AN INTRODUCTION TO ELECTRIC MACHINES

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CONTENTS

1 INTRODUCTION

The aim of this part of the module is to provide an introduction to two main types of electrical machine: the ac synchronous machine and the dc machine. We will also briefly discuss the transformer. The discussion will focus on the construction, principle of operation and their characteristics.

Section 2 covers some fundamental material on magnetic fields including Faraday's law of induction and magnetic force calculation. Section 3 presents a review of the construction and principle of operation of synchronous generators and the transformer. Section 4 covers dc machines, focusing on the relationship between torque and size and the calculation of K_T and K_E. There is also a brief discussion of the different types of dc motor and their characteristics.

There are many good books on the subject. In addition to the books on the reading list, the following is a list of some old and new ones for reference:

- A. E. Fitzgeral, C. Kingley and S. D. Uman, Electric Machinary, McGraw-Hill, 2003.
- T. Kenjo, Electric Motors and their Control, Oxford University Press, 1991.

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Finally, I want to apologise in advance for any typographical errors in these notes. Please let me know if you spot any.

Xize Niu

2 MAGNETIC FIELDS

When current passes through a coil, a magnetic field is generated as illustrated in Fig. 2.1. The magnetic field can be visualised by for example sprinkling iron filing on a non-magnetic plate covering the coil. The direction of the magnetic field is determined using the right hand rule, with the thumb pointing to the north from which the magnetic flux ϕ is assumed to emerge. The unit for flux is [Weber], or [Wb] which is equivalent to [Vs] (see Farady's law). Flux linkage is defined as

$$\lambda = N\phi \tag{0.1}$$

where N is the number of turns of the coil. Flux density B, measured in [Tesla] or [T], is related to ϕ by

$$\phi = \int B.dA \tag{0.2}$$

When the flux is constant over the area A, the flux density can be calculated simply as

$$B = \frac{\phi}{A} \tag{0.3}$$

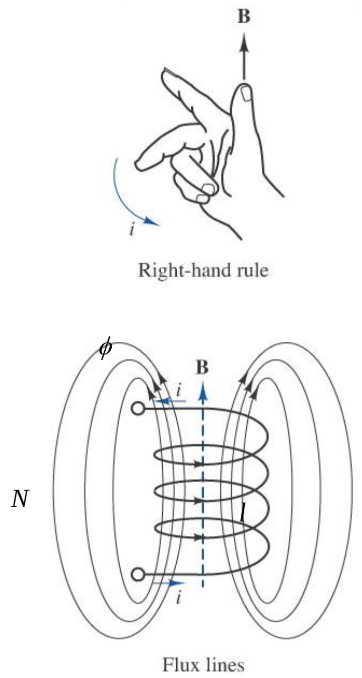


Fig. 2.1. Magnetic field of current currying coil

The amount of flux and the magnetic flux density can be increased by either increasing the current i or the number of the turns of the coil. The product Ni is known as the magnetomotive force or mmf,

$$F = Ni \tag{0.4}$$

The amount of mmf per unit length of the flux path l is known the magnetic field intensity H ,

$$\oint H.dl = Ni = F \tag{0.5}$$

In machines, we often can justifiably assume that H is constant over sections of the path and the above equation becomes,

$$F = Ni = \sum_k H_k l_k \tag{0.6}$$

2.1 FARADAY'S LAW

When flux linking a coil varies in time, as illustrated in Fig. 2.2, it induces a voltage or an electromotive force (emf) according to Faraday's law of induction,

$$E = -\frac{d\lambda}{dt} = -\frac{dN\phi}{dt} \tag{0.7}$$

The negative sign is known as Lenz's law, and it arises from the law of conservation of energy. It indicates that the emf E is in such a direction such that it tends to generate a current whose magnetic flux opposed the movement of the magnet. The greater the generated electric current and power, the harder it will be to move the magnet, as expected.

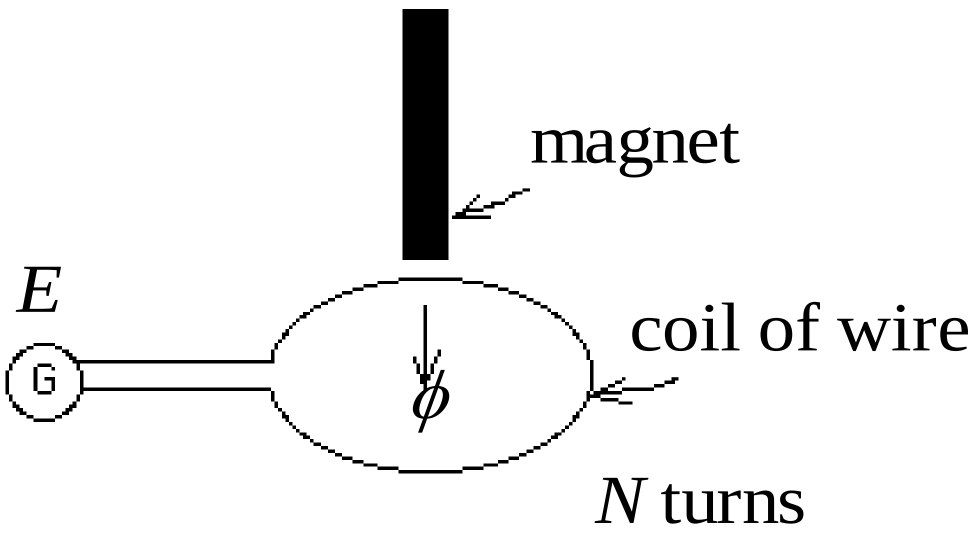


Fig. 2.2 When flux varies inside a coil it induces an emf according to Faraday's law

Faraday's law in equation (0.7) can be difficult to apply in practice. Fortunately in many applications including electric machines a simplified version can be derived. In Figure 2.3 consider a coil, with N turns, moving with speed u across flux lines generated by a magnet. The flux lines are perpendicular to the surface of the page and the side L of the coil cuts the flux lines. The area overlapping with the magnet, and the flux, reduce as the coil moves to the left. Assuming that the magnet flux density B inside the coil is uniform, i.e., constant, then the flux ϕ in the coil will be the product of flux density B times the overlap area $A = Lx$ between the magnet and coil,

$$\phi = BLx \tag{0.8}$$

According to equation (2.7) an emf will be generated in the coil,

$$E = -\frac{dN\phi}{dt} = -\frac{dNBLx}{dt} = -NBL\frac{dx}{dt} \tag{0.9}$$

Noting that the speed if the coil is given by

$$u = -\frac{dx}{dt} \tag{0.10}$$

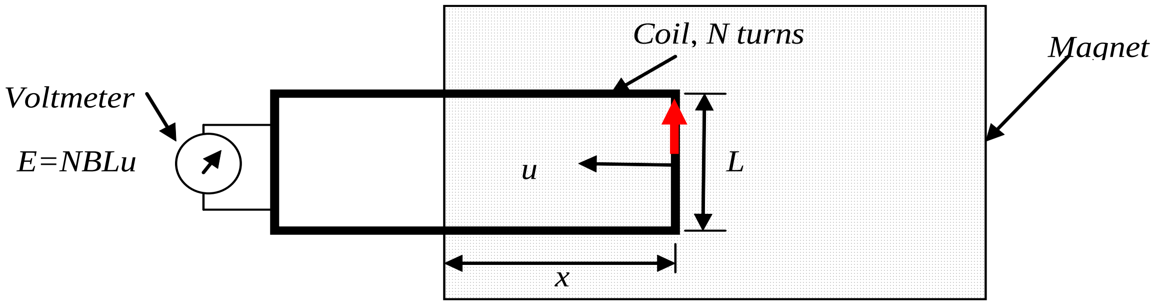
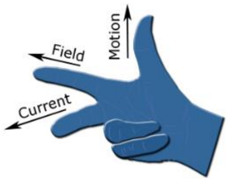


Fig. 2.3 EMF generated in a coil moving in a magnetic field.

Substituting (2.10) into (2.9) we obtain,

$$E = NBLu \tag{0.11}$$

The direction of the emf is determined by Fleming's right-hand rule,



The red arrow in Fig. 2.3 shows the direction of induced current by the emf if the coil is closed.

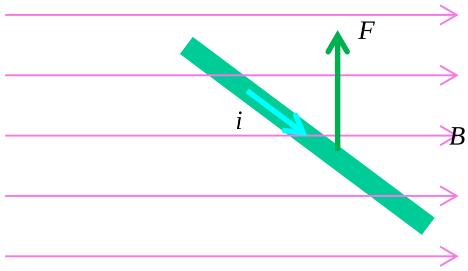
Note that the sides of the coils leading to the voltmeter do not contribute to generating emf as they do not cut the flux lines.

2.2 LORENZ'S FORCE EQUATION

When a current carrying conductor of length L is placed in a magnetic field it experiences a force

$$F = BiL \tag{2.12}$$

The direction of the force is determined using the left hand rule as illustrated in Fig. 2.4 below. The magnetic force on the conductor can be visualized to be due to the interaction between the flux lines of the conductor and the applied field as illustrated in Figure 2.5. Figure 2.5a shows the applied field flux lines (going up) and Figure 2.5b shows the flux lines of the magnetic field produced by the current in the conductor on its own (current is coming out of the page). Figure 2.5c shows the resultant of adding the two fluxes in Figures 2.5a and b; the flux on the right of the conductor is strengthened while the flux on the left is weakened. The magnetic force could be visualized by thinking of the flux lines as stretched rubber bands that have been bent by the conductor, and you could see that the "bands" are pushing the conductor to the left.



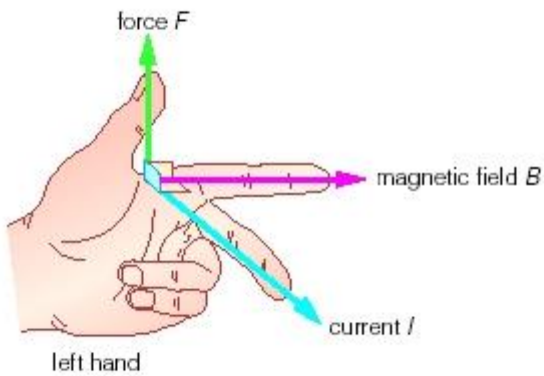


Figure 2.4 – Magnetic force on a current carrying conductor.

