

# SESA2023 Week 2: Thermodynamics

This week we will mostly refresh our thermodynamics of part 1, but will also introduce a few new concepts.

## 2.1 Learning outcomes

After completing this section you should be able to:

- Explain the meaning of a thermodynamic equilibrium
- Use the two property rule and know its limitations
- Explain the meaning and use the first law of thermodynamics
- Explain the difference between a real, ideal, and a perfect gas
- Relate mass fractions and molar fractions to partial pressures of a mixture of gases
- Calculate the properties of mixtures of gases
- Apply the Steady Flow Energy Equation to the components of a jet engine
- Calculate the entropy difference between two thermodynamic states
- Explain how entropy is generated in compressors and turbines, and how it affects their performance
- Use isentropic efficiency to perform calculations on real compressors and turbines

## 2.2 Thermodynamic fundamentals

We will start by looking at a thermodynamic system, where the word system means that there is no mass transfer through any boundaries. Within this system, we will define a number of important concepts.

### 2.2.1 Thermodynamic equilibrium and the Zeroth Law of Thermodynamics

A system is considered to be in a thermodynamic equilibrium when all properties are uniform and unchanging throughout the system. When this is the case, then the properties fully describe the *state* of a system. Thermodynamic equilibrium requires all of the following equilibria:

- Thermal equilibrium: a uniform temperature, and therefore no tendency for heat transfer.
- Mechanical equilibrium: all forces are balanced, and therefore no tendency to accelerate.
- Phase equilibrium: no tendency for phase changes.

- Chemical equilibrium: no tendency for chemical change.

Note that these should all be viewed on a macroscopic scale, and not on a microscopic (molecular) scale. For example, individual molecules will continuously change their kinetic energy due to collisions with other molecules, but on a macroscopic scale they average out and we can measure a single temperature.

The **Zeroth Law of Thermodynamics** relates to the first of the four listed equilibria:

*If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.*

Basically stating that if body A is in thermal equilibrium with body C (i.e., having the same temperature), and body B is in thermal equilibrium with body C, then body A and body B will have the same temperature.

### 2.2.2 Thermodynamic properties

Properties that define the state of a system are called state properties. There is an almost endless list of state properties, but the ones that we will encounter the most are listed in table 2.1 with their SI units. We distinguish between different types of properties: intensive, extensive, and specific properties. Intensive properties do not depend on the amount of material. For example, the temperature of 1 kg of water is the same as the temperature of 2 kg of water (if it is in the same thermodynamic state). Extensive properties *do* depend on the amount of material. For example, the internal energy (in Joules) of 2 kg of water is double that of the internal energy of 1 kg of water. If an extensive property is divided by the mass of the material, then it becomes a specific property. For example:

$$\begin{aligned} v &= \frac{V}{m} \\ u &= \frac{U}{m} \\ &\text{etc.} \end{aligned}$$

The specific internal energy will therefore get the units J/kg, while the extensive property internal energy had the unit J. Because all specific properties are *per kg of material*, they do not depend on the amount of material anymore, and are therefore automatically also intensive properties. This is why all specific properties are also listed as intensive properties in table 2.1.

Note that by convention, extensive properties are indicated with an upper case, while specific properties are indicated with a lower case letter. The exception to this is for temperature and pressure, but these properties don't have an extensive counterpart. A word of warning about the specific heats  $c_V$  and  $c_P$ : It is very uncommon to use the extensive version, so much so, that sometimes you will find the capitalized version (e.g.,  $C_P$ ) to indicate the specific version. So be extra careful to always check the units when using them.

Finally, we can also define molar properties. They are similar to the specific properties, except the extensive properties is divided by the number of moles instead of by the mass. Molar volume for example, will then have the units  $\text{m}^3 \text{mol}^{-1}$ .

### 2.2.3 Two-property rule (or state postulate)

The two-property rule states that: *"The state of a simple compressible system is completely specified by two independent, intensive properties."*

This statement is more formally known as the state postulate. The postulate implies two important conditions for applying the two-property rule, namely that it needs to be a simple compressible system, and that the two properties need to be independent. A system is called simple in the absence of any electrical, magnetic, gravitational, motion, and surface tension effects. Two properties are independent when one property can be kept constant, while the changing the other one.

