SESA6085 – Advanced Aerospace Engineering Management

Lecture 10

2024-2025

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Competing Risk Models

Competing Risk Models

- In some cases, it is not possible to use a single failure distribution to model failure data
- This can occur if a component fails through a number of different mechanisms
- Semi-conductors, for example, are affected by humidity which can cause failures due to:
  - Large increases in current
  - Mechanical stresses due to volume expansion

Competing Risk Models

- In such cases competing risk models can be used to model failure data
  - Also known as a compound model, series system model or multi-risk model
- Such models can only be used if:
  - The failure modes are completely independent
  - The component fails when the first mode is encountered
  - Each mode has it's own distribution

Competing Risk Models

- Let's construct such a model
- We have a component with n independent failure modes
- For each mode we have a distribution function, $F_i(t)$
- Failure of the component occurs when the component reaches a failed state for any mode
  – Does this sound familiar?
- Recall that this is identical to our definition of a series system

Competing Risk Models

- For a series system our probability of failure is given by:

$$P[X = 0] = 1 - P[X_1 = 1]P[X_2 = 1] \cdots P[X_n = 1]$$

- Hence the failure distribution of our competing risk model is given by:

$$F(t) = 1 - [1 - F_1(t)][1 - F_2(t)] \cdots [1 - F_n(t)]$$

$$F(t) = 1 - \prod_{i=1}^{n} R_i$$

- Hence the reliability of our model is:

$$R(t) = \prod_{i=1}^{n} R_i$$

Competing Risk Models

- Take a model defined by two different failure distributions

$$R(t) = R_1(t)R_2(t)$$

- The definition of a density function is:

$$f(t) = F'(t) = (1 - R(t))'$$

- Differentiating $1 - R(t)$ with respect to time, t gives:

$$f(t) = f_1(t)R_2(t) + f_2(t)R_1(t)$$

Competing Risk Models

- Expanding this out to *n* models we see:

$$f(t) = f_1(t)R_2(t)\cdots R_n(t) + \cdots + f_n(t)R_1(t)\cdots R_{n-1}(t)$$

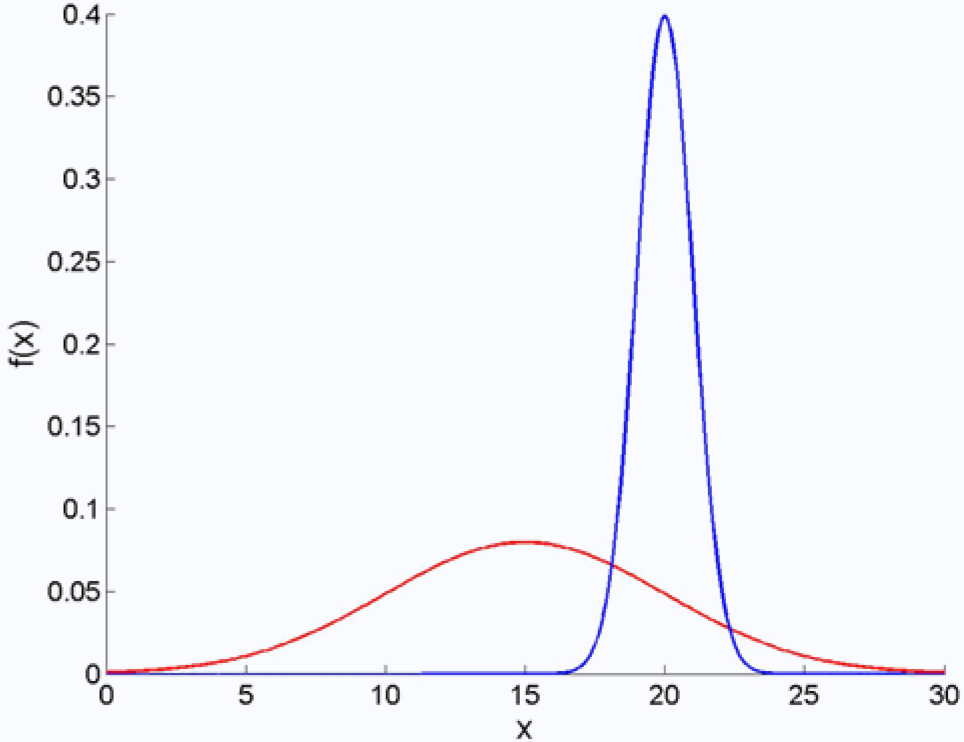
- Or alternatively:

$$f(t) = R(t)\left[\sum_{i=1}^{n} \frac{f_i(t)}{R_i(t)}\right]$$

- Which we can then use to find the parameters

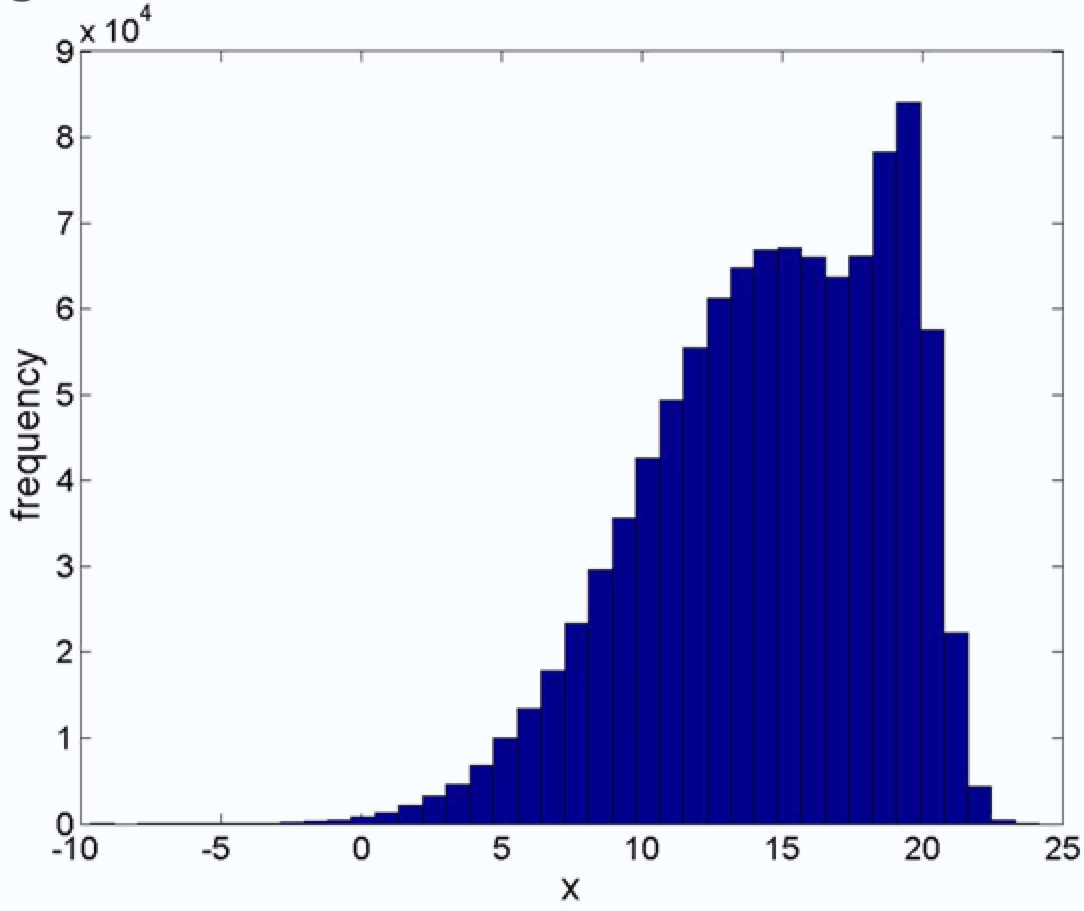
Competing Risk – An Example

- Consider the case where we have a two component system
  - $\mu_1 = 15.0$ & $\sigma_1 = 5$ $\mu_2 = 20.0$ & $\sigma_2 = 1$
- The PDFs for the individual components look like this…



