Introduction to Logic

Logical Sentences

Colorful Block World:

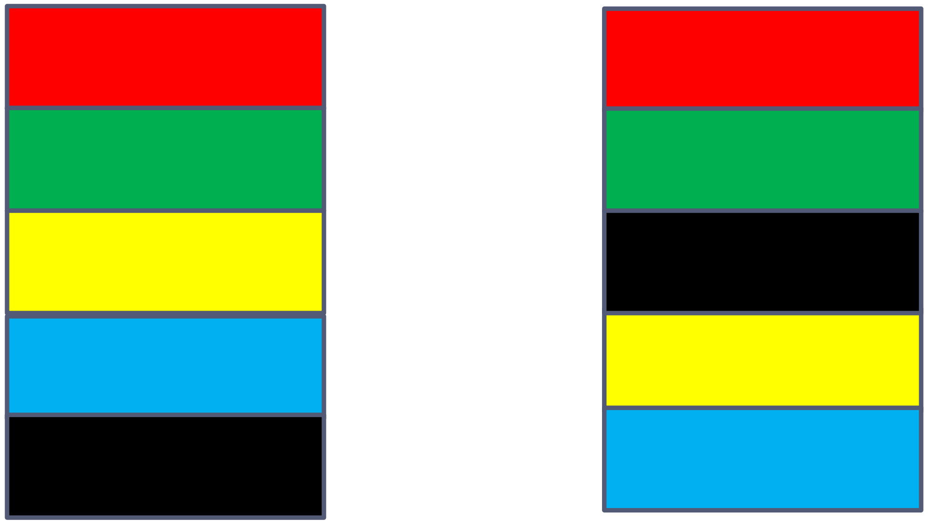
- A block cannot be two colors at once
- There can be only one block on another
- If there are 5 blocks with different color → there are 120 possibility arrangements.
- If we can not see the arrangement, and have others to inform some partial information, can we decide the exact block arrangement?

Logical Sentences (2)

- Here where logic comes in!!!
  - Each informant writes logic sentence, exactly what they know
  - Using logic theory we combine the 'partial' information
  - Draw logical conclusion, including some that may not be known to any one of the informants
- Example for Block World
- The red block is on the green block.
- The green block is somewhere **above** the blue block.
- The green block is **not** on the blue block.
- The yellow block is on the green block **or** the blue block.
- There is **some** blocks on the black block.

Logical Entailment

- Ideally, when we have enough sentences, we know exactly how things stand → but not always the case
- Set of logical sentences doesn't determine a unique world; but some sentences are true in every world that satisfies the given sentences → Logical Conclusion
- Example of Logical Conclusion from Logical Sentences of Block World:



Logical Proof

- Logical Conclusion from all possible sets is impractical
- Use **Logical Reasoning:** the application of reasoning rules to derive logical conclusions and produce a *logical proofs*
- ***Logical Proofs*** sequences of reasoning steps that leads from premises to conclusions
- Example:

Premises		Conclusion
• **The red block is on the green block.** • The yellow block is on the green block **or** the blue block.	⇒	• **The yellow block is on the blue block.**

