$e_1 = x^0$
$e_2 = x^1$
$e_3 = x^2$
$e_4 = x^3$
$a = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$
$b = \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4$
$c = \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4$
$a' = \alpha_1' e_1 + \alpha_2' e_2 + \alpha_3' e_3 + \alpha_4' e_4$
$a, b, c, a' \in \mathbb{V}$
$\lambda, \mu, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \alpha_1', \alpha_2', \alpha_3', \alpha_4' \in \mathbb{F}$

1. $(a + b) + c = a + (b + c)$

$(a + b) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4) = (\alpha_1 + \beta_1)e_1 + (\alpha_2 + \beta_2)e_2 + (\alpha_3 + \beta_3)e_3 + (\alpha_4 + \beta_4)e_4$

$(a+b)+c = (\alpha_1 + \beta_1)e_1 + (\alpha_2 + \beta_2)e_2 + (\alpha_3 + \beta_3)e_3 + (\alpha_4 + \beta_4)e_4 + (\gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4) = (\alpha_1 + \beta_1 + \gamma)e_1 + (\alpha_2 + \beta_2 + \gamma)e_2 + (\alpha_3 + \beta_3 + \gamma)e_3 + (\alpha_4 + \beta_4 + \gamma)e_4$

$(b + c) = (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4) + (\gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4) = (\beta_1 + \gamma_1)e_1 + (\beta_2 + \gamma_2)e_2 + (\beta_3 + \gamma_3)e_3 + (\beta_4 + \gamma_4)e_4$

$a+(b+c) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) + (\beta_1 + \gamma_1)e_1 + (\beta_2 + \gamma_2)e_2 + (\beta_3 + \gamma_3)e_3 + (\beta_4 + \gamma_4)e_4 = (\alpha_1 + \beta_1 + \gamma)e_1 + (\alpha_2 + \beta_2 + \gamma)e_2 + (\alpha_3 + \beta_3 + \gamma)e_3 + (\alpha_4 + \beta_4 + \gamma)e_4$

$\implies (a + b) + c = a + (b + c) = a + b + c$

2. $a + b = b + a$

$(a + b) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4) = (\alpha_1 + \beta_1)e_1 + (\alpha_2 + \beta_2)e_2 + (\alpha_3 + \beta_3)e_3 + (\alpha_4 + \beta_4)e_4$

$(b + a) = (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4) + (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = (\alpha_1 + \beta_1)e_1 + (\alpha_2 + \beta_2)e_2 + (\alpha_3 + \beta_3)e_3 + (\alpha_4 + \beta_4)e_4$

$\implies a + b = b + a$

3. $a + 0 = a$

$a + 0 = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) + (0e_1 + 0e_2 + 0e_3 + 0e_4) = (\alpha_1 + 0)e_1 + (\alpha_2 + 0)e_2 + (\alpha_3 + 0)e_3 + (\alpha_4 + 0)e_4 = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$\implies a + 0 = a$

4. $\exists a' : a + a' = 0$

$a + a' = 0$

$(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) + (\alpha_1' e_1 + \alpha_2' e_2 + \alpha_3' e_3 + \alpha_4' e_4) = (0e_1 + 0e_2 + 0e_3 = 0e_4)$

$(\alpha_1 + \alpha_1')e_1 + (\alpha_2 + \alpha_2')e_2 + (\alpha_3 + \alpha_3')e_3 + (\alpha_4 + \alpha_4')e_4 = (0e_1 + 0e_2 + 0e_3 = 0e_4)$

$$\equiv \begin{vmatrix} \alpha_1' & + & \alpha_1' & = & 0 \\ \alpha_2 & + & \alpha_2' & = & 0 \\ \alpha_3 & + & \alpha_3' & = & 0 \\ \alpha_4 & + & \alpha_4' & = & 0 \end{vmatrix} \rightarrow \begin{vmatrix} \alpha_1' & = & -\alpha_1 \\ \alpha_2' & = & -\alpha_2 \\ \alpha_3' & = & -\alpha_3 \\ \alpha_4' & = & -\alpha_4 \end{vmatrix}$$

$\implies a' = -\alpha_1 e_1 + -\alpha_2 e_2 + -\alpha_3 e_3 + -\alpha_4 e_4 = -(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = -a$

$\implies \exists\, a' : a + a' = 0$

5. $\quad 1a = a$

$1a = 1(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = (1\alpha_1)e_1 + (1\alpha_2)e_2 + (1\alpha_3)e_3 + (1\alpha_4)e_4 = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$\implies 1a = a$

