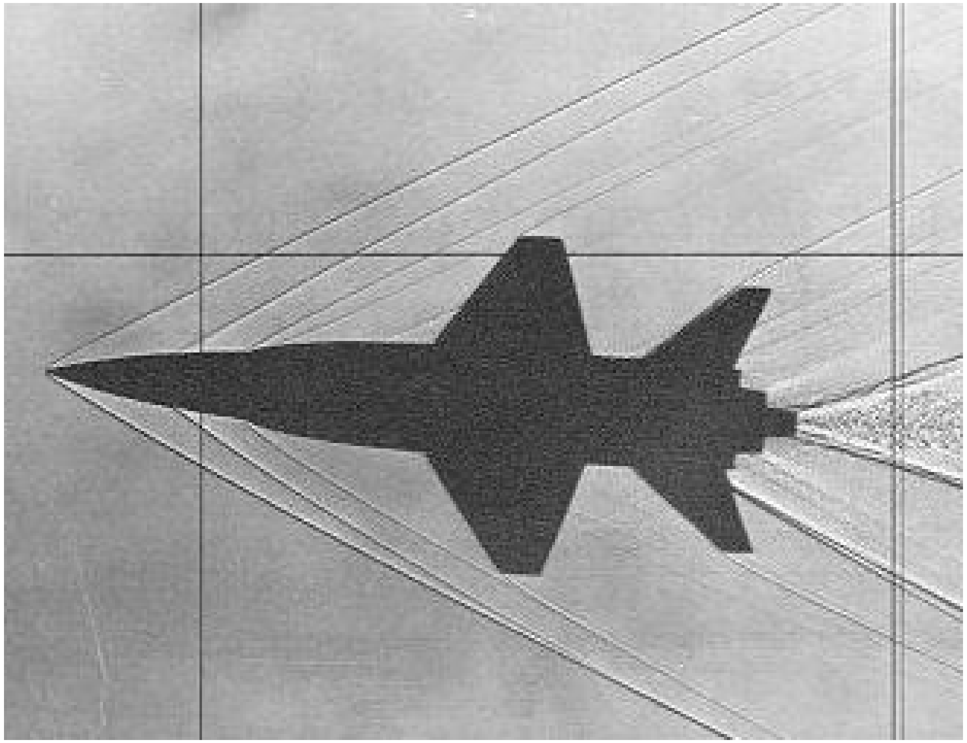
SESA3029 Aerothermodynamics

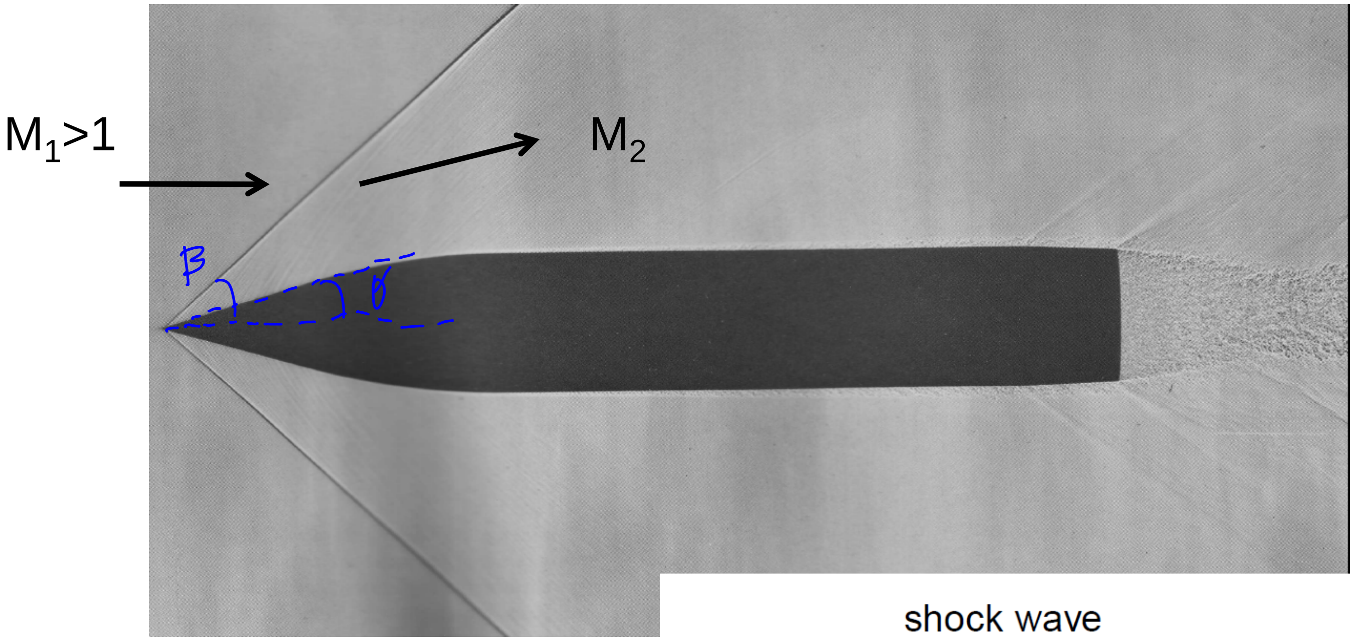


Lecture 2.1

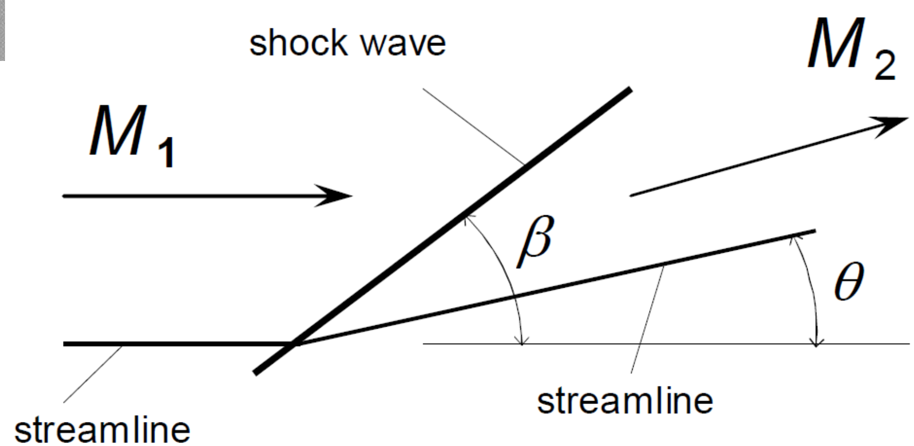
