RSA for Bayes Filters and POMDPs

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1 Definitions

- $b_i(s)$ is the listener's current belief in s
- $b^0(s \mid u)$ is the primitive interpretation of utterance u
- $b_i^d(s \mid u) = \dfrac{O^{d-1}(u \mid s)b(s)}{\sum\limits_{s'} O^{d-1}(u \mid s')b(s')}$ would be the listener's updated belief if they interpreted u with RSA of depth d
- $O_i^d(u \mid s) = \dfrac{e^{\alpha \ln(b_i^d(s|u))}}{\sum\limits_{u'} e^{\alpha \ln(b_i^d(s|u'))}} = \dfrac{\left(b_i^d(s \mid u)\right)^\alpha}{\sum\limits_{u'} \left(b_i^d(s \mid u')\right)^\alpha}$ is the probability of the speaker saying u to communicate s with RSA of depth d.

2 Desired behavior

We hope that using the listener's current belief will allow utterances to have context-dependent meaning. As an example, we would like for a single, when spoken under belief b_i, to be evidence for s_0, but when spoken under b_1 act as evidence against s_0. Equivalently, we wish to find $b_i, b_j, b^0, d, \alpha, u, s_0, s_1$ s.t.

$$(1) \quad \frac{O_i^d(u \mid s_0)}{O_i^d(u \mid s_1)} > 1, \qquad \frac{O_j^d(u \mid s_0)}{O_j^d(u \mid s_1)} < 1$$

Substituting the speaker formulas gives

$$\frac{\dfrac{\left(b_i^d(s_0 \mid u)\right)^\alpha}{\sum\limits_{u'} \left(b_i^d(s_0 \mid u')\right)^\alpha}}{\dfrac{\left(b_i^d(s_1 \mid u)\right)^\alpha}{\sum\limits_{u'} \left(b_i^d(s_1 \mid u')\right)^\alpha}} > 1, \quad \frac{\dfrac{\left(b_j^d(s_0 \mid u)\right)^\alpha}{\sum\limits_{u'} \left(b_j^d(s_0 \mid u')\right)^\alpha}}{\dfrac{\left(b_j^d(s_1 \mid u)\right)^\alpha}{\sum\limits_{u'} \left(b_j^d(s_1 \mid u')\right)^\alpha}} < 1 \tag{2}$$

$$\frac{\left(b_i^d(s_0 \mid u)\right)^\alpha \sum\limits_{u'} \left(b_i^d(s_1 \mid u')\right)^\alpha}{\left(b_i^d(s_1 \mid u)\right)^\alpha \sum\limits_{u'} \left(b_i^d(s_0 \mid u')\right)^\alpha} > 1, \quad \frac{\left(b_j^d(s_0 \mid u)\right)^\alpha \sum\limits_{u'} \left(b_j^d(s_1 \mid u')\right)^\alpha}{\left(b_j^d(s_1 \mid u)\right)^\alpha \sum\limits_{u'} \left(b_j^d(s_0 \mid u')\right)^\alpha} < 1 \tag{3}$$

$$\frac{\left(b_i^d(s_0 \mid u)\right)^\alpha}{\left(b_i^d(s_1 \mid u)\right)^\alpha} > \frac{\sum\limits_{u'} \left(b_i^d(s_0 \mid u')\right)^\alpha}{\sum\limits_{u'} \left(b_i^d(s_1 \mid u)\right)^\alpha}, \quad \frac{\left(b_j^d(s_0 \mid u)\right)^\alpha}{\left(b_j^d(s_1 \mid u)\right)^\alpha} < \frac{\sum\limits_{u'} \left(b_j^d(s_0 \mid u')\right)^\alpha}{\sum\limits_{u'} \left(b_j^d(s_1 \mid u)\right)^\alpha} \tag{4}$$

2.1 Unraveled recursion formula

$$O_i^d(u \mid s) = \frac{\left(b_i^d(s \mid u)\right)^\alpha}{\sum\limits_{u'} \left(b_i^d(s \mid u')\right)^\alpha} \tag{5}$$

$$= \frac{\left(\dfrac{O^{d-1}(u \mid s) b(s)}{\sum\limits_{s'} O^{d-1}(u \mid s') b(s')}\right)^\alpha}{\sum\limits_{u'} \left(\dfrac{O^{d-1}(u' \mid s) b(s)}{\sum\limits_{s'} O^{d-1}(u' \mid s') b(s')}\right)^\alpha} \tag{6}$$

3 Strategies

Write out desired behavior explicitly, find solution.

Uncurl recursion to see the effects of different initial belief after multiple steps.

Run simulations until a good example is found. Will need to write out theory of why that example works afterwards.