Module 9 test

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1. Find the following indefinite integrals

(a) $\int x(1+x^2)^5\,dx$

a) $\frac{1}{12}(1+x^2)^6 + k$

(b) $\int \sinh^4 x \cosh x\,dx$

$t = \sinh x$

$\frac{dt}{dx} = \cosh x$

$\int t^4 \frac{\cosh x}{\cosh x}\,dt$

$\int t^4\,dt$

$\frac{t^5}{5} + k = \frac{\sinh^5 x}{5} + k$

2. Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos x}\,dx$

$t = 1+\cos x$

$\frac{dt}{dx} = -\sin x$

$\int_2^1 \frac{\sin x}{t}\,\frac{dt}{-\sin x}$

$-1\int_2^1 \frac{1}{t}\,dt$

$-[\ln t]_2^1$

$-\ln\frac{1}{2}$

$= \ln 2$

3. Use the substitution $u = \sqrt{x}$ to evaluate $\int_0^4 e^{\sqrt{x}}\,dx$

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\int_0^2 e^u \frac{du}{\frac{1}{2}x^{-1/2}}$

$\int 2\sqrt{x}\,e^u$

$\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$

$2\int ue^u\,du = 2\left[(u-1)e^u\right]_0^2$

$= 16.778$

4. Find the magnitude of the area enclosed between the curve $y = \sqrt{x+4}$, the x-axis and the lines $x = 0$ and $x = 5$.

$\int^5$

$$\int_0^5 \sqrt{x+4}\ dx \qquad t = x+4,\quad \frac{dt}{dx} = 1$$

$$\int_4^9 \sqrt{t}\ dt$$

$$\left[\frac{2t^{3/2}}{3}\right]_4^9 = \frac{38}{3}$$

5. The area bounded by the curve $y = x(x-1)$, the x-axis and the lines $x = 0$ and $x = 1$ is rotated about the x-axis through one complete revolution.

(i) Find the volume of the solid of revolution.

$$V = \pi \int_0^1 x^2(x-1)^2\ dx$$

$$\int_0^1 x^4 - 2x^3 + x^2\ dx$$

$$\pi\left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3}\right]_0^1 = \pi/30$$

(ii) Find the coordinates of the centre of gravity of this solid.

$$y = x(x-1) \qquad x\ 0 \to 1$$

$$\frac{\pi}{V}\int_0^1 x^5 - 2x^4 + x^3\ dx$$

$$\bar{X} = 30\pi\left[\frac{x^6}{6} - \frac{2x^5}{5} + \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$$

$$(1/2,\ 0)$$