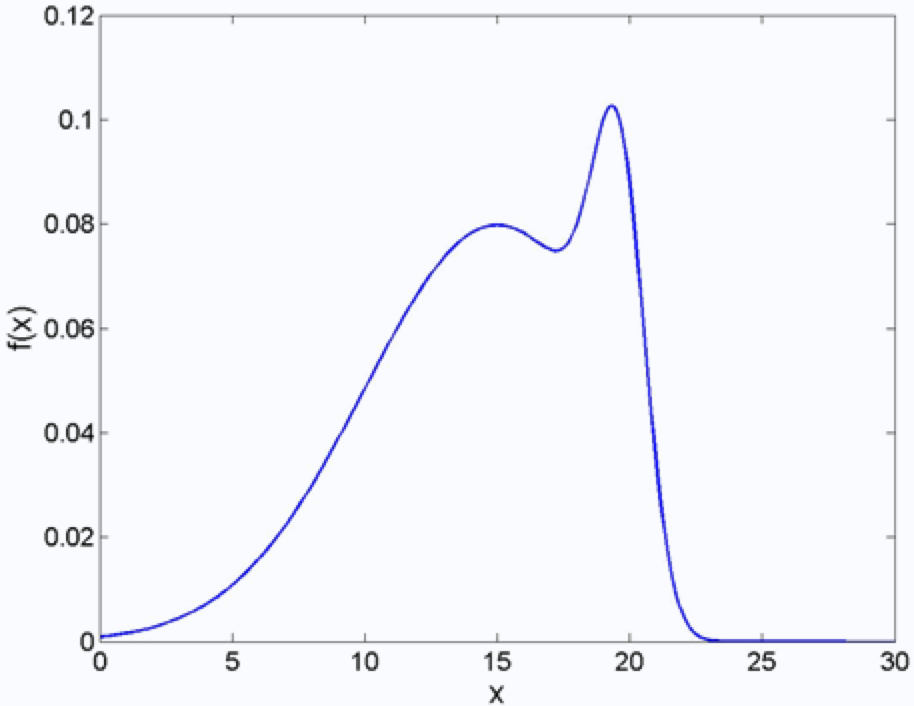
Competing Risk – An Example

- If we run a Monte Carlo Analysis on the system what would the histogram of failures look like?



Competing Risk – An Example

- We can define an analytical function of the PDF for our competing risk model using our known component data
- This could be derived using MLE but requires us to know the number of risks and nature of their distributions

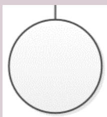
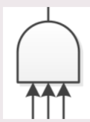
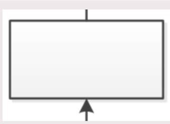
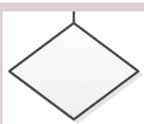


Fault Tree Analysis

Fault Tree Analysis (FTA)

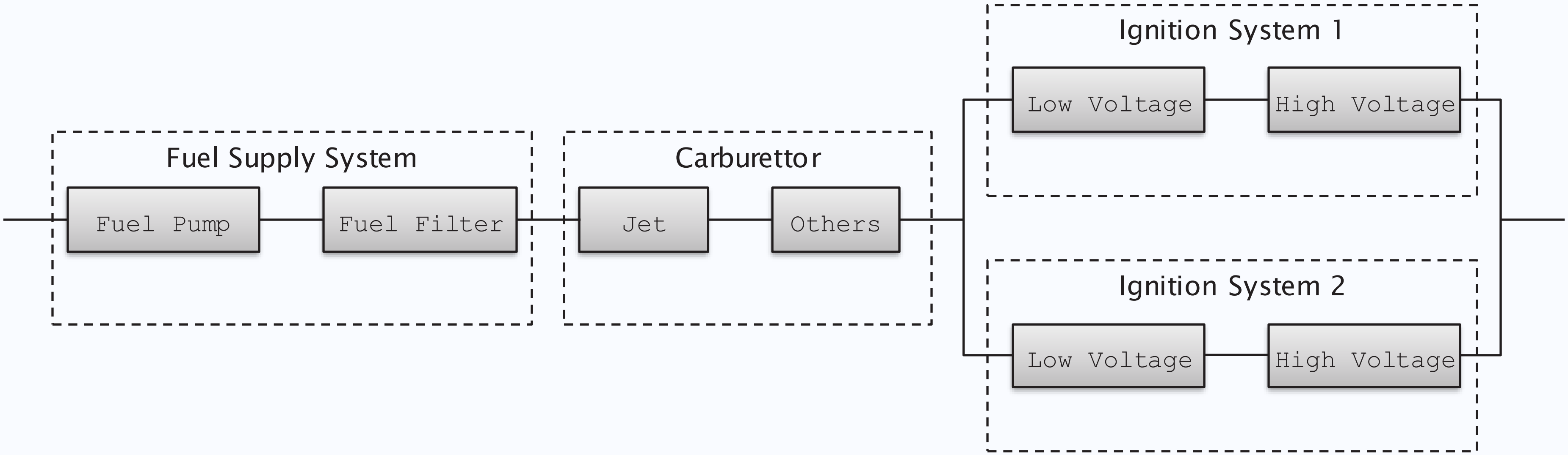
- FTA is a reliability (as well as safety) design analysis technique.
- It starts from consideration of system failure effects.
- The analysis proceeds by determining how these ‘top events’ can be caused by individual or combined lower level failures or events.

Some FTA Symbols

Symbol	Description
	**Basic Event:** A basic fault event that requires no further development. It is s-independent of other events
	**And Gate:** Failure (next higher event) will occur if all inputs fail (parallel redundancy)
	**Or Gate:** Failure (next higher level) will occur if any input fails (series reliability)
	**Combination Event:** An event that results from the combination of basic events through the input logic gates
	**Undeveloped Event:** It is s-dependent upon lower events, but not developed downwards

