SESA1015 Astronautics

Launch Vehicles

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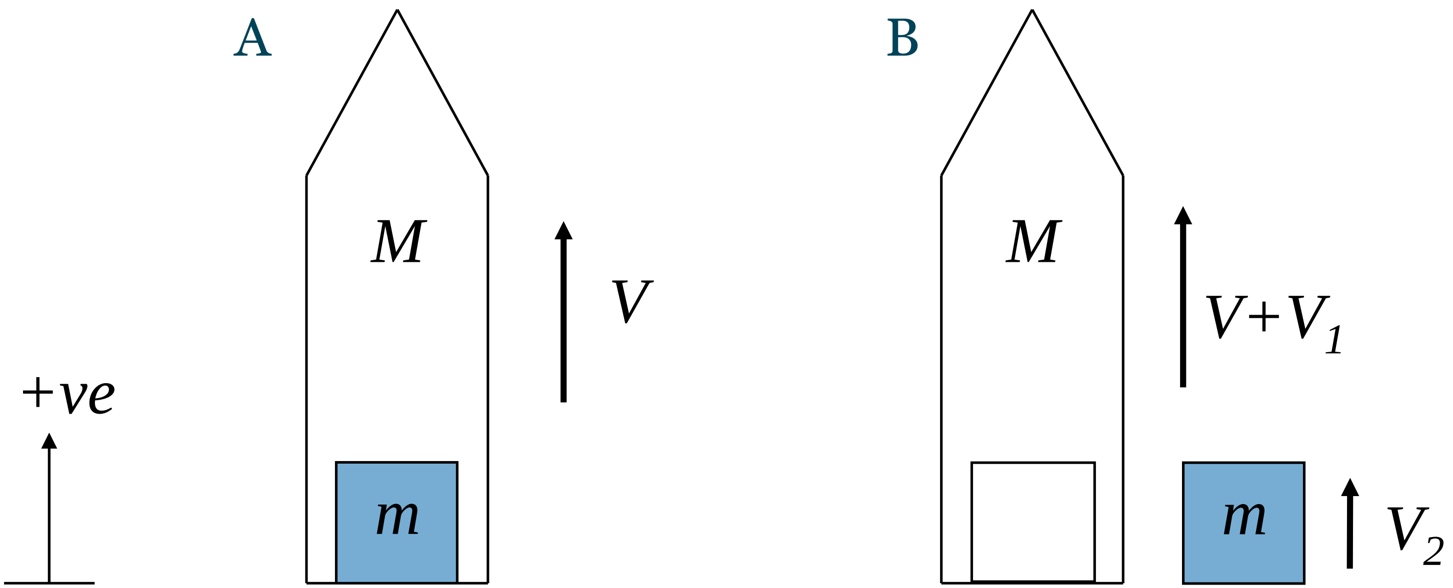
Conservation of Momentum

Basic System, relative to an observer



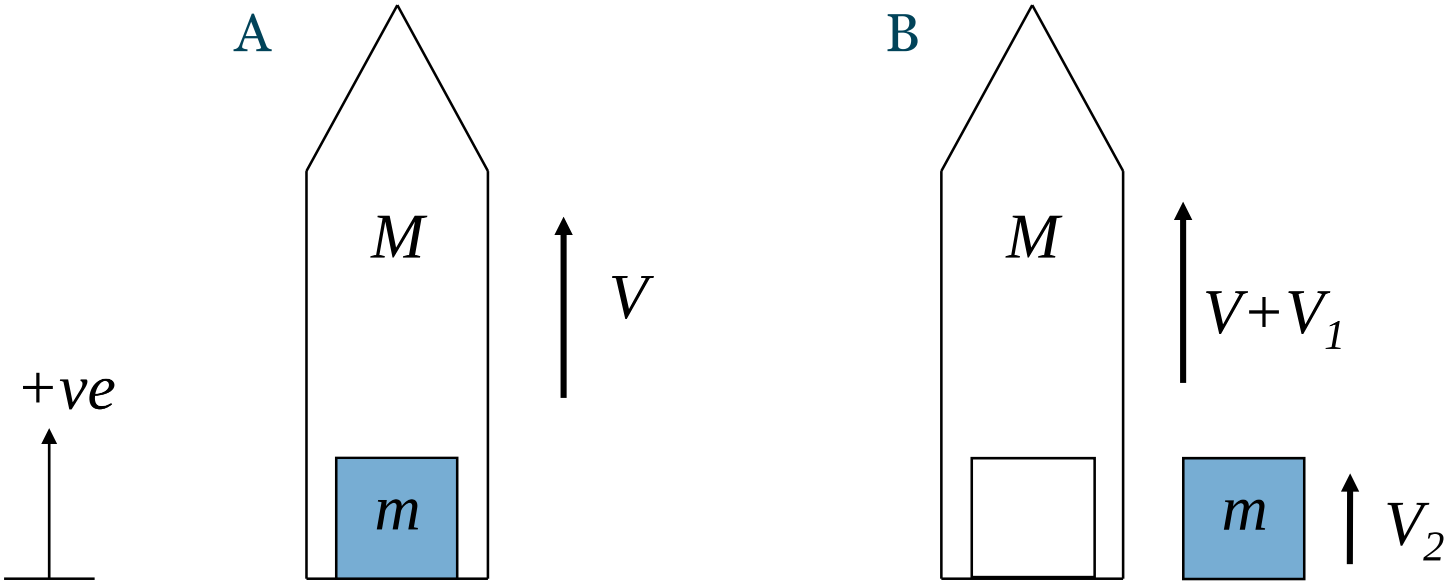
$$(M+m)V = M(V+V_1) + mV_2$$

$$MV + mV = MV + MV_1 + mV_2$$

$$mV = MV_1 + mV_2$$

Conservation of Momentum

Basic System, relative to an observer



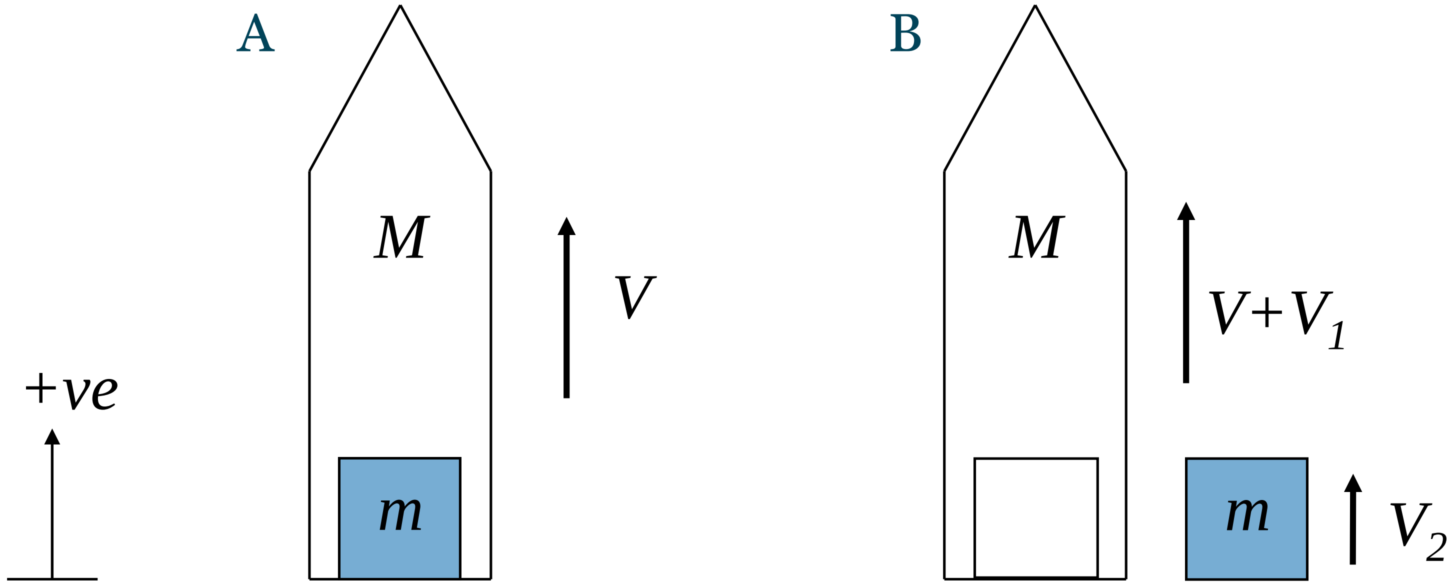
$$mV = MV_1 + mV_2$$

$$MV_1 = mV - mV_2 \quad \rightarrow V_1 = \frac{m}{M}\left(V - V_2\right)$$

To increase V_1 we need to increase the mass ratio and make V_2 as large and as negative as possible

Conservation of Momentum

Exhaust Velocity, relative to vehicle



$$V_1 = \frac{m}{M}\underbrace{(V - V_2)}_{V_{ex}}$$

In very simple terms: $V_1 = \dfrac{m}{M} V_{ex}$

However for rockets the burnt mass and velocity change is continuously happening over the time of burn, so this linear approximation is not accurate, (i.e. *M* is constantly reducing!)