Property	Intensive	Extensive	Specific
Temperature	$T$ (K)		
Pressure	$P$ (Pa)		
Volume	$v$ (m <sup>3</sup> kg <sup>-1</sup> )	$V$ (m <sup>3</sup> )	$v$ (m <sup>3</sup> kg <sup>-1</sup> )
Internal Energy	$u$ (J kg <sup>-1</sup> )	$U$ (J)	$u$ (J kg <sup>-1</sup> )
Enthalpy	$h$ (J kg <sup>-1</sup> )	$H$ (J)	$h$ (J kg <sup>-1</sup> )
Entropy	$s$ (J kg <sup>-1</sup> K <sup>-1</sup> )	$S$ (J K <sup>-1</sup> )	$s$ (J kg <sup>-1</sup> K <sup>-1</sup> )
Specific heat at constant volume	$c_V$ (J kg <sup>-1</sup> K <sup>-1</sup> )	$C_V$ (J K <sup>-1</sup> )	$c_V$ (J kg <sup>-1</sup> K <sup>-1</sup> )
Specific heat at constant pressure	$c_P$ (J kg <sup>-1</sup> K <sup>-1</sup> )	$C_P$ (J K <sup>-1</sup> )	$c_P$ (J kg <sup>-1</sup> K <sup>-1</sup> )

Table 2.1: Common thermodynamic properties

For example, when considering an ideal gas, both internal energy and enthalpy are a function of temperature only. Temperature, internal energy, and enthalpy are therefore not independent, and knowing for example the temperature and enthalpy does not fully specify the state of the system. In that case a third property (e.g. pressure or specific volume) is required to specify the state of the system.

When considering real substances, an example where the independence requirement is important is during phase changes (think for example of evaporating rocket propellant or oxidizer!). When a system is partially liquid and partially vapour, then temperature and pressure are directly coupled and therefore not independent.

## 2.2.4 Thermodynamic processes and the First Law of Thermodynamics

The change of a system from one equilibrium state to another is called a thermodynamic process. To fully specify a process, not only the start and end state should be known, but also the path. The path determines the amount of work ( $W$ ) and heat transfer ( $Q$ ) that is done during the process. Two different paths between state A and B can have very different work output, as shown with the shaded areas in figure 2.1.

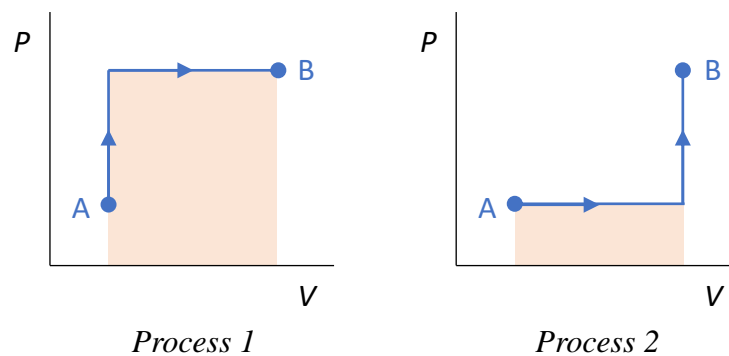


Figure 2.1: Two example processes with the work done indicated.

Processes can take any path, but we generally make use of just a few model processes:

- Isothermal (constant temperature)
- Isobaric (constant pressure)
- Isochoric (constant volume)
- Adiabatic (no heat transfer)

Gas	Specific gas constant $\text{kJ kg}^{-1} \text{K}^{-1}$
Air	0.287
N <sub>2</sub>	0.297
O <sub>2</sub>	0.260
H <sub>2</sub>	4.120

Table 2.2: Specific gas constant examples

- Isentropic (constant entropy, valid when a process is adiabatic *and* reversible)
- Isenthalpic (constant enthalpy)

For propulsion applications, the isobaric and isentropic processes are the most important, as they form the basis for the analysis of diffusers, nozzles, compressors, turbines, and combustion.

The **First Law of Thermodynamics** provides a strict rule of what can be achieved in a process. It uses conservation of energy to relate work and heat transfer to the change in internal energy of a system:

$$\Delta U = Q - W \quad (2.1)$$

The sign convention of this equation is such that the heat transfer  $Q$  is positive when heat is added to the system, and the work  $W$  is positive when the system is doing work (i.e., work output from the system).

