```

4.3. Composition Axiom

```
M|Z = { fₙ ∘ ... ∘ f₁ | all fᵢ: Xᵢ → Xᵢ₊₁ | Z ∧ ∀ i, j: fᵢ ≅ fⱼ | Z }

Then for all f, g, h ∈ M|Z:

For f, g, h ∈ M|Z,
where:
f:V → W | Z
g:Y → V | Z
h:X → Y | Z

(f ∘ g) ∘ h = f ∘ (g ∘ h)
```

Example:

```
For f, g, h ∈ M|Z,

where:
f:she → you | Human
g:he → she | Human
h:man → he | Human

(f ∘ g) ∘ h = f ∘ (g ∘ h)
```

5. Conceptual Topos

5.1. Category Level: Initial Object

Definition:

```
Let Concept be a category where Ob(Concept) are lexical / conceptual
objects.
Then "" ∈ Ob(Concept) is Initial Object if:

∀ X ∈ Ob(Concept), ∃ unique morphism:

u_X : "" → X | X

such that:

∀ f: X → Y | Z,
f ∘ u_X = u_Y
```

Monoid Level: Unit in M|Z

```
Recall:

M|Z = { fₙ ∘ ... ∘ f₁ | all fᵢ: Xᵢ → Xᵢ₊₁ | Z ∧ ∀ i, j: fᵢ ≅ fⱼ | Z }

Now, define:

"" ∈ Ob(Concept)

and identity morphism under Z-frame:

e|Z_"" := id_"" | Z

Then for all f ∈ M|Z:

e|Z_"" ∘ f = f
f ∘ e|Z_"" = f
```

5.2. Finite Limits

Terminal Object Conceptual Topos defines a terminal object as the Z-frame identity:

```
id_Z := Z → Z | Z


Any morphism f: X → Z | Z factors uniquely through id_Z.

This realizes the semantic universal target:

∀ X ∈ Ob(C ∪ D), ∃! f_terminal: X → Z | Z
```

Example:

```
she → human | Human
me → human | Human
```

Note: Pullbacks, Products, Equalizers under construction

5.3. Exponentials

Conceptual Topos models exponentials via semantic shift operators.

Definition

```
For any objects A, B:

B^A exists such that:
Hom(X ⊗ A, B) ≅ Hom(X, B^A)
```

Construction via σ operator

Semantic shift operators:

```
σ(X). >>(A, B)
or
σ(X). >(A, B)

act as internal exponential morphisms:

        (A, B) ≅ B^A
```

Definition: Semantic Shifting Morphism (σ)

```
σ: D(Xₙ₋₁ | X) → D(Xₙ₋₁ | X)

such that σ ⊕ f ∈ M|Z if and only if type compatibility holds:

∀ A, B, (A → B) ∘ σ(X) is valid if:

( A >> X or X >> A )
and
( B >> X or X >> B )

Definition: Subsumption
A >> X ≡ A ⊑ X

Definition: SubsumedBy
X >> A ≡ X ⊑ A

Example:
king → king >> human → human
  ⇒ king >> human  → valid
```

```
human → human >> queen → queen
  ⇒ human >> queen → valid
```

Example

```
σ(Human). >>(puppy → dog → mammal | Canine, Human)
≅ girl → she → mammal | Human
```

This shift realizes an internal semantic transformation corresponding to exponential behavior.

5.4. Definition of Ω

Let **Ω** be an object in the Concept category, representing the **semantic truth space**.

```
For any subobject (conceptual inclusion):

m: M ↪ X

there exists a unique characteristic morphism:

χ_m: X → Ω
```

such that the following diagram commutes:

```
M ────→ X
|       |
|       v χ_m
|       Ω
```

Interpretation in Conceptual Topology

- **Ω** encodes **semantic entailment / membership / inclusion**.
- **Z-frame membership** is naturally mapped to Ω:

```
χ_Z : X → Ω
```

interpreted as:
"Does X conceptually belong to Z-frame Z?"

Examples

Example 1: Dog in Pet Z-frame

χ_Pet(Dog) = True

Example 2: Apple in Pet Z-frame

χ_Pet(Apple) = False

Example 3: Innocent in Body Z-frame (after rupture)

χ_Body("innocent") = True / False depending on whether the semantic projection is coherent under Z-Frame.

Relation to Rupture

Semantic rupture can be lifted to Ω as:

σ(Not)(f: A → B | Z) ⇒ rupture(A,B,Z) ⇒ χ_Z(f) = False

Thus, **negation** and **semantic discontinuity** become **Ω-classifiable**.

5.5. Conceptual Topos as Fibered Topos over Z-frame

Conceptual Topos is structured as a **fibered topos** over the semantic base space **Z-frame**.

Z-frame as Fibered Structure

- Let π: C ∪ D → Z be the semantic projection.
- Each fiber $\pi^{-1}(Z)$ forms a category of morphic chains **D(C_{n-1} | Z)**.
- Morphisms of the form:

```
X → Y | Z ≡ X → Y in fiber over Z
```

correspond to morphisms within the fibered structure over Z.

Initial Object and Codomain Projection

- The **Initial Object ""** serves as the semantic origin.
- It projects into the codomain via:

```
"" → | X ≡ "" → π(X)
```

```
   ""
    ↓ u_X
  X ────→ π(X) (in Z-frame)
```

```
Fiber π⁻¹(Z_X):
    "" → X → Y
```

Thus, semantic generation naturally occurs anchored in Z-frame.

Semantic Flow Closure

- Semantic flows:

```
X → Y | Z
```

are closed within the fiber over Z, corresponding to the codomain Z of the semantic projection π.

- Rupture and negation are classified by **Ω**:

```
χ_Z: X → Ω
```

Conclusion

Conceptual Topos is a **fibered topos** over Z-frame:

```
π: C ∪ D → Z
```

with:

- Initial Object "" → codomain π(X)
- Morphic Chains as fibers $\pi^{-1}(Z)$
- Ω as subobject classifier in Z
- σ operator inducing internal exponential morphisms.

Conceptual Topos Named as 概念位相論 / Conceptual Topology

This theory, named 概念位相論 or Conceptual Topoloy, was proposed by **No Name Yet Exist**.

GitHub: https://github.com/No-Name-Yet-Exist/Conceptual-Topology
Note: https://note.com/xoreaxeax/n/n3711c1318d0b
Zenodo: https://zenodo.org/records/15455079

This is Version: 1.0
