Endogenous Fragility of Supply Chains and Correlated Disruption Risk *

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Abstract

I model the endogenous formation of supply chains in the presence of correlated disruptions. The incentives of firms to diversify the supply chain risk are concave in the correlation between the disruption events among producers of their input goods. This concavity has consequences for the endogenous formation of the supply chain. If upstream producers are highly diversified, their disruption risk might be correlated, which, in turn, reduces diversification incentives for downstream firms. Because of this mechanism, a small increase in the correlation of risk among upstream producers, due to, for example, offshoring or climate disruptions to economic activities, can generate under-diversification throughout the production network. This creates large welfare losses. Finally, I show that firms gaining more information on their supply chain risk exacerbates such losses.

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In August 2020, hurricane Laura hit one of the world's largest petrochemical districts, in the U.S. states of Louisiana and Texas. As polymer producers in the area were forced to halt production, up to 15% of the country's polypropene (PE) and polypropene (PP) producers were unable to source polymer inputs, which in turn caused shortages across the economy (Vakil, 2021). As in this case, agglomeration of economic activity can increase the correlation between disruptions among suppliers of crucial goods for the economy. In the face of such correlated risk, how do downstream producers make sourcing decisions? And, do these decisions yield supply chains resilient to such correlated disruptions?

In this paper, I study the feedback between the risk of a disruption in sourcing inputs and the endogenous formation of supply chains. A widespread approach to mitigate risk is to diversify it by multisourcing. This practice consists of procuring the same inputs from multiple suppliers, sometimes redundantly (Zhao and Freeman, 2019). Yet, when deciding how many suppliers from which to source, a firm faces decreasing marginal benefits in risk reduction, because each additional supplier's failure to deliver is increasingly likely to be correlated with that of the firm's current suppliers. In the presence of marginal costs of sourcing, for example, contractual costs or higher prices, the uncertainty behind the correlation of a firm's potential suppliers might induce it to diversify risk less than socially optimal. The wedge between endogenous firm decisions and social optimality arises because downstream firms would be willing to compensate their suppliers for increased diversification of inputs. This under-diversification can generate aggregate fragility in production networks. To understand the relationship between the firm's diversification decisions and supply chain fragility, I study the properties of a stylised production game. In the equilibrium of the game, correlation in the risk of disruption among suppliers generates fragility via two channels. First, it directly introduces endogenous correlation in downstream firms' risk, which amplifies through the production network. This increases the probability of cascading failures, in which the entire production network is unable to produce. Second, it indirectly affects firms' decisions by reducing the expected marginal gain from adding a source of input goods. The latter channel leads to firms diversifying increasingly less, such that small increases in the expected disruption probability can yield fragile production networks.

The role that production networks play in determining economic outcomes has been long recognised. As far back as Leontief (1936), economists have studied how networks in production can act as aggregators of firm-level activity. Following a foundational paper by Hulten (1978), which showed that the first order impact of a productivity shock to an industry is independent of the production network structure, macroeconomics has since de-emphasised this role (Baqaee and Farhi, 2019, p. 2). However, more recently, Baqaee and Farhi (2019) illustrated how the structure of the production network can aggregate micro shocks via second order effects.[1] Furthermore, the degree of competition in an industry also interacts with the production network to aggregate shocks, which can lead to cascading failures (Baqaee, 2018). Once established that production networks play a central role in aggregating shocks, two natural questions arise. First, which networks can we expect to observe, given that firms endogenously and strategically choose suppliers? Second, are these endogenous network formations responsible for the growth or fragility that large economies display? These questions fuelled a number of recent papers studying endogenous production network formation. Focusing on growth, Acemoglu and Azar (2020) show that endogenous production networks can be a channel through which firms' increased productivity lowers costs throughout the supply chain and allows for sustained economic growth. In parallel, a vast literature dealt with studying the role of endogenous production networks and firm incentives in determining fragile or resilient economies. Erol and Vohra (2014) showed that in networks with strategic link formation, systemic endogenous fragility arises if the shocks experienced by firms are correlated. Later work, by Amelkin and

[1] These results build on a vast literature and recent literature (e.g. Gabaix (2011); Acemoglu et al. (2012); Carvalho et al. (2020); Baqaee and Farhi (2019); Carvalho and Tahbaz-Salehi (2019))

Vohra (2020), shows that uncertainty in the time of production is crucial in determining whether production networks in equilibrium are sparse and, hence fragile. Finally, Elliott, Golub and Leduc (2022) illustrate how complexity in the production process can also be a key driver of endogenous fragility in production networks. [2]

A less understood link is that between the correlation of risk within the supply chain, how firms deal with it, and the consequences this has on the economy. Kopytov et al. (2021) studied the effect of uncertainty in endogenous production network formation on firms' productivity and business cycles. They find that higher uncertainty can lead to lower economic growth. In contrast, this paper focuses on the role of uncertainty in generating endogenous fragility to cascading failures using a more stylised production network model, akin to that studied by Elliott, Golub and Leduc (2022). In line with the existing literature, in the model, small idiosyncratic shocks can be massively amplified. The degree of amplification depends on the equilibrium behaviour of firms. This phenomenon holds true in vertical economies producing simple goods. This paper extends the analysis of production network formation to an environment in which firms aim to minimise risk while accounting for correlation between suppliers. To do so, I develop a tractable analytical framework that describes the propagation of idiosyncratic shocks through the supply chain when firms make sourcing decisions endogenously in an imperfect information environment. The model describes the evolution of risk through the supply chain as a dynamical system over its depth. The social planner solution shows that endogenous fragility can impose large welfare losses. Importantly, these losses might be discontinuous: an arbitrarily small increase in the correlation of risk among basal firms can generate large welfare losses. Finally, I study a benchmark case where firms have perfect information over idiosyncratic risk. In this case, despite each individual firm being able to achieve a smaller disruption

[2]The literature on production networks is vast and it is unfortunately impossible to give a fair overview in this introduction. For a more comprehensive review of the literature, I refer the reader to Carvalho and Tahbaz-Salehi (2019) and Amelkin and Vohra (2020)

risk, the supply chain is maximally fragile and there is a high probability of large disruptions.

The remainder of the paper is structured as follows. Section 1 discusses the assumptions on the supply chain disruptions and the problem of the firm, and establishes the results that allow the firm to make sourcing decisions. Section 2 derives the law of propagation of the disruption events through the supply chain. Section 3 establishes the firm's optimal sourcing strategy and how this endogenously determines the fragility of the supply chain. These results are then compared, in Section 4, to the social planner solution to determine the welfare losses induced by the firm's endogenous decisions. Finally, in Section 5, the role of imperfect information is isolated by solving the model under perfect information.

1 Model

1.1 Production Technology and the Firm Objective

The economy produces $K + 1$ goods, indexed by $k \in \{0, 1, \dots K\}$. Each firm produces a single good and each good is produced by m_k firms. Production of the *basal good* $k = 0$ does not require any input, yet, it is at risk of random exogenous disruptions in the production process. A *disrupted* basal firm is unable to deliver its good as input to downstream producers. The economy is vertical as each downstream good $k > 0$ requires only good $k - 1$ as input. If a firm producing good k is unable to source its input good $k - 1$, the firm is itself *disrupted* and hence unable to deliver downstream. In other words, the i-th firm producing good k, indexed by (k, i), is able to produce if at least one of its suppliers is able to deliver, namely, not all of its suppliers are disrupted. To avoid being disrupted, the firm chooses which firms to source from, among the producers of its input good. In other words, letting $\mathcal{D}_k$ be the random set of disrupted firms in layer k and $\mathcal{S}_{k,i}$ the set of suppliers of firm (k, i), we can say that $(k, i) \in \mathcal{D}_k$ if and only if all of its suppliers $(k - 1, j) \in \mathcal{S}_{k,i}$ are in $\mathcal{D}_{k-1}$. I refer to the set of the firm's suppliers $\mathcal{S}_{k,i}$ as its *sourcing strategy*. The disruption events are random and the probability that a firm is disrupted can be written as

$$P_{k,i} := \mathbb{P}\big((k,i) \in \mathcal{D}_k\big) = \mathbb{P}\big(\mathcal{S}_{k,i} \subset \mathcal{D}_{k-1}\big). \tag{1}$$

Figure 1 illustrates this mechanism.

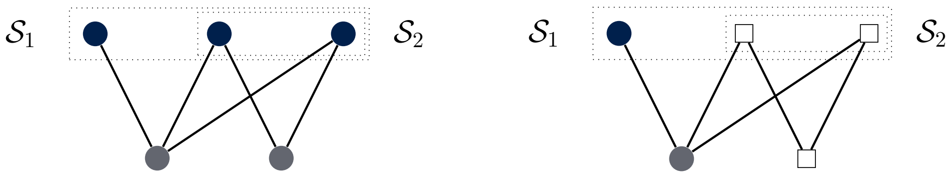

Figure 1: The supply chain is depicted in the left panel. The left firm is sourcing its input good from all three suppliers, $\mathcal{S}_1$, while the right firm only from the latter two, $\mathcal{S}_2$. As a disruption occurs, some upstream firms are unable to supply the input good (white box). Unlike the left firm, the right firm is unable to source its inputs and is hence disrupted.

