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# 1 Theorie

## 1.1 Operational Amplifier in General

The operational amplifier (=OA) is a DC-coupled difference amplifier and a general circuit is shown in (@@).

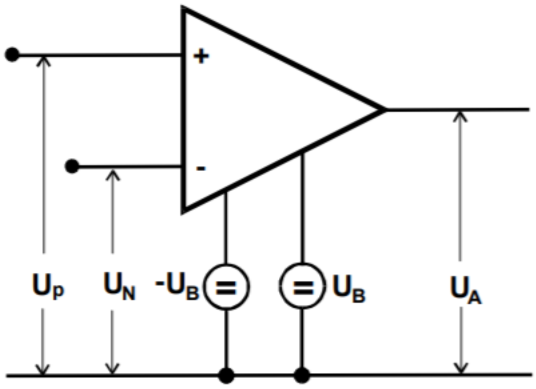


**Figure 1:** General principle of an operational amplifier (operating voltages $\pm U_B$ are often left out)

[1]

With the amplification value $V$, the output voltage is

$$U_A = V(U_p - U_N) \tag{1}$$

as long as it does not exceed the operating voltage $\pm U_B$. If it does, then $U_A = \pm U_B$ applies, depending on whether the negative or positive value is exceeded. $V$ usually is a very large value, so minimal differences of the input voltages are enough to exceed the operating range $\pm U_B$. Furthermore $U_A$ is in phase with the "+" - input making this the non-inverting input, and out of phase with the "-" - input making this the inverting input.
For an ideal OA some characteristics are: infinite neutral amplification $V$ and input resistances $r_e$, and an output resistance $r_a$ of zero. However for the real OA these values do not apply, but instead $V$ is large compared to one and depends on the frequency, $r_e$ take up large and $r_a$ small values. For an almost-ideal OA these conditions may result only in small discrepancies on measurements compared to idealistic calculations.

## 1.2 Linear Amplifier

To get an useful operating range for the OA a negative-feedback-branch is needed, meaning that some part of $U_A$ is send back to the inverting input. In figure (2) a linear amplifier is shown. The negative feedback reduces the total amplification of the circuit but expands the operating range. Calculations for an ideal OA ($V = \infty = r_e$), using

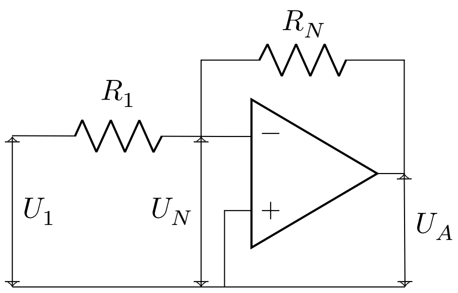


**Figure 2:** Negative feedback inverting linear amplifier

Kirchhoff's current law, lead to

$$\frac{U_1}{R_1} + \frac{U_A}{R_N} = 0 \tag{2}$$

$$V' = -\frac{R_N}{R_1} \; . \tag{3}$$

However, the non-idealistic characteristics will have a small influence on this equation. If the impact of a finite neutral amplification $V$ on the amplification value $V'$ is calculated through with equation (1) and figure (**??**) a connection between $V$ and $V'$ is found

$$\frac{1}{V'} \approx \frac{1}{V} + \frac{R_1}{R_N} \; . \tag{4}$$

Now it becomes clear that for $V \gg \frac{R_N}{R_1}$ equations (3) and (4) are about the same. Concluding this, the larger the negative feedback the smaller the amplification value and the more precisely equation (3) applies. Also this leads to the OA working more stable at lower amplifications.
In figure (3) the frequency responses between an OA with and without negative feedback are compared to visualise the expanded bandwith by negative feedback.

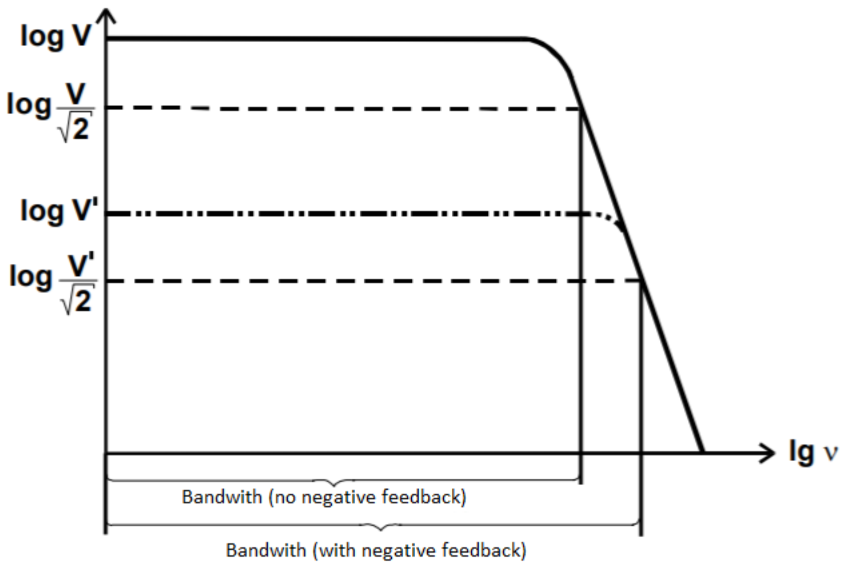


**Figure 3:** Frequency response of the linear amplifier
[1]

Furthermore the product of bandwith and $V'$ is a constant and describes the transit frequency where the amplification drops down to 1. This value is independent of the negative feedback.

## 1.3 Electrometer Amplifier

To measure high-resistance voltage sources a large input resistance $r_e$ is needed. Because that is not the case for the linear amplifier, where $r_e \approx R_1$ ($U_N \approx 0$), the circuit is modified to the electrometer amplifier. Here the input voltage is directly connected to the non-inverting input of the OA. For the ideal OA is $r_e = \infty$ and for the real OA $r_e \approx 20\,\mathrm{G\Omega}$.

