UNIVERSITY OF Southampton

Chapter 5: Mission Analysis

Lecture 9 – Orbital energy

Professor Hugh Lewis

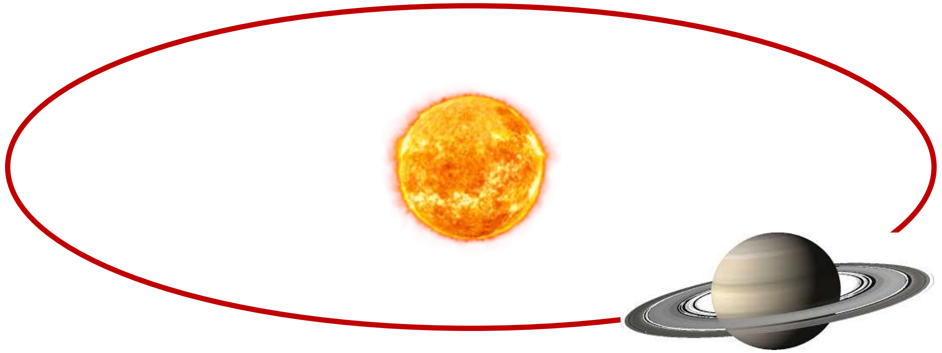
Overview of lecture 9

- This lecture takes a simple insight – that orbital energy is conserved – and uses it to develop one of the most important equations in orbital mechanics: the energy equation
  - The orbital energy is derived in terms of:
    - The kinetic energy
    - The potential energy

  - A quick quiz (embedded in the lecture recording if watching via Panopto) enables you to test your understanding
  - Two activities are provided to further your understanding
  - A worked example is provided in the next lecture

Orbital energy

- The gravitational force is a conservative force (a function of position only) and so the energy is conserved



- Hence, the energy is:

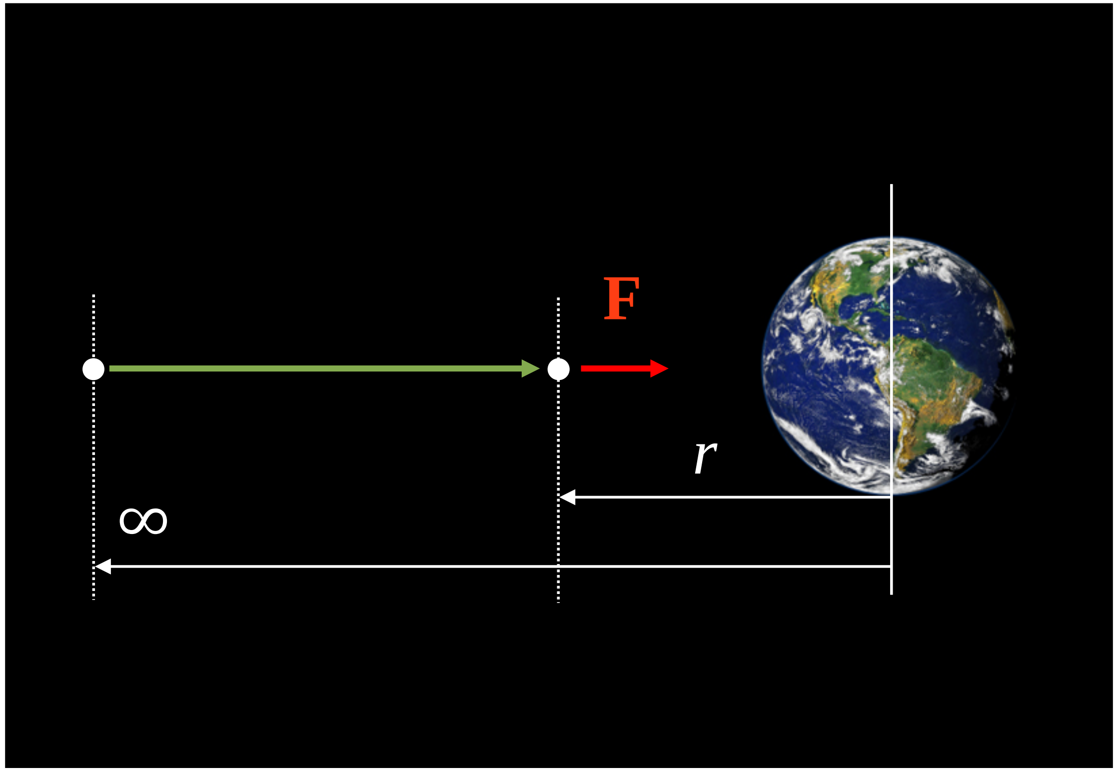
$$\boxed{\mathrm{KE} + \mathrm{PE} = \text{constant}}$$