If a firm is not disrupted, it obtains a profit π. Implementing a given sourcing strategy costs the firm $C\big(\,|\mathcal{S}_{k,i}|\,\big)$. The cost C is assumed to be increasing in the number $|\mathcal{S}_{k,i}|$ of suppliers. The problem of firm (k,i) is then to maximise the expected profit[3]

$$\Pi_{k,i}(\mathcal{S}_{k,i}) = \Big(1 - \mathbb{P}\big(\mathcal{S}_{k,i} \subseteq \mathcal{D}_{k-1}\big)\Big)\,\pi - C\big(\,|\mathcal{S}_{k,i}|\,\big) \tag{2}$$

by picking a sourcing strategy $\mathcal{S}_{k,i}$. Before moving on with the solution of the model, it is useful to discuss the assumptions presented in this section. The production game is highly stylised: first, firms do not adjust prices but only quantities, such that failure to produce only arises in case no input is sourced; second, they are able to obtain profits by simply producing; third, contracting with new suppliers has a cost. There are both theoretical and empirical reasons behind these choices. Theoretically, a simpler model allows us to isolate the interplay between the variables of interest: correlation in the risk of suppliers, supply chain opacity, and endogenous production network fragility. Empirically, these assumptions capture well the rationale behind firms' multisourcing. There is strong evidence that firms, first, when faced with supply chain shocks, adjust

[3]The expectation is taken over the random set $\mathcal{D}_{k-1}$.

quantities rather than prices in the short run (Jiang, Rigobon and Rigobon, 2022; Lafrogne-Joussier, Martin and Mejean, 2022; di Giovanni and Levchenko, 2010; Macchiavello and Morjaria, 2015), second, that production shutdowns can have significant costly (Hameed and Khan, 2014; Barrot and Sauvagnat, 2016), and third, that fostering relationships with suppliers is costly, but important in guaranteeing operational performance (Cousins and Menguc, 2006). The model establishes a link between these issues faced by firms when choosing a sourcing strategy and the fragility of the production network.

1.2 Imperfect Information and Ex-Ante Symmetry

The supply chain is opaque: firms cannot observe the sourcing decisions of their potential suppliers before making their own. Furthermore, firms do not know how risky individual basal producers are, nor how their risk is correlated. Yet, firms know the distribution from which the probabilities of disruption in the basal layer are drawn. To motivate this assumption, recall the introductory example of Hurricane Laura. A downstream firm producing PP, might not be able to trace back the production steps from its input to individual polymer producers in Louisiana or Texas, and, hence, the exact exposure of its production process to hurricanes. Yet, it can estimate the aggregate risk the polymer industry faces in the region. Given this information about the basal layer and their own depth k in the production network, firms can derive the distribution of risk among their suppliers and make sourcing decisions based on it. By symmetry, the risk of two firms downstream sourcing from the same number of suppliers is ex-ante identical, albeit possibly correlated. The following two assumptions formalise this idea. Introduce

$$X_{k,j} := \begin{cases} 1 & \text{if } (k,j) \text{ is disrupted and} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Assumption 1. *Fix an arbitrary measure ν over $[0, 1]$. The probabilities $P_{0,j}$ of disruptions in the basal layer are sampled from ν. I assume ν is observed by all firms, while $P_{0,j}$ are hidden.*

Going back to the example of polymer producers, under this assumption, downstream PP producers understand how hurricane risk can impact the production of their input good, via ν, yet, they cannot estimate the risk that individual polymer producers face since they do not observe $P_{0,j}$.

Assumption 2. *If there are multiple sourcing strategies that yield the same expected profit, the firm chooses one with equal probability.*

Proposition 1. *Under these assumptions, in each downstream layer $k \geq 1$, disruption events*

$$X_{k,1}, X_{k,2}, X_{k,3} \dots X_{k,m_k},$$

are exchangeable, that is, their distribution is invariant under permutation.

Proposition 1, proven in Appendix B.1, asserts that, from the point of view of the firm, all suppliers are ex-ante identical, yet their risk might be correlated. Hence, the profit of the firm depends exclusively on *how many* suppliers it chooses, rather than *which* suppliers it chooses. Then, a firm producing good k can first infer the distribution of the number $D_{k-1} := |\mathcal{D}_{k-1}|$ of disrupted firms among its potential suppliers and then choose the optimal number $s_{k,i} := |\mathcal{S}_{k-1,i}|$ of firms from which to source its input good. Furthermore, by symmetry, all firms in layer k choose the same number s_k of sources, that is,

$$s_{k,i} = s_k \text{ for all } i. \tag{4}$$

As a result, the sourcing strategies $\mathcal{S}_{k,i}$ and $\mathcal{S}_{k,j}$ of any two firms i and j are such that their disruption probabilities $P_{k,i}$ and $P_{k,j}$ are identically distributed[4].

[4]This approach is widely used in the study of random graphs, see, for example, Kallenberg (2005); Diaconis and Janson (2007)

2 Disruptions Propagation

Building on the mechanisms behind firms' disruptions introduced above, this section studies how these disruptions propagate through the supply chain. To do so, I consider the case in which the number m_k of firms in each layer k grows large. To study the limit, it is first necessary to characterise how the sourcing relations $\mathcal{S}_{k,j}$ form as the number of firms in each layer increases.

Assumption 3. *As a new firm is introduced in layer k, it starts establishing relations with its s_k suppliers. As soon as it pairs with a supplier, a new firm is introduced among the producers of its input good $k-1$, which, in turn, selects its sources. This procedure continues recursively until all firms realise their sourcing strategy s_k.*

Indexing by n the n-th step of this procedure, this section focuses on the limit as $n \to \infty$. Every new firm introduced in the basal layer has a disruption probability that is ν-distributed, hence, the new firm is ex-ante identical to existing firms. This ensures that, as $n \to \infty$, Assumption 1 is satisfied and the downstream sourcing decisions $s_1, s_2 \ldots$ are unaffected. This, allows us to simply consider the problem of the representative firm in layer k.

To analytically characterise the disruption propagation through the production network, the only missing piece is the distribution ν of the disruption probabilities in the basal layer. As mentioned in the previous section, I assume that basal firms fail with a not necessarily independent probability P_0. We can model this by assuming that P_0 follows a Beta distribution.

Assumption 4. *The probability of a disruption in the basal layer follows*

$$P_0 \sim \nu_0 \equiv \textit{Beta for all } j. \tag{5}$$

The Beta distribution allows to flexibly model shocks that might happen due to the spacial or technological proximity of basal producers, which cannot be diversified. Consider, for example, how oil extraction plants must be located near oil reserves and are hence all subject to correlated weather shocks that might force them to shut down. In this case, despite the small expected probability that an individual firm is disrupted, as a hurricane is a rare occurrence, disruptions are highly correlated, as when a hurricane occurs most of them are disrupted. To keep track of the expected disruption probability and the correlation of risk through the layers, I introduce the following alternative parametrisation of the Beta distribution.

Definition 1. *Let μ and ρ be respectively the mean and the overdispersion of a Beta distribution with shape parameters α and β, defined by*

$$\mu := \frac{\beta}{\alpha+\beta} \text{ and } \rho := \frac{1}{1+\alpha+\beta}. \tag{6}$$

I write $P \sim Beta(\mu, \rho)$.

Given Assumption 4, the following result links the probabilities of experiencing disruption from upstream suppliers of k to downstream producers $k+1$.

Definition 2. *A random variable Y follows a BetaPower distribution, with mean μ, overdispersion ρ, and power s if it can be written as $Y = X^s$ where X follows a Beta distribution with mean μ and overdispersion ρ.*

Proposition 2. *If the disruption probability P_k among suppliers of good k follows a BetaPower distribution, so does the downstream probability P_{k+1}.*

The proof is provided in Appendix B.2. Proposition 2 guarantees that the distribution of disrupted firms will remain in the same distribution family as risk amplifies through the production network. This result allows us to describe disruption propagation in the supply chain by mapping the evolution of the parameters μ_k and ρ_k through the layers. Furthermore, it allows firms to estimate μ_k and ρ_k and use this to determine the optimal sourcing strategy s_{k+1}. It is useful at this point to give an interpretation of μ_k and ρ_k in the context of our model. The parameter μ_k is the average failure probability of firms in layer k. The parameter ρ_k tracks the degree of correlation in the disruption of firms operating in layer k. I illustrate this in Figure 2. This figure shows the distribution of the disruption probability P_{k+1} among downstream firms in the case the firm has a single supplier (dotted lines) or two suppliers (solid line). For low overdispersion, $\rho_k = 0.01$, the suppliers' disruptions are weakly correlated and the downstream disruption probability is concentrated around the average μ_k. If firms contract an additional supplier, the distribution of failures decreases and remains concentrated around the average. As ρ_k increases, the suppliers' disruption events become more correlated and the downstream disruption probabilities become fat-tailed, that is, a significant fraction of firms is likely to be disrupted and, as a consequence, diversification is ineffective. If firms contract an additional supplier the average disruption probability decreases, but a large disruption probability remains.