Oblique shock waves

X-15 M=3.5

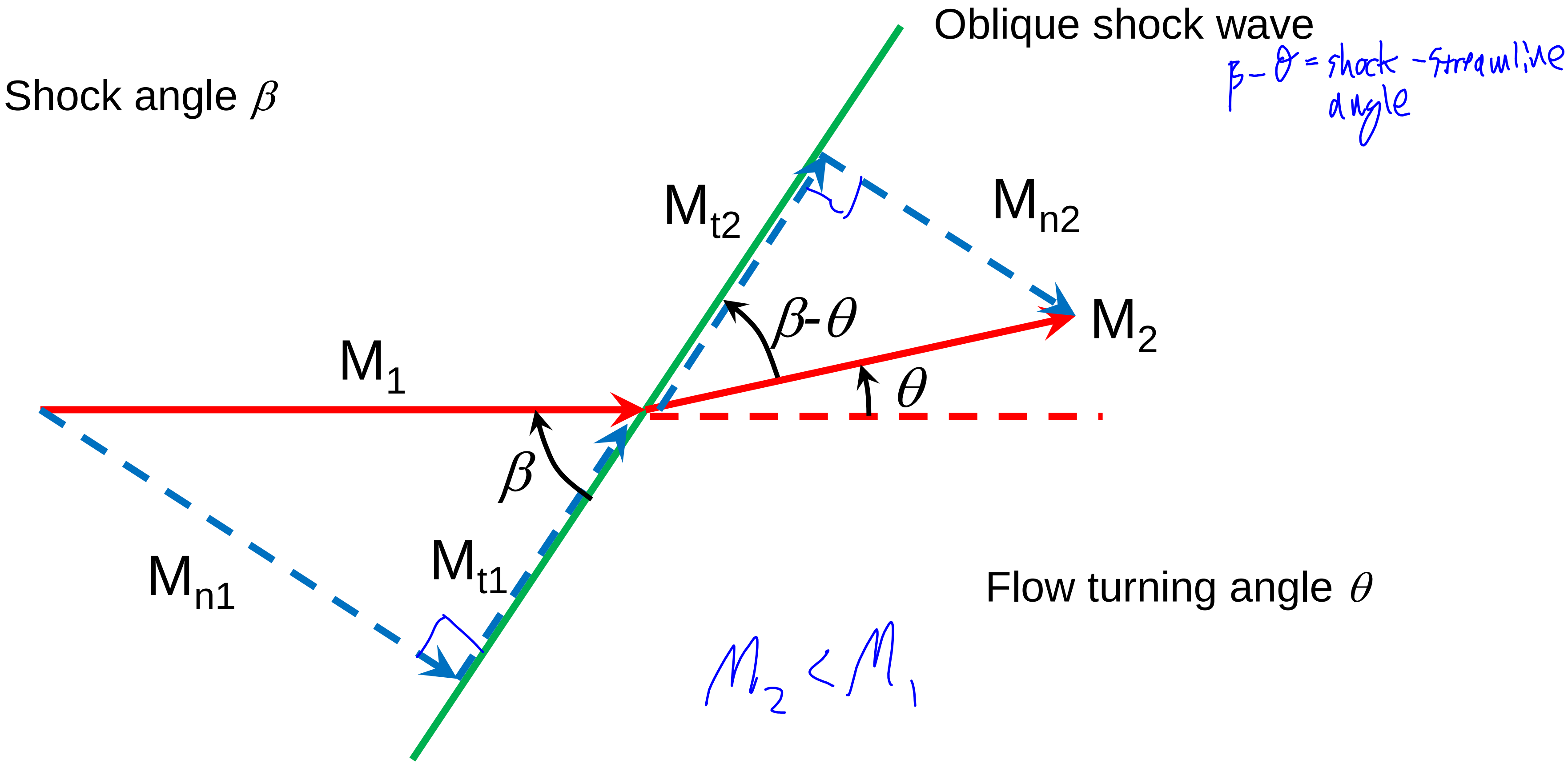
Oblique shocks are generated by turning a supersonic flow



