1 118

$\overrightarrow{CA} = \overrightarrow{a},\ \overrightarrow{CB} = \overrightarrow{b},\ |\overrightarrow{b}| = 2,\ (\overrightarrow{a}, \overrightarrow{b})_e = \frac{2\pi}{3}$
$|AB| = ?,\ m_B \perp l_C$

$\overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a}$

$\overrightarrow{BM} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC}) = \frac{1}{2}(-\overrightarrow{AB} + (-\overrightarrow{CB})) =$

$= \frac{1}{2}(-(\overrightarrow{b} - \overrightarrow{a}) + (-\overrightarrow{b})) = \frac{1}{2}(\overrightarrow{a} - 2\overrightarrow{b})$

$\overrightarrow{CL} = \overrightarrow{CA} + \overrightarrow{AL}$

$CL \equiv l_C,\ L \in AB \implies \frac{|AL|}{|LB|} = \frac{|CA|}{|CB|} = \frac{|\overrightarrow{a}|}{|\overrightarrow{b}|}$

$\implies |AL| = \frac{|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|}|AB|$

$\implies \overrightarrow{AL} = \frac{|\overrightarrow{a}|}{|\overrightarrow{a}|+|\overrightarrow{b}|}(\overrightarrow{b} - \overrightarrow{a}) = \frac{|\overrightarrow{a}|\overrightarrow{b} - |\overrightarrow{a}|\overrightarrow{a}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$

$\overrightarrow{CL} = \overrightarrow{a} + \frac{|\overrightarrow{a}|\overrightarrow{b} - |\overrightarrow{a}|\overrightarrow{a}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$
$= \frac{\cancel{|\overrightarrow{a}|\overrightarrow{a}} + |\overrightarrow{b}|\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b} - \cancel{|\overrightarrow{a}|\overrightarrow{a}}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$
$= \frac{|\overrightarrow{b}|\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+|\overrightarrow{b}|}$

$|\overrightarrow{b}| = 2 \implies \overrightarrow{CL} = \frac{2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+2}$

$m_B \perp l_C \implies \overrightarrow{BM}\overrightarrow{CL} = 0$

$\frac{1}{2}(\overrightarrow{a} - 2\overrightarrow{b})\frac{2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}}{|\overrightarrow{a}|+2} = 0 \mid \quad 2(|\overrightarrow{a}| + 2)$

$(\overrightarrow{a} - 2\overrightarrow{b})(2\overrightarrow{a} + |\overrightarrow{a}|\overrightarrow{b}) = 0$

$2|\overrightarrow{a}|^2 + \overrightarrow{a}\overrightarrow{b}|\overrightarrow{a}| - 4\overrightarrow{a}\overrightarrow{b} - 2|\overrightarrow{a}||\overrightarrow{b}|^2$

$\overrightarrow{a}\overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\cos(\overrightarrow{a}, \overrightarrow{b})_e = |\overrightarrow{a}|\cancel{2}\frac{-1}{\cancel{2}} = -|\overrightarrow{a}|$

$2|\overrightarrow{a}|^2 - |\overrightarrow{a}|^2 + 4|\overrightarrow{a}| - 8|\overrightarrow{a}| = 0$

$|\overrightarrow{a}|^2 - 4|\overrightarrow{a}| = 0$

$|\overrightarrow{a}|(|\overrightarrow{a}| - 4) = 0$

$|\overrightarrow{a}| = 0 (|\overrightarrow{a}| > 0) \quad |\overrightarrow{a}| - 4 = 0$

$\implies |\overrightarrow{a}| = 4$

$|\overrightarrow{AB}|^2 = \overrightarrow{AB}^2 = (\overrightarrow{b} - \overrightarrow{a})^2 = \overrightarrow{b}^2 - 2\overrightarrow{a}\overrightarrow{b} + \overrightarrow{a}^2$

$= 4 + 8 + 16 = 28$

$\implies |\overrightarrow{AB}| = \sqrt{28} = 2\sqrt{7} = |AB|$

2 136

$OCS \quad K = O\overrightarrow{e_1}\overrightarrow{e_2},\ A_1(x_1, y_1),\ A_2(x_2, y_2),\ A_3(x_3, y_3),\ A_1 \nparallel A_2 \nparallel A_3 \nparallel A_1$

