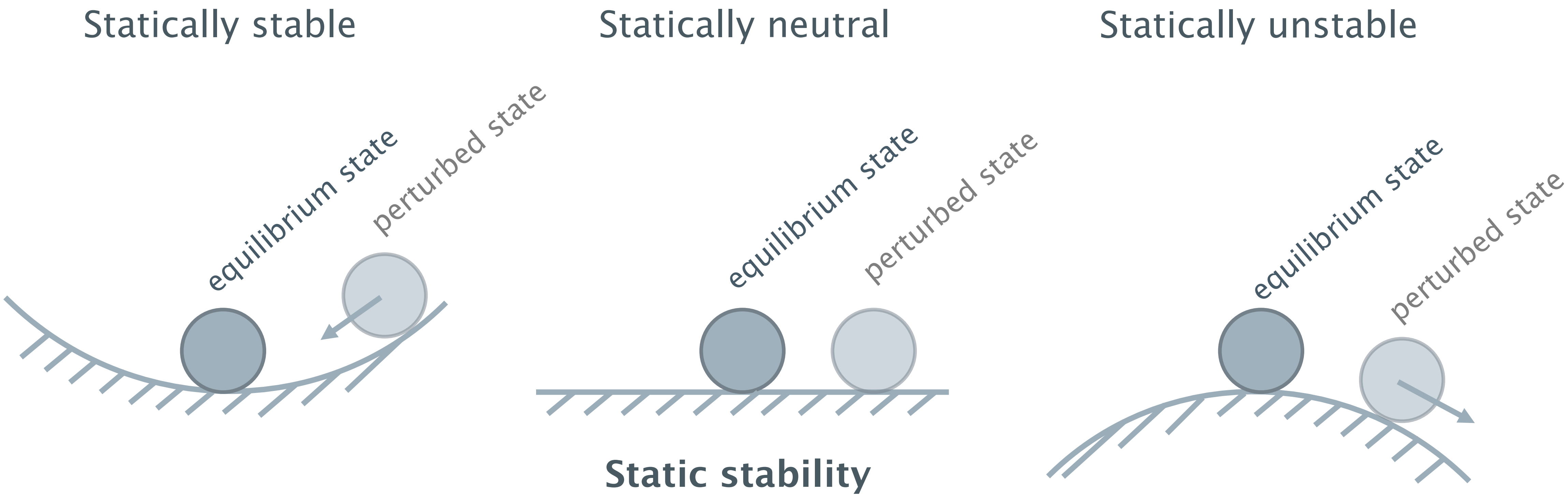
SESA2025 Mechanics of Flight
Longitudinal Static Stability

Lecture 2.1

Static* stability

Illustrations of static stability



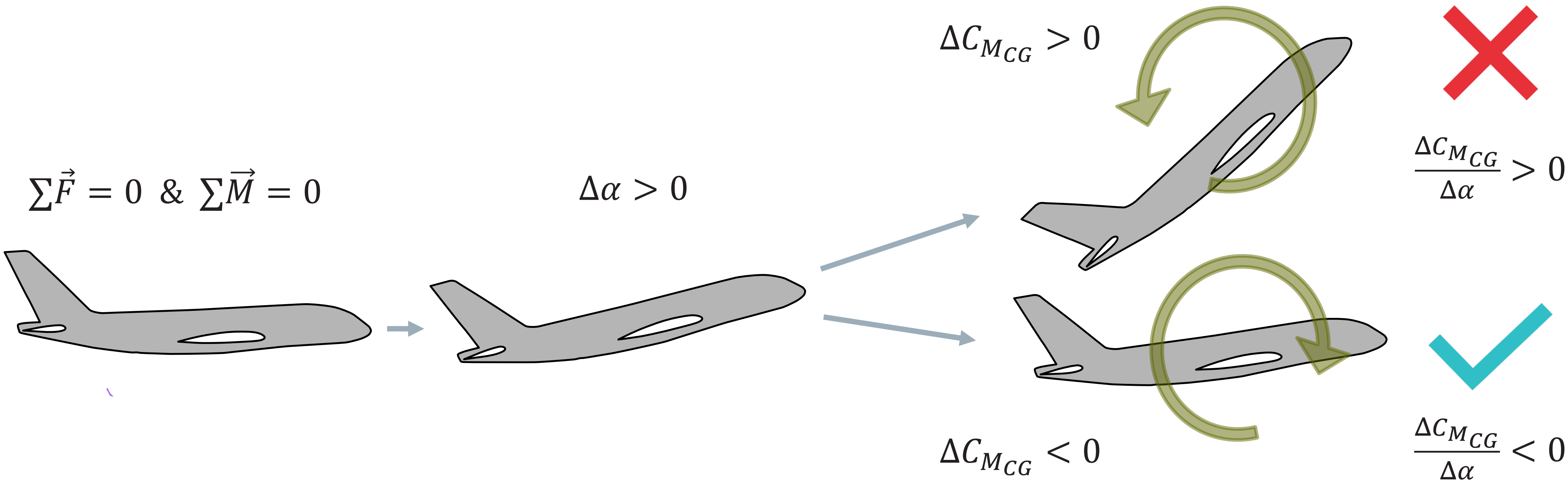
Static stability

if a system is disturbed from its equilibrium state by a small perturbation then the set of forces and moments so caused **initially** tend to return the system to its original state

*Static as opposed to Dynamic (see Part B)

Longitudinal static stability

Consider a positive angle of attack perturbation



Longitudinal static stability

Consider a negative angle of attack perturbation (downgust)