- The kinetic energy is:

$$\mathrm{KE} = \frac{mV^2}{2}$$

Orbital energy

- Potential energy

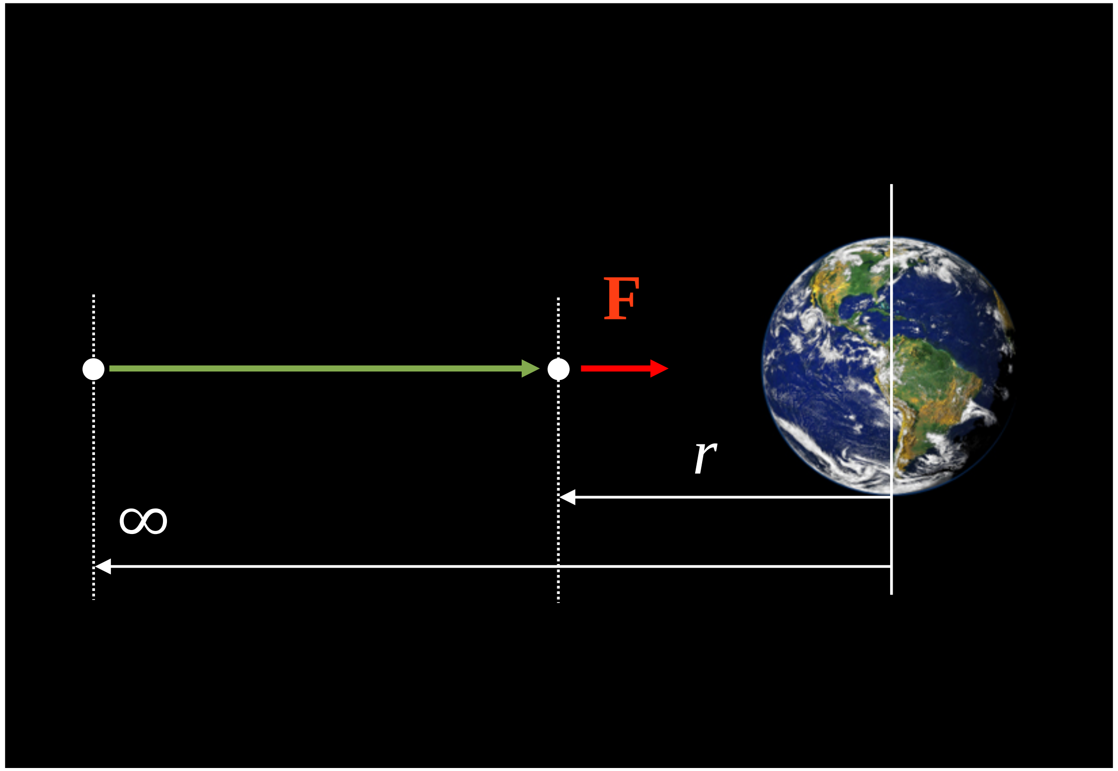


- The gravitational potential energy is zero when the satellite is infinitely far from the Earth.
- For the inverse-square law gravity field the potential energy is the work done by the gravitational force in bringing the satellite from infinity to a distance r from the centre of the Earth:

$$\mathrm{PE} = \int_{\infty}^{r} \left(\frac{\mu m}{r^2}\right) dr$$

Orbital energy

- Potential energy and conservation of energy



- Hence, the potential energy is:

$$\text{PE} = -\frac{\mu m}{r}$$

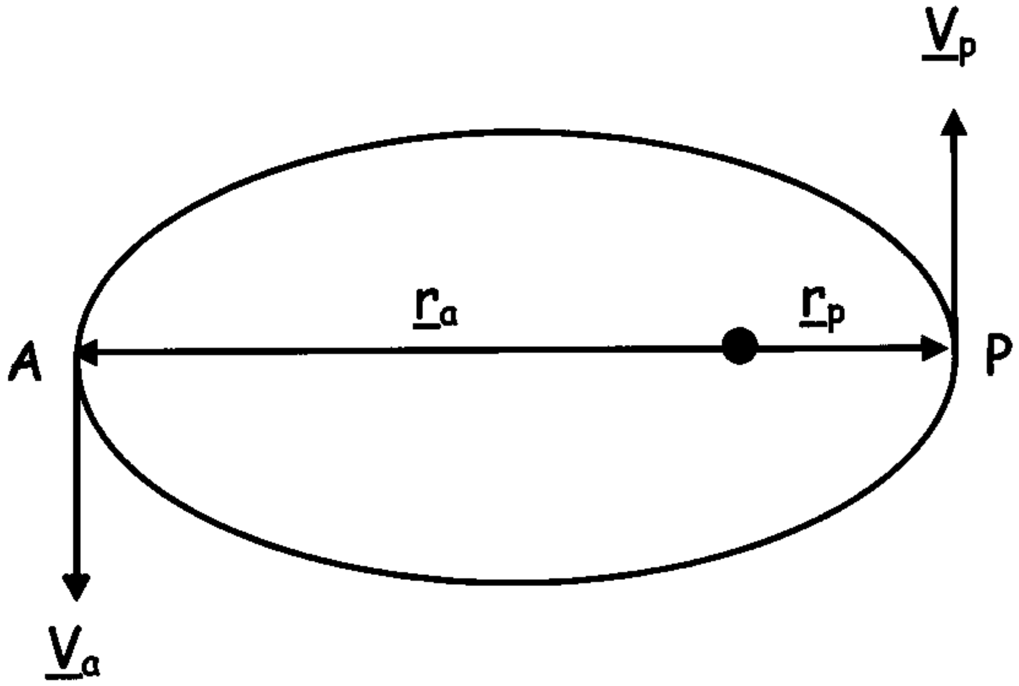
- For unit mass we can write the specific energy as:

$$\text{KE} + \text{PE} = \frac{V^2}{2} - \frac{\mu}{r} = \varepsilon$$

- Where ε is constant

Orbital energy

- What is the value of ε?



- We already know that the orbital angular momentum (moment of momentum) is conserved, so:

$$\mathbf{h}_p = \mathbf{h}_a$$

- Hence we can write:

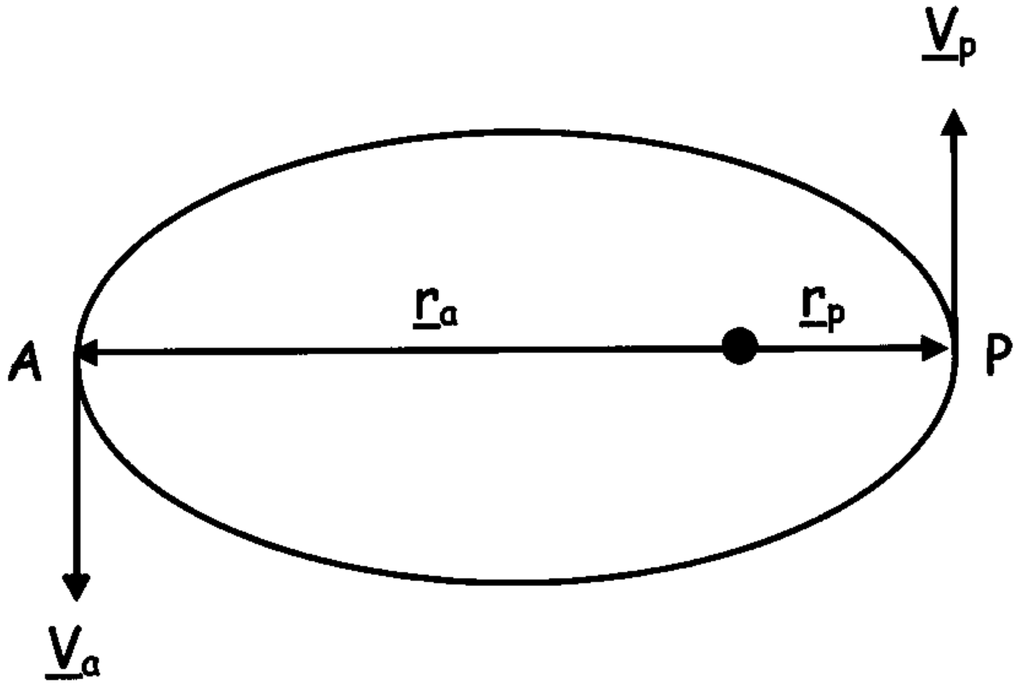
$$\mathbf{r}_p \times \mathbf{V}_p = \mathbf{r}_a \times \mathbf{V}_a$$

$$r_p V_p \sin 90° = r_a V_a \sin 90°$$

- And $$\frac{V_p}{V_a} = \frac{r_a}{r_p}$$

Orbital energy

- What is the value of ε?



