SESA6085 – Advanced Aerospace Engineering Management

Lecture 8

2024-2025

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Recap

Reliability Modelling

- In our last lecture we considered the basic building blocks of reliability modelling
  - Series systems
  - Parallel systems
  - m-out-of-n systems
  - Block diagrams & block diagram decomposition
  - Common failure modes
- Today we will consider
  - m-out-of-n balanced systems
  - Active and inactive redundancy
  - Multi-state components

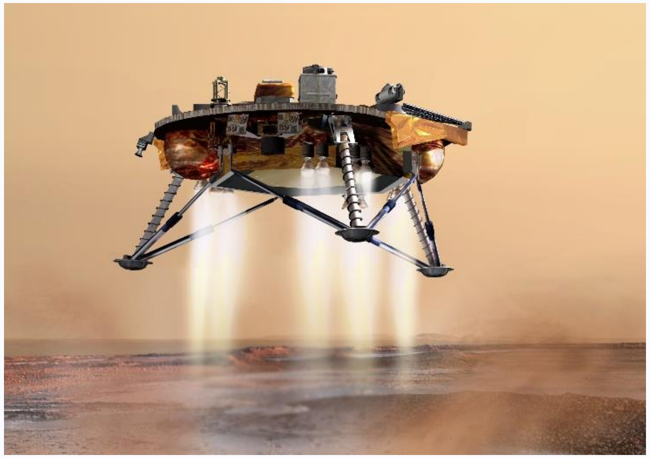
m-out-of-n Balanced Systems

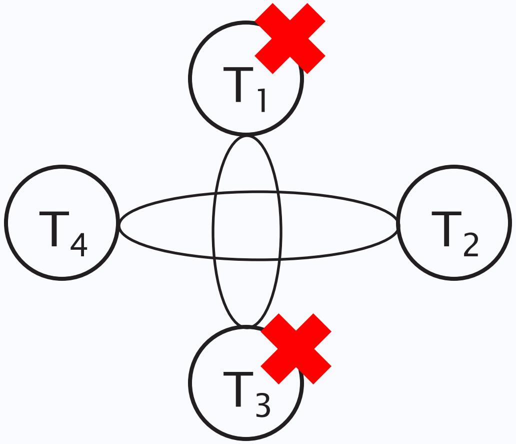
m-out-of-*n* Balanced Systems

- Lets now return to our parallel *m*-out-of-*n* system
- Consider a system with *n* components operating in parallel
- Previously in our standard *m*-out-of-*n* redundant system if one of these components failed the others took over
- However, in some cases if one component fails another of the parallel components must be forced to shut down
- We refer to this as a *m*-out-of-*n* balanced system
  - A component must be shut down to maintain balance

m-out-of-*n* Balanced System Example

- A mars lander has a series of thrusters used to slow its descent
- If one of these thrusters fails then the opposite thruster should be disabled so as to maintain the balance of the vehicle
- In our 4 thruster example, if T1 fails
- Then T3 should be shut down





m-out-of-n Balanced Systems

- Let's assume that all components in such a system are identical and function with probability p
- Let's define the status of the i^{th} engine as X_i where,

$$X_i = \begin{cases} 1 & \text{if the ith engine functions} \\ 0 & \text{if the ith engine fails} \end{cases}$$

- Hence,

$$P(X_i = 1) = p$$

$$P(X_i = 0) = 1 - p$$

m-out-of-*n* Balanced Systems

- Let's also define Y,

$$Y = \begin{cases} 1 & \text{the system operates successfully} \\ 0 & \text{the system fails} \end{cases}$$

- Finally let's define the successful operation of a pair of components as $X^{(i)}$
- The success of a pair of components is given by,

$$P\left(X^{(i)} = 1\right) = p^2$$

- Therefore the failure of a pair of components is,

$$P\left(X^{(i)} = 0\right) = 1 - p^2$$

m-out-of-*n* Balanced Systems

- System unreliability occurs when *n* component pairs fail

$$P(Y = 0) = \prod_{i=1}^{n} P(X^{(i)} = 0)$$

- The reliability of the system is then

$$R = 1 - P(Y = 0) = 1 - \prod_{i=1}^{n} P(X^{(i)} = 0)$$

m-out-of-*n* Balanced System Example

- Let's revisit our descent thruster system
- We have 4 thrusters which are paired together, p=0.95
- Calculate the reliability of a 2-out-of-4 balanced system
- From our previous equation

