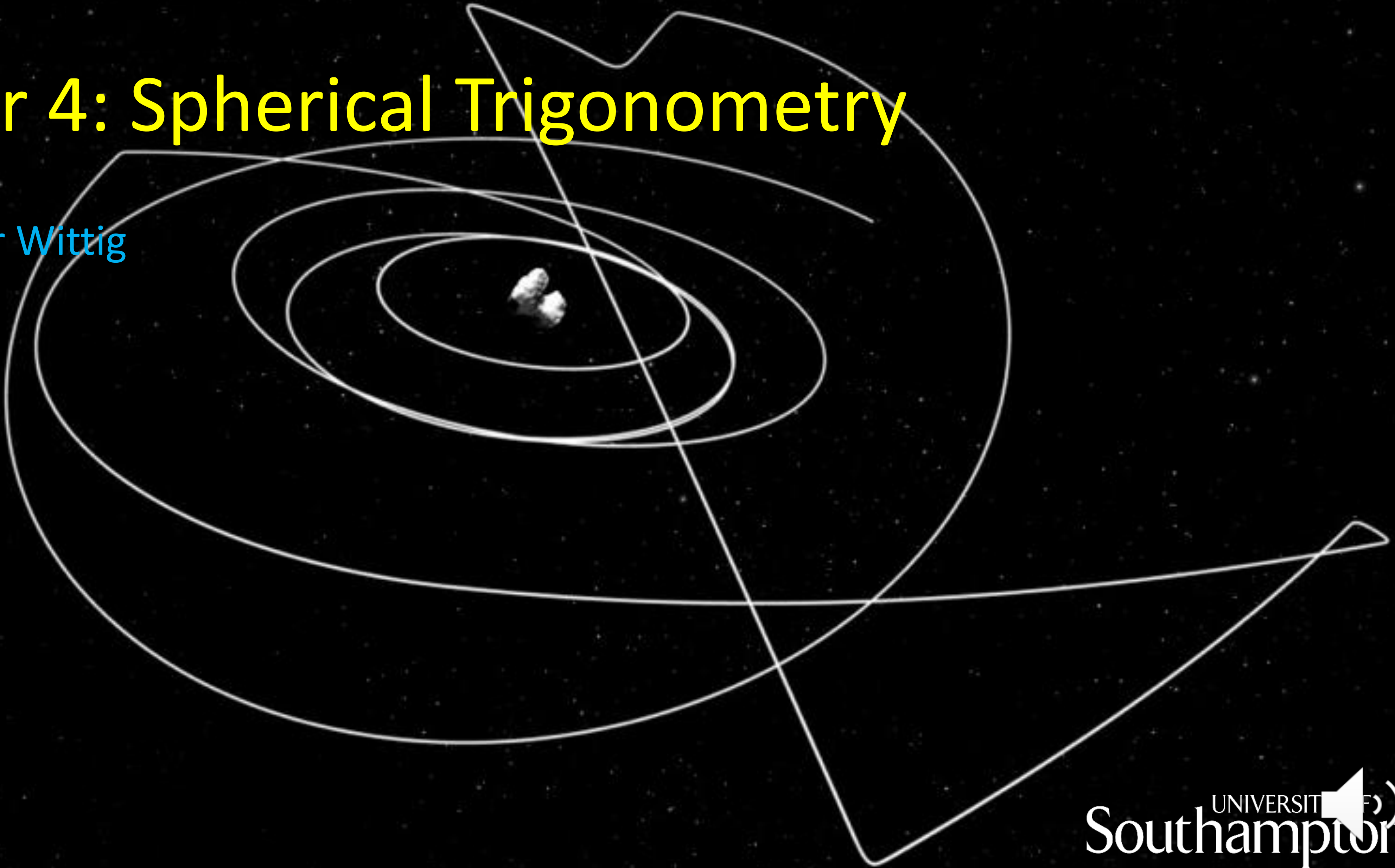
Chapter 4: Spherical Trigonometry

Dr. Alexander Wittig



Rosetta lander deployment orbit (ESA)