### 2.2.5 Ideal gas equation of state

In order to be able to do calculations, we will often assume an ideal gas. An ideal gas is described by the ideal gas equation of state, which you will typically either see in the molar form

$$PV = n\bar{R}T, \quad (2.2)$$

where  $n$  is the number of moles and  $\bar{R}$  the universal gas constant, or the forms based on mass, which is more common in engineering

$$PV = mRT, \quad (2.3)$$

where  $m$  is the mass of the gas, and  $R$  the specific gas constant. The specific gas constant is different for each gas, and can be looked up in tables. Some examples are given in table 2.2 The latter equation can also be written using specific volume  $v = V/m$  or density  $\rho = m/V$  as

$$Pv = RT \quad (2.4)$$

$$P = \rho RT. \quad (2.5)$$

We will mainly be using the mass based equations, but you can switch between the different forms, because they are ultimately the same equations just represented differently. The universal and specific gas constants are related by

$$R = \frac{\bar{R}}{M}, \quad (2.6)$$

with  $M$  the molar mass of the gas. The mass and number of moles are related by

$$m = nM. \quad (2.7)$$

The ideal gas model is based on the following assumptions:

- All collisions are elastic

- The volume occupied by the molecules is negligible
- Intermolecular forces are negligible
- Molecules carry only translational kinetic energy (i.e., no energy is stored as vibrations or rotation)

Although for real gases these assumptions are never fully satisfied, the ideal gas model is often still very accurate. For example, using the ideal gas equation to calculate the density of air at room temperature (293.15 K) and at 100 °C (373.15 K), both at 1 bar pressure, gives 1.1886 kg/m<sup>3</sup> and 0.8863 kg/m<sup>3</sup>, respectively. The densities of *real* air at these conditions are 1.1888 kg/m<sup>3</sup> and 0.8860 kg/m<sup>3</sup>, a difference less than 0.1% in both cases.

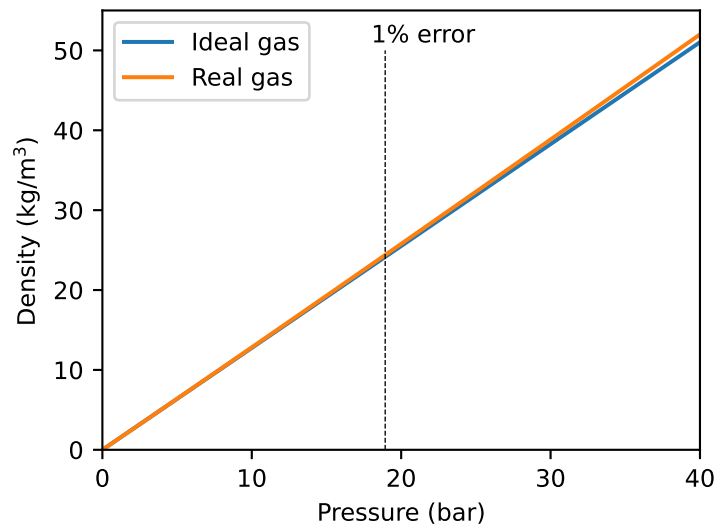


Figure 2.2: Density of air as a function of pressure at a temperature of 20 °C.

The main limitations to be aware of are when (i) approaching phase transitions, and (ii) at high pressures. At high pressures, the volume occupied by the molecules becomes non-negligible, and near phase transitions intermolecular forces become significant. In quantitative terms, the pressure should be much smaller than the critical pressure for the ideal gas equation to be accurate. The critical pressure of a gas can again be looked up in tables. Air, for example, has a critical pressure of approximately 38 bar. Figure 2.2 shows the real density of air compared to that calculated using the ideal gas law as a function of pressure. When the pressure approaches 20 bar, the error in density is still only 1%.

