SESA2025 Mechanics of Flight
Trim Drag & Performance Optimisation

Lecture 1.5

Total aircraft induced drag

Dependence on CG location

- Given

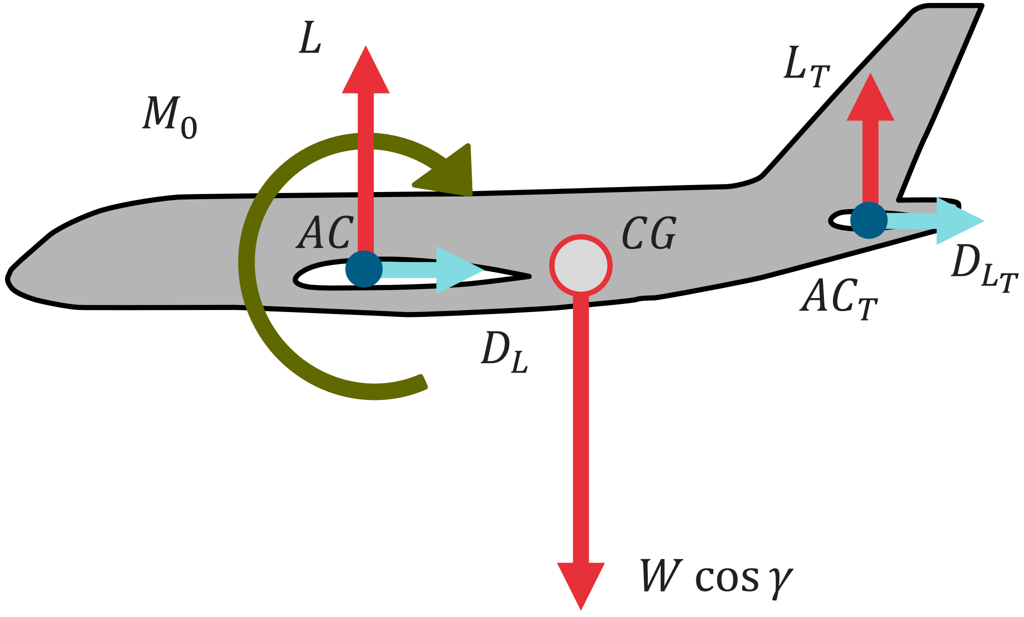
$$C_{M_0} + C_{L^*}(h - h_0) - C_{L_T}\,K = 0$$

