UNIVERSITY OF Southampton

Chapter 5: Mission Analysis

Lecture 11 – Orbital transfers

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Overview of lecture 11

- This lecture is focused on orbital transfers – changing from one orbit to another
  - Calculations for orbital transfers make multiple uses of the energy equation
    - See lecture 9 for the theory and lecture 10 for a worked example of the use of the energy equation
  - To gain some insights into orbital transfers we will look at a video showing the deployment of the RemoveDebris spacecraft from the International Space Station
    - The key will be to focus on the different motions that are visible in the video
  - We will make some assumptions about the "burns" that cause these orbital transfers:
    - Principally, that they are impulsive
  - At the end of the lecture we look at an important orbital transfer – the Hohmann transfer
    - An activity that should help with your understanding is explained in lecture 12 and there is a worked example in lecture 13

Orbital transfers

- Deployment of RemoveDebris from the ISS
  - https://www.youtube.com/watch?v=Gb1u9LGjTaM (watch from about 50 seconds)

RemoveDebris

Nanoracks
Kaber deployer

Orbital transfers

- Deployment of RemoveDebris from the ISS
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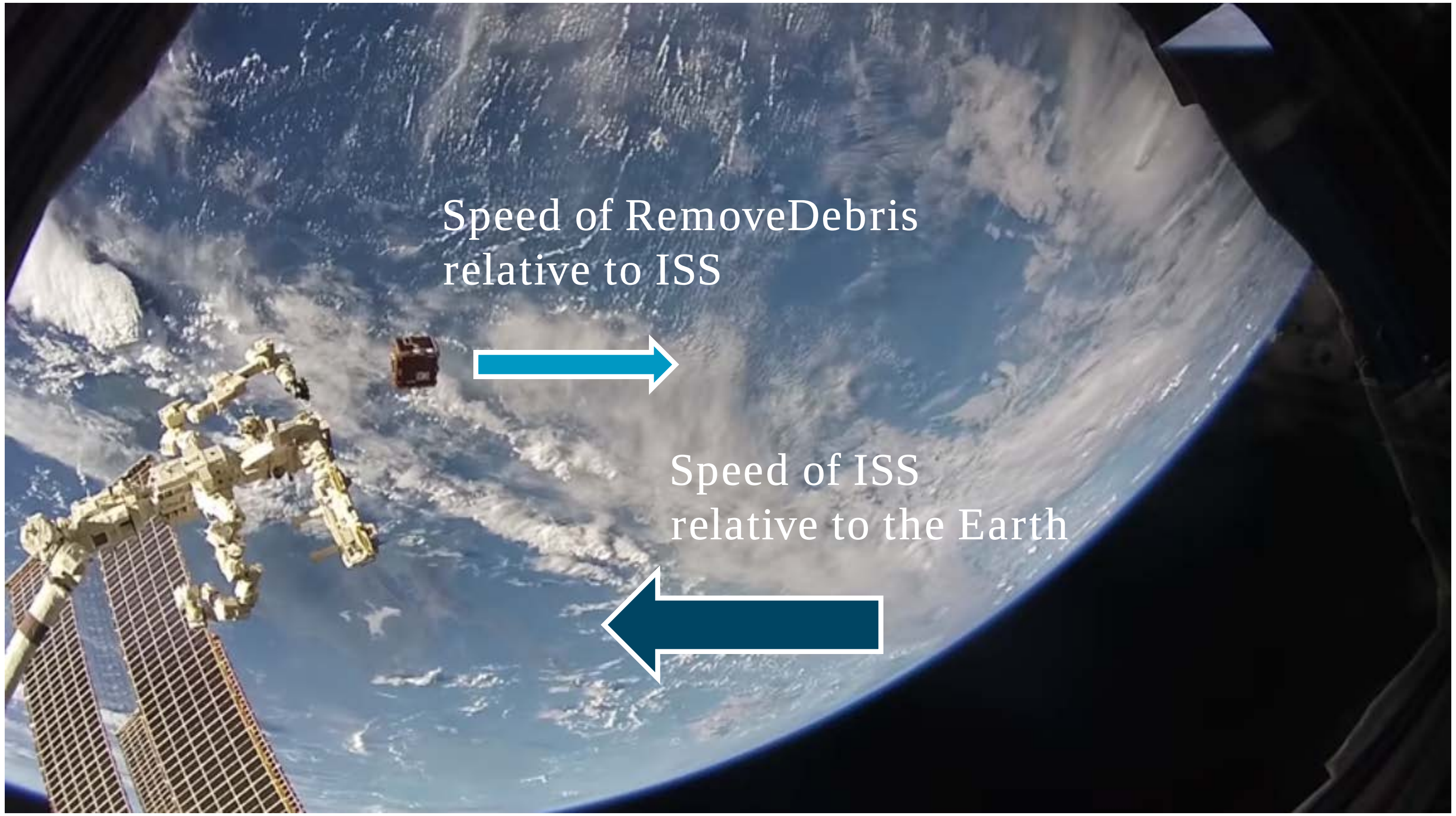
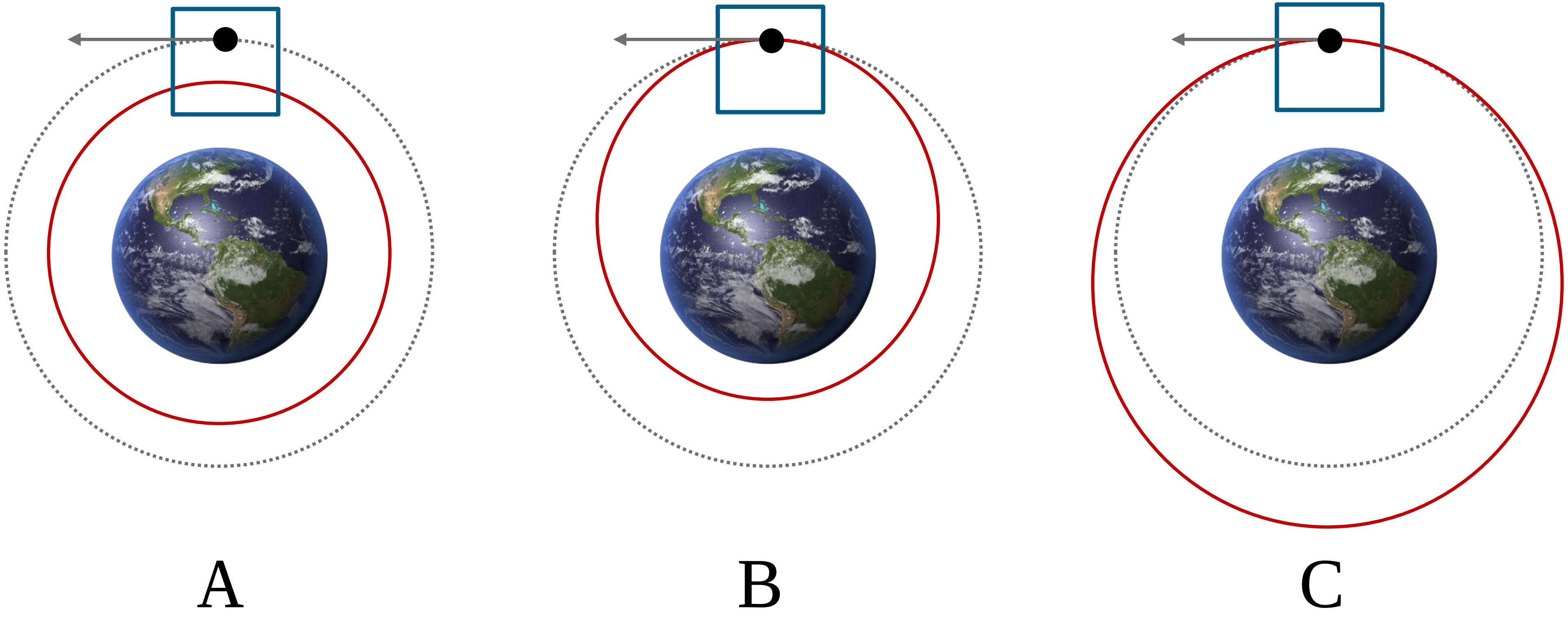