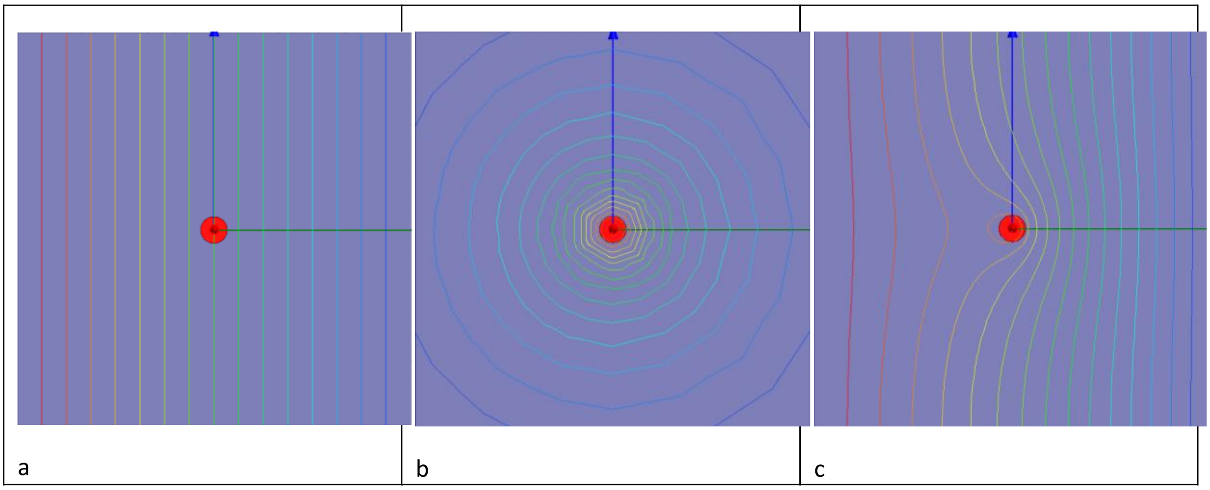


Figure 2.5 – Magnetic flux lines of a) applied field, b) field of the current, c) resultant field.

3 SYNCHRONOUS GENERATORS

Electricity is mainly generated at power stations by large electric generators driven usually by steam or gas turbine as prime movers, as illustrated in Fig. 3.1. Steam for the steam turbines is usually produced using heat from nuclear reactors, coal fire and oil. But any source of heat could be used e.g. renewable sources such as biomass (e.g. burning rubbish) or solar heat, but usually at smaller scale compared to nuclear, coal, gas and oil.

Diesel engines are normally used to drive standby and portable generator units. Active research is also on going to develop other types of heat engine prime movers such as the organic Rankine cycle engine and sterling engine for domestic scale electricity generators. Organic cycle Rankine use a low boiling temperature fluid (refrigerant) and hence it is suited for use with low grade (low temperature) heat from say solar or geothermal sources. Alternatively, low grade heat can be used to preheat water, before it is fed into gas or oil boilers to produce steam.

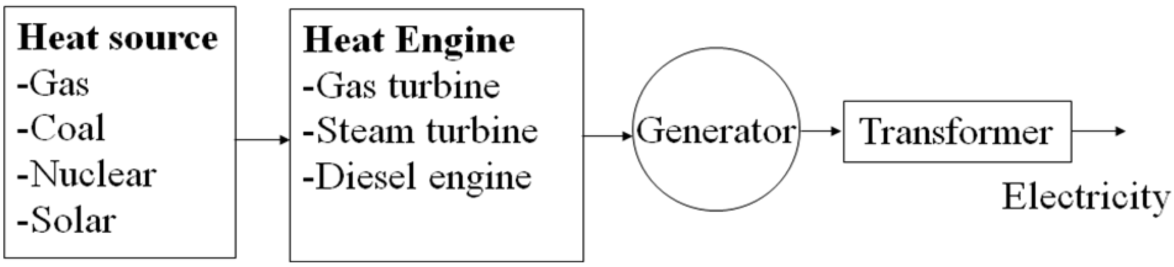


Fig. 3.1 - Block diagram of electric generators driven by heat engines

The large synchronous generators used in power stations are usually driven by steam or gas turbines, and hence they are commonly known as turbo-alternators. A typical alternator has an approximate diameter of 0.7 m, and an active length of 7 m (see Figs. 3.2-3.4).

Fig. 3.2 – Turbo-alternator driven by a steam turbine

Fig. 3.3 – Assembly of a turbo alternator

Fig. 3.4 – Winding the rotor of a turbo-alternator

Generators may also be driven by renewable energy sources such as wind, wave, tidal and marine current energy sources as illustrated in Figure 3.5 or used for exhaust energy recovery as illustrated in Figure 3.6. Due to the slow speed of wind and marine turbines, it is necessary to use a gearbox to step up the speed, in order to reduce the size of the generator. The generator power density (per per unit mass or volume) increases with its speed, i.e. the faster the generator the more power it can generate. The reason for this will become clearer when we discuss DC machines. Due to the variable nature of wind and marine current, it is necessary to have a power conditioning systems, the function of which is outside the scope of this module.

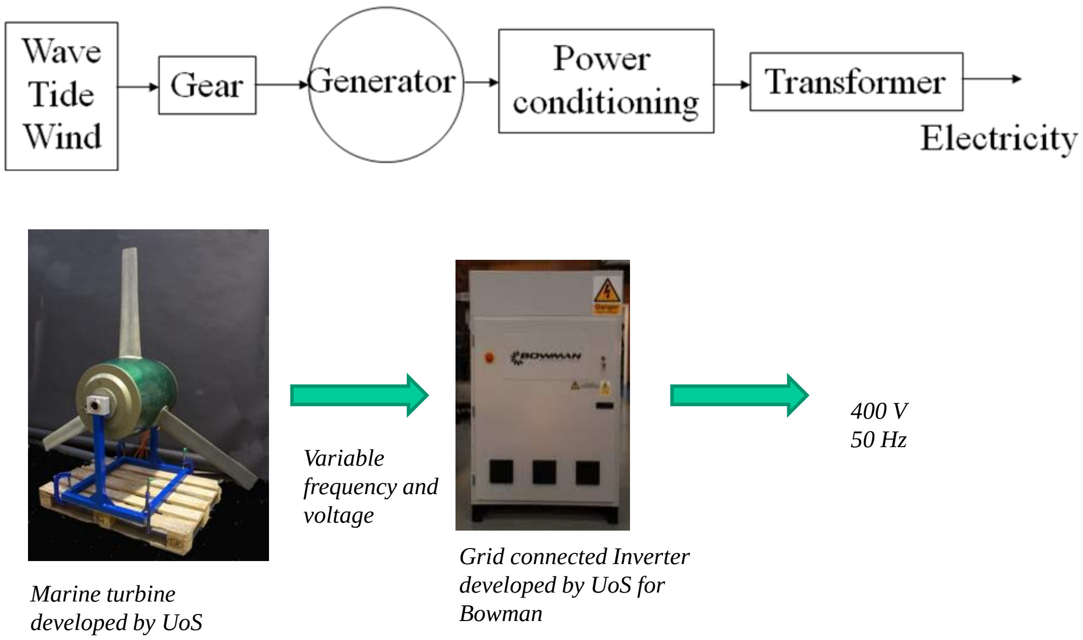


Fig. 3.5 Block diagram of an electricity generation system driven by wind and marine power

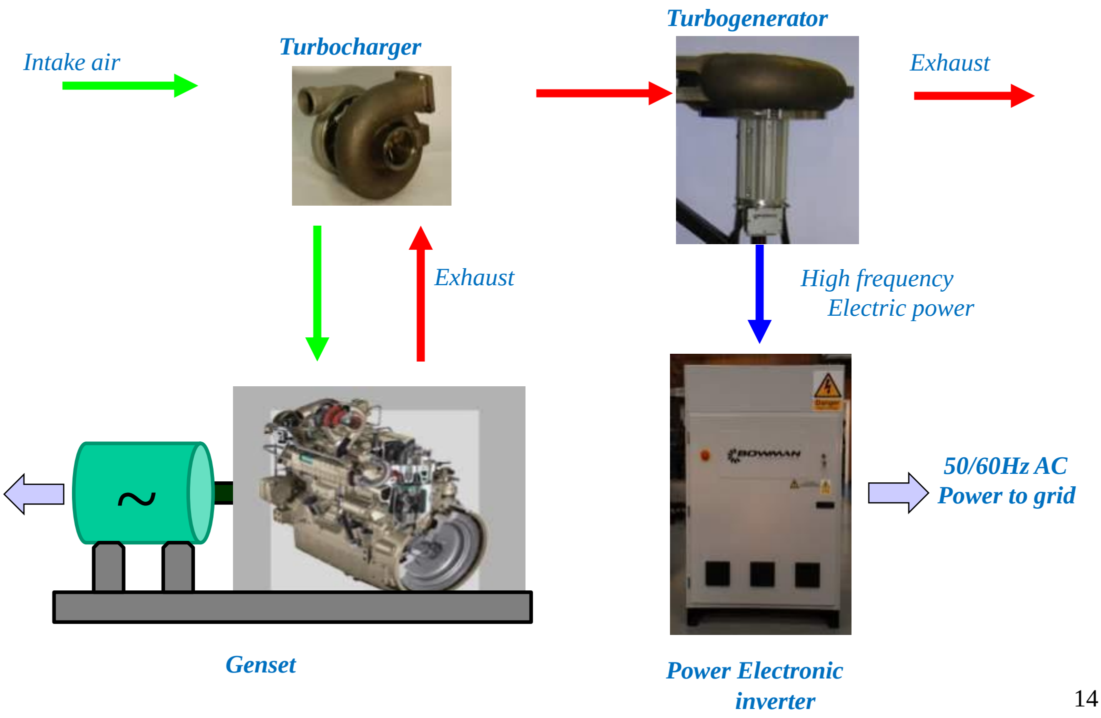


Fig. 3.6 – Exhaust energy recovery

3.1 PRINCIPLE OF OPERATION

A cross-section of a 3 phase synchronous machine is shown in Fig. 3.7. It comprises a rotating magnet or electromagnet inside 3 sets of coils located in slots in a stack of steel laminations (to stop the flow of eddy currents). The three sets of coils A, B and C are called 3 phases.

