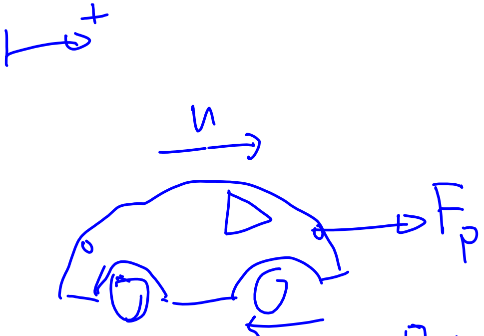
1. INTRODUCTION: THE CONTROL PROBLEM

- Control can be defined as the operation of maintaining the state of a given system at a required point.
- Control systems are required in many situations ranging from relatively simple applications such as maintaining the temperature in our homes or the speed of our cars through to more complex ones including full authority flight control, autonomous robots and intensive care facilities.

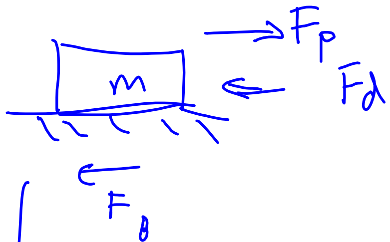
  - Reduce workload
  - Perform tasks people can't
  - Reduce the effect of disturbances
  - Reduce the effect of plant variations
  - Stabilise unstable systems
  - Improve performance and/or efficiency
  - Improve linearity
  - Reduce operating cost



$F_d = -Bu$

friction coefficient

simplification



modelling F_p

pedal

$F_p = k_c \theta$

pedal constant

$0 \le \theta \le \theta_{max}$

$$ma = \Sigma F = F_T + F_d + F_g$$

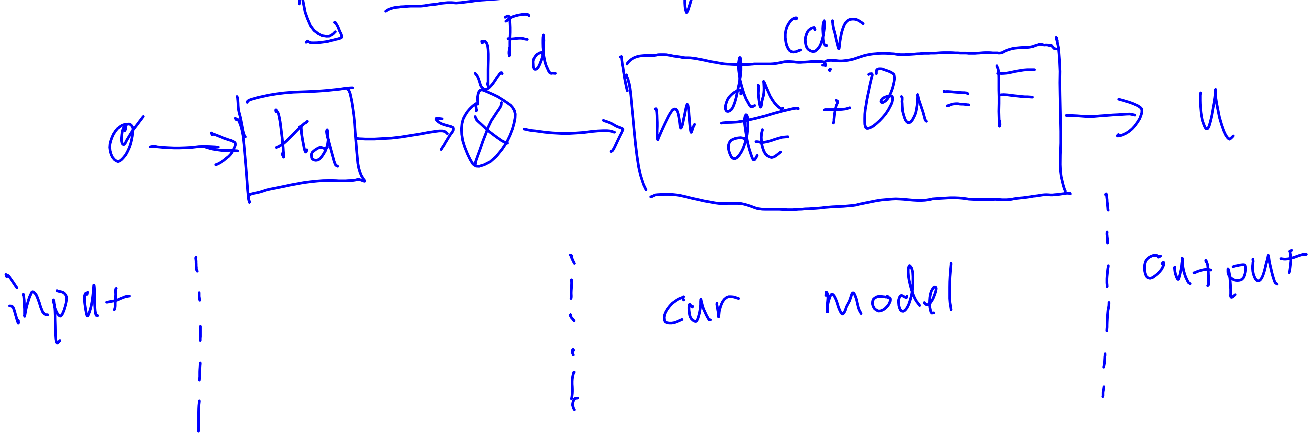
$$m\frac{du}{dt} = -Bu + F_p + F_d$$

$$= -Bu + k_c\theta + F_d$$

$\theta \rightarrow$ input variable (user input)

$u \rightarrow$ output variable (thing we want to control)

block diagram



Solving

① assume $F_d = 0$ (disturbance force)

$\therefore m\frac{du}{dt} + \beta u = k_e \theta$

② assume θ = constant
(simple function $\theta(t) = k$)

$$\underbrace{\frac{m}{\beta}}\ \underbrace{\frac{du}{dt}} + \underbrace{u} = \underbrace{\frac{k_\theta}{\beta}}\ \theta$$

units m/s^2

units are m/s

$\therefore$ units are m/s

$\therefore$ unit is s
(τ is time constant)

rewrite:

$$\tau\frac{du}{dt} + u = f_c\theta$$

$$\tau\left[c(t)\,e^{-\frac{t}{\tau}} + c(t)\,e^{-\frac{t}{\tau}}\left(\frac{-1}{\tau}\right)\right]$$
$$+ c(t)\,e^{-t/\tau} = -$$

$$\tau\frac{du}{dt} + u = 0$$

$$\frac{du}{dt} = -\frac{u}{\tau}$$

$$u = C(t)\ e^{-t/\tau}$$

Particular solution: $u =$

Examples (video):

