

$$a) \lim_{n \rightarrow \infty} \frac{2n^3+3n+5}{-3n^3+4n+7}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n+5}{-3n^3+4n+7}$$

$$\lim_{n \rightarrow \infty} \frac{n^3(2+\frac{3}{n^2}+\frac{5}{n^3})}{n^3(-3+\frac{4}{n^2}+\frac{7}{n^3})}$$

$$\lim_{n \rightarrow \infty} \frac{2+\frac{3}{n^2}+\frac{5}{n^3}}{-3+\frac{4}{n^2}+\frac{7}{n^3}}$$

$$\lim_{n \rightarrow \infty} \frac{2+3\frac{1}{n}+\frac{5}{n^2}}{-3+4\frac{1}{n}+\frac{7}{n^2}}$$

$$\frac{1}{n} \rightarrow 0 \implies \lim_{n \rightarrow \infty} \frac{2+3 \times 0 + 5 \times 0}{-3+4 \times 0 + 7 \times 0}$$

$$\implies \lim_{n \rightarrow \infty} \frac{2n^3+3n+5}{-3n^3+4n+7} = \frac{-2}{3}$$

$$6) \lim_{n \rightarrow \infty} \frac{3n^4+4^n n^2+(-3)^n}{2n^3+5^n}$$

$$b) \lim_{n \rightarrow \infty} \frac{n^2+3n+3}{n^2-3n-4}^{3n+3}$$

$$\lim_{n \rightarrow \infty} \frac{n^2+3n+3}{n^2-3n-4}^{3n+3}$$

$$\frac{n^2+3n+3}{n^2-3n-4} = \frac{n^2(1+\frac{3}{n}+\frac{3}{n^2})}{n^2(1-\frac{3}{n}-\frac{4}{n^2})} = \frac{1+3\frac{1}{n}+3\frac{1}{n^2}}{1-3\frac{1}{n}-4\frac{1}{n^2}}$$

$$\frac{1}{n} \rightarrow 0 \implies \frac{1+3\frac{1}{n}+3\frac{1}{n^2}}{1-3\frac{1}{n}-4\frac{1}{n^2}} \rightarrow 1$$

$$\lim_{n \rightarrow \infty} \left[1 + \left(\frac{n^2+3n+3}{n^2-3n-4} - 1 \right) \right]^{3n+3}$$

$$\lim_{n \rightarrow \infty} \left[1 + \left(\frac{n^2+3n+3}{n^2-3n-4} - \frac{n^2-3n-4}{n^2-3n-4} \right) \right]^{3n+3}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{6n+7}{n^2-3n-4} \right)^{3n+3}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{n^2-3n-4}{6n+7}} \right]^{\frac{6n+7}{n^2-3n-4} 3n+3}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{n^2-3n-4}{6n+7}} \right]^{\frac{18n^2+39n+21}{n^2-3n-4}}$$

$$\left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{n^2-3n-4}{6n+7}} \rightarrow e$$

$$\frac{18n^2+39n+21}{n^2-3n-4} = \frac{n^2(18+\frac{39}{n}+\frac{21}{n^2})}{n^2(1-\frac{3}{n}-\frac{4}{n^2})} = \frac{18+39\frac{1}{n}+21\frac{1}{n^2}}{1-3\frac{1}{n}-4\frac{1}{n^2}}$$

$$\frac{1}{n} \rightarrow 0 \implies \frac{18+39\frac{1}{n}+21\frac{1}{n^2}}{1-3\frac{1}{n}-4\frac{1}{n^2}} \rightarrow 18$$

$$\implies \lim_{n \rightarrow \infty} \frac{n^2+3n+3}{n^2-3n-4}^{3n+3} = e^{18}$$

$$\text{r) } \lim_{n \rightarrow \infty} \frac{n^2+3n+3}{n^2-3n-4}^{\frac{1}{3n+3}}$$

$$\lim_{n \rightarrow \infty} \frac{n^2+3n+3}{n^2-3n-4}^{\frac{1}{3n+3}}$$

$$\frac{n^2+3n+3}{n^2-3n-4} = \frac{\pi^2(1+\frac{3}{n}+\frac{3}{n^2})}{\pi^2(1-\frac{3}{n}-\frac{4}{n^2})} = \frac{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}-4\frac{1}{n}\frac{1}{n}}$$

$$\frac{1}{n} \rightarrow 0 \implies \frac{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}{1-3\frac{1}{n}-4\frac{1}{n}\frac{1}{n}} \rightarrow 1$$

