

# SESA2023 Week 4: Ramjets

This week we look at ramjets. We will investigate the working principles of a ramjet, analyse ideal and non-ideal ramjets, and finally have a brief look into scramjets.

## 4.1 Learning outcomes

After completing this section you should be able to:

- Explain the operating principles for ramjets.
- Explain why ramjets can operate at higher Mach numbers than gas turbine engines.
- Analyse the three components of an ideal ramjet, obtaining the temperature and pressure at each component inlet and outlet.
- Calculate the fuel consumption and thrust produced by an ideal ramjet given a set of operating conditions.
- Analyse a non-ideal ramjet using stagnation pressure ratios and combustion efficiency.
- Draw the ideal and non-ideal ramjet processes on a T-s diagram.
- Explain why there is a lower and an upper speed limit for the operation of a ramjet.
- Explain the difference between a ramjet and a scramjet.
- Explain why a scramjet can operate at higher Mach numbers than a ramjet.
- Explain why a scramjet needs an even higher speed to operate than a ramjet.

## 4.2 Ramjets

At high supersonic velocities, ramjets are more suitable as a propulsion system than gas turbine engines. The kinetic energy of the air flowing into the engine is high enough to reach large enough compression ratios without the need of a compressor. This also removes the need of a turbine to drive the compressor, so the operating temperatures can be much higher because the turbine blades do not limit the upper temperature in the combustion chamber.

### 4.2.1 Operating principles

#### Diffuser

In a typical ramjet operation, the air intake is supersonic, and needs to be converted to subsonic conditions in the diffuser. Figure 4.1 shows a schematic inlet for a ramjet. A series of oblique shocks (orange lines) slow the air flow down, followed by a normal shock (green line), after which the flow will be subsonic. As we've seen in week 3, once the flow is subsonic, the air can be slowed down further

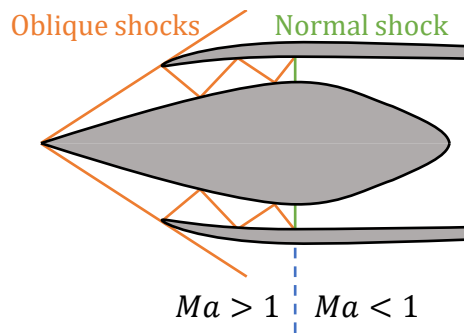


Figure 4.1: Inlet geometry of a ramjet.

by increasing the area. Controlling the positioning of shocks is one of the key design considerations for a ramjet intake, with the aim of recovering as much pressure as possible from the incoming air.

The SR-71 Blackbird is a good example of this, operating effectively as a ramjet at supersonic speeds, where the inlet geometry is changed during flight for optimal positioning of the shock waves.

### Combustion

Combustion takes place at low subsonic velocities, and because there is no turbine after the combustion chamber, fuel can be burned close to the stoichiometric ratio. This results in higher combustion temperatures up to around 2,400 K.

### Nozzle

The nozzle will use the high enthalpy (pressure and temperature) generated in the combustion chamber to accelerate the combustion products and thereby generate thrust. Because the ramjet operates at supersonic velocities, and the combustion happens at subsonic velocities, a converging-diverging nozzle is used to generate a high-velocity supersonic jet.

### Operating range

The standard optimal operating range of a ramjet is around Mach 3 to Mach 4. At lower velocities, the reduction of the pressure ratio results in a large loss in efficiency. At higher velocities, it becomes harder to generate subsonic conditions in the combustion chamber. The large deceleration can result in large pressure losses and will also result in higher stagnation temperatures, eventually reaching material limits of the ramjet at high Mach numbers.