$\sum \vec{F} = 0$ & $\sum \vec{M} = 0$

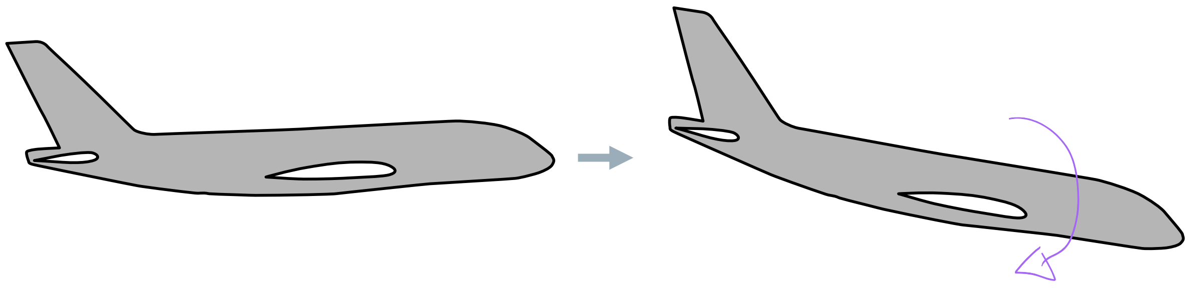
$\Delta\alpha < 0$

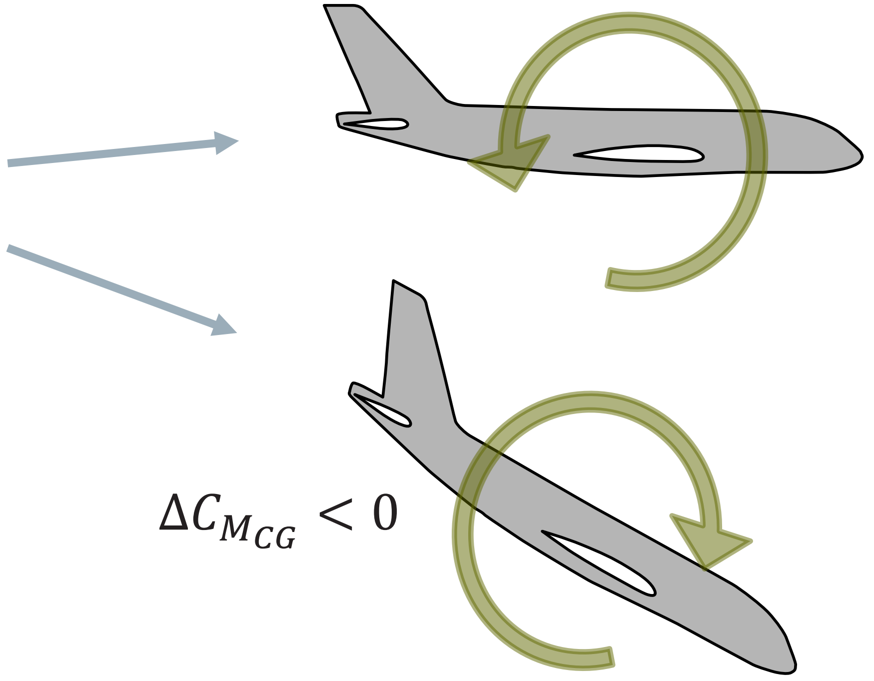
positive moment for restoration

$\Delta C_{M_{CG}} > 0$

since $\Delta\alpha < 0$ and $\Delta C_{M_{CG}} > 0$ then

$\dfrac{\Delta C_{M_{CG}}}{\Delta\alpha} < 0$





$\Delta C_{M_{CG}} < 0$

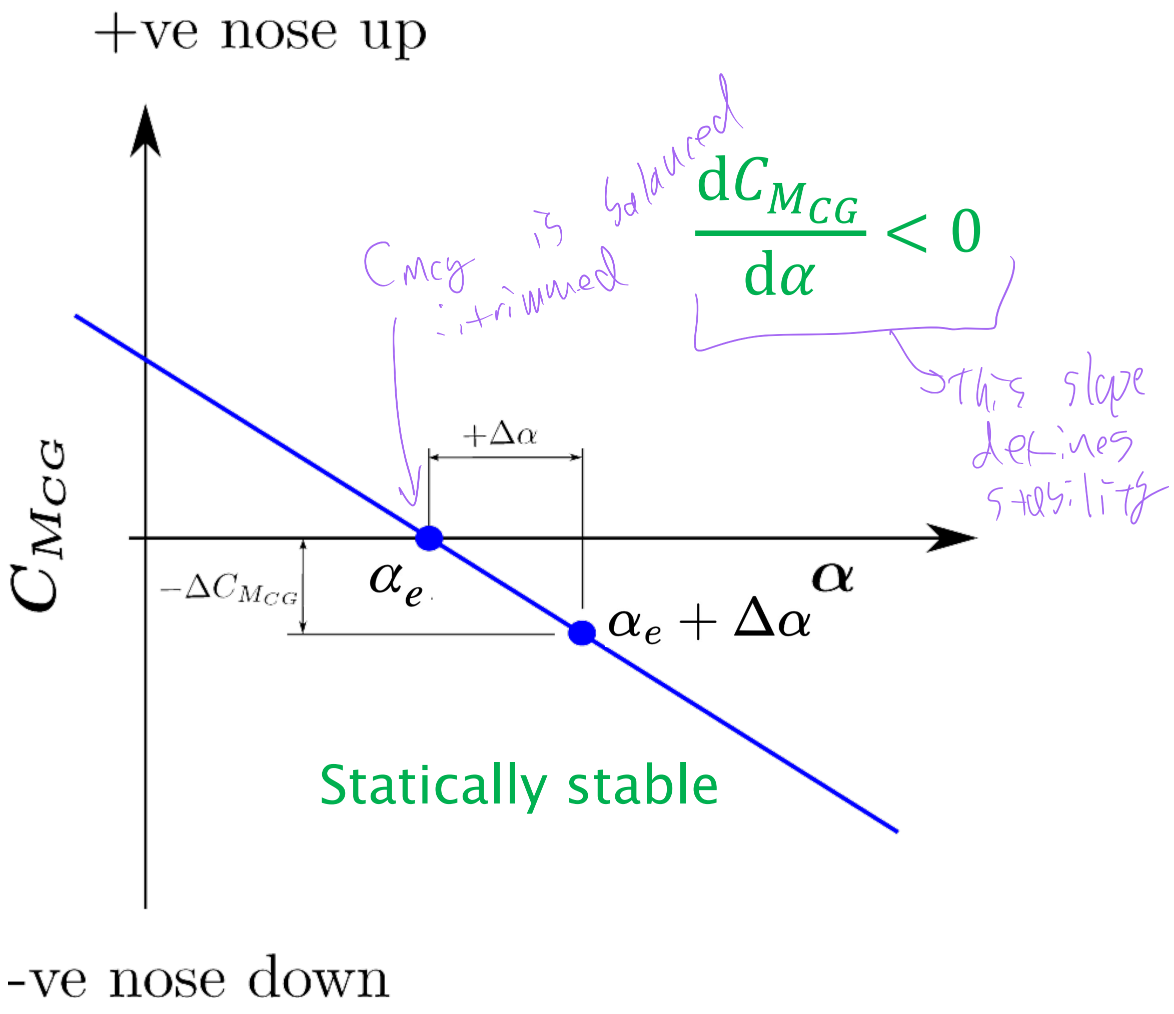
$\dfrac{\Delta C_{M_{CG}}}{\Delta\alpha} > 0$

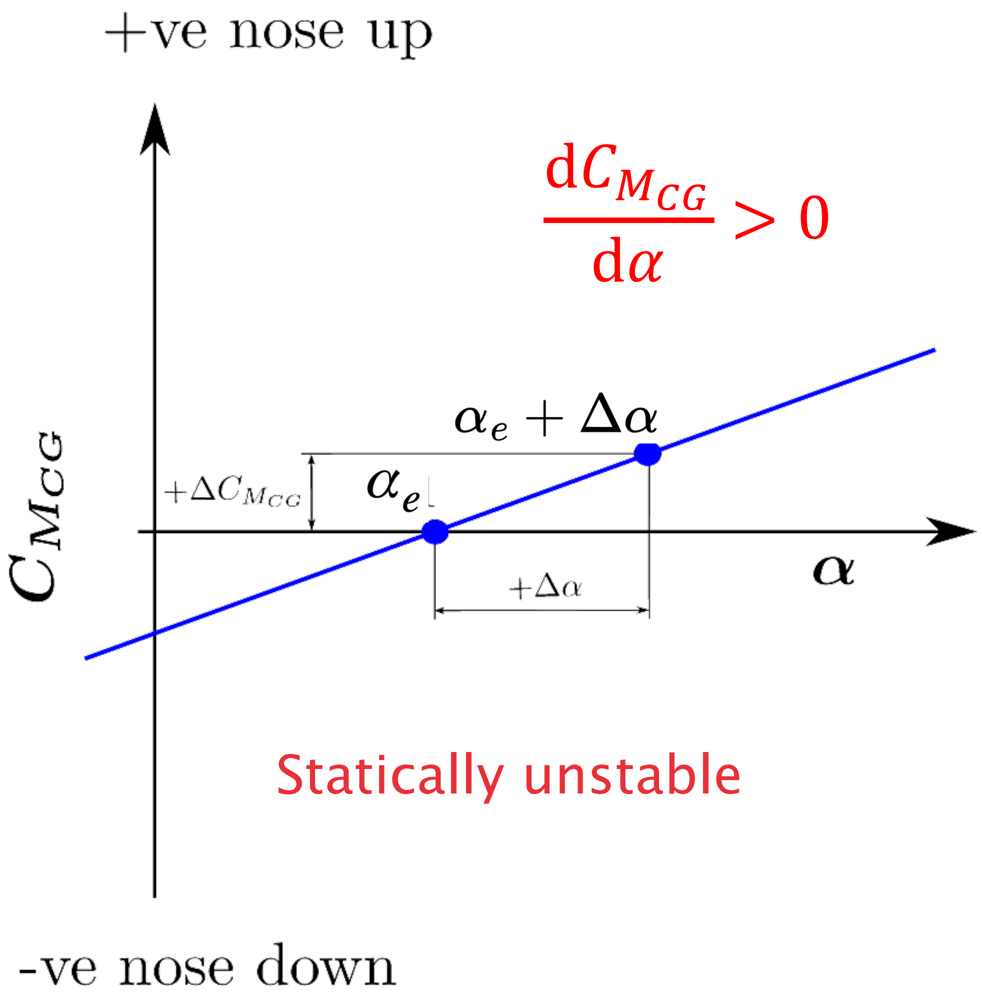
- So we want $\dfrac{\Delta C_{M_{CG}}}{\Delta\alpha} < 0$ for a stable aircraft
- For small perturbations

$$\frac{\Delta C_{M_{CG}}}{\Delta\alpha} < 0 \quad \xRightarrow{\Delta\alpha \to 0} \quad \frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} < 0$$

