



# SESA 6071 Spacecraft Propulsion

## Week One - Introduction to Propulsion

Charlie Ryan (c.n.ryan@soton.ac.uk)

*Office: 176 / 2039*

*Associate Professor in Astronautics*

## ***Module Aims:***

- To gain an understanding of the various types of propulsion systems applicable for launch vehicles and in-space manoeuvring of spacecraft and to be able to analyse and evaluate their main operational and performance characteristics.
- To be able to determine what particular propulsion system is suitable for a particular mission, allowing for application of the knowledge within industrial and academic settings.

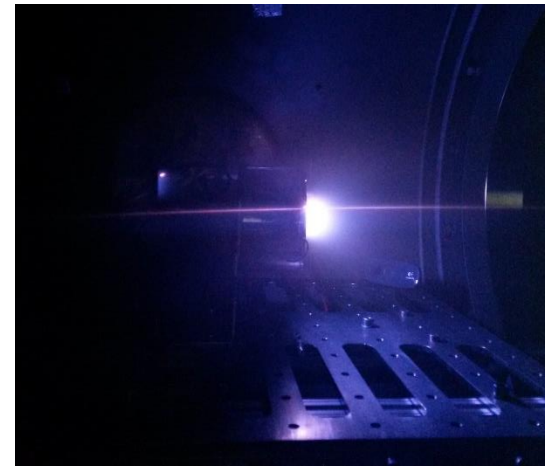
**Module will be assessed in three ways:**

## 1: Exam (70%)

- Answer all questions in the exam (four questions).
- Some mathematical, some descriptive.
- Will likely be an in person exam.

## 2: Lab on electric propulsion (20%)

- Coursework from lab in weeks eight and nine.
- Testing of small Hall Effect thruster, measuring thrust.
- Hopefully this will be in person (in groups of six)
  - (if not, we will use data from a previous year)
- Check your timetable for your slot
- More details in week 7: Introduction to EP Lab
- Coursework submission week before Christmas



**Module will be assessed in three ways:**

### 3: Online Blackboard tests

- Summative, worth 10% of final mark
- Four online tests
- At the end of weeks two, four, eight and ten.

- Sutton & Biblarz “Rocket Propulsion Elements”, John Wiley and Sons Ltd. Ninth Edition.
  - Available as library eBook.
- Fortescue, Swinerd & Stark “Spacecraft System Engineering” John Wiley and Sons Ltd.
  - Chapter 6, Propulsion Systems; Chapter 7, Launch Vehicles.
  - Available as library ebook.
- Dan M. Goebel “Fundamentals of Electric Propulsion: Ion and Hall Thrusters” JPL Space Science Technology Series
  - Freely available online.
- Course notes from others;
- ‘Space propulsion’ course notes from MIT (Martinez Sanchez, 2012, 2005) (Lozano, 2015)
  - Freely available online.

## Dr Charlie Ryan

- **PhD, Queen Mary, University of London**, specializing in electrospray thrusters for CubeSats, 2011
- **Two Postdoc Researcher positions**, Queen Mary (electrospray CubeSat thrusters), Surrey Space Centre (Hall Effect Thrusters)
- **Associate Professor in Astronautics**, University of Southampton, since October 2015

<https://www.southampton.ac.uk/people/5xg2p2/doctor-charlie-ryan>



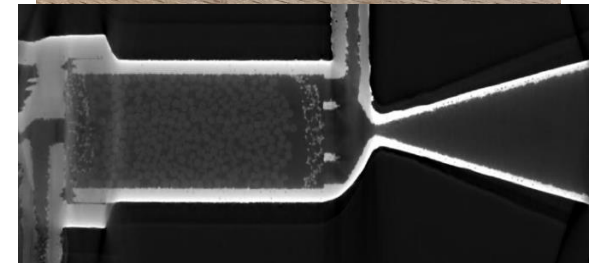
Research interests: the physics and applications of small satellite propulsion (electrospray thrusters, Hall Effect thrusters, green propellant chemical thrusters).

Contact me via email!: [c.n.ryan@soton.ac.uk](mailto:c.n.ryan@soton.ac.uk)

# My propulsion experience - chemical



40 N hybrid chemical thruster



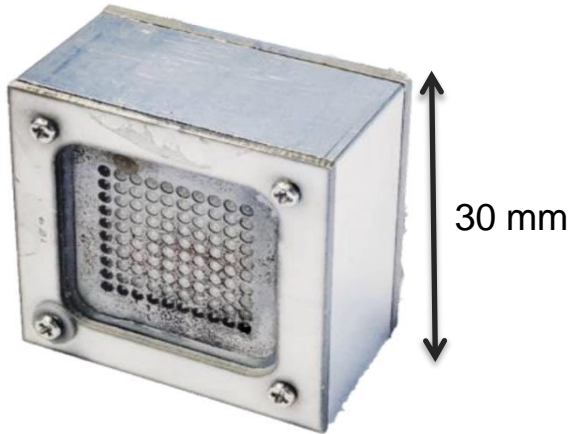
1 N peroxide thruster



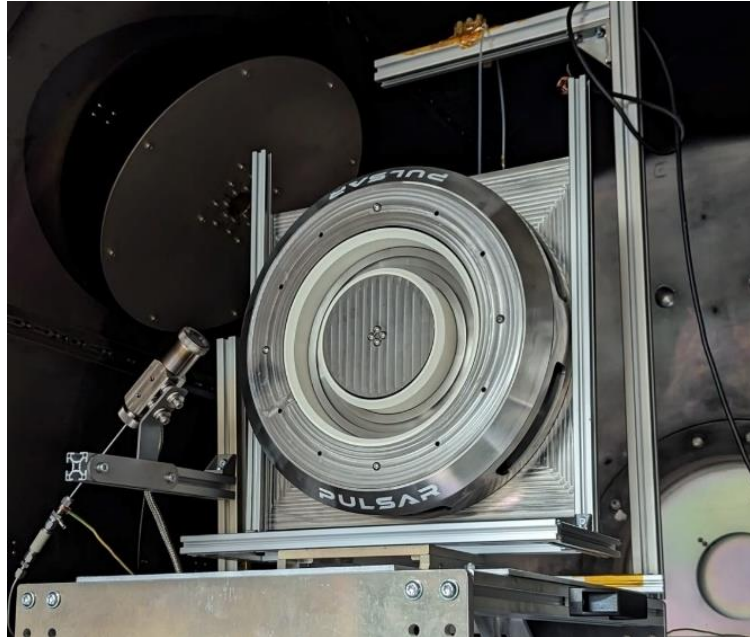
Rotating Detonation Engine



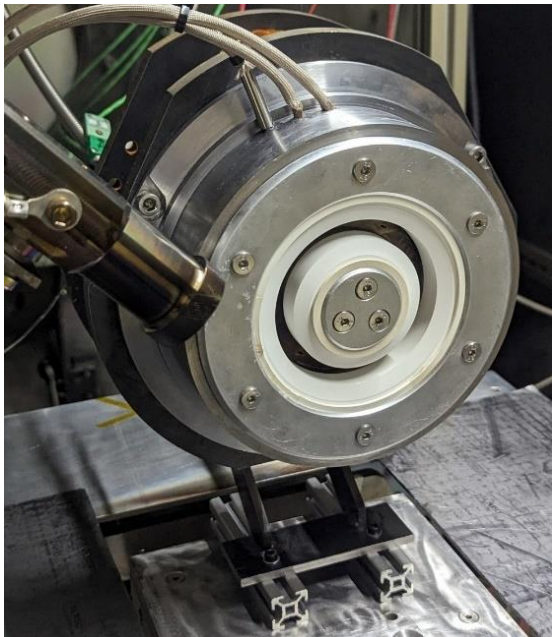
# My propulsion experience - electric



Electrospray thruster

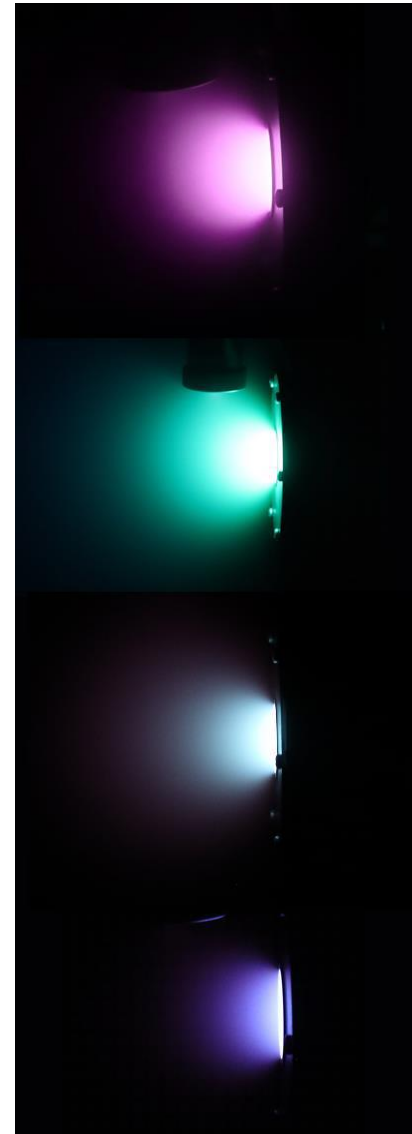


10 kW Hall thruster



1.5 kW Hall thruster

Hall thruster operation on  
different propellants





Every week there will be three hours of contact time, consisting of lectures:

- Monday, 3pm – 4pm, building 58, Room 1009.
- Thursdays, 10am – 12, building 46, 2005.

Lectures will also be recorded.

Will also have some guest speakers from industry (in later weeks)

# Week One Learning Outcomes

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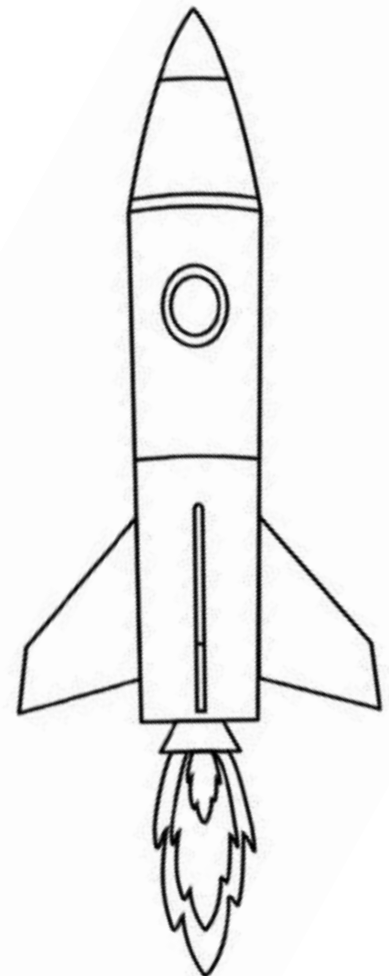
## Introduction to Spacecraft Propulsion

- Be able to describe the key propulsion types, and their potential applications.
- Understand and derive the fundamental propulsion parameters (specific impulse, rocket equation, characteristic velocity, etc.), and understand why their propulsion differs.
- Understand the typical change in velocity requirements for different missions.
- Understand why launch vehicles typically have several stages, and be able to apply the rocket equation for this situation.
- Be able to apply a simplified equation for calculating the trajectory of a launch vehicle.

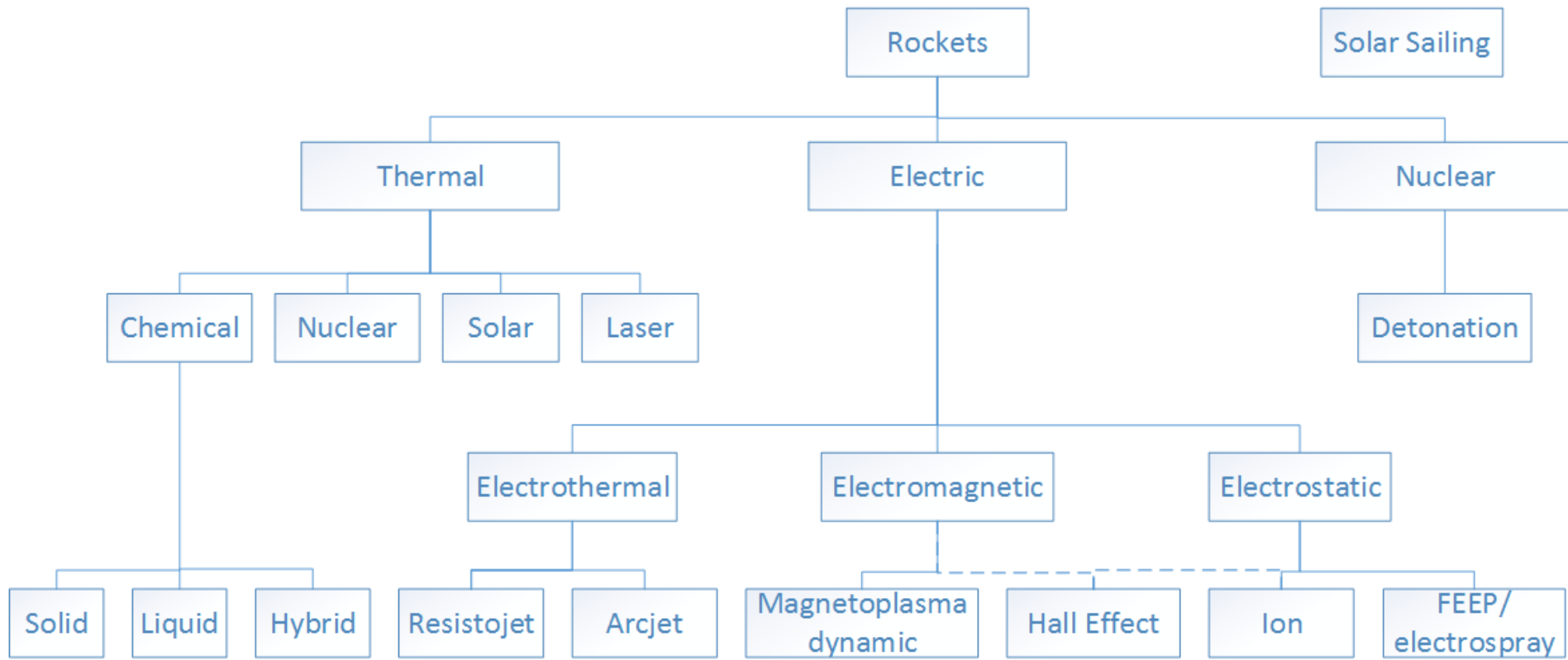
## What is rocket propulsion?



