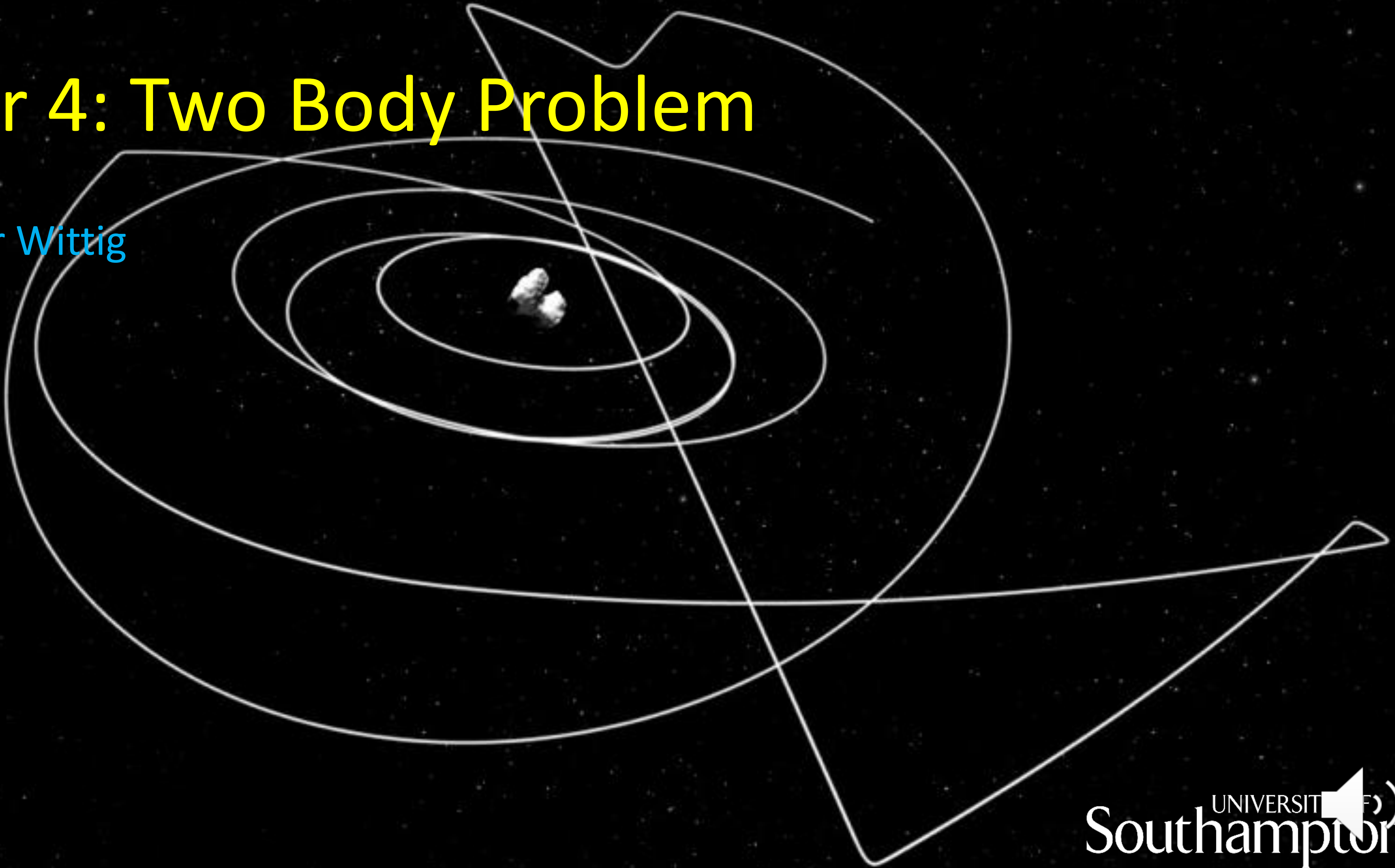
Advanced Astronautics (SESA3039)