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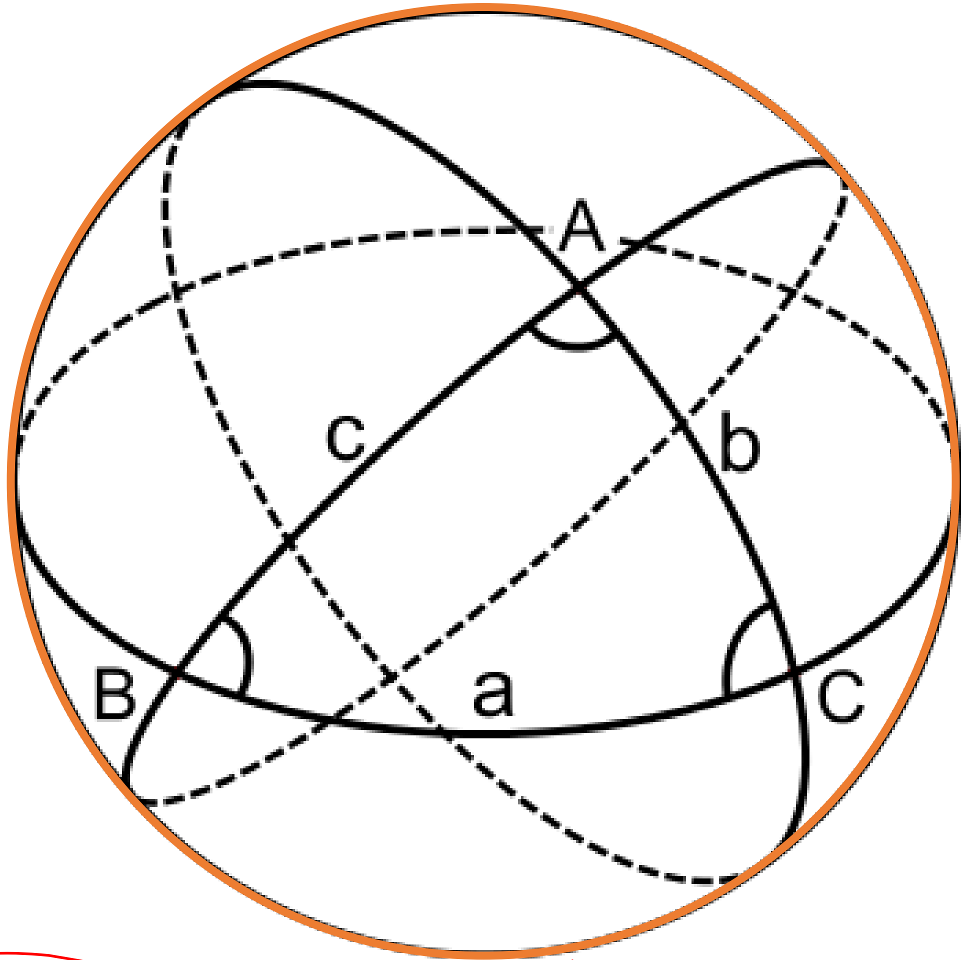


Cosine Rules

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Complementary Cosine Rules

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a,$$
$$\cos B = -\cos C \cos A + \sin C \sin A \cos b,$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c.$$

Sine Rules

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

CAREFUL WITH QUADRANT DISAMBIGUATION!

Distances on a sphere

One application: distances on a sphere (e.g. Earth):