The Rocket Equation

Derived by Tsiolkovsky in 1903.

Newtons second law:

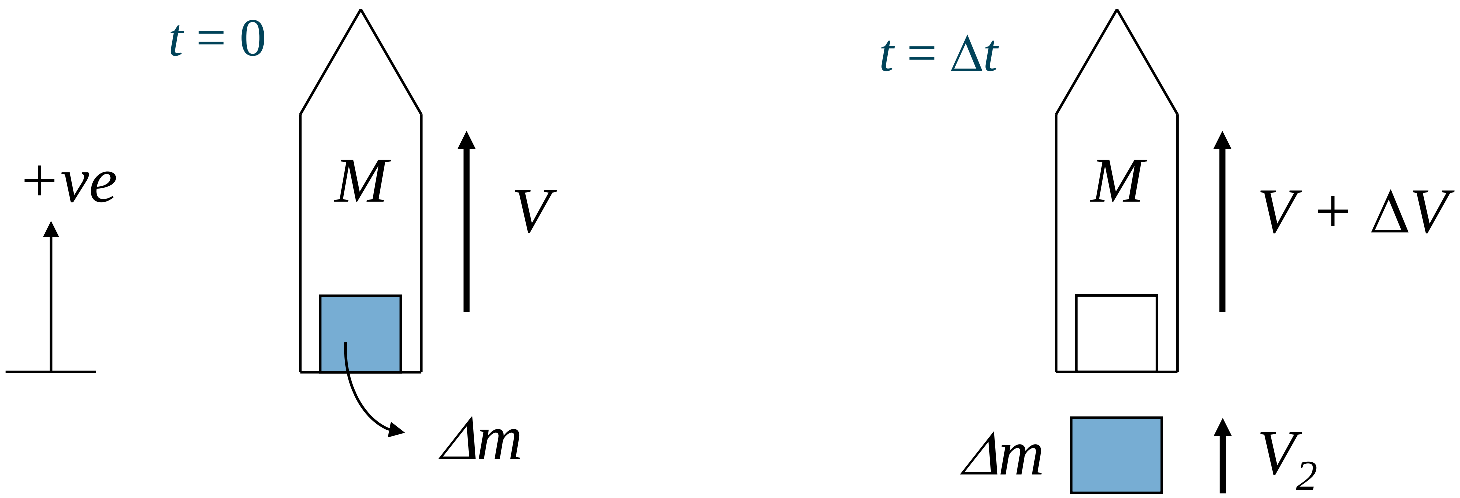
$$F = ma = \frac{d}{dt}(mv)$$

Or equivalently:

$$\sum F_{ext} = \lim_{\Delta t \to 0} \frac{\text{Momentum @ B} - \text{Momentum @ A}}{\Delta t}$$

The Rocket Equation

A Rocket in Space



Momentum

$$(M + \Delta m)V \qquad\qquad M(V + \Delta V) + \Delta m V_2$$

Therefore from Newton's second law:

$$F_{ext} = \lim_{\Delta t \to 0} \frac{\left(M(V + \Delta V) + \Delta m V_2\right) - \left((M + \Delta m)V\right)}{\Delta t}$$

The Rocket Equation

Equation of motion:

$$F_{ext} = \lim_{\Delta t \to 0} \frac{\left(M\left(V + \Delta V\right) + \Delta m V_2\right) - \left(\left(M + \Delta m\right) V\right)}{\Delta t}$$

As there are no external forces being applied $F_{ext} = 0$ and as Δt tends to 0 (i.e. we make the time step really small) the equation becomes:

$$0 = \frac{\left(M\left(V + dV\right) + dmV_2\right) - \left(\left(M + dm\right)V\right)}{dt}$$

$$= \frac{MV + MdV + dmV_2 - \left(MV + dmV\right)}{dt}$$

$$= \frac{MV + MdV + dmV_2 - MV - dmV}{dt} = M\frac{dV}{dt} + \frac{dm}{dt}V_2 - \frac{dm}{dt}V$$

The Rocket Equation

Rearranging:

$$0 = M\frac{dV}{dt} + \frac{dm}{dt}V_2 - \frac{dm}{dt}V \quad \rightarrow \quad M\frac{dV}{dt} = \frac{dm}{dt}\left(V - V_2\right)$$

From before we defined the exhaust velocity relative to the vehicle to be: $V_{ex} = V - V_2$

So the Rocket Equation becomes:

$$\frac{dV}{dt} = V_{ex}\frac{1}{M}\frac{dm}{dt} \qquad \left\{ \text{Previously: } V_1 = \frac{m}{M}V_{ex} \right\}$$

Which is the differential equation version of the previous simple conservation of momentum equation. This differential equation needs to be integrated between limits to determine the total velocity change from the rocket burn… but…

What is… $\dfrac{dm}{dt}$?

The Rocket Equation

Mass

$\frac{dm}{dt}$ is the fuel mass ejected over the time *dt*, i.e. the *mass flow rate*

$\frac{dM}{dt}$ is the change in mass of the rocket

Therefore:

$$\frac{dM}{dt} = -\frac{dm}{dt}$$

So the equation becomes:

$$\frac{dV}{dt} = -V_{ex}\frac{1}{M}\frac{dM}{dt}$$

The Rocket Equation

The Solution

Therefore, integrating…

$$\int_{u}^{v} dv = -V_{ex} \int_{M_0}^{M_b} \frac{1}{M} dM$$

$$v - u = -V_{ex} \left[\ln M \right]_{M_0}^{M_b}$$

Where:

- u is the initial velocity
- v is the final velocity
- M_0 is the initial rocket mass
- M_b is the final rocket mass or 'burnout mass'

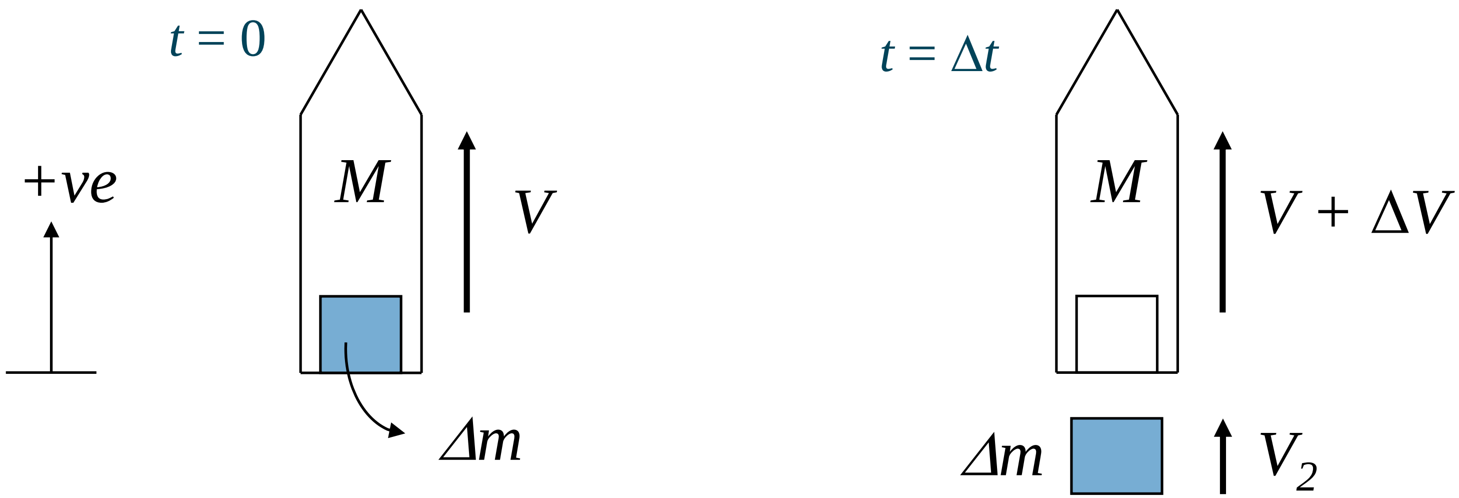
So the final solution is:

$$\boxed{\Delta V = V_{ex} \ln\left(\frac{M_0}{M_b}\right)} \quad (3.1)$$

This is the *Rocket Equation*

The Rocket Equation

A Rocket in Space



Momentum

$$(M + \Delta m)V \qquad\qquad M(V + \Delta V) + \Delta m V_2$$

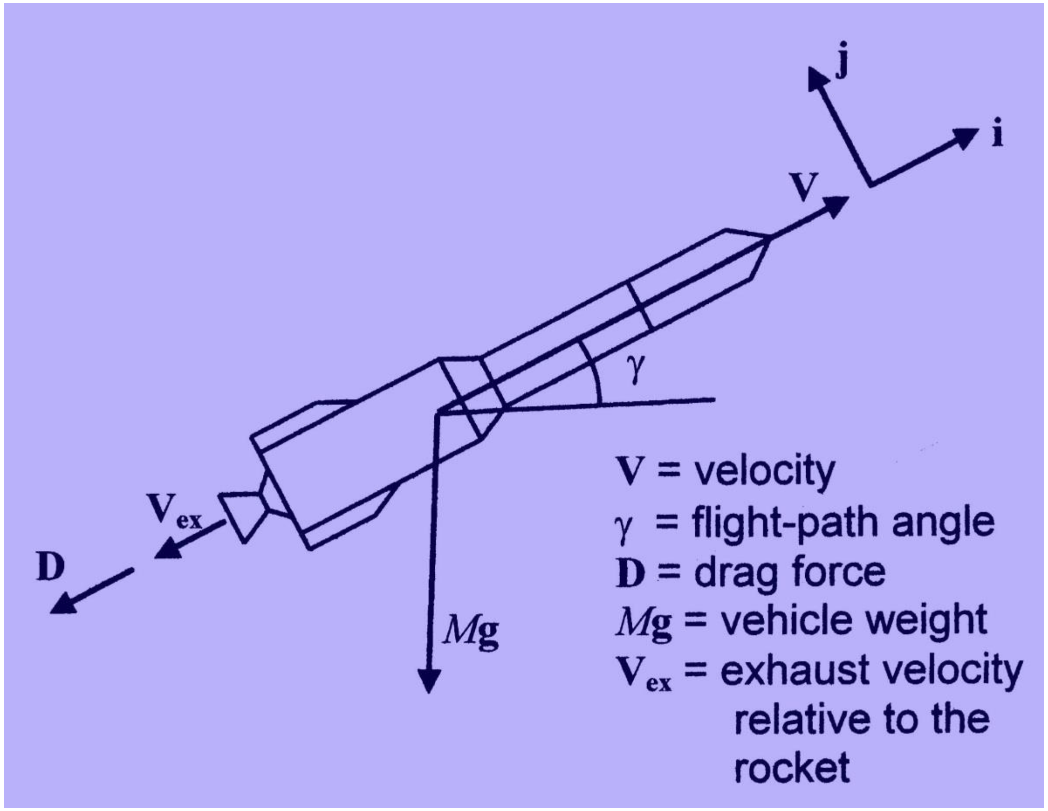
Therefore from Newton's second law:

$$F_{ext} = \lim_{\Delta t \to 0} \frac{\left(M(V + \Delta V) + \Delta m V_2\right) - \left((M + \Delta m)V\right)}{\Delta t} \rightarrow \boxed{\Delta V = V_{ex} \ln\left(\frac{M_0}{M_b}\right)}$$

Launch Vehicles

The Rocket Equation Applied to Launch Vehicles

If we apply the rocket equation derivation to launch vehicles then there are external forces acting on the rocket… for example drag and rocket weight. If we assume the rocket is thrusting along the vehicles velocity vector then…



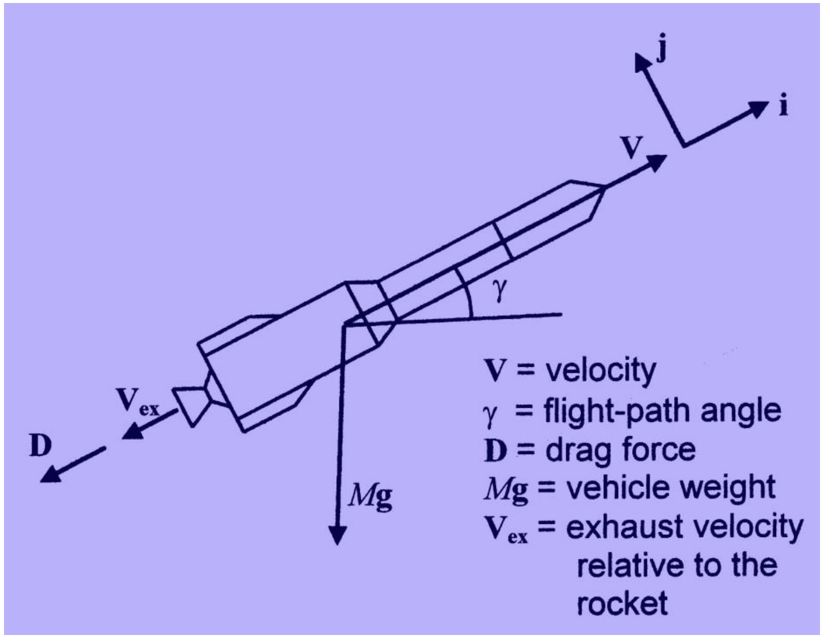
Where the rocket drag force is given by:

$$D = \frac{1}{2}\rho V^2 S C_D$$

Launch Vehicles

The Rocket Equation Applied to Launch Vehicles

We can derive the equation of motion in the *i* direction:



$$\frac{\left(M\left(V+dV\right)+dmV_{2}\right)-\left(\left(M+dm\right)V\right)}{dt}=F_{ext}=-Mg\sin\gamma-D$$

Launch Vehicles

The Rocket Equation Applied to Launch Vehicles

$$\frac{\left(M\left(V + dV\right) + dmV_2\right) - \left(\left(M + dm\right)V\right)}{dt} = -Mg\sin\gamma - D$$

$$M\frac{dV}{dt} + \frac{dm}{dt}\left(V_2 - V\right) = -Mg\sin\gamma - D$$

$$M\frac{dV}{dt} = -V_{ex}\frac{dM}{dt} - Mg\sin\gamma - D$$

Which is the same as the rocket equation with the added drag and mass terms.

$$\frac{dV}{dt} = -V_{ex}\frac{1}{M}\frac{dM}{dt} - g\sin\gamma - \frac{D}{M}$$

Launch Vehicles

The Rocket Equation Applied to Launch Vehicles

Integrate:

$$\int_u^v dV = -V_{ex} \int_{M_0}^{M_b} \frac{1}{M} dM - \int_0^t g \sin \gamma dt - \int_0^t \frac{D}{M} dt$$

Therefore:

$$\Delta V = \underbrace{V_{ex} \ln\left(\frac{M_0}{M_b}\right)}_{\Delta V_{ideal}} - \underbrace{\int_0^t g \sin \gamma dt}_{\text{Gravity}} - \underbrace{\int_0^t \frac{D}{M} dt}_{\text{Drag}} \qquad (3.2)$$

Which can be written as:

$$\Delta V = \Delta V_{ideal} - \Delta V_g - \Delta V_D \qquad (3.3)$$

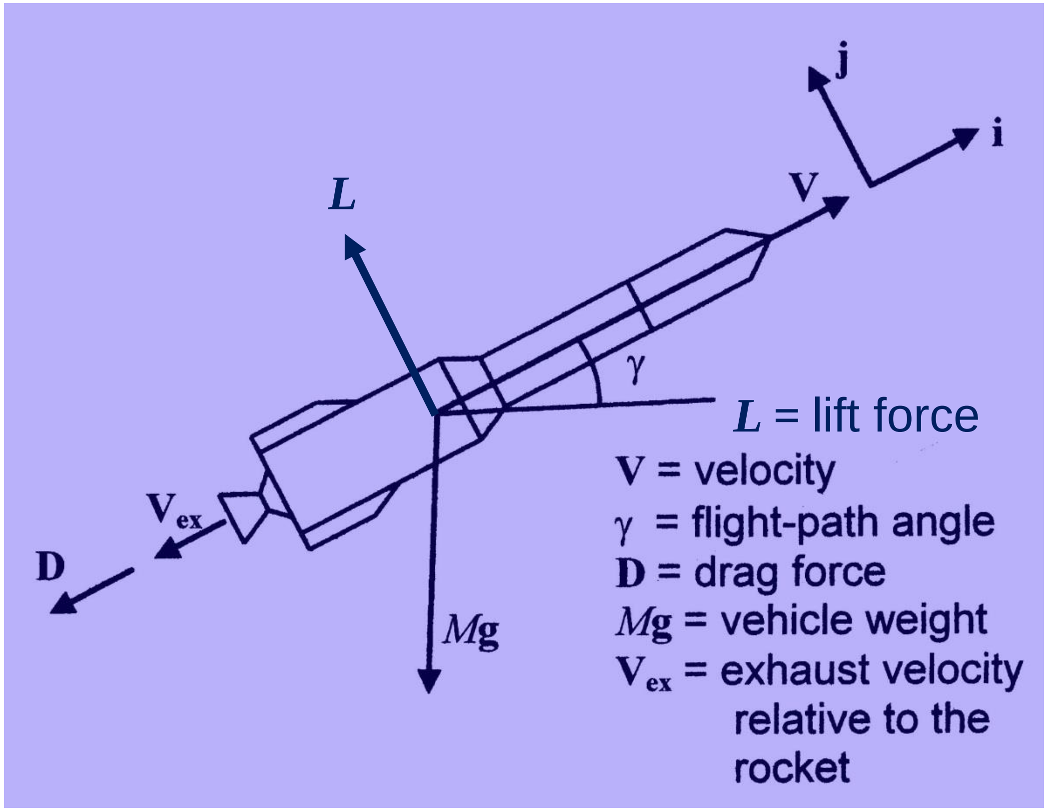
Where: ΔV_g = 'gravity loss' ; ΔV_D = 'drag loss'

ΔV required to achieve LEO ~ 7.8 km/s

Launch Vehicles

The Rocket Equation Applied to Launch Vehicles – Using vector notation

This analysis can be done in a more advanced way using vector notation analysing both the ***i*** and ***j*** directions in one equation.