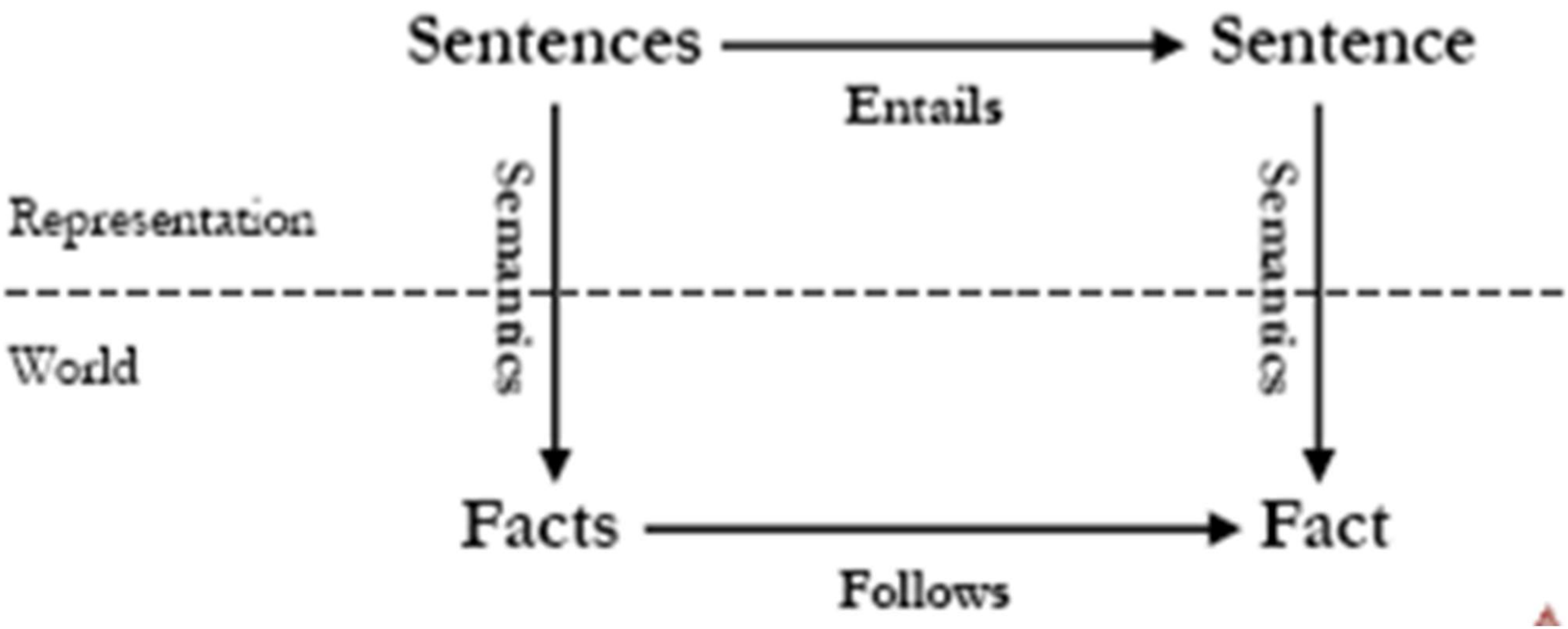
Proof: **The yellow block is on the blue block.**

1. *A block cannot be two colors at once*
2. *There can be only one block on another*
3. The red block is on the green block.
4. The yellow block is on the green block **or** the blue block.
5. (2,3) The yellow block is not on the green block.
6. (4,5) **The yellow block must be on the blue block.**

And so on....

