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$$\overrightarrow{CA} = \vec{a}, \overrightarrow{CB} = \vec{b}, |\vec{b}| = 2, (\vec{a}, \vec{b})_e = \frac{2\pi}{3}$$

$$|AB| = ?, m_B \perp l_C$$

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$

$$\overrightarrow{BM} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC}) = \frac{1}{2}(-\overrightarrow{AB} + (-\overrightarrow{CB})) =$$

$$= \frac{1}{2}(-(\vec{b} - \vec{a}) + (-\vec{b})) = \frac{1}{2}(\vec{a} - 2\vec{b})$$

$$\overrightarrow{CL} = \overrightarrow{CA} + \overrightarrow{AL}$$

$$CL \equiv l_C, L \in AB \implies \frac{|AL|}{|LB|} = \frac{|CA|}{|CB|} = \frac{|\vec{a}|}{|\vec{b}|}$$

$$\implies |AL| = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|} |AB|$$

$$\implies \overrightarrow{AL} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|} (\vec{b} - \vec{a}) = \frac{|\vec{a}|\vec{b} - |\vec{a}|\vec{a}}{|\vec{a}| + |\vec{b}|}$$

$$\overrightarrow{CL} = \vec{a} + \frac{|\vec{a}|\vec{b} - |\vec{a}|\vec{a}}{|\vec{a}| + |\vec{b}|}$$

$$= \frac{|\vec{a} + \vec{a}| |\vec{b}| \vec{a} + |\vec{a}| \vec{b} - |\vec{a} + \vec{a}| \vec{a}}{|\vec{a}| + |\vec{b}|}$$

$$= \frac{|\vec{b}| \vec{a} + |\vec{a}| \vec{b}}{|\vec{a}| + |\vec{b}|}$$

$$|\vec{b}| = 2 \implies \overrightarrow{CL} = \frac{2\vec{a} + |\vec{a}|\vec{b}}{|\vec{a}| + 2}$$

$$m_B \perp l_C \implies \overrightarrow{BM} \overrightarrow{CL} = 0$$

$$\frac{1}{2}(\vec{a} - 2\vec{b}) \frac{2\vec{a} + |\vec{a}|\vec{b}}{|\vec{a}| + 2} = 0 \mid 2(|\vec{a}| + 2)$$

$$(\vec{a} - 2\vec{b})(2\vec{a} + |\vec{a}|\vec{b}) = 0$$

$$2|\vec{a}|^2 + \vec{a} \vec{b} |\vec{a}| - 4\vec{a} \vec{b} - 2|\vec{a}| |\vec{b}|^2$$

$$\vec{a} \vec{b} = |\vec{a}| |\vec{b}| \cos(\vec{a}, \vec{b})_e = |\vec{a}| 2 \frac{-1}{2} = -|\vec{a}|$$

$$2|\vec{a}|^2 - |\vec{a}|^2 + 4|\vec{a}| - 8|\vec{a}| = 0$$

$$|\vec{a}|^2 - 4|\vec{a}| = 0$$

$$|\vec{a}|(|\vec{a}| - 4) = 0$$

$$|\vec{a}| = 0 (|\vec{a}| > 0) \mid |\vec{a}| - 4 = 0$$

$$\implies |\vec{a}| = 4$$

$$\begin{aligned}
|\overrightarrow{AB}|^2 &= \overrightarrow{AB}^2 = (\vec{b} - \vec{a})^2 = \vec{b}^2 - 2\vec{a}\vec{b} + \vec{a}^2 \\
&= 4 + 8 + 16 = 28 \\
\Rightarrow |\overrightarrow{AB}| &= \sqrt{28} = 2\sqrt{7} = |AB|
\end{aligned}$$

2 136

$$OCS \quad K = O\vec{e_1}\vec{e_2}, A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3), A_1 \nparallel A_2 \nparallel A_3 \nparallel A_1$$

$$S \triangle A_1 A_2 A_3 = ? \frac{1}{2} |(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)|$$

$$K' = O\vec{e_1}\vec{e_2}\vec{e_3}; e_3 = e_1 \times e_2 \Rightarrow A_1(x_1, y_1, 0), A_2(x_2, y_2, 0), A_3(x_3, y_3, 0)$$

$$S \triangle A_1 A_2 A_3 = \frac{1}{2} |\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3}|$$

$$\overrightarrow{A_1 A_2}(x_2 - x_1, y_2 - y_1, 0), \overrightarrow{A_1 A_3}(x_3 - x_1, y_3 - y_1, 0)$$

