10 PRINCIPLES OF TURBOMACHINERY (WEEK 10)

10.1 LEARNING OBJECTIVES

After working through the content for this week of the module you will be able to:

- Perform calculations involving flow and work coefficients, and to explain the effect of different coefficient values.
- Construct independent non-dimensional groups that characterise the operation of turbomachines, and justify the choice of variables.
- Use the *specific speed* to select appropriate types of turbomachine for different applications.
- Give physical explanations for the main features of characteristic maps for high speed compressors and turbines.

10.2 FLOW AND WORK COEFFICIENTS

The axial flow velocity V_x and the enthalpy change Δh_0 due to the work done on (or by) the flow are key variables for analysing and discussing axial-flow turbomachine performance. Conventionally these variables are non-dimensionalised using the blade speed U giving the *flow coefficient*:

$$\phi = \frac{V_x}{U}$$

and the *work coefficient*, also known as the *stage loading coefficient*

$$\psi = \frac{\Delta h_0}{U^2}$$

10.2.1 Flow Coefficient

The ratio of meridional velocity and the blade speed ϕ is related to the relative angle at which the flow approaches the blade. A stage designed with a low value of ϕ generally implies highly staggered blades with the relative flow angles close to tangential. High values imply low stagger and flow angles closer to axial.

In order to keep the incidence angle of the flow approaching the blades close to zero, it is necessary to operate a turbomachine in a narrow range of flow coefficient.

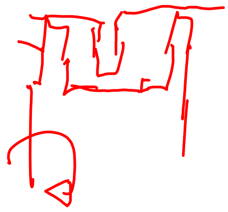
In a multi-stage axial-flow compressor, the flow radius and hence blade speed is approximately constant through the machine, however the flow density increases. In order to maintain the axial velocity and thereby keep the flow coefficient close to the design value, it is necessary to reduce the blade height on each stage.

10.2.2 Stage Loading Coefficient

The stage loading coefficient measures the amount of work done in a stage. It is usual to calculate the coefficient using magnitude of the enthalpy change Δh_0 so that the stage loading coefficient is positive both for turbines and for compressors. From Euler's equation we can write (assuming constant radius)

$$\Delta h_0 = U \Delta V_\theta$$

The compressor stages alternate between rotating and stationary blades, to make full use of the rotational velocity imparted during compression



This can be written as $\psi = \Delta V_\theta / U$ where ΔV_θ, is the change through the rotor alone, since only the rotor does work. The last equation shows that high loading implies a high ΔV_θ, and hence blades with a large amount of flow turning.

High stage loading is desirable as it means that few stages are needed to produce a given enthalpy change (hence pressure change), however, we will see that this is limited by the effects of increased stage loading on efficiency.

10.2.3 Parameter Selection

Experience has allowed designers to choose combinations of work and flow coefficients that give satisfactory performance. Figure 1 shows contours of efficiency for actual turbines. There is no diagram equivalent to Figure 1 for compressors however guideline values of flow and work coefficients are discussed below, with $\Delta h_0/U^2$ in the range 0.35 to 0.5.

The speed of sound increases due to the temperature rise through a compressor, so that, for fixed meridional radius and V_x/U, the blade Mach number is greater at compressor entry, and the front of the compressor or fan may be closed to the limit of being choked. The choking limit at entry to the fan and the engine mass flow requirements then determine a minimum fan area. To reduce noise from high speed flow, and to limit damage caused by bird strikes and blade failure, the relative Mach number onto the fan tip is usually kept below Mach 1.6, with an axial Mach number around 0.6 at the face of the fan. For the fan with pressure ratio 1.5 considered for the NEA, we will assume a relative Mach number of 1.3 at the fan tip in our design calculations.

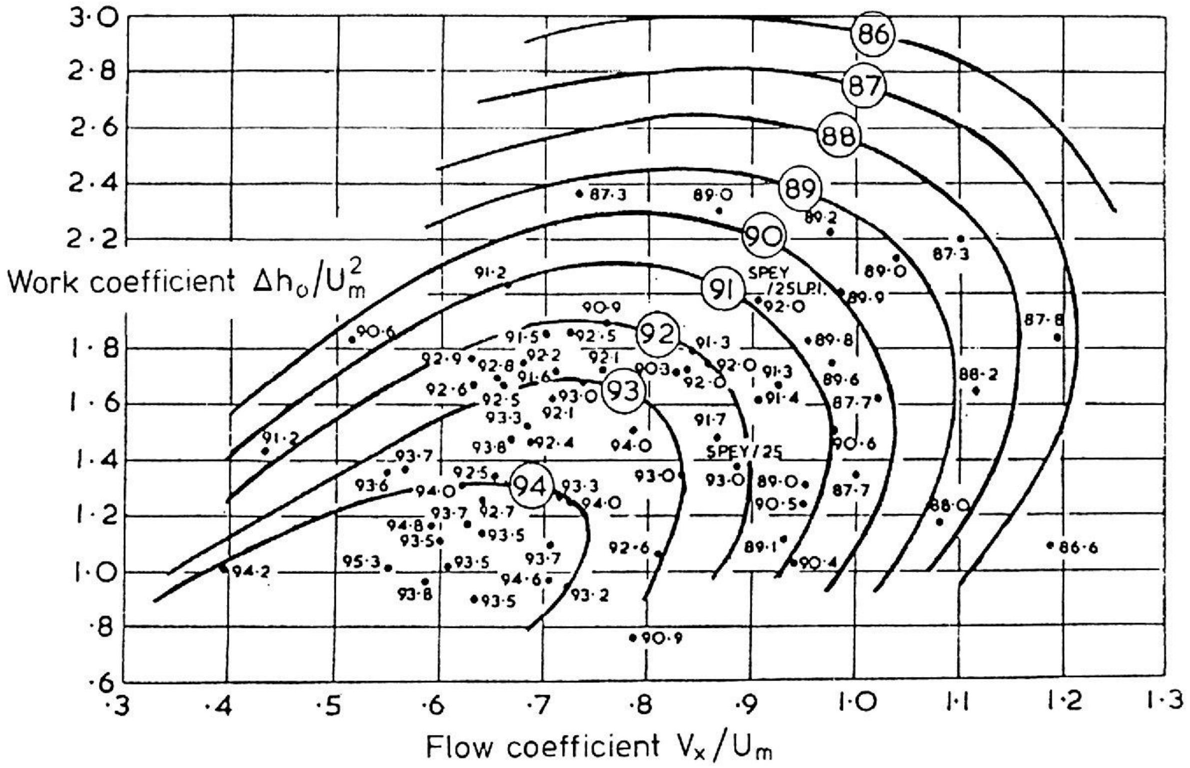


Figure 1. Variation of measured stage efficiency with stage loading and flow coefficient for axial-flow turbines (after Smith, 1965).

