SEMESTER 1 EXAMINATIONS 2016-17

TITLE Aerothermodynamics

DURATION 120 MINS

This paper contains **FOUR** Questions

Answer **THREE** questions on this paper. Each question is worth 30 marks out of a total of 90.

An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow **and** normal shock tables (11 sides) are provided. (In reading from tables, nearest values are acceptable unless explicitly stated otherwise.)

An oblique shock chart is provided.

Note that a formula sheet is provided at the end of this paper

Only University approved calculators may be used.

A foreign language direct 'Word to Word' translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), γ=1.4, ρ=1.225 kg/m^3 and μ=1.79x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

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Q.1 Figure Q.1 opposite shows a converging-diverging nozzle that generates a uniform Mach 3.0 flow in a test section. A test plate and a shock generator are inserted into the test section, with their upper surfaces aligned with the main flow. The leading edge of the shock generator has an included angle of 13°. The upstream stagnation pressure is p_0=500 kPa and the stagnation temperature is T_0=420 K.

(i) Calculate the pressure and temperature in the throat.

[3 marks]

(ii) Calculate the pressure and temperature in the test section.

[3 marks]

(iii) Calculate the pressure and temperature immediately after the oblique shock.

[6 marks]

(iv) Sketch and explain the flow pattern on the test plate, assuming inviscid flow.

[2 marks]

(v) Calculate the pressure and temperature after the interaction of the shock with the test plate.

[6 marks]

(vi) Sketch and explain the flow pattern on the test plate when the viscosity of the fluid is taken into account.

[4 marks]

(vii) For a leading-edge angle of 26° find the Mach number after the oblique shock from the shock generator and sketch the expected flow pattern in the wall interaction region, assuming inviscid flow. Explain your answer, providing supporting calculations where appropriate.

[6 marks]

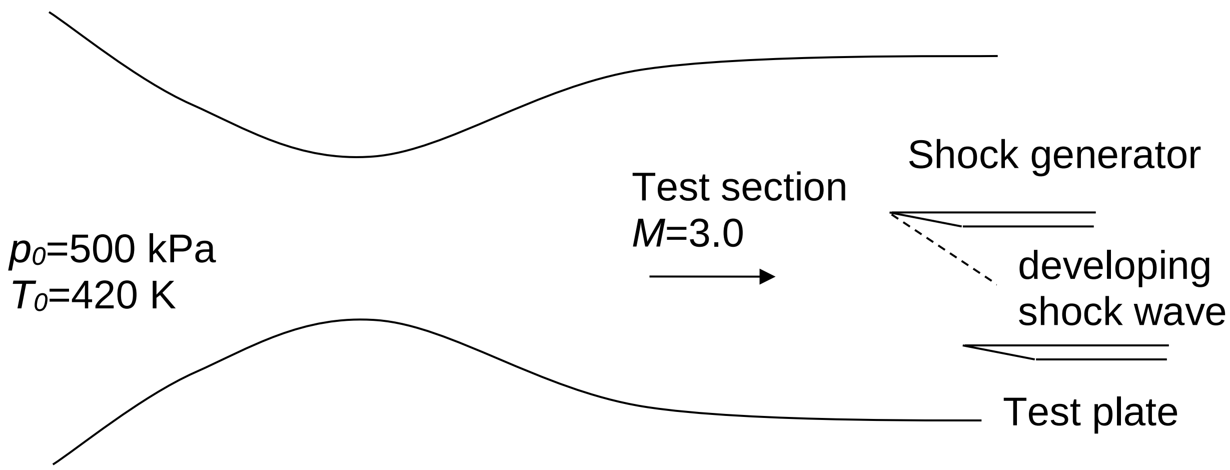

Figure Q.1

TURN OVER

Q.2

(i) Starting from the compressible potential flow equation

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

derive Ackeret's relation linking surface slope to pressure coefficient

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

[10 marks]

(ii) Derive an expression for the lift slope ($dC_L/d\alpha$) of a flat plate and sketch the variation with Mach number, paying particular attention to the limiting cases $M\to1$ and $M\to\infty$. Is the behaviour for $M\to1$ physical?

[6 marks]

(iii) Find the drag coefficient on the aerofoil sketched in figure Q.2 below, when it is placed at an angle of attack of 0° in a Mach 3.0 flow stream. The aerofoil has a chord c=1.0 m, a flat lower surface and an upper surface shape given by y=0.2x(1-x) m, with x measured from the leading edge along the chord line.

[10 marks]

(iv) Discuss whether this would be a suitable aerofoil for a supersonic aircraft?

[4 marks]

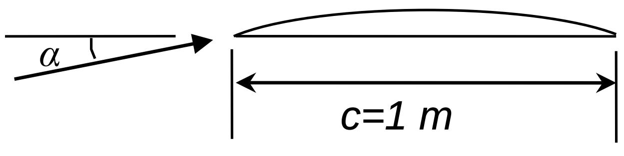

Figure Q.2

Q.3 Consider a prototype method-of-characteristics calculation for a minimum length nozzle, shown on Figure Q.3, using only two characteristic waves. The exit Mach number is M_e=1.5. The two characteristics emanating from point 0 are associated with flow angles θ of 3° and θ_{max} respectively. The centreline has y=0 and the x co-ordinate is measured relative to point 0, such that the point 0 has co-ordinates (x=0, y=1).