- Saab Gripen pilot induced oscillation
  - https://www.youtube.com/watch?v=k6yVU_yYtEc
  - On 2 February 1989, the first prototype JAS 39-1 crashed on its sixth flight, when attempting to land in Linköping. The crash was the result of *pilot-induced oscillation*. Extremely gusty winds were also a contributing factor.
  - Issues with dynamic stability, effect of disturbances
- Rockets are inherently unstable
  - https://www.youtube.com/watch?v=gp_D8r-2hwk
  - On June 4, 1996 an unmanned Ariane 5 rocket launched by the European Space Agency exploded after its lift-off from Kourou, French Guiana. Reason was a software bug in the control system which regulated the flight.
  - Issues with processing of the response of the rocket by the control system

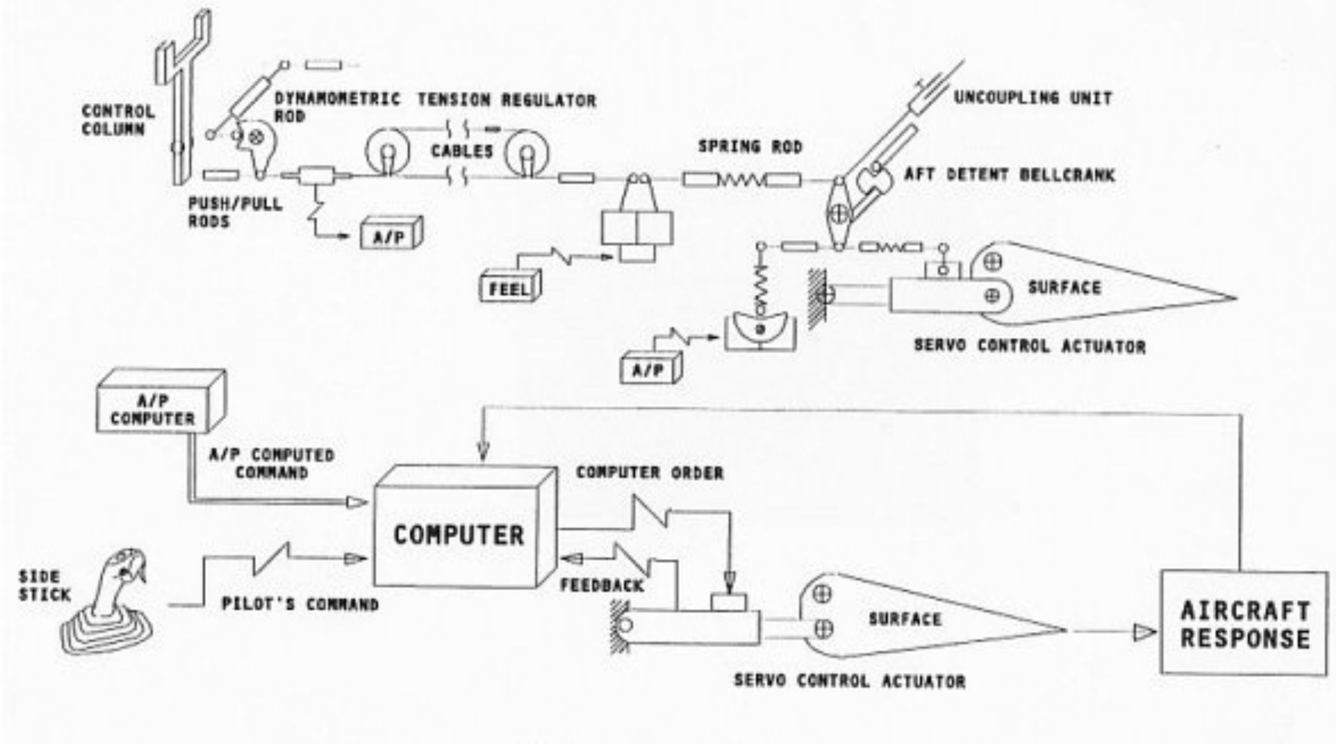
1.1 Definitions

System	An arrangement of components in such a manner as to form an entire unit.
Control System	A system where the components are connected so as to regulate itself or another system.
Input (excitation)	The stimulus or excitation applied to a control system.
Output (response)	The response from a system. Often the variable which is to be controlled.
Dynamic	A system where the output varies with time or is dependant on the past values of the input. A system which is not dynamic is said to be static.

