

RSA for Bayes Filters and POMDPs

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1 Definitions

- $b_i(s)$ is the listener's current belief in s
- $b^0(s \mid u)$ is the primitive interpretation of utterance u
- $b_i^d(s \mid u) = \frac{O^{d-1}(u \mid s)b(s)}{\sum_{s'} O^{d-1}(u \mid s')b(s')}$ would be the listener's updated belief if they interpreted u with RSA of depth d
- $O_i^d(u \mid s) = \frac{e^{\alpha \ln(b_i^d(s|u))}}{\sum_{u'} e^{\alpha \ln(b_i^d(s|u'))}} = \frac{(b_i^d(s \mid u))^\alpha}{\sum_{u'} (b_i^d(s \mid u'))^\alpha}$ is the probability of the speaker saying u to communicate s with RSA of depth d .

2 Desired behavior

We hope that using the listener's current belief will allow utterances to have context-dependent meaning. As an example, we would like for a single, when spoken under belief b_i , to be evidence for s_0 , but when spoken under b_1 act as evidence against s_0 . Equivalently, we wish to find $b_i, b_j, b^0, d, \alpha, u, s_0, s_1$ s.t.

$$(1) \quad \frac{O_i^d(u \mid s_0)}{O_i^d(u \mid s_1)} > 1, \quad \frac{O_j^d(u \mid s_0)}{O_j^d(u \mid s_1)} < 1$$

Substituting the speaker formulas gives

$$(2) \quad \frac{\frac{(b_i^d(s_0 | u))^\alpha}{\sum_{u'} (b_i^d(s_0 | u'))^\alpha}}{\frac{(b_i^d(s_1 | u))^\alpha}{\sum_{u'} (b_i^d(s_1 | u'))^\alpha}} > 1, \quad \frac{\frac{(b_j^d(s_0 | u))^\alpha}{\sum_{u'} (b_j^d(s_0 | u'))^\alpha}}{\frac{(b_j^d(s_1 | u))^\alpha}{\sum_{u'} (b_j^d(s_1 | u'))^\alpha}} < 1$$

$$(3) \quad \frac{(b_i^d(s_0 | u))^\alpha \sum_{u'} (b_i^d(s_1 | u'))^\alpha}{(b_i^d(s_1 | u))^\alpha \sum_{u'} (b_i^d(s_0 | u'))^\alpha} > 1, \quad \frac{(b_j^d(s_0 | u))^\alpha \sum_{u'} (b_j^d(s_1 | u'))^\alpha}{(b_j^d(s_1 | u))^\alpha \sum_{u'} (b_j^d(s_0 | u'))^\alpha} < 1$$

$$(4) \quad \frac{(b_i^d(s_0 | u))^\alpha}{(b_i^d(s_1 | u))^\alpha} > \frac{\sum_{u'} (b_i^d(s_0 | u'))^\alpha}{\sum_{u'} (b_i^d(s_1 | u))^\alpha}, \quad \frac{(b_j^d(s_0 | u))^\alpha}{(b_j^d(s_1 | u))^\alpha} < \frac{\sum_{u'} (b_j^d(s_0 | u'))^\alpha}{\sum_{u'} (b_j^d(s_1 | u))^\alpha}$$

2.1 Unraveled recursion formula