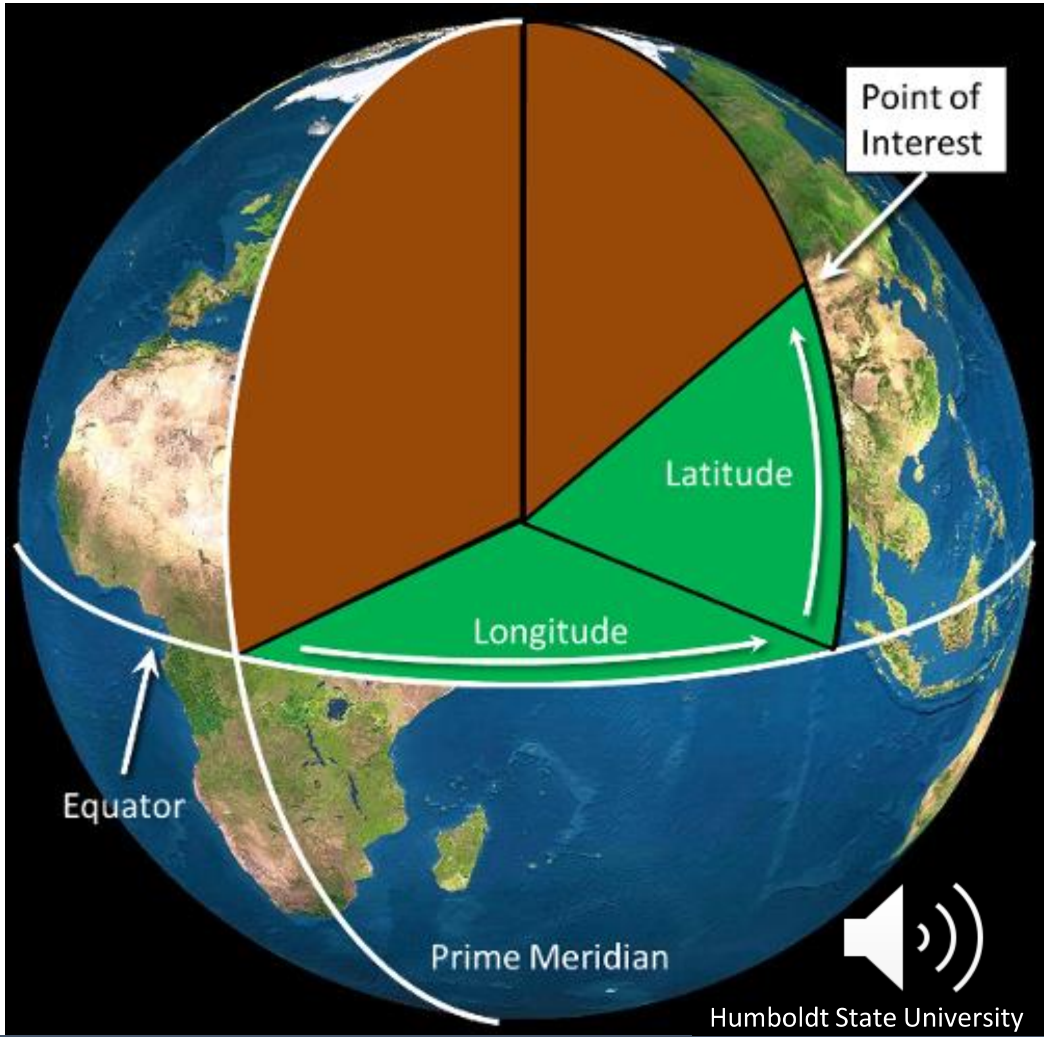
- Points located by latitude and longitude
- Prime meridian = 0 longitude
- Equator = 0 latitude

Question: given two points on Earth, what's the shortest distance between them?

$$\Phi_1, \lambda_1$$

$$\Phi_2, \lambda_2$$

$$d = ???$$



Humboldt State University

Memorise $\cos(90-a) = \sin a$

identities

Distances on a sphere

- From law of cosines:

$$\begin{aligned}\cos\delta &= \cos\widehat{\Phi}_1\cos\widehat{\Phi}_2 + \sin\widehat{\Phi}_1\sin\widehat{\Phi}_2\cos\Delta\lambda \\ &= \sin\Phi_1\sin\Phi_2 + \cos\Phi_1\cos\Phi_2\cos\Delta\lambda\end{aligned}$$

Distances on a sphere

- From law of cosines:

$$\cos\delta = \cos\widehat{\Phi}_1 \cos\widehat{\Phi}_2 + \sin\widehat{\Phi}_1 \sin\widehat{\Phi}_2 \cos\Delta\lambda$$
$$= \sin\Phi_1 \sin\Phi_2 + \cos\Phi_1 \cos\Phi_2 \cos\Delta\lambda$$

- Then:

$$d = R_E\delta \qquad (R_E = 6371\ \text{km})$$

Error from Earth triaxiality: < 0.5%

Distances on a sphere

- From law of cosines:

$$\cos\delta = \cos\widehat{\Phi}_1 \cos\widehat{\Phi}_2 + \sin\widehat{\Phi}_1 \sin\widehat{\Phi}_2 \cos\Delta\lambda$$
$$= \sin\Phi_1 \sin\Phi_2 + \cos\Phi_1 \cos\Phi_2 \cos\Delta\lambda$$