The principle of operation can be illustrated as shown in Fig. 3.8, which shows the variation of flux and emf in one of the phases as a function of rotor position. A sinusoidal or alternating current emf is generated. The current flow alternates between positive and negative. Most of the electricity is generated, transmitted and distributed in the form of alternating current or AC, rather than direct current or DC (see later). The main reason for this is that AC can be readily transformed to higher voltages by a device called a transformer. AC is also easier to switch off as it falls naturally to zero.

Note for every revolution of the rotor there is one electrical cycle and hence the electrical frequency equals to the number of revolutions per second

$$f = rev/s$$

Or the speed in rpm will be given by

$$rpm = 60f \tag{0.12}$$

The above equation applies to a two pole machine. In general the speed and frequency relationship is given by

$$rpm = \frac{120f}{N_p} \tag{0.13}$$

$$rpm = \frac{120f}{N_p} \tag{3.2}$$

where N_p is the number of poles.

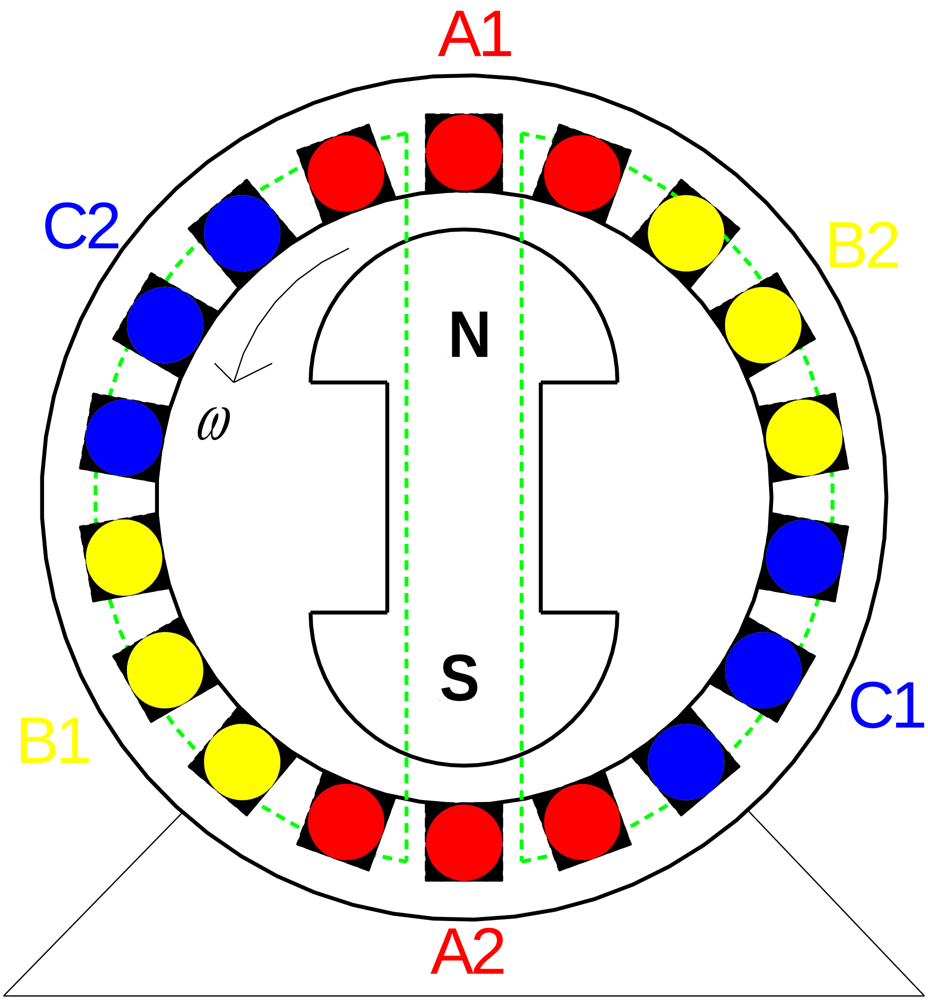


Fig. 3.7 - Schematic diagram of a cross-section of a three phase synchronous generator

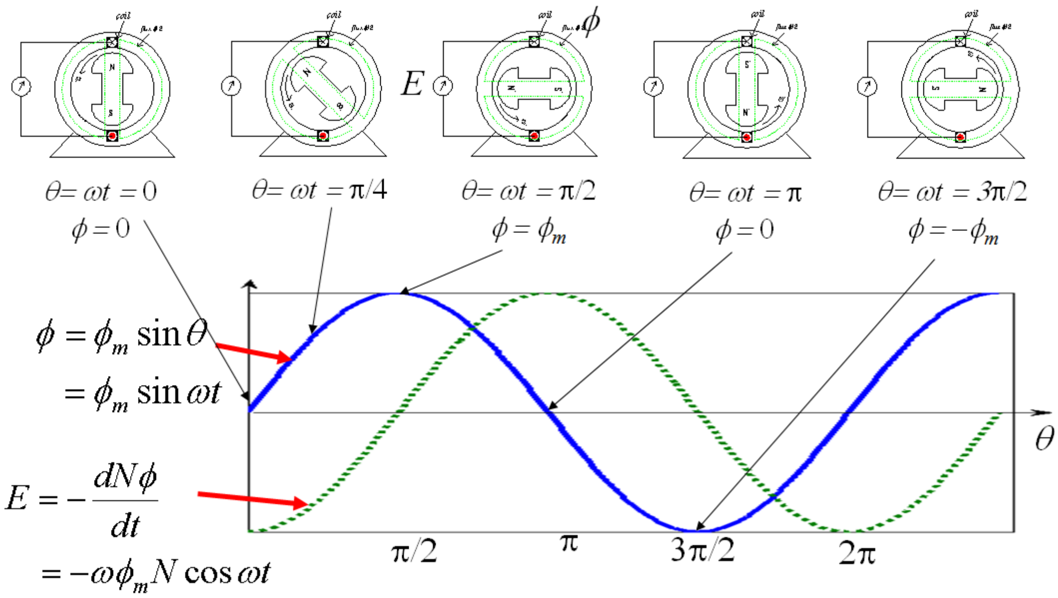
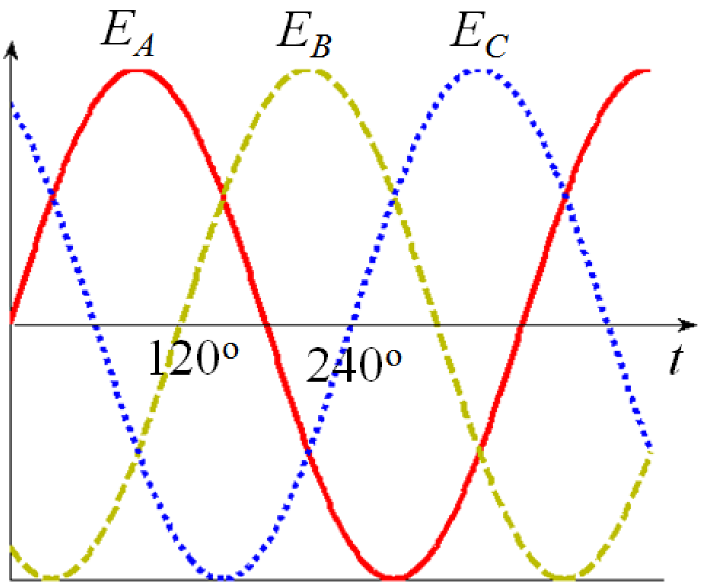


Fig. 3.8 – Operation of a single phase ac generator

Since the axis of the 3 phase coils are shifted in space by 120 degrees, the emfs will also be shifted by 120 phase angles as illustrated in Fig. 3.9. It can be readily shown that the three phase emfs add up to zero at any instant in time.

The phase coils are normally connected in either star (or Y) or Delta (Δ) as shown illustrated in Fig. 3.10. Note that there will be no circulating current and no need for a wire connecting the neutral point of the star as the emf add up to zero.



$$E_A = E_m \sin(2\pi f t)$$

$$E_B = E_m \sin(2\pi f t - \frac{2\pi}{3}) \qquad f = 50 \text{ Hz}$$

$$E_C = E_m \sin(2\pi f t - \frac{4\pi}{3})$$

Fig. 3.9 - EMF waveforms generated in a 3 phase generator

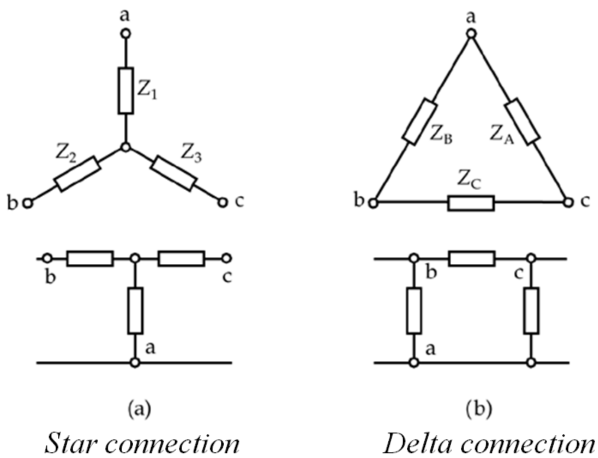


Fig. 3.10 - Three-phase generator winding connections

3.2 EXAMPLE

The peak value of the air gap flux density is about 0.8 T. Show that a 2-pole alternator running at 3000 rpm produces a sinusoidal voltage with a frequency of 50 Hz. Calculate the phase peak voltage, assuming 2 turns per phase coil.

Using equation (3.2) we can readily show that the frequency will be 50 Hz when the alternator speed is 3000 rpm.

$$rpm = \frac{120 f}{N_p} \Rightarrow f = \frac{rpm \times N_p}{120} = \frac{3000 \times 2}{120} = 50 \text{ Hz}$$

The emf can be calculated using equation Faraday's law

$E = NBLu$

The peripheral speed is given by $u = \omega \frac{D}{2}$, where D is he diameter of the rotor. Note also that $L = 2L_{rotor}$, where L_{rotor} is the rotor active length, because each coil has two sides cutting the flux lines emanating radially from the rotor surface.

