Курсова задача №2

Иво Стратев

21 декември 2017 г.

$f(x) = \frac{x^2-4}{x}.e^{-\frac{5}{3x}} = (x - \frac{4}{x})e^{-\frac{5}{3x}}$

1. Допустимо множество

$x \in \mathbb{R}\backslash\{0\}$

$\forall x \in \mathbb{R}\backslash\{0\}\ e^{-\frac{5}{3x}} > 0 \implies f(x) = 0 \iff \frac{x^2-4}{x} = 0 \iff x^2 - 4 = 0 \iff x = \pm 2$

2. $f'(x) = (1 + \frac{4}{x^2})e^{-\frac{5}{3x}} + (x - \frac{4}{x})e^{-\frac{5}{3x}}\frac{5}{3x^2} =$

$= (1 + \frac{4}{x^2} + \frac{5}{3x} - \frac{20}{3x^3})e^{-\frac{5}{3x}}$

$\forall x \in \mathbb{R}\backslash\{0\}\ e^{-\frac{5}{3x}} > 0 \implies f'(x) = 0 \iff 1 + \frac{4}{x^2} + \frac{5}{3x} - \frac{20}{3x^3} = 0 \iff$

$\frac{3x^3+12x+5x^2-20}{3x^3} = 0 \iff 3x^3 + 5x^2 + 12x - 20 = 0$

Нека $p(x) = 3x^3 + 5x^2 + 5x^2 - 20 \implies sign(f(x)) = sign(p(x)).sign(x^3)$

$p(1) = 3 + 5 + 12 - 20 = 20 - 20 = 0 \implies (x - 1) \mid p(x)$

$$
\begin{array}{rl}
 & 3x^3 + 5x^2 + 12x - 20 \ :\ x - 1 = 3x^2 + 8x + 20 \\
- & \\
 & \underline{3x^3 - 3x^2 \qquad\qquad\qquad} \\
 & \qquad 8x^2 + 12x - 20 \\
 & - \\
 & \qquad \underline{8x^2 - 8x \qquad\quad} \\
 & \qquad\qquad 20x - 20 \\
 & \qquad - \\
 & \qquad\qquad \underline{20x - 20}
\end{array}
$$

0

$\Longrightarrow\ p(x) = (x-1)(3x^2+8x+20)$

Нека $q(x) = 3x^2+8x+20$

$D(q) = 64 - 240 < 0\ \wedge\ 3 > 0 \Longrightarrow \forall x \in \mathbb{R}\ q(x) > 0 \Longrightarrow$

$sign(f'(x)) = sign(x-1).sign(x^3)$

$f'(x)$ $\quad + \quad 0 \quad - \quad 1 \quad +$ $\quad\Longrightarrow\quad$ $f'(x)$ $\quad \nearrow \quad 0 \quad \searrow \quad 1 \quad \nearrow$

3.

x	$-\infty$	$0-0$	0	$0+0$	1	∞
$f(x)$	$-\infty \nearrow$	$\infty \nearrow$	$\|\|$	$0 \searrow$	$-\frac{3}{e^{\frac{5}{3}}}$	$\infty \nearrow$
$f'(x)$	$+$	$+$	$\|\|$	$-$	0	$+$

$f(1) = (1-4)e^{-\frac{5}{3}} = -\frac{3}{e^{\frac{5}{3}}} \approx -0.56663$

$\forall x \in (0.5,\ 1)\ sign(f(x)) < 0\ \wedge\ \forall x \in (1,\ 1.5)\ sign(f(x)) > 0\ \wedge\ f(1) = 0 \Longrightarrow$

$f(1)$ е локален минимум.