## 4.3 Ideal ramjet

The ideal ramjet consists of three ideal processes and four states, as indicated in figure 4.2. The three ideal processes are:

- 1-2 isentropic compression (diffuser)
- 2-3 constant pressure combustion
- 3-4 isentropic expansion (nozzle)

Further assumptions are

- All processes are adiabatic
- Perfect gas (constant  $\gamma$ ,  $c_p$ )

- Fully expanded exhaust ( $P_4 = P_A$ )
- Combustion occurs at negligible kinetic energy

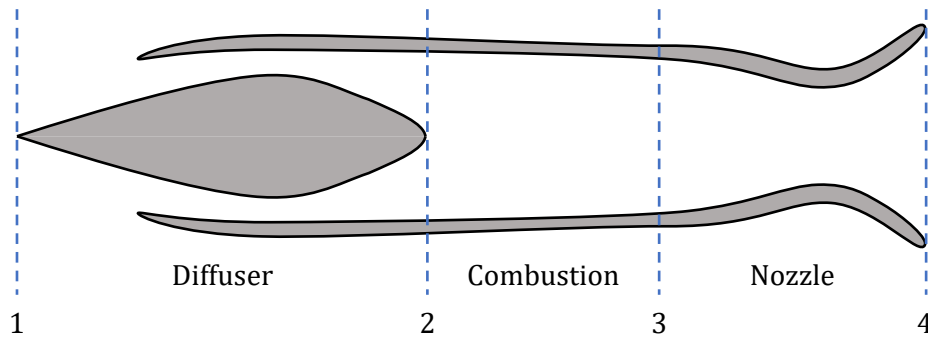


Figure 4.2: Schematic drawing of a ramjet.

Typical operating parameters will include a Mach number (defining the inlet velocity), an ambient pressure ( $P_A$ ) and temperature ( $T_A$ ), the fuel lower calorific value ( $LCV$ ), and a maximum temperature ( $T_{max}$ ). The ambient pressure and temperature can alternatively also be obtained from altitude tables when the flight altitude is known.

We will now explain how to use the operating conditions to calculate the specific thrust produced by the ideal ramjet. Recalling the thrust equation from week 1

$$F = \dot{m}_a [(1 + f) V_j - V] + A_j (P_j - P_A), \quad (4.1)$$

with  $\dot{m}_a$  the mass flow rate of air,  $f$  the fuel-air ratio,  $V_j$  the jet velocity,  $V$  the aircraft velocity,  $A_j$  the area of the exhaust plane, and  $P_j$  the pressure at the exhaust plane. Because the exhaust is fully expanded,  $P_j = P_A$ , and we can therefore write the specific thrust as

$$\frac{F}{\dot{m}_a} = (1 + f) V_j - V. \quad (4.2)$$

The unknowns are the fuel-air ratio and the jet velocity, which we can obtain by analysing the three processes in a ramjet. For each process, we start with a steady flow energy equation (SFEE) to work out the unknown values. because state 2 and 3 have negligible kinetic energy, the properties at those states are equal to the stagnation properties. We will therefore write the standard SFEE

$$q - w_x = h_{out} + \frac{1}{2} V_{out}^2 - h_{in} - \frac{1}{2} V_{in}^2 \quad (4.3)$$

using the stagnation enthalpies:

$$q - w_x = h_{0,out} - h_{0,in}, \quad (4.4)$$

recalling from week 3 that  $h_0 = h + \frac{1}{2} U^2$ .

#### 4.3.1 Diffuser

The diffuser is adiabatic and no work is done, so the SFEE simplifies to

$$0 = h_{0,2} - h_{0,1}, \quad (4.5)$$

which we can also express using temperatures because of the perfect gas assumption:

$$0 = c_p (T_{0,2} - T_{0,1}), \quad (4.6)$$

or

$$T_{0,2} = T_{0,1}. \quad (4.7)$$

Because we neglect kinetic energy in the combustion chamber, the static temperature  $T_2$  is equal to the stagnation temperature  $T_{0,2}$ . We therefore know the the temperature at state 2 is equal to the stagnation temperature at the diffuser inlet, which we can write as a function of the inlet Mach number:

$$T_2 = T_{0,1} = T_1 \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right). \quad (4.8)$$

