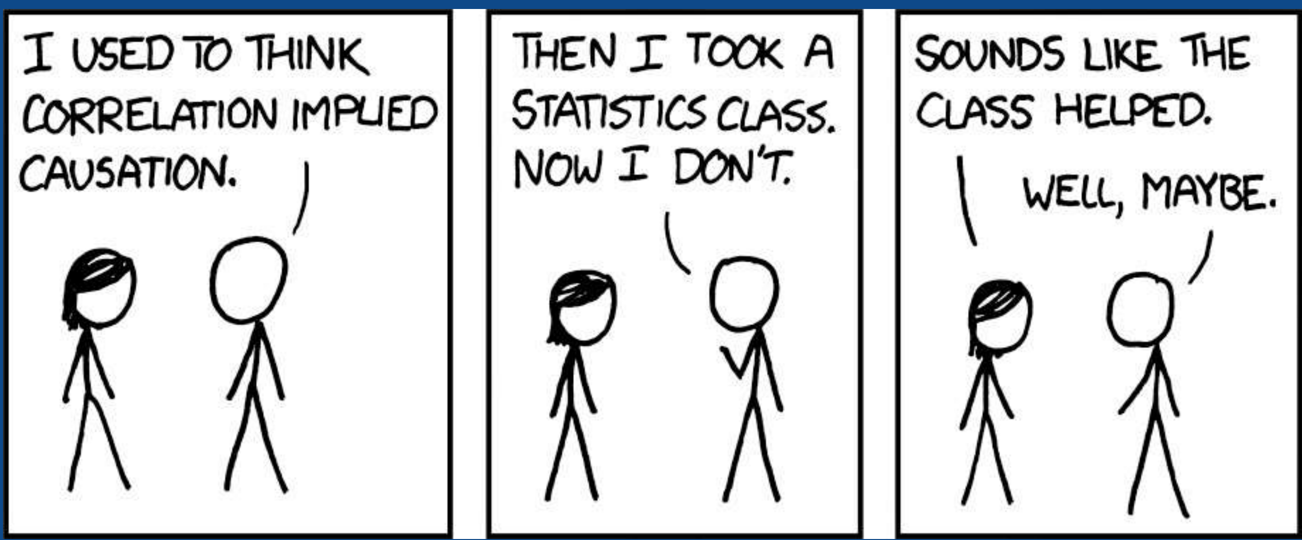

I USED TO THINK CORRELATION IMPLIED CAUSATION.
THEN I TOOK A STATISTICS CLASS. NOW I DON'T.
SOUNDS LIKE THE CLASS HELPED.
WELL, MAYBE.

T-3

$$\xi \sim p(x) = \begin{cases} \frac{e^{-x/\theta}}{\theta}, & x \geq 0 \\ 0, & x < 0 \end{cases} \qquad \theta > 0$$

$n = 3$

$$\tilde{\theta}_1 = \bar{x} = \frac{1}{3}\sum_{i=1}^{3} x_i$$

$$\tilde{\theta}_2 = \frac{x_{min} + x_{max}}{2}$$

$$\tilde{\theta}_3 = x_{(2)}$$

Посчитаем хар-ки ξ:

$$M[\xi] = \int_0^{+\infty} x \cdot \frac{e^{-\frac{x}{\theta}}}{\theta} dx =$$

$$= \left\{ t = \frac{x}{\theta} \right\} = \int_0^{+\infty} t e^{-t} \theta \, dt =$$

$$= -\theta \int_0^{+\infty} t \, d(e^{-t}) = -\theta \Big(\cancel{t e^{-t}} \Big|_0^{+\infty} \to 0 -$$

$$+ \int_0^{+\infty} e^{-t} d(-t) \Big) = -\theta e^{-t} \Big|_0^{+\infty} =$$

$$= \underline{\theta}$$

$$\underline{M[\xi^2]} = \int_0^{+\infty} x^2 \frac{e^{-\frac{x}{\theta}}}{\theta} dx =$$