- Newton's third Law: for every action there is an equal and opposite reaction
- Can be classified by;
  - Energy source (chemical, nuclear, electric)
  - The function (launch vehicle, upper stage, attitude control, orbit station keeping, deep space)
  - Type of propellant
  - Etc...

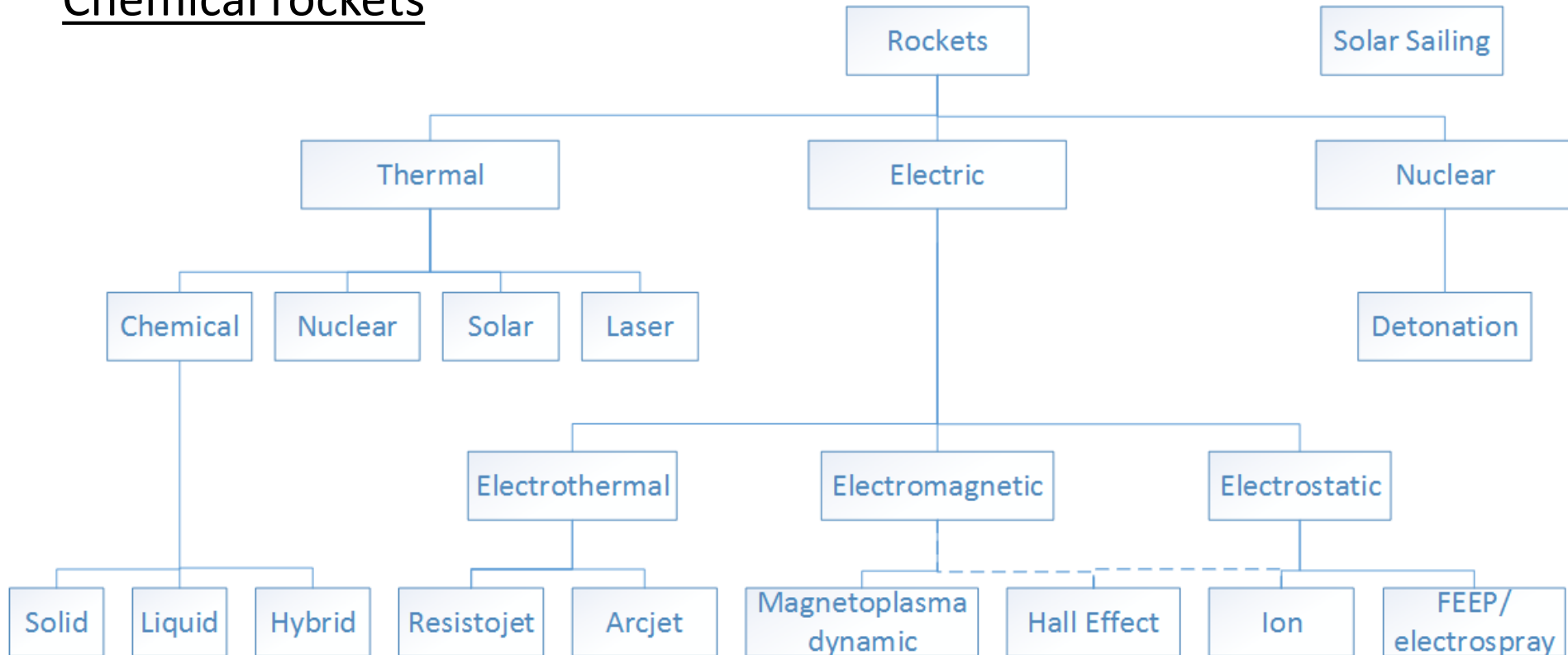


# Rocket Propulsion Family Tree



# Propulsion Types

## Chemical rockets



## Chemical Rockets:

- Utilise the energy generated by chemical reactions to produce hot products at between 700 and 3900 °C.
- Hot gases are expanded through a supersonic nozzle to exit velocities of between 1.5 and 4.5 km/s.
- Fuel and an Oxidizer required.





# Chemical rocket types



## Ariane 5 ECA

Wikipedia

### Solid

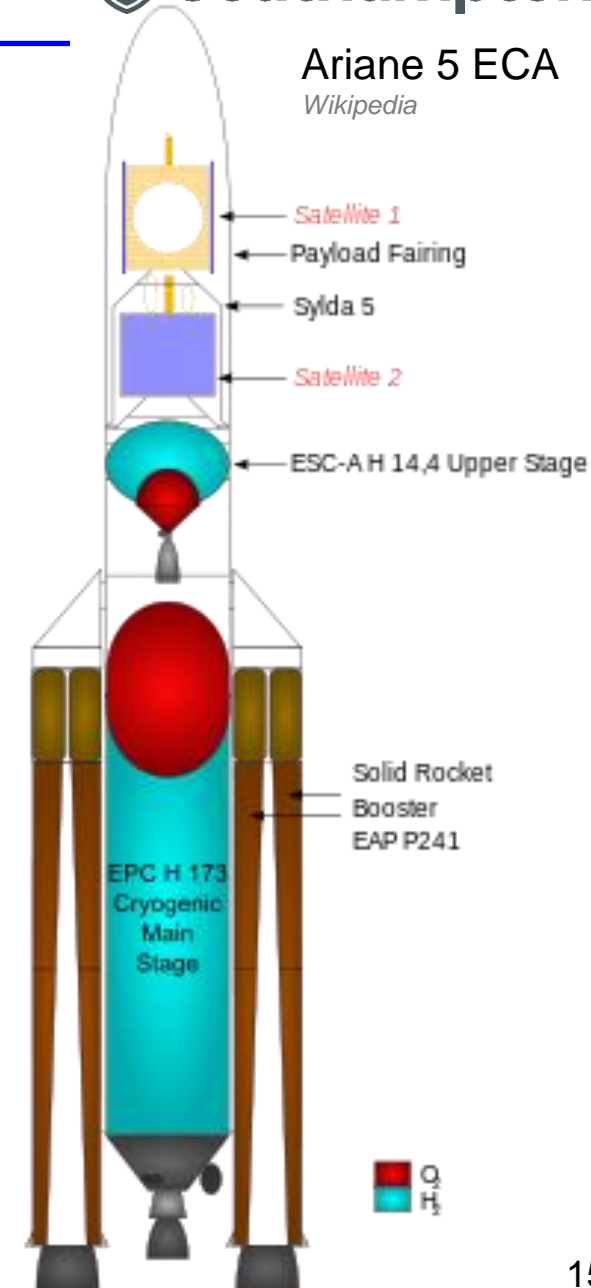
- Fuel and Oxidizer is contained within a solid *grain*.
- Once ignited they cannot be stopped.
- Solid propellants have high thrust density but low performance.
- Used in missiles, launcher boosters, and apogee boost motors.

### Liquid

- Liquid fuel and oxidizer.
- Enable repeated firing and variable thrust operation.
- Generally high performance and high thrust.
- Complex configuration.

### Hybrid

- Liquid oxidizer, solid fuel.
- Better performance than solid, simpler than liquid.
- Less utilized and developed, issues with length.

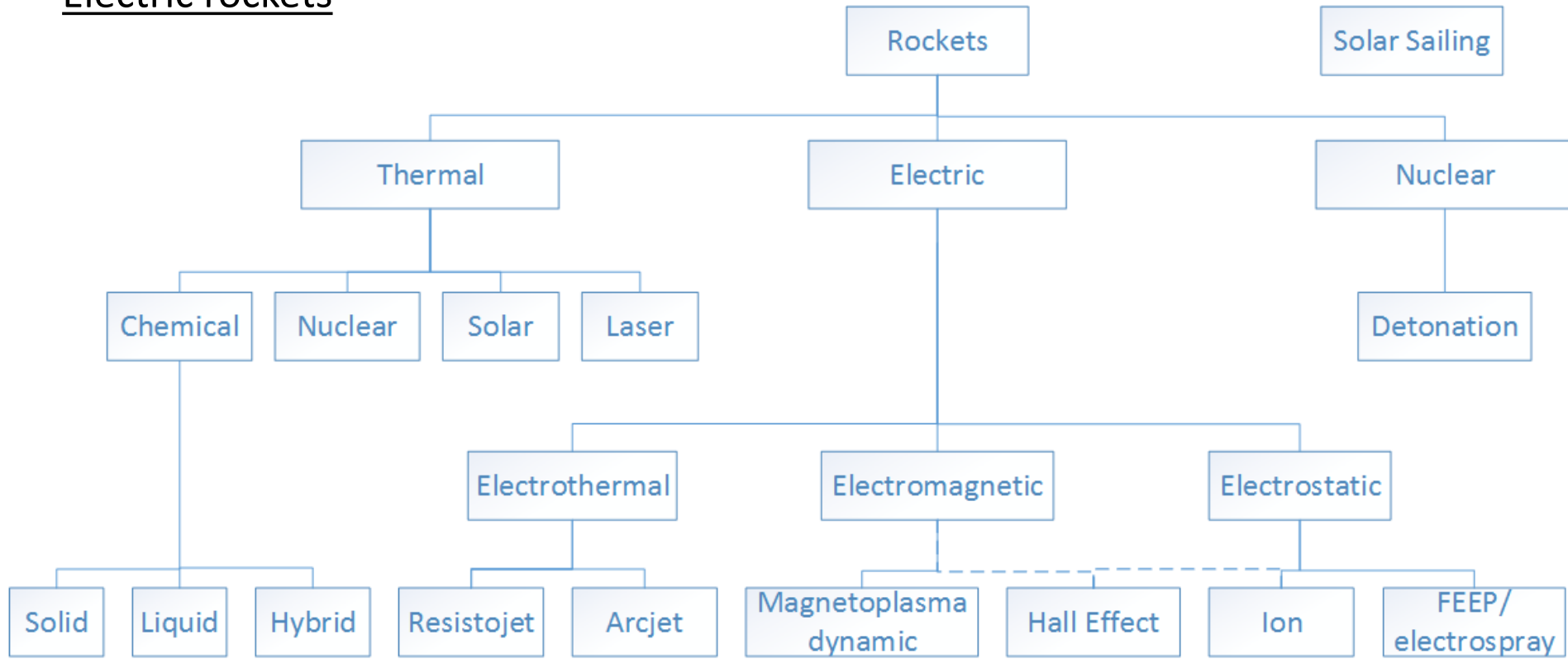


# Chemical rocket types

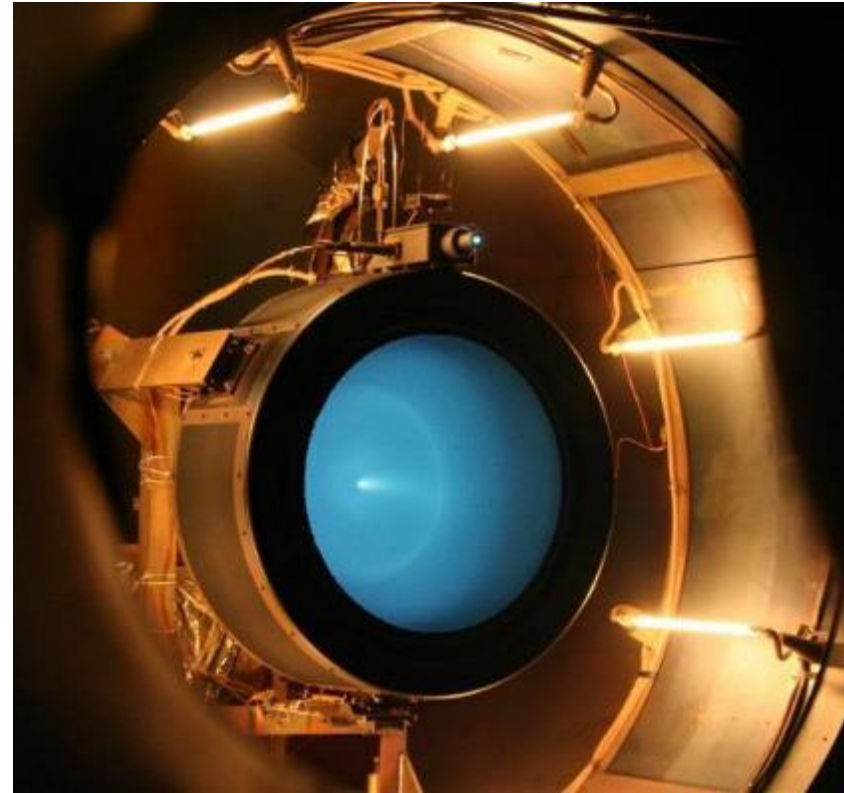


# Propulsion Types

## Electric rockets



- Utilizes electrical energy to accelerate propellant.
  - No combustion.
- High exhaust velocity (up to 60,000 m/s) enables spacecraft using electric propulsion to use less propellant.
- But can be physically complex.
- Large amount of power required for higher thrusts.



