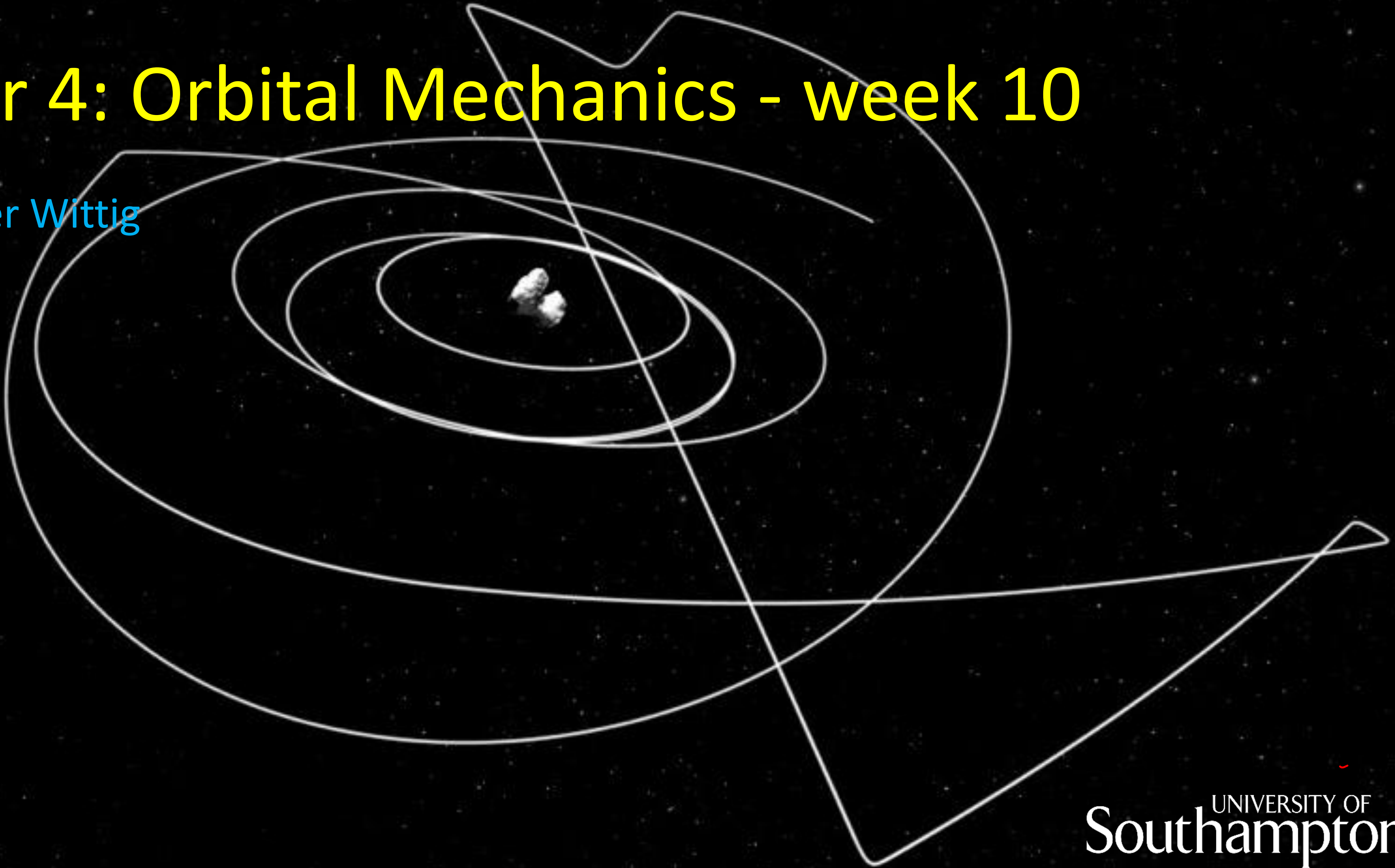
Advanced Astronautics (SESA3039)

Rosetta lander deployment orbit (ESA)



Chapter 4: Orbital Mechanics - week 10

Dr. Alexander Wittig

Southampton AERO ASTRO

UNIVERSITY OF Southampton

Chapter 4 Roadmap

- Week 8: Orbital Motion
  - Math Basics
  - Spherical Trigonometry
  - Keplerian Motion from First Principles
- Week 9: Orbit Properties
  - Constants of Motion
  - Eccentricity Vector
  - Conic Sections
- **Week 10: Orbit Representation**
  - Coordinates
  - Dates & Times
  - Orbital Elements
- Week 11: Time Dependence
  - Eccentric Anomaly
  - Hyperbolic Anomaly
  - Kepler’s Equations

Questions

- Any questions on previous weeks content?

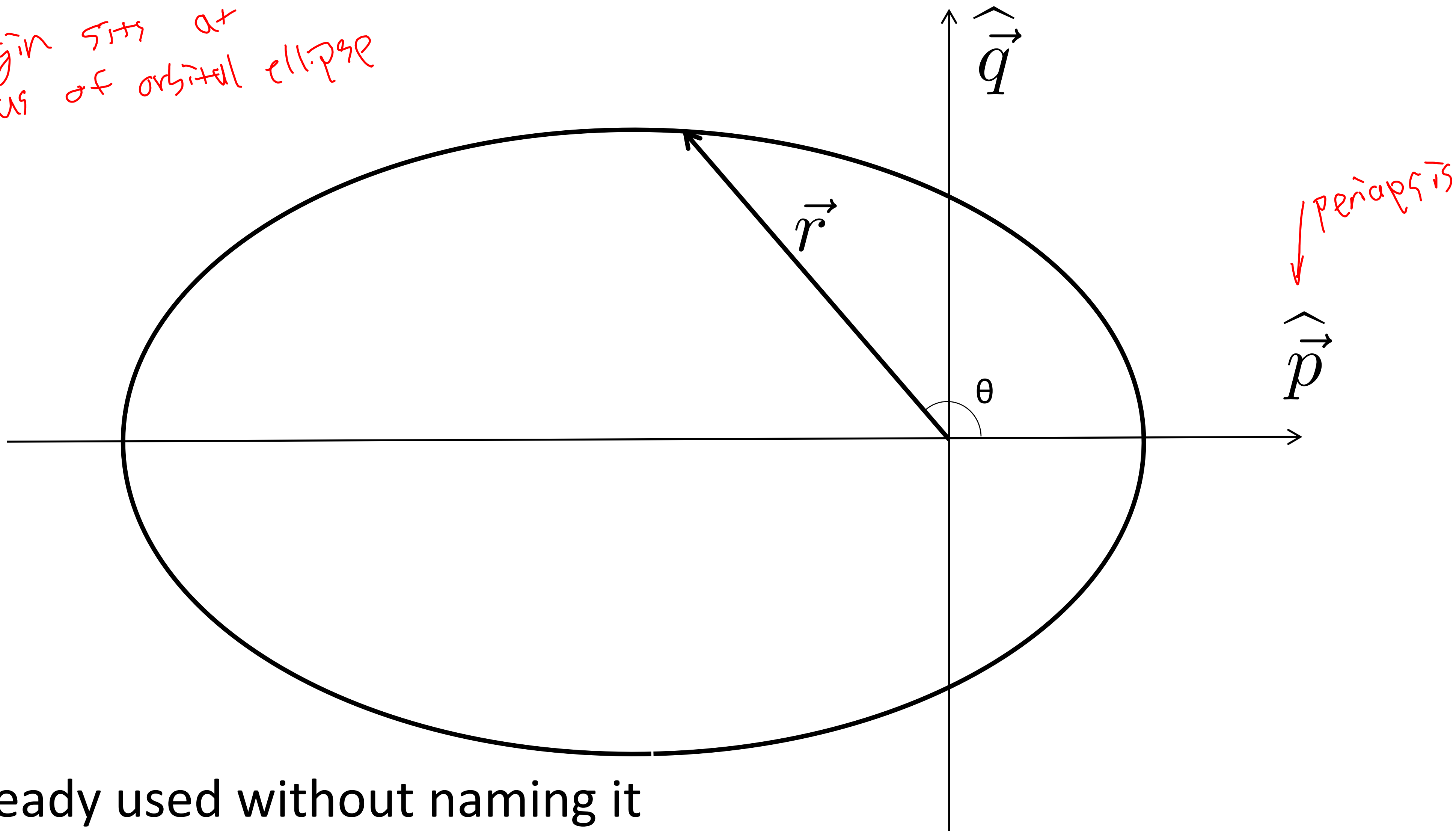
Coordinate Systems

Learning outcomes

- Ability to understand, apply, and convert coordinate systems used in astrodynamics
- Define orbit representation and orientation
- Choose appropriate of coordinate frame for different problems
- Convert between coordinate systems

