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S.1.1

**Exercise 5.1.1:** Design context-free grammars for the following languages:

- \* a) The set  $\{0^n 1^n \mid n \geq 1\}$ , that is, the set of all strings of one or more 0's followed by an equal number of 1's.

$$\begin{array}{l} S \rightarrow 01 \\ S \rightarrow 0S1 \end{array} \quad \text{atau} \quad S \rightarrow 01 \mid 0S1$$

- \*! b) The set  $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$ , that is, the set of strings of  $a$ 's followed by  $b$ 's followed by  $c$ 's, such that there are either a different number of  $a$ 's and  $b$ 's or a different number of  $b$ 's and  $c$ 's, or both.

$$\begin{array}{l} S \rightarrow AB \\ A \rightarrow aA \quad B \rightarrow bBc \\ A \rightarrow \epsilon \quad B \rightarrow E \\ \quad \quad B \rightarrow cD \\ S \rightarrow CD \\ C \rightarrow aCb \quad D \rightarrow cD \\ C \rightarrow E \quad D \rightarrow \epsilon \\ C \rightarrow aA \\ E \rightarrow bE \quad E \rightarrow b \end{array} \quad \text{atau} \quad \begin{array}{l} S \rightarrow AB \mid CD \\ A \rightarrow aA \mid \epsilon \\ B \rightarrow bBc \mid E \mid cD \\ C \rightarrow aCb \mid E \mid aA \\ D \rightarrow cD \mid \epsilon \\ E \rightarrow bE \mid b \end{array}$$

Kasus

- $i = j \rightarrow j \neq k$
- $i \neq j \rightarrow j = k$
- $i \neq j \rightarrow j \neq k$

- ! c) The set of all strings of  $a$ 's and  $b$ 's that are *not* of the form  $ww$ , that is, not equal to any string repeated.

$$\begin{array}{l} S \rightarrow \epsilon \quad A \rightarrow a \\ S \rightarrow A \quad A \rightarrow aB \\ S \rightarrow B \quad B \rightarrow b \\ \quad \quad B \rightarrow bA \end{array} \quad \text{atau} \quad \begin{array}{l} S \rightarrow \epsilon \mid A \mid B \\ A \rightarrow a \mid aB \\ B \rightarrow b \mid bA \end{array}$$

- !! d) The set of all strings with twice as many 0's as 1's.

$$\begin{array}{lllll} S \rightarrow 001 & S \rightarrow 00S1 & S \rightarrow 0S01 & S \rightarrow S001 & S \rightarrow 001S \\ S \rightarrow 010 & S \rightarrow 01S0 & S \rightarrow 0S10 & S \rightarrow S010 & S \rightarrow 010S \\ S \rightarrow 100 & S \rightarrow 10S0 & S \rightarrow 1S00 & S \rightarrow S100 & S \rightarrow 100S \end{array}$$

atau

abau

$$S \rightarrow 001 \mid 010 \mid 100 \mid S001 \mid S010 \mid S100 \mid 0S01 \mid 0S10 \mid 1S00 \mid 00S1 \mid 01S0 \mid 10S0 \mid 001S \mid 010S \mid 001S \mid$$

5.1.2

**Exercise 5.1.2:** The following grammar generates the language of regular expression  $0^*1(0+1)^*$ :

$$\begin{aligned} S &\rightarrow A1B \\ A &\rightarrow 0A \mid \epsilon \\ B &\rightarrow 0B \mid 1B \mid \epsilon \end{aligned}$$

Give leftmost and rightmost derivations of the following strings:

\* a) 00101.

Left Most:

$$\begin{aligned} S &\rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 001B \\ &\quad \downarrow \\ 00101 &\leftarrow 00101B \leftarrow 0010B \end{aligned}$$

Right Most:

$$\begin{aligned} S &\rightarrow A1B \rightarrow A10B \rightarrow A101B \rightarrow A101 \\ &\quad \downarrow \\ 00101 &\leftarrow 00A101 \leftarrow 0A101 \end{aligned}$$

b) 1001.