### 2.2.6 Specific heats

In many applications, including propulsion, we typically want to relate temperature (a directly measurable quantity) to energy. More specifically, we want to relate *changes* in temperature to *changes* in energy. The two forms of energy that we are mainly interested in are the specific internal energy  $u$  and the specific enthalpy  $h$ . You might recall that the enthalpy is the internal energy plus the pressure energy

$$H = U + PV, \quad (2.8)$$

or in specific form

$$h = u + \frac{P}{\rho} = u + Pv. \quad (2.9)$$

How changes of these two forms of energy are related to changes in temperature is given by the specific heats  $c_V$  and  $c_P$ . The specific heat at constant volume is defined as

$$c_V \equiv \left( \frac{\partial u}{\partial T} \right)_V, \quad (2.10)$$

and the specific heat at constant pressure is defined as

$$c_P \equiv \left( \frac{\partial h}{\partial T} \right)_P. \quad (2.11)$$

We will often make use of the ratio of specific heats, which is defined as

$$\gamma \equiv \frac{c_P}{c_V} \quad (2.12)$$

Note that these are definitions, and are generally true without making any assumptions. For a *real* gas, both  $c_V$  and  $c_P$  will depend on the thermodynamic state of a system, and therefore depend on other thermodynamic properties (e.g., temperature, pressure, volume).

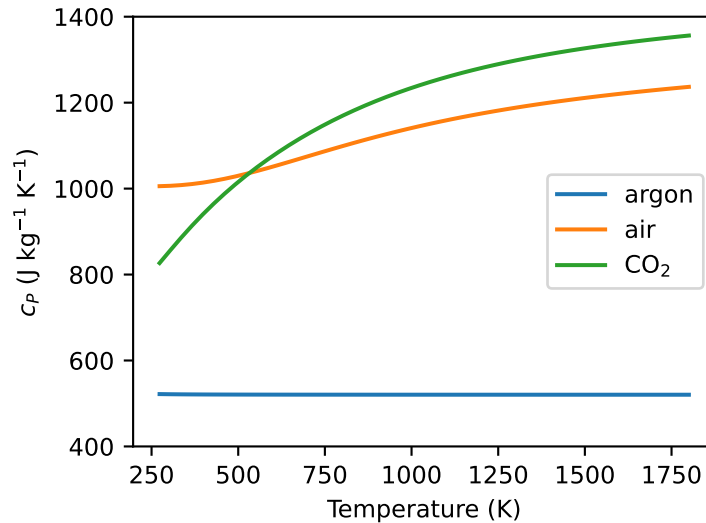


Figure 2.3: Specific heat at constant pressure as a function of temperature.

For an *ideal* gas, both the internal energy and the enthalpy are a function of temperature only, and therefore the specific heats are also a function of temperature only:

$$c_V = c_V(T) \quad (2.13)$$

$$c_P = c_P(T) \quad (2.14)$$

A *perfect* gas is a gas where we can assume  $c_V$  and  $c_P$  to be true constants:

$$c_V = \text{constant} \quad (2.15)$$

$$c_P = \text{constant} \quad (2.16)$$

Under the perfect gas assumption, the specific heats can be used to relate the change in internal energy or the change in enthalpy for finite changes in temperature:

$$\Delta u = c_V(T_2 - T_1) \quad (2.17)$$

$$\Delta h = c_P(T_2 - T_1) \quad (2.18)$$

The perfect gas model is a good assumption for small changes in temperature. When a change in temperature can be considered ‘small’ will depend on the type of gas. Monoatomic gases (e.g., He, Ar) have constant  $c_V$  and  $c_P$  values over a relatively large range, while for gases consisting of complex molecules you will find changes in the specific heats over smaller temperature ranges. Figure 2.3 shows how  $c_P$  changes with temperature for three different gases. Note that the monoatomic gas argon has a constant  $c_P$  value over the full temperature range. The  $c_P$  value for air is reasonably constant near room temperature, but has a significant increase when temperature is increased more. This is something to be aware of specifically when analysing combustion processes, because it involves complex molecules and very large changes in temperature.

## 2.3 Mixtures

Because we will be analysing combustion processes, a basic understanding of mixtures of gases is required. We start with a rather obvious relation, the total mass  $m$  of a mixture given the masses of the components  $m_a$  and  $m_b$ :

$$m = m_a + m_b, \quad (2.19)$$

or more generally for a mixture containing  $k$  components

$$m = \sum_{i=1}^k m_i \quad (2.20)$$

Similarly, for the total number of moles we have:

$$n = n_a + n_b, \quad (2.21)$$

or more generally

$$n = \sum_{i=1}^k n_i \quad (2.22)$$

Two quantities often used for mixtures are the mass fraction  $x$

$$x_i = \frac{m_i}{m}, \quad (2.23)$$

and the molar fraction  $y$

$$y_i = \frac{n_i}{n}. \quad (2.24)$$

Mass fractions and molar fractions can be related to each other as

$$x_i = y_i \frac{M_i}{M}, \quad (2.25)$$

where  $M_i$  is the molar mass of the component  $i$ , and  $M$  is the average molar mass  $M = m/n$  of the mixture.

