SESA3029 Aerothermodynamics

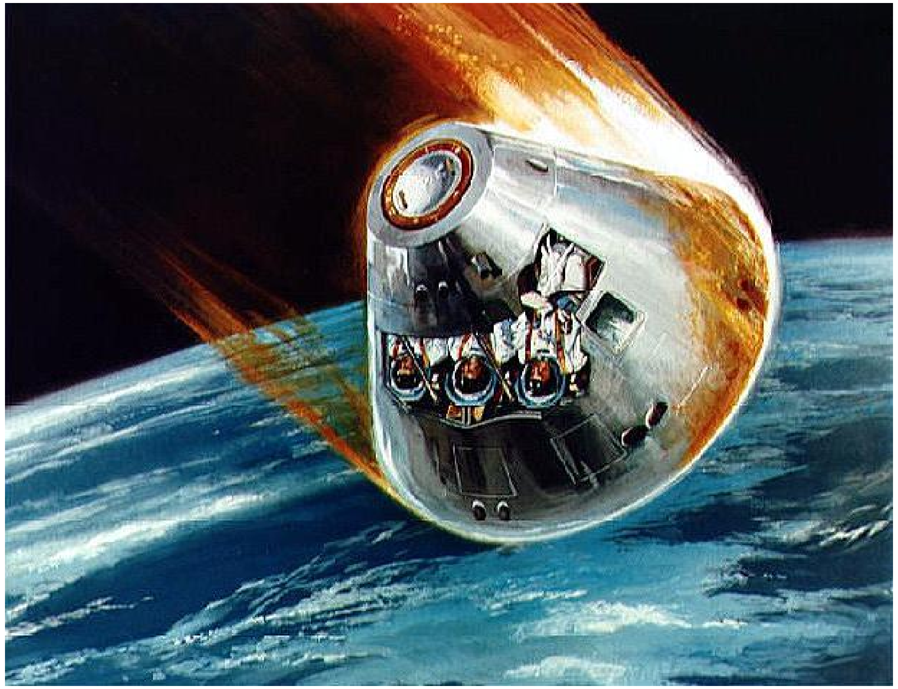


Figure: NASA Artist’s impression

Lecture 5.1

Conduction heat transfer

SESA3029: Lecture structure and background reading for heat transfer

Lecture	Topic	Bergman et al., 7th ed. Incropera et al., 8th ed.	Anderson, Comput. Fluid Dynamics, 1995	Online tests
5.1	Conduction heat transfer	1.2.1, 2.1, 3.1		
5.2	Convective heat transfer	1.2.2, 6.1		
5.3	Turbulent flow, Reynolds' analogy	6.3, 6.5, 7.2		
5.4	Radiation heat transfer	1.2.3, 12.1, 12.2		
5.5	Heat diffusion eq., 1D finite differences	2.3, 2.4, 4.4	4.2	
5.6	Transient 1D finite differences	5.3, 5.10	4.3, 4.4	#4 - 12/12/23
5.7	2D finite differences	4.4, 4.5		5.1 - 5.6
5.8	Heat exchangers -log-mean method	11.2, 11.3		
5.9	Heat exchangers -NTU method	11.4		
Tutorial	Test#4 Q&A, heat exchanger examples	11.5, 8.6		
Revision 3	12/1/24, Exam problems for 5			#5 - 9/1/24, 5.7 - 5.9 #6 - 11/1/24, 5.8 - 5.9

Textbooks:

Bergman, Lavine, Incropera, Dewitt. Fundamentals of heat and mass transfer. 7th edition. Wiley & Sons, 2011.

Incropera, Dewitt, Bergman, Lavine. Principles of heat and mass transfer. 8th edition. Wiley & Sons, 2017.

Anderson. Computational Fluid Dynamics. McGraw-Hill, 1995.