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}(x-\frac{4}{x})e^{-\frac{5}{3x}} = \left[\lim_{x\to-\infty}(x-\frac{4}{x})\right].\left[\lim_{x\to-\infty} e^{-\frac{5}{3x}}\right] =$

$= \left[(\lim_{x\to-\infty} x) - \left(\lim_{x\to-\infty} \frac{4}{x}\right)\right].\left[e^{\lim_{x\to-\infty} -\frac{5}{3x}}\right] = [(\lim_{x\to-\infty} x) - 0].e^0 =$

$= \lim_{x\to-\infty} x = -\infty$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}(x-\frac{4}{x})e^{\frac{5}{3x}} = \left[\lim_{x\to\infty}(x-\frac{4}{x})\right].\left[\lim_{x\to\infty} e^{-\frac{5}{3x}}\right] =$

$= \left[(\lim_{x\to\infty} x) - \left(\lim_{x\to\infty} \frac{4}{x}\right)\right].\left[e^{\lim_{x\to\infty} -\frac{5}{3x}}\right] = [(\lim_{x\to\infty} x) - 0].e^0 =$

$= \lim_{x\to\infty} x = \infty$

$\lim_{x\to0-0} f(x) = \lim_{x\to0-0}(x-\frac{4}{x})e^{-\frac{5}{3x}} = \left[\lim_{x\to0-0}(x-\frac{4}{x})\right].\left[\lim_{x\to0-0} e^{-\frac{5}{3x}}\right] =$

$= \left[(\lim_{x\to0-0} x) - \left(\lim_{x\to0-0} \frac{4}{x}\right)\right].\left[e^{\lim_{x\to0-0} -\frac{5}{3x}}\right] = [0^- - (-\infty)].[e^{\infty}] =$

$= \infty.\infty = \infty$

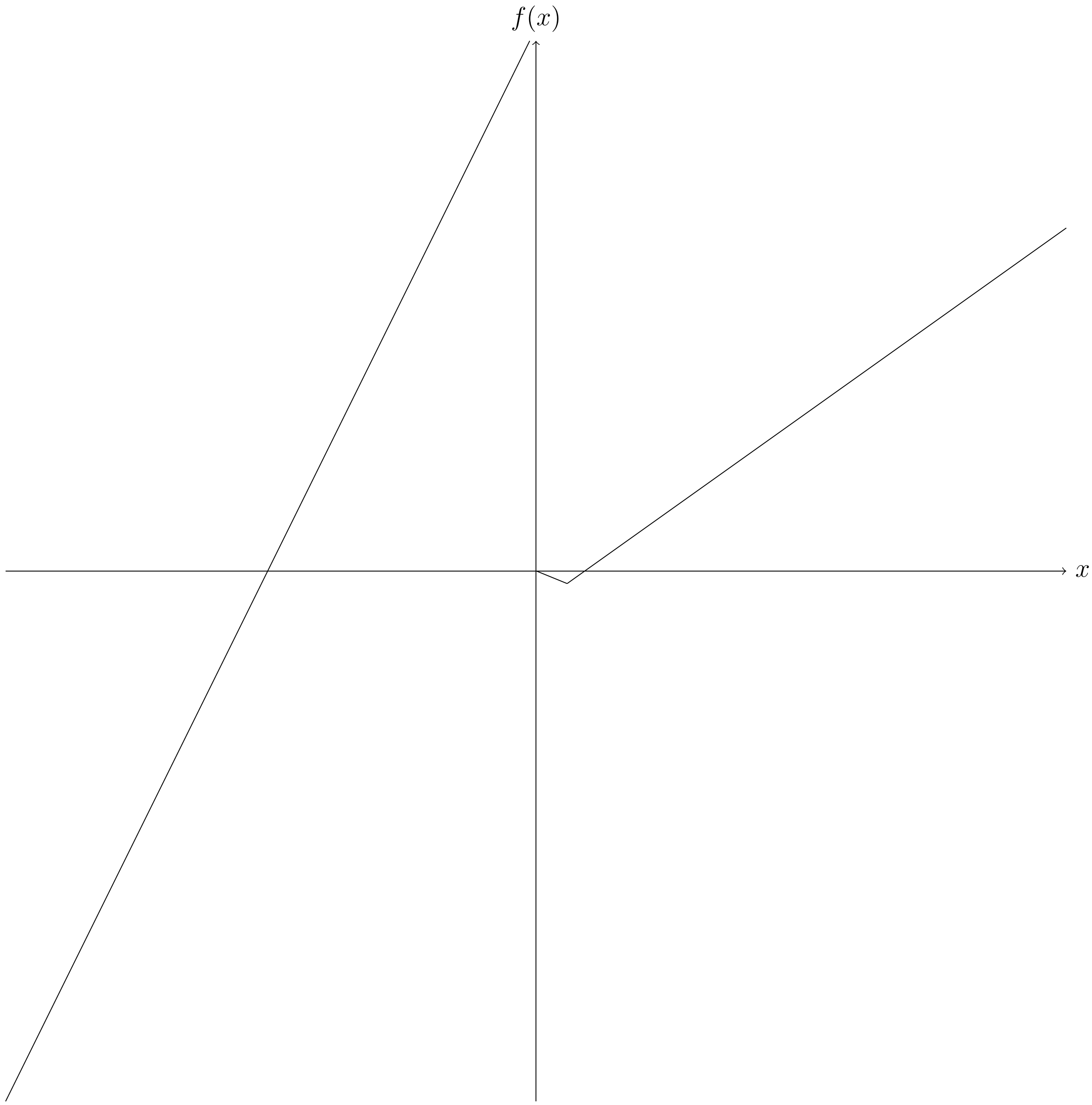
$\lim_{x\to0+0} f(x) = \lim_{x\to0+0}(x-\frac{4}{x})e^{-\frac{5}{3x}} = \left[\lim_{x\to0+0} \frac{x}{e^{\frac{5}{3x}}}\right] - 4\left[\lim_{x\to0+0} \frac{\frac{1}{x}}{e^{\frac{5}{3x}}}\right]$

