# FEEG 2005
# Structures: Lecture 5

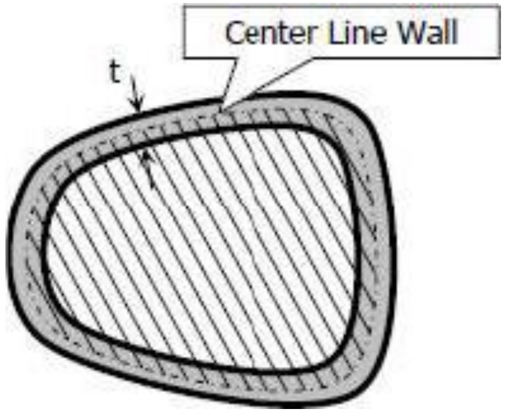
Rectangular Strips and Arbitrary Open Sections Under Torsion

# Summary of last lecture

- We developed the theory for torsion of arbitrary closed thin-walled sections:
  - 1) Stress:

$$\boxed{\tau = \frac{T}{2A_m t}}$$

**NOTE:** $A_m$ is the area enclosed by the median line!




  - 2) Deformation (Twist):

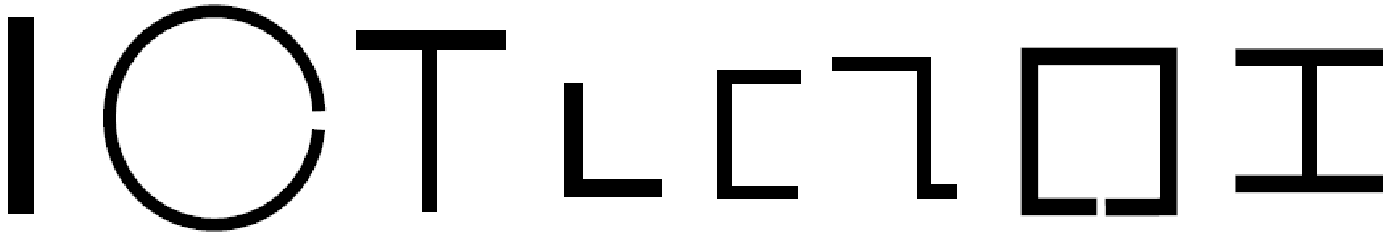
$$\boxed{\frac{d\phi}{dx} = \frac{T}{4A_m^2} \oint \frac{1}{Gt} ds}$$

If G is constant:

$$\boxed{\frac{d\phi}{dx} = \frac{T}{GJ}} + \boxed{J = \frac{4A_m^2}{\oint \frac{ds}{t}}}$$

# This lecture

- What about torsion of thin-walled open sections?
  - 1) Stress: $\tau = ?$
  - 2) Deformation (twist): $\phi = ?$



- These sections are made of mostly rectangular strips, if we can derive a theory for a single rectangular strip, we can generalise this for arbitrary sections.

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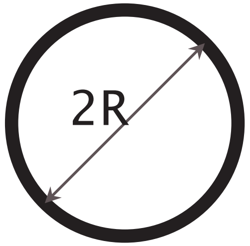
# Question

- For a constant applied torque T at the end of a bar and similar thickness t, which cross section does twist more?

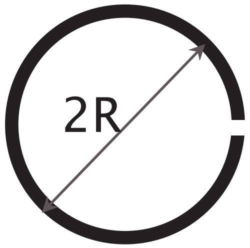
least twist

1. Tube with radius R

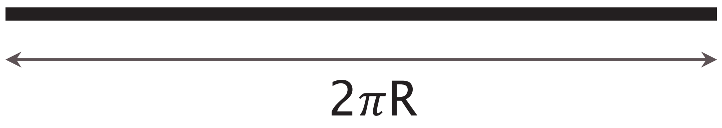



both same

2. Tube with radius R and a slit along the length




3. A rectangle with length $2\pi R$



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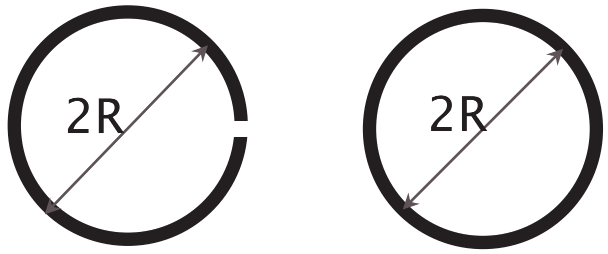
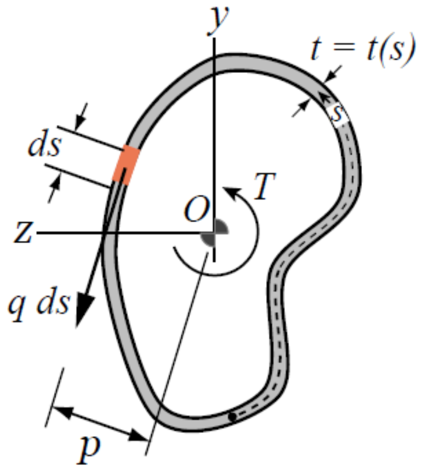
# Question