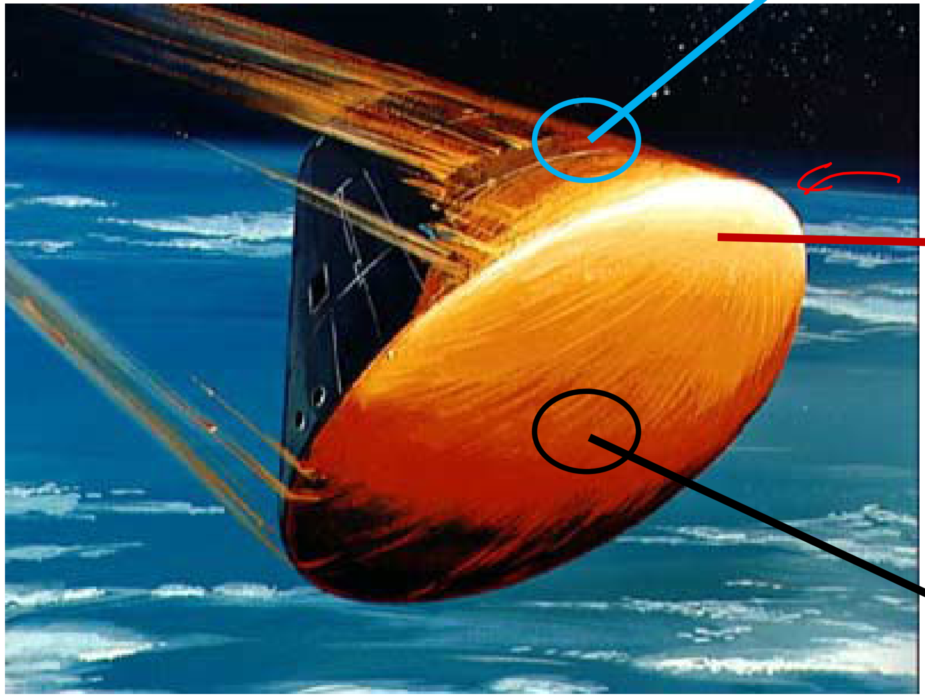
Convective heat transfer

U_e, T_e

Boundary layer

$u=0 \; T=T_{wall}$

this region could have a temp ~18000 K (Perfect assumption)



bright spots

Radiation heat transfer

These all contribute to heat transfer

Figure: NASA Artist's impression

In real life temps are closer to 10000 K

T_{inside} $T_{outside}$

very much not in thermal equalibrium

Conduction heat transfer

Heat transfer rate

used more often for us

$$\dot{Q} \quad (\mathrm{W=J/s}) \qquad\qquad \dot{q} = \frac{\dot{Q}}{A} \quad (\mathrm{W/m^2})$$

heat

heat flux

diffusion and brownian motion

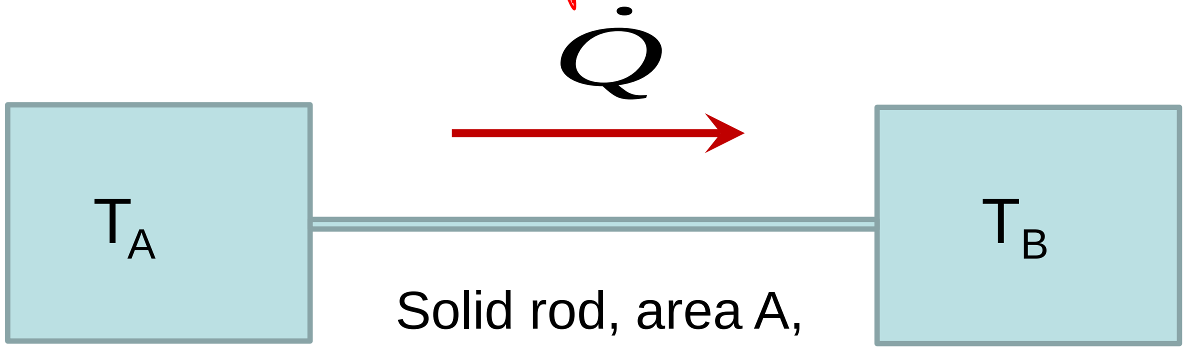
The heat transfer rate is driven by temperature differences:

- **conduction heat transfer** is the transfer of energy through a stationary medium e.g. in solids the vibration of molecules and in fluids additionally by molecular motions and collisions
- **radiation heat transfer** is the transfer of energy by disorganised photon emission

different from diffusion!