(i) Copy the sketch into your answer book and identify complex, simple and uniform flow regions.

[3 marks]

(ii) Define the Riemann invariant connecting points 0 and 3 and show that the turning angle is given by $\theta_{max} = \nu(M_e)/2$.

[5 marks]

(iii) At points 1 and 3 a symmetry condition applies. Find the x co-ordinate of point 1.

[6 marks]

(iv) Explain how waves are cancelled at point 4 and hence show how the flow angle and Mach number at point 4 are related to those of point 2.

[4 marks]

(v) Find the Mach number and flow angle at points 1, 2 and 3 and the *(x,y)* co-ordinates of point 2.

[12 marks]

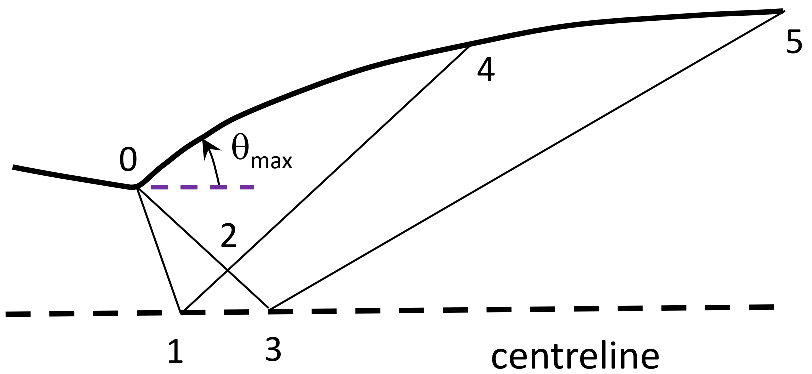

Figure Q.3

TURN OVER

Q.4

(i) Define the Nusselt number (Nu), Stanton number (St), Prandtl number (Pr) and Reynolds number (Re) and show they are related by

$$\mathrm{Nu} = \mathrm{St}\,\mathrm{Re}\,\mathrm{Pr}\,.$$

[4 marks]

(ii) A flat plate with sharp leading edge is tested in a wind tunnel. The plate has length 30cm, width 10cm and is kept at the constant temperature 50°C. The plate is exposed to a tangential flow of air (Pr=0.7) with velocity 60m/s and temperature 20°C. A trip wire is installed on the plate surface 5cm behind the leading edge and across the entire plate width.

(a) Estimate the Reynolds number at x=5 cm and confirm that the boundary layer satisfies $\mathrm{Re}_x < 300{,}000$, thereby remaining laminar until the trip wire.

[2 marks]

(b) Provide an estimation of the total heat flux through the plate surface section from the leading edge to x=5cm.

[7 marks]

(c) Estimate the total heat flux through the surface of the entire plate. Assume that transition occurs abruptly at x=5cm, rendering the boundary layer downstream of the trip wire fully turbulent.

[8 marks]

(iii) Show that the one-dimensional isentropic Euler equations

$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = 0, \text{ with } q = \begin{pmatrix} \rho \\ \rho u \end{pmatrix}, \quad f(q) = \begin{pmatrix} \rho u \\ \rho u^2 + p \end{pmatrix}$$

and equation of state $p = C\rho^{\gamma}$, $C = \text{const.}$ can be written as

$$\frac{\partial q}{\partial t} + A\frac{\partial q}{\partial x} = 0, \text{ with } A = \begin{pmatrix} 0 & 1 \\ a^2 - u^2 & 2u \end{pmatrix}$$

and hence show that the characteristic speeds are given by $u-a, u+a$. Note that the relation $a^2 = \gamma \dfrac{p}{\rho}$ holds true as usual.

[9 marks]

END OF PAPER (Formula sheet overleaf)

TURN OVER

Useful Formulae

Perfect gas equation of state:

$$p = \rho R T$$

Sound speed in a perfect gas:

$$a^2 = \gamma R T$$

Adiabatic flow:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Mach angle:

$$\sin\mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta + \mu)_A + (\theta + \mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta - \mu)_B + (\theta - \mu)_P\right]$$

$$x_p = \frac{x_B \tan\alpha_{BP} - x_A \tan\alpha_{AP} + y_A - y_B}{\tan\alpha_{BP} - \tan\alpha_{AP}}$$

$$y_P = y_A + \left(x_P - x_A\right)\tan\alpha_{AP}$$

Velocity potential equation:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

Linearised pressure coefficient:

$$C_p=-2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation:

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

Ackeret formula:

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

Laminar pipe flow:

$$\mathrm{Nu}=4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}=3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x=0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu}=0.022\,\mathrm{Pr}^{0.5}\,\mathrm{Re}^{0.8}$$

Turbulent boundary layer:

$$\mathrm{Nu}_x=0.029\mathrm{Re}_x^{0.8}\mathrm{Pr}^{0.6}$$