Домашна работа 3, №45342, Martin, 1, I, Информатика

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1 Задача 1.

$$\begin{vmatrix} -1 & -5 & -5 & -5 & -5 & \dots & -5 \\ -8 & 9+n.1 & 9 & 9 & 9 & \dots & 9 \\ -8 & 9 & 9+1.2 & 9 & 9 & \dots & 9 \\ -8 & 9 & 9 & 9+2.3 & 9 & \dots & 9 \\ -8 & 9 & 9 & 9 & 9+3.4 & \dots & 9 \\ & & & \dots & & & \\ -8 & 9 & 9 & 9 & 9 & \dots & 9+(n-1).n \end{vmatrix} =$$

$$= \begin{vmatrix} -1 & -5 & -5 & -5 & -5 & \dots & -5 \\ -\frac{49}{5} & n.1 & 0 & 0 & 0 & \dots & 0 \\ -\frac{49}{5} & 0 & 1.2 & 0 & 0 & \dots & 0 \\ -\frac{49}{5} & 0 & 0 & 2.3 & 0 & \dots & 0 \\ -\frac{49}{5} & 0 & 0 & 0 & 3.4 & \dots & 0 \\ & & & \dots & & & \\ -\frac{49}{5} & 0 & 0 & 0 & 0 & \dots & (n-1).n \end{vmatrix} = \Delta$$

$$\varsigma_{ij} = \begin{cases} 1 & i \geq j \\ 0 & i < j \end{cases}$$

$$\Delta = \begin{vmatrix} -1-49.(\frac{1}{n}+\varsigma_{n2}\sum_{i=2}^{n}\frac{1}{(i-1)i}) & -5 & -5 & -5 & -5 & \dots & -5 \\ 0 & n.1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1.2 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 2.3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 3.4 & \dots & 0 \\ & & & \dots & & & \\ 0 & 0 & 0 & 0 & 0 & \dots & (n-1).n \end{vmatrix}$$

$\Delta = [-1-49.(\frac{1}{n}+\varsigma_{n2}\sum_{i=2}^{n}\frac{1}{(i-1)i})]n!^2$

$\forall n \in \mathbb{N} \quad \frac{1}{n}+\varsigma_{n2}\sum_{i=2}^{n}\frac{1}{(i-1)i} \stackrel{?}{=} 1$

$n=1 \implies \frac{1}{1}+0=1$

$n=2 \implies \frac{1}{2}+\frac{1}{1.2}=\frac{2}{2}=1$

$n=k \implies \frac{1}{k}+\varsigma_{k2}\sum_{i=2}^{k}\frac{1}{(i-1)i}=1$

$n=k+1 \implies \frac{1}{k+1}+\varsigma_{(k+1)2}\sum_{i=2}^{k+1}\frac{1}{(i-1)i}=$

$=\frac{1}{k+1}+\varsigma_{k2}\sum_{i=2}^{k}\frac{1}{(i-1)i}+\frac{1}{k(k+1)}=$

$=\frac{1}{k+1}-\frac{1}{k}+\frac{1}{k}+\varsigma_{k2}\sum_{i=2}^{k}\frac{1}{(i-1)i}+\frac{1}{k(k+1)}=$

$=\frac{1}{k+1}-\frac{1}{k}+1+\frac{1}{k(k+1)}=\frac{k-(k+1)+k(k+1)+1}{k(k+1)}=$

$=\frac{(k+1)-(k+1)+k(k+1)}{k(k+1)}=\frac{k(k+1)}{k(k+1)}=1$

$\implies \forall n \in \mathbb{N} \quad \frac{1}{n}+\varsigma_{n2}\sum_{i=2}^{n}\frac{1}{(i-1)i}=1$

$\implies \Delta = (-1-49)n!^2=-50.n!^2$

2 Задача 2.