- What's the ratio of the twist angle in a tube with a slit to the twist angle of a tube without slit ($\frac{\phi_{open}}{\phi_{closed}}$=?) if R/t=10? =?
- 300
- 10
- 0.1
- 0.003

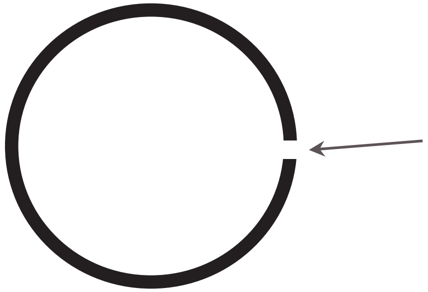


# Open sections vs closed sections

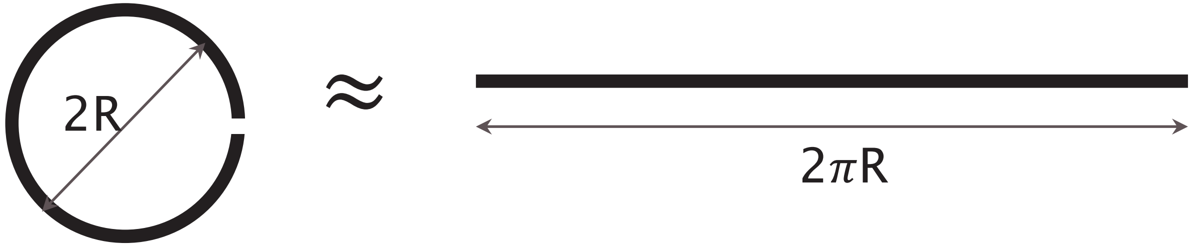



- Last lecture: shear flow is constant in closed sections. ($q = \tau t = constant$)

- But if the section is not closed, $q = 0$ at the slit.



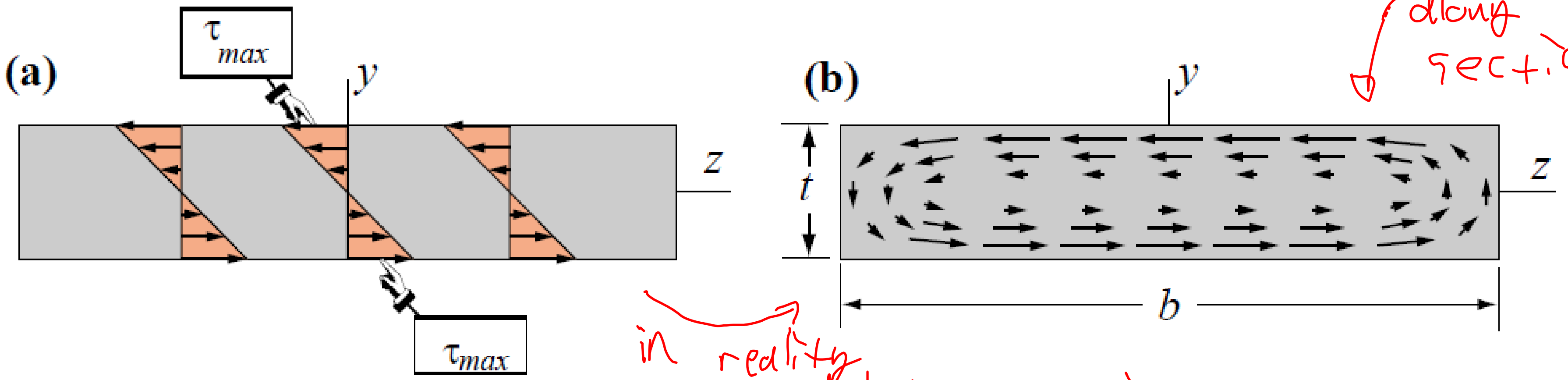

- We can estimate the torsion of open sections with thin rectangular sections.