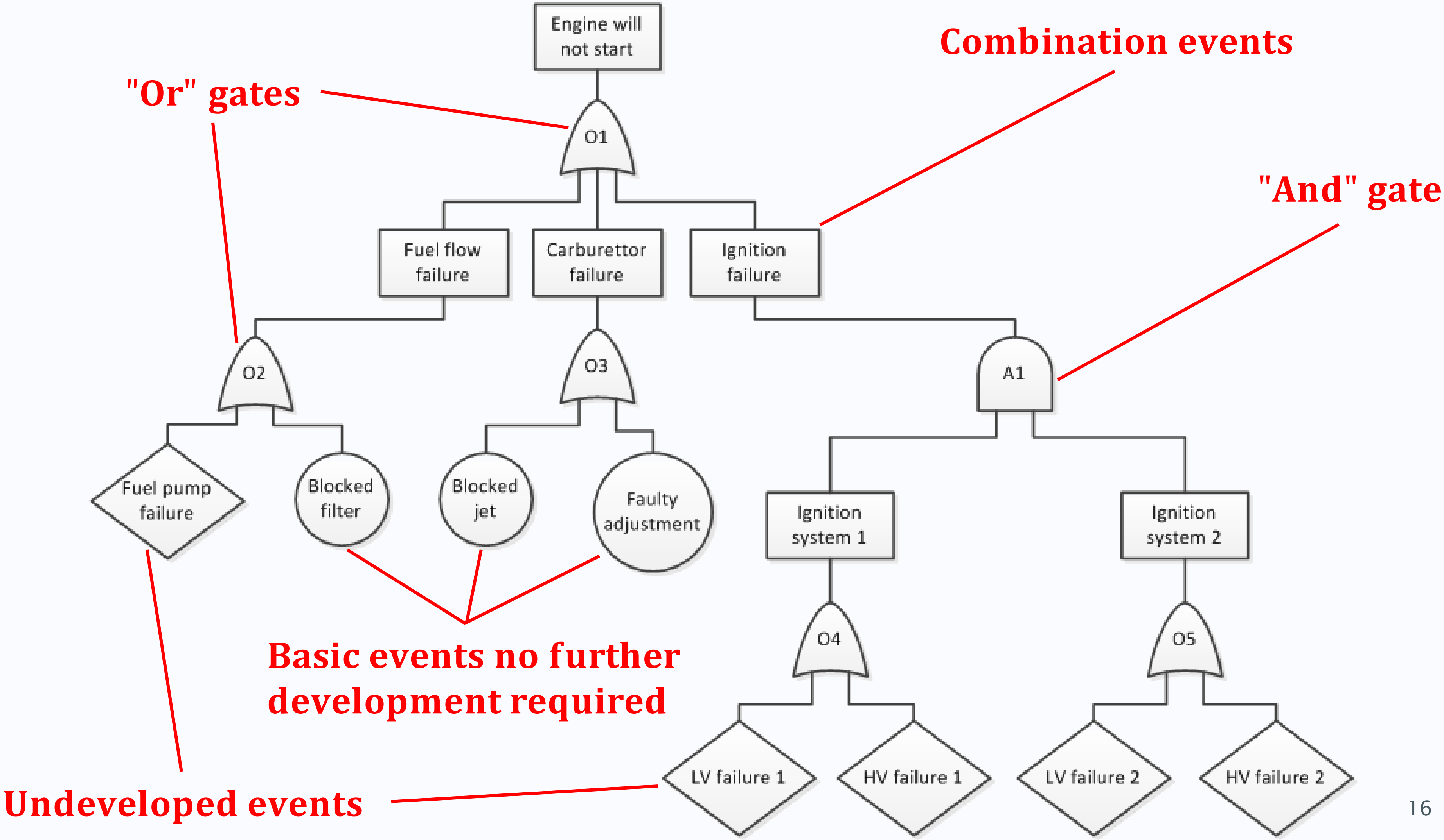
RBD vs FTA

- Consider the RBD of an aircraft engine…



- How can we build an FTA for engine failing to start?

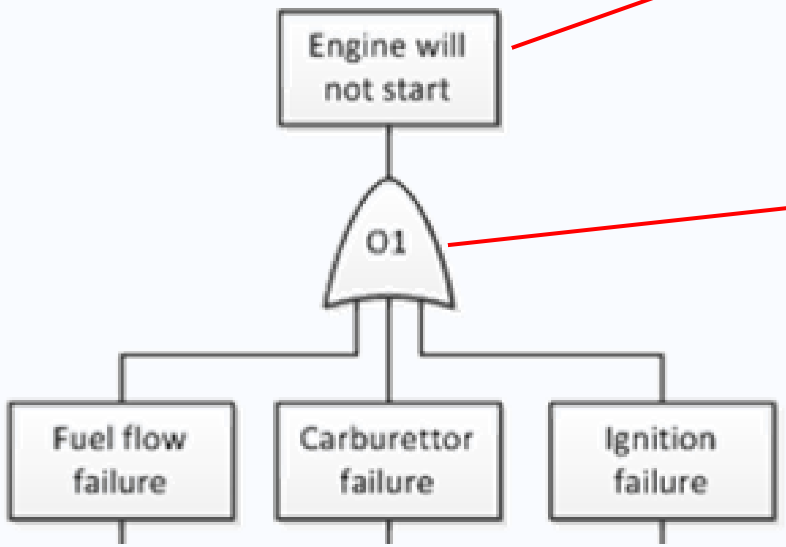
FTA for an Engine (Incomplete)



What is the Difference?

- RBD: We build the model by concentrating on the success of the system
- FTA: We build the hierarchy in terms of failures
- While an RBD considers the entire system the FTA may disregard parts of the system not contributing to a fault
  - RBD of an aircraft includes engine, wing, gear etc.
  - FTA for an ignition failure may not include anything outside of the engine sub-system and perhaps only some engine components

Building a FTA for an Engine



We start with an ultimate failure i. e. that the engine will not start

How do these events combine to cause the failure?

We define the events which may lead to that failure

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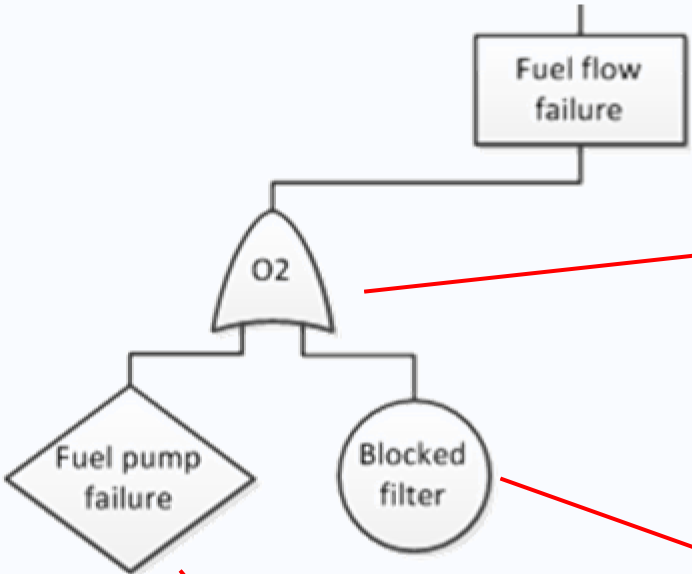
Building a FTA for an Engine