$A = M_e(\Lambda)$

$$A = \begin{pmatrix} -12 & 2 & 3 \\ 6 & -13 & -6 \\ -9 & 6 & 0 \end{pmatrix}$$

$$f_A(\lambda) = \begin{vmatrix} -12-\lambda & 2 & 3 \\ 6 & -13-\lambda & -6 \\ -9 & 6 & -\lambda \end{vmatrix} = \begin{vmatrix} -12-\lambda & 2 & 3 \\ 6 & -13-\lambda & -6 \\ 3+\lambda & 4 & -3-\lambda \end{vmatrix} =$$

$$= \begin{vmatrix} -12-\lambda & 2 & 3 \\ 9+\lambda & -9-\lambda & -9-\lambda \\ 3+\lambda & 4 & -3-\lambda \end{vmatrix} = (9+\lambda) \begin{vmatrix} -12-\lambda & 2 & 3 \\ 1 & -1 & -1 \\ 3+\lambda & 4 & -3-\lambda \end{vmatrix} =$$

$$= (9+\lambda) \begin{vmatrix} -10-\lambda & 0 & 1 \\ 1 & -1 & -1 \\ 7+\lambda & 0 & -7-\lambda \end{vmatrix} = (9+\lambda)(7+\lambda) \begin{vmatrix} -10-\lambda & 0 & 1 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix} =$$

$$= (9+\lambda)(7+\lambda) \begin{vmatrix} -9-\lambda & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = (9+\lambda)^2(7+\lambda) \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} =$$

$$= (9+\lambda)^2(7+\lambda) \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -(9+\lambda)^2(7+\lambda)$$

$$f_A(\lambda) = 0 = -(9+\lambda)^2(7+\lambda)$$

$$\implies \lambda_{1,2} = -9,\ \lambda_3 = -7$$

$$\lambda_{1,2} = -9$$

$$\begin{pmatrix} -3 & 2 & 3 \\ 6 & -4 & -6 \\ -9 & 6 & 9 \end{pmatrix} \to \begin{pmatrix} -3 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -3 & 2 & 3 \end{pmatrix}$$

$v_1 = (1,0,1)$
$v_2 = (2,3,0)$

$$\begin{pmatrix} -5 & 2 & 3 \\ 6 & -6 & -6 \\ -9 & 6 & 7 \end{pmatrix} \to \begin{pmatrix} -5 & 2 & 3 \\ 1 & -1 & -1 \\ -9 & 6 & 7 \end{pmatrix} \to \begin{pmatrix} -3 & 0 & 1 \\ 1 & -1 & -1 \\ -3 & 0 & 1 \end{pmatrix} \to$$

$$\to \begin{pmatrix} -3 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -3 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix} \to \begin{pmatrix} -3 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}$$

$v_3 = (1,-2,3)$

$v_1 = (1, 0, 1)$
$v_2 = (2, 3, 0)$
$v_3 = (1, -2, 3)$

$$M_v(\Lambda) = \begin{pmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -7 \end{pmatrix}$$

3 Задача 3.

$\mathbb{V} = \mathbb{R}^2$ е линейното пространство над $\mathbb{R}$

$A = \begin{pmatrix} 7 & 6 \\ 6 & 9 \end{pmatrix} \in M_2(\mathbb{V}),\ v = (3, -1) \in \mathbb{V}$

$\forall x = (x_1, x_2),\ y = (y_1, y_2) \in \mathbb{V}$

$[x, y]_1 := xAy^t = 7x_1y_1 + 6x_2y_1 + 6x_1y_2 + 9x_2y_2$

$[x, y]_2 := -xAy^t = -7x_1y_1 - 6x_2y_1 - 6x_1y_2 - 9x_2y_2$

$[x, y] : \mathbb{V} \times \mathbb{V} \to \mathbb{R}$ е скаларно произведение ако:

1. $\forall a, b, c \in \mathbb{V}\ [a + b, c] = [a, c] + [b, c]$

2. $\forall a, b \in \mathbb{V}\ , \forall \lambda \in \mathbb{R}\ [\lambda a, b] = \lambda[a, b]$

3. $\forall a, b \in \mathbb{V}\ [a, b] = [b, a]$

4. $\forall a \in \mathbb{V}\ [a, a] \geq 0,\ [a, a] = 0 \iff a = \theta$

$[x, y]_2$ е скаларно произведение ако $\forall a \in \mathbb{V} \implies [a, a]_2 \geq 0$

$v \in \mathbb{V} \implies [v, v]_2$ трябва да е ≥ 0

$[v, v]_2 = -(7.9 - 6.3 - 6.3 + 9) = -(8.9 - 4.9) = -(4.9) = -36$

$[v, v]_2 < 0 \implies [x, y]_2$ не е скаларно произведение

$\forall a, b, c \in \mathbb{V}\ [a + b, c]_1 = (a + b)Ac^t = aAc^t + bAc^t$

$\forall a, b \in \mathbb{V}\ , \forall \lambda \in \mathbb{R}\ [\lambda a, b]_1 = (\lambda a)Ab^t = \lambda(aAb^t) = \lambda[a, b]_1$