# Assumptions: torsion of rectangular strips

- Assume the rectangular strip is thin (thickness 10x smaller than length). Therefore:
  1. Shear stress across the thickness can be approximated as a linear variation $\tau = \tau_{max}\frac{y}{t/2}$
  2. Shear stress distribution along the strip is constant except for areas close to the edges ($z = \pm b/2$) where it decreases

shear stress along cross section



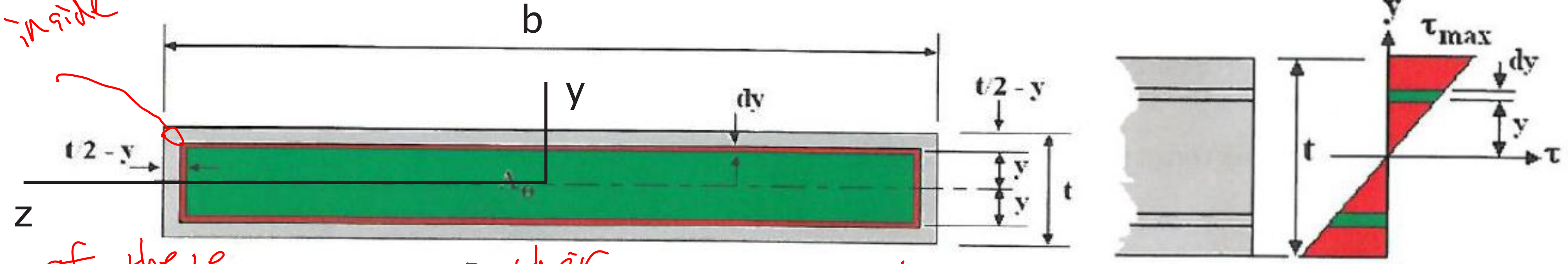

in reality
linear model less effective at corners

approximated using

# Solid Rectangular strip made of co-centric closed tubes

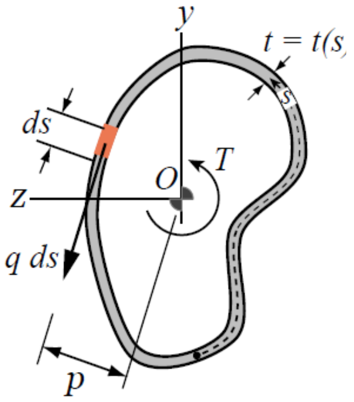
- Consider the rectangular strip as a concentric series of closed tubes that all have the same angle of twist $\phi$ (compatibility):

(hollow) tiny closed section inside



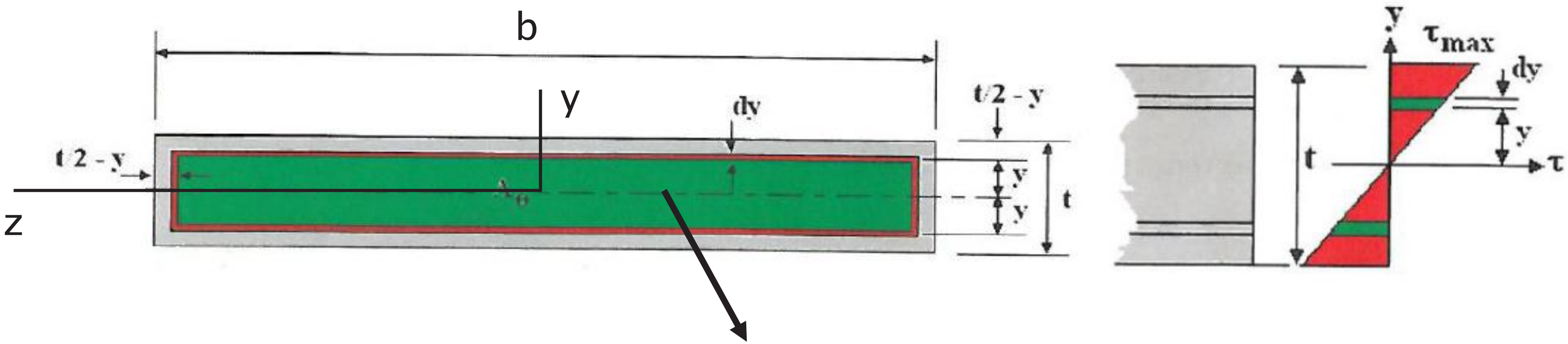

make lots of these closed loops → sum up their effect (integration) → solved

- We can use closed section equation $\tau = \dfrac{T}{2A_m t}$ for the assumed concentric tubes.