Model problem:

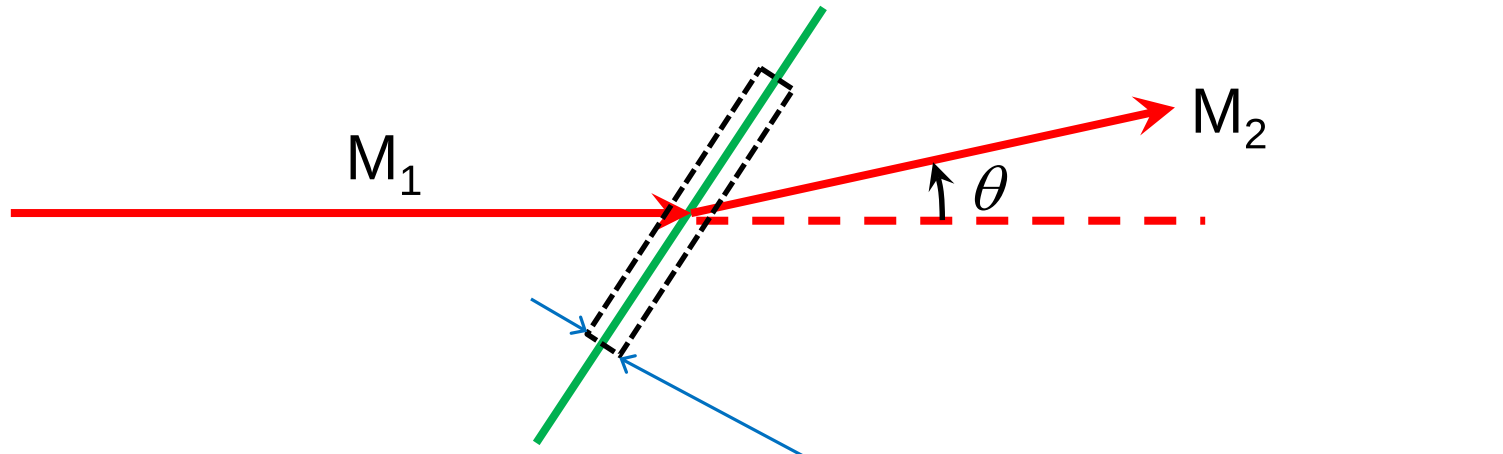


Analysis



$$M_{n1} = M_1 \sin\beta \qquad M_{n2} = M_2 \sin(\beta - \theta)$$

$$M_{t1} = M_1 \cos\beta \qquad M_{t2} = M_2 \cos(\beta - \theta)$$



Mass conservation (applied along normal to surface)

$$\dot{m} = \rho_1 V_{n1} A = \rho_2 V_{n2} A \qquad (1)$$

normal components!

Newton II for momentum normal to shock

$$\dot{m}\left(V_{n2} - V_{n1}\right) = \left(p_1 - p_2\right) A \qquad (2)$$

Newton II for momentum parallel to shock

$$\dot{m}\left(V_{t2} - V_{t1}\right) = 0 \qquad (3)$$

Energy:

$$T_0 = \text{const} \qquad (4)$$

$$\dot{m} = \rho_1 V_{n1} A = \rho_2 V_{n2} A \qquad (1)$$

$$\dot{m}\left(V_{t2} - V_{t1}\right) = 0 \qquad (3)$$

$$\dot{m}\left(V_{n2} - V_{n1}\right) = \left(p_1 - p_2\right) A \qquad (2)$$

$$T_0 = \text{const} \qquad (4)$$

basically same equations as normal shock case

Deductions:

The tangential flow is unaffected by the shock ($V_{t1}=V_{t2}$)

Equations (1), (2) and (4) are the same as for the normal shock derivation i.e. we can apply the normal shock jump relations based on M_{n1}, e.g.

$$\mathrm{M}_{n2}^2 = \frac{2 + (\gamma - 1)\mathrm{M}_{n1}^2}{2\gamma \mathrm{M}_{n1}^2 - (\gamma - 1)} \qquad \text{with} \quad \mathrm{M}_{n1} = \mathrm{M}_1 \sin\beta$$

and then

$$\mathrm{M}_2 = \frac{\mathrm{M}_{n2}}{\sin(\beta - \theta)}$$

divide them together

$$\mathrm{M}_{n1} = \mathrm{M}_1 \sin\beta$$
$$\mathrm{M}_{t1} = \mathrm{M}_1 \cos\beta$$

$$\mathrm{M}_{n2} = \mathrm{M}_2 \sin(\beta-\theta)$$
$$\mathrm{M}_{t2} = \mathrm{M}_2 \cos(\beta-\theta)$$

