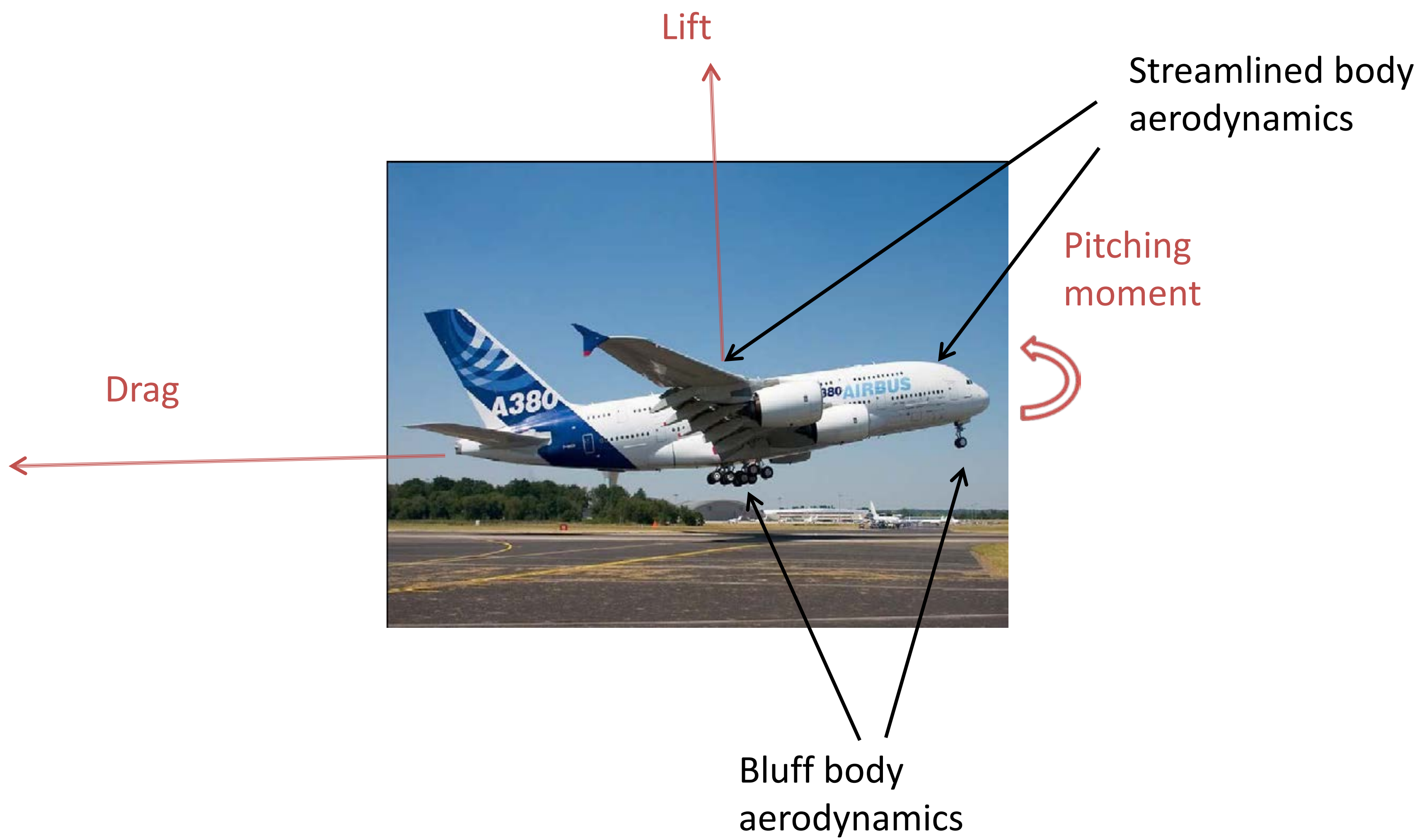
SESA3029 Aerothermodynamics

Lecture 1.2: Basic aerodynamics



Dimensionless numbers

- Mach number $$M = \frac{U}{a}$$
  - Ratio of velocity to speed of sound
- Reynolds number $$\mathrm{Re} = \frac{\rho U L}{\mu}$$
  - Ratio of the order of magnitude of inertia to viscous terms in the governing equations
- Knudsen number $$Kn = \frac{\lambda}{L}$$
  - Ratio of mean free path to flow scale
  - $\lambda = 8\times10^{-8}$ m for air at STP
  - ($Kn<0.01$ continuum flow, $Kn>1$ free molecule flow)

Example

- A380 in cruise at 10km
  - Mean chord= 10.4 m
  - Mach 0.85
  - U=300 m/s ρ=0.414 kg/m^3 μ=1.45$\times$10^{-5} Nsm^{-2}

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{0.414 \times 300 \times 10.4}{1.45 \times 10^{-5}} = 8.9 \times 10^{7}$$

Vorticity definition

$$\boldsymbol{\omega} = \nabla \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \mathbf{e}_r & r\mathbf{e}_\theta & \mathbf{e}_z \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial z \\ v_r & rv_\theta & v_z \end{vmatrix}$$