- The process is then repeated for each sub-failure…



Our new top level failure is a failure in the fuel flow

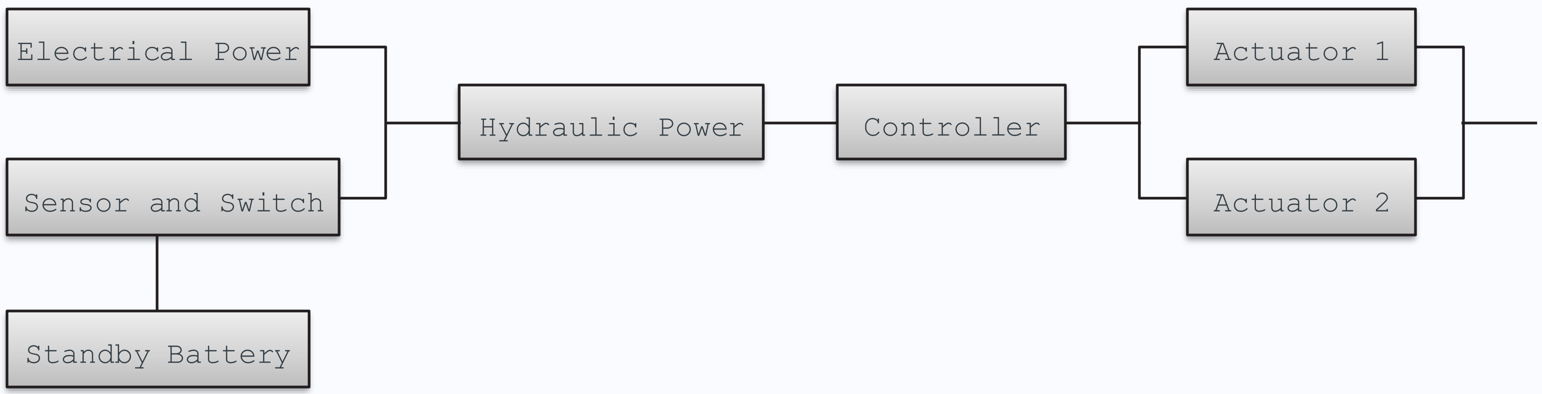
Once again we define how these events must combine to cause the failure

We identify events which may lead to this failure

In this case we have failure data for the pump as a whole so we will not develop the pump failure further

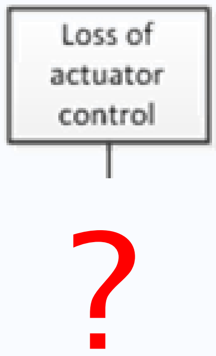
Example #1

- A control system consists of an electrical power supply, a standby battery supply which is activated by a sensor and switch if the main supply fails, a hydraulic power pack, a controller, and two actuators acting in parallel
- Lets consider the RBD for this system:



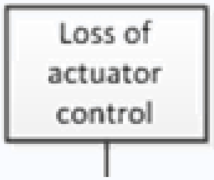
Example #1

- Lets construct a FTA diagram for this system
- Consider the ultimate failure mode as the "loss of actuator control"

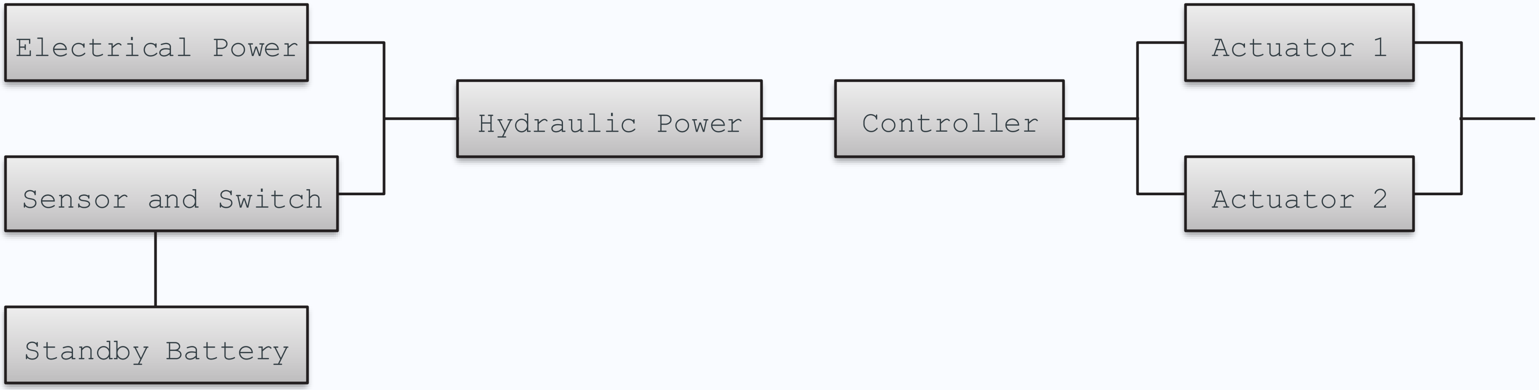


Example #1

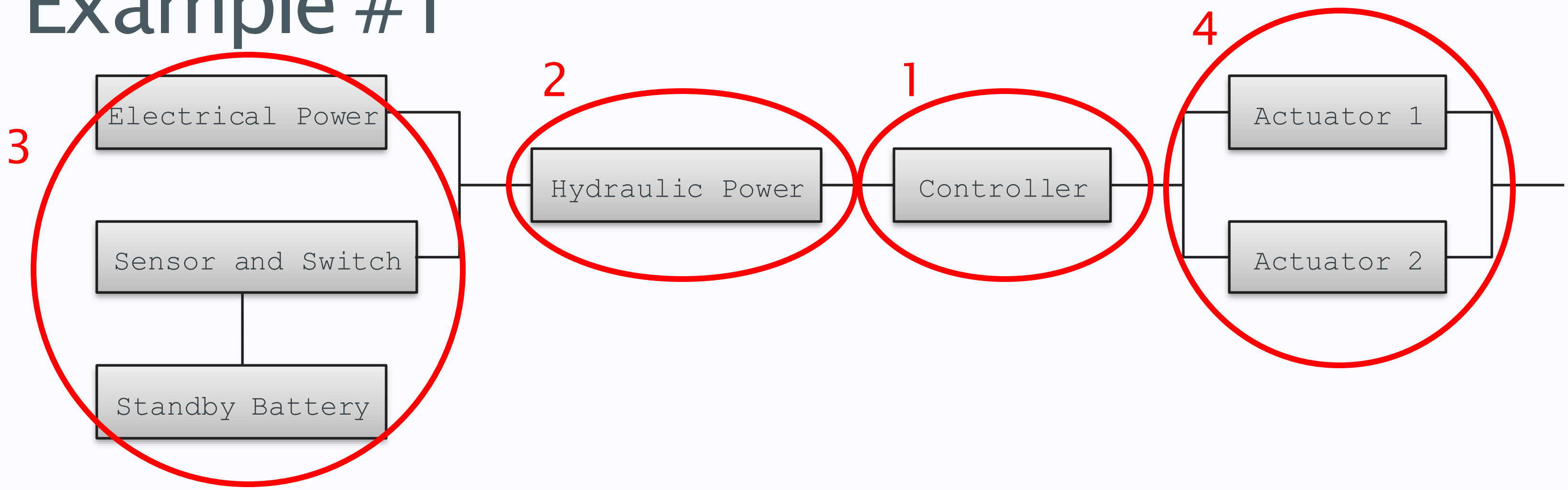
- What comes next?