$S \triangle A_1A_2A_3 =?\ \frac{1}{2}|(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)|$

$K' = O\overrightarrow{e_1}\overrightarrow{e_2}\overrightarrow{e_3};\ e_3 = e_1 \times e_2 \implies A_1(x_1, y_1, 0),\ A_2(x_2, y_2, 0),\ A_3(x_3, y_3, 0)$

$S \triangle A_1A_2A_3 = \frac{1}{2}|\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3}|$

$\overrightarrow{A_1A_2}(x_2 - x_1, y_2 - y_1, 0),\ \overrightarrow{A_1A_3}(x_3 - x_1, y_3 - y_1, 0)$

$\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3} \left(\begin{vmatrix} y_2 - y_1 & 0 \\ y_3 - y_1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & x_2 - x_1 \\ 0 & x_3 - x_1 \end{vmatrix}, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$

$\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3} \left(0, 0, \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right)$

$|\overrightarrow{A_1A_2} \times \overrightarrow{A_1A_3}| = \sqrt{0 + 0 + [(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)]^2} =$

$= |(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)|$

$\implies S \triangle A_1A_2A_3 = \frac{1}{2}|(x_1 - x_2)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)|$

$\implies S \triangle A_1A_2A_3 = \frac{1}{2} \left| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} \right| = \frac{1}{2}\varepsilon \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix};\ \varepsilon = \pm 1$

3 137 (Th Shal)

$A, B, C, D \in \lambda \in S_2,\ S \triangle ABC =?\ S \triangle DAB + S \triangle DBC + S \triangle DCA$

$A(a_1, a_2),\ B(b_1, b_2),\ C(c_1, c_2),\ D(d_1, d_2)$

$\lambda' \in S_3;\ A(a_1, a_2, 0),\ B(b_1, b_2, 0),\ C(c_1, c_2, 0),\ D(d_1, d_2, 0) \in \lambda' \implies$

$signS \triangle ABC = signS \triangle DAB = signS \triangle DBC = signS \triangle DCA = \varepsilon;\ \varepsilon = \pm 1$

$S \triangle ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$\overrightarrow{AB}(b_1-a_1, b_2-a_2, 0),\ \overrightarrow{AC}(c_1-a_1, c_2-a_2, 0)$

$S \triangle ABC = \frac{1}{2}\begin{vmatrix} b_1-a_1 & b_2-a_2 \\ c_1-a_1 & c_2-a_2 \end{vmatrix} = \frac{1}{2}[(b_1-a_1)(c_2-a_2)-(b_2-a_2)(c_1-a_1)] =$

$= \frac{1}{2}(b_1c_2 - b_1a_2 - a_1c_2 + \cancel{a_1a_2} - b_2c_1 + b_2a_1 + a_2c_1 - \cancel{a_1a_2}) =$

$= \frac{1}{2}(b_1c_2 - b_1a_2 - a_1c_2 - b_2c_1 + b_2a_1 + a_2c_1)$

$\overrightarrow{DA}(a_1-d_1, a_2-d_2, 0),\ \overrightarrow{DB}(d_1-b_1, d_2-b_2, 0)$

$S \triangle DAB = \frac{1}{2}\begin{vmatrix} a_1-d_1 & a_2-d_2 \\ b_1-d_1 & b_2-d_2 \end{vmatrix}$

$\overrightarrow{DC}(c_1-d_1, c_2-d_2, 0), \quad S \triangle DBC = \frac{1}{2}\begin{vmatrix} b_1-d_1 & b_2-d_2 \\ c_1-d_1 & c_2-d_2 \end{vmatrix}$

$S \triangle DCA = \frac{1}{2}\begin{vmatrix} c_1-d_1 & c_2-d_2 \\ a_1-d_1 & a_2-d_2 \end{vmatrix}$

$S \triangle DAB + S \triangle DCA =$

$= \frac{1}{2}[(a_1-d_1)(b_2-d_2)-(a_2-d_2)(b_1-d_1)+(c_1-d_1)(a_2-d_2)-(c_2-d_2)(a_1-d_1)] =$

$= \frac{1}{2}[(a_1-d_1)(b_2-\cancel{d_2}-c_2+\cancel{d_2})+(a_2-d_2)(c_1-\cancel{d_1}-b_1+\cancel{d_1})] =$

