UNIVERSITY OF SOUTHAMPTON SESA3030W1

SEMESTER 1 ASSESSMENT PAPER 2020/21

TITLE: AEROSPACE CONTROL DESIGN

DURATION – 24 hours

This paper contains **FOUR** questions.

Answer **ALL** questions in **Section A** and **only TWO** questions in **Section B.**

Section A carries 40% of the total marks for the exam paper and you should aim to spend about 60 minutes on it.

Section B carries 60% of the total marks for the exam paper and you should aim to spend about 120 minutes on it.

An outline marking scheme is shown in brackets to the right of each question. Note that marks will only be awarded when appropriate working is given. All solutions should be hand-written and all steps should be shown to receive full credit. Provide explanations for every answer and indicate the unit(s) used in **ALL** calculations.

This examination is recommended to take approximately 3 hours of working time. You are advised to allocate your time accordingly. Your answer file may be submitted at any time before the due time. Please allow time to complete the submission process.

Results from the lecture material (anything on the Blackboard module site) can be used wherever needed. Any resources accessed from the internet must be properly cited.

The supplementary file 'SESA3030_exam_pars.pdf' gives specific data for each student. You must use the numbers corresponding to your Student ID number.

SECTION A

A1.

(i) Give a definition of the transfer function. The output of a system given by transfer function $G(s)$ to a unit step input $u(s) = 1/s$ is expressed in the time domain for $t \geq 0$ as:

$$y(t) = -0.4 + e^{-2t} - 0.6e^{-5t}$$

Determine the transfer function $G(s)$.

[4 marks]

(ii) Consider the following second-order ordinary differential equation of a rotational system:

$$J\frac{d^2\theta(t)}{dt^2} + D\frac{d\theta(t)}{dt} + MgL\sin\theta(t) = T(t)$$

where $\theta(t)$ is the time-dependent rotation angle. The system parameters are the moment of inertia J, viscous damping D, mass M, length L, and g is the gravitational constant. The driving torque is $T(t)$. Find the transfer function of this system linearised about the equilibrium point $\theta = 0$ expressed as $G(s) = \delta\theta(s)/T(s)$, where $\delta\theta$ is a small variation of the angle about the equilibrium. Express the natural frequency (ω_n), damping (ζ), peak time (T_p), settling time (T_s), and percentage overshoot (PO) in terms of the system parameters. Express the condition for the system to be critically damped in terms of the system parameters.

[6 marks]

TURN OVER

(iii) Determine the gain and phase of the system:

$$G(s) = \frac{b_1 s^2 + b_2 s + 1}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

at frequency $\omega = 1$ rad/sec. Determine the DC gain of this system. Assume $a_1 = a_3$ and $a_2 = a_4$.

[4 marks]

(iv) For the system shown in Figure A1-1, find the value of A that results in 30% overshoot to a step input.

[4 marks]

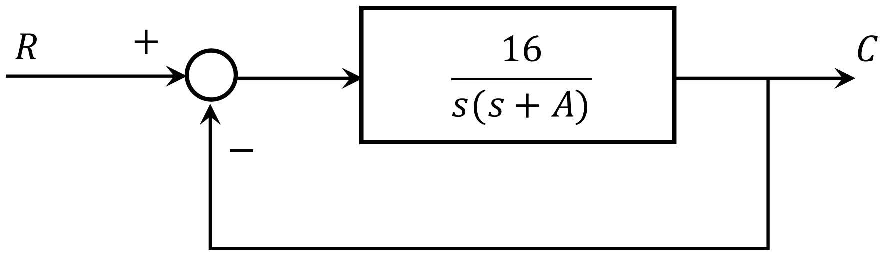

Figure A1-1

(v) Find the time domain function corresponding to the Laplace transform:

$$F(s) = \frac{10}{s(s+3)(s+7)}$$

[4 marks]

(vi) Give definitions for (a) the root locus, (b) gain margin (GM), (c) phase margin (PM), (d) gain crossover, and (e) phase crossover.

[3 marks]

TURN OVER

(vii) A system is described by the differential equation:

$$3\ddot{y} + 6\dot{y} - 45y = 2\dot{u} - 5u$$

Find the transfer function $G(s) = Y(s)/U(s)$ and its poles and zeros. Determine whether the system is stable.

[4 marks]

(viii) Write the input-output relationship of a PID controller in the time domain and determine the corresponding transfer function. Briefly describe the role of each term in the PID controller.