- The tailplane lift is

CG cont

$$C_{L_T} = \frac{C_{M_0} + C_{L^*}(h - h_0)}{K}$$

D_L, D_{L_T} ⟶ induced drag



UNIVERSITY OF Southampton

Total aircraft induced drag

Dependence on CG location

- Given

$$C_{M_0} + C_{L^*}(h - h_0) - C_{L_T} K = 0$$

- The tailplane lift is

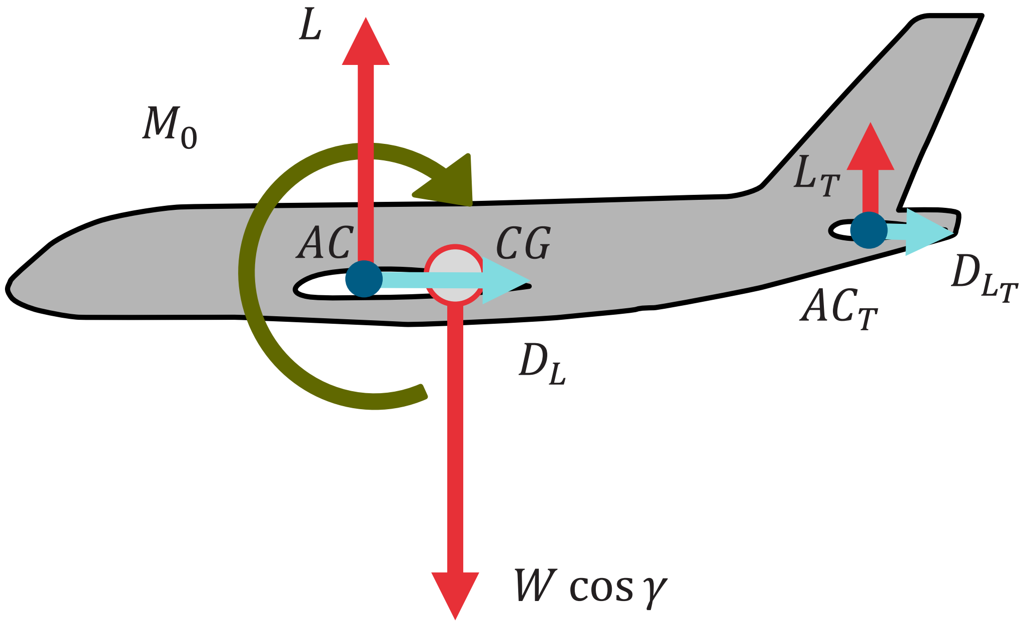
$$C_{L_T} = \frac{C_{M_0} + C_{L^*}(h - h_0)}{K}$$

- The total lift is still the same

$$L^* = L + L_T$$

how do these vary with CG pos and what CG pos gives lowest drag?

D_L, D_{L_T} → induced drag



Total aircraft induced drag

Dependence on CG location

h

- Given

$$C_{M_0} + C_{L^*}(h - h_0) - C_{L_T}\,K = 0$$

- The tailplane lift is

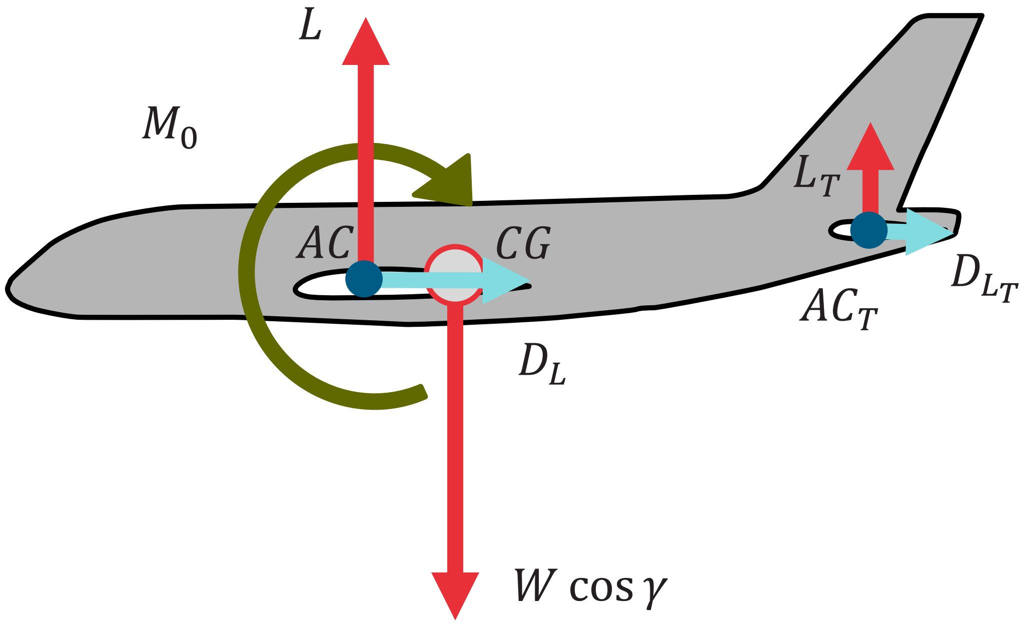
$$C_{L_T} = \frac{C_{M_0} + C_{L^*}(h - h_0)}{K}$$

- The total lift is still the same

$$L^* = L + L_T$$

- What is the optimal CG location? (lecture 1.6)
- What is the optimal split L_T/L^*? (today)