Orbital transfers

- Quick quiz:
  - Which of the red orbits best describes the orbit of RemoveDebris immediately after deployment from the ISS?
    - Assume deployment in the box; ISS orbit is grey (with velocity vector shown)



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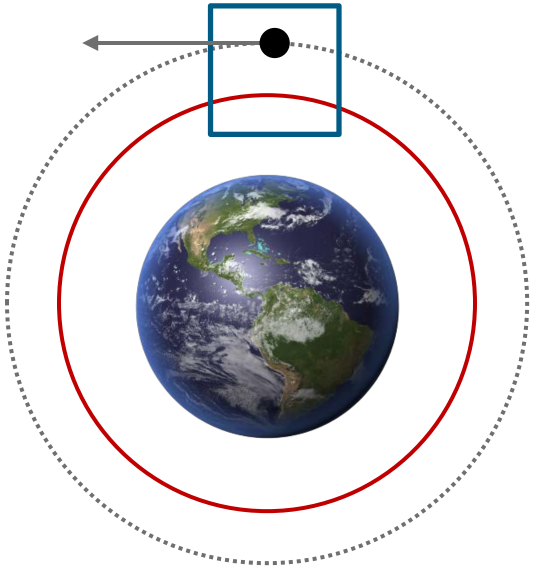
Orbital transfers