1.2 Elements of aerospace control systems

- Consider the components of an elevator actuation system for a large modern aircraft

- An electrical signal derived from the pilot's controls stick (or autopilot) drives a servo valve that is mechanically coupled to the elevator and thus determines its position

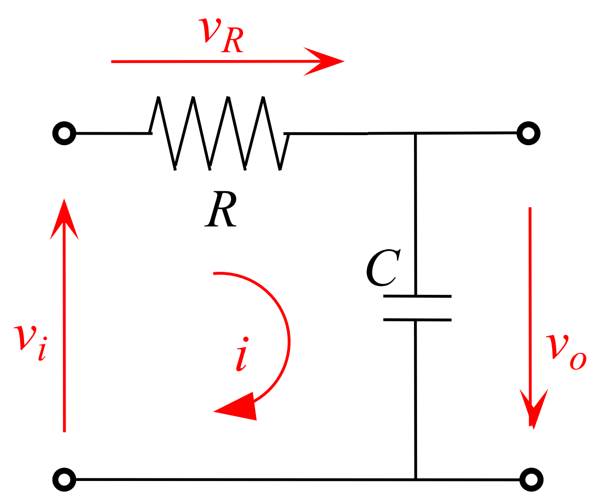


FLY BY WIRE PHILOSOPHY

https://aviation.stackexchange.com/questions/21690/what-is-fly-by-wire

1.2(a) System dynamics

System dynamics are a unified way of modelling and representing the dynamic behaviour of linear systems. The electrical signal from the pilot's stick will have some dynamics, for example:



- Applying Kirchhoff's laws:

$$v_i = v_0 + v_R$$

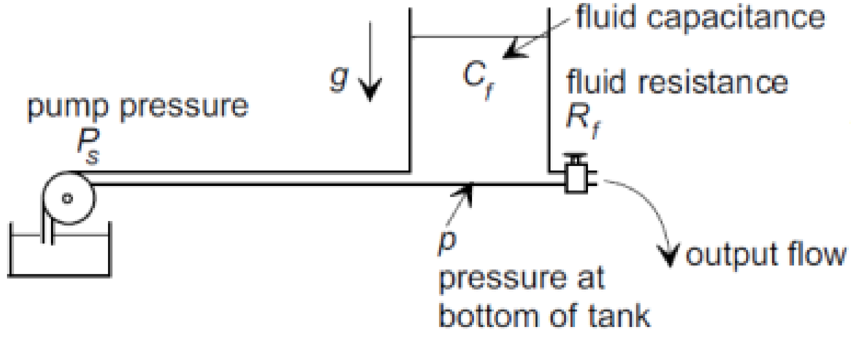
- But: $v_R = iR$

$$v_o = \frac{1}{C}\int i dt \quad \rightarrow \quad i = C\frac{dv_o}{dt}$$

- Combining the expressions we obtain → $CR\dfrac{dv_o}{dt} = -v_o + v_i$

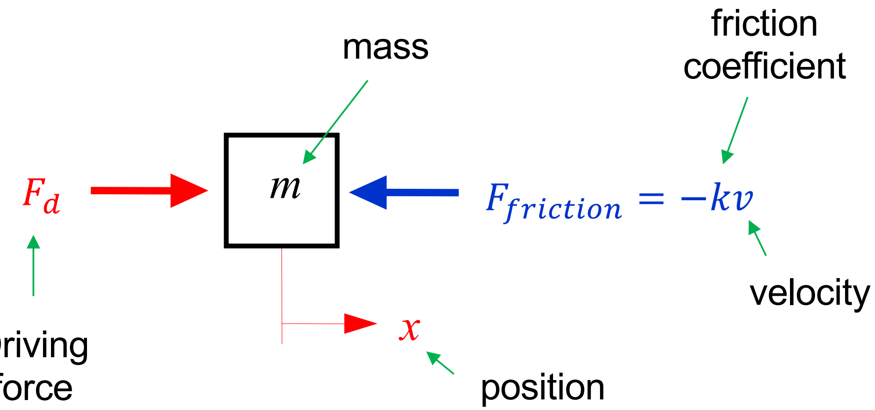
Similarly we could describe the hydraulic part of the elevator system in a similar manner:

$$R_f C_f \frac{dp}{dt} + p = P_s(t)$$



And also the mechanical components:

$$m\frac{d^2x}{dt^2} = m\frac{dv}{dt} = -kv + F_d(t)$$



For the different energy domains we can recognise that we have used the common form of the ODE to describe each subsystem

$$\begin{pmatrix} voltage \\ pressure \\ acceleration \end{pmatrix} \text{ and } \begin{pmatrix} current \\ volume\ flow\ rate \\ force \end{pmatrix} \text{ in the } \begin{pmatrix} electrical \\ fluidic \\ mechanical \end{pmatrix} \text{ domains}$$