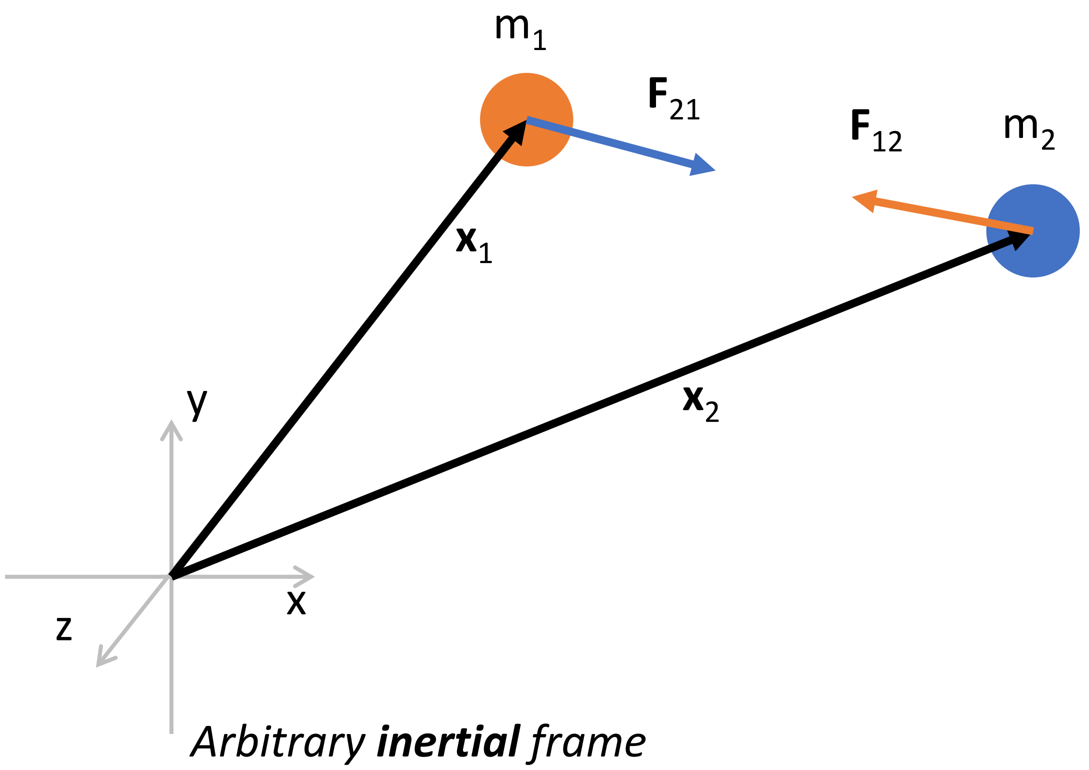
Chapter 4: Two Body Problem

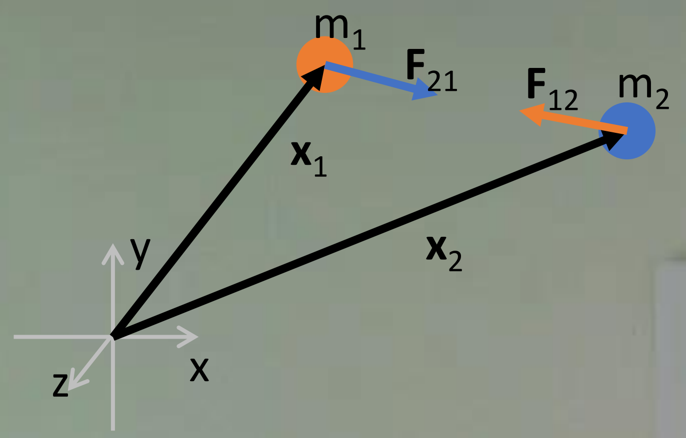
Dr. Alexander Wittig



Rosetta lander deployment orbit (ESA)

Aim: derive the full motion of two gravitating masses from first principles!




