PR 4

Raden Francisco Trianto B.
13522091

8.25 The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

(a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;

Diketahui : $\mu = 7$ years
$\sigma = 1$ years

(a) $n = 9$

$P(6{,}4 < \bar{X} < 7{,}2) = P(-1{,}8 < Z < 0{,}6) = 0{,}6898$

$z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{6{,}4 - 7}{1/\sqrt{9}}$

$z_1 = -1{,}8$

$z_2 = \frac{\bar{x}_2 - \mu}{\sigma/\sqrt{n}} = \frac{7{,}2 - 7}{1/\sqrt{9}}$

$z_2 = 0{,}6$

Peluang adalah 0,6898

(b) the value of x to the right of which 15% of the means computed from random samples of size 9 would fall.

(b) $n = 9$

$P = 0{,}15 \approx P(Z < 1{,}04)$

$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$1{,}04 = \frac{\bar{X} - 7}{1/\sqrt{9}}$

$\bar{X} = 7{,}35$

8.31 Consider Case Study 8.2 on page 238. Suppose 18 specimens were used for each type of paint in an experiment and $\bar{x}_A - \bar{x}_B$, the actual difference in mean drying time, turned out to be 1.0.

(a) Does this seem to be a reasonable result if the two population mean drying times truly are equal?

two population mean drying times truly are equal? Make use of the result in the solution to Case Study 8.2.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)}}$$

$$z = \frac{1 - (\mu - \mu)}{\sqrt{\left(\frac{\sigma^2}{18}\right) + \left(\frac{\sigma^2}{18}\right)}}$$

$$z = 3$$

$$P(Z > 3) = 0,0013$$

Jika dua mean populasi drying times sama, maka peluang bahwa perbedaan antara mean dua sampel bernilai 1 adalah 0.0013. Sehingga asumsi kedua mean populasi adalah sama sangat tidak mungkin.

(b) If someone did the experiment 10,000 times under the condition that $\mu_A = \mu_B$, in how many of those 10,000 experiments would there be a difference $\bar{x}_A - \bar{x}_B$ that was as large as (or larger than) 1.0?

$$\text{jumlah selisih} > 1 = \underset{n}{(10.000)} \cdot \underset{p}{(0,0013)}$$

$$= 13 \text{ eksperimen}$$

8.49 A normal population with unknown variance has a mean of 20. Is one likely to obtain a random sample of size 9 from this population with a mean of 24 and a standard deviation of 4.1? If not, what conclusion would you draw?

$\mu = 20$

$n = 9$

$\bar{x} = 24$

$s = 4,1$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{24 - 20}{4,1/\sqrt{9}} = 2,927$$

degree of freedom $= n - 1 = 9 - 1 = 8$

tidak, $\mu > 20$

tidak, $\mu > 20$

9.27 Consider the situation of Case Study 9.1 on page 281 with a larger sample of metal pieces. The diameters are as follows: 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 1.01, 1.03, 0.99, 1.00, 1.00, 0.99, 0.98, 1.01, 1.02, 0.99 centimeters. Once again the normality assumption may be made. Do the following and compare your results to those of the case study. Discuss how they are different and why.

(a) Compute a 99% confidence interval on the mean diameter.

Diketahui: $n = 16$

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$= \frac{1{,}01 + 0{,}97 + 1{,}03 + 1{,}04 + 0{,}99 + 0{,}98 + 1{,}01 + 1{,}03 + 0{,}99 + 1 + 1 + 0{,}99 + 0{,}98 + 1{,}01 + 1{,}02 + 0{,}99}{16}$$

$$\bar{X} = 1{,}0025$$

$$S = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}}$$

$$= 0{,}0202$$

degree of freedom $= n - 1$

$= 16 - 1$

$= 15$

$1 - \alpha = 0{,}99$

$\alpha = 0{,}01$

$t_{0{,}005} = 2{,}947$

Interval: $\left(\bar{x} - t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}\right)$

$$1.0025 - 2.947 \times \frac{0{,}0202}{\sqrt{16}} < \mu < 1{,}0025 + 2.947 \, \frac{0{,}0202}{\sqrt{16}}$$

