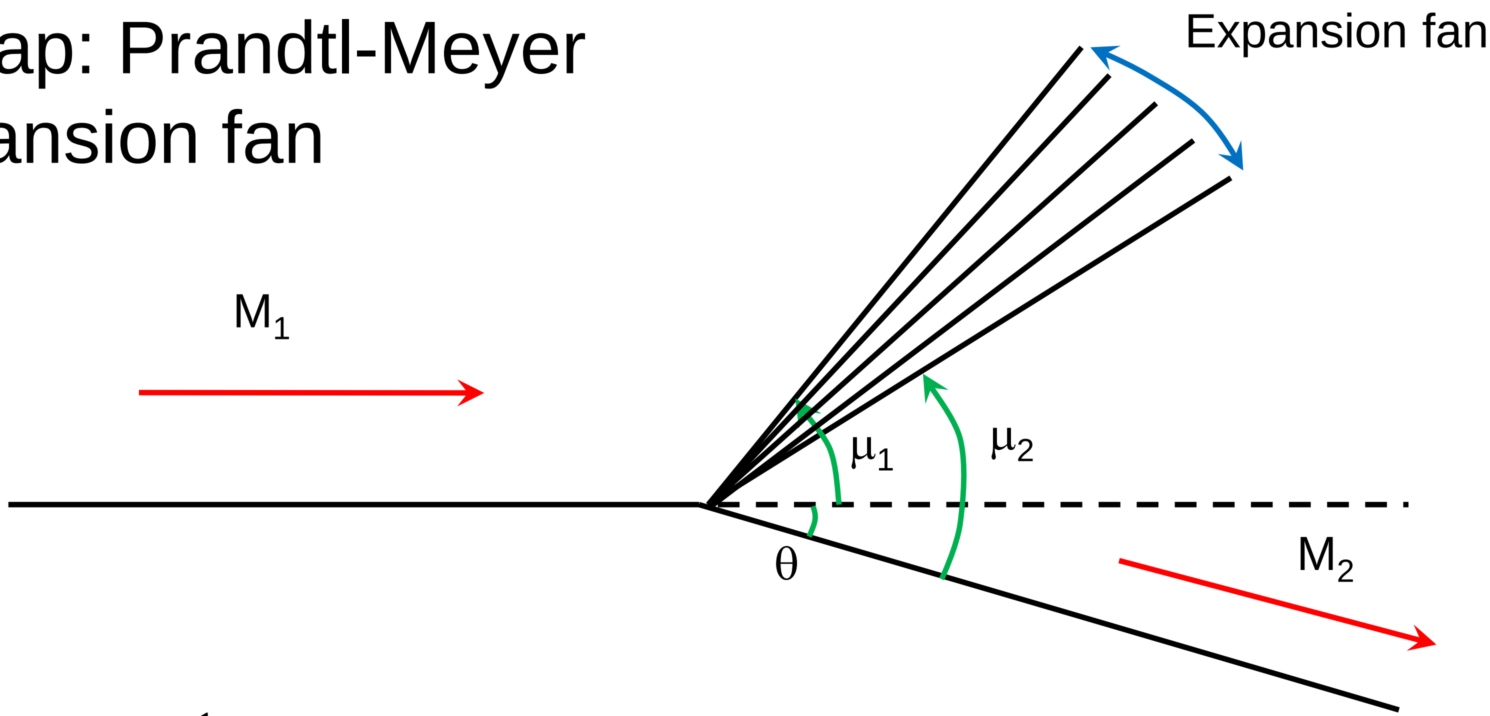
SESA3029 Aerothermodynamics

Lecture 2.5

Shock-expansion method

Recap: Prandtl-Meyer expansion fan

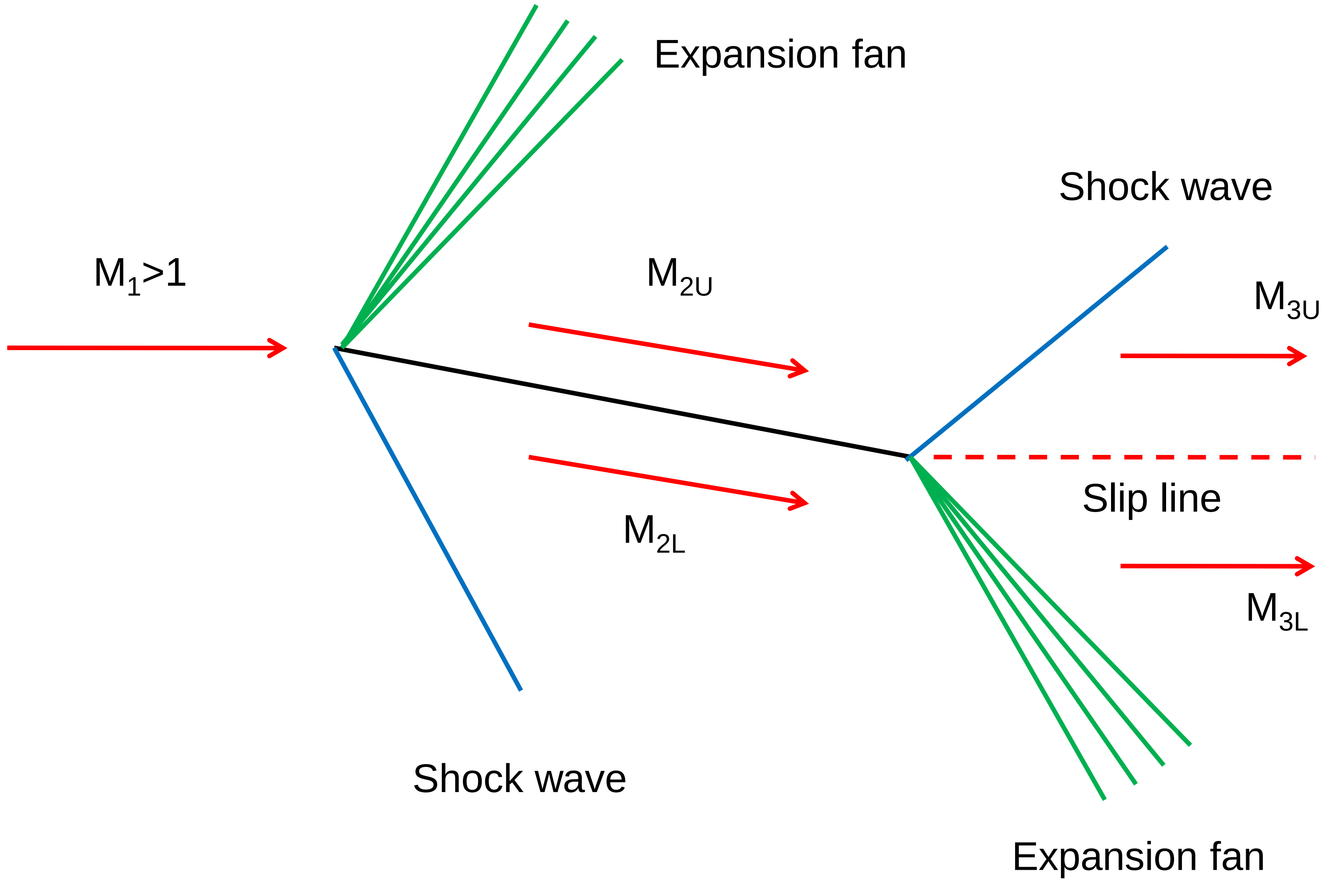


$$\sin \mu_1 = \frac{1}{M_1}$$

$$\sin \mu_2 = \frac{1}{M_2}$$

$$\theta = \nu(M_2) - \nu(M_1)$$

Flow past a flat plate at incidence



Example

- $M_1=2$, $p_1=30\ kN/m^2$, 1m chord plate at 10 degrees incidence
  - Find M_{2U}, M_{2L}, p_{2U}, p_{2L}
  - Find lift, drag and pitching moment
  - Find centre of pressure and aerodynamic centre
  - Convert to coefficient form

Expansion fan (1 to 2U)