Substituting we get the peak value of the emf,

$$E = 2NBL_{rotor}\omega \frac{D}{2} = NBL_{rotor}\omega D \tag{0.14}$$

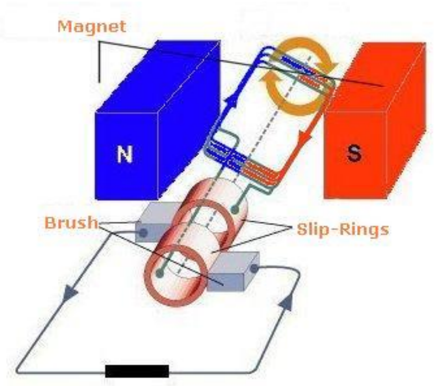
Note that $\omega = 2\pi \times rpm$ in radians per second. Substituting the values we get

$$E = 2 \times 0.8 \times 7 \times \frac{3000 \times 2\pi}{60} \times 0.7 = 2463 \text{ V}$$

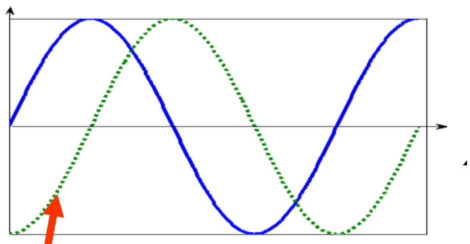
The rms value of the emf will be $\frac{2473}{\sqrt{2}} = 1742 \text{ V}$.

3.3 AC GENERATOR WITH A ROTATING COIL

Rotating the coil, as illustrated in Fig. 3.8, is not actually very common, but it is done in some small bicycle dynamos. The main reason for not using the rotating coil arrangement is that slip rings will be needed to connect the stationary generator terminals to the terminals of the rotating coil. Also when the coil is on the inside, it will not be as well cooled as a coil on the outside, and hence the rated current and power of a moving coil generator will be smaller than a rotating magnet version. Note that the generated emf is proportional to the rotation speed.



$$\phi = \phi_m \sin\theta$$
$$= \phi_m \sin\omega t$$



$$E = -\frac{dN\phi}{dt}$$
$$= -\omega\phi_m N\cos\omega t$$

Fig. 3.8 - Rotating coil AC generator

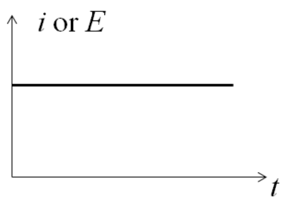
3.4 AC VS DC

A mentioned earlier electricity is mainly generated, transmitted and distributed in AC form rather than DC. See the figures below for definition of AC and DC and the circuit symbols of AC and DC sources.

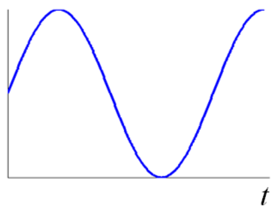
There are several reasons for this:

- AC is natural to electrical generators and motors.
- Easier to switch-off as the current naturally falls to zero every half cycle.
- AC voltage *v* can easily and efficiently be stepped up (increased) using a transformer.
    - Since power *p=vi*, then by increasing *v*, the current *i* is reduced.
    - This reduces the size of the cables and transmission lines needed to carry the current.

- Direct Current DC Source
    - Current and voltage does not change direction
    - Batteries, fuel cells, solar PV, thermocouples.



- Alternating Current AC Source
    - Current and voltage change (alternate) direction usually sinusoidally
    - Synchronous generators, piezoelectric devices.



Electrical power is generated, transmitted and distributed in the form of alternating current (AC).

AC Waveform

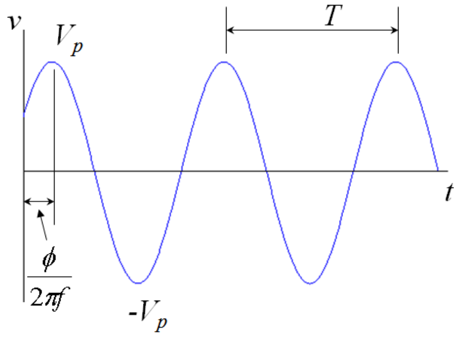
Peak Value = V_p
Peak to peak value = $V_{pp}=2V_p$

Period $= T$ [s]

Frequency $= f = \dfrac{1}{T}$ [Hz]

Angular frequency
$\omega = 2\pi f$ [rad/s]

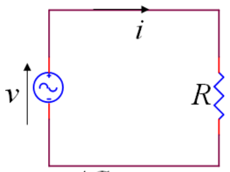
Phase ϕ



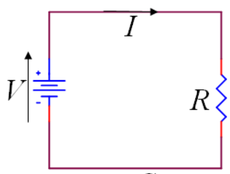
Formula
$$v = V_p \cos(2\pi f t - \phi) = V_p \cos(\omega t - \phi)$$

Root Mean Square (rms)

It is the equivalent DC voltage V or current I that generates the same mean amount of power loss P in a resistor.



AC circuit



Equiv. DC circuit

Power in DC circuit

$$\mathrm{P} = VI = I^2R = \frac{V^2}{R}$$

Instantaneous power in AC circuit

$p=vi$

Mean power in AC circuit

$$\mathrm{P} = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T vi\,dt$$

But $v=iR$

$$\mathrm{P} = \frac{1}{T}\int_0^T \frac{v^2}{R}dt = \frac{V^2}{R}$$

The rms value of v

$$V = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

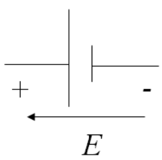
Let $v = V_p \cos(2\pi f t - \phi)$

You can show that the rms voltage is

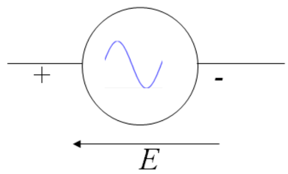
$$V = \frac{V_p}{\sqrt{2}}$$

Similarly rms current

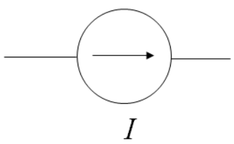
$$I = \frac{I_p}{\sqrt{2}}$$



DC voltage source
Voltage is constant



AC voltage source



DC or AC current source

3.5 THE TRANSFORMER

As discussed earlier the generator voltage is normally transformed to a higher value using a transformer. For given power, increasing the voltage reduces the current and the size of transmission lines and cables needed to transmit the electrical power. In the UK AC electricity is transformed to 400 kV before it is transmitted on high voltage overhead lines on large towers. It is transformed down at sub-stations near load centres. Three-phase power is normally supplied at 400 V to industrial premises. But domestic premises are normally supplied with single phase power at 230 V.

Figure 3.9 shows a diagram of a typical single phase transformer. It comprises two coils or windings wound around a laminated steel core (the core is laminated to reduce eddy currents). The winding connected to the source is known as the primary winding. The winding connected to the load is known as the secondary winding.

When the primary winding is connected to the source, magnetic flux will be generated. The flux will mostly flow through the steel core, which is an easier path to travel through compared to air, and will link with the secondary winding. If the source was AC, the flux will also be alternating and an emf will be generated in the secondary winding. Electric power will therefore be transmitted from the primary to the secondary via the magnetic field.

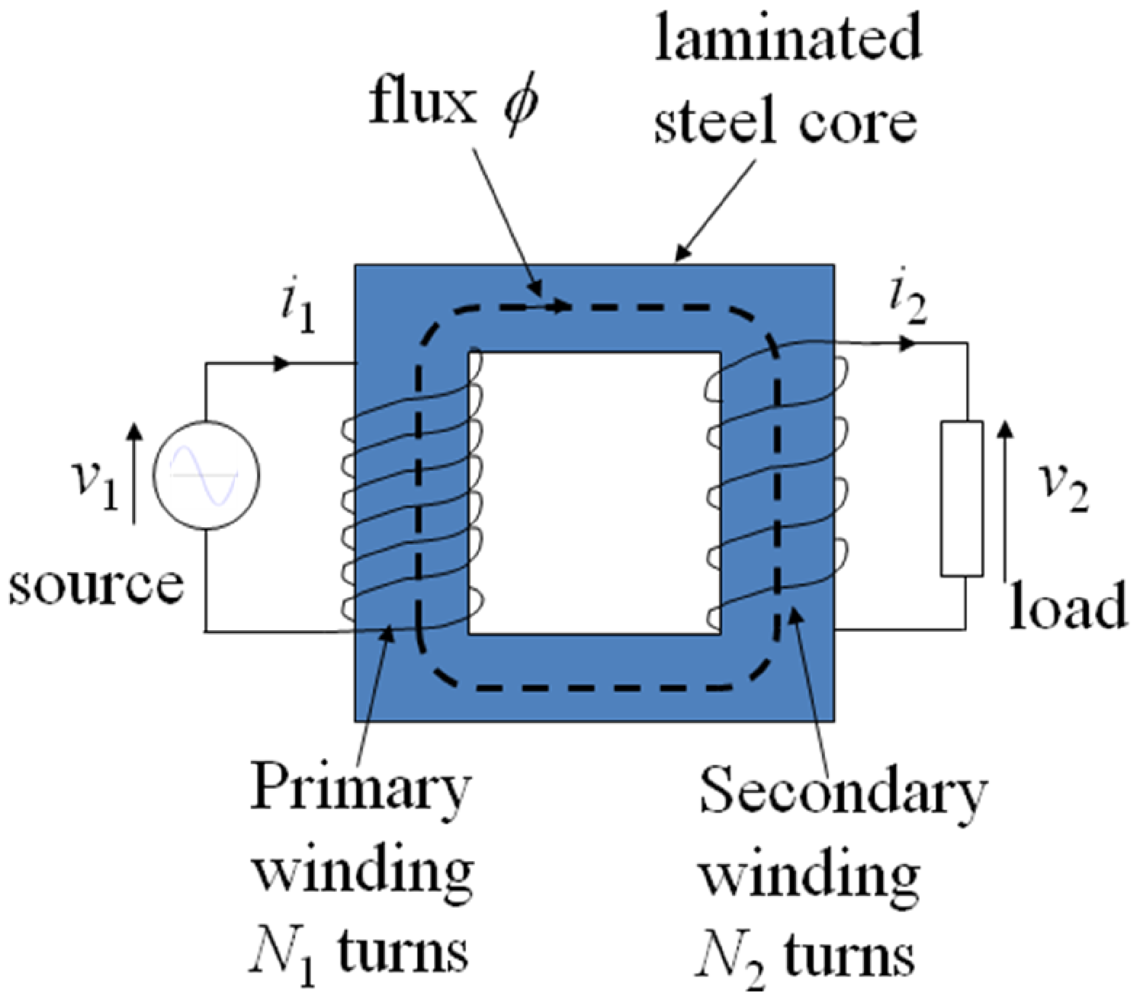


Figure 3.9 Single-phase transformer

According to Faraday's law (neglecting voltage drop across the windings' resistance and leakage reactance).

$$v_1 = -\frac{dN_1\phi}{dt}$$
$$v_2 = -\frac{dN_2\phi}{dt} \tag{3.4}$$

which results in the following relationship between the primary and secondary voltages

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \tag{3.5}$$

i.e. the winding voltage is proportional to the number of turns. By adjusting the number of turns a transformer may be used to transform the voltage up or down.