Свойства 1. и 2. са вярни от свойства на матриците

$\forall a = (a_1, a_2), b = (b_1, b_2) \in \mathbb{V}\ [b, a]_1 = 7b_1a_1 + 6b_2a_1 + 6b_1a_2 + 9b_2a_2 =$

$= 7a_1b_1 + 6a_1b_2 + 6a_2b_1 + 9a_2b_2 = 7a_1b_1 + 6a_2b_1 + 6a_1b_2 + 9a_2b_2 = [a, b]_1$

Свойство 3. е вярно от факта, че $\mathbb{V}$ е Л.П. над $\mathbb{R}$, което е числово поле

$\forall a = (a_1, a_2) \in \mathbb{V}\ [a, a]_1 = 7a_1a_1 + 6a_2a_1 + 6a_1a_2 + 9a_2a_2 =$

$= 7a_1^2 + 12a_1a_2 + 9a_2^2 = G(a)$

$a = \theta \implies G(a) = 0$

$a \neq \theta$, Б.О.О. $a_2 \neq 0$

$7a_1^2 + 12a_1a_2 + 9a_2^2 = 0 \,|\, \frac{1}{a_2^2}$

$7\left(\frac{a_1}{a_2}\right)^2 + 12\frac{a_1}{a_2} + 9 = 0$

$H(a) = 7\left(\frac{a_1}{a_2}\right)^2 + 12\frac{a_1}{a_2} + 9$

$D_H = 4.4.3.3 - 4.7.9 = 36(4-7) = -3.36 = -108$

$D_H < 0,\ 7 > 0 \implies \forall a;\ a_2 \neq 0\ H(a) > 0$

$\implies G(a) > 0\ \forall a;\ a_2 \neq 0$

$\implies \forall a = (a_1, a_2) \in \mathbb{V}\ [a, a]_1 \geq 0,\ [a, a]_1 = 0 \iff a = \theta$

$\implies \forall x,\ y \in \mathbb{V}\ [x, y]_1$ е скаларно произведение

$\implies [v, v]_1 = G(v) = 7.9 - 12.3 + 9 = 7.9 - 4.9 + 9 = (8-4)9 = 36$

$|v| = \sqrt{[v, v]_1} = \sqrt{36} = 6$

4 Задача 4.

$a_1 = (-22, 17, -27, -8), \quad a_2 = (-2, -6, 4, -4),$
$a_3 = (-15, -2, -6, -9), \quad a_4 = (-3, -1, -1, -3)$

$\mathbb{U} = l(a_1, a_2, a_3, a_4)$

$$
\begin{pmatrix} -22 & 17 & -27 & -8 \\ -2 & -6 & 4 & -4 \\ -15 & -2 & -6 & -9 \\ -3 & -1 & -1 & -3 \end{pmatrix}
\begin{array}{c} -1 \\ -\frac{1}{2} \\ -2 \\ -2 \end{array}
\to
\begin{array}{c} -4 \\ \\ -9 \\ -3 \end{array}
\begin{pmatrix} 22 & -17 & 27 & 8 \\ 1 & 3 & -2 & 2 \\ 30 & 4 & 12 & 18 \\ 6 & 2 & 2 & 6 \end{pmatrix}
\to
$$

$$
\to
\begin{pmatrix} 18 & -29 & 35 & 0 \\ 1 & 3 & -2 & 2 \\ 21 & -23 & 30 & 0 \\ 3 & -7 & 8 & 0 \end{pmatrix}
3
\to
\begin{array}{c} -6 \\ -1 \\ -7 \\ \\ \end{array}
\begin{pmatrix} 18 & -29 & 35 & 0 \\ 3 & 9 & -6 & 6 \\ 21 & -23 & 30 & 0 \\ 3 & -7 & 8 & 0 \end{pmatrix}
\to
$$

$$
\to
\begin{pmatrix} 0 & 13 & -13 & 0 \\ 0 & 16 & -14 & 6 \\ 0 & 26 & -26 & 0 \\ 3 & -7 & 8 & 0 \end{pmatrix}
\begin{array}{c} \frac{1}{13} \\ \frac{1}{2} \\ \frac{1}{26} \\ \\ \end{array}
\to
\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 8 & -7 & 3 \\ 0 & 1 & -1 & 0 \\ 3 & -7 & 8 & 0 \end{pmatrix}
\to
$$

$$
\to
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 8 & -7 & 3 \\ 0 & 1 & -1 & 0 \\ 3 & -7 & 8 & 0 \end{pmatrix}
\to
\begin{array}{c} -7 \\ \\ 8 \end{array}
\begin{pmatrix} 0 & 8 & -7 & 3 \\ 0 & 1 & -1 & 0 \\ 3 & -7 & 8 & 0 \end{pmatrix}
\to
$$

$$
\to
\begin{pmatrix} 0 & 1 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ 3 & 1 & 0 & 0 \end{pmatrix}
$$