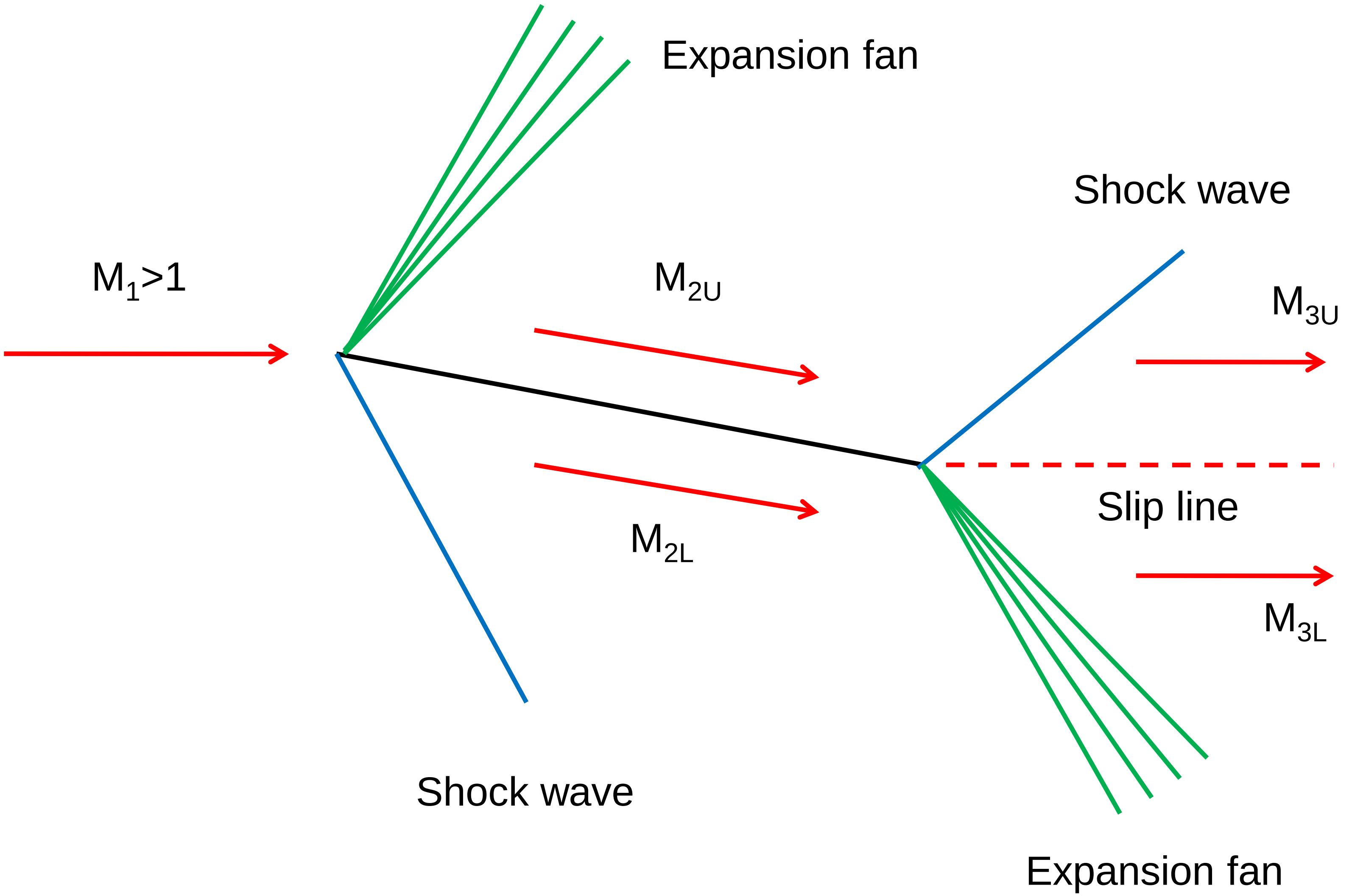
Isentropic-flow table (γ = 1.4):					
M	p / p_0	ρ / ρ_0	T / T_0	ν (deg.)	A / A^*
2.0000	0.1278	0.2300	0.5556	26.3798	1.6875

- Isentropic process,
  - M_1=2 IFT gives $\nu(M_1)$=26.38 deg., p_1/p_0=0.1278
  - P-M equation $\theta=\nu(M_2)- \nu(M_1)$
  - Hence $\nu(M_2)$=36.38 deg.