$$\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3} \left(\begin{vmatrix} y_2 - y_1 & 0 \\ y_3 - y_1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & x_2 - x_1 \\ 0 & x_3 - x_1 \end{vmatrix}, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$$

$$\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3} \left(0, 0, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$$

$$|\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3}| = \sqrt{0 + 0 + [(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)]^2} =$$

$$= |(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)|$$

$$\Rightarrow S \triangle A_1 A_2 A_3 = \frac{1}{2} |(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)|$$

$$\Rightarrow S \triangle A_1 A_2 A_3 = \frac{1}{2} \left| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right| = \frac{1}{2} \varepsilon \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}; \varepsilon = \pm 1$$

3 137 (Th Shal)

$$A, B, C, D \in \lambda \in S_2, S \triangle ABC = ? S \triangle DAB + S \triangle DBC + S \triangle DCA$$

$$A(a_1, a_2), B(b_1, b_2), C(c_1, c_2), D(d_1, d_2)$$

$$\lambda' \in S_3; A(a_1, a_2, 0), B(b_1, b_2, 0), C(c_1, c_2, 0), D(d_1, d_2, 0) \in \lambda' \Rightarrow$$

$$\text{sign} S \triangle ABC = \text{sign} S \triangle DAB = \text{sign} S \triangle DBC = \text{sign} S \triangle DCA = \varepsilon; \varepsilon = \pm 1$$

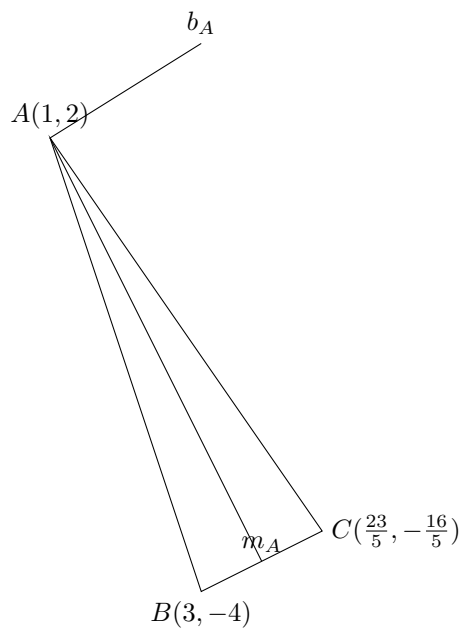
$$S \triangle ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$\begin{aligned}
& \overrightarrow{AB}(b_1 - a_1, b_2 - a_2, 0), \overrightarrow{AC}(c_1 - a_1, c_2 - a_2, 0) \\
& S \triangle ABC = \frac{1}{2} \begin{vmatrix} b_1 - a_1 & b_2 - a_2 \\ c_1 - a_1 & c_2 - a_2 \end{vmatrix} = \frac{1}{2} [(b_1 - a_1)(c_2 - a_2) - (b_2 - a_2)(c_1 - a_1)] = \\
& = \frac{1}{2} (b_1 c_2 - b_1 a_2 - a_1 c_2 + \cancel{a_1 a_2} - b_2 c_1 + b_2 a_1 + a_2 c_1 - \cancel{a_1 a_2}) = \\
& = \frac{1}{2} (b_1 c_2 - b_1 a_2 - a_1 c_2 - b_2 c_1 + b_2 a_1 + a_2 c_1) \\
& \overrightarrow{DA}(a_1 - d_1, a_2 - d_2, 0), \overrightarrow{DB}(d_1 - b_1, d_2 - b_2, 0) \\
& S \triangle DAB = \frac{1}{2} \begin{vmatrix} a_1 - d_1 & a_2 - d_2 \\ b_1 - d_1 & b_2 - d_2 \end{vmatrix} \\
& \overrightarrow{DC}(c_1 - d_1, c_2 - d_2, 0), \quad S \triangle DBC = \frac{1}{2} \begin{vmatrix} b_1 - d_1 & b_2 - d_2 \\ c_1 - d_1 & c_2 - d_2 \end{vmatrix} \\
& S \triangle DCA = \frac{1}{2} \begin{vmatrix} c_1 - d_1 & c_2 - d_2 \\ a_1 - d_1 & a_2 - d_2 \end{vmatrix} \\
& S \triangle DAB + S \triangle DCA = \\
& = \frac{1}{2} [(a_1 - d_1)(b_2 - d_2) - (a_2 - d_2)(b_1 - d_1) + (c_1 - d_1)(a_2 - d_2) - (c_2 - d_2)(a_1 - d_1)] = \\
& = \frac{1}{2} [(a_1 - d_1)(b_2 - \cancel{d_2} - c_2 + \cancel{d_2}) + (a_2 - d_2)(c_1 - \cancel{d_1} - b_1 + \cancel{d_1})] = \\
& = \frac{1}{2} [(a_1 - d_1)(b_2 - c_2) + (a_2 - d_2)(c_1 - b_1)] \\
& S \triangle DAB + S \triangle DCA + S \triangle DBC = \\
& = \frac{1}{2} [(a_1 - d_1)(b_2 - c_2) + (a_2 - d_2)(c_1 - b_1) + (b_1 - d_1)(c_2 - d_2) - (b_2 - d_2)(c_1 - d_1)] = \\
& = \frac{1}{2} (a_1 b_2 - a_1 c_2 - \cancel{d_1 b_2} + \cancel{d_1 c_2} + a_2 c_1 - a_2 b_1 - \cancel{d_2 c_1} + \cancel{d_2 b_1} + b_1 c_2 - b_1 d_2 - \cancel{d_1 c_2} + \\
& \cancel{d_1 d_2} - b_2 c_1 + \cancel{b_2 d_1} + \cancel{d_2 c_1} - \cancel{d_2 d_1}) = \\
& = \frac{1}{2} (a_1 b_2 - a_1 c_2 + a_2 c_1 - a_2 b_1 + b_1 c_2 - b_2 c_1) = \\
& = \frac{1}{2} (b_1 c_2 - b_1 a_2 - a_1 c_2 - b_2 c_1 + b_2 a_1 + a_2 c_1) \\
& \implies S \triangle ABC = S \triangle DAB + S \triangle DBC + S \triangle DCA
\end{aligned}$$

