$$A = a_n \to -\infty \quad if \quad \forall \, C < 0 \quad \exists \, v \, ; \, n > v ; a_n < C$$

$$\overline{A} = \exists C \ge 0 \quad \forall n \, ; \, a_n \ge C$$

$$a_n \downarrow \Longrightarrow \exists v ; a_v \ge C; a_{v+1} < C$$

$$\implies \forall n > v \quad a_n < C \implies conflict$$

$$\implies a_n \to -\infty$$