[3 marks]

(ix) Reduce the block diagram shown in Figure A1-2 to a single block and write the overall transfer function $Y(s)/U(s)$. Show all of the steps performed.

[6 marks]

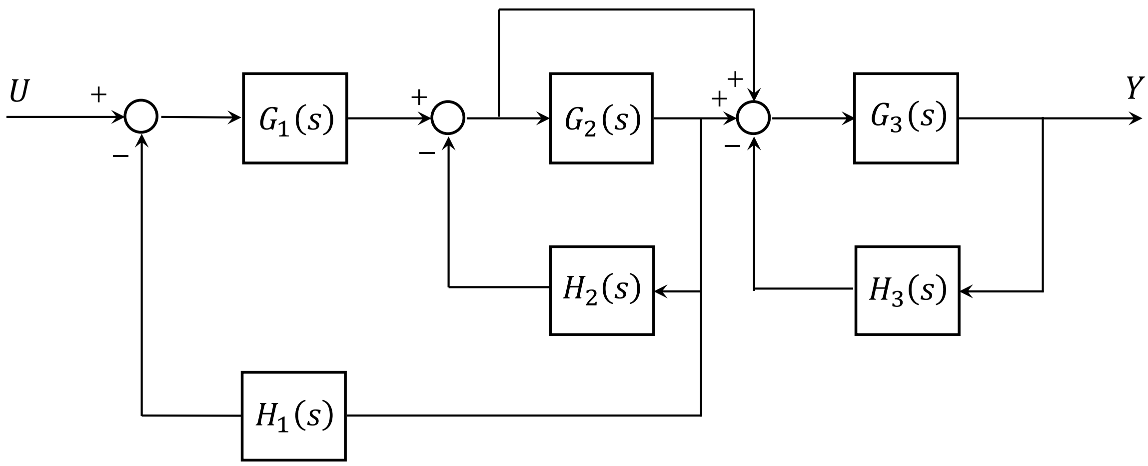

Figure A1-2

(x) What is the effect of adding an extra pole in the forward path in a feedback system? What might an additional pole represent?

[2 marks]

TURN OVER

SECTION B

B1. Figure B.1 shows a block diagram of a multiple-input, single-output (MISO) system.

[30 marks total]

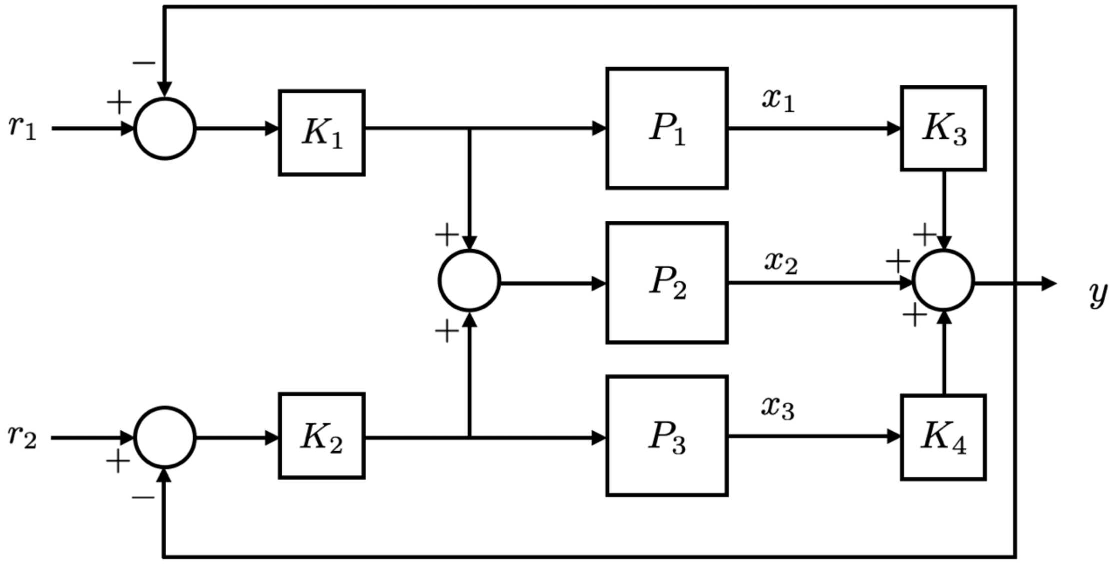

Figure B1-1

(i) Find the state-space system, i.e. matrices A, B, and C that describe this system. Please consult the spreadsheet SESA3030_exam_pars.pdf to determine what belongs in blocks P_1, P_2, and P_3.