$$= \left\{ t = \frac{x}{\theta} \right\} = \int_0^{+\infty} t^2 \theta^2 e^{-t} dt =$$

$$= -\theta^2 \int_0^{+\infty} t^2 d(e^{-t}) = -\theta^2 \Big(\cancel{t^2 e^{-t}} \Big|_0^{+\infty} \to 0 -$$

$$-\int_0^{+\infty} e^{-t}\,2t\,dt = -2\theta^2\int_0^{+\infty} t\,d(e^{-t}) =$$

$$= -2\theta^2\left(\overset{0}{\cancel{te^{-t}\Big|_0^{+\infty}}} - \int_0^{+\infty} e^{-t}\,dt\right) =$$

$$= -2\theta^2\int_0^{+\infty} e^{-t}\,d(-t) = -2\theta^2 e^{-t}\Big|_0^{+\infty} = \underline{2\theta^2}$$

$$\underline{D[\xi]} = M[\xi^2] - M^2[\xi] = 2\theta^2 - \theta^2 =$$

$$= \theta^2$$

$\tilde{\theta}_1$:

<u>несмещ:</u>

$$M[\tilde{\theta}_1] = M\left[\frac{1}{3}\sum_{i=1}^{3} X_i\right] =$$

$$= \frac{1}{3}M\left[\sum X_i\right] = \frac{1}{3}\sum M[X_i] =$$

$$= \frac{1}{3}\sum M[\xi] = \frac{1}{\cancel{3}}\cdot\cancel{3}\theta = \theta$$

Несмещ.

Эффективность:

$$D[\tilde{\theta}_1] = D[\tfrac{1}{3}\sum_{i=1}^{3} X_i] = \tfrac{1}{9}\sum D[X_i] =$$

$$= \tfrac{1}{9}\sum D[\xi] = \tfrac{1}{9}\, 3\cdot\theta^2 =$$

$$= \frac{\theta^2}{3}$$

$\tilde{\theta}_2$:

Несмещ:

$$M[\tilde{\theta}_2] = \tfrac{1}{2} M[X_{min} + X_{max}] =$$

$$= \tfrac{1}{2}\left(M[X_{min}] + M[X_{max}]\right)$$

$$\begin{cases} \Phi_{min}(y) = 1 - (1 - F(y))^n \\ \Psi_{max}(z) = (F(z))^n \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \varphi_{min}(y) = n(1 - F(y))^{n-1} F'(y) \\ \varphi_{max}(z) = n(F(z))^{n-1} F'(z) \end{cases}$$

$$F'(x) = p(x)$$

$$F(x) = P(\xi < x) = \int_{-\infty}^{x} p(x)\,dx$$

$$F(x) = \int_0^x \frac{e^{-t/\theta}}{\theta}\,dt = -\int_0^x e^{-\frac{t}{\theta}}\,d\left(-\frac{t}{\theta}\right) =$$

$$= -e^{-t/\theta}\Big|_0^x = \underbrace{1 - e^{-x/\theta}}$$

$$\underline{M[X_{min}]} = \int_{-\infty}^{+\infty} x\,\varphi(x)\,dx =$$

$$= \int_{0}^{+\infty} x \cdot 3\left(1 - \left(1 - e^{-x/\theta}\right)^{3-1} \frac{e^{-x/\theta}}{\theta}\,dx =\right.$$

$$= \frac{3}{\theta} \int_{0}^{+\infty} x\left(e^{-\frac{2x}{\theta}}\right) e^{-x/\theta}\,dx =$$

$$= \frac{3}{\theta} \int_{0}^{+\infty} x\, e^{-3x/\theta}\,dx =$$

$$= -\cancel{\frac{3}{\theta}}\,\cancel{\frac{\theta}{3}} \int_{0}^{+\infty} x\, d\left(e^{-3x/\theta}\right) = -\Big(\cancelto{0}{x e^{-\frac{3x}{\theta}}\Big|_0^{+\infty}} -$$

$$- \int_{0}^{+\infty} e^{-3x/\theta}\,dx = -\frac{\theta}{3} \int_{0}^{+\infty} e^{-3x/\theta}\,d\left(\frac{-3x}{\theta}\right) =$$

