Climate Tipping Points and Optimal Emissions

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2. Large focus on stochastic tipping point, via jump processes (Lin and Wijnbergen, 2023; Van den Bremer and Van der Ploeg, 2021)
3. In climate models most tipping points are caused by bifurcations (Ashwin and Von Der Heydt, 2020; Ashwin et al., 2012)

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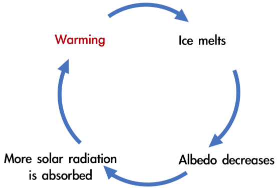


Castlefield
Petersfield
Great Northern
Manchester Art Gallery
Manchester Aquatics Centre
Whitworth Art Gallery
Manchester Museum
40
AMAZING CRAFTSMANSHIP

Temperature

- S: solar radiation
- T: temperature

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Temperature

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- T: temperature
- M: carbon concentration

$$S_0(1-\lambda(T)) = \eta\sigma T^4 - (G_0 + G_1 \log M/M^{\mathrm{p}})$$

Temperature

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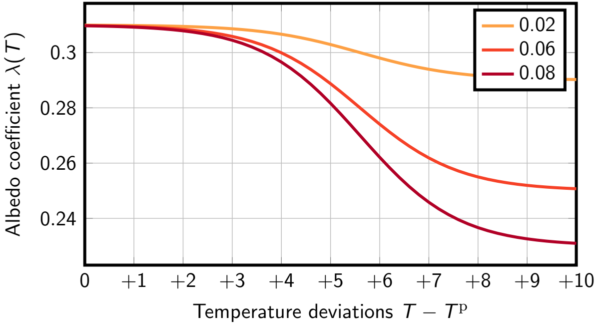
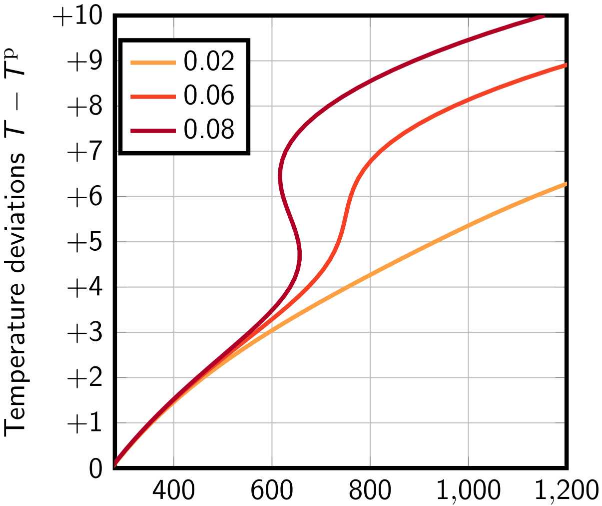
$$\epsilon \, \mathrm{d}T = \big(S_0(1-\lambda(T)) - \eta\sigma T^4 + G_0 + G_1 \log M/M^{\mathrm{p}}\big)\mathrm{d}t + \sigma_T \mathrm{d}W$$

Albedo loss

$$\lambda(T) = \lambda_1 - L(T)\Delta\lambda$$



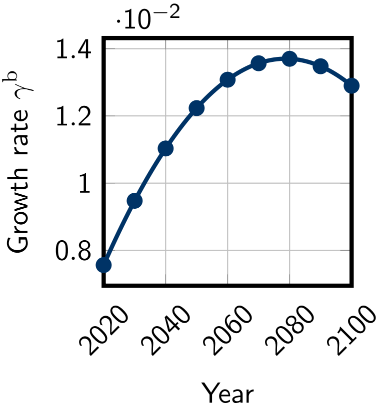
Albedo loss



0.02
0.06
0.08
Temperature deviations $T - T^{\mathrm{p}}$
+10
+9
+8
+7
+6
+5
+4
+3
+2
+1
0
400
600
800
1,000
1,200

