а) $\lim_{n\to\infty} \frac{2n^3+3n+5}{-3n^3+4n+7}$

$\lim_{n\to\infty} \frac{2n^3+3n+5}{-3n^3+4n+7}$

$\lim_{n\to\infty} \frac{\cancel{n^3}(2+\frac{3}{n^2}+\frac{5}{n^3})}{\cancel{n^3}(-3+\frac{4}{n^2}+\frac{7}{n^3})}$

$\lim_{n\to\infty} \frac{2+\frac{3}{n^2}+\frac{5}{n^3}}{-3+\frac{4}{n^2}+\frac{7}{n^3}}$

$\lim_{n\to\infty} \frac{2+3\frac{1}{n}\frac{1}{n}+5\frac{1}{n}\frac{1}{n}\frac{1}{n}}{-3+4\frac{1}{n}\frac{1}{n}+7\frac{1}{n}\frac{1}{n}\frac{1}{n}}$

$\frac{1}{n} \to 0 \implies \lim_{n\to\infty} \frac{2+3\times0+5\times0}{-3+4\times0+7\times0}$

$\implies \lim_{n\to\infty} \frac{2n^3+3n+5}{-3n^3+4n+7} = \frac{-2}{3}$

б) $\lim_{n\to\infty} \frac{3n^4+4^n n^2+(-3)^n}{2n^3+5^n}$

$\lim_{n\to\infty} \frac{3n^4+4^n n^2+(-3)^n}{2n^3+5^n}$

$\lim_{n\to\infty} \frac{\cancel{n^4}\left(3+\frac{4^n}{n^2}+\frac{(-3)^n}{n^4}\right)}{\cancel{n^3}\left(2+\frac{5^n}{n^3}\right)}$

$\lim_{n\to\infty} \frac{n\left(3+\frac{4^n}{n^2}+\frac{(-3)^n}{n^4}\right)}{2+\frac{5^n}{n^3}}$

$n \to \infty\, a^n \prec n^b, a, b \in \mathbb{R} \implies \frac{4^n}{n^2} \to 0, \frac{(-3)^n}{n^4} \to 0, \frac{5^n}{n^3} \to 0$

$\implies \lim_{n\to\infty} \frac{n\left(3+\frac{4^n}{n^2}+\frac{(-3)^n}{n^4}\right)}{2+\frac{5^n}{n^3}} \to \frac{n3}{2} = n \to \infty$

$\lim_{n\to\infty} \frac{3n^4+4^n n^2+(-3)^n}{2n^3+5^n} = \infty$

в) $\lim_{n\to\infty} \frac{n^2+3n+3}{n^2-3n-4}^{3n+3}$

$\lim_{n\to\infty} \frac{n^2+3n+3}{n^2-3n-4}^{3n+3}$

$\frac{n^2+3n+3}{n^2-3n-4} = \frac{\cancel{n^2}(1+\frac{3}{n}+\frac{3}{n^2})}{\cancel{n^2}(1-\frac{3}{n}-\frac{4}{n^2}} = \frac{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}\frac{1}{n}-4\frac{1}{n}\frac{1}{n}}$

$\frac{1}{n} \to 0 \implies \frac{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}\frac{1}{n}-4\frac{1}{n}\frac{1}{n}} \to 1$

$\lim_{n\to\infty} \left[1+\left(\frac{n^2+3n+3}{n^2-3n-4}-1\right)\right]^{3n+3}$

$\lim_{n\to\infty} \left[1+\left(\frac{n^2+3n+3}{n^2-3n-4}-\frac{n^2-3n-4}{n^2-3n-4}\right)\right]^{3n+3}$

$\lim_{n\to\infty}\left(1+\frac{6n+7}{n^2-3n-4}\right)^{3n+3}$

$\lim_{n\to\infty}\left[\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{n^2-3n-4}{6n+7}}\right]^{\frac{6n+7}{n^2-3n-4}3n+3}$

$\lim_{n\to\infty}\left[\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{n^2-3n-4}{6n+7}}\right]^{\frac{18n^2+39n+21}{n^2-3n-4}}$

$\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{n^2-3n-4}{6n+7}}\to e$

$\frac{18n^2+39n+21}{n^2-3n-4}=\frac{\cancel{n^2}(18+\frac{39}{n}+\frac{21}{n^2})}{\cancel{n^2}(1-\frac{3}{n}-\frac{4}{n})}=\frac{18+39\frac{1}{n}+21\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}\frac{1}{n}-4\frac{1}{n}\frac{1}{n}}$

$\frac{1}{n}\to 0 \implies \frac{18+39\frac{1}{n}+21\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}\frac{1}{n}-4\frac{1}{n}\frac{1}{n}}\to 18$