NASA gridded ion thruster

NASA

# Electric Rocket Types

Arcjet  
*NASA*

## Electrothermal

- Utilizes electrical energy to heat propellant
- Simple but performance and thrust are relatively low

## Electrostatic

- Uses electrical energy to ionize and then accelerate propellant through an electric field
- Very good performance, low thrust, but costly

## Electromagnetic

- Uses electrical energy to ionize propellant, and then accelerate it using magnetic fields (right hand rule)
- Efficiency is low until at very high power

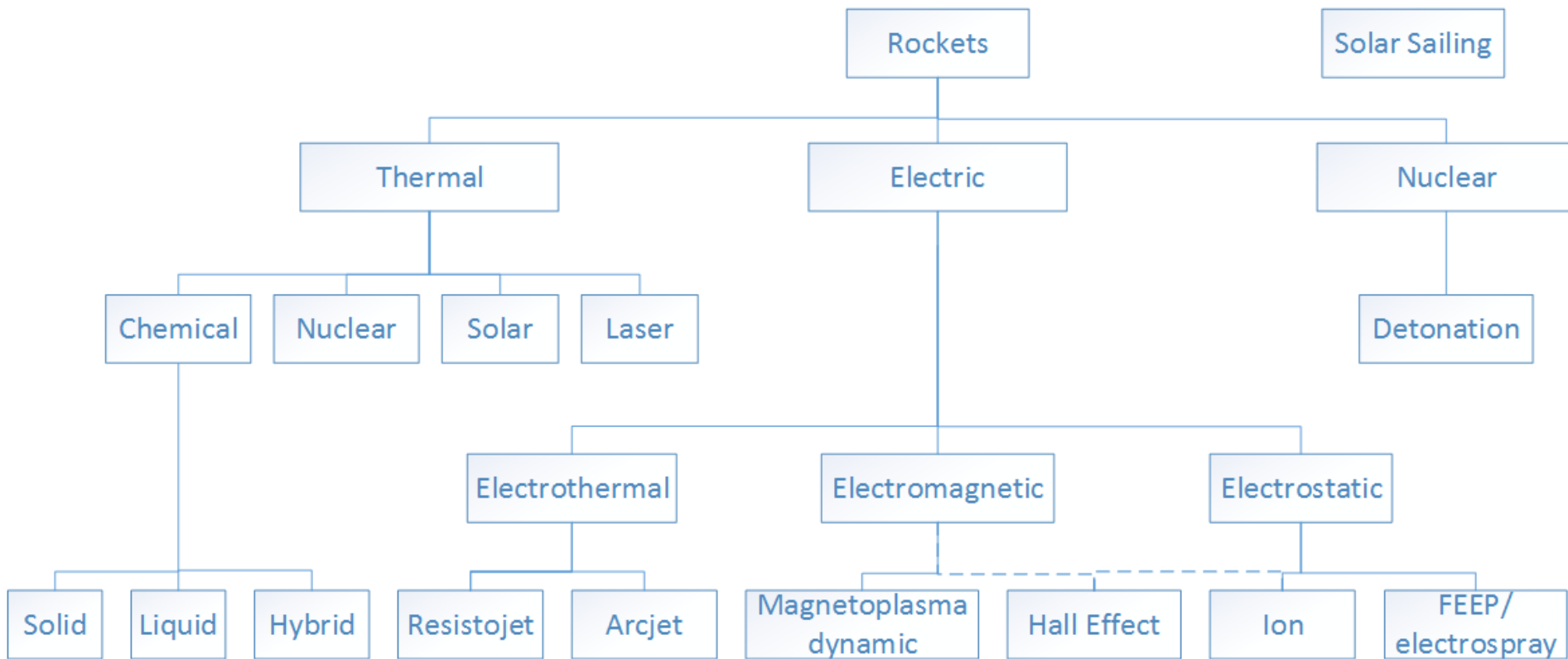


Hall Effect Thruster

*Aerojet Rocketdyne*

# Propulsion Types

## Nuclear rockets





# Nuclear Rockets

- Two (main) types;
  - Using nuclear fission to heat propellant, and expel it through a nozzle.
  - Using nuclear detonations to accelerate rocket.
- Both types developed, but halted (dangerous!).
- High performance and high thrust.



Project Orion nuclear detonation rocket

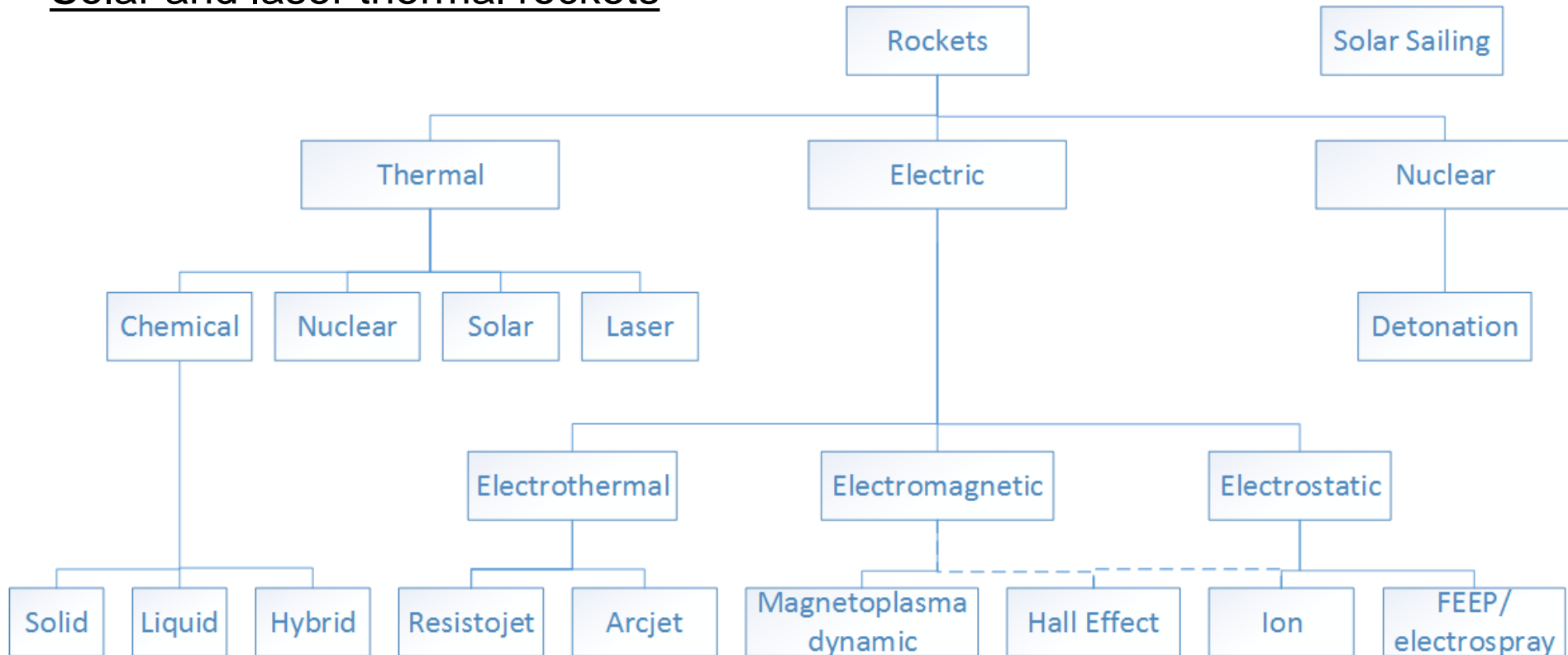


NERVA nuclear thermal rocket

NASA

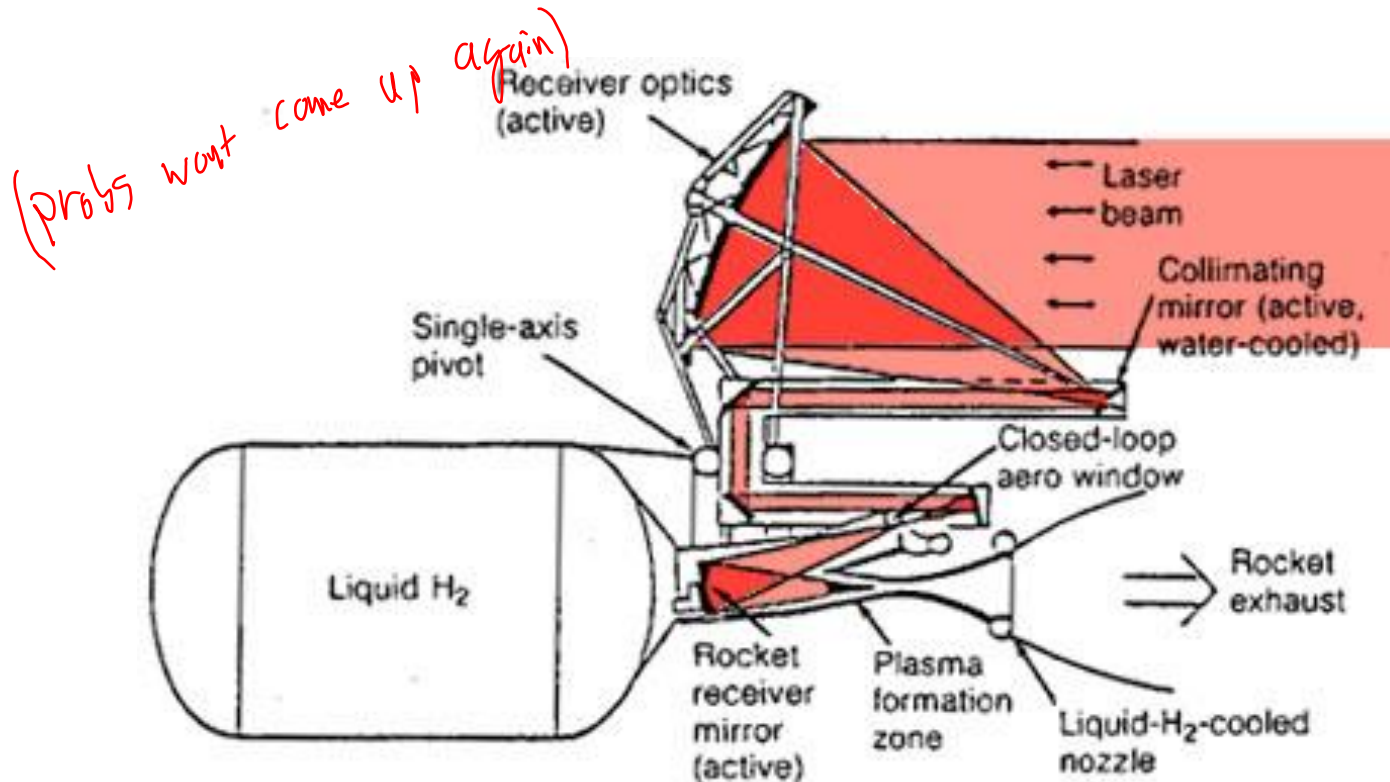
# Propulsion Types

## Solar and laser thermal rockets



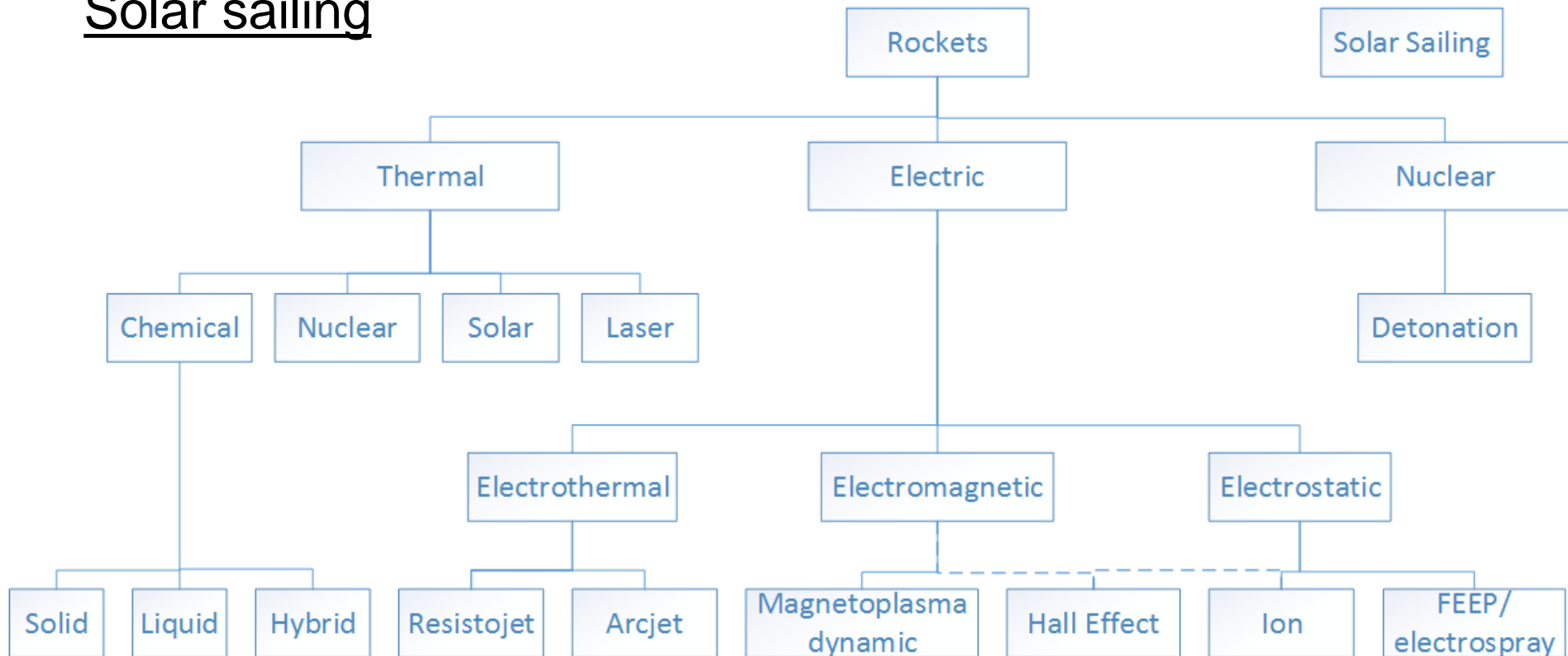
# Solar and laser thermal rockets

- A solar thermal rocket uses large diameter optics to concentrate the sun's radiation
- Laser thermal propulsion works similarly but employing a laser
- Good performance, low thrust
- Not tested in space



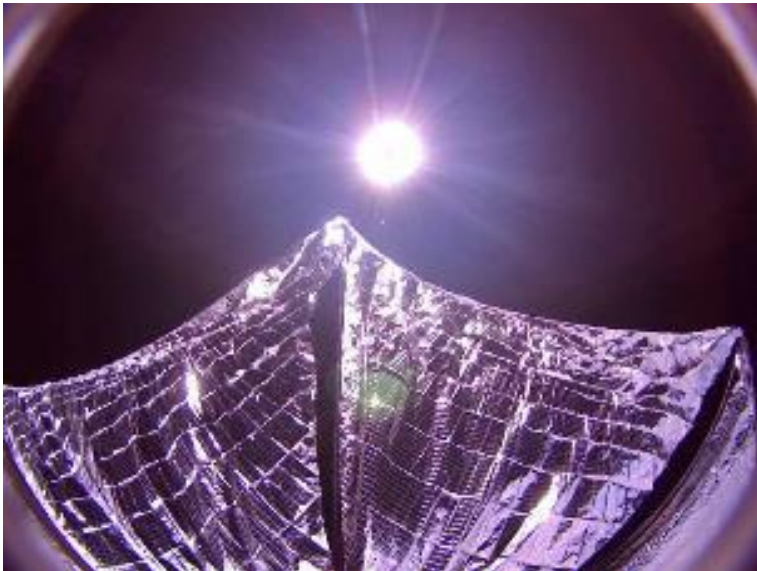
Laser thermal rocket

## Solar sailing



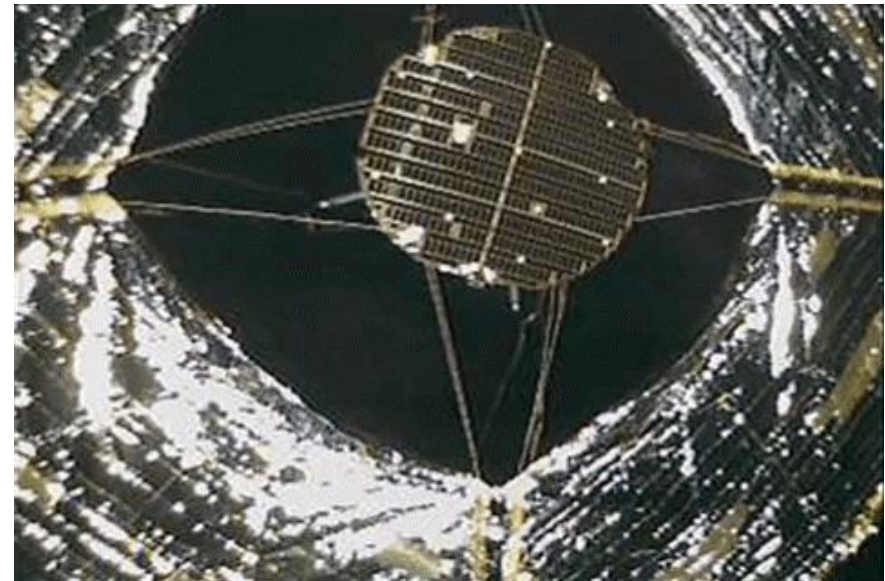
# Solar Sailing

- Using the momentum gained by Sunlight photon to propel spacecraft.
- Can offer good performance, with no fuel needed, but low thrust.
- Some have flown – IKAROS, NanoSail-D2, LightSail-1.
- Difficult to create large lightweight structure and control it.