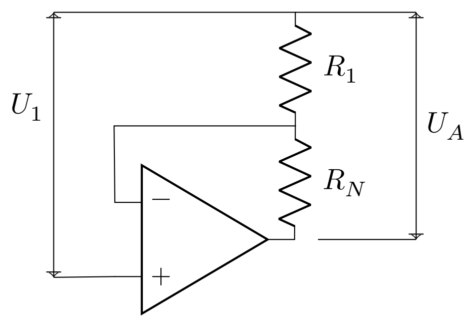


**Figure 4:** Non-inverting electrometer amplifier.

For an ideal electrometer amplifier the amplification is

$$V' = \frac{R_N + R_1}{R_1}$$

### 1.4 Inverting Integrator

With the circuit in figure (5) it is possible to integrate the input voltage $U_1$. Using Kirchhoff's law on branch point $A$, that

$$\int I_C dt = CU_A$$

and that

$$U_1 = U_0 \mathrm{sin}\omega t$$

it follows for the output voltage

$$U_A = \frac{U_0}{\omega RC} \mathrm{cos}\omega t \ .$$

This means the output voltage is antiproportional to the frequency.

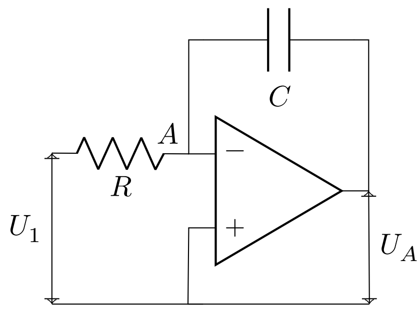


**Figure 5:** Inverting integrator

### 1.5 Inverting Differentiator

With the same considerations as with the inverting integrator, for the inverting differentiator follows the output voltage

$$U_A = -\omega RCU_0 \mathrm{cos}\omega t$$

with $U_A$ being proportional to the frequency.

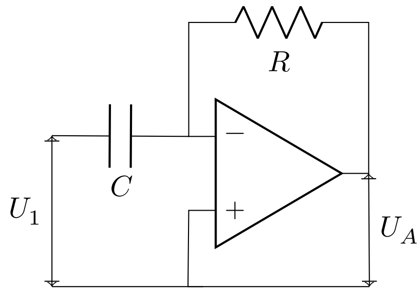


**Figure 6:** Inverted differentiator

### 1.6 Schmitt-Trigger

In contrast to the previous amplifiers the Schmitt-Trigger uses positive feedback. In figure (7) the circuit is shown. If a certain input voltage is reached the output voltage will jump immediately to $\pm U_B$, depending on whether the input voltage is positive or negative. This certain voltage is adjusted by the ratio of the resistances and it follows

$$U_A = \begin{cases} +U_B & : U_1 > +\frac{R_1}{R_P}U_B \\ -U_B & : U_1 < -\frac{R_1}{R_P}U_B \end{cases}$$

Notice that as long as $U_1$ does not exceed one of the two trigger voltages $U_A$ will stay at its current value. The difference $2U_B\frac{R_1}{R_P}$ between the two trigger values is called switching hysteresis.

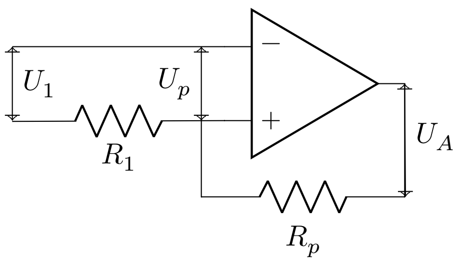


**Figure 7:** Schmitt-Trigger

## 2 Execution

For the first task the input- and output voltage in dependence of the frequency of a linear amplifier, as in figure (2), are measured with an oscilloscope for the two different resistance ratios $\frac{100\,\mathrm{k\Omega}}{10\,\mathrm{k\Omega}}$ and $\frac{20\,\mathrm{k\Omega}}{0.2\,\mathrm{k\Omega}}$. The frequency is varied from about $1\,\mathrm{kHz}$ to $1\,\mathrm{MHz}$.

Next, the terminal voltage of the used NF-Generator is measured firstly with the same linear amplifier used before and then with an additional resistance $R = 100\,\mathrm{k\Omega}$ switched in front of $R_1$. The same is done for an electrometer amplifier, as in figure(4). Both resistance ratios are $\frac{20\,\mathrm{k\Omega}}{0.2\,\mathrm{k\Omega}}$.

Then for the inverting differentiator and inverting differentiator, as in figure (5) and (6), the output voltage in dependence of the frequency is measured from 50 Hz to 1000 Hz. Additionally a sin-, triangular- and rectangular voltage are integrated/differentiated and thermal prints of input- and output voltage are taken with the oscilloscope. It is $C = 1\,\mathrm{\mu F}$ and $R = 200\,\Omega$.

For the last part a Schmitt-Trigger, as in figure (7), is build up and the trigger voltage for a resistance ratio of $\frac{10\,\mathrm{k\Omega}}{100\,\mathrm{k\Omega}}$ is measured, as well as the value $2U_B$. Adding to this a thermal print is taken.

Furthermore it is important to keep the input voltage as low as possible for the measurements, because the nearer the OA works at his voltage limits the lesser the precision of the measurement will be.

# 3 Evaluation

All calculations and plots are done by Python 3.6.1.

## 3.1 Inverting Linear Amplifier

Note that the used amplifier inverts the signal, transforming a positive input to a negative output and vice versa, but for this evaluation only the absolute amplification and voltages are used.
For the first part the frequency response of the input voltage $U_1$ for a resistance ratio of 10 and 100 were measured. The grade of amplification is adjusted by choosing a corresponding ratio of $\frac{\mathrm{R_N}}{\mathrm{R_1}}$ with $\mathrm{R_N} = 100\,\mathrm{k\Omega}$ and $\mathrm{R_1} = 10\,\mathrm{k\Omega}$ for a theoretical amplification of -10, and $\mathrm{R_N} = 20\,\mathrm{k\Omega}$ and $\mathrm{R_1} = 0.2\,\mathrm{k\Omega}$ for -100. These are theoretical because due to equation (4) the neutral amplification $V$ can have an impact on $V'$ and can be approximated with it. For this $V'$ is taken from the measurement at low frequencies. In tabular (1) and tabular (2) are the measured data of frequency $\nu$, input voltage $U_1$ and output voltage $U_A$.

