

$$z \in \mathbb{C}; \quad z + \frac{1}{z} = 2 \cos \varphi$$

$$z^{81} + \frac{1}{z^{81}} =? 2 \cos (81 \varphi)$$

$$z + \frac{1}{z} = 2 \cos \varphi \mid \times z$$

$$z^2 + 1 - 2z \cos \varphi = 0$$

$$D = 4 \cos^2 \varphi - 4$$

$$D = 4(\cos^2 \varphi - 1)$$

$$\sqrt{D} = 2\sqrt{\cos^2 \varphi - 1} = 2\sqrt{-\sin^2 \varphi}$$

$$z_{1,2} = \frac{2 \cos \varphi \pm 2\sqrt{-\sin^2 \varphi}}{2}$$

$$z_{1,2} = \cos \varphi \pm \sqrt{-1} \sqrt{\sin^2 \varphi}$$

$$z_{1,2} = \cos \varphi \pm \imath \sin \varphi$$

$$\frac{1}{z_{1,2}} = z_{1,2}^{-1} = \cos -\varphi \pm \imath \sin -\varphi = \cos \varphi \mp \imath \sin \varphi$$

$$z^{81} = \cos 81 \varphi \pm \imath \sin 81 \varphi$$

$$\frac{1}{z^{81}} = \cos 81 \varphi \mp \imath \sin 81 \varphi$$

$$z^{81} + \frac{1}{z^{81}} = \cos 81 \varphi \pm \imath \sin 81 \varphi + \cos 81 \varphi \mp \imath \sin 81 \varphi$$

$$z^{81} + \frac{1}{z^{81}} = 2 \cos (81 \varphi)$$