Lightsail-A

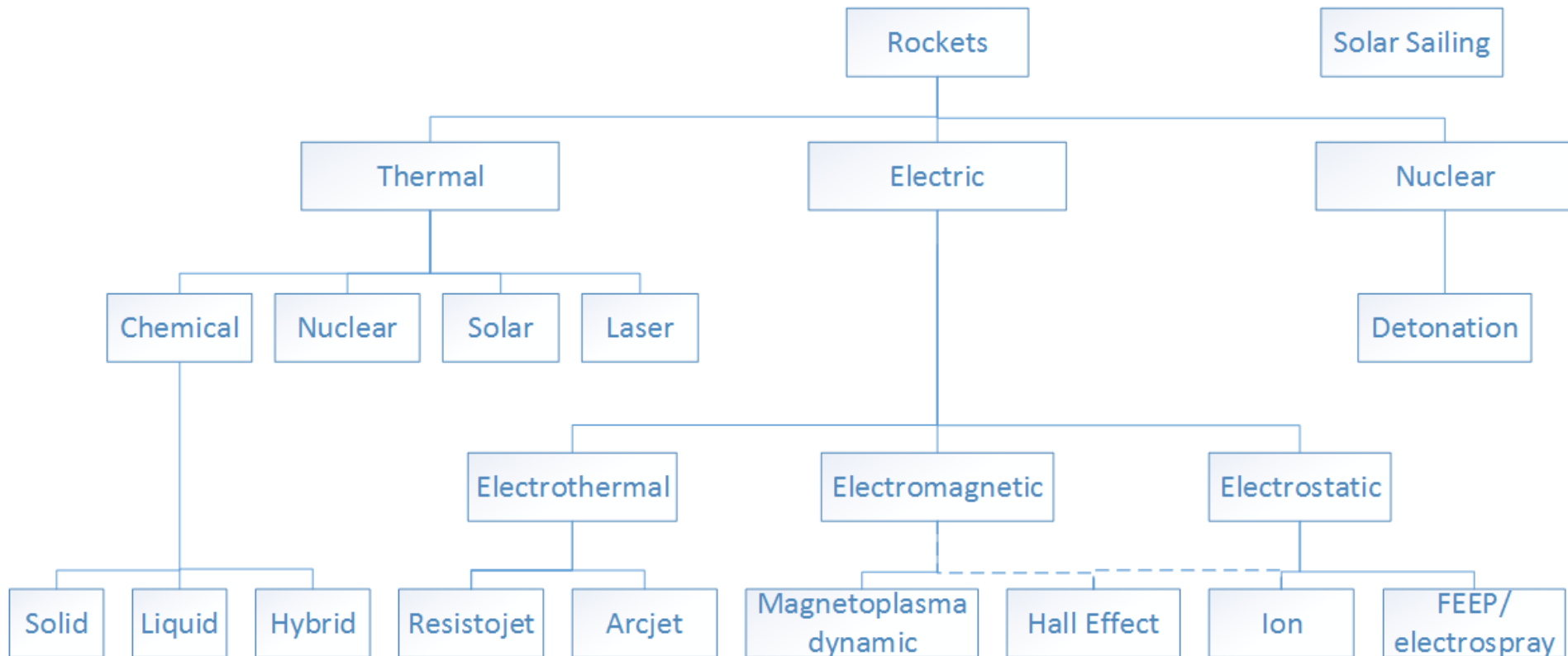
<http://www.planetary.org/explore/projects/lightsail-solar-sailing/>  
*The Planetary Society*



IKAROS solar sail spacecraft

JAXA

# Propulsion Family Tree

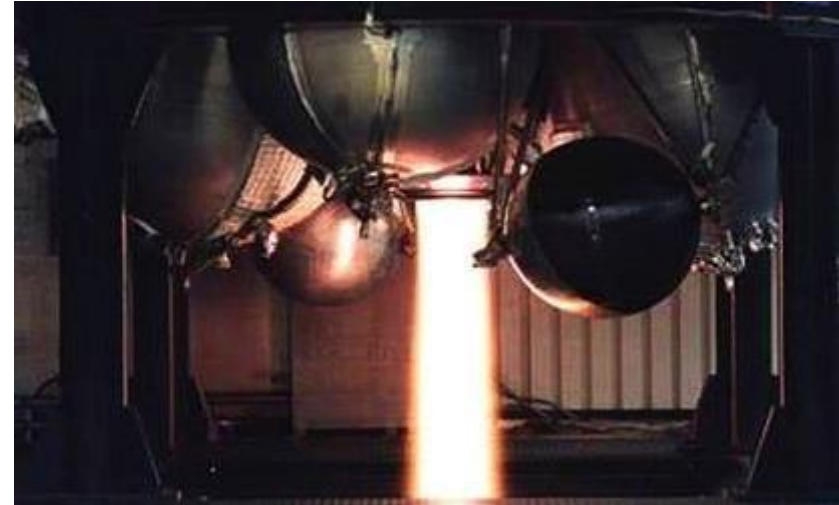




# Rocket applications



Soyuz launch



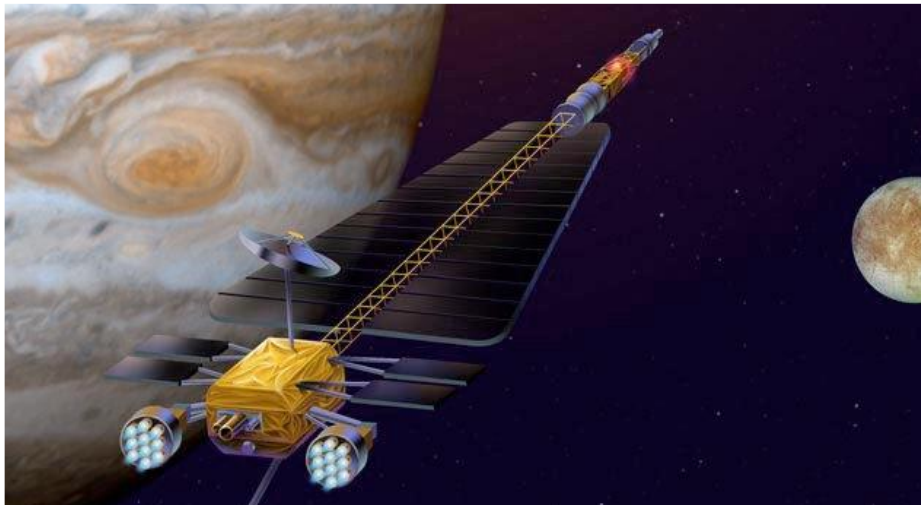
Hot fire testing of Ariane V upper stage  
*Airbus Defence and Space*

Trident missile



Mars Curiosity Sky Crane  
*Mars.nasa.gov*

# Rocket applications

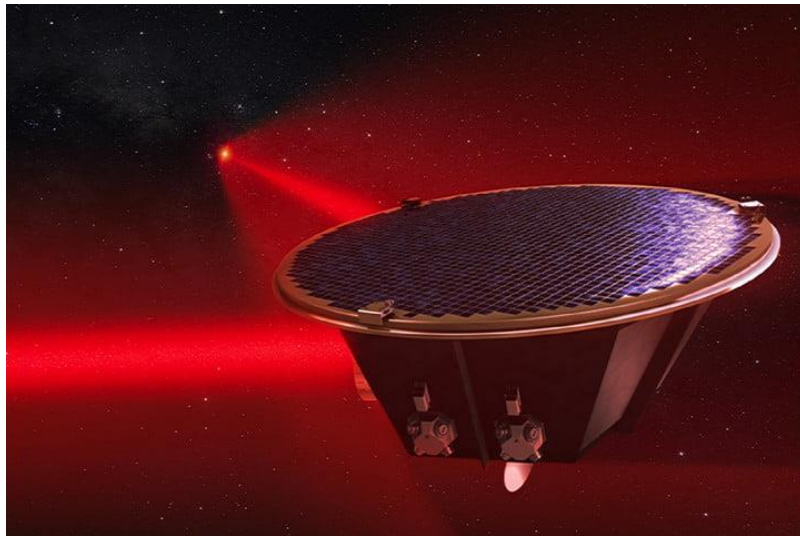


Mission to Jupiter

*Nasa*



Dawn  
spacecraft  
*NASA*



LISA gravitational mission

*ESA*

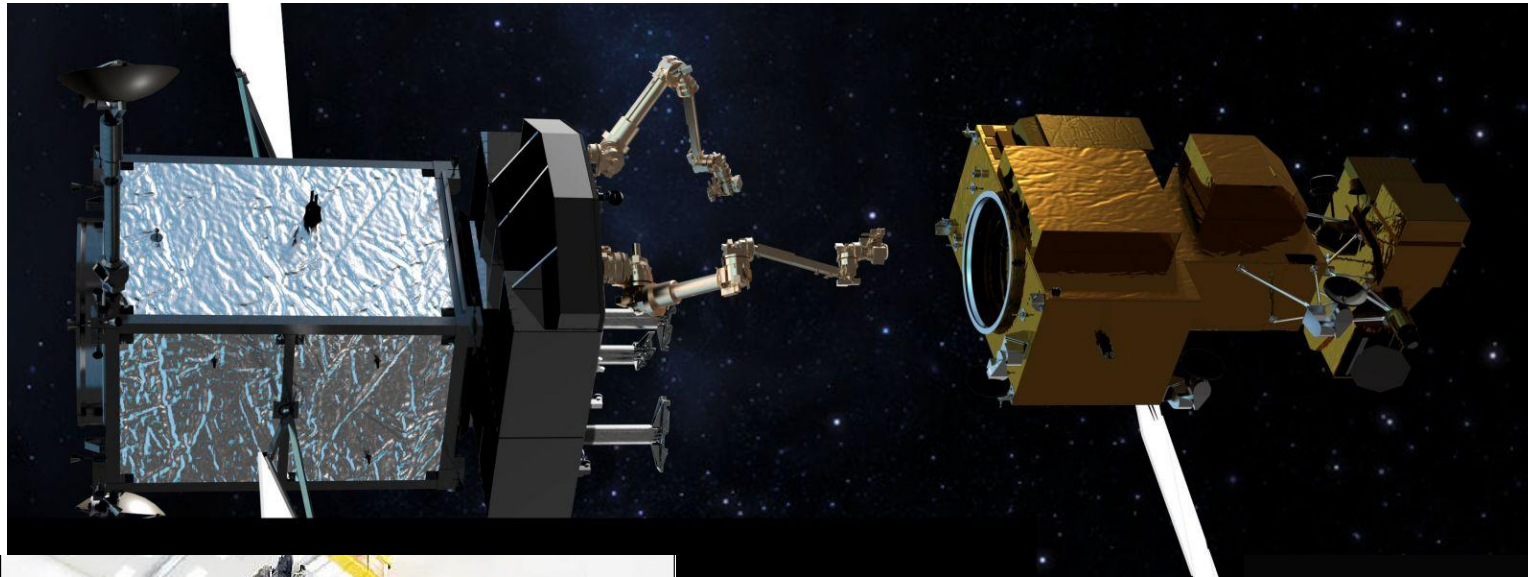


Goce satellite

*ESA*



# Rocket applications

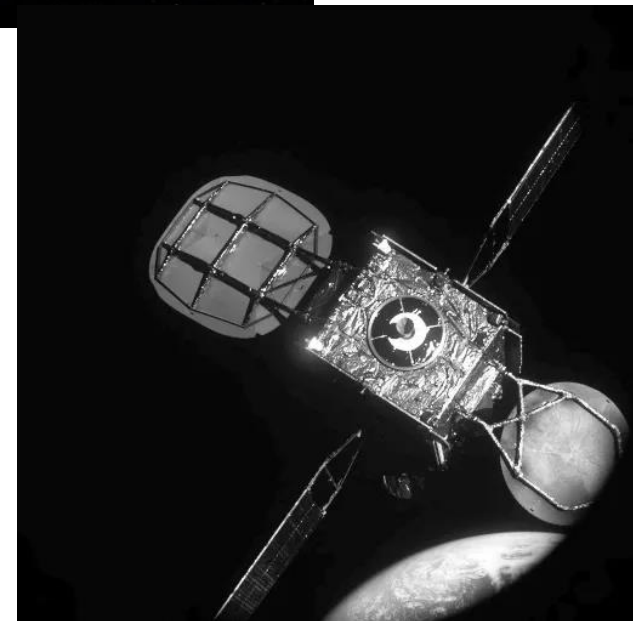


Satellite  
refuelling  
*Space Systems  
Loral*



Boeing  
Geostationary  
satellite  
*Boeing*

1<sup>st</sup> in space  
refuelling in 2020  
*Northrop Gruman*



The total impulse delivered by the rocket is the thrust integrated over the burn time;

$$I_t = \int_0^t F dt \quad \text{——— (1.1)}$$

Given constant thrust this simplifies to  $I_t = Ft$ .