$$R = 1 - (1 - p^2)^2$$
$$R = 0.9905$$

m-out-of-*n* Balanced System

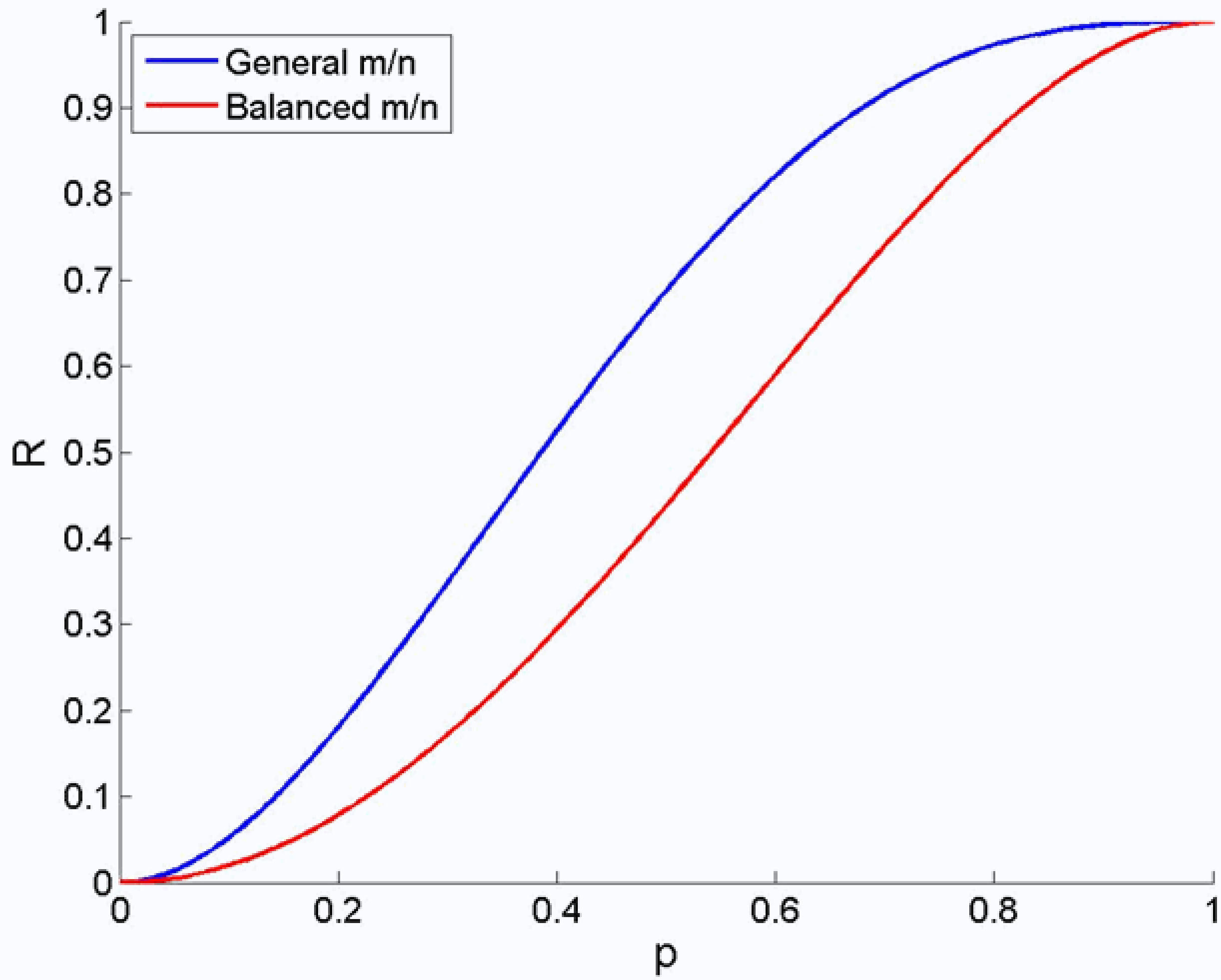
- Is a balanced *m*-out-of-*n* system more reliable than our basic *m*-out-of-*n* redundant system?
- The reliability for our 2-out-of-4 redundant system is:

$$R = 6p^2 - 8p^3 + 3p^4$$

$$R = 0.9995$$

- A balanced redundant system is never as reliable as a standard redundant system except when p=1.0