Perifocal Coordinate Frame

origin sits at
focus of orbital ellipse

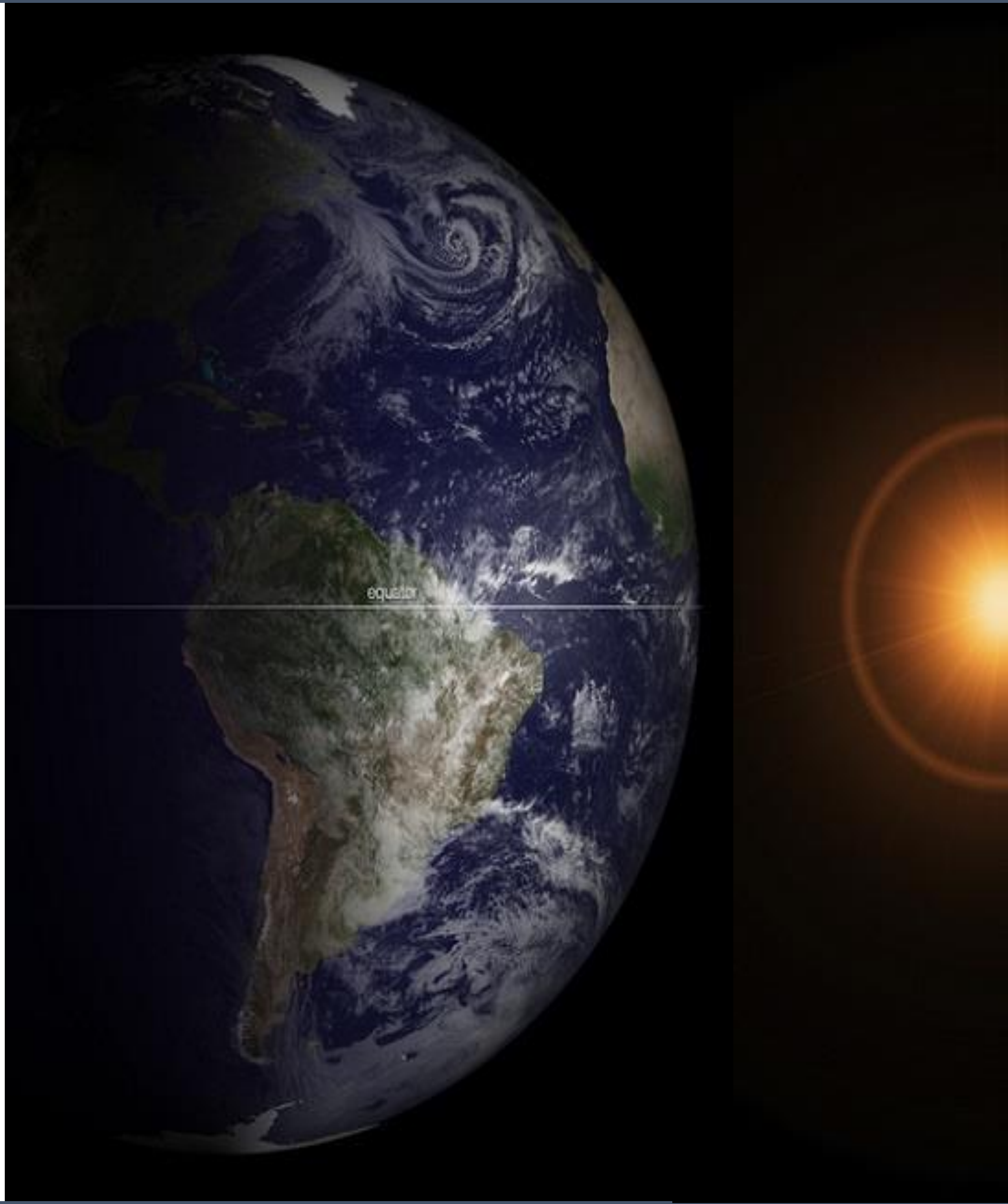


- Already used without naming it
- Abstract frame: no connection to physical world

Physical coordinate system

- Connect abstract mathematical system to physical world
- Need to define reference directions
- Requirements

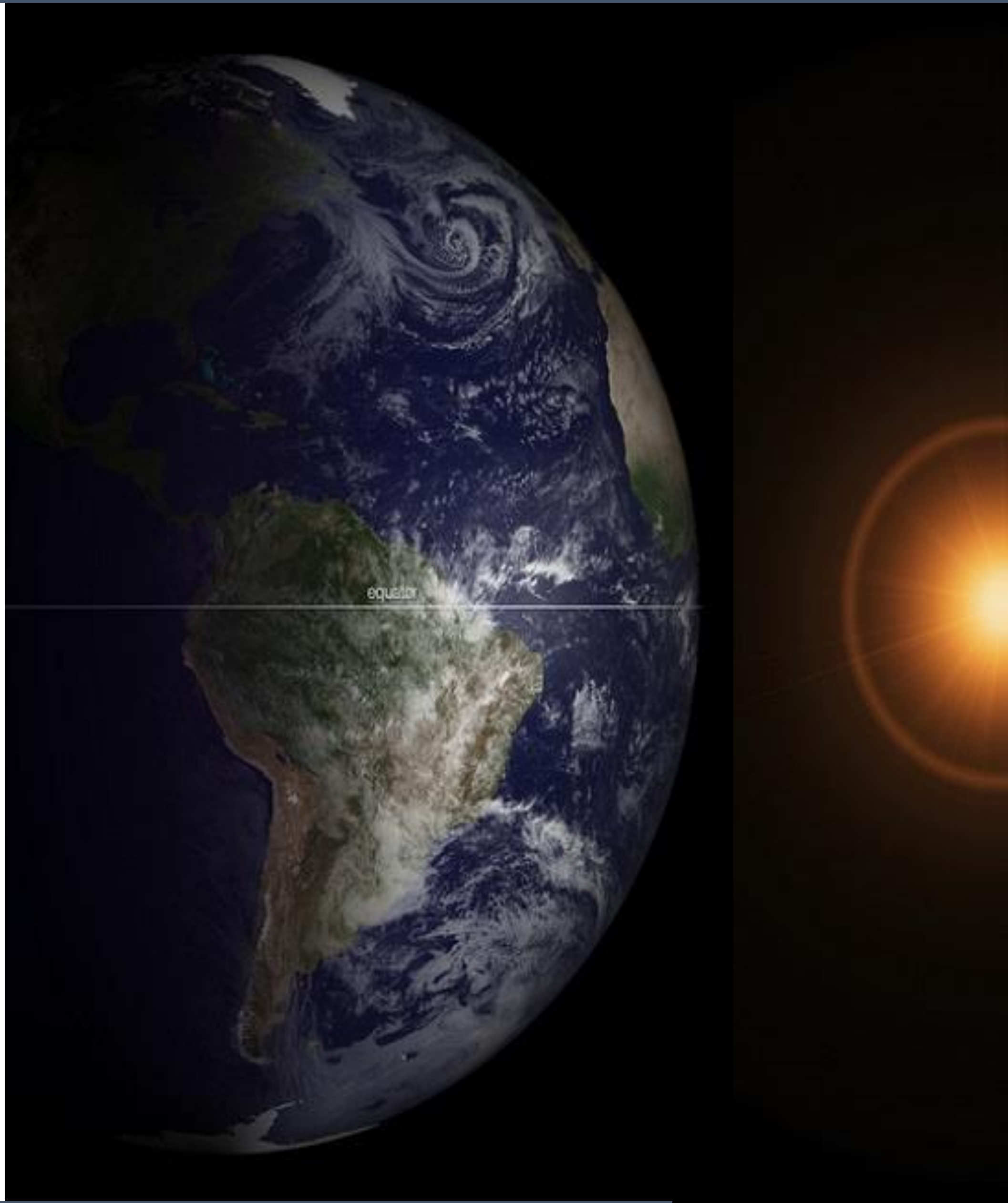
???



Physical coordinate system

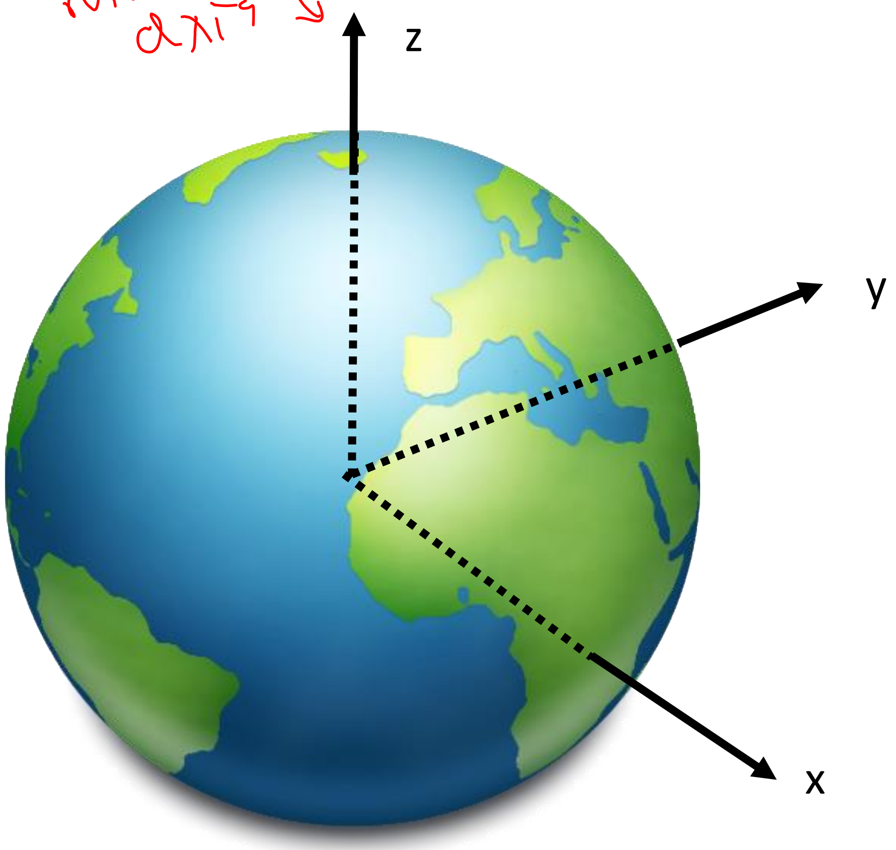
- Connect abstract mathematical system to physical world
- Need to define reference directions
- Requirements:
  - Measurable for all → can be easily determined within problems scope
  - Fixed in time

fixed (in problems scope) we want it fixed to some refrence point



First attempt: Earth Centered Earth Fixed

→ axis rotate with earth



perspective would suck for satellites, though geosatellites could use it.

observations onto earth would be helped by this system.

if perspective is on surface:

- useful since ground is fixed to axis

if perspective is outside:

- horrible

Icons: Delikate Icons, kyo-tux, Oxygen icon set

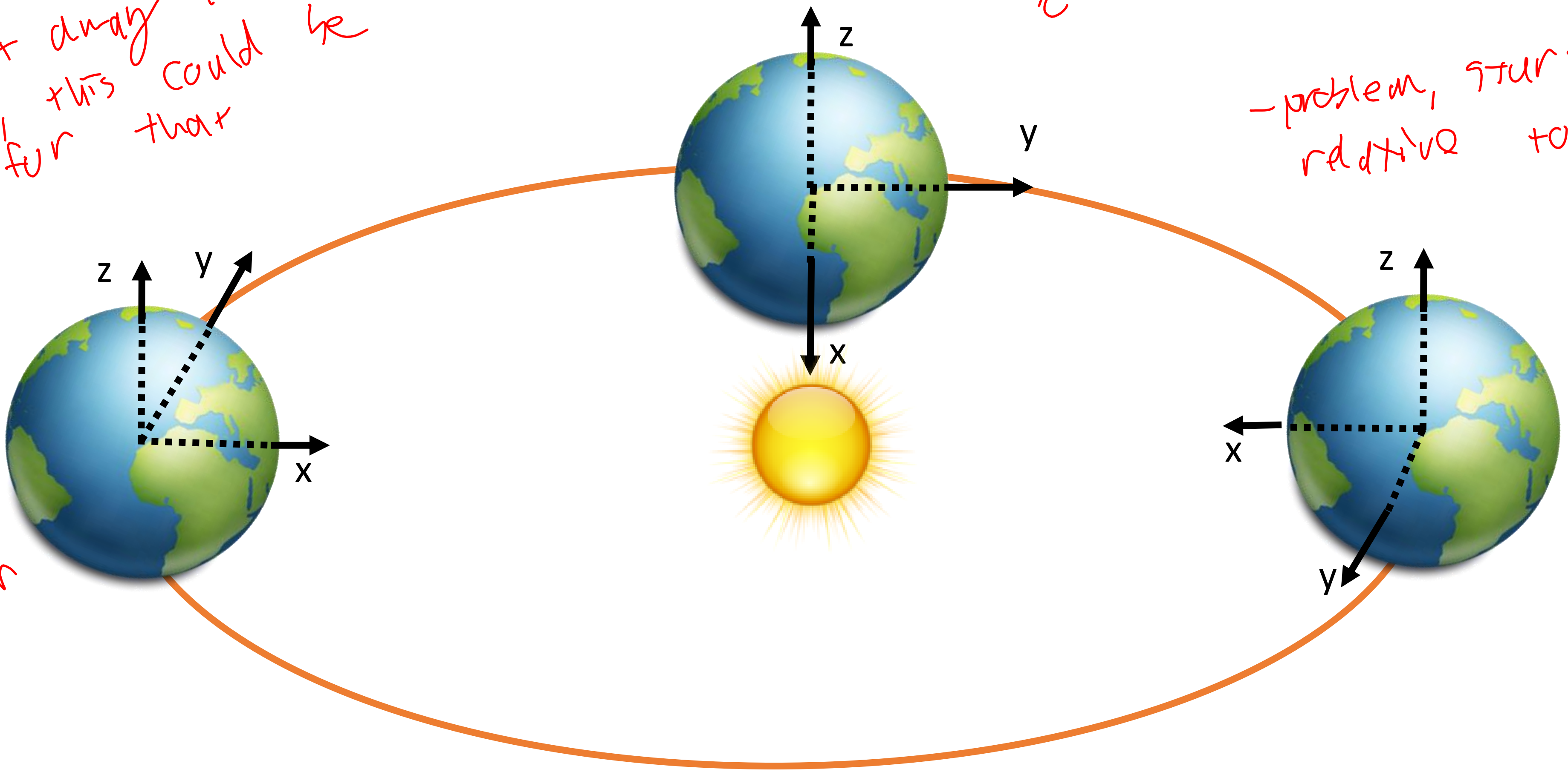
Earth Centered Sun Pointing

Second attempt: Earth Centered Sun Pointing

slower rotation of z axis (365x slower)

Telescope missions must point away from the sun, this could be used for that

- problem, stars move relative to coordinates



Solar panels could benefit from this

Icons: Delikate Icons, kyo-tux, Oxygen icon set

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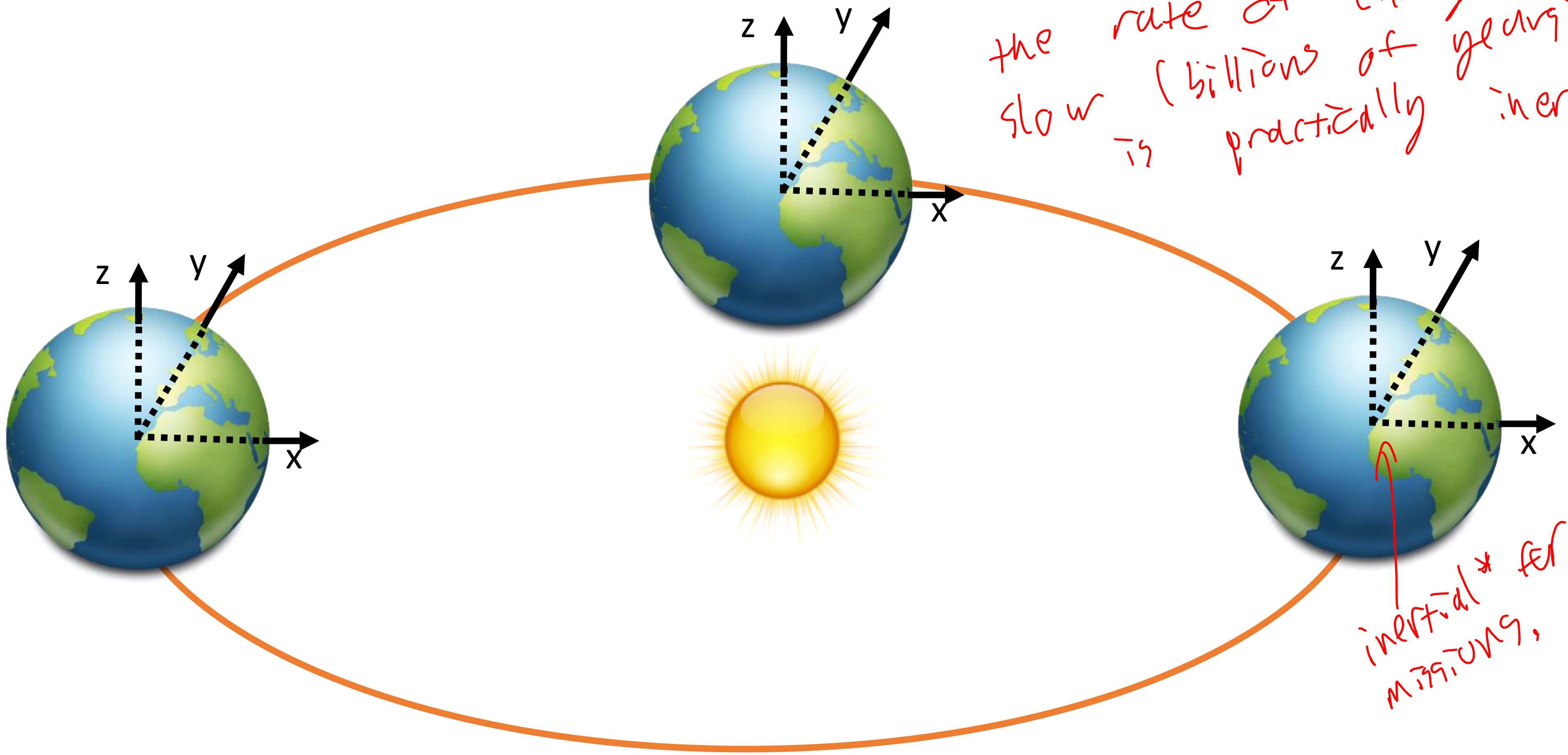
Third attempt: Earth Centered Inertial

x, y, z dir? are directionally fixed on really really far stars.

the rate of change is so slow (billions of years) so is practically inertial.



inertial* for earth orbit missions,

Icons: Delikate Icons, kyo-tux, Oxygen icon set

Fixing directions in (nearly empty) space

- How to define fixed directions if not using Sun?
- Requirement: must be determinable from everywhere!

How are other physical constants defined?

Fixing directions in (nearly empty) space

- How to define fixed directions if not using Sun?
- Requirement: must be determinable from everywhere!

How are other physical constants defined?

Examples:

  - 1 sec = 9,192,631,770 hyperfine structure transitions of caesium-133
  - 1 m = distance light in vacuum travels in 1/ 299,792,458 sec
  - Bad: 1 kg = mass of prototype kg in Paris

Fixing directions in (nearly empty) space

- How to define fixed directions if not using Sun?
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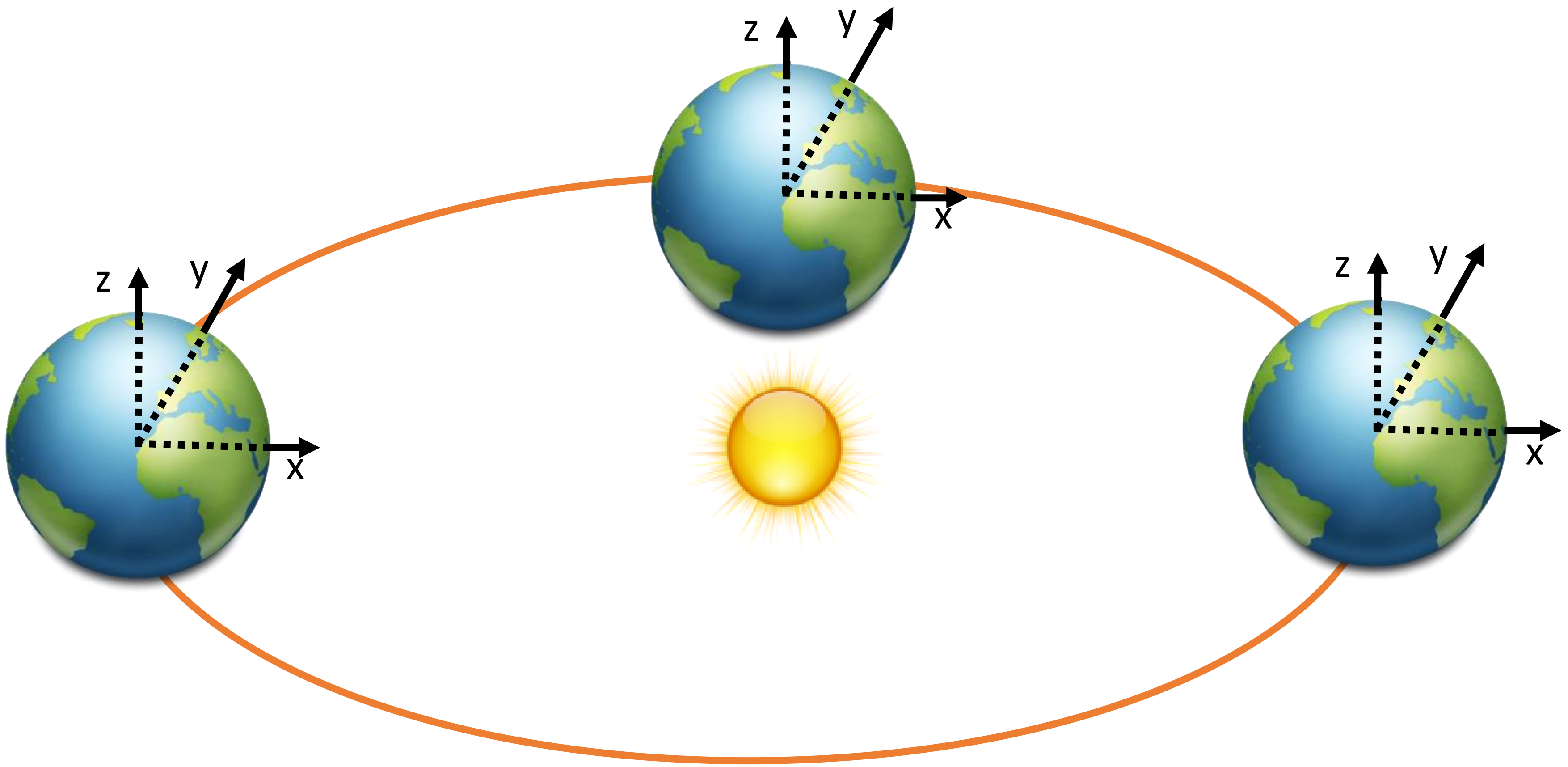