$v_1 = (0, 1, 0, 3)$
$v_2 = (0, 1, -1, 0)$
$v_3 = (3, 1, 0, 0)$

$g_1 = v_1 = (0, 1, 0, 3), \quad (g_1, g_1) = 1 + 9 = 10$

$g_2 = v_2 + \lambda g_1 \;|(\quad , g_1)\; \lambda \in \mathbb{R}$

$0 = (v_2, g_1) + \lambda (g_1, g_1)$

$\lambda = -\frac{(v_2, g_1)}{(g_1, g_1)} = -\frac{1}{10}$

$\implies g_2 = (0, 1, -1, 0) + (0, -\frac{1}{10}, 0, -\frac{3}{10}) = (0, \frac{9}{10}, -1, -\frac{3}{10})$

$(g_2, g_2) = \frac{81}{100} + \frac{100}{100} + \frac{9}{100} = \frac{190}{100} = \frac{19}{10}$

$g_3 = v_3 + \mu_1 g_1 + \mu_2 g_2$

$g_3 = v_3 + \mu_1 g_1 + \mu_2 g_2 \;|(\quad , g_1)$

$0 = (v_3, g_1) + \mu_1 (g_1, g_1) + 0$

$\mu_1 = -\frac{(v_3,g_1)}{(g_1,g_1)} = -\frac{1}{10}$

$g_3 = v_3 + \mu_1 g_1 + \mu_2 g_2 \;|(\quad , g_2)$

$0 = (v_3, g_2) + 0 + \mu_2(g_2, g_2)$

$\mu_2 = -\frac{(v_3,g_2)}{(g_2,g_2)} = -\frac{9}{\cancel{10}}\frac{\cancel{10}}{19} = -\frac{9}{19}$

$\implies g_3 = (3,1,0,0) + -\frac{1}{10}(0,1,0,3) + -\frac{9}{19}(0, \frac{9}{10}, -1, -\frac{3}{10}) =$

$= (3,1,0,0) + (0, -\frac{1}{10}, 0, -\frac{3}{10}) + (0, -\frac{81}{190}, \frac{9}{19}, \frac{27}{190}) =$

$= (3, \frac{9}{10}, 0, -\frac{3}{10}) + (0, -\frac{81}{190}, \frac{9}{19}, \frac{27}{190}) = (3, \frac{9.19-81}{190}, \frac{9}{19}, \frac{27-3.19}{190}) =$

$= (3, \frac{90}{190}, \frac{9}{19}, -\frac{30}{190}) = (3, \frac{9}{19}, \frac{9}{19}, -\frac{3}{19})$

$b_1 = g_1 = (0,1,0,3)$

$b_2 = 10g_2 = (0, 9, -10, -3)$

$b_3 = \frac{19}{3}g_3 = (19, 3, 3, -1)$

$a = (1, 7, -15, -5) \in \mathbb{R}^4$

$a_0 \in \mathbb{U},\ h \in \mathbb{U}^\perp$

$dimU = 3,\ \mathbb{U}^\perp = \{\forall v \in \mathbb{R}^4 \,|\, \forall u \in \mathbb{U}\ (u, v) = 0\}$

$\forall u \in \mathbb{U}\ \exists \lambda_i \in \mathbb{R},\ i = 1,2,3\,;\ u = \sum_{i=1}^{3} \lambda_i b_i,\ u \neq \theta$

$\forall u' \in \mathbb{U}\ \exists \mu_i \in \mathbb{R},\ i = 1,2,3\,;\ u' = \sum_{i=1}^{3} \mu_i b_i$

$(u, u') = \sum_{i=1}^{3} \lambda_i \mu_i (b_i, b_i),\ (u, u') = 0 \iff u' = \theta$