Curl operator

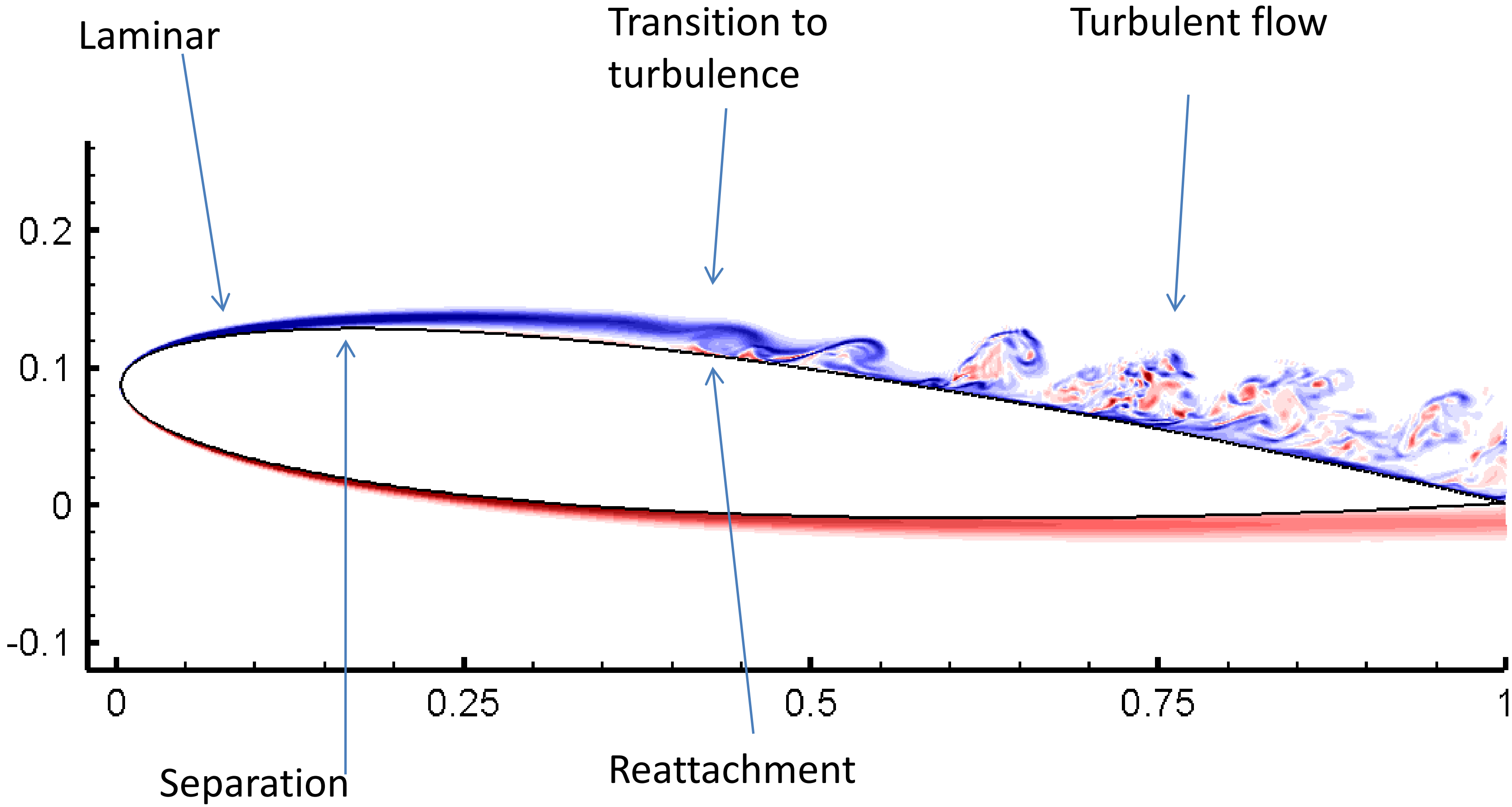
Cartesian

Cylindrical polar