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$$b_A : x - 2y + 3 = 0, \quad m_A : 2x + y - 4 = 0, \quad S \triangle ABC = ?, \quad B(3, -4)$$

$$b_A \perp m_A \implies m_A \equiv b_A \angle (BAC)_e \implies AB = AC \implies B = OImC$$



$$m_A \cap b_A = A(x_A, y_A)$$

$$\begin{aligned} x_A - 2y_A + 3 &= 0 \\ 2x_A + y_A - 4 &= 0 \quad |2 \end{aligned}$$

$$\begin{aligned} x_A - 2y_A + 3 &= 0 \\ + \\ 4x_A + 2y_A - 8 &= 0 \end{aligned}$$

$$5x_A = 5, \quad x_A = 1$$

$$2 + y_A - 4 = 0, \quad y_A = 2$$

$$\implies (x_A, y_A) = (1, 2)$$

$$m_A \equiv h_A \perp BC \implies BC : x - 2y + c = 0$$

$$B(3, -4) \in BC \implies 3 + 8 + c = 0, \quad c = -11$$

$$\implies BC : x - 2y - 11 = 0$$

$$BC \cap m_A = M(x_M, y_M)$$

$$\begin{array}{rcl} x_M - 2y_M - 11 & = & 0 \\ 2x_M + y_M - 4 & = & 0 \mid 2 \end{array}$$

$$\begin{array}{rcl} x_M - 2y_M - 11 & = & 0 \\ + & & \\ 4x_M + 2y_M - 8 & = & 0 \end{array}$$

$$5x_M - 19 = 0, \quad x_M = \frac{19}{5}$$

$$\frac{19}{5} - 2y_M - 11 = 0 \mid 5$$

$$19 - 10y_M - 55 = 0$$

$$-10y_M - 36 = 0$$

$$y_M = -\frac{36}{10} = -\frac{18}{5}$$

$$\implies (x_M, y_M) = \left(\frac{19}{5}, -\frac{18}{5}\right)$$

$$(x_M, y_M) = \left(\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2}\right)$$

$$\frac{19}{5} = \frac{3 + x_C}{2} \quad - \quad \frac{18}{5} = \frac{-4 + y_C}{2}$$

