Climate Tipping Points and Optimal Emissions

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2. Large focus on stochastic tipping point, via jump processes (Lin and Wijnbergen, 2023; Van den Bremer and Van der Ploeg, 2021)
3. In climate models most tipping points are caused by bifurcations (Ashwin and Von Der Heydt, 2020; Ashwin et al., 2012)

In this paper I look at one such tipping point: **ice-albedo feedback**

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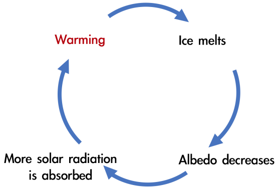


Castlefield
Petersfield
Great Northern
Manchester Art Gallery
Manchester Aquatics Centre
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AMAZING CRAFTSMANSHIP

Temperature

- S solar radiation
- T temperature

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$$S_0(1-\lambda(T)) = \eta\sigma T^4 - (G_0 + G_1 \log M/M^{\mathrm{p}})$$

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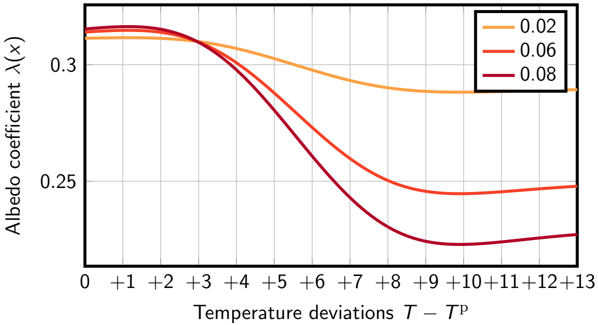
$$\epsilon \, \mathrm{d}T = \big(S_0(1-\lambda(T)) - \eta\sigma T^4 + G_0 + G_1 \log M/M^{\mathrm{p}}\big)\mathrm{d}t + \sigma_T \mathrm{d}W$$

Albedo loss

$$\lambda(T) = \lambda_1 - (1 - L(T))\Delta\lambda$$



Albedo loss

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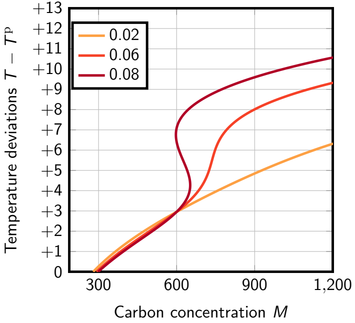
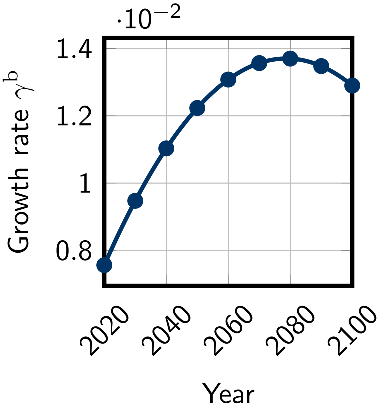


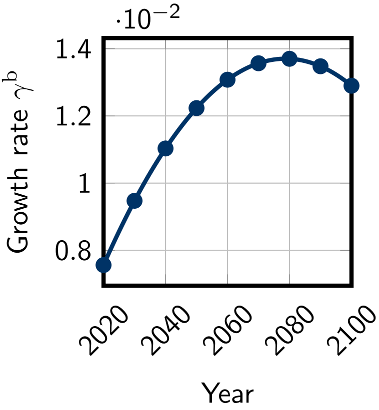
Carbon Concentration

Let γ^{b} be the growth rate of carbon concentration M under the business-as-usual scenario.

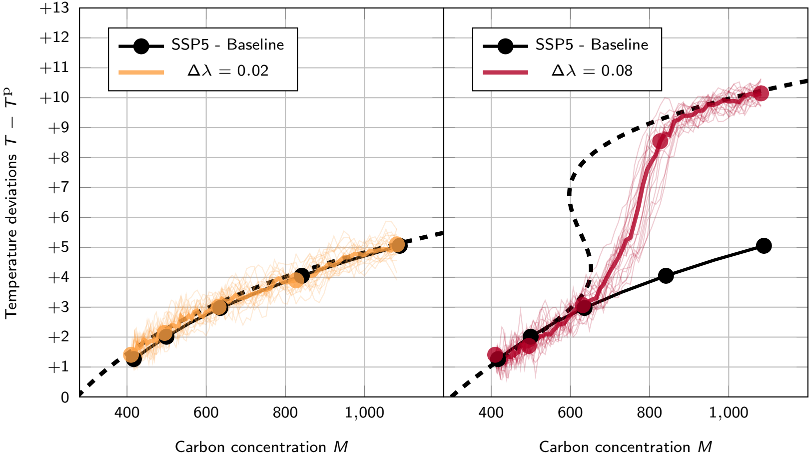


Carbon Concentration

$$\mathrm{d}m = \frac{\mathrm{d}M}{M} = \left(\gamma^{\mathrm{b}} - \boldsymbol{\alpha}\right) \mathrm{d}t$$