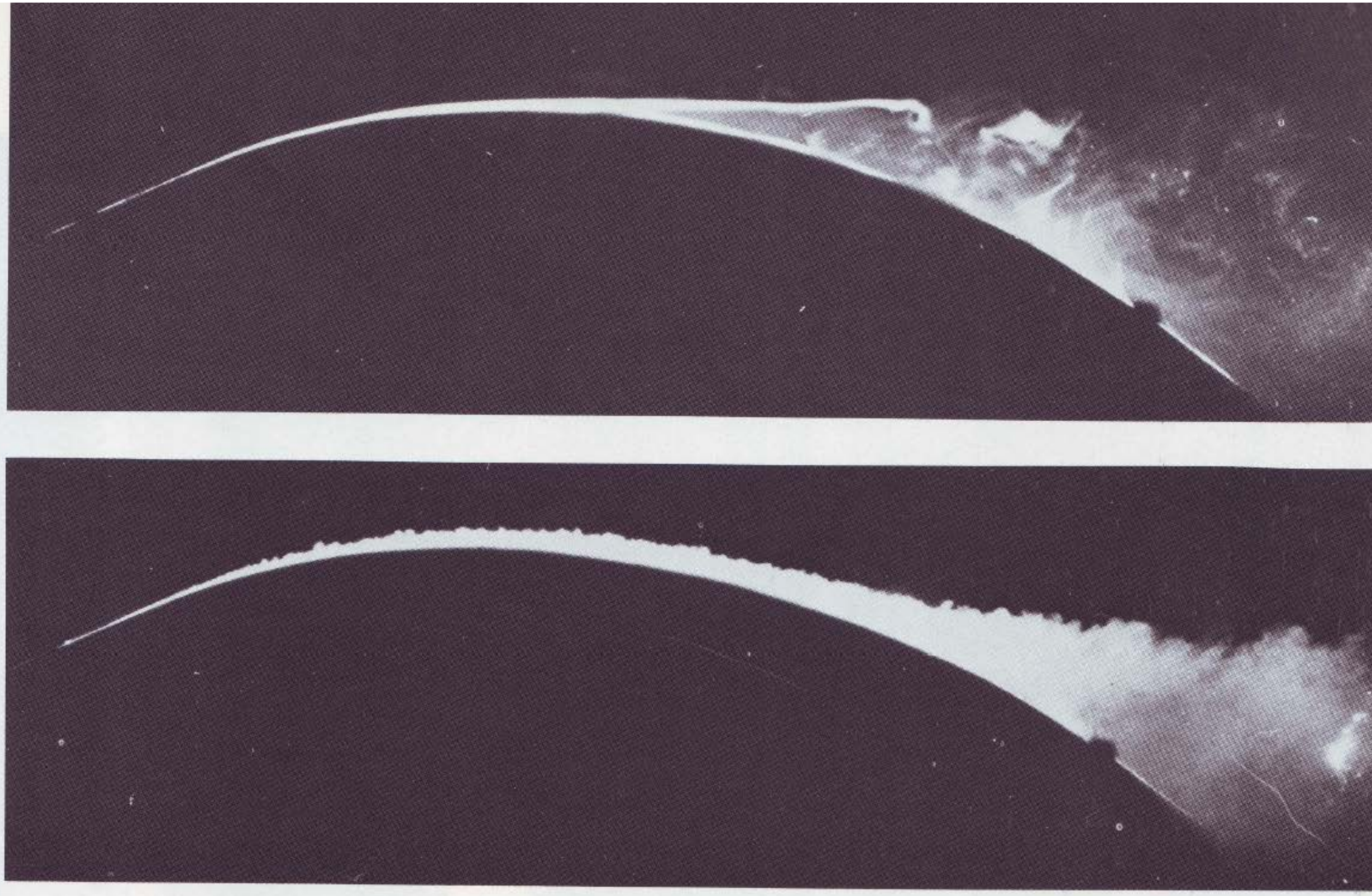
Can write out components using rules of determinants