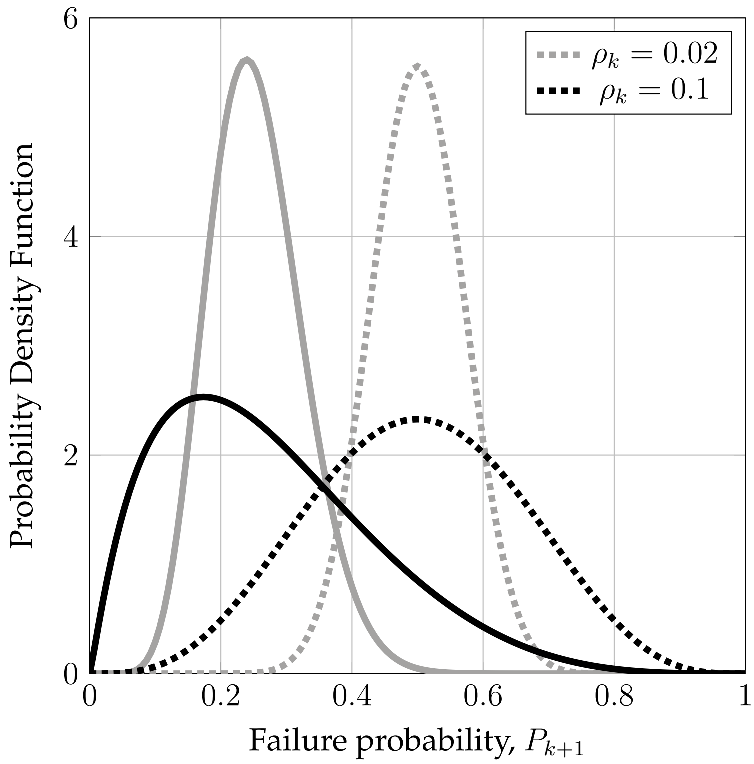

Figure 2: Distribution of disruption probabilities of downstream firms for different levels of upstream correlation ρ_k, in the cases of single sourcing (dotted) and multisourcing (solid). In both cases $\mu_k = 1/2$.

Having established the link between the disruptions of layer k to layer $k+1$, I now turn to the analysis of how these propagate through the whole supply chain, before studying how firms make decisions endogenously. The following result recursively connects downstream distributions with upstream sourcing decisions and initial conditions.

Proposition 3. *The average disruption probability between one layer k and the next $k+1$ depends on the sourcing strategy s_{k+1} via*

$$\mu_{k+1} = \begin{cases} \eta(s_{k+1}, S_k)\ \mu_k & \text{if } s_{k+1} > 0, \\ 1 & \text{otherwise,} \end{cases} \tag{7}$$

where $S_k := \prod_{j=1}^{k} s_j$ is the diversification level up to layer k and η is the "risk reduction factor", which is given by

$$\begin{aligned} \eta(s_{k+1}, S_k) &= \left(\mu_0 \frac{1-\rho_0}{\rho_0} + S_k\right)^{\overline{S_k s_{k+1}}} \Big/ \left(\frac{1-\rho_0}{\rho_0} + S_k\right)^{\overline{S_k s_{k+1}}} \\ &= \left(\frac{\mu_0\ \frac{\rho_0}{1-\rho_0} + S_k}{\frac{\rho_0}{1-\rho_0} + S_k}\right)\left(\frac{\mu_0\ \frac{\rho_0}{1-\rho_0} + S_k + 1}{\frac{\rho_0}{1-\rho_0} + S_k + 1}\right)\cdots\left(\frac{\mu_0\ \frac{\rho_0}{1-\rho_0} + S_k s_{k+1} - 1}{\frac{\rho_0}{1-\rho_0} + S_k s_{k+1} - 1}\right). \end{aligned} \tag{8}$$

This is proven in Appendix B.3. The risk reduction factor $\eta(s_{k+1}, S_k)$ governs how the firm's choice s_{k+1}, the choices along the firm's production chain S_k, and the basal conditions μ_0, ρ_0 affect the expected number of disruptions downstream. This interplay is illustrated in the following figures.

Figure 3 shows how the risk reduction factor varies with basal correlation ρ_0 for different sourcing strategies s_k, fixing the upstream diversification of $S_k = 2$. If correlation ρ_0 in the basal layer grows, to obtain a given level of risk reduction η, firms producing good k need to source more suppliers. If $\rho_0 \to 1$, diversification becomes impossible, as $\eta \to 1$ and $\mu_{k+1} \to \mu_k$ for any sourcing strategy s_{k+1}.

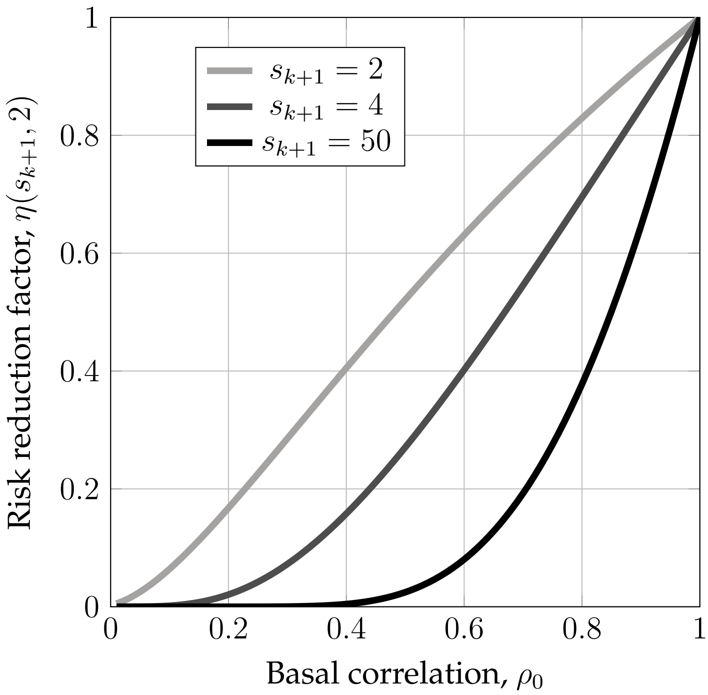

Figure 3: Risk reduction factor μ_{k+1}/μ_k at different basal overdispersion levels ρ_0 and for different sourcing strategies s_{k+1}. $S_k = 2$.

As the above, Figure 4 shows the response of the risk reduction factors to different levels of basal correlation, but instead of varying the strategy s_k of the firm, it varies the level of upstream diversification S_k. For low levels of basal correlation ρ_0, more upstream diversification S_k allows downstream producers to achieve lower risk with fewer suppliers. Yet, there is a level of basal correlation after which more diversification is detrimental for the downstream firm, as this high upstream diversification simply exacerbates tail risk. This represents a crucial externality the upstream suppliers impose on downstream producers. For a low level of correlation, sourcing downstream represents a positive externality downstream. This externality shrinks as correlation increases until it becomes a negative externality. Section 4 explores the welfare consequences of this mechanism.

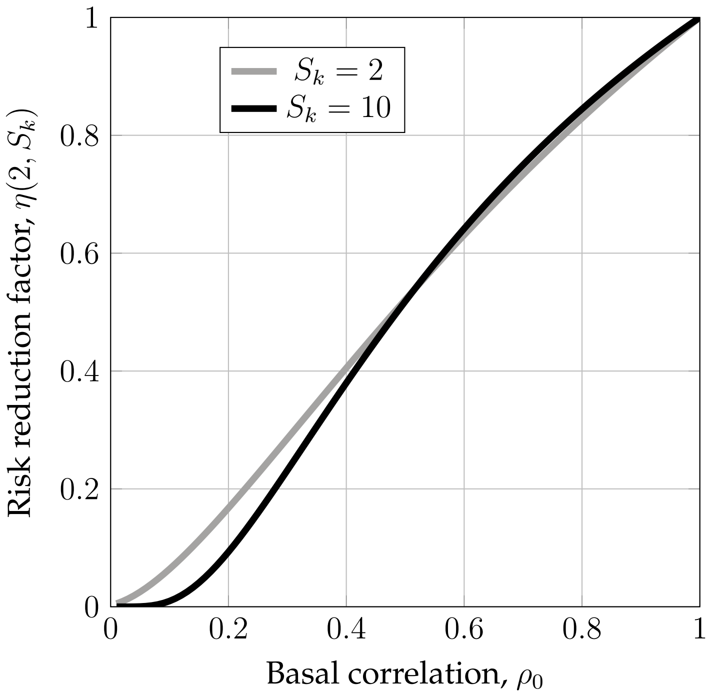

Figure 4: Risk reduction factor μ_{k+1}/μ_k at different basal overdispersion levels ρ_0 and for different upstream diversification levels strategies S_k. $s_{k+1} = 2$.