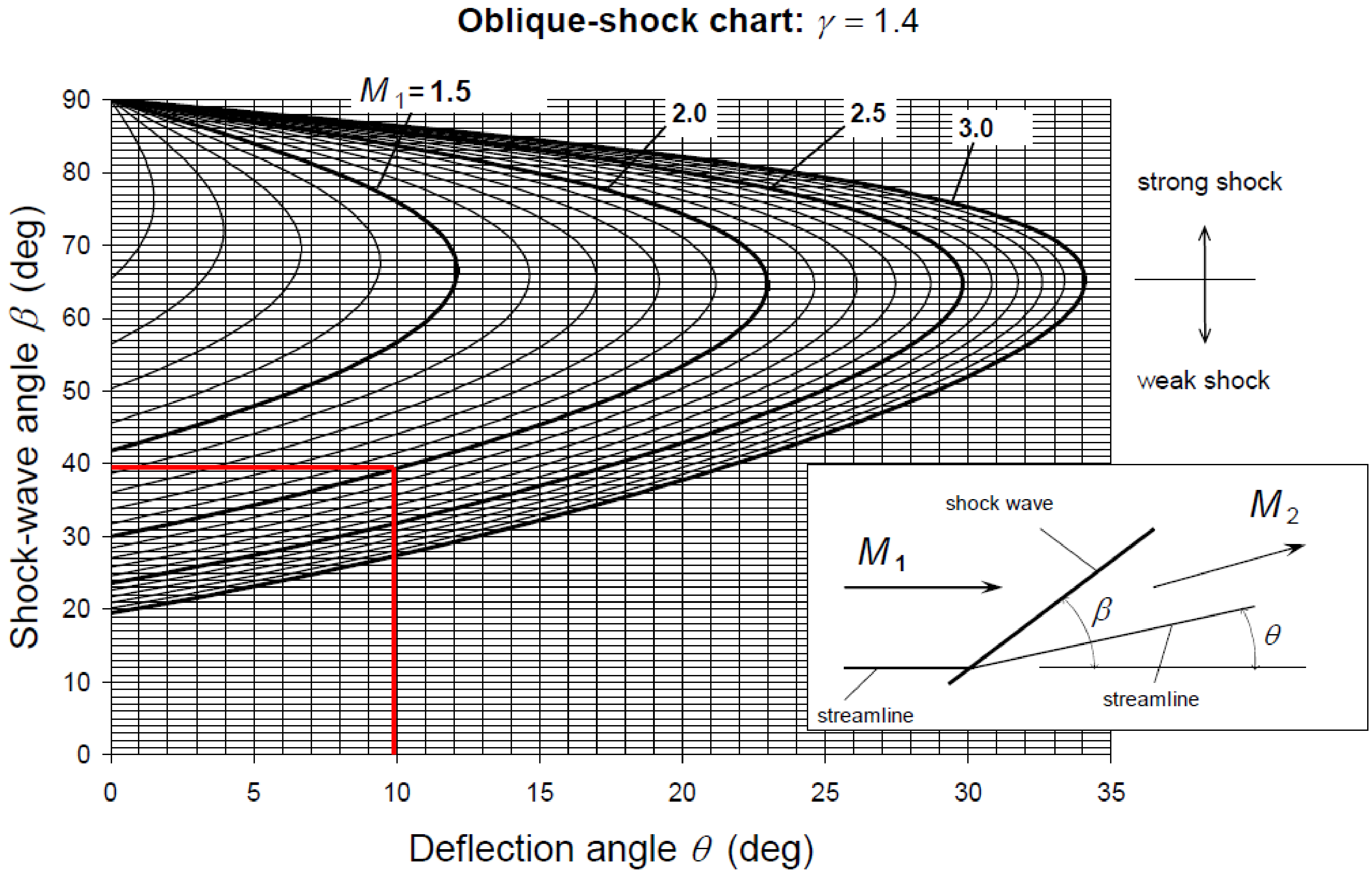
2.3800	0.0706	0.1505	0.4688	36.2607	2.3593
2.4000	0.0684	0.1472	0.4647	36.7465	2.4031

Interpolation from IFT: M_{2U}=2.385, p_{2U}/p_0=0.0701
Hence p_{2U}=16.5 kN/m^2

Flow past a flat plate at incidence



Oblique shock (1 to 2L)



Shock angle 39 deg.

