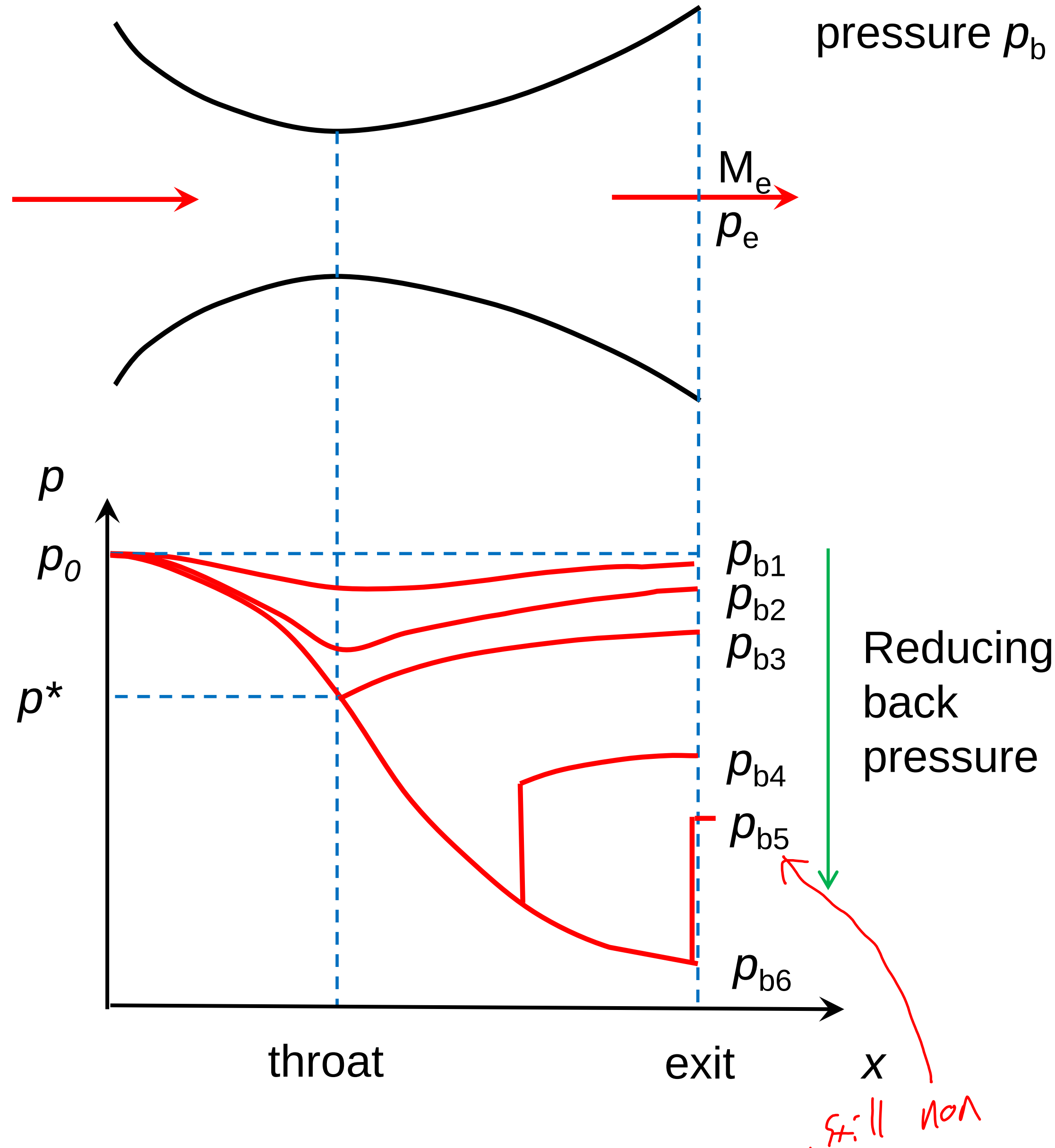
SESA3029 Aerothermodynamics

Lecture 2.7 Under/over-expanded nozzles and supersonic wind tunnels

Effect of back pressure

Environment is at 'back' pressure p_b

Reservoir conditions p_0, T_0



p_{b1} – low subsonic throughout (effectively incompressible)

p_{b2} –subsonic throughout (compressible)

p_{b3} –sonic at throat, subsonic in diverging section

p_{b4} –shock in diverging section

p_{b5} –shock at exit

p_{b6} –design condition (supersonic exit)