Longitudinal static stability

Pitching moment vs angle of attack





Southampton

Contributions to pitching moment

Pitching moment vs angle of attack

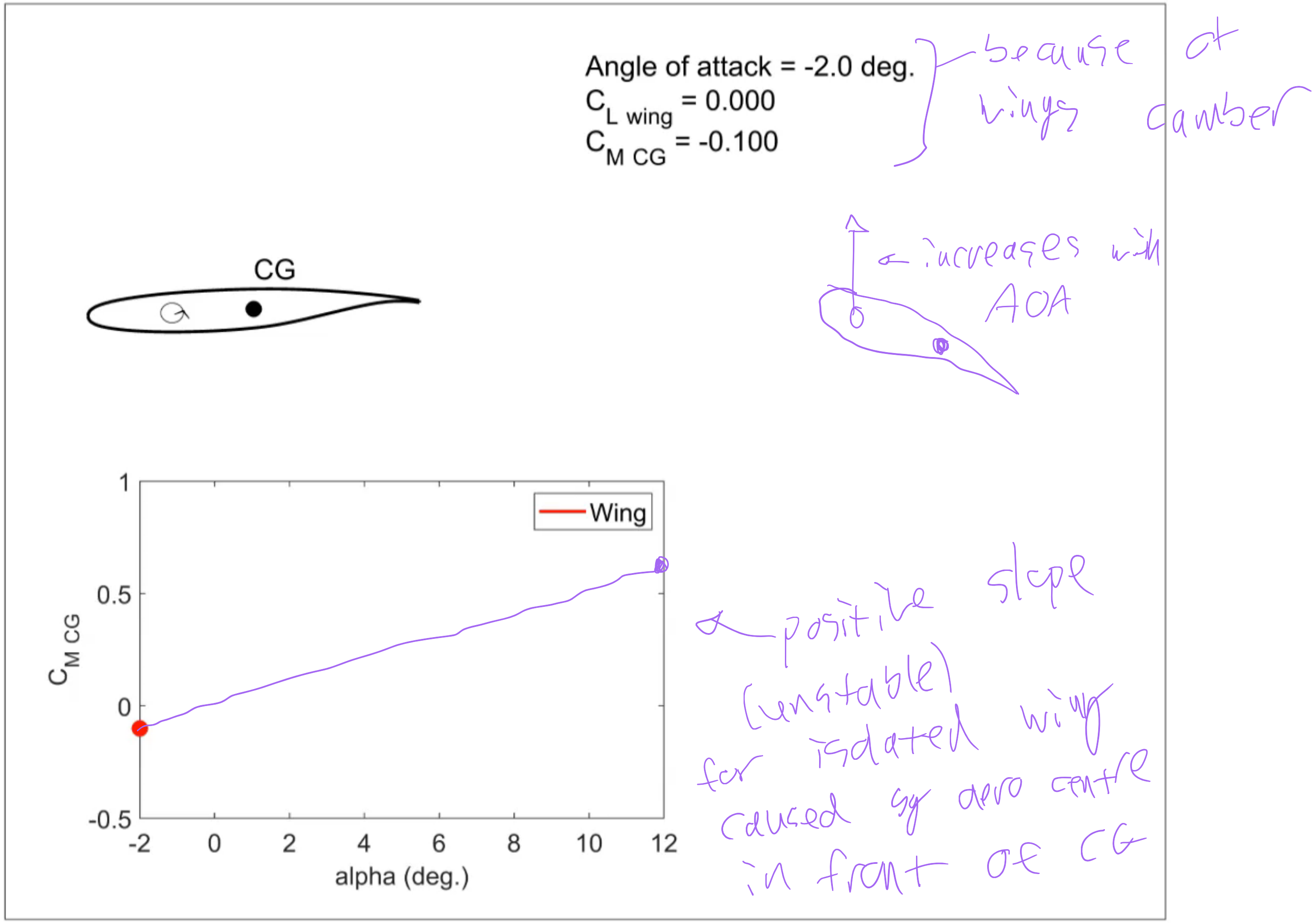
The Boeing 787

- Main contributions
  - Wing/fuselage lift
  - Wing pitching moment
  - Tailplane lift
- Minor contributions
  - Power effects
  - Drag forces
  - Other components
- Location matters!