Knowing the temperature change, we can now use the isentropic relations to obtain the pressure  $P_2$  in the combustion chamber

$$\frac{P_2}{P_A} = \frac{P_2}{P_1} = \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} = \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right)^{\frac{\gamma}{\gamma-1}}. \quad (4.9)$$

### 4.3.2 Combustion

In the combustion chamber we have a different mass flow rate going in (only air) compared to the mass flow rate going out (air + fuel), so we need to take mass flow rates into account in the SFEE:

$$\dot{Q} - \dot{W}_x = (\dot{m}_a + \dot{m}_f) h_{0,3} - \dot{m}_a h_{0,2} \quad (4.10)$$

The heat addition is equal to the mass flow rate of fuel multiplied by the LCV of the fuel:  $\dot{Q} = \dot{m}_f \text{LCV}$ . We further know that no work is done in the combustion chamber, and the stagnation enthalpies are equal to the static enthalpies:

$$\dot{m}_f \text{LCV} = (\dot{m}_a + \dot{m}_f) h_3 - \dot{m}_a h_2 \quad (4.11)$$

or, after dividing by  $\dot{m}_a$ :

$$f \times \text{LCV} = (1 + f) h_3 - h_2 = (1 + f) c_p T_3 - c_p T_2 \quad (4.12)$$

We can solve this for the fuel-air ratio and obtain

$$f = \frac{T_3 - T_2}{\frac{\text{LCV}}{c_p} - T_3}. \quad (4.13)$$

For a given maximum temperature  $T_{max} = T_3$ , this provides the fuel-air ratio. Alternatively, for a given fuel-air ratio, the same analysis can be used to determine the combustion outlet temperature  $T_3$ . The combustion outlet pressure is the same as the inlet, because the combustion is a constant pressure process for an ideal ramjet:  $P_3 = P_2$

### 4.3.3 Nozzle

The nozzle is effectively the inverse of the diffuser, adiabatic and no shaft work done, so we have

$$0 = h_4 + \frac{1}{2} V_4^2 - h_3, \quad (4.14)$$

noting that we must take into account the exit kinetic energy, with  $V_4 = V_j$ . Solving for  $V_j$  and using the perfect gas assumption we have

$$V_j = \sqrt{2c_p(T_3 - T_4)} \quad (4.15)$$

We can obtain the unknown temperature  $T_4$  from the isentropic relation

$$\frac{T_4}{T_3} = \left( \frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{P_A}{P_3} \right)^{\frac{\gamma-1}{\gamma}} \quad (4.16)$$

giving the jet velocity

$$V_j = \sqrt{2c_p T_3 \left[ 1 - \left( \frac{P_A}{P_3} \right)^{\frac{\gamma-1}{\gamma}} \right]}, \quad (4.17)$$

which can now be used in equation (4.2) to obtain the specific thrust of the ideal ramjet.

## 4.4 Non-ideal ramjets

Real ramjet engines have losses in all components, that all either reduce the thrust, or the fuel efficiency of the engine. There are two main sources of losses: pressure losses and heat losses. Pressure losses are the result of friction, which reduces the pressure and increases the entropy. We quantify the pressure loss for each stage of a ramjet using stagnation pressure ratios for the diffuser, combustion chamber, and nozzle:

$$\Gamma_d = \frac{P_{0,2}}{P_{0,1}} \quad (4.18)$$

$$\Gamma_c = \frac{P_{0,3}}{P_{0,2}} \quad (4.19)$$

$$\Gamma_n = \frac{P_{0,4}}{P_{0,3}} \quad (4.20)$$

The stagnation pressure ratios will take a value between 0 and 1, with  $\Gamma = 1$  being equal to the ideal ramjet. Using the ratios, the stagnation pressure at the nozzle exit can be directly related to the stagnation pressure at the diffuser inlet:

$$P_{0,4} = P_{0,3}\Gamma_n = P_{0,2}\Gamma_c\Gamma_n = P_{0,1}\Gamma_d\Gamma_c\Gamma_n \quad (4.21)$$