Fixing directions in (nearly empty) space

- How to define fixed directions if not using Sun?
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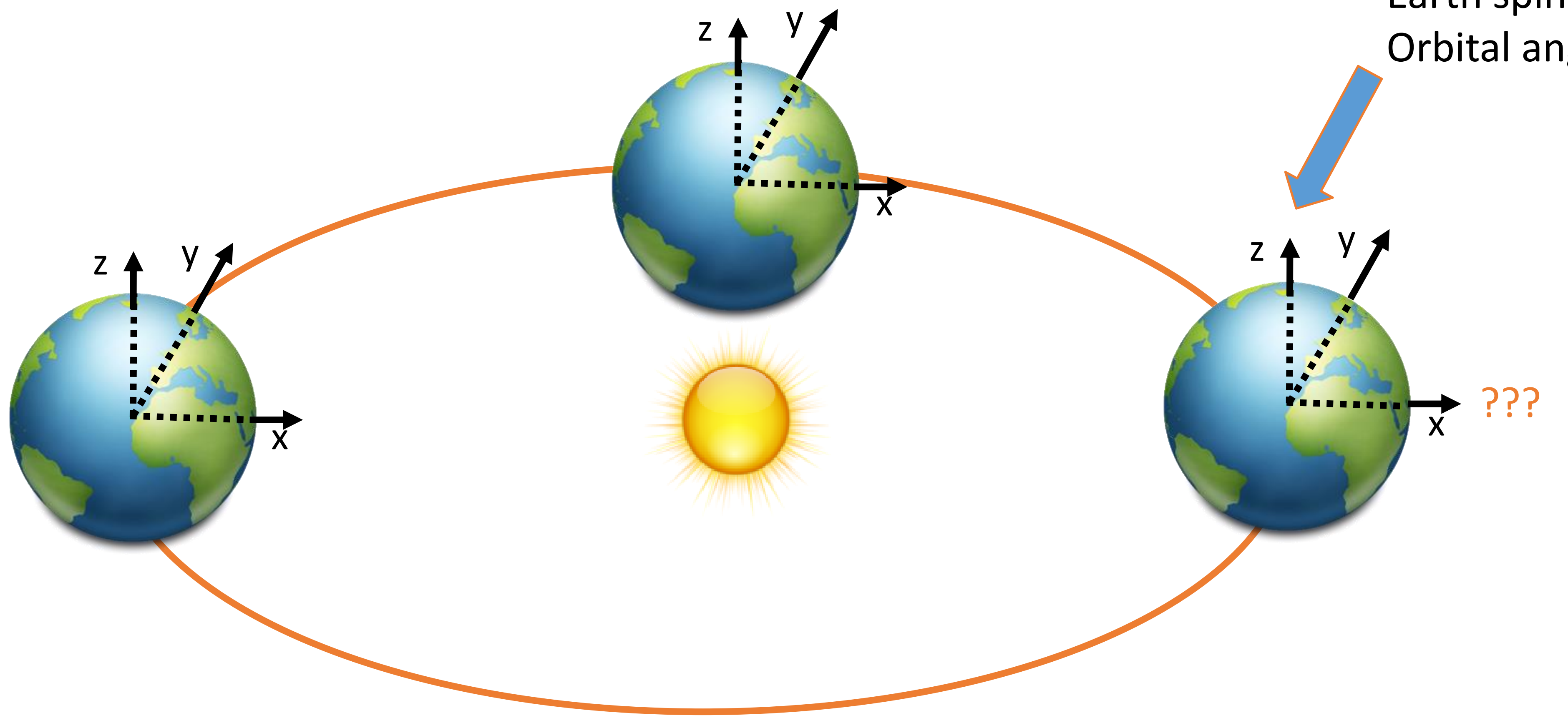
mesarable without using refrance external to earth

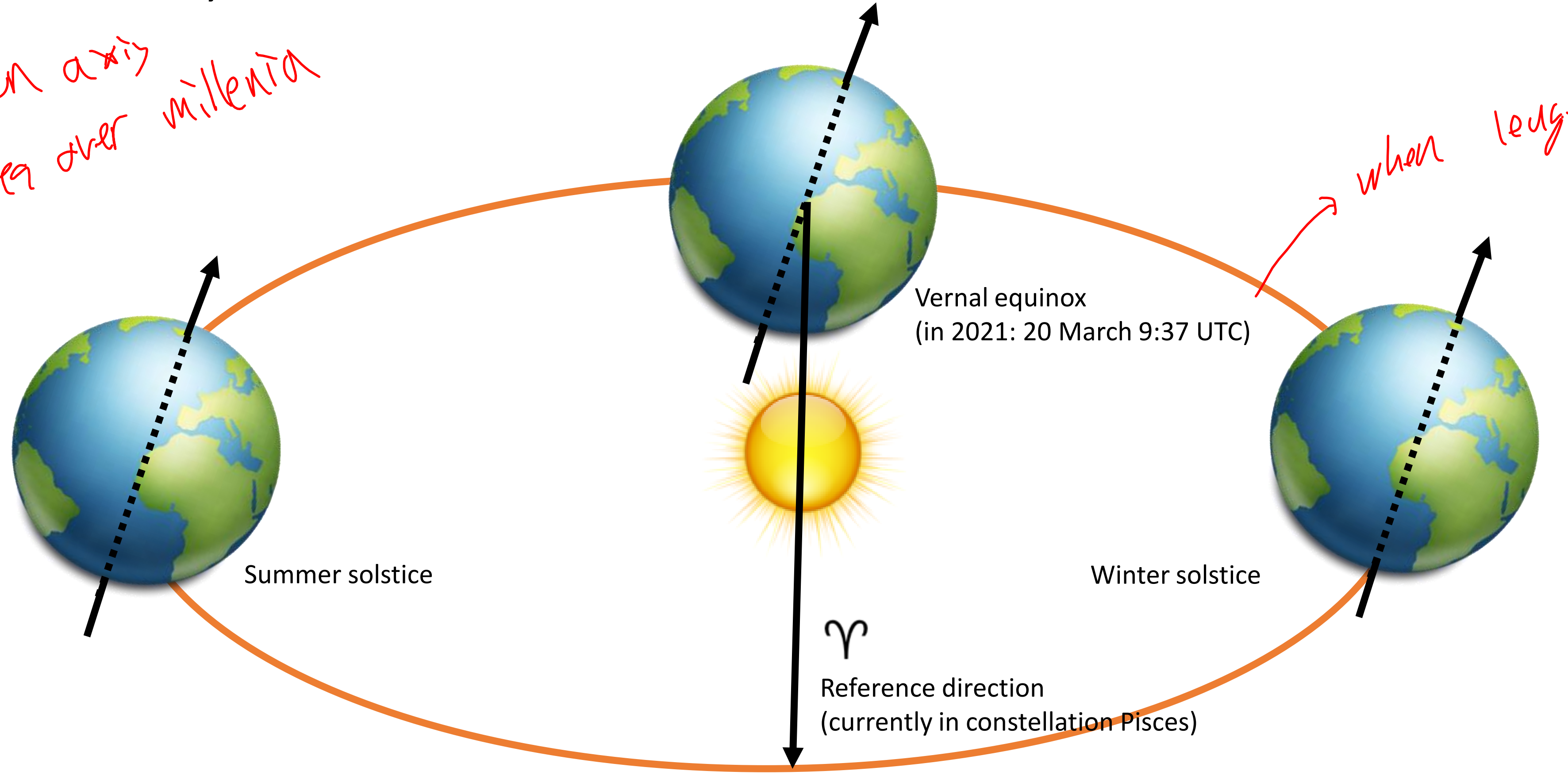
Earth spin-axis or
Orbital angular momentum



- Reference direction for x axis: direction of sun on vernal equinox ♈ used in past
- Not constant, moves over millennia

rotation axis wobbles over millenia

How many more reference directions do we need?



Celestial Coordinates

Reference Frame = reference directions + reference origin

- Reference directions
  - Earth orbital angular momentum (normal to ecliptic)
  - Earth spin axis (normal to equatorial plane)
  - vernal equinox

- Origins
  - Earth
  - Solar system barycenter
  - Other celestial bodies (planets, asteroids, …)
    Each celestial body has its own reference plane!

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not really constant

modern definition: Quasar locations

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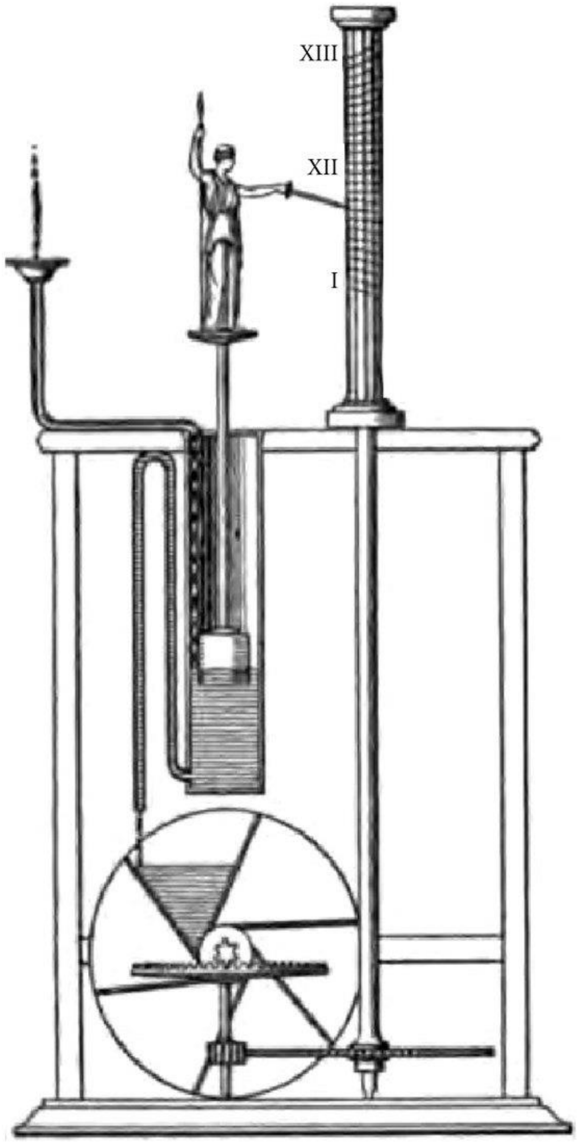
see Chapter 4 Video Lecture 7

Dates and Times

Ghasemzadeh, Nima & Zafari, A. (2011). A Brief Journey into the History of the Arterial Pulse. Cardiology research and practice. 2011. 164832. 10.4061/2011/164832.