Same trig relations hold for velocity, so

$$V_{n1} = V_{t1} \tan\beta \qquad V_{n2} = V_{t2} \tan(\beta-\theta)$$

We know $V_{t1}=V_{t2}$

$$\frac{V_{n1}}{V_{n2}} = \frac{\tan\beta}{\tan(\beta-\theta)}$$

From mass conservation

$$\frac{\tan\beta}{\tan(\beta-\theta)} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2\sin^2\beta}{2+(\gamma-1)M_1^2\sin^2\beta}$$

can find relationship between β and θ

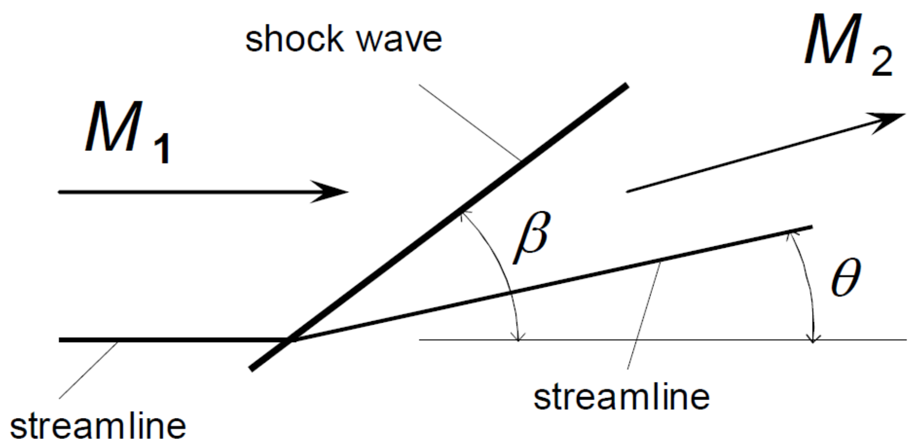
$$\frac{\tan\beta}{\tan(\beta-\theta)} = \frac{(\gamma+1)M_1^2\sin^2\beta}{2+(\gamma-1)M_1^2\sin^2\beta}$$

uses tan(A − B) relation.