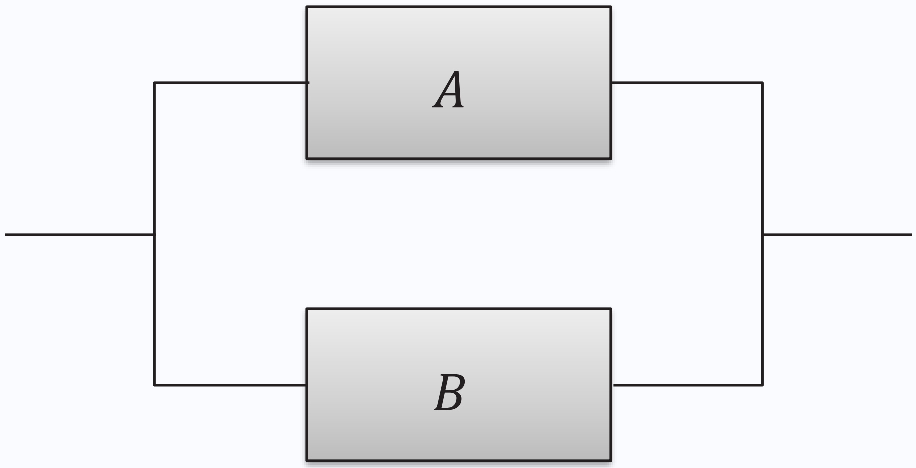
m-out-of-*n* Balanced System



Active & Inactive Redundancy

Redundancy

- Recall our traditional parallel redundant system



- Is such a system completely realistic?

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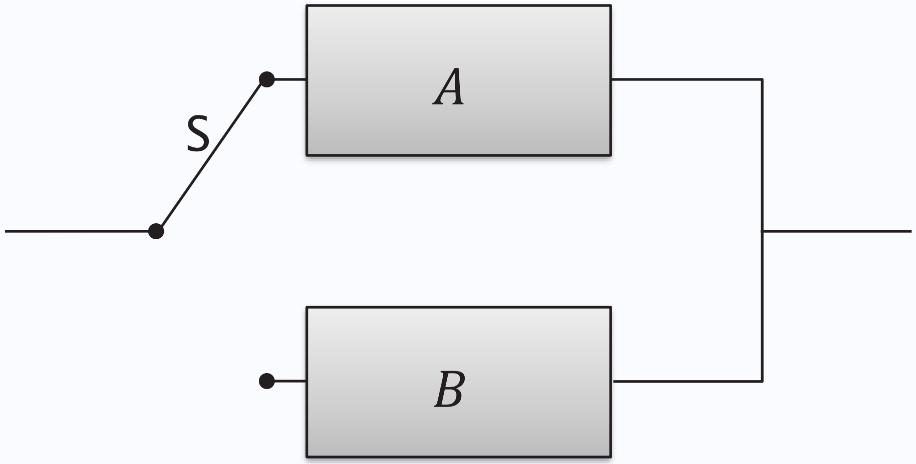
Active & Inactive Redundancy

- Our classic parallel redundant system is an example of an active redundant system
- In such a system all of the components are active and sharing the load
- An inactive redundant system is one whereby other component(s) start operating when the main component fails

