$$\text{Algebric form of: } \left(\frac{8-4\sqrt{3}\imath}{2+6\sqrt{3}\imath}\right)^{342} = ?$$

$$\left(\frac{8-4\sqrt{3}\imath}{2+6\sqrt{3}\imath}\right)^{342} = \left(\frac{(8-4\sqrt{3}\imath)(2-6\sqrt{3}\imath)}{(2+6\sqrt{3}\imath)(2-6\sqrt{3}\imath)}\right)^{342} =$$

$$= \left(\frac{16-48\sqrt{3}\imath-8\sqrt{3}\imath-72}{4+108}\right)^{342} = \left(\frac{\cancel{-56}-\cancel{56}\sqrt{3}\imath}{\cancel{112}}\right)^{342} =$$

$$= \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\imath\right)^{342} = z^{342}$$

$$|z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{4}{4}} = 1$$

$$z = |z|\left(\cos\varphi + \imath\sin\varphi\right)$$

$$z^n = |z|^n\left(\cos n\varphi + \imath\sin n\varphi\right)$$

$$z^{342} = 1^{342}\left(\cos \cancel{342}\frac{7}{\cancel{6}}\pi + \imath\sin \cancel{342}\frac{7}{\cancel{6}}\pi\right)$$

$$z^{342} = \cos 399\pi + \imath\sin 399\pi$$

$$z^{342} = \cos\left(398\pi+\pi\right) + \imath\sin\left(398\pi+\pi\right)$$

$$z^{342} = \cos\pi + \imath\sin\pi = 1$$

$$\Longrightarrow \left(\frac{8-4\sqrt{3}\imath}{2+6\sqrt{3}\imath}\right)^{342} = 1$$