Latihan Gauss-Jordan

Sunday, September 10, 2023 4:09 PM

1. Selesaikan SPL berikut dengan metode eliminasi Gauss-Jordan

(a)
$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\quad - 3w &= -3 \end{aligned}$$

(b) SPL dalam bentuk matriks augmented

$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

(a)
$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix} \xrightarrow[R_4 - 3R_1]{R_2 - 2R_1,\ R_3 + R_1} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[R_2 - 3R_2]{R_4 - 3R_2,\ R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solusinya:

$x = s - 1$
$y = 2r$
$z = r$
$w = s$
$r, s \in R$

$y - 2z = 0$
$y = 2z = 2r$

$x - w = -1$
$x = w - 1 = s - 1$

$z = r$, $w = s$

(b)
$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 2 & 4 & -5 & 6 & -5 & -1 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \end{bmatrix} \xrightarrow[R_2 - R_1]{R_1 / 2} \begin{bmatrix} 1 & 2 & -5/2 & 3 & -5/2 & -1/2 \\ 0 & 0 & -5 & 0 & 17 & 29 \\ 0 & 0 & -2 & 0 & 7 & 12 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3 \cdot -1/2} \begin{bmatrix} 1 & 2 & -5/2 & 3 & -5/2 & -1/2 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & -5 & 0 & 17 & 29 \end{bmatrix} \xrightarrow[R_3 + 5R_1]{R_1 + 5/2 R_2} \begin{bmatrix} 1 & 2 & 0 & 3 & -45/4 & -31/2 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & -1/2 & -1 \end{bmatrix} \xrightarrow{R_4 (-2)} \begin{bmatrix} 1 & 2 & 0 & 3 & -45/4 & -31/2 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow[R_2 + 7/2 R_3]{R_1 + 45/4 R_3} \begin{bmatrix} 1 & 2 & 0 & 3 & 0 & 7 \\ 0 & 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad b$

$x_5 = 2$
$x_3 = 8$
$x_1 + 2x_2 + 3x_4 = 7$
$x_1 = 7 - 2x_2 - 3x_4$
$= 7 - 2r - 3s$
$x_2 = r$
$x_4 = s$

Solusi:
$x_1 = 7 - 2r - 3s$
$x_2 = r$
$x_3 = 8$
$x_4 = s$
$x_5 = 2$
$r, s \in R$

2. Tentukan balikan matriks berikut (jika ada)

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 1 & 0 \\ 2 & 3 & -2 & 6 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$

Catatan: k1, k2, k3, dan k4 tidak sama dengan nol

(a) $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 \end{array}\right]$

$\downarrow R_3/2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right]$

$\xleftarrow[R_2 + R_1]{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right]$

$\downarrow$

A^{-1}

$A \cdot A^{-1} = I$

$[A|I] = [I|A^{-1}]$

$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \end{bmatrix}$ //

(b) $\begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & -5 \end{bmatrix}$

$\left[\begin{array}{cccc|cccc} 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1/2} \left[\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$

$\downarrow R_2 - R_1$

$\left[\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 4 & 12 & 0 & -1/2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow -R_4} \left[\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 4 & 12 & 0 & -1/2 & 1 & 0 & 0 \end{array}\right]$

$\downarrow R_1 + 2R_2$
$R_4 - 4R_2$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 8 & 10 & 1/2 & 0 & 0 & -2 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & -20 & -1/2 & 1 & 0 & 4 \end{array}\right] \xrightarrow{R_3/2} \left[\begin{array}{cccc|cccc} 1 & 0 & 8 & 10 & 1/2 & 0 & 0 & -2 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & -4 & -20 & -1/2 & 1 & 0 & 4 \end{array}\right]$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & -20 & -1/2 & 1 & 0 & 4 \end{array}\right] \quad \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & -4 & -20 & -1/2 & 1 & 0 & 4 \end{array}\right]$$

$R_1 - 0R_3$, $R_2 - 4R_3$, $R_4 + 4R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 10 & 1/2 & 0 & -4 & -2 \\ 0 & 1 & 0 & 5 & 0 & 0 & -2 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -20 & -1/2 & 1 & 2 & 4 \end{array}\right]$$

$R_4 \times (-20)$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 10 & 1/2 & 0 & -4 & -2 \\ 0 & 1 & 0 & 5 & 0 & 0 & -2 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$R_1 - 10R_4$, $R_2 - 5R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/2 & -3 & 0 \\ 0 & 1 & 0 & 0 & -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

Solusi

$$A^{-1} = \begin{bmatrix} 1/4 & 1/2 & -3 & 0 \\ -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 1/40 & -1/20 & -1/10 & -1/5 \end{bmatrix}$$