Heat losses are the main concern in the combustion stage, which result from incomplete combustion and heat transfer to the environment. We quantify these losses using a single combustion efficiency parameter  $\eta_{comb}$ , effectively reducing  $\dot{Q}$ , the heat added to the gas:

$$\dot{Q} = \eta_{comb} \times LCV \times \dot{m}_f \quad (4.22)$$

Finally, the nozzle exit pressure  $P_j$  is not necessarily the same as the ambient pressure  $P_A$ , so the full equation for thrust must be used:

$$F = \dot{m}_a [(1 + f)V_j - V] + (P_j - P_A)A_j \quad (4.23)$$

The value of the exit pressure depends on the design and the operating conditions, and cannot be calculated directly using the stagnation pressure ratios given above.

Figure 4.3 shows the operation of an ideal and a non-ideal ramjet on a T-s diagram. The ideal processes (1 – 2<sub>i</sub> – 3<sub>i</sub> – 4<sub>i</sub>) are coloured green, and the non-ideal processes (1 – 2 – 3 – 4) are coloured orange. Note that the temperatures  $T_2$  and  $T_3$  are the same for the ideal and non-ideal ramjet (which we will show below), and that the stagnation pressure loss always results in an increase in entropy. In this figure, the jet pressure is equal to the ambient pressure, but this is not necessarily the case for a non-ideal ramjet. It could be either at a higher pressure (over-expanded) or lower (under-expanded).

We will now analyse the SFEE for the diffuser, combustion, and nozzle separately and see how the stagnation pressure ratios and combustion efficiency influence the pressures, temperatures, and thrust produced. We assume a known inlet Mach number  $Ma_1$ , temperature  $T_1$ , and the ambient pressure  $P_A$ , as well as a known jet exit pressure  $P_4$ .

### 4.4.1 Diffuser

The SFEE for the diffuser is written as

$$0 = h_{0,2} - h_{0,1} = h_2 - h_1 - \frac{1}{2} V_1^2 \quad (4.24)$$

Assuming a perfect gas, we can express this as

$$0 = c_p(T_{0,2} - T_{0,1}) = c_p(T_2 - T_1) - \frac{1}{2} V_1^2 \quad (4.25)$$

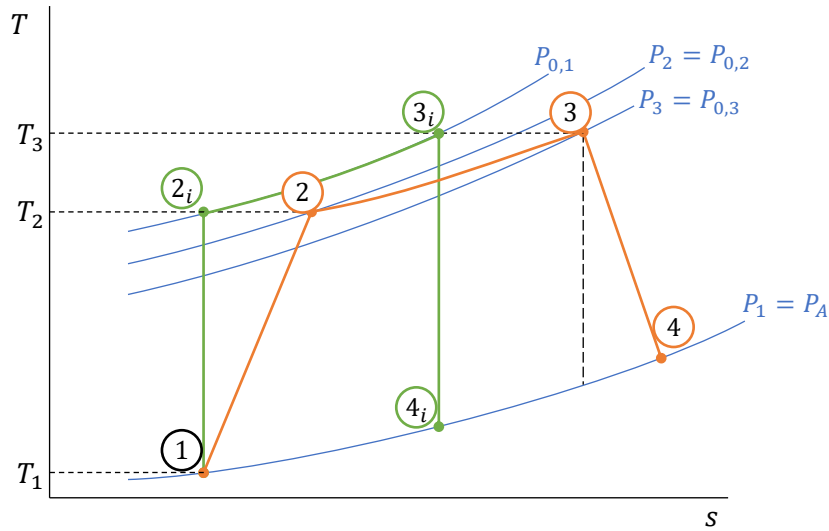


Figure 4.3: T-s diagram of an ideal and non-ideal ramjet.