Ancient:



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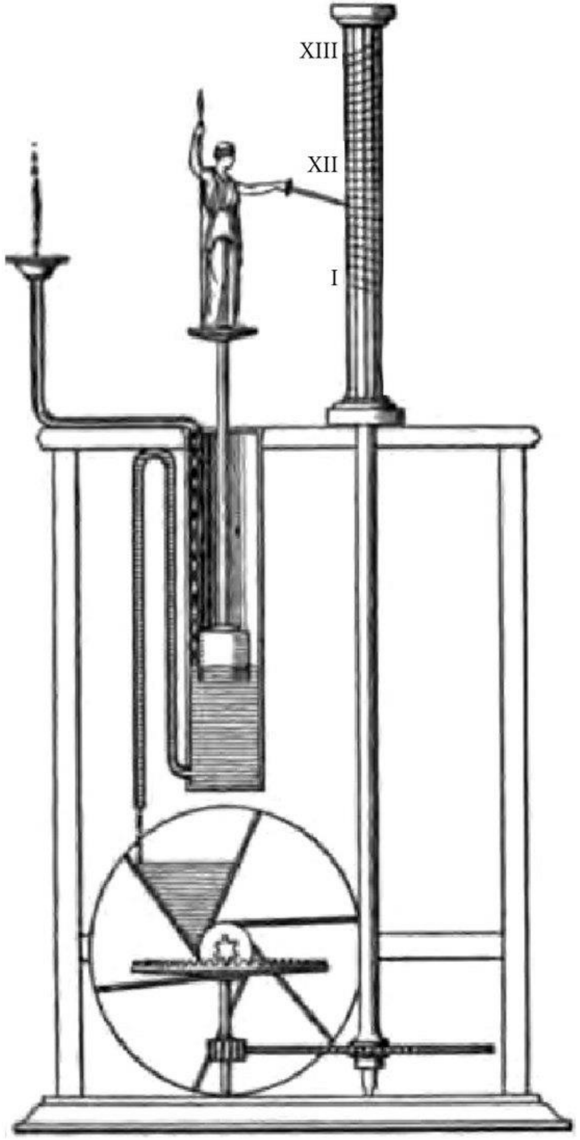
Ancient:

- Sun Dials
- Candles
- Water clock

rates



Ancient:

- Sun Dials
- Candles
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rates

Precision Time Measurement:

A Brief History Of Time Measurement

Ancient:

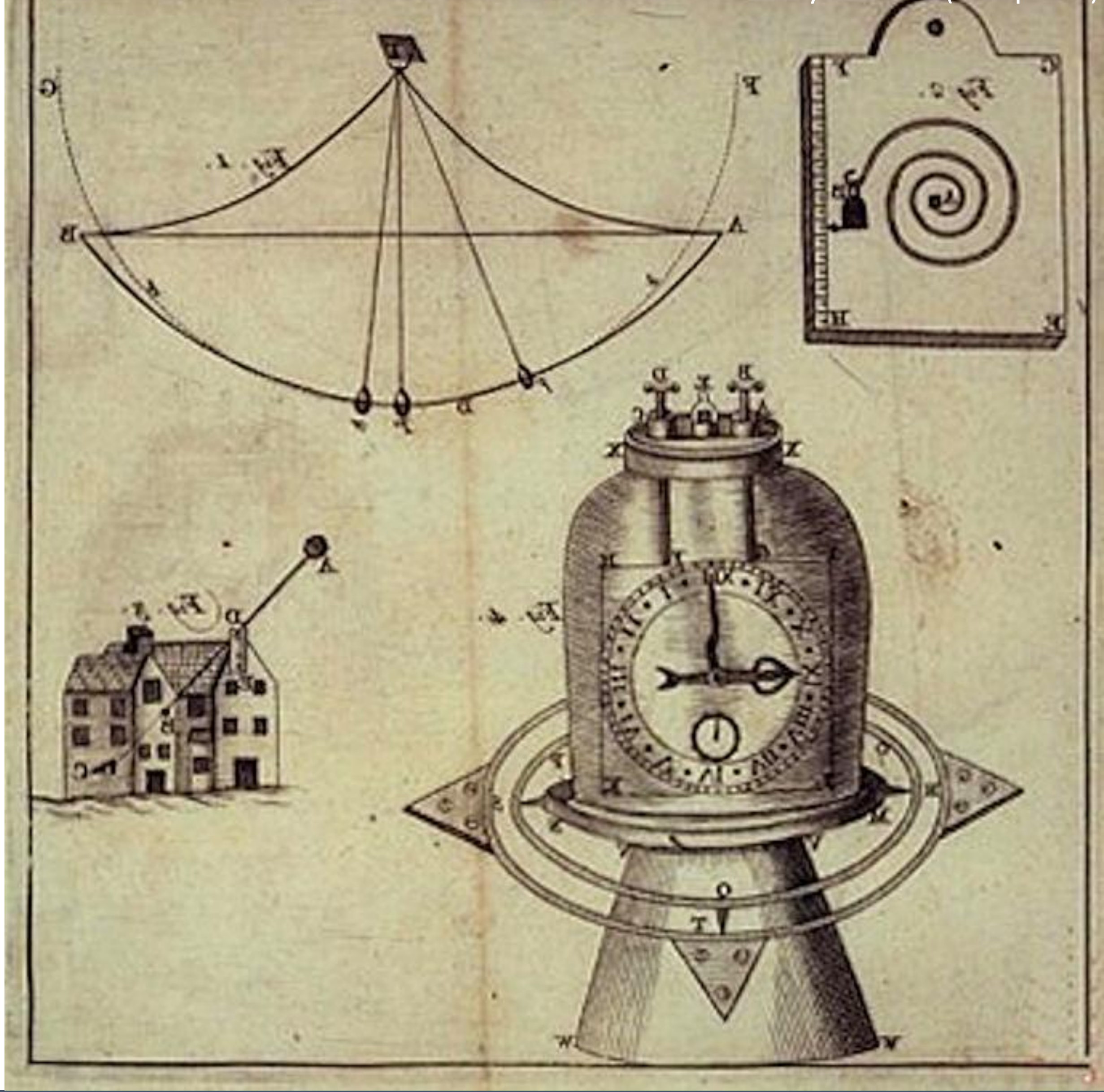
- Sun Dials
- Candles
- Water clock

rates

Precision Time Measurement:

- Pendulum clock
- Marine chronometer
- Quartz clock
- Atomic clock

frequencies



Marine Chronometer of Jeremy Thacker (Wikipedia)

How long is a day?

How can we define one day?

1 day is…

How long is a day?

How can we define one day?

1 day is…

- 24h * 60 min/h * 60 sec/min = 86400 s
  SI day
- time between noon (Sun zenith) on consecutive days
  Historical definition of 1 second
- something else? arbitrary?
  Actual days we use!

Incompatible! Choose 1 out of 3.

How long is a day?

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see Chapter 4 Video Lecture 8

Incompatible! Choose 1 out of 3.

How long is a year?

How can we define one year?

1 year is…

How long is a year?

How can we define one year?

1 year is…

- time between two consecutive vernal equinoxes
  Tropical Year

- 365 days, but sometime also 366.
  Calendar Year

- Earth orbital period around the Sun
  Sidereal Year

spin axis wobbles so these are (slightly) different

Incompatible! Choose 1 out of 3.

How long is a year?

How can we define one year?

1 year is…

- time between two consecutive vernal eq…
  Tropical Year
- 365 days, but someti…
  Calendar Year
- Earth orbit… …eriod around the Sun
  Sidereal Year

see Chapter 4 Video Lecture 8

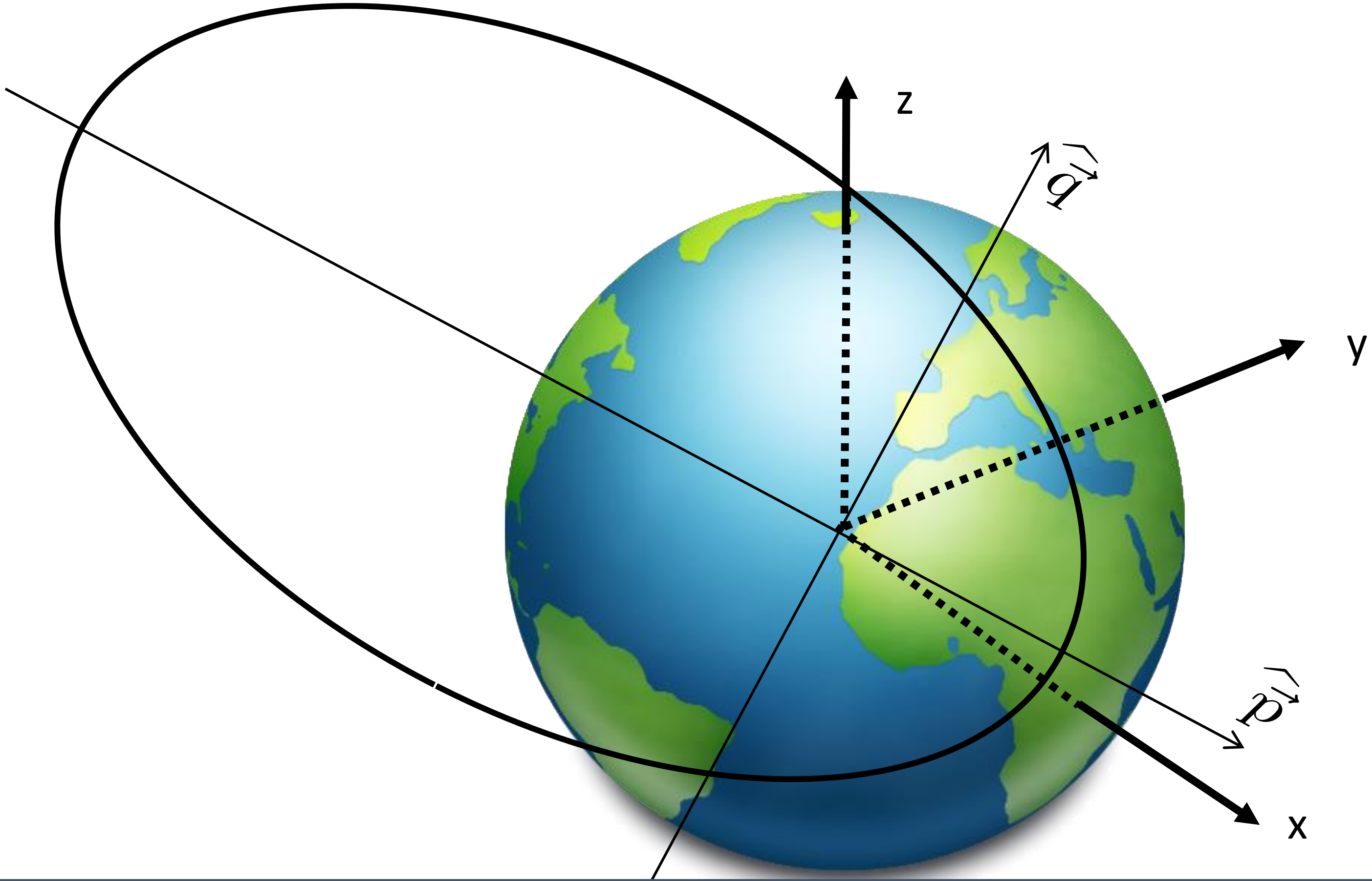
Incompatible! Choose 1 out of 3.