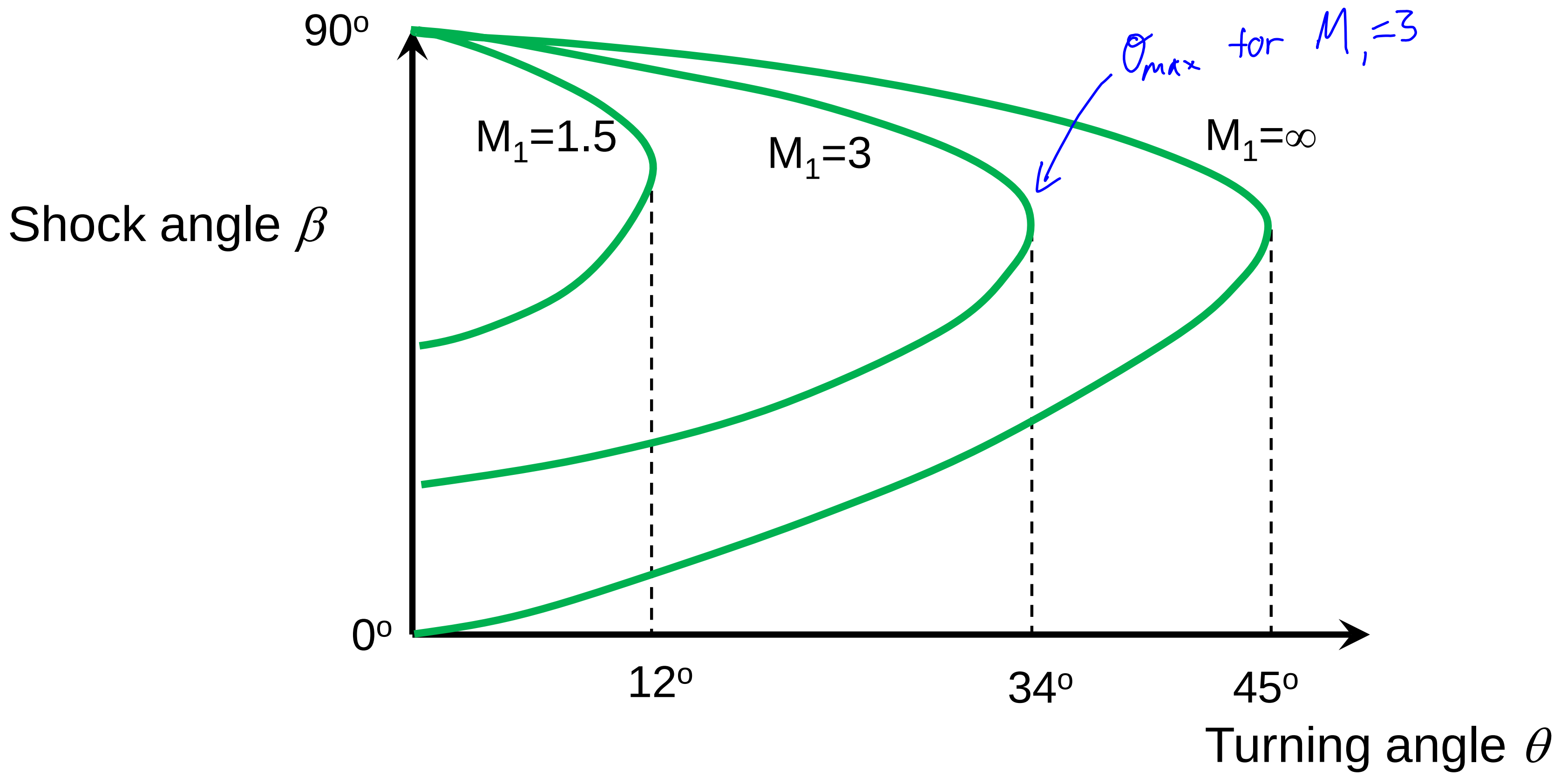
Expand tan(β-θ)and rearrange

$$\boxed{\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right]}$$

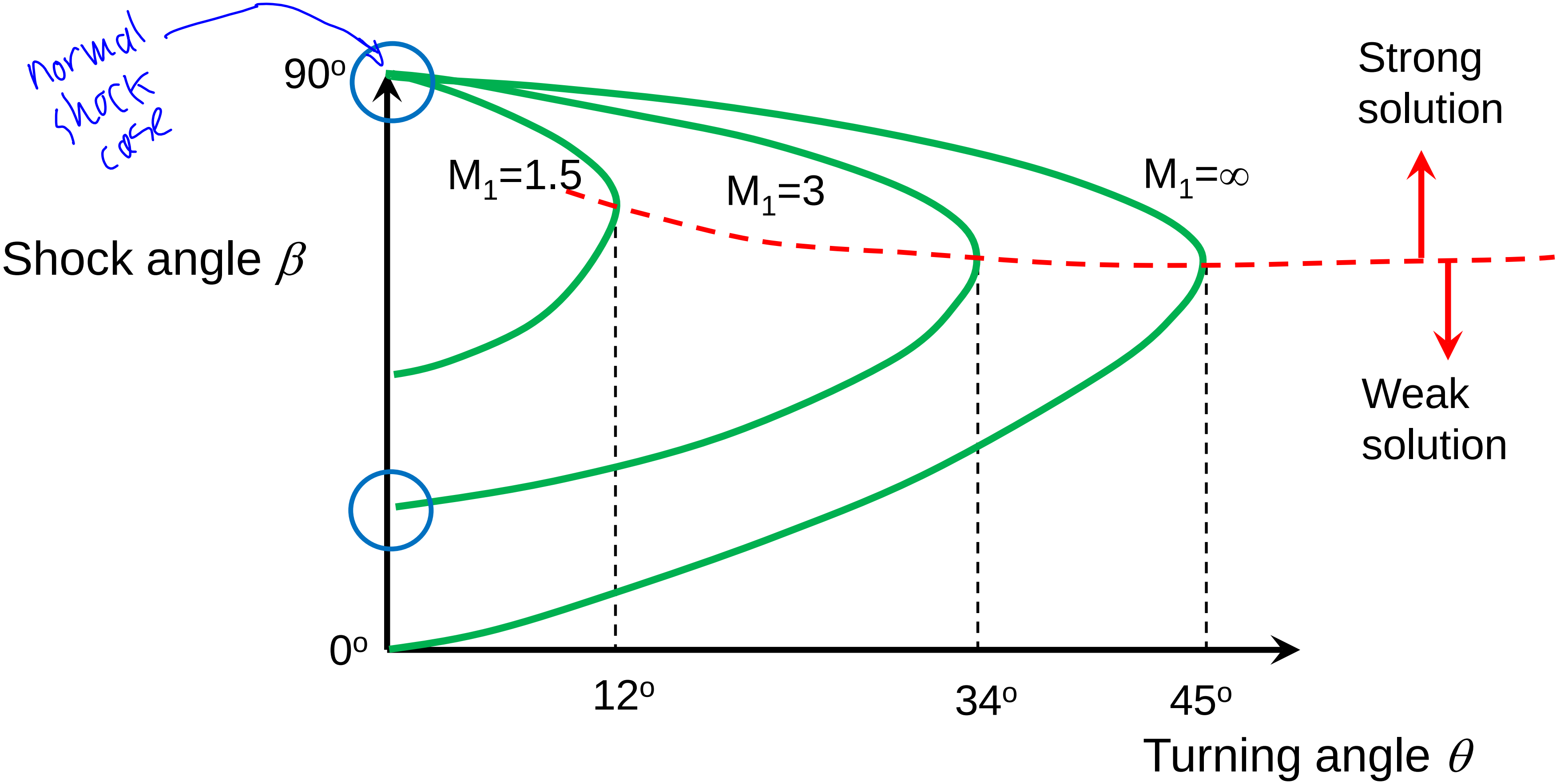
Oblique shock relation: gives turning angle θ in terms of M_1 and the shock angle β



Oblique shock chart (γ=1.4, simplified)



