$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 =?\, n^2(2n^2 - 1), n = 1, 2, 3, \ldots$

$n = 1 \implies 1^3 =?\, 1^2(2(1^2) - 1)$
$1^3 = 1$
$1^2(2(1^2) - 1) = 1(2 - 1) = 1$
$1 = 1$

$n = k \implies 1^3 + 3^3 + 5^3 + \cdots + (2k-1)^3 = k^2(2k^2 - 1)$

$n = k + 1 \implies 1^3 + 3^3 + 5^3 + \cdots + (2(k+1)-1)^3 =?\, (k+1)^2(2(k+1)^2 - 1)$

$1^3 + 3^3 + 5^3 + \cdots + (2k-1)^3 + (2(k+1) - 1)$
$= k^2(2k^2 - 1) + (2k + 2 - 1)^3$
$= 2k^4 - k^2 + (2k+1)^3$
$= 2k^4 - k^2 + 8k^3 + 12k^2 + 6k + 1$
$= 2k^4 + 8k^3 + 11k^2 + 6k + 1$

$(k+1)^2(2(k+1)^2 - 1)$
$= (k^2 + 2k + 1)(2(k^2 + 2k + 1) - 1)$
$= (k^2 + 2k + 1)(2k^2 + 4k + 1)$
$= 2k^4 + 4k^3 + k^2 + 4k^3 + 8k^2 + 2k + 2k^2 + 4k + 1$
$2k^4 + 8k^3 + 11k^2 + 6k + 1$

$\implies 1^3 + 3^3 + 5^3 + \cdots + (2(k+1)-1)^3 = (k+1)^2(2(k+1)^2 - 1)$
$\implies 1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1), n = 1, 2, 3, \ldots$