$$= -\frac{\theta}{3}\, e^{-\frac{3x}{\theta}}\Big|_0^{+\infty} = \frac{\theta}{3}$$

$$M[X_{max}] = \int_{-\infty}^{+\infty} x\,\varphi(x)\,dx =$$

$$= \int_0^{+\infty} x \cdot 3\left(1 - e^{-\frac{x}{\theta}}\right)^{3-1} \frac{e^{-\frac{x}{\theta}}}{\theta}\,dx =$$

$$= \frac{3}{\theta} \int_0^{+\infty} x e^{-\frac{x}{\theta}} \left(1 - 2e^{-\frac{x}{\theta}} + e^{-\frac{2x}{\theta}}\right) dx =$$

$$= 3\left(\int_0^{+\infty} x e^{-\frac{x}{\theta}} \frac{dx}{\theta} - 2\int_0^{+\infty} x e^{-\frac{2x}{\theta}} \frac{dx}{\theta} +\right.$$

$$\left. + \int_0^{+\infty} x e^{-\frac{3x}{\theta}} \frac{dx}{\theta}\right) = 3\theta - 3\int_0^{+\infty} e^{-\frac{2x}{\theta}}\,dx$$

$$+ \int_0^{+\infty} e^{-\frac{3x}{\theta}}\,dx = 3\theta - \frac{3\theta}{2} + \frac{\theta}{3} =$$

$$= \frac{18\theta - 9\theta + 2\theta}{6} = \frac{11\theta}{6}$$

$$M[\tilde{\theta}_2] = \frac{1}{2}\left(\frac{\theta}{3} + \frac{11\theta}{6}\right) =$$

$$= \frac{1}{2} \cdot \frac{13\theta}{6} = \frac{13}{12}\theta \Rightarrow \text{смещ.}$$

Исправим оценку: $\tilde{\theta}_2' = \frac{12}{13}\tilde{\theta}_2$

$$D[\tilde{\theta}_2'] = D\left[\cancel{\frac{1}{2}} \frac{\overset{6}{\cancel{12}}}{13}(X_{min} + X_{max})\right] =$$

$$= \frac{36}{169} D[X_{min} + X_{max}] =$$

$$= \{ D[a\xi + b\eta] = a^2 D[\xi] +$$
$$+ 2ab\, \mathrm{cov}(\xi, \eta) + b^2 D[\eta] \} =$$

$$= \frac{36}{169}\Big(D[X_{min}] + D[X_{max}] +$$

$$+ 2\,cov\left(X_{min}, X_{max}\right)\Big)$$

Найдем все слагаемые по отдельности:

$$D[X_{min}] = M[X_{min}^2] - M^2[X_{min}]$$

$$\underline{M[X_{min}^2]} = \int_0^{+\infty} x^2\, 3\left(1-\left(1-e^{-\frac{x}{\theta}}\right)^2 \frac{e^{-\frac{x}{\theta}}}{\theta}\, dx$$

$$= -\int_0^{+\infty} x^2 e^{-\frac{3x}{\theta}}\, d\left(-\frac{3x}{\theta}\right) = -\int_0^{+\infty} x^2\, d\left(e^{-\frac{3x}{\theta}}\right) =$$

$$= \overset{0}{\cancel{-x^2 e^{-\frac{3x}{\theta}}\Big|_0^{+\infty}}} + 2\int_0^{+\infty} e^{-3x/\theta}\, x\, dx =$$

$$= -\frac{2\theta}{3}\int_0^{+\infty} x\, d\left(e^{-3x/\theta}\right) =$$

$$= -\frac{2\theta}{3}\left(\cancel{x e^{-\frac{3x}{\theta}}} \rightarrow 0 - \int_0^{+\infty} e^{-\frac{3x}{\theta}} dx\right) =$$

$$= -\frac{2\theta^2}{9} e^{-\frac{3x}{\theta}}\Big|_0^{+\infty} = \frac{2\theta^2}{9}$$

