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$\vec{CA} = \vec{a}$, $\vec{CB} = \vec{b}$, $|\vec{b}| = 2$, $(\vec{a}, \vec{b})_e = \frac{2\pi}{3}$

$|AB| = ?$, $m_B \perp l_C$

$\vec{AB} = \vec{b} - \vec{a}$

$\vec{BM} = \frac{1}{2}(\vec{BA} + \vec{BC}) = \frac{1}{2}(-\vec{AB} + (-\vec{CB})) =$

$= \frac{1}{2}(-(\vec{b} - \vec{a}) + (-\vec{b})) = \frac{1}{2}(\vec{a} - 2\vec{b})$

$\vec{CL} = \vec{CA} + \vec{AL}$

$CL \equiv l_C$, $L \in AB \implies \frac{|AL|}{|LB|} = \frac{|CA|}{|CB|} = \frac{|\vec{a}|}{|\vec{b}|}$

$\implies |AL| = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}|AB|$

$\implies \vec{AL} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}(\vec{b} - \vec{a}) = \frac{|\vec{a}|\vec{b} - |\vec{a}|\vec{a}}{|\vec{a}| + |\vec{b}|}$

$\vec{CL} = \vec{a} + \frac{|\vec{a}|\vec{b} - |\vec{a}|\vec{a}}{|\vec{a}| + |\vec{b}|}$

$= \frac{|\vec{a}|\vec{a} + |\vec{b}|\vec{a} + |\vec{a}|\vec{b} - |\vec{a}|\vec{a}}{|\vec{a}| + |\vec{b}|}$

$= \frac{|\vec{b}|\vec{a} + |\vec{a}|\vec{b}}{|\vec{a}| + |\vec{b}|}$

$|\vec{b}| = 2 \implies \vec{CL} = \frac{2\vec{a} + |\vec{a}|\vec{b}}{|\vec{a}| + 2}$

$m_B \perp l_C \implies \vec{BM}\,\vec{CL} = 0$

$\frac{1}{2}(\vec{a} - 2\vec{b})\frac{2\vec{a} + |\vec{a}|\vec{b}}{|\vec{a}| + 2} = 0 \mid 2(|\vec{a}| + 2)$

$(\vec{a} - 2\vec{b})(2\vec{a} + |\vec{a}|\vec{b}) = 0$

$2|\vec{a}|^2 + \vec{a}\,\vec{b}|\vec{a}| - 4\vec{a}\,\vec{b} - 2|\vec{a}||\vec{b}|^2$

$\vec{a}\,\vec{b} = |\vec{a}||\vec{b}|\cos(\vec{a}, \vec{b})_e = |\vec{a}|2\frac{-1}{2} = -|\vec{a}|$

$2|\vec{a}|^2 - |\vec{a}|^2 + 4|\vec{a}| - 8|\vec{a}| = 0$

$|\vec{a}|^2 - 4|\vec{a}| = 0$

$|\vec{a}|(|\vec{a}| - 4) = 0$

$|\vec{a}| = 0(|\vec{a}| > 0)$ $|\vec{a}| - 4 = 0$

$\implies |\vec{a}| = 4$

$$|\overrightarrow{AB}|^2 = \overrightarrow{AB}^2 = (\overrightarrow{b} - \overrightarrow{a})^2 = \overrightarrow{b}^2 - 2\overrightarrow{a}\,\overrightarrow{b} + \overrightarrow{a}^2$$

$$= 4 + 8 + 16 = 28$$

$$\implies |\overrightarrow{AB}| = \sqrt{28} = 2\sqrt{7} = |AB|$$

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$OCS \quad K = O\overrightarrow{e_1}\overrightarrow{e_2}, \ A_1(x_1, y_1), \ A_2(x_2, y_2), \ A_3(x_3, y_3), \ A_1 \nparallel A_2 \nparallel A_3 \nparallel A_1$

$S \triangle A_1 A_2 A_3 =? \ \frac{1}{2}(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)$

$K' = O\overrightarrow{e_1}\overrightarrow{e_2}\overrightarrow{e_3}; \ e_3 = e_1 \times e_2 \implies A_1(x_1, y_1, 0), \ A_2(x_2, y_2, 0), \ A_3(x_3, y_3, 0)$

$S \triangle A_1 A_2 A_3 = \frac{1}{2}|\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3}|$

$\overrightarrow{A_1 A_2}(x_2 - x_1, y_2 - y_1, 0), \ \overrightarrow{A_1 A_3}(x_3 - x_1, y_3 - y_1, 0)$

$\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3} \left(\begin{vmatrix} y_2 - y_1 & 0 \\ y_3 - y_1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & x_2 - x_1 \\ 0 & x_3 - x_1 \end{vmatrix}, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$

$\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3} \left(0, 0, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$

$|\overrightarrow{A_1 A_2} \times \overrightarrow{A_1 A_3}| = \sqrt{0 + 0 + [(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)]^2} =$

$= (x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)$

$\implies S \triangle A_1 A_2 A_3 = \frac{1}{2}(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)$