**Table 1:** Data for $\frac{\mathrm{R_N}}{\mathrm{R_1}} = 10$

| $\nu$ / kHz | $U_1$ / mV | $U_A$ / V |
|---|---|---|
| 1.50 | 125 | 1.25 |
| 15.00 | 269 | 2.65 |
| 20.00 | 265 | 2.61 |
| 22.00 | 265 | 2.57 |
| 30.00 | 265 | 2.53 |
| 40.00 | 265 | 2.45 |
| 47.00 | 265 | 2.41 |
| 50.00 | 265 | 2.37 |
| 55.44 | 265 | 2.29 |
| 62.24 | 265 | 2.17 |
| 67.55 | 265 | 2.09 |
| 73.78 | 261 | 1.93 |
| 79.16 | 261 | 1.85 |
| 85.42 | 261 | 1.73 |
| 91.59 | 261 | 1.65 |
| 96.81 | 261 | 1.57 |
| 102.00 | 261 | 1.49 |
| 115.00 | 261 | 1.37 |
| 129.00 | 261 | 1.25 |
| 141.00 | 261 | 1.13 |
| 160.00 | 261 | 1.01 |
| 176.00 | 261 | 0.92 |
| 207.00 | 261 | 0.80 |
| 230.00 | 261 | 0.72 |
| 259.00 | 261 | 0.64 |
| 309.00 | 261 | 0.56 |
| 369.00 | 261 | 0.48 |
| 471.00 | 261 | 0.40 |
| 572.00 | 261 | 0.36 |
| 607.00 | 261 | 0.32 |
| 718.00 | 261 | 0.28 |
| 945.00 | 261 | 0.28 |
| 1,000.00 | 261 | 0.28 |

**Table 2:** Data for $\frac{\mathrm{R_N}}{\mathrm{R_1}} = 100$

| $\nu$ / kHz | $U_1$ / mV | $U_A$ / V |
|---|---|---|
| 0.20 | 23.0 | 2.00 |
| 2.00 | 23.0 | 2.00 |
| 5.00 | 23.3 | 1.77 |
| 10.00 | 24.1 | 1.55 |
| 15.00 | 25.3 | 1.27 |
| 20.00 | 25.3 | 1.07 |
| 25.00 | 25.3 | 0.88 |
| 30.00 | 25.3 | 0.72 |
| 35.00 | 25.3 | 0.64 |
| 40.00 | 25.3 | 0.57 |
| 45.00 | 25.3 | 0.51 |
| 50.00 | 25.3 | 0.47 |
| 55.00 | 25.7 | 0.43 |
| 60.00 | 25.7 | 0.40 |
| 65.00 | 25.7 | 0.35 |
| 70.00 | 25.7 | 0.33 |
| 75.00 | 25.7 | 0.31 |
| 90.00 | 25.7 | 0.26 |
| 110.00 | 25.7 | 0.22 |
| 130.00 | 25.7 | 0.19 |
| 180.00 | 25.7 | 0.13 |
| 250.00 | 25.7 | 0.10 |
| 330.00 | 25.7 | 0.07 |
| 450.00 | 25.7 | 0.06 |
| 560.00 | 25.7 | 0.04 |
| 700.00 | 25.7 | 0.04 |
| 800.00 | 25.7 | 0.03 |
| 900.00 | 25.7 | 0.03 |
| 950.00 | 25.7 | 0.03 |
| 1,000.00 | 25.7 | 0.03 |

Looking at the values for the low frequencies, it can be seen that $V'_{10} = 10$ and $V'_{100} = \frac{2}{0.023}$. Therefore $V'_{10} = \frac{\mathrm{R_N}}{\mathrm{R_1}} = \frac{100\,\mathrm{k\Omega}}{10\,\mathrm{k\Omega}}$ satisfies for low frequencies, but $V'_{100} = \frac{\mathrm{R_N}}{\mathrm{R_1}} = \frac{20\,\mathrm{k\Omega}}{0.2\,\mathrm{k\Omega}}$ does not. Now with equation (4) $V$ can be estimated using $V'_{100} = \frac{2}{0.023}$ and $\frac{\mathrm{R_N}}{\mathrm{R_1}} = 100$ to

$$V = 666,67\ .$$

In figure (8) and figure (9) is a graphical representation of the data. The fit curve is a fit

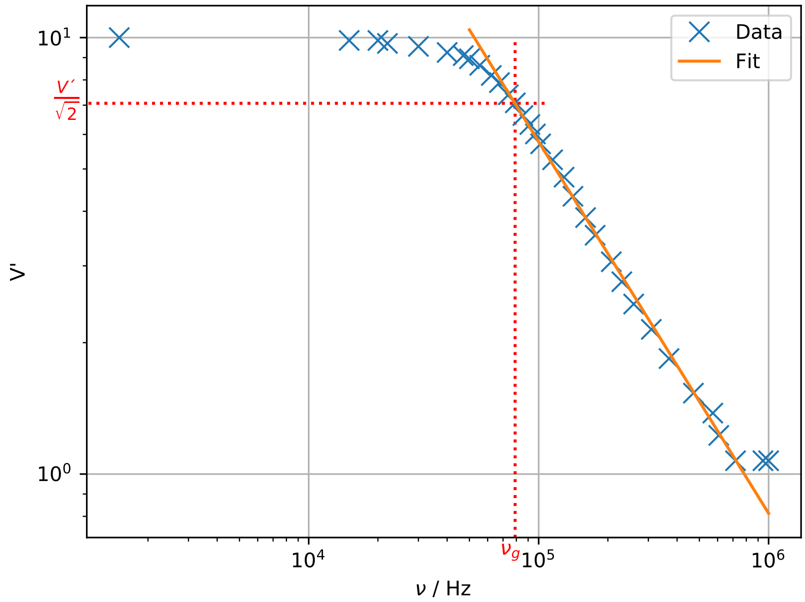


**Figure 8:** Graphical representation of frequency response for $\frac{\mathrm{R_N}}{\mathrm{R_1}} = 10$ $(V'_{10})$

of the function

$$f(x) = a \cdot x^b$$

and the parameters for $V'_{10}$ are

$$a = (1.05 \pm 0.11) \cdot 10^5$$
$$b = -0.852 \pm 0.009\ .$$

In addition the value for $\frac{V'_{10}}{\sqrt{2}} = \frac{10}{\sqrt{2}}$ and its corresponding frequency $\nu'_{g10} = (7.9 \pm 1.4) \cdot 10^4 Hz$ are marked to compare these to the values for $V'_{100}$.