- Now we can use the specific orbital energy:

$$\frac{V^2}{2} = \varepsilon + \frac{\mu}{r}$$

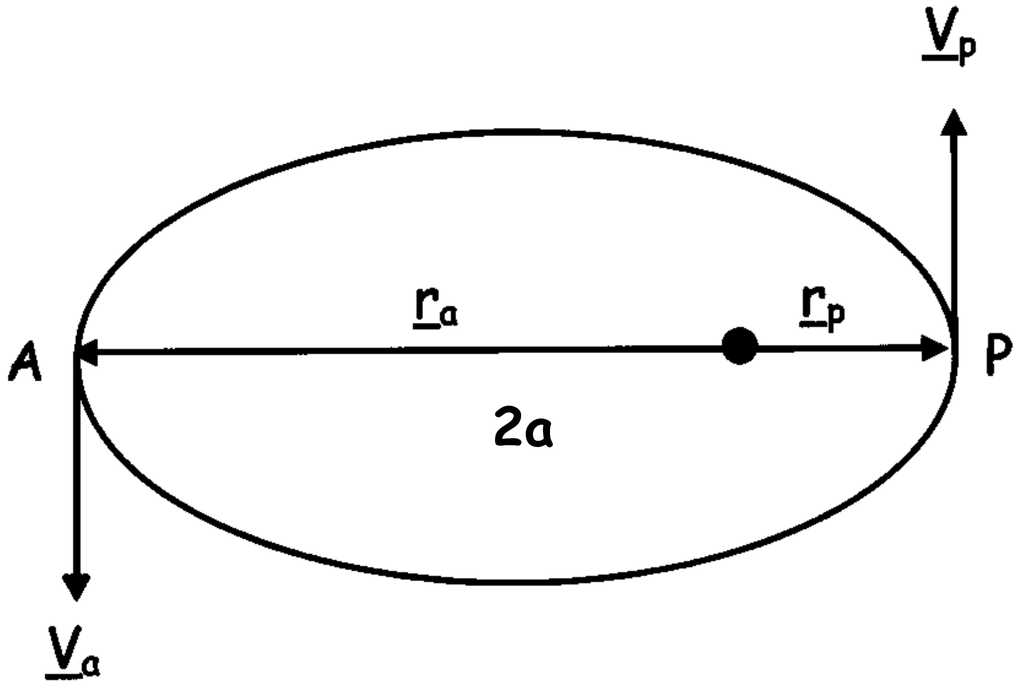
- Hence we can write:

$$\frac{\frac{1}{2}V_p^2}{\frac{1}{2}V_a^2} = \left(\frac{r_a}{r_p}\right)^2 = \frac{\varepsilon + \frac{\mu}{r_p}}{\varepsilon + \frac{\mu}{r_a}}$$

- Simplifying gives…

Orbital energy

- What is the value of ε?



Remember: $\mu = GM$

- Simplifying gives…

$$\varepsilon = -\frac{\mu}{r_a + r_p} = -\frac{\mu}{2a}$$

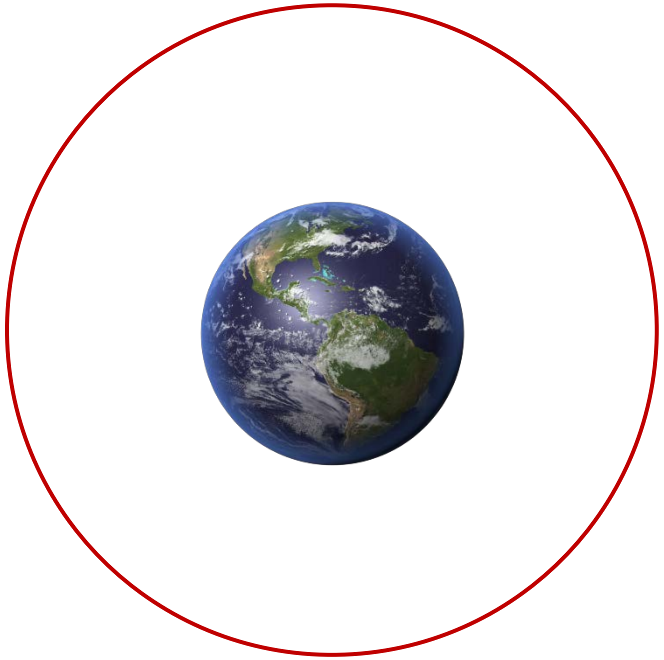
- Resulting in the orbital energy equation:

$$\boxed{\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}}$$

- This applies for all conics (circle, ellipse, parabola, hyperbola)