$$\underline{D[X_{min}]} = \frac{2\theta^2}{9} - \frac{\theta^2}{9} = \frac{\theta^2}{9}$$

$$D[X_{max}] = M[X^2_{max}] - M^2[X_{max}]$$

$$\underline{M[X^2_{max}]} = \int_{-\infty}^{+\infty} x^2 \varphi(x)\, dx =$$

$$= \int_0^{+\infty} x^2 \cdot 3\left(1 - e^{-x/\theta}\right)^{3-1} \frac{e^{-x/\theta}}{\theta} dx =$$

$$= \int_0^{+\infty} 3x^2 \frac{e^{-x/\theta}}{\theta}\,dx - 2\int_0^{+\infty} x^2 \cdot 3\,\frac{e^{-\frac{2x}{\theta}}}{\theta}\,dx +$$

$$+ \int_0^{+\infty} 3x^2 \frac{e^{-3x/\theta}}{\theta}\,dx = \ldots = \frac{85}{18}\theta^2$$

$$\underline{D[X_{max}]} = \frac{85}{18}\theta^2 - \frac{121}{36}\theta^2 =$$

$$= \frac{170-121}{36}\theta^2 = \frac{49}{36}\theta^2$$

?

$$\operatorname{cov}(X_{min}, X_{max}) = M[X_{min} \cdot X_{max}] - M[X_{min}]\,M[X_{max}]$$

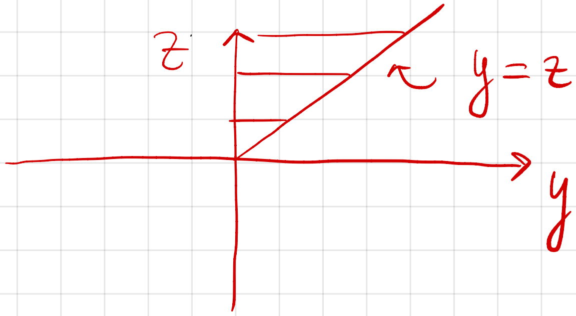
$$M[X_{min} X_{max}] = \iint\limits_{-\infty}^{+\infty} x y \, p(x,y)\, dx\, dy$$

совместная плотность

совм. распр.

$$K(y,z) = \begin{cases} y > z; & (F(z))^n \\ y \le z; & F^n(z) - (F(z) - F(y))^n \end{cases}$$

$y \leqslant z$:



$$\mathcal{H}(y,z) = \frac{\partial^2 K}{\partial y\,\partial z} = \Big(nF^{n-1}(z)F'(z) -$$

$$- n\big(F(z)-F(y)\big)^{n-1}\cdot F'(z)\Big)'_y =$$

$$= +n(n-1)\big(F(z)-F(y)\big)^{n-2}F'(y)\cdot F'(z) =$$

$$= 6\left(\cancel{1} - e^{-z/\theta} - \cancel{1} + e^{-y/\theta}\right) e^{-\frac{y-z}{\theta}}\cdot\frac{1}{\theta^2}$$

$$= 6\left(e^{-y/\theta} - e^{-z/\theta}\right) e^{\frac{-z-y}{\theta^2}}\cdot\frac{1}{\theta^2}$$

$$M[X_{min}\cdot X_{max}] = \iint\limits_{y\leqslant z} yz\,\mathcal{H}(y,z)\,dy\,dz =$$

$$= 6\int_0^{+\infty} z\,dz \int_0^z y\left(e^{-y/\theta} - e^{-z/\theta}\right)e^{\frac{-z-y}{\theta}}\frac{1}{\theta^2}\,dy$$

$$J_1 = \int_0^z y e^{-\frac{z-2y}{\theta}} dy = -\frac{\theta}{2}\int_0^z y\, d\left(e^{-\frac{z-2y}{\theta}}\right) =$$

$$= -\frac{\theta}{2}\left(y e^{-\frac{z-2y}{\theta}}\Big|_0^z - \int_0^z e^{-\frac{z-2y}{\theta}} dy\right) =$$