Recognizing that  $T_2 = T_{0,2}$ , we can write

$$T_2 = T_{0,1} = T_1 \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right) \quad (4.26)$$

Note that we've not made any assumptions on the pressure, so the diffuser outlet temperature  $T_2$  is the same for an ideal and a non-ideal ramjet. The pressure at the diffuser outlet is

$$P_2 = P_{0,2} = \Gamma_d P_{0,1} \quad (4.27)$$

where the stagnation pressure ratio  $\Gamma_d$  appears. The stagnation pressure at the inlet is given by

$$P_{0,1} = P_1 \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} = P_1 \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (4.28)$$

and so

$$P_2 = \Gamma_d P_A \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (4.29)$$

revealing a lower diffuser outlet pressure compared to the ideal ramjet.

#### 4.4.2 Combustion

In the combustion chamber we need to consider both the combustion efficiency and the pressure loss. The SFEE for the combustion chamber is

$$\dot{Q} = (\dot{m}_a + \dot{m}_f) h_3 - \dot{m}_a h_2. \quad (4.30)$$

Including the combustion efficiency  $\eta_{comb}$  we get

$$\dot{m}_f LCV \eta_{comb} = (\dot{m}_a + \dot{m}_f) h_3 - \dot{m}_a h_2. \quad (4.31)$$

or

$$f \times LCV \eta_{comb} = (1+f) h_3 - h_2 = (1+f) c_p T_3 - c_p T_2. \quad (4.32)$$

Solving for  $f$  gives

$$f = \frac{T_3 - T_2}{\frac{LCV \eta_{comb}}{c_p} - T_3}, \quad (4.33)$$

which shows that for a fixed  $T_3$ , a lower combustion efficiency increases the fuel consumption, or for a fixed air-fuel-ratio the combustion outlet temperature  $T_3$  will be lower. Note that again no assumptions on the pressure have been made, so the pressure has no direct influence on  $f$  or  $T_3$ . The pressure at the combustion chamber outlet will be

$$P_3 = P_{0,3} = \Gamma_c P_{0,2} \quad (4.34)$$

and therefore

$$P_3 = \Gamma_d \Gamma_c P_A \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (4.35)$$

#### 4.4.3 Nozzle

The nozzle is the component where we will see the effect of the drop in stagnation pressure on the propulsion. We start with the SFEE for the nozzle

$$0 = h_4 + \frac{1}{2} V_4^2 - h_3 \quad (4.36)$$

with  $V_4 = V_j$  and assuming a perfect gas we get

$$V_j^2 = 2c_p(T_3 - T_4) \quad (4.37)$$

We know  $T_3$  from the combustion process, so we only need to find  $T_4$ . Writing the SFEE for the nozzle in terms of stagnation temperatures

$$0 = c_p(T_{0,4} - T_{0,3}), \quad (4.38)$$

we see that  $T_{0,4} = T_{0,3} = T_3$ . We can now calculate  $T_4$  using

$$T_4 = T_{0,4} \left( \frac{P_4}{P_{0,4}} \right)^{\frac{\gamma-1}{\gamma}} \quad (4.39)$$

with  $T_{0,4} = T_3$ ,  $P_4$  the known jet exit pressure, and  $P_{0,4}$  given by

$$P_{0,4} = \Gamma_n P_{0,3} = \Gamma_n \Gamma_d \Gamma_c P_A \left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (4.40)$$

Working our way back through the equations, we can see that the reduction in  $P_{0,4}$  due to the stagnation pressure ratios results in an increase of  $T_4$ , which in turn results in a decrease of the jet velocity and a decrease in thrust.

### 4.5 Scramjets

Ramjets have an upper limit for operation at high Mach numbers. At too large velocities, slowing the air down to subsonic speeds would result in a pressure ratio, and therefore temperature, that is too high. The high temperature would hinder efficient combustion, and would put too much stress on materials. In a scramjet (supersonic combustion ramjet), the combustion occurs at supersonic speeds. Because the flow needs to decelerate less, combustion can occur at lower temperatures.