$\Rightarrow 0{,}9876 < \mu < 1{,}0174$

(b) Compute a 99% prediction interval on the next diameter to be measured.

$$\bar{X} - t_{a/2} \cdot S \cdot \sqrt{1 + \frac{1}{n}} < X_0 < \bar{X} + t_{a/2} \cdot S \cdot \sqrt{1 + \frac{1}{n}}$$

$$1{,}0025 - 2{,}947 \cdot 0{,}0202 \cdot \sqrt{1 + \frac{1}{16}} < X_0 < 1{,}0025 + 2{,}947 \cdot 0{,}0202 \cdot \sqrt{1 + \frac{1}{16}}$$

$0{,}9411 < X_0 < 1{,}0639$

(c) Compute a 99% tolerance interval for coverage of the central 95% of the distribution of diameters.

$1-\gamma = 0,99$ $\quad$ $1-\alpha = 0,95$

$\gamma = 0,01$ $\quad$ $\alpha = 0,05$

$k = 3,421$

$\bar{x} - (k \cdot s) < x < \bar{x} + (k \cdot s)$

$1,0025 - (3,421 \cdot 0,0202) < x < 1,0025 + (3,421 \cdot 0,0202)$

$0,9334 < x < 1,0716$

9.49 Two different brands of latex paint are being considered for use. Fifteen specimens of each type of paint were selected, and the drying times, in hours, were as follows:

Paint A					Paint B				
3.5	2.7	3.9	4.2	3.6	4.7	3.9	4.5	5.5	4.0
2.7	3.3	5.2	4.2	2.9	5.3	4.3	6.0	5.2	3.7
4.4	5.2	4.0	4.1	3.4	5.5	6.2	5.1	5.4	4.8

Assume the drying time is normally distributed with $\sigma_A = \sigma_B$. Find a 95% confidence interval on $\mu_B - \mu_A$, where μ_A and μ_B are the mean drying times.

$n_A = 15$ $\quad$ $n_B = 15$ $\quad$ $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\quad$ $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$

$\bar{x}_A = 3,82$ $\quad$ $\bar{x}_B = 4,94$

$s_A = 0,7794$ $\quad$ $s_B = 0,7538$ $\quad$ $v = v_1 + v_2$

$= 15-1 + 15-1$

$s_p = 0,7667$ $\quad$ $t_{0,025} = 2,048$ $\quad$ $v = 28$

$(4,94 - 3,82) - (2,048)(0,7667)\sqrt{\frac{1}{15} + \frac{1}{15}} < \mu_B - \mu_A < (4,94 - 3,82) - (2,048)(0,7667)\sqrt{\frac{1}{15} + \frac{1}{15}}$

$1,12 - 0,57 < \mu_B - \mu_A < 1,12 + 0,57$

$0,55 < \mu_B - \mu_A < 1,69$

9.53 (a) A random sample of 200 voters in a town is selected, and 114 are found to support an annexation suit. Find the 96% confidence interval for the fraction of the voting population favoring the suit.

Diketahui : $n = 200$
$X = 114$

Proportion $\hat{p} = \frac{X}{n} = \frac{114}{200} = 0,57$

$\hat{q} = 1 - \hat{p} = 1 - 0,57 = 0,43$

$1 - \alpha = 0,96$
$\alpha = 0,04$
$\frac{\alpha}{2} = 0,02$

$t_{0,02} = 2,055$

Interval $\rightarrow = \hat{p} \pm t_{\alpha/2} \sqrt{\frac{\hat{p}.\hat{q}}{n}}$

$= 0,57 \pm 2,055 . \sqrt{\frac{0,57 . 0,43}{200}}$

$= 0,57 \pm 0,0719$

$= [0,4981 , 0,6419]$

$\Rightarrow 0,4981 < p < 0,6419$

(b) What can we assert with 96% confidence about the possible size of our error if we estimate the fraction of voters favoring the annexation suit to be 0.57?

$\hat{p} = 0,57$ $\quad \hat{q} = 1 - 0,57 = 0,43$

$E \leq t_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}.\hat{q}}{n}}$

$= t_{0,02} \sqrt{\frac{0,57 . 0,43}{200}}$

$= 0,0719$

$E \leq 0{,}0719$