We also talk about **convective heat transfer (**really the convection of internal energy in a fluid) as the method by which the surface temperature (or heat transfer rate) under a moving fluid is obtained graphical error $\dot{Q}$

Conduction rate in a solid



Solid rod, area A, connecting two systems

$\dot{Q}$ Should be zero for equal temperatures and a reasonable model is that it is proportional to T_A-T_B

Also expected to be proportional to A, inversely proportional to rod length L and dependent on properties of the rod, so simplest model is:

$$\boxed{\dot{Q} = \frac{kA}{L}(T_A - T_B)}$$

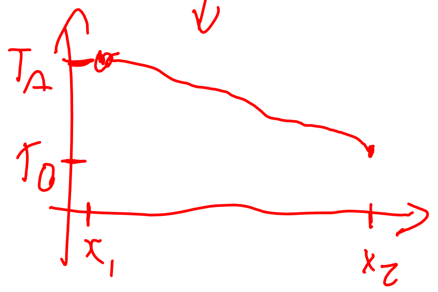
simple (but reasonably good) linear model

k = thermal conductivity

$$\dot{Q} = \frac{kA}{L}(T_A - T_B)$$

$T_A > T_B$

negative gradient



By shrinking the bar and the size of the temperature difference we can also write a differential form

$$\dot{Q} = -kA\frac{dT}{dx}$$

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{T_B - T_A}{L}$$

(Taking x in the direction of the heat transfer)

$$\boxed{\dot{q} = -k\frac{\mathrm{d}T}{\mathrm{d}x}}$$

This is **Fourier's law** of heat conduction

k is called the thermal conductivity, with units W/(mK)

3D vector/tensor generalisation $\dot{\mathbf{q}} = -k\nabla T$ $\quad \dot{q}_i = -k\frac{\partial T}{\partial x_i}$

heat flux vector

1 D form

Electrical resistance analogy

Multimaterial conductive medium

$$\dot{Q} = \frac{kA}{L}(T_A - T_B)$$

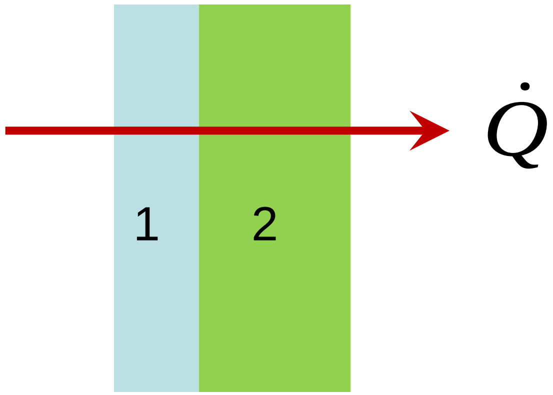
$$I = \frac{V}{R}$$

$I = \frac{1}{R} \times V$

Heat transfer rate ~ current
temperature ~ voltage

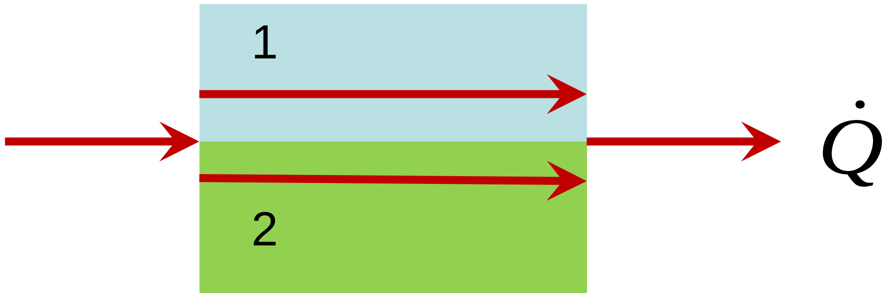
resistance $\overbrace{R = \frac{L}{kA}}$

