$a = \alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4$
$b = \beta_1 x^3 + \beta_2 x^2 + \beta_3 x^1 + \beta_4$
$c = \gamma_1 x^3 + \gamma_2 x^2 + \gamma_3 x^1 + \gamma_4$
$a' = \alpha'_1 x^3 + \alpha'_2 x^2 + \alpha'_3 x^1 + \alpha'_4$
$a, b, c, a' \in \mathbb{V}$
$\lambda, \mu, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \alpha'_1, \alpha'_2, \alpha'_3, \alpha'_4 \in \mathbb{F}$

1. $(a + b) + c = a + (b + c)$

$(a+b) = (\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) + (\beta_1 x^3 + \beta_2 x^2 + \beta_3 x^1 + \beta_4) = (\alpha_1 + \beta_1)x^3 + (\alpha_2 + \beta_2)x^2 + (\alpha_3 + \beta_3)x^1 + (\alpha_4 + \beta_4)$

$(a+b)+c = (\alpha_1 + \beta_1)x^3 + (\alpha_2 + \beta_2)x^2 + (\alpha_3 + \beta_3)x^1 + (\alpha_4 + \beta_4) + (\gamma_1 x^3 + \gamma_2 x^2 + \gamma_3 x^1 + \gamma_4) = (\alpha_1 + \beta_1 + \gamma)x^3 + (\alpha_2 + \beta_2 + \gamma)x^2 + (\alpha_3 + \beta_3 + \gamma)x^1 + (\alpha_4 + \beta_4 + \gamma)$

$(b+c) = (\beta_1 x^3 + \beta_2 x^2 + \beta_3 x^1 + \beta_4) + (\gamma_1 x^3 + \gamma_2 x^2 + \gamma_3 x^1 + \gamma_4) = (\beta_1 + \gamma_1)x^3 + (\beta_2 + \gamma_2)x^2 + (\beta_3 + \gamma_3)x^1 + (\beta_4 + \gamma_4)$

$a+(b+c) = (\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) + (\beta_1 + \gamma_1)x^3 + (\beta_2 + \gamma_2)x^2 + (\beta_3 + \gamma_3)x^1 + (\beta_4 + \gamma_4) = (\alpha_1 + \beta_1 + \gamma)x^3 + (\alpha_2 + \beta_2 + \gamma)x^2 + (\alpha_3 + \beta_3 + \gamma)x^1 + (\alpha_4 + \beta_4 + \gamma)$

$\implies (a + b) + c = a + (b + c) = a + b + c$

2. $a + b = b + a$

$(a+b) = (\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) + (\beta_1 x^3 + \beta_2 x^2 + \beta_3 x^1 + \beta_4) = (\alpha_1 + \beta_1)x^3 + (\alpha_2 + \beta_2)x^2 + (\alpha_3 + \beta_3)x^1 + (\alpha_4 + \beta_4)$

$(b+a) = (\beta_1 x^3 + \beta_2 x^2 + \beta_3 x^1 + \beta_4) + (\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = (\alpha_1 + \beta_1)x^3 + (\alpha_2 + \beta_2)x^2 + (\alpha_3 + \beta_3)x^1 + (\alpha_4 + \beta_4)$

$\implies a + b = b + a$

3. $a + 0 = a$

$a + 0 = (\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) + (0x^3 + 0x^2 + 0x^1 + 0) = (\alpha_1 + 0)x^3 + (\alpha_2 + 0)x^2 + (\alpha_3 + 0)x^1 + (\alpha_4 + 0) = \alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4$

$\implies a + 0 = a$

4. $\exists\, a' : a + a' = 0$

$a + a' = 0$
$(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) + (\alpha'_1 x^3 + \alpha'_2 x^2 + \alpha'_3 x^1 + \alpha'_4) = (0x^3 + 0x^2 + 0x^1 = 0)$
$(\alpha_1 + \alpha'_1)x^3 + (\alpha_2 + \alpha'_2)x^2 + (\alpha_3 + \alpha'_3)x^1 + (\alpha_4 + \alpha'_4) = (0x^3 + 0x^2 + 0x^1 = 0)$

$$\equiv \left| \begin{array}{cccc} \alpha_1' & + & \alpha_1' & = & 0 \\ \alpha_2 & + & \alpha_2' & = & 0 \\ \alpha_3 & + & \alpha_3' & = & 0 \\ \alpha_4 & + & \alpha_4' & = & 0 \end{array} \right. \rightarrow \left| \begin{array}{ccc} \alpha_1' & = & -\alpha_1 \\ \alpha_2' & = & -\alpha_2 \\ \alpha_3' & = & -\alpha_3 \\ \alpha_4' & = & -\alpha_4 \end{array} \right.$$

