SESA2024 Astronautics

Chapter 5: Mission Analysis

Lecture 13 – Hohmann transfer worked example

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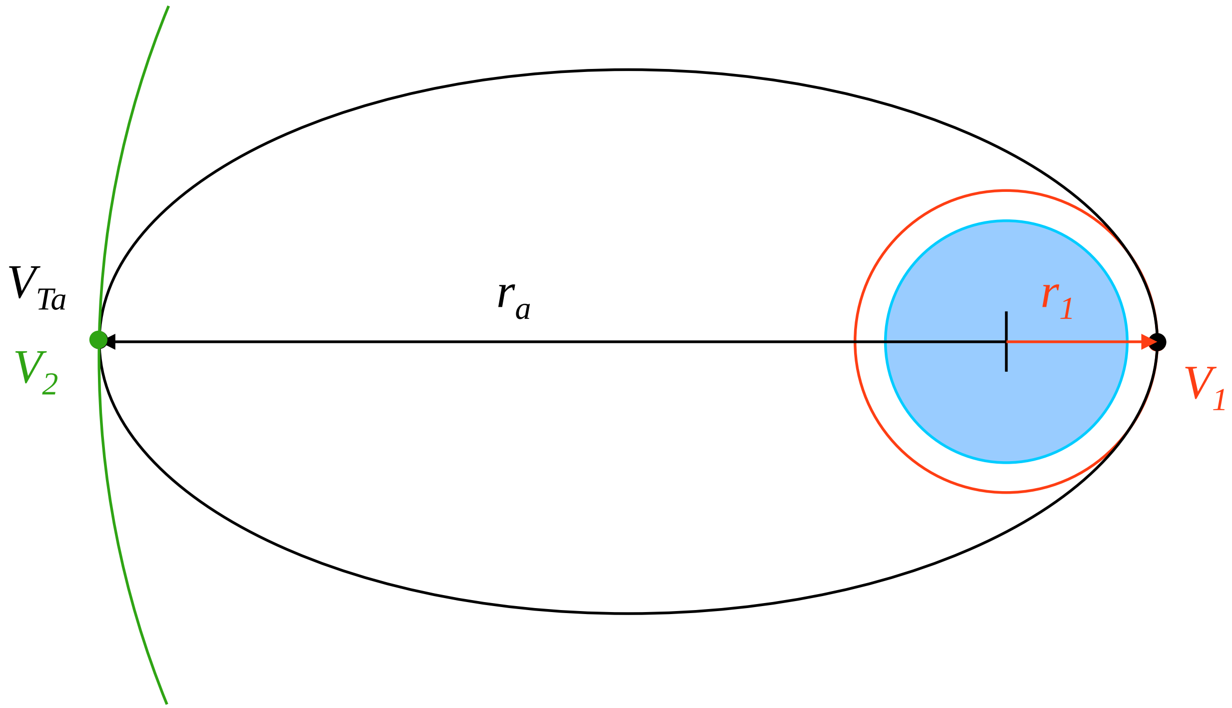
Overview of lecture 13

- This lecture presents a worked solution to a question about a Hohmann transfer
- The energy equation is needed for the answer (see lecture 9), as is an understanding of impulsive orbital transfers (see lecture 11)
- The question appears on the next slide:
  - If you would like to attempt the question before continuing the lecture, simply pause or stop the recording
    - If you keep the Panopto player open you may still be able to see the solution even if you pause the recording on the question slide. If you prefer, you can minimise the Panopto window, open the lecture slides and advance to the question
  - Then resume the recording to see an explanation of the full worked solution

Orbital transfers

- Example calculation
  - Calculate the ΔV and transfer time from a circular 200 km altitude parking orbit to GEO. Estimate the fuel mass for the 2nd burn if $V_{ex} = 2.5$ km/s. Take $R_E = 6378$ km, $r_{GEO} = 42{,}164$ km $\mu_E = 398{,}600$ km^3/s^2.



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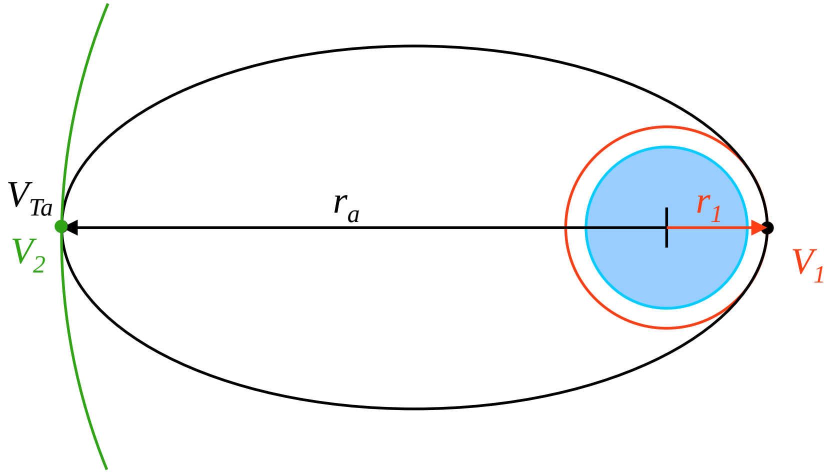
Orbital transfers