Launch Vehicles

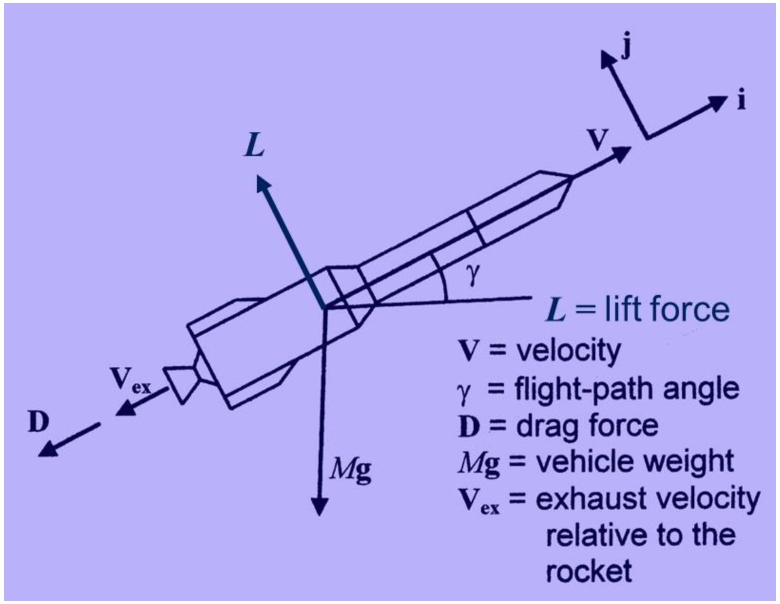
The Rocket Equation Applied to Launch Vehicles – Using vector notation

Newton's second law for systems with momentum inflow and outflow (jets, rockets etc):

$$\frac{d}{dt}(M\mathbf{V}) = \mathbf{F}_{ext} + \{\text{rate of momentum inflow}\} - \{\text{rate of momentum outflow}\}$$

So:

$$\frac{d}{dt}(M\mathbf{V}) = \mathbf{D} + M\mathbf{g} + \mathbf{L} + \{0\} - \{\sigma(\mathbf{V} + \mathbf{V}_{ex})\}$$



Using the $\boldsymbol{i}, \boldsymbol{j}$ reference system:

$$\dot{M}\mathbf{V} + M\dot{\mathbf{V}} = -D\mathbf{i} - Mg\sin\gamma\mathbf{i} - Mg\cos\gamma\mathbf{j} + L\mathbf{j} - \sigma(V - V_{ex})\mathbf{i}$$

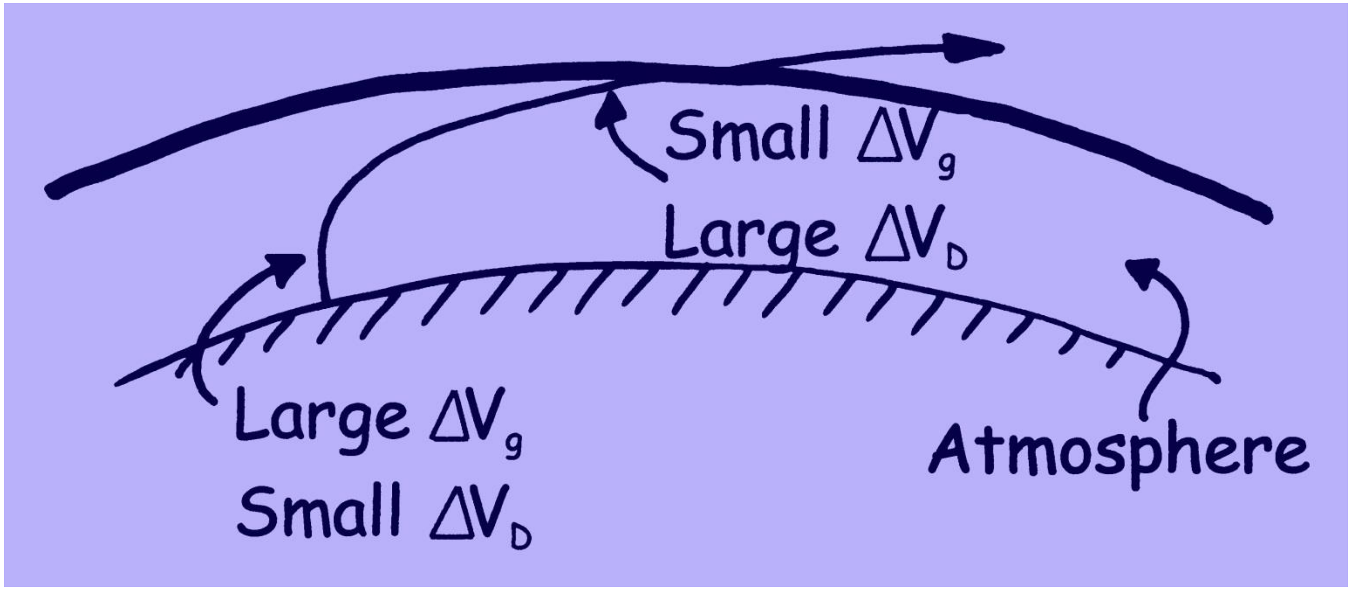
Where:

$$\dot{M} = -\sigma \qquad \dot{\mathbf{V}} = \dot{V}\mathbf{i} + V\dot{\gamma}\mathbf{j}$$

Launch Vehicles

Ascent Optimisation

Realistic launches are a trade off between the gravity loss and the drag loss

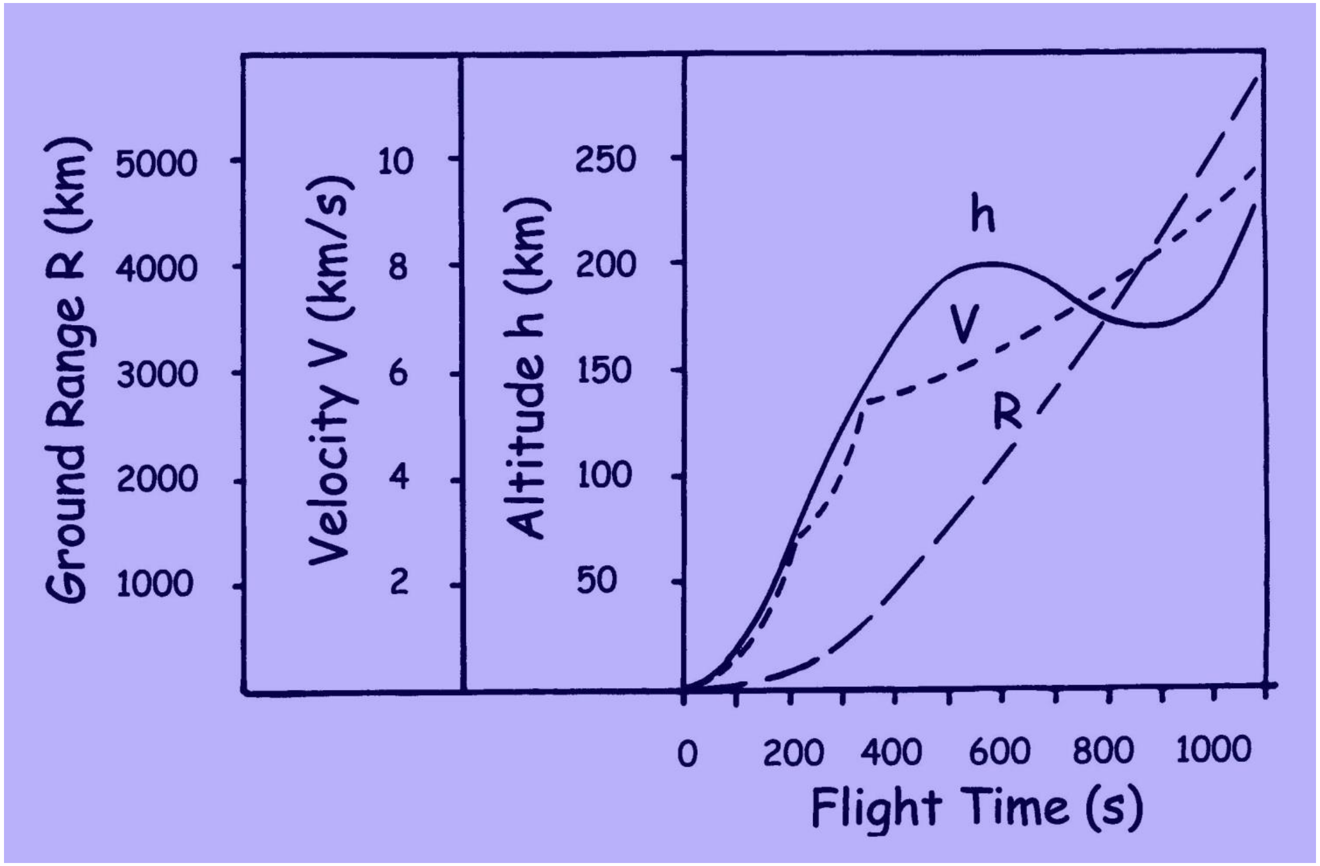


Trajectory Optimisation

Typically, ΔV_g ~ 0.75 to 1.5 km/s, ΔV_D ~ 0.2 to 0.3 km/s

Launch Vehicles

Ascent Profile



Ascent profile for typical 'classical' 3-stage expendable launch vehicle - Ariane 44LP taken as an example

