Practice 1

16 October 2021 15:01

p576 — 8.3.8

26 Differentiate the function f where $f(x)$ is

(a) $(5x + 3)^9$ (b) $(4x - 2)^7$

(c) $(1 - 3x)^6$ (d) $(3x^2 - x + 1)^3$

(e) $(4x^3 - 2x + 1)^6$ (f) $(1 + x - x^4)^5$

a) $y = (5x+3)^9$

$\frac{dy}{dx} = 9 \times 5(5x+3)^8$

$= 45(5x+3)^8$ ✓

e) $y = (4x^3 - 2x + 1)^6$

$\frac{dy}{dx} = (12x^2 - 2)6(4x^3 - 2x + 1)^6$

$= 12(6x^2 - 1)(4x^3 - 2x + 1)^6$ ✓

27 Differentiate the function f where $f($

(a) $(2x + 4)^7(3x - 2)^5$ (b) $(5x$

(c) $(\frac{1}{2}x + 2)^2(x + 3)^4$

(d) $(x^2 + x + 1)^2(x^3 + 2x^2 + 1)^4$

d) $f(x) = (x^2+x+1)^2(x^3+2x^2+1)^4$

$2(2x+1)(x^2+x+1)$ $\quad 4(3x^2+4x)(x^3+2x^2+1)^3$

$f'(x) = 2(2x+1)(x^2+x+1)(x^3+2x^2+1)^4 + 4(3x^2+4x)(x^3+2x^2+1)^3(x^2+x+1)^2$

$= 2(x^3+2x^2+1)^3(x^2+x+1)[(2x+1)(x^3+2x^2+1) + 2(3x^2+4x)(x^2+x+1)]$?

32 Differentiate

(a) $x\sqrt{(4 + x^2)}$ (b) $x\sqrt{(9 - x^2)}$

(c) $(x + 1)\sqrt{(x^2 + 2x + 3)}$ (d) $x^{2/3} - x^{1/4}$

(e) $\sqrt[3]{(x^2 + 1)}$ (f) $x(2x - 1)^{1/3}$

a) $y = x(4+x^2)^{1/2}$

$1 \quad \frac{2x}{2}(4+x^2)^{-1/2}$

$\dot{y} = (4+x^2)^{1/2} + x^2(4+x^2)^{-1/2}$ ✓

e) $y = (x^2+1)^{1/3}$

$\dot{y} = \frac{2x}{3}(x^2+1)^{-2/3}$ ✓

33 Differentiate

(a) $1/(x + 3)^2$ (b) $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

(c) $x/\sqrt{(x^2 - 1)}$ (d) $(2x + 1)^2/(3x^2 + 1)^3$

b) $y = (x^{1/2} + x^{-1/2})^2 = x + x^{-1} + 2$

$\dot{y} = 1 - x^{-2}$ ✓

d) $y = \frac{(2x+1)^2}{(3x^2+1)^3}$ $\quad \frac{u}{v} \to \frac{v\dot{u} - u\dot{v}}{v^2}$

$18x(3x^2+1)^2 \quad 4(2x+1)$

$\dot{y} = \frac{(3x^2+1)^3 4(2x+1) - (2x+1)^2 18x(3x^2+1)^2}{(3x^2+1)^6}$

$= \frac{4(2x+1)(3x^2+1) - 18x(2x+1)}{(3x^2+1)^4}$ $-12x^2$

$= \frac{2(2x+1)(6x^2+2-9x)}{(3x^2+1)^4}$ ✗

Test 3.1

18 October 2021 10:09

1)

i) $\lim_{h \to 0} f'(x) = \frac{f(h+x) - f(x)}{h}$

ii) $f(x) = 3x + 4$

$= \frac{3(h+x) + 4 - (3x+4)}{h}$

$= \frac{3h}{h}$

$= 3$ ✓

2)

i) $2x^3 + 3x - 4$

$f'(x) = 6x^2 + 3$ ✓

ii) $3x^{-2} \to -6x^{-3}$ ✓

iii) $(1-x^2)^6 \to -12x(1-x^2)^5$ ✓

iv) $3x^2 \cos(x^3+1)$ ✓

v) $\frac{e^x}{x+1}$ $e^x \to e^x$, $x+1 \to 1$ $\quad \frac{u}{v} \to \frac{\dot{u}v - u\dot{v}}{v^2}$

$\frac{e^x(x+1) - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$ ✓

vi) $y = x^2 \ln x$

$\dot{y} = 2x \ln x + \frac{1}{x}x^2$

$= 2x \ln x + x$ ✓

vii) $\ln(x + \sqrt{x^2 - 1})$ $\quad \sqrt{x^2-1} \to 1 + 2x0.5(x^2-1)^{-0.5}$

$\dot{y} = \frac{1 + x(x^2-1)^{-1/2}}{x + \sqrt{x^2-1}}$ ✓ (but not simplified)

viii) $y = (\tan^3(1+x^2))^3$

$\dot{y} = 2x\sec^6(1+x^2)$ x ✗ $\quad 6x\tan^2(1+x^2)\sec^2(1+x^2)$

3) approximating $x^3 = 7$ $\quad x_0 = 2$ $\quad$ Newton raphson style

$x^3 - 7 = f(x)$

$3x^2 = f'(x)$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1.9583$

$x_2 = 1.9356$

$x_3 = 1.924$

$x_{\ldots} = 1.913$ ✓

4. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ when

(i) $f(x, y) = x^4y^2 + x\sin y + x^2 + y^2 + 2$

$$\frac{df}{dx} = 4x^3y^2 + \sin y + 2x + \cancel{y^2}$$

$$\frac{df}{dy} = 2x^4y + x\cos y + 2y$$

(ii) $f(x, y) = \sin^2(x + y)$

$$\frac{df}{dx} = 2\sin(x+y)\cos(x+y) = \sin\left(2[x+y]\right)$$

$$\frac{df}{dy} = 2\sin(x+y)\cos(x+y) = \sin\left(2[x+y]\right)$$