10.2.4 Example: Determining blade speed

Consider a turbojet engine in flight at sea level (T=288 K, p=1.01 bar) and Mach 0.8. Due to flow divergence in the engine inlet, the flow enters the first compressor rotor at Mach 0.6. Assume there is no inlet guide vane and therefore no swirl in the flow entering the first rotor. If the designer has specified a mid-span flow coefficient of 0.55 for the first stage, determine the mid-span blade speed.

$$T_{02} = T_{atm}\left(1+\frac{(\gamma-1)}{2}M^2\right) = 288.\left(1+\frac{(1.4-1)}{2}0.8^2\right) = 324.9\ K$$

$$\frac{V_x}{\sqrt{c_p T_{02}}} = \sqrt{(\gamma-1)}M_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{-\frac{1}{2}} = 0.3665$$

$$V_x = 0.3666\sqrt{1005\times 324.9} = 209.4\ m/s$$

$$\phi = \frac{V_x}{U} = 0.55$$

$$U = \frac{V_x}{\phi} = 380.8\ m/s$$

10.2.5 Example: selection of number of stages

For the engine operating as in the preceding example, with mid-span blade speed of 380.8 m/s, the multi-stage compressor has an overall total pressure ratio of 10:1. Assuming that the designer has decided that the upper limit for the mid-span stage loading coefficient should be 0.4, determine the minimum number of stages required.

Treat the compressor as isentropic and treat air as a perfect gas.

$$\psi = \frac{\Delta h_{0,stage}}{U^2} \le 0.4$$

or

$$\Delta h_{0,stage} \le 0.4\times 380.8^2 = 57.99\ kJ/kg$$

The overall enthalpy change for the whole compressor is

$$\Delta h_{0,overall} = c_p(T_{03}-T_{02}) = c_p T_{02}\left(\frac{p_{03}}{p_{02}}^{\frac{\gamma-1}{\gamma}} - 1\right) = 303.9\ kJ/kg$$

The minimum number of stages is found by assuming every stage has the maximum allowed loading coefficient:

$$n_{min} = \frac{\Delta h_{0,overall}}{\Delta h_{0,stage}} = 5.24 \rightarrow\ n_{stages} = 6$$

It not possible to have a fractional number of stages so it is necessary to round the value up to 6 stages. Note that a compressor with only 5 stages would require the stage loading coefficient to be greater than 0.4.

10.3 Dimensional analysis

The flow in two *geometrically similar* turbomachines is dynamically similar if the streamline patterns of the flows are geometrically similar. i.e. the streamline patterns scale in the same way as the geometry. This implies that the velocity triangles are similar *everywhere* in the two machines. If the flows are dynamically similar then *all* dimensionless groups are the same for the two machines.

The trick to dimensional analysis it to identify which groups are most meaningful and convenient to work with in practice. We have already introduced the flow and work coefficients, which are a fundamental part of the vocabulary of turbomachinery engineers. The method for developing groupings that are useful, rather than merely having no dimensions, is to perform a "mental experiment":

- Perform a "mental experiment" to decide what variables (e.g. Ω, D, $\dot{m}$, μ, ρ, T, … etc) will affect the streamline pattern. Think whether changing that variable alone, keeping the other independent variables that you have already chosen constant, will affect the streamlines. If in doubt include the variable and then you will have to decide by experiment whether it is really significant.
- Form these variables into dimensionless groups, these are the independent groups. Then any other dimensionless groups involving other variables that we are interested in, e.g. Δh_0, W_x, η, etc (the dependent variables) are functions of the groups of independent variables.

The compressor and turbines in aircraft engines feature high Mach number flows (certainly above M=0.3), so it is relevant to consider compressibility of the working fluid. However, to introduce dimensional analysis of turbomachines we will first consider the simpler situation where the fluid is incompressible. The incompressible analysis leads to a number of observations that are also true for compressible flow systems in aircraft, but it is also directly applicable for low-Mach turbomachines/pumps, and to hydraulic turbines used in hydro-power generation, where the flow of liquid water is effectively incompressible.

10.3.1 Incompressible Flow Machines

In incompressible (but viscous) flow with no cavitation[1] the streamline pattern is affected by:

- The volumetric flow rate, Q
- The rotational speed Ω.
- The machine size (diameter) D.
- The fluid density ρ, and,
- The fluid viscosity μ.

There are thus 5 independent variables and 3 dimensions M, L, T and so the rules of dimensional analysis tell us we can form only two groups. There are many ways of forming the groups which are perfectly acceptable in theory but the ones conventionally chosen for convenience are

$$\frac{Q}{\Omega D^3} \text{ and } \frac{\rho \Omega D^2}{\mu}$$

[1] Cavitation is a phenomenon that occurs in liquids where the pressure at some point in the flow is low enough for gas bubbles to form (either due to the liquid vaporising or dissolved gases coming out of solution). This breaks the assumption of constant density/incompressibility.

$Q/\Omega D^3$ is a type of flow coefficient since Q/D^2 is a measure of flow velocity and ΩD is a measure of blade speed. This group determines the angle of attack of the fluid on the blades.

$\rho\Omega D^2/\mu$ is a type of Reynolds number since ΩD is also a measure of flow speed once the flow coefficient is fixed. This group determines the thickness of the boundary layers and hence the streamline pattern within and close to the solid boundaries (i.e. blades, hub and casing).

If we are interested in the pressure rise of a compressor (or the pressure drop of a turbine), Δp, this is a dependent variable and we can form a dimensionless group involving it, this is conventionally $\Delta p/\rho\Omega^2 D^2$ and say

$$\frac{\Delta p}{\rho\Omega^2 D^2} = fn\left(\frac{Q}{\Omega D^3}, \frac{\rho\Omega D^2}{\mu}\right) = fn(\phi,\mathrm{Re}).$$

Similarly the efficiency η is a function of the same two groups

$$\eta = fn(\phi,\mathrm{Re}),$$

and so is a power coefficient, involving the shaft power W_x,

$$\frac{W_x}{\rho\Omega^3 D^5} = fn(\phi,\mathrm{Re}).$$

A work coefficient can be constructed out of these variables by noting that the enthalpy change in an incompressible flow through a turbomachine is equal to $\Delta p/\rho$, and non-dimensionalising by $(\Omega D)^2$, giving

$$\frac{\Delta p}{\rho\Omega^2 D^2} = fn(\phi,\mathrm{Re}).$$

Dimensional analysis can tell us nothing about the exact form of these functional relationships. To obtain this we must resort to experiment. For example, for a family of geometrically similar pumps or compressors, the pressure coefficient might be found to vary with ϕ and Re as shown in Figure 2.

