$|\overrightarrow{a}| = 1,\ |\overrightarrow{b}| = 2,\ \measuredangle(\overrightarrow{a}, \overrightarrow{b}) = \frac{\pi}{3}$

$\overrightarrow{CA} = \overrightarrow{a}(\overrightarrow{b} \times \overrightarrow{a})$

$\overrightarrow{CB} = \overrightarrow{a} + \overrightarrow{b}$

$\overrightarrow{CD} = \overrightarrow{a} \times \overrightarrow{b}$

$|\overrightarrow{AH}| =?\ S_{\triangle ABC} =?\ V_{ABCD} =?$

$\overrightarrow{CA} = \begin{vmatrix} \overrightarrow{a}\overrightarrow{a} & \overrightarrow{a}\overrightarrow{b} \\ \overrightarrow{a} & \overrightarrow{b} \end{vmatrix} = (\overrightarrow{a}\overrightarrow{a})\overrightarrow{b} - (\overrightarrow{a}\overrightarrow{b})\overrightarrow{a}$

$\overrightarrow{a}^2 = |\overrightarrow{a}|^2 = 1^2 = 1$

$\overrightarrow{b}^2 = |\overrightarrow{b}|^2 = 2^2 = 4$

$\overrightarrow{a}\overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\cos\measuredangle(\overrightarrow{a}, \overrightarrow{b}) = 2\cancel{2}\frac{1}{\cancel{2}} = 1$

$\Longrightarrow \overrightarrow{CA} = \overrightarrow{b} - \overrightarrow{a}$

$\overrightarrow{AH} = \overrightarrow{CH} - \overrightarrow{CA}$

$\overrightarrow{AH} = \lambda\overrightarrow{CB} - \overrightarrow{CA} \quad |\overrightarrow{CB}$

$0 = \lambda\overrightarrow{CB}^2 - \overrightarrow{CA}\overrightarrow{CB}$

$\lambda = \frac{\overrightarrow{CA}\overrightarrow{CB}}{\overrightarrow{CB}^2} = \frac{(\overrightarrow{b}-\overrightarrow{a})(\overrightarrow{a}+\overrightarrow{b})}{(\overrightarrow{a}+\overrightarrow{b})^2} = \frac{\overrightarrow{b}^2-\overrightarrow{a}^2}{\overrightarrow{a}^2+2\overrightarrow{a}\overrightarrow{b}+\overrightarrow{b}^2} = \frac{4-1}{1+2+4} = \frac{3}{7}$

$\overrightarrow{AH} = \frac{3}{7}(\overrightarrow{a} + \overrightarrow{b}) - \overrightarrow{b} + \overrightarrow{a} = \frac{3}{7}\overrightarrow{a} + \overrightarrow{a} + \frac{3}{7}\overrightarrow{b} - \overrightarrow{b} = \frac{10}{7}\overrightarrow{a} - \frac{4}{7}\overrightarrow{b}$

$\overrightarrow{AH}^2 = (\frac{10}{7}\overrightarrow{a} - \frac{4}{7}\overrightarrow{b})^2 = \frac{100}{49}\overrightarrow{a}^2 - \frac{80}{7}\overrightarrow{a}\overrightarrow{b} + \frac{16}{7}\overrightarrow{b}^2 = \frac{100}{49} - \frac{160}{49} + \frac{64}{49} = \frac{4}{49}$

$|\overrightarrow{AH}| = \sqrt{\overrightarrow{AH}^2} = \sqrt{\frac{4}{49}} = \frac{2}{7}$

$S_{\triangle ABC} = \frac{1}{2}|\overrightarrow{CA} \times \overrightarrow{CB}|$

$= \frac{1}{2}|(\overrightarrow{b} - \overrightarrow{a}) \times (\overrightarrow{a} + \overrightarrow{b})|$

$= \frac{1}{2}|(\overrightarrow{b} - \overrightarrow{a}) \times \overrightarrow{CB}|$

$= \frac{1}{2}|\overrightarrow{b} \times \overrightarrow{CB} - \overrightarrow{a} \times \overrightarrow{CB}|$

$= \frac{1}{2}|\overrightarrow{b} \times (\overrightarrow{a} + \overrightarrow{b}) - \overrightarrow{a} \times (\overrightarrow{a} + \overrightarrow{b})|$

$= \frac{1}{2}|\overrightarrow{b} \times \overrightarrow{a} + \overrightarrow{b} \times \overrightarrow{b} - \overrightarrow{a} \times \overrightarrow{a} - \overrightarrow{a} \times \overrightarrow{b}|$

$= \frac{1}{2}|-\overrightarrow{a}\times\overrightarrow{b}+0-0-\overrightarrow{a}\times\overrightarrow{b}|$

$= \frac{1}{2}|-2(\overrightarrow{a}\times\overrightarrow{b})|$

$= \frac{1}{2}|-2|\sqrt{(\overrightarrow{a}\times\overrightarrow{b})^2}$

$= \sqrt{(|\overrightarrow{a}||\overrightarrow{b}|\sin\measuredangle(\overrightarrow{a},\overrightarrow{b}))^2}$

$= \sqrt{|\overrightarrow{a}|^2|\overrightarrow{b}|^2\sin^2\measuredangle(\overrightarrow{a},\overrightarrow{b})}$

$= \sqrt{4\frac{3}{4}} = \sqrt{3}$

$V_{ABCD} = \frac{1}{6}|\overrightarrow{CA}\overrightarrow{CB}\overrightarrow{CD}|$

$= \frac{1}{6}|(\overrightarrow{CA}\times\overrightarrow{CB})\overrightarrow{CD}|$

$= \frac{1}{6}|-2(\overrightarrow{a}\times\overrightarrow{b})(\overrightarrow{a}\times\overrightarrow{b})|$

$= \frac{1}{6}|-2||(\overrightarrow{a}\times\overrightarrow{b})^2|$

$= \frac{1}{3}4\frac{3}{4} = 1$