3 Firm Optimal Diversification and Equilibrium

The mechanics of disruption propagation, derived in the previous section, determines the firm's desired sourcing strategy and, as a consequence, their optimal sourcing strategy. This section derives such optimal strategies. Importantly, due to Proposition 1, all firms in a given layer are identical before the shock and so is their optimisation problem. We can hence focus on the problem of the representative firm in layer $k + 1$: to choose how many suppliers in layer k to source from, based on the inferred distribution of their probability of experiencing a disruption event. This, in turn, is fully determined by average disruption probability μ_0, the correlation ρ_0 of the disruptions, and the sourcing strategies $\{s_1, s_2, \ldots s_k\}$ of the representative firms upstream. Henceforth, I assume firms face quadratic costs of sourcing, with cost parameter c, such that expected profits (2) can be written as

$$\Pi_k(s) = \left(1 - \mathbb{E}_s\left[P_k\right]\right)\pi - \frac{c}{2}s^2, \tag{9}$$

The optimisation problem of the firm is to then choose the optimal sourcing strategy

$$s_k = \arg\max_{s\in\{0,1,2,\ldots\}} \Pi_k(s). \tag{10}$$

3.1 Limit Case: Uncorrelated Disruptions

Before turning towards the general framework, I first analyse a limit case in which suppliers' risk is not correlated, that is $\rho_0 \to 0$. This limit case gives a useful interpretation of the incentives behind multisourcing and allows us to establish a benchmark against which to study the introduction of correlated shocks.

Proposition 4. *If risk among basal firms is uncorrelated, that is $\rho_0 \to 0$, disruption events in layer k are independent and happen with probability*

$$\mu_{k+1} = \mu_k^{s_{k+1}}. \tag{11}$$

Proof. Follows immediately from $P_k \to \mu_k$ as $\rho_0 \to 0$. □

As in this case $\mathbb{E}_s[P_{k+1}] = \mu_k^s$, profits (9) are given by $\Pi_{k+1}(s) = \left(1 - \mu_k^s\right)\pi - \frac{c}{2}s^2$. Using this, we can derive the optimal sourcing strategy s_k of a firm producing good k. A firm with s suppliers contracts an extra one only if doing so yields a positive marginal profit

$$\begin{aligned}\Delta\Pi_{k+1}(s) &\coloneqq \Pi_{k+1}(s+1) - \Pi_{k+1}(s) \\ &= \mu_k^s(1-\mu_k)\pi - c\left(s + \frac{1}{2}\right).\end{aligned} \tag{12}$$

It is easy to check that $\Delta\Pi_{k+1}(s)$ is strictly decreasing and that it has a unique root. Hence, the optimal number of suppliers s_{k+1} is the smallest integer s for which the expected marginal profit is negative, that is $\Delta\Pi_{k+1}(s) < 0$.

Definition 3. *Let $\tilde{s}_{k+1}$ be the unique real root of $\Delta\Pi_{k+1}$. I refer to this quantity as the "desired sourcing strategy" of the firm.*

The optimal sourcing strategy is then given by

$$s_{k+1} = \begin{cases} \lceil \tilde{s}_{k+1} \rceil & \text{if } \tilde{s}_{k+1} > 0 \text{ and} \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Proposition 5. *Introduce the threshold*

$$\mu^0 := 1 - rc \tag{14}$$

where $rc := \frac{c/2}{\pi}$ is the real marginal costs of an additional supplier. If the average disruption probability μ_k is larger than μ_0, the downstream firm does not source any inputs, that is $s_{k+1} = 0$.

Proof. Suppose a firm optimally does not source any inputs. This implies that the marginal benefit of adding the first supplier is negative, namely $\Delta\Pi_{k+1}(0) < 0$, which yields the desired inequality. □

As expected, the desired $\tilde{s}_{k+1}$ and the optimal sourcing strategy s_{k+1} are determined by the upstream average disruption probability μ_k and the real marginal costs of contracting a new supplier rc. Figure 5 illustrates the effect these two conditions have on the optimal sourcing strategy. First, higher real marginal costs rc reduce the firm's number of sources. Second, as the the upstream average disruption probability μ_k increases, initially, the firm seeks higher diversification, until a level above which the desired sourcing strategy starts decreasing steeply.

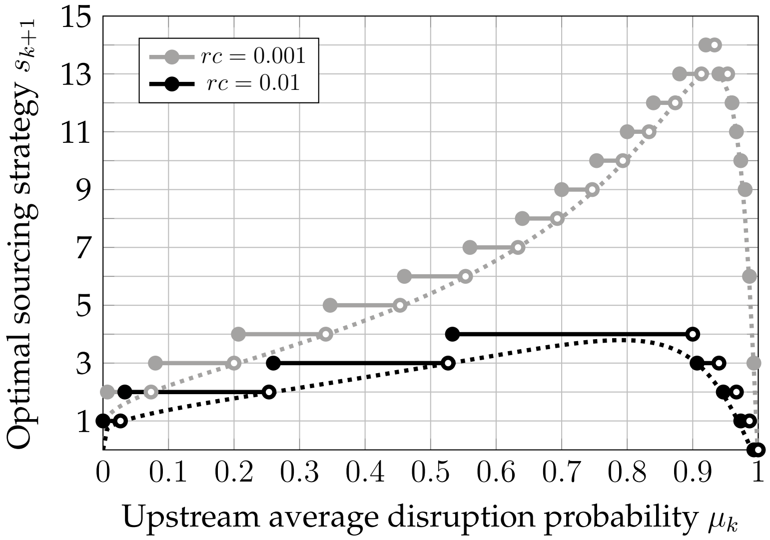

Figure 5: The desired $\tilde{s}_{k+1}$ (dotted) and optimal s_{k+1} sourcing strategy (solid) as a function of the upstream average disruption probability μ_k

Having studied how risk affects the firm's optimal sourcing, I now look at the opposite channel, that is, how the firm sourcing strategy affects risk propagation. To do so, we think of the average disruption probability

$$\mu_{k+1} = \mu_k^{s_{k+1}(\mu_k)} \tag{15}$$

from suppliers to downstream producers as a dynamical system, not in time but in layers $k \in \{0, 1, 2 \dots\}$ of the supply chain. Given a *basal condition* μ_0, a fixed point $\bar{\mu}$ of the map (15) is then a level of disruption probability $\bar{\mu}$ such that all firms downstream of a layer $\bar{k}$ single-source, namely $s_l = 1$ for all $l \geq \bar{k}$, and hence all share the same disruption probability $\mu_l \equiv \bar{\mu}$. When looking at the production network through this lens, a natural question arises: which basal levels of disruption probabilities μ_0 are not endogenously diversified by the production network, that is $\bar{\mu} \geq \mu_0$? To answer this, first I characterise the downstream disruption probability $\bar{\mu}$.

Proposition 6. *The downstream disruption probability $\bar{\mu}$ satisfies*

$$\bar{\mu}\,(1 - \bar{\mu}) \leq 3rc. \tag{16}$$

Proof. A steady state is attained at level $\bar{k}$ if $s_{\bar{k}} = 1$. This implies that the marginal benefit of multisourcing is negative $\Delta\Pi_{\bar{k}}(1) \leq 0$. This yields the desired inequality. $\square$

Corollary 6.1. *Introduce the critical threshold*

$$\mu^c := \frac{1}{2} + \sqrt{\frac{1}{4} - 3rc}. \tag{17}$$

If $\mu_0 > \mu^c$, the endogenous supply chain is unable to diversify risk, that is $\bar{\mu} \geq \mu_0$.

This result links the firm real marginal costs of sourcing rc and the production network risk. As relative marginal costs increase, the capacity of the production network to endogenously diversify decreases and firms' under-diversification yields endogenous fragility. Notice that, comparing the threshold μ^c of endogenous diversification with the threshold μ^0 of firm shutdown, illustrated in Figure 6, for some levels of basal probability of disruption μ_0, despite no firm shutting down production $\mu_0 < \mu^0$, the production network as a whole is still unable to endogenously diversify risk $\mu_0 > \mu^c$. This is true even in this special case, where the firms' risk is uncorrelated. In the next section, I introduce correlation risk $\rho > 0$ and investigate how doing so changes the dynamics illustrated here.