currently neglected

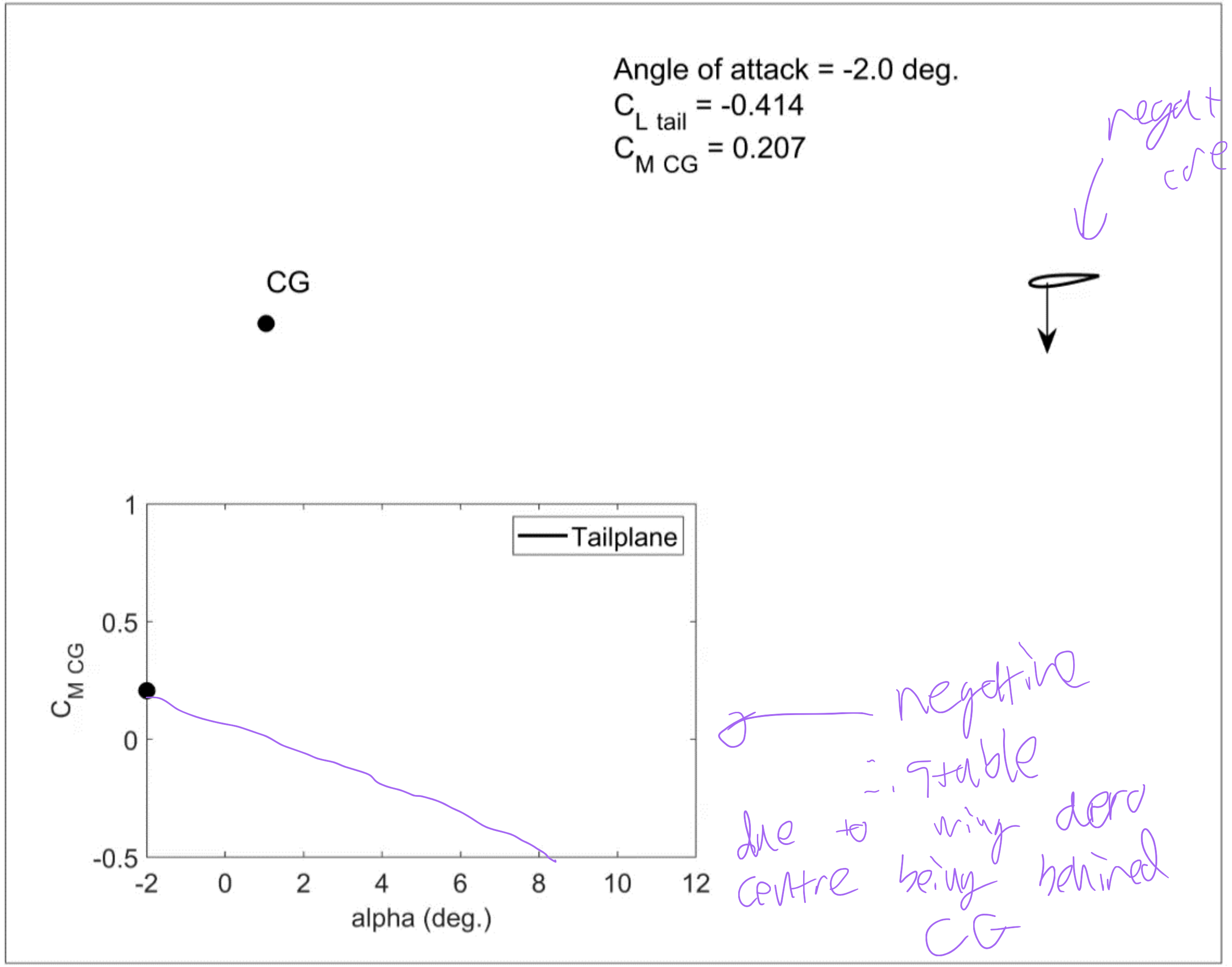
Contributions to pitching moment

Wing Alone



Contributions to pitching moment

Tailplane

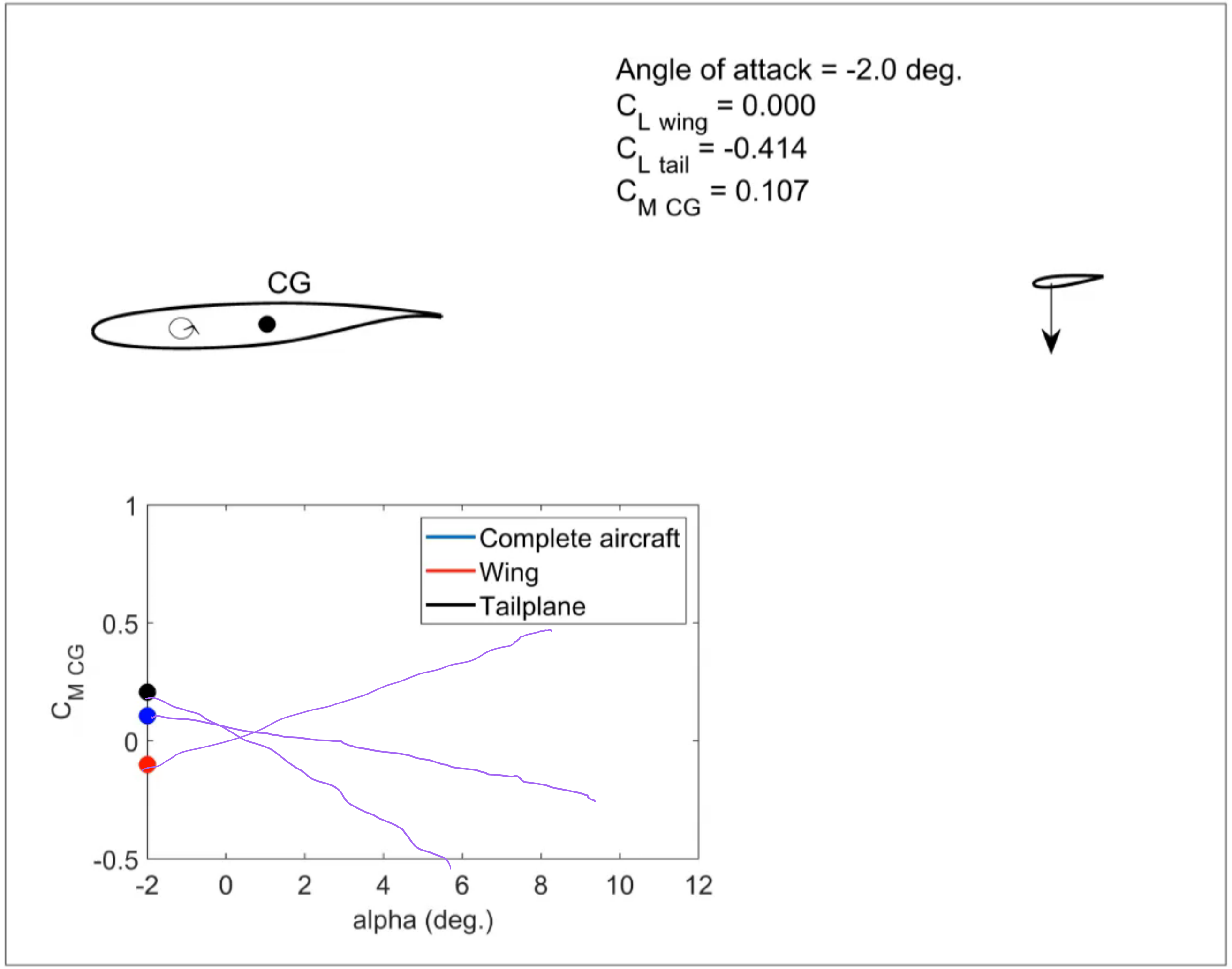


negative setting angle creates positive moment

negative ∴ stable due to wing aero centre being behind CG

Contributions to pitching moment

Complete aircraft

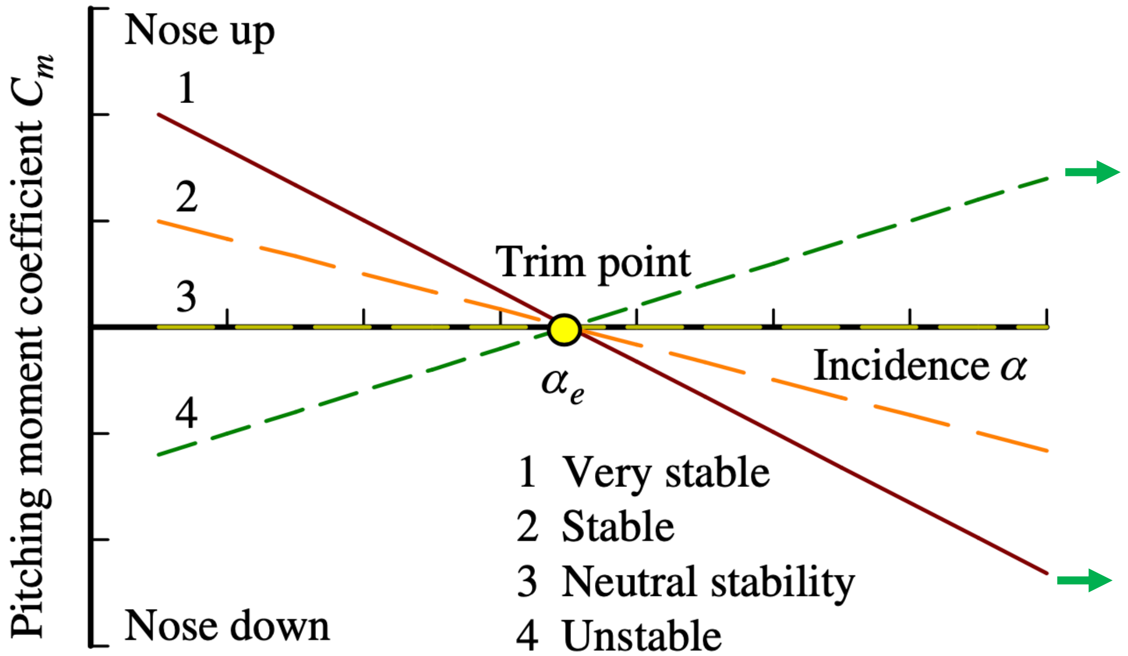


UNIVERSITY OF Southampton

Longitudinal static stability

Total Aircraft

- Restoring pitch moment is greater for a more stable aircraft (pitch stiffness)
- Stability/Manoeuvrability trade-off



