Question 5.

Let $X_i \overset{iid}{\sim} N(0, \sigma^2)$ for $i = 1, 2, \cdots, n$, where σ^2 is unknown.

1. Find $\hat{\sigma}_{ML}$, the MLE of σ, and show that $\hat{\sigma}_{ML}$ is a consistent estimator of σ.
   Solution: (**5 marks**)

2. Consider $T_n = T(X_1, \cdots, X_n) = \sqrt{\pi/2}\sum_{i=1}^{n} |X_i|/n$. Show that T_n is an unbiased and consistent estimator of σ. (**5 marks**)
Solution:

3. Compute asymptotic relative efficiency of T_n with respect to $\hat{\sigma}_{ML}$,

$$\lim_{n\to\infty} \frac{\mathbb{V}(\hat{\sigma}_{ML})}{\mathbb{V}(T_n)}.$$

Do you choose $\hat{\sigma}_{ML}$ or T_n to make a confidence interval for σ? Explain using the asymptotic relative efficiency (**5 marks**)

Solution: