Automata notes - Evolutionary game theory

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1 Notes

We can represent a strategy over m states and n actions as a pair (Σ, X) where,

$$\Sigma \in \mathbf{2}^{m \times n}, \; X \in \mathbf{2}^{m \times n \times m} \tag{1}$$

A game can be played between two strategies as long as the number of actions in the same.

Assume two strategies (Σ_g, X_g) and (Σ_y, X_y), with a different number of states, m_g and m_y. A state can be represented with a binary vector $g \in \mathbf{2}^{m_g}$. We can define a state evolution tensor as a map from an action to a state, $\mathbf{T}_g : \mathbf{2}^n \mapsto \mathbf{2}^{m_g}$ as,

$$\mathbf{T}_g \coloneqq (T_g)_{j,k} = g_i \; (X_g)^i{}_{j,k} \tag{2}$$

The response of the other player is then $\Sigma^T y$. This yields the evolution,

$$g' = \mathbf{T}_g(\Sigma^T y) \tag{3}$$