"Thermal resistance"



$$R = R_1 + R_2$$

series



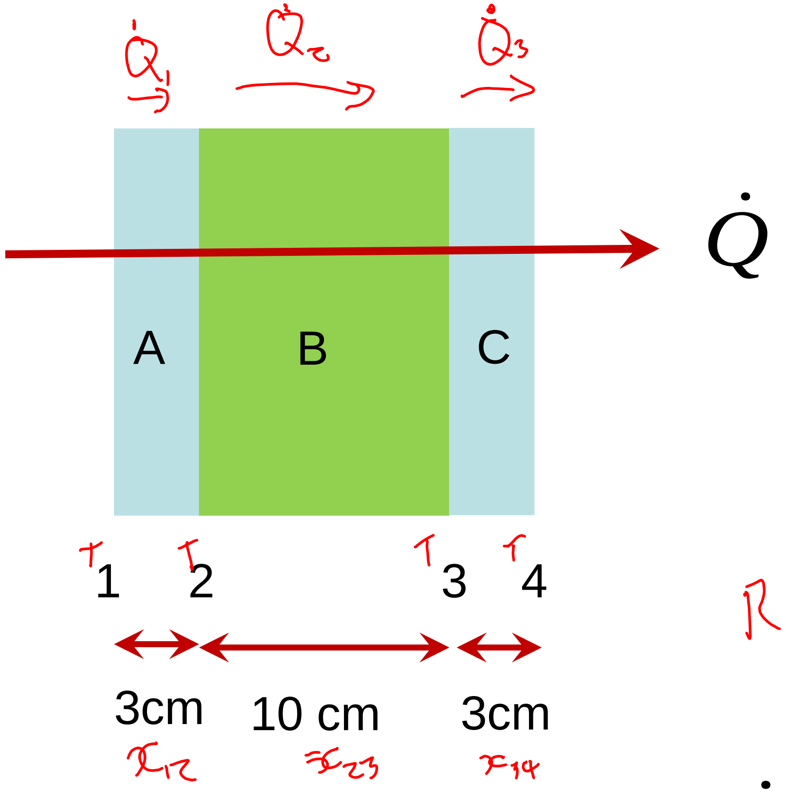
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

parallel

since no losses

Example: insulated plane block

$\dot{Q} = \dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3$



$\dot{Q}_1 = \frac{k_{12}A}{x_{12}}(T_1 - T_2)$

$\dot{Q}_2 = \frac{k_{23}A}{x_{23}}(T_2 - T_3)$

$\dot{Q}_3 = \frac{k_{34}A}{x_{34}}(T_3 - T_4)$

can be used to prove resistance model

$$T_1 = 150\,^{\circ}C$$

$$T_4 = 10\,^{\circ}C$$

$$k_A = k_C = 0.048\,\mathrm{W/(mK)}$$

$$k_B = 0.69\,\mathrm{W/(mK)}$$

$R_{12} = R_{34} = \frac{x_{12}}{k_{12}A}$ $\quad R_{23} = \frac{x_{23}}{k_{23}A}$

$R_{14} = 2R_{12} + R_{23}$

Find: $\dot{q} = \dfrac{\dot{Q}}{A}$, T_2 and T_3

$\dot{q}_{14} = A\left(\frac{2x_{12}}{k_A A} + \frac{x_{23}}{k_B A}\right)(T_1 - T_4)$

$= \left(2\frac{x_{12}}{k_A} + \frac{x_{23}}{k_B}\right)(T_1 - T_4)$

$= 1.3949\ W/m^2$

Also, compare with the heat transfer rate for the block alone (without A and C)

$\Delta T_{12} = \Delta T_{34} = 100.36\frac{W}{m^2}$ $T_2 = 87C$ $\quad T_3 = 72C$ $\quad$ Then T_1 and T_4 can be found