- For every M_1 there is a maximum turning angle θ_{max}

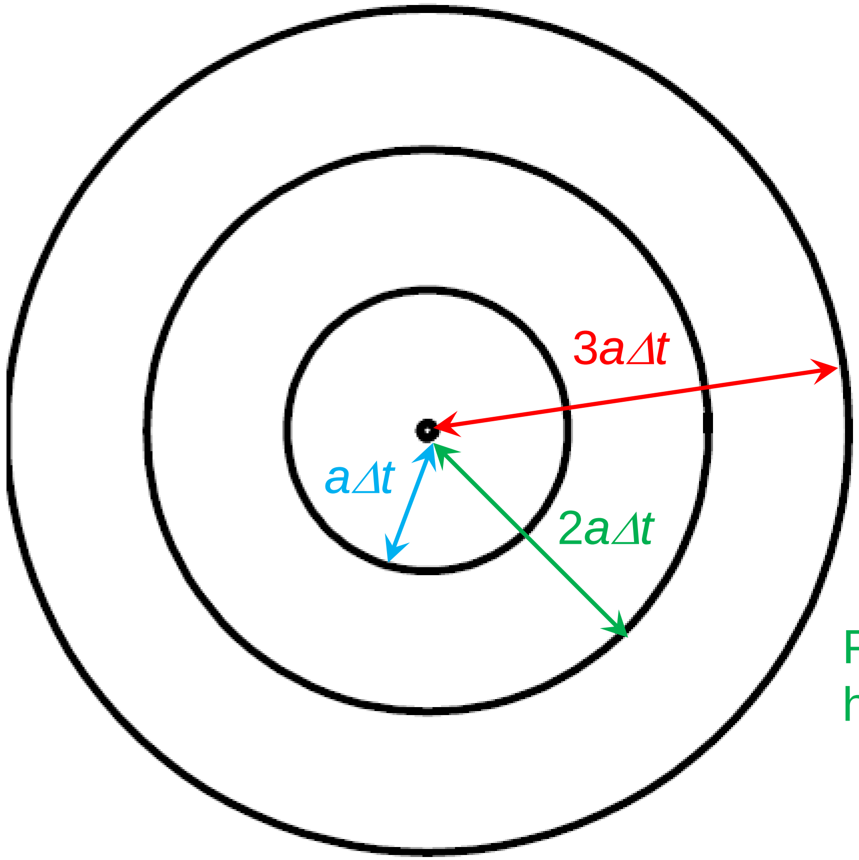


- For any $\theta < \theta_{max}$ there are two solutions
  - High β (strong shock solution), Lower β (weak shock solution)
- At $\theta = 0°$ there are two solutions (e.g. for M_1=3)
  - $\beta = \sin^{-1}(1/M)$: Mach wave
  - $\beta = 90°$: Normal shock

Stationary sound source (M=0)