- Quick quiz: ANSWER
  - Let’s look at option A first



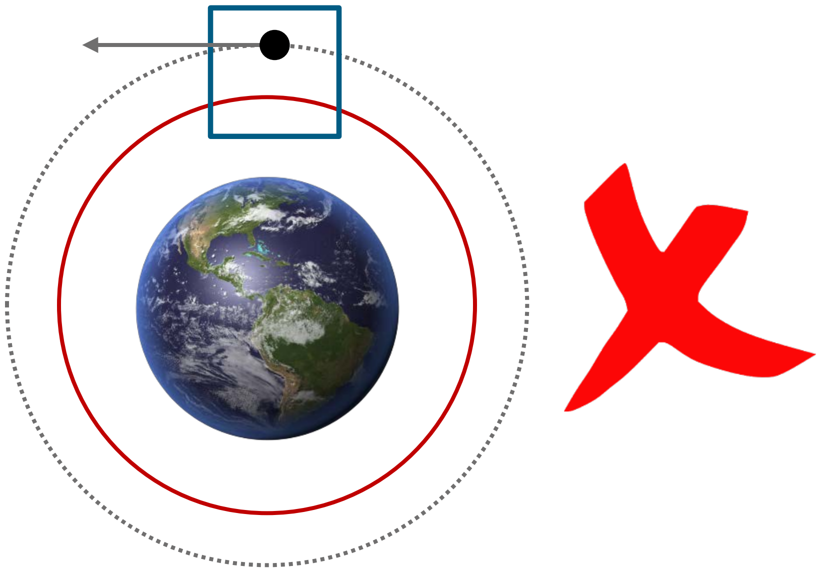
A

- We can see in the video that RemoveDebris is initially in contact with the ISS (i.e. in the same orbit)

- RemoveDebris is given an <u>impulse</u>. At that exact moment, the position of RemoveDebris is unchanged but its speed is different…

Orbital transfers

- Quick quiz: ANSWER
  - Let’s look at option A first



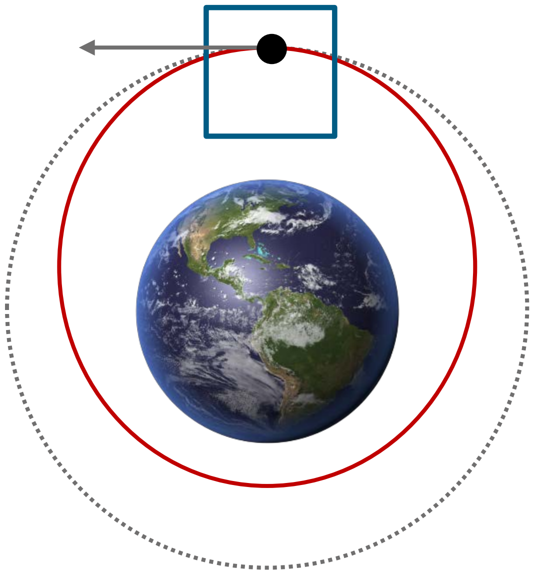
A

- ...This means that the new orbit for RemoveDebris must have that position in common with the orbit of the ISS.

- Option A does not have any points in common with the ISS orbit, so A cannot be correct.

