PR 2

Raden Francisco Trianto B.
13522091

3.11) Diketahui : 7 set TV, terdiri dari 5 benar, 2 rusak. Diambil 3 set.

cara pengambilan 3 dari 7 set $= \binom{7}{3} = 35$ cara

Sehingga peluang distributif x

$$f(x) = \frac{\binom{2}{x}\binom{5}{3-x}}{35}, \quad x = 0, 1, 2$$

$$f(0) = \frac{\binom{2}{0}\binom{5}{3}}{35} = \frac{2}{7}$$

$$f(1) = \frac{\binom{2}{1}\binom{5}{2}}{35} = \frac{4}{7}$$

$$f(2) = \frac{\binom{2}{2}\binom{5}{1}}{35} = \frac{1}{7}$$

3.27) Diketahui : $f(x) \begin{cases} \frac{1}{2000} \exp(-x/2000), & x \geq 0 \\ 0, & x < 0 \end{cases}$

a. Untuk $x \geq 0$, $F(x) = \int_0^x \frac{1}{2000} \exp(-t/2000)\, dt$

$$= -\exp(-t/2000) \Big|_0^x$$

$$= 1 - \exp(-t/2000)$$

Sehingga $F(x) = \begin{cases} 0, & x < 0 \\ 1 - \exp(-t/2000), & x \geq 0 \end{cases}$

b. $X > 1000$

$$P(X > 1000) = 1 - F(1000)$$
$$= 1 - \left[1 - \exp\left(\frac{-1000}{2000}\right)\right]$$
$$= 0{,}6065$$

c. $X < 2000$

$$P(X < 2000) = F(2000)$$
$$= 1 - \exp\left(\frac{-2000}{2000}\right)$$
$$= 0{,}6321$$

3.49) Diketahui : X, jumlah kegagalan mesin (1/2/3)
Y, jumlah pemanggilan teknisi

a. $g(x) = \sum_{y} f(x, y)$

$$g(1) = f(1,1) + f(1,3) + f(1,5)$$
$$= 0{,}05 + 0{,}05 + 0{,}00$$
$$= 0{,}10$$

$$g(2) = f(2,1) + f(2,3) + f(2,5)$$
$$= 0{,}05 + 0{,}10 + 0{,}20$$
$$= 0{,}35$$

$$g(3) = f(3,1) + f(3,3) + f(3,5)$$
$$= 0{,}10 + 0{,}35 + 0{,}10$$
$$= 0{,}55$$

b. $h(y) = \sum_{x} f(x, y)$

$$h(1) = f(1,1) + f(2,1) + f(3,1)$$
$$= 0{,}05 + 0{,}05 + 0{,}10$$
$$= 0{,}20$$

$$h(3) = f(1,3) + f(2,3) + f(3,3)$$
$$= 0{,}05 + 0{,}10 + 0{,}35$$
$$= 0{,}50$$

$$h(5) = f(1,5) + f(2,5) + f(3,5)$$
$$= 0{,}00 + 0{,}20 + 0{,}10$$
$$= 0{,}30$$

c. $P(Y=3 \mid X=2) = \dfrac{f(2,3)}{g(2)} =$

$= \dfrac{0{,}1}{0{,}05 + 0{,}10 + 0{,}20}$

$= 0{,}2857$

4.51)

$x \backslash y$	0	1	2	$f.(x)$
0	0	$\frac{1}{35}$	$\frac{3}{70}$	$\frac{1}{14}$
1	$\frac{3}{70}$	$\frac{9}{35}$	$\frac{9}{70}$	$\frac{3}{7}$
2	$\frac{9}{70}$	$\frac{9}{35}$	$\frac{3}{70}$	$\frac{3}{7}$
3	$\frac{3}{70}$	$\frac{1}{35}$	0	$\frac{1}{14}$
$f_1(x)$	$\frac{3}{14}$	$\frac{4}{7}$	$\frac{3}{14}$	1

$E(x) = \sum_x x\, f(x)$

$= 0 f(0) + 1 . f_1(1) + 2 f_1(2) + 3 . f(3)$

$= \frac{3}{7} + \frac{6}{7} + \frac{3}{14}$

$= \frac{3}{2}$

$E(Y) = \sum_Y Y f(Y)$

$= 0 . \frac{3}{14} + 1 . \frac{4}{7} + 2 . \frac{3}{14}$

$= \frac{4}{7} + \frac{6}{14}$

$= 1$

$E(XY) = \sum_{x=0}^{3} \sum_{y=0}^{2} x . y . f(x,y) = \frac{9}{7}$

Koefisien Kolerasi $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ $\quad \sigma_X = E(X^2) - \mu_X \quad \sigma_Y = E(Y^2) - \mu_Y$

$$E(X^2) = \sum_X x^2 f(x)$$
$$= 1 \cdot 9/7 + 4 \cdot 3/7 + 9 \cdot 1/14$$
$$= \frac{39}{14}$$

$$E(Y^2) = \sum_Y y^2 f(y)$$
$$= 1 \cdot 4/7 + 4 \cdot 3/14 -$$
$$= \frac{20}{14}$$

$$\sigma(x) = \sqrt{\frac{15}{28}}$$

$$\sigma(y) = \sqrt{\frac{3}{7}}$$

$$\sigma_{XY} = E(XY) - \sigma_X \sigma_Y$$
$$= 9/7 - \left(\sqrt{\frac{15}{28}}\right)\left(\sqrt{\frac{3}{7}}\right)$$
$$= 9/7 - \frac{3}{14}\sqrt{5}$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = -\frac{1}{\sqrt{5}} \simeq -0{,}44721$$

4.67) $E[g(x,y)] = E(x/y^3 + x^2 . y)$

$= E(x/y^3) + E(x^2 y)$

$E(x/y^3) = \int_1^2 \int_0^1 \frac{2x(x+2y)}{7y^3} \, dx \, dy$

$= \frac{2}{7} \int_0^1 \left(\frac{1}{3y^3} + \frac{1}{y^2} \right) dy$

$= \frac{15}{84}$

$E(x^2 y) = \int_1^2 \int_0^1 \frac{2x^2 y (x+2y)}{7} \, dx \, dy$

$= \frac{2}{7} \int_0^1 y \left(\frac{1}{4} + \frac{2y}{3} \right) dy$

$= \frac{139}{252}$

$E[g(x,y)] = \frac{15}{84} + \frac{139}{252}$

$= \frac{46}{63}$

4.75) $\mu = 900$

$\sigma = 50$

$\mu - k\sigma = 700$

$900 - k.50 = 700$

$k = 4$

Teorema Chebyshev :

$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

$P(\mu - 4\sigma < X < \mu + 4\sigma) \geq 1 - \frac{1}{16}$

$P(700 < X < 1100) \geq 0,9375$

Sehingga $P(X \leq 700) \leq 0,03125$