### 2.3.1 Partial pressure and partial volume

For an *ideal gas*, we can use Dalton's law of partial pressure, which states that the pressure of a mixture is equal to the sum of the pressures that each component would exert if it existed in the same volume and at the same temperature:

$$P = P_a + P_b, \quad (2.26)$$

or more generally

$$P = \sum_{i=1}^k P_i(V, T), \quad (2.27)$$

where  $P_i$  is the partial pressure of component  $i$ . If we now define the pressure fraction  $P_f = P_i/P$ , then we can relate the partial pressure to the mole fraction using the ideal gas relation:

$$\frac{P_i}{P} = \frac{\frac{n_i \bar{R} T}{V}}{\frac{n \bar{R} T}{V}} = \frac{n_i}{n} = y_i \quad (2.28)$$

giving the expression for partial pressure

$$P_i = y_i P. \quad (2.29)$$

In a similar way to the partial pressure, we can define the partial volume, which is the volume a component would occupy if it would exist at the same pressure and temperature as the mixture:

$$V = \sum_{i=1}^k V_i(P, T) \quad (2.30)$$

If we now define the volume fraction  $V_f = V_i/V$ , then we can also relate the partial volume to the mole fraction using the ideal gas relation:

$$\frac{V_i}{V} = \frac{\frac{n_i \bar{R} T}{P}}{\frac{n \bar{R} T}{P}} = \frac{n_i}{n} = y_i, \quad (2.31)$$

from which it follows that

$$\frac{P_i}{P} = \frac{V_i}{V} = \frac{n_i}{n} = y_i. \quad (2.32)$$

### 2.3.2 Properties of mixtures

In an ideal gas, all molecules behave independently, as if no other molecules are present in the system. This results in Dalton's law for partial pressures above (an intensive property), but we can also use it to calculate the extensive properties of mixtures. Noting that extensive properties depend on the amount of material, we can write the following for the internal energy

$$U = U_a + U_b = m_a u_a + m_b u_b \quad (2.33)$$

We can write this more generally, and the same for other extensive properties:

$$U = \sum_{i=1}^k m_i u_i \quad (2.34)$$

$$H = \sum_{i=1}^k m_i h_i \quad (2.35)$$

$$S = \sum_{i=1}^k m_i s_i \quad (2.36)$$

If we want to convert extensive properties to intensive properties, we simply divide by the total mass, which turns the masses into mass fractions:

$$u = \sum_{i=1}^k x_i u_i \quad (2.37)$$

$$h = \sum_{i=1}^k x_i h_i \quad (2.38)$$

$$s = \sum_{i=1}^k x_i s_i \quad (2.39)$$



Similarly we can obtain the specific heats of mixtures by

$$c_V = \sum_{i=1}^k x_i c_{V,i} \quad (2.40)$$

$$c_P = \sum_{i=1}^k x_i c_{P,i} \quad (2.41)$$

## 2.4 Steady Flow Energy Equation (SFEE)

The steady flow energy equation is the First Law of Thermodynamics applied to a control volume. It considers conservation of energy by accounting for heat transferred to the control volume and work done by the fluid, as well as the energy flowing in and out of the control volume ('carried' by the fluid). Figure 2.4 illustrates this concept. Whatever is added as heat, and what is removed as work, must be

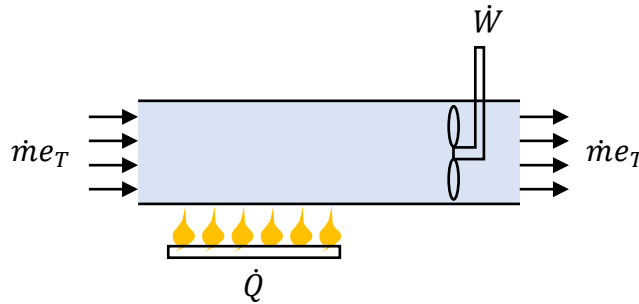


Figure 2.4: Schematic illustration of the Steady Flow Energy Equation.

equal to the change of the energy carried by the fluid entering and exiting the control volume. Based on this we can write the SFEE as

