

Question 8.

Consider a sequence of n independent trials, each resulting in one of $k + 1$ outcomes $1, 2, \dots, k + 1$. Outcome j occurs with probability p_j on any given trial. Let Y_j be the number of trials resulting in outcome j . Consider testing the simple null hypothesis $p_j = \pi_j$ for $j = 1, 2, \dots, k + 1$. Find the likelihood ratio statistic λ_n and show that $-2 \log(\lambda_n) - Q_n \xrightarrow{P} 0$ where

$$Q_n = \sum_{j=1}^{k+1} \frac{(Y_j - n\pi_j)^2}{n\pi_j}.$$

Using the asymptotic equivalence of $-2 \log(\lambda_n)$ and Q_n find the asymptotic distribution of Q_n . (*Hint:* Use the Taylor expansion $f(x) = x \log(x/x_0) = (x - x_0) + (x - x_0)^2/2x_0 + o[(x - x_0)^2]$.) (15 marks)

Solution:

