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SESA2025 Mechanics of Flight
Lateral approximations

Lecture 3.9

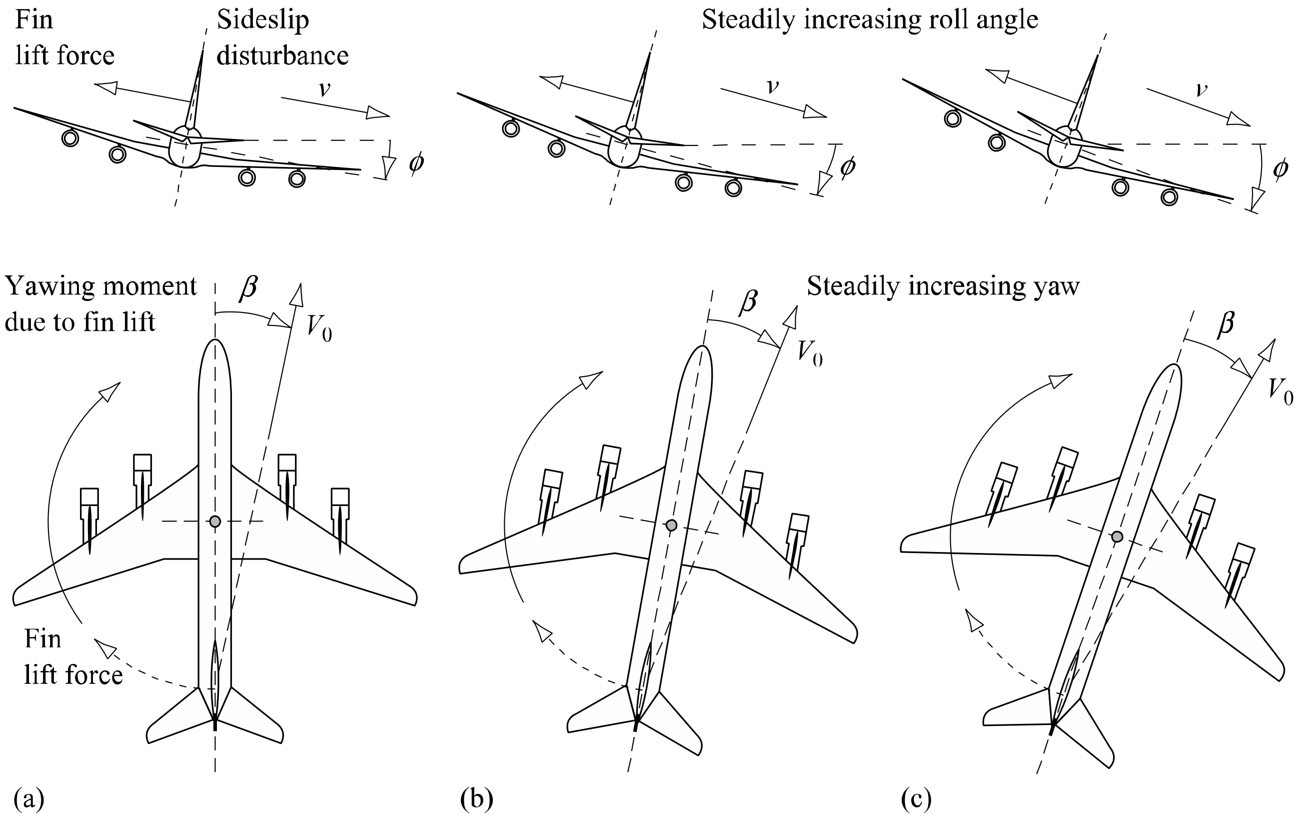
Spiral Mode (Slow Process)

In case of low wing with insufficient dihedral or sweep

- Slow process due to yaw damping & roll damping

Forces and moments in spiral divergence

- Sideslip causes side-force on fin in turn causing yaw, and aircraft enters a curved path. Extra velocity on outer wing causes roll leading to further sideslip and divergence. Dihedral or sweep will lead to opposite rolling moment tending to stabilise motion.



Cook 2007 *Flight Dynamics* 2nd edition, Elsevier

Slow spiral approximation

very slow changes in attitude (true, atleast initially)

Slow to develop, so we can set:

$$\dot{v} = \dot{p} = \dot{r} = 0$$

assume

Ignore aerodynamic side-force terms

Derivation Not Examinable

since the side slip forces are much smaller than other forces acting it can be neglected

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 \\ 0 & -I_{xz} & I_{zz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cancel{\mathring{Y}_v} & \cancel{\mathring{Y}_p} & \cancel{\mathring{Y}_r} - mU_\infty & mg \\ \mathring{L}_v & \mathring{L}_p & \mathring{L}_r & 0 \\ \mathring{N}_v & \mathring{N}_p & \mathring{N}_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

Derivation Not Examinable

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What's left?

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -mU_\infty & mg \\ \mathring{L}_v & \mathring{L}_p & \mathring{L}_r & 0 \\ \mathring{N}_v & \mathring{N}_p & \mathring{N}_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

Substitute

$$\phi = \phi_0 e^{\lambda t}$$

Look for zero determinant for non-trivial solutions

$$\begin{vmatrix} 0 & 0 & mU_\infty & -mg \\ -\mathring{L}_v & -\mathring{L}_p & -\mathring{L}_r & 0 \\ -\mathring{N}_v & -\mathring{N}_p & -\mathring{N}_r & 0 \\ 0 & -1 & 0 & \lambda \end{vmatrix} = 0$$

Derivation Not Examinable

What's left?

$$\begin{vmatrix} 0 & 0 & mU_\infty & -mg \\ -\mathring{L}_v & -\mathring{L}_p & -\mathring{L}_r & 0 \\ -\mathring{N}_v & -\mathring{N}_p & -\mathring{N}_r & 0 \\ 0 & -1 & 0 & \lambda \end{vmatrix} = 0$$

Expand out

$$mU_\infty \begin{vmatrix} -\mathring{L}_v & -\mathring{L}_p & 0 \\ -\mathring{N}_v & -\mathring{N}_p & 0 \\ 0 & -1 & \lambda \end{vmatrix} + mg \begin{vmatrix} -\mathring{L}_v & -\mathring{L}_p & -\mathring{L}_r \\ -\mathring{N}_v & -\mathring{N}_p & -\mathring{N}_r \\ 0 & -1 & 0 \end{vmatrix} = 0$$

Keep going

$$mU_\infty\lambda\left(\mathring{L}_v\mathring{N}_p - \mathring{L}_p\mathring{N}_v\right) + mg\left(\mathring{L}_v\mathring{N}_r - \mathring{L}_r\mathring{N}_v\right) = 0$$

Derivation
Not Examinable

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What's left?

Keep going

$$mU_\infty\lambda\left(\mathring{L}_v\mathring{N}_p - \mathring{L}_p\mathring{N}_v\right) + mg\left(\mathring{L}_v\mathring{N}_r - \mathring{L}_r\mathring{N}_v\right) = 0$$

So we have:

important bits

$$\lambda = -\frac{g}{U_\infty}\frac{\left(\mathring{L}_v\mathring{N}_r - \mathring{L}_r\mathring{N}_v\right)}{\left(\mathring{L}_v\mathring{N}_p - \mathring{L}_p\mathring{N}_v\right)}$$

Now due to the similarity in numerator and denominator we can write:

$$\lambda = -\frac{g}{U_\infty}\frac{\left(\mathring{L}_v\mathring{N}_r - \mathring{L}_r\mathring{N}_v\right)}{\left(\mathring{L}_v\mathring{N}_p - \mathring{L}_p\mathring{N}_v\right)} = -\frac{g}{U_\infty}\frac{(L_vN_r - L_rN_v)}{(L_vN_p - L_pN_v)}$$

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Stability condition

So we now have the slow spiral mode approximation:

$$\lambda = -\frac{g}{U_\infty}\frac{\left(\mathring{L}_v\mathring{N}_r - \mathring{L}_r\mathring{N}_v\right)}{\left(\mathring{L}_v\mathring{N}_p - \mathring{L}_p\mathring{N}_v\right)}$$

Navion $\lambda = -0.0097$
(exact $\lambda = -0.0087$)

$L_v = \frac{\partial L}{\partial v}$

The denominator is positive for conventional aircraft

therefore, for a stable aircraft (ie $\lambda < 0$) we need

$$L_v N_r > L_r N_v$$

this approximations still quite useful

dihedral of wing and wing position (high wing vs low wing)

fin

wing

fin

main contributions to responses

yaw moment with yaw rate

More on the physical origin of aerodynamic derivatives later

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Dutch Roll Mode

Lateral equivalent of short period oscillation mode

Due to weaker directional stability:

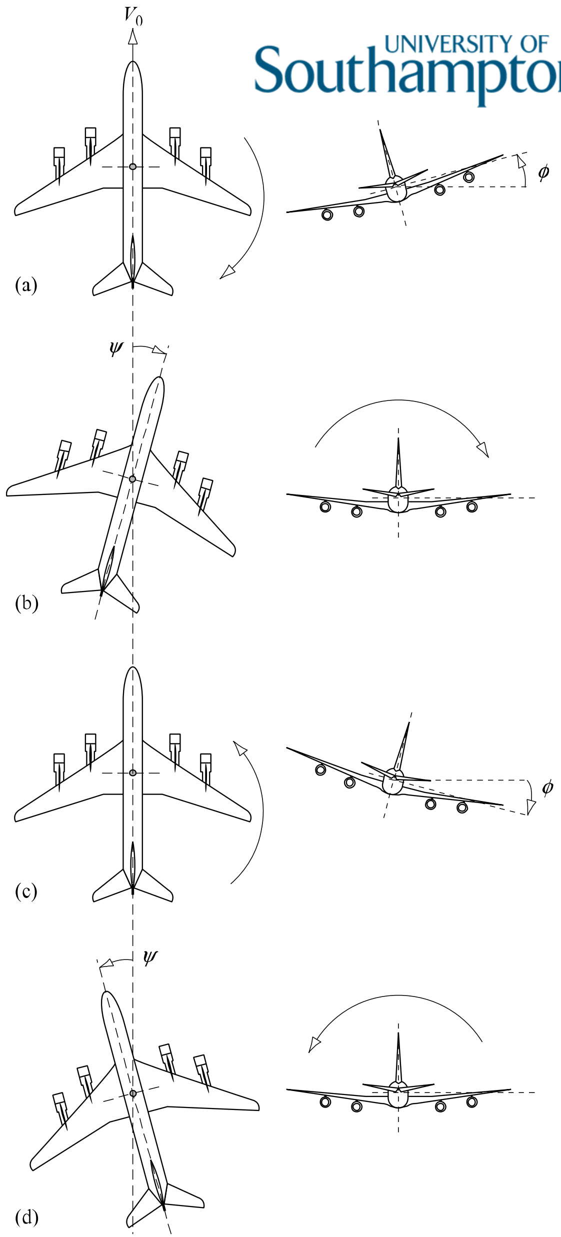
- fin less effective than tailplane at damping

Associated with flying quality:

- provoking nausea

Consider a disturbance from straight-level flight

- Primary effect: Oscillation in yaw
(yaw rate r vs sideslip v)
- Secondary effect: Typical yaw–roll motion



