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$\overrightarrow{CA} = \overrightarrow{a}$, $\overrightarrow{CB} = \overrightarrow{b}$, $|\overrightarrow{b}| = 2$, $(\overrightarrow{a}, \overrightarrow{b})_e = \frac{2\pi}{3}$
$|AB| = ?$, $m_B \perp l_C$

$\overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a}$
$\overrightarrow{BM} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC}) = \frac{1}{2}(-\overrightarrow{AB} + (-\overrightarrow{CB})) =$
$= \frac{1}{2}(-(\overrightarrow{b} - \overrightarrow{a}) + (-\overrightarrow{b})) = \frac{1}{2}(\overrightarrow{a} - 2\overrightarrow{b})$
$\overrightarrow{CL} = \overrightarrow{CA} + \overrightarrow{AL}$

$CL \equiv l_C$, $L \in AB \implies \frac{|AL|}{|LB|} = \frac{|CA|}{|CB|} = \frac{|\overrightarrow{a}|}{|\overrightarrow{b}|}$

$\implies |AL| = \frac{|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|}|AB|$

$\implies \overrightarrow{AL} = \frac{|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|}(\overrightarrow{b} - \overrightarrow{a}) = \frac{|\overrightarrow{a}|\overrightarrow{b} - |\overrightarrow{a}|\overrightarrow{a}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$

$\overrightarrow{CL} = \overrightarrow{a} + \frac{|\overrightarrow{a}|\overrightarrow{b} - |\overrightarrow{a}|\overrightarrow{a}}{|\overrightarrow{a}|+|\overrightarrow{b}|} = \frac{\overrightarrow{a}|\overrightarrow{a}| + |\overrightarrow{b}|\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b} - \overrightarrow{a}|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|} = \frac{|\overrightarrow{b}|\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$

$|\overrightarrow{b}| = 2 \implies \overrightarrow{CL} = \frac{2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+2}$

$m_B \perp l_C \implies \overrightarrow{BM}\,\overrightarrow{CL} = 0$
$\frac{1}{2}(\overrightarrow{a} - 2\overrightarrow{b})\frac{2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+2} = 0 \mid 2(|\overrightarrow{a}| + 2)$
$(\overrightarrow{a} - 2\overrightarrow{b})(2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}) = 0$
$2|\overrightarrow{a}|^2 + \overrightarrow{a}\,\overrightarrow{b}|\overrightarrow{a}| - 4\overrightarrow{a}\,\overrightarrow{b} - 2|\overrightarrow{a}||\overrightarrow{b}|^2$
$\overrightarrow{a}\,\overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\cos(\overrightarrow{a}, \overrightarrow{b})_e = |\overrightarrow{a}|2\frac{-1}{2} = -|\overrightarrow{a}|$
$2|\overrightarrow{a}|^2 - |\overrightarrow{a}|^2 + 4|\overrightarrow{a}| - 8|\overrightarrow{a}| = 0$
$|\overrightarrow{a}|^2 - 4|\overrightarrow{a}| = 0$
$|\overrightarrow{a}|(|\overrightarrow{a}| - 4) = 0$
$|\overrightarrow{a}| = 0 (|\overrightarrow{a}| > 0) \quad |\overrightarrow{a}| - 4 = 0$
$\implies |\overrightarrow{a}| = 4$

$|\overrightarrow{AB}|^2 = \overrightarrow{AB}^2 = (\overrightarrow{b} - \overrightarrow{a})^2 = \overrightarrow{b}^2 - 2\overrightarrow{a}\,\overrightarrow{b} + \overrightarrow{a}^2$
$= 4 + 8 + 16 = 28$
$\implies |\overrightarrow{AB}| = \sqrt{28} = 2\sqrt{7} = |AB|$