Orbital Elements

Embedding Frames

- We have orbital shape in perifocal frame
- Need to embed perifocal frame in celestial frame

"top down" view

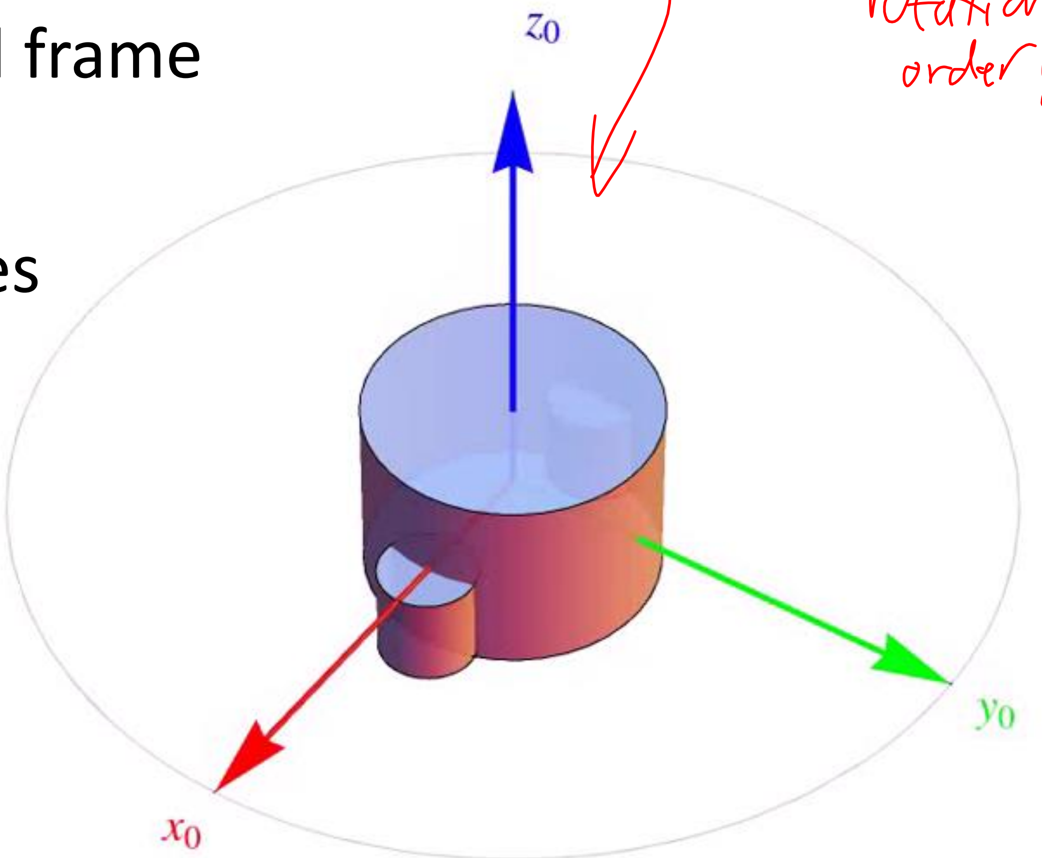


What information is needed?

Orbit orientation in space

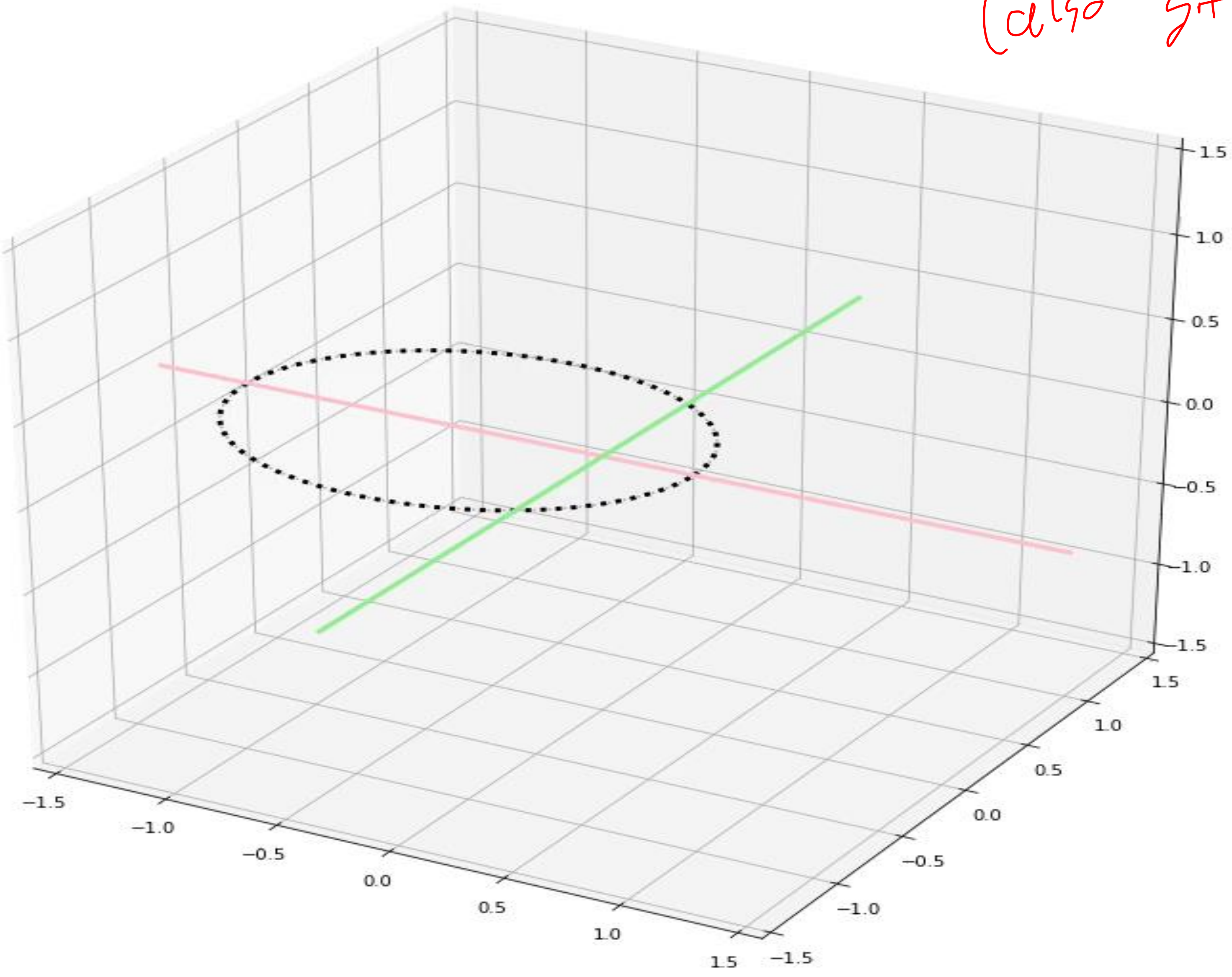
- Shape (e) and size (p) of ellipse fixed in perifocal frame
- Orientation of perifocal relative to fixed frame
- 3 angles needed
- Examples: Pitch, roll, yaw or Euler angles

gif → useful for rotation order!