The input power to the primary,

$$p_1 = v_1 i_1 \tag{3.6}$$

The output power from the secondary

$$p_2 = v_2 i_2 \tag{3.7}$$

The efficiency of the transformer can be calculated as

$$\eta = \frac{p_2}{p_1} \times 100 \tag{3.8}$$

Ideally, the efficiency should be 100%. But in practice there will be losses in the transformer winding and laminations. In practice for very large transformers, the efficiency can approach 99%.

In an *ideal* transformer with no losses the efficiency is 100% and

$$p_1 = p_1$$
$$v_1 i_1 = v_2 i_2 \tag{3.10}$$

From equations (3.5) and (3.10) we get

$$\frac{i_2}{i_1} = \frac{v_1}{v_2} = \frac{N_1}{N_2} \tag{3.11}$$

The current is inversely proportional to the number of turns, i.e. stepping up the voltage reduces the current.

EXAMPLE

A SINGLE-PHASE TRANSFORMER HAS 400 PRIMARY TURNS AND 1000 SECONDARY TURNS. THE NET CROSS-SECTIONAL AREA OF THE CORE IS 60 CM^2. IF THE PRIMARY WINDING IS

CONNECTED TO A 50 HZ SUPPLY OF 500 V, CALCULATE THE VOLTAGE INDUCED IN THE SECONDARY WINDING AND THE PEAK FLUX DENSITY IN THE CORE.

SOLUTION

Assuming sinusoidal flux (which is approximately true in most transformer with little saturation),

$$\phi = \phi_m \sin(2\pi f t)$$

For a winding with N turns the generated emf will be given by

$$E = -\frac{dN\phi}{dt} = -2\pi f \phi_m N \cos(2\pi f t)$$

The peak and rms values of the emf will be given by

$$E_p = 2\pi f \phi_m N$$

$$E_{rms} = \frac{E_p}{\sqrt{2}} = \frac{2\pi f \phi_m N}{\sqrt{2}} = 4.44\, f \phi_m N \qquad (3.12)$$

The rms values of primary voltages will be given by

$$V_1 = 4.44\, f \phi_m N_1$$

Substituting values in the above equation we can find the peak flux density

$$\phi_m = \frac{V_1}{4.44\, f N_1} = \frac{500}{4.44 \times 50 \times 400} = 0.00563\, Wb$$

The peak flux density can be by dividing peak flux by the core cross-sectional area

$$B = \frac{\phi_m}{A} = \frac{0.00563}{60 \times 10^{-4}} = 0.938\, T$$

The flux density in steel should not exceed 1.5 T to avoid magnetic saturation.

We can calculate the secondary voltage using equation (3.5) or (3.12)

$V_2 = 4.44\, f \phi_m N_2$ = 1250 V.

4 DC MACHINE

If we replace the slip rings in the AC generator in Fig. 3.5 with one split ring as shown in Fig. 4.1, the emf measured at the terminal will become DC. The split ring is known as a commutator. But using only one coil and a commutator with only two sections results in a pulsating emf. To produce a smoother emf, several coils are used and connected to a commutator with several segments as illustrated in Fig. 4.2. Note that the DC emf is proportional to the speed of rotation:

$$E = K_E \omega \tag{4.1}$$

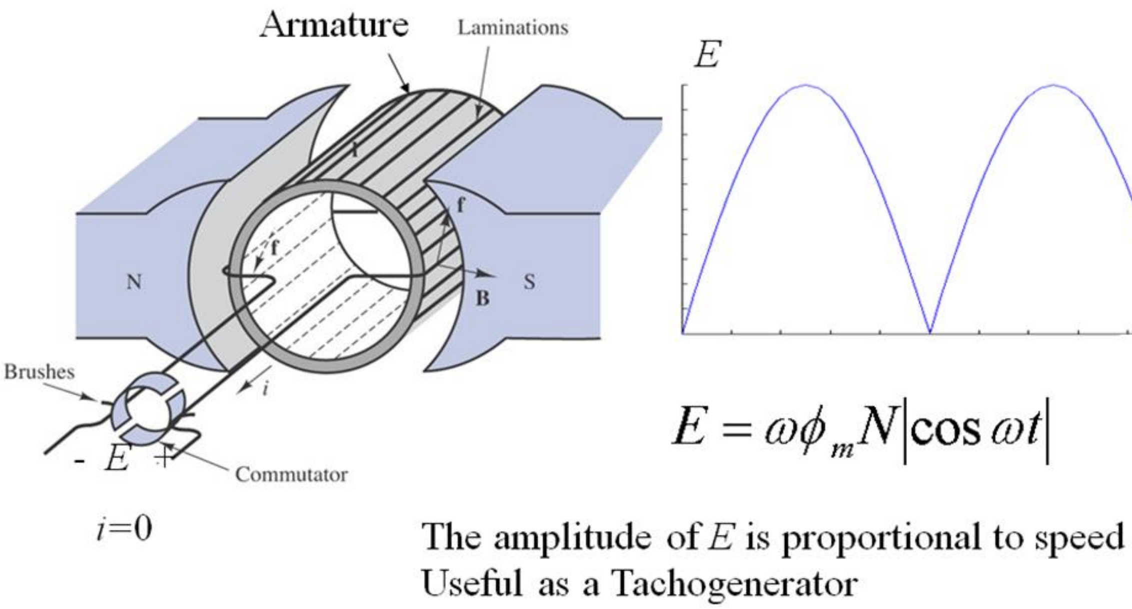


Fig. 4.1 - A DC generator with one coil

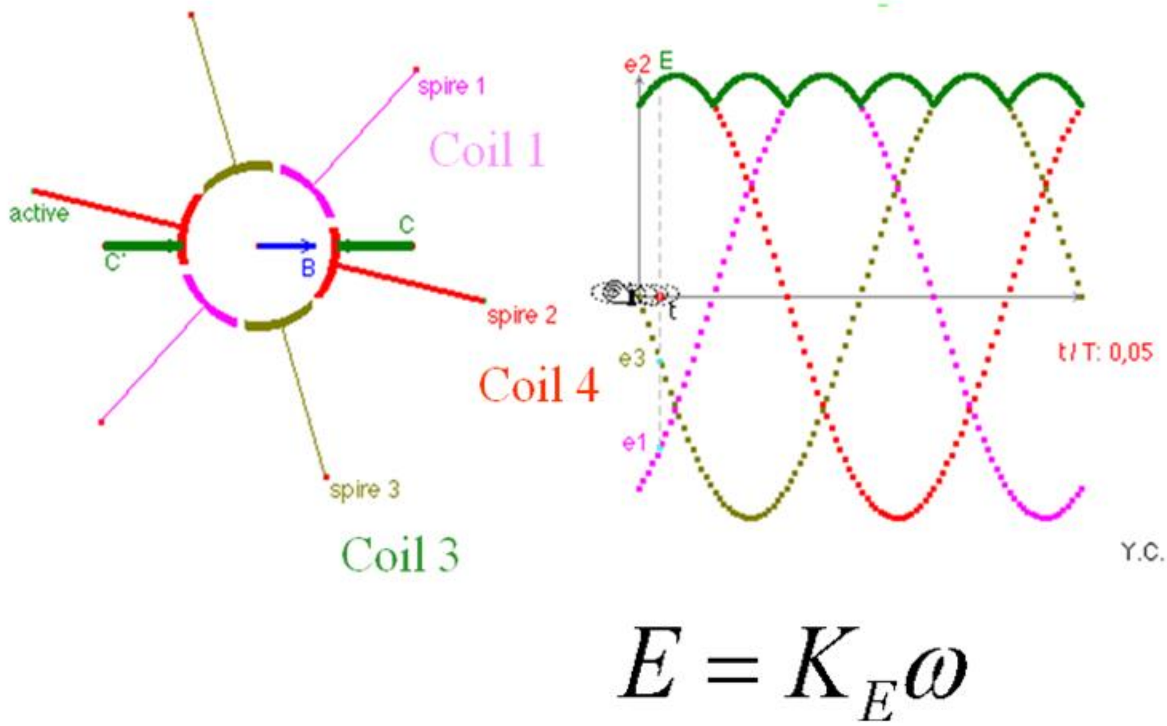


Fig. 4.2 – Commutator and emf of a multi-coil DC machine

Consider the generator in Fig. 4. 3 which is supplying current to a load. The current in the coil will generate a magetic field such that the coil will have a north pole at the bottom side and a south pole at the top. If the coil is rotated clockwise, the coil's south pole will be brought closer to the stator's south pole (also the north will be moved closer to the north) and this will be felt as a counter torque that opposes the motion. The greater the current supplied by the generator the stronger its magnetic poles will become and the harder it will be to rotate the generator, as we should expect according the law of conservation of energy.

