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SESA2025 Mechanics of Flight
Longitudinal aerodynamic derivatives

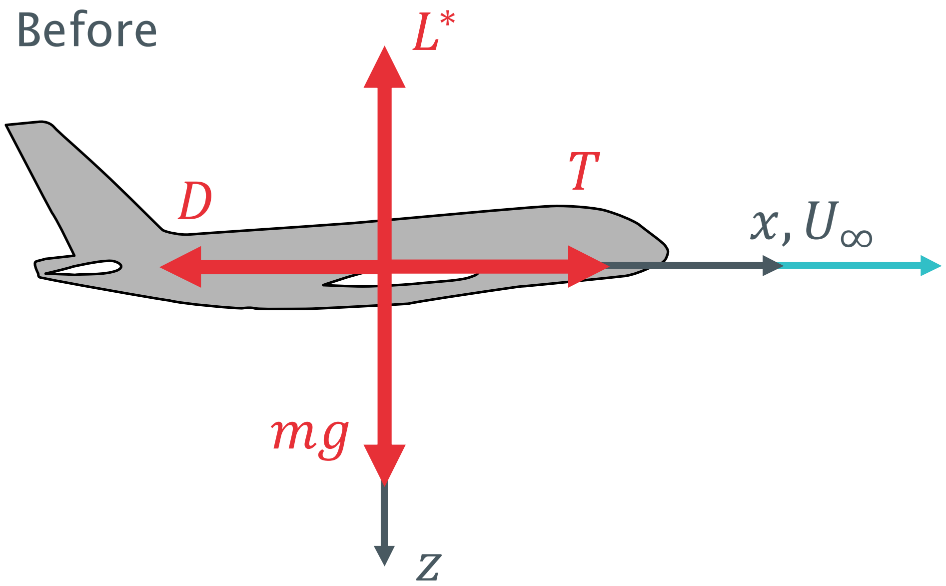
Lecture 4.1

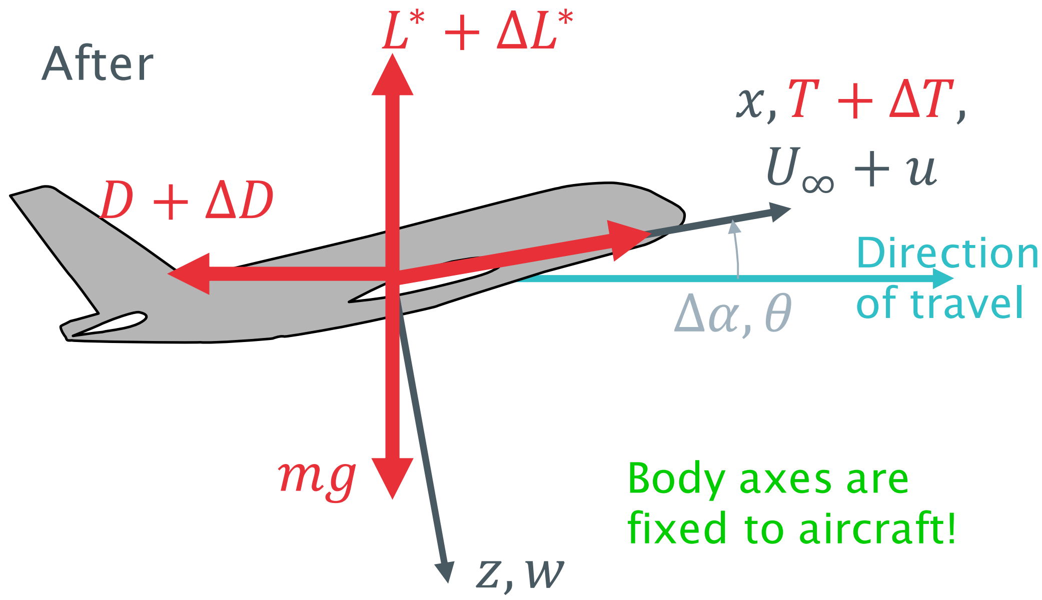
Aerodynamic derivative estimates

- Derive simple estimates of key derivatives
  - This lecture: longitudinal
  - Next lecture: lateral
- Improve physical understanding of aircraft motions
- Relates the dynamics back to aircraft design features

Longitudinal perturbation

Consider a change in pitch, thrust and drag, but with direction of motion unchanged (and $q = 0$, $\dot{w} = 0$)