Objectives of Logic Reasoning

- Explicit representation of knowledge
- Draw conclusion from existing knowledge



Human Logic

Information/
Premises

conclusion

reasoning

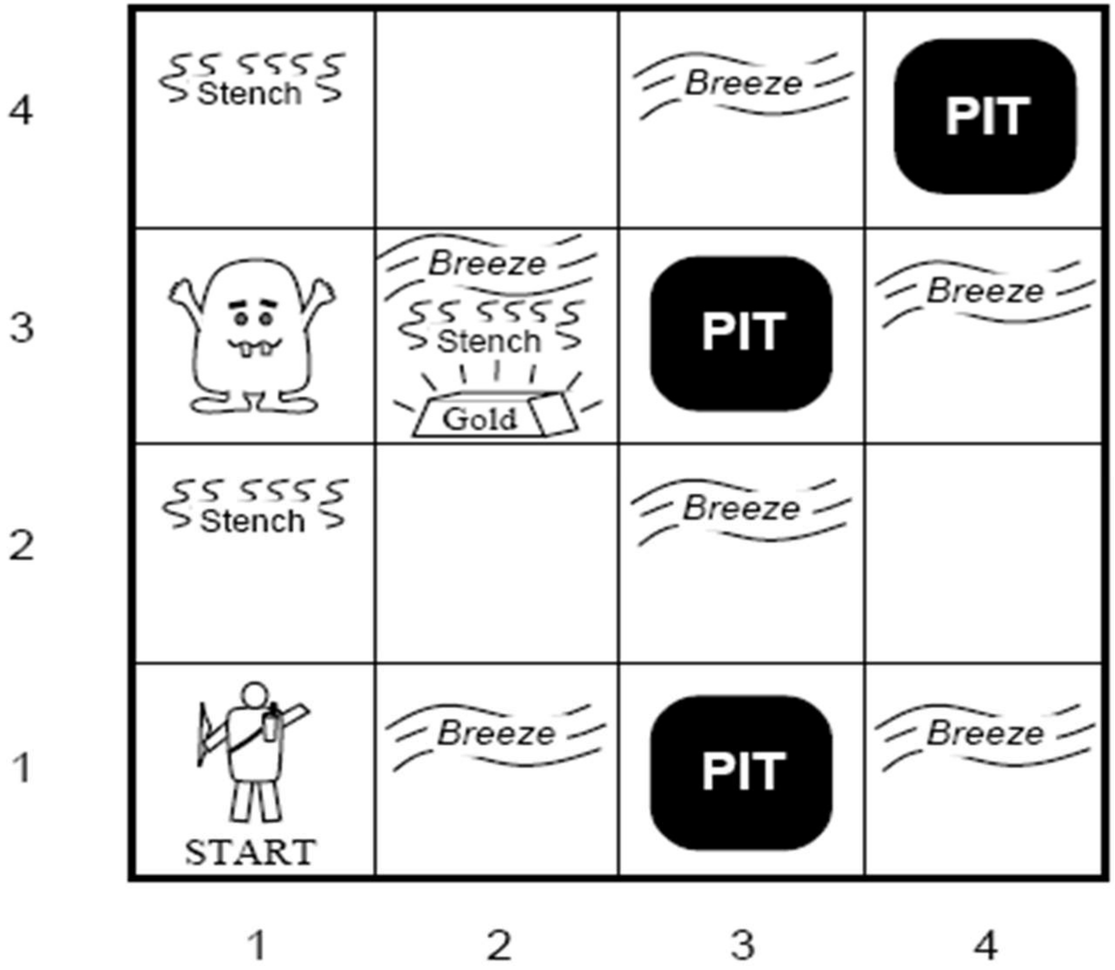
- Human: information processors
- Example: information of 5 blocks in a stack → determine exact arrangement

Proof: **The yellow block is on the blue block.**

1. *A block cannot be two colors at once*
2. *There can be only one block on another*
3. The red block is on the green block.
4. The yellow block is on the green block **or** the blue block.
5. (2,3) The yellow block is not on the green block.
6. (4,5) **The yellow block must be on the blue block.**

And so on....

Other Example: The Wumpus World (1)



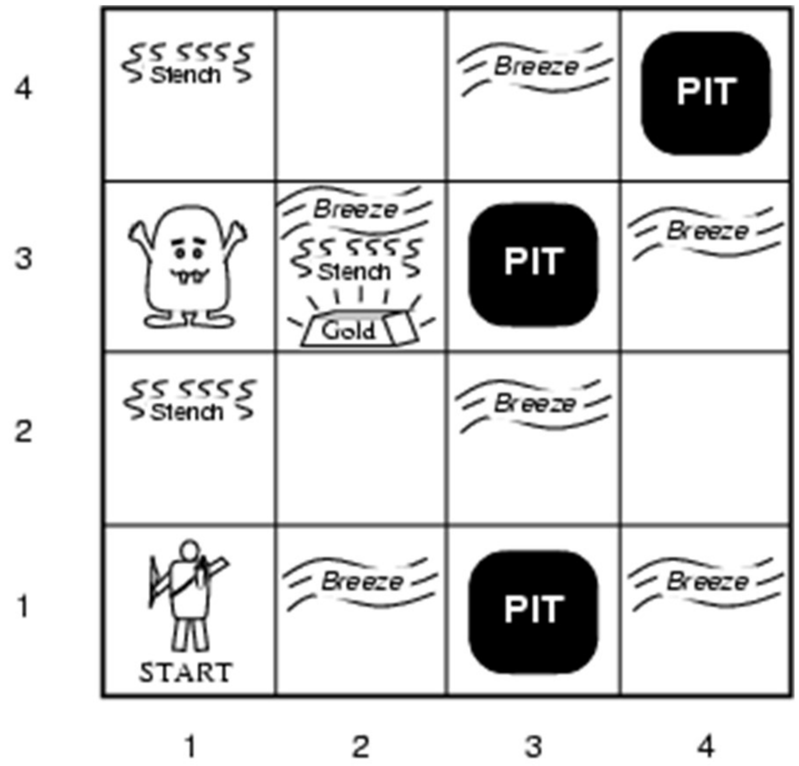
- Cave with rooms
- Wumpus eats anyone who enters its room
- Wumpus can be shot by an agent, but the agent has only one arrow
- Pit will trap anyone, except for the wumpus
- Agent can find gold heap