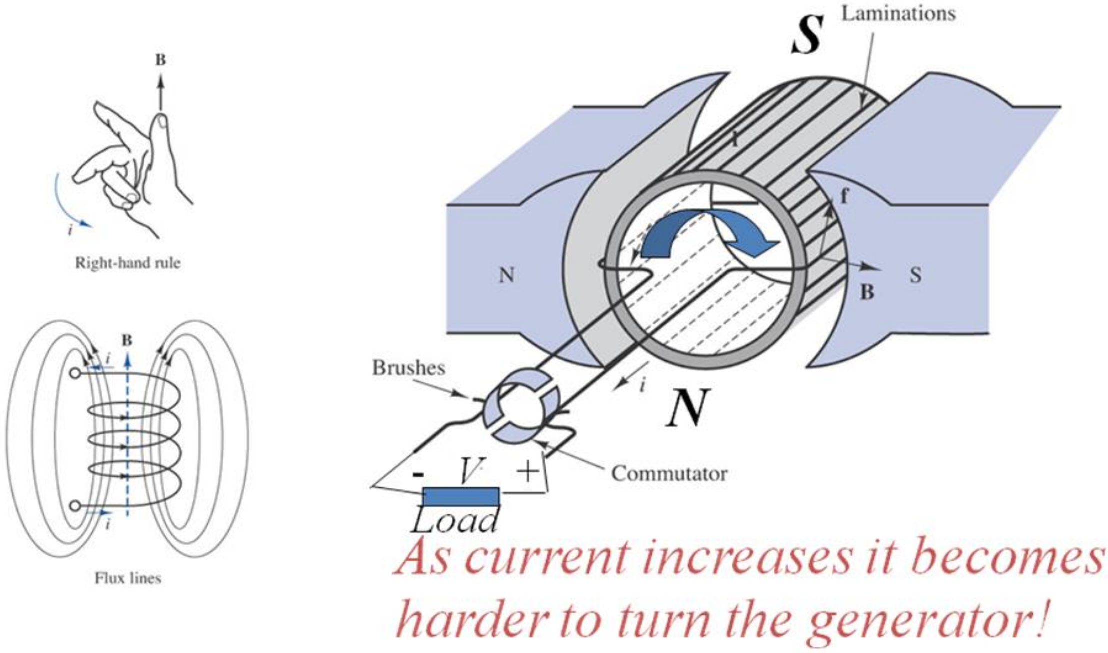


Fig. 4. 3 – Electromechanical energy conversion by a DC machine

The equivalent circuit of a DC generator is shown in Fig. 4.4. The coil obviously has a resistance and an inductance and the voltage source represents the emf generated in the coil, which is proportional to speed according to equation (4.1).

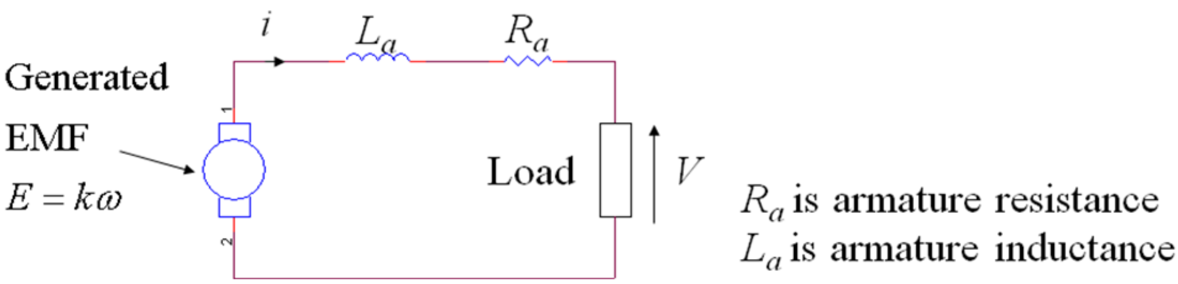


Fig. 4.4 – Equivalent circuit of a DC generator connected to a load

During steady state operation with constant current the voltage across the inductance will be zero and often the inductance is removed from the equivalent circuit as shown in Fig. 4.5.

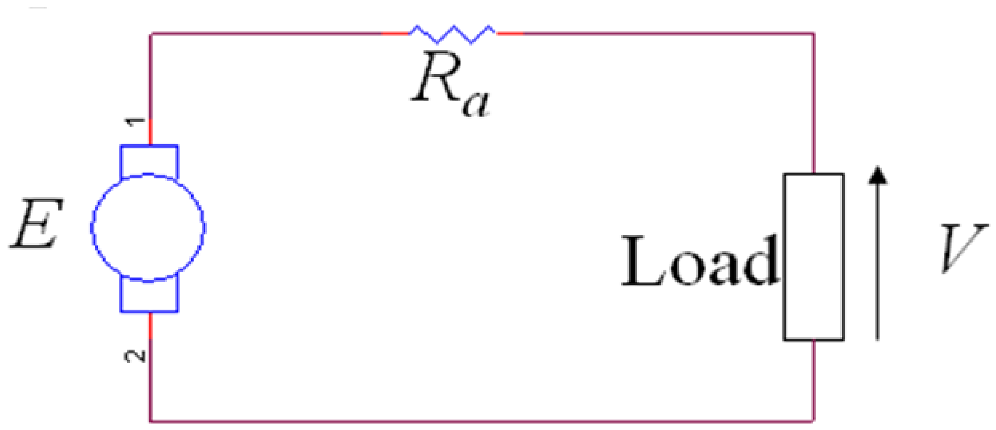


Fig. 4.5 – Steady state constant current equivalent circuit of a DC generator conenected to a load

4.1 DC MOTOR

If we replace the load in Fig. 4.3 with a battery to supply current to the load as shown in Fig. 4.6, then the interaction between the rotor coil poles and the stator poles will cause the rotor to rotate counter clockwise. The commutator ensures that the current in the conductor under a stator pole is always in the same direction so that unidirectional torque is produced.

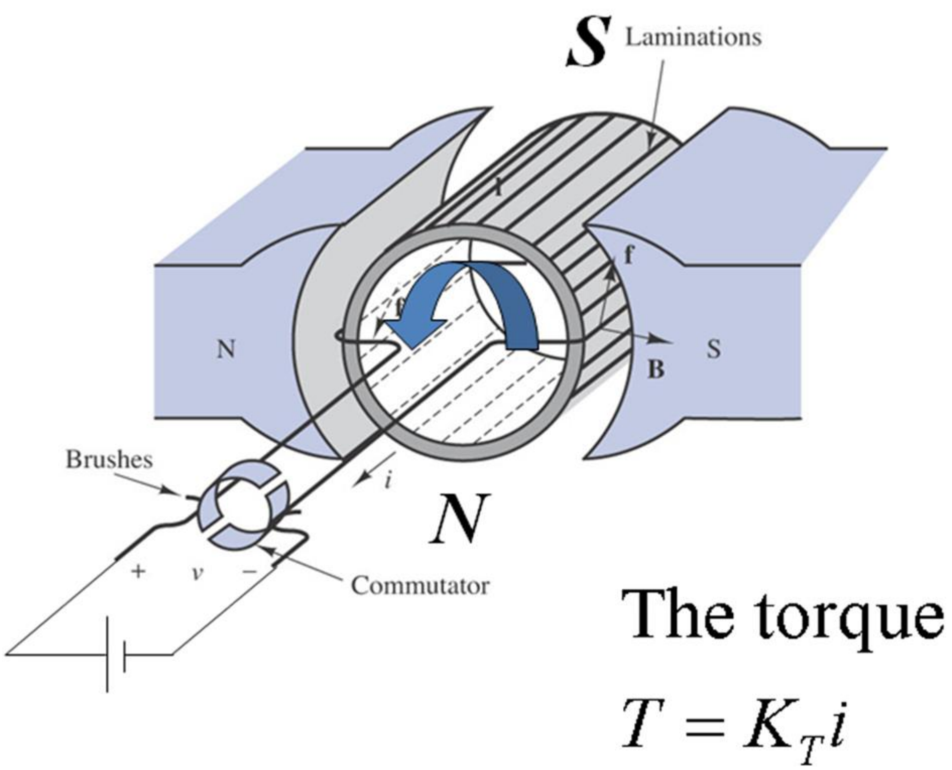


Fig. 4.6 – DC Motor

The equivalent circuit of a DC motor is similar to that of the generator as shown in Fig. 4.7. Again, if the current is constant we can ignore the inductance and the equivalent circuit will reduce to that shown in Fig. 4.8.

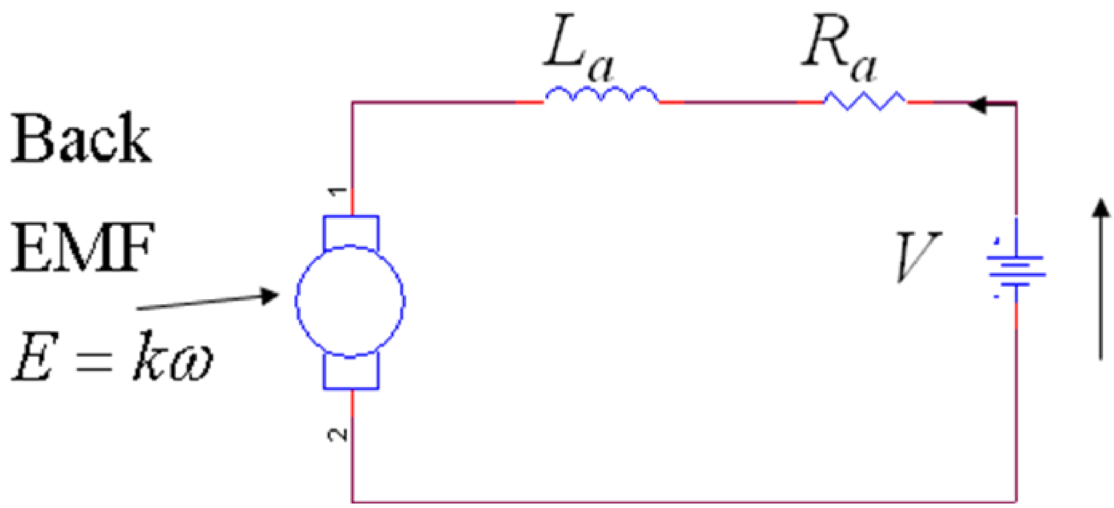


Fig. 4.7 – Equivalent circuit of a DC motor connected to a battery

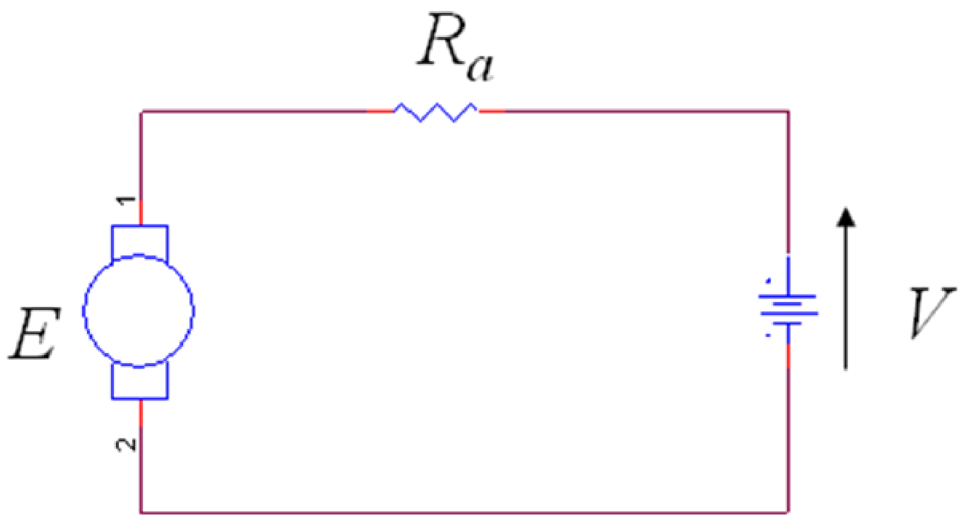


Fig. 4.8 – Equivalent circuit of a DC motor with a constant current

The current drawn by a DC motor is proportional to the torque produced as expected:

$$T = K_T i \tag{4.2}$$

In fact we will show that

$$K_T = K_E \tag{4.3}$$

4.2 CONSTRUCTION

Figure 4.9 shows some photos of a small radial gap DC machine and its components. The arc magnets are fitted in a steel housing. The armature comprises a stack of steel laminations and 3 coils. The terminals of the coils are connected to the commutator segments in a lap (parallel) or wave (series) configuration.

