1.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \,, \quad \mathbf{B} = \begin{pmatrix} 4 & 0 & 1 \end{pmatrix} \,, \quad \mathbf{C} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix} \,, \quad \mathbf{D} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \,,$$

write down the order (in the form 
$$m \times n$$
) of each of the above matrices.

A:  $Z \neq Z$ 

B:  $Z \neq Z$ 

C:  $Z \neq Z$ 

D:  $Z \neq Z$ 

$$Z \neq Z$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix}$$

State which of the following exist and express as a single matrix those which do:

(i) 
$$\mathbf{A} + \mathbf{B}^T$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

(ii) 
$$2\mathbf{D} - \mathbf{A}$$

$$2\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 3 & -1 \end{pmatrix}$$

(v) 
$$\mathbf{C}^T\mathbf{C}$$

$$\begin{pmatrix} z & 1 \\ 1 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 7 & -3 \\ 2 & 1 & 6 \\ -3 & 0 & 6 \end{pmatrix}$$

State the condition for a matrix  ${\bf A}$  to be skew-symmetric.

For general matrices express each of the following in equivalent terms

e.g. 
$$(\mathbf{A}\mathbf{B}^T)^T = (\mathbf{B}^T)^T \mathbf{A}^T = \mathbf{B}\mathbf{A}^T$$

(i) 
$$(\mathbf{B}^T)^T = \mathcal{S}^{\checkmark}$$

(i) 
$$(\mathbf{B}^T)^T = \mathcal{B}^T A \times$$

(ii)  $(\mathbf{B}\mathbf{A}^T)^T = \mathcal{B}^T A \times$ 

(iii)  $(\mathbf{B}\mathbf{A}^T)^T = \mathcal{B}^T A \times$ 

$$(\mathcal{B}\mathcal{A}^{\mathsf{T}})^{\mathsf{T}} = (\mathcal{A}^{\mathsf{T}})^{\mathsf{T}}\mathcal{B}^{\mathsf{T}} = \mathcal{A}\mathcal{B}^{\mathsf{T}}$$

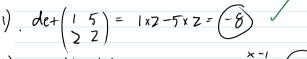
Assuming the required sums and products between the matrices P are Q exist, are the following relations always satisfied?

(i) 
$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P}$$

$$\mathbf{y} \in \mathbf{S}$$

(ii) 
$$\mathbf{PQ} = \mathbf{QP}$$

For the matrix  $\begin{pmatrix} 4 & 1 & 5 \\ -2 & 3 & -1 \\ -1 & 2 & 2 \end{pmatrix}$ 



i) 
$$\det \begin{pmatrix} 1 & 5 \\ 2 & 2 \end{pmatrix} = 1 \times 2 - 5 \times 2 = \begin{pmatrix} -8 \end{pmatrix}$$
  
ii)  $\det \begin{pmatrix} 4 & 5 \\ -2 & -1 \end{pmatrix} = -4 - -10 = 6 \xrightarrow{\times -1} \begin{pmatrix} -6 \\ \end{pmatrix}$   
1ii)  $\det \begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix} = -4 - -3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ 

(i) the minor of the element in the second row and first column

(ii) the cofactor of the element in the third row and second column

- (iii) the cofactor of the element in the first row and third column
- Evaluate directly the determinant in question 5 by expanding in terms of elements of the second row.

7. If  $\mathbf{A} = \begin{pmatrix} 4 & 2 & 1 \\ 1 & -2 & 3 \\ 0 & 1 & -1 \end{pmatrix}$  determine adj  $\mathbf{A}$ .

$$\begin{pmatrix}
-2 & 3 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
1 & 3 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
1 & -2 \\
0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
2 & 1 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
1 & 3 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
1 & +7 \\
0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
2 & 1 \\
-7 & 3
\end{pmatrix}
\begin{pmatrix}
4 & 1 \\
1 & 7
\end{pmatrix}
\begin{pmatrix}
4 & 2 \\
1 & -7
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & + | & 1 \\
+3 & -4 & -4 & 4 \\
9 & -11 & -10
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 3 & 9 \\
1 & -4 & -11 \\
1 & -4 & -10
\end{pmatrix}$$