And for each we have written a differential equation of the form:

$$\tau \frac{dy}{dt} + y = u(t)$$

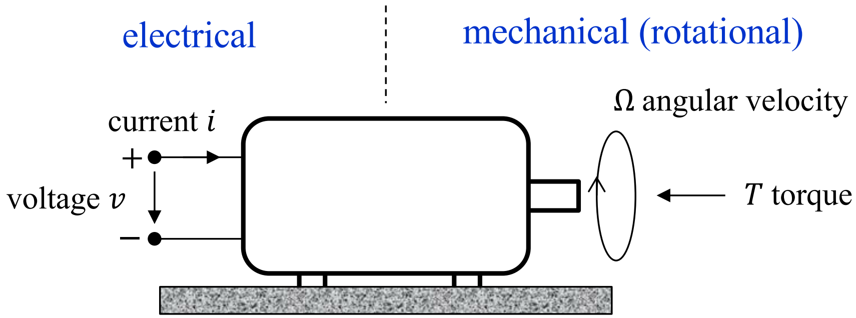
Where:

$u(t)$ is the system **input**

$y(t)$ is the system **output**, and

τ obtained by normalisation is a system **parameter** (the *time-constant*)

We will also look at **transducers**, devices that convert energy from one domain to another. For example an electric motor converts an electric current to rotation:

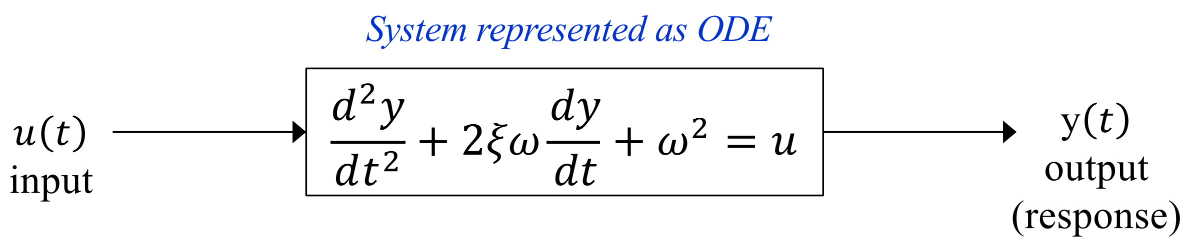


Transducers can be bi-directional. The power P can be transmitted in either direction, i.e. the motor can also act as a generator.

$$\begin{matrix} P = vi \\ \text{electrical} \end{matrix} \qquad \left\{ \begin{matrix} transduction \\ \Rightarrow motor \Rightarrow \\ \Leftarrow generator \Leftarrow \end{matrix} \right\} \qquad \begin{matrix} P = T\Omega \\ \text{rotational} \end{matrix}$$

1.2(b) Linear systems theory

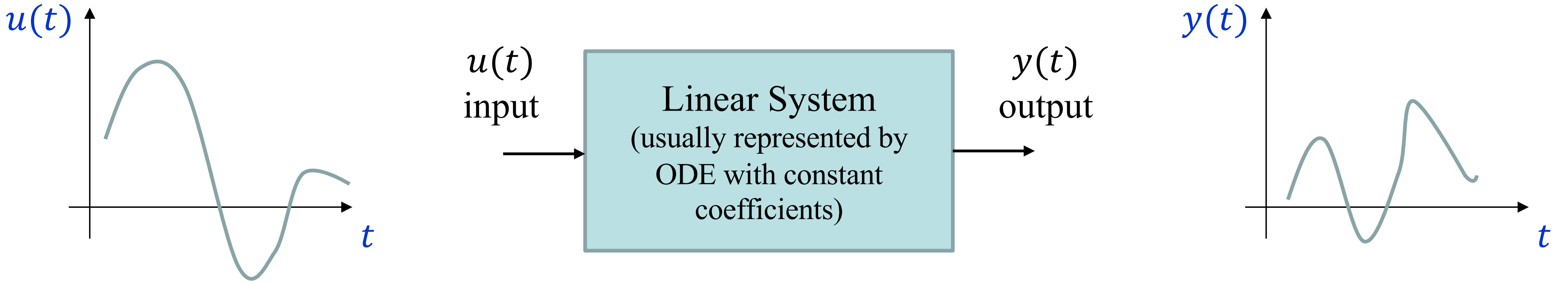
A generalised method for describing the dynamic response of systems of ODEs with constant coefficients independent of time.