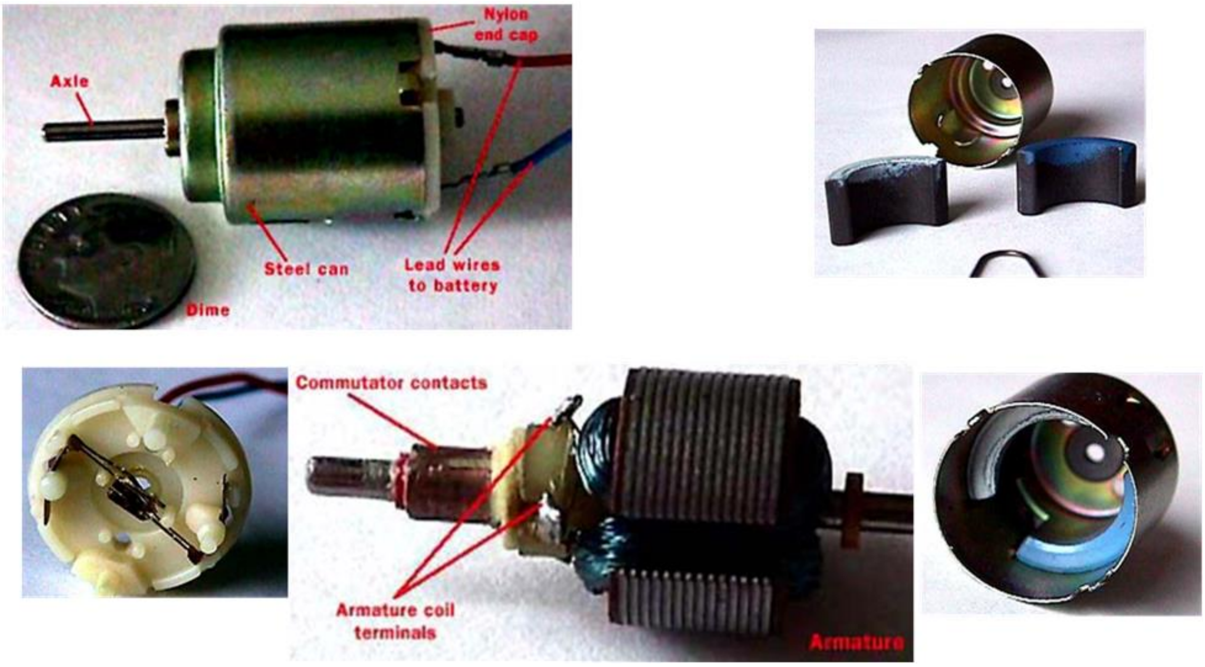


Fig. 4.9 - Photograph of a small DC machine and its components

4.3 K_T AND K_E

In the following two sections we use Faraday's law and Lorenz's force equation to calculate the emf and torque constants of a DC machine.

4.3.1 EMF AND K_E

Each coil will have 2 conductors. The conductors may be connected in series or in parallel. Assume we have a total of Z conductors, and that the conductors are connected such that we have a parallel paths. The number of conductors in series in each parallel path will be $\frac{Z}{a}$.

Each conductor will have an emf generated across it according to equation (2.11) given by

$$E_c = BLu = BL\omega\frac{D}{2} \tag{4.4}$$

The total emf will be equal to the sum of emf across the conductors in series:

$$E = \frac{Z}{a}BL\omega\frac{D}{2} \tag{4.5}$$

Noting the flux per pole is given by

$$\phi = \frac{\pi}{N_p} BLD \tag{4.6}$$

Substituting (4.6) into (4.5)

$$E = \frac{ZN_p}{2a\pi}\phi\omega \tag{4.7}$$

Comparing (4.7) and (4.1) we get

$$K_E = \frac{ZN_p}{2a\pi}\phi \tag{4.8}$$

4.3.2 TORQUE AND K_T

We can calculate the force on each conductor in terms of the current through it and its length using the following equation

$$F_c = Bi_c L \tag{4.9}$$

The torque due to the force on each conductor will be given by

$$T_c = F_c \frac{D}{2} = Bi_c L \frac{D}{2} \tag{4.10}$$

The total torque is then given by

$$T = ZT_c = ZBi_c L \frac{D}{2} \tag{4.11}$$

The current in each conductor will be equal to the total current flowing into the machine divided by the number of parallel paths, i.e.,

$$i_c = \frac{i}{a} \tag{4.12}$$

Substituting (4.12) into (4.11),

$$T = ZB\frac{i}{a}L\frac{D}{2} \tag{4.13}$$

Substituting (4.6) into (4.13) we get

$$T = \frac{ZN_p}{2a\pi}\phi i \tag{4.14}$$

From (4.2) and (4.14) we get

$$K_T = \frac{z}{a} BL \frac{D}{2} = \pi r^2 \frac{ZN_p}{2a\pi} \varphi \qquad (4.15)$$

Equation (4.3) follows from (4.15) and (4.8).

EXAMPLE

A permanent magnet DC motor draws an armature current of 20 A from a 230 V supply. The motor has a rotor diameter of 0.2 m, an active length of 0.2 m and an average gap flux density of 0.5 T. The motor armature winding has a total of 200 conductors with two parallel paths. The resistance of the armature is 0.5 Ω. Calculate torque, speed, mechanical output power and efficiency of the machine. The total of friction, windage and core losses is 200 W.

Since we have two parallel paths, i.e. $a = 2$, then the current in each conductor is $i_c = \frac{i}{a} = \frac{20}{2} = 10$ A.

The torque produced by the machine is given by

$$T = Zi_c BL \frac{D}{2} = 200 \times 10 \times 0.5 \times 0.2 \times \frac{0.2}{2} = 20 \text{ Nm}$$

The back emf is given by

$$E = V - iR = 230 - 20 \times 0.5 = 220V = \frac{Z}{a} BL\omega \frac{D}{2} = \frac{200}{2} \times 0.5 \times 0.2\omega \frac{0.2}{2} = \omega$$

Therefore the speed of the motor is in rpm= $\omega \frac{60}{2\pi} = 220 \frac{60}{2\pi} = \frac{6600}{\pi}$ rpm

Electromagnetic output power= $T\omega = EI = 20 \times 220 = 4400W$

Output mechanical power $P_o = EI - coreloss - frictionloss - windageloss = 4400 - 200 = 4200$ W

Input electrical power= $P_i = Vi = 230 \times 20 = 4600$ W

Efficiency= $\frac{P_o}{P_i} \times 100\% = \frac{4200}{4600} \times 100 = 91\%$

4.4 RELATIONSHIP BETWEEN TORQUE AND SIZE

We can write equation (4.11) as follows

$$T = 2B\left(\frac{Zi_c}{\pi D}\right)\left(\pi \frac{D^2 L}{4}\right) = 2BAV_R \qquad (4.16)$$

The variable $A = \frac{Zi_c}{\pi D}$ is known as the electric loading. It is basically the rated current per unit length of the perimeter of the machine, which is found to be similar for machines of similar cooling regardless of their size.

Similar the magnet loading B is also similar for machines of different sizes. Hence the volume V_R of the rotor of the machine, and hence its total volume is proportional to its torque capability.

Since power $p = T\omega$, we can conclude the volume of a machine is proportional to its its power divided by its speed or

$$V_R \propto \frac{p}{\omega} \tag{4.17}$$

For given power, the faster the machine the smaller it needs to be. Therefore, it is desirable to speed up a machine using a gearbox to keep its size small.

4.5 WOUND FIELD MACHINES AND THEIR CHARACTERISTICS

In the previous sections we described PM DC machines. However, most large machines use wound fields as illustrated in Fig. 4.10. The field and armature windings can be connected in series, shunt (paralle) or in a combination of both (compund), or they can be seperately excited as illustrated in Fig. 4.11. The torque speed characteristics depend on the type of field connection used as shown in Fig. 4.12 – see if you can derive these characteristics from basic equations of emf and torque and equivalent circuit.

