

10.17 A new curing process developed for a certain type of cement results in a mean compressive strength of 5000 kilograms per square centimeter with a standard deviation of 120 kilograms. To test the hypothesis that $\mu = 5000$ against the alternative that $\mu < 5000$, a random sample of 50 pieces of cement is tested. The critical region is defined to be $\bar{x} < 4970$.

- (a) Find the probability of committing a type I error when H_0 is true.
 (b) Evaluate β for the alternatives $\mu = 4970$ and $\mu = 4960$.

① $n = 50$
 $\mu = 5000$
 $\sigma = 120$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

$$= \frac{4970 - 5000}{120/\sqrt{50}}$$

$$= -1,77$$

$$P = P(Z < -1,77) = 0,0384$$

② jika $\mu = 4970$

$$Z = \frac{4970 - 4970}{120/\sqrt{50}}$$

$$= 0$$

$$\beta = P(Z > 0)$$

$$= 0,5$$

jika $\mu = 4960$

$$Z = \frac{4970 - 4960}{120/\sqrt{50}}$$

$$= 0,59$$

$$\beta = P(Z > 0,59)$$

$$= 0,2776$$

10.21 An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a P -value in your answer.

$$H_0: \mu = 800$$

$$H_1: \mu \neq 800$$

$$\sigma = 40$$

$$n = 30$$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{788 - 800}{40/\sqrt{30}}$$

$$= -1,64$$

$$P_{\text{value}} = 2P(Z = 1,64)$$

$$= 2 \cdot (0,0505)$$

$$= 0,101$$

Sehingga rata-rata tidak jauh berbeda dari 800 untuk $\alpha < 0,101$

10.41 A study was conducted by the Department of Zoology at Virginia Tech to determine if there is a significant difference in the density of organisms at two different stations located on Cedar Run, a secondary stream in the Roanoke River drainage basin. Sewage from a sewage treatment plant and overflow from the Federal Mogul Corporation settling pond enter the stream near its headwaters. The following data give the density measurements, in number of organisms per square meter, at the two collecting stations:

Number of Organisms per Square Meter			
Station 1		Station 2	
5030	4980	2800	2810
13,700	11,910	4670	1330
10,730	8130	6890	3320
11,400	26,850	7720	1230
860	17,660	7030	2130
2200	22,800	7330	2190
4250	1130		
15,040	1690		

Can we conclude, at the 0.05 level of significance, that the average densities at the two stations are equal? Assume that the observations come from normal populations with different variances.

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

$$\alpha = 0,05$$

$$s^2 = \frac{\left(\frac{7874 \cdot 329^2}{16} + \frac{2479 \cdot 503^2}{12} \right)^2}{\frac{\left(\frac{7874 \cdot 329^2}{16} \right)^2}{15} + \frac{\left(\frac{2479 \cdot 503^2}{12} \right)^2}{11}}$$

$$= 19$$

degree of freedom = 19

Critical regions $t < -2,093$ atau $t > 2,093$

$$t = \frac{9997.500 - 4120833}{\sqrt{7874 \cdot 329^2 / 16 + 2479 \cdot 503^2 / 12}}$$

$$= 2,76$$

Hasil hipotesa yang benar adalah H_1
dengan $\mu_1 \neq \mu_2$

10.61 In a winter of an epidemic flu, the parents of 2000 babies were surveyed by researchers at a well-known pharmaceutical company to determine if the company's new medicine was effective after two days. Among 120 babies who had the flu and were given the medicine, 29 were cured within two days. Among 280 babies who had the flu but were not given the medicine, 56 recovered within two days. Is there any significant indication that supports the company's claim of the effectiveness of the medicine?

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2$$

$$\hat{p} = \frac{29 + 56}{120 + 280}$$

$$= 0,2125$$

$$Z = \frac{\frac{29}{120} - \frac{56}{280}}{\sqrt{(0,2125)(0,7875)\left(\frac{1}{120} + \frac{1}{280}\right)}}$$

$$= 0,93$$

$$p = P(Z > 0,93)$$

$$= 0,1762$$

Sehingga H_0 ditolak, tidak ada bukti signifikan yang menunjukkan obat lebih efektif

10.77 An experiment was conducted to compare the alcohol content of soy sauce on two different production lines. Production was monitored eight times a day. The data are shown here.

Production line 1:

0.48 0.39 0.42 0.52 0.40 0.48 0.52 0.52

Production line 2:

0.38 0.37 0.39 0.41 0.38 0.39 0.40 0.39

Assume both populations are normal. It is suspected that production line 1 is not producing as consistently as production line 2 in terms of alcohol content. Test the hypothesis that $\sigma_1 = \sigma_2$ against the alternative that $\sigma_1 \neq \sigma_2$. Use a P -value.

$$H_0 : \sigma_1 = \sigma_2$$

$$H_1 : \neq$$

$$H_0 : \sigma_1 = \sigma_2$$

$$H_1 : \sigma_1 \neq \sigma_2$$

$$f = \frac{0,0533^2}{0,0125^2}$$

$$= 19,67$$

$$P_{\text{value}} = 2 P(f > 19,67)$$

$$= 2 (0,0004)$$

$$= 0,0008$$

Sehingga production line 1 tidak sekonstan
production line 2

10.93 To determine current attitudes about prayer in public schools, a survey was conducted in four Virginia counties. The following table gives the attitudes of 200 parents from Craig County, 150 parents from Giles County, 100 parents from Franklin County, and 100 parents from Montgomery County:

Attitude	County			
	Craig	Giles	Franklin	Mont.
Favor	65	66	40	34
Oppose	42	30	33	42
No opinion	93	54	27	24

Test for homogeneity of attitudes among the four counties concerning prayer in the public schools. Use a P -value in your conclusion.

H_0 : homogeneous

H_1 : tidak homogenous

Attitude	County				total
	Craig	Giles	Franklin	Mont.	
Favor	65	66	40	34	205
Oppose	42	30	33	42	147
No opinion	93	54	27	24	198
total	200	150	100	100	550

$$\chi^2 = \frac{(65 - 74,5)^2}{74,5} + \frac{(66 - 55,9)^2}{55,9} + \dots + \frac{(24 - 36)^2}{36}$$

$$= 31,17$$

$P_{\text{value}} = P(\chi^2 > 31,17) < 0,001$
dengan derajat kebebasan 6

Sehingga H_0 gagal dan
attitude tidak homogenous