Example

For a Laval nozzle with a design exit Mach number of M_e=2.2 find p_{b6}, p_{b3} and p_{b5} in terms of p_0

Isentropic-flow table (γ = 1.4):

M	p/p_0	ρ/ρ_0	T/T_0	ν (deg.)	A/A^*
2.2000	0.0935	0.1841	0.5081	31.7325	2.0050
2.2200	0.0906	0.1800	0.5036	32.2494	2.0409
2.2400	0.0878	0.1760	0.4991	32.7629	2.0777
2.2600	0.0851	0.1721	0.4947	33.2730	2.1153
2.2800	0.0825	0.1683	0.4903	33.7796	2.1538

Design condition M_e=2.2, A_e/A^*=2.005 and p_{b6}/p_0=0.0935

For same area ratio there is a subsonic solution

Isentropic-flow table (γ = 1.4):					
M	p/p_0	ρ/ρ_0	T/T_0	ν (deg.)	A/A^*
0.3000	0.9395	0.9564	0.9823	n/a	2.0351
0.3200	0.9315	0.9506	0.9799	n/a	1.9219
0.3400	0.9231	0.9445	0.9774	n/a	1.8229
0.3600	0.9143	0.9380	0.9747	n/a	1.7358
0.3800	0.9052	0.9313	0.9719	n/a	1.6587

Linear interpolation for $A_e/A^*=2.005$ gives $M_e=0.305$ and $p_{b3}/p_0=0.937$

