Latihan Gauss-Jordan

Sunday, September 10, 2023 3:17 PM

Latihan

Selesaikan SPL berikut dengan metode eliminasi Gauss-Jordan

(a) $3x_1 + x_2 + x_3 + x_4 = 0$
$5x_1 - x_2 + x_3 - x_4 = 0$

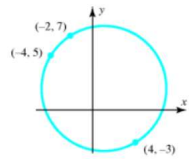
(b) $2I_1 - I_2 + 3I_3 + 4I_4 = 9$
$I_1 - 2I_3 + 7I_4 = 11$
$3I_1 - 3I_2 + I_3 + 5I_4 = 8$
$2I_1 + I_2 + 4I_3 + 4I_4 = 10$

(c) $x - y + 2z - w = -1$
$2x + y - 2z - 2w = -2$
$-x + 2y - 4z + w = 1$
$3x - 3w = -3$

(d) SPL dalam bentuk matriks augmented

$$\begin{bmatrix} 3 & -1 & 0 & 4 & 1 \\ 2 & 0 & 3 & 3 & -1 \end{bmatrix}$$

(e) Carilah koefisien a, b, c, dan d yang memenuhi persamaan lingkaran $ax^2 + ay^2 + bx + cy + d = 0$



(a) $\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{R_1/3} \begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{R_2 - 5R_1} \begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & -8/3 & 2/3 & -8/3 & 0 \end{bmatrix}$

$\downarrow R_2 \times -3/8$

Matriks Eselon Baris $\longrightarrow \begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1 & -1/4 & 1 & 0 \end{bmatrix}$

$* \; X_2 - 1/4 X_3 + X_4 = 0$
$X_2 + X_4 = 1/4 X_3$
$X_3 = 4X_2 + 4X_4$

$* \; X_1 + 1/3 X_2 + 1/3 X_3 + 1/4 X_4 = 0$
$X_1 = -1/3 X_2 - 1/3 (4X_2 + 4X_4) - 1/4 X_4$

$X_2 = r \quad X_4 = s \quad X_3 = 4r + 4s$

$X_1 = -5/3 X_2 - 19/12 X_4 = -5/3 r - 19/12 s$

Solusinya : $X_1 = -5/3 r - 19/12 s$, $X_2 = r$, $X_3 = 4r + 4s$, $X_4 = s$
$r, s \in R$

(b)

$$\begin{bmatrix} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix} \xrightarrow[R_3 - 3R_1 \; R_4 - 2R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -11 \end{bmatrix}$$

$\downarrow R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -11 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & -1 & 7 & -10 & -13 \end{bmatrix} \xrightarrow[R_4 + R_2]{R_3 + 3R_2} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -11 \\ 0 & 0 & 31 & -46 & -58 \\ 0 & 0 & 15 & -20 & -24 \end{bmatrix} \xrightarrow{R_3 - 2R_4} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -11 \\ 0 & 0 & 1 & -6 & -10 \\ 0 & 0 & 15 & -20 & -24 \end{bmatrix}$$

$| \; R_4 - 15R_3$

$\begin{bmatrix} 0 & 0 & 15 & -20 & -24 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 15 & -20 & -24 \end{bmatrix}$ $\begin{bmatrix} 0 & -1 & 7 & -10 & -13 \end{bmatrix}$

$\downarrow R_4 - 15R_3$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -11 \\ 0 & 0 & \underline{1} & -6 & -10 \\ 0 & 0 & 0 & 70 & 126 \end{bmatrix} \xrightarrow{R_4/70} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -11 \\ 0 & 0 & \underline{1} & -6 & -10 \\ 0 & 0 & 0 & \underline{1} & 9/5 \end{bmatrix} \longrightarrow$$ Matriks Eselon Baris

* $X_4 = 9/5$
* $X_3 - 6X_4 = -10$
  $X_3 = -10 + 6(9/5)$
  $X_3 = 4/5$

* $X_2 + 8X_3 - 10X_4 = -11$
  $X_2 = -11 - 8(4/5) + 10(9/5)$
  $= -11 - 32/5 + 18$
  $= 7 - 32/5$
  $X_2 = 3/5$

* $X_1 - 2X_3 + 7X_4 = 11$
  $X_1 = 11 + 2(4/5) - 7(9/5)$
  $= 11 + 8/5 - 63/5$
  $= \frac{55 + 8 - 63}{5}$
  $= 0$

Solusinya : $X_1 = 0$, $X_2 = 3/5$, $X_3 = 4/5$, $X_4 = 9/5$