Figure 4.4 shows a schematic drawing of a scramjet. Note that the diffuser is purely converging, and the nozzle is purely diverging, as required for supersonic conditions. The main challenges for operation are mixing the fuel and the air at supersonic speeds and flame stability. Due to their design, scramjets will only work at or near hypersonic velocities, and therefore require another means of propulsion to achieve the required velocity, for example a booster rocket.

The projected upper limit for scramjets is around Mach 12-15. The NASA X-43A currently holds the record for fastest aircraft with an air-breathing propulsion system, reaching Mach 9.6 using a scramjet engine. It was released from a Boeing B52 Stratofortress, together with a Pegasus rocket as a booster to reach operational speed.

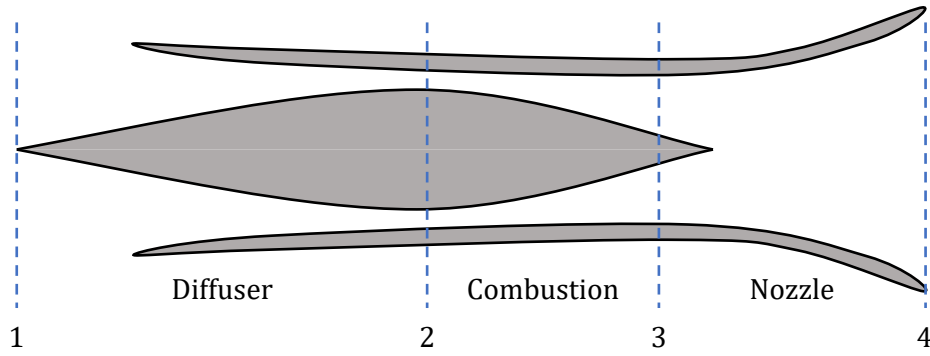


Figure 4.4: Schematic drawing of a scramjet.

## Examples

### 4.6 Ideal ramjet example question

An ideal ramjet operates at Mach 5 at an altitude of 50,000 ft. The temperature at the exit of the combustion chamber is 2000 K, burning a fuel with a lower calorific value of  $41,000 \text{ kJ kg}^{-1}$ . The ramjet generates a thrust of 10 kN. Use standard air properties  $\gamma = 1.4$  and  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ . Answer the following questions:

- Show that the stagnation-to-static pressure ratio at the ramjet inlet is equal to the stagnation-to-static pressure ratio at the nozzle exit.
- Calculate the stagnation-to-static temperature ratio at the nozzle exit, and hence the temperature at the nozzle exit.
- Calculate the flight speed and the jet speed.
- Calculate the mass flow rate of air and fuel.
- Calculate the thrust specific fuel consumption.

#### Solution

a) The stagnation-to-static pressure ratio at the inlet is

$$\frac{P_{0,1}}{P_1} \quad (4.41)$$

Because the inlet pressure is equal to the ambient pressure, and the stagnation pressure is equal to the pressure at the combustion chamber inlet we can write

$$\frac{P_{0,1}}{P_1} = \frac{P_2}{P_A} \quad (4.42)$$

The stagnation-to-static pressure ratio at the nozzle exit is

$$\frac{P_{0,4}}{P_4} \quad (4.43)$$

Because the nozzle exit is equal to the ambient pressure, and the stagnation pressure is equal to the pressure at the combustion chamber outlet, we can write

$$\frac{P_{0,4}}{P_4} = \frac{P_3}{P_A} \quad (4.44)$$



Finally, because the combustion is a constant-pressure process, we know that  $P_2 = P_3$ , so we have

$$\frac{P_{0,1}}{P_1} = \frac{P_2}{P_A} = \frac{P_3}{P_A} = \frac{P_{0,4}}{P_4} \quad (4.45)$$

**b)** Because the nozzle is isentropic, we can relate the temperature ratio to the pressure ratio

$$\frac{T_{0,4}}{T_4} = \left( \frac{P_{0,4}}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{P_{0,1}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \quad (4.46)$$