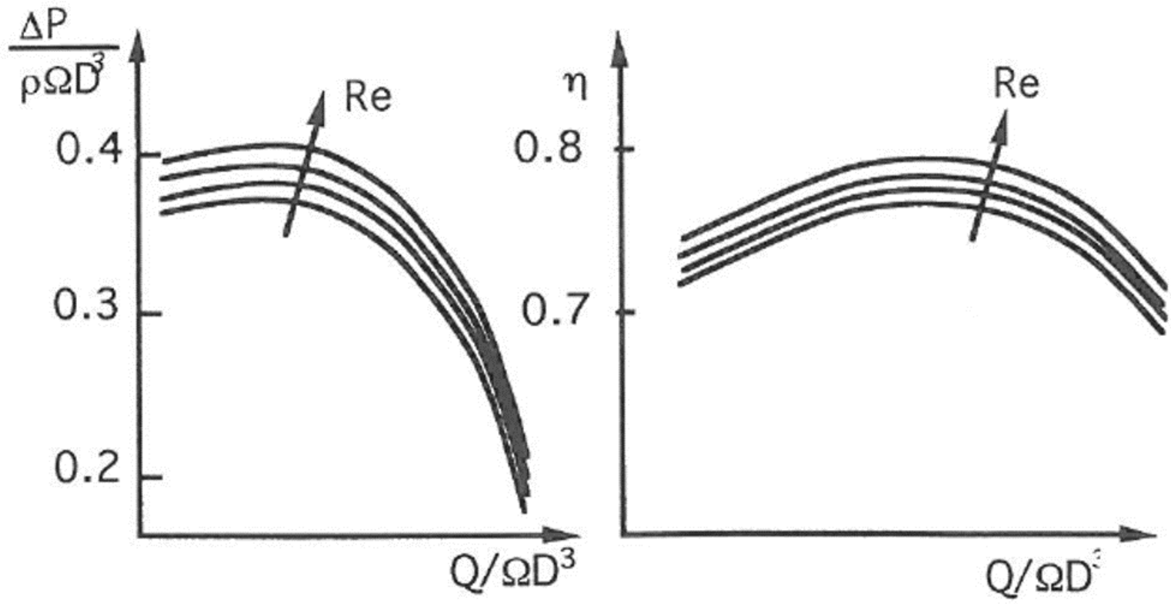


Figure 2. Typical effects of Reynolds number on the performance of a pump or fan.

By experiment it is usually found that the effects of Re on the performance of turbomachines is small provided that, Re is greater than about 2×10^5. This is because at high Re the boundary layers on the blades are predominantly turbulent and are very thin so they have little effect on the streamline pattern. In practice Re is above 2×10^5 in all except very small low speed machines (e.g. computer cooling fans) and so we can usually neglect the effects of viscosity (Re) on the flow. It is important to realise that this is only justified by experiment and not (at this stage) by theory.

Efficiency is the variable most likely to be affected by Reynolds number and will usually increase slightly as Re is increased. However, typically an order of magnitude change in Re (say from 2×10^5 to 2×10^6) will produce only one or two percent change in efficiency, as illustrated in Figure 2.

Note that if cavitation occurs in machines with liquid as the working fluid then another dimensionless group involving the vapour pressure of the fluid also becomes important.

10.4 Specific speed and shape selection

There is a wide variety of different types of turbomachine, and we require some means of selecting the appropriate type for a given application. Here we develop a dimensionless group that indicates what type of machine is suitable for a given application. The group is known as *specific speed*, however the name is misleading as it is not a speed and it is perhaps more helpful to think of it as a shape factor.

At the outset of the process of choosing a machine for a certain application we will usually know the flow rate, the pressure rise or drop, the fluid density and the preferred rotational speed (e.g. 3000 rpm), but not the size (D) or shape of the optimum machine for the duty. The specific speed is a mean of using the information we know at this stage to choose the most suitable type of machine.

In general, machines which are designed to have a low flow coefficient ϕ tend to have a high stage loading ψ (both ϕ and ψ are based on the inlet conditions) and very short closely spaced blades, e.g. a vacuum cleaner fan. Machines with a high flow coefficient tend to have a low stage loading and long widely spaced blades, e.g. a windmill or propeller.

If we sketch the ψ, ϕ characteristics of different machines on the same axes we tend to get curves as shown in Figure 3 where each curve is characteristic of a certain type of machine.

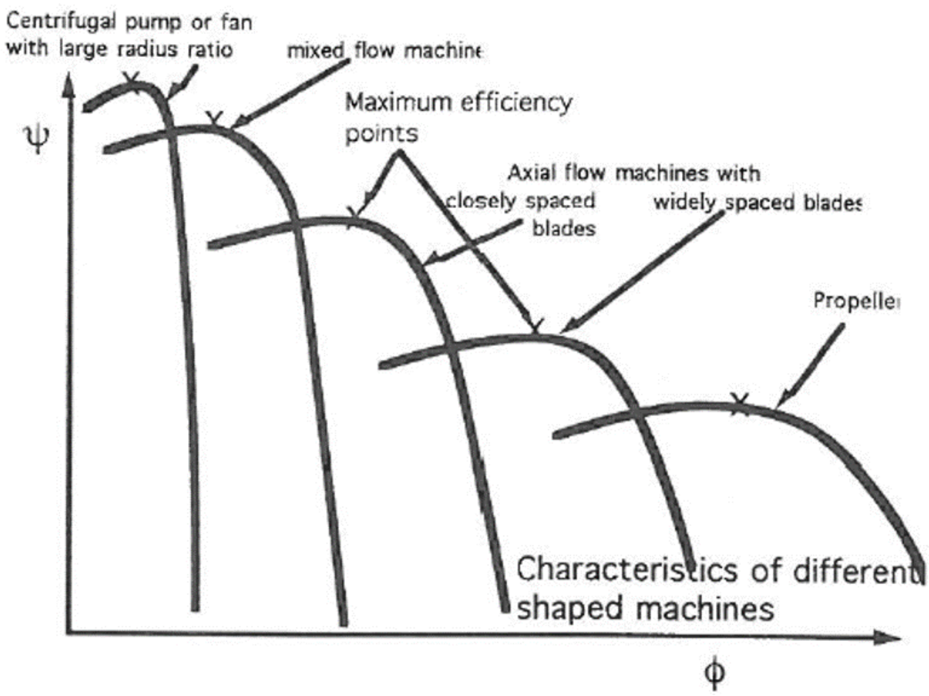


Figure 3. Concept of specific speed.