$$38 = 15 + 5x_C \quad - \quad 36 = -20 + 5y_C$$

$$x_C = \frac{23}{5} \quad y_C = -\frac{16}{5}$$

$$\implies (x_C, y_C) = \left(\frac{23}{5}, -\frac{16}{5}\right)$$

$$C\left(\frac{23}{5}, -\frac{16}{5}\right), \quad A(1, 2)$$

$$S \triangle ABC = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2}$$

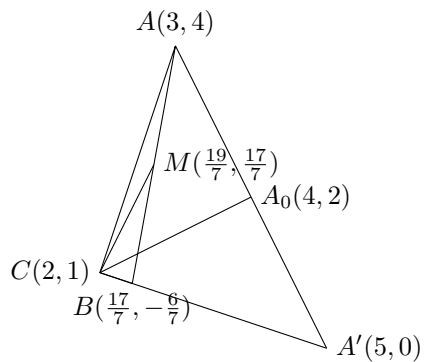
$$\overrightarrow{AB}(2, -6), \quad \overrightarrow{AC}\left(\frac{18}{5}, -\frac{26}{5}\right)$$

$$S \triangle ABC = \frac{1}{2} \varepsilon \left\| \begin{pmatrix} 2 & -6 \\ \frac{18}{5} & -\frac{26}{5} \end{pmatrix} \right\|, \quad \varepsilon = 1, \quad \overrightarrow{AB}, \overrightarrow{AC} \in S^-$$

$$S \triangle ABC = \frac{1}{2} \left(-\frac{52}{5} + \frac{108}{5}\right) = \frac{56}{10} = \frac{38}{5}$$

5 207

$b_C : x - 2y = 0$, $m_C : 2x - y - 3 = 0$, $S \triangle ABC = ?$, $A(3, 4)$



$$b_C \cup m_C = C(x_C, y_C)$$

$$\begin{array}{rcl} x_C - 2y_C & = & 0 \mid 2 \\ 2x_C - y_C - 3 & = & 0 \end{array}$$

$$\begin{array}{rcl} -2x_C + 4y_C & = & 0 \\ + & & \\ 2x_C - y_C - 3 & = & 0 \\ y_C = 1, x_C = 2 & \implies & C(2, 1) \end{array}$$

$$AA' = \begin{cases} z A(3, 4) \\ \perp b_C : x - 2y = 0 \end{cases}$$

$$AA' \perp b_C \implies AA' : 2x + y + c_{AA'} = 0$$

$$A z AA' \implies c_{AA'} = -10$$

$$\begin{array}{l} \implies AA' : 2x + y - 10 = 0 \\ AA' \cup b_C = A_0(x_0, y_0) \end{array}$$

$$\begin{array}{rcl} 2x_0 + y_0 - 10 & = & 0 \mid 2 \\ x_0 - 2y_0 & = & 0 \end{array}$$

$$\begin{array}{rcl} -x_0 + 2y_0 - 20 & = & 0 \\ + & & \\ x_0 - 2y_0 & = & 0 \end{array}$$

$$5x_0 = 20 \implies x_0 = 4$$

$$-2y_0 = 4 \implies y_0 = 2$$

$$\implies A_0(4, 2)$$

$$A'(x_{A'}, y_{A'})$$

$$A_0(\frac{x_A+x_{A'}}{2}, \frac{y_A+y_{A'}}{2})$$

$$4 = \frac{3+x_{A'}}{2} \quad 2 = \frac{4+y_{A'}}{2}$$

$$8 = 3 + x_{A'} \quad 4 = 4 + y_{A'}$$

$$x_{A'} = 5 \quad y_{A'} = 0$$

$$\implies A'(5, 0)$$

$$CA' : \begin{cases} z \ C(2, 1) \\ z \ A'(5, 0) \end{cases}$$

$$\implies CA' : \begin{vmatrix} x-5 & y \\ 2-5 & 1 \end{vmatrix} = 0$$

$$CA' : x + 3y - 5 = 0$$

$$B(x_B, y_B) \ z \ CA'$$

$$x_B + 3y_B - 5 = 0$$

$$M(x_M, y_M), \ \overrightarrow{OM}(x_M, y_M), \ \overrightarrow{OA}(3, 4), \ \overrightarrow{OB}(x_B, y_B)$$

$$AM = MB \implies \overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$$

$$x_M = \frac{3+x_B}{2} \quad y_M = \frac{4+y_B}{2}$$

$$2x_M = 3 + x_B \quad 2y_M = 4 + y_B$$

$$x_B = 2x_M - 3 \quad y_B = 2y_M - 4$$

$$2x_M - y_M - 3 \quad = \quad 0$$

$$x_B + 3y_B - 5 \quad = \quad 0$$

$$2x_M - y_M - 3 \quad \quad \quad = \quad 0$$

$$2x_M - 3 + 6y_B - 12 - 5 \quad = \quad 0$$

$$\begin{array}{rcl} 2x_M - y_M - 3 & = & 0 \mid -1 \\ 2x_M + 6y_M - 20 & = & 0 \end{array}$$