Using the same isentropic relation, we can equally relate the stagnation-to-static pressure ratio at the diffuser inlet to the temperature ratio

$$\frac{T_{0,4}}{T_4} = \left( \frac{P_{0,4}}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{P_{0,1}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{0,1}}{T_1} \quad (4.47)$$

Because we know the Mach number at the diffuser inlet, we have a direct equation for the stagnation-to-static temperature ratio

$$\frac{T_{0,4}}{T_4} = \frac{T_{0,1}}{T_1} = 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \quad (4.48)$$

so we can calculate the ratio

$$\frac{T_{0,4}}{T_4} = 1 + \frac{1.4-1}{2} 5^2 = 6.0 \quad (4.49)$$

The stagnation temperature  $T_{0,4}$  at the nozzle exit is equal to the temperature  $T_3$  at the combustion chamber outlet, so we can calculate the temperature at the nozzle exit

$$T_4 = T_3 \times \frac{T_4}{T_{0,4}} = 2000/6 = 333.33 \text{ K} \quad (4.50)$$

**c)** To obtain the flight speed, we first need to know the speed of sound at 50,000 ft. To calculate the speed of sound, we need to know the local temperature, which we can obtain from the altitude tables. At 50,000 ft, we find

$$\frac{T}{T_{\text{std}}} = 0.7519 \quad (4.51)$$

with  $T_{\text{std}} = 288.15 \text{ K}$ , we have

$$T_1 = 0.7519 \times 288.15 = 216.66 \text{ K} \quad (4.52)$$

So we get the speed of sound

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 216.66} = 295.05 \text{ m/s} \quad (4.53)$$

and thus the flight speed

$$V = a_1 \times \text{Ma}_1 = 295.05 \times 5 = 1475.25 \text{ m/s} \quad (4.54)$$

The jet speed we obtain by solving the SFEE for the nozzle

$$0 = h_4 + \frac{1}{2} V_4^2 - h_3 \quad (4.55)$$

$$0 = c_P (T_4 - T_3) + \frac{1}{2} V_4^2 \quad (4.56)$$

with  $V_4 = V_j$  and  $c_P = \gamma R / (\gamma - 1) = 1004.5 \text{ J kg}^{-1} \text{ K}^{-1}$  we have

$$V_j = \sqrt{2 c_P (T_3 - T_4)} = \sqrt{2 \times 1004.5 \times (2000 - 333.33)} = 1829.85 \text{ m/s} \quad (4.57)$$

**d)** In order to find the mass flow rate of air and fuel, we first calculate the temperature at the inlet of the combustion chamber  $T_2$ :

$$T_2 = T_1 \left( 1 + \frac{\gamma - 1}{2} \text{Ma}_1^2 \right) = 216.66 \times \left( 1 + \frac{1.4 - 1}{2} 5^2 \right) = 1299.96 \text{ K} \quad (4.58)$$

We can use  $T_2$  to calculate the fuel-air-ratio

$$f = \frac{T_3 - T_2}{\frac{LCV}{c_p} - T_3} = \frac{2000 - 1299.96}{\frac{4.1 \times 10^7}{1004.5} - 2000} = 0.01803 \quad (4.59)$$

Using our equation for specific thrust

$$\frac{F}{\dot{m}_a} = (1 + f) V_j - V \quad (4.60)$$

we can solve for the mass flow rate of air

$$\dot{m}_a = \frac{F}{(1 + f) V_j - V} = \frac{10,000}{(1 + 0.018) 1829.85 - 1475.25} = 25.80 \text{ kg/s} \quad (4.61)$$

and the mass flow rate of fuel

$$\dot{m}_f = f \times \dot{m}_a = 0.01803 \times 25.8 = 0.465 \text{ kg/s} \quad (4.62)$$

e) Using the definition of the thrust specific fuel consumption

$$TSFC = \frac{\dot{m}_f}{F} = \frac{0.47}{10,000} = 4.65 \times 10^{-5} \text{ kg s}^{-1} \text{ N}^{-1} \quad (4.63)$$