Assume small angles: $\theta = \Delta\alpha = \frac{w}{U_\infty}$

$$\Delta(U_\infty^2) = (U_\infty + u)^2 + w^2 - U_\infty^2 =_1 2uU_\infty$$

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Changes in thrust

In general we can take

$$\frac{\partial}{\partial U}$$

$$T = c_t U^{\Lambda}$$

$$\frac{\partial T}{\partial U} = c_t \Lambda U^{\Lambda - 1} = \frac{c_t \Lambda U^{\Lambda}}{U}$$

$$\frac{\partial T}{\partial U} = \frac{\Lambda T}{U} = \frac{\Lambda D}{U}$$

So finally

$$\Delta T = \frac{\partial T}{\partial U} u$$

$$\Delta T = \Lambda D \frac{u}{U_\infty}$$

velocity independent

For a jet engine: $T = const.$, so $\Lambda = 0$

For a piston engine: $TU = const.$, so $\Lambda = -1$

at a set power setting, if U increases thrust decreases

Changes in drag and lift

these change

First let's look at drag:

$$\Delta D = \Delta\left(\boxed{C_D}\tfrac{1}{2}\rho \boxed{U_\infty^2} S\right)$$

taylor series

$$\Delta D = C_D \tfrac{1}{2}\rho S \Delta(U_\infty^2) + \tfrac{1}{2}\rho U_\infty^2 S \Delta(C_D)$$

$$\Delta D = \tfrac{1}{2}\rho U_\infty^2 S\left(C_D \frac{2u}{U_\infty} + C_{D_\alpha}\frac{w}{U_\infty}\right)$$

Similarly for lift:

$$\Delta L^* = \Delta\left(C_{L^*}\tfrac{1}{2}\rho U_\infty^2 S\right)$$

$$\Delta L^* = \tfrac{1}{2}\rho U_\infty^2 S\left(C_{L^*}\frac{2u}{U_\infty} + C_{L^*_\alpha}\frac{w}{U_\infty}\right)$$

α changes $\rightarrow$ C_L changes $\downarrow$ C_D changes

Recall, small angles: $\Delta\alpha = \frac{w}{U_\infty}$ and $\Delta(U_\infty^2) =_1 2uU_\infty$

Recall the derivative definitions

Dimensional

$$\Delta X_a = \mathring{X}_u u + \mathring{X}_w w + \ldots$$

$$\Delta Z_a = \mathring{Z}_u u + \mathring{Z}_w w + \ldots$$

Dimensionless

$$\frac{\Delta X_a}{\frac{1}{2}\rho U_\infty^2 S} = X_u \left(\frac{u}{U_\infty}\right) + X_w \left(\frac{w}{U_\infty}\right) + \ldots$$

$$\frac{\Delta Z_a}{\frac{1}{2}\rho U_\infty^2 S} = Z_u \left(\frac{u}{U_\infty}\right) + Z_w \left(\frac{w}{U_\infty}\right) + \ldots$$

Out-of-balance forces in x-direction

$$\frac{\Delta X_a}{\frac{1}{2}\rho U_\infty^2 S} = X_u\left(\frac{u}{U_\infty}\right) + X_w\left(\frac{w}{U_\infty}\right) + \dots$$

Take the balance from the change in forces (after–before):