The Grumman X-29

The Airbus A380

Longitudinal static stability

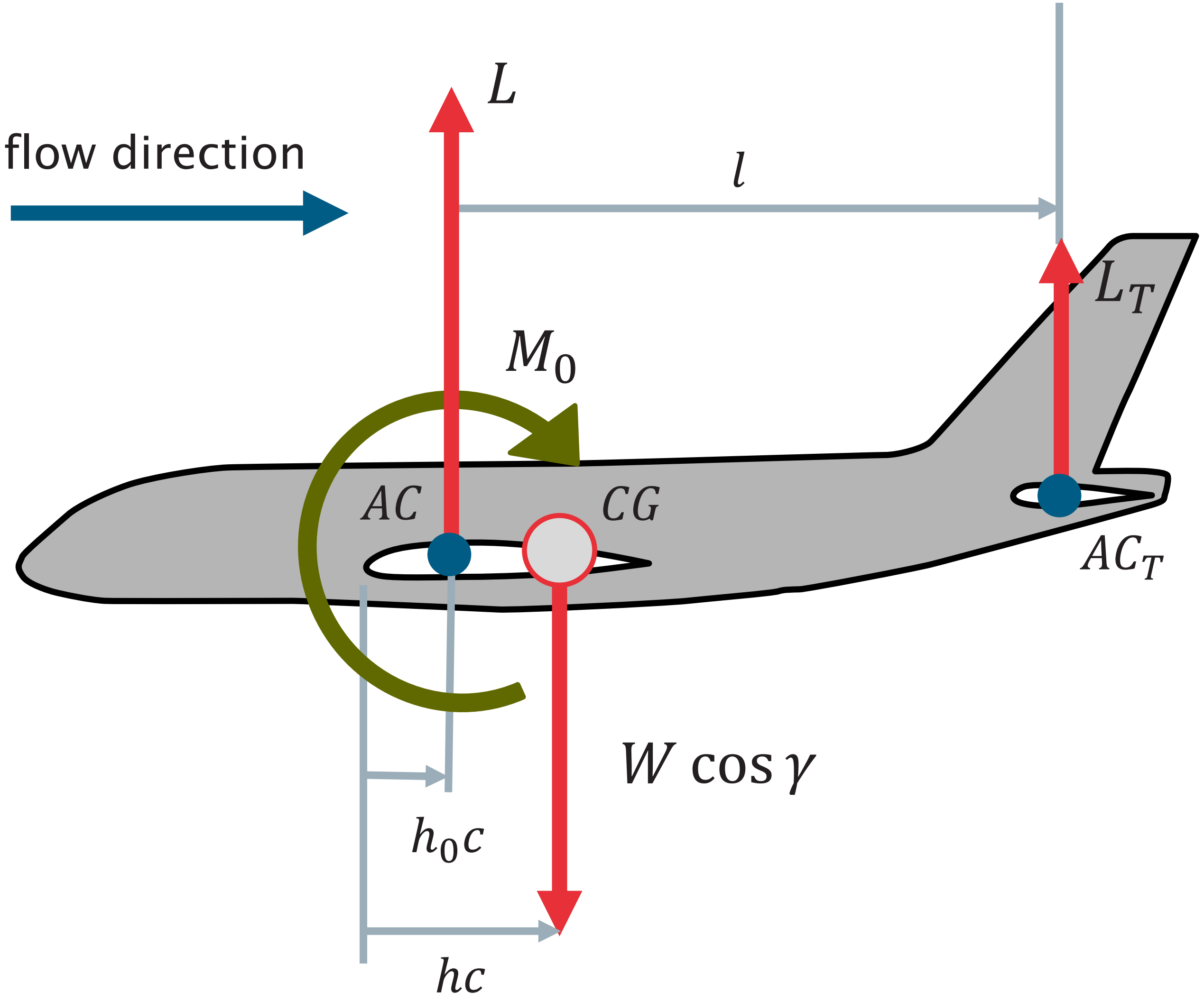
Moments about CG

- Force balance

$$C_{L^*} = C_L + C_{L_T}\frac{S_T}{S} = C_W(\cos\gamma)$$

- Moment balance

$$C_{M_{CG}} = C_{M_0} + C_{L^*}(h - h_0) - C_{L_T}K = 0$$



Longitudinal static stability

Moment derivative

- differentiate moment with respect to α

independent of α

$$\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} = \frac{\mathrm{d}}{\mathrm{d}\alpha}\left(C_{M_0} + C_{L^*}(h - h_0) - C_{L_T}K\right) \quad \text{with } K \stackrel{\text{def}}{=} \frac{S_T l}{Sc}$$

$$\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} = 0 + \frac{\mathrm{d}C_{L^*}}{\mathrm{d}\alpha}(h - h_0) - \frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha}K$$

$$\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} = \boxed{C_{L^*_\alpha}(h - h_0)} - \boxed{C_{L_{T,\alpha}}K}$$

with $C_{L^*_\alpha} = C_{L_\alpha} + C_{L_{T,\alpha}}\dfrac{S_T}{S}$

and $C_L = C_{L_\alpha}(\alpha - \alpha_0)$ with $C_{L_\alpha} = a_0\dfrac{\pi Ae}{\pi Ae + a_0}$

positive

depends on relative location of CG and AC

positive/negative for tailplane/foreplane

stable contribution if $l > 0$ (tail)

unstable contribution if $h - h_0 > 0$ (AC ahead of CG) (main wing)

Longitudinal static stability

Neutral point

can be negative

where CG moves far enough to change stable → unstable

- Neutral point, $h = h_n$, as CG position where

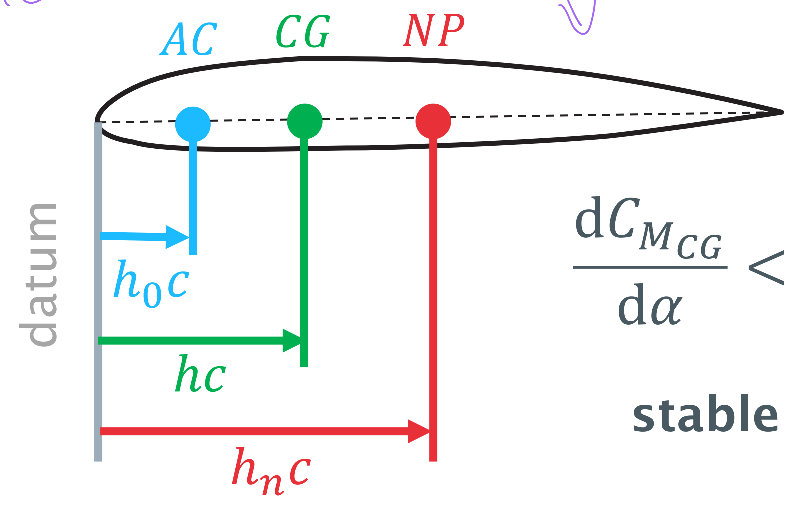
$$\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} = C_{L_\alpha^*}(h_n - h_0) - C_{L_{T,\alpha}} K = 0$$