We wish to choose a combination of ϕ and ψ which eliminates D, so that we can evaluate it before a machine is chosen, but whose value is typical of the optimum type of machine for the duty. It turns out that the combination

$$N_s = \frac{\phi^{1/2}}{\psi^{3/4}} = \frac{Q^{1/2}\Omega}{(\Delta P/\rho)^{3/4}}$$

is such that lines of $N_s = const.$, as sketched on Figure 8, pass through the optimum performance point of only one type of machine. N_s is called the *Specific Speed* but it would be more correctly called the *shape factor*. In the past it was evaluated using strange combinations of units (gallons/hr, rpm, and ft head) but nowadays it should always be evaluated from SI units with the speed in radians/sec.

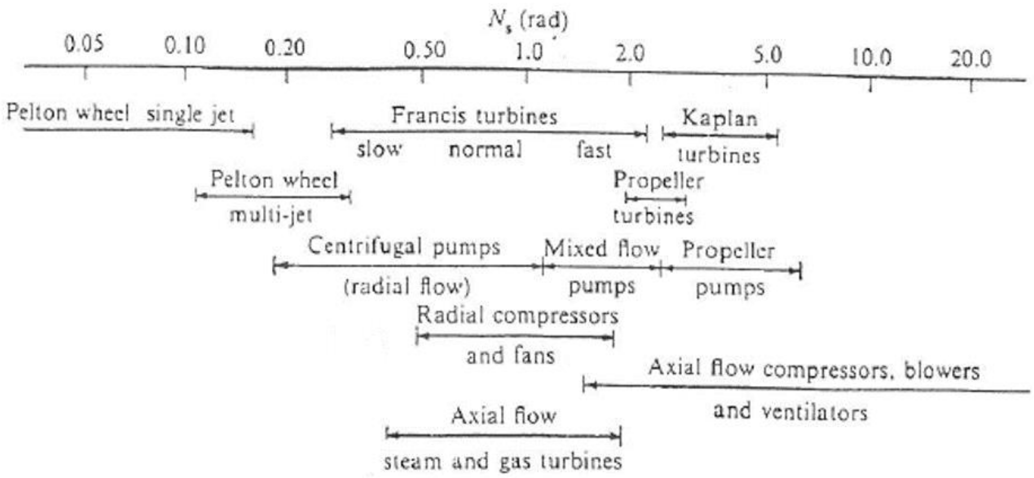


Figure 4. Specific speed ranges for various types of turbomachinery.