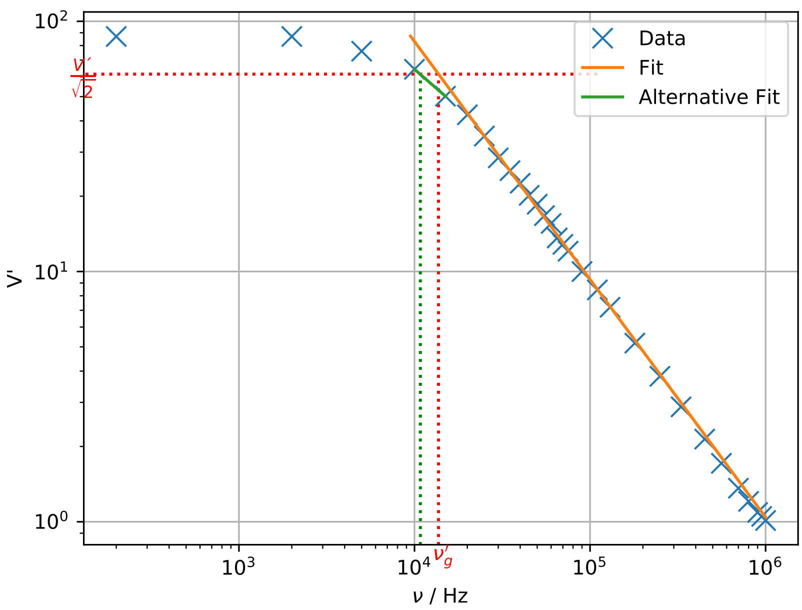


**Figure 9:** Graphical representation of frequency response for $\frac{\mathrm{R_N}}{\mathrm{R_1}} = 100$ ($V'_{100}$)

The parameters for $V'_{100}$ are

$$a = (5.3 \pm 0.5) \cdot 10^5$$
$$b = -0.951 \pm 0.009 \ .$$

Here the marked value for $\frac{V'_{100}}{\sqrt{2}} = \frac{2}{0.023 \cdot \sqrt{2}}$ with its frequency $\nu'_{g100} = (1.37 \pm 0.18) \cdot 10^4 Hz$ can now be compared to the previous ones to verify the condition of the product of bandwidth and $V'$ being constant.

$$\nu'_{g10} V'_{10} = (7.9 \pm 1.4) \cdot 10^5 Hz$$
$$\nu'_{g100} V'_{100} = (1.19 + / - 0.16) \cdot 10^6 Hz$$
$$\Delta = (51 \pm 31)\%$$

Looking at figure (9) it is clear, that the intersections of $\frac{V'_{100}}{\sqrt{2}}$ with the fitted curve and the direct connection between the data (visualized in green) are about 3 kHz apart. This is why an alternative calculation $\nu'_{g100(alt)} V'_{100}$ with $\nu'_{g100(alt)} = 10.8\,\mathrm{kHz}$ (approximated through the graph) is done.

$$\nu'_{g10} V'10 = (7.9 \pm 1.4) \cdot 10^5 Hz$$
$$\nu'_{g100(alt)} V'100 = 9.39 \cdot 10^5 Hz$$
$$\Delta_{alt} = (19 \pm 21)\%$$

### 3.2 Terminal Voltage

The terminal voltage is measured with the inverting linear amplifier, $\nu = 100\,\mathrm{Hz}$ and a resistance ratio of $\frac{20\,\mathrm{k\Omega}}{0.2\,\mathrm{k\Omega}} = 100$ to If a resistance of $100\,\mathrm{k\Omega}$ is switched in front of the amplifier, then the resistance ratio changes to $\frac{20\,\mathrm{k\Omega}}{100.2\,\mathrm{k\Omega}} = 100$ and it follows

$$U_T R = 15.7\,\mathrm{mV}$$

For the same procedure for the non-inverting electrometer (same resistance ratio and frequency) the results are

$$U_T = 3.8\,\mathrm{V}$$
$$U_{TR} = 3.8\,\mathrm{V}\ .$$

Note that the absolute values are different due to the amplification done by the oscilloscope to get a visible signal without too much distortion, and only the relation between the values is important. The additional resistance has no effect on the electrometer but changes the outcome on the linear amplifier completely. This is because the linear amplifier has a low input resistance ($\mathrm{R}_1$) and switching an additional one in series changes the total input resistance a lot. Therefore linear amplifier is not suitable for measuring high-resistance voltage sources. However the electrometer has a relative high input resistance ($20\,\mathrm{G\Omega}$) and does not have these problems and can be used for measuring high-resistance voltage sources.

### 3.3 Inverting Integrator

In tabular (3) are the measured data for the inverting integrator.

**Table 3:** Measurement Data for the inverting integrator

| $\nu$ / Hz | $U_A$ / V |
|---|---|
| 50 | 3.14 |
| 70 | 2.25 |
| 90 | 1.81 |
| 110 | 1.49 |
| 130 | 1.29 |
| 150 | 1.13 |
| 170 | 1.01 |
| 190 | 0.92 |
| 250 | 0.72 |
| 300 | 0.6 |
| 400 | 0.52 |
| 500 | 0.44 |
| 700 | 0.36 |
| 1,000 | 0.32 |
| 3,000 | 0.32 |

In figure (10) the data is visualized and the condition of $U_A \sim \frac{1}{\omega}$ can be verified. The operating range ends approximately where the output voltage reaches a plateau value at about $\nu = 1\,\mathrm{k\Omega}$.