m1
F21
F12
m2
x1
x2
y
x
z

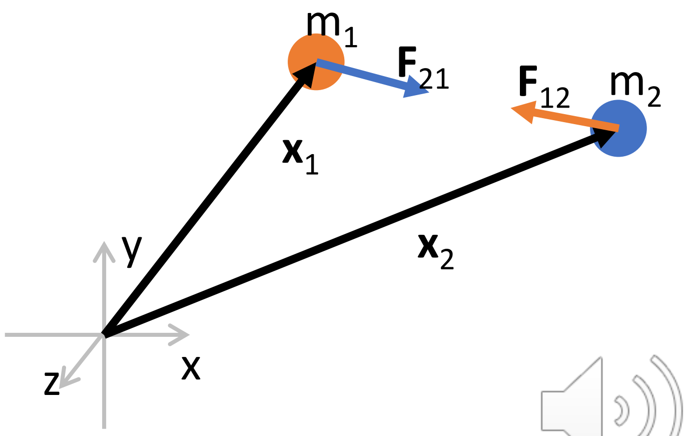
Newton's equations of motion

- Forces and accelerations:

$$\vec{F}_{21} = \frac{G m_1 m_2}{|\vec{x}_2 - \vec{x}_1|^3} (\vec{x}_2 - \vec{x}_1) \qquad \vec{F}_{21} = m_1 \vec{a}_1 = m_1 \ddot{\vec{x}}_1$$

$$\vec{F}_{12} = \frac{G m_1 m_2}{|\vec{x}_1 - \vec{x}_2|^3} (\vec{x}_1 - \vec{x}_2) \qquad \vec{F}_{12} = m_2 \vec{a}_2 = m_2 \ddot{\vec{x}}_2$$

Note: $\vec{F}_{21} = -\vec{F}_{12}$ (actio = reactio)



Newton's equations of motion

- Forces and accelerations:

$$\vec{F}_{21} = \frac{G m_1 m_2}{|\vec{x}_2 - \vec{x}_1|^3}(\vec{x}_2 - \vec{x}_1) \qquad \vec{F}_{21} = m_1 \vec{a}_1 = m_1 \ddot{\vec{x}}_1$$

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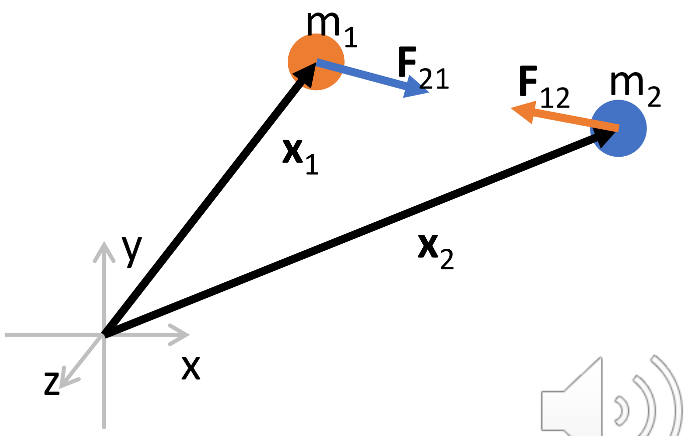
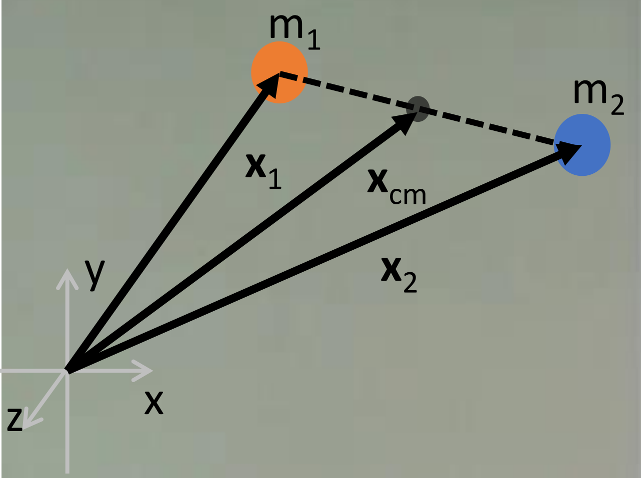
12 degrees of freedom!