$\implies \mathbb{U}^\perp = \{\mathbb{R}^4 \cap \mathbb{U}\} \cup \{\theta\},\ \mathbb{U} < \mathbb{R}^4 \implies dim\mathbb{U}^\perp = 4 - 3 + 0 = 1$

$\implies \mathbb{U}^\perp \cap \mathbb{U} = \{\{\mathbb{R}^4 \cap \mathbb{U}\} \cup \{\theta\}\} \cap \mathbb{U} = \{\theta\}$

$\implies R^4 = \mathbb{U} \oplus \mathbb{U}^\perp$

$\implies a = a_0 + h \implies h = a - a_0$

$a_0 \in \mathbb{U} \implies \exists \rho_i \in \mathbb{R},\ i = 1,2,3\,;\ a_0 = \sum_{i=1}^{3} \rho_i b_i$

$h \in \mathbb{U}^{\perp} \implies (h, b_i) = 0,\ i = 1, 2, 3$

$\implies (a - a_0, b_i) = 0,\ i = 1, 2, 3$

$\implies (a - \sum_{j=1}^{3} \rho_j b_j, b_i) = 0,\ i = 1, 2, 3$

$\implies (a, b_i) - \sum_{j=1}^{3} \rho_j (b_j, b_i) = 0,\ i = 1, 2, 3$

$\implies \rho_i = \frac{(a,b_i)}{(b_i,b_i)},\ i = 1, 2, 3\ ((b_j, b_i) = 0 \iff i \neq j,\ j = 1, 2, 3)$

$\rho_1 = \frac{(a,b_1)}{(b_1,b_1)} = -\frac{8}{10} = -\frac{4}{5}$

$\rho_2 = \frac{(a,b_2)}{(b_2,b_2)} = \frac{7.9+10.15+3.5}{81+100+9} = \frac{228}{190} = \frac{12}{10} = \frac{6}{5}$

$\rho_3 = \frac{(a,b_3)}{(b_3,b_3)} = \frac{19+21-45+5}{19^2+9+9+1} = \frac{40-40}{361+19} = \frac{0}{380} = 0$
$\implies a_0 = -\frac{4}{5} b_1 + \frac{6}{5} b_2 = -\frac{4}{5}(0, 1, 0, 3) + \frac{6}{5}(0, 9, -10, -3) =$

$= (0, -\frac{4}{5}, 0, -\frac{12}{5}) + (0, \frac{54}{5}, -\frac{60}{5}, -\frac{18}{5}) = (0, \frac{50}{5}, -\frac{60}{5}, -\frac{30}{5}) =$

$= (0, 10, -12, -6)$

$\implies h = a - a_0 = (1, 7, -15, -5) - (0, 10, -12, -6) = (1, -3, -3, 1)$

5 Задача 5.

$M_e(\varphi) = A$

$$A = \begin{pmatrix} 1 & -9 & 3 \\ -9 & 1 & -3 \\ 3 & -3 & -7 \end{pmatrix}$$

$f_{\varphi}(\lambda) = f_A(\lambda) = -\lambda^3 + trA\lambda^2 - (\Delta_{23} + \Delta_{13} + \Delta_{12})\lambda + detA$

$trA = 1 + 1 - 7 = -5$

$$\Delta_{23} = \begin{vmatrix} 1 & -3 \\ -3 & -7 \end{vmatrix} = -7 - 9 = -16$$

$$\Delta_{13} = \begin{vmatrix} 1 & 3 \\ 3 & -7 \end{vmatrix} = -7 - 9 = -16$$

$$\Delta_{12} = \begin{vmatrix} 1 & -9 \\ -9 & 1 \end{vmatrix} = 1 - 81 = -80$$

$$detA = \begin{vmatrix} 1 & -9 & 3 \\ -9 & 1 & -3 \\ 3 & -3 & -7 \end{vmatrix} = -7 + 9.9 + 9.9 - 9 - 9 + 7.9.9 =$$

$= 9^3 - 5^2 = (9.3)^2 - 5^2 = (27-5)(27+5) = 22.32 = 704$

$\implies f_\varphi(\lambda) = -\lambda^3 + (-5)\lambda^2 - (-16-16-80)\lambda + 704 =$

$= -\lambda^3 - 5\lambda^2 + 112\lambda + 704$

Потенциални корени на f_φ :

$\pm 1,\ \pm 2,\ \pm 4,\ \pm 8,\ \pm 11,\ \pm 16,\ \pm 22,\ \pm 32,\ \pm 44,\ \pm 64,\ \pm 88,\ \pm 176,\ \pm 352,\ \pm 704$