D_L, D_{L_T} $\longrightarrow$ induced drag

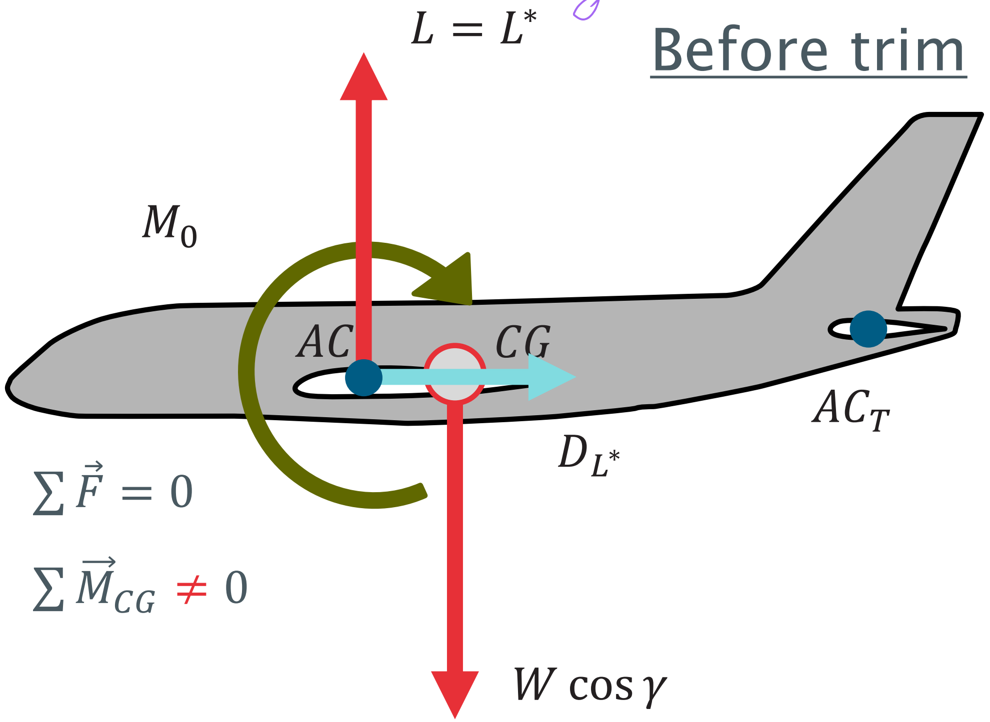


Defining trim drag

Consider these two configurations

no tail load

Before trim



$\sum \vec{F} = 0$

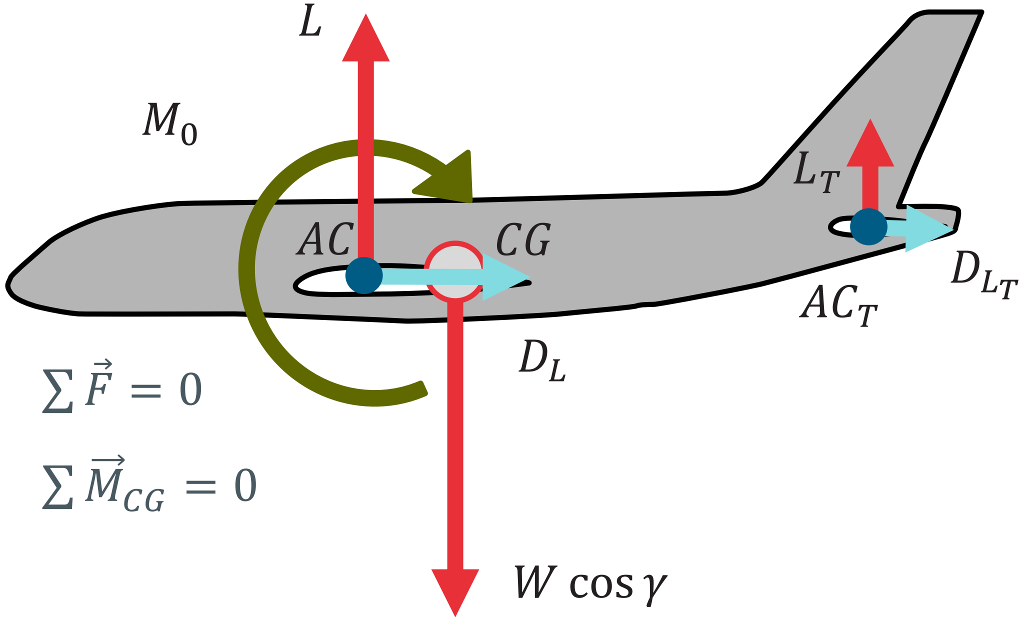
$\sum \vec{M}_{CG} \neq 0$

Lift and drag

$L = L^* = W\cos\gamma$ constant

$D_{before} = D_0 + \boxed{D_{L^*}}$ → induced drag

After trim



$\sum \vec{F} = 0$

$\sum \vec{M}_{CG} = 0$

Lift and drag

$L + L_T = L^* = W\cos\gamma$

$D_{after} = D_0 + \boxed{D_L + D_{L_T}}$ → induced drag

Trim drag

Definition

- More formally:

$$D_m = \overbrace{D_{\text{after}}}^{\text{drag at no trim}} - \overbrace{D_{\text{before}}}^{\text{drag under trim}} = D_L + D_{L_T} - D_{L^*}$$

- Change of total drag after trim, w.r.t reference (untrimmed) case
  - increased induced drag from tailplane
  - decreased induced drag from main wing
- So that:

$$D_{\text{after}} = \overbrace{D_0}^{\text{a constant}} + \boxed{D_{L^*}} + \boxed{D_m}$$

only depends on the weight

depends on CG location

Trim drag

Positive or negative?

- Is it positive, negative, zero?
- If induced drag varied linearly with lift, i.e. $D = f(L)$