$\lim_{x\to 0+0} \frac{x}{e^{\frac{5}{3x}}} = \lim_{x\to 0+0} \frac{1}{\frac{1}{x} e^{\frac{5}{3x}}} = \frac{1}{\left[\lim_{x\to 0+0} \frac{1}{x}\right]\left[\lim_{x\to 0+0} e^{\frac{5}{3x}}\right]} = \frac{1}{\infty.\infty} = \frac{1}{\infty} = 0$

$\lim_{x\to 0+0} \frac{\frac{1}{x}}{e^{\frac{5}{3x}}} = \frac{\infty}{\infty} \implies \lim_{x\to 0+0} \frac{\frac{1}{x}}{e^{\frac{5}{3x}}} = \lim_{x\to 0+0} \frac{-\frac{1}{x^2}}{e^{\frac{5}{3x}}.\left(-\frac{5}{3x^2}\right)} =$

$= \lim_{x\to 0+0} \frac{3}{5e^{\frac{5}{3x}}} = \frac{3}{5} \frac{1}{e^{\lim_{x\to 0+0} \frac{5}{3x}}} = \frac{3}{5} \frac{1}{e^{\infty}} = \frac{3}{5} \frac{1}{\infty} = \frac{3}{5}.0 = 0 \implies$

$\lim_{x\to 0+0} f(x) = 0 - 4.0 = 0 - 0 = 0$



4. $f''(x) = \left[\left(1 + \frac{4}{x^2} + \frac{5}{3x} - \frac{20}{3x^3}\right) e^{-\frac{5}{3x}}\right]' =$

$= \left(-\frac{8}{x^3} - \frac{5}{3x^2} + \frac{20}{x^4}\right) e^{-\frac{5}{3x}} + \left(1 + \frac{4}{x^2} + \frac{5}{3x} - \frac{20}{3x^3}\right) e^{-\frac{5}{3x}}.\left(\frac{5}{3x^2}\right) =$

$= e^{-\frac{5}{3x}} \left(-\frac{8}{x^3} - \frac{5}{3x^2} + \frac{20}{x^4} + \frac{5}{3x^2} + \frac{20}{3x^4} + \frac{25}{9x^3} - \frac{100}{9x^5}\right) =$

$= e^{-\frac{5}{3x}} \left(-\frac{47}{9x^3} + \frac{80}{3x^4} - \frac{100}{9x^5}\right) = e^{-\frac{5}{3x}} \left(-\frac{47x^2-240x+100}{9x^5}\right)$

$\forall x \in \mathbb{R}\backslash\{0\}\ e^{-\frac{5}{3x}} > 0 \implies f''(x) = 0 \iff \frac{47x^2-240x+100}{9x^5} = 0 \iff$

$47x^2 - 240x + 100 = 0$

Нека $g(x) = 47x^2 - 240x + 100 \implies sign(f''(x)) = sign(g(x)).sign(x^5) = sign(x).sign(g(x))$