- Solve for h_n

$$h_n = h_0 + K\frac{C_{L_{T,\alpha}}}{C_{L_\alpha^*}}$$

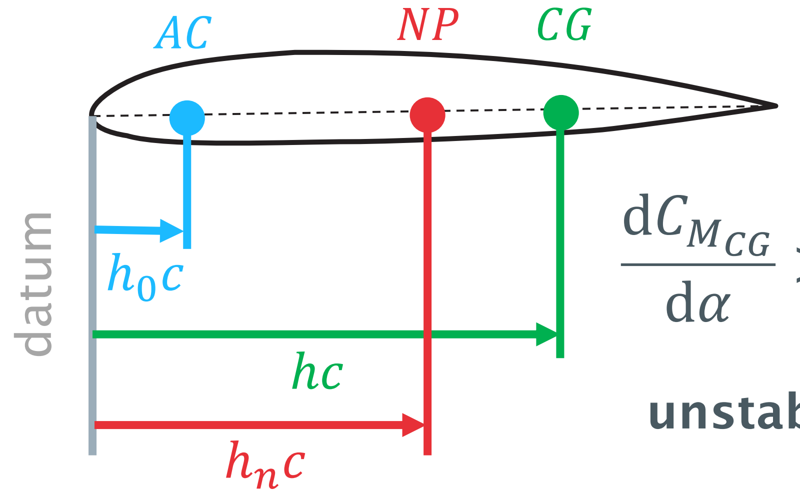
- In units of the MAC

positive



$$\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} < 0$$

stable ✓



$$\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha} > 0$$

unstable ✗

Longitudinal static stability

Static stability margin

- Given CG position, define **static stability margin**

$$\boxed{H_S} \stackrel{\text{def}}{=} (h_n - h) = h_0 - h + K\frac{C_{L_{T,\alpha}}}{C_{L^*_\alpha}}$$

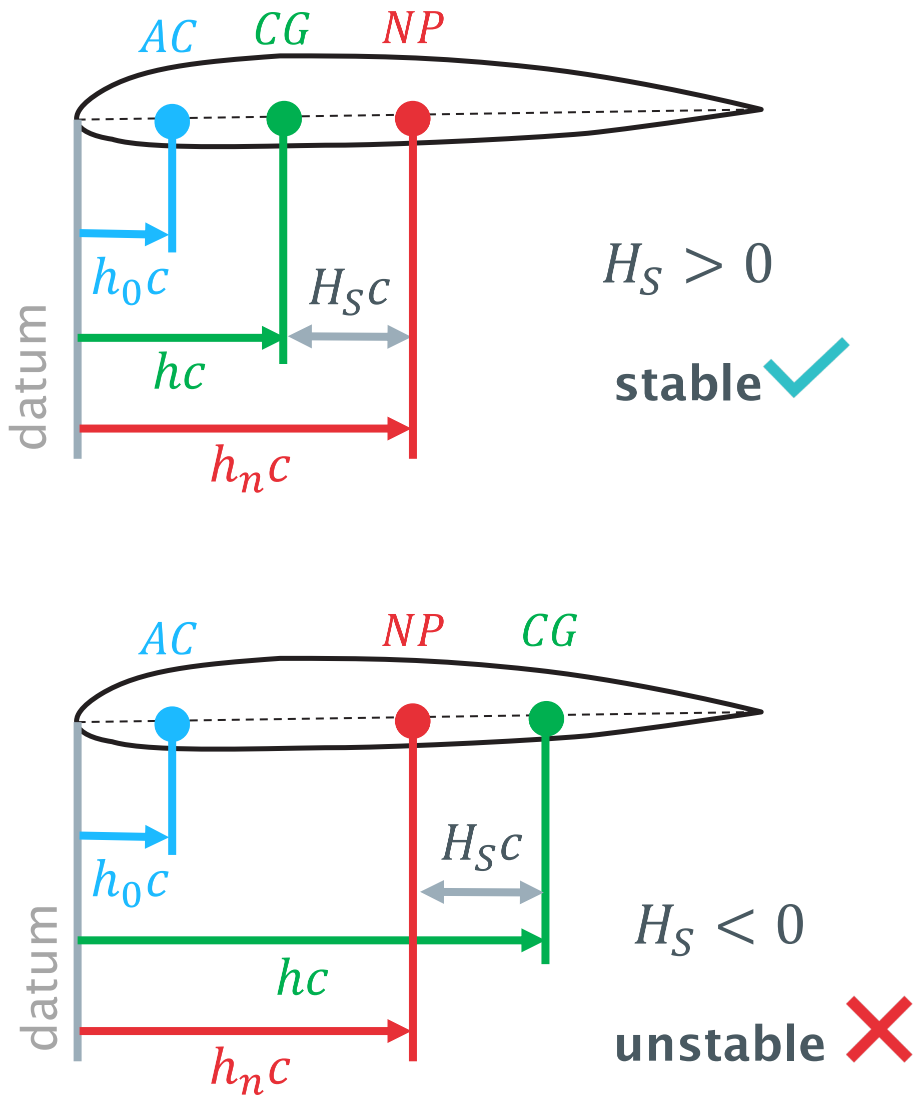
(non dimensional)

- Measure of the stability of the aircraft

$$H_S \propto -\frac{\mathrm{d}C_{M_{CG}}}{\mathrm{d}\alpha}$$

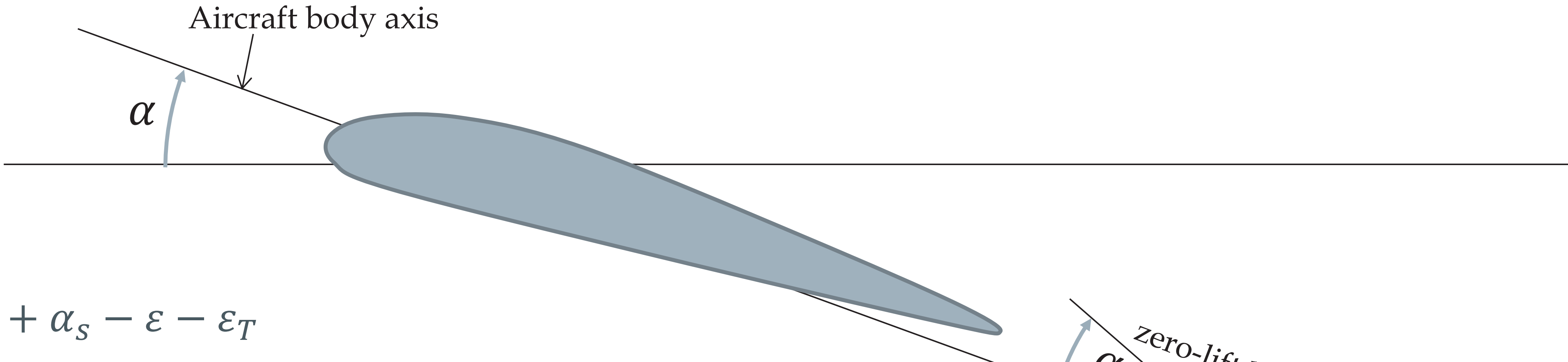
- Stability constraint on the CG location

