1 Афинни операции с вектори

1.1 Вектор

$M(x_1, x_2, x_3),\ N(y_1, y_2, y_3)$
$\overrightarrow{MN}(y_1 - x_1, y_2 - x_2, y_3 - x_3)$
$|\overrightarrow{MN}| = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2}$

1.2 Умножение на вектор със скалар

$\overrightarrow{a}(a_1, a_2, a_3),\ |a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
$\lambda \in \mathbb{R}\ \overrightarrow{b} = \lambda\overrightarrow{a},\ \overrightarrow{b}(\lambda a_1, \lambda a_2, \lambda a_3)$
$|\overrightarrow{b}| = \sqrt{(\lambda a_1)^2 + (a_2\lambda)^2 + (a_3\lambda)^2} = \sqrt{\lambda^2(a_1^2 + a_2^2 + a_3^2)} = |\lambda||\overrightarrow{a}|$
$\overrightarrow{b} \uparrow\uparrow \overrightarrow{a},\ \lambda > 0$
$\overrightarrow{b} \uparrow\downarrow \overrightarrow{a},\ \lambda < 0$
$\overrightarrow{a} = \overrightarrow{MN},\ -\overrightarrow{a} = -\overrightarrow{MN} = \overrightarrow{NM},\ \lambda = -1$

1.3 Събиране на вектори

$\overrightarrow{AC}(a_1, a_2, a_3),\ \overrightarrow{CB}(b_1, b_2, b_3)$
$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{AB}$
$\overrightarrow{AB}(a_1 + b_1, a_2 + b_2, a_3 + b_3)$

1.4 Среда на отсечка

$A(a_1, a_2, a_3),\ B(b_1, b_2, b_3)$
$AM = MB = \frac{1}{2}AB$

$M(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2})$

1.5 Медицентър

$A(a_1, a_2, a_3),\ B(b_1, b_2, b_3),\ C(c_1, c_2, c_3)$

$M(\frac{a_1+b_1+c_1}{3}, \frac{a_2+b_2+c_2}{3}, \frac{a_3+b_3+c_3}{3})$

2 Скаларно произведение

$\overrightarrow{a}, \overrightarrow{b} \neq \overrightarrow{0}$

2.1

$\overrightarrow{a}\,\overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\cos(\overrightarrow{a}, \overrightarrow{b}) \in \mathbb{R}$

2.2

$\overrightarrow{a}\overrightarrow{b} = \overrightarrow{b}\overrightarrow{a}$

2.3

$(\overrightarrow{a} + \overrightarrow{b})\overrightarrow{c} = \overrightarrow{a}\overrightarrow{c} + \overrightarrow{b}\overrightarrow{c}$

2.4

$(\lambda\overrightarrow{a})\overrightarrow{b} = \lambda(\overrightarrow{a}\overrightarrow{b})$

2.5

$\overrightarrow{a}^2 = \overrightarrow{a}\overrightarrow{a} = |\overrightarrow{a}|^2$

2.6

$\overrightarrow{a}\overrightarrow{b} = 0 \iff \overrightarrow{a} \perp \overrightarrow{b}$

2.7

$\cos(\overrightarrow{a}, \overrightarrow{b}) = \frac{\overrightarrow{a}\overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|}$

2.8

$\overrightarrow{a}(a_1, a_2, a_3),\ \overrightarrow{b}(b_1, b_2, b_3) \implies \overrightarrow{a}\overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3$

3 Векторно произведение

$\overrightarrow{a}, \overrightarrow{b} \neq \overrightarrow{0} \implies \exists! \quad \overrightarrow{a} \times \overrightarrow{b}$

3.1

$|\overrightarrow{a} \times \overrightarrow{b}| = |\overrightarrow{a}||\overrightarrow{b}|\sin(\overrightarrow{a}, \overrightarrow{b})$

3.2

$\overrightarrow{a} \times \overrightarrow{b} \perp \overrightarrow{a},\ \overrightarrow{a} \times \overrightarrow{b} \perp \overrightarrow{b}$

3.3

$\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{a} \times \overrightarrow{b} \in S^+$

3.4 Свойства

3.4.1

$\overrightarrow{a} \times \overrightarrow{b} = -\overrightarrow{b} \times \overrightarrow{a}$

3.4.2

$(\overrightarrow{a} + \overrightarrow{b}) \times \overrightarrow{c} = \overrightarrow{a} \times \overrightarrow{c} + \overrightarrow{b} \times \overrightarrow{c}$

3.4.3

$(\lambda\overrightarrow{a}) \times (\mu\overrightarrow{b}) = \lambda\mu(\overrightarrow{a} \times \overrightarrow{b})$

3.4.4

$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{0} \iff \overrightarrow{a} \parallel \overrightarrow{b}$

3.4.5

Лице на успоредник, построен върху $\overrightarrow{a}, \overrightarrow{b}$ взети с общо начало

$S = |\overrightarrow{a} \times \overrightarrow{b}|$

3.4.6

Лице на триъгълник, построен върху $\overrightarrow{a}, \overrightarrow{b}$ взети с общо начало

$S = \frac{|\overrightarrow{a} \times \overrightarrow{b}|}{2}$

3.4.7

$\sin(\overrightarrow{a}, \overrightarrow{b}) = \frac{\overrightarrow{a} \times \overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|}$