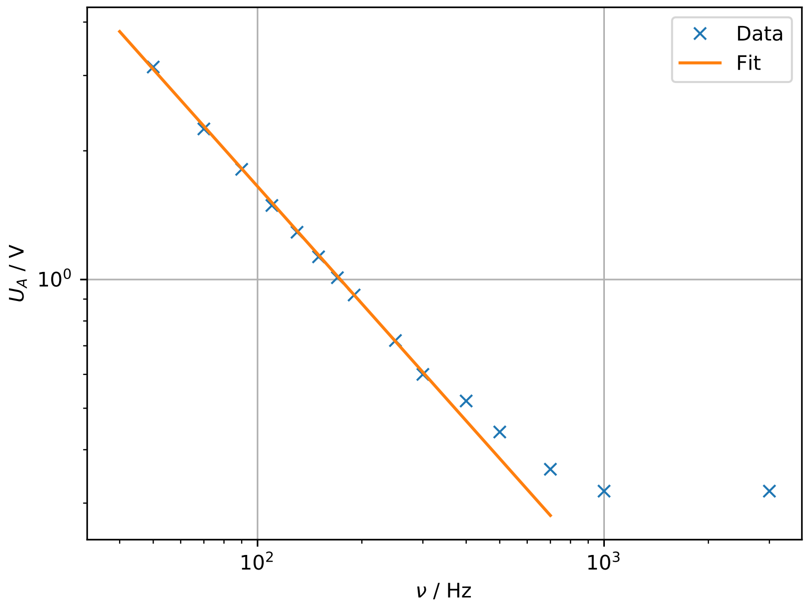


**Figure 10:** Frequency response of the inverting integrator where $U_A \sim \frac{1}{\omega}$

In figure (11), (12) and (13) is a visualisation of how the inverting integrator works for a sin voltage, triangular voltage and rectangular voltage.