- What events will cause the actuator to fail?
- Note: Actuator control loss is not the same as failure of only an actuator



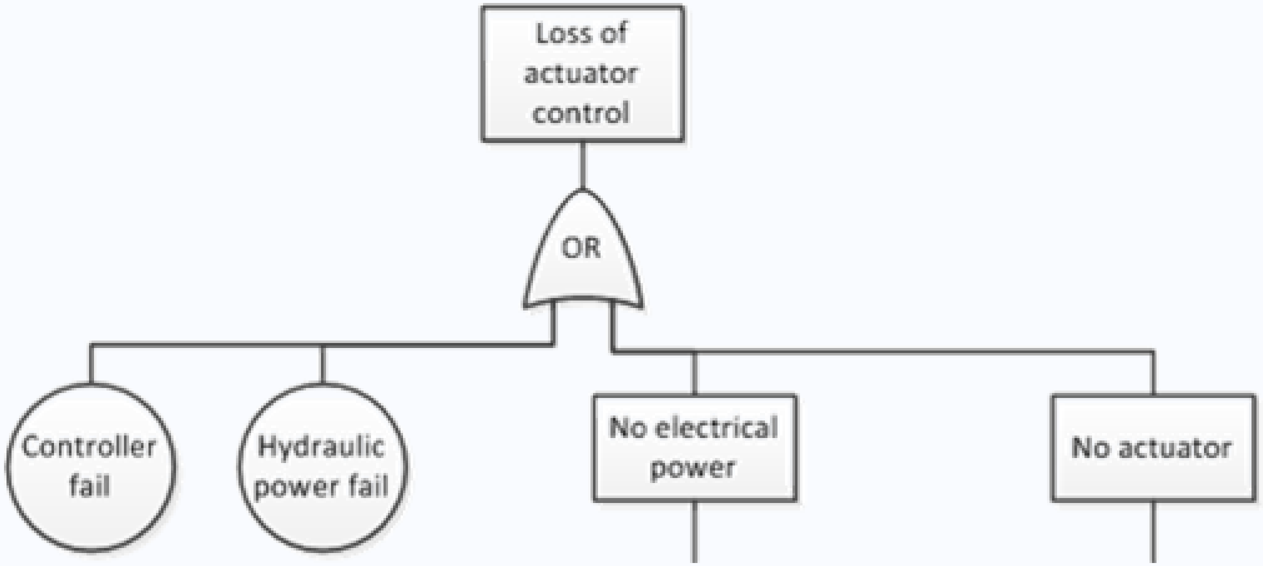
Example #1



- Actuator control could be lost due to:
    1. Controller failure
    2. Hydraulic power failure
    3. Electrical power failure
    4. Actuator failure
- How should these events be combined to cause a failure?

Example #1

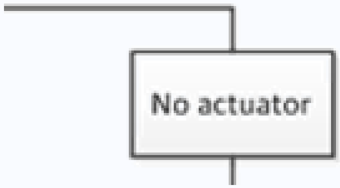
- Which gives use the following FTA diagram



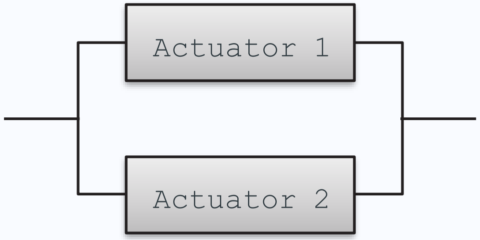
- What’s next?

Example #1

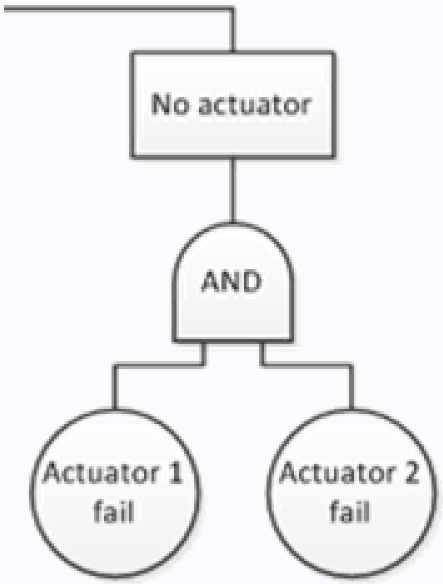
- Let’s consider the actuator failure…



- What events will cause this failure?
    1. Failure of actuator #1
    2. Failure of actuator #2
- How do these events combine?