$= \frac{1}{2}[(a_1-d_1)(b_2-c_2)+(a_2-d_2)(c_1-b_1)]$

$S \triangle DAB + S \triangle DCA + S \triangle DBC =$

$= \frac{1}{2}[(a_1-d_1)(b_2-c_2)+(a_2-d_2)(c_1-b_1)+(b_1-d_1)(c_2-d_2)-(b_2-d_2)(c_1-d_1)] =$

$= \frac{1}{2}(a_1b_2 - a_1c_2 - \cancel{d_1b_2} + \cancel{d_1c_2} + a_2c_1 - a_2b_1 - \cancel{d_2c_1} + \cancel{d_2b_1} + b_1c_2 - \cancel{b_1d_2} - \cancel{d_1c_2} + \cancel{d_1d_2} - b_2c_1 + \cancel{b_2d_1} + \cancel{d_2c_1} - \cancel{d_2d_1}) =$

$= \frac{1}{2}(a_1b_2 - a_1c_2 + a_2c_1 - a_2b_1 + b_1c_2 - b_2c_1) =$

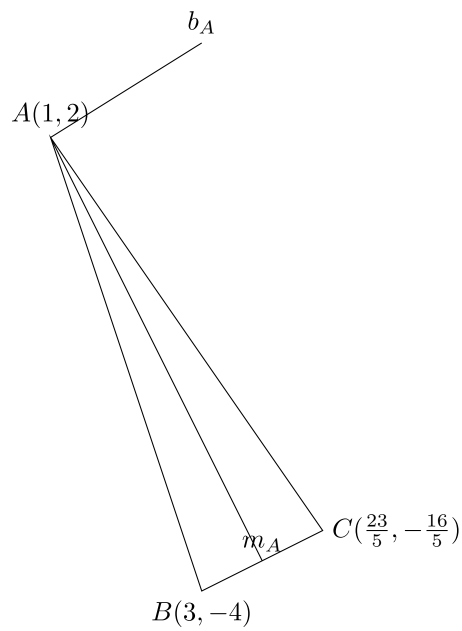
$= \frac{1}{2}(b_1c_2 - b_1a_2 - a_1c_2 - b_2c_1 + b_2a_1 + a_2c_1)$

$\implies S \triangle ABC = S \triangle DAB + S \triangle DBC + S \triangle DCA$

4 206

$b_A : x - 2y + 3 = 0,\ m_A : 2x + 7 - 4 = 0,\ S \triangle ABC = ?,\ B(3, -4)$

$b_A \perp m_A \implies m_A \equiv b_A \sphericalangle (BAC)_e \implies AB = AC \implies B = OImC$



$m_A \cap b_A = A(x_A, y_A)$

$$\begin{aligned} x_A - 2y_A + 3 &= 0 \\ 2x_A + y_A - 4 &= 0 \,|2 \end{aligned}$$

$$\begin{aligned} x_A - 2y_A + 3 &= 0 \\ + & \\ 4x_A + 2y_A - 8 &= 0 \end{aligned}$$

$5x_A = 5,\ x_A = 1$

$2 + y_A - 4 = 0,\ y_A = 2$

$\implies (x_A, y_A) = (1, 2)$

$m_A \equiv h_A \perp BC \implies BC : x - 2y + c = 0$

$B(3,-4)\ z\ BC \implies 3 + 8 + c = 0,\ c = -11$

$\implies BC : x - 2y - 11 = 0$

$BC \cup m_A = M(x_M, y_M)$

$$\begin{aligned} x_M - 2y_M - 11 &= 0 \\ 2x_M + y_M - 4 &= 0 \;|2 \end{aligned}$$

$$\begin{aligned} x_M - 2y_M - 11 &= 0 \\ + & \\ 4x_M + 2y_M - 8 &= 0 \end{aligned}$$