Pulses of sound emitted every Δt

Picture is at time $t=3\Delta t$

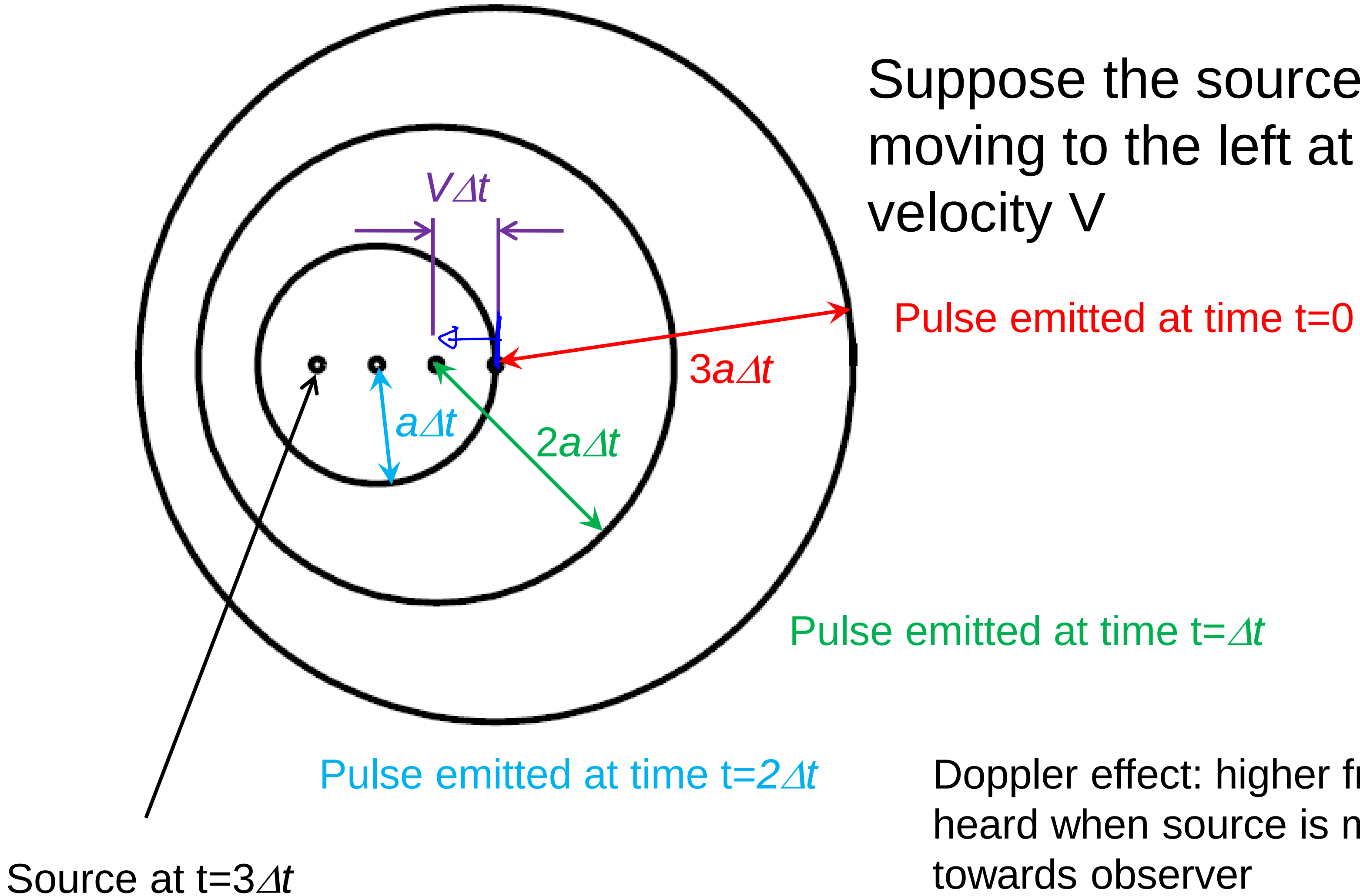


Pulse emitted at time t=0 has travelled $3a\Delta t$

Pulse emitted at time $t=\Delta t$ has travelled $2a\Delta t$

Pulse emitted at time t= $2\Delta t$ has now travelled $a\Delta t$

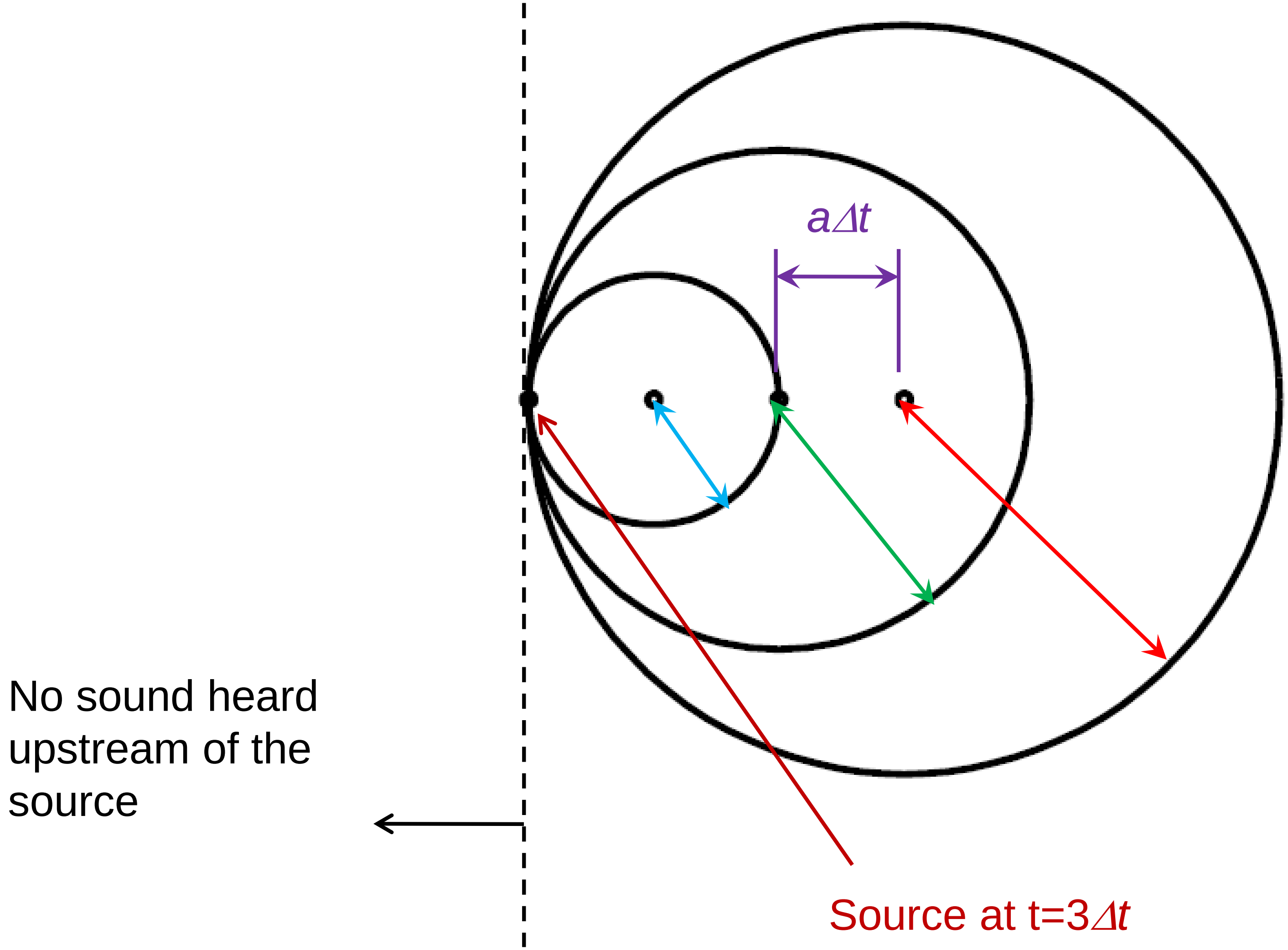
Subsonic sound source (M<1)