$D(g) = 240^2 - 47.400 = 400(24.6 - 47) =$

$= 400(144 - 47) = 400.97 = 20^2.97 \implies \sqrt{D(g)} = 20\sqrt{97} \implies$

$x_{1,\,2} = \frac{240 \pm 20\sqrt{97}}{2.47} = \frac{120 \mp 10\sqrt{97}}{47} = \frac{10}{47}(12 \mp \sqrt{97}) \approx 2.5532 \mp 2.0955 \implies$

$x_1 \approx 0.4577$, $\quad x_2 \approx 4.6487 \implies$

$sign(f''(x)) = sign(x).sign(-(x - x_1)(x + x_2)) = -sign(x).sign((x - x_1)(x - x_2))$

$$\implies \quad f'(x) \quad \begin{array}{ccccccccc} + & 0 & - & x_1 & + & x_2 & - \end{array}$$

$$\implies \quad f'(x) \quad \begin{array}{ccccccccc} \smile & 0 & \frown & x_1 & \smile & x_2 & \frown \end{array}$$

5.

x	$-\infty$	$0-0$	0	$0+0$	x_1	1	x_2	∞
$f(x)$	$-\infty\ \nearrow \smile$	$\infty\ \nearrow \smile$	$\vert\vert$	$0\ \searrow \frown$	$f(x_1)$	$-\frac{3}{e^{\frac{5}{3}}}$	$f(x_2)$	$\infty\ \nearrow \frown$
$f'(x)$	$+$	$+$	$\vert\vert$	$-$	$f'(x_1)$	0	$f'(x_2)$	$+$
$f''(x)$	$+$	$+$	$\vert\vert$	$-$	0	$+$	0	$-$

$f(x_1) = f(\frac{10}{47}(12 - \sqrt{97})) = \left(\frac{10}{47}(12 - \sqrt{97}) - \frac{94}{5(12-\sqrt{97})}\right) e^{-\frac{47}{6(12-\sqrt{97})}} \approx -0.21709732$

$f(x_2) = f(\frac{10}{47}(12 + \sqrt{97})) = \frac{48}{235}(-11 + 3\sqrt{97})e^{-2+\frac{\sqrt{97}}{6}} \approx 2.64686745142$

$f'(x_1) = f'(\frac{10}{47}(12 - \sqrt{97})) = -\frac{2}{25}(277 + 30\sqrt{97})e^{-2-\frac{\sqrt{97}}{6}} \approx -1.20051176$

$f'(x_2) = f'(\frac{10}{47}(12 + \sqrt{97})) = \frac{2}{25}(30\sqrt{97} - 277)e^{\frac{\sqrt{97}}{6}-2} \approx 1.0321711097$

