Astronautics (SESA2024)

Section 9: Space telecommunications – suggested solutions.

1.

Below about 500 MHz, microwave radiation is absorbed by electrons in the ionosphere, and above about 10 GHz it is absorbed by molecular gases in the atmosphere (water vapour and molecular oxygen). As a consequence, an atmospheric window is formed between about 100 MHz and 30 GHz, where the attenuation due to these sources is not significant, so allowing space-based communications at these frequencies. However, another source of attenuation, due to precipitation in the troposphere, can produce significant losses (> 10 dB) at frequencies above around 10 GHz. The advantage of using the higher frequency end of this window is that there is wider bandwidth available, which means higher data rates. The disadvantage is that if it rains hard at the ground station the high frequency signal from the S/C can be significantly attenuated – see 'absorption due to rain' in notes.

2 & 3. To calculate wavelength, use $\lambda = c/f$, with c in m/s and f in Hz. For example, at L-Band, $\lambda = 3\times10^8/1.5\times10^9 = 0.2\,\text{m}$.

To calculate the gain of a parabolic antenna of diameter D, use,

$G = \eta\left(\dfrac{\pi D}{\lambda}\right)^2$. So, for example, a 3 m dish at L-Band gives $G = 0.5\left(\dfrac{3\pi}{0.2}\right)^2 \approx 1110.$ In dBs, then, this becomes $G = 10\log_{10}(1110) = 30.45\,\text{dB}$.

To calculate the 3 dB beamwidth of a parabolic dish of diameter D, use $\theta_{3dB} \approx 72(\lambda/D)$, where the angle is expressed in degrees. For example, a 3 m dish at L-Band therefore gives $\theta_{3dB} \approx 72(0.2/3) = 4.8^\circ$. Required results are tabulated below:

Band	**Frequency (Hz)**	**Wavelength (m)**	**Gain* (dB)**	**θ_{3dB}* (degrees)**
L	1.5×10^9	0.2	30.45	4.8
C	4×10^9	0.075	38.97	1.8
X	8×10^9	0.0375	44.99	0.9
Ku	14×10^9	0.0214	49.87	0.51

*****For a 3 m diameter parabolic dish.

4.

Three types of digital modulation:

Amplitude Shift Keying (ASK): Amplitude of carrier is modulated, one amplitude signifying '0' and another amplitude corresponding to a '1'. The illustration in the notes shows an example with '0' denoted by zero amplitude.

Frequency Shift Keying (FSK): Frequency of carrier is changed, to distinguish between bits – see illustration.

Phase Shift Keying (PSK): Phase of carrier changes at boundary between dissimilar bit values. In the illustration, a phase change of 180^o is shown. PSK is the most commonly used technique in digital communications.

5. Work through and understand the derivation of the link budget equation given in the notes. Notice the introduction of noise (in the form of noise density N_0) near the end of the proof, and the identification of received power P_R with the carrier wave power C. This highlights the fact that the receiving antenna not only receives power from the transmitter, but also from other interfering sources, as illustrated in the notes.

   The ***Equivalent Isotropic Radiated Power (EIRP)*** of a satellite is the radiated power required of an imaginary isotropic radiator at the S/C's position, which would have the same effect as the S/C (in terms of power flux – W/m^2) at the ground station receiver.

   The ***(G_R/T_R) ratio*** of a receiver can be shown to be directly proportional to the received signal-to-noise ratio, and so is a commonly used figure of merit for the sensitivity of a receiver.

6. Advantages and disadvantages of a large communications antenna on a spacecraft:

   Advantages
   - High gain
   - Low level of electrical/radiated power required, relative to a small antenna

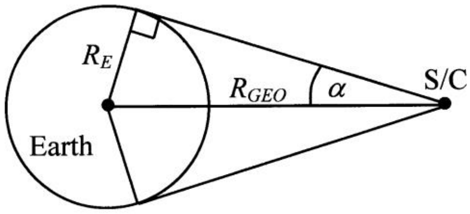
   Disadvantages
   - Possible difficulties of accommodation on S/C
   - Potential difficulties in accommodating antenna within launch vehicle fairing envelope
   - Potential for shadowing of arrays, obscuration of payload field of view, etc.
   - Small 3 dB beamwidth, and so more stringent pointing required
   - Etc.