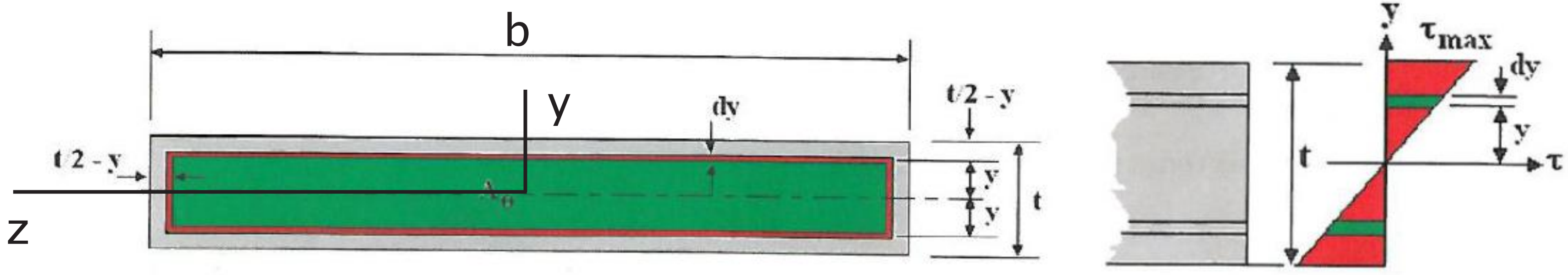
# Area enclosed by the median line for each concentric close tube

- $A_m = 2y\left[b - 2\left(\frac{t}{2} - y\right)\right] = 2yb \overbrace{- 2yt + 2y^2}^{\text{tiny because } b>>y \text{ and } b>>t} \approx 2yb$




$A_m \approx 2yb$

# Max shear stress in a thin rectangle section




- Replacing $T$ by $dT$ and t by $dy$ in $\tau = \frac{T}{2A_m t}$ as the infinitesimal torque and infinitesimal thickness of this closed section, we can write:

$$\tau = \frac{dT}{2(2yb)dy} \Rightarrow \; dT = 4\tau yb\, dy$$

- $T = \int dT = \int_0^{t/2} 4\tau yb\, dy = \int_0^{t/2} 4\tau_{max} \frac{y}{t/_2} yb\, dy = \frac{8b\tau_{max}}{t} \int_0^{t/2} y^2\, dy =$

$$\left[\frac{8b\tau_{max}}{t}\frac{y^3}{3}\right]_0^{t/2} = \frac{\tau_{max}}{t}\boxed{\frac{bt^3}{3}} = \frac{\tau_{max}}{t} J \Rightarrow \tau_{max} = \frac{\pm Tt}{J} \text{ where } J = \frac{bt^3}{3}$$

geometry based

This J is only for rectangular sections

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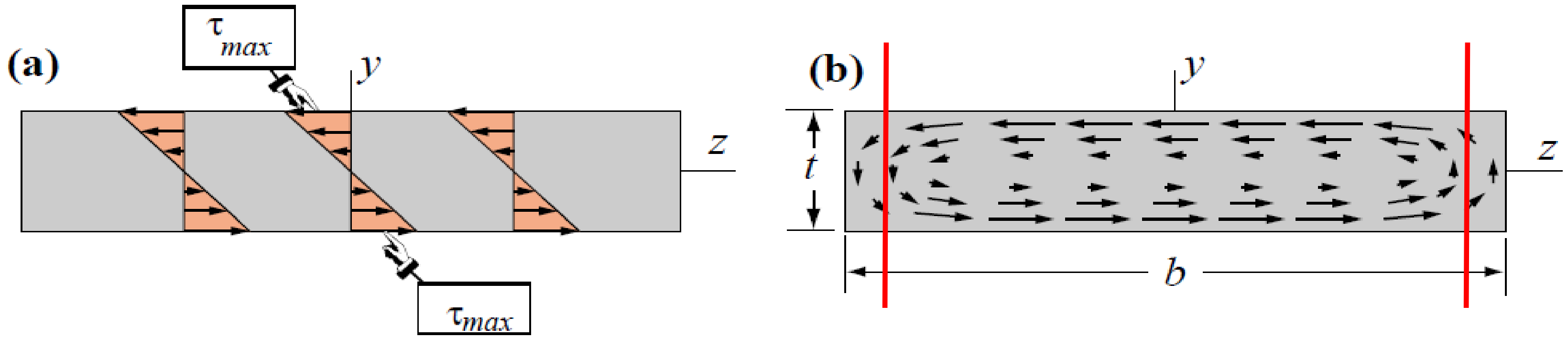
# Shear stress in rectangular strips