Business-as-usual dynamics



AK Economy (Hambel et al., 2021)

- y log-output
- k log-capital
- I investment in capital
- B abatement expenditure
- C consumption

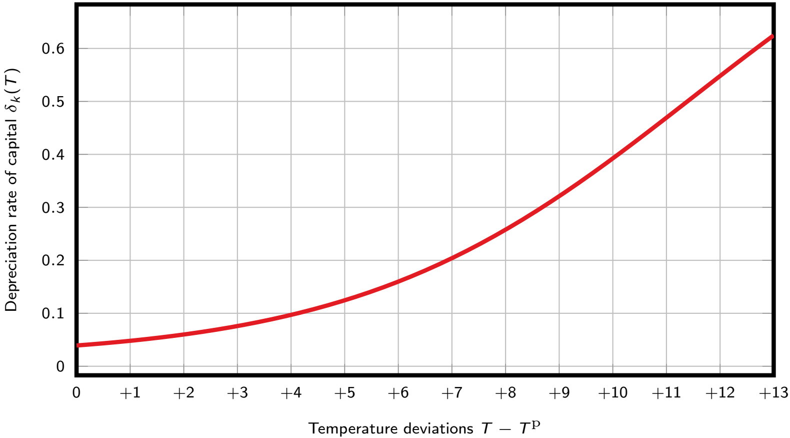
$$y = \log(A) + k \text{ s.t.}$$
$$Y = I + B + C$$

Climate change slows growth

- k log-capital
- T temperature
- I investment in capital
- B abatement expenditure

$$\mathrm{d}k = \left(\frac{I}{K} - \delta_k(T) - \frac{\kappa}{2} \left(\frac{I}{K} + \frac{B}{K} \right)^2 \right) \mathrm{d}t$$

Climate damages



Making everything a rate...

- Y output
- B abatement expenditure
- C consumption
- χ consumption rate
- ε emissivity rate

$$\chi := \frac{C}{Y} \text{ and } \frac{\omega}{2}\varepsilon^2 := \frac{B}{Y}$$

Emissivity rate ε to abatement α

- M CO_2 concentration
- E^{B} BaU emissions
- $\delta_m(M)$ natural decay of CO_2
- γ^{B} BaU growth of M

$$\varepsilon = 1 - \frac{M}{E^{\mathrm{B}}} \left(\delta_m(M) + \gamma^{\mathrm{B}} - \alpha\right)$$

Putting it all back into capital...

$$\frac{\mathrm{d}k}{\mathrm{d}t} = \overbrace{A(1-\chi) - \frac{A\kappa}{2}(1-\chi)^2}^{\text{Standard consumption problem } \phi(\chi)} - \underbrace{\frac{A\omega}{2}\varepsilon^2}_{\text{abatement}} - \overbrace{\delta_k(T)}^{\text{climate change}}$$

But we really care about output...

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \overbrace{\varrho + \phi(\chi)}^{\text{Economic growth}} - \underbrace{\frac{A\omega}{2}\varepsilon^2}_{\text{abatement}} - \overbrace{\delta_k(T)}^{\text{climate change}}$$

Social Planner

The social planner at time τ is trying to maximise

$$U(\tau, \alpha, \chi) = \mathbb{E} \int_{\tau}^{\infty} f(C(t), U(t, \alpha, \chi)) \mathrm{d}t$$

where $f(C, U)$ is the Epstein-Zin aggregator.

Why Epstein-Zin?

I flip a coin today. You get 2€ every day if it is head and 1€ every day if it is tails.

Each day I flip a coin. That day you get 2€ if it is head and 1€ if it is tails.

Value Function

Looking for $V(t, \chi, \alpha) = \sup_{\chi,\alpha} U(t, \alpha, \chi)$, which satisfies

$$-\partial_t V = f(\chi Y, V) + \nabla V \cdot (\text{drift of state}) + \partial_T^2 V \, \sigma_T^2$$

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No really... how?

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2. Optimal emissions paths matter, not only total emissions (would you walk on the edge of a ditch)
3. TODO: solve this HJB numerically?

Thank you!

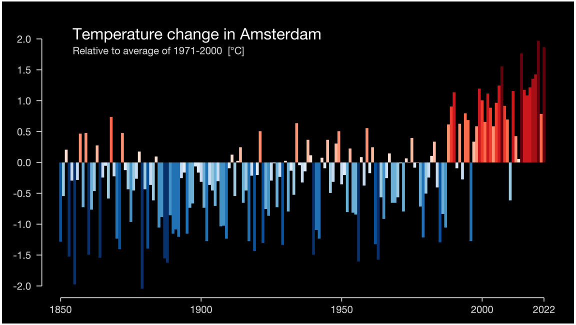

Figure: Ed Hawkins, Berkeley Earth data