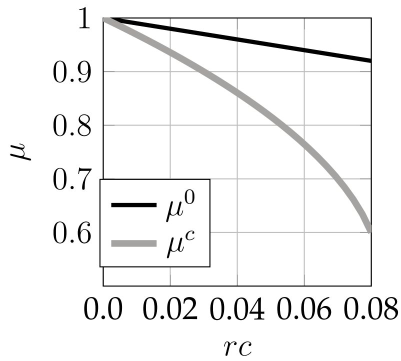

Figure 6

3.2 Optimal Sourcing with Correlated Distributions

If disruption events are not independent, that is $\rho_0 > 0$, the risk among suppliers throughout the production network is correlated, which affects the firm's optimisation incentives. In this case, the problem of a firm in layer $k+1$ is still to choose the number of suppliers $s_{k+1} \in \{0, 1, 2 \ldots\}$ that maximises the profits Π, but, by Proposition 3, the firm's disruption probability is given by the average disruption probability of its suppliers μ_k multiplied by a factor $\eta(s_{k+1}, S_k)$

which depends on the upstream diversification S_k. As in the limit case analysed in the previous section, the firm will increase diversification as long as the expected increase in profits obtained by adding an additional supplier outweighs the costs of contracting that additional supplier. These expected marginal profits are given by

$$\Delta\Pi_{k+1}(s_{k+1}) = \Big(\eta(s_{k+1}, S_k) - \eta(s_{k+1}+1, S_k)\Big)\,\mu_k\pi - c\left(s_{k+1} + \frac{1}{2}\right). \tag{18}$$

The characterisation of the optimal sourcing strategy is analogous to the limit case without the correlation discussed above. $\tilde{s}_{k+1} \in \mathbb{R}$ is the desired sourcing strategy for which the marginal benefits and marginal costs of diversification are equal, such that $\Delta\Pi_{k+1}(\tilde{s}_{k+1}) = 0$. As the marginal profits are strictly decreasing in the number of suppliers (see Appendix B.5), the firm will, as in the limit case, choose its optimal sourcing strategy as $s_{k+1} = \lceil \tilde{s}_{k+1} \rceil$ if $\tilde{s}_{k+1} > 0$ and chooses not to source any inputs otherwise. Figure 7 illustrates how the optimal sourcing strategy s_{k+1} changes with upstream correlation ρ_k for different levels of relative costs rc. As upstream correlation increases, the firm increases its sources to diversify risk. Yet, for large levels of correlation, the disruption of an additional source of the input good is likely correlated to a disruption among the firm's existing suppliers, which reduces the firm's incentive to multisource. As disruptions among suppliers become perfectly correlated, $\rho_k \to 1$, the firm sources from a single supplier.

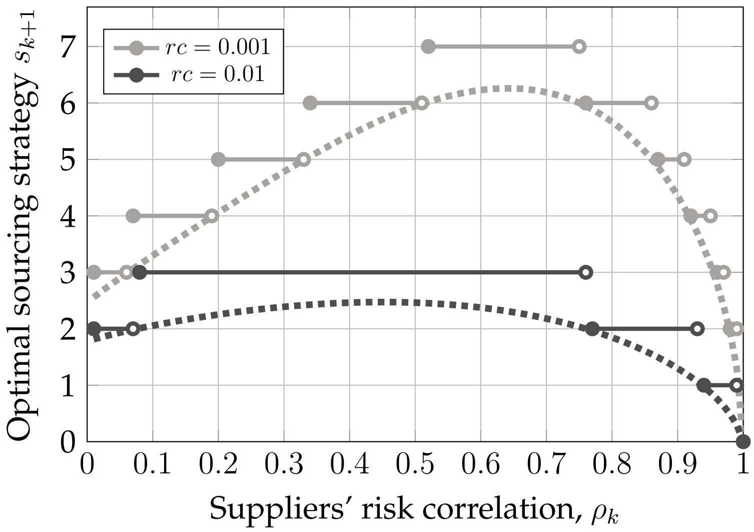

Figure 7: The desired $\tilde{s}_{k+1}$ (dotted) and optimal s_{k+1} sourcing strategy (solid) as a function of the upstream correlation ρ_k

To study the ramifications this endogenous channel has on the supply chain formation and its fragility, in the following I analyse the propagation of risk through the layers. As above, we can view the mapping of the probability of disruption between layers as a dynamical system through the layers of the production network. A steady state of the dynamical system is then an average disruption probability $\bar{\mu} := \mu_{\bar{k}}$ in some layer $\bar{k}$ such that all downstream layers $l \geq \bar{k}$ have the same average disruption probability $\mu_l \equiv \bar{\mu}$. This can occur in two cases. Either the firm in layer $\bar{k}$ does not source, that is $s_{\bar{k}} = 0$, or it single sources, that is $s_{\bar{k}} = 1$. The former case is trivial: the production network shuts down and all downstream firms do not produce, such that $\bar{\mu} = 1$. In the latter case, by single sourcing, the average disruption probability in layer $\bar{k}$ is the average disruption probability among the suppliers $\bar{k} - 1$, as the risk reduction factor $\eta(s_k, S_{k-1}) = 1$ if $s_k = 1$. Because the layers are symmetric, the firms in the downstream layer $\bar{k} + 1$ face the same problem as those in layer $\bar{k}$, such that they endogenously single source, that is $s_{\bar{k}+1} = 1$. Inductively, this holds true for all $l \geq \bar{k}$, hence $\mu_l \equiv \bar{\mu}$. Hereafter, I refer to the situation in which the downstream average disruption probability is greater than the basal one, that is $\bar{\mu} \geq \mu_0$, as *endogenous fragility*. Figure 8 shows the downstream average disruption probability $\bar{\mu}$ as a function of basal average disruption probability μ_0, for cases in which basal correlation ρ_0 is low or high. In both cases for large possible initial levels of basal average disruption probability μ_0 the supply chain is endogenously resilient, as $\bar{\mu} < \mu_0$. But, as in the uncorrelated cases studied above, there is a threshold of average basal disruption probability $\mu_0 > \mu^c$ for which the firm is endogenously fragile and $\bar{\mu} \geq \mu_0$. The threshold effect is discontinuous. At $\mu_0 \equiv \mu^c$ an arbitrarily small increase in μ_0 can lead to discontinuously large downstream failure probabilities $\bar{\mu}$.

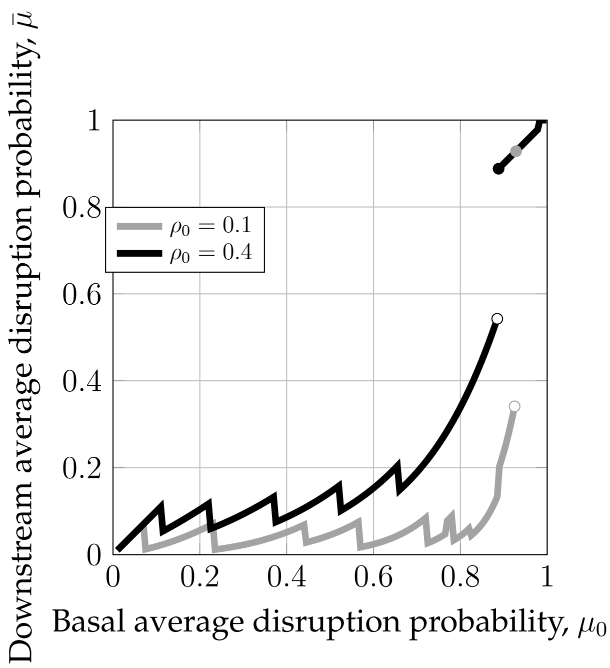

Figure 8: Downstream average disruption probability as a function of basal average disruption probability for low and high basal levels of correlation ρ_0

The threshold μ^c is decreasing in the basal level of correlation ρ_0, as illustrated in Figure 9. This implies that a small increase in basal correlation leads to discontinuous increases in the downstream average disruption probability. This results highlights an additional channel to that studied by Elliott, Golub and Leduc (2022) by which supply chains can be endogenously fragile: even if the expected failure probability μ_0 of basal producers remains unchanged, an increase in the correlation of their disruptions ρ_0, can endogenously induce large fragilities. This result highlights how phenomena that can lead to increases in risk among upstream producers, such as offshoring, climate disruptions, and economic agglomeration, can generate under-diversification and endogenous fragility, even as they leave individual producer's risk unchanged.

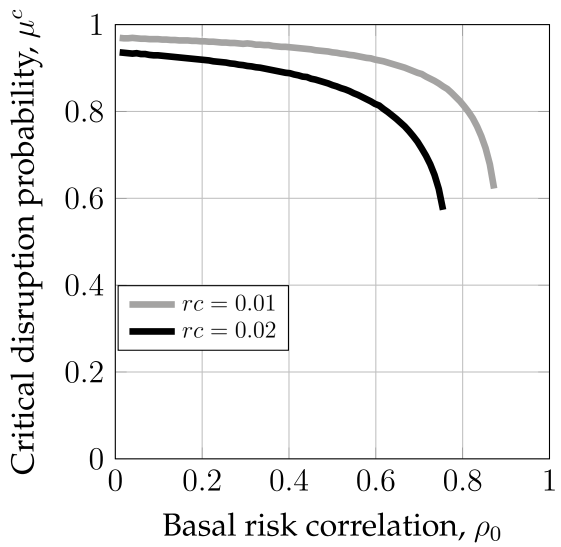

Figure 9: Critical level of basal average disruption probability μ^c as a function of the basal correlation.