$$((T_1(f))(a))(x) = f(a) + f'(a)(x-a) \implies$$

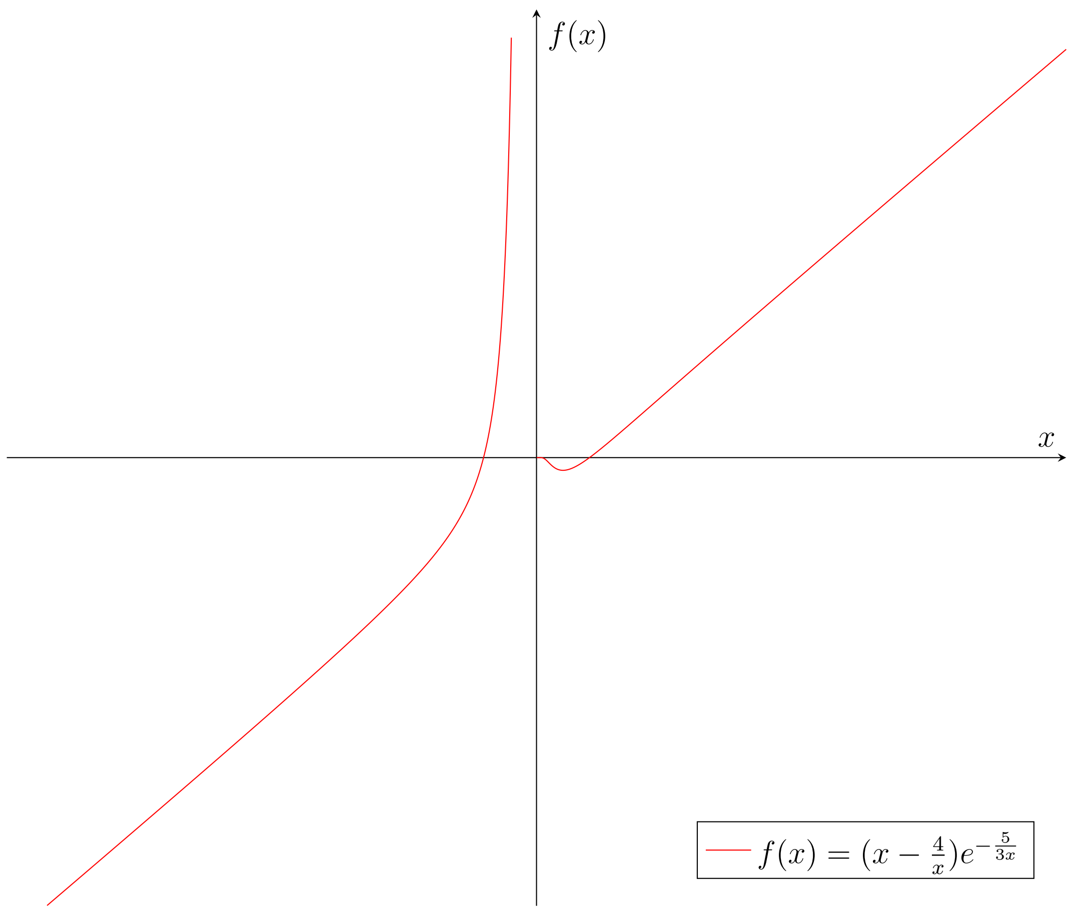
$$((T_1(f))(x_1))(x) = f(x_1) + f'(x_1)(x-x_1)$$

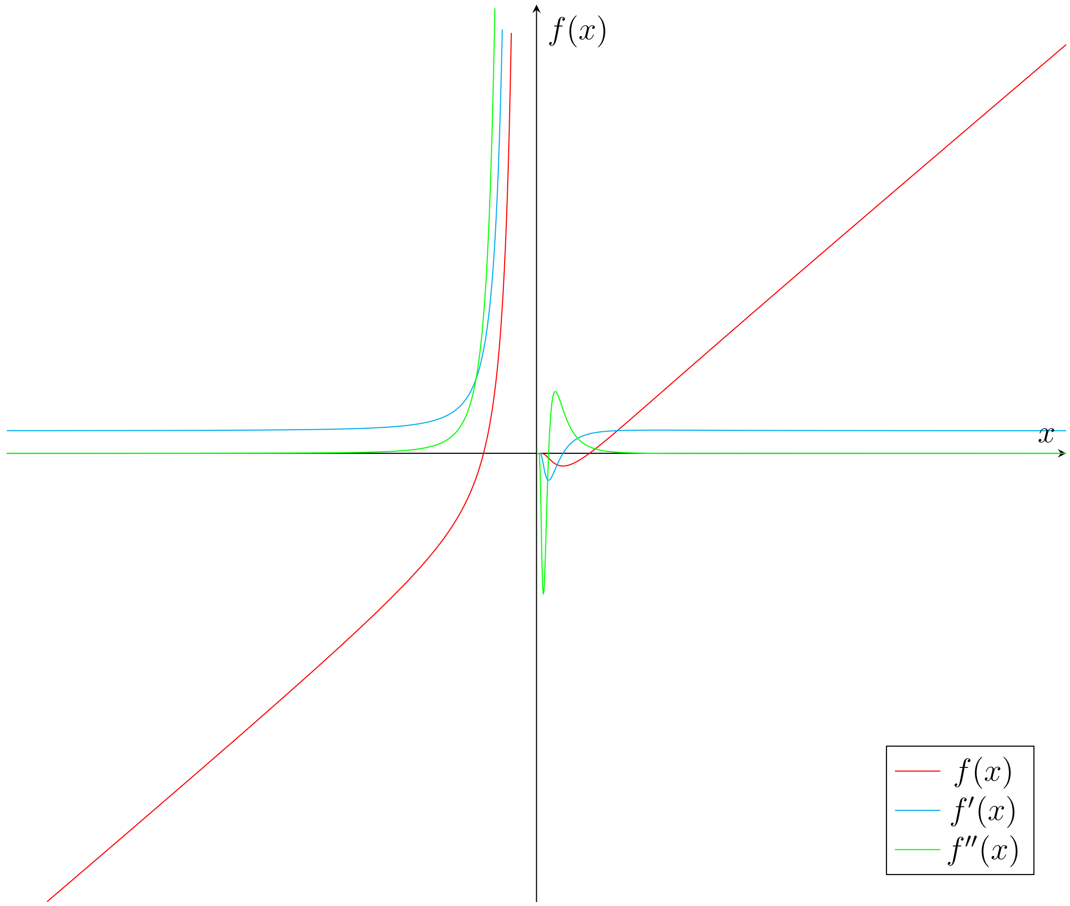
$$((T_1(f))(x_2))(x) = f(x_2) + f'(x_2)(x-x_2)$$