Oblique shock calculation

$M_{n1}=M_1 \sin\beta=2.\sin(39)=1.259$

NST gives $p_{2L}/p_1=1.683$ (interpolation)

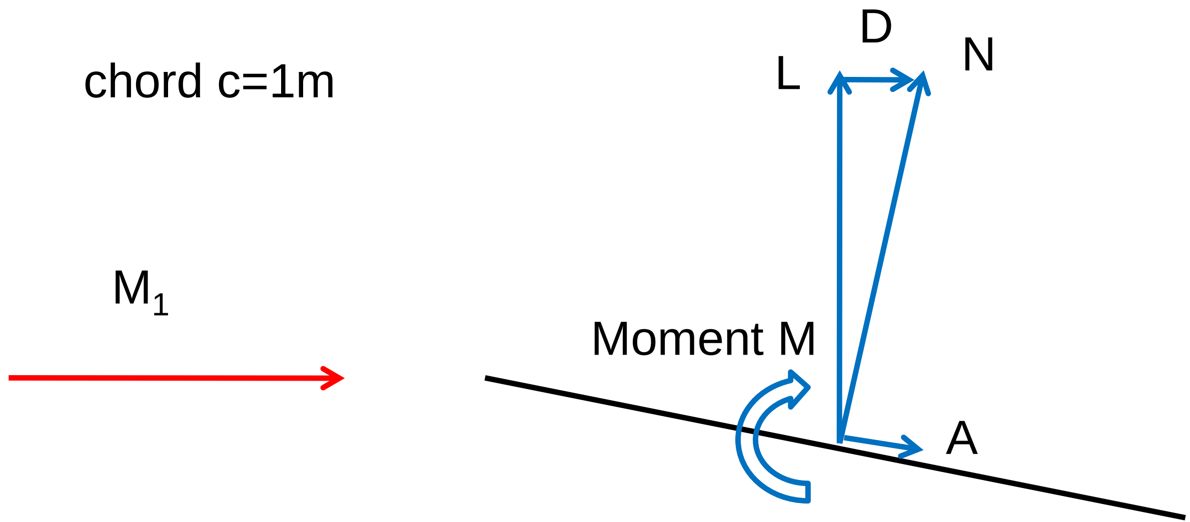
Hence $p_{2L}=50.5$ kN/m^2

For $M_{n1}=1.259$ NST gives $M_{n2}=0.8077$ (interpolation)

Hence $M_{2L}=0.8077/\sin(39-10)=1.67$

Aerodynamic forces

chord c=1m



L=Ncosα-Asinα
D=Acosα+Nsinα

Here A=0 (flat plate);

Pressure is constant along both upper and lower surfaces (centre of pressure is at the half chord)

$N=(p_{2L}-p_{2U})c=(50.5-16.5)$ =34.0 kN/m

L=Ncosα=34cos10=33.5 kN/m

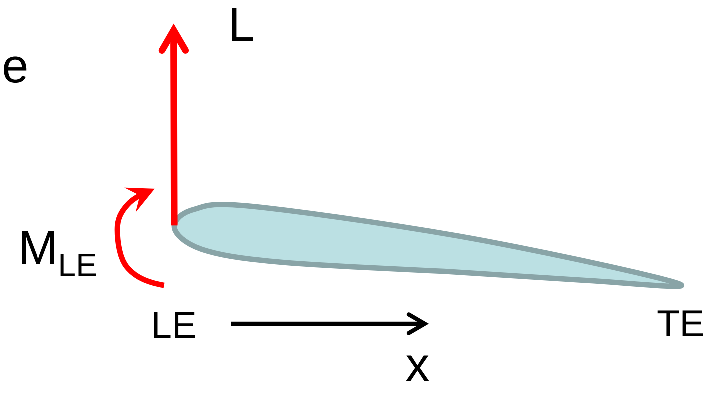
D=Nsin α=34sin10=5.9 kN/m (Non zero!)