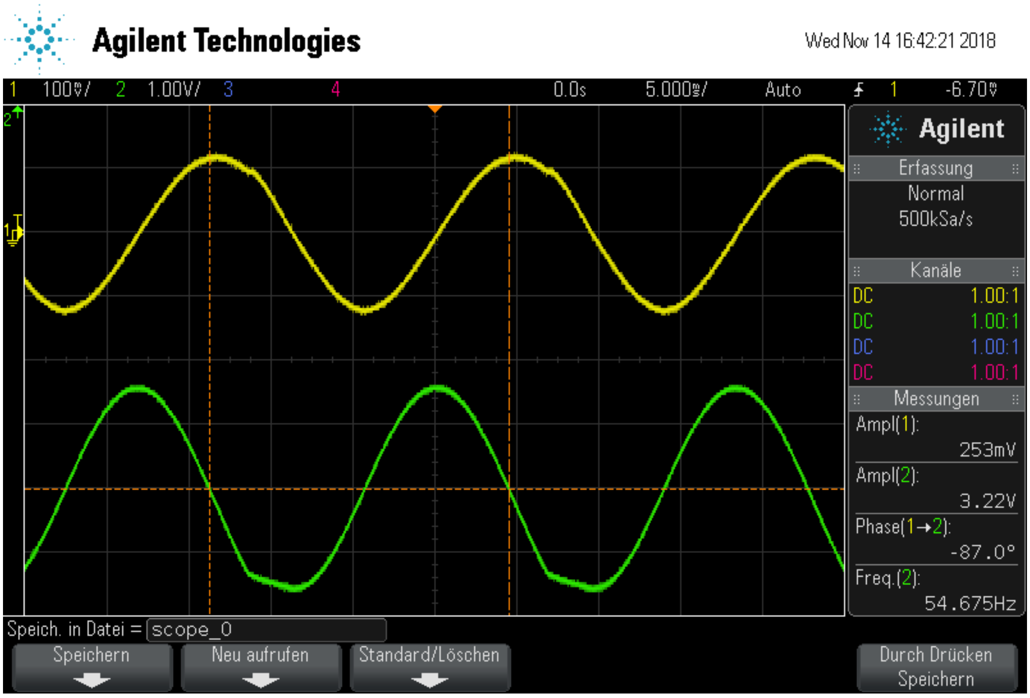


**Figure 11:** Sin voltage integrated

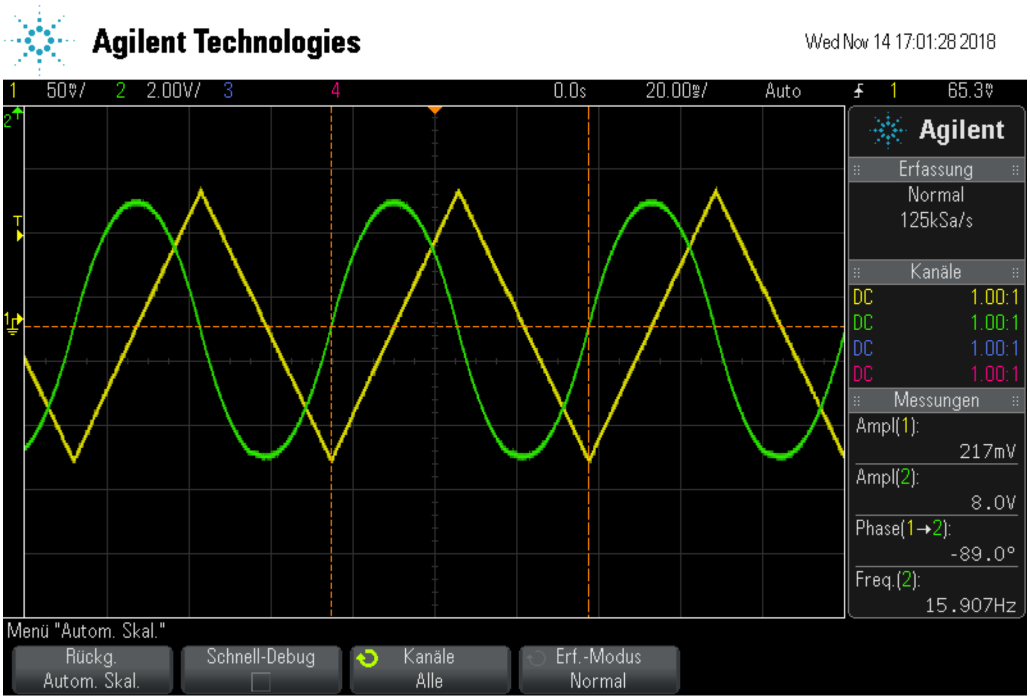


**Figure 12:** Triangular voltage integrated

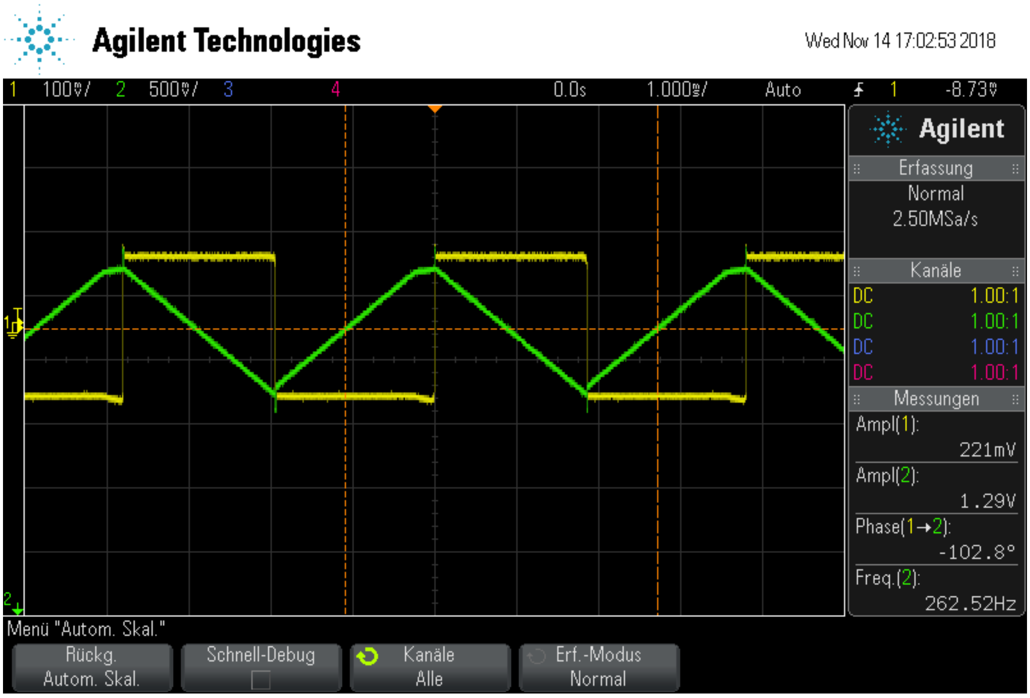


**Figure 13:** Rectangular voltage integrated

## 3.4 Inverting Differentiator

In tabular (4) are the measured data for the inverting differentiator.

**Table 4:** Measurement Data for the inverting differentiator

| $\nu$ / Hz | $U_A$ / mV |
|---|---|
| 50 | $9.4 \cdot 10^{-2}$ |
| 70 | 0.139 |
| 90 | 0.159 |
| 110 | 0.205 |
| 130 | 0.237 |
| 150 | 0.269 |
| 170 | 0.306 |
| 190 | 0.338 |
| 210 | 0.366 |
| 230 | 0.406 |
| 250 | 0.422 |
| 270 | 0.458 |
| 300 | 0.507 |
| 350 | 0.591 |
| 400 | 0.71 |
| 500 | 0.9 |
| 750 | 1.33 |
| 1,000 | 1.71 |

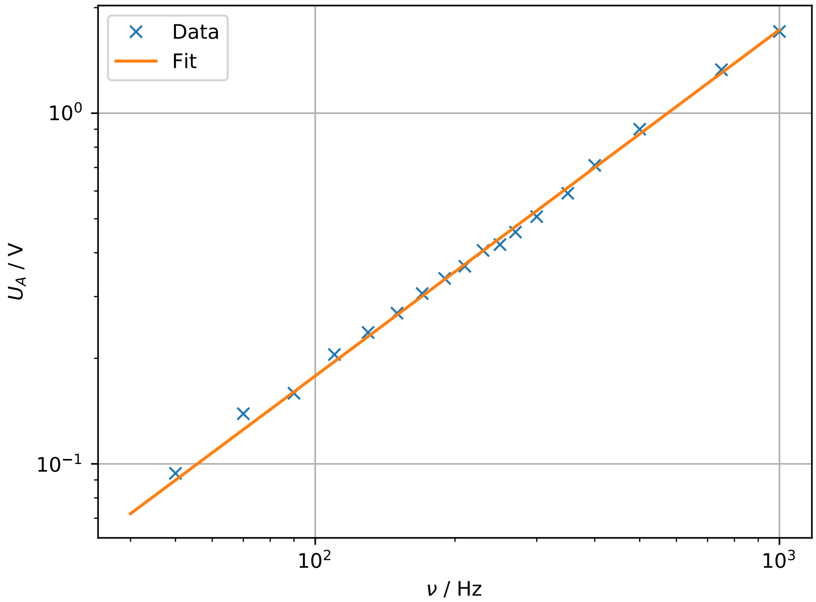


**Figure 14:** Frequency response of the inverting differentiator where $U_A \sim \omega$

As no meaningful discrepancy of the fitted curve and the data points is seen, the condition

$U_A \sim \omega$ applies for the whole operating range that has been examined.
In figure (15), (16) and (17) is a visualisation of how the inverting differentiator works for a sin voltage, triangular voltage and rectangular voltage.