The specific impulse  $I_{sp}$  is the total impulse per unit weight of the propellant;

$$I_{sp} = \frac{\int_0^t F dt}{g_0 \int_0^t \dot{m} dt} \quad \text{——— (1.2)}$$

Where  $g_0$  is the standard acceleration of gravity ( $9.81\text{m/s}^2$ ), and  $\dot{m}$  the propellant mass flow rate.

For constant thrust and propellant flow this simplifies to;

$$I_{sp} = \frac{I_t}{g_0 m_p} \quad \text{———— (1.3)}$$

where  $m_p$  is the total mass of propellant expelled.

The effective exhaust velocity  $c$  is defined as;

$$c = \frac{F}{\dot{m}} \quad \text{———— (1.4)}$$

It is described as “effective” as the actual exhaust velocity is not uniform across the rocket nozzle exit.

The  $I_{sp}$  can be defined as;

$$I_{sp} = \frac{I_t}{g_0 m_p} = \frac{Ft}{g_0 \dot{m}t} = \frac{\dot{m}ct}{g_0 \dot{m}t}$$

$$I_{sp} = \frac{c}{g_0} \quad \text{———— (1.5)}$$

Typical  $I_{sp}$  values;

- Chemical Liquid  $\text{H}_2/\text{O}_2$   $I_{sp} \sim 450$  s
- Solid  $I_{sp} \sim 260$  s
- Cold gas  $I_{sp} \sim 70$  s
- Gridded ion thruster  $I_{sp} \sim 3000$  s



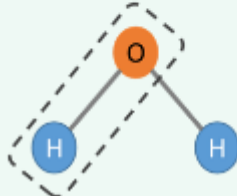


# Maximum chemical performance

Take the chemical reaction;



The bond energies involved are;

$\text{H}_2$		436 kJ/mol
$\text{O}_2$		498 kJ/mol
$\text{H}_2\text{O}$		OH bond: 428 kJ/mol OH-H bond: 498.7 kJ/mol

- *What is the energy per kg released??*
- *What is the specific impulse??*

# Maximum chemical performance

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- What is the energy per kg released??

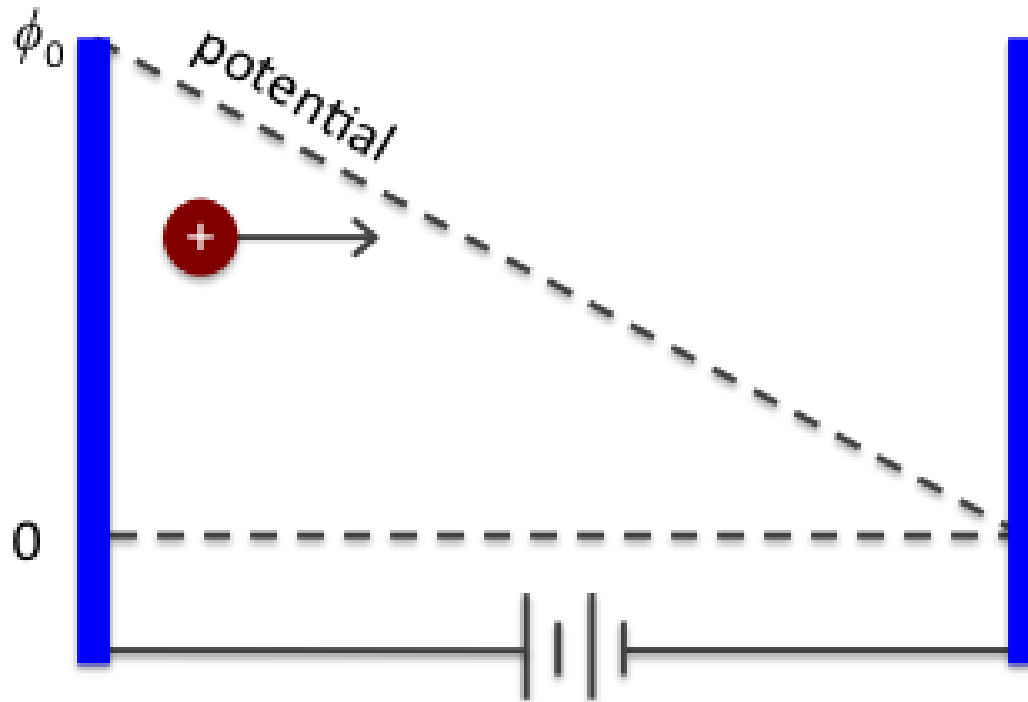
# Maximum chemical performance

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- What is the resulting specific impulse??  
[Assume 90 % efficiency]

# Electrical performance

In a similar fashion we can calculate the acceleration of a charged water molecule through an electric field



Taking an energy balance;

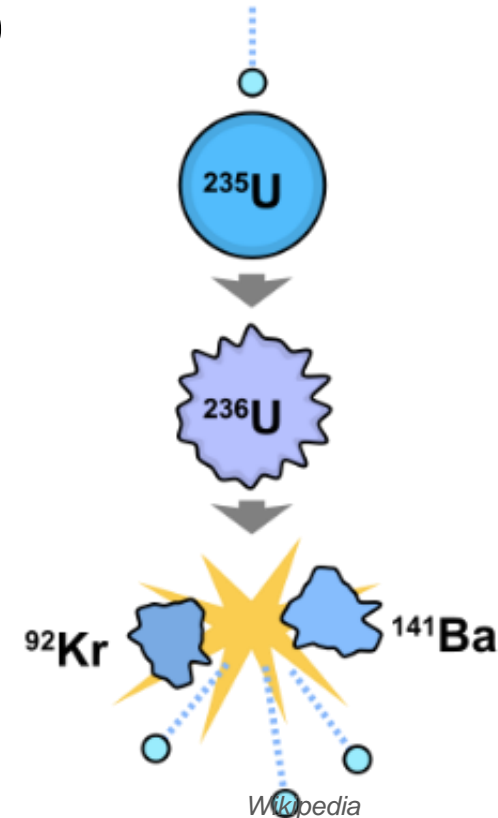
- What is the voltage required to accelerate the charged water molecule to 500 seconds, assuming 90 % efficiency?

- Alternatively, what  $I_{sp}$  would be obtained with an applied voltage of 1000 Volts?

# Nuclear thermal performance

The energy released in one Uranium-235 fission event is 180 MeV.

- What is the specific impulse assuming using water as the propellant and 90 % efficiency?





An efficiency can be defined as the fraction of the total source power which is transformed into kinetic (or jet) power;

$$\eta_T = \frac{\dot{m} c^2}{2P_{in}} \quad \text{———— (1.6)}$$

Rearranging, the input power is;

$$P_{in} = \frac{\dot{m} c^2}{2\eta_T} = \frac{Fc}{2\eta_T}$$

For a chemical rocket with 10 Newtons of thrust, an  $I_{sp}$  of 200 seconds and an efficiency of 90 % the input power is 10.9 kW.

For an electrical thruster this power must come from a **power source**. If solar panels are used, this equates to 32 m<sup>2</sup> array.

Consider the specific power (power over spacecraft mass);

$$\frac{P_{in}}{m} = \frac{F}{m} \frac{c}{2\eta_T} = a \frac{c}{2\eta_T} \quad \text{———— (1.7)}$$

Where  $a$  – acceleration from propulsion system.

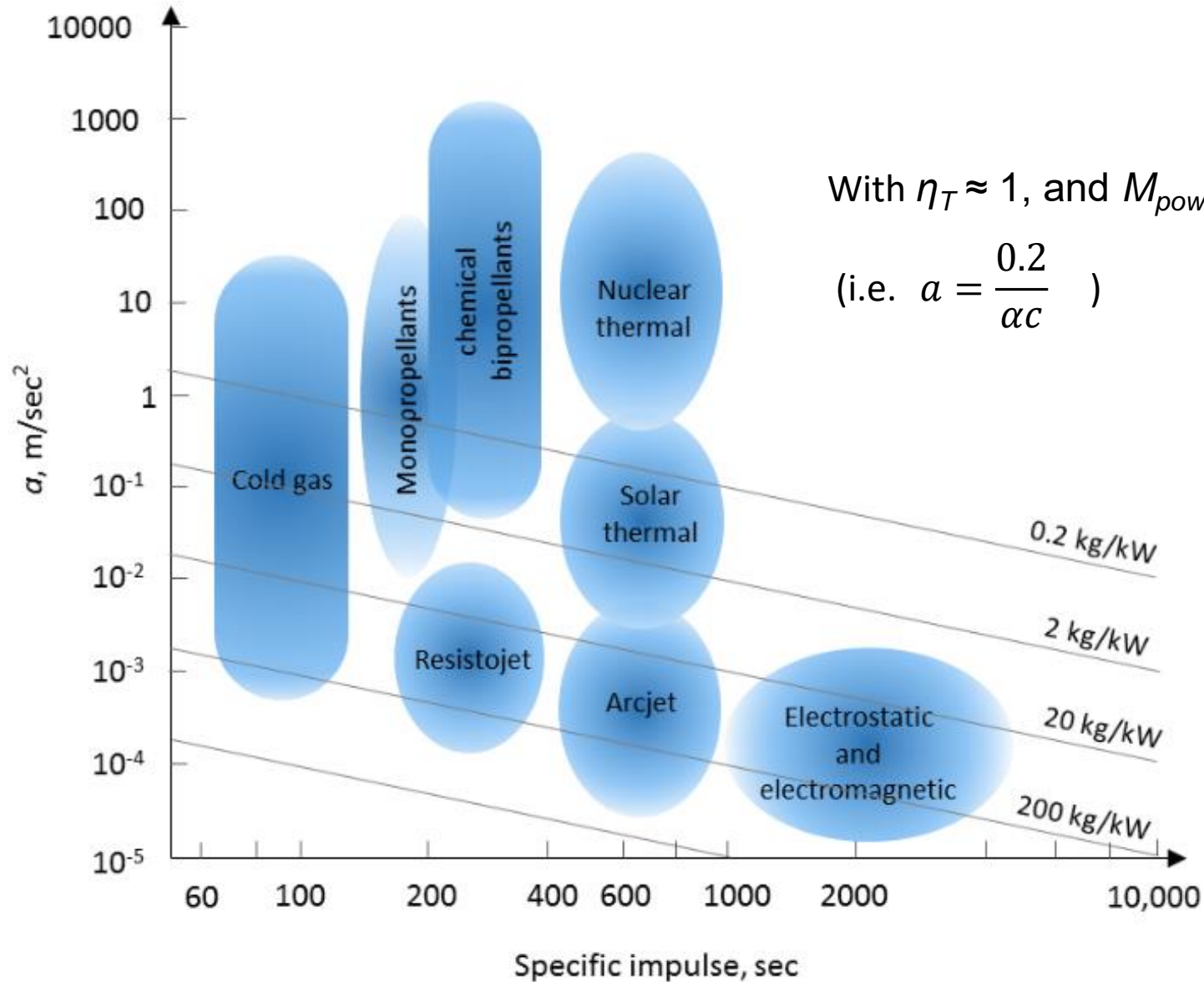
For a fixed specific power, a high effective exhaust speed means a low acceleration.

We can define a specific power plant mass as;

$$\alpha = \frac{M_{pow}}{P_{in}} \quad \text{———— (1.8)}$$

Where  $M_{pow}$  is the power plant mass of the thruster.

# Propulsion performance comparison



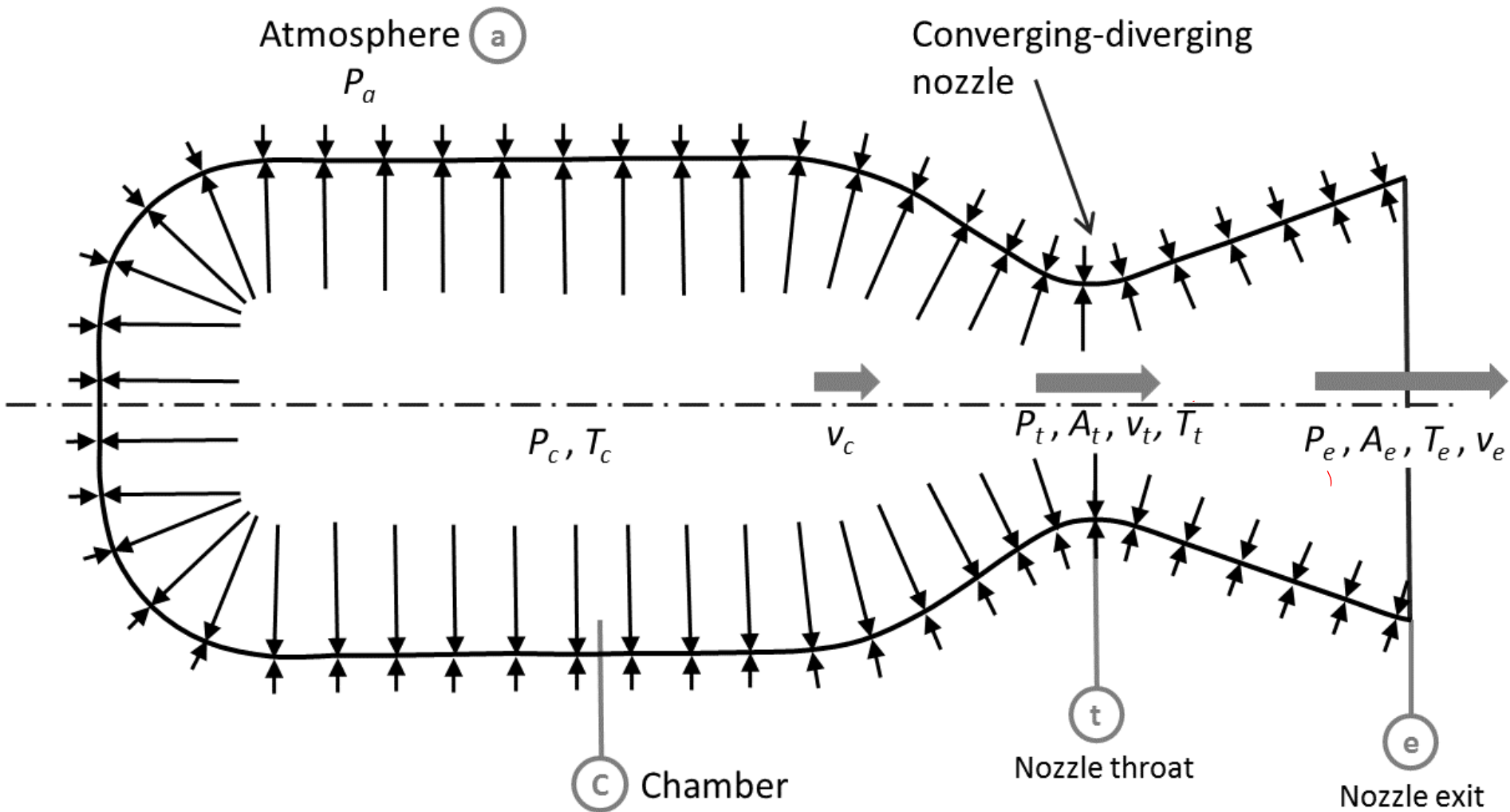
# Typical performance values

Type	Specific Impulse, seconds	Thrust range, N	Total efficiency, $\eta_T$	Power required, W	Typical propellants	
Chemical bipropellant	200 - 450	$\leq 10$ Mega N	0.8		Liquid or solid propellants	Thermal (chemical)
Chemical monopropellant	150 - 250	0.03 – 100	0.9		$N_2H_4$	
Thermal nuclear fission	500 – 860	$\leq 10$ Mega N	0.5		$H_2$	Thermal (Nuclear)
Resistojet - electrothermal	150 – 350	0.01 – 10	0.4		$N_2H_4$ , $NH_3$ , $H_2$	Electric
Ion Thruster - electrostatic	1500 – 8000	$10^{-5}$ – 0.5	0.65		Xe	
Hall Thruster	1500 – 2000	$10^{-5}$ – 2	0.55		Xe	