$$= -\frac{\theta}{2} z e^{-\frac{3z}{\theta}} - \frac{\theta^2}{4}\int_0^z d\left(e^{-\frac{z-2y}{\theta}}\right) =$$

$$= -\frac{\theta}{2} z e^{-3z/\theta} - \frac{\theta^2}{4} e^{-\frac{3z}{\theta}} + \frac{\theta^2}{4} e^{-\frac{z}{\theta}}$$

$$J_2 = -\int_0^z y e^{-\frac{2z-y}{\theta}} dy = \theta \int_0^z y\, d\left(e^{-\frac{2z-y}{\theta}}\right) =$$

$$= \theta\left(z e^{-3z/\theta} + \theta \int_0^z d\left(e^{-\frac{2z-y}{\theta}}\right)\right) =$$

$$= \theta\left(z e^{-3z/\theta} + \theta e^{-\frac{3z}{\theta}} - \theta e^{-2z/\theta}\right) =$$

$$= \theta z e^{-3z/\theta} + \theta^2 e^{-3z/\theta} - \theta^2 \cdot e^{-2z/\theta}$$

Tongai:

$$M[X_{min} \cdot X_{max}] = \frac{6}{\theta^2} \int_0^{+\infty} \Big[z\, e^{-\frac{3z}{\theta}} \cdot$$

$$\cdot \left(-\frac{\theta}{2} z - \frac{\theta^2}{4} + \theta z + \theta^2 \right) -$$

$$- z\, e^{-\frac{2z}{\theta}} \theta^2 + z \frac{\theta^2}{4} e^{-z/\theta} \Big] dz =$$

$$= \frac{6}{\theta^2} \int_0^{+\infty} z e^{-\frac{3z}{\theta}} \left(\frac{1}{2}\theta z + \frac{3\theta^2}{4} \right) dz -$$

$$- \frac{6}{\theta^2} \int_0^{+\infty} z e^{-\frac{2z}{\theta}} \theta^2 dz + \frac{6}{\theta^2} \int_0^{+\infty} z \frac{\theta^2}{4} \cdot$$

$$\cdot e^{-z/\theta}\,dz = \frac{3}{\theta}\int_0^{+\infty} z^2 e^{-\frac{3z}{\theta}}\,dz +$$

$$+ \frac{9}{2}\int_0^{+\infty} z e^{-\frac{3z}{\theta}}\,dz - 6\int_0^{+\infty} z e^{-\frac{2z}{\theta}}\,dz +$$

$$+ \frac{3}{2}\int_0^{+\infty} z e^{-z/\theta}\,dz = -\int_0^{+\infty} z^2\, d\left(e^{-\frac{3z}{\theta}}\right)$$

$$- \frac{3\theta}{2}\int_0^{+\infty} z\, d\left(e^{-\frac{3z}{\theta}}\right) + 3\theta\int_0^{+\infty} z\, d\left(e^{-\frac{2z}{\theta}}\right)$$

$$- \frac{3\theta}{2}\int_0^{+\infty} z\, d\left(e^{-z/\theta}\right) =$$

$$= \frac{2\theta^2}{9} + \frac{\theta^2}{2} - \cancel{\frac{3\theta^2}{2}} + \cancel{\frac{3\theta^2}{2}} =$$

$$= \frac{4\theta^2 + 9\theta^2}{18} = \frac{13}{18}\theta^2$$

$$\underline{\mathrm{cov}(X_{min}, X_{max})} = \frac{13}{18}\theta^2 - \frac{11\theta^2}{18} =$$

$$= \frac{1}{9}\theta^2$$

$$\underline{D[\tilde{\theta}_2']} = \frac{36}{169}\left(\frac{\theta^2}{9} + \frac{49}{36}\theta^2 + \frac{2}{9}\theta^2\right) =$$

$$= \frac{\cancel{36}}{169}\left(\frac{4\theta^2 + 49\theta^2 + 8\theta^2}{\cancel{36}}\right) =$$

$$= \frac{61}{169}\theta^2$$

$\tilde{\theta}_3$:

несмещ:

$$M[\tilde{\theta}_3] = ?$$

Поймем, как распределена

k-ая пор. статистика:

$$\xi_k = order_k(\xi_1 \ldots \xi_n)$$

Найдем $F(x)$:

$$F_{X_{(k)}}(x) = P(X_{(k)} < x)$$

событие $X_{(k)} < x$

$$\Updownarrow$$

$$\left[\begin{array}{l} k \text{ эл-тов выборки} < x \\ k+1 \text{ эл-т. выборки} < x \\ \ldots \\ n \text{ эл-во выборки} < x \end{array}\right.$$

$$\underline{P}(X_k < x) = \underline{P}(\xi < x) = F_\xi(x)$$

$$\underline{P}(X_k \geqslant x) = \underline{P}(\xi \geqslant x) = 1 - F_\xi(x)$$

$$F_{X_{(k)}}(x) = \sum_{m=k}^{n} C_n^m F_\xi^m(x)\left(1 - F_\xi(x)\right)^{n-m}$$

Теперь хотим найти плотность.

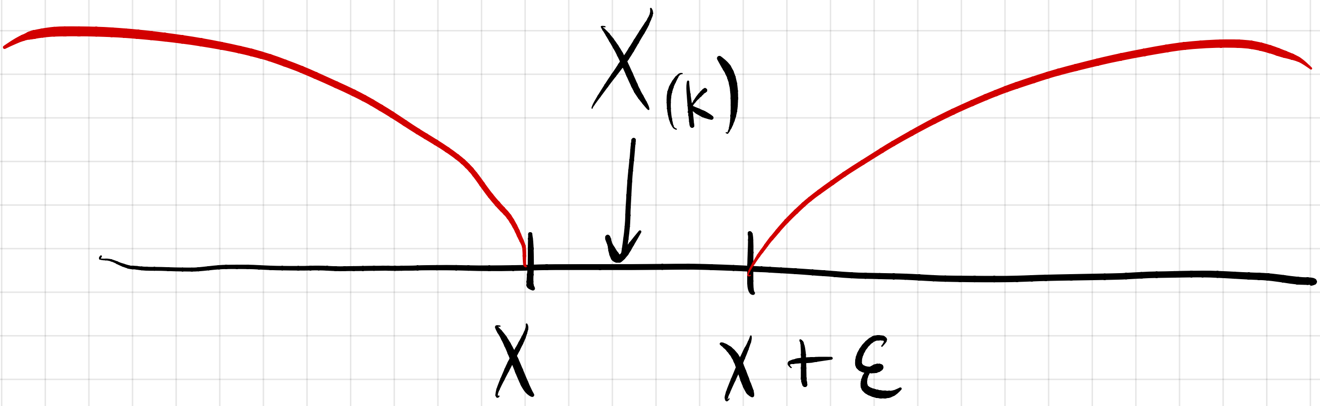
Дифференцировать $F_{X_{(k)}}(x)$ – плохая идея!

Попробуем по определению производной:

$$f_{X_{(k)}}(x) = F'_{X_{(k)}}(x) = \lim_{\varepsilon \to 0} \frac{F(x+\varepsilon) - F(x)}{\varepsilon} =$$

$$= \lim_{\varepsilon \to 0} \frac{P\left(X_{(k)} \in [x, x+\varepsilon)\right)}{\varepsilon}$$

Хотим разложить в ряд по ст. ε. Оставив только $\sim \varepsilon$.



Когда такое возможно?