The Wumpus World (2)

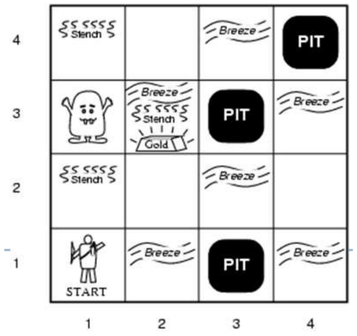
- Sensors: sensor to capture Stench, Breeze, Glitter, Bump, Scream
  Actuators: motor to move Left, Right, Forward, hands to Grab, Release, and Shoot arrow
- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow

The Wumpus World (3)

- Agent doesn't have information about all of the states in Wumpus World. It only has 'basic knowledge/ premises':
  - Squares adjacent to wumpus are smelly
    Squares adjacent to pit are breezy
    Glitter iff gold is in the same square
    Shooting kills wumpus if you are facing it
    Shooting uses up the only arrow
    Grabbing picks up gold if in same square
    Releasing drops the gold in same
- When agent percept a state in a room, it will try to reason new facts/ states, this is how agent will step by step collecting all of the states of wumpus world in order to achieve its goal
- Reasoning has to be done by Agent → by deducting the premises with percepted fact.



The Wumpus World (3)



- Percept [2, 1] : [None, Breeze, None, None, None]
- No stench in [2,1] : No wumpus in [3,1] and [2,2] → deduction from premises and fact
- Breeze in [2,1]: there must be a pit in [3,1] or [2,2] → deduction from premises and fact
- <u>Set action</u>: go back to [1,1] and forward to [1,2]

- Percept [1,2] : [Stench, None, None, None, None]
- Stench in [1,2] : there must be a wumpus in [1,3] or [2,2] or [1,1] → deduction from premises and fact
- No wumpus in [1,1] and No stench in [2,1] → wumpus in [1,3] → deduction from premises, fact that is perceived, and previously deducted facts
- No breeze in [1,2]: No pit in [1,3] and [2,2] → pit in [1,3] and [2,2] OK → deduction from premises, fact that is perceived, and previously deducted facts
- <u>Set action</u>: go to [2,2]

Reasoning by Pattern

- We need to familiar with reasoning “atoms” → able to recognize some reasoning steps as immediately obvious
- Aristotle’s great contribution to philosophy:
  - what makes a step of a proof immediately obvious is its form rather than its content → structure of facts matters!
  - Proofs by patterns (forms) than contents
- This is what we called rule of inference
- Proof by contents: informal proof

Reasoning by Pattern (2)

- Example:

All Accords are Hondas.

All Hondas are Japanese.

Therefore, *all Accords are Japanese*.

All borogoves are slithy toves.

All slithy toves are mimsy.

Therefore, all borogoves are mimsy

- Pattern:
  - All x are y.
  - All y are z.

∴ Therefore, all x are z.

Unsound Pattern

- Pattern
  - All x are y.
  - Some y are z.
  - Therefore, some x are z.

- Good Instance
  - All Toyotas are Japanese cars.
  - Some Japanese cars are made in America.
  - Therefore, some Toyotas are made in America.

- Not-So-Good Instance
  - All Toyotas are cars.
  - Some cars are Porsches.
  - Therefore, some Toyotas are Porsches.

Deduction Reasoning

- All men are mortal.
  Socrates is a man.

  ∴ Therefore, Socrates is mortal.

- If it is raining then the streets are wet.

  It is raining.

  ∴ Therefore, the streets are wet.

→ Truth preserving

Induction

I have seen 1000 black ravens.

I have never seen a raven that is not black.

∴Therefore, every raven is black.

The sun has risen every day so far;

∴ Therefore, the sun will rise tomorrow.