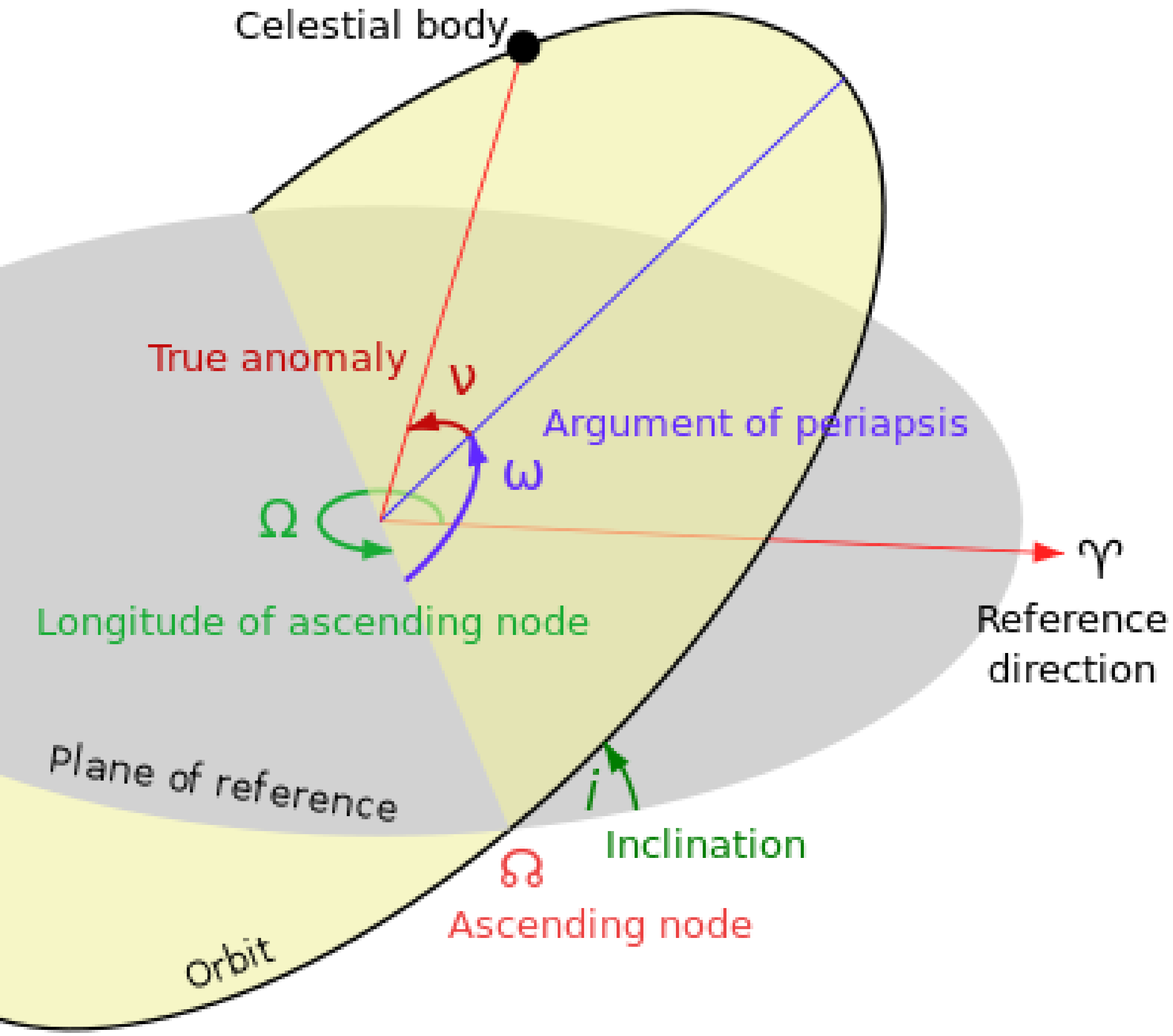
z-x-z Euler angles

(also git)



Classical Keplerian Elements

Wikipedia



- Ω
  Longitude of ascending node,
  Right ascension of ascending node,
  RAAN
- i
  Inclination
- ω
  Argument of perigee/periapsis

Keplerian Elements: Function

a	semi-major axis	size	orbit shape
e	eccentricity	shape	
i	inclination		orbit orientation
Ω	ascending node		
ω	periapsis		
θ,ν	true anomaly		position on orbit

Static, orbit geometry

changes object position

Keplerian Elements: Ranges

elliptic

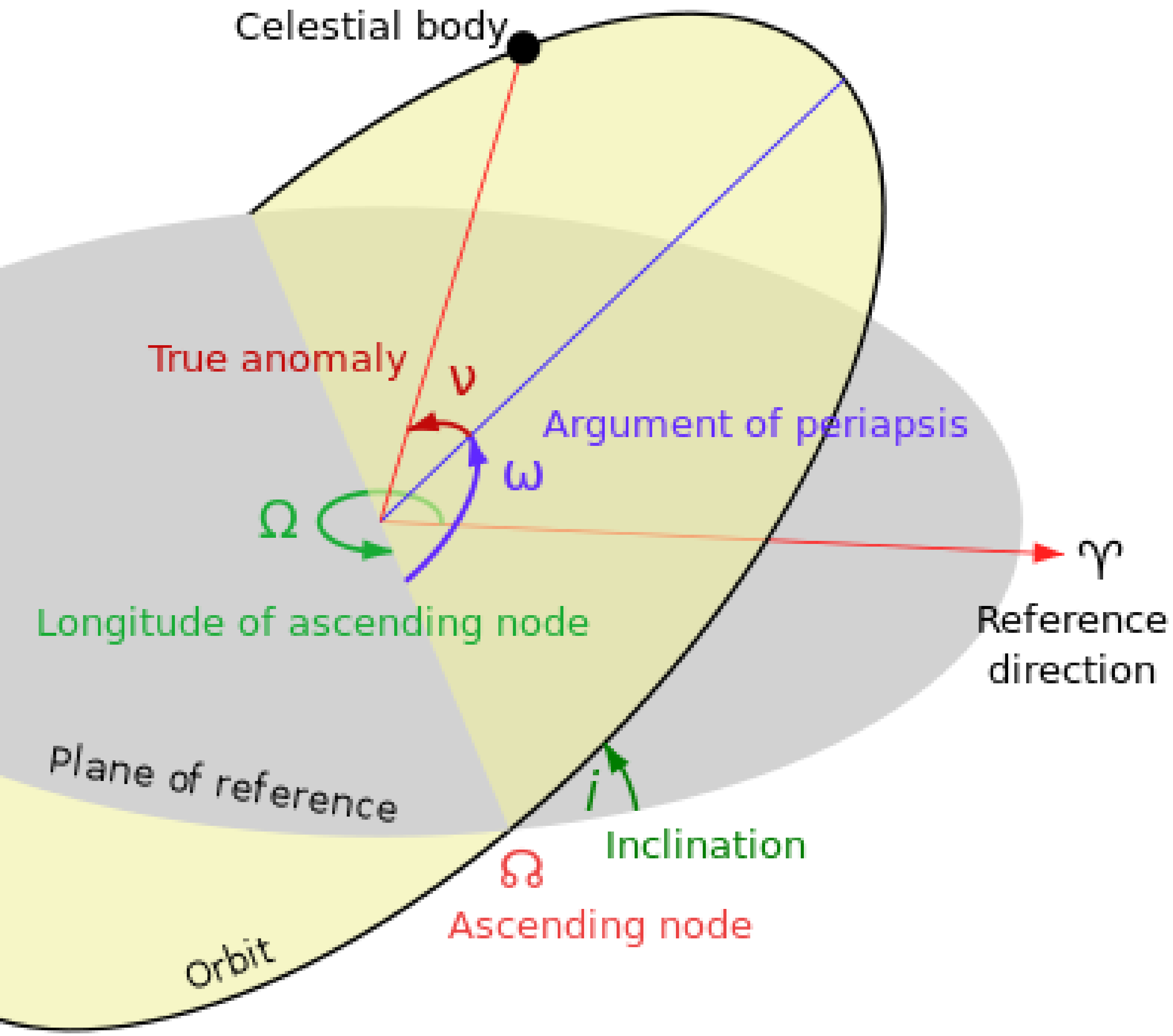
undefined, parabolic

hyperbolic

a	semi-major axis	$(0, \infty)$, -, $(-\infty, 0)$
e	eccentricity	$[0, 1)$, 1, $(1, \infty)$
i	inclination	$[0^\circ, 180^\circ)$
Ω	ascending node	$[0^\circ, 360^\circ)$
ω	periapsis	$[0^\circ, 360^\circ)$
θ,ν	true anomaly	$[0^\circ, 360^\circ)$

Geometric Understanding

Wikipedia



Important:

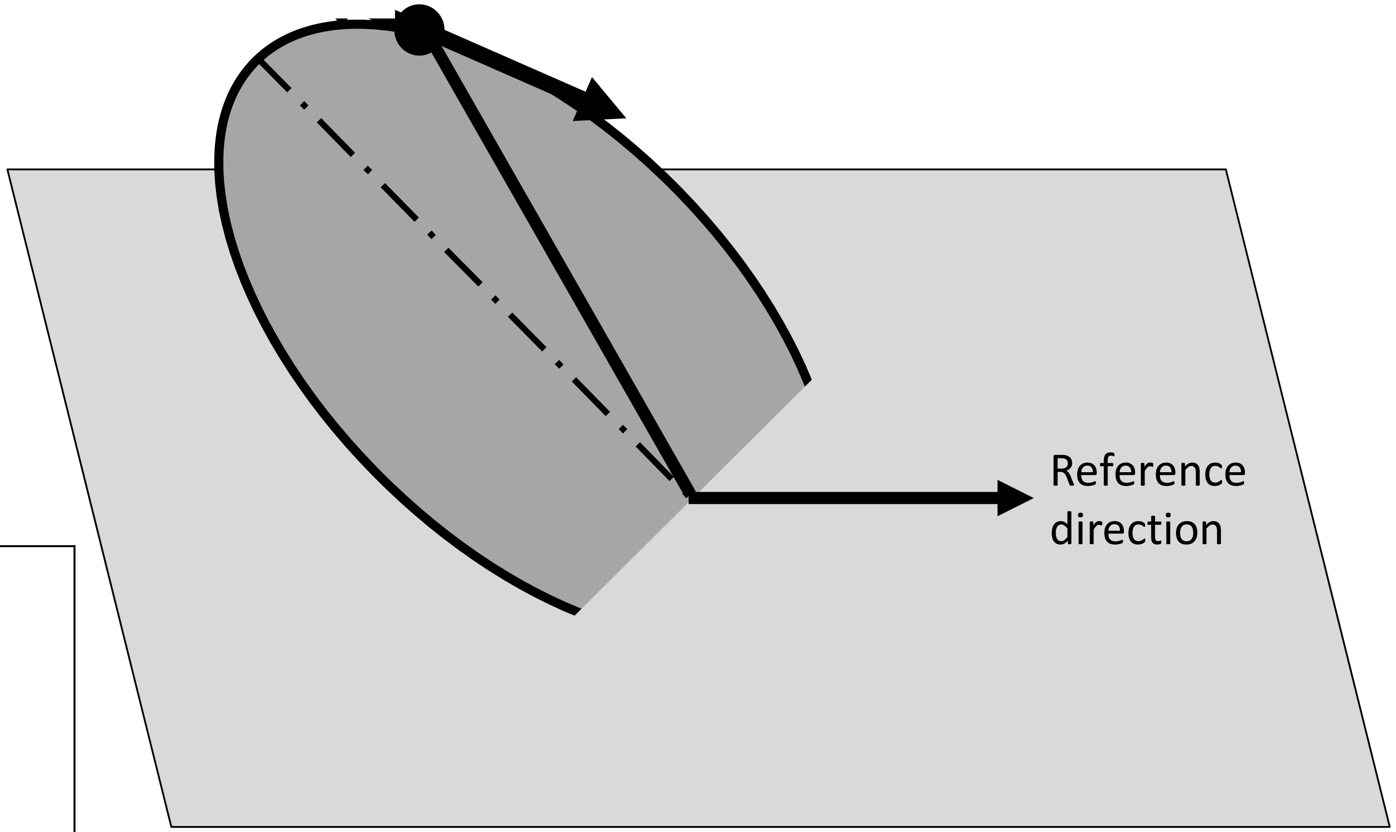
- visualize elements
- intuitive understanding
- connect geometry & elements

Problem solving:

- Identify geometry
- Don’t blindly calculate from equations
- Sketch orbits

See problem sheet!