Orbital energy

- Circular orbits



- For circular orbits

$$r = a$$

- Which means we can write

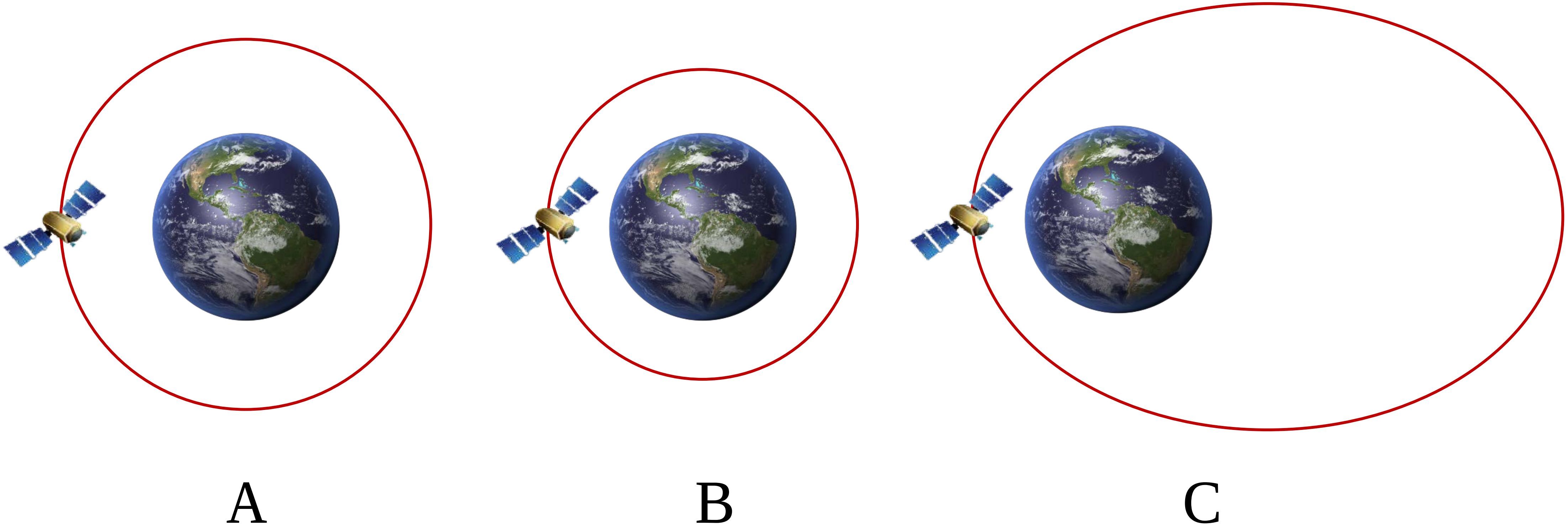
$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2r}$$

- And the energy equation simplifies to:

$$V = \sqrt{\frac{\mu}{r}}$$

Orbital energy

- Quick quiz:
  - Which satellite has the greatest speed?



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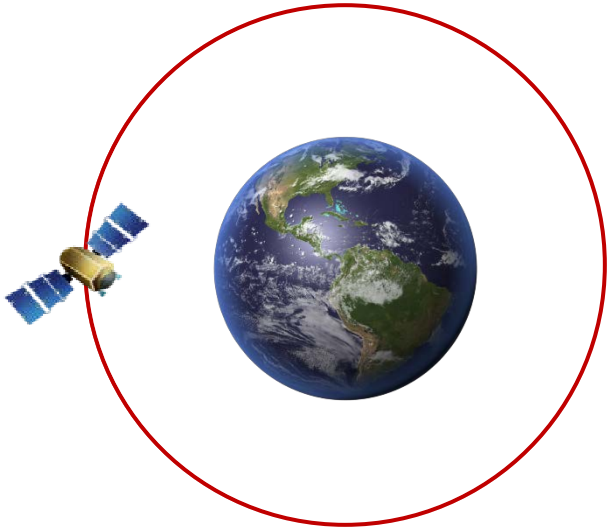
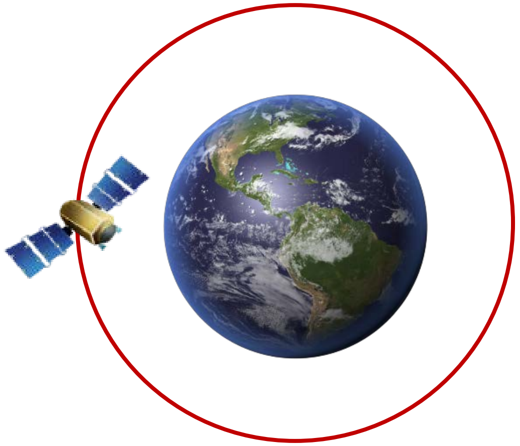
Orbital energy