The boundary layer

(low-speed flow over an airfoil with a laminar separation bubble)



Contours of vorticity show rotational flow region;   Re=50,000 in this example



156. Comparison of laminar and turbulent boundary layers. The laminar boundary layer in the upper photograph separates from the crest of a convex surface (cf. figure 38), whereas the turbulent layer in the second photograph remains attached; similar behavior is shown below for a sharp corner. (Cf. figures 55-58 for a sphere.) Titanium tetrachloride is painted on the forepart of the model in a wind tunnel. *Head 1982*

From Van Dyke 'An Album of Fluid Motion'

Potential flow

If a flow is irrotational (ω=curl **u**=0) we can always write $\mathbf{u} = \nabla\phi$

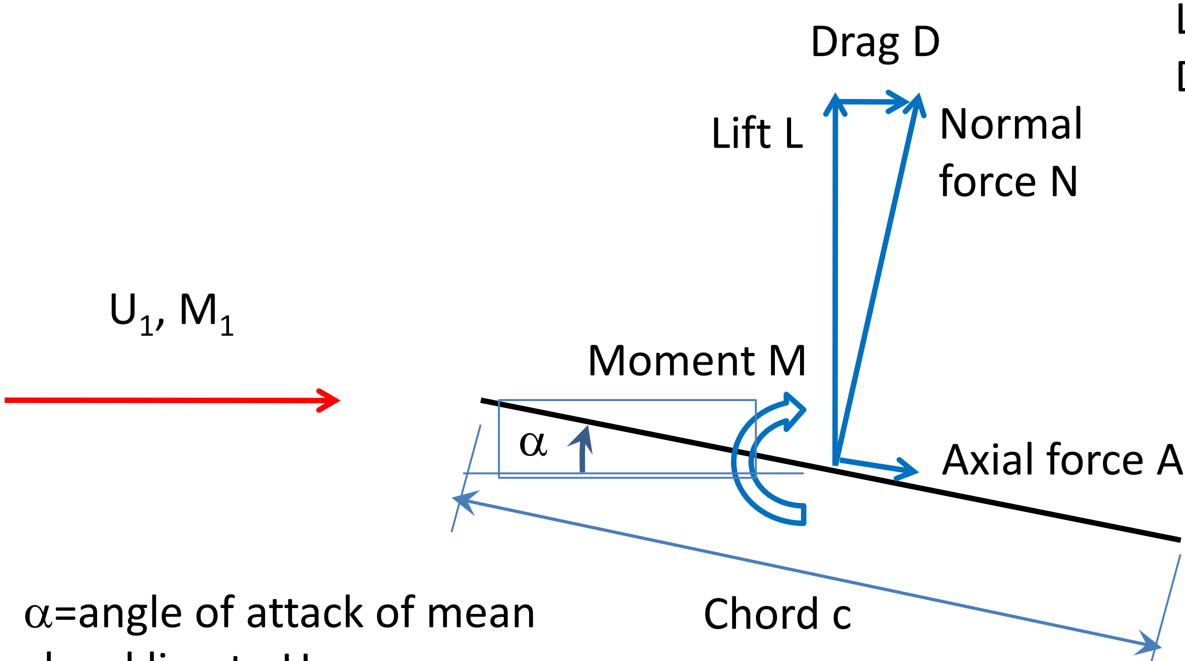
with velocity components $u = \dfrac{\partial\phi}{\partial x} \quad v = \dfrac{\partial\phi}{\partial y}$

