

**Questions 2.**

An experimenter wishes to compare the number of bacteria of types A and B in a water source. A total of  $n$  independent water samples are taken from the source, and counts are made for each sample. Let  $X_i$  denote the number of type A bacteria and  $Y_i$  denote the number of type B bacteria for sample  $i$ . Assume that the two bacteria types are sparsely distributed within a water sample so that  $X_1, X_2, \dots, X_n$  and  $Y_1, Y_2, \dots, Y_n$  can be considered independent random samples from Poisson distributions with means  $\lambda_1$  and  $\lambda_2$ , respectively.

1. Find the MLE of  $\tau = \lambda_1/(\lambda_1 + \lambda_2)$  and show that it is consistent. **(7 marks)**

**Solution:**

2. Using asymptotic properties of the maximum likelihood estimators and the  $\delta$ -method find an approximate pivotal quantity and make a  $100 \times (1 - \alpha)\%$  confidence interval for  $\tau$ . **(8 marks)**

**Solution:**