- For a rectangular strip the shear stress is: $\boxed{\tau = 2y\frac{T}{J}}$ $(0 \leq y \leq t/2)$
  where: $J = \frac{bt^3}{3}$

- The max shear occurs at the outer surface $(y = t/2)$: $\boxed{\tau_{max} = \pm\frac{Tt}{J}}$

# Shear stress and twist angle of a rectangular strip

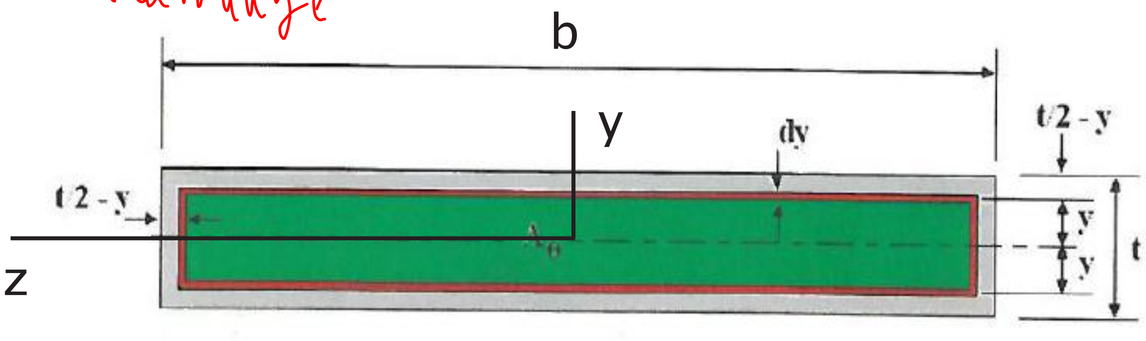
rate of change of twist along beam

- Using our previous eq. for the rate of twist of a closed tube:

$$\frac{d\phi}{dx} = \frac{1}{2A_m G}\oint \tau ds \quad \Rightarrow \quad \phi = \frac{L}{2A_m G}\oint \tau ds \quad \text{or} \quad \tau = \frac{2A_m G\phi}{L\oint ds}$$

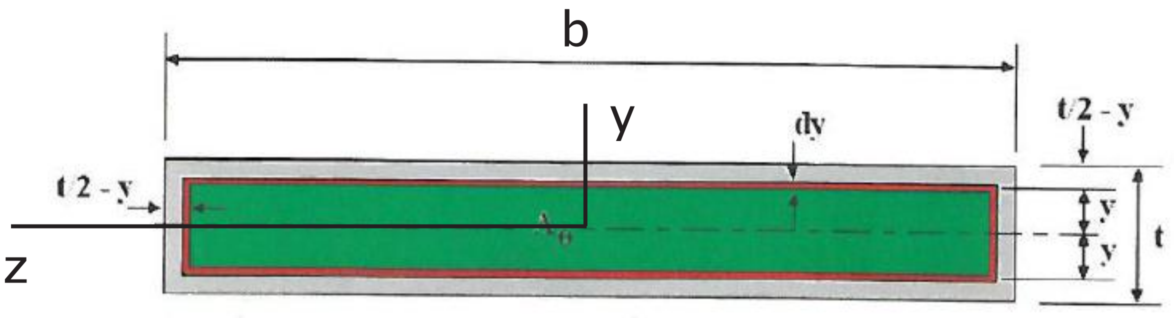
$\frac{d\phi}{dx} = \text{const}$ rearrange

- Using the geometry of the thin strip:
  - The perimeter $\oint ds$ is approximately $2b$
  - The area $A_m$ is approximately $2yb$




- $\tau = \frac{2A_m G\phi}{L\oint ds} = \frac{4byG\phi}{2Lb} = 2yG\frac{\phi}{L}$ where $0 \leq y \leq t/2$