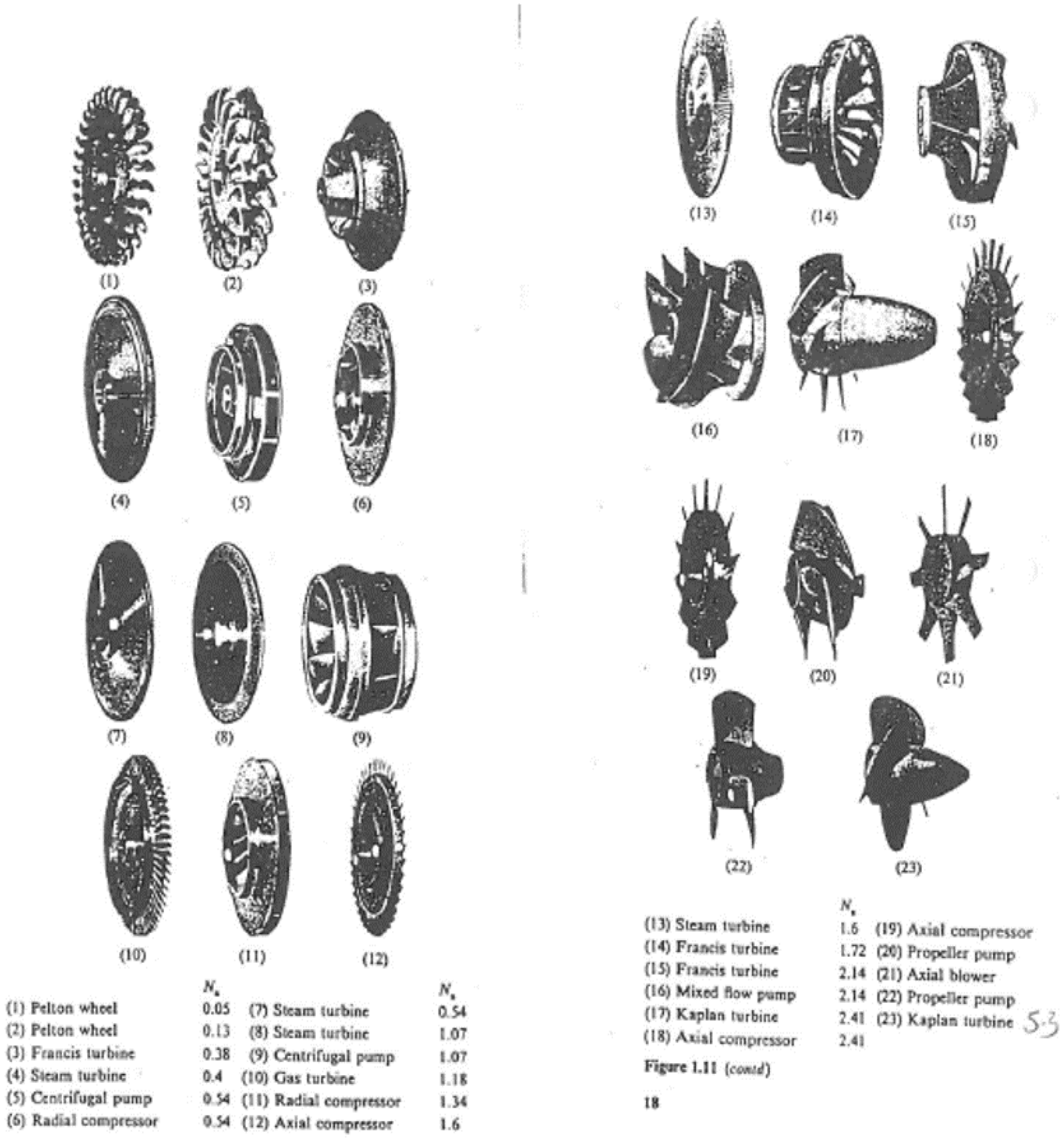


Figure 5. Some rotor designs and their dimensionless specific speeds

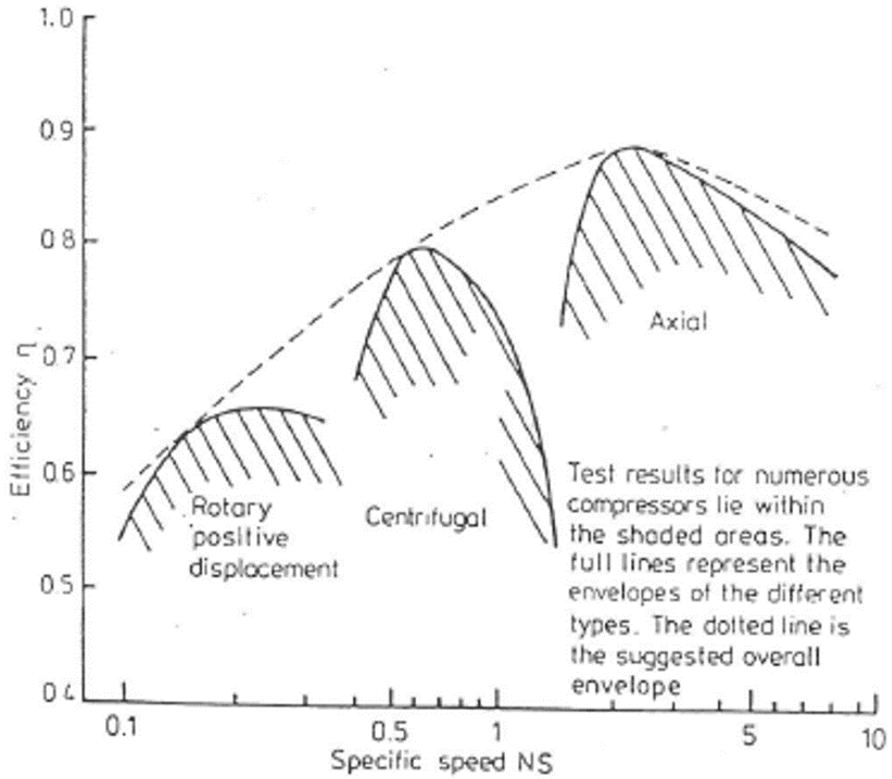


Figure 6. Variation of compressor efficiency with specific speed.

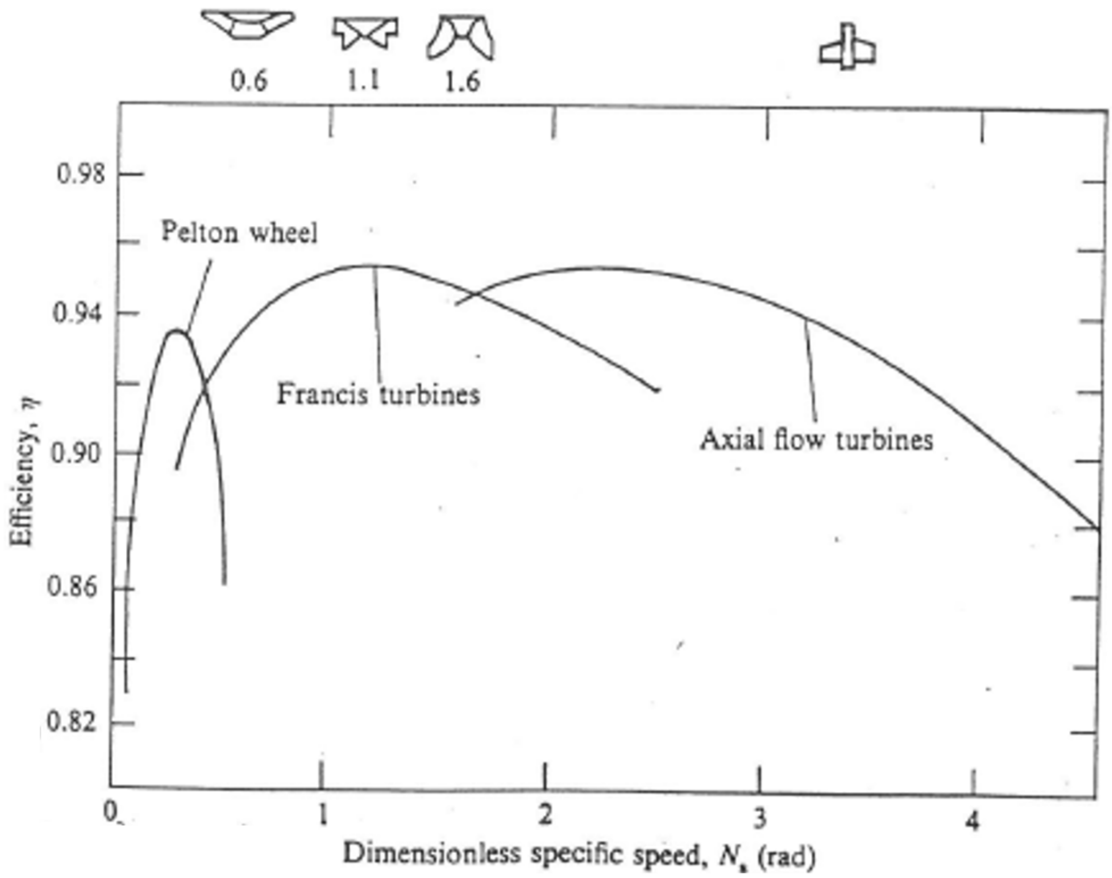


Figure 7. Variation of hydraulic turbine runner design with dimensionless specific speed

