SEMESTER 1 ASSESSMENT 2021-2022

TITLE: AEROTHERMODYNAMICS

DURATION: 2 HOURS

Attempt all THREE questions.

Questions 1 is worth 40 marks and questions 2 and 3 are worth 30 marks each (total 100 marks). An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow and normal shock tables (11 sides) are provided. In reading from tables, nearest values are acceptable unless explicitly stated otherwise.

An oblique shock chart is provided.

A formula sheet is included at the end of this paper

Only University approved calculators may be used.

A foreign language direct ‘Word to Word’ translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), c_p=1004.5 J/(kg K), γ=1.4, ρ=1.225 kg/m^3 and μ=1.8x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

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Q.1

(i) A pitot-static probe measures a static pressure of 84.8 kPa and a stagnation pressure of 122 kPa. A temperature gauge adjacent to the static pressure transducer measures a temperature of 243 K. Find the velocity of the flow in m/s.

[4 marks]

(ii) A converging-diverging nozzle has a design exit Mach number of 4.5 at a static pressure of 12 kPa. Find the pressure in the nozzle throat in kPa.

[4 marks]

(iii) A Mach 1.5 flow expands around a 15° corner forming an expansion fan. Find the angle of the trailing Mach line relative to the oncoming flow.

[4 marks]

(iv) The critical Mach number of an aerofoil is found to be 0.64. Compute the minimum value of the surface pressure coefficient that would be measured on the same aerofoil in a wind tunnel operating at low speed.

[6 marks]

Q1 parts (v), (vi) and (vii) refer to a flat plate of chord c=12 cm placed at a positive angle of incidence α=15° to an oncoming supersonic flow at Mach number M_1=1.9 and p_1=50 kPa, as sketched in Figure Q.1 below. The leading edge of the plate is located a distance h above the wind tunnel wall, with h sufficiently large that the reflected wave from the wall doesn't interfere with the plate.

(v) Determine the normal force per unit width on the plate according to the shock-expansion method.

[10 marks]

(vi) Determine the normal force per unit width on the plate according to Ackeret's method.

[6 marks]

(vii) Determine whether the leading edge shock undergoes a regular or Mach reflection at the wind tunnel wall and explain why.

[6 marks]

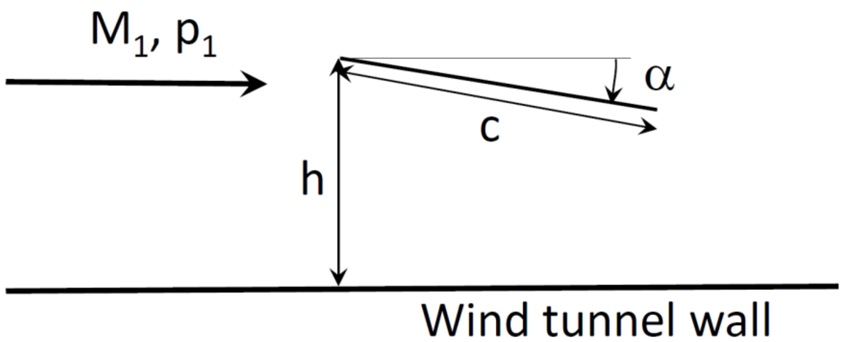

Figure Q.1 (not to scale)

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Q.2 A symmetric planar (2D) dual-bell nozzle is designed using a Method of Characteristics minimum-length method to have two operating points. One is for low altitude supersonic operation where the pressure at an intermediate point 'i', as sketched in Figure Q.2 below, is equal to the environment pressure p_b and one at high altitude, where there is an additional expansion from 'i' to the exit 'e'.

(i) Considering a high-altitude design case with M_i=1.6 and M_e=2.7, find the turning angle of the flow at points 't' (the throat) and 'i' and sketch the characteristic lines (start with 3 lines from each expansion fan).

[12 marks]

(ii) Sketch the Mach number and pressure distributions along the centreline of the nozzle, using a full range of values for the environment pressure p_b. Include additional sketches and commentary to explain important flow features. You can assume that the stagnation properties upstream of the nozzle throat are fixed. No calculations are necessary.

