SESA2024 Astronautics

Chapter 5: Mission analysis – Exercises

Assume the following constants where necessary:
Earth radius, R_E = 6378 km
Earth gravitational constant, μ_E = 398 600 km^3/sec^2

1. Given the conservation of moment of momentum (or orbital angular momentum)

   $$m\mathbf{h} = \mathbf{r} \times m\mathbf{V}$$

   show that the orbital motion takes place in an invariant plane. Also show that

   $$r_p V_p = r_a V_a \qquad (1)$$

   where the subscripts *p* and *a* refer to periapsis and apoapsis values respectively.

2. In the two-body problem, the specific energy (energy per unit mass) is conserved

   $$\frac{1}{2}V^2 - \frac{\mu}{r} = \varepsilon = \text{constant.}$$

   Use this, and the conservation of orbital angular momentum (equation (1) above), to show that

   $$\varepsilon = -\frac{\mu}{2a}.$$

3. The total delta-V for a Hohmann transfer is given in Chapter 5 of the notes. If we define V_1 as the circular orbit speed in the lower orbit, and $x = r_1 / r_2$ as the ratio of orbital radii, show that the total delta-V is given by

   $$\Delta V = V_1\left[\left(\frac{2}{x+1}\right)^{1/2}(1-x) + x^{1/2} - 1\right].$$

4. An unidentified object is observed at its apogee height of 1000 km by radar to have a speed of 5.589 km/sec. Is the object likely to be a ballistic missile, an orbiting satellite, or a space probe escaping from Earth?

5. Prove that the distance of the apoapsis of an elliptic orbit from the Earth's centre is

   $$r_{apoapsis} = a(1+e) \ .$$

6. Discuss the advantages and disadvantages of the Hohmann transfer for interplanetary missions.

7. A spacecraft is deployed into a circular, equatorial orbit of altitude 300 km. It is transferred to a geostationary Earth orbit (GEO) using a Hohmann transfer.

   Estimate the magnitude of each of the delta-Vs required.

   The initial mass of the spacecraft in the low orbit is 2500 kg. This comprises a satellite, plus an attached rocket stage to execute the first burn. After this burn, the rocket stage's dry mass of 136 kg is discarded in transfer orbit. The second manoeuvre is executed using the satellite's integrated propulsion system. Use the data below, the constants at the top of page 1, and the 'field free rocket equation' (chapter 3 in the notes) to estimate the final mass deployed in GEO.

   GEO radius, R_{GEO} = 42164 km.
   Effective exhaust velocity of rocket stage and the satellite's propulsion system, V_{ex} = 3 km/sec.

8. A space probe is launched to fly-by a comet. The probe is launched into a circular orbit about the Sun, which is coplanar with, and of the same orbit radius as the Earth's orbit. The probe has a chemical rocket engine that is to be fired impulsively at the appropriate time in order to intercept the comet.

   The comet's orbit is in a plane inclined at 90^o to the Earth's orbit plane, and has elements of $a = 30 \times 10^8$ km and $e = 0.9$. The comet's path intersects the Earth's orbit plane at perihelion (its closest approach to the Sun), and it is at this point that the probe must encounter the comet (see Figure 1).

   - Calculate the speed, and distance from the Sun's centre of the comet when it passes through perihelion
   - To intercept the comet, the probe is injected into a Hohmann transfer orbit. Calculate the delta-V required to enter the transfer orbit.
   - How many days before the comet's perihelion must the probe's engine be fired?
   - Neglecting the gravitational influence of the comet, calculate the speed of the comet relative to the probe at fly-by.

   Earth orbit radius about the Sun, $r_E = 1.5 \times 10^8$ km
   Sun's gravity constant, $\mu_{Sun} = 1.3 \times 10^{11}$ km^3/sec^2

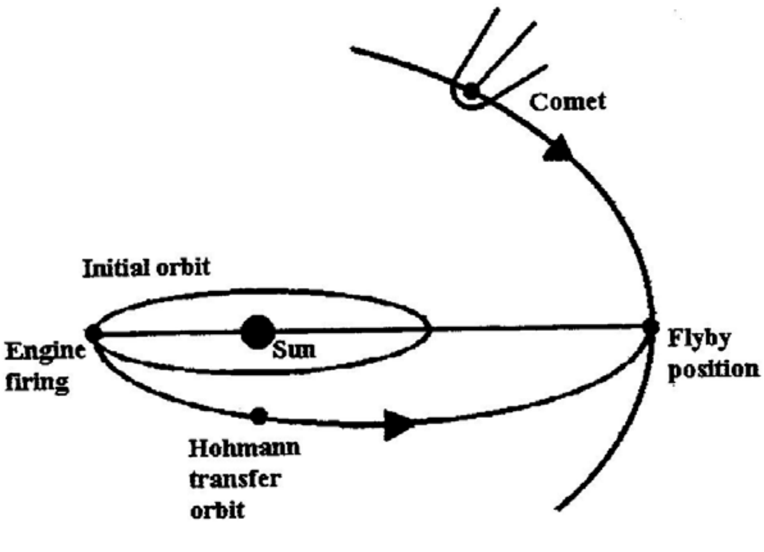

Figure 1