6. $\quad \lambda(a + b) = \lambda a + \lambda b$

$\lambda(a + b) = \lambda[(\alpha_1 + \beta_1)e_1 + (\alpha_2 + \beta_2)e_2 + (\alpha_3 + \beta_3)e_3 + (\alpha_4 + \beta_4)e_4] = \lambda(\alpha_1 + \beta_1)e_1 + \lambda(\alpha_2 + \beta_2)e_2 + \lambda(\alpha_3 + \beta_3)e_3 + \lambda(\alpha_4 + \beta_4)e_4 = (\lambda\alpha_1 + \lambda\beta_1)e_1 + (\lambda\alpha_2 + \lambda\beta_2)e_2 + (\lambda\alpha_3 + \lambda\beta_3)e_3 + (\lambda\alpha_4 + \lambda\beta_4)e_4$

$\lambda a = \lambda(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = \lambda\alpha_1 e_1 + \lambda\alpha_2 e_2 + \lambda\alpha_3 e_3 + \lambda\alpha_4 e_4$

$\lambda b = \lambda(\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4) = \lambda\beta_1 e_1 + \lambda\beta_2 e_2 + \lambda\beta_3 e_3 + \lambda\beta_4 e_4$

$\lambda a + \lambda b = (\lambda\alpha_1 e_1 + \lambda\alpha_2 e_2 + \lambda\alpha_3 e_3 + \lambda\alpha_4 e_4) + (\lambda\beta_1 e_1 + \lambda\beta_2 e_2 + \lambda\beta_3 e_3 + \lambda\beta_4 e_4) = (\lambda\alpha_1 + \lambda\beta_1)e_1 + (\lambda\alpha_2 + \lambda\beta_2)e_2 + (\lambda\alpha_3 + \lambda\beta_3)e_3 + (\lambda\alpha_4 + \lambda\beta_4)e_4$

$\implies \lambda(a + b) = \lambda a + \lambda b$

7. $\quad (\lambda + \mu)a = \lambda a + \mu a$

$(\lambda + \mu)a = (\lambda + \mu)(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = [(\lambda + \mu)\alpha_1]e_1 + [(\lambda + \mu)\alpha_2]e_2 + [(\lambda + \mu)\alpha_3]e_3 + [(\lambda + \mu)\alpha_4]e_4 = (\lambda\alpha_1 + \mu\alpha_1)e_1 + (\lambda\alpha_2 + \mu\alpha_2)e_2 + (\lambda\alpha_3 + \mu\alpha_3)e_3 + (\lambda\alpha_4 + \mu\alpha_4)e_4$

$\lambda a = \lambda(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = \lambda\alpha_1 e_1 + \lambda\alpha_2 e_2 + \lambda\alpha_3 e_3 + \lambda\alpha_4 e_4$

$\mu a = \mu(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = \mu\alpha_1 e_1 + \mu\alpha_2 e_2 + \mu\alpha_3 e_3 + \mu\alpha_4 e_4$

$\lambda a + \mu a = (\lambda \alpha_1 e_1 + \lambda \alpha_2 e_2 + \lambda \alpha_3 e_3 + \lambda \alpha_4 e_4) + (\mu \alpha_1 e_1 + \mu \alpha_2 e_2 + \mu \alpha_3 e_3 + \mu \alpha_4 e_4) = (\lambda \alpha_1 + \mu \alpha_1)e_1 + (\lambda \alpha_2 + \mu \alpha_2)e_2 + (\lambda \alpha_3 + \mu \alpha_3)e_3 + (\lambda \alpha_4 + \mu \alpha_4)e_4$

$\implies (\lambda + \mu)a = \lambda a + \mu a$

8. $\lambda(\mu a) = \lambda \mu a$

$\mu a = \mu(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = \mu \alpha_1 e_1 + \mu \alpha_2 e_2 + \mu \alpha_3 e_3 + \mu \alpha_4 e_4$

$\lambda(\mu a) = \lambda(\mu \alpha_1 e_1 + \mu \alpha_2 e_2 + \mu \alpha_3 e_3 + \mu \alpha_4 e_4) = \lambda(\mu \alpha_1 e_1 + \mu \alpha_2 e_2 + \mu \alpha_3 e_3 + \mu \alpha_4 e_4) = \lambda \mu \alpha_1 e_1 + \lambda \mu \alpha_2 e_2 + \lambda \mu \alpha_3 e_3 + \lambda \mu \alpha_4 e_4) = \lambda \mu(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4) = \lambda \mu a$

$\implies \lambda(\mu a) = \lambda \mu a$