Left most:

$$\begin{aligned} S &\rightarrow A1B \rightarrow 1B \rightarrow 10B \rightarrow 100B \\ &\quad \downarrow \\ 1001 &\leftarrow 1001B \end{aligned}$$

Right most:

$$\begin{aligned} S &\rightarrow A1B \rightarrow A10B \rightarrow A100B \rightarrow A1001B \\ &\quad \downarrow \\ 1001 &\leftarrow A1001 \end{aligned}$$

c) 00011.

Leftmost:

$$\begin{aligned} S &\rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 000A1B \\ &\quad \downarrow \\ 00011 &\leftarrow 00011B \leftarrow 0001B \end{aligned}$$

Rightmost:

$$\begin{aligned} S &\rightarrow A1B \rightarrow A11B \rightarrow A11 \rightarrow 0A11 \\ &\quad \downarrow \\ 00011 &\leftarrow 000A11 \leftarrow 00A11 \end{aligned}$$

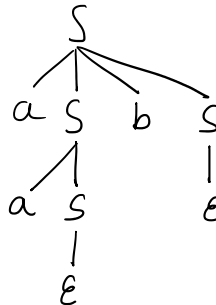
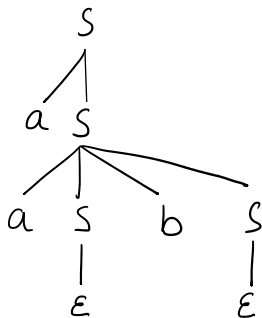
5.4.1

\* **Exercise 5.4.1:** Consider the grammar

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

This grammar is ambiguous. Show in particular that the string  $aab$  has two:

a) Parse trees.



b) Leftmost derivations.

$$S \rightarrow aS \rightarrow aaSbS \rightarrow aabS \rightarrow aab$$

$$S \rightarrow aSbS \rightarrow aaSbS \rightarrow aabS \rightarrow aab$$

c) Rightmost derivations.

$$S \rightarrow aS \rightarrow aaSbS \rightarrow aaSb \rightarrow aab$$

$$S \rightarrow aSbS \rightarrow aSb \rightarrow aaSb \rightarrow aab$$

5.4.7

**Exercise 5.4.7:** The following grammar generates *prefix* expressions with operands  $x$  and  $y$  and binary operators  $+$ ,  $-$ , and  $*$ :

$$E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$$

a) Find leftmost and rightmost derivations, and a derivation tree for the string  $+*-xyxy$ .

Leftmost :

$$E \rightarrow +EE \rightarrow +*EEE \rightarrow +*-EEEE \rightarrow +*-xEEE$$

$$\downarrow$$

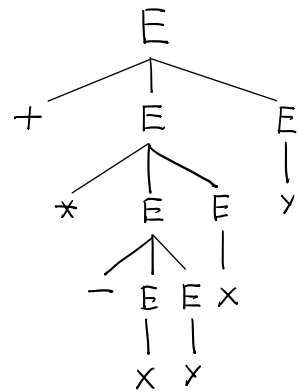
$$+*-xyxy \leftarrow +*-xyxE \leftarrow +*-xyEE$$

Rightmost :

$$E \rightarrow +EE \rightarrow +Ey \rightarrow +*EEy \rightarrow +*Exy$$

$$\downarrow$$

$$+*-xyxy \leftarrow +*-Eyxxy \leftarrow +*-EExy$$



! b) Prove that this grammar is unambiguous.

Metode Induksi

(i) Basis : untuk panjang kata 1, hasil pasti  $x$  atau  $y$

$$w = x, \quad E \rightarrow x$$

$$w = y, \quad E \rightarrow y$$

(ii) untuk panjang kata  $n$ , asumsikan hanya terdapat 1 leftmost derivation

$$\{ E \rightarrow +w; E \rightarrow *w; E \rightarrow -w \}, \quad w \in \text{String dengan panjang } (n-2)$$

(iii) Pembuktian : untuk String dengan panjang  $(n+1)$

$$E \rightarrow +Ew \mid *Ew \mid -Ew, w \in \text{String dengan panjang } (n-2)$$

untuk String dengan panjang  $(n+1)$ , dapat dilihat hanya memiliki 1 buah leftmost dan rightmost derivation yang mungkin.

$$E \rightarrow +xw \mid +yw \mid *xw \mid *yw \mid -xw \mid -yw, w \in \text{String dengan panjang } (n-2)$$

maka terbukti unambiguous