$5x_M - 19 = 0,\ x_M = \frac{19}{5}$

$\frac{19}{5} - 2y_M - 11 = 0\ |5$

$19 - 10y_M - 55 = 0$

$-10y_M - 36 = 0$

$y_M = -\frac{\cancel{36}}{\cancel{10}} = -\frac{18}{5}$

$\implies (x_M, y_M) = (\frac{19}{5}, -\frac{18}{5})$

$(x_M, y_M) = (\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2})$

$\frac{19}{5} = \frac{3 + x_C}{2} \quad -\frac{18}{5} = \frac{-4 + y_C}{2}$

$38 = 15 + 5x_C \quad -36 = -20 + 5y_C$

$x_C = \frac{23}{5} \quad y_C = -\frac{16}{5}$

$\implies (x_C, y_C) = (\frac{23}{5}, -\frac{16}{5})$

$C(\frac{23}{5}, -\frac{16}{5}),\ A(1,2)$

$S \triangle ABC = |\overrightarrow{AB} \times \overrightarrow{AC}|$

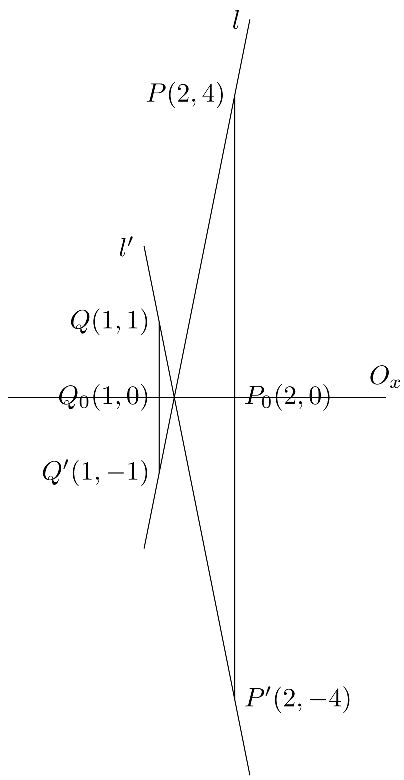
$\overrightarrow{AB}(2, -6),\ \overrightarrow{AC}(\frac{18}{5}, -\frac{26}{5})$

$S \triangle ABC = \varepsilon \left| \begin{vmatrix} 2 & -6 \\ \frac{18}{5} & -\frac{26}{5} \end{vmatrix} \right|, \varepsilon = 1,\ \overrightarrow{AB}, \overrightarrow{AC} \in S^-$

$S \triangle ABC = -\frac{52}{5} + \frac{108}{5} = \frac{56}{5}$

5 218

$P(2,4),\ Q(1,1),\ l\ z\ P,\ l'\ z\ Q;\quad l,\ l'?$



$O_x : x = 0$

$Q_0\ z\ O_x \implies Q_0(1,0)$

$Q'(x_{Q'}, y_{Q'})$

$Q_0(\frac{x_Q+x_{Q'}}{2}, \frac{y_Q+y_{Q'}}{2})$

$\implies 1 = \frac{1+x_{Q'}}{2}, \quad 0 = \frac{1+y_{Q'}}{2}$

$\implies x_{Q'} = 1,\ y_{Q'} = -1 \implies Q'(1,-1)$

$P_0\ z\ O_x \implies P_0(2,0)$

$P'(x_{P'}, y_{P'})$

$P_0(\frac{x_P+x_{P'}}{2}, \frac{y_P+y_{P'}}{2})$

$\implies 2 = \frac{2+x_{P'}}{2}, \quad 0 = \frac{4+y_{P'}}{2}$

$\implies x_{P'} = 2,\ y_{P'} = -4 \implies P'(2,-4)$

$l \begin{cases} z\ P(2,4) \\ z\ Q'(1,-1) \end{cases}$

$\implies l : \begin{vmatrix} x-1 & y+1 \\ 2-1 & 4+1 \end{vmatrix} = 0$

$\begin{vmatrix} x-1 & y+1 \\ 1 & 5 \end{vmatrix} = 0$

$l : 5x - 5 - y - 1 = 0$

$l : 5x - y - 6 = 0$

$l' \begin{cases} z\ Q(1,1) \\ z\ P'(2,-4) \end{cases}$

$\implies l' : \begin{vmatrix} x-1 & y-1 \\ 2-1 & -4-1 \end{vmatrix} = 0$

$\begin{vmatrix} x-1 & y-1 \\ 1 & -5 \end{vmatrix} = 0$

$l' : -5x + 5 - y + 1 = 0$

$l' : -5x - y + 6 = 0$