1 Глава 3 Уравнения на права и равнина в пространството

1.1 Задача 1.

Да се намери точка M', ортогонално симетрична
на точката $M(1,1,2)$ относно равнината ε, определена с точките
$M_1(5,10,0),\ M_2(4,0,-7),\ M_3(2,4,-5)$. Да се определят и директорните
косинуси в посока от M към M'
Решение:

$$\varepsilon \begin{cases} z\ M_1(5,10,0) \\ z\ M_2(4,0,-7) \\ z\ M_3(2,4,-5) \end{cases}$$

$$\varepsilon : \begin{vmatrix} x-5 & y-10 & z \\ -1 & -10 & -7 \\ -3 & -6 & -5 \end{vmatrix} = 0$$

$$\varepsilon : 50(x-5) + 21(y-10) + 6z - 30z - 42(x-5) - 5(y-10) = 0$$

$$\varepsilon : 8(x-5) + 16(y-10) - 24z = 0 \mid \tfrac{1}{8}$$

$$\varepsilon : x - 5 + 2(y-10) - 3z = 0$$

$$\varepsilon : x + 2y - 3z - 25 = 0$$

$$N_\varepsilon(1,2,-3) \perp \varepsilon$$

$$g \begin{cases} z\ M(1,1,2) \\ \parallel N_\varepsilon(1,2,-3) \end{cases}$$

$$g \begin{cases} x = 1 + \lambda \\ y = 1 + 2\lambda \\ z = 2 - 3\lambda \end{cases}$$

$$g \cap \varepsilon = M_0(x_0, y_0, z_0)$$

$$1 + \lambda_0 + 2 + 4\lambda_0 - 6 + 9\lambda_0 - 25 = 0$$

$$14\lambda_0 = 28 \implies \lambda_0 = 2 \implies M_0(3,5,-4)$$

$$M'(x', y', z')$$

$$M_0\left(\tfrac{x_M + x'}{2}, \tfrac{y_M + y'}{2}, \tfrac{z_M + z'}{2}\right)$$

$3 = \frac{1+x'}{2} \quad 5 = \frac{1+y'}{2} \quad -4 = \frac{2+z'}{2}$

$x' = 5 \quad y' = 9 \quad z = -10$

$\implies M'(5, 9, -10)$

$\overrightarrow{MM'}(4, 8, -12) \parallel \overrightarrow{q}(1, 2, -3)$

$\implies \overrightarrow{n_q} = \frac{\overrightarrow{q}}{|\overrightarrow{q}|}$

$|\overrightarrow{q}| = \sqrt{1+4+9} = \sqrt{14}$

$\overrightarrow{n_q}(\frac{1}{14}, \frac{2}{14}, -\frac{3}{14})$

Директроните косинуси съвпадат с кординатите на $\overrightarrow{n_q}$

1.2 Задача 2.

Да се намери точка M', ортогонално симетрична
на точката $M(-1, 1, 2)$ относно правата
$g \begin{cases} x - y + 1 = 0 \\ x - z - 2 = 0 \end{cases}$
и да се намери разтоянието от M до g
Решение:

$$g \begin{cases} x = 0 + \lambda \\ y = 1 + \lambda \qquad \lambda \in \mathbb{R} \\ z = -2 + \lambda \end{cases}$$

$$g \begin{cases} z\ G(0, 1, -2) \\ \parallel \overrightarrow{g}(1, 1, 1) \end{cases}$$

$$\varepsilon \begin{cases} z\ M(-1, 1, 2) \\ \perp \overrightarrow{g}(1, 1, 1) \end{cases}$$