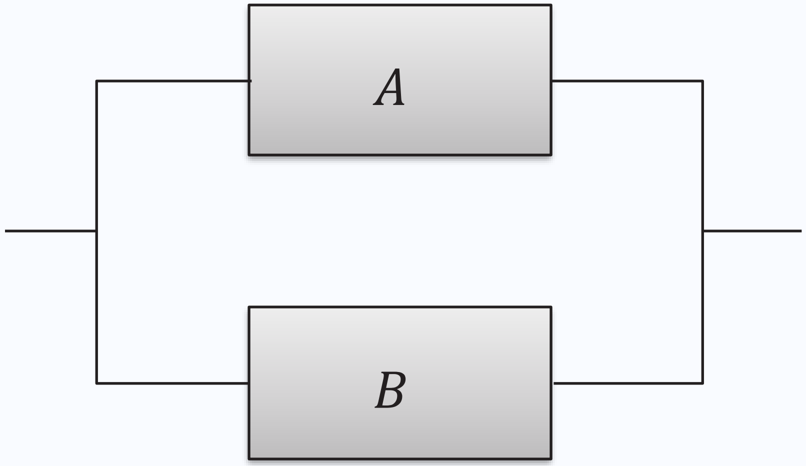
Inactive Redundancy



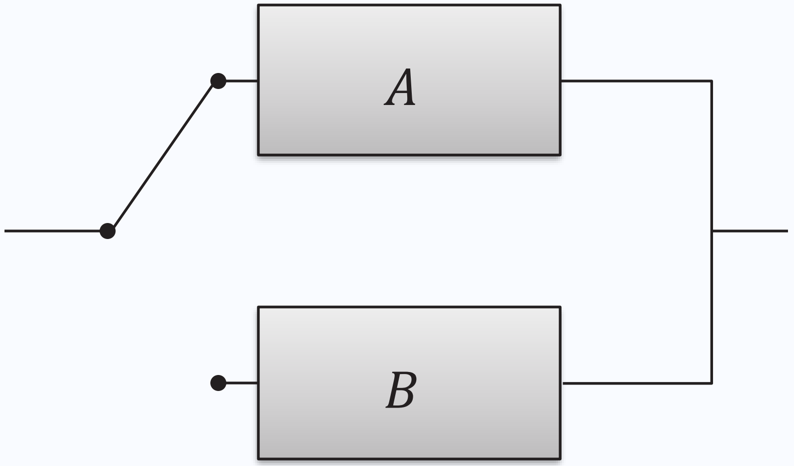
- Above is an RBD representation of such a system including a switch, denoted S
- There is no difference between an active and inactive redundant system if the switch is perfect and all components have the same failure rate

Inactive Redundancy

- Let’s assume we have a system with two components in parallel, in one configuration the components are in an active redundant arrangement and in the other they are in a inactive redundant arrangement



Active redundancy



Inactive redundancy

Inactive Redundancy

- For the active redundant configuration:

$$R_{\text{active}} = 1 - P(\bar{A}\bar{B}) = 1 - P(\bar{A})P(\bar{B}|\bar{A})$$

- Which we've seen previously, given each component's reliability is p:

$$R_{\text{active}} = 2p - p^2$$

- For the inactive redundant configuration:

$$R_{\text{inactive}} = 1 - P(\bar{A}\bar{B}) = 1 - P(\bar{A})P(\bar{B}|\bar{A})$$

- The formulae appear to be the same, but are they?

Inactive Redundancy

- Why are they different?
- The key difference is in the term $P(\bar{B}|\bar{A})$
- For an active redundant system this reduces to $P(\bar{B})$ as a failure in component A is independent from B and is assumed to have been operating since t=0
- For an inactive redundant system $P(\bar{B}|\bar{A})$ is always a dependent probability as B does not start until A fails
  - $P(\bar{B}|\bar{A})$ is therefore a function of time

Standby Definitions

- Before proceeding lets define the different types of standby systems:
- **Hot** standby:
  - Standby components have the same failure rate as the primary component
- **Warm** standby:
  - Standby components have a smaller failure rate than the primary component but greater than zero
- **Cold** standby:
  - Standby components don’t fail in standby and have a zero failure rate until activated

Standby Definitions

- If the primary component has a failure rate of λ
- Then:
  - $\lambda_{hot} = \lambda$
  - $\lambda_{cold} = 0$
  - $\lambda_{warm} < \lambda$