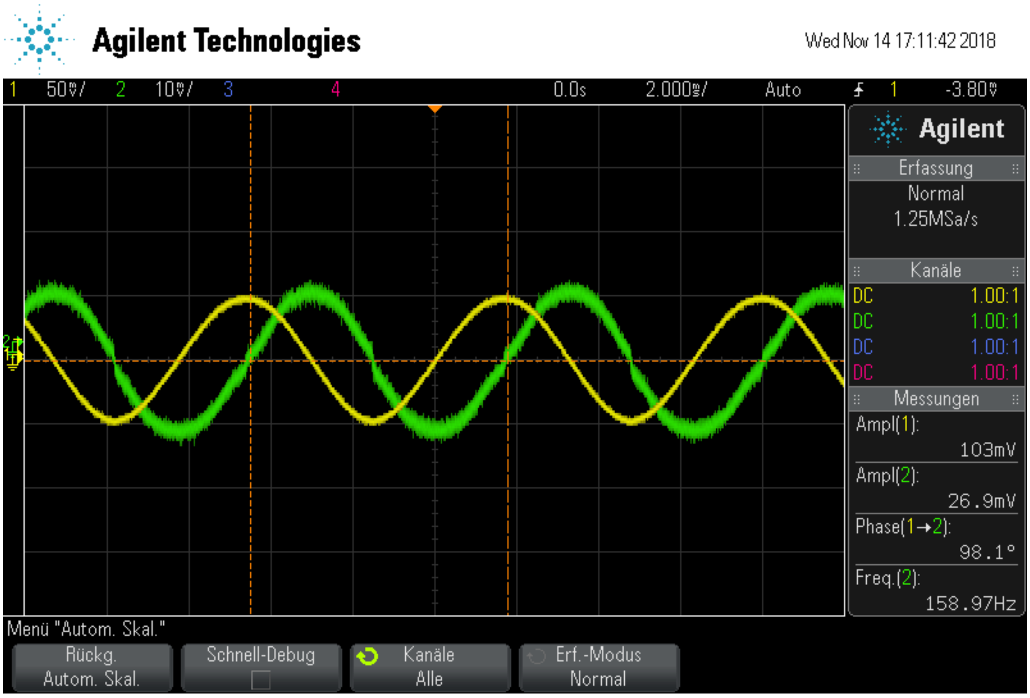


**Figure 15:** Sin voltage differentiated

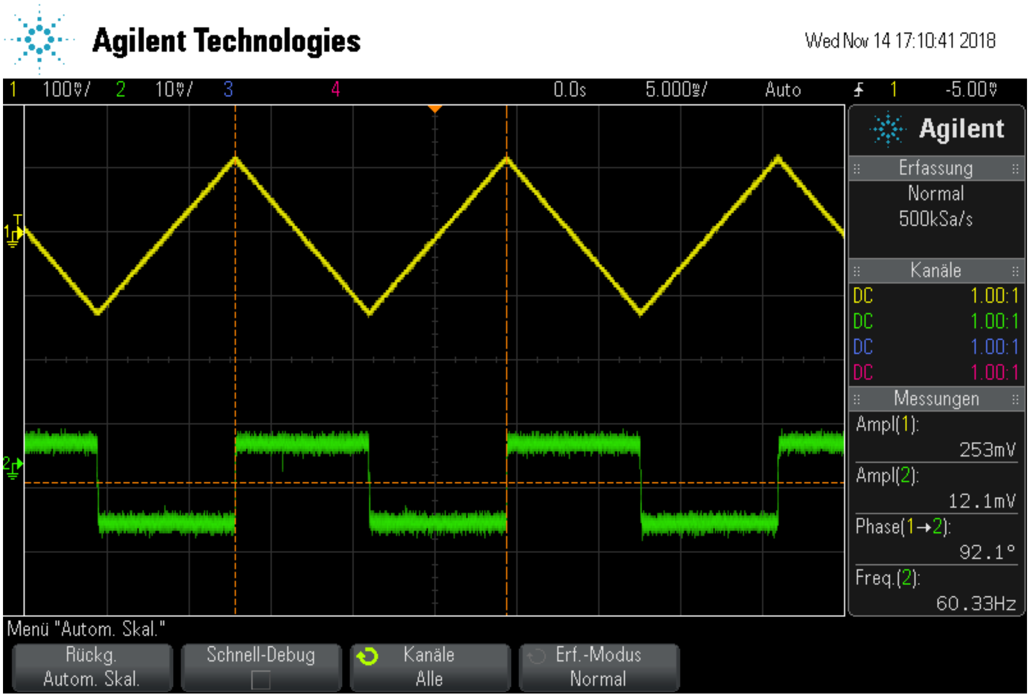


**Figure 16:** Triangular voltage differentiated

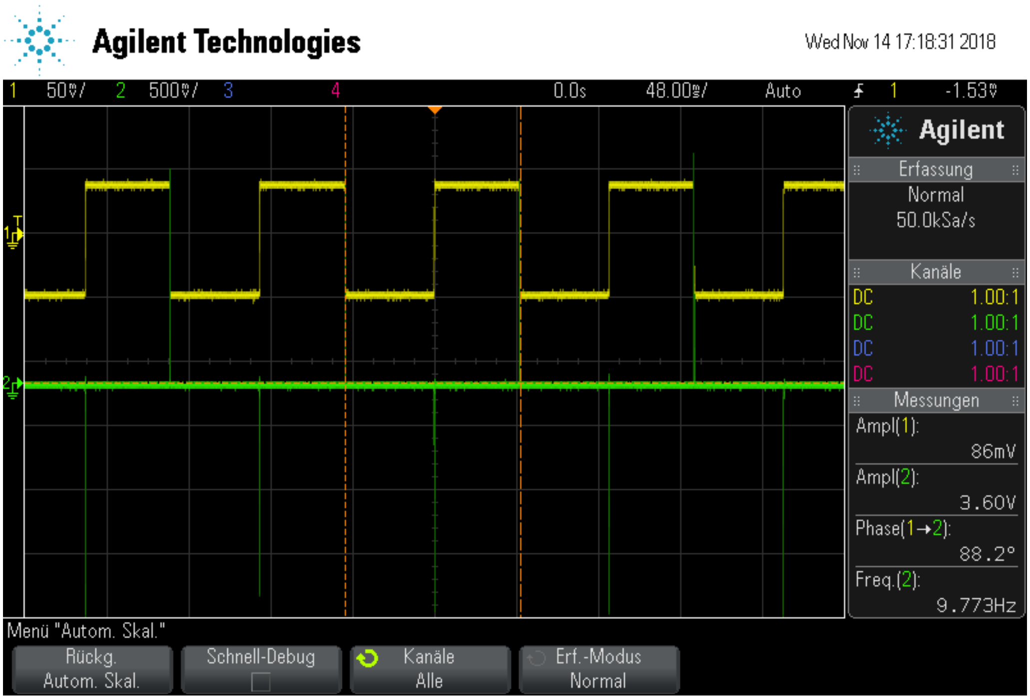


**Figure 17:** Rectangular voltage differentiated (notice the green spikes)

### 3.5 Schmitt-Trigger

For a resistance ratio of $\frac{\mathrm{R}_1}{\mathrm{R}_\mathrm{P}} = \frac{10\,\mathrm{k\Omega}}{100\,\mathrm{k\Omega}}$ the measurement for the trigger voltage $U_{B(trigger)}$ and the voltage $2U_B$ gives

$$U_{B(trigger)} = 1.405\,\mathrm{V}$$
$$2U_B = 26.9\,\mathrm{V}\ .$$

The difference between $\frac{\mathrm{R}_1}{\mathrm{R}_\mathrm{P}} U_B = 1.345\,\mathrm{V}$ and $U_{B(trigger)}$ is 4.5 %. To see the Schmitt-Trigger in action the trigger effect is visualised in figure (18)