1) $k-1$ эл-т $< x$

1 эл-т $\in [x, x+\varepsilon]$

$n-k$ эл-т $\geqslant x+\varepsilon$

2) $k-2 \quad < x$

$2 \quad \in [x, x+\varepsilon]$

$n-k \quad \geqslant x+\varepsilon$

$$P(X_1 \in [x, x+\varepsilon], X_2 \in [x, x+\varepsilon]) =$$

$$= f_\xi(x)\cdot\varepsilon\cdot f_\xi(x)\cdot\varepsilon \sim \underbrace{\varepsilon^2}$$

не берем такие слагаемые

Значит, рассм. только первый случай:

$$\lim_{\varepsilon\to 0}\left[C_n^1 P(X_{(k)}\in[x, x+\varepsilon))\, C_{n-1}^{k-1} P(\xi < x)\,\cdot\right.$$

$$\left.\cdot\, C_{n-k}^{n-k} P(\xi \geqslant x+\varepsilon)\right]\cdot\frac{1}{\varepsilon} = f_{X_{(k)}}(x)$$

$$\frac{P(X_{(k)}\in[x, x+\varepsilon))}{\varepsilon} = f_\xi(x)$$

$$= \boxed{C_n^1 f_\xi(x)\, C_{n-1}^{k-1} F_\xi^{k-1}(x) \left(1 - F_\xi(x)\right)^{n-k}}$$

$$f_{X_{(2)}}(x) = 3 \cdot \frac{e^{-x/\theta}}{\theta}\, 2\left(1 - e^{-x/\theta}\right) \cdot$$

$$\cdot \left(1 - (1 - e^{-x/\theta}\right) =$$

$$= \frac{6}{\theta}\left(e^{-2x/\theta} - e^{-3x/\theta}\right)$$

$$M[\tilde{\theta}_3] = M[X_{(2)}] =$$

$$= \int_0^{+\infty} x f(x)\, dx = \int_0^{+\infty} \frac{6x}{\theta}\left(e^{-2x/\theta} - e^{-3x/\theta}\right) dx$$

$$= -3\int_0^{+\infty} x\, d\left(e^{-2x/\theta}\right)dx + 2\int_0^{+\infty} x\, d\left(e^{-\frac{3x}{\theta}}\right)dx =$$

$$= 3\theta\int_0^{+\infty} e^{-2x/\theta}\, dx - 2\theta\int_0^{+\infty} e^{-3x/\theta}\, dx =$$

$$= +\frac{3\theta}{2} - \frac{2\theta}{3} = \frac{5}{6}\theta$$

$\Downarrow$

смещ.

Исправим оценку: $\tilde{\theta}_3' = \frac{6}{5}\tilde{\theta}_3$

$$D[\tilde{\theta}_3'] = \frac{36}{25} D[\tilde{\theta}_3] = \frac{36}{25}\left(M[\tilde{\theta}_3^2] - M^2[\tilde{\theta}_3]\right)$$

$$M[\tilde{\theta}_3^2] = \int_0^{+\infty} x^2 \frac{6}{\theta}\left(e^{-2x/\theta} - e^{-3x/\theta}\right)dx =$$

$$= -3\int_0^{+\infty} x^2 e^{-2x/\theta}\, d\left(\frac{-2x}{\theta}\right) +$$

$$+ 2\int_0^{+\infty} x^2 e^{-3x/\theta}\, d\left(-\frac{3x}{\theta}\right) = -\frac{4\theta^2}{9} + \frac{3\cdot 2\theta^2}{4} =$$

$$= \frac{3\theta^2}{2} - \frac{4\theta^2}{9} = \frac{19}{18}\theta^2$$

$$D[\tilde{\theta}_3'] = \frac{36}{25}\left(\frac{19}{18}\theta^2 - \frac{25}{36}\theta^2\right) =$$

$$= \frac{\cancel{36}}{25}\cdot\frac{13}{\cancel{36}}\theta^2 = \frac{13}{25}\theta^2$$

Сравним эффективности:

$$D[\tilde{\theta}_1] \vee D[\tilde{\theta}_2] \vee D[\tilde{\theta}_3']$$

$$\frac{\theta^2}{3} \vee \frac{61}{169}\theta^2 \vee \frac{13}{25}\theta^2$$

$$\Downarrow$$

$$\frac{\theta^2}{3} < \frac{61}{169}\theta^2 < \frac{13}{25}\theta^2$$

$$\Downarrow$$

$\tilde{\theta}_1$ – наиболее эффективно.

б) Исслед. оценки с помощью К-Р.