$$((T_1(f))(1))(x) = f(1) + f'(1)(x-a) = f(1) + 0 = -\frac{3}{e^{\frac{5}{3}}} \approx -0.5666268085$$

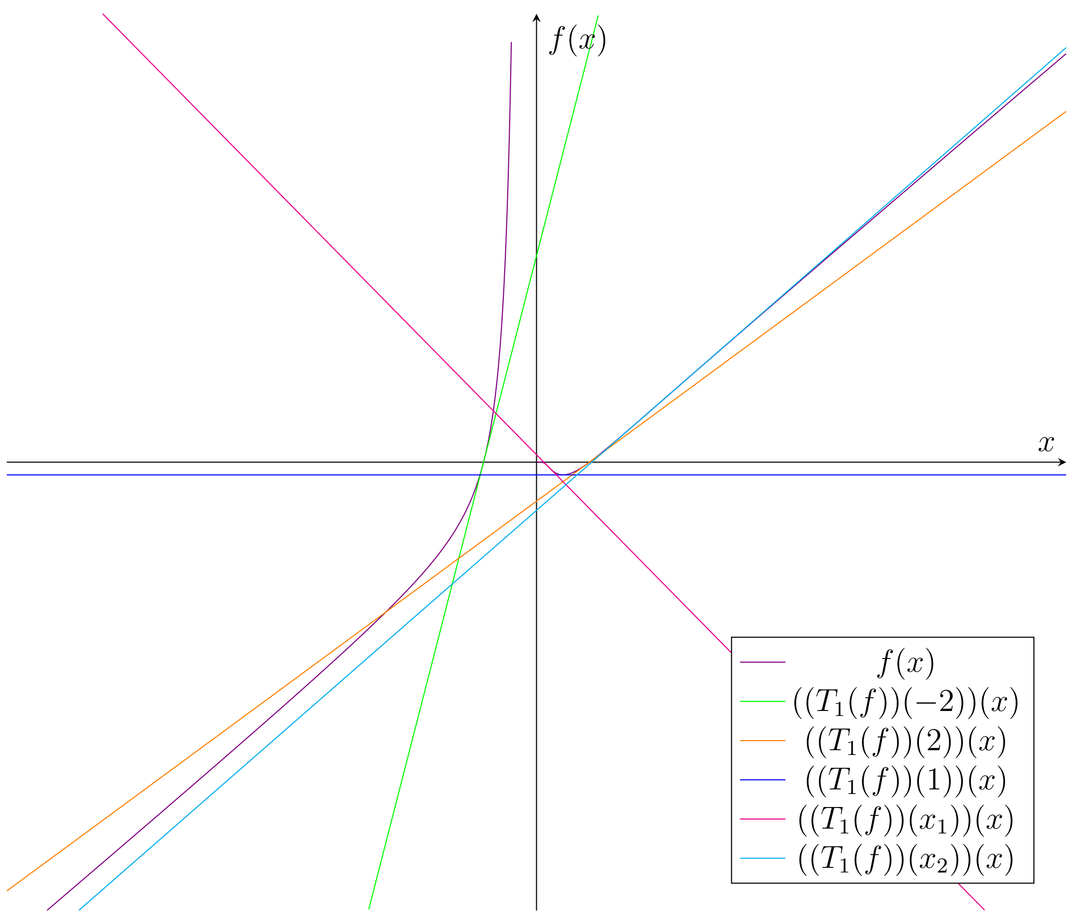
$$((T_1(f))(-2))(x) = f(-2) + f'(-2)(x+2) = 0 + (1+1-\tfrac{5}{6}+\tfrac{5}{6})e^{\frac{5}{6}} = 2e^{\frac{5}{6}}(x+2)$$