  - $D_{L^*} = D_L + D_{L_T}$ for any split of $L^* = L + L_T$
- But induced drag varies quadratically with lift, i.e. $D = f(L^2)$
  - and therefore, $D_{L^*} \neq D_L + D_{L_T}$ in general
- Example (with arbitrary units):

thought experiment

$D = \frac{L^2}{100}$

using model (quadratic)

L	L_T	L^*	D_L	D_{L_T}	$D_L + D_{L_T}$
10	2	12	1	0.04	1.04
9	3	12	0.81	0.09	0.9

move CG

can see $D_L + D_{L_T}$ change!

Trim drag

Rewrite into coefficient form

- Normalise trim drag using qS:

$$D_m = D_L + D_{L_T} - D_{L^*}$$

$\div \frac{1}{2}\rho V^2 S_x$

$\downarrow S_T$ not S

- To obtain:

$$C_{D_m} = C_{D_L} + C_{D_{L_T}}\frac{S_T}{S} - C_{D_{L^*}}$$

we want to get this in terms of C_L so we can do the optimisation problem

- Where

$$C_{D_L} = \frac{D_L}{\frac{1}{2}\rho V^2 S} \qquad C_{D_{L_T}} = \frac{D_{L_T}}{\frac{1}{2}\rho V^2 S_T} \qquad C_{D_{L^*}} = \frac{D_{L^*}}{\frac{1}{2}\rho V^2 S}$$

Trim drag

Introduce expressions for lift-induced drag

approximations being made (assuming entirely induced drag)

- Using

$$C_{D_L} \approx \frac{C_L^2}{\pi A e} \qquad C_{D_{L_T}} \approx \frac{C_{L_T}^2}{\pi A_T e_T} \qquad C_{D_{L^*}} \approx \frac{C_{L^*}^2}{\pi A e}$$