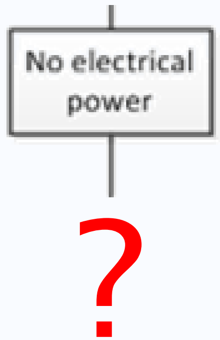


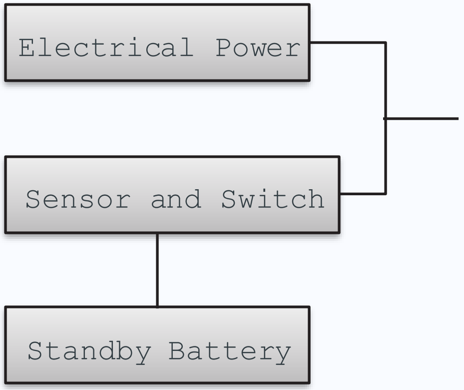
Example #1



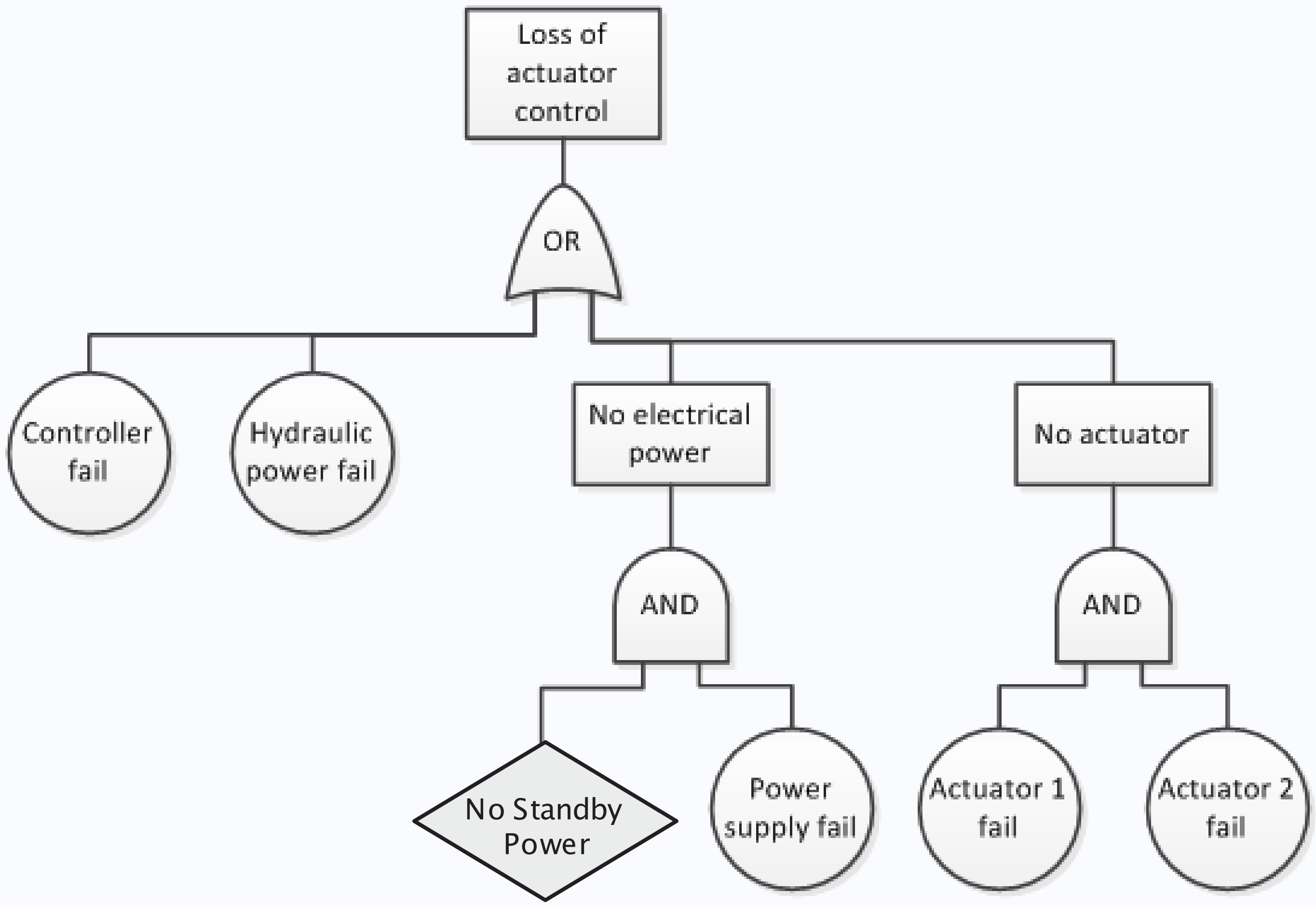
Example #1

- Any thoughts on the expansion of the failure of the electrical power?