A Simple Standby System

- Let's take a simple standby system defined by 2 components
- Then the reliability of the system at a time, t, is equal to the reliability of unit A until t and the probability that A fails at a time, τ, $0 < \tau < t$ and the standby unit functions successfully from τ to t

$$R_{sb}(t) = R_A(t) + \int_{\tau=0}^{t} f_A(\tau) R_B(t-\tau) d\tau$$

A Graphical Interpretation

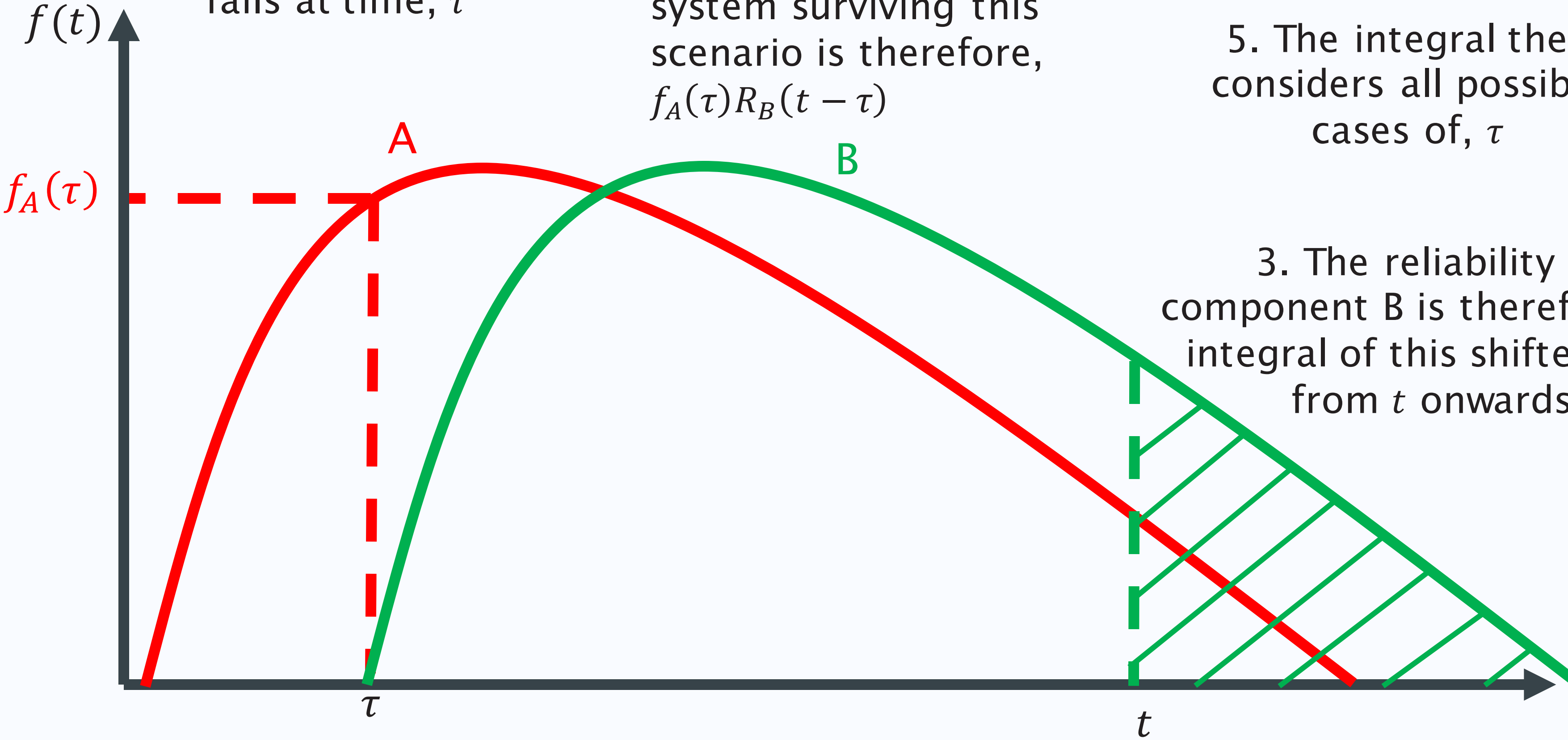
1. Component A fails at time, τ

2. Component B starts working at time, τ, it's PDF is therefore shifted by τ

3. The reliability of component B is therefore the integral of this shifted PDF from t onwards

4. The probability of the system surviving this scenario is therefore, $f_A(\tau)R_B(t-\tau)$

5. The integral then considers all possible cases of, τ



A Simple Standby System

- If we assume the PDFs of both components are defined by an exponential i.e.