[18 marks]

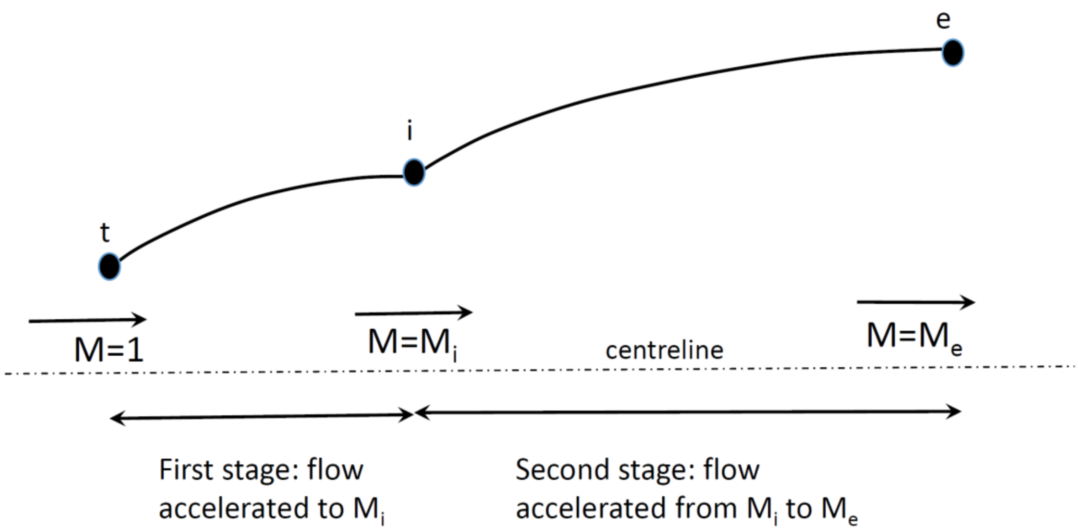

Figure Q.2 (not to scale)

Q.3 One end of a rod of diameter 10 mm and length 18 cm is inserted into a fixture maintained at 200°C. The rod is made of steel with thermal conductivity 14.8 W/(m K) and thermal diffusivity 3.63x10^{-6} m^2/s. Covered with an insulated sleeve, the rod reaches a uniform temperature throughout its length. When the sleeve is removed, the rod is subjected to ambient air at T_∞=25°C such that the convective heat transfer coefficient is 30 W/(m^2 K). The explicit finite difference (FD) method is used to approximate the temperature in the rod over time.

(i) Show that for an equidistant 1D grid in space and constant time steps the FD equation for an interior discretization node is

$$T_i^{n+1} = \left(1 - 2F - FB\right)T_i^n + F\left(T_{i+1}^n + T_{i-1}^n\right) + FBT_\infty,$$

while for the free end node it reads

$$T_I^{n+1} = \left(1 - 2F - FB - 2FD\right)T_I^n + 2FT_{I-1}^n + \left(FB + 2FD\right)T_\infty.$$

[14 marks]

(ii) Verify that for a spatial increment of Δx=3 cm and a time step of Δt=60 s the non-dimensional parameters read *F*=0.2420, *B*=0.7297, *D*=0.0608 and apply the FD method to estimate the time required for the mid-length of the rod to reach 110°C. Use linear interpolation to improve the result.

[10 marks]

(iii) A computer code implementing the FD method has produced the following output for Δx=2.25 cm:

t	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
168.75	150.4	133.7	129.5	128.8	128.6	128.2	127.2	125.4
202.50	145.9	125.9	120.0	118.6	118.3	117.8	116.9	115.3
236.25	142.2	119.3	111.6	109.5	109.0	108.5	107.6	106.2
270.00	139.0	113.6	104.4	101.5	100.6	100.1	99.3	98.0

Make use of this data and your estimate from (ii) to extrapolate the solution for $\Delta t \to 0$ and $\Delta x \to 0$. Comment on the two results.

[6 marks]

End of questions (formula sheet on next page)

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Useful Formulae

Perfect gas equation of state

$$p = \rho RT$$

Sound speed in a perfect gas

$$a^2 = \gamma RT$$

Adiabatic flow

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

Mach angle:

$$\sin \mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta + \mu)_A + (\theta + \mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta - \mu)_B + (\theta - \mu)_P\right]$$

$$x_p = \frac{x_B \tan \alpha_{BP} - x_A \tan \alpha_{AP} + y_A - y_B}{\tan \alpha_{BP} - \tan \alpha_{AP}}$$

$$y_P = y_A + (x_P - x_A) \tan \alpha_{AP}$$

Velocity potential equation:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

Linearised pressure coefficient

$$C_p=-2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

Ackeret formula:

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

Laminar pipe flow:

$$\mathrm{Nu}=4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}=3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x=0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu}=0.022\,\mathrm{Pr}^{0.5}\,\mathrm{Re}^{0.8}$$

Turbulent boundary layer:

$$\mathrm{Nu}_x=0.029\mathrm{Re}_x^{0.8}\mathrm{Pr}^{0.6}$$

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