Subbing into the other equation

- To obtain:

$$C_{D_m} = \frac{C_L^2 - C_{L^*}^2}{\pi A e} + \frac{C_{L_T}^2}{\pi A_T e_T}\frac{S_T}{S}$$

Trim drag

Rewrite further

$$C_{D_m} = \frac{\left(C_{L^*} - C_{L_T}\frac{S_T}{S}\right)^2 - C_{L^*}^2}{\pi A e} + \frac{C_{L_T}^2}{\pi A_T e_T}\frac{S_T}{S}$$

$$= \frac{\left(C_{L_T}\frac{S_T}{S}\right)^2 - 2C_{L^*}C_{L_T}\frac{S_T}{S}}{\pi A e} + \frac{C_{L_T}^2}{\pi A_T e_T}\frac{S_T}{S}$$

$$= \boxed{\frac{C_{L^*}^2}{\pi A e}}\frac{C_{L_T}^2}{C_{L^*}^2}\frac{S_T}{S}\frac{S_T}{S} - 2\boxed{\frac{C_{L^*}^2}{\pi A e}}\frac{C_{L_T}}{C_{L^*}}\frac{S_T}{S} + \boxed{\frac{C_{L^*}^2}{\pi A e}}\frac{C_{L_T}^2}{C_{L^*}^2}\frac{\pi A e}{\pi A_T e_T}\frac{S_T}{S}$$

$$= C_{D_{L^*}}\left(\left(1 + \frac{S\pi A e}{S_T \pi A_T e_T}\right)\left(\frac{C_{L_T}}{C_{L^*}}\frac{S_T}{S}\right)^2 - 2\frac{C_{L_T}}{C_{L^*}}\frac{S_T}{S}\right)$$

$$\boxed{\begin{aligned} C_{L^*} &= C_L + C_{L_T}\frac{S_T}{S} \\ C_L &= C_{L^*} - C_{L_T}\frac{S_T}{S} \end{aligned}}$$

$\boxed{\times \frac{C_{L^*}^2}{C_{L^*}^2}}$ and $\boxed{\times \frac{\pi A e}{\pi A e}}$ for the second term

collect

$$\boxed{C_{D_{L^*}} = \frac{C_{L^*}^2}{\pi A e}}$$

Trim drag

Rewrite further

- Now introduce

$$\sigma = \left(1 + \frac{S\pi A e}{S_T \pi A_T e_T}\right)$$

same variable
reping plane
geometry

- To obtain

$$C_{D_m} = C_{D_{L^*}} \left(\sigma \left(\frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \right)^2 - 2 \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \right)$$

- And finally

$$C_{D_m} = C_{D_{L^*}} \boxed{\frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S}} \left(\sigma \boxed{\frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S}} - 2 \right)$$

fraction of lift due to tailplane

- Aircraft weight coefficient C_W is fixed
- Total lift coefficient C_{L^*} is fixed
- Initial induced drag $C_{D_{L^*}}$ is fixed too
- Trim drag influenced by:
  - tailplane/wing lift split
  - wing/tail surface area
  - wing/tail aspect ratio and span efficiency

$$\frac{C_{L_T} \frac{S_T}{S}}{C_{L^*}} = \frac{L_T}{L^*}$$

Performance optimisation

Minimum trim drag

- Plot and minimise trim drag (graphically)

$$C_{D_m} = C_{D_{L^*}} \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \left(\sigma \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} - 2 \right)$$

- Or by differentiating and setting to zero

$$\frac{\mathrm{d}C_{D_m}}{\mathrm{d}C_{L_T}} = C_{D_{L^*}} \left(2\sigma \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} - 2 \right) = 0$$