To get p_{b5} we add a normal shock at the exit i.e. from M=2.2

NST for M=2.2 gives $p_{b5}/p_{b6}=5.480$

$p_{b5}=p_{b6} \times p_{b5}/p_{b6} = 0.0935p_0 \times 5.480 = 0.512p_0$

Flow regimes

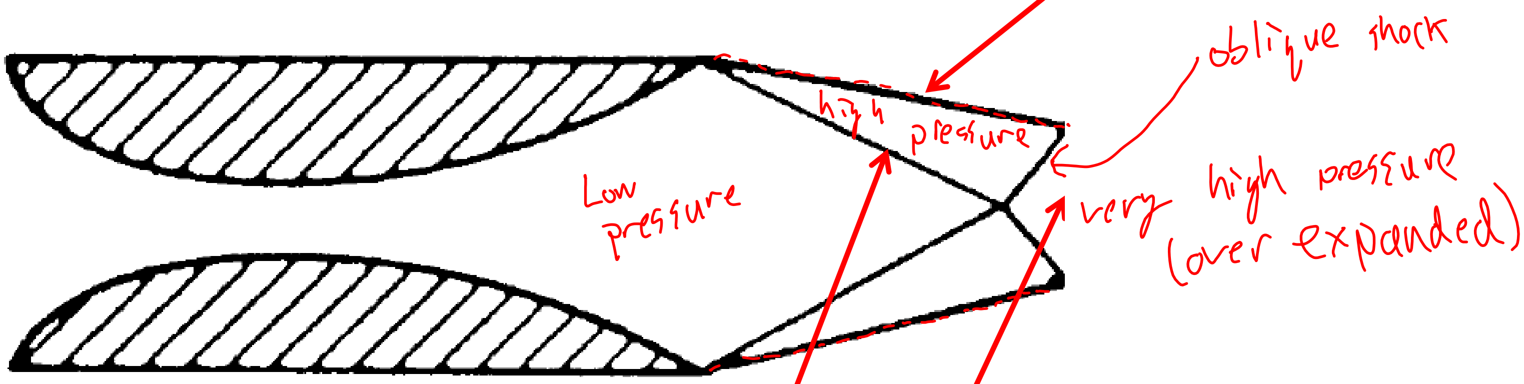
trash

- $p_b > p_{b3}$ Subsonic flow
- $p_{b5} < p_b < p_{b3}$ Shock wave in diverging section
- $p_b < p_{b5}$ Supersonic flow at exit
- $p_{b6} < p_b < p_{b5}$ Flow needs to recompress after exit to get to ambient conditions (‘over-expanded’ case)
- $p_b < p_{b6}$ Flow needs to expand further to get to ambient conditions (‘under-expanded’ case)

How does the adjustment in pressure occur?

Over-expanded nozzle/jet

Slip line (edge of jet fluid)

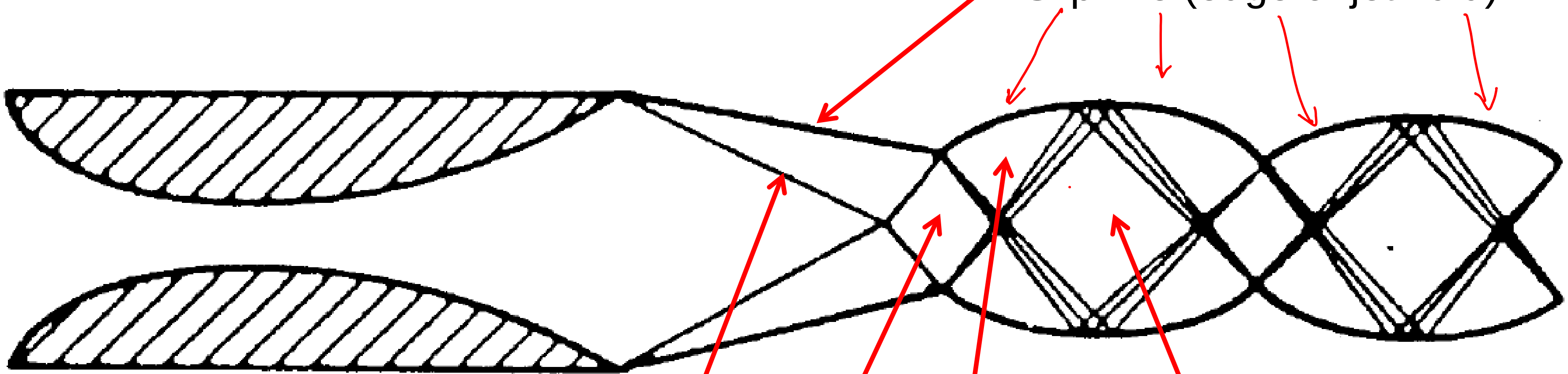


Flow needs to recompress, so an oblique shock forms, with turning angle so that pressure is continuous across the slip line

Shocks cross. Now the pressure in this region is too high

Over-expanded nozzle/jet

Slip line (edge of jet fluid)



Flow needs to recompress, so an oblique shock forms, with turning angle so that pressure is continuous across the slip line