*After Sutton, Rocket Propulsion Elements, Table 2-1.*

# Definitions and fundamentals

## A Thermal Rocket



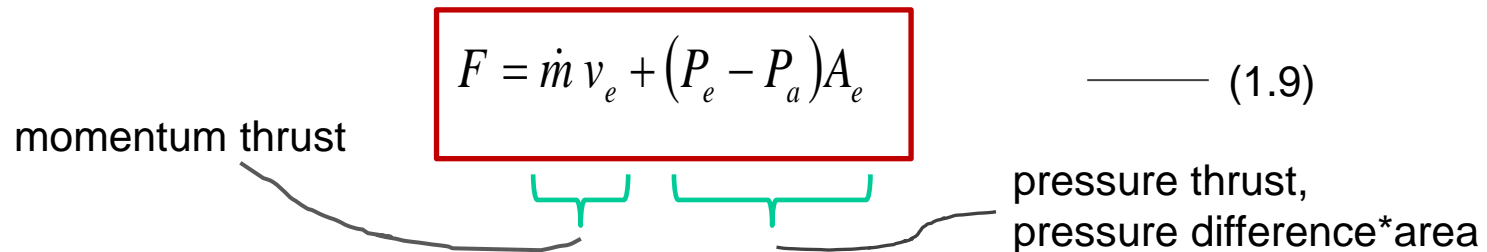
# Definitions and fundamentals

As a result of an imbalance in exhaust and atmospheric pressure, the force from a thermal rocket is composed of two parts;

$$F = \dot{m} v_e + (P_e - P_a) A_e \quad \text{--- (1.9)}$$

momentum thrust

pressure thrust,  
pressure difference\*area



Generally the second term accounts for 10 - 30% of the thrust.

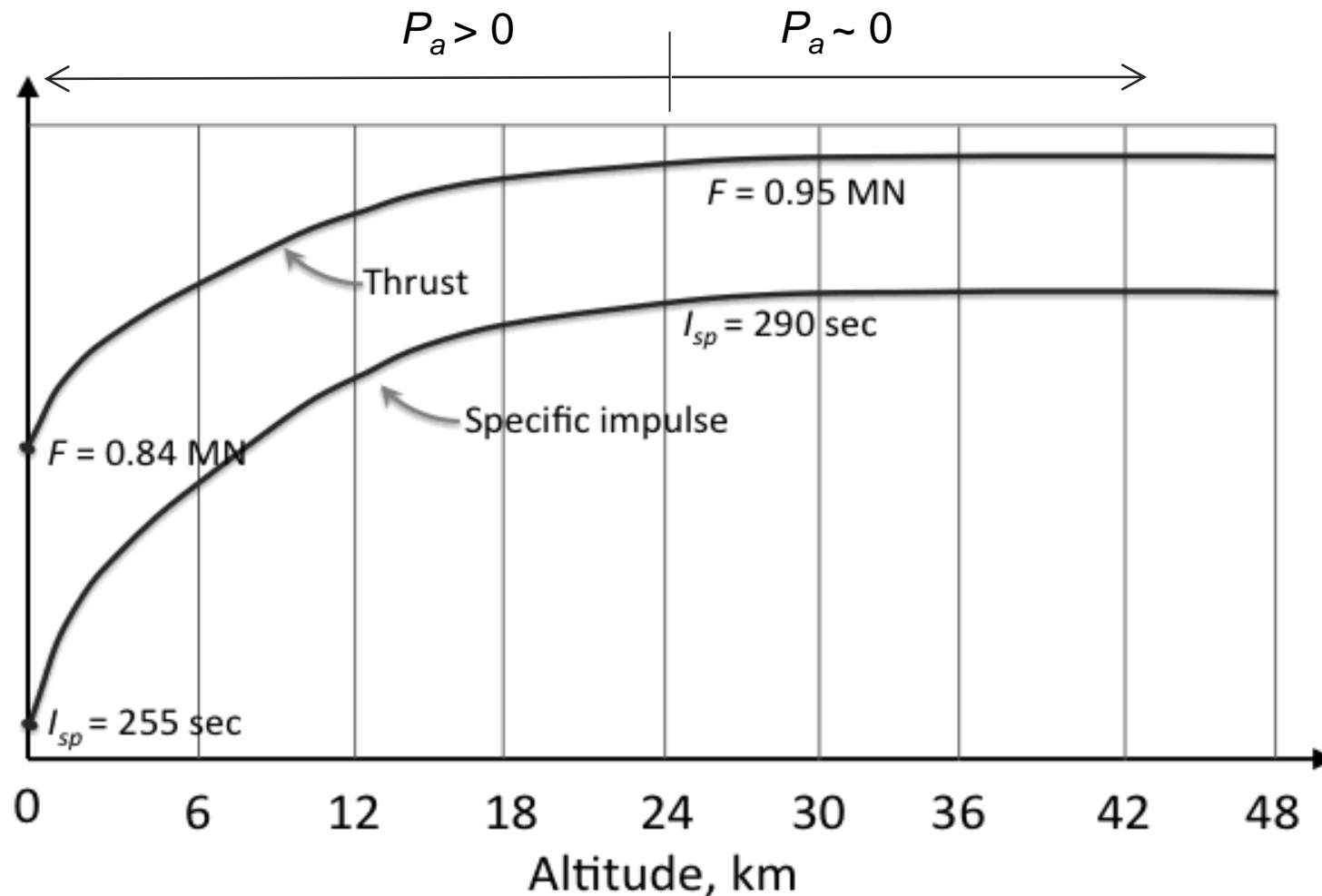
The effective exhaust velocity is;

$$c = v_e + (P_e - P_a) A_e / \dot{m} \quad \text{--- (1.10)}$$

Therefore the specific impulse is;

$$I_{sp} = \frac{1}{g_0} (v_e + (P_e - P_a) A_e / \dot{m}) \quad \text{--- (1.11)}$$

# Definitions and fundamentals



*Sutton, Fig. 2-2*

$$F = \dot{m} v_e + (P_e - P_a) A_e$$

$$I_{sp} = \frac{1}{g_0} \left( v_e + (P_e - P_a) A_e / \dot{m} \right)$$



A further parameter applied to thermal rockets is the characteristic velocity  $c^*$ ;

$$c^* = \frac{P_c A_t}{\dot{m}} \quad \text{———— (1.12)}$$

$c^*$  is used to compare different chemical propulsion systems as it is easily determined from measured data.

- An easy method to compare the efficiency of combustion.
  - Just need a pressure transducer, throat size, and a flow meter.
- Like  $I_{sp}$  but independent of nozzle characteristics.

- Typical values

Solid rocket  $\sim 1500$  m/s

liquid  $\text{H}_2/\text{O}_2$  bipropellant rocket  $\sim 2500$  m/s

## Example

*A solid rocket motor was tested at sea level. Calculate the propellant flow rate,  $c^*$ ,  $v_e$ , effective exhaust velocity, pressure thrust and specific impulse at sea level. Determine the pressure thrust, total thrust and specific impulse at 20,000m.*

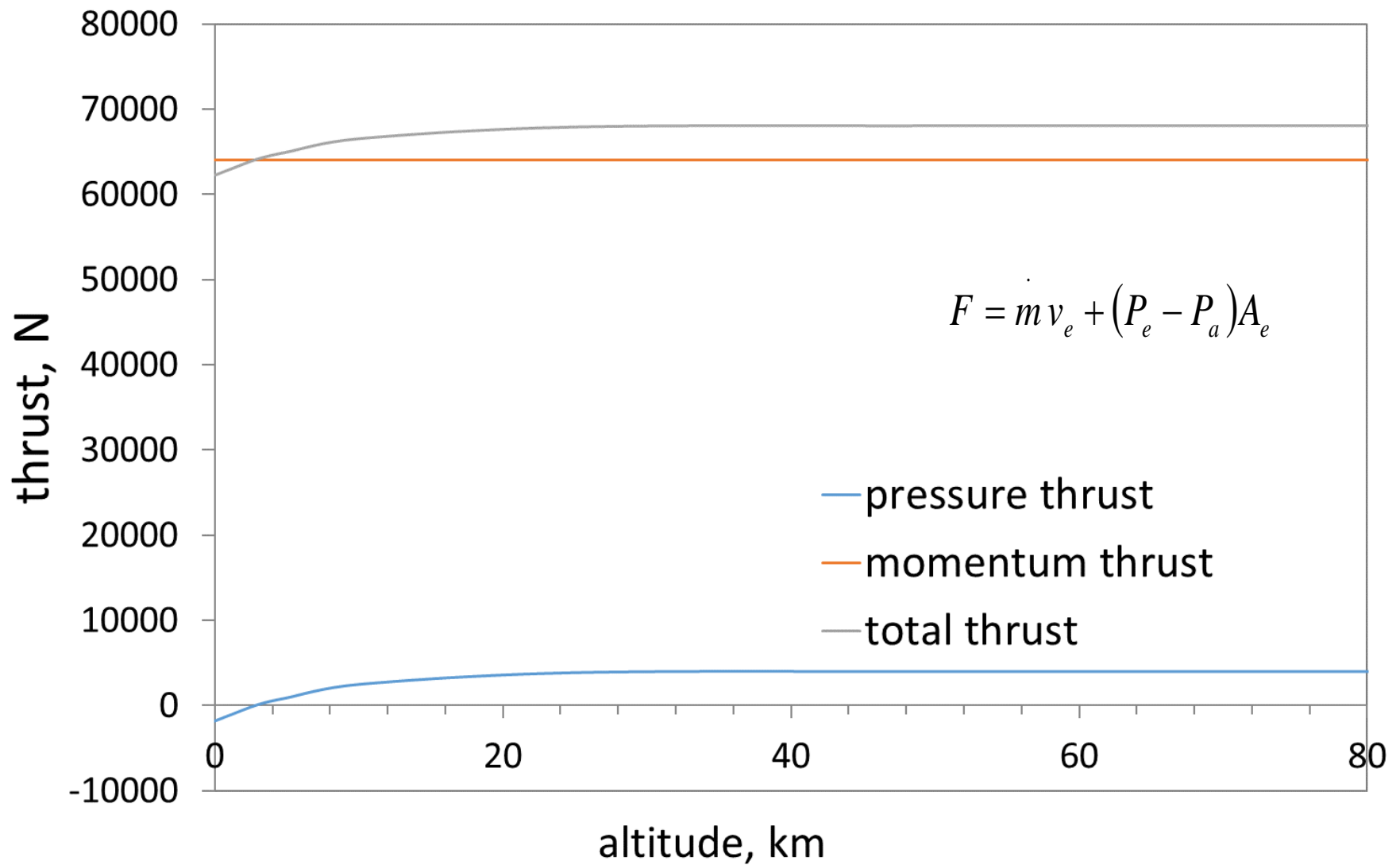
<i>Burn duration</i>	<i>40 seconds</i>	<i>Nozzle exit pressure</i>	<i>70.0 kPa</i>
<i>Initial mass</i>	<i>1200 kg</i>	<i>Nozzle throat diameter</i>	<i>8.55 cm</i>
<i>Mass after test</i>	<i>215 kg</i>	<i>Nozzle exit diameter</i>	<i>27.03 cm</i>
<i>Sea-level thrust</i>	<i>62,250 N</i>	<i>Atmos. Pressure at 20km</i>	<i>5.5 kPa</i>
<i>Chamber pressure 7.00 MPa</i>			

*[at sea level; propellant flow rate = 24.63 kg/s  
 $c^* = 1632$  m/s,  $v_e = 2600$  m/s  
 $c = 2528$  m/s, pressure thrust = -1779 N  
specific impulse = 258 seconds]*

*At 20 km; pressure thrust = 3701 N  
specific impulse = 280 seconds]*

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## Example calculations



# Tsiolkovsky Rocket Equation

The propellant mass fraction is;

$$\mu = \frac{\text{Propellant mass}}{\text{initial mass}} = \frac{M_p}{M_0} \quad \text{——— (1.13)}$$

For a well designed rocket,  $\mu = 0.8 - 0.85$ .