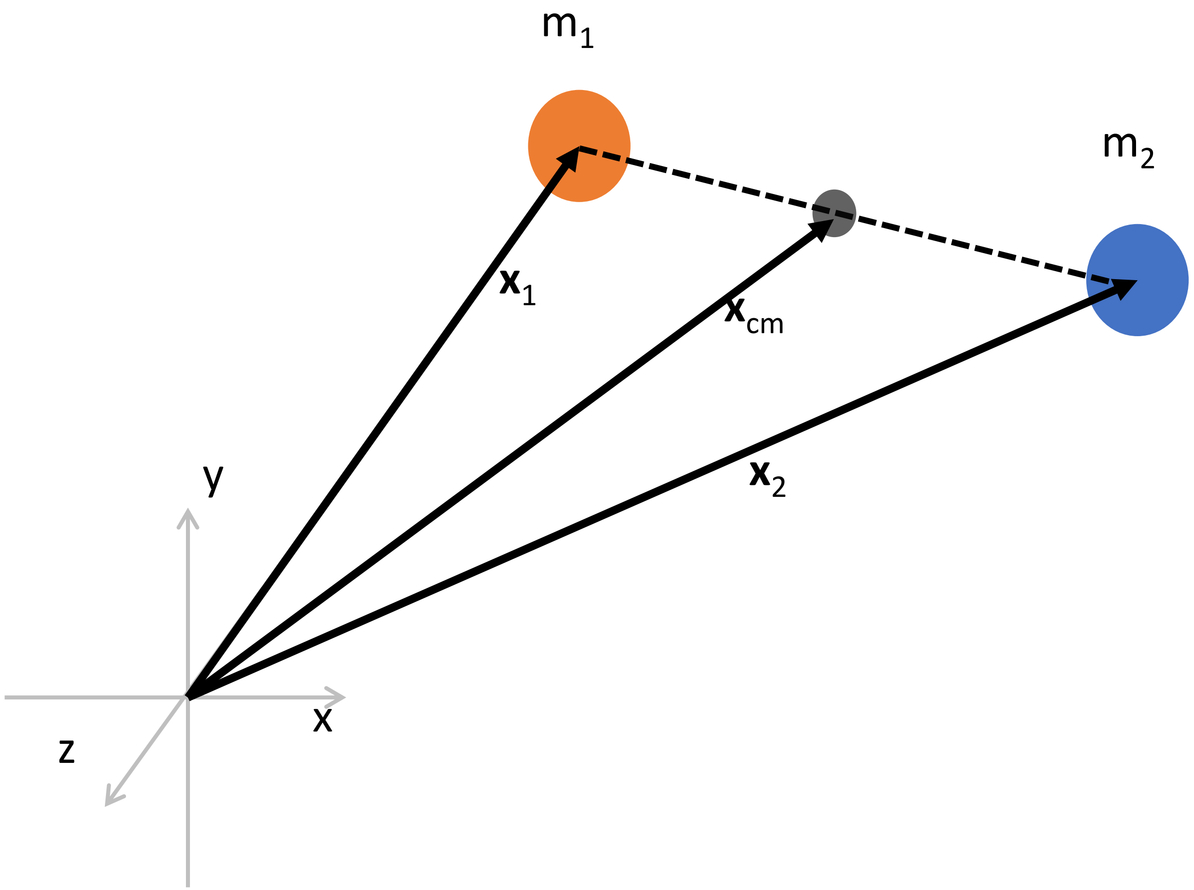
$$\vec{F}_{21} = m_1 \vec{a}_1 = m_1 \ddot{\vec{x}}_1 \qquad \vec{F}_{21} = \frac{G m_1 m_2}{|\vec{x}_2 - \vec{x}_1|^3}(\vec{x}_2 - \vec{x}_1)$$

$$\vec{F}_{12} = m_2 \vec{a}_2 = m_2 \ddot{\vec{x}}_2 \qquad \vec{F}_{12} = \frac{G m_2 m_1}{|\vec{x}_1 - \vec{x}_2|^3}(\vec{x}_1 - \vec{x}_2)$$

$$|\vec{F}_{21}| = \frac{G m_1 m_2}{|\vec{x}_2 - \vec{x}_1|^3} |\vec{x}_2 - \vec{x}_1| = \frac{G m_1 m_2}{|\vec{x}_2 - \vec{x}_1|^2}$$