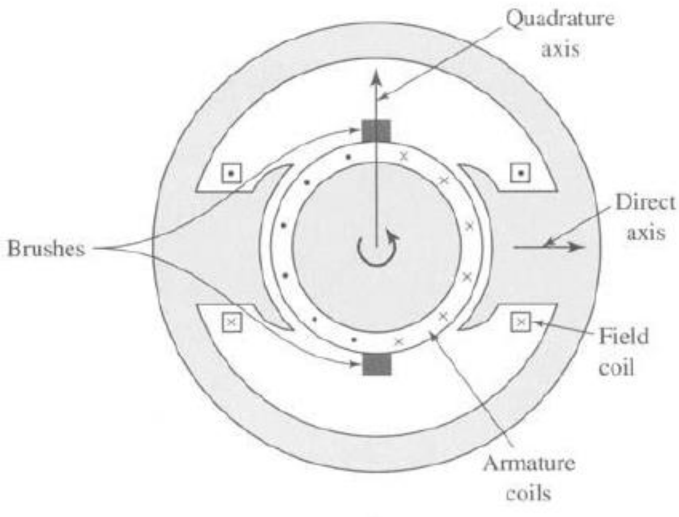


Fig. 4.10 – Schematic diagram of a wound field DC machine

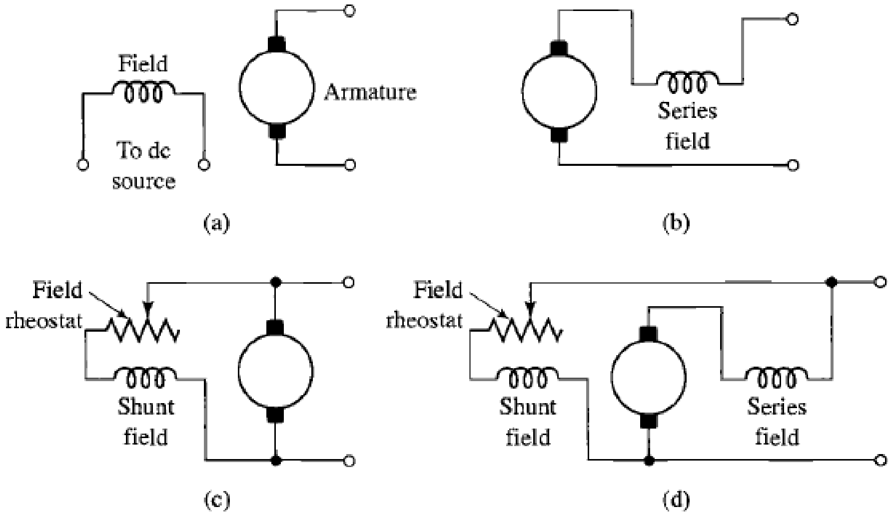
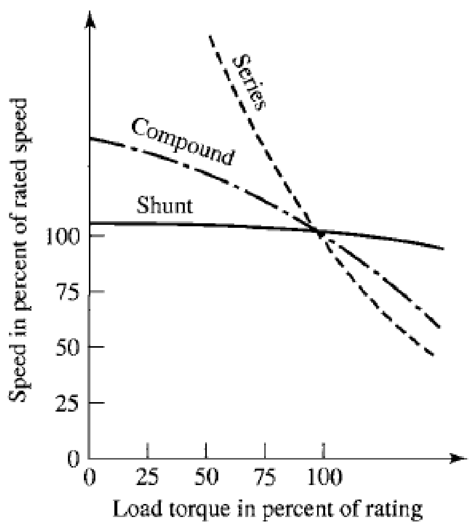


Fig. 4.11 – Field connections of DC machines: (a) separate excitation, (b) series excitation, (c) shunt, (d) compound.



4.12 – Speed torque characteristics of DC motors.

4.6 SPEED CONTROL

The speed of a DC motor can be controlled by varying the terminal voltage. If a constant voltage battery is used, the speed of the motor can be varied by inserting a variable resistor in series. But obviously this is not efficient as much of power will be dissipated in the resistor. The alternative is to use transistors to chop the voltage using dc/dc converters as shown in Fig. 4.13, which is more efficient.

To reverse the direction of rotation, the polarity of the motor terminal voltage needs to be reversed. This can be achieved using more transistors, e.g. H-Bridge as shown in Fig 4.14: use T1 and T4 for rotation in one direction and T3 and T2 for rotation in the other direction. We shall discuss this briefly in the lectures.

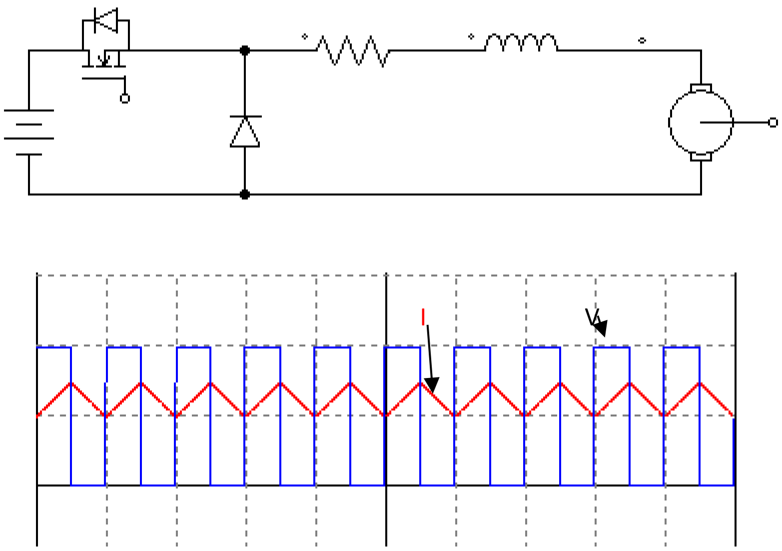


Fig. 4.13 - DC chopper used to control speed of a DC motor

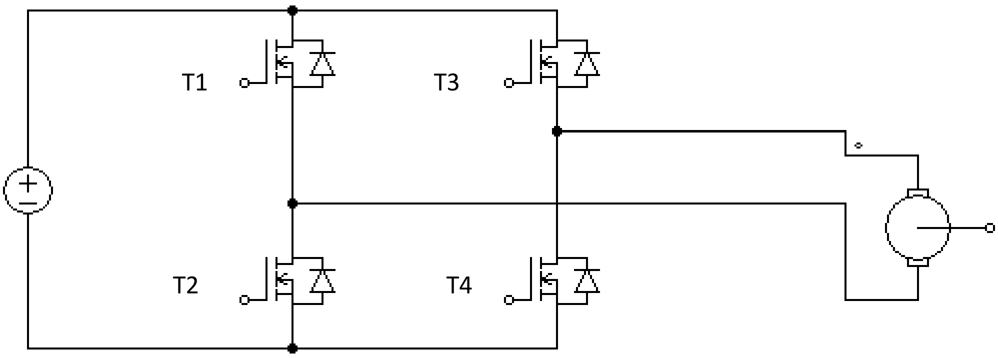


Fig. 4.14 Four quadrant H-Bridge

4.6.1 EXAMPLE: SPEED CONTROL USING A SERIES RESISTANCE

A permanent magnet dc motor takes an armature current of 20 A from a 230 V supply. The motor has a rotor diameter of 0.2 m, an active length of 0.2 m and an average gap flux density of 0.5 T. The motor armature winding has a total of 200 conductors with two parallel paths. The resistance of the armature is 0.5 Ω. Calculate the speed of the motor and calculate the resistance needed in series with the armature to halve the speed if the load torque is constant.

Using the equation (4.7) and substituting the numbers gives $E_1 = 220V$. When the speed is halved $E_2 = 110V$. Using the circuit diagram in Figure 4.8 with an additional resistance between the battery and motor armature resistance and recognizing that the current remains constant- since the torque remains constant - it is possible to calculate the required resistance to half the speed to be 5.5 ohms.

EXAMPLE

A permanent magnet DC motor is used to drive a fan. The power needed to drive the propeller is proportional to the cube of its speed. The speed of the motor is controlled using a DC chopper as shown in Figure 4.15. The armature resistance of the motor is 0.1 Ω. Sketch the waveforms of the voltage V and the current I when the duty cycle of the transistor (ratio or on time to total switching period) is 80%. Assume the switching frequency is high and the current is continuous.

When the duty cycle of the transistor is 100% (kept on all the time), the motor speed was 3600 rpm, and its current was 15 A. Calculate the speed of the motor when the duty cycle of the transistor was reduce to 40%. The transistor and diode on voltages are 0.6 V each.

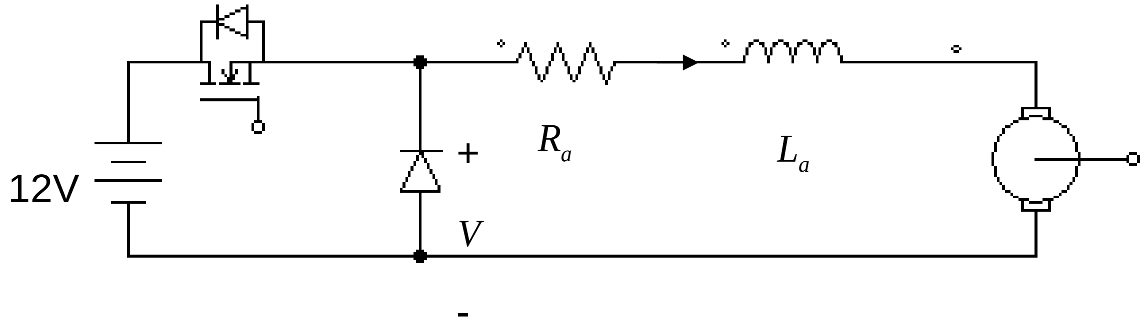


Figure 4.15

$$V = IR_a + K_E\omega$$

The voltage $V = V_{battery} - V_{transistor} = 12 - 0.6 = 11.4$V when the transistor is on. Substituting into the fist equation gives

$$K_E = K_T = \frac{V - IR_a}{\omega} = \frac{11.4 - 15\times 0.1}{\frac{3600\times 2\pi}{60}} = 0.026\,\text{V/rad/s.}$$

The torque produced in this case will be

$$T = K_T I = 0.026\times 15 = 0.39\,\text{Nm.}$$

The power produced by the motor

$$P = T\omega = k\omega^3 \propto \omega^3$$

This gives

$$k = \frac{T}{\omega^2} = \frac{0.39}{\left(\frac{3600\times 2\pi}{60}\right)^2} = 2.7\times 10^{-6}\,\text{Nm/rad}^2\text{/s}^2.$$

When the duty cycle reduces to 40% the terminal voltage becomes

$V = 0.4 \times 11.4 = 4.56$ V.

Recognizing that $I = \dfrac{T}{K_T} = \dfrac{k\omega^2}{K_T}$ then the first equation becomes:

$$V = R_a \frac{k\omega^2}{K_T} + K_E\omega$$

Substituting and arranging we get a quadratic equation in ω,

$$0.1 \times \frac{2.7 \times 10^{-6}}{0.026}\omega^2 + 0.026\omega - 4.56 = 0$$

$$10^{-5}\omega^2 + 0.026\omega - 4.56 = 0$$

$$\omega^2 + 2600\omega - 4.56 \times 10^5 = 0$$

This gives

$\omega = \dfrac{-2600 + \sqrt{2600^2 + 4 \times 4.56 \times 10^5}}{2} = 165$ rad/s=1575 rpm.