Take the force as;

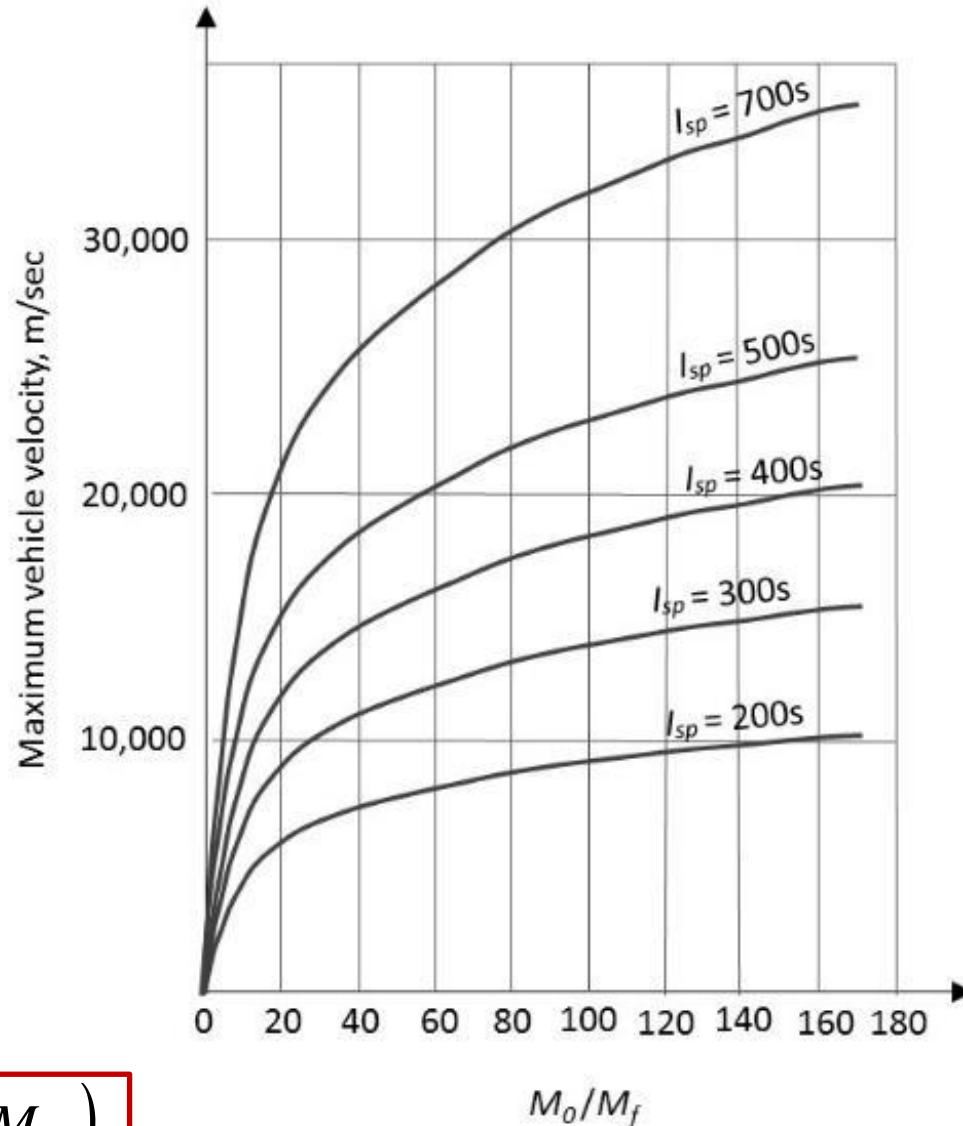
$$F = m du/dt = - dm/dt c$$

Which on integration leads to;

$$\Delta V = c \ln(M_0/M_f) \quad \text{——— (1.14)}$$

Taking  $M_f$  as the final mass.

# Tsiolkovsky Rocket Equation



After Sutton, *Rocket Propulsion Elements*, Figure 4-2.

$$\Delta V = I_{sp} g_0 \ln(M_0/M_f)$$

# Tsiolkovsky Rocket Equation

## Example

An upper-stage (IUS) is being designed to boost a new cable TV satellite from low-altitude parking orbit to geostationary orbit. The change in velocity  $\Delta V$  for the first burn of the Hohmann transfer is 3.34 km/s, and the effective exhaust velocity of biprop engine is 3000 m/s. If the mass of just the structure of the IUS, without propellant, is 100 kg and the satellite mass is 1000 kg, what mass of propellant should be loaded into the upper-stage?

Alternatively, a gridded ion thruster has been suggested, which has an exhaust velocity of 30,000m/s. Assuming the same mass of the upper stage, what mass of propellant is required?

Alternatively a 100 kg astronaut on EVA is trying to return to the space station, requiring a 1 km/s change in velocity manoeuvre. He is trying to do it by throwing 0.5 kg spanners at 20 m/s. How spanners will he need to get back to the space station?

*[for c value of 3000m/s; propellant mass = 2249 kg  
for c value of 30,000m/s; propellant mass = 129 kg  
for astronaut, would need  $1.0 \times 10^{24}$  spanners]*



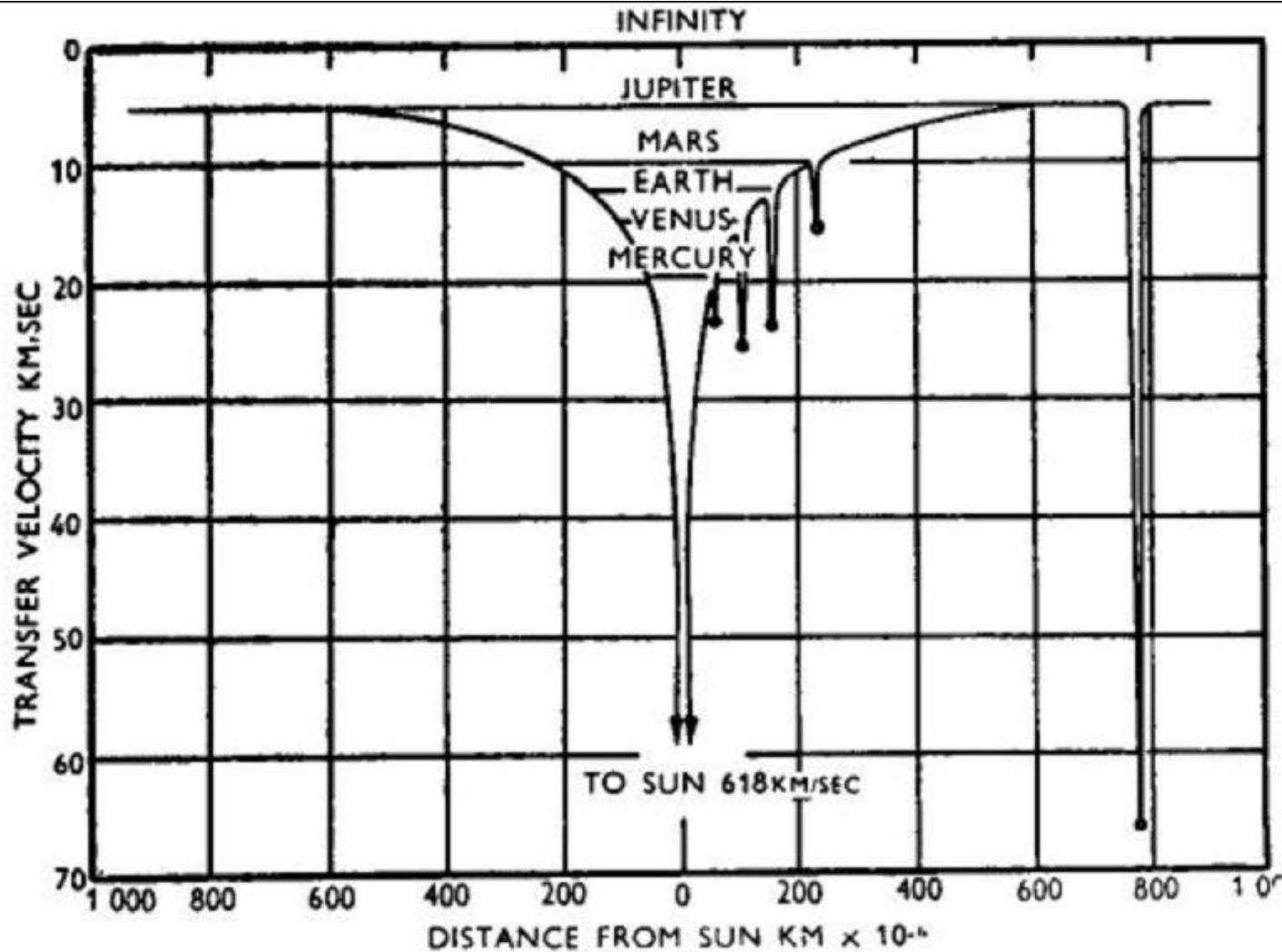
# Typical $\Delta V$ 's

Manoeuvre	Delta-V, km/s
Launch into LEO (including drag and gravity loss)	9.5
LEO to GEO, impulsive, no plane change	3.95
LEO to GEO, low thrust, no plane change	4.71
LEO to lunar orbit, impulsive	3.9
LEO to lunar orbit, low thrust	~8
LEO to Mars orbit, impulsive	5.7
Station keeping, GEO	50m/s per year
Station keeping, LEO	<25m/s per year

Source: *Tudelft.nl*

$$\Delta V = I_{sp} g_0 \ln(M_0 / M_f)$$

# Typical $\Delta V$ 's



**Fig. 9. Energy Diagram of the Solar System, in terms of Required Transfer Velocities.**

A typical launch vehicle rocket engine has an  $I_{sp} \sim 450\text{s}$ , whilst to reach LEO requires  $\sim 9.5\text{km/s } \Delta V$ .

Applying the ideal Rocket Equation gives a mass ratio of  $M_0/M_f = 8.6$ .

$$\eta = \frac{M_p}{M_0} = 1 - \frac{M_f}{M_0} = 88\%$$

$\sim 90\%$  of the mass of the rocket is fuel, with no more than 10% of the mass left over for the body, engines and payload.

- Very difficult to achieve

→ Solved by staging!

# Rocket staging



## Launch vehicle

## Parameters

## Payload to orbit

### Single Stage

$$M_{structure} = 250\text{kg}$$

$$M_{propellant} = 1500\text{kg}$$

$$\text{Engine } I_{sp} = 480\text{s}$$

$$\Delta V_{required} = 8000 \text{ m/s}$$

$$I_{sp} = 480 \text{ s}$$

$$M_{payload} = 84\text{kg}$$



### Two Stage

$$M_{structure} = 140\text{kg}$$

$$M_{propellant} = 750\text{kg}$$

$$\text{Engine } I_{sp} = 480\text{s}$$

$$\Delta V_{required} = 8000 \text{ m/s}$$

### Stage 2

$$I_{sp} = 480 \text{ s}$$

### Stage 1

$$I_{sp} = 480 \text{ s}$$

$$M_{payload} = 175\text{kg}$$

$$M_{structure} = 140\text{kg}$$

$$M_{propellant} = 750\text{kg}$$

$$\text{Engine } I_{sp} = 480\text{s}$$

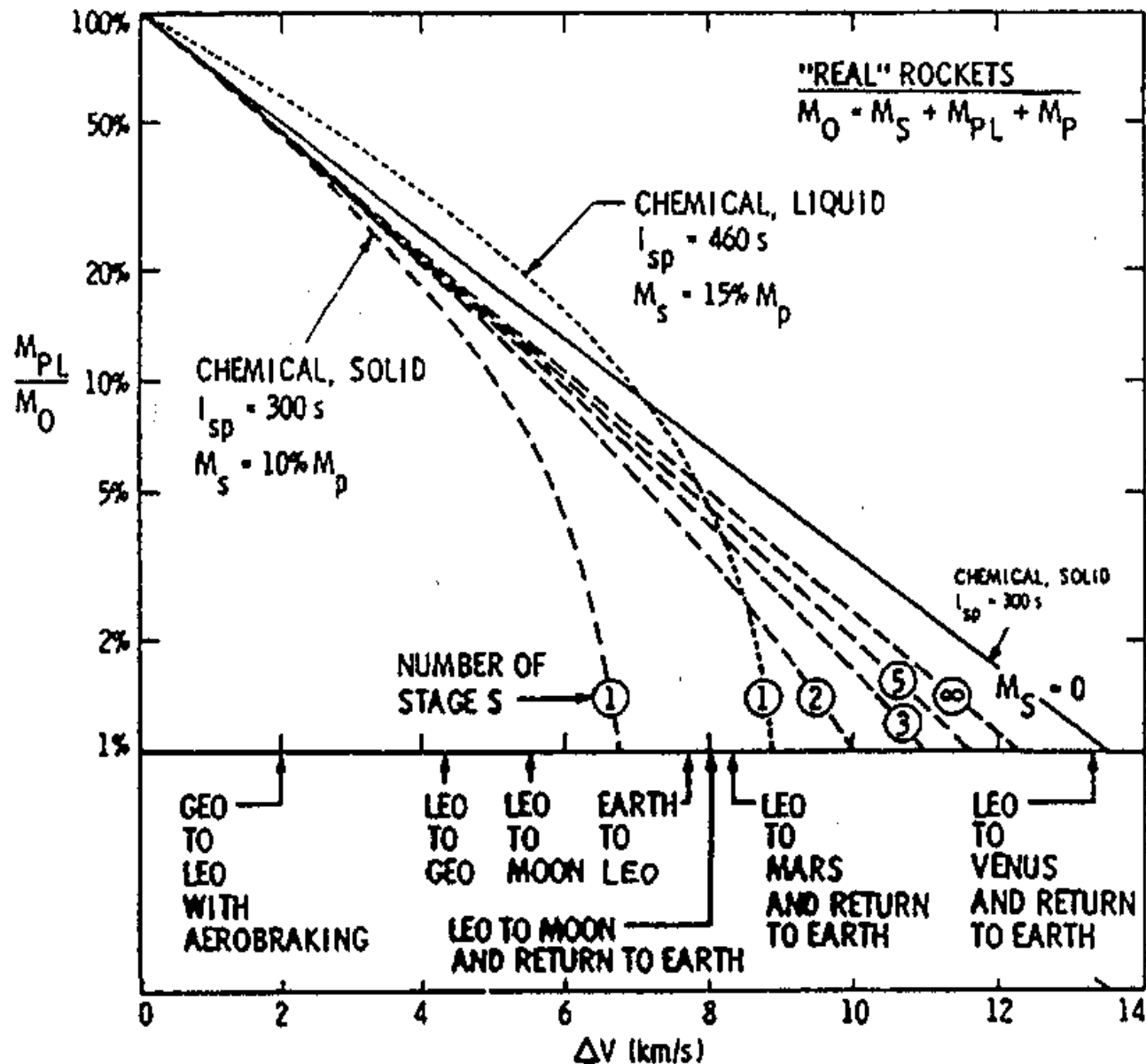
To obtain the total  $\Delta V$  for a multi-stage rocket, we must add the  $\Delta V$  for each stage.

