

6.1.1

Exercise 6.1.1: Suppose the PDA $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$ has the following transition function:

↓
Start
State

1. $\delta(q, 0, Z_0) = \{(q, XZ_0)\}.$
2. $\delta(q, 0, X) = \{(q, XX)\}.$
3. $\delta(q, 1, X) = \{(q, X)\}.$
4. $\delta(q, \epsilon, X) = \{(p, \epsilon)\}.$
5. $\delta(p, \epsilon, X) = \{(p, \epsilon)\}.$
6. $\delta(p, 1, X) = \{(p, XX)\}.$
7. $\delta(p, 1, Z_0) = \{(p, \epsilon)\}.$

Starting from the initial ID (q, w, Z_0) , show all the reachable ID's when the input w is:

* a) 01.

$$(q, 01, Z_0) \vdash (q, 1, XZ_0) \vdash (p, 1, Z_0) \vdash (p, \epsilon, \epsilon)$$

b) 0011.

$$(q, 0011, Z_0) \vdash (q, 011, XZ_0) \vdash (q, 11, XXZ_0) \vdash (q, 1, XXZ_0) \vdash (p, 1, XZ_0) \vdash (p, 1, Z_0) \vdash (p, \epsilon, \epsilon)$$

c) 010.

$$(q, 010, Z_0) \vdash (q, 10, XZ_0) \vdash (q, 0, XZ_0) \vdash (q, \epsilon, XXZ_0) \vdash (p, \epsilon, XZ_0) \vdash (p, \epsilon, Z_0)$$

6.2.1

Exercise 6.2.1: Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

* a) $\{0^n 1^n \mid n \geq 1\}.$

$$\text{PDA} : (\{q, p\}, \{0, 1\}, \{X, Z_0\}, \delta, q, Z_0)$$

$$\delta(q, 0, Z_0) = \{(q, XZ_0)\}$$

$$\delta(q, 0, X) = \{(q, X)\}$$

$$\delta(q, 1, X) = \{(p, \epsilon)\}$$

$$\delta(p, 1, X) = \{(p, \epsilon)\}$$

b) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

$$PDA : (\{q, p\}, \{0, 1\}, \{A, B, Z_0\}, \delta, q, Z_0)$$

$$\delta(q, 0, Z_0) = \{(q, AZ_0)\}$$

$$\delta(q, 1, Z_0) = \{(q, BZ_0)\}$$

$$\delta(q, 0, A) = \{(q, AA)\}$$

$$\delta(q, 1, B) = \{(q, BB)\}$$

$$\delta(q, 1, A) = \{(q, \epsilon)\}$$

$$\delta(q, \epsilon, Z_0) = \{(p, Z_0)\}$$

c) The set of all strings of 0's and 1's with an equal number of 0's and 1's.

$$PDA : (\{q, p\}, \{0, 1\}, \{X, Y, Z_0\}, \delta, q, Z_0)$$

$$\delta(q, 0, Z_0) = \{(q, XZ_0)\}$$

$$\delta(q, 0, X) = \{(q, XX)\}$$

$$\delta(q, 0, Y) = \{(q, \epsilon)\}$$

$$\delta(q, 1, Z_0) = \{(q, YZ_0)\}$$

$$\delta(q, 1, X) = \{(q, \epsilon)\}$$

$$\delta(q, 1, Y) = \{(q, YY)\}$$

$$\delta(q, \epsilon, Z_0) = \{(p, \epsilon)\}$$

6.2.5

Exercise 6.2.5: PDA $P = (\{q_0, q_1, q_2, q_3, f\}, \{a, b\}, \{Z_0, A, B\}, \delta, q_0, Z_0, \{f\})$ has the following rules defining δ :

