

$$\begin{aligned}
& \text{Algebraic form of: } \left( \frac{8-4\sqrt{3}\iota}{2+6\sqrt{3}\iota} \right)^{342} = ? \\
& \left( \frac{8-4\sqrt{3}\iota}{2+6\sqrt{3}\iota} \right)^{342} = \left( \frac{(8-4\sqrt{3}\iota)(2-6\sqrt{3}\iota)}{(2+6\sqrt{3}\iota)(2-6\sqrt{3}\iota)} \right)^{342} = \\
& = \left( \frac{16-48\sqrt{3}\iota-8\sqrt{3}\iota-72}{4+108} \right)^{342} = \left( \frac{-56-56\sqrt{3}\iota}{112} \right)^{342} = \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}\iota \right)^{342} \\
& |z| = \sqrt{\operatorname{Re}(z)^2 + \operatorname{Im}(z)^2} \\
& \cos \varphi = \frac{\operatorname{Re}(z)}{|z|}, \sin \varphi = \frac{\operatorname{Im}(z)}{|z|} \\
& z = |z| (\cos \varphi + \iota \sin \varphi) \\
& z^n = |z|^n (\cos n\varphi + \iota \sin n\varphi) \\
& z = -\frac{1}{2} - \frac{\sqrt{3}}{2}\iota \\
& |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{4}{4}} = 1 \\
& \cos \varphi = \frac{-1}{2}, \sin \varphi = \frac{-\sqrt{3}}{2} \implies \varphi = \frac{5}{3} \\
& z = 1 \left( \cos \frac{5}{3}\pi + \iota \sin \frac{5}{3}\pi \right) \\
& z^{342} = 1^{342} \left( \cos 342 \frac{5}{3}\pi + \iota \sin 342 \frac{5}{3}\pi \right) \\
& z^{342} = \cos 560\pi + \iota \sin 560\pi \\
& z^{342} = \cos 0\pi + \iota \sin 0\pi = 1 \\
& \implies \left( \frac{8-4\sqrt{3}\iota}{2+6\sqrt{3}\iota} \right)^{342} = 1
\end{aligned}$$