$\implies a' = -\alpha_1 x^3 + -\alpha_2 x^2 + -\alpha_3 x^1 + -\alpha_4 = -(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = -a$
$\implies \exists\, a' : a + a' = 0$

5. $1a = a$

$1a = 1(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = (1\alpha_1)x^3 + (1\alpha_2)x^2 + (1\alpha_3)x^1 + (1\alpha_4) = \alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4$

$\implies 1a = a$

6. $\lambda(a + b) = \lambda a + \lambda b$

$\lambda(a + b) = \lambda[(\alpha_1 + \beta_1)x^3 + (\alpha_2 + \beta_2)x^2 + (\alpha_3 + \beta_3)x^1 + (\alpha_4 + \beta_4)] = \lambda(\alpha_1 + \beta_1)x^3 + \lambda(\alpha_2 + \beta_2)x^2 + \lambda(\alpha_3 + \beta_3)x^1 + \lambda(\alpha_4 + \beta_4) = (\lambda\alpha_1 + \lambda\beta_1)x^3 + (\lambda\alpha_2 + \lambda\beta_2)x^2 + (\lambda\alpha_3 + \lambda\beta_3)x^1 + (\lambda\alpha_4 + \lambda\beta_4)$

$\lambda a = \lambda(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = \lambda\alpha_1 x^3 + \lambda\alpha_2 x^2 + \lambda\alpha_3 x^1 + \lambda\alpha_4$

$\lambda b = \lambda(\beta_1 x^3 + \beta_2 x^2 + \beta_3 x^1 + \beta_4) = \lambda\beta_1 x^3 + \lambda\beta_2 x^2 + \lambda\beta_3 x^1 + \lambda\beta_4$

$\lambda a + \lambda b = (\lambda\alpha_1 x^3 + \lambda\alpha_2 x^2 + \lambda\alpha_3 x^1 + \lambda\alpha_4) + (\lambda\beta_1 x^3 + \lambda\beta_2 x^2 + \lambda\beta_3 x^1 + \lambda\beta_4) = (\lambda\alpha_1 + \lambda\beta_1)x^3 + (\lambda\alpha_2 + \lambda\beta_2)x^2 + (\lambda\alpha_3 + \lambda\beta_3)x^1 + (\lambda\alpha_4 + \lambda\beta_4)$

$\implies \lambda(a + b) = \lambda a + \lambda b$

7. $(\lambda + \mu)a = \lambda a + \mu a$

$(\lambda + \mu)a = (\lambda + \mu)(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = [(\lambda + \mu)\alpha_1]x^3 + [(\lambda + \mu)\alpha_2]x^2 + [(\lambda + \mu)\alpha_3]x^1 + [(\lambda + \mu)\alpha_4] = (\lambda\alpha_1 + \mu\alpha_1)x^3 + (\lambda\alpha_2 + \mu\alpha_2)x^2 + (\lambda\alpha_3 + \mu\alpha_3)x^1 + (\lambda\alpha_4 + \mu\alpha_4)$

$\lambda a = \lambda(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = \lambda\alpha_1 x^3 + \lambda\alpha_2 x^2 + \lambda\alpha_3 x^1 + \lambda\alpha_4$

$\mu a = \mu(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = \mu\alpha_1 x^3 + \mu\alpha_2 x^2 + \mu\alpha_3 x^1 + \mu\alpha_4$

$\lambda a + \mu a = (\lambda\alpha_1 x^3 + \lambda\alpha_2 x^2 + \lambda\alpha_3 x^1 + \lambda\alpha_4) + (\mu\alpha_1 x^3 + \mu\alpha_2 x^2 + \mu\alpha_3 x^1 + \mu\alpha_4) = (\lambda\alpha_1 + \mu\alpha_1)x^3 + (\lambda\alpha_2 + \mu\alpha_2)x^2 + (\lambda\alpha_3 + \mu\alpha_3)x^1 + (\lambda\alpha_4 + \mu\alpha_4)$

$\implies (\lambda + \mu)a = \lambda a + \mu a$

8. $\lambda(\mu a) = \lambda\mu a$

$\mu a = \mu(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = \mu\alpha_1 x^3 + \mu\alpha_2 x^2 + \mu\alpha_3 x^1 + \mu\alpha_4$

$\lambda(\mu a) = \lambda(\mu\alpha_1 x^3 + \mu\alpha_2 x^2 + \mu\alpha_3 x^1 + \mu\alpha_4) = \lambda(\mu\alpha_1 x^3 + \mu\alpha_2 x^2 + \mu\alpha_3 x^1 + \mu\alpha_4) = \lambda\mu\alpha_1 x^3 + \lambda\mu\alpha_2 x^2 + \lambda\mu\alpha_3 x^1 + \lambda\mu\alpha_4) = \lambda\mu(\alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x^1 + \alpha_4) = \lambda\mu a$

$\implies \lambda(\mu a) = \lambda\mu a$