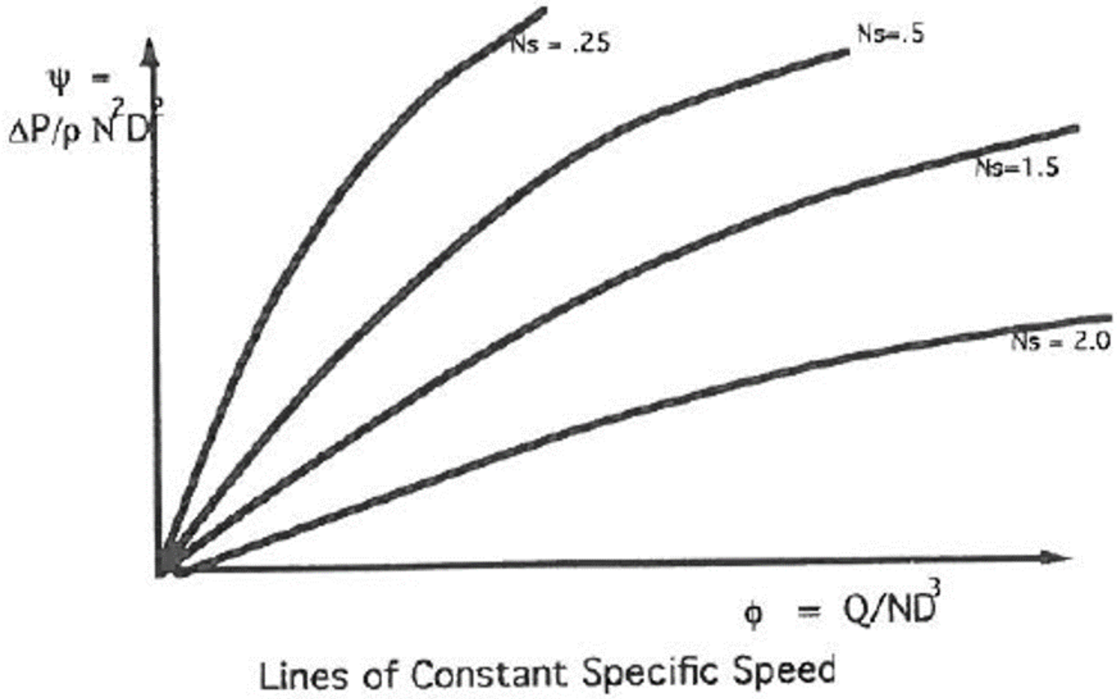


Figure 8. Lines of constant specific speed.

Low values of N_S (e.g. < 0.4) are characteristic of centrifugal machines with short blades and large values of the ratio of inlet to outlet radius, low values of N_S (e.g. > 2.0) are typical of axial flow machines with few blades of large aspect ratio. The attached Figures show the range of N_S for many practical machines.

Specific speed is used mainly for incompressible flow machines, both turbines and pumps and fans, but it is also applicable to compressible flow machines. Since the volume flow rate is not constant through these it must be specified either at inlet or outlet of the machine. The inlet value is usually used.

Large gas turbines invariably use both axial compressors and axial turbines, even though the pressure ratio that can be supported by one axial flow rotor is much smaller than the pressure ratio required for efficient engine operation. Axial-flow turbomachinery is preferred because it offers the best efficiencies and because several stages can be used in series to achieve the required pressure change, while the overall weight remains acceptable. Smaller gas turbines used in some helicopters combine multi-stage centrifugal compressors with axial turbines. The centrifugal compressors are preferred due to lower specific speed requirements, however there can be substantial pressure loss in the flow between the stages of the centrifugal compressor.

10.5 DIMENSIONAL ANALYSIS FOR COMPRESSIBLE FLOW MACHINES

Compressible flow dimensional analysis is considerably more complicated. If the density changes then the volume flow rate changes through the machine and the streamline pattern depends on the local volume flow rate. Hence making the velocity triangles similar at one point (e.g. the blade inlet) does not ensure that they are similar at other points (e.g. the blade exit). To fully define the streamline pattern we must specify the ratio of volumetric flow at outlet to that at inlet in addition to the variables used for incompressible flow. The change in volume flow rate through the machine depends on the pressure change and on the compressibility of the fluid, $d\rho/dp$. The pressure change is chosen to be a dependent variable, determined by the independent variables, but we must now include a measure of compressibility as one of these independent variables. This is chosen conventionally to be the speed of sound c since $(dp/d\rho)_s = c^2$.

For simplicity we restrict the analysis to a perfect gas for which

$$c = \sqrt{\gamma RT} = \sqrt{(\gamma - 1)c_pT}$$

and so we choose γ and c_pT_1 as two independent variables. Since the volume flow rate is no longer constant, it is better to use the mass flow rate $\dot{m}$ and the inlet density ρ_1 as two of the other independent variables in order to fix the volume flow rate at inlet. In practice it is customary to choose p_1 rather than ρ_1 on the grounds that is more easily measured. This is justifiable since ρ_1 is fixed by γ, c_p, T_1 and p_1 and we have already chosen the first 3 of these as independent variables.

Our mental experiment, with a little experience, will tell us that if we fix the above variables to define the inlet flow and the rotational speed of the machine, (which is by convention given the symbol N rather than Ω), then the flow pattern is fixed everywhere within it. Hence the independent variables are: $\dot{m}$, γ, c_pT_1, D, N, p_1. Where we assume at the outset that the effects of viscosity can be neglected.

With these independent variables we can form three dimensionless groups which *by convention* are chosen to be

$$\gamma, \frac{\dot{m}\sqrt{c_pT_1}}{D^2p_1}, \frac{ND}{\sqrt{c_pT_1}}.$$

Note that any combination of these groups would form a perfectly valid alternative group but the above groups are chosen for convenience.

- γ is a measure of the gas properties only. It is unlikely to change for any one machine.
- $\dot{m}\sqrt{c_pT_1}/D^2p_1$ is a dimensionless mass flow rate that you have come across before in compressible flow. In many (most) cases it is defined using p_{01} and T_{01} rather than p_1 and T_1 because it is easier to measure the stagnation values at machine inlet.
- $ND/\sqrt{c_pT_1}$ is a type of Mach number based on blade speed. It is a measure of the importance of compressibility in the machine.

Any other dimensionless group involving a dependent variable that we are interested in is a function of the above 3 groups. e.g.

$$\frac{p_{02}}{p_{01}} = fn_1\left(\gamma, \frac{\dot{m}\sqrt{c_pT_{01}}}{D^2p_{01}}, \frac{ND}{\sqrt{c_pT_{01}}}\right)$$

$$\frac{T_{02}}{T_{01}} = fn_2\left(\gamma, \frac{\dot{m}\sqrt{c_pT_{01}}}{D^2p_{01}}, \frac{ND}{\sqrt{c_pT_{01}}}\right)$$

$$\eta = fn_3\left(\gamma, \frac{\dot{m}\sqrt{c_pT_{01}}}{D^2p_{01}}, \frac{ND}{\sqrt{c_pT_{01}}}\right)$$

$$\frac{W_x}{\dot{m}c_pT_{01}} = fn_4\left(\gamma, \frac{\dot{m}\sqrt{c_pT_{01}}}{D^2p_{01}}, \frac{ND}{\sqrt{c_pT_{01}}}\right)$$

It is very seldom that we test a machine or a model with a different gas to its usual one and so γ is usually dropped from the list of independent groups. An exception is for steam turbines where model turbines are sometimes tested in air which has a different γ to steam.

