SESA3029 Aerothermodynamics

Lecture 1.4

Pitot probe in compressible flow

Adiabatic flow

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

Shock jump relations

$$\frac{\rho_2}{\rho_1} = \frac{M_1^2 (\gamma + 1)}{2 + (\gamma - 1) M_1^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma\left(M_1^2 - 1\right)}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{p_2 / p_1}{\rho_2 / \rho_1}$$

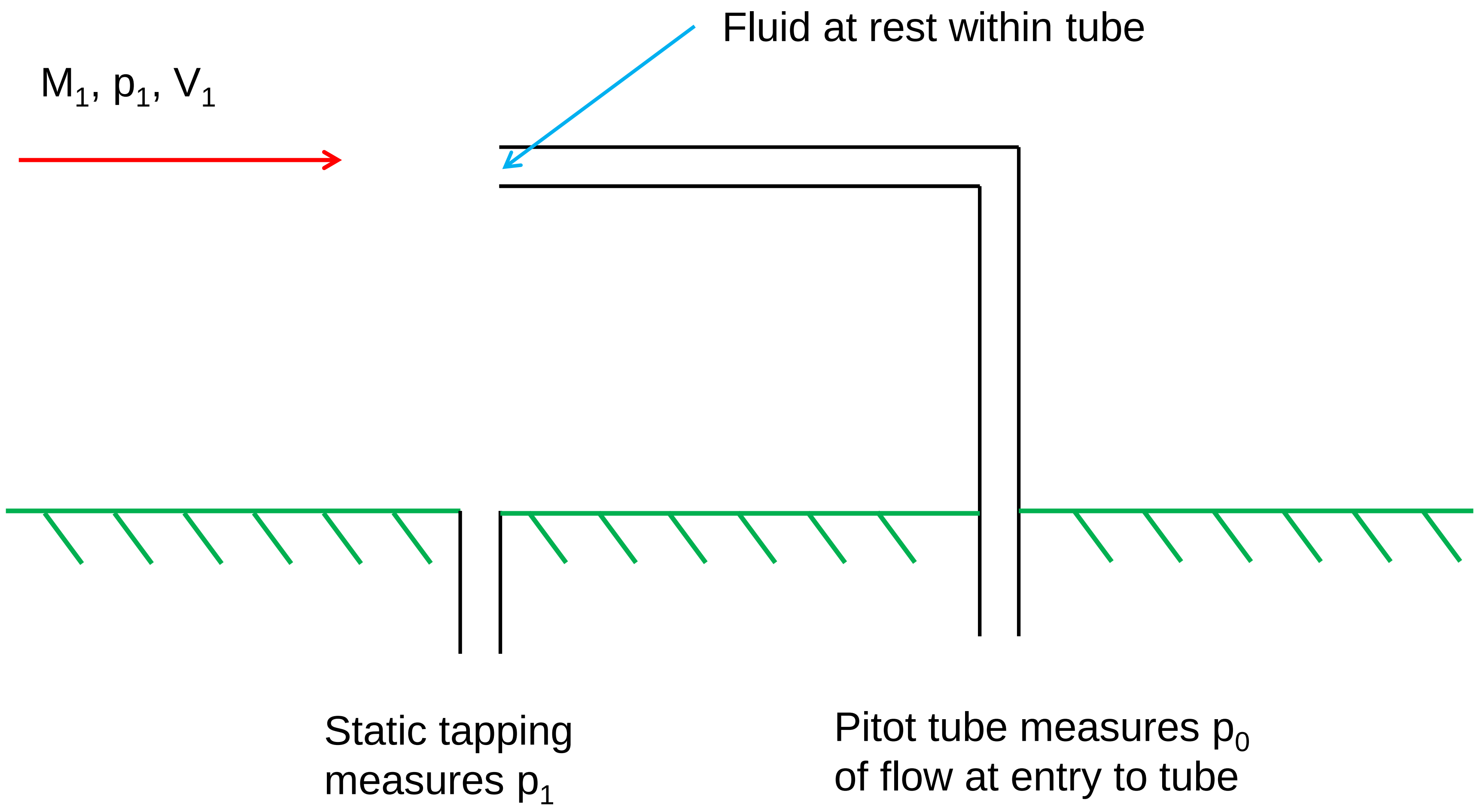
$$M_2^2 = \frac{2 + (\gamma - 1) M_1^2}{2\gamma M_1^2 - (\gamma - 1)}$$