[10 marks]

(ii) Suppose we simplify the system such that $K_2 = K_3 = K_4 = 0$. Draw the reduced block diagram.

[2 marks]

TURN OVER

(iii) Consider the new transfer function P_2 given below. Plot the Bode phase and magnitude plots for this system.

$$P_2(s) = \frac{100(s+1)}{(s+0.1)^2(s+10)(s+100)}$$

Hint: set $s = 0$ to determine the correct gain for this system.

[10 marks]

(iv) Estimate the gain and phase margin of the system.

[4 marks]

(v) Based on your results from B1-(iv), would you consider designing a controller for this system? Why or why not?

[2 marks]

(vi) Returning to B1-(i) and the block diagram in Figure B1-1, how would you assess the stability of the system if you could not ignore the second input as done in B1-(ii)?

[2 marks]

TURN OVER

B2. Suppose you have a gas turbine engine for spool speed regulation with the following dynamics:

$$G_p(s) = \frac{1}{(s+3)(0.5s+2)}$$

[30 marks total]

(i) Find the closed-loop transfer function if you use a PI controller.

[4 marks]

(ii) Find the steady-state error to a step input.

[4 marks]

(iii) Find the steady-state error to a ramp input.

[4 marks]

(iv) Instead of PI control, assume you have a velocity feedback control as shown in the block diagram in Figure B2-1 where $H_1(s) = 2s$. Find the new closed-loop transfer function.

[4 marks]

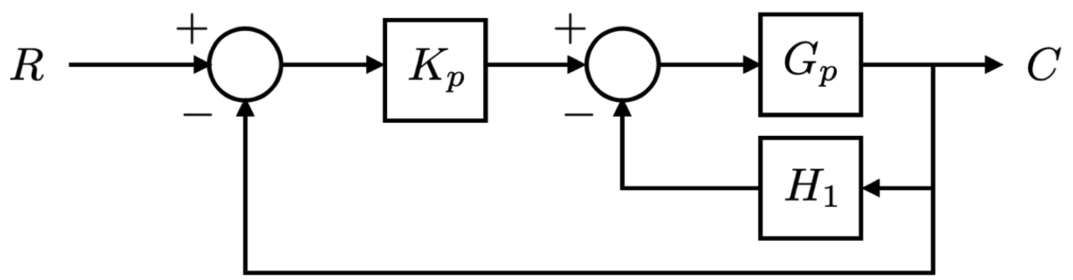

Figure B2-1

(v) Using the Routh-Hurwitz criterion, identify whether the system is always stable or not.

[4 marks]

TURN OVER

(vi) Find the state-space representation of the closed-loop system in B2-(iv) for arbitrary K_p.

[7 marks]

(vii) Suppose we have acceleration feedback instead of velocity feedback in B2-(iv). How does this affect the system and how can this be achieved in practice?

[3 marks]

TURN OVER

B3. The relevant differential equations for beam-bending are fourth-order. Suppose you have four poles in the left-half s-plane. Two poles lie on the real axis and the other two poles form a complex-conjugate pair. The real part of the complex-conjugate pair is in between the two poles on the real axis.

[30 marks total]

(i) Without knowing the precise location of the poles, draw the root locus diagram for this system. You must explain each step using the rules derived in lecture. Make sure the poles are spaced sufficiently apart for clarity.

[8 marks]

(ii) Consult the spreadsheet SESA3030_exam_pars.pdf for your pole locations and characteristic equation. Find the gain K for which the system becomes unstable.

[6 marks]

(iii) Suppose you use a phase-lead compensator to stabilise the system. Draw the new root locus diagram.

[6 marks]

(iv) Why does the phase-lead compensator delay the onset of instability for larger gains? Use you root locus plot from B3-(iii) to explain what is happening.

[2 marks]

(v) Suppose you try to remove the least stable pole from your system by setting $G_c(s) = K(s+1)$. Is this a good idea? Why or why not?

[3 marks]

TURN OVER

(vi) Let's say PID control was used to the stabilise the system instead of a phase-lead compensator. How many zeros and poles does this add to the root locus plot?

[3 marks]

(vii) What is the advantage of PID control over a phase-lead compensator for this system?

[2 marks]

TURN OVER

END OF PAPER

