$$\omega_0, \omega_1, \ldots, \omega_{71} = \sqrt[72]{1}$$

$$\omega_k = \cos\frac{2k\pi}{72} + \imath\sin\frac{2k\pi}{72}$$

$$\omega_0^{389} + \omega_1^{389} + \cdots + \omega_{71}^{389} = ?$$

$$\omega_0^{389} + \omega_1^{389} + \cdots + \omega_{71}^{389} = \sum_{i=0}^{71} \omega_i^{389}$$

$$\omega_1 \in \mathbb{C}$$

$$\Longrightarrow \omega_1^k = \cos\frac{2k\pi}{72} + \imath\sin\frac{2k\pi}{72} = \omega_k$$

$$\Longrightarrow \sum_{i=0}^{71} \omega_i^{389} = \sum_{i=0}^{71} \left(\omega_1^{389}\right)^i = \frac{\left(\omega_1^{389}\right)^{72} - 1}{\omega_1^{389} - 1}$$

$$= \frac{\left(\cos\frac{2\times\cancel{72}\times 389\pi}{\cancel{72}} + \imath\sin\frac{2\times\cancel{72}\times 389\pi}{\cancel{72}}\right) - 1}{\omega_1^{389} - 1}$$

$$= \frac{(\cos 2\times 389\pi + \imath\sin 2\times 389\pi) - 1}{\omega_1^{389} - 1}$$

$$= \frac{(\cos\pi + \imath\sin\pi) - 1}{\omega_1^{389} - 1} = \frac{1 + 0\imath - 1}{\omega_1^{389} - 1} = \frac{0}{\omega_1^{389} - 1}$$

$$\omega_1^{389} = \cos\frac{2\pi 389}{72} + \imath\sin\frac{2\pi 389}{72} \neq 1$$

$$\Longrightarrow \omega_1^{389} - 1 \neq 0$$

$$\Longrightarrow \frac{0}{\omega_1^{389} - 1} = 0$$

$$\Longrightarrow \omega_0^{389} + \omega_1^{389} + \cdots + \omega_{71}^{389} = 0$$