$$\Delta X_a = \Delta T - \Delta D + L^* \frac{w}{U_\infty}$$

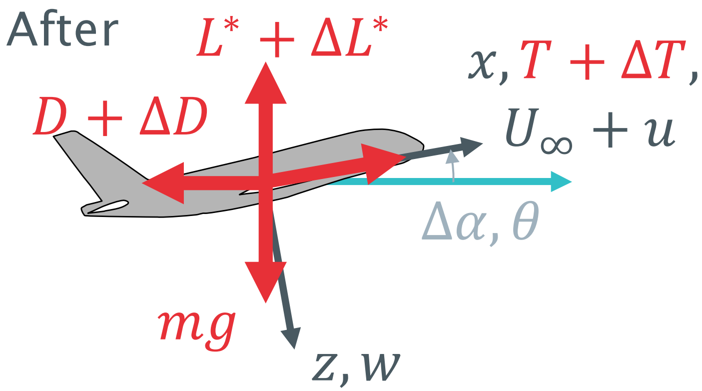
derived from a bunch of ommitted maths

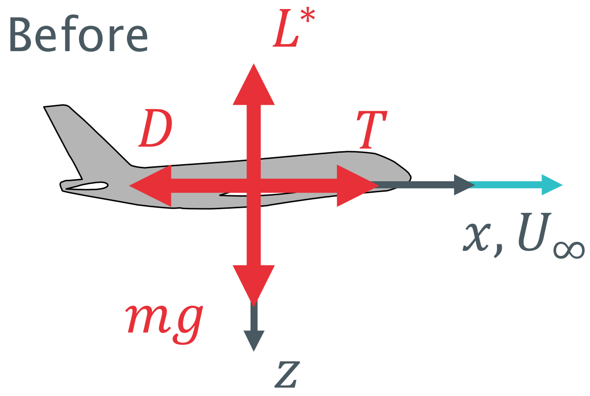
$$\frac{\Delta X_a}{\frac{1}{2}\rho U_\infty^2 S} = \Lambda C_D \frac{u}{U_\infty} - \left(C_D \frac{2u}{U_\infty} + C_{D_\alpha} \frac{w}{U_\infty}\right) + C_{L^*} \frac{w}{U_\infty}$$

Collect all terms and compare with the definition to obtain:

$$X_u = (\Lambda - 2) C_D$$

$$X_w = C_{L^*} - C_{D_\alpha}$$





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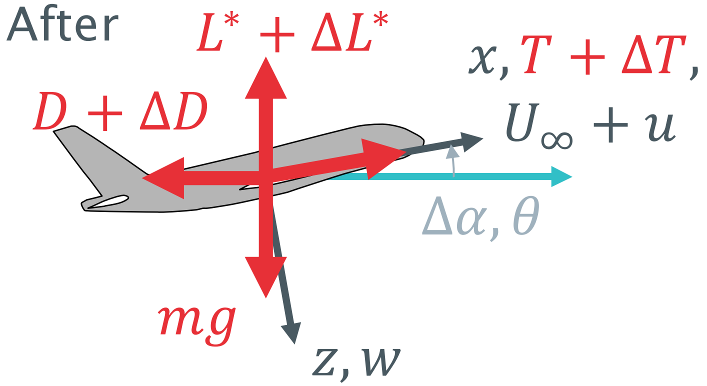
Out-of-balance forces in z-direction

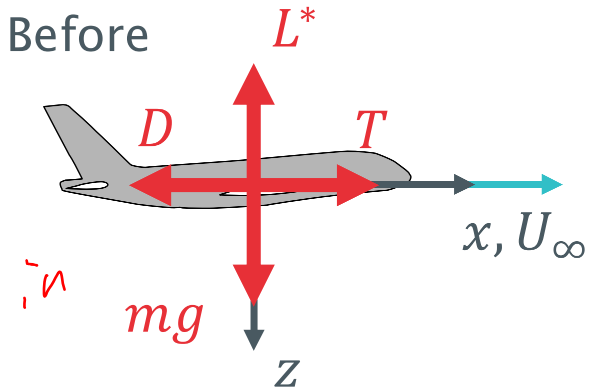


$$\frac{\Delta Z_a}{\frac{1}{2}\rho U_\infty^2 S} = Z_u\left(\frac{u}{U_\infty}\right) + Z_w\left(\frac{w}{U_\infty}\right) + \dots$$

Take the balance from the change in forces (after–before):

$$\Delta Z_a = -\Delta L^* - D\frac{w}{U_\infty}$$

$$\frac{\Delta Z_a}{\frac{1}{2}\rho U_\infty^2 S} = -\left(C_{L^*}\frac{2u}{U_\infty} + C_{L_\alpha^*}\frac{w}{U_\infty}\right) - C_D\frac{w}{U_\infty}$$



Collect all terms and compare with the definition to obtain:

$$Z_u = -2C_{L^*}$$

increase in speed leads to inc in lift (makes sense)

$$Z_w = -(C_{L_\alpha^*} + C_D)$$

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Moment Derivatives

Starting from the pitching moment equation (Lecture 2.1 from Part A) it can be shown that:

$M_w \propto -H_s C_{L_\alpha^*}$ where H_s is the static margin

$M_u = 0$ (which is a good approximation at low Mach numbers)

mach tuch

M_q Pitch damping derivative due to change in tailplane angle of attack changes with pitch rate (see Lecture 2.2 from Part A)

$$\Delta\alpha_{T_q} \approx \frac{ql}{U_\infty}$$