Today

Velocity measurement using pitot probe, considering three cases:

- incompressible,
- compressible subsonic,
- compressible supersonic

Pitot probe


Fluid at rest within tube
M_1, p_1, V_1
Static tapping measures p_1
Pitot tube measures p_0 of flow at entry to tube

Incompressible flow

Bernoulli equation: $$p_1 + \frac{1}{2}\rho V_1^2 = p_0$$

In terms of a pressure coefficient $$C_{p_0} \equiv \frac{p_0 - p_1}{\frac{1}{2}\rho V_1^2} = 1$$ (stagnation condition)

Rearrange for $$\boxed{V_1 = \sqrt{\frac{2(p_0 - p_1)}{\rho}}}$$

Compressible subsonic flow

Isentropic flow

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}$$

Rearrange for

$$M_1^{\cancel{2}} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

and

$$\boxed{V_1 = a_1 M_1 = \sqrt{\frac{2\gamma R T_1}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}}$$

$a = \sqrt{\gamma R T}$

T_1 can be estimated by altitude, or measured

Question: looks very different, shouldn't this be consistent with the incompressible flow result for $M_1 \rightarrow 0$?

$$V_1 = a_1 M_1 = \sqrt{\frac{2\gamma RT}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \qquad \frac{p_0}{p_1} = \left(1 + \underbrace{\frac{\gamma - 1}{2} M_1^2}_{\varepsilon}\right)^{\overbrace{\frac{\gamma}{\gamma-1}}^{a}}$$

For small M the isentropic relation is of the form

(here we are showing that the supersonic equation converges towards the incompressible version as speed decreases.

$$\frac{p_0}{p_1} = (1+\varepsilon)^a = 1 + a\varepsilon + \frac{a(a-1)}{2}\varepsilon^2 + \dots$$

Binomial expansion

$$\frac{p_0 - p_1}{p_1} = \frac{\gamma}{2} M_1^2 + O\left(M_1^4\right)$$

$\rho R T = P$

$$\frac{p_0 - p_1}{\frac{1}{2}\rho_1 V_1^2} = \frac{p_1}{\frac{1}{2}\rho_1 V_1^2}\left(\frac{\gamma}{2} M_1^2 + O\left(M_1^4\right)\right)$$

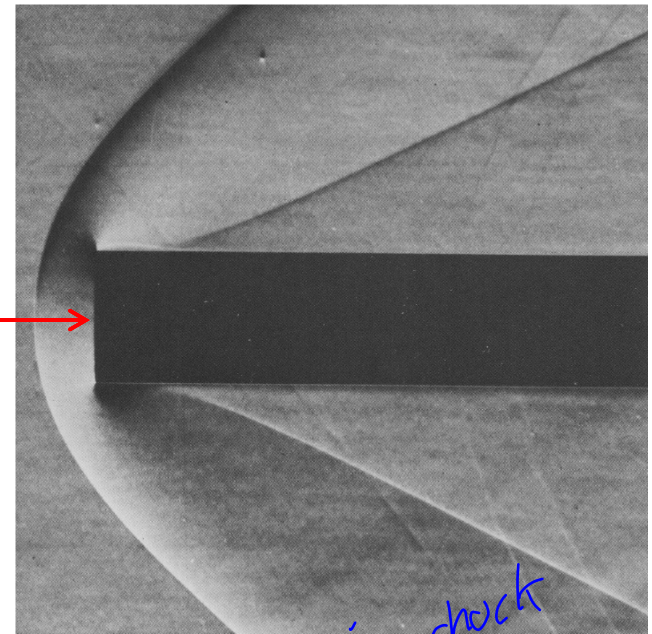
$$= \frac{2}{\gamma M_1^2}\left(\frac{\gamma}{2} M_1^2 + O\left(M_1^4\right)\right) = 1 + O\left(M_1^2\right)$$