$$H_S \stackrel{\text{def}}{=} (h_n - h) > 0$$



Tail plane Lift

Effective angle of attack

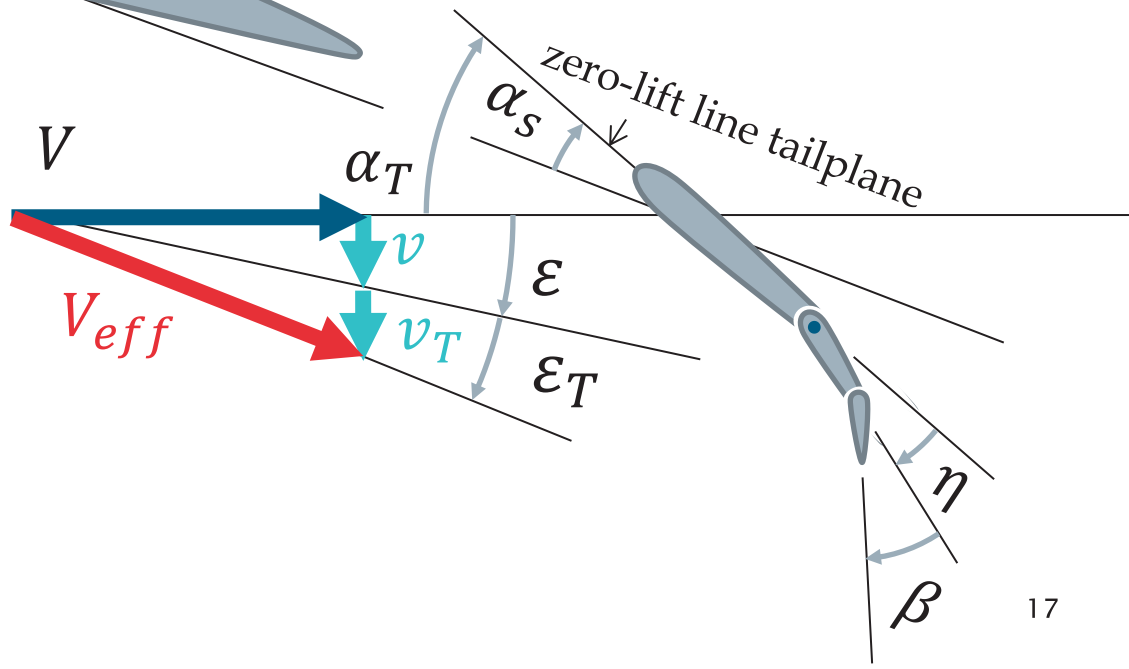


$$\alpha_{T_{eff}} = \alpha + \alpha_s - \varepsilon - \varepsilon_T$$

$$\alpha_{T_{eff}} = \alpha + \alpha_s - \varepsilon_\alpha(\alpha - \alpha_0) - \frac{C_{L_T}}{\pi A_T e_T}$$

$$\alpha_{T_{eff}} = \boxed{(1 - \varepsilon_\alpha)}\,\alpha + \varepsilon_\alpha \alpha_0 + \alpha_s - \frac{C_{L_T}}{\pi A_T e_T}$$

$$\frac{\partial \alpha_{T_{eff}}}{\partial \alpha} = (1 - \varepsilon_\alpha)$$



Tail plane Lift

And its rate of change with angle of attack – Stick-fixed

Flight parameters
Design parameters
Control parameters

- Stick-fixed (see Lecture 1.2):

$$C_{L_T} = a_1 \alpha_{T_{eff}} + a_2 \eta \;= a_1 \left((1-\varepsilon_\alpha)\alpha + \varepsilon_\alpha \alpha_0 + \alpha_s - \frac{C_{L_T}}{\pi A_T e_T} \right) + a_2 \eta$$

q d

- Solve for C_{L_T}:

$$C_{L_T} = a_1 \frac{\pi A_T e_T}{\pi A_T e_T + a_1} \left((1-\varepsilon_\alpha)\alpha + \varepsilon_\alpha \alpha_0 + \alpha_s \right) + a_2 \frac{\pi A_T e_T}{\pi A_T e_T + a_1} \eta$$

really simple

- Introducing $k \stackrel{\text{def}}{=} a_1 \frac{\pi A_T e_T}{\pi A_T e_T + a_1} = a_1 \frac{1}{1 + \frac{a_1}{\pi A_T e_T}}$

$$C_{L_T} = k \left((1-\varepsilon_\alpha)\alpha + \varepsilon_\alpha \alpha_0 + \alpha_s + \frac{a_2}{a_1}\eta \right) \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}\alpha}} \boxed{C_{L_{T,\alpha}} \stackrel{\text{def}}{=} \frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha} = k(1-\varepsilon_\alpha)}$$

Tail plane Lift

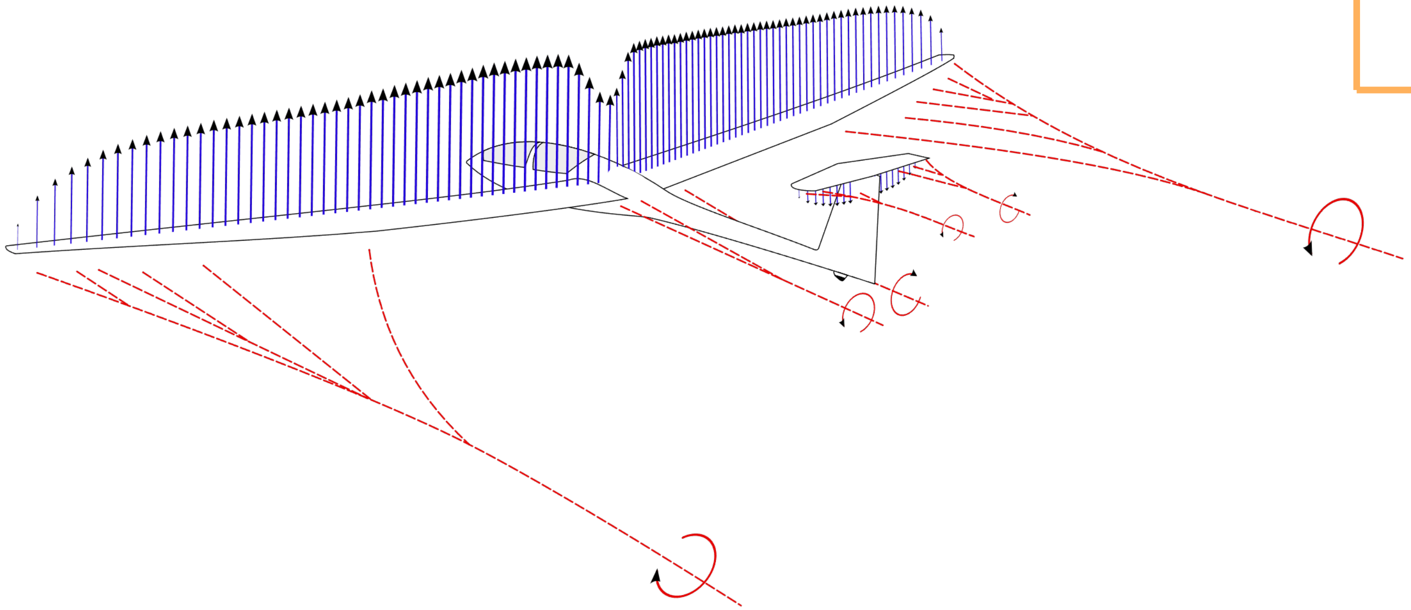
Analysis of the rate of change

$$\frac{dC_{L_T}}{d\alpha} = C_{L_{T,\alpha}} = k(1-\varepsilon_\alpha) = \boxed{\boxed{\boxed{a_1}\frac{\pi A_T e_T}{\pi A_T e_T + a_1}}(1-\varepsilon_\alpha)}$$

2D airfoil

3D tailplane

3D tailplane + downwash



UNIVERSITY OF Southampton

Tail plane Lift

And its rate of change with angle of attack – Stick-free

Flight parameters
Design parameters
Control parameters

- Stick-free (see Lecture 1.2):

$$C_{L_T} = \overline{a_1}\alpha_{T_{eff}} + \overline{a_3}\beta = \overline{a_1}\left((1-\varepsilon_\alpha)\alpha + \varepsilon_\alpha\alpha_0 + \alpha_s - \frac{C_{L_T}}{\pi A_T e_T}\right) + \overline{a_3}\beta$$

(usually) $\bar{a}_1 < a_1$

- Solve for C_{L_T}:

$$C_{L_T} = \overline{a_1}\frac{\pi A_T e_T}{\pi A_T e_T + \overline{a_1}}\left((1-\varepsilon_\alpha)\alpha + \varepsilon_\alpha\alpha_0 + \alpha_s\right) + \overline{a_3}\frac{\pi A_T e_T}{\pi A_T e_T + \overline{a_1}}\beta$$