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m} e_T - \sum_{in} \dot{m} e_T, \quad (2.42)$$

where  $e_T$  is the specific total energy carried by the fluid. The total energy consists of the specific internal energy  $u$ , the specific pressure energy  $e_p$ , the specific kinetic energy  $e_k$  and the specific potential energy  $e_g$ :

$$e_T = u + e_p + e_k + e_g \quad (2.43)$$

the last three energy term we can express as

$$e_p = \frac{p}{\rho} \quad (2.44)$$

$$e_k = \frac{1}{2} V^2 \quad (2.45)$$

$$e_g = gz \quad (2.46)$$

noting that here we've used  $V$  for velocity, not volume. Using the definition of enthalpy, we will almost always combine the internal energy and the pressure energy

$$h = u + \frac{p}{\rho}, \quad (2.47)$$

so we can write the more useful form of the SFEE

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m} \left( h + \frac{V^2}{2} + gz \right) - \sum_{in} \dot{m} \left( h + \frac{V^2}{2} + gz \right). \quad (2.48)$$

In this module the potential energy term can (almost) always be neglected, so we will do that from now onward. The kinetic energy term can be neglected in some cases, but can be important for engine inlets (when flying at high velocities), and is always important for nozzles, because we use the high outlet velocity for propulsion!

### 2.4.1 SFEE simplifications for propulsion applications

We will now look at simplifications of the SFEE that we usually make when analysing a gas turbine. They will also apply for ramjets, but just without the compressor and the turbine. What we will show here are basically the ‘simplest’ forms and the most ideal processes. This should get you started in being able to analyse various propulsion systems. Later in the module, and also a little later in this chapter, you will make adjustments so your calculations can resemble reality more closely.

The first thing to note is that we would usually apply the SFEE to individual components of a gas turbine: diffuser, compressor, burner, turbine, turbine, nozzle. For now, we will look at a turbojet engine, where each component has a single inlet and a single outlet. With that, we only have a single mass flow rate  $\dot{m}$ , so we can divide both sides of the SFEE by  $\dot{m}$

$$q - w = \left( h + \frac{V^2}{2} \right)_{out} - \left( h + \frac{V^2}{2} \right)_{in}, \quad (2.49)$$

with  $q$  and  $w$  the specific heat and work, respectively. Note that for the burner, there are technically two inlets (one for fuel and one for air), but for small fuel-air ratios (see notes week 1) the mass flow rate of fuel can be neglected. We will now apply equation 2.49 to the gas turbine schematically drawn in figure 2.5. Note the numbers between each component, which we will use to indicate the states.

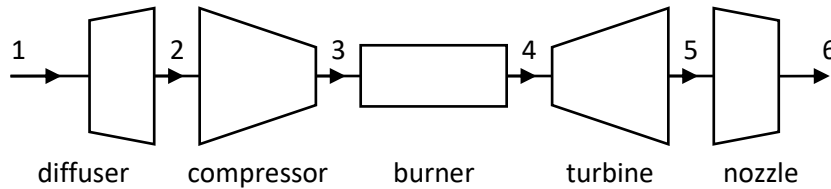


Figure 2.5: Schematic diagram of a turbojet engine.

#### Diffuser

This is a passive component, where no work is done and no heat is transferred. Air typically flows in at a high velocity, but is slowed down significantly such that the outlet kinetic energy  $\frac{1}{2} V_2^2$  can be neglected:

$$0 = h_2 - h_1 - \frac{1}{2} V_1^2 \quad (2.50)$$

#### Compressor

The compressor is doing work on the fluid, increasing its pressure and temperature. This is an adiabatic component (no heat transfer) and the kinetic energy can be neglected at the inlet and the outlet:

$$w_c = -w_{23} = h_3 - h_2 \quad (2.51)$$

#### Burner

The combustion is modelled as a constant pressure heat addition process. No work is done, and kinetic energy can be neglected:

$$q_{34} = h_4 - h_3 \quad (2.52)$$

#### Turbine

The turbine extracts work from the fluid, decreasing its pressure and temperature. Like the compressor, it is an adiabatic component and kinetic energies can be neglected.