$\implies \lim_{n\to\infty}\frac{n^2+3n+3}{n^2-3n-4}^{3n+3}=e^{18}$

г) $\lim_{n\to\infty}\frac{n^2+3n+3}{n^2-3n-4}^{\frac{1}{3n+3}}$

$\lim_{n\to\infty}\frac{n^2+3n+3}{n^2-3n-4}^{\frac{1}{3n+3}}$

$\frac{n^2+3n+3}{n^2-3n-4}=\frac{\cancel{n^2}(1+\frac{3}{n}+\frac{3}{n^2})}{\cancel{n^2}(1-\frac{3}{n}-\frac{4}{n^2})}=\frac{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}\frac{1}{n}-4\frac{1}{n}\frac{1}{n}}$

$\frac{1}{n}\to 0 \implies \frac{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}\frac{1}{n}-4\frac{1}{n}\frac{1}{n}}\to 1$

$\lim_{n\to\infty}\left[1+\left(\frac{n^2+3n+3}{n^2-3n-4}-1\right)\right]^{\frac{1}{3n+3}}$

$\lim_{n\to\infty}\left[1+\left(\frac{n^2+3n+3}{n^2-3n-4}-\frac{n^2-3n-4}{n^2-3n-4}\right)\right]^{\frac{1}{3n+3}}$

$\lim_{n\to\infty}\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{1}{3n+3}}$

$\lim_{n\to\infty}\left[\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{n^2-3n-4}{6n+7}}\right]^{\frac{6n+7}{n^2-3n-4}\frac{1}{3n+3}}$

$\lim_{n\to\infty}\left[\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{n^2-3n-4}{6n+7}}\right]^{\frac{6n+7}{3n^3-6n^2-21n-12}}$

$\left(1+\frac{6n+7}{n^2-3n-4}\right)^{\frac{n^2-3n-4}{6n+7}}\to e$

$\frac{6n+7}{3n^3-6n^2-21n-12}=\frac{\cancel{n}(6+\frac{7}{n})}{n^{\cancel{3}}(3-\frac{6}{n}-\frac{21}{n^2}-\frac{12}{n^3})}=\frac{1}{n}\frac{1}{n}\left(\frac{6+7\frac{1}{n}}{3-6\frac{1}{n}-21\frac{1}{n}\frac{1}{n}-12\frac{1}{n}\frac{1}{n}\frac{1}{n}}\right)$

$\frac{1}{n}\to 0 \implies \frac{1}{n}\frac{1}{n}\left(\frac{6+7\frac{1}{n}}{3-6\frac{1}{n}-21\frac{1}{n}\frac{1}{n}-12\frac{1}{n}\frac{1}{n}\frac{1}{n}}\right)\to 0$

$\implies \lim_{n\to\infty} \frac{n^2+3n+3}{n^2-3n-4}^{\frac{1}{3n+3}} = e^0 = 1$

Д) $\lim_{n\to\infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}}$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}}$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}} \frac{\sqrt{n^4+3n^2+4}+\sqrt{n^4-n^3+1}}{\sqrt{n^4+3n^2+4}+\sqrt{n^4-n^3+1}}$

$\lim_{n\to\infty} \frac{n(\sqrt{n^4+3n^2+4}+\sqrt{n^4-n^3+1})}{(\sqrt{n^4+3n^2+4})^2-(\sqrt{n^4-n^3+1})^2}$

$\lim_{n\to\infty} \frac{n\sqrt{n^4}(\sqrt{1+\frac{3}{n^2}+\frac{4}{n^4}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^4}})}{(n^4+3n^2+4)-(n^4-n^3+1)}$

$\lim_{n\to\infty} \frac{n^3(\sqrt{1+\frac{3}{n^2}+\frac{4}{n^4}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^4}})}{n^3+3n^2+3}$

$\lim_{n\to\infty} \frac{\cancel{n^3}(\sqrt{1+\frac{3}{n^2}+\frac{4}{n^4}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^4}})}{\cancel{n^3}(1+\frac{3}{n}+\frac{3}{n^3})}$

$\lim_{n\to\infty} \frac{\sqrt{1+3\frac{1}{n}\frac{1}{n}+4\frac{1}{n}\frac{1}{n}\frac{1}{n}\frac{1}{n}}+\sqrt{1-\frac{1}{n}+\frac{1}{n}\frac{1}{n}\frac{1}{n}\frac{1}{n}}}{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}\frac{1}{n}}$

$\frac{1}{n} \to 0 \implies \lim_{n\to\infty} \frac{\sqrt{1+3\times0+4\times0}+\sqrt{1-0+0}}{1+3\times0+3\times0}$

$\implies \lim_{n\to\infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}} = 2$

e) $\lim_{n\to\infty} \frac{(2n)!!}{(2n)^n}$

$\lim_{n\to\infty} \frac{(2n)!!}{(2n)^n}$

$\lim_{n\to\infty} \frac{\cancel{2n}(n!)}{\cancel{2n}(2n)^{n-1}}$

$\lim_{n\to\infty} \frac{\displaystyle\prod_{k=1}^{n} k}{\displaystyle\prod_{i=1}^{n-1} 2n}$

$$\prod_{k=1}^{n} k \prec \prod_{i=1}^{n-1} 2n \implies \lim_{n\to\infty} \frac{(2n)!!}{(2n)^n} = 0$$