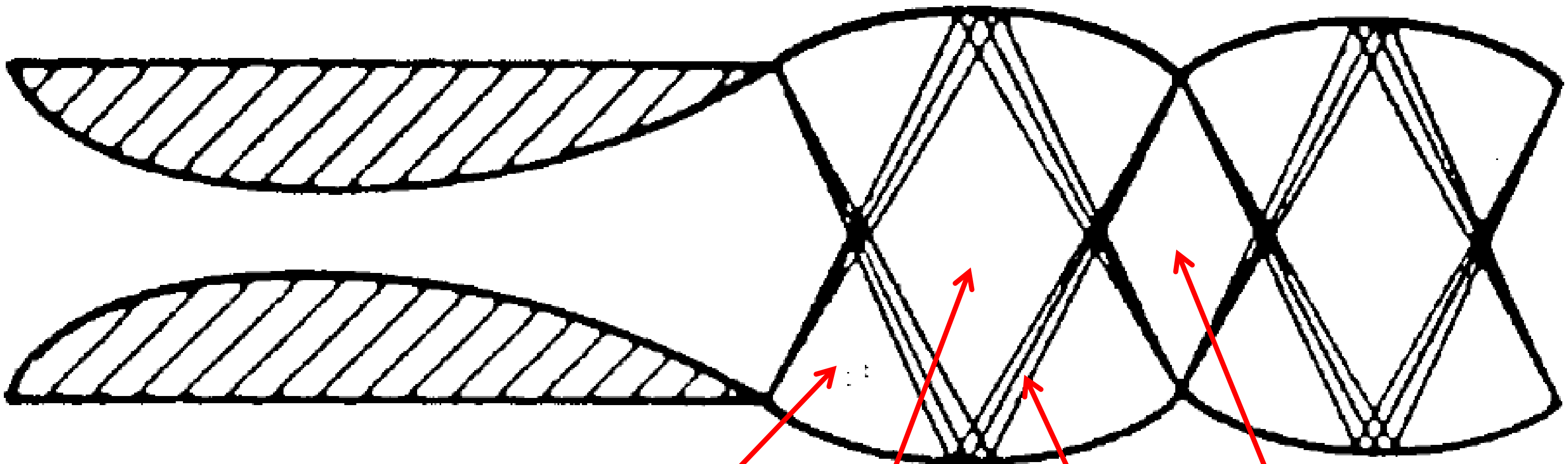
Expansion fan forms so the pressure matches the ambient in this region

Expansion waves cross – now the pressure is too low and the flow must recompress.

Shocks cross. Now the pressure in this region is too high

Back where we started. The expansion waves reflect from the slip line as compression waves.

Under-expanded nozzle/jet



Expansion fan forms so the pressure matches the ambient in this region

Expansion waves cross – now the pressure is too low and the flow must recompress.

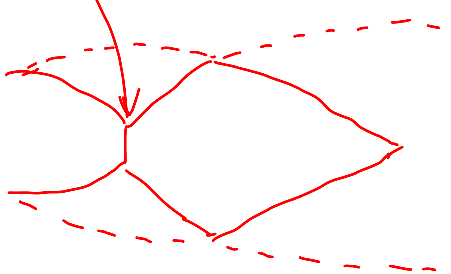
The expansion waves reflect from the slip line as compression waves.