$$-w_t = -w_{45} = h_5 - h_4 \quad (2.53)$$

An important aspect is that all the work that the turbine delivers is used to drive the compressor:

$$w_t = w_c = h_4 - h_5 = h_3 - h_2 \quad (2.54)$$

### Nozzle

A nozzle is effectively the inverse of a diffuser. It is an adiabatic component and no work is done. The inlet kinetic energy can be neglected, but we need to account for the kinetic energy at the outlet:

$$0 = h_6 - h_5 + \frac{1}{2} V_6^2 \quad (2.55)$$

### 2.4.2 Perfect gas and reversibility assumption

To do calculations on the above processes, we can further assume a perfect gas, and that all processes are reversible. The perfect gas assumption enables us to directly relate any enthalpy change to temperature changes using

$$h_2 - h_1 = c_p(T_2 - T_1). \quad (2.56)$$

The reversibility assumption means that all adiabatic processes are also isentropic processes, for which we can use the perfect gas isentropic relationships to relate pressure changes to temperature changes:

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \quad (2.57)$$

## 2.5 Entropy

Real turbines, compressors, nozzles and diffusers are not isentropic due to irreversibilities. This means that the isentropic relations are not valid anymore to relate changes in pressure to changes in temperature and density. In this section we will look at how we can take non-isentropic effects into account in our calculations. We will start with some fundamental concepts of entropy and how to calculate changes in entropy.

### 2.5.1 Definition of entropy

Macroscopically, the *change* in entropy is defined as the relation between heat transfer and temperature

$$dS = \frac{dQ}{T}, \quad (2.58)$$

which can also be written in specific form as

$$ds = \frac{dq}{T}. \quad (2.59)$$

Entropy is a state variable (or state property), just like for example internal energy and enthalpy. This means that if we have knowledge of two states, we can calculate the change in entropy between these two states.

In some cases, a change in entropy can be directly calculated using equation 2.58 or 2.59. For example for heat transfer at a constant temperature, resulting in  $\Delta S = Q/T$ , or if the relation between the heat transfer and temperature is known such that the equation can be integrated directly. For other cases, we can derive relations with other properties, known as the first and second ‘TdS equations’.

### 2.5.2 Changes in entropy: $TdS$ equations

Here we derive the  $TdS$  equations starting from three fundamental equations. They are the differential form of the First Law of Thermodynamics, the equation for displacement work, and the definition of entropy:

$$dU = dQ - dW \quad (2.60)$$

$$dW = pdV \quad (2.61)$$

$$dQ = TdS \quad (2.62)$$

Combining and rearranging these equations gives us the ‘first  $TdS$  equation’:

$$TdS = dU + pdV \quad (2.63)$$

If we now include the definition of enthalpy

$$H = U + pV \quad (2.64)$$

and write it in differential form

$$dH = dU + pdV + Vdp, \quad (2.65)$$

then we can substitute  $dU$  in the first  $TdS$  equation by  $dH - pdV - Vdp$ , which is called the ‘second  $TdS$  equation’:

$$TdS = dH - Vdp \quad (2.66)$$

Note that so far no assumptions are made, and so these equations are generally true. We will now further develop these equations so they can be used for finite changes in entropy.

### 2.5.3 Finite changes in entropy

To make the equations above more practical, we want to be able to calculate the difference in entropy between two states, given directly measurable properties. We start with the specific form of the second  $TdS$  equation:

$$Tds = dh - vdp \quad (2.67)$$

Using the definition of  $c_p$  we can write

$$dh = c_p dT \quad (2.68)$$

and assuming an ideal gas, we can write

$$v = \frac{RT}{P} \quad (2.69)$$

Using both these relations we can rewrite equation 2.67 as

$$Tds = c_p dT - \frac{RT}{P} dP \quad (2.70)$$

which, after dividing by the temperature becomes

$$ds = c_p \frac{dT}{T} - R \frac{dP}{P} \quad (2.71)$$

If we now assume a perfect gas, where  $c_p$  is a true constant, we can integrate this equation between state 1 and state 2:

$$\int_{s_1}^{s_2} ds = c_p \int_{T_1}^{T_2} \frac{dT}{T} - R \int_{P_1}^{P_2} \frac{dP}{P} \quad (2.72)$$

which equals

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \quad (2.73)$$

This equation can be used directly when you have knowledge of the temperature and pressure of two states to calculate the difference in entropy between those states. Two similar equations can be constructed using the same assumptions:

$$s_2 - s_1 = c_V \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right) \quad (2.74)$$

$$s_2 - s_1 = c_V \ln \left( \frac{P_2}{P_1} \right) + c_P \ln \left( \frac{v_2}{v_1} \right) \quad (2.75)$$