10.6 TURBOMACHINERY CHARACTERISTICS

10.6.1 High Speed Compressor Characteristics

The performance of a high speed compressor (single stage or multistage) is usually presented as a graph of pressure ratio or efficiency against dimensionless mass flow rate with curves drawn for different constant dimensionless speed lines.

For low non-dimensional mass flow rate (corresponding to low flow coefficient and high angles of incidence), the constant speed lines terminate at the surge (or stall) line where the compressor becomes unstable and violent oscillations in the flow may occur. It is not safe to operate a high speed compressor beyond this point. The efficiency may be plotted similarly as in Figure 9 or may be plotted as contours of efficiency on the pressure ratio characteristic.

The pressure ratio curves become very steep at high speeds and that the compressor ceases to produce a pressure increase above a particular non-dimensional mass flow rate due to choking of the flow.

The maximum pressure ratio of a single stage axial compressor is about 2, limited by shock waves. For a single stage centrifugal compressor the limit is about 10. Greater pressure ratios can be achieved by constructing multi-stage compressors. If all n stages have the same pressure ratio, the overall pressure ratio is given by $(p_{out}/p_1)_{overall} = (p_{out}/p_{in})^n$. A difficulty with multi-stage compressors, is that a small change in the flow conditions in one stage may “snowball” and cause increasingly large changes in the subsequent stages. Multistage axial flow compressors become very difficult to start if the overall pressure ratio is greater than about 10 because the volume flow change through the machine is completely mis-matched to the change in flow area at low speeds and low pressure ratios. This is one reason why aircraft engines have separate low and high pressure compressors that can turn at different speeds as the engine comes up to speed.

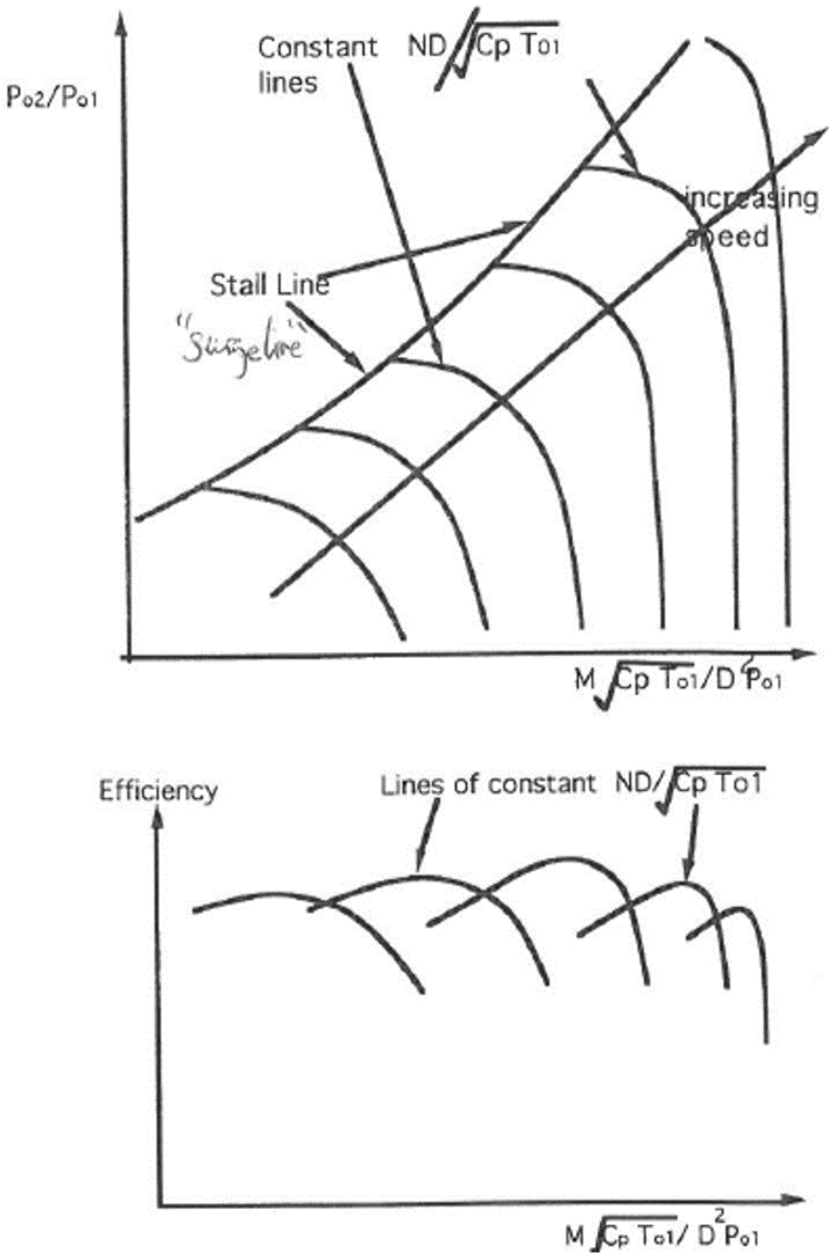


Figure 9. High speed compressor characteristics

10.6.2 Turbine Characteristics

These are plotted using the same variables as compressors but the behaviour is very different. Because the boundary layers are much more stable, turbines can operate with a high pressure ratio (p_{in}/p_{out}) per stage which may lead to choking in the blade rows and so to a constant dimensionless mass flow. Also the lift on a turbine blade is much less dependent on the angle of attack than that on a compressor and so the performance is not so much affected by speed. Typical pressure ratio:mass flow characteristics are as shown in Figure 10-Figure 11.

