$$A = a_n \to -\infty \quad if \quad \forall C < 0 \quad \exists \upsilon \,;\, n > \upsilon ; a_n < C$$

$$\overline{A} = \exists C \geq 0 \quad \forall n \,;\, a_n \geq C$$

$$a_n \downarrow \implies a_{n+1} < a_n \implies \exists \upsilon \,;\, a_\upsilon \geq C ; a_{\upsilon+1} < C$$

$$\implies \forall n > \upsilon \quad a_n < C \implies conflict$$

$$\implies a_n \to -\infty$$