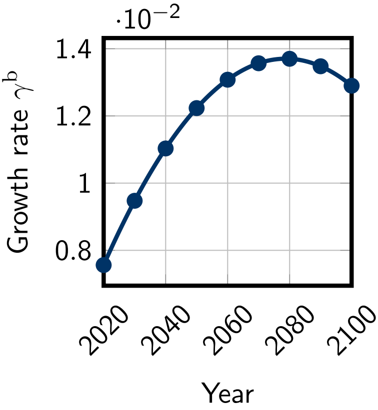
Carbon Concentration

γ^{b}: growth rate of carbon concentration M under the business-as-usual scenario.

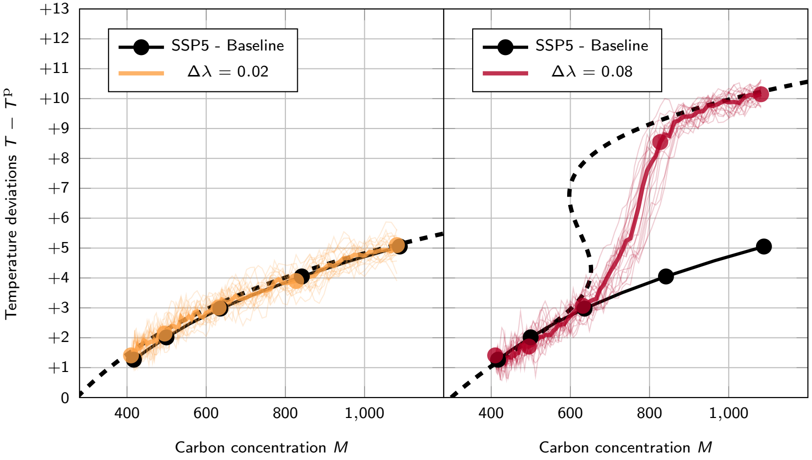


Carbon Concentration

$$\mathrm{d}m = \frac{\mathrm{d}M}{M} = \left(\gamma^{\mathrm{b}} - \boldsymbol{\alpha}\right)\mathrm{d}t$$



Business-as-usual dynamics



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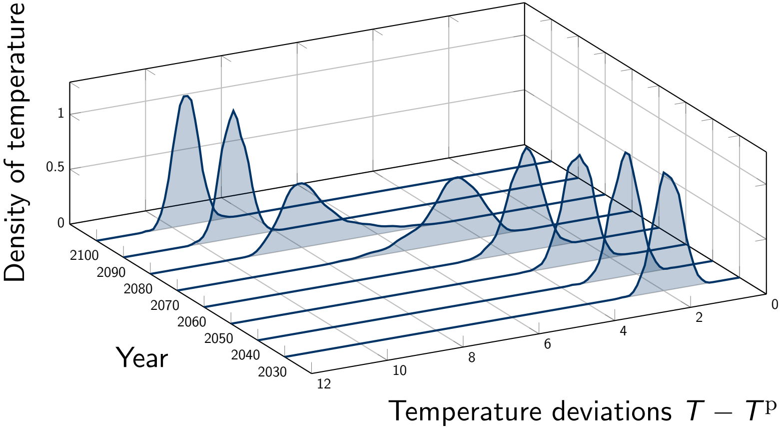

Economy... without climate change

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \varrho - \delta_k + \phi(1 - \chi)$$

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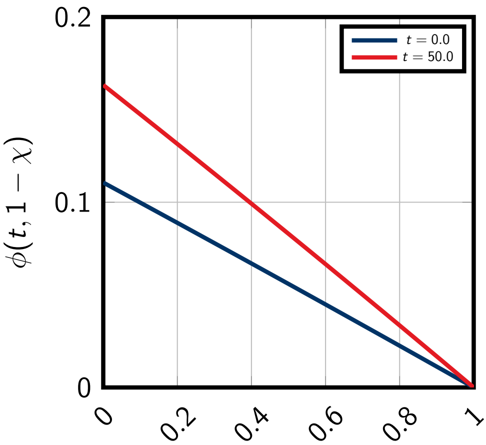