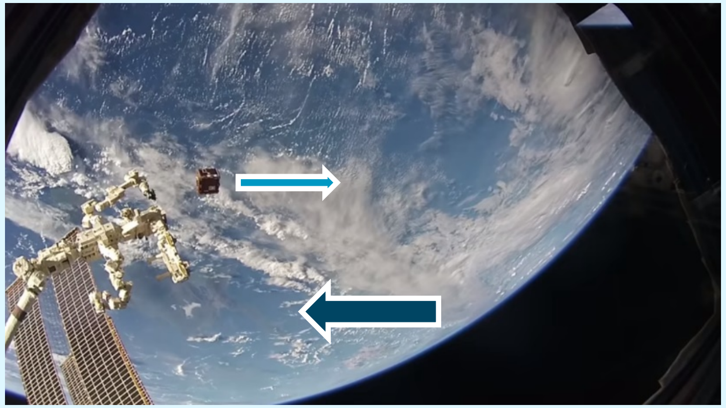
Orbital transfers

- Quick quiz: ANSWER
  - Let’s look at option B next



B

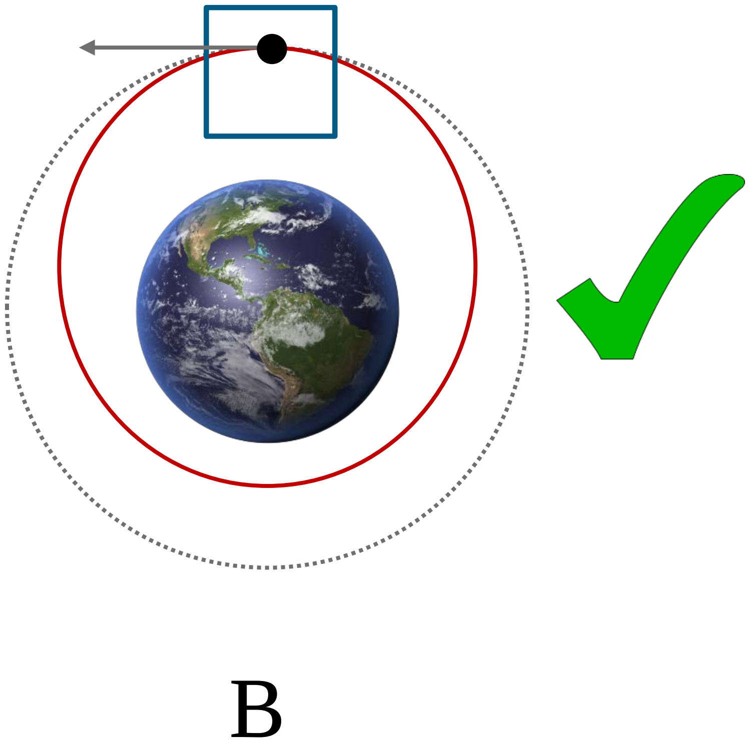
- We know that at the moment of deployment the two orbits must intersect.
- We also know that the impulse provided to RemoveDebris is in a direction that is at 180° to the velocity of the ISS



- The impulse is a force. It is non-conservative so it removes energy from the orbit…

Orbital transfers

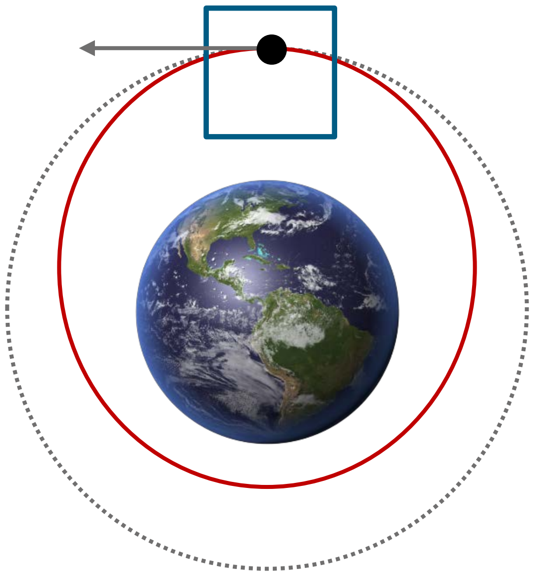
- Quick quiz: ANSWER
  - Let's look at option B next



- The combination of three factors:
  - The ISS and RemoveDebris orbits must intersect;
  - Energy is removed from the orbit of RemoveDebris; and
  - The Earth must be at the focus of the orbit
- ...means that the orbit of RemoveDebris must be:
  - Smaller (i.e. smaller semi-major axis); and
  - Elliptical
- Option B matches this description and is the correct answer (the orbit in option C is larger)

Orbital transfers

- Quick quiz:
  - Which of these descriptions best describes the motion of RemoveDebris relative to the ISS?



A RemoveDebris moves ahead and above the ISS

B RemoveDebris moves behind and above the ISS

C RemoveDebris moves ahead and below the ISS

D RemoveDebris moves behind and below the ISS

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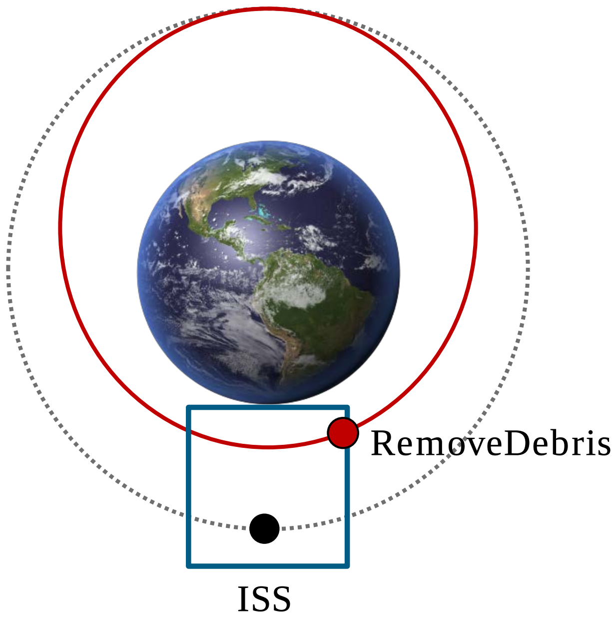
Orbital transfers

- Quick quiz: ANSWER
  - To answer this we look at where the ISS and RemoveDebris will be after ~half an orbit