i.e. consistent with i/c flow result

Compressible supersonic flow

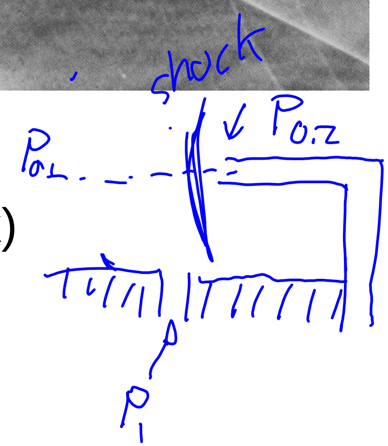
Photograph of M_1=2 flat-faced cylinder (comparable to entry to pitot probe)

(centre of normal shock has $v=0$)



Flow characteristics:

- Uniform isentropic flow to shock wave ($M=M_1$)
- Normal shock jump relations (gives M_2 straight after shock)
- Isentropic flow from after shock to stagnation ($M<1$)



shock means

$P_{02} \neq P_{01}$

Compressible supersonic flow

Analysis method:

• Normal shock jump relations (give M_2 p_2 straight after shock)

• Isentropic flow from after shock to stagnation (p_2 to $p_{0,2}$)

$$\frac{p_{0,2}}{p_1} = \frac{p_2}{p_1}\frac{p_{0,2}}{p_2}$$

From normal shock relations

From isentropic flow relations

from pitot tube

$$\frac{p_{0,2}}{p_1} = \left(1+\frac{2\gamma\left(M_1^2-1\right)}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}} \quad \text{with} \quad M_2^2 = \frac{2+\left(\gamma-1\right)M_1^2}{2\gamma M_1^2-\left(\gamma-1\right)}$$

can be estimated from altitude

$$\frac{p_{0,2}}{p_1} = \left(1 + \frac{2\gamma\left(M_1^2 - 1\right)}{\gamma + 1}\right)\left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \quad \text{with} \quad M_2^2 = \frac{2 + (\gamma - 1) M_1^2}{2\gamma M_1^2 - (\gamma - 1)}$$

Rearranges to

$$\boxed{\frac{p_{0,2}}{p_1} = \left[\frac{(\gamma + 1)^{\gamma+1}\left(\frac{1}{2} M_1^2\right)^{\gamma}}{2\gamma M_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}}}$$

The Rayleigh pitot formula

Tabulated as "$p_{0,2}/p_1$" in NST
(only to be used for normal shock configuration)

At M_1=1 both formulae reduce to

$$\frac{p_0}{p_1} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma-1}} = 1.893$$

(for γ=1.4)

Example

p_1=80 kPa
T_1=2°C
$p_{0,2}$=400 kPa

Find M_1 and V_1

Normal-shock table (γ = 1.4)**:**

M_{n1}	M_{n2}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{02}/p_{01}	"p_{02}/p_1"
1.8000	0.6165	3.6133	2.3592	1.5316	0.8127	4.6695
1.8200	0.6121	3.6978	2.3909	1.5466	0.8038	4.7618
1.8400	0.6078	3.7832	2.4224	1.5617	0.7948	4.8552
1.8600	0.6036	3.8695	2.4537	1.5770	0.7857	4.9497
1.8800	0.5996	3.9568	2.4848	1.5924	0.7765	5.0452

M_1=1.8705 $V_1=M_1a_1$=621.8 m/s

Pitot probe summary

If $\dfrac{p_0}{p_1} > \left(\dfrac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$ Supersonic flow (use NST)

If $\dfrac{p_0}{p_1} < \left(\dfrac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$ Subsonic flow (use IFT)

Reading: Anderson Section 8.7 pages 548-553