$z \in \mathbb{C}; \quad z + \frac{1}{z} = 2\cos\varphi$
$z^{81} + \frac{1}{z^{81}} =? 2\cos(81\varphi)$
$z + \frac{1}{z} = 2cos\varphi| \times z$
$z^2 + 1 - 2z\cos\varphi = 0$
$D = 4\cos^2\varphi - 4$
$D = 4(\cos^2\varphi - 1)$
$\sqrt{D} = 2\sqrt{\cos^2\varphi - 1} = 2\sqrt{-\sin^2\varphi}$
$z_{1,2} = \frac{\cancel{2}\cos\varphi \pm \cancel{2}\sqrt{-\sin^2\varphi}}{\cancel{2}}$
$z_{1,2} = \cos\varphi \pm \sqrt{-1}\sqrt{\sin^2\varphi}$
$z_{1,2} = \cos\varphi \pm \imath\sin\varphi$
$\frac{1}{z_{1,2}} = {z_{1,2}}^{-1} = \cos -\varphi \pm \imath\sin -\varphi = \cos\varphi \mp \imath\sin\varphi$
$z^{81} = \cos 81\varphi \pm \imath\sin 81\varphi$
$\frac{1}{z^{81}} = \cos 81\varphi \mp \imath\sin 81\varphi$
$z^{81} + \frac{1}{z^{81}} = \cos 81\varphi \pm \cancel{\imath\sin 81\varphi} + \cos 81\varphi \mp \cancel{\imath\sin 81\varphi}$
$z^{81} + \frac{1}{z^{81}} = 2\cos(81\varphi)$