Launch Vehicles

Some numbers…

From equation (3.3):

$$\Delta V = V_{ex} \ln\left(\frac{M_0}{M_b}\right) - \Delta V_g - \Delta V_D$$

ΔV → 7.8 km/s; ΔV_g → 0.75 km/s; ΔV_D → 0.2 km/s

$$V_{ex} \ln\left(\frac{M_0}{M_b}\right) = 8.75 \text{ km/s}$$

Assume V_{ex} for our rocket is 3 km/s and we want to put 1000kg into LEO

$$3 \ln\left(\frac{1000 + M_f}{1000}\right) = 8.75 \quad \rightarrow \quad M_f = 17500 \text{ kg}$$

Launch Vehicles

Single Stage To Orbit (SSTO) Vehicle

From equation (3.3):

$$\Delta V + \Delta V_g + \Delta V_D = V_{ex}\, ln\left(\frac{M_p + M_s + M_f}{M_p + M_s}\right)$$

Where: M_P = p/l mass
M_S = structure mass
M_f = fuel mass

Define: $p = M_P/M_0$ (payload fraction)
s = M_S/M_f (propellant tankage structural efficiency)

Then:

$$\Delta V + \Delta V_g + \Delta V_D = V_{ex} \ln\left(\frac{1+s}{p+s}\right)$$

Launch Vehicles

Single Stage To Orbit (SSTO) Vehicle

To achieve LEO, we require $\Delta V = 7.8$ km/s.

Also assume that $s = 0.1$, $\Delta V_g = \Delta V_D = 0$ (no losses) and $p = 0$ (no p/l !)

Then : $7.8 \text{ km/s} = V_{ex} \ln(11)$

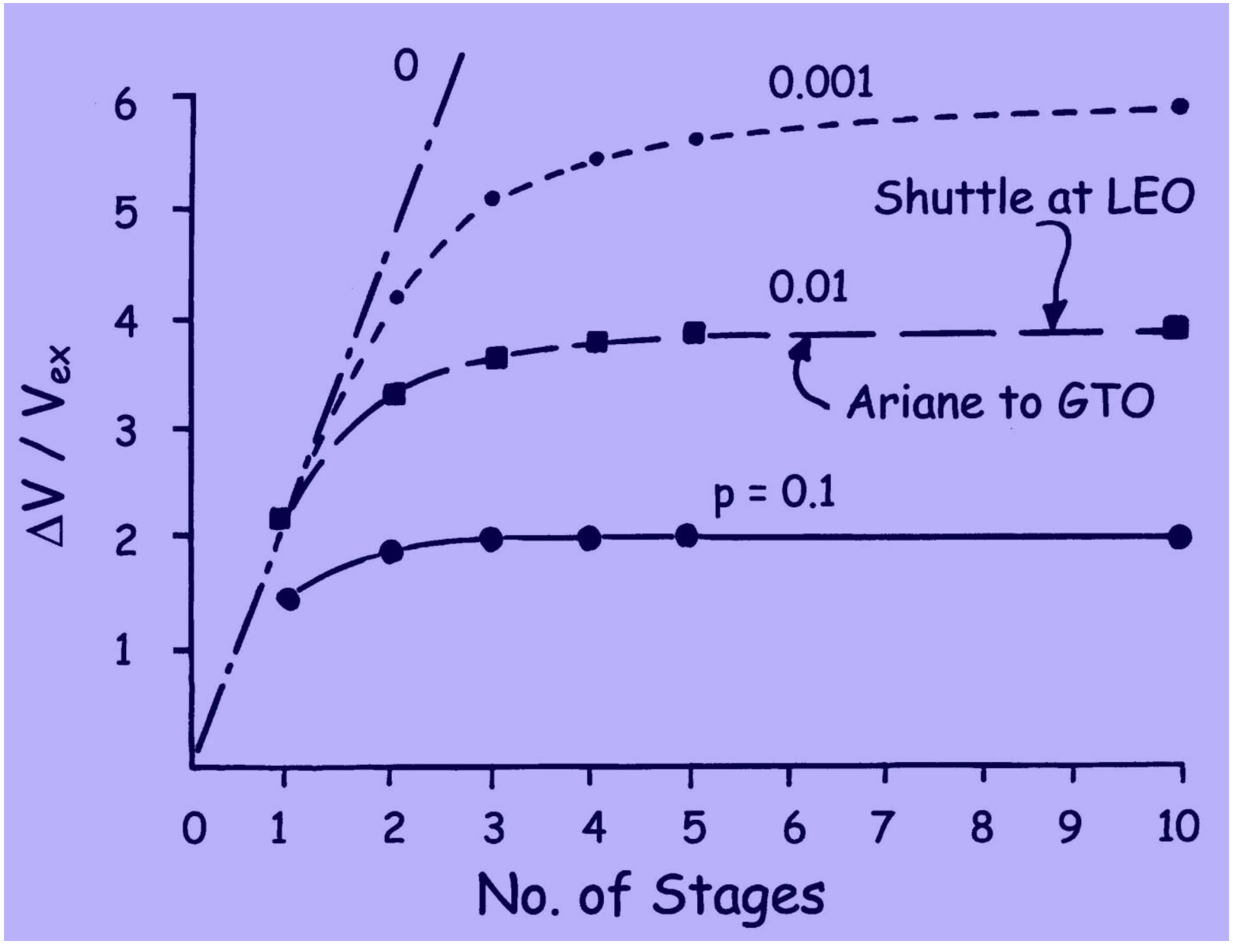
or $V_{ex} = 3.25 \text{ km/s}$

Even under these ideal conditions (no p/l, no losses), SSTO only achievable with high energy propellants.

Since mass of propellant tankage is large, significant performance benefits result from progressive shedding of mass on ascent → staging.

Launch Vehicles

Staging



Variation of Velocity Increment with Number of Stages

Launch Vehicles

Multi Stage Vehicle

From previously:

$$\Delta V_{ideal} = V_{ex} \ln\left(\frac{M_0}{M_b}\right) = V_{ex} \ln\left(\frac{M_p + M_s + M_f}{M_p + M_s}\right)$$

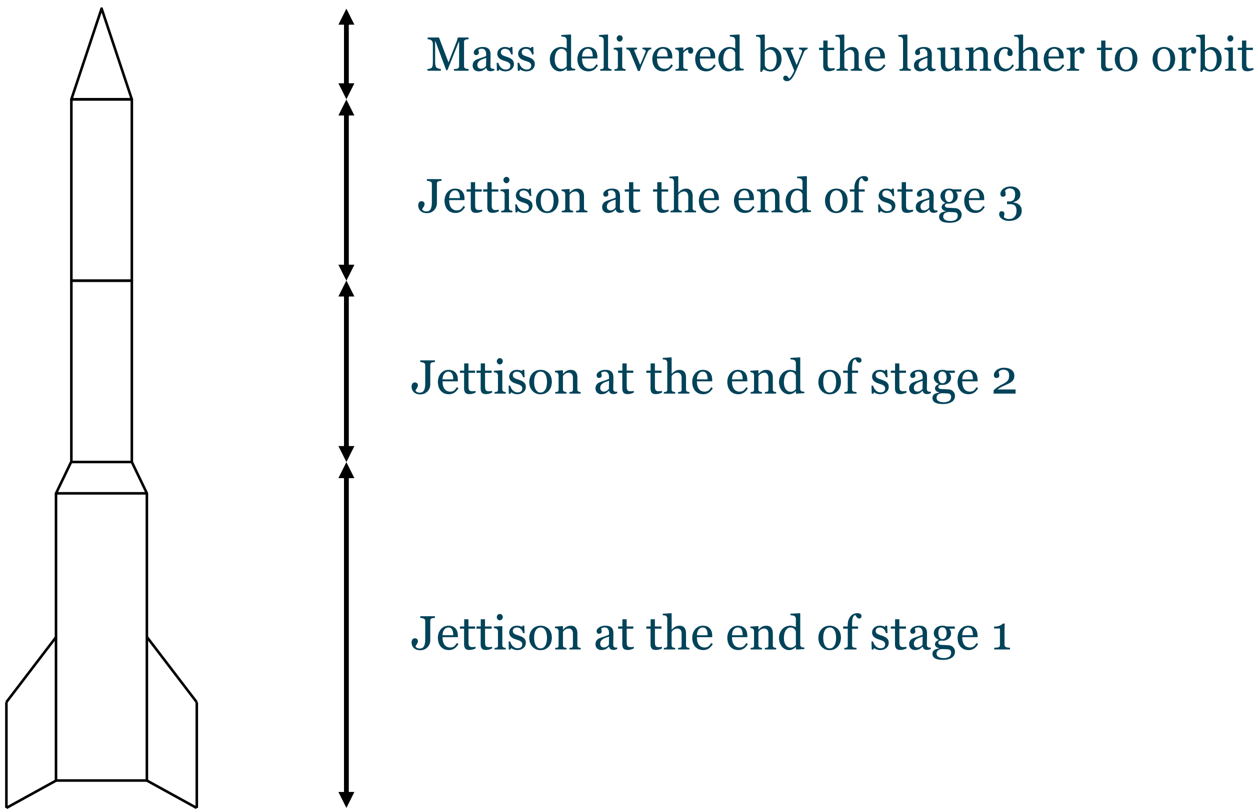
$$= V_{ex} \ln\left(\frac{1+s}{p+s}\right)$$