Exercise: Determine Elements



e:	0	0.6	1.0	1.5
i:	0°	30°	-30°	90°
Ω:	0°	45°	135°	225°
ω:	0°	270°	180°	90°
ν:	20°	200°	160°	340°

Transformations

- Forward: Cartesian position and velocity vectors from Keplerian Elements
- Backwards: Keplerian Elements from position and velocity vectors

Transformations

- Forward: Cartesian position and velocity vectors from Keplerian Elements

$$\vec{K} \longrightarrow \vec{r}_{perifocal} \longrightarrow \vec{r}_{reference}$$

- Backwards: Keplerian Elements from position and velocity vectors

$$\vec{r}_{reference} \longrightarrow \vec{K}$$

Transformations

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see Chapter 4 Video Lecture 9

Singularities

All 3-angle formulations have **gimbal lock**: singularities where physical state is ill defined

Problematic Keplerian elements:

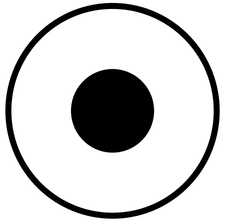
Singularities

All 3-angle formulations have **gimbal lock**:
singularities where physical state is ill defined

Problematic Keplerian elements:

- $h = 0$ no orbital plane

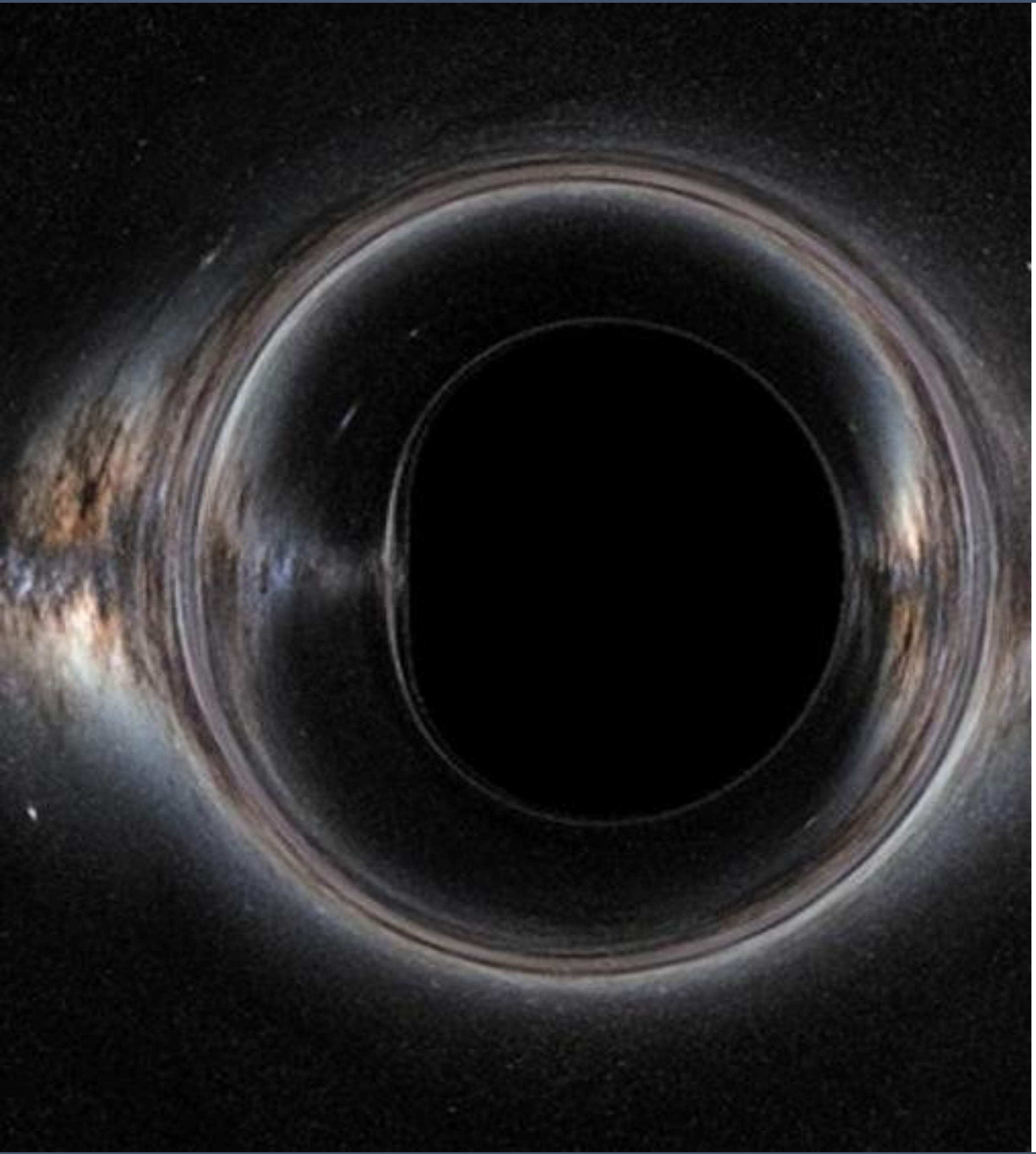
- $e = 0$ no periapsis



- $i = 0$ no ascending node

NASA

Keplerian Elements Singularities



- *e = 0* and *i = 0* very common cases!
- "Coordinate singularity": problem in coordinates, physics is fine
- Jumps in representation for smooth physics break ODE formulation

- Solution: different elements
  - shift singularities elsewhere (Equinoctial elements, spherical, flight elements)
  - canonical coordinates (Delaunay elements, Poincaré elements)

Keplerian Elements Singularities

see Chapter 4 Video Lecture 9

- $e = 0$ and $i = 0$ very common cases!
- “Coordinate singu[illegible] roblem in coordinates [illegible]
- Jump[illegible] ation for smooth [illegible] ODE formulation
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Summary & Further Reading

- Various reference frames and times for different purposes
  - Perifocal frame, Celestial reference frame
  - Civilian Time, Solar Time, Atomic Time
  - Sidereal year, Tropical year, Calendar year
- Defined either using vernal equinox (low accuracy)
  or fixed radio source such as quasars (high accuracy)
- Orientation of perifocal frame in Cartesian frame via Euler angles
- Keplerian elements describe orbit geometry
- Conversion between KEP and r, v in reference frame
- Keplerian elements are singular

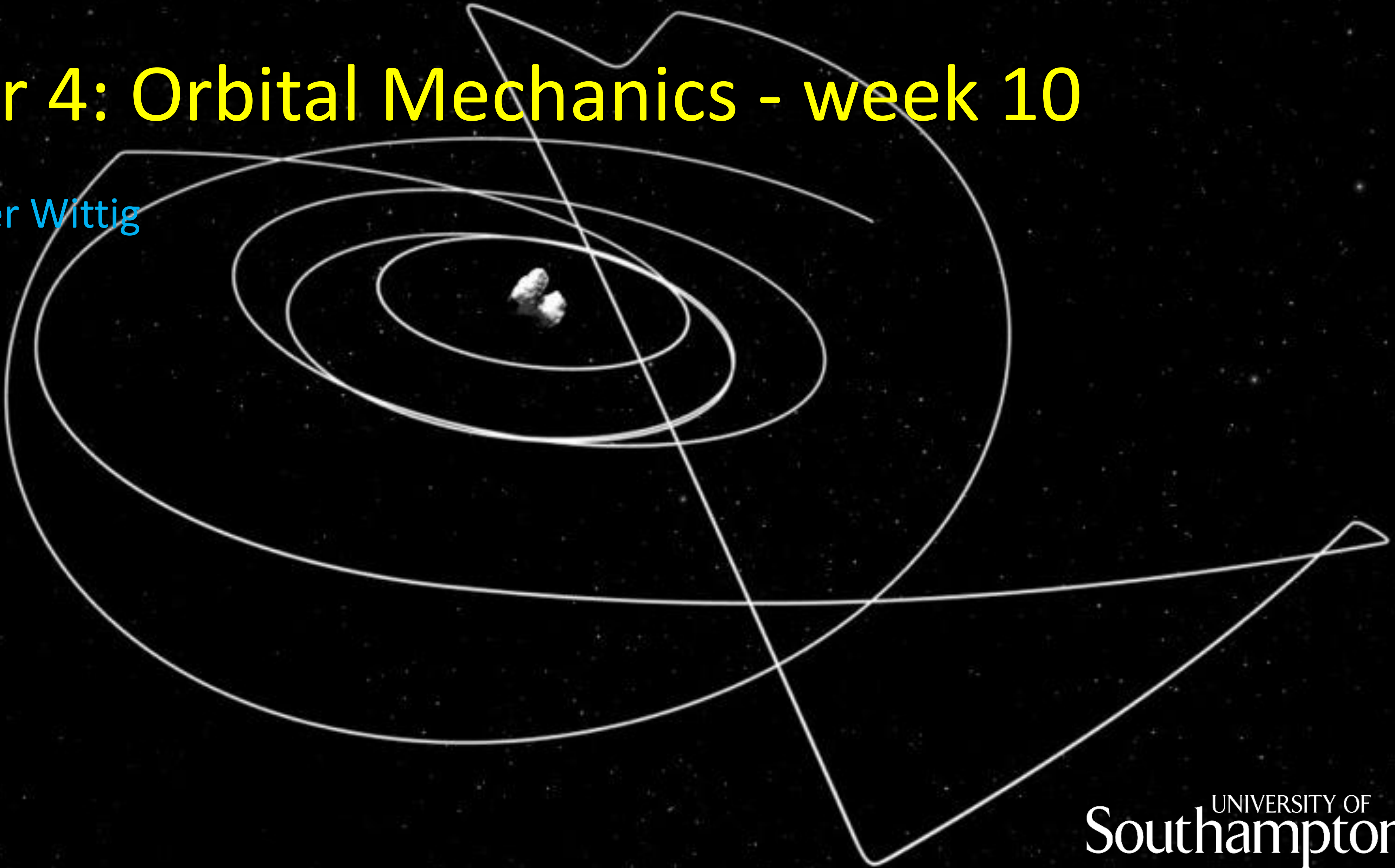
Further reading

- Curtis, Chapter 4
- Vallado, Chapter 2

Advanced Astronautics (SESA3039)

Chapter 4: Orbital Mechanics - week 10

Dr. Alexander Wittig



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