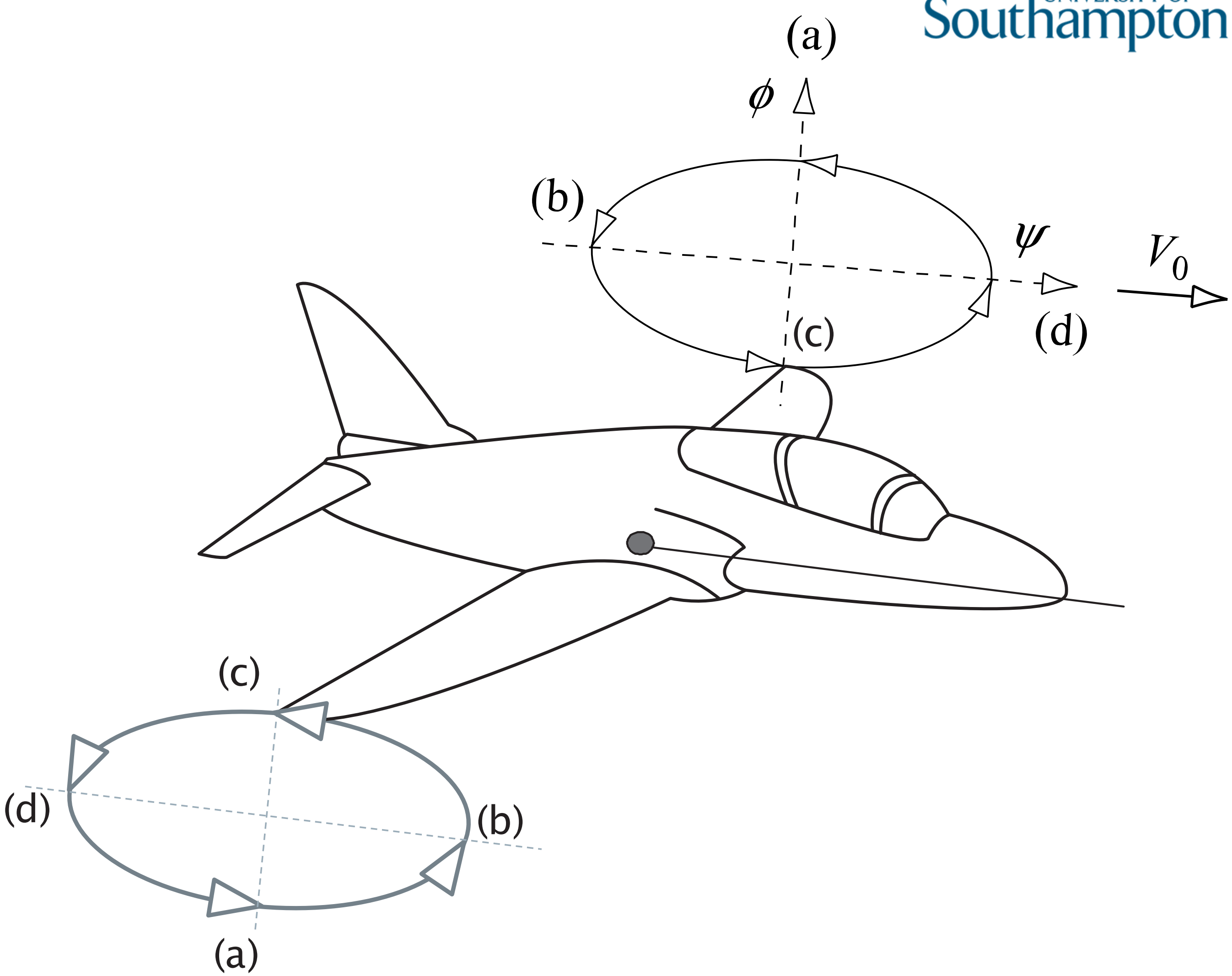
Cook 2007 *Flight Dynamics* 2nd edition, Elsevier

Dutch Roll Mode

Oscillation in yaw
(Primary effect)