- Resulting in the minimum trim drag

$$\left(C_{D_m}\right)_{min} = \frac{-C_{D_{L^*}}}{\sigma} < 0$$

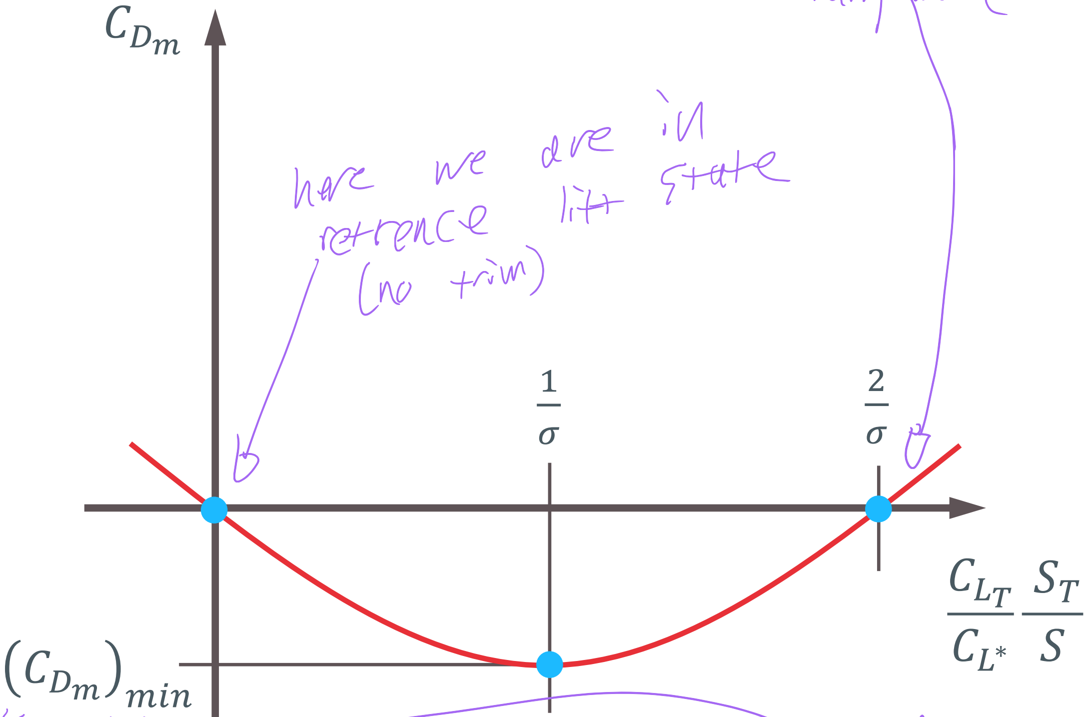
hence the trimmed drag is less than if only main wing (def of C_{D_m})

$1 = \sigma \left(\frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \right)$

$\left(\frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \right) = \frac{1}{\sigma}$

$C_{D_{m\,min}} = \left(C_{D_{L^*}} \frac{1}{\sigma} \left(\sigma \frac{1}{\sigma} - 2 \right) \right)$

$= -\frac{C_{D_{L^*}}}{\sigma}$



here we are in reference lift state (no trim)

overloaded the tailplane

$$\left. \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \right|_{opt} = \frac{1}{\sigma} > 0$$