- Example calculation



- The radius of the parking orbit is r_1

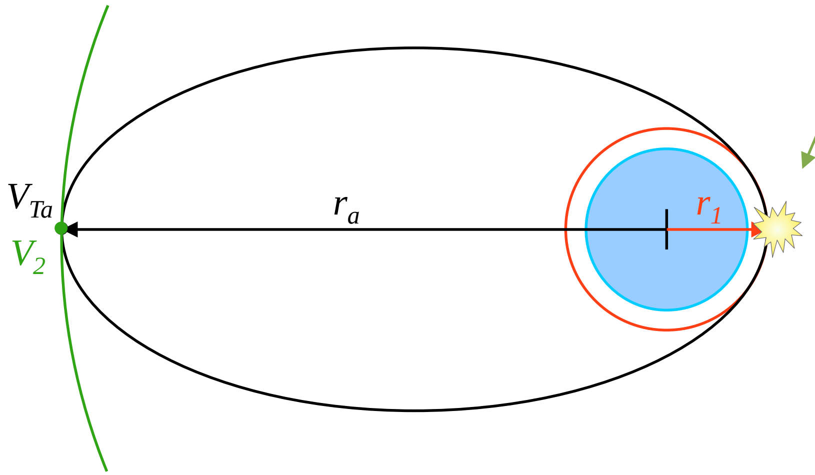
$$r_1 = 6378 + 200 = 6578 \text{ km}$$

- This is also the perigee of the elliptical transfer orbit.
- The radius of the circular GEO orbit is $r_2 = r_{GEO}$
- This is also the apogee of the elliptical transfer orbit
- Hence:

$$a_T = (6578 + 42164)/2 = 24371 \text{ km}$$

Orbital transfers

- Example calculation



- We can now use the energy equation twice, where the parking orbit and the transfer orbit intersect:
  - On the circular parking orbit:

$$V_1 = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{398600}{6578}} = 7.784 \text{ km/s}$$

  - At the perigee of the elliptical transfer orbit:

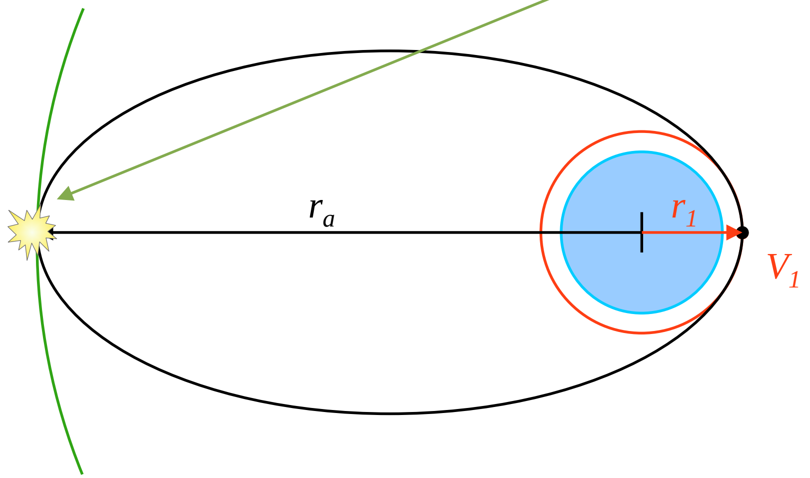
$$V_{Tp} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_T}\right)} = \sqrt{398600\left(\frac{2}{6578} - \frac{1}{24371}\right)}$$

$$= 10.239 \text{ km/s}$$

- The difference is the ΔV_{Tp} for the first burn

Orbital transfers

- Example calculation



- We can now use the energy equation twice, where the transfer orbit and the GEO orbit intersect:
  - At the apogee of the elliptical transfer orbit:

$$V_{Ta} = \sqrt{\mu\left(\frac{2}{r_{GEO}} - \frac{1}{a_T}\right)} = \sqrt{398600\left(\frac{2}{42164} - \frac{1}{24371}\right)}$$

$$= 1.597 \text{ km/s}$$

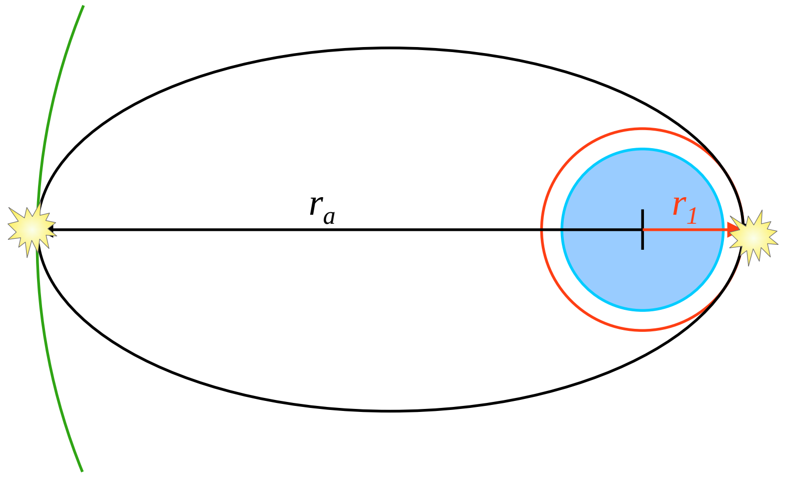
  - On the circular GEO orbit:

$$V_{GEO} = \sqrt{\frac{\mu}{r_{GEO}}} = \sqrt{\frac{398600}{42164}} = 3.075 \text{ km/s}$$