Where: $p = M_P/M_0$ (payload fraction)
$s = M_S/M_f$ (propellant tankage structural efficiency)

Launch Vehicles

Multi Stage Vehicle

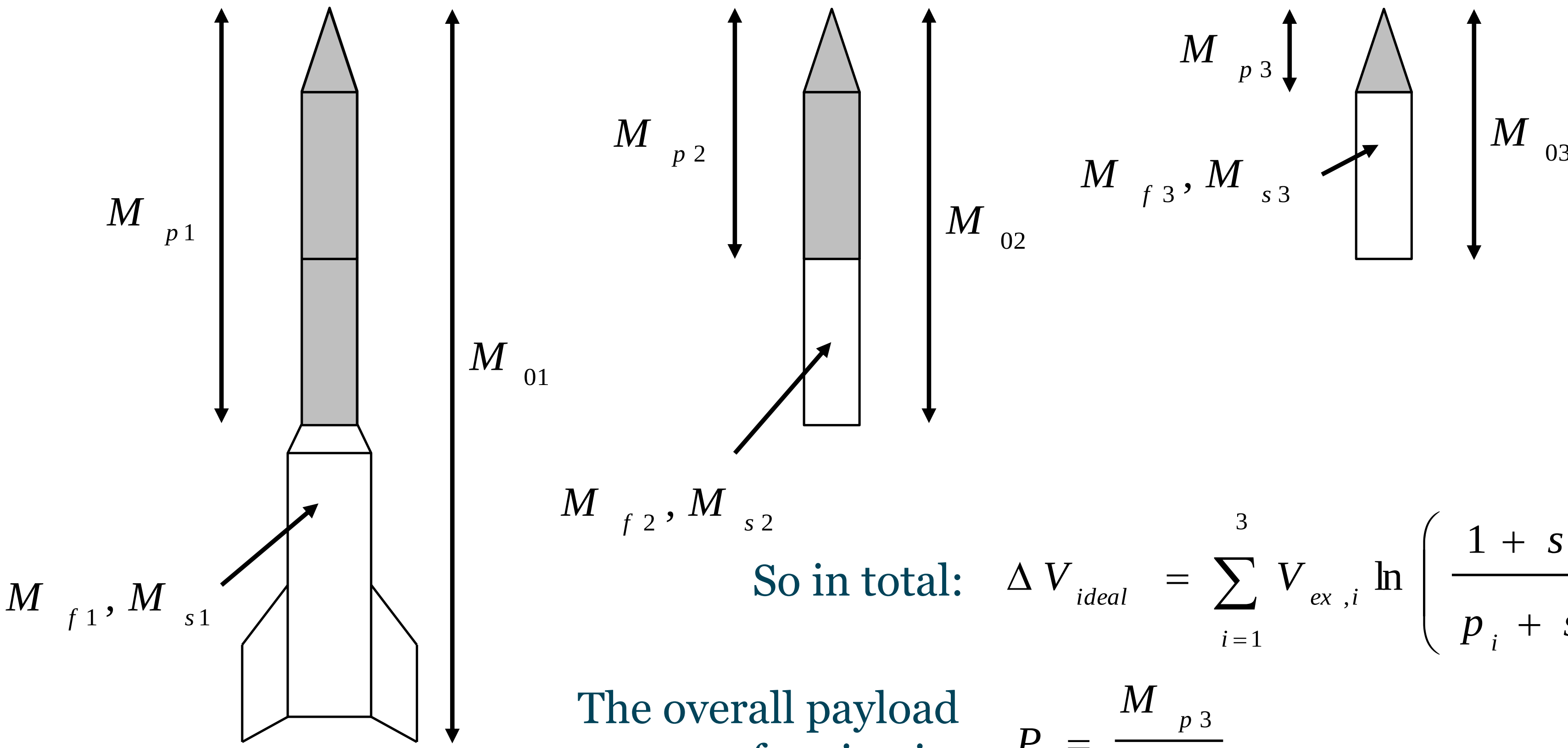
Consider a three stage vehicle:



Launch Vehicles

Multi Stage Vehicle

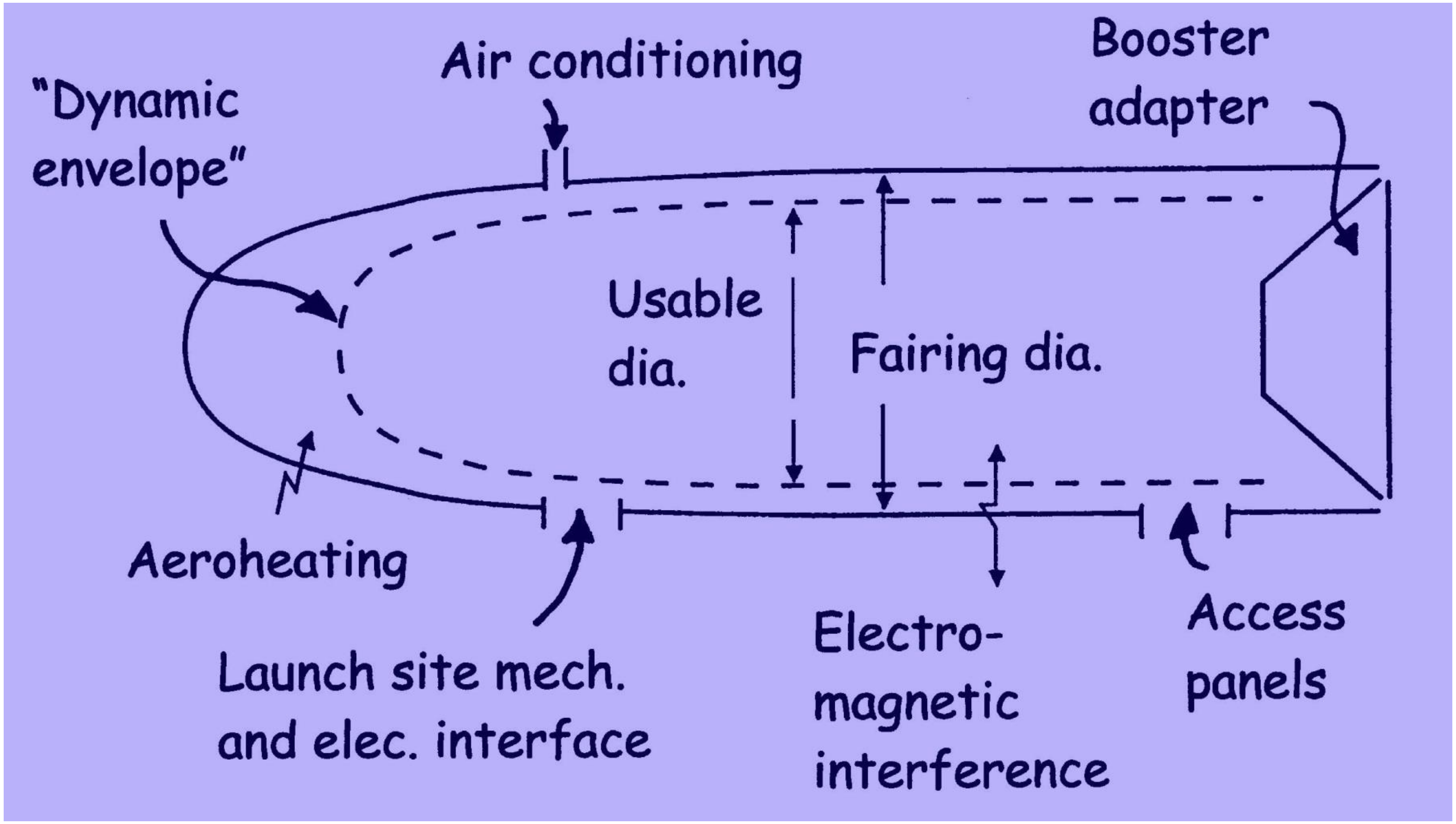
For stage 1: For stage 2: For stage 3:



So in total: $$\Delta V_{ideal} = \sum_{i=1}^{3} V_{ex,i} \ln\left(\frac{1+s_i}{p_i+s_i}\right)$$

The overall payload fraction is: $$P = \frac{M_{p3}}{M_{01}}$$

Launch Vehicle Interface



Typical Launch System Fairing

Launch Vehicle Interface

Impacts of Launcher Selection on Spacecraft System

- main input to S/C structure design
- major impact on:
    - S/C mass
    - S/C volume
    - S/C stowed configuration
    - S/C mass distribution
      (constraint on centre of mass position)
- input to design of S/C deployment mechanisms