- not always *truth preserving*
- for hypothesis → machine learning

Abduction

If a person has a cold, then he has a runny nose;
Jack has a runny nose;
Therefore Jack has a cold

If there is no fuel, the car will not start.

If there is no spark, the car will not start.

There is spark.

The car will not start.

Therefore, there is no fuel.

- reasoning from effects to causes
- possibility of wrong conclusion
- practical reasoning → diagnosis

Analogy

- The flow in a pipe is proportional to its diameter.
- Wires are like pipes.
- Therefore, the current in a wire is proportional to diameter

We infer a conclusion based on similarity of two situations

Formal Logic

Introduction

- Natural Language works well
- But sentences in natural language can be:
  - complex
  - ambiguous
  - failing to understand the meaning → reasoning errors
- Example

The cherry blossoms in the Spring.

The cherry blossoms in the Spring sank.

There's a girl in the room with a telescope

Reasoning in Natural Language

- Good ('correct') reasoning

  Water is better than tea.

  Tea is better than soda.

  Therefore, water is better than soda.

- How about this?

  Bad grade is better than nothing.

  Nothing is better than good grade.

  Therefore, bad grade is better than good grade.

  → We need Logic

What is Formal Logic

- A formal language
  - Syntax: what expressions are legal
  - Semantics: what legal expressions mean
  - Proof system: a way of manipulating syntactic expressions to get other syntactic expressions (which will tell us something new)

Formal Logic vs Formal Mathematics

- Formal Mathematics (Algebra)
  - Formal language for encoding information
  - Legal transformations
  - Automation
- Logic
  - Formal language for encoding information
  - Legal transformations
  - Automation

Algebra Problem

- *Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?*

$$x - 3y = 0$$
$$x + y = 12$$
$$-4y = -12$$
$$y = 3$$
$$x = 9$$

Logic Problem

- If it is raining, the ground is wet.
  If the ground is wet, it is slippery.
  It is raining.
  Prove that it is slippery.

Formalization

- Simple Sentences:
  - It is raining. p
  - The ground is wet. q
  - It is slippery. r

- Premises:
  - If it is raining, the ground is wet. $p \Rightarrow q$
  - If the ground is wet, it is slippery. $q \Rightarrow r$
  - It is raining. p

Proof

- **Premis:**
  1. $p \Rightarrow q$
  2. $q \Rightarrow r$
  3. p
- **Proof:**
  4. MP 1,3: q
  5. MP 2,4: r

Modus Ponens (MP)

$$\frac{\begin{array}{l}\varphi \Rightarrow \psi \\ \varphi\end{array}}{\psi}$$

Another example of Formal Logic

- *If Mary loves Pat, then Mary loves Quincy. If it is Monday and raining, then Mary loves Pat or Quincy. If it is Monday and raining, does Mary love Quincy?*

- *If it is Monday, does Mary love Pat?*

- *Mary loves only one person at a time. If it is Monday, does Mary love Pat?*

Formalization

- *Simple sentences*

Mary loves Pat.	p
Mary loves Quincy.	q
It is Monday.	m
It is raining	r

- *Premises*

  $p \rightarrow q$

  $m \wedge r \rightarrow p \vee q$

- *Question:*
  - *does Mary love Pat?*
  - *does Mary love Quincy?*

Rule of Inference

- Propositional Resolution
  - If *pi* on the left hand side of one sentence is the same as *qj* in the right hand side of the other sentence, it is okay to drop the two symbols, with the proviso that *only one* such pair may be dropped.
  - If a constant is repeated on the same side of a single sentence, all but one of the occurrences can be deleted.
- Example

$$
\begin{array}{ccc}
p & \Rightarrow & q \\
 & \Rightarrow & p \\
\hline
 & \Rightarrow & q
\end{array}
\qquad\qquad
\begin{array}{ccc}
p & \Rightarrow & q \\
q & \Rightarrow & \\
\hline
p & \Rightarrow &
\end{array}
\qquad\qquad
\begin{array}{ccc}
p & \Rightarrow & q \\
q & \Rightarrow & r \\
\hline
p & \Rightarrow & r
\end{array}
$$