drag minimised due to less induced drag on main wing

Performance optimisation

$$\frac{(C_{D_m})_{min}}{C_{D_{L^*}}} = -\frac{1}{\sigma}$$

Minimum trim drag

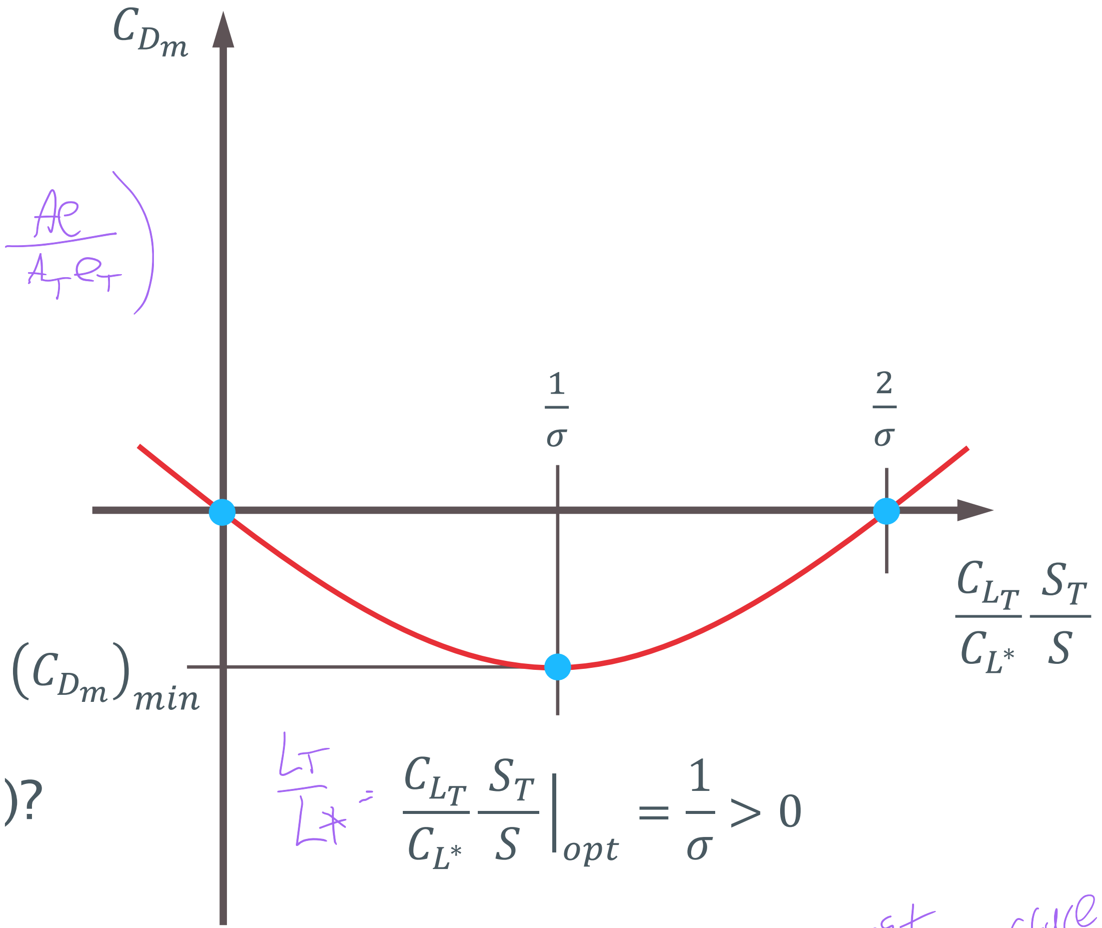
- Consider

$$\sigma = \left(1 + \frac{S\pi Ae}{S_T \pi A_T e_T}\right) = \left(1 + \frac{S}{S_T}\frac{Ae}{A_T e_T}\right)$$

- Assume $\frac{S}{S_T} = 6$ and $\frac{Ae}{A_T e_T} = 1$, then $\sigma = 7$
- Then, the optimal split it

$$\frac{C_{L_T}}{C_{L^*}}\frac{S_T}{S}\Big|_{opt} = \frac{L_T}{L^*}\Big|_{opt} = \frac{1}{7} \approx 14.3\%$$

- What happens if A_T increases (at constant S_T)?



↓ σ decreases

↓ $\frac{1}{\sigma}$ increases → $\frac{L_T}{L^*}$ increases

want to load the most efficient lifting surface!