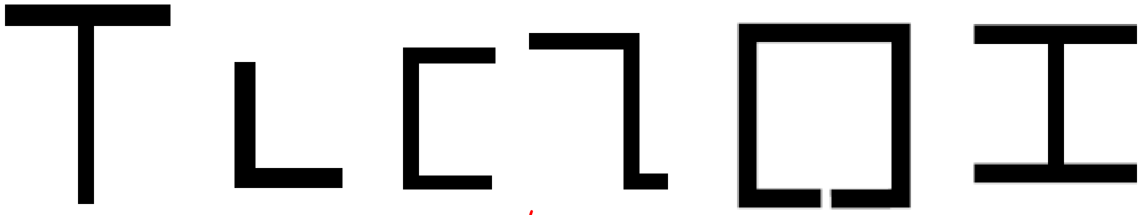
# Twist of a rectangular strip



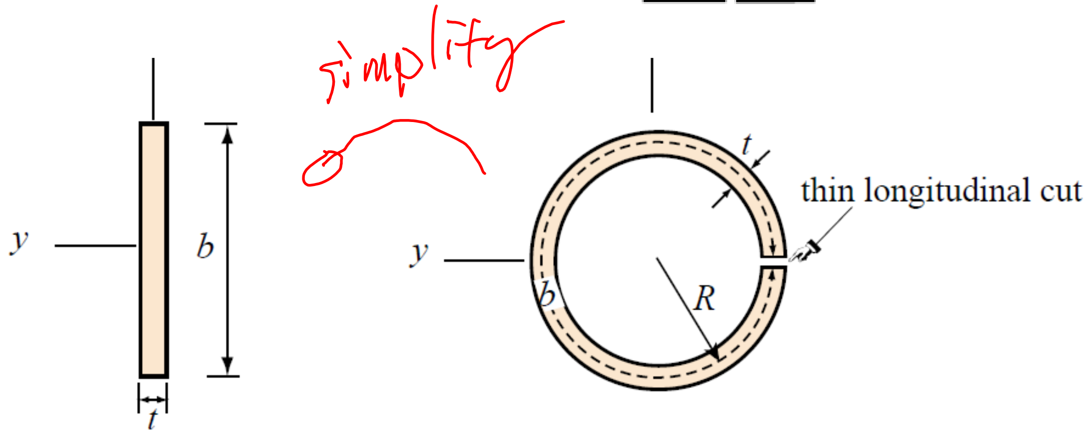

- Combine these equations:
- $T = \int_0^{t/2} 4\tau by\, dy$ and $\tau = 2yG\frac{\phi}{L}$
- $T = \int_0^{t/2} 4\left(2yG\frac{\phi}{L}\right) by\, dy = \left[8b\frac{y^3}{3}G\frac{\phi}{L}\right]_0^{t/2}$
- $T = G\frac{bt^3}{3}\frac{\phi}{l}$ or $\boxed{\phi = \frac{TL}{GJ}}$ ← equation same as last lecture J is what changes (duh)
- where $\boxed{J = \frac{bt^3}{3}}$ is the torsional constant for a thin rectangular strip

# Extension to arbitrary open sections

- Most opens sections are made of a series of rectangular strips:



- Narrow curved sections will have similar shear stress distribution




Narrow rectangle and slotted tube (cut-along) tube sections can be treated by the same method

- Assumptions: All rectangular strips in the arbitrary section undergo the same twist '$\phi$' which is constant over the section (compatibility)

# Shear Stress and Twist for Arbitrary Open Sections

just add all
the rectangle equivilent cross section
elements in yd

- For one element:

$$J = \frac{bt^3}{3}$$

$$\boxed{\tau = 2y\frac{T}{J}}$$

$$\boxed{\tau_{max} = \pm\frac{Tt}{J}}$$

$$\boxed{\phi = \frac{TL}{GJ}}$$

For N elements:

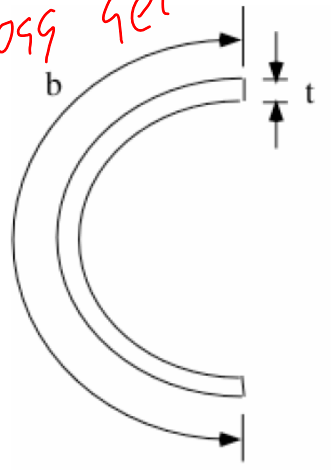
$$J \approx \sum_i^N \frac{b_i t_i^3}{3}$$