$f_\varphi(1) = -1 - 5 + 112 + 704 \neq 0$

$f_\varphi(-1) = 1 - 5 - 112 + 704 \neq 0$

$f_\varphi(2) = -8 - 20 + 242 + 704 \neq 0$

$f_\varphi(-2) = 8 - 20 - 242 + 704 \neq 0$

$f_\varphi(4) = -64 - 80 + 448 + 704 \neq 0$

$f_\varphi(-4) = 64 - 80 - 448 + 704 \neq 0$

$f_\varphi(8) = -64.8 - 5.64 + 112.8 + 704 \neq 0$

$f_\varphi(-8) = 64.8 - 5.64 - 112.8 + 704 = 8.64 - 5.64 - 14.64 + 11.64 = (19-19).64 = 0$

$\implies \lambda_1 = -8 \implies (\lambda + 8)$ е делител на $f_\varphi(\lambda)$

$$
\begin{array}{l}
-\lambda^3 - 5\lambda^2 + 112\lambda + 704 : \lambda + 8 = -\lambda^2 + 3\lambda + 88 \\
- \\
-\lambda^3 - 8\lambda^2 \\
\hline
\qquad 3\lambda^2 + 112\lambda + 704 \\
\qquad - \\
\qquad 3\lambda^2 + 24\lambda \\
\hline
\qquad\qquad 88\lambda + 704 \\
\qquad\qquad - \\
\qquad\qquad 88\lambda + 704 \\
\hline
\qquad\qquad\quad 0
\end{array}
$$

$\implies f_\varphi(\lambda) = (\lambda + 8)(-\lambda^2 + 3\lambda + 88)$

$f_{\varphi_1}(\lambda) = -\lambda^2 + 3\lambda + 88$

$f_{\varphi_1}(-8) = -64 - 24 + 88 = 0$

$\implies \lambda_2 = -8$

$\lambda_2 + \lambda_3 = 3 \quad -8 + \lambda_3 = 3$

$\lambda_2.\lambda_3 = -88 \quad -8.\lambda_3 = -88$

$\implies \lambda_3 = 11$

$\lambda_{1,2} = -8$

$$\begin{pmatrix} 9 & -9 & 3 \\ -9 & 9 & -3 \\ 3 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & -3 & 1 \end{pmatrix}$$

$v_1 = (1, 0, -3)$

$v_2 = (0, 1, 3)$

$g_1 = v_1 = (0, 1, 3)$

$g_2 = v_2 + \mu g_1 \,|(\quad , g_1)$

$0 = (v_2, g_1) + \mu(g_1, g_1)$

$\mu = -\frac{(v_2, g_1)}{(g_1, g_1)} = -\frac{-9}{10} = \frac{9}{10}$

$g_2 = (0, 1, 3) + (\frac{9}{10}, 0, -\frac{27}{10}) = (\frac{9}{10}, 1, -\frac{3}{10})$

$\lambda_3 = 11$

$$\begin{pmatrix} -10 & -9 & 3 \\ -9 & -10 & -3 \\ 3 & -3 & -18 \end{pmatrix} \rightarrow \begin{pmatrix} -10 & -9 & 3 \\ -9 & -10 & -3 \\ 1 & -1 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -19 & 3 - 10.6 \\ 0 & -19 & -3 - 9.6 \\ 1 & -1 & -6 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -19 & -57 \\ 1 & -1 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & -3 \\ 1 & -1 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & -3 \\ 1 & 0 & -3 \end{pmatrix}$$

$v_3 = (3, -3, 1)$

$f_1 = (1, 0, -3), \quad e_1' = \frac{1}{|f_1|} f_1 = (\frac{1}{\sqrt{10}}, 0, -\frac{3}{\sqrt{10}})$

$f_2 = (9, 10, 3), \quad e_2' = \frac{1}{|f_2|} f_2 = (\frac{9}{\sqrt{190}}, \frac{10}{\sqrt{190}}, \frac{3}{\sqrt{190}})$

$f_3 = (3, -3, 1), \quad e_3' = \frac{1}{|f_3|} f_3 = (\frac{3}{\sqrt{19}}, -\frac{3}{\sqrt{19}}, \frac{1}{\sqrt{19}})$

$$M_{e'}(\varphi) = \begin{pmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$