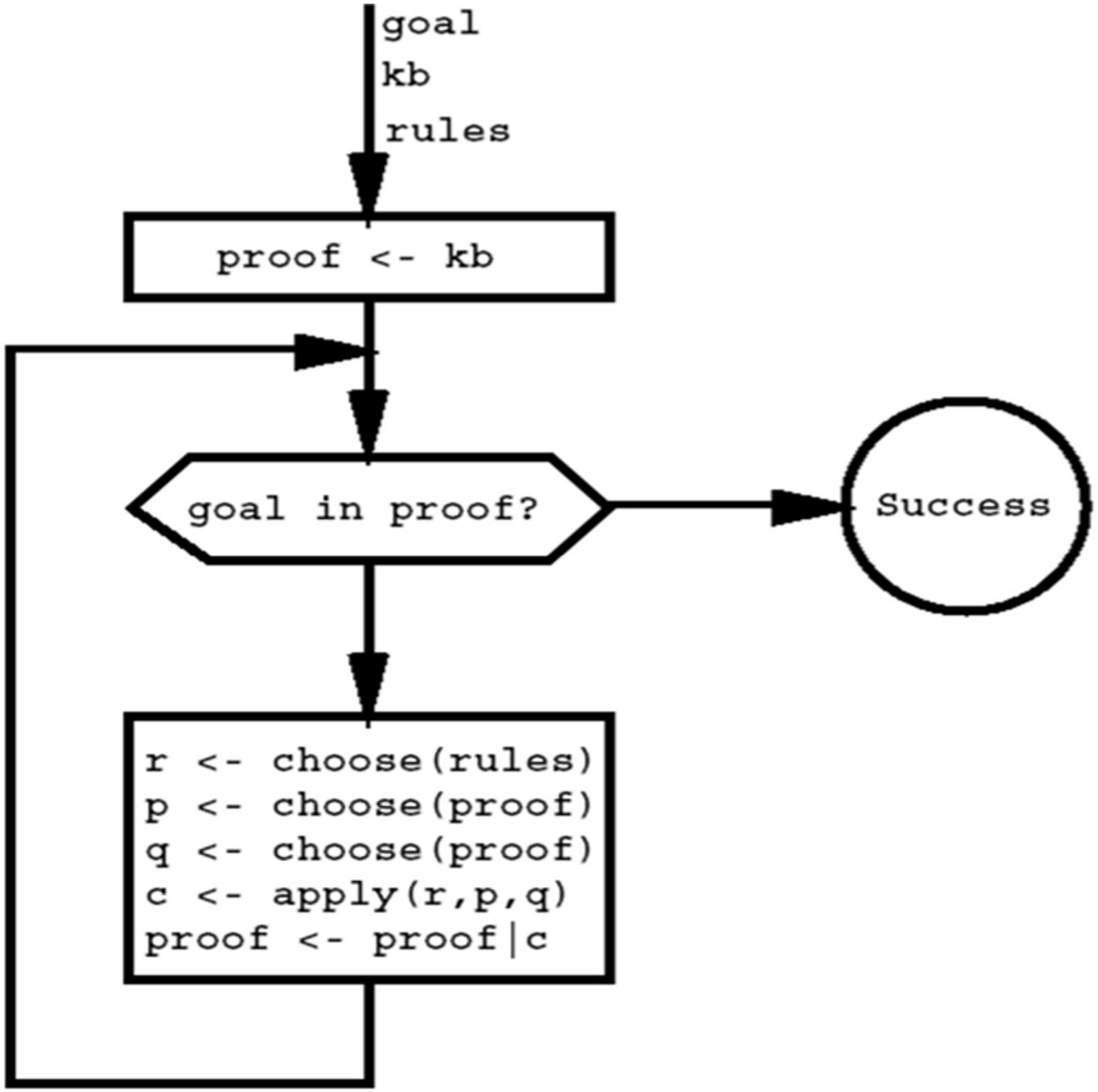
Logic Problem Revisited

- *If Mary loves Pat, then Mary loves Quincy. If it is Monday, then Mary loves Pat or Quincy. If it is Monday, does Mary love Quincy?*

$$
\begin{array}{ccc}
p & \Rightarrow & q \\
m & \Rightarrow & p \vee q \\
\hline
m & \Rightarrow & q \vee q \\
m & \Rightarrow & q
\end{array}
$$