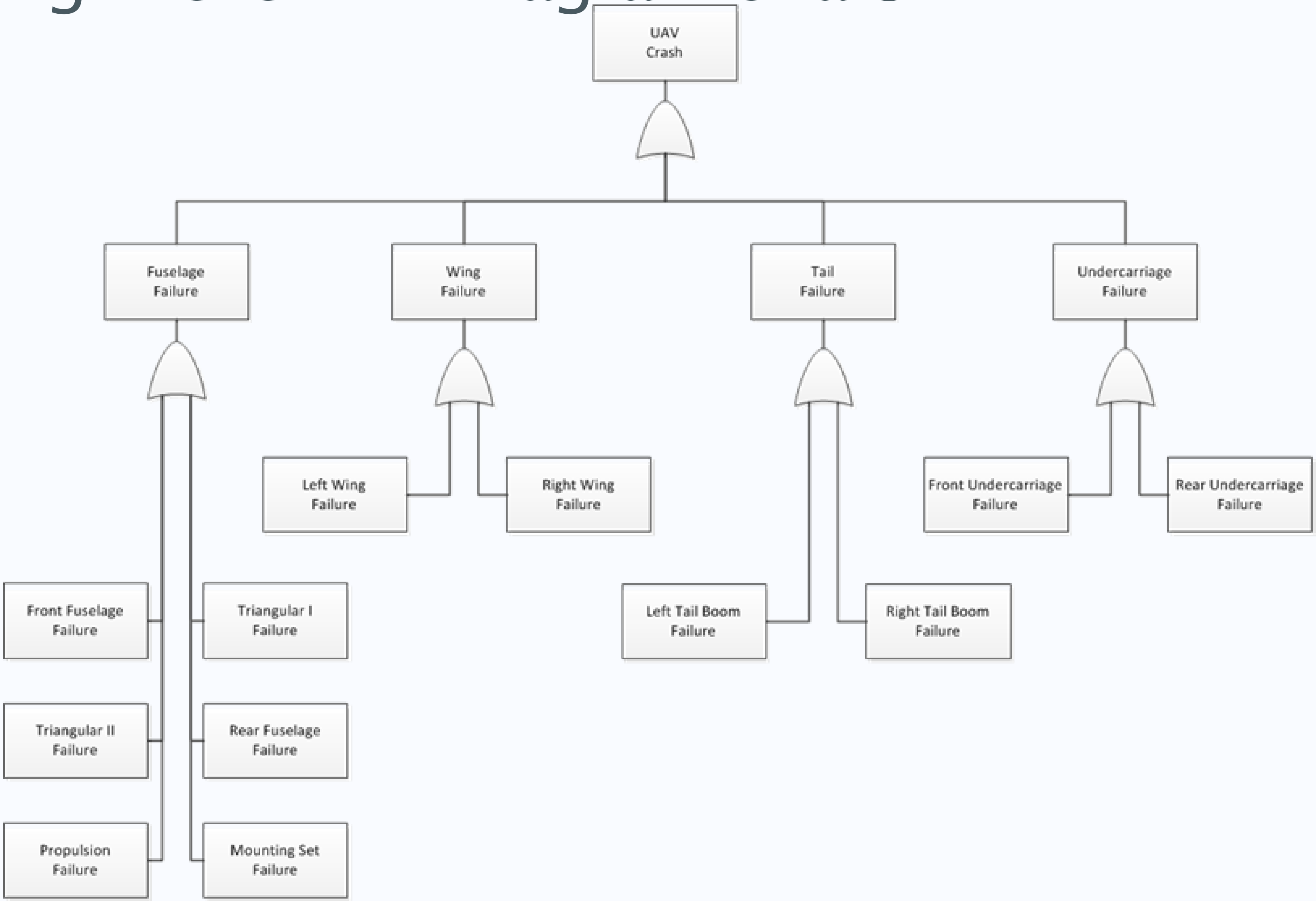
Example #1



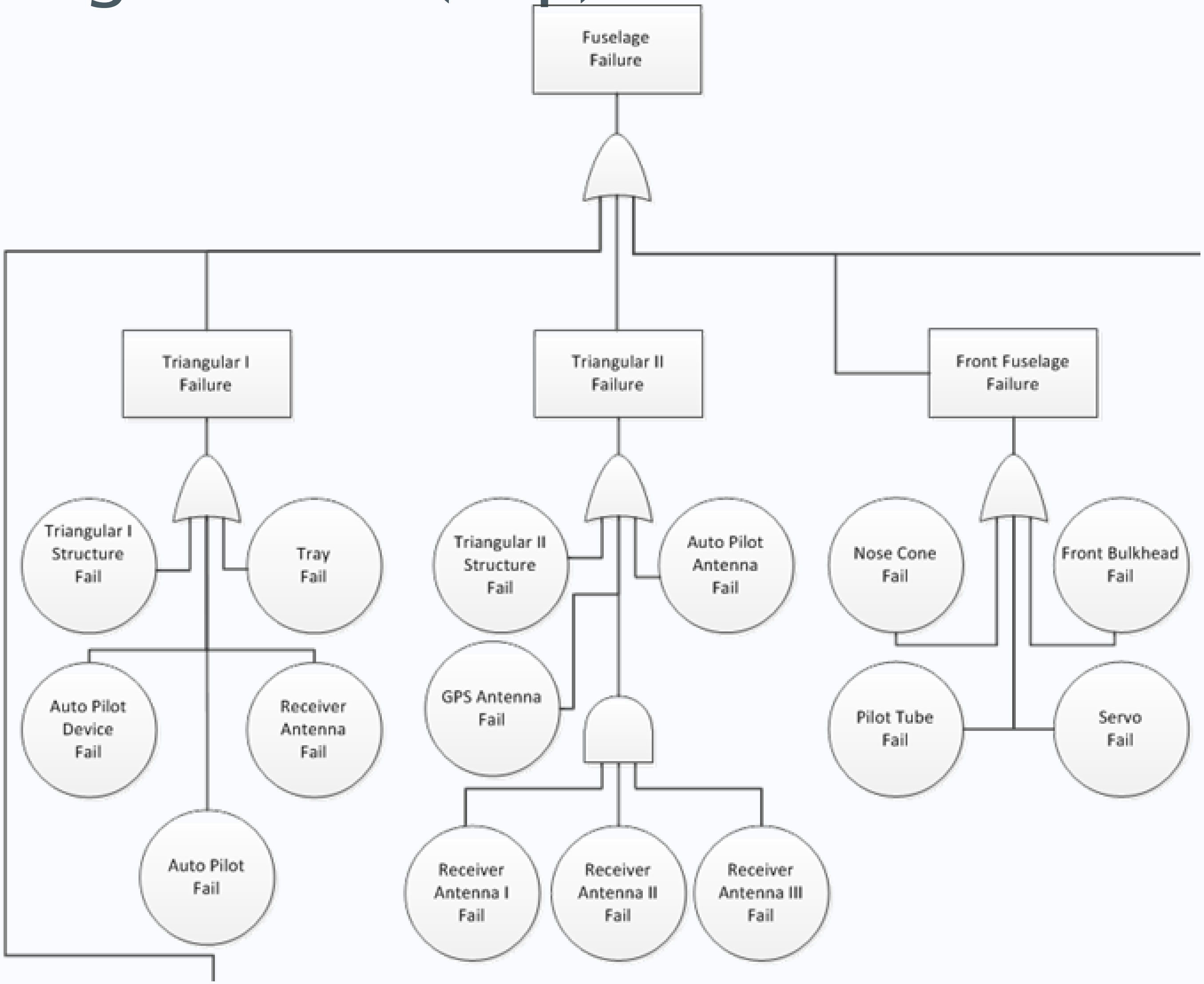
N.B. Standby power is left undeveloped

What About for a UAV?