3.4.8

$(\overrightarrow{a} \times \overrightarrow{b})(\overrightarrow{a} \times \overrightarrow{b}) = \overrightarrow{a}^2\overrightarrow{b}^2 - (\overrightarrow{a}\overrightarrow{b})^2$

3.4.9

$\overrightarrow{a}(a_1, a_2, a_3),\ \overrightarrow{b}(b_1, b_2, b_3)$

$\implies \overrightarrow{a} \times \overrightarrow{b} \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right)$

4 Двойно векторно произведение

4.1

$(\overrightarrow{a} \times \overrightarrow{b}) \times \overrightarrow{c} = \begin{vmatrix} \overrightarrow{a}\overrightarrow{c} & \overrightarrow{b}\overrightarrow{c} \\ \overrightarrow{a} & \overrightarrow{b} \end{vmatrix}$

4.2

$\overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c}) = \begin{vmatrix} \overrightarrow{a}\overrightarrow{c} & \overrightarrow{a}\overrightarrow{b} \\ \overrightarrow{c} & \overrightarrow{b} \end{vmatrix}$

5 Смесено произведение

$\overrightarrow{a}\overrightarrow{b}\overrightarrow{c} = (\overrightarrow{a} \times \overrightarrow{b})\overrightarrow{c} = \overrightarrow{a}(\overrightarrow{b} \times \overrightarrow{c}) \in \mathbb{R}$

5.1

$\overrightarrow{a}\overrightarrow{b}\overrightarrow{c} = 0 \iff \overrightarrow{a} \parallel \overrightarrow{b} \parallel \overrightarrow{c} \parallel \overrightarrow{a}$

5.2

$\overrightarrow{a}\overrightarrow{b}\overrightarrow{c} = \overrightarrow{b}\overrightarrow{c}\overrightarrow{a} = \overrightarrow{c}\overrightarrow{a}\overrightarrow{b}$

5.3

$\overrightarrow{a}\overrightarrow{b}\overrightarrow{c} = -\overrightarrow{b}\overrightarrow{a}\overrightarrow{c} = -\overrightarrow{a}\overrightarrow{c}\overrightarrow{b} = -\overrightarrow{c}\overrightarrow{b}\overrightarrow{a}$

5.4

$(\overrightarrow{a_1} + \overrightarrow{a_2})\overrightarrow{b}\overrightarrow{c} = \overrightarrow{a_1}\overrightarrow{b}\overrightarrow{c} + \overrightarrow{a_2}\overrightarrow{b}\overrightarrow{c}$

5.5

$(\lambda\overrightarrow{a})\overrightarrow{b}\overrightarrow{c} = \overrightarrow{a}(\lambda\overrightarrow{b})\overrightarrow{c} = \overrightarrow{a}\overrightarrow{b}(\lambda\overrightarrow{c}) = \lambda(\overrightarrow{a}\overrightarrow{b}\overrightarrow{c})$

5.6

Обем на паралепипед, построен върху $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ взети с общо начало

$V = |\overrightarrow{a}\overrightarrow{b}\overrightarrow{c}|$

5.7

Обем на тетраедър, построен върху $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ взети с общо начало

$V = |\frac{\overrightarrow{a}\overrightarrow{b}\overrightarrow{c}}{6}|$

5.8

$\overrightarrow{a}(a_1, a_2, a_3),\ \overrightarrow{b}(b_1, b_2, b_3),\ \overrightarrow{c}(c_1, c_2, c_3)$

$$\Longrightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

5.9

$$(\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c})^2 = \begin{vmatrix} \overrightarrow{a}^2 & \overrightarrow{a}\,\overrightarrow{b} & \overrightarrow{a}\,\overrightarrow{c} \\ \overrightarrow{b}\,\overrightarrow{a} & \overrightarrow{b}^2 & \overrightarrow{b}\,\overrightarrow{c} \\ \overrightarrow{c}\,\overrightarrow{a} & \overrightarrow{c}\,\overrightarrow{b} & \overrightarrow{c}^2 \end{vmatrix}$$

6 Права в равнина

$g : Ax + By + C = 0$ - Общо уравнение на права в равнина

6.1

$\overrightarrow{p}(-B, A) \parallel g$

6.2

$\overrightarrow{q}(A, B) \perp g$

6.3

$h \parallel g \iff h : Ax + By + C_h = 0$

6.4

$l \perp g \iff l : -Bx + Ay + C_l = 0$

6.5 Нормално уравнение на права в равнина

$g : \frac{Ax+By+C}{\sqrt{A^2+B^2}} = 0$

6.6 Разтояние от точка до права

$M(x_M, y_M) \implies d = \frac{Ax_M+By_M+C}{\sqrt{A^2+B^2}}$

6.7 уравнение на ъглополовиящи

$g_1 : A_1x + B_1y + C_1 = 0,\ g : A_2x + B_2y + C_2 = 0$

$\implies b_1, b_2 = \frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}} \pm \frac{A_2x+B_2y+C_2}{\sqrt{A_2^2+B_2^2}}$

6.8 Уравниение на права минаваща през две точки

$A(x_A, y_A),\ B(x_B, y_B)$

$$l : \begin{vmatrix} x - x_A & y - y_A \\ x_B - x_A & y - y_B \end{vmatrix} = 0$$