where ϕ is a scalar known as the velocity potential
- this works because of the vector identity $\nabla\times\left(\nabla\phi\right) = 0$

Circulation: $$\Gamma = -\oint_{\text{closed circuit}} \mathbf{u}.\mathrm{d}\mathbf{s} = -\iint_{\text{surface}} \boldsymbol{\omega}.\mathrm{d}\mathbf{S}$$

Circular cylinder with circulation
(Kutta-Joukowski theorem): $$L = \rho U_\infty \Gamma$$

Aerodynamic forces



α=angle of attack of mean chord line to U_1

Lift is defined normal to U_1
Drag is defined tangential to U_1

$$L = N\cos\alpha - A\sin\alpha$$

$$D = A\cos\alpha + N\sin\alpha$$

Force and moment coefficients

- Wing-based (L=lift, D=drag, M=pitching moment, S=wing planform area)

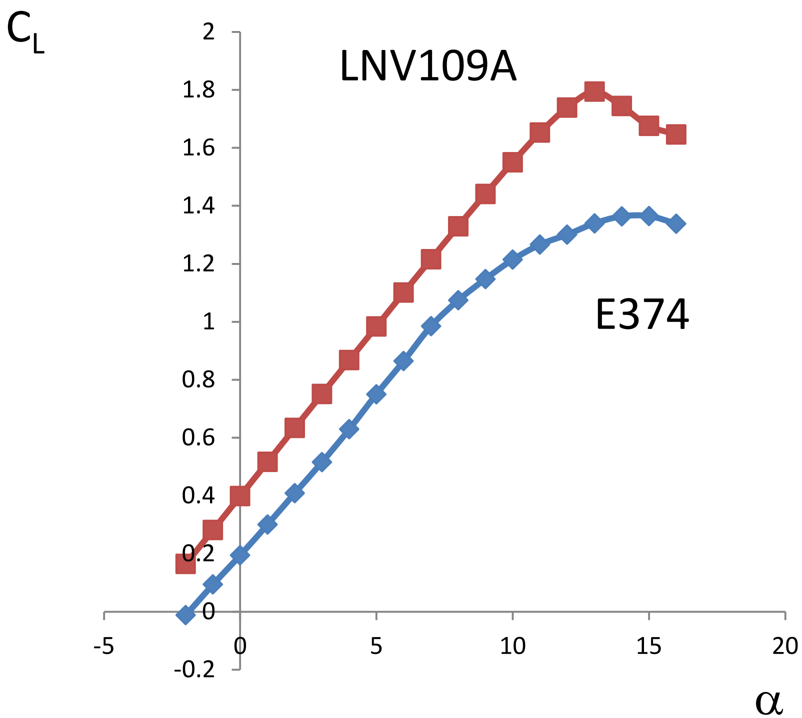
$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} \quad C_D = \frac{D}{\frac{1}{2}\rho_\infty U_\infty^2 S} \quad C_M = \frac{M}{\frac{1}{2}\rho_\infty U_\infty^2 Sc}$$

