<u>Questions 1.</u>

Suppose $X_i \overset{iid}{\sim} F$, for $i = 1, 2, \cdots, n$. Let $\hat{F}_n(t) = \frac{1}{n}\sum_{i=1}^{n} \varepsilon(t - X_i)$ where $\varepsilon(x) = 1$ if $x \geq 0$ and $= 0$ otherwise.

1. Suppose t is fixed. Find $\mathbb{E}\left[\hat{F}_n(t)\right]$ and $\mathbb{V}\left[\hat{F}_n(t)\right]$. **(5 marks)**
   <u>Solution:</u>

2. Suppose $\alpha_n \to \infty$ and $\alpha_n/\sqrt{n} \to 0$. Show that for any fixed t

$$P\left[\sqrt{\frac{n}{\alpha_n}}|\hat{F}_n(t) - F(t)| > \varepsilon\right] \to 0 \text{ as } n \to \infty,\ \forall \varepsilon > 0,$$

suggesting that $|\hat{F}_n(t) - F(t)| \xrightarrow{P} 0$ as fast as $n^{-1/2}$. **(5 marks)**

Solution:

3. Suppose $\{t_i\}_{i=1}^{k_n}$ is a fixed sequence of t_i's where $k_n = o(\alpha_n)$, i.e. $k_n/\alpha_n \to 0$. Use part (b) to show that

$$P\left[\sqrt{\frac{n}{\alpha_n}} \max_{1\leq i\leq k_n} |\hat{F}_n(t_i) - F(t_i)| > \varepsilon\right] \to 0 \text{ as } n \to \infty,\ \forall \varepsilon > 0.$$

(5 marks)

<u>Solution:</u>