Question 4.

1. Consider a family of probability mass functions $\mathcal{F} = \{f_\theta(x) : \theta \in \Theta \subset \mathbb{R}\}$ on a sample space $\mathcal{X}$. Suppose $f_\theta(x)$ is twice differentiable with respect to θ, for all $x \in \mathcal{X}$. Let $X_i \overset{iid}{\sim} f_\theta(x)$, for $i = 1, 2, \cdots, n$. Suppose $T_n = T(X_1, \cdots, X_n)$ is a sufficient statistic for θ. Show that the Fisher information of T_n is equal to the Fisher information contained in the whole sample $\vec{X} = (X_1, \cdots, X_n)$.
   Solution: **(8 marks)**

2. Let $I(\eta)$ be the amount of information in X about $\eta = h(\theta)$. Using the Chain Rule and the derivative of an inverse function show that

$$I(\eta) = \mathbb{E}_\theta \left\{ \left[\frac{\partial}{\partial \eta} \ln f_\theta(X) \right]^2 \right\} = \frac{I(\theta)}{[h'(\theta)]^2}$$

Solution: (**7 marks**)