$$\begin{array}{lll} \delta(q_0, a, Z_0) = (q_1, AAZ_0) & \delta(q_0, b, Z_0) = (q_2, BZ_0) & \delta(q_0, \epsilon, Z_0) = (f, \epsilon) \\ \delta(q_1, a, A) = (q_1, AAA) & \delta(q_1, b, A) = (q_1, \epsilon) & \delta(q_1, \epsilon, Z_0) = (q_0, Z_0) \\ \delta(q_2, a, B) = (q_3, \epsilon) & \delta(q_2, b, B) = (q_2, BB) & \delta(q_2, \epsilon, Z_0) = (q_0, Z_0) \\ \delta(q_3, \epsilon, B) = (q_2, \epsilon) & \delta(q_3, \epsilon, Z_0) = (q_1, AZ_0) & \end{array}$$

Note that, since each of the sets above has only one choice of move, we have omitted the set brackets from each of the rules.

* a) Give an execution trace (sequence of ID's) showing that string bab is in $L(P)$.

$$(q_0, bab, Z_0) \vdash (q_2, ab, BZ_0) \vdash (q_3, b, Z_0) \vdash (q_1, b, AZ_0)$$

$$(q_0, bab, z_0) \vdash (q_2, ab, Bz_0) \vdash (q_3, b, z_0) \vdash (q_1, b, Az_0) \\ \vdash (q_1, \epsilon, z_0) \vdash (q_0, \epsilon, z_0) \vdash (f, \epsilon, \epsilon)$$

b) Give an execution trace showing that abb is in $L(P)$.

$$(q_0, abb, z_0) \vdash (q_1, bb, AAz_0) \vdash (q_1, b, Az_0) \vdash (q_1, \epsilon, z_0) \\ \vdash (q_0, \epsilon, z_0) \vdash (f, \epsilon, \epsilon)$$

c) Give the contents of the stack after P has read b^7a^4 from its input.

$$(q_0, b^7a^4, z_0) \vdash (q_2, b^6a^4, Bz_0) \vdash (q_2, b^5a^4, BBz_0) \vdash \dots \vdash (q_2, a^4, B^7z_0) \\ \vdash (q_3, a^3, B^6z_0)$$

Stack

B
B
B
B
B
B
B
z ₀

Hasil setelah membaca b^7a^4

! d) Informally describe $L(P)$.

$L(P)$ adalah bahasa yang diterima PDA

yaitu semua string dengan jumlah "b" dua

kali lebih dari jumlah "a".

6.2.6

Exercise 6.2.6: Consider the PDA P from Exercise 6.1.1.

a) Convert P to another PDA P_1 that accepts by empty stack the same language that P accepts by final state; i.e., $N(P_1) = L(P)$.

$L(P)$ = semua string yang diawali dengan 0 dan diakhiri dengan 1

PDA $P_1 : (\{q, p\}, \{0, 1\}, \{x, z_0\}, \delta, q, z_0)$

$$\delta(q, 0, z_0) = \{(q, xz_0)\}$$

$$\delta(q, 0, x) = \{(q, x)\}$$

...

$$\delta(q, 0, x) = \{ (q, x) \}$$

$$\delta(q, 1, x) = \{ (p, \epsilon) \}$$

$$\delta(p, 1, z_0) = \{ (p, z_0) \}$$

$$\delta(p, 0, z_0) = \{ (q, xz_0) \}$$

$$\delta(p, \epsilon, z_0) = \{ (p, \epsilon) \}$$

b) Find a PDA P_2 such that $L(P_2) = N(P)$; i.e., P_2 accepts by final state what P accepts by empty stack.

$$\text{PDA } P_2 : (\{q, p\}, \{0, 1\}, \{z_0\}, \delta, q, z_0)$$

$$\delta(q, 0, z_0) = (q, z_0)$$

$$\delta(q, 1, z_0) = (p, z_0)$$

$$\delta(p, 0, z_0) = (q, z_0)$$

$$\delta(p, 1, z_0) = (p, z_0)$$

$$\delta(p, \epsilon, z_0) = (p, \epsilon)$$