Launch System Characteristics

Ariane 5

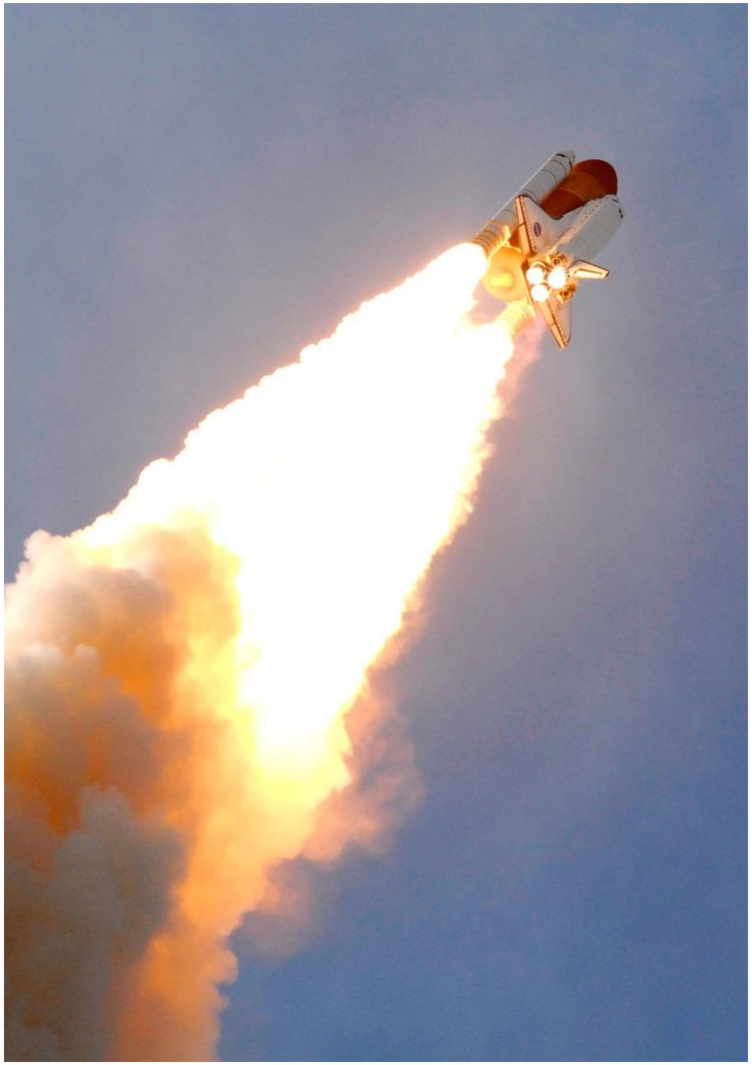
Launch System Characteristics

Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Ariane 5 G (generic) ECA (evolution)	 21000 21000	 9500 9500	 6600 10000	4.6	~8.5 to ~13.0

(+) = Approximate values

Launch System Characteristics

Space Shuttle



Launch System Characteristics

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	LEO	LEO Polar	GTO	Diam.	Length
Ariane 5 G (generic) ECA (evolution)	 21000 21000	 9500 9500	 6600 10000	4.6	~8.5 to ~13.0
Shuttle	24400	---	5900	4.5	18.0

(+) = Approximate values

Launch System Characteristics

Space Shuttle

Launch System Characteristics

US ELV (Expendable Launch Vehicle) – Delta 2

Launch System Characteristics

US ELV (Expendable Launch Vehicle) – Titan III

Launch System Characteristics

US ELV (Expendable Launch Vehicle) – Atlas 5

Launch System Characteristics

Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Ariane 5				4.6	~8.5 to ~13.0
G (generic)	21000	9500	6600		
ECA (evolution)	21000	9500	10000		
Shuttle	24400	---	5900	4.5	18.0
Delta II	5000	3000	2000	2.5	4.8
Titan III	11000	---	5000	3.6	12.4
Atlas V	20500	17300	8700	4.2	12.2

(+) = Approximate values

Launch System Characteristics

Russian ELV: Proton

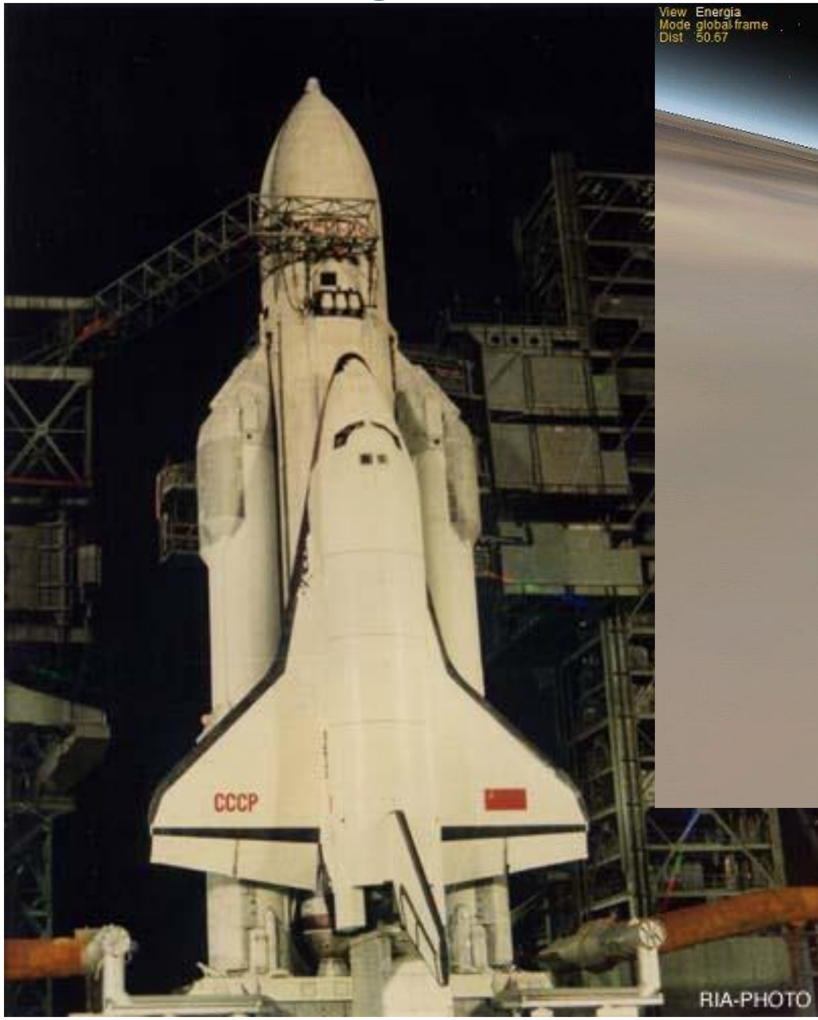
Launch System Characteristics

Russian ELV: Energia

University of Southampton

Launch System Characteristics

Russian ELV: Energia



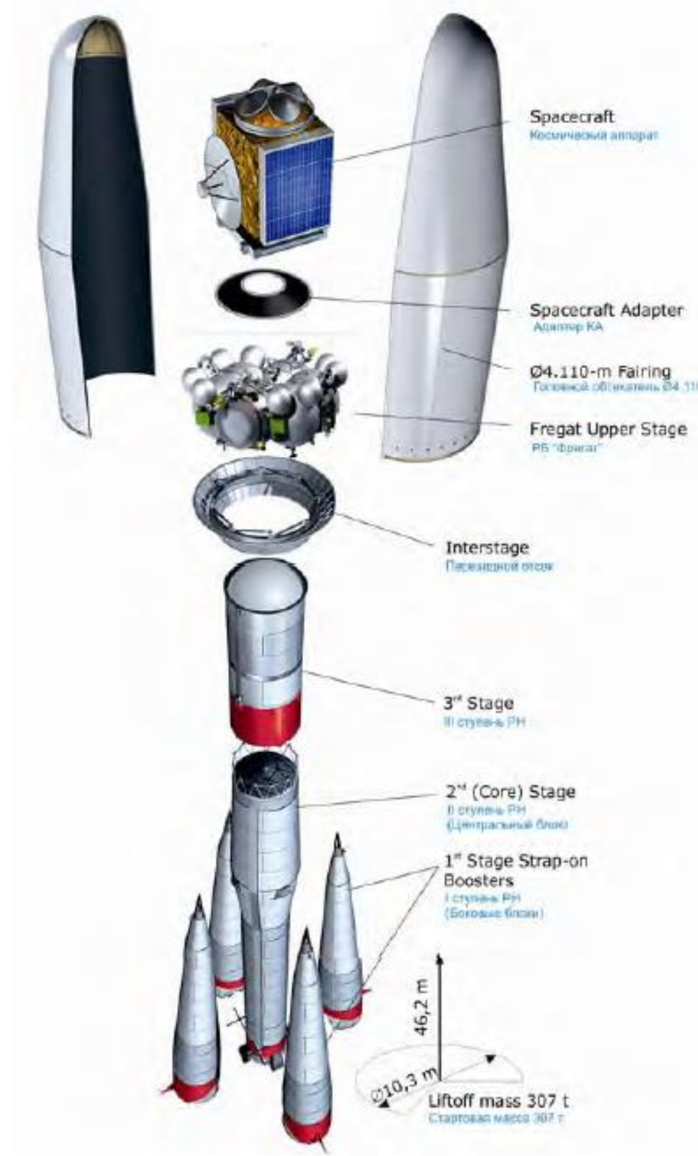
Launch System Characteristics

Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Ariane 5				4.6	~8.5 to ~13.0
G (generic)	21000	9500	6600		
ECA (evolution)	21000	9500	10000		
Shuttle	24400	---	5900	4.5	18.0
Delta II	5000	3000	2000	2.5	4.8
Titan III	11000	---	5000	3.6	12.4
Atlas V	20500	17300	8700	4.2	12.2
Proton	20000	---	5500	3.3	7.5
Energia	90000	72000	---	5.5	37