- Quick quiz 1: ANSWER
  - First, look at A and B



A



B

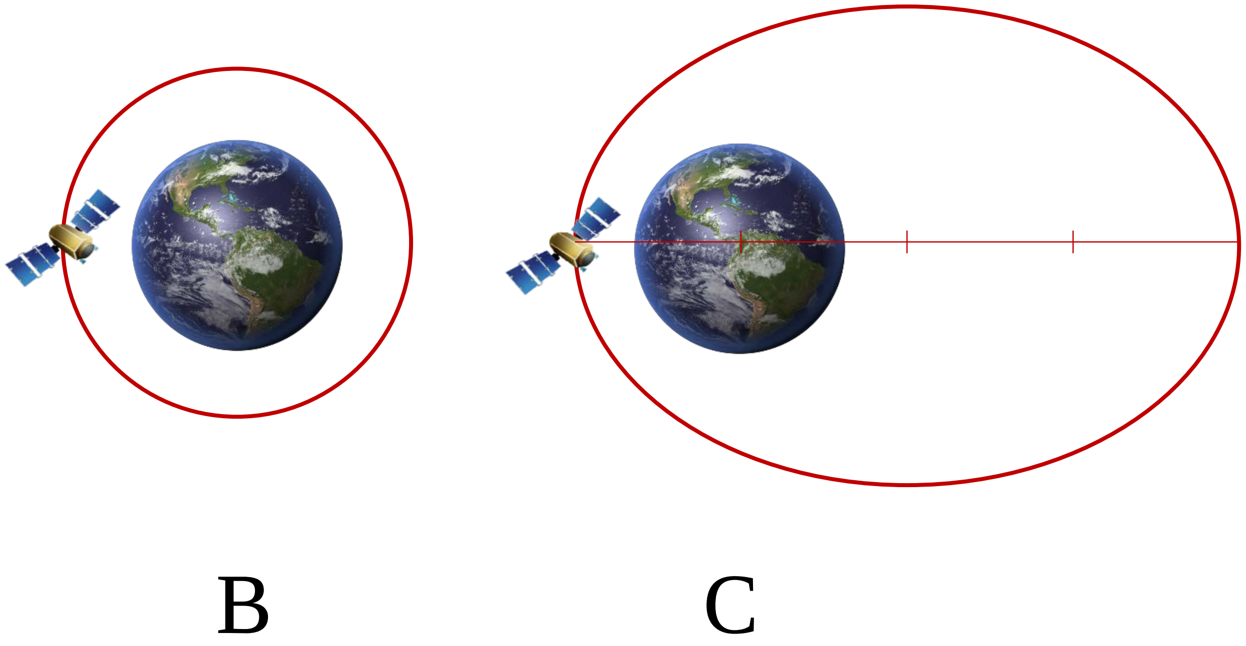
- For circular orbits:

$$V = \sqrt{\frac{\mu}{r}}$$

- And $r_A > r_B$ so $V_A < V_B$
- So the answer is NOT A

Orbital energy

- Quick quiz 1: ANSWER
  - Now, look at B and C



- Orbit C is elliptical not circular
- Comparing the orbit radii:

$$r_C \approx r_B$$

- But comparing their semi-major axes we can also see that:

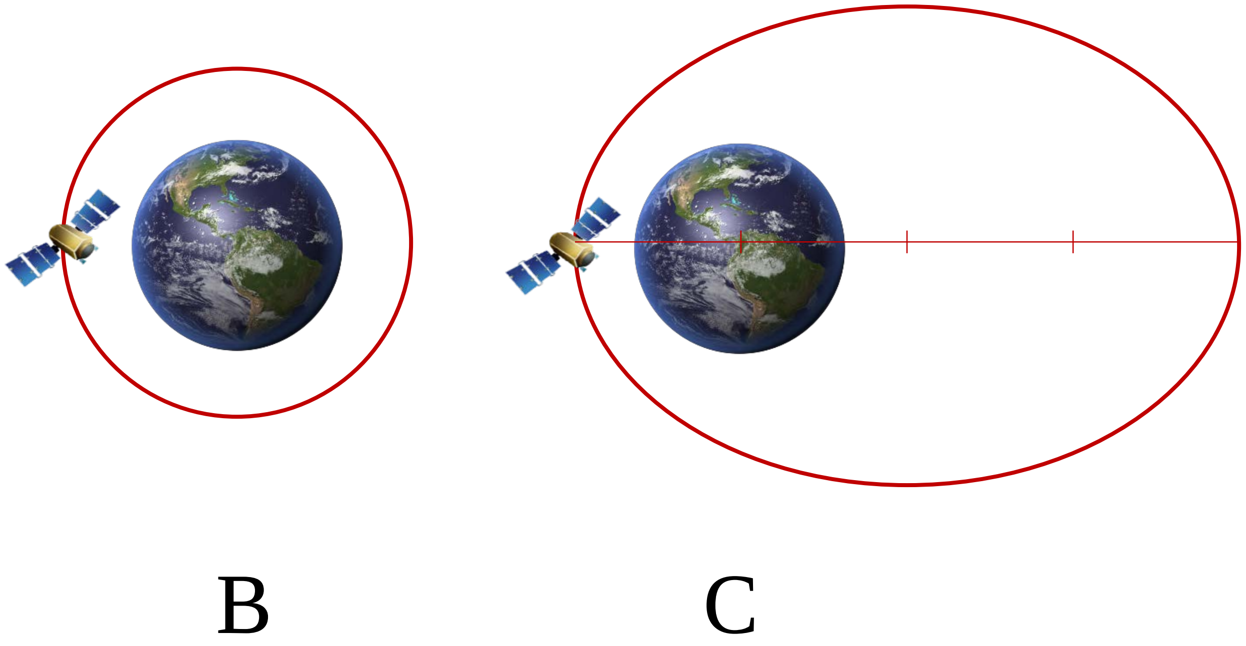
$$a_C > a_B$$

- In fact, $a_C \approx 2r_C$ and we can use the full energy equation to determine the speed of satellite C

$$\frac{V_C^2}{2} - \frac{\mu}{r_C} = -\frac{\mu}{2(2r_C)}$$

Orbital energy

- Quick quiz 1: ANSWER
  - Now, look at B and C



- Rearranging and simplifying gives the speed of satellite C:

$$V_C = \sqrt{\frac{3\mu}{2r_C}} > \sqrt{\frac{\mu}{r_C}}$$

- And because $r_C = r_B$ we can finally say:

$$V_C > V_B$$

- So the answer is C

Recap of lecture 9

- This lecture showed that for any point on an orbit:
  - The orbital energy is conserved:

$$\text{KE} + \text{PE} = \text{constant}$$

  - With an understanding of the kinetic energy and the potential energy, we defined the energy equation:

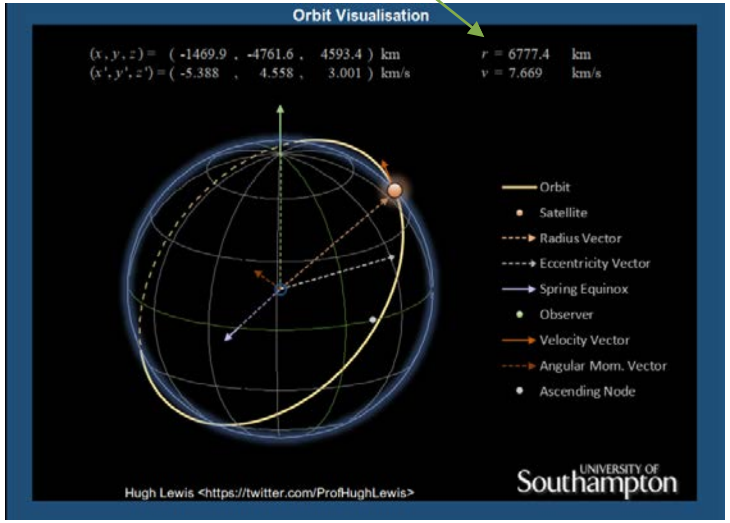
$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