   Power-gain tradeoff: Looking at the link budget equation, we can see that the characteristics of the transmitter (in this case, the satellite) are defined by the EIRP, which is given by EIRP = $P_T G_T$. Therefore, for a particular case, the link quality is assured for a particular value of the satellite EIRP – for example, for an interplanetary mission an EIRP of, say, 65 dBW (~ 3.16 × 10^6 W) may be required. If we express the equation for EIRP in dBs, we have in this case,

   $$\text{EIRP} = 65\,\text{dBW} = (P_T)_{dB} + (G_T)_{dB}.$$

   Hence, to produce an EIRP of 65 dBW, we can choose how it should be distributed between the radiated power of the transmitter P_T, and the gain of the transmitter G_T. This choice, which is driven by consideration of the advantages and disadvantages as listed above, is referred to as the power-gain tradeoff. So for our interplanetary spacecraft example, we could decide to have a large dish size (i.e. large gain) so that the power is small. This would be good generally since power generation on board such a spacecraft may be limited (due to the great distance from the Sun), but large dishes have there disadvantages as well (as itemised above). So the choice of how much P_T to have, and how much G_T to have, to make up the required EIRP, is the subject of a detailed tradeoff involving payload and subsystem issues.

7. From the geometry in the diagram, we have $\sin\alpha = R_E / R_{GEO} = 1/6.611$. Hence $\alpha = 8.7^\circ$, and the beamwidth for global coverage is $\theta_{3dB} = 2\alpha = 17.4^\circ = 72(\lambda / D)$,



At a frequency of 1.5 GHz, the wavelength is 0.2 m, so the dish size in m is $D = 72(0.2/17.4) \approx 0.83$ m. Finally, the gain in dBs is $G = 10\log_{10}\left(0.5\left[\frac{0.83\pi}{0.2}\right]^2\right) \approx 19.3\,\text{dB}.$

8. The BER is the probability that a data bit will be received incorrectly – for example, a BER of 10^{-1} means that you can expect one bit in 10 to be wrong. The chances of a bit being incorrectly received is strongly related to the amount of energy that is attributed to that bit in transmission, but it is also influenced by the level of noise power (interference) in which it is embedded on reception. Hence, there is a relationship between the BER and the E_b / N_0 ratio, where E_b is the energy per bit, and N_0 is the noise density. An example of this relationship is illustrated for PSK modulation in the notes. For the relationship between BER and C/N_0, see derivation in the notes.

9. Consider each term in the link budget equation:

C/N0:

$\frac{C}{N_0} = \frac{E_b}{N_0}\frac{1}{t_b} = \frac{E_b}{N_0}R_b$, where $\frac{E_b}{N_0} = 10\,\text{dB}$, and R_b is the bit rate (data rate in bps). Therefore, in dBs, we have

$$\left(\frac{C}{N_0}\right)_{dB} = 10 + (R_b)_{dB}.$$

EIRP:

This is given in dBs as EIRP = 65 dBW.

(GR/TR):

$$G_R = \eta\left(\frac{\pi D}{\lambda}\right)^2 = 0.5\left(\frac{\pi 70}{\lambda}\right)^2$$

where $\lambda = c/f = (3\times10^8)/(8.44\times10^9) = 0.0355\,\text{m}$. Therefore,

$$\left(\frac{G_R}{T_R}\right)_{dB} = 10\log_{10}\left(\frac{1.9139\times10^7}{28}\right) = 58.35\,\text{dB}.$$

Free space loss LFS:

$L_{FS} = 20\log_{10}\left(\dfrac{4\pi\rho}{\lambda}\right)$, where ρ = 40 AU = 40(1.5 × 10^8) km = 6 × 10^9 km = 6 × 10^{12} m $\Rightarrow$ $L_{FS} = 306.53$ dB.

L_A:

This is given as $L_A = 0$ dB.

k:

Finally, Boltzmann's constant in dBs is $(k)_{dB} = -228.60$ dB.

Therefore link budget equation becomes

$10 + (R_b)_{dB} = 65.00 + 58.35 - 306.53 - 0 + 228.60 \;\Rightarrow\; (R_b)_{dB} = 35.42\,\text{dB}$

$\Rightarrow\; R_b = 3483\,\text{bits/sec.}$

Hence time to transmit data

$= 2\times10^9/3483 \approx 5.742\times10^5$ sec ≈ 159.5 hours ≈ 6.6 days.

To show relationship between P_T and D:

$EIRP = P_T G_T = 65\,\text{dBW} \;\Rightarrow\; 10\log_{10} P_T G_T = 65.$ Hence,

$$P_T G_T = 10^{6.5} \;\Rightarrow\; P_T \eta\left(\frac{\pi D}{\lambda}\right)^2 = 10^{6.5} \;\Rightarrow\; P_T D^2 = 10^{6.5}\lambda^2/(\eta\pi^2) \approx 810.$$

Power-gain tradeoff:

Using this relationship we can produce data like that below to produce a plot:

P_T (W)	D (m)	
810	1	
202	2	
90	3	
50	4	etc.

The power-gain tradeoff involves arguments like those discussed in Q6. Given the distance from the Sun, it is likely that the designer would go for a large dish, with a relatively small power requirement – bearing in mind some of the disadvantages of having a large dish. Perhaps a dish around 3 to 4 m in diameter would be preferred. Final definitive sizing would require detailed analysis of the issues.