Suppose the source is moving to the left at velocity V

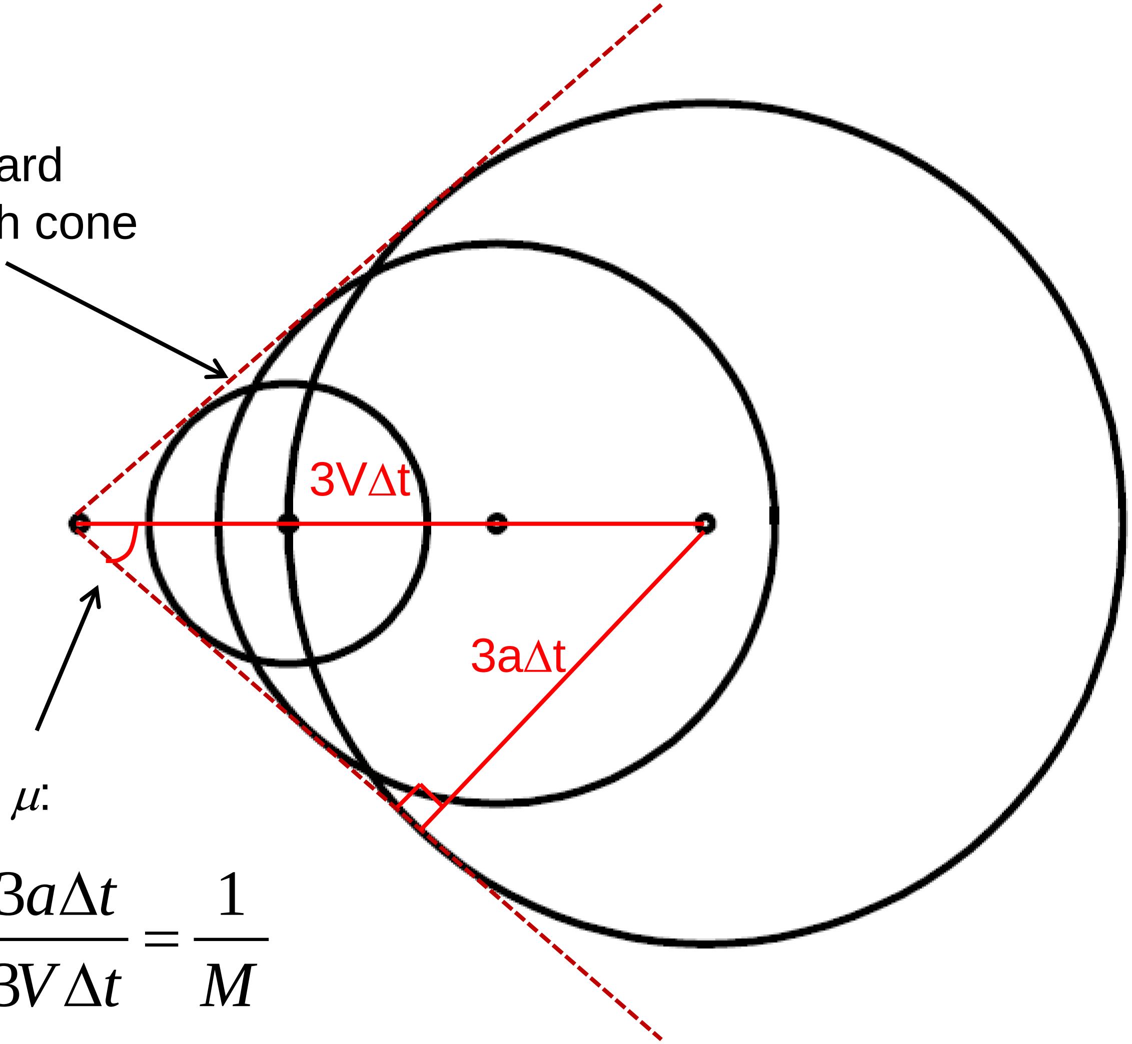
Doppler effect: higher frequency heard when source is moving towards observer

Sonic sound source (M=1, V=a)



Supersonic sound source (M>1)

Sound only heard
within the Mach cone



Mach angle μ:

$$\sin \mu = \frac{3a\Delta t}{3V\Delta t} = \frac{1}{M}$$