4 Social Planner Problem

To establish a benchmark to which one can compare the competitive equilibrium analysed above, in this section I solve the model from the perspective of a social planner. The social planner attempts to, on the one hand, minimise the number of firms expected to fail, and, on the other, minimise the number of costly sourcing relations. To develop a useful benchmark, I define a social planner problem that can be meaningfully compared to the decentralised firm's problem by making the following two assumptions.

Assumption 5. *The social planner knows the distribution of failure in the basal layer* $P_0 \sim$ *Beta*(μ_0, ρ_0) *and makes a decision before* P_0 *is realised.*

Assumption 6. *As in the firm problem, I consider the limit in which the number of firms producing each good goes to infinity, that is* $n \to \infty$*. This allows the social planner to recursively, from the last layer* K *upwards, assign suppliers* $\mathcal{S}_{k,i}$ *such that there are sufficiently many firms so that no two firms share suppliers* $\mathcal{S}_{k,i} \cap \mathcal{S}_{k,j} = \emptyset$*.*

To understand the intuition behind Assumption 6, consider the possible supplier overlap illustrated in Figure 10: if a supplier has multiple downstream clients (dashed box), the social planner can always rewire a link towards a supplier without downstream clients (solid box). By doing so, the social planner can "diversify away" all the correlation that arises due to the network structure. Hence, the only source of risk in the model is represented by the shutdowns experienced by firms in the basal layer, which happen with non-idiosyncratic probabilities P_0 (Assumption 5). Combining Assumptions 5 and 6, the social planner problem is then to maximise average expected payoffs

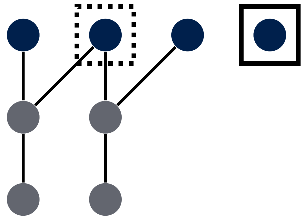

Figure 10

$$W(\{\mathcal{S}_{k,i}\}) \coloneqq \frac{1}{K}\sum_{k=0}^{K}\lim_{n\to\infty}\frac{1}{m_k(n)}\sum_{i=1}^{m_k(n)}\Big(1-\mathbb{P}\big(\mathcal{S}_{k,i}\subset\mathcal{D}_{k-1}\big)\Big)\,\pi-\frac{c}{2}\left|\mathcal{S}_{k,i}\right|^2, \tag{19}$$

by choosing a sourcing strategy $S_{k,i} \subseteq \{1, 2, \ldots\}$ for each firm in each layer such that $S_{k,i} \cap S_{k,j}$ is empty for all i, j. The social planner problem can be further simplified by noticing that, given that all firms in layer k are identical if establishing an additional path from a firm in layer k to a basal firm has positive marginal benefits, then it has positive marginal benefits for all firms in layer k which share the same number of paths to basal firms. Hence, as in the decentralised firms' problem, the social planner can choose the optimal number of sources in each layer, let the firms source at random, and finally disentangle any overlapping paths. Using this, the social planner problem can be formulated recursively, by letting V_k be the maximal average welfare from layer k to the last layer K. This can be recursively defined as

$$V_k(P_{k-1}) = \max_{s_k}\left\{\big(1-\mathbb{E}\big[P_{k-1}^{s_k}\big]\big)\,\pi-\frac{c}{2}s_k^2+\mathbb{E}\big[V_{k+1}(P_k)\big]\right\} \tag{20}$$

where the state $P_{k-1} \sim \text{BetaPower}(\mu_0, \rho_0, s_1 s_2 \ldots s_{k-1})$ is the fraction of disrupted firms, which evolves as

$$P_k = P_{k-1}^{s_k}. \tag{21}$$

The average welfare in layer $K+1$ is given by $V_{K+1}(P_K) = 0$, since firms in the last layer are never sources to other firms, and an initial state condition $P_0 \sim \text{Beta}(\mu_0, \rho_0)$. This problem can be solved using standard backward induction techniques (see Appendix C). The optimum average social welfare (19) can then be written as

$$V_1(P_0) = \max_{s_1, s_2, \dots s_{K-1}} W(\{s_1, s_2, \dots s_{K-1}\}). \tag{22}$$

Letting $\{s_k^p\}_{k=1}^K$ be the socially optimal sourcing strategies sequence and $\{\mu_k^p\}_{k=1}^K$ be the expected disruption in each layer given by such sourcing strategies, we can compute the change in downstream risk compared to the decentralised case. Figure 11 shows this difference $\overline{\mu} - \overline{\mu}^p$ for the same two cost regimes. If pairing costs are low, the social planner achieves marginally lower risk levels of downstream risk for most initial conditions. If initial basal correlation ρ_0 is sufficiently large and the average basal disruption probability μ_0 is sufficiently low, the firms over-diversify compared to the socially optimum $\overline{\mu} < \overline{\mu}^p$. If relative pairing costs are high, the social planner is able to diversify risk around the critical threshold μ_c, such that the decentralised equilibrium induces inefficiently high levels of average downstream disruption probability, that is $\overline{\mu} > \overline{\mu}^p$. This result implies that the cascading failures that occur around the critical threshold are fully attributable to firms' endogenous under-diversification motives and are hence inefficient.

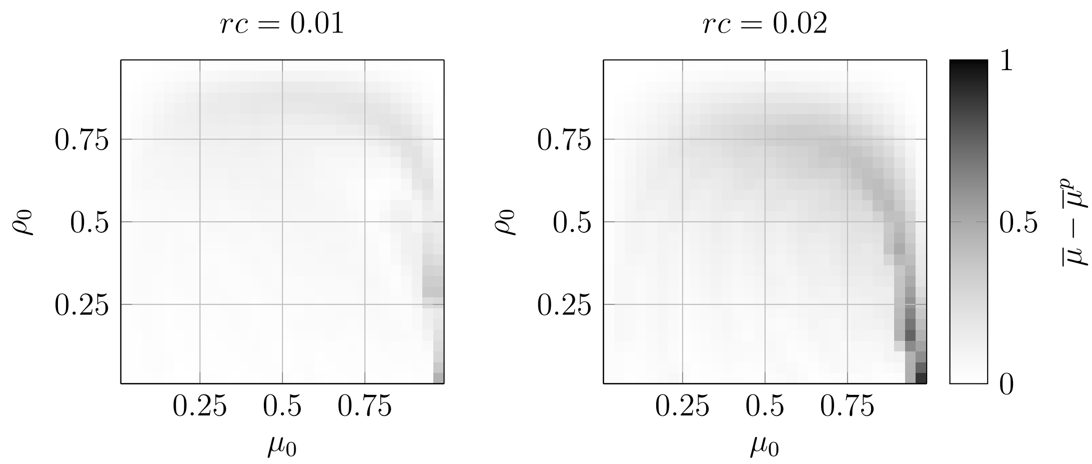

Figure 11: Change in downstream expected failure probability between the firms' $\overline{\mu}$ and the social planner $\overline{\mu}^p$ equilibrium, given different initial conditions μ_0 and ρ_0 in a low (left) and a high (right) relative pairing costs regime.

The differences between the firms' sourcing strategies and the social optimum generate welfare losses in the production network. Letting W be the average firm profit in the decentralised case and W^p be the average profit achieved by the social planner, Figure 12 illustrates the welfare loss due to the firms' diversification decisions $W - W^p$. The welfare loss is largest around the critical value μ^c, where the production network is endogenously fragile. At these levels of risk, firms' upstream firms' diversification incentives are weak, which creates large downstream resilience externalities. Crucially, both an increase in basal risk μ_0 and an increase in basal correlation ρ_0 can generate discontinuous welfare losses.