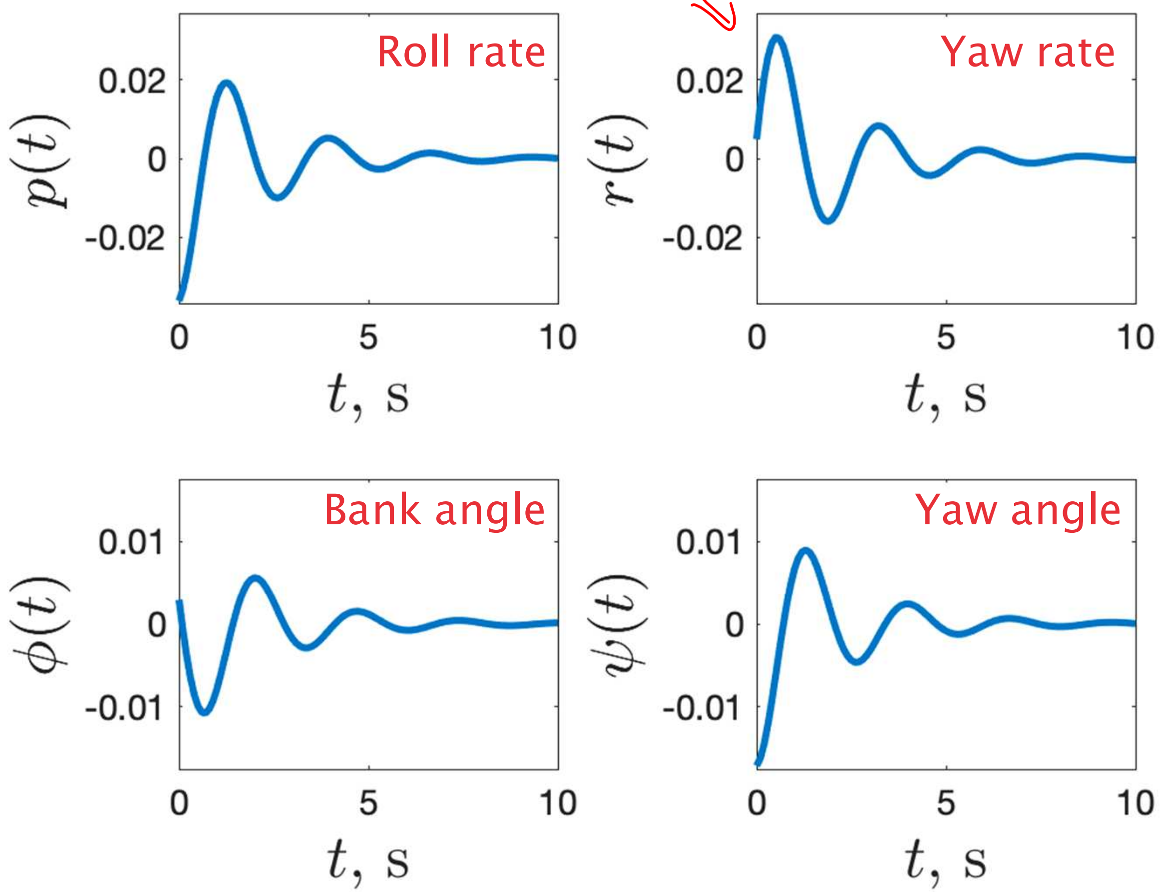
Typical yaw-roll motion
(secondary effect)

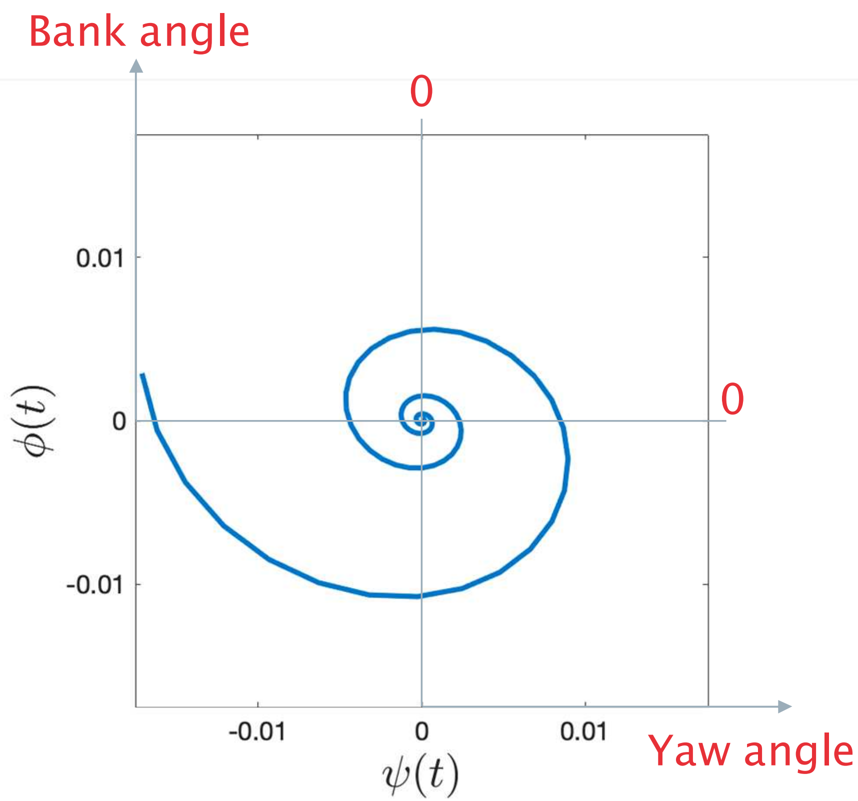
Due to the oscillation in yaw
- ⇒ wingtip moving back & forth
- ⇒ oscillatory differential lift: wingtip moving forward generates more lift.
- ⇒ oscillatory roll motion