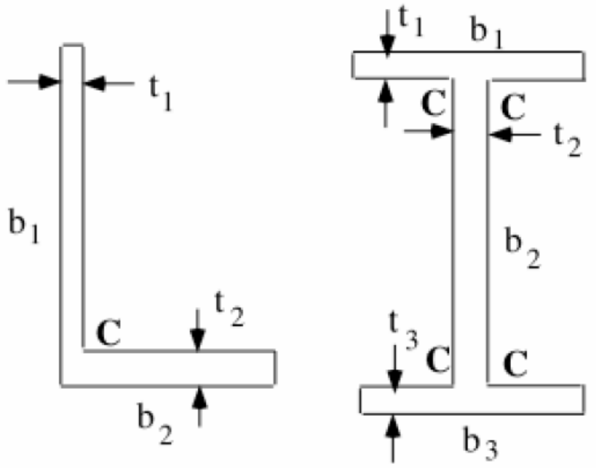
$$\boxed{\tau_i = 2y_i\frac{T}{J}} \qquad 0 \leq y \leq t/2$$

$$\boxed{\tau_{i,max} = \pm\frac{Tt_i}{J}}$$

$$\boxed{\phi = \frac{TL}{GJ}}$$

Angle of twist is the same for all elements.



# What happens if the section is not 'thin-walled'?

- The coefficient of the torsional constant J must be modified from 1/3 to account for this!
- Using an exact analytical solution (not covered here) we obtain:

Shear stress: $\tau_{max} = \pm \frac{Tt}{J_\alpha}$ Angle of twist: $\phi = \frac{TL}{GJ_\beta}$

Where: $J_\alpha = \alpha b t^3$ $J_\beta = \beta b t^3$

equivilent to what we've just done

| $b/t$ | 1.0 | 1.2 | 1.5 | 2.0 | 2.5 | 3.0 | 4.0 | 5.0 | 6.0 | 10.0 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.208 | 0.219 | 0.231 | 0.246 | 0.258 | 0.267 | 0.282 | 0.291 | 0.299 | 0.312 | 1/3 |
| $\beta$ | 0.141 | 0.166 | 0.196 | 0.229 | 0.249 | 0.263 | 0.281 | 0.291 | 0.299 | 0.312 | 1/3 |

derivation too hard, correction factors Imao

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# Question

- What's the ratio of the twist angle in a tube with a slit to the twist angle of a tube without slit ($\frac{\phi_{open}}{\phi_{closed}}$=?) if R/t=10? =?
- 300
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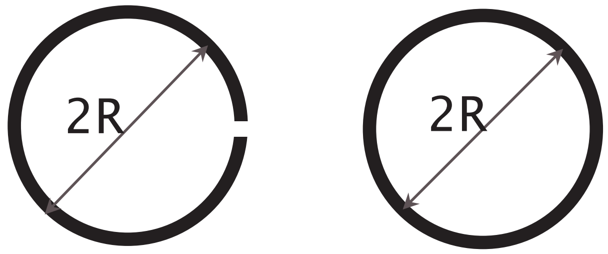
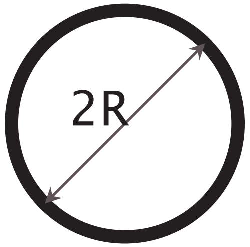
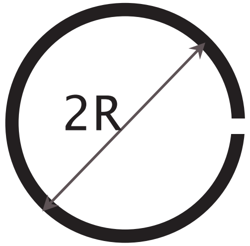


# Open circular section vs closed circular section




Close section




Open section

- $\phi_{CS} = \frac{TL}{GJ_{CS}}$ where$J_{CS} = \frac{4A_m^2}{\oint \frac{ds}{t}}$
- $J_{CS} = \frac{4A^2}{\oint \frac{ds}{t}} = \frac{4\left(\pi R^2\right)^2 t}{2\pi R}$=$2\pi R^3 t$
- $\phi_{OS} = \frac{TL}{GJ}$ where$J_{OS} \approx \sum_i^N \frac{b_i t_i^3}{3}$
- $J_{OS} = \frac{bt^3}{3} = \frac{2\pi R t^3}{3}$

$$\frac{\phi_{open}}{\phi_{closed}} = \frac{J_{CS}}{J_{OS}} = \frac{2\pi R^3 t}{\frac{2}{3}\pi R t^3} = 3\left(\frac{R}{t}\right)^2$$

If $R/t$=10 then $J_{CS}/J_{OS}$=300 ⇒The OS CIRCULAR tube will twist 300 times more (twist angle $\phi$) than the CS CIRCULAR tube when subjected to the same torsional moment *T!*

# Question

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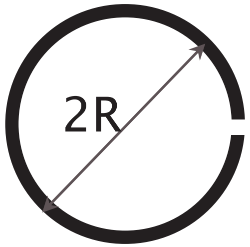
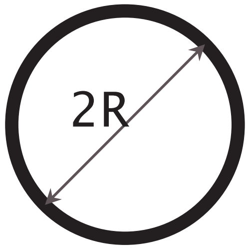
- Why $J_{OS}$ open sections is significantly smaller than $J_{CS}$ closed sections?