$$f(t) = \lambda \mathrm{e}^{-\lambda t}$$

$$R(t) = \mathrm{e}^{-\lambda t}$$

- And their respective hazard rates are λ_A & λ_B then:

$$R_{sb}(t) = \mathrm{e}^{-\lambda_A t} + \int_{\tau=0}^{t} \lambda_A \mathrm{e}^{-\lambda_A \tau} \mathrm{e}^{-\lambda_B (t-\tau)} d\tau$$

A Simple Standby System

- Which equates to:

$$R_{sb}(t) = \mathrm{e}^{-\lambda_A t} + \frac{\lambda_A \mathrm{e}^{-\lambda_B t}}{\lambda_A - \lambda_B}\left(1 - \mathrm{e}^{-(\lambda_A - \lambda_B)t}\right)$$

- Or if the components are identical $\lambda_A = \lambda_B$:

$$R_{sb}(t) = (1 + \lambda t)\mathrm{e}^{-\lambda t}$$

Multi-unit Standby Systems

- Let's extend the system to include *n* components with *n*-1 in standby
- In such a system if component one fails, component two takes over, when this unit fails component three takes over…
- In such a case, with identical failure rates the reliability is:

$$R_{sb}(t) = \mathrm{e}^{-\lambda t}\left[1 + \lambda t + \frac{(\lambda t)^2}{2!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!}\right]$$

Standby System Example

- A system consists of two components with constant hazard rate, $\lambda = 0.5 \times 10^{-6}$
- These components can be in either an active or inactive redundant system
- Compare the reliabilities of each system, which is more reliable?

Standby System Example

- The reliability of the parallel system is:

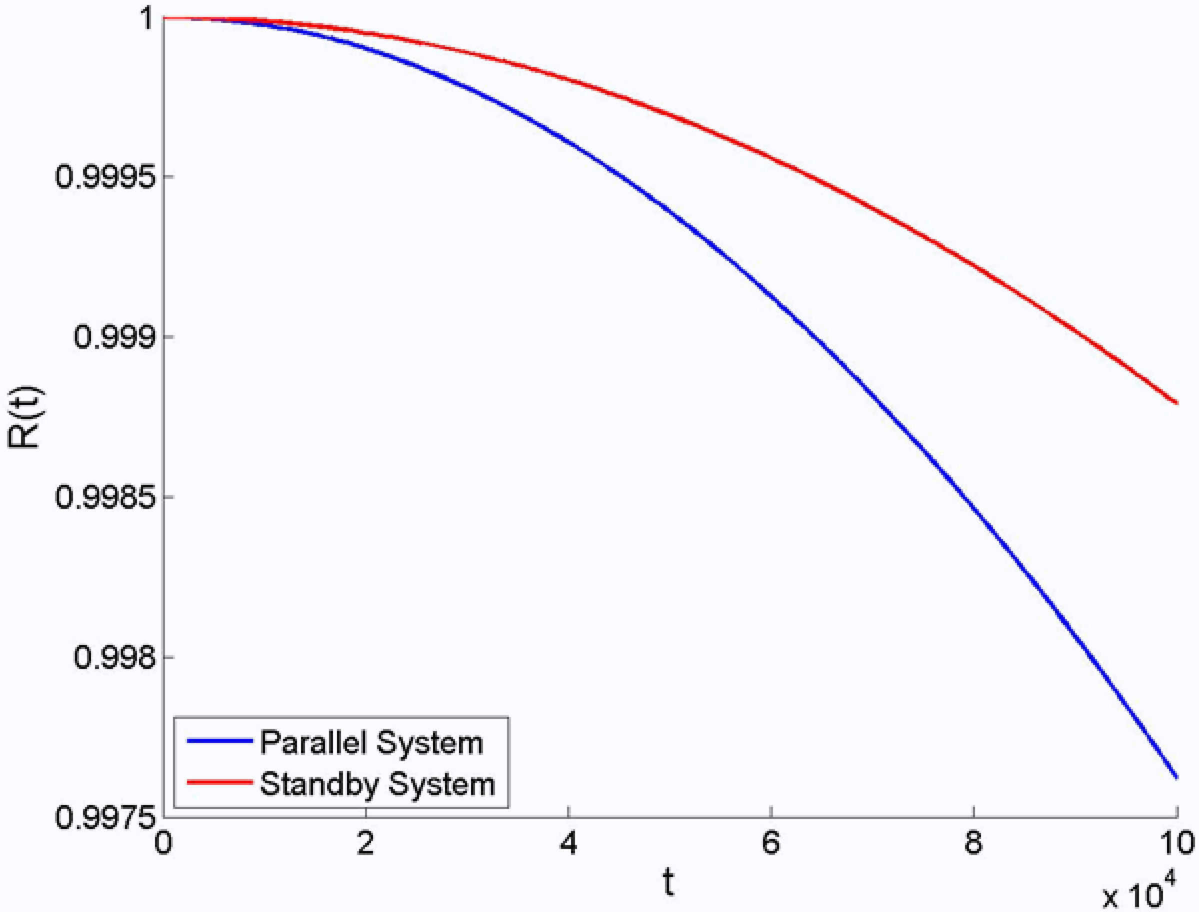
$$R_p(t) = 1 - (1 - \mathrm{e}^{-\lambda t})^2$$