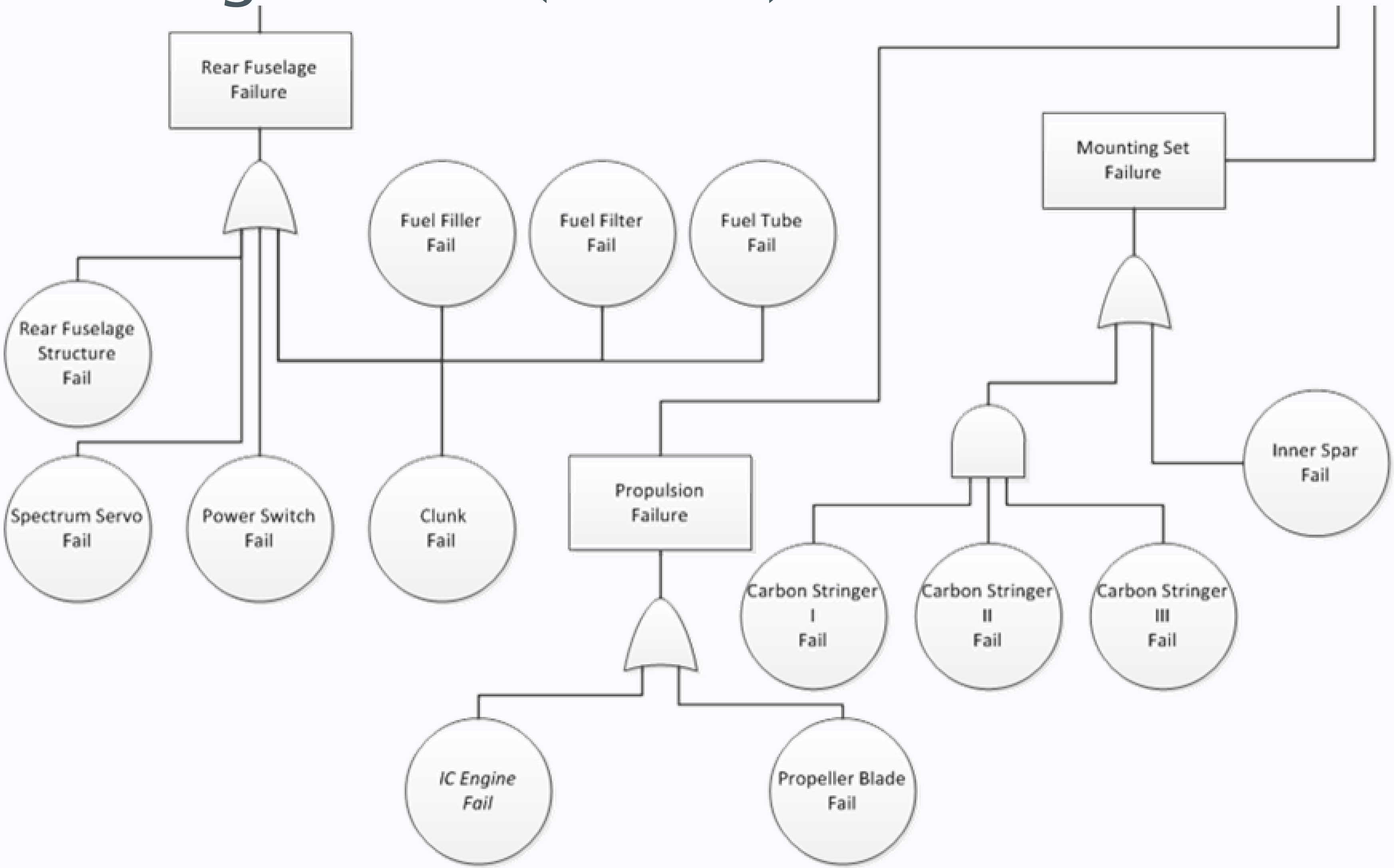
High Level FTA Diagram of a UAV



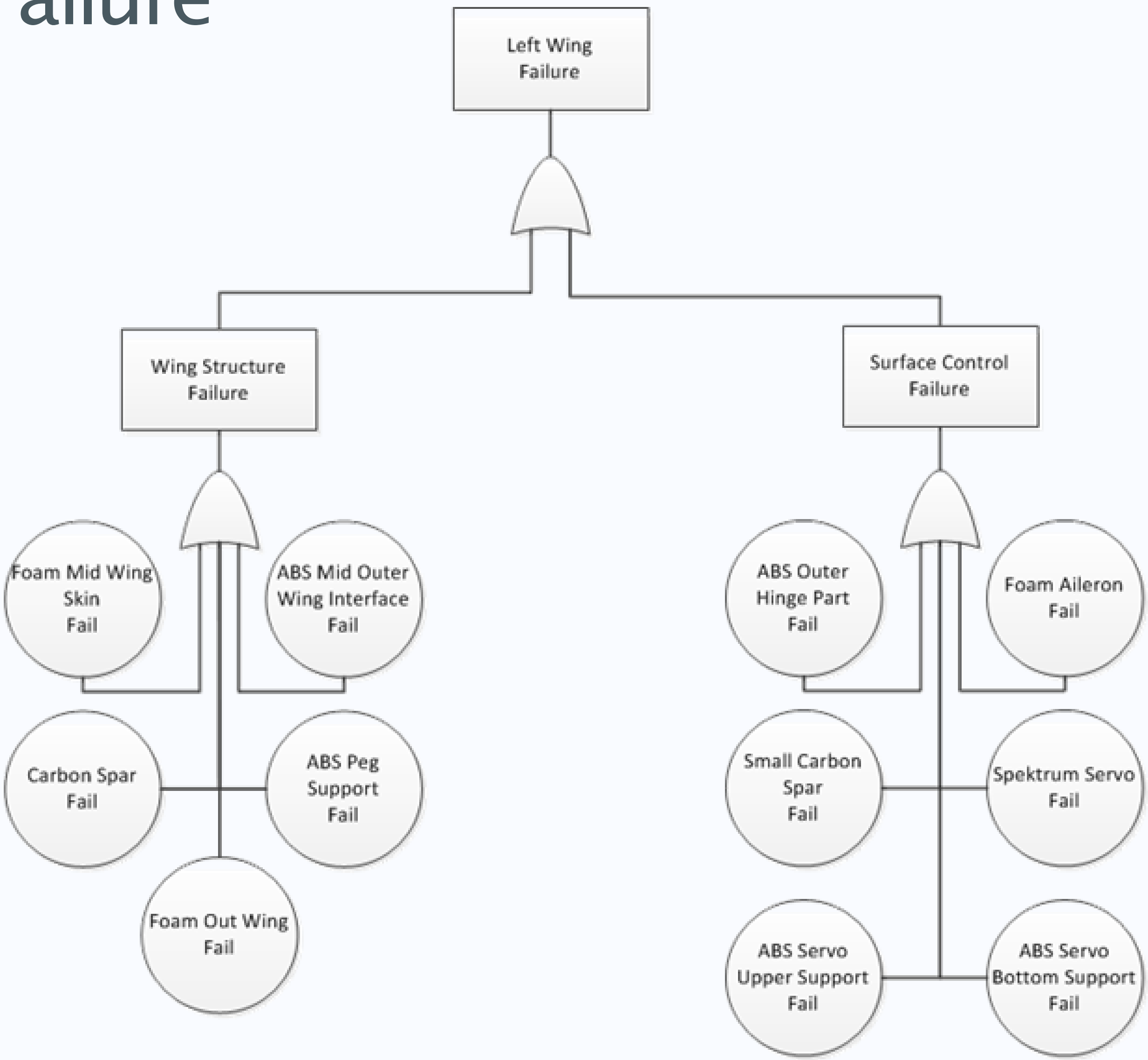
Fuselage Failure (Top)



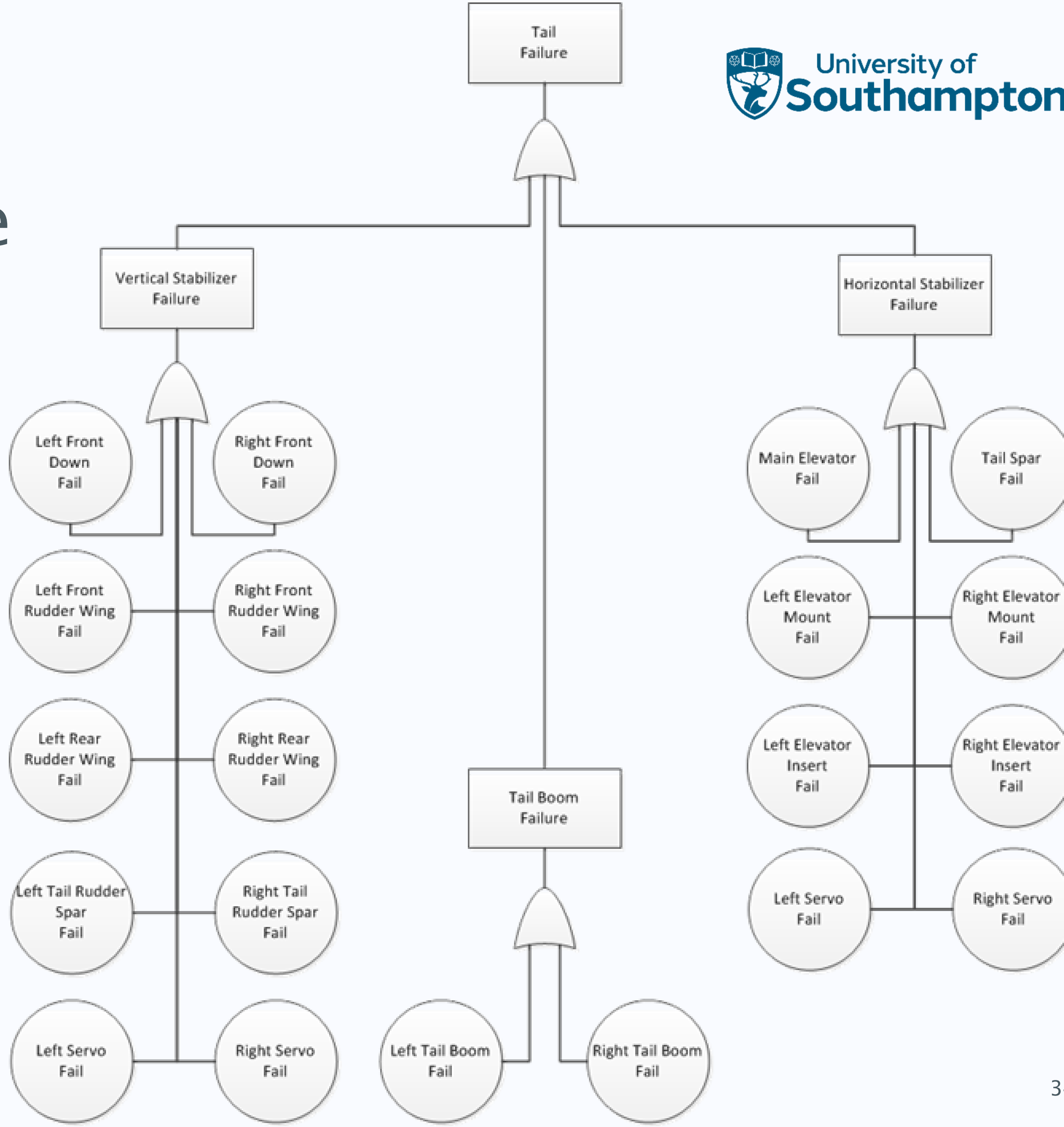
Fuselage Failure (Bottom)



Wing Failure



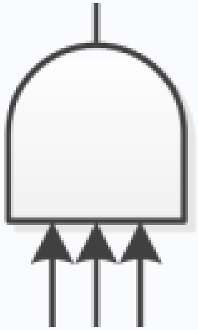
Tail Failure



Performing a Fault Tree Analysis

- As well as illustrating logical connections between failure events FTA can be used to quantify event probabilities
- Failure probabilities can be assigned to the failure events and used to calculate top level failure probabilities

  – P(A and B) = P(A)P(B)



  – P(A or B) = P(A)+P(B)-P(A)P(B)

Performing a Fault Tree Analysis

- A different FTA may need to be constructed for different top level events as the logical connections may change
- Although FTA's from sub-failures may be repeatedly used in multiple analyses e.g.
  - The FTA for our "no electrical power branch" may be used in multiple FTAs if the same power supply provides power to multiple components
- FTAs can be created and solved using specialist software e.g. Relex

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