

$$\begin{aligned}
& \omega_0, \omega_1, \dots, \omega_{71} = \sqrt[72]{1} \\
& \omega_k = \cos \frac{2k\pi}{72} + i \sin \frac{2k\pi}{72} \\
& \omega_0^{389} + \omega_1^{389} + \dots + \omega_{71}^{389} = ? \\
& \omega_0^{389} + \omega_1^{389} + \dots + \omega_{71}^{389} = \sum_{i=0}^{71} \omega_i^{389} \\
& \omega_1 \in \mathbb{C} \\
& \implies \omega_1^k = \cos \frac{2k\pi}{72} + i \sin \frac{2k\pi}{72} = \omega_k \\
& \implies \sum_{i=0}^{71} \omega_i^{389} = \sum_{i=0}^{71} (\omega_1^{389})^i = \frac{(\omega_1^{389})^{72} - 1}{\omega_1^{389} - 1} \\
& = \frac{(\cos \frac{2 \times 72 \times 389\pi}{72} + i \sin \frac{2 \times 72 \times 389\pi}{72}) - 1}{\omega_1^{389} - 1} \\
& = \frac{(\cos 2 \times 389\pi + i \sin 2 \times 389\pi) - 1}{\omega_1^{389} - 1} \\
& = \frac{(\cos \pi + i \sin \pi) - 1}{\omega_1^{389} - 1} = \frac{1 + 0i - 1}{\omega_1^{389} - 1} = \frac{0}{\omega_1^{389} - 1} \\
& \omega_1^{389} = \cos \frac{2\pi \cdot 389}{72} + i \sin \frac{2\pi \cdot 389}{72} \neq 1 \\
& \implies \omega_1^{389} - 1 \neq 0 \\
& \implies \frac{0}{\omega_1^{389} - 1} = 0 \\
& \implies \omega_0^{389} + \omega_1^{389} + \dots + \omega_{71}^{389} = 0
\end{aligned}$$