- Using our previous equation, the reliability of the standby system is:

$$R_{sb}(t) = \mathrm{e}^{-\lambda t}[1 + \lambda t]$$

Standby System Example

- Clearly the standby system offers greater reliability



Multistate Components

Multistate Components

- Until now we've only considered components which can be in either one of two states – working or failed
- However, there can be instances where a component can fail in more than one mode
- Such systems are called multistate systems
- They are special in that adding additional components may increase or reduce the reliability of the system depending on the dominant failure mode

Multistate Components

- Consider a diode in an electronic system
- A diode allows current in one direction but not in another
- The resistance is therefore zero forward but infinite in reverse
- Diode can therefore have three states:
    1. Operate normally
    2. Fail due to an infinite resistance in both directions (an open circuit)
    3. Fail due to a zero resistance in both directions (a short circuit)

Multistate Series System

- Let's consider a system defined by n diodes in series
- For each diode we have three states:
  - x - Component works correctly
  - $\bar{x}_s$ - Fails due to a short circuit
  - $\bar{x}_o$ - Fails due to an open circuit
- When in series:
  - The whole system fails if any component is $\bar{x}_o$
  - The whole system fails if all components are $\bar{x}_s$

Multistate Series System

- For the case with one component (*n*=1), the reliability is:

$$R = P(x) = 1 - P(\bar{x}_s) - P(\bar{x}_o)$$

- For a two component system (*n*=2), the reliability is:

$$R = 1 - P(\bar{x}_{o1} + \bar{x}_{o2} + \bar{x}_{s1}\bar{x}_{s2})$$

$$R = 1 - [P(\bar{x}_{o1}) + P(\bar{x}_{o2}) - P(\bar{x}_{o1}\bar{x}_{o2}) + P(\bar{x}_{s1}\bar{x}_{s2})]$$

- Defining the probabilities of failures as q_o and q_s we obtain the general expression:

$$R = \prod_{i=1}^{n}(1 - q_{oi}) - \prod_{i=1}^{n} q_{si}$$

Multistate Series System

- Unlike a standard series system the reliability of a multistate system will reach a maximum for an optimum number of components
- Derivatives of the above expression can be taken to calculate an optimum number of components
- A number of components greater than or less than this value will result in a suboptimal system reliability
- N.B. the presented formulae may change if the failure modes of the entire system alter

Multistate Parallel System

- Using our previous notation, the reliability for a system with two components in parallel is:

$$R = 1 - P(\bar{x}_{o1}\bar{x}_{o2} + \bar{x}_{s1} + \bar{x}_{s2})$$

$$R = 1 - [P(\bar{x}_{o1})P(\bar{x}_{o2}) + P(\bar{x}_{s1}) + P(\bar{x}_{s2}) - P(\bar{x}_{s1})P(\bar{x}_{s2})]$$

$$R = 1 - [q_{o1}q_{o2} + q_{s1} + q_{s2} - q_{s1}q_{s2}]$$

- Or for *n* components in parallel:

$$R = \prod_{i=1}^{n}(1 - q_{si}) - \prod_{i=1}^{n} q_{oi}$$

Multistate Parallel System

- If all components are identical then the optimum number of components for maximum reliability is 1 if $q_s > q_o$
- As with the series system derivatives can be taken with respect to n to calculate an optimal number of components
- N.B. Once again the above formula holds for the failures of a diode and similar cases but may need to be derived again for other cases

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