  - A worked example is provided in lecture 10

Activity 1

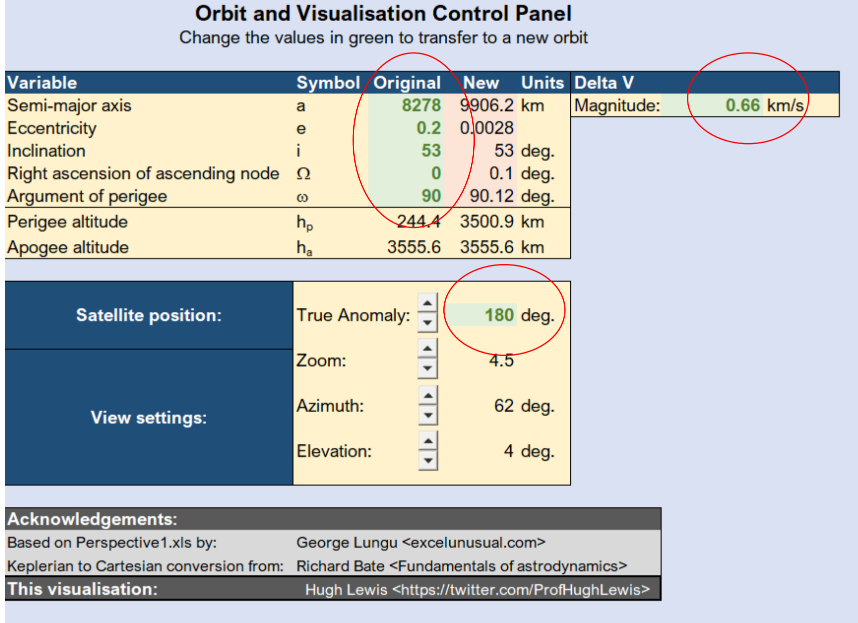
- Activity using the orbit visualisation tool:
  1. Visit “Heavens Above” (https://www.heavens-above.com/orbit.aspx?satid=24960&lat=-35.2809&lng=149.13&loc=Canberra&alt=567&tz=AEST) to find the orbital elements of the following satellite:
     – Molniya-1T
  2. Enter the orbital elements into the orbit visualisation tool (you will need to calculate the semi-major axis and also adjust the zoom value to see the full orbit using the up and down arrow buttons)
  3. Change the true anomaly (using the appropriate buttons) and observe how the speed changes

The satellite speed is shown here



Activity 2

- Activity using the orbit visualisation tool:
  - Use the “Delta-V” version of the visualisation tool
    - You can change the original orbit and the magnitude of the delta-V (which can be negative too)
    - The orbital elements of the new orbit are displayed in the control panel on the left
    - The delta-V is applied at the current position of the satellite in the direction of the velocity vector (or in the opposite direction if the delta-V is negative)
    - The new orbit and the original orbit are coplanar



Orbit and Visualisation Control Panel

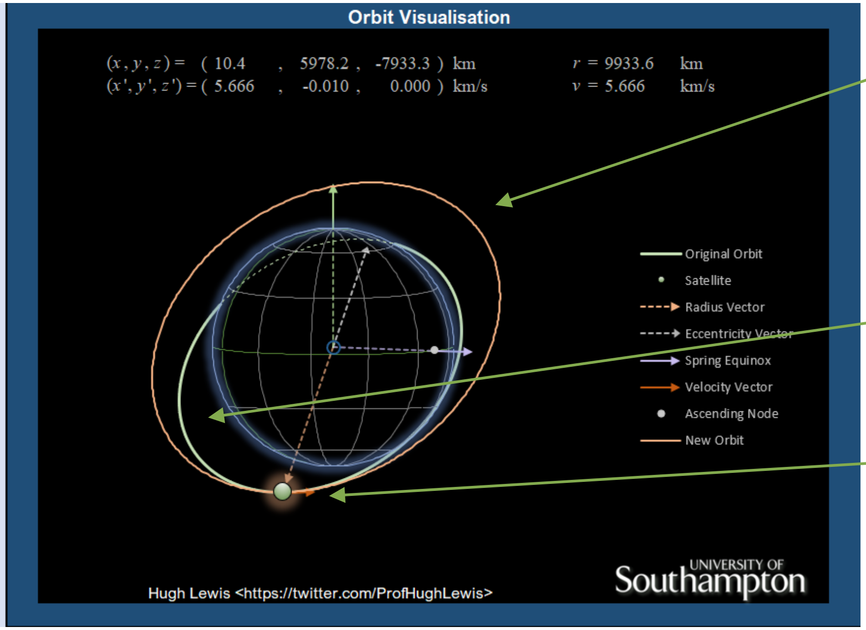
Change the values in green to transfer to a new orbit

Variable	Symbol	Original	New	Units
Semi-major axis	a	8278	9906.2	km
Eccentricity	e	0.2	0.0028	
Inclination	i	53	53	deg.
Right ascension of ascending node	Ω	0	0.1	deg.
Argument of perigee	ω	90	90.12	deg.
Perigee altitude	h_p	244.4	3500.9	km
Apogee altitude	h_a	3555.6	3555.6	km

Delta V	
Magnitude:	0.66 km/s

Satellite position:	True Anomaly:	180 deg.
View settings:	Zoom:	4.5
	Azimuth:	62 deg.
	Elevation:	4 deg.

Acknowledgements:	
Based on Perspective1.xls by:	George Lungu <excelunusual.com>
Keplerian to Cartesian conversion from:	Richard Bate <Fundamentals of astrodynamics>
This visualisation:	Hugh Lewis <https://twitter.com/ProfHughLewis>



The new orbit (after the application of the delta-V) is shown in orange

The original orbit is shown in light green

The delta-V is applied at the current position of the satellite