$$\begin{array}{rcl} -2x_M + y_M + 3 & = & 0 \\ + & & \\ 2x_M + 6y_M - 20 & = & 0 \\ 7y_M = 17 & \implies & y_M = \frac{17}{7} \end{array}$$

$$2x_M = \frac{17}{7} + 3$$

$$2x_M = \frac{38}{7} \implies x_M = \frac{19}{7}$$

$$x_B = \frac{38}{7} - 3 = \frac{17}{7}$$

$$y_B = \frac{34}{7} - 4 = \frac{6}{7}$$

$$\implies B(\frac{17}{7}, \frac{6}{7})$$

$$S \triangle ABC = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2}$$

$$\overrightarrow{AB}(-\frac{4}{7}, -\frac{22}{7})$$

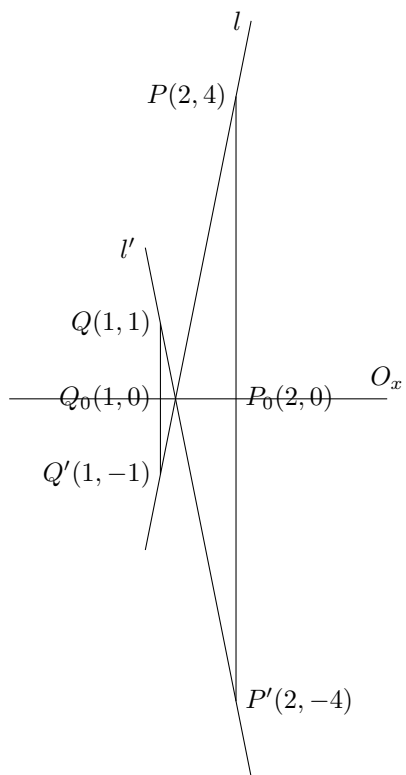
$$\overrightarrow{AC}(-1, -3)$$

$$\overrightarrow{AB}, \overrightarrow{AC} \in S^- \implies S \triangle ABC > 0$$

$$S \triangle ABC = \frac{1}{2} \left\| \begin{vmatrix} -\frac{4}{7} & -1 \\ -\frac{22}{7} & -3 \end{vmatrix} \right\| = \frac{1}{2} \left| \frac{17}{7} - \frac{22}{7} \right| = \frac{1}{2} \left| -\frac{5}{7} \right| = \frac{5}{14}$$

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$$P(2, 4), Q(1, 1), l \perp P, l' \perp Q; \quad l, l'?$$



$$O_x : x = 0$$

$$Q_0 \text{ } z \text{ } O_x \implies Q_0(1, 0)$$

$$Q'(x_{Q'}, y_{Q'})$$

$$Q_0\left(\frac{x_Q + x_{Q'}}{2}, \frac{y_Q + y_{Q'}}{2}\right)$$

$$\implies 1 = \frac{1 + x_{Q'}}{2}, \quad 0 = \frac{1 + y_{Q'}}{2}$$

$$\implies x_{Q'} = 1, \quad y_{Q'} = -1 \implies Q'(1, -1)$$

$$P_0 \text{ } z \text{ } O_x \implies P_0(2, 0)$$

$$P'(x_{P'}, y_{P'})$$

$$P_0\left(\frac{x_P + x_{P'}}{2}, \frac{y_P + y_{P'}}{2}\right)$$

$$\implies 2 = \frac{2 + x_{P'}}{2}, \quad 0 = \frac{4 + y_{P'}}{2}$$

$$\implies x_{P'} = 2, y_{P'} = -4 \implies P'(2, -4)$$

$$l \begin{cases} z \ P(2, 4) \\ z \ Q'(1, -1) \end{cases}$$

$$\implies l : \begin{vmatrix} x-1 & y+1 \\ 2-1 & 4+1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y+1 \\ 1 & 5 \end{vmatrix} = 0$$

$$l : 5x - 5 - y - 1 = 0$$

$$l : 5x - y - 6 = 0$$

$$l' \begin{cases} z \ Q(1, 1) \\ z \ P'(2, -4) \end{cases}$$

$$\implies l' : \begin{vmatrix} x-1 & y-1 \\ 2-1 & -4-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 \\ 1 & -5 \end{vmatrix} = 0$$

$$l' : -5x + 5 - y + 1 = 0$$

$$l' : -5x - y + 6 = 0$$