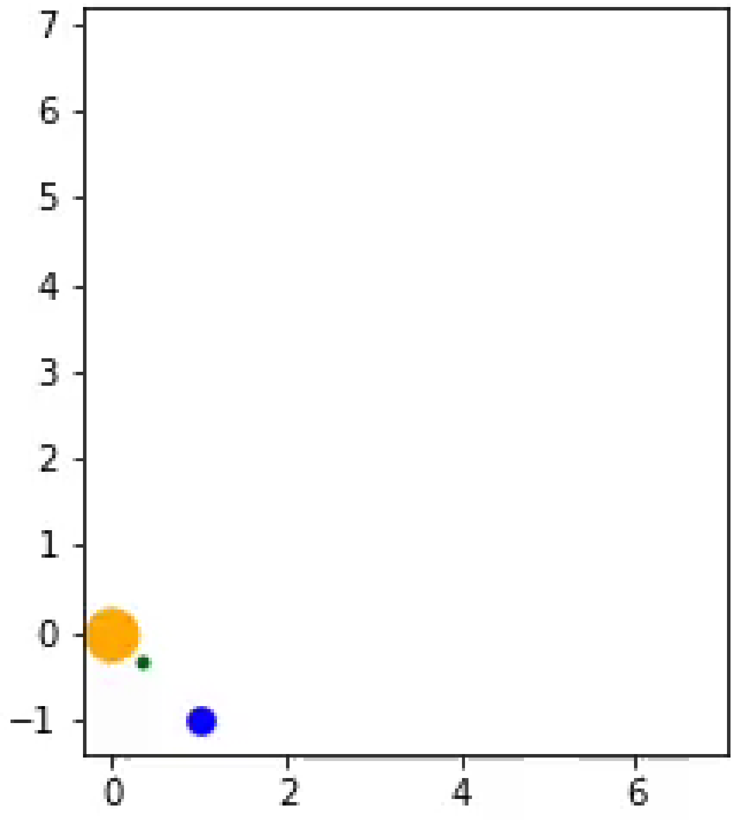
Center of mass / Barycenter

$$\vec{x}_{cm} = \frac{m_1 \vec{x}_1 + m_2 \vec{x}_2}{m_1 + m_2}$$



- 6 degrees of freedom
- Moves uniformly at constant velocity
- Coordinate frame attached at CM is inertial!

Center of mass / Barycenter



$$m_1+m_2$$

$$\dot{\vec{x}}_{cm} = \frac{m_1\dot{\vec{x}}_1 + m_2\dot{\vec{x}}_2}{m_1+m_2}$$

$$\ddot{\vec{x}}_{cm} = \frac{m_1\ddot{\vec{x}}_1 + m_2\ddot{\vec{x}}_2}{m_1+m_2} = \frac{\vec{F}_{21}+\vec{F}_{12}}{m_1+m_2} = \vec{0}$$

Relative distance

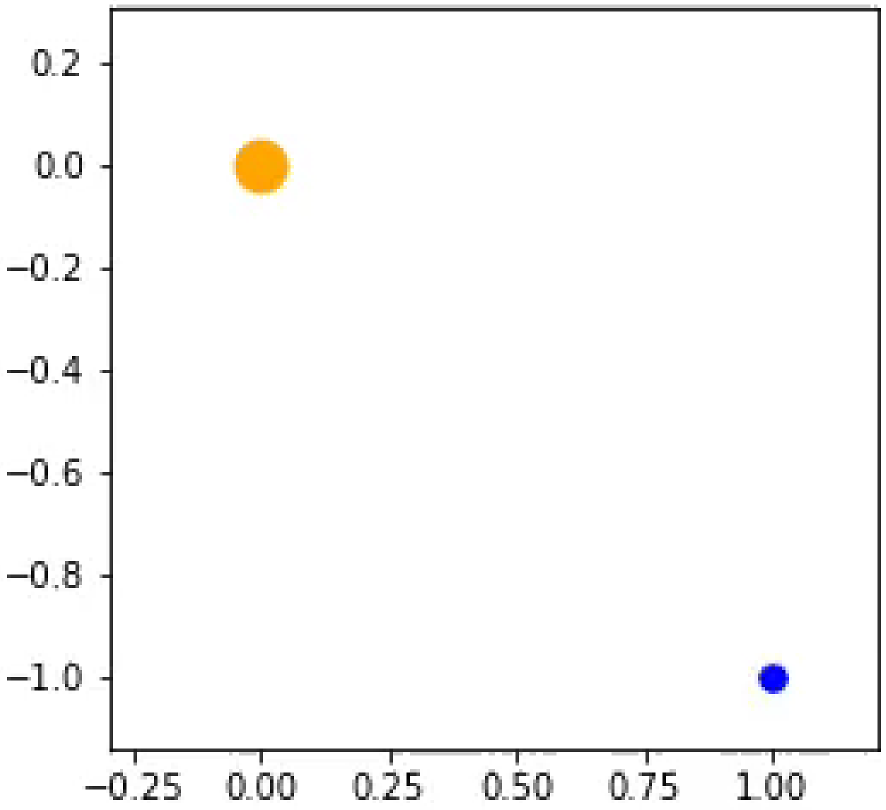
- Equation of motion exactly like central force problem with sum of masses:

$$\vec{d} = \vec{x}_2 - \vec{x}_1$$

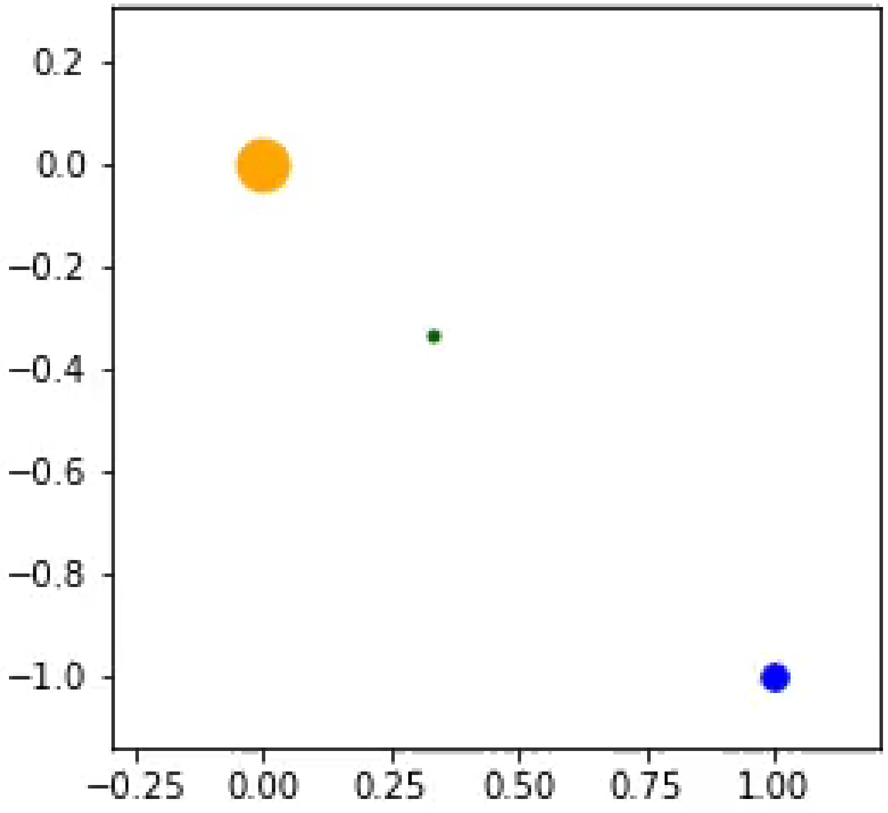
$$\ddot{\vec{d}} = -\frac{G(m_1 + m_2)}{|\vec{d}|^3}\vec{d} = -\frac{\mu}{|\vec{d}|^3}\vec{d}$$

- Solution of relative motion is a conic section (ellipse, parabola, hyperbola)
- BUT: fixing one body is then not an inertial frame!

Relative distance



Relative distance



Center of mass is moving along ellipse because frame not inertial!
Requires fictitious forces generating motion of CM!

Full solution to 2BP

- Center of mass: moves uniformly
- Relative distance: moves on ellipse
- Convert back from x_{cm} and d to x_1 and x_2:

$$\vec{x}_1 = \vec{x}_{cm} - \frac{m_2}{m_1 + m_2}\vec{d}$$

$$\vec{x}_2 = \vec{x}_{cm} + \frac{m_1}{m_1 + m_2}\vec{d}$$

Observations:

- Both bodies on ellipses around barycenter
- Heavy mass moves closer to barycenter than light mass

$$\vec{x}_1 = \vec{x}_{cm} - \frac{m_2}{m_1 + m_2}\vec{d}$$

$$\vec{x}_2 = \vec{x}_{cm} + \frac{m_1}{m_1 + m_2}\vec{d}$$

$$\mu = G(m_1 + m_2)$$

If $m_1 \ggg m_2$:

- $\mu \approx Gm_1$
- $\vec{x}_{cm} \approx \vec{x}_1$

Restricted Two-Body Problem

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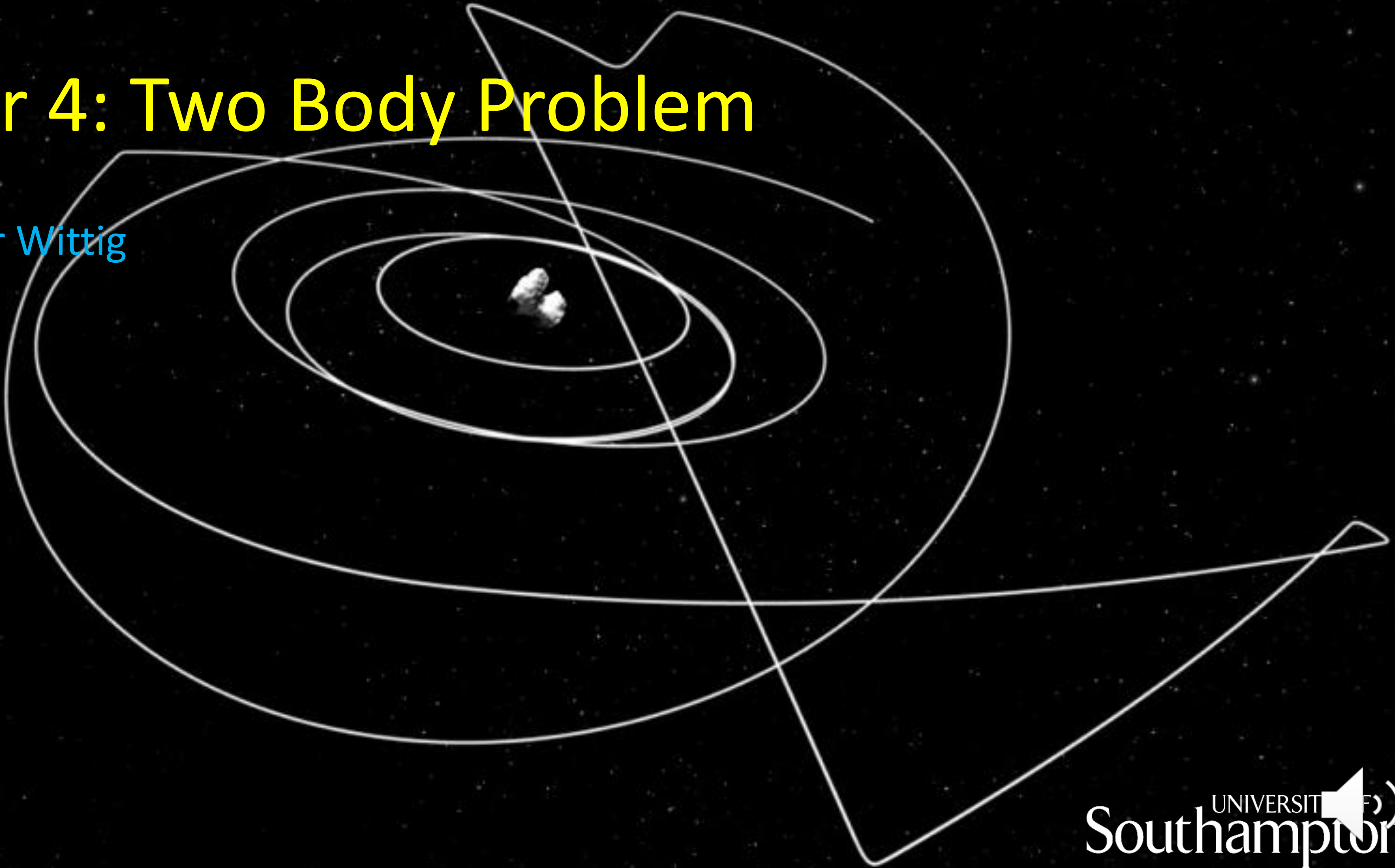

Rosetta lander deployment orbit (ESA)