1 118

$\overrightarrow{CA} = \overrightarrow{a},\ \overrightarrow{CB} = \overrightarrow{b},\ |\overrightarrow{b}| = 2,\ (\overrightarrow{a}, \overrightarrow{b})_e = \frac{2\pi}{3}$
$|AB| =?,\ m_B \perp l_C$

$\overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a}$

$\overrightarrow{BM} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC}) = \frac{1}{2}(-\overrightarrow{AB} + (-\overrightarrow{CB})) =$

$= \frac{1}{2}(-(\overrightarrow{b} - \overrightarrow{a}) + (-\overrightarrow{b})) = \frac{1}{2}(\overrightarrow{a} - 2\overrightarrow{b})$

$\overrightarrow{CL} = \overrightarrow{CA} + \overrightarrow{AL}$

$CL \equiv l_C,\ L \in AB \implies \frac{|AL|}{|LB|} = \frac{|CA|}{|CB|} = \frac{|\overrightarrow{a}|}{|\overrightarrow{b}|}$

$\implies |AL| = \frac{|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|}|AB|$

$\implies \overrightarrow{AL} = \frac{|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|}(\overrightarrow{b} - \overrightarrow{a}) = \frac{|\overrightarrow{a}|\overrightarrow{b} - |\overrightarrow{a}|\overrightarrow{a}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$

$\overrightarrow{CL} = \overrightarrow{a} + \frac{|\overrightarrow{a}|\overrightarrow{b} - |\overrightarrow{a}|\overrightarrow{a}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$
$= \frac{\cancel{|\overrightarrow{a}|\overrightarrow{a}} + |\overrightarrow{b}|\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b} - \cancel{|\overrightarrow{a}|\overrightarrow{a}}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$
$= \frac{|\overrightarrow{b}|\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$

$|\overrightarrow{b}| = 2 \implies \overrightarrow{CL} = \frac{2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+2}$

$m_B \perp l_C \implies \overrightarrow{BM}\overrightarrow{CL} = 0$

$\frac{1}{2}(\overrightarrow{a} - 2\overrightarrow{b})\frac{2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+2} = 0 \mid \quad 2(|\overrightarrow{a}| + 2)$

$(\overrightarrow{a} - 2\overrightarrow{b})(2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}) = 0$

$2|\overrightarrow{a}|^2 + \overrightarrow{a}\overrightarrow{b}|\overrightarrow{a}| - 4\overrightarrow{a}\overrightarrow{b} - 2|\overrightarrow{a}||\overrightarrow{b}|^2$

$\overrightarrow{a}\overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\cos(\overrightarrow{a}, \overrightarrow{b})_e = |\overrightarrow{a}|\cancel{2}\frac{-1}{\cancel{2}} = -|\overrightarrow{a}|$

$2|\overrightarrow{a}|^2 - |\overrightarrow{a}|^2 + 4|\overrightarrow{a}| - 8|\overrightarrow{a}| = 0$

$|\overrightarrow{a}|^2 - 4|\overrightarrow{a}| = 0$

$|\overrightarrow{a}|(|\overrightarrow{a}| - 4) = 0$

$|\overrightarrow{a}| = 0 (|\overrightarrow{a}| > 0) \quad |\overrightarrow{a}| - 4 = 0$

$\implies |\overrightarrow{a}| = 4$

$|\overrightarrow{AB}|^2 = \overrightarrow{AB}^2 = (\overrightarrow{b} - \overrightarrow{a})^2 = \overrightarrow{b}^2 - 2\overrightarrow{a}\overrightarrow{b} + \overrightarrow{a}^2$

$= 4 + 8 + 16 = 28$

$\implies |\overrightarrow{AB}| = \sqrt{28} = 2\sqrt{7} = |AB|$

2 136

$OCS \quad K = O\overrightarrow{e_1}\overrightarrow{e_2},\ A_1(x_1, y_1),\ A_2(x_2, y_2),\ A_3(x_3, y_3),\ A_1 \not\equiv A_2 \not\equiv A_3 \not\equiv A_1$

$S \triangle A_1A_2A_3 =?\ \frac{1}{2}|(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)|$

$K' = O\overrightarrow{e_1}\overrightarrow{e_2}\overrightarrow{e_3};\ e_3 = e_1 \times e_2 \implies A_1(x_1, y_1, 0),\ A_2(x_2, y_2, 0),\ A_3(x_3, y_3, 0)$

$S \triangle A_1A_2A_3 = \frac{1}{2}|\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3}|$

$\overrightarrow{A_1A_2}(x_2 - x_1, y_2 - y_1, 0),\ \overrightarrow{A_1A_3}(x_3 - x_1, y_3 - y_1, 0)$