Проверим модель на регулярность:

1) $$p(x,\theta) = \begin{cases} \dfrac{e^{-\frac{x}{\theta}}}{\theta}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

дифф. по $\theta > 0$

2) $$\frac{\partial}{\partial\theta}\int_0^{+\infty} p(x,\theta)\,dx = \frac{\partial}{\partial\theta}\int_0^{+\infty} -e^{-\frac{x}{\theta}}\,d\left(-\frac{x}{\theta}\right)$$

$$= \frac{\partial}{\partial\theta}\left(-e^{-\frac{x}{\theta}}\Big|_0^{+\infty}\right) = \frac{\partial}{\partial\theta}(1) = 0 \quad //$$

$$\int_0^{+\infty} \frac{\partial}{\partial\theta}\left(\frac{e^{-x/\theta}}{\theta}\right)dx = \int_0^{+\infty} \frac{+\frac{1}{\theta^2}e^{-x/\theta}\theta - e^{-x/\theta}}{\theta^2}\,dx$$

$$= \int_0^{+\infty}\left(\frac{e^{-x/\theta}}{\theta^3} - \frac{e^{-x/\theta}}{\theta^2}\right)dx =$$

$$= -\int_0^{+\infty} \frac{x e^{-\frac{x}{\theta}}}{\theta^2}\,d\left(-\frac{x}{\theta}\right) + \int_0^{+\infty} \frac{e^{-x/\theta}}{\theta}\,d\left(-\frac{x}{\theta}\right) =$$

$$= -\frac{1}{\theta}\int_0^{+\infty} e^{-x/\theta}\,d\left(-\frac{x}{\theta}\right) - \frac{1}{\theta} =$$

$$= -\frac{1}{\theta}\cdot(-1) - \frac{1}{\theta} = 0$$ //

$0 = 0 \Rightarrow$ модель регулярна //

Посчитаем информацию Фишера:

$$I(\theta) = M\left[\left(\frac{\partial \ln p(x,\theta)}{\partial \theta}\right)^2\right] =$$

$$= \int_{-\infty}^{+\infty} \left(\frac{\partial \ln p}{\partial \theta}\right)^2 p(x,\theta)\,dx =$$

$$= \int_{0}^{+\infty} \left(\frac{x}{\theta^2} - \frac{1}{\theta}\right)^2 \frac{e^{-x/\theta}}{\theta}\,dx =$$

$$= \int_0^{+\infty} \frac{x^2}{\theta^5} e^{-\frac{x}{\theta}} dx - 2\int_0^{+\infty} \frac{x}{\theta^4} e^{-\frac{x}{\theta}} dx +$$

$$+ \int_0^{+\infty} \frac{e^{-x/\theta}}{\theta^3} dx = \cancel{\frac{2}{\theta^2}} - \cancel{\frac{2}{\theta^2}} + \frac{1}{\theta^2} = \frac{1}{\theta^2}$$

<u>Нерав. К-р:</u>

$$D[\tilde{g}(\vec{X}_n)] \geqslant \frac{g'^2(\theta)}{nI(\theta)}$$

$\tilde{\theta}_1$:

$$\boxed{g(\theta) = \theta}$$

откуда это?

$$\underset{\theta^2/3}{\overset{\shortparallel}{D[\tilde{\theta}_1]}} \geqslant \frac{1^2}{3\,\frac{1}{\theta^2}} = \frac{\theta^2}{3}$$

нижняя граница

дисперсии оценки.

$\Downarrow$

$\tilde{\theta}_1$ — эффективна по Крамеру – Рао

Замечание: К–Р работает потому что:

1) модель регулярна.
2) $\exists I(\theta)$
3) $\tilde{\theta}_1$ – несмещ.
4) $\tilde{\theta}_1$ – регулярна ?

Т.к. $D[\tilde{\theta}_2] > D[\tilde{\theta}_1]$

$D[\tilde{\theta}_3] > D[\tilde{\theta}_1]$

$\Downarrow$

$\tilde{\theta}_2$ и $\tilde{\theta}_3$ – не явл. эфф.

по К.-Р.