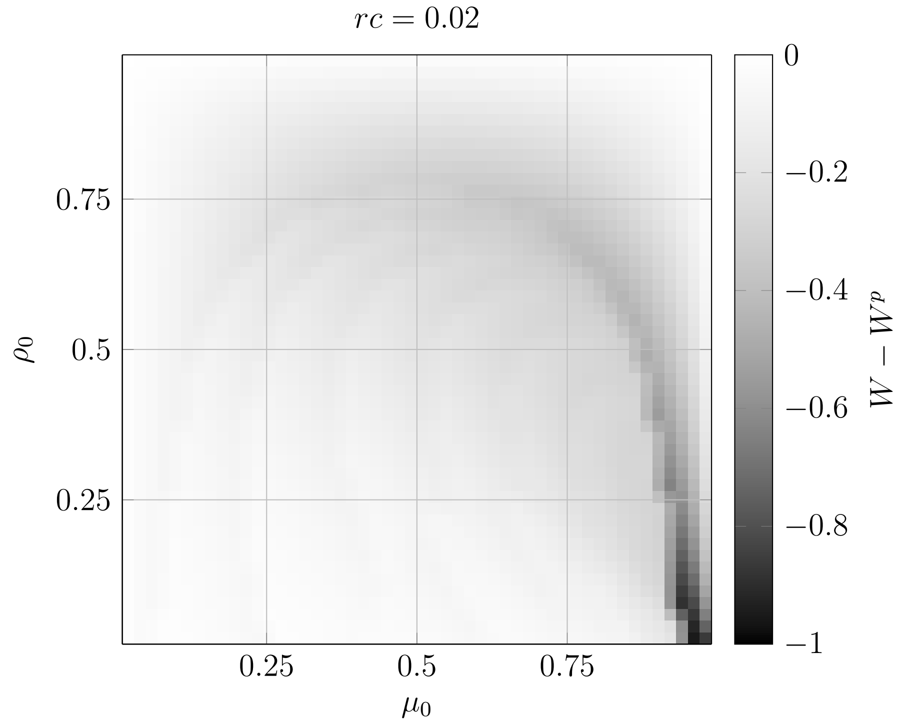

Figure 12: Welfare loss of the decentralised equilibrium $W - W^p$, given different initial conditions μ_0 and ρ_0.

5 The Role of Opacity

So far I assumed that firms cannot observe the realisation of the supply chain and the basal disruption probabilities P_0 when making sourcing decisions. To understand how this assumption affects optimal decisions and fragility within the supply chain, I now analyse the supply chain under perfect information. The following assumption clarifies what is meant by perfect information in the context of the model.

Assumption 7. *In a regime of perfect information, each firm i in level k is able to perfectly estimate the disruption probability of each potential supplier and the full correlation structure of the disruption events.*

Under this perfect information regime, the firm can assign correct probabilities to its own disruption risk

$$\mathbb{P}\Big(\mathcal{S}_{k,i} \subset \mathcal{D}_{k-1}\Big) \text{ for all possible sourcing strategies } \mathcal{S}_{k,i}.$$

The firm can hence rank suppliers by the marginal reduction in risk they provide and source from the "safest" s_k desired suppliers. As all firms downstream are ex-ante identical, the marginal benefits of diversification experienced by firm (i, j) are the same as those of all other firms in layer k, which implies that, in equilibrium, all firms in layer k will employ the same sourcing strategy, given that they are ex-ante identical. This outcome is beneficial for any single firm, but detrimental to the stability of the production network. The following two propositions formalise this.

Proposition 7. *Compared to the opaque scenario, for the same number of sources, firm (k, i) is (weakly) less likely to be disrupted.*

Proof. Given the same number of sources, the firm with perfect information minimises its disruption risk with fewer constraints than in the opaque scenario. $\square$

Proposition 8. *Under perfect information, the supply chain is maximally fragile: either all firms fail or none do.*

Proof. Without loss of generality, assume basal firms are sorted by their disruption risk $P_{0,1} > P_{0,2} > \ldots$. Suppose a firm in layer $k = 1$ chooses to contract $s_{1,i}$ suppliers. In choosing which suppliers to source from, the optimal choice is then to pick the first $s_{1,i}$-th basal producers, namely

$$\mathcal{S}_{1,i} = \{1, 2, \ldots s_{1,i}\}. \tag{23}$$

By symmetry, this is also the optimal choice of all other firms in layer $k = 1$, such that $\mathcal{S}_{1,i} = \mathcal{S}_{1,j}$ for all i, j. This further implies that disruption events in layer $k = 1$ are perfectly correlated, $X_{1,i} = X_{1,j}$ for all i, j, as each two firms share all suppliers. This in turn implies that, regardless of the diversification strategy, all firms downstream $k > 1$ experience perfectly correlated distribution events. Hence

$$X_{k,i} = X_{k,j} \text{ for all firms } i, j \text{ and layers } k > 0. \tag{24}$$

$\square$

Supply chain opacity, despite preventing firms from implementing an optimal diversification strategy, leads to more resilient supply chains. Hence, policy efforts to improve supply chain resilience via transparency might backfire if not paired with efforts to coordinate diversification of firms' sources.

6 Conclusion

Deeper and more globalised supply chains are more vulnerable to widespread, correlated disruptions. This paper studies how firms diversify sourcing risk when disruptions are correlated, and how this affects the endogenous formation of supply chains. I show that, as disruption correlation rises, a firm sources its input good from more suppliers to diversify the risk of a disruption. Yet, there is a level of correlation after which the expected risk reduction of adding an additional supplier is small, hence, the firm starts reducing its number of suppliers and lowers its diversification. This mechanism can create two forms of externalities imposed by upstream suppliers onto downstream producers. When firms upstream choose to under-diversify, this generates increased disruption risk downstream. When firms upstream choose to over-diversify, this generates a high correlation in their disruption risk, which has consequences for downstream diversification incentives. Due to this, I show that the supply chain is endogenously fragile to disruption correlation, that is, small increases in the correlation among upstream disruptions can trigger large under-diversification throughout the supply chain. This can leave the economy highly susceptible to small disruption events.

To construct a welfare benchmark for these results, I then solve the equivalent social planner problem. I show that a social planner can design a supply chain that is resilient to such correlated shocks. This result has two consequences. First, it illustrates that the fragility derives from the individual firm's diversification strategy and the risk externalities it induces downstream. It is hence entirely endogenous to the supply chain formation. Second, it implies the presence of large and discontinuous welfare losses. Finally, I study the role of imperfect information on supply chain formation. I show that, in the presence of perfect information on the structure and risk of the supply chain, firms are better able to individually diversify risk, yet, in doing so, they choose identical suppliers, which renders the supply chain vulnerable to small upstream disruptions. This suggests that recent efforts in increasing firms' visibility of the supply chain, despite reducing individual firms' supply chain risk, might have the unintended consequence of making supply chains more fragile.

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A Notation and Distributions

This appendix introduces standard notation and definitions that will be used throughout the following appendices.

For $x \in \mathbb{R}$ and $n \in \mathbb{N}$, I denote the rising factorial as

$$x^{\overline{n}} := \underbrace{x(x+1)(x+2)\ldots(x+(n-1))}_{n \text{ terms}}. \tag{25}$$

For non-integer exponents $n \in \mathbb{R}$, the definition (25) can be extended as

$$x^{\overline{n}} := \frac{\Gamma(x+n)}{\Gamma(x)}, \tag{26}$$

where Γ is the gamma function.

Two properties of the rising factorial that are used below but not proven are the additive property of the exponent

$$x^{\overline{n+m}} = x^{\overline{n}}(x+m)^{\overline{m}}, \tag{27}$$

and that it is strictly increasing in its base

$$\frac{\partial x^{\overline{n}}}{\partial x} > 0. \tag{28}$$

B Omitted Proofs

This appendix contains the proofs omitted from the paper.

B.1 Proof of Proposition 1

Proving Proposition 1, requires the following Lemma.

Lemma 8.1. *If the disruption events among upstream firms are exchangeable, then the probability that a downstream firm is disrupted depends only on the number of suppliers it picks.*

Proof. Consider the sequence of disruption events among upstream firms

$$X_{k,1}, X_{k,2}, X_{k,3} \ldots . \tag{29}$$

We assume the sequence to be exchangeable, that is,

$$X_{k,1}, X_{k,2}, X_{k,3} \ldots \stackrel{d}{=} X_{k,\sigma(1)}, X_{k,\sigma(2)}, X_{k,\sigma(3)} \ldots , \tag{30}$$

for an arbitrary permutation of its indices σ. Fix two arbitrary finite subsets of disruptions $\mathcal{A} = \{X_{k,a_1}, X_{k,a_2} \ldots X_{k,a_n}\}$ and $\mathcal{B} = \{X_{k,b_1}, X_{k,b_2} \ldots X_{k,b_n}\}$ of size n. Here $a_i, b_i \in [m]$ are the indices of the original sequence corresponding to the i-th index of the subset. Let σ be a permutation that takes elements of $\mathcal{A}$ to $\mathcal{B}$, namely,

$$\sigma(\mathcal{A}) = \mathcal{B} \text{ and } \sigma(\mathcal{A}^c) = \mathcal{B}^c. \tag{31}$$

Then the probability distribution over $\mathcal{A}$ is

$$\begin{aligned}
\mathbb{P}(\mathcal{A}) &= \mathbb{P}(\mathcal{A} \text{ and } \mathcal{A}^c \text{ taking any value}) \\
&= \mathbb{P}(\sigma(\mathcal{A}) \text{ and } \sigma(\mathcal{A}^c) \text{ taking any value}) \\
\text{then by exchangeability, } &= \mathbb{P}(\mathcal{B} \text{ and } \mathcal{B}^c \text{ taking any value}) = \mathbb{P}(\mathcal{B}).
\end{aligned} \tag{32}$$

□

Now we can prove Proposition 1

Proof. The proof is done by induction. The base case $k = 0$ follows from Assumption 1, as the disruption probabilities are ν-distributed.