- The difference is the ΔV_{Ta} for the second burn

Orbital transfers

- Example calculation



- For the first burn:

$$\Delta V_{Tp} = V_{Tp} - V_1 = 10.239 - 7.784\ \text{km/s}$$
$$= 2.455\ \text{km/s}$$

- For the second burn:

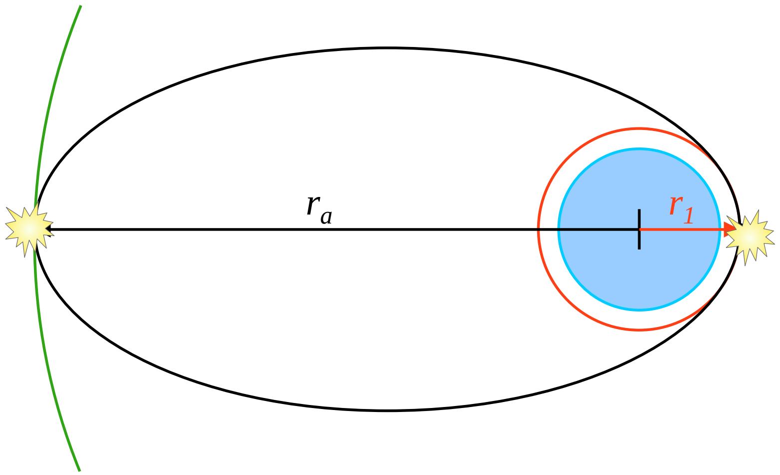
$$\Delta V_{Ta} = V_{GEO} - V_{Ta} = 3.075 - 1.597\ \text{km/s}$$
$$= 1.478\ \text{km/s}$$

- Hence the total ΔV is:

$$\Delta V = \Delta V_{Tp} + \Delta V_{Ta} = 2.455 + 1.478\ \text{km/s}$$
$$= 3.933\ \text{km/s}$$

Orbital transfers

- Example calculation

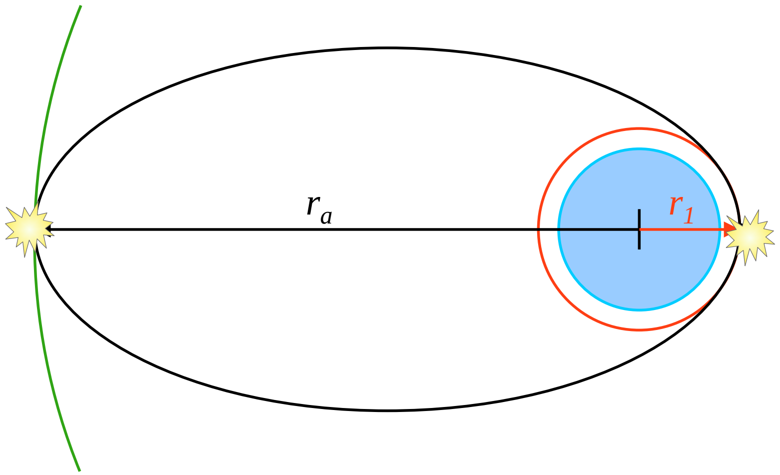


- The transfer time from the circular parking orbit to the GEO circular orbit is the tie taken to move from the perigee to the apogee of the elliptical transfer orbit
  - I.e. it is half the orbit period for the elliptical transfer orbit
- The orbit period is: $\tau = 2\pi\sqrt{\frac{a_T^3}{\mu}}$
- Hence the transfer time is $t = \tau/2$

$$t = \pi\sqrt{\frac{24371^3}{398600}} = 18931.77 \text{ sec} = 5.259 \text{ hrs}$$

Orbital transfers

- Example calculation



- We use the rocket equation to estimate the fuel mass for the second burn:

$$\Delta V_{Ta} = V_{ex} \ln\left(\frac{M_0}{M_b}\right)$$

- Let $x = M_0/M_b = \exp(\Delta V_{Ta}/V_{ex})$

$$x = \exp(\Delta V_{Ta}/V_{ex}) = \exp(1.477/2.5) = 1.805$$

- As $M_b = M_0 - M_f$ we can write:

$$\frac{M_f}{M_0} = \frac{x-1}{x} = \frac{1.805-1}{1.805} = 0.45$$

- So, finally we have $M_f = 0.45 M_0$