Shocks cross. Back where we started - pattern can continue downstream

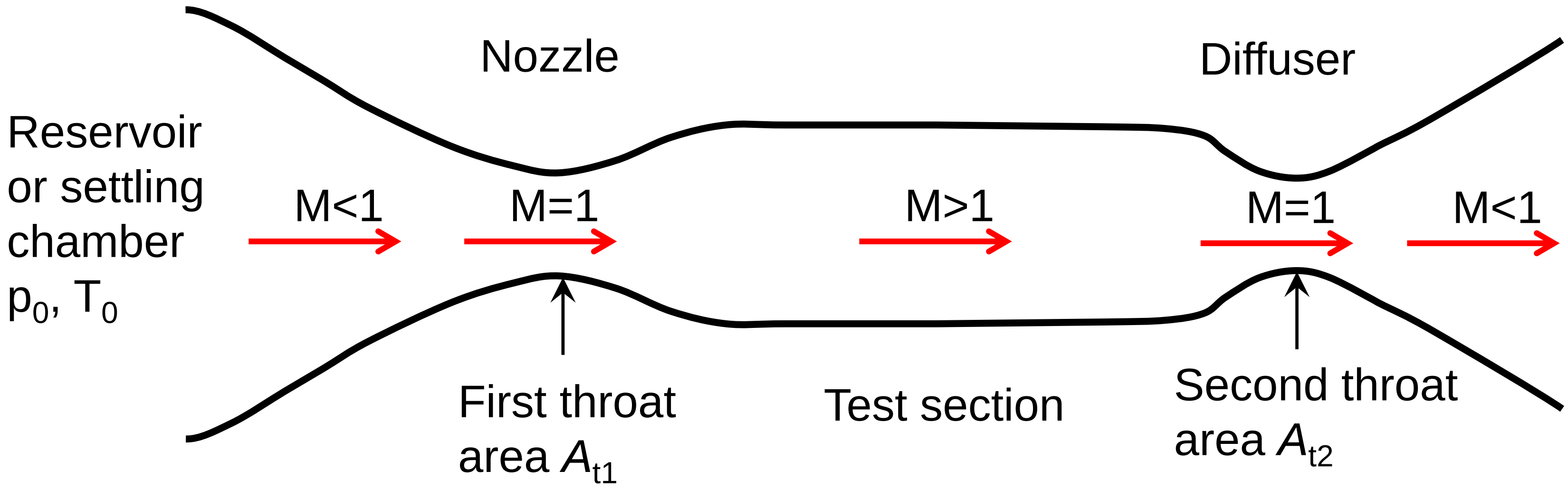
Notes

- After the first cycle over- and under-expanded jets follow the same mechanism.
- Slip line follows a barrel shape
- Shock waves follow a diamond pattern
- Turning angle may be too great to get a regular shock crossing. go get normal mach stem type mechanism
  - Instead we can get a Mach disk.

look at irl image



Supersonic wind tunnel: ideal configuration



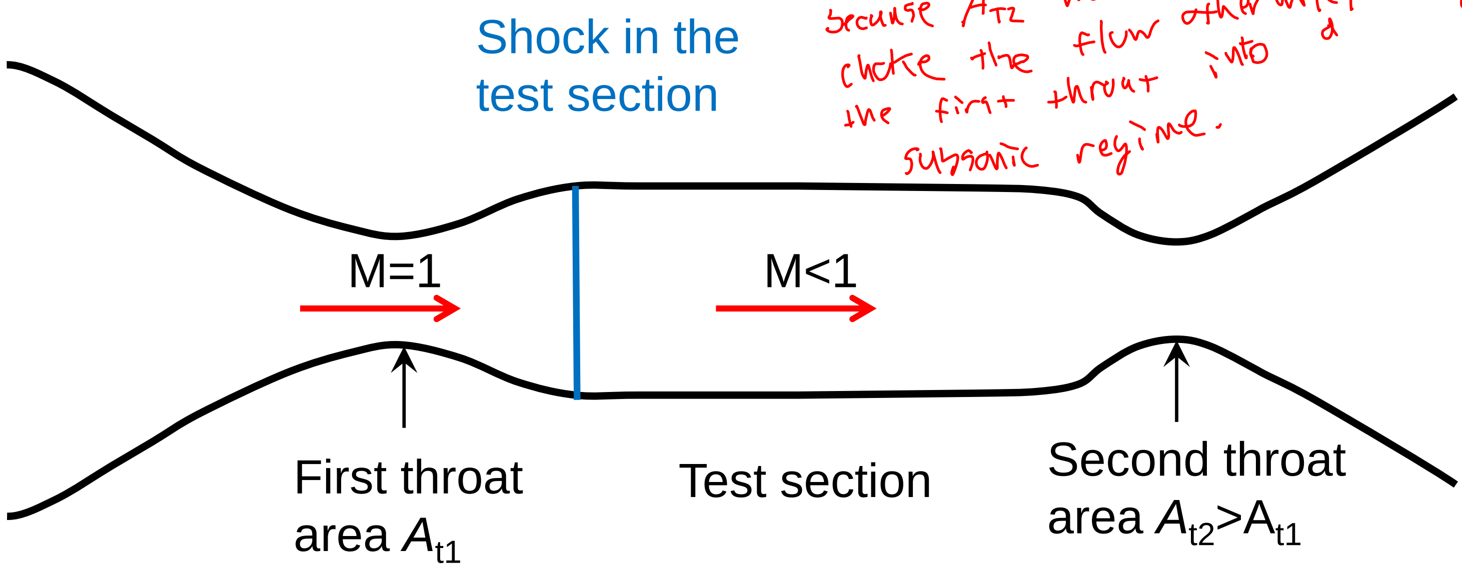
An efficient diffuser section is used to get the M>1 test section for a lower stagnation pressure compared to an open jet configuration