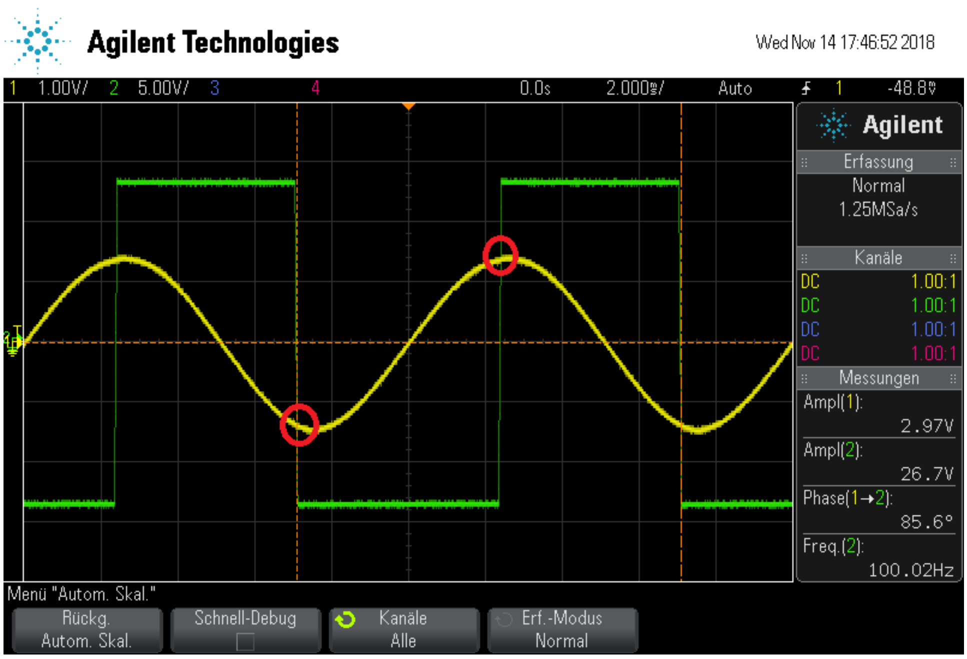


**Figure 18:** Visualisation of the trigger effect at a Schmitt-Trigger. (yellow=input, green=output)

The red circles mark the spots where the input voltage reaches $\pm\frac{\mathrm{R}_1}{\mathrm{R}_\mathrm{P}} U_B$. Notice that, once triggered, the output voltage keeps the same as long as the input voltage does not reach the opposing trigger voltage. Inbetween lies the switch hysterese.

## 4 Discussion

The frequency responses measured with the linear amplifier cover the expected form of the theory. For $V'_{10}$ at low frequencies the neutral amplification has to be very large, because the amplification of the measurement and the expected value from the resistance ratio are exactly the same. This means that the used precision of the measurement in millivolts was not enough to get a difference between the resistance ratio and $V'$, as it is supposed in equation (4). In contrast to that for $V'_{100}$ a neutral amplification of $V = 666.67$ was approximated. This value is quite low for the neutral amplification of an operation amplifier that usually are very large values of about $10^5$ or even higher. Therefore this is probably not a precise approximation.

The product of bandwidth and $V'$ is supposed to be constant, but had differences between the two measurements, respectively between the two fitted curves, of $\varDelta = (51 \pm 31)\%$. That is a relatively big range, caused by the errors of the fit, with a possible minimum of $\varDelta_{min} = 20\,\%$. Looking at the figure of $V'_{100}$ it was already mentioned, that the fitted curve does not cover the data in the inspected range that precisely.
To clarify that the real frequency $\nu'_g$ is probably shifted to the left the alternative calculation, where $\frac{V'}{\sqrt{2}}$ exactly cuts the direct connection between the data, lead to $\varDelta = (19 \pm 21)\%$. This is still a relatively big range but obviously closer to the condition of a constant product.
Further the product $\nu'_{g100}V'_{100} = (1.19 + / - 0.16) \cdot 10^6 Hz$ is the transit frequency, but looking at the graph it is clear that the fitted curve cuts the point where $V'_{100} = 1$ much more left. This is one more reason to expect the actual $\nu'_g$ to be shifted to the left.
For the transit frequency $\nu'_{g10}V'_{10} = (7.9 \pm 1.4) \cdot 10^5 Hz$ for $V'_{10}$ in contrast the calculated value and the graph cover each other nicely. Concluding this it is verified, that the operational amplifier works more stable at low amplifications.

The different measurements of the terminal voltage verify as expected, that the linear amplifier is not suited for measuring high-resistance voltage sources, as the output signal is easily influenced by changes of the input resistance. However the electrometer amplifier showed no difference, whether there was an additional input resistance or not, and therefore is suited much better for these kinds of measurement.

The conditions, $U_A \sim \frac{1}{\omega}$ for the inverting integrator and $U_A \sim \omega$ for the inverting differentiator, were both verified. For the inverting integrator the operating range could be limited to about $< 3\,\text{kHz}$ but for the inverting differentiator no such limit was found, probably because not enough frequency range was covered in the measurement.
The thermal prints of both show very nicely how the circuit is able to integrate/differentiate (and invert) different input signals.

For the last part with the Schmitt-Trigger, the trigger function, adjusted by the resistance ratio, and its relation to the operating voltage $\pm U_B$ were verified and the thermal print very nicely underlines and visualises the effect.

## References

[1] URL: `http://129.217.224.2/HOMEPAGE/PHYSIKER/BACHELOR/FP/SKRIPT/V51.pdf` (visited on 11/21/2018).