$$((T_1(f))(2))(x) = f(2) + f'(2)(x-2) = 0 + (1+1+\tfrac{5}{6}-\tfrac{5}{6})e^{\frac{5}{6}} = 2e^{-\frac{5}{6}}(x-2)$$

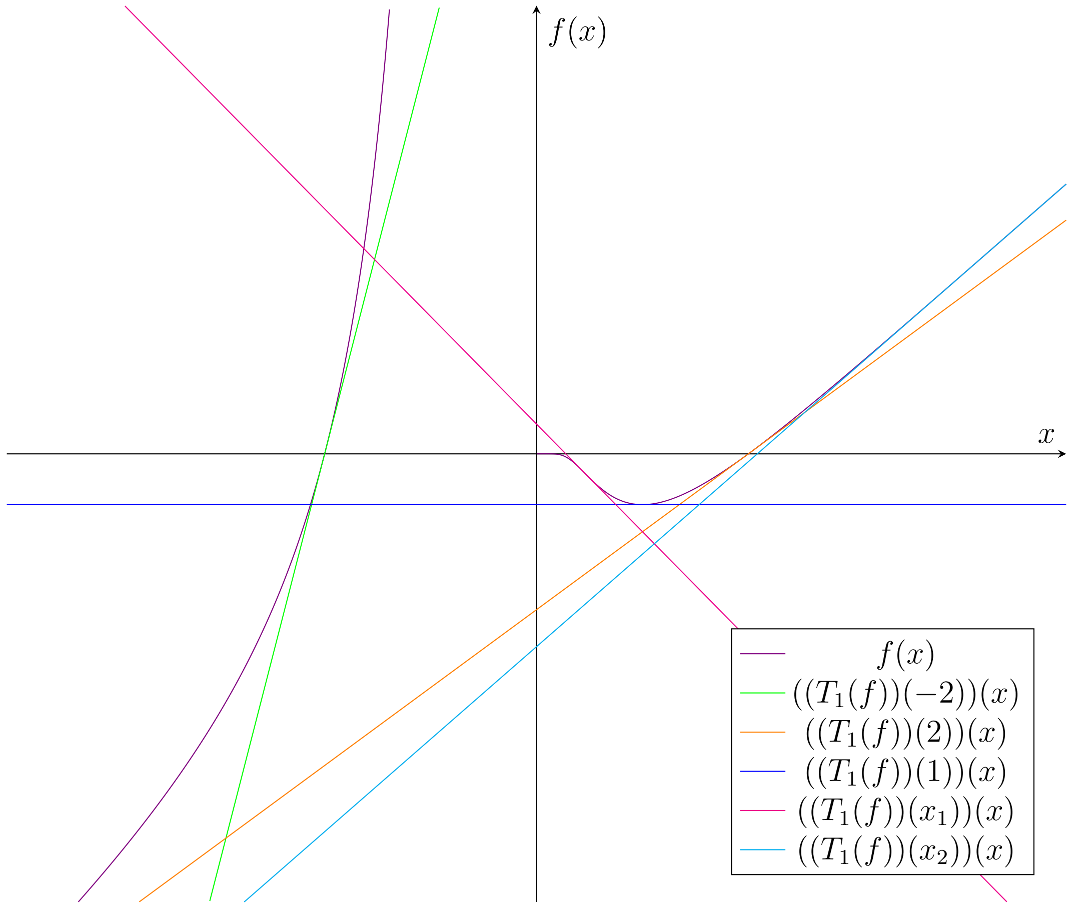




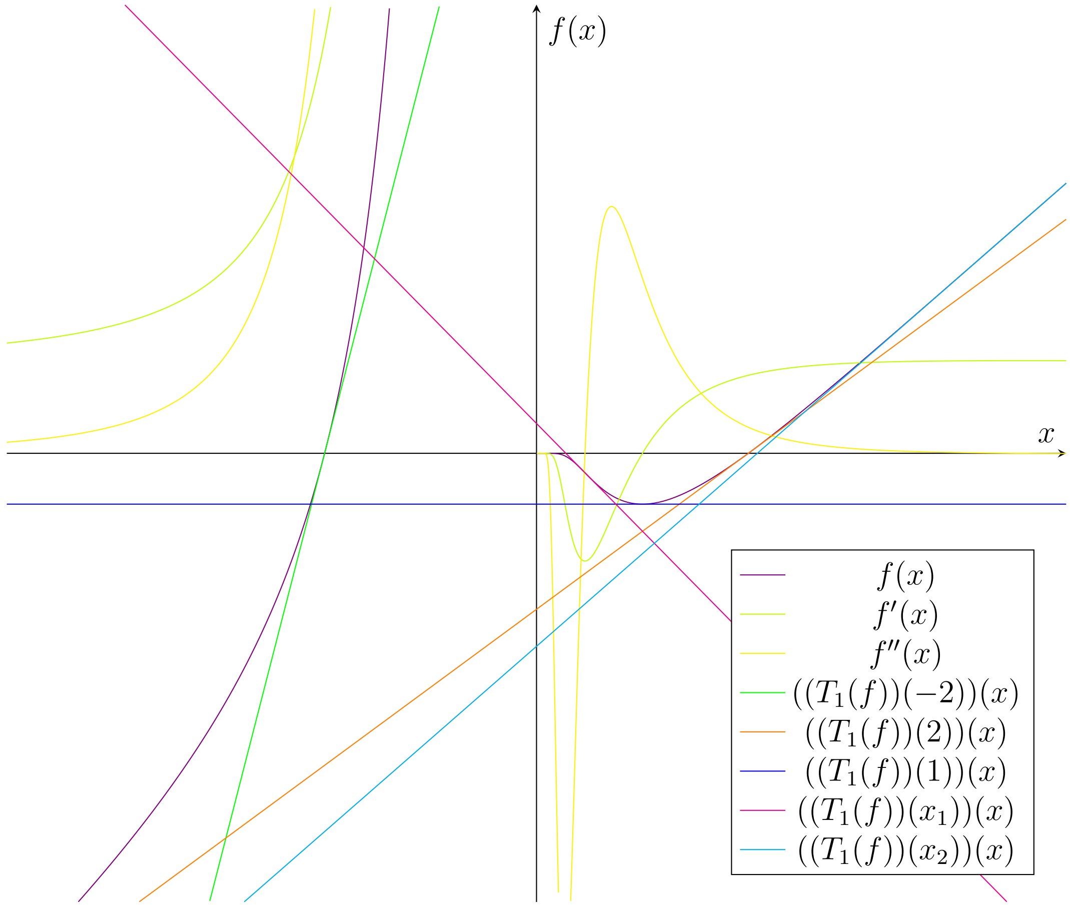
$f(x)$
x
$f(x)$
$f'(x)$
$f''(x)$



$f(x)$
x
$f(x)$
$((T_1(f))(-2))(x)$
$((T_1(f))(2))(x)$
$((T_1(f))(1))(x)$
$((T_1(f))(x_1))(x)$
$((T_1(f))(x_2))(x)$



$f(x)$
x
$f(x)$
$((T_1(f))(-2))(x)$
$((T_1(f))(2))(x)$
$((T_1(f))(1))(x)$
$((T_1(f))(x_1))(x)$
$((T_1(f))(x_2))(x)$


$f(x)$
x
$f(x)$
$f'(x)$
$f''(x)$
$((T_1(f))(-2))(x)$
$((T_1(f))(2))(x)$
$((T_1(f))(1))(x)$
$((T_1(f))(x_1))(x)$
$((T_1(f))(x_2))(x)$