Computational Logic

Automated Reasoning



Formal Logic vs Computational Logic

- Formal Logic
  - Syntax, semantics, correctness and completeness
  - Emphasis on minimal sets of rules to simplify analysis
  - These rules are not always easy to implement or efficient
- Computational Logic
  - Syntax, semantics, correctness, completeness
  - Also concerned with efficiency
  - Emphasis of different languages and different sets of rules
  - Attention to those that better suited to automation

Application of Automated Reasoning

- Mathematics

Group Axioms

$$(x \times y) \times z = x \times (y \times z)$$

$$x \times e = x$$

$$e \times x = x$$

$$x \times x^{-1} = e$$

Theorem

$$x^{-1} \times x = e$$

Tasks:

Proof Checking

Proof Generation

Application of Automated Reasoning (2)

- Deductive Database Systems

parent

art	*bob*
art	*bea*
bea	*coe*

parent(art,bob)
parent(art,bea)
parent(bob,coe)

Queries

```
query(X,Z) :- parent(X,Y) & parent(Y,Z)
```

Constraints

```
illegal :- parent(X,X)
illegal :- parent(X,Y) & parent(Y,X)
```

Application of Automated Reasoning (3)

- Data Integration

name	employee	location	telephone
John		MJH222	7238086
Jane		Cedar12	7257493
Jill	John	MJH222	
Jerry	Jane	420-032	7256777

$$employee(x,y) \Leftarrow manager(y,x)$$

name	manager	office	phone
John	Jill	MJH222	38086
Jane	Jerry	Cedar12	57493
Jill		MJH222	
Jerry		420-032	56777

Application of Automated Reasoning (4)

- Program Verification

Program

Specification:

$$\forall i.\forall j.(i < j \Rightarrow sort(L)_i < sort(L)_j)$$

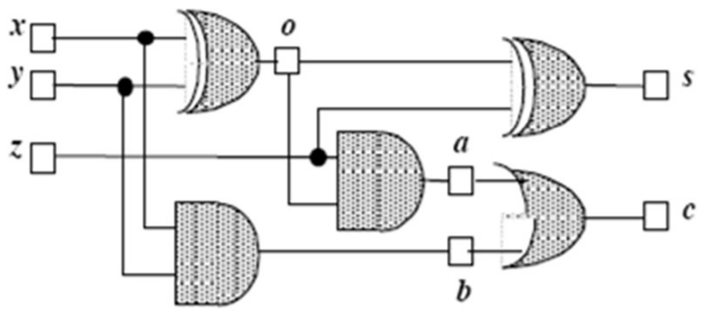
Tasks:

Partial Evaluation
Verification
Proof of Termination
Complexity Analysis

Application of Automated Reasoning (5)

- Hardware Engineering

Circuit:



Premises:

$$o \Leftrightarrow (x \wedge \neg y) \vee (\neg x \wedge y)$$

$$a \Leftrightarrow z \wedge o$$

$$b \Leftrightarrow x \wedge y$$

$$s \Leftrightarrow (o \wedge \neg z) \vee (\neg o \wedge z)$$

$$c \Leftrightarrow a \vee b$$

Applications:

Simulation
Configuration
Diagnosis
Test Generation

Conclusion:

$$x \wedge y \Rightarrow \neg c$$

Application of Automated Reasoning (6)

- Constraint Satisfaction System

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

Application of Automated Reasoning (7)

- Regulations and Business Rules

Using the language of logic, it is possible to define new relations.

Office mates are people who share an office.

```
officemate(X,Y) :- office(X,Z) ∧ office(Y,Z)
```

This includes the property of legality / illegality.

Managers and subordinates may not be office mates.

```
illegal :- manages(X,Y) ∧ officemate(X,Y)
```

Review

- Reasoning: information → conclusion
  - Deduction, Induction, Abduction, Analogy
  - Which one is truth preserving?
- Formal Logic
  - Formal language → syntax, semantics, proof systems
  - Encode information, legal transformation
- Computational Logic → Automated Reasoning
  - Formal language → syntax, semantics, proof systems
  - Encode information, legal transformation, efficiency
  - Propositional logic: proposition, interrelationship*
  - Relational logic: object, interrelationship*

*: in the following courses

THANK YOU