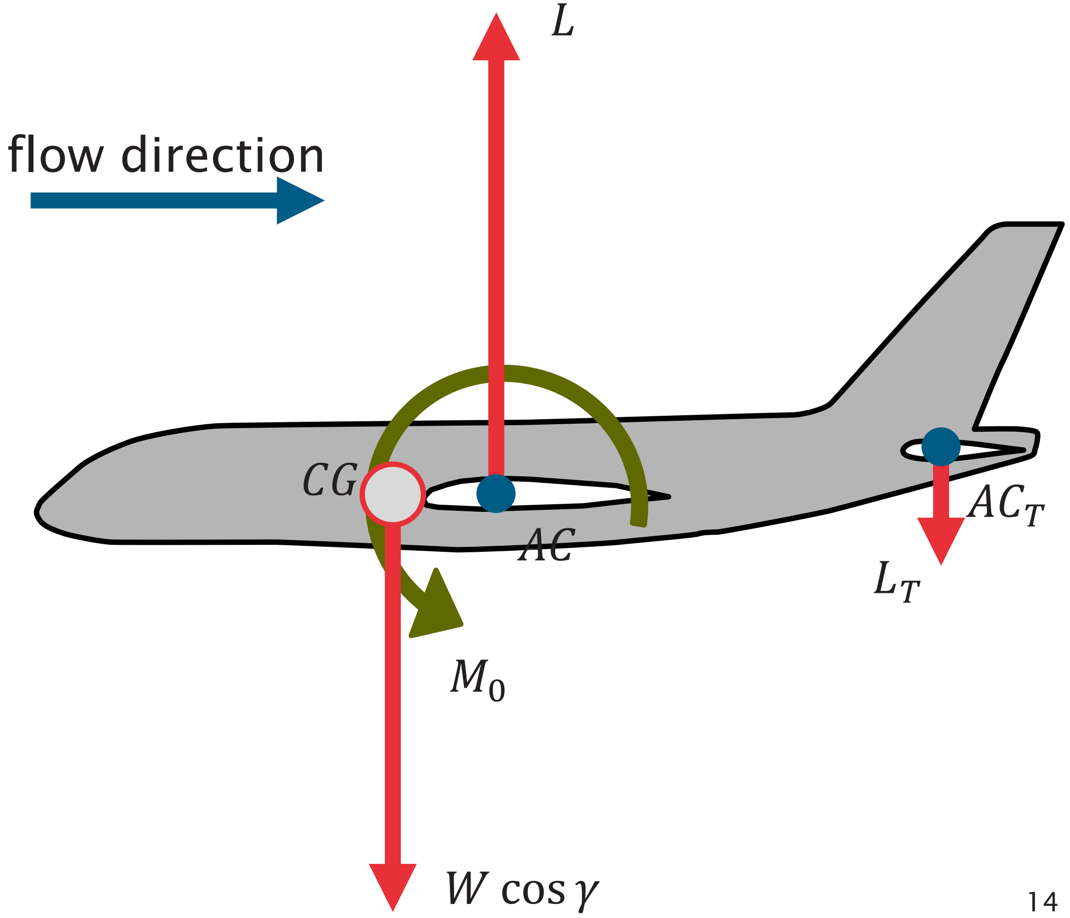
Trim drag

Exam question from 2021/22

(ii) Define trim drag and sketch its dependence with the tailplane lift coefficient.

Explain from a physical perspective why the minimum trim drag can be negative.

Using similar arguments explain why trim drag is positive when the tailplane lift is negative.



Drag

Overall aircraft drag

- The overall aircraft drag after trim can be thus expressed as

$$D_{after} = D_{before} + D_m = D_0 + D_{L^*} + D_m$$

- Or in non-dimensional form

$$C_D = C_{D_0} + C_{D_{L^*}} + C_{D_m}$$

- where
  - C_{D_0} is the (constant) zero-lift drag
  - $C_{D_{L^*}} \approx \frac{C_{L^*}^2}{\pi A e}$ is the lift dependent drag before trim (approx. as induced drag)
  - $C_{D_m} = C_{D_{L^*}} \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} \left(\sigma \frac{C_{L_T}}{C_{L^*}} \frac{S_T}{S} - 2 \right)$ or $\frac{-C_{D_{L^*}}}{\sigma}$ is the (minimum) trim drag