Adapted from Cook 2007 *Flight Dynamics* 2nd edition, Elsevier yaw-roll is 90° out of phase

Dutch roll eigenvector (Navion)





Counter-clockwise motion of wing tip, as seen when looking outward along the port wing

Dutch roll mode approximation

note that a pilot may make modes worse, by putting in control inputs that resonate making pertibations worse.

The primary effects are due to yaw-rate and sideslip.

Therefore ignore roll!!!

Drop the roll rate and roll angle equations, ignore p, ϕ and Y_r

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 \\ 0 & -I_{xz} & I_{zz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \mathring{Y}_v & \mathring{Y}_p & \cancel{\mathring{Y}_r} - mU_\infty & mg \\ \mathring{L}_v & \mathring{L}_p & \mathring{L}_r & 0 \\ \mathring{N}_v & \mathring{N}_p & \mathring{N}_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

$p=0$

$\phi=0$

Usual procedure leading to:

$$\begin{vmatrix} m\lambda - \mathring{Y}_v & mU_\infty \\ -\mathring{N}_v & I_{zz}\lambda - \mathring{N}_r \end{vmatrix} = 0$$

Dutch roll mode approximation

$$\begin{vmatrix} m\lambda - \mathring{Y}_v & mU_\infty \\ -\mathring{N}_v & I_{zz}\lambda - \mathring{N}_r \end{vmatrix} = 0$$

expand determinant to get non trivial solution

Keep going

$$\left(m\lambda - \mathring{Y}_v\right)\left(I_{zz}\lambda - \mathring{N}_r\right) + mU_\infty \mathring{N}_v = 0$$

$$mI_{zz}\lambda^2 - \left(m\mathring{N}_r + I_{zz}\mathring{Y}_v\right)\lambda + \mathring{Y}_v\mathring{N}_r + mU_\infty\mathring{N}_v = 0$$

Compare with

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

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Dutch roll mode approximation

$\mathring{N}_r$ = yaw moment with respect to yaw rate

Undamped natural frequency

$$\omega_n^2 = \frac{\mathring{Y}_v \mathring{N}_r + mU_\infty \mathring{N}_v}{mI_{zz}}$$

Dutch roll approximation

$\mathring{Y}_v$ = sideslip force w respect to sideslip motion

Damping ratio

$$\zeta = \frac{-1}{2\omega_n}\left(\frac{\mathring{N}_r}{I_{zz}} + \frac{\mathring{Y}_v}{m}\right)$$

Y_v and N_r both negative
Fin contributes to N_r
Fin, wing dihedral and fuselage contribute to Y_v

if this was too large you would be forced to be aligned with the cross wind, causing issues

Accuracy of node approximations (Navion)

Results of approximations compared with exact solutions.

	Approximation	Exact
Roll damping	$\lambda = -8.4117$	$\lambda = -8.4442$
Slow spiral	$\lambda = -0.0097$	$\lambda = -0.0087$
Dutch roll	$\lambda = -0.5079 \pm \mathrm{i}2.1081$	$\lambda = -0.4871 \pm i2.3381$

Note for complex eigenvalues:

$\lambda = \sigma \pm i\omega$

where $\omega_n = \sqrt{\sigma^2 + \omega^2}$ and $\zeta = -\sigma/\omega_n$

for the Dutch roll approximation:

$\omega_n = 2.1684$ and $\zeta = 0.2342$