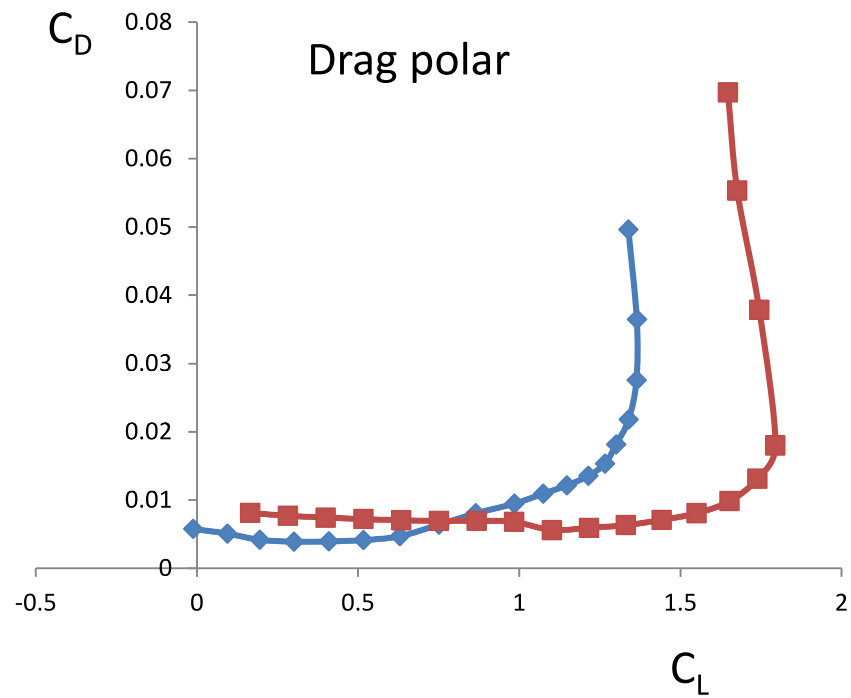
- Section-based (c=chord)

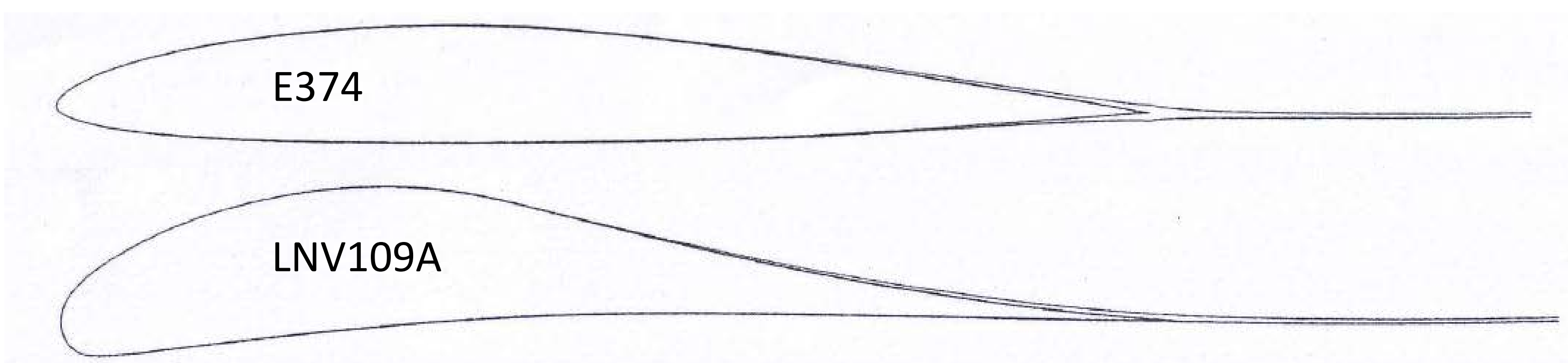
$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 c} \quad C_D = \frac{D}{\frac{1}{2}\rho_\infty U_\infty^2 c} \quad C_M = \frac{M}{\frac{1}{2}\rho_\infty U_\infty^2 c^2}$$