Starting issues

turning it on

If $A_{t2}<A_{t1}$ we will never get supersonic flow in the test section. Therefore consider only $A_{t2}>A_{t1}$

because A_{t2} would choke the flow otherwise, forcing the first throat into a subsonic regime.



If A_{t2} is too small we have a problem starting this configuration – the second throat must be large enough to accommodate the mass flow with a shock in position as shown

It is also possible that the first gets choked, to prevent that we increase mass flow rate.

Example

For an M=3 wind tunnel, find A_{t2}/A_{t1} sufficient to swallow a shock sitting in the test section.

Isentropic-flow table (γ = 1.4):					
M	p/p_0	ρ/ρ_0	T/T_0	ν (deg.)	A/A^*
3.0000	0.0272	0.0762	0.3571	49.7573	4.2346

For M=3 in test section we know (IFT) A_{test}/A_{t1}=4.23

Normal-shock table (γ = 1.4):						
M_{n1}	M_{n2}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{02}/p_{01}	"p_{02}/p_1"
3.0000	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610

A normal shock in the test section would have (NST) M_2=0.475 and p_{02}/p_{01}=0.3283

Recall the result for mass flow rate

$$\dot{m} = \frac{p_0}{\sqrt{RT_0}} A^* \sqrt{\gamma} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Stagnation conditions — Throat area — Gas properties

If p_0 drops we need a bigger A to let the mass flow pass through the second throat at the required rate

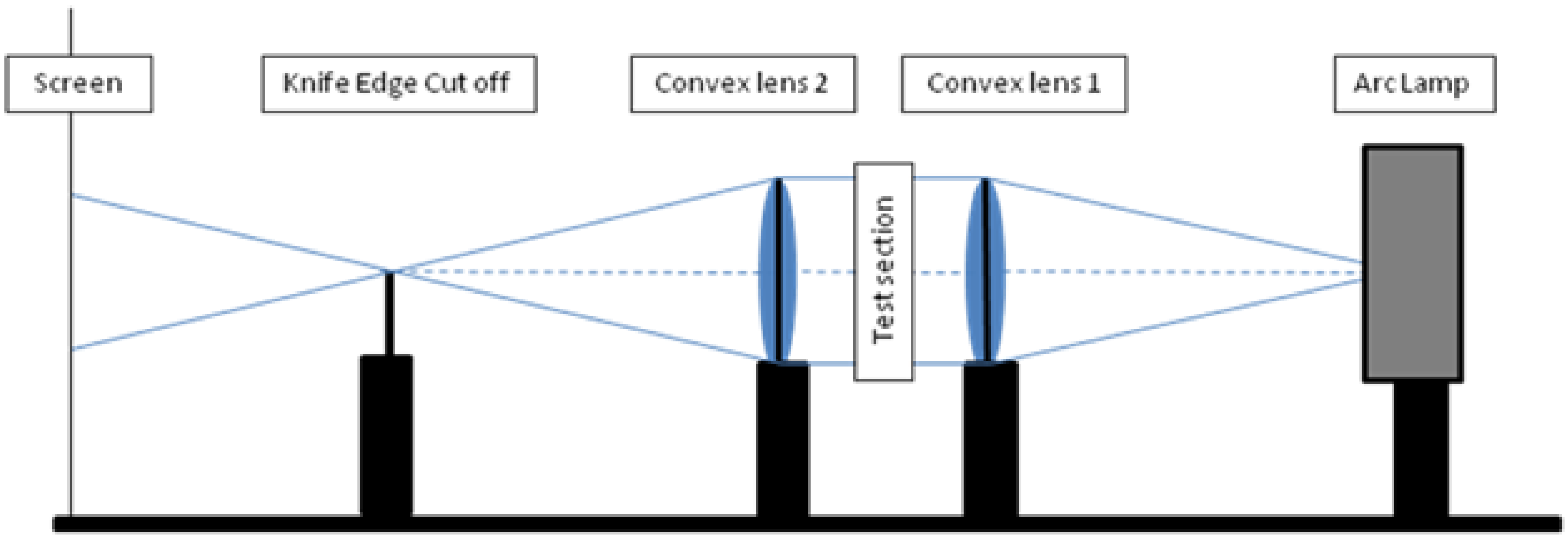
In this case we need $A_{t2}/A_{t1} > p_{01}/p_{02} = 1/0.3283 = 3.046$