$$M_{LE} = \int_0^c \left(p_{2U} - p_{2L}\right) x \,\mathrm{d}x$$

$$= \left[-34 \frac{x^2}{2} \right]_0^1 \quad = -17 \text{ kN}$$

Centre of pressure and aerodynamic centre

Forces and moments about leading edge



Moment coefficient about any point

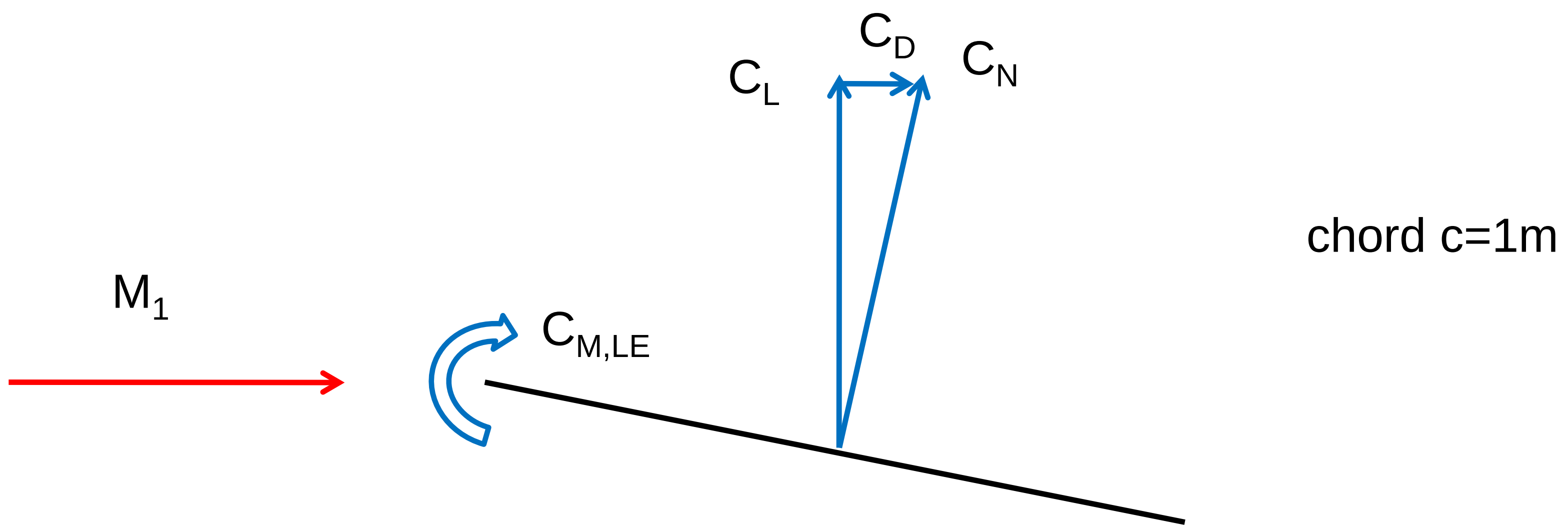
$$C_{M,x} = C_{M,LE} + \bar{x} C_L \quad \Rightarrow \quad \bar{x}_{cp} = -\frac{C_{M,LE}}{C_L}$$

(point where force acts with no moment)

$$\frac{\mathrm{d}C_{M,x}}{\mathrm{d}C_L} = \frac{\mathrm{d}C_{M,LE}}{\mathrm{d}C_L} + \bar{x} \quad \Rightarrow \quad \bar{x}_{ac} = -\frac{\mathrm{d}C_{M,LE}}{\mathrm{d}C_L}$$

(point where C_M is independent of C_L)

Aerodynamic coefficients



$$C_L = \frac{L}{\frac{1}{2}\rho_1 V_1^2 c} = \frac{2L}{\gamma p_1 M_1^2 c} = \frac{2\times 33,500}{1.4\times 30,000\times 2^2} = 0.40$$

$$C_D = \frac{D}{\frac{1}{2}\rho_1 V_1^2 c} = \frac{2D}{\gamma p_1 M_1^2 c} = \frac{2\times 5,900}{1.4\times 30,000\times 2^2} = 0.070$$

$$C_{M,LE} = \frac{M_{LE}}{\frac{1}{2}\rho_1 V_1^2 c^2} = \frac{2M_{LE}}{\gamma p_1 M_1^2 c^2} = \frac{2\times(-17,000)}{1.4\times 30,000\times 2^2\times 1^2} = -0.20$$

$$\bar{x}_{CP} = \frac{-C_{M,LE}}{C_L} = \frac{0.2}{0.4} = 0.5$$

$$\bar{x}_{AC} = \frac{-dC_{M,LE}}{dC_L} = 0.5$$

(assuming linear variation of coefficients with incidence)

Key points

- In supersonic inviscid flow we have non-zero drag – this is called wave drag.
- Centre of pressure and aerodynamic centre are at the half chord position for a flat plate in supersonic flow
  - Compared to quarter chord location for incompressible flow
  - Movement of aerodynamic centre of a wing as we pass through Mach 1 has control implications