- Forces and moments come from the integrated effects of pressure and surface shear stress

Airfoil performance (Re=$3x10^6$ XFOIL results)

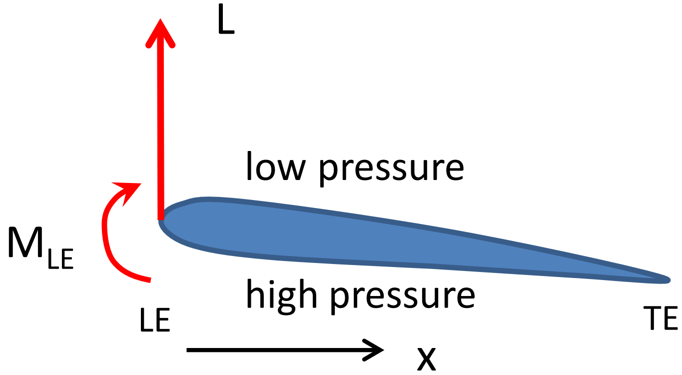


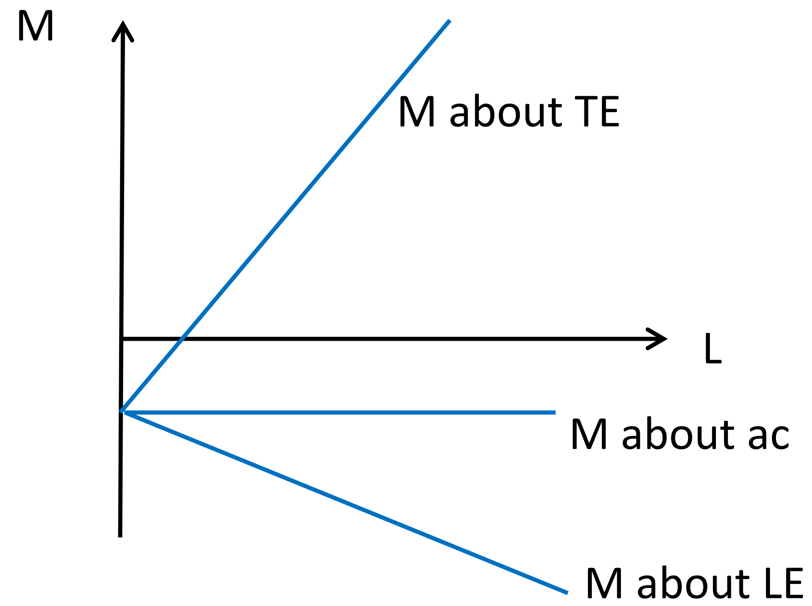




Moments, centre of pressure (cp) and aerodynamic centre (ac)

We can represent forces and moments about any point e.g. leading edge (LE)





About any x $M_x = M_{LE} + xL$

$$\text{let } \bar{x} = x / c$$

$$C_{M,x} = C_{M,LE} + \bar{x} C_L \quad \Rightarrow \quad \bar{x}_{cp} = -\frac{C_{M,LE}}{C_L}$$

(point where lift acts with no moment)

$$\frac{\mathrm{d}C_{M,x}}{\mathrm{d}C_L} = \frac{\mathrm{d}C_{M,LE}}{\mathrm{d}C_L} + \bar{x} \quad \Rightarrow \quad \bar{x}_{ac} = -\frac{\mathrm{d}C_{M,LE}}{\mathrm{d}C_L}$$

(point where C_M is independent of C_L)

Results from thin aerofoil theory

- Incompressible inviscid flow around a 2D thin aerofoil
- Kutta condition (zero loading at trailing edge)
- Kutta-Joukowski theorem $L=\rho U_{\infty}\Gamma$

$$\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} = 2\pi \qquad \bar{x}_{ac} = \frac{1}{4}$$