- We know (from the previous quiz answer) that the orbit of RemoveDebris will be smaller than the orbit of the ISS.
- We also know from Kepler's 3rd Law that the orbital period is only a function of the size of the orbit: smaller orbits have shorter orbital periods.
- In other words, RemoveDebris will make one orbit in a shorter time than the ISS, so it will appear to move ahead of the ISS.
- This means that RemoveDebris moves ahead and below the ISS (<u>option C is correct</u>)

Orbital transfers

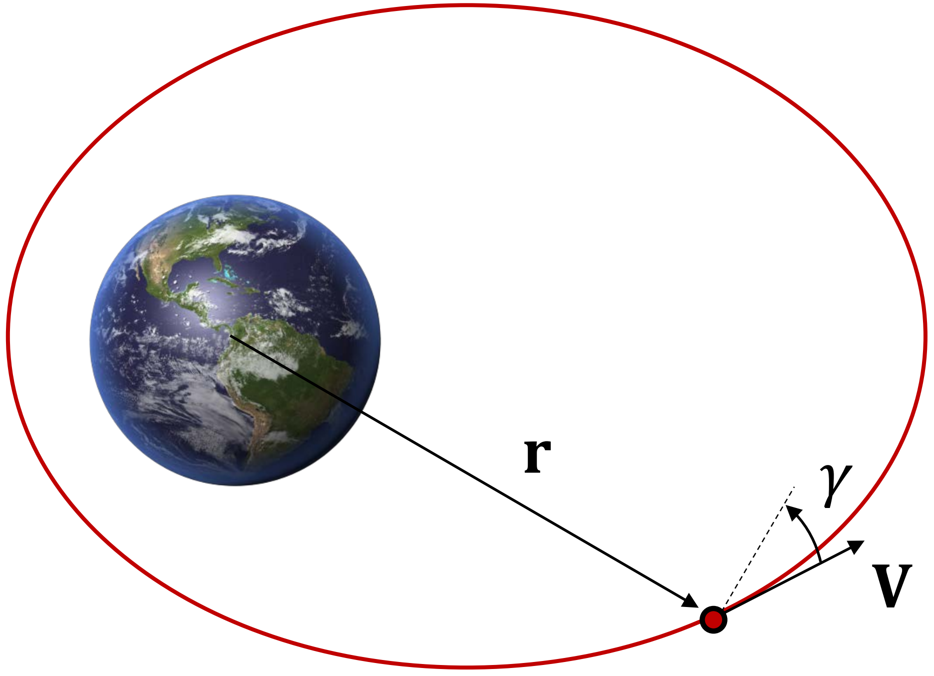
- Co-planar orbit transfers
  - To transfer between one orbit and another, a rocket motor* is fired to change the vehicle's speed, i.e. impart a ΔV.
  - In spacecraft mission analysis one of the main objectives is to determine the ΔV that must be imparted to the spacecraft to achieve its journey from launch site to its destination.
  - We normally try to minimise the ΔV.
  - To make the calculations easier we assume that:

a) The manoeuvre is a high thrust, short duration event, so the ΔV is acquired while the spacecraft has not moved very far along the orbit. This is usually referred to as an <u>impulsive manoeuvre</u>.

Orbital transfers

- Co-planar orbit transfers
  - (Continued...) to make the calculations easier we assume that:

b) The flight path angle γ is small

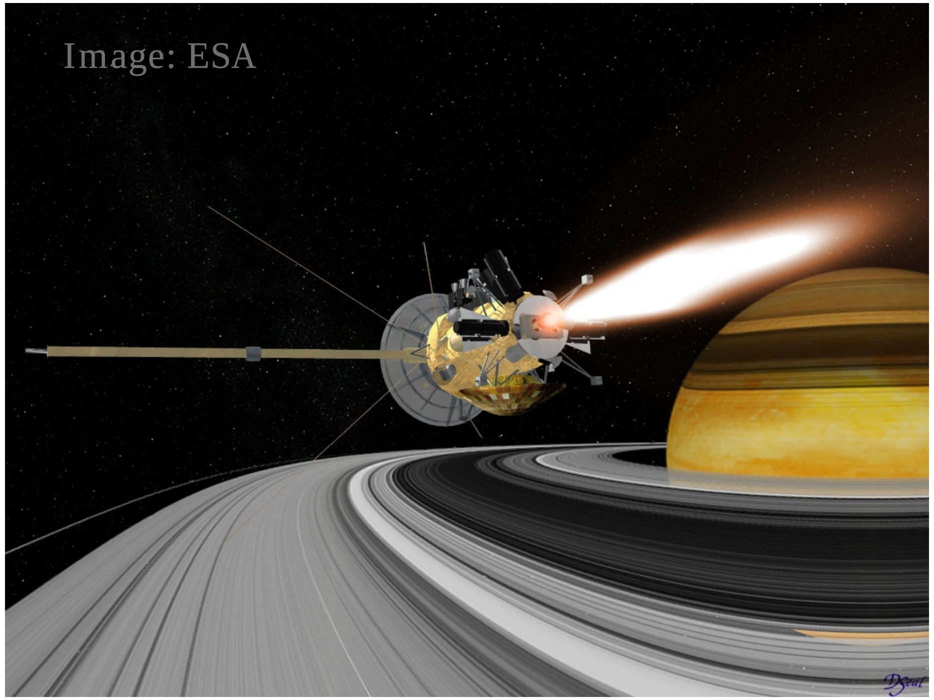


  - The combination of (a) and (b) gives a negligible gravity loss term so that:

$$\Delta V = V_{ex} \ln\left(\frac{M_0}{M_b}\right) \text{ where } M_b = M_0 - M_f$$

Orbital transfers

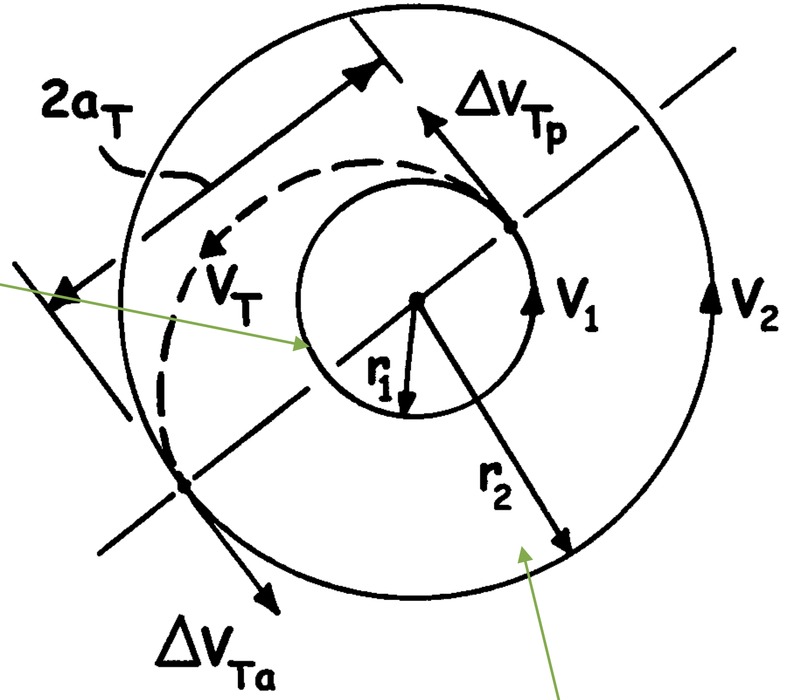
- Co-planar orbit transfers
  - During an impulsive manoeuvre the point at which the engine fires becomes common to both the old orbit and the new one
    - This implies that single manoeuvres can only achieve transfer between intersecting orbits. Transfer between non-intersecting orbits requires at least two engine firings.
  - To calculate the ΔV we can use the energy equation to calculate the spacecraft velocity in each orbit at this common point. The ΔV is the magnitude of the vector difference between them.



Orbital transfers

- The Hohmann Transfer
  - This is a two-impulse strategy giving minimum ΔV between two coplanar circular orbits.
  - Initially the spacecraft is in the small circular orbit, r_1
    - Here the speed is: $$V_1 = \sqrt{\frac{\mu}{r_1}}$$
  - From here, apply a tangential ΔV_{TP} to inject the spacecraft into an elliptical transfer orbit with:

$$2a_T = r_1 + r_2$$ → the apogee of the transfer orbit is r_2



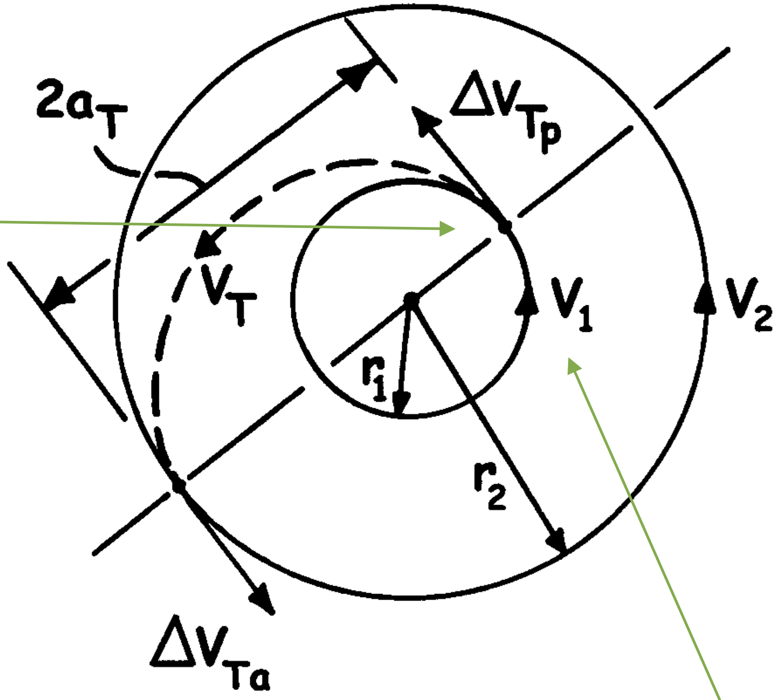
Orbital transfers

- ## The Hohmann Transfer
  - The speed at the perigee of the elliptical transfer orbit with semi-major axis a_T is given by:

$$V_{Tp}^2 = \mu\left(\frac{2}{r_1} - \frac{1}{a_T}\right)$$

  - So the ΔV_{TP} needed to inject the spacecraft into this elliptical transfer orbit is the difference in the speed at the point that is common to both orbits:

$$\Delta V_{Tp} = V_{Tp} - V_1$$



Note: ΔV_{Tp} is applied in the same direction as V_1

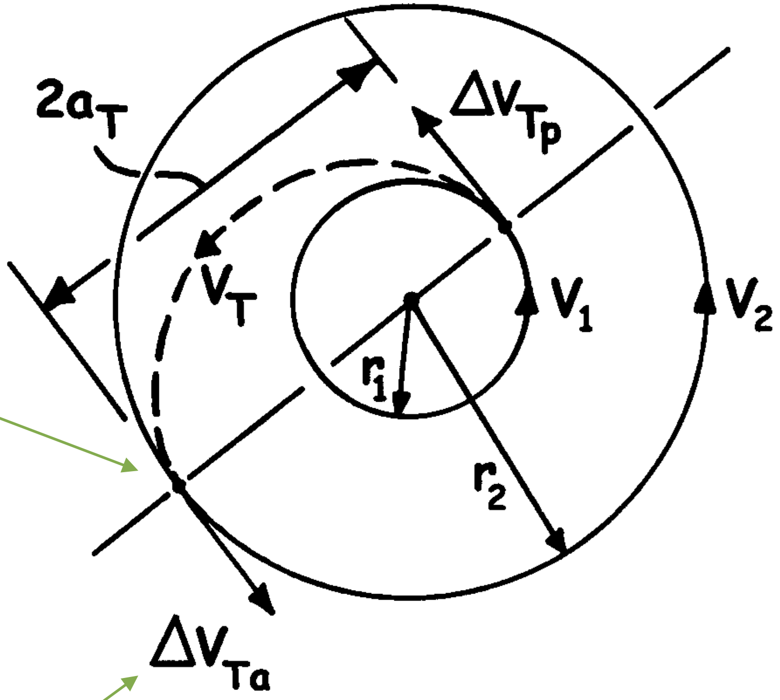
Orbital transfers

- ## The Hohmann Transfer
  - Now we look at the apogee of the elliptical transfer orbit. The speed at this point is:

$$V_{Ta}^2 = \mu\left(\frac{2}{r_2} - \frac{1}{a_T}\right)$$

  - From here, apply a tangential ΔV_{Ta} to inject the spacecraft into a circular orbit with radius r_2 and speed given by:

$$V_2 = \sqrt{\frac{\mu}{r_2}}$$



Note: ΔV_{Ta} is applied in the same direction as V_2

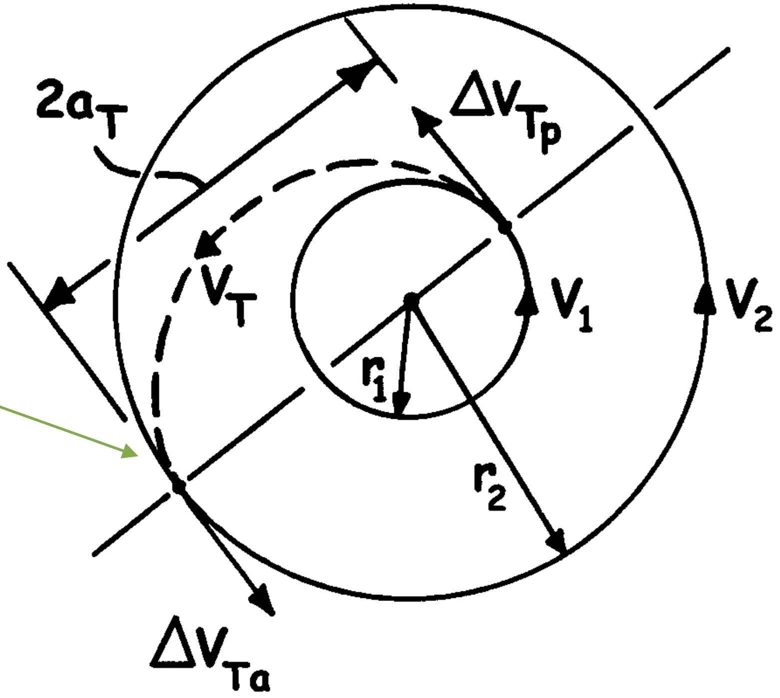
Orbital transfers

- The Hohmann Transfer
  - So the ΔV_{Ta} needed to inject the spacecraft into this circular orbit is the difference in the speed at the point that is common to both orbits:

$$\Delta V_{Ta} = V_2 - V_{Ta}$$

  - And the total ΔV for the Hohmann transfer is:

$$\Delta V = \Delta V_{Tp} + \Delta V_{Ta}$$

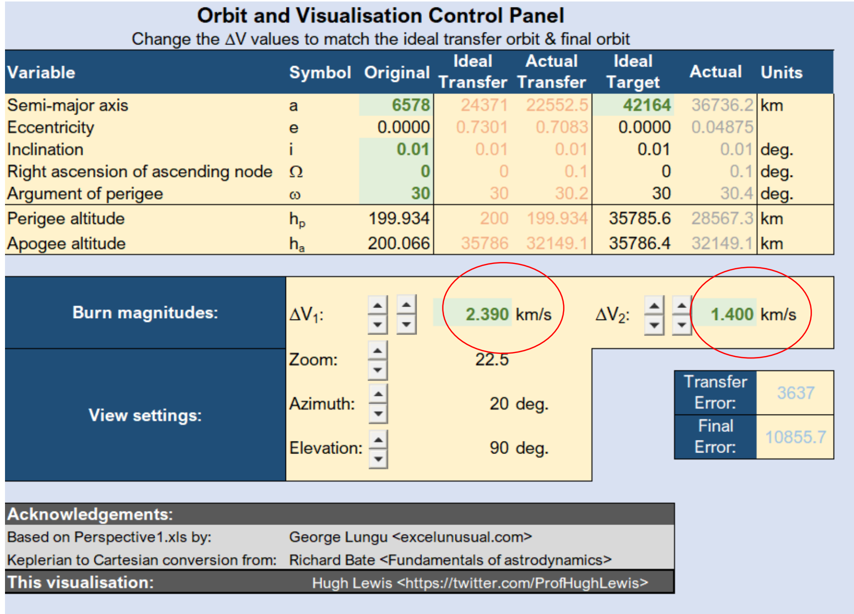


Recap of lecture 11

- This lecture focused on orbital transfers – changing from one orbit to another
  - We assumed that orbital transfers are impulsive:
    - The burn is high thrust and short duration so the ΔV is acquired while the spacecraft has not moved very far along the orbit
    - The old and new orbits must intersect
    - The point where the engine fires is common to the old and new orbits
    - For non-intersecting orbits we need two burns
  - We can use the energy equation to calculate the speed on the old and new orbits at the point where the engine fires
    - The ΔV is then the difference between these two values
  - At the end of the lecture we looked at the Hohmann transfer – which is a two-burn strategy to transfer between non-intersecting circular orbits
    - An activity that should help with your understanding is explained in lecture 12 and there is a worked example in lecture 13

Activity

- Activity using the orbit visualisation tool:
  - Use the "Hohmann Transfer" version of the visualisation tool
    - The aim is to adjust the delta-V values for the two burns so that the actual transfer orbit matches the ideal transfer orbit, and the actual final orbit matches the ideal final orbit (with a small margin of error)
    - You can change the initial orbital elements (except the eccentricity) and the final orbit semi-major axis. In the example shown, the transfer is from a 200 km parking orbit to GEO



Orbit and Visualisation Control Panel

Change the ΔV values to match the ideal transfer orbit & final orbit

Variable	Symbol	Original	Ideal Transfer	Actual Transfer	Ideal Target	Actual	Units
Semi-major axis	a	6578	24371	22552.5	42164	36736.2	km
Eccentricity	e	0.0000	0.7301	0.7083	0.0000	0.04875	
Inclination	i	0.01	0.01	0.01	0.01	0.01	deg.
Right ascension of ascending node	Ω	0	0	0.1	0	0.1	deg.
Argument of perigee	ω	30	30	30.2	30	30.4	deg.
Perigee altitude	h_p	199.934	200	199.934	35785.6	28567.3	km
Apogee altitude	h_a	200.066	35786	32149.1	35786.4	32149.1	km

Burn magnitudes:	ΔV_1:	2.390 km/s	ΔV_2:	1.400 km/s
View settings:	Zoom:	22.5	Transfer Error:	3637
	Azimuth:	20 deg.	Final Error:	10855.7
	Elevation:	90 deg.		

Acknowledgements:
Based on Perspective1.xls by: George Lungu <excelunusual.com>
Keplerian to Cartesian conversion from: Richard Bate <Fundamentals of astrodynamics>
This visualisation: Hugh Lewis <https://twitter.com/ProfHughLewis>