$$\lim_{n \rightarrow \infty} \left[1 + \left(\frac{n^2+3n+3}{n^2-3n-4} - 1 \right) \right]^{\frac{1}{3n+3}}$$

$$\lim_{n \rightarrow \infty} \left[1 + \left(\frac{n^2+3n+3}{n^2-3n-4} - \frac{n^2-3n-4}{n^2-3n-4} \right) \right]^{\frac{1}{3n+3}}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{1}{3n+3}}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{n^2-3n-4}{6n+7}} \right]^{\frac{6n+7}{n^2-3n-4} \frac{1}{3n+3}}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{n^2-3n-4}{6n+7}} \right]^{\frac{6n+7}{3n^3-6n^2-21n-12}}$$

$$\left(1 + \frac{6n+7}{n^2-3n-4} \right)^{\frac{n^2-3n-4}{6n+7}} \rightarrow e$$

$$\frac{6n+7}{3n^3-6n^2-21n-12} = \frac{\pi(6+\frac{7}{n})}{\pi^3(3-\frac{6}{n}-\frac{21}{n^2}-\frac{12}{n^3})} = \frac{1}{n} \frac{1}{n} \left(\frac{6+7\frac{1}{n}}{3-6\frac{1}{n}-21\frac{1}{n}\frac{1}{n}-12\frac{1}{n}\frac{1}{n}\frac{1}{n}} \right)$$

$$\frac{1}{n} \rightarrow 0 \implies \frac{1}{n} \frac{1}{n} \left(\frac{6+7\frac{1}{n}}{3-6\frac{1}{n}-21\frac{1}{n}\frac{1}{n}-12\frac{1}{n}\frac{1}{n}\frac{1}{n}} \right) \rightarrow 0$$

$$\implies \lim_{n \rightarrow \infty} \frac{n^2+3n+3}{n^2-3n-4}^{\frac{1}{3n+3}} = e^0 = 1$$

$$\text{II) } \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}}$$

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}}$$

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}} \frac{\sqrt{n^4+3n^2+4}+\sqrt{n^4-n^3+1}}{\sqrt{n^4+3n^2+4}+\sqrt{n^4-n^3+1}}$$

$$\lim_{n \rightarrow \infty} \frac{n(\sqrt{n^4+3n^2+4}+\sqrt{n^4-n^3+1})}{(\sqrt{n^4+3n^2+4})^2-(\sqrt{n^4-n^3+1})^2}$$

$$\lim_{n \rightarrow \infty} \frac{n\sqrt{n^4}(\sqrt{1+\frac{3}{n^2}+\frac{4}{n^4}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^4}})}{(n^4+3n^2+4)-(n^4-n^3+1)}$$

$$\lim_{n \rightarrow \infty} \frac{n^3(\sqrt{1+\frac{3}{n^2}+\frac{4}{n^4}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^4}})}{n^3+3n^2+3}$$

$$\lim_{n \rightarrow \infty} \frac{n^3(\sqrt{1+\frac{3}{n^2}+\frac{4}{n^4}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^4}})}{n^3(1+\frac{3}{n}+\frac{3}{n^3})}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{1+3\frac{1}{n}+4\frac{1}{n}\frac{1}{n}\frac{1}{n}}+\sqrt{1-\frac{1}{n}+\frac{1}{n}\frac{1}{n}\frac{1}{n}}}{1+3\frac{1}{n}+3\frac{1}{n}\frac{1}{n}}$$

$$\frac{1}{n} \rightarrow 0 \implies \lim_{n \rightarrow \infty} \frac{\sqrt{1+3 \times 0 + 4 \times 0} + \sqrt{1-0+0}}{1+3 \times 0 + 3 \times 0}$$

$$\implies \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^4+3n^2+4}-\sqrt{n^4-n^3+1}} = 2$$

$$\text{e) } \lim_{n \rightarrow \infty} \frac{(2n)!!}{(2n)^n}$$

$$\lim_{n \rightarrow \infty} \frac{(2n)!!}{(2n)^n}$$

$$\lim_{n \rightarrow \infty} \frac{2n(n!)}{2n(2n)^{n-1}}$$

$$\lim_{n \rightarrow \infty} \frac{\prod_{k=1}^n k}{\prod_{i=1}^{n-1} 2n}$$

$$\prod_{k=1}^n k \prec \prod_{i=1}^{n-1} 2n \implies \lim_{n \rightarrow \infty} \frac{(2n)!!}{(2n)^n} = 0$$