- Noting that each stage has a different initial and final mass
- And that the  $I_{sp}$  might be different per stage

$$\Delta V_{total} = \Delta V_{stage\ 1} + \Delta V_{stage\ 2} + \dots + \Delta V_{stage\ n}$$

$$\begin{aligned} \Delta V_{total} = & I_{sp\ stage\ 1} g_0 \ln \left( \frac{M_{0\ stage\ 1}}{M_{f\ stage\ 1}} \right) \\ & + I_{sp\ stage\ 2} g_0 \ln \left( \frac{M_{0\ stage\ 2}}{M_{f\ stage\ 2}} \right) \\ & + \dots I_{sp\ stage\ n} g_0 \ln \left( \frac{M_{0\ stage\ n}}{M_{f\ stage\ n}} \right) \end{aligned} \quad \text{--- (1.15)}$$

# Rocket staging



# Rocket staging



Senat.fr

Ariane 5  
2 stages  
+ boosters



bbc.co.uk

Falcon 9  
2 stages



United Launch  
Alliance

Delta IV  
2 stages  
+ boosters



Univertoday.com

Soyuz 2-3  
2 stages  
+ boosters



PSLV  
4 stages  
+ boosters

# Launch vehicle dynamics

We can envisage a vehicle having the following forces acting upon it;

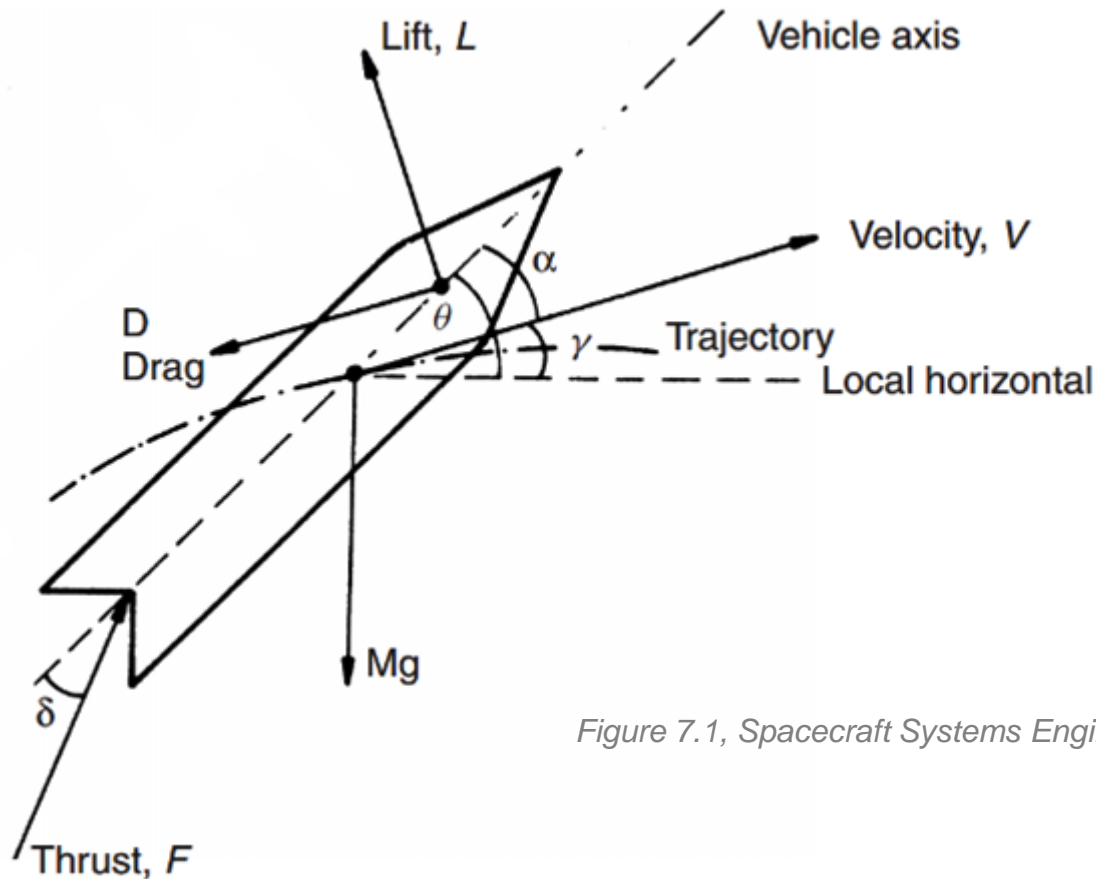


Figure 7.1, *Spacecraft Systems Engineering*, 4<sup>th</sup> Ed.



Parallel to the flight direction we have;

$$M \left( \frac{dV}{dt} \right) = F \cos(\alpha - \delta) - Mg \sin(\gamma) - D \quad \text{—— (1.16)}$$

Where drag is in the normal form;

$$D = \frac{1}{2} \rho V^2 A C_D$$

For small values of  $\alpha$  and  $\delta$  equation (1.16) becomes;

$$\frac{dV}{dt} = \frac{F}{M} - g \sin(\gamma) - \frac{D}{M}$$

After some rearrangement this leads to;

$$\frac{dV}{dt} = \frac{c \mu / t_p}{1 - \mu t / t_p} - g \sin(\gamma) - \frac{D / M_0}{1 - \mu t / t_p}$$

Which on integration leads to;

$$V = V_0 - \bar{c} \ln(1 - \mu) - \int_0^t g \sin(\gamma) dt - \int_0^t \frac{D / M_0}{1 - \mu t / t_p} dt$$

$$V = V_0 + \bar{c} \ln\left(\frac{M_0}{M_f}\right) - \bar{g} \sin(\gamma) t - \int_0^t \frac{D / M_0}{1 - \mu t / t_p} dt$$

Where  $\bar{c}$  and  $\bar{g}$  are time averaged.

Under the assumption of zero initial velocity this can be written as;

$$\Delta V = \Delta V_{ideal} - \Delta V_g - \Delta V_D \quad \text{—— (1.17)}$$

$$\Delta V_{ideal} = \bar{c} \ln \left( \frac{M_0}{M_f} \right),$$

$$\Delta V_g = \int_0^{t_b} g \sin(\gamma) dt,$$

$$\Delta V_D = \int_0^{t_b} \frac{D/M_0}{1 - \mu t/t_p} dt.$$

If drag is small the equation can be simplified to;

$$\Delta V = \bar{c} \ln \left( \frac{M_0}{M_f} \right) - \bar{g} t \sin(\gamma) \quad \text{—— (1.18)}$$

This equation can be integrated to give the horizontal and vertical components;

$$y = \bar{c} t \left[ 1 - \frac{\ln(M_0/M_f)}{(M_0/M_f - 1)} \right] \sin(\gamma) - \frac{1}{2} \bar{g} t^2 \quad \text{—— (1.19)}$$

$$x = \bar{c} t \left[ 1 - \frac{\ln(M_0/M_f)}{(M_0/M_f - 1)} \right] \cos(\gamma) \quad \text{—— (1.20)}$$

## Example

A simple single stage rocket has the following characteristics.

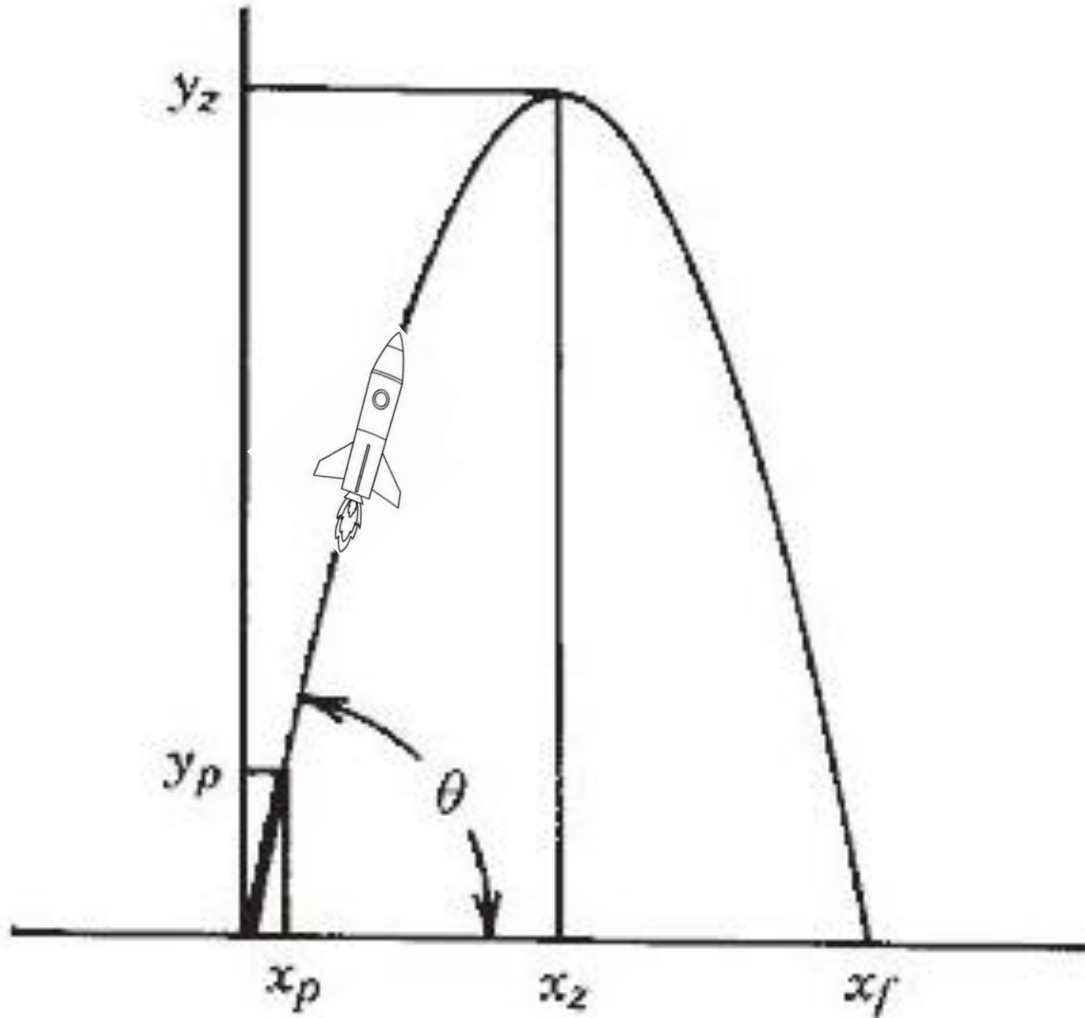
Launch mass	40 Kg	Propellant mass	4 Kg
Effective $I_{sp}$	120 seconds	Launch angle, $\theta$	80 degrees
Burn time (with constant thrust)	1.0 seconds		

Drag may be neglected as velocities are low. Assume gravity remains it's sea level value, and start and stop transients can be ignored.

Calculate the initial acceleration, the maximum trajectory height, and the angle at engine cut-off.

*[force = 4709 N, initial acceleration along path =  $117 \text{ ms}^{-2}$ , horizontal burn out velocity = 21.5m/s, vertical burn out velocity = 112.3 m/s, angle at engine cut-off = 79.14 degrees. Horizontal burn out distance= 10.6 m, vertical burn out distance = 55.1 m, maximum trajectory altitude = 698.3 m]*

## Example



*Sutton, Rocket Propulsion Elements, page 115.*

## Example calculations

# Launch vehicle flight profile

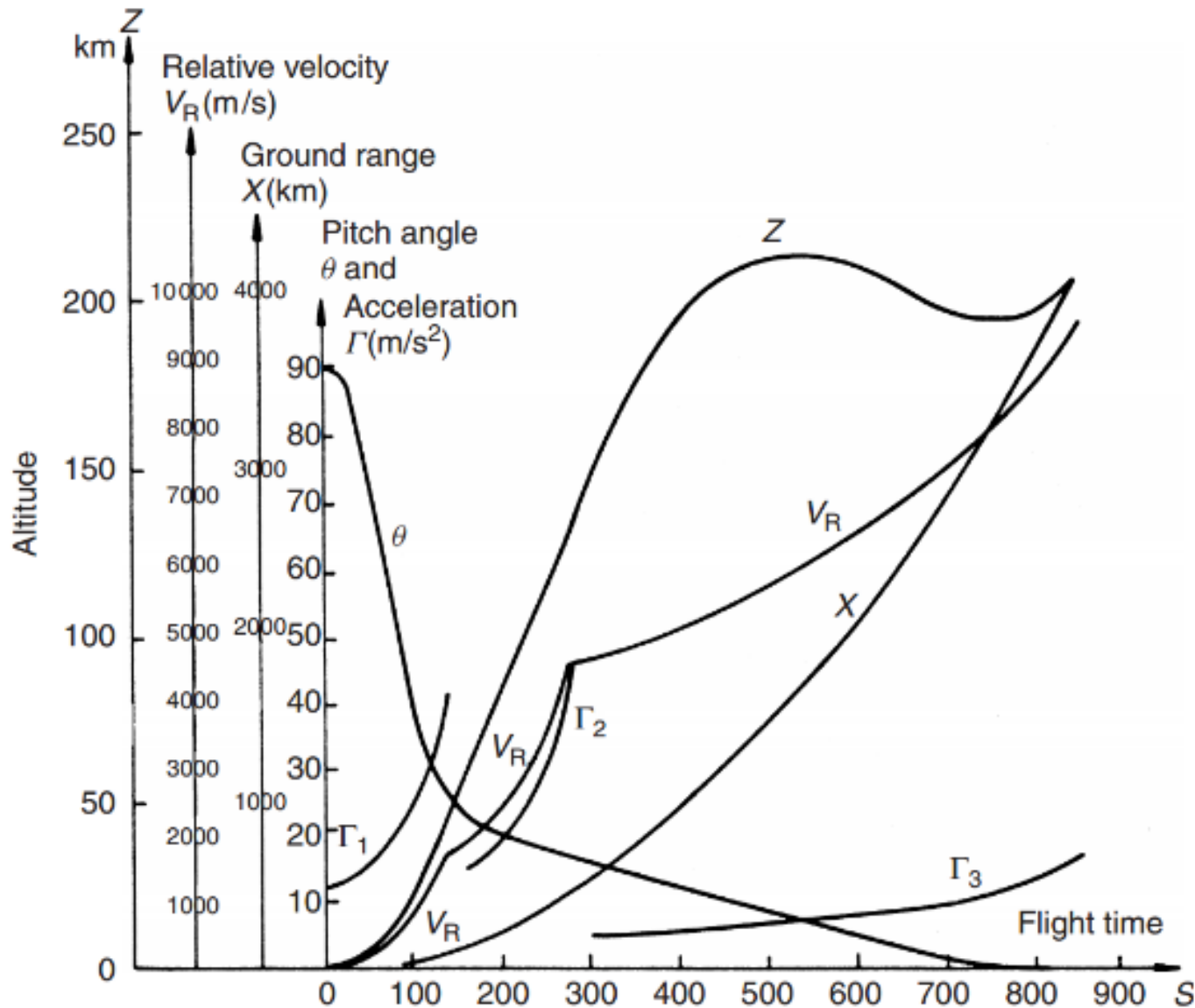


Figure 7.3, *Spacecraft Systems Engineering*, 4<sup>th</sup> Ed.