For slightly higher A_{t2} the shock gets 'swallowed' by the second throat, leaving supersonic flow in the test section

After we have established supersonic flow we would ideally reduce A_{t2} to make the diffuser more efficient

Flow visualisation

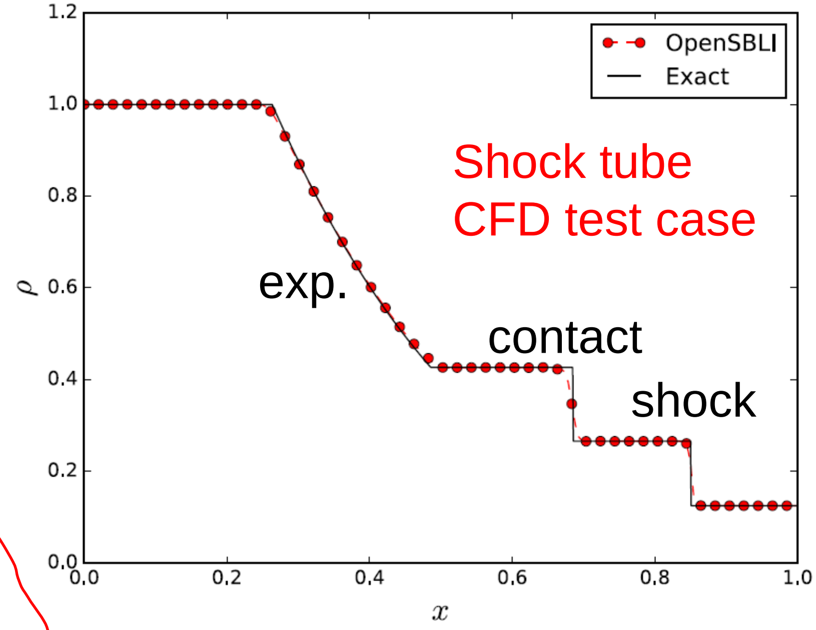
- Light is refracted by density variations
  - Shadowgraph: shine a parallel beam of light though test section onto screen/camera
    - Image intensity proportional to second derivative of density annoying
  - Schlieren: use lenses to focus light beam onto a knife edge that cuts refracted light
    - Image intensity proportional to density gradient. often preferred



Ludwieg tube

- Shock tube = long straight tube with diaphragm separating high pressure gas from low pressure (or vacuum)
- Bursting the diaphragm leads to a shock wave and contact discontinuity moving from left to right and an expansion fan moving right to left
- Add a ~~con-di~~ diverging nozzle to accelerate the flow behind the contact discontinuity to supersonic = relatively inexpensive supersonic/hypersonic wind tunnel
- Short duration (of order 0.1s): can run until the expansion wave reflects back from the end of the pressurised tube

(Add image of pressure wave with time)



experiment TU Braunschweig

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