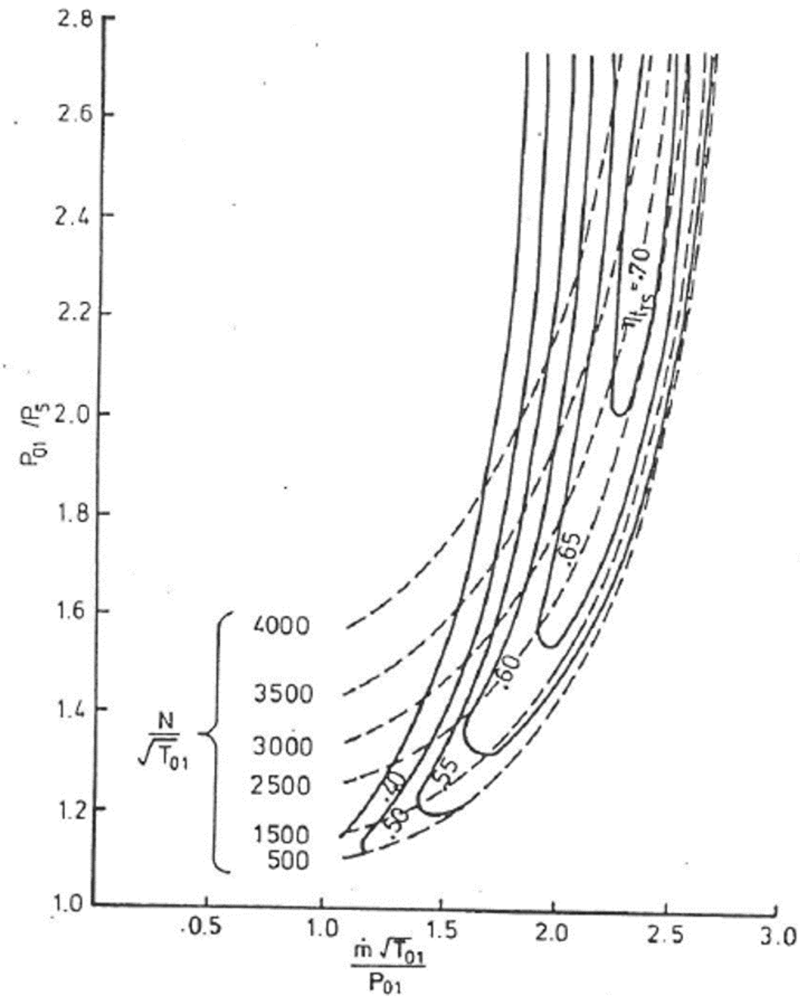


Figure 10. The pressure ratio and efficiency of a radial turbine.

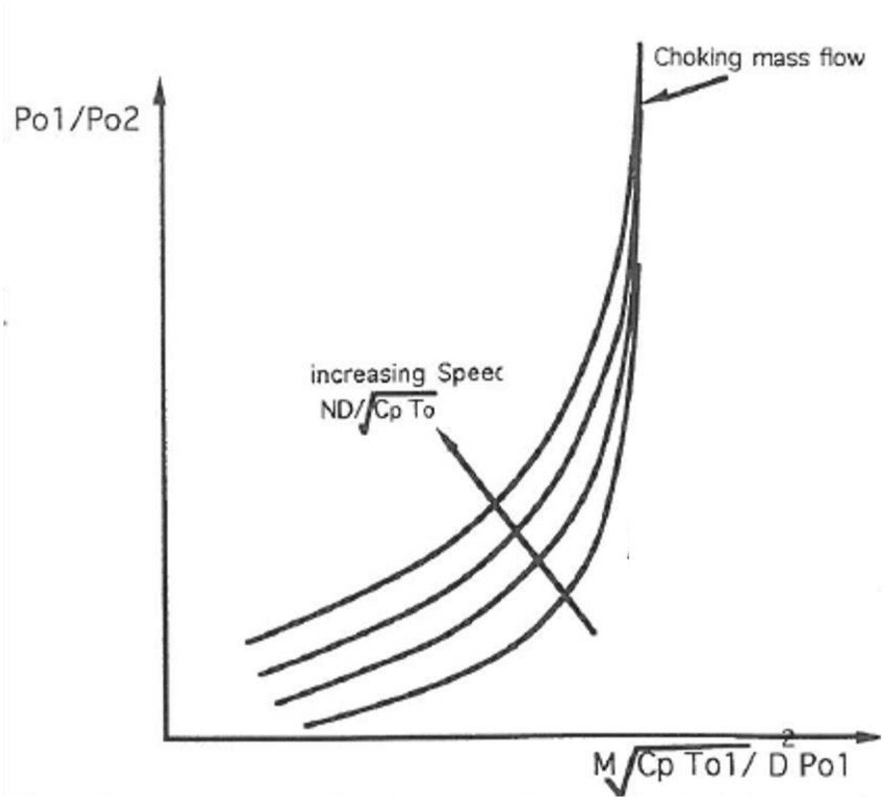


Figure 11. Typical turbine characteristics for a machine which chokes in the first stator row.

Most high pressure ratio turbines operate with their blades either choked or near choked so that their dimensionless mass flow is independent of speed and of pressure ratio. In this case the mass flow can only be varied by changing the stagnation conditions at inlet. e.g. the flow through steam turbines is varied by throttling the steam so as to drop its pressure before it enters the turbine. Turbines do not become unstable at high pressure ratios. Pressure ratios of up to 4:1 (or greater with some loss of efficiency) can be achieved in a single stage and overall pressure ratios of the order of 100:1 can be achieved in a multistage turbine.

10.6.3 Alternative Groups

The true dimensionless groups described above can be applied to all geometrically similar machines and to different working fluids. However, in practice dimensional analysis is usually used to present the performance of a single machine with varying inlet conditions and varying speed but with the same fluid. In this case c_p, D and γ do not vary and it is customary to leave them out of the groups and to plot the performance using the *dimensional* groups

$$\frac{\dot{m}\sqrt{T_1}}{p_1} \; or \; \frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$$

$$\frac{N}{\sqrt{T_1}} \; or \; \frac{N}{\sqrt{T_{01}}}$$

The shape of the characteristics is not altered by this. Perhaps even more common is to make the groups have physically meaningful units such as kg/s and rpm by plotting them as

$$\frac{\dot{m}\sqrt{T_{01}/T_{0,ref}}}{p_{01}/p_{0,ref}} \; and \; \frac{N}{\sqrt{T_{01}/T_{0,ref}}}$$

where $p_{0,ref}$ and $T_{0,ref}$ are usually standard atmospheric conditions.

The groups are then always proportional to the true dimensionless groups but having physical units they give more "feel" to the performance charts.

10.7 SUMMARY

This chapter has defined a number of non-dimensional parameters with which you need to be conversant:

- Flow coefficient -- the non-dimensional meridional flow velocity.
- Work coefficient (or stage loading coefficient) -- the non-dimensional shaft work of the stage or device.
- Specific speed -- a measure of the flow rate versus pressure rise that gives an indication of which type of turbomachine will be most efficient. Low values imply positive displacement devices, and radial turbomachines. High values imply axial flow turbomachines and propellers.

Compressor, turbine and nozzle performance is usually reported in the form of a characteristic map, including contours of non-dimensional speed and efficiency on axes of pressure ratio versus non-dimensional mass flow rate. The operating region of a compressor is bounded by the choke limit (negative incidence) on one side and the stall or surge limit (positive incidence) on the other.