Algebric form of: $\left(\frac{8-4\sqrt{3}\imath}{2+6\sqrt{3}\imath}\right)^{342} = ?$

$$\left(\frac{8-4\sqrt{3}\imath}{2+6\sqrt{3}\imath}\right)^{342} = \left(\frac{(8-4\sqrt{3}\imath)(2-6\sqrt{3}\imath)}{(2+6\sqrt{3}\imath)(2-6\sqrt{3}\imath)}\right)^{342} =$$

$$= \left(\frac{16-48\sqrt{3}\imath-8\sqrt{3}\imath-72}{4+108}\right)^{342} = \left(\frac{\cancel{-56}\cancel{-56}\sqrt{3}\imath}{\cancel{112}}\right)^{342} = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\imath\right)^{342}$$

$$|z| = \sqrt{Re(z)^2 + Im(z)^2}$$

$$\cos\varphi = \frac{Re(z)}{|z|}, \sin\varphi = \frac{Im(z)}{|z|}$$

$$z = |z|\left(\cos\varphi + \imath\sin\varphi\right)$$

$$z^n = |z|^n\left(\cos n\varphi + \imath\sin n\varphi\right)$$

$$|z| = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{4}{4}} = 1$$

$$\cos\varphi = \frac{-1}{2}, \sin\varphi = \frac{-\sqrt{3}}{2} \implies \varphi = \frac{5}{3}$$

$$z = 1\left(\cos\tfrac{5}{3}\pi + \imath\sin\tfrac{5}{3}\pi\right)$$

$$z^{342} = 1^{342}\left(\cos\cancel{342}\tfrac{5}{\cancel{3}}\pi + \imath\sin\cancel{342}\tfrac{5}{\cancel{3}}\pi\right)$$

$$z^{342} = \cos 560\pi + \imath\sin 560\pi$$

$$z^{342} = \cos 0\pi + \imath\sin 0\pi = 1$$

$$\implies \left(\frac{8-4\sqrt{3}\imath}{2+6\sqrt{3}\imath}\right)^{342} = 1$$