Economy... with climate change

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \varrho - \delta_k + \phi(1 - \boldsymbol{\chi}) - d(T) - \frac{\omega}{2}\left(1 - \frac{E(\boldsymbol{\alpha})}{E^{\mathrm{B}}}\right)^2$$

Social Planner

The social planner at time t is trying to maximise

$$U(t, X, \alpha, \chi) = \mathbb{E}_X \int_t^\infty f\Big(\underbrace{\chi e^{y}}_{C}, U(t, X, \alpha, \chi)\Big)\, \mathrm{d}t$$

where $f(C, U)$ is the Epstein-Zin aggregator and $X = (T, m, y)$.

Why Epstein-Zin?

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I flip a coin today. You get 2€ every day if it is head and 1€ every day if it is tails.

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I flip a coin today. You get 2€ every day if it is head and 1€ every day if it is tails.

Each day I flip a coin. That day you get 2€ if it is head and 1€ if it is tails.

Value Function

We are looking for $V(t, X) = \sup_{\chi,\alpha} U(t, X, \alpha, \chi)$, which satisfies

$$
\begin{aligned}
-\partial_t V = \sup_{\chi,\alpha} \Big\{ & f(\chi e^{y}, V) + \\
& \nabla_X V \cdot \mathrm{drift}(X, \chi, \alpha) \Big\} + \\
& \Delta_X V \cdot \mathrm{noise}
\end{aligned}
$$

Terminal condition

1. **Assume** that at some point τ (e.g. 500 years) abatement is free, hence $\mathrm{d}m = 0$.

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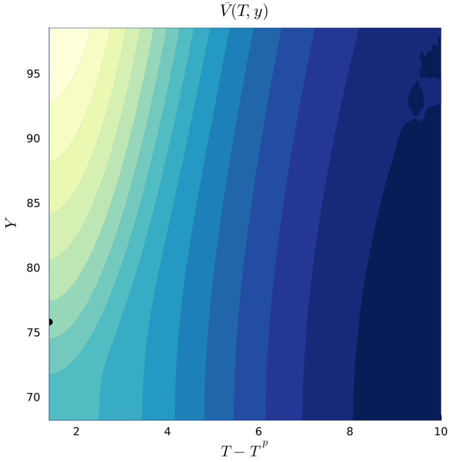
2. Solve the "terminal" problem

$$0 = \sup_{\chi} f(\chi e^{y}, \overline{V}) + \partial_y \overline{V} \times \Big(\phi(t, 1 - \chi) - \delta_k - d(T) \Big)$$

Terminal condition



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Given $\overline{V}$, numerically simulate backwards in time until today $t = 0$ using $\partial_t V$. But there are complications...

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1. Cannot use first order conditions to find optimal χ, α;
2. Large state space X around 180 million points

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1. Optimal emissions prescribes steep reductions compared to business as usual
2. Unlike previous work, timing matters: reduce emissions early
3. As a consequence the social cost of carbon is increasing rapidly

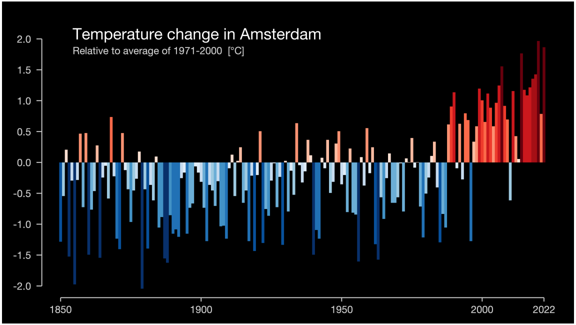

Figure: Ed Hawkins, Berkeley Earth data