From $C_{M_H} = 0$

$$\overline{a_1} \stackrel{\text{def}}{=} a_1 - a_2\frac{b_1}{b_2}$$

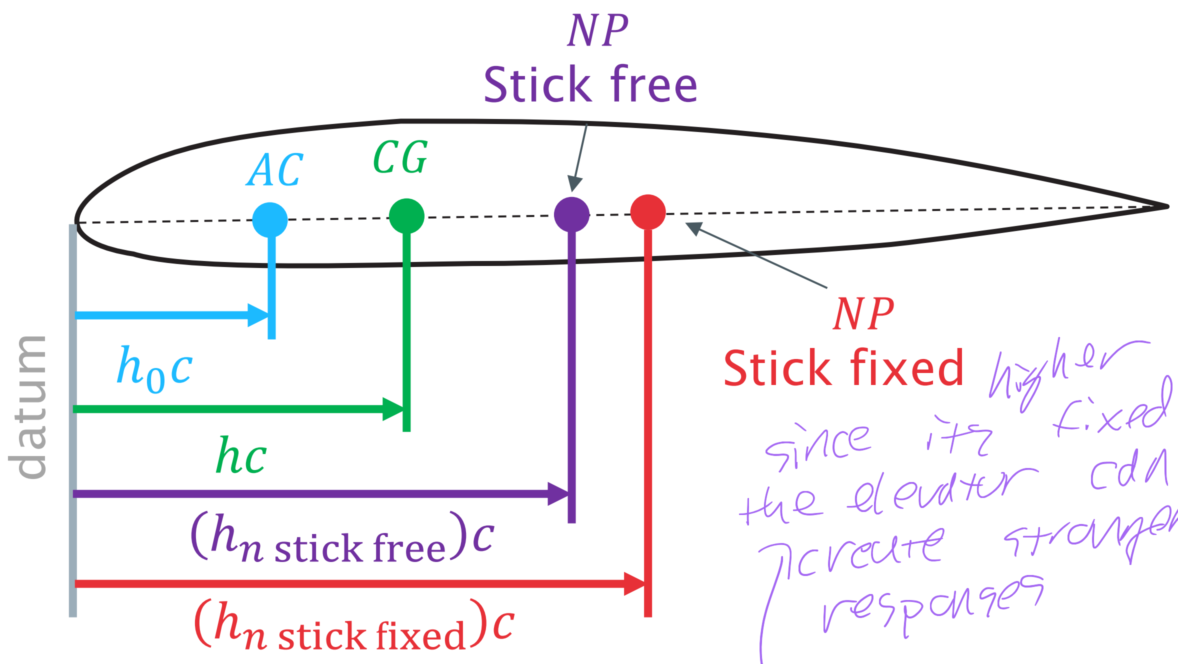
$$\overline{a_3} \stackrel{\text{def}}{=} a_3 - a_2\frac{b_3}{b_2}$$

- Introducing $\bar{k} \stackrel{\text{def}}{=} \overline{a_1}\frac{\pi A_T e_T}{\pi A_T e_T + \overline{a_1}}$

$$C_{L_T} = \bar{k}\left((1-\varepsilon_\alpha)\alpha + \varepsilon_\alpha\alpha_0 + \alpha_s + \frac{\overline{a_3}}{\overline{a_1}}\beta\right) \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}\alpha}} \boxed{C_{L_{T,\alpha}} \stackrel{\text{def}}{=} \frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha} = \bar{k}(1-\varepsilon_\alpha)}$$

Tail plane Lift

Stick-fixed/free – effect on stability margin



higher since it's fixed the elevator can create stronger responses

- The neutral point

$$h_n = h_0 + K\frac{C_{L_{T,\alpha}}}{C_{L^*_\alpha}}$$

- Or

$$h_n = h_0 + K\frac{C_{L_{T,\alpha}}}{C_{L_\alpha} + C_{L_{T,\alpha}}\frac{S_T}{S}}$$

$C_{L_{T,\alpha}} = k(1-\varepsilon_\alpha)$ for stick-fixed

$C_{L_{T,\alpha}} = \bar{k}(1-\varepsilon_\alpha)$ for stick-free

- An aircraft with a conventional elevator is less stable to fly stick-free than stick-fixed

$$H_{s\ \text{stick fixed}} > H_{s\ \text{stick free}}$$

lower because it floats, reducing overall response force

Exam question

From 21-22

(i) Explain why an aircraft with a conventional tailplane is less stable when flying in stick-free conditions with respect to flying in stick-fixed conditions. Use appropriate equations to justify your reasoning.

Then explain why this effect is likely to be more significant when the elevator chord length is large relative to the tailplane chord length.

[7 marks]

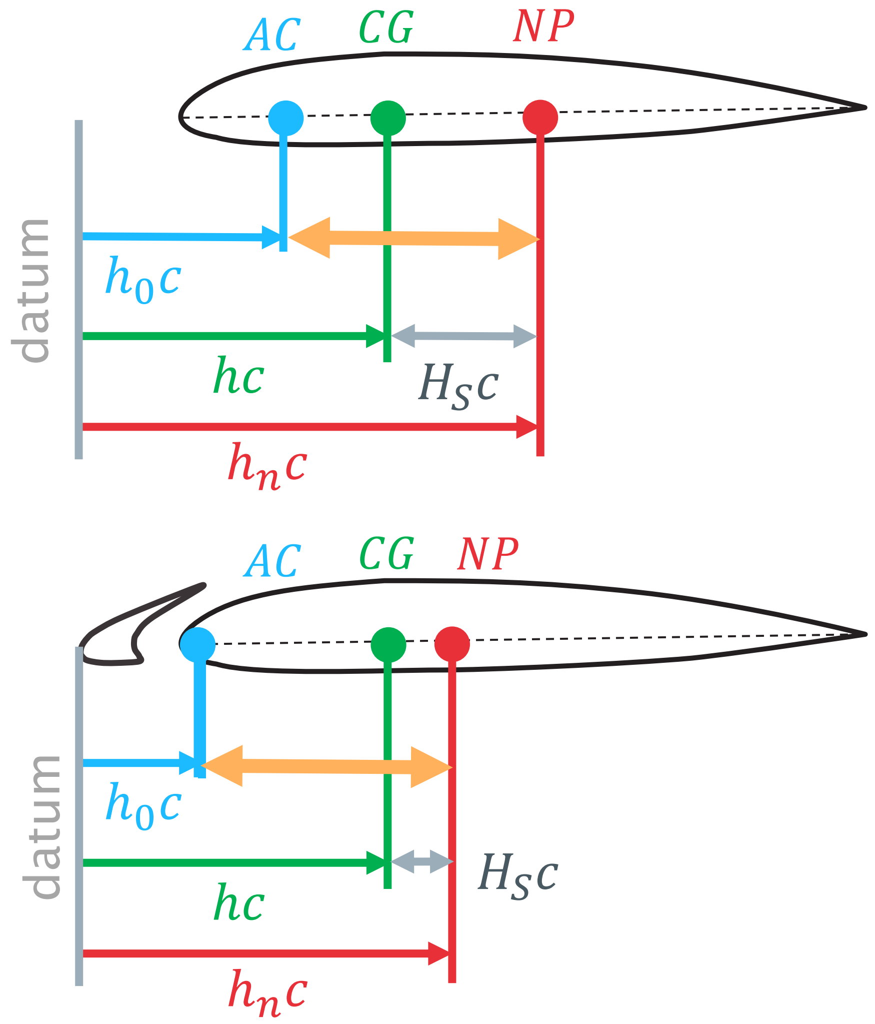
Longitudinal static stability

Influence of high-lift devices - **Slats**

- Neutral point is at fixed distance from AC

$$h_n = h_0 + K\frac{C_{L_{T,\alpha}}}{C_{L^*_\alpha}}$$

- AC shifting forward: $\Delta h_0 < 0$
- Neutral point gets closer to CG
- Smaller static margin



Longitudinal static stability

Influence of high-lift devices - **Flaps**

- Neutral point is at fixed distance from AC

$$h_n = h_0 + K\frac{C_{L_{T,\alpha}}}{C_{L_\alpha^*}}$$

- AC shifting backwards: $\Delta h_0 > 0$
- Neutral point gets farther from CG
- Higher static margin