Assume that for some layer $k-1$ the disruption events $\{X_{k-1,i}\}_i^{m_{k-1}} = 1$ are exchangeable. By Lemma 8.1 the downstream expected profits $\Pi_{k,i}(\mathcal{S})$ depend only the number of suppliers $|\mathcal{S}|$. By symmetry and Assumption 2, all firms in layer k are then selecting a random subset of supplier from layer $k-1$ with equal probability, which in turn determines their disruption risk $X_{k,i}$. This construction is independent of the downstream firm index i, hence

$$X_{k,1}, X_{k,2} \ldots, X_{k,m}$$

are exchangeable. $\square$

B.2 Proof of Proposition 2

Proof. A firm producing good $k+1$ sources from s_{k+1} suppliers, hence, its disruption event is given by

$$X_{i,k+1} = X_{j_1,k} X_{j_2,k} \ldots X_{j_{s_{k+1}},k}, \tag{33}$$

where $\{j_1, \ldots j_{s_{k+1}}\}$ is an arbitrary subset of suppliers and $X_{j,k}$ are exchangeable Bernoulli trials with a P_k success probability, where

$$P_k \sim \text{BetaPower}.$$

Conditional on the underline distribution P_k of disruption probabilities, the trials $X_{j,k}$ are independent and identically distributed. Hence, we have

$$\begin{aligned} P_{k+1} = \mathbb{E}[X_{i,k+1}] &= \mathbb{E}\left[\prod_{l=1}^{s_k} X_{j_l,k}\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[\prod_{l=1}^{s_k} X_{j_l,k} \middle| P_k = p_{j_l}\right]\right] \\ \text{by conditional independence,} \quad &= \mathbb{E}\left[\mathbb{E}\left[X_{j_l,k} \middle| P_k = p_{j_l}\right]^{s_k}\right] \\ \text{by independence of the draws,} \quad &= \mathbb{E}\left[\mathbb{E}\left[X_{j_l,k} \middle| P_k = p_{j_l}\right]\right]^{s_k} = P_k^{s_k}. \end{aligned} \tag{34}$$

$\square$

B.3 Mapping of risk across layers

This section derives the risk reduction factor η.

Lemma 8.2. *If $P_{k-1} \sim$ BetaPower(m, α, β, S) for some integer S, then*

$$P_k \sim \textit{BetaPower}(m, \alpha, \beta, Ss_k) \tag{35}$$

where s_k is the choice of suppliers in layer k.

Proof. Follows from the definition of BetaPower. □

Proposition 9. *The expected probability of disruption faced by a firm is given by*

$$\mathbb{E}\Big[P_k\Big] = \frac{B\left(\mu_0 \frac{1-\rho_0}{\rho_0} + S_{k-1}s_k, (1-\mu_0)\frac{1-\rho_0}{\rho_0}\right)}{B\left(\mu_0 \frac{1-\rho_0}{\rho_0}, (1-\mu_0)\frac{1-\rho_0}{\rho_0}\right)}. \tag{36}$$

Proof. It follows from rewriting the moment generating function of the beta distribution as

$$M(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{B(\alpha+n, \beta)}{B(\alpha, \beta)} \tag{37}$$

and noticing that $P_k \sim \text{BetaPower}(\alpha, \beta, S_{k-1}s_k)$. □

To simplify notation, let $r_0 = \frac{1-\rho_0}{\rho_0}$ and

$$\eta(s, S) = \frac{(\mu_0 r_0 + S)^{\overline{S(s-1)}}}{(r_0 + S)^{\overline{S(s-1)}}} \tag{38}$$

which satisfies the recursion

$$\eta(s+1, S) = \eta(s, S)\frac{(\mu_0 r_0 + Ss)^{\overline{S}}}{(r_0 + Ss)^{\overline{S}}}. \tag{39}$$

Another property of (38) which will be use later is

$$\frac{\partial \eta}{\partial s} = \eta(s, S)S\left(\psi(\mu_0 r_0 + Ss) - \psi(r_0 + Ss)\right). \tag{40}$$

Corollary 9.1. *From equation (40) and the fact that ψ is increasing over positive values, it follows that η is decreasing in s.*

Using Proposition (9), the coefficient η allows us to write the propagation of risk recursively

$$\mu_{k+1} = \eta(s_{k+1}, S_k)\ \mu_k. \tag{41}$$

B.4 Limit case $\rho_0 \to 0$

For the following proof I only consider non-trivial values of upstream risk $\mu < \mu^0$. If $\mu \geq \mu^0$, no firm has suppliers and the supply chain is by definition stable.

Lemma 9.1. *A fixed point of the law of motion $g(\bar{\mu}) = \bar{\mu}$, is attained iff $\tilde{g}(\bar{\mu}) \geq \bar{\mu}$.*

Proof. By definition $\tilde{g}(\bar{\mu}) = \bar{\mu}^{\tilde{s}(\bar{\mu})}$ and $0 \leq \bar{\mu} \leq 1$. Hence $\tilde{g}(\bar{\mu}) \geq \bar{\mu} \iff \tilde{s}(\bar{\mu}) \in (0, 1]$. By definition $s(\bar{\mu}) = \lceil \tilde{s}(\bar{\mu}) \rceil = 1$, which implies that $g(\bar{\mu}) = \bar{\mu}$. □

Now we can prove Corollary 6.1.

Proof. We seek μ, such that $\tilde{g}(\mu) \geq \mu$, which then implies that $g(\mu) = \overline{\mu}$. This will be the case if $\tilde{s}(\mu) \in (0, 1]$. This is the case if $\Delta\Pi(1) \leq 0$ and $\Delta\Pi(0) > 0$, which yields the desired inequality. □

B.5 General Case, $\rho_0 > 0$

This appendix proves the existence of an optimal sourcing in the case $\rho > 0$.

Proof. It is sufficient to show that $\Delta\Pi$ is strictly decreasing in s when $\rho_0 > 0$. It is convenient to rewrite η (38) as

$$\eta(s) = \frac{\Gamma(r_0 + S)}{\Gamma(\mu_0\, r_0 + S)} \frac{\Gamma(\mu_0\, r_0 + Ss)}{\Gamma(r_0 + Ss)}. \tag{42}$$

Then

$$\Delta\Pi(s) = \big(\eta(s) - \eta(s+1)\big)\pi\mu - c\left(s + \frac{1}{2}\right), \tag{43}$$

hence

$$\begin{aligned}\Delta\Pi'(s) = -c - \mu\pi S\Bigg(&\eta(s+1)\Big(\psi(\mu_0 r_0 + S(s+1)) - \psi(r_0 + S(s+1))\Big) - \\ &\eta(s)\Big(\psi(\mu_0 r_0 + Ss) - \psi(r_0 + Ss)\Big)\Bigg).\end{aligned} \tag{44}$$

Then $\Delta\Pi'(s) < 0$, since ψ is increasing. Finally notice that $\Delta\Pi(-1/2) = (\eta(-1/2) - \eta(1/2))\pi\mu < 0$ and $\lim_{s\to\infty} \Delta\Pi(s) = \infty$. $\square$

C Solution of the Social Planner Problem

First, notice that the terminal condition V_K is linear in P_{K-1}, hence

$$\mathbb{E}\big[V_K(P_{K-1})\big] = V_K\big(\mathbb{E}[P_{K-1}]\big). \tag{45}$$

In turn, this implies that V_k is linear for all k. Hence we can rewrite the value to be a function of the state space S,

$$V_k(S_{k-1}) = \max_s \left\{ \Big(1 - \mathbb{E}\big[\text{BetaPower}(\mu_0, \rho_0, S_{k-1}\ s)\big]\Big)\pi - \frac{c}{2}s^2 + V_k(S_{k-1}\ s_k) \right\}. \tag{46}$$

We can find V_k numerically. Let $\Omega = [m] \times [m^K]$ for some $m \in \mathbb{N}$ and

$$l(s, S) \coloneqq \Big(1 - \mathbb{E}\big[\text{BetaPower}(\mu_0, \rho_0, S_{k-1}\ s)\big]\Big)\pi - \frac{c}{2}s^2. \tag{47}$$

Then, by means of backward induction we obtain a recursive expression for V_1, namely

$$\begin{aligned}
V_K(S) &= \max_s l(\Omega), \\
V_{k-1}(S) &= \max_s l(\Omega) + V_K(S\ s), \\
&\ \vdots \\
V_1(S) &= \max_s l(\Omega) + V_2(S\ s).
\end{aligned} \tag{48}$$