$\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3} \left(\begin{vmatrix} y_2 - y_1 & 0 \\ y_3 - y_1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & x_2 - x_1 \\ 0 & x_3 - x_1 \end{vmatrix}, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$

$\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3} \left(0, 0, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$

$|\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3}| = \sqrt{0 + 0 + [(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)]^2} =$

$= |(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)|$

$\implies S \triangle A_1A_2A_3 = \frac{1}{2}|(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)|$

$\implies S \triangle A_1A_2A_3 = \frac{1}{2} \left| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right| = \frac{1}{2}\varepsilon \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix};\ \varepsilon = \pm 1$

3 137 (Th Shal)

$A, B, C, D \in \lambda \in S_2,\ S \triangle ABC =?\ S \triangle DAB + S \triangle DBC + S \triangle DCA$

$A(a_1, a_2),\ B(b_1, b_2),\ C(c_1, c_2),\ D(d_1, d_2)$

$\lambda' \in S_3;\ A(a_1, a_2, 0),\ B(b_1, b_2, 0),\ C(c_1, c_2, 0),\ D(d_1, d_2, 0) \in \lambda' \implies$

$signS \triangle ABC = signS \triangle DAB = signS \triangle DBC = signS \triangle DCA = \varepsilon;\ \varepsilon = \pm 1$

$S \triangle ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$\overrightarrow{AB}(b_1-a_1, b_2-a_2, 0),\ \overrightarrow{AC}(c_1-a_1, c_2-a_2, 0)$

$S\triangle ABC = \frac{1}{2}\begin{vmatrix} b_1-a_1 & b_2-a_2 \\ c_1-a_1 & c_2-a_2 \end{vmatrix} = \frac{1}{2}[(b_1-a_1)(c_2-a_2)-(b_2-a_2)(c_1-a_1)] =$

$= \frac{1}{2}(b_1c_2 - b_1a_2 - a_1c_2 + \cancel{a_1a_2} - b_2c_1 + b_2a_1 + a_2c_1 - \cancel{a_1a_2}) =$

$= \frac{1}{2}(b_1c_2 - b_1a_2 - a_1c_2 - b_2c_1 + b_2a_1 + a_2c_1)$

$\overrightarrow{DA}(a_1-d_1, a_2-d_2, 0),\ \overrightarrow{DB}(d_1-b_1, d_2-b_2, 0)$

$S\triangle DAB = \frac{1}{2}\begin{vmatrix} a_1-d_1 & a_2-d_2 \\ b_1-d_1 & b_2-d_2 \end{vmatrix}$

$\overrightarrow{DC}(c_1-d_1, c_2-d_2, 0),\quad S\triangle DBC = \frac{1}{2}\begin{vmatrix} b_1-d_1 & b_2-d_2 \\ c_1-d_1 & c_2-d_2 \end{vmatrix}$

$S\triangle DCA = \frac{1}{2}\begin{vmatrix} c_1-d_1 & c_2-d_2 \\ a_1-d_1 & a_2-d_2 \end{vmatrix}$

$S\triangle DAB + S\triangle DCA =$

$= \frac{1}{2}[(a_1-d_1)(b_2-d_2)-(a_2-d_2)(b_1-d_1)+(c_1-d_1)(a_2-d_2)-(c_2-d_2)(a_1-d_1)] =$

$= \frac{1}{2}[(a_1-d_1)(b_2-\cancel{d_2}-c_2+\cancel{d_2})+(a_2-d_2)(c_1-\cancel{d_1}-b_1+\cancel{d_1})] =$

$= \frac{1}{2}[(a_1-d_1)(b_2-c_2)+(a_2-d_2)(c_1-b_1)]$

$S\triangle DAB + S\triangle DCA + S\triangle DBC =$

$= \frac{1}{2}[(a_1-d_1)(b_2-c_2)+(a_2-d_2)(c_1-b_1)+(b_1-d_1)(c_2-d_2)-(b_2-d_2)(c_1-d_1)] =$

$= \frac{1}{2}(a_1b_2 - a_1c_2 - \cancel{d_1b_2} + \cancel{d_1c_2} + a_2c_1 - a_2b_1 - \cancel{d_2c_1} + \cancel{d_2b_1} + b_1c_2 - \cancel{b_1d_2} - \cancel{d_1c_2} + \cancel{d_1d_2} - b_2c_1 + \cancel{b_2d_1} + \cancel{d_2c_1} - \cancel{d_2d_1}) =$

$= \frac{1}{2}(a_1b_2 - a_1c_2 + a_2c_1 - a_2b_1 + b_1c_2 - b_2c_1) =$

$= \frac{1}{2}(b_1c_2 - b_1a_2 - a_1c_2 - b_2c_1 + b_2a_1 + a_2c_1)$

$\Longrightarrow\ S\triangle ABC = S\triangle DAB + S\triangle DBC + S\triangle DCA$