(+) = Approximate values

Launch System Characteristics

Russian ELV: Soyuz



Launch System Characteristics

Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Ariane 5 G (generic) ECA (evolution)	 21000 21000	 9500 9500	 6600 10000	4.6	~8.5 to ~13.0
Shuttle	24400	---	5900	4.5	18.0
Delta II	5000	3000	2000	2.5	4.8
Titan III	11000	---	5000	3.6	12.4
Atlas V	20500	17300	8700	4.2	12.2
Proton	20000	---	5500	3.3	7.5
Energia	90000	72000	---	5.5	37
Soyuz	4850	4400	3250	3.7	6.5

(+) = Approximate values
(*) = air launched

Launch System Characteristics

Air Launch - Pegasus

Tristar

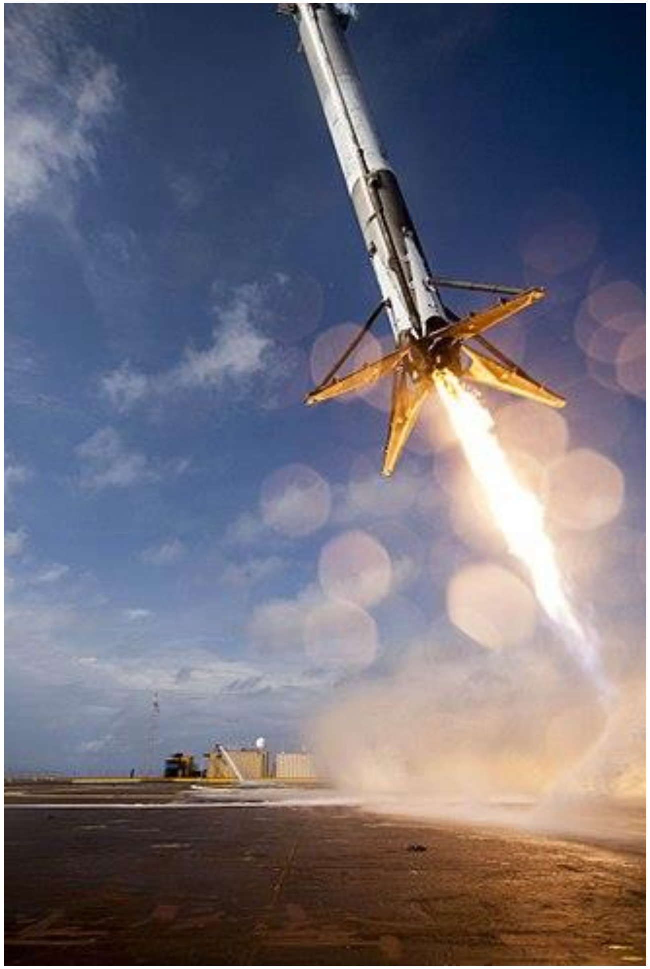
Launch System Characteristics

Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Ariane 5				4.6	~8.5 to ~13.0
G (generic)	21000	9500	6600		
ECA (evolution)	21000	9500	10000		
Shuttle	24400	---	5900	4.5	18.0
Delta II	5000	3000	2000	2.5	4.8
Titan III	11000	---	5000	3.6	12.4
Atlas V	20500	17300	8700	4.2	12.2
Proton	20000	---	5500	3.3	7.5
Energia	90000	72000	---	5.5	37
Soyuz	4850	4400	3250	3.7	6.5
Pegasus(*)	455	265	125	1.2	1.9

(+) = Approximate values
(*) = air launched

Launch System Characteristics

SpaceX Commercial Systems – Falcon 9



Launch System Characteristics

SpaceX Commercial Systems

SpaceX Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Falcon 9	22,800*		8,300*	4.6	~6.6 to ~11.4

* From SpaceX website. However, typical payloads in the Falcon 9 class are below 6800 kg.

Falcon 9 stats as of 11th Sept 2020:

92 launches, 53 total landings, 38 reflown rockets

(+) = Approximate values

Launch System Characteristics

SpaceX Commercial Systems – Falcon Heavy

Launch System Characteristics

SpaceX Commercial Systems

SpaceX Launcher	Payload(+) into orbit (kg)			Fairing size (m)	
	LEO	LEO Polar	GTO	Diam.	Length
Falcon 9	22,800*		8,300*	4.6	~6.7 to ~11
Falcon Heavy	63,800*		26,700*		

* From SpaceX website.

Falcon Heavy stats as of 11th Sept 2020:

3 launches, 7 total landings, 4 reflown rockets

(+) = Approximate values

Launch System Characteristics

Future Air-launch systems

'Virgin Orbit', launcher one
(~500 kg to SSO)
Maiden flight 25th May 2020 - unsuccessful

Stratolaunch
(6,000 – 8,000 kg to LEO)

Launch System Characteristics

Future Air-launch systems

Stratolaunch – largest aircraft (by wingspan) ever to fly

Launch System Characteristics

Second Generation Reusable LV – X37

Launch System Characteristics

Future – Single Stage to Orbit

Basic statistics from Ariane 1 launcher:

1st stage – 120 metric tons of propellant
(55% of the initial mass of the rocket)

Accelerates the vehicle to 1.5 km/s
(20% of the orbital velocity at 30km altitude)

2/3 of this is oxidiser (by mass)
– which is burnt within the Earth atmosphere

Launch System Characteristics

Second Generation Reusable LV – X34 (air launch)

Launch System Characteristics

Future – Reaction Engines ‘Skylon’

‘Sabre’ Engine

Combines elements of rocket and gas turbine technology.

Enables the launcher to be accelerated to mach 5.5 using an air-breathing mode, before switching to the rocket mode, to accelerate the vehicle to the orbital velocity.

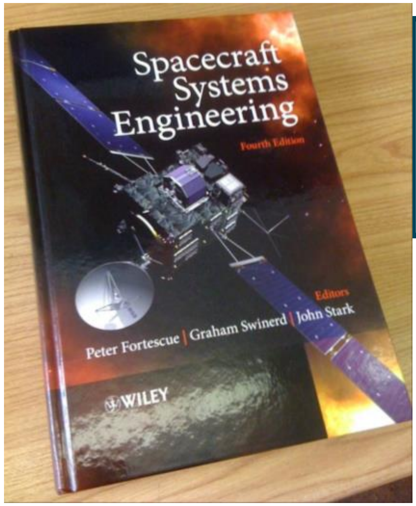
(Theoretically!)

Launch Vehicles Summary

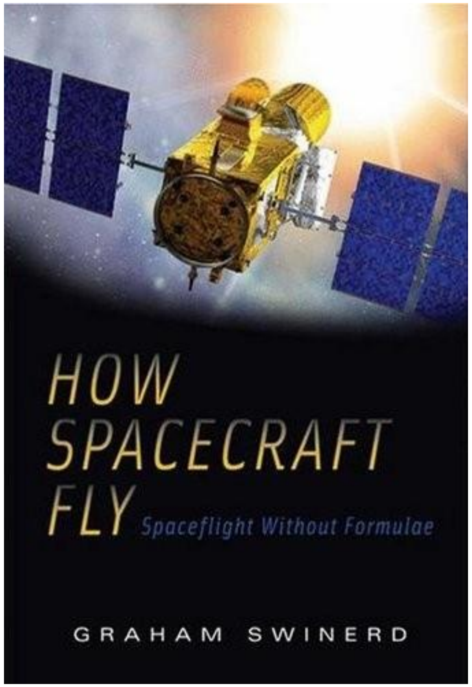
Key points:

Conservation of Momentum	• Basic definition of exhaust velocity and its link to the location of the observer • Important performance conclusion of this basic approach • The limitations of this approach
The Rocket Equation	• The derivation of the rocket equation by applying Newton's second law
Launch Vehicles	• How the rocket equation is modified for launch vehicles • The fundamental trade-offs required for trajectory optimisation • Key performance limitations outlined by numerical examples • The need for staging, the diminishing returns and how to apply the rocket equation to multi-stage vehicles
Launch Vehicle Interface	• Important features of the launch vehicle interface • Impacts of launcher selection
Launch System Characteristics	• Overview of typical launch system characteristics • Knowledge of future areas of development in the field

Launch Vehicles Summary



Read Chapter 7 of Fortescue, Stark & Swinerd



Read Chapter 5 of 'How S/C Fly'