Задача 4.

$$P : \begin{cases} x \in [2, 3] \\ y = x^2 \end{cases}$$

$l_P = \int_2^3 \sqrt{1 + (2x)^2}\,\mathrm{d}x =$

$= \int_2^3 \sqrt{1 + 4x^2}\,\mathrm{d}x =$

$= x\sqrt{1 + 4x^2}|_2^3 - \int_2^3 x\,\mathrm{d}\sqrt{1 + 4x^2} =$

$= 3\sqrt{1 + 36} - 2\sqrt{1 + 16} - \int_2^3 \frac{x4x\cancel{2}}{\cancel{2}\sqrt{1+4x^2}} =$

$= 3\sqrt{37} - 2\sqrt{17} - \int_2^3 \frac{4x^2+1-1}{\sqrt{1+4x^2}} =$

$= 3\sqrt{37} - 2\sqrt{17} - \int_2^3 \frac{\cancel{4x^2+1}}{\sqrt{\cancel{1+4x^2}}} + \int_2^3 \frac{1}{\sqrt{1+(2x)^2}}\,\mathrm{d}x =$

$= 3\sqrt{37} - 2\sqrt{17} - l_P + \frac{1}{2}\int_2^3 \frac{1}{\sqrt{1+(2x)^2}}\,\mathrm{d}x2x =$

$= 3\sqrt{37} - 2\sqrt{17} - l_P + \frac{1}{2}\ln|2x + \sqrt{1 + (2x)^2}||_2^3 =$

$= 3\sqrt{37} - 2\sqrt{17} - l_P + \frac{1}{2}\ln(6 + \sqrt{37}) - \frac{1}{2}\ln(4 + \sqrt{17}) \implies$

$2l_P = 3\sqrt{37} - 2\sqrt{17} + \frac{1}{2}\ln(6 + \sqrt{37}) - \frac{1}{2}\ln(4 + \sqrt{17}) \implies$

$l_P = \frac{3}{2}\sqrt{37} - \sqrt{17} + \frac{1}{4}\ln(6 + \sqrt{37}) - \frac{1}{4}\ln(4 + \sqrt{17})$