### 2.5.4 Isentropic efficiency

The two main components where we are interested in how closely they resemble an isentropic process are compressors and turbines. When a process is not isentropic, there will be a drop in efficiency. For a compressor this means that more work needs to be done to reach a specific pressure, while for a turbine this means that the work output for a given pressure drop will be less compared to an isentropic process. In both cases, the entropy at the outlet is higher than the entropy at the inlet. As we will see, this also means that in both cases the temperature at the outlet is higher than the outlet temperature of the isentropic process. The outlet temperature of a real process is higher because of irreversibility effects (e.g., friction, turbulence) which always result in the generation of heat.

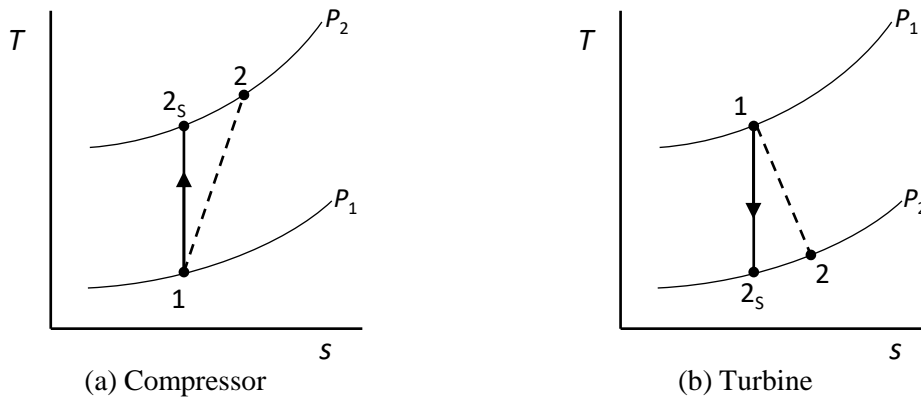


Figure 2.6: Isentropic (solid line) and real (dashed line) process for a compressor and a turbine.

#### Isentropic efficiency of a compressor

For a compressor, the isentropic efficiency is defined as

$$\eta_c = \frac{\text{isentropic work input}}{\text{actual work input}} = \frac{w_{c,s}}{w_c} = \frac{h_{2,s} - h_1}{h_2 - h_1}, \quad (2.76)$$

where the subscript  $c$  stands for compressor, and the subscript  $s$  is used to refer to the isentropic process values. When we assume a perfect gas, this equation can also be written using only temperatures:

$$\eta_c = \frac{c_P(T_{2,s} - T_1)}{c_P(T_2 - T_1)} = \frac{T_{2,s} - T_1}{T_2 - T_1} \quad (2.77)$$

Figure 2.6(a) shows the isentropic and real process of a compressor on a T-s diagram. Note that the outlet entropy and temperature of the real process (2) is higher than that of the isentropic process (2<sub>s</sub>).

**Isentropic efficiency of a turbine**

The isentropic efficiency of a turbine is defined similar to the compressor, except the work terms have switched:

$$\eta_t = \frac{\text{actual work output}}{\text{isentropic work output}} = \frac{w_t}{w_{t,s}} = \frac{h_1 - h_2}{h_1 - h_{2,s}}, \quad (2.78)$$

where the subscript  $t$  indicates the turbine. Also here, if we assume a perfect gas, we can write

$$\eta_t = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_{2,s})} = \frac{T_1 - T_2}{T_1 - T_{2,s}} \quad (2.79)$$

The isentropic and real process is shown in figure 2.6(b). Note that again the outlet entropy and temperature of the real process (2) is higher than that of the isentropic process (2<sub>s</sub>).

You will have noticed that the isentropic efficiency of a compressor and a turbine look very similar. It is therefore easy to confuse the two, and accidentally use the wrong equation. A good way to avoid making mistakes is to do a few simple checks:

- The isentropic efficiency is always less than 1
- The outlet temperature of the real process is always higher than the outlet temperature of the isentropic process
- The work done by a real compressor is always higher than the work done by an isentropic compressor
- The work done by a real turbine is always less than the work done by an isentropic turbine