$\implies \varepsilon : 1(x+1) + 1(y-1) + 1(z-2) = 0$

$\varepsilon : x + y + z - 2 = 0$

$\varepsilon \cap g = M_0(x_0, y_0, z_0)$

$$\begin{aligned}
y_0 &= x_0+1\\
z_0 &= x_0-2\\
x_0+y_0+z_0-2 &= 0
\end{aligned}$$

$$\begin{aligned}
y_0 &= x_0+1\\
z_0 &= x_0-2\\
x_0+x_0+1+x_0-2-2 &= 0
\end{aligned}$$

$$\begin{aligned}
y_0 &= x_0+1\\
z_0 &= x_0-2\\
3x_0 &= 3
\end{aligned}$$

$$\begin{aligned}
y_0 &= 2\\
z_0 &= -1\\
x_0 &= 1
\end{aligned}$$

$\Longrightarrow\ M_0(1,2,-1)$

$M'(x',y',z')$

$M_0(\frac{x_M+x'}{2},\frac{y_M+y'}{2},\frac{z_M+z'}{2})$

$1=\frac{-1+x'}{2}\quad 2=\frac{1+y'}{2}\quad -1=\frac{2+z'}{2}$

$x'=3\quad y'=3\quad z=-4$

$\Longrightarrow\ M'(3,3,-4)$

$d(M,g)=d(M,M_0)=|\overrightarrow{MM_0}|\ (\varepsilon\cap g=M_0,\ \varepsilon\ z\ M,\ \varepsilon\perp g)$

$\overrightarrow{MM_0}(2,1,-3)$

$|\overrightarrow{MM_0}|=\sqrt{4+1=9}=\sqrt{14}$

1.3 Задача 3.

Да се намери трансферзалата на правите

$$a\begin{cases}x=3+\lambda\\ y=-1+2\lambda\quad\lambda\in\mathbb{R}\\ z=4\lambda\end{cases}\qquad b\begin{cases}x=-2+3\mu\\ y=-1\qquad\quad\mu\in\mathbb{R}\\ z=4-5\mu\end{cases}$$

минаваща през точката $P(1,1,1)$

Решение:

$$a\begin{cases}z\ A(3,-1,0)\\ \parallel\ \overrightarrow{a}(1,2,4)\end{cases}$$

$$\alpha\begin{cases} z\ P(1,1,1) \\ z\ A(3,-1,0) \\ \parallel\ \overrightarrow{a}(1,2,4) \end{cases}$$

$$\overrightarrow{AP}(-2,2,1)$$

$$\overrightarrow{AP}\times\overrightarrow{a}\left(\begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix}, \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix}, \begin{vmatrix} -2 & 2 \\ 1 & 2 \end{vmatrix}\right)$$
$$\overrightarrow{AP}\times\overrightarrow{a}(6,9,-6) \parallel \overrightarrow{q}(2,3,-2)$$

$$\alpha\begin{cases} z\ P(1,1,1) \\ \perp\ \overrightarrow{q}(2,3,-2) \end{cases}$$
$$\alpha : 2(x-1)+3(y-1)-2(z-1)=0$$

$$\alpha : 2x-\cancel{2}+3y-3-2z+\cancel{2}=0$$

$$\alpha : 2x+3y-2z-3=0$$

$$b\begin{cases} z\ B(-2,-1,4) \\ \parallel\ \overrightarrow{b}(3,0,-5) \end{cases}$$

$$\beta\begin{cases} z\ P(1,1,1) \\ z\ B(-2,-1,4) \\ \parallel\ \overrightarrow{b}(3,0,-5) \end{cases}$$

$$\overrightarrow{BP}(3,2,-3)$$

$$\overrightarrow{BP}\times\overrightarrow{b}\left(\begin{vmatrix} 2 & -3 \\ 0 & -5 \end{vmatrix}, \begin{vmatrix} -3 & 3 \\ -5 & 3 \end{vmatrix}, \begin{vmatrix} 3 & 2 \\ 3 & 0 \end{vmatrix}\right)$$
$$\overrightarrow{BP}\times\overrightarrow{b}(-10,6,-6) \parallel \overrightarrow{w}(-5,3,-3)$$

$$\beta\begin{cases} z\ P(1,1,1) \\ \perp\ \overrightarrow{w}(-5,3,-3) \end{cases}$$
$$\beta : -5(x-1)+3(y-1)-3(z-1)=0$$

$$\beta : -5x+5+3y-\cancel{3}-3z+\cancel{3}=0$$

$$\beta : -5x+3y-3z+5=0 \mid -1$$

$$\beta : 5x-3y+3z-5=0$$

$$t\begin{cases}2x+3y-2z-3=0\\5x-3y+3z-5=0\end{cases}$$