- Then:

$$d = R_E\delta \qquad (R_E = 6371\ \mathrm{km})$$

- Numerically more stable:

$$\sin^2\left(\frac{\delta}{2}\right) = \sin^2\left(\frac{\Delta\Phi}{2}\right) + \cos\Phi_1 \cos\Phi_2 \sin^2\left(\frac{\Delta\lambda}{2}\right)$$

(Haversine formula)

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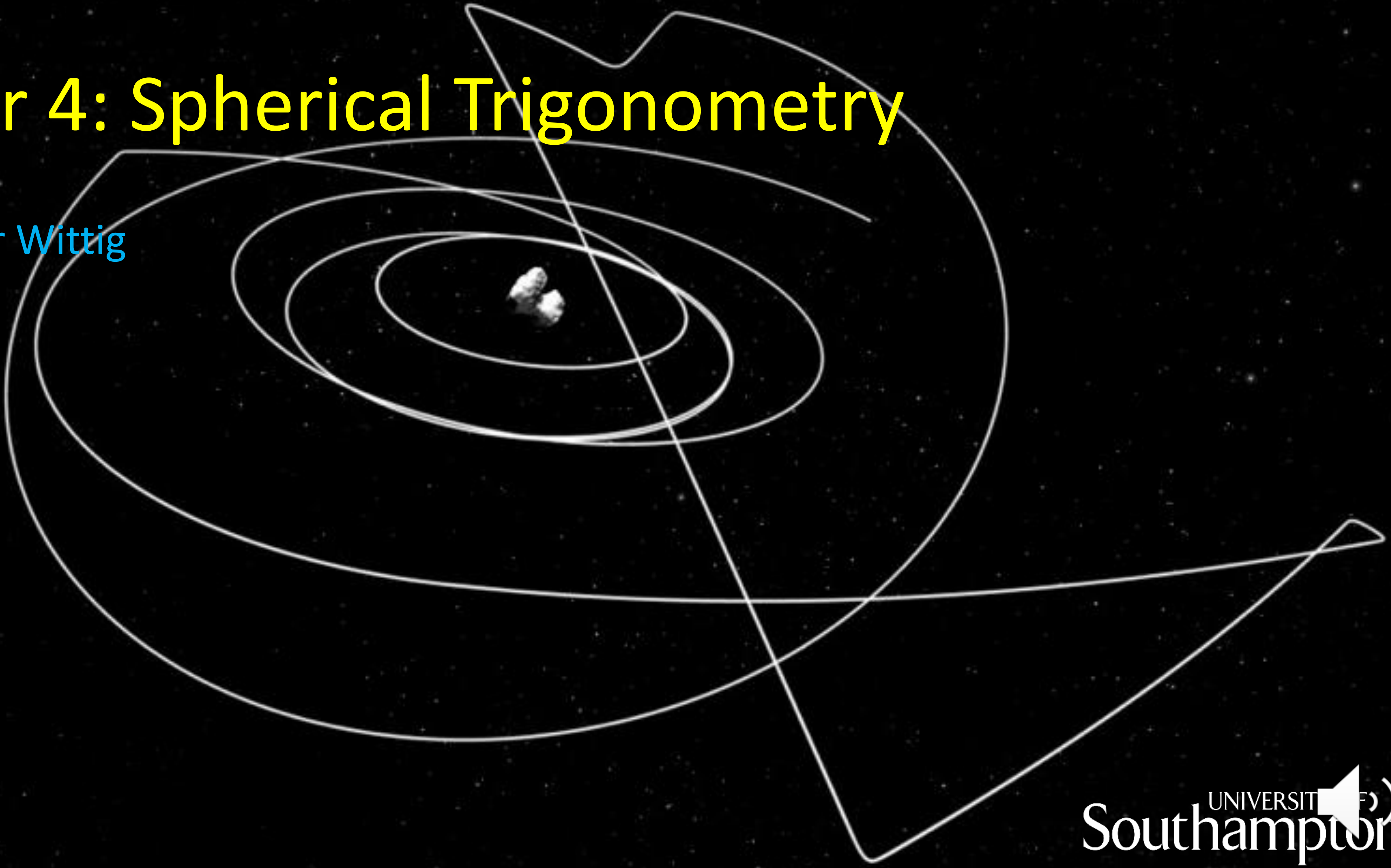

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