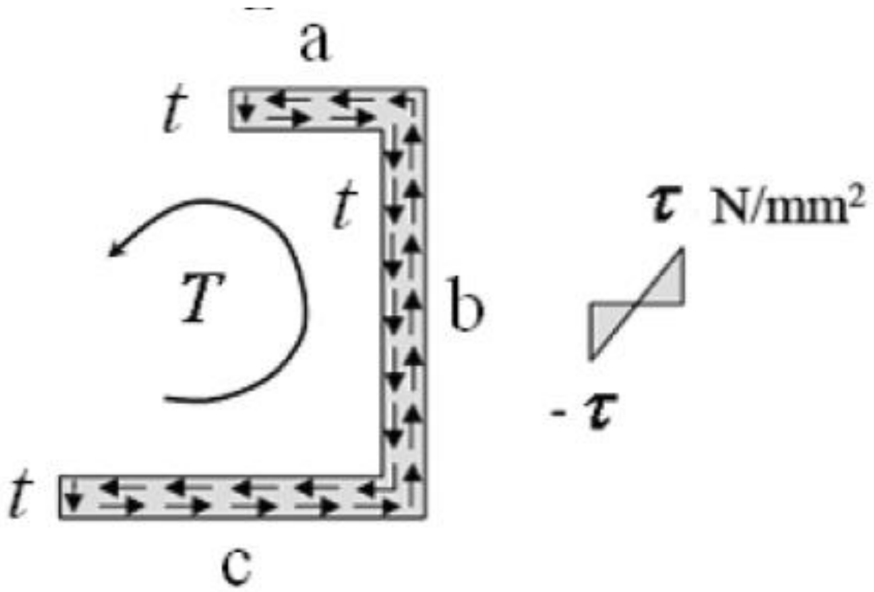
# Summary: Torsion in Thin-Walled Sections

| | Closed Sections: | Open Sections: |
|---|---|---|
| 1) Stress: | $\boxed{\tau_{max} = \frac{T_{max}}{2A_m t_{min}}}$ | $\boxed{\tau_{i,max} = \pm\frac{Tt_i}{J}}$ |
| 2) Deformation: | $\boxed{\phi = \frac{TL}{GJ}}$ | $\boxed{\phi = \frac{TL}{GJ}}$ |
| Torsional Constant: | $\boxed{J = \frac{4A_m^2}{\oint \frac{ds}{t}}}$ | $\boxed{J \approx \sum_i^N \frac{b_i t_i^3}{3}}$ |

- Closed sections are much better at supporting torsion!

# Next: Example 1/2 – Have a go first!

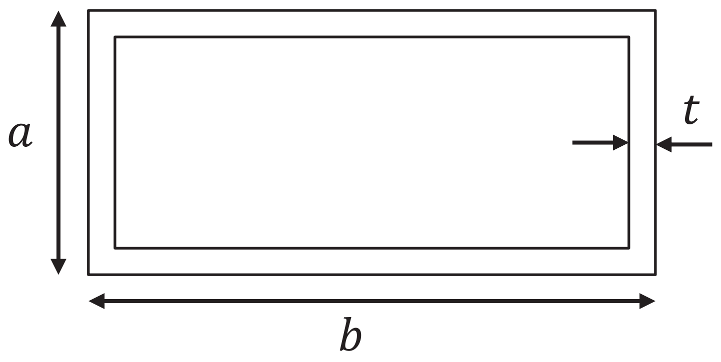
- For the cross section shown calculate the maximum shear stress and angle of twist if a torsional moment of 30 Nm is applied to the section. The geometrical parameters of the section are: a = 40mm, b = 100mm, c=80mm, t=3mm. The section is made of steel (G = 79 GPa) and is 1m long.



# Next: Example 2/2 – Have a go first!

- For the thin-walled rectangular section shown a torsional moment of 50 Nm is applied, consider two cases: 1) where the section is fully closed and 2) a small slit is cut in one of the corners (i.e. the section is open).
- What is the maximum shear stress and rate of twist for each case?
- The geometrical parameters of the section are: a = 35mm, b = 75mm, t=2mm. The section is made of steel (G = 79 GPa).

Case 1




Case 2