

# Домашна работа 3, № 45342, Група 3

Иво Стратев

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## 1 Задача 19

$$f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = \frac{x}{x^2+1}$$

$$\forall w \in \mathbb{R} \ w^2 + 1 > 0 \implies$$

$f$  е непрекъсната в  $\mathbb{R}$

$$f(x) = 0 \iff x = 0$$

### 1.1 Инекция

Нека  $x_1, x_2 \in \mathbb{R}, x_1 \neq x_2$

$$\text{Допс. } f(x_1) = f(x_2)$$

$$f(x_1) = \frac{x_1}{x_1^2+1}$$

$$f(x_2) = \frac{x_2}{x_2^2+1}$$

$$f(x_1) = f(x_2) \implies$$

$$\frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \implies$$

$$x_1(x_2^2 + 1) = x_2(x_1^2 + 1) \implies$$

$$x_1x_2^2 + x_1 = x_2x_1^2 + x_2 \implies$$

$$x_1x_2(x_2 - x_1) = x_2 - x_1 \iff x_2 - x_1 = 0 \implies$$

$$x_2 = x_1 \implies \nexists \implies f \text{ е инекция}$$

### 1.2 Сюрекция

$$\forall h \in \mathbb{R} \ h = \text{sign}(h)|h| \implies$$

$$f(x) = \frac{x}{x^2+1} = \frac{\text{sign}(x)|x|}{x^2+1}$$

$$\text{Нека } g : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} : g(x) = \frac{|x|}{x^2+1}$$

$$\forall u \in \mathbb{R} \setminus \{0\} \ |u| < u^2 + 1 \mid \frac{1}{u^2+1} \implies$$

$$\frac{|u|}{u^2+1} < 1 \mid \text{sign}(u) \implies$$

$$\begin{cases} \frac{u}{u^2+1} < 1, & u > 0 \\ \frac{u}{u^2+1} > -1, & u < 0 \end{cases} \implies$$

$$\begin{cases} f(x) < 1, & x > 0 \\ f(x) = 0, & x = 0 \\ f(x) > -1, & x < 0 \end{cases} \implies$$

$$\forall t \in \mathbb{R} \quad -1 < f(t) < 1 \implies$$

$$f \text{ не е сюрекция} \implies$$

$$f \text{ не е биекция}$$