Allows us to ask questions such as:

- What is the response of the system to a step input, a sinusoid or a short pulse?
- Is the system stable?
- How does the system response change with the value of a coefficient?

We will often use **block diagrams** to represent the input/output relationships of (linear) systems:



We can use these to predict the system response to various inputs if we have a model for the dynamics.

If we know the input/output behaviour we can estimate the dynamics – **system identification**

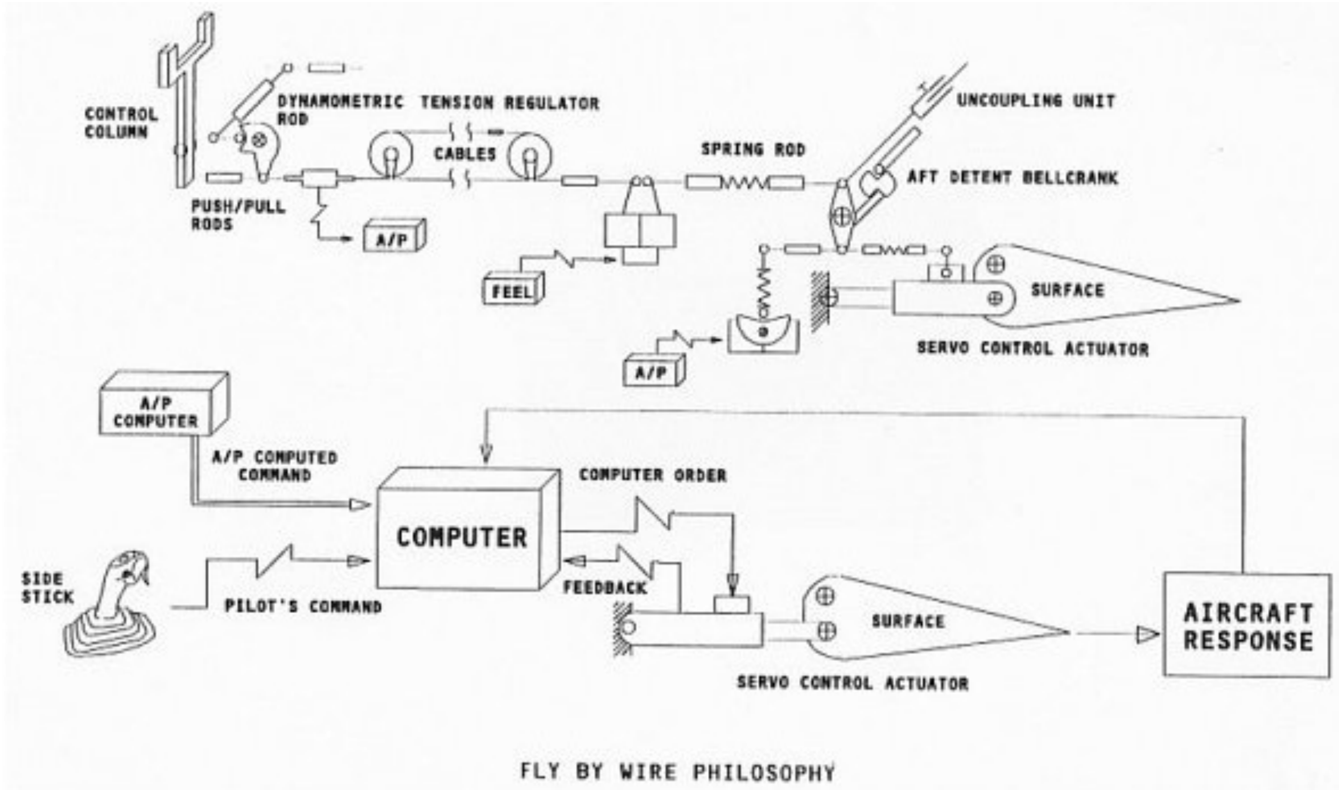
We can also determine how the system behaves at different frequencies – the **frequency response**

1.2(c) Feedback control theory

- Consider the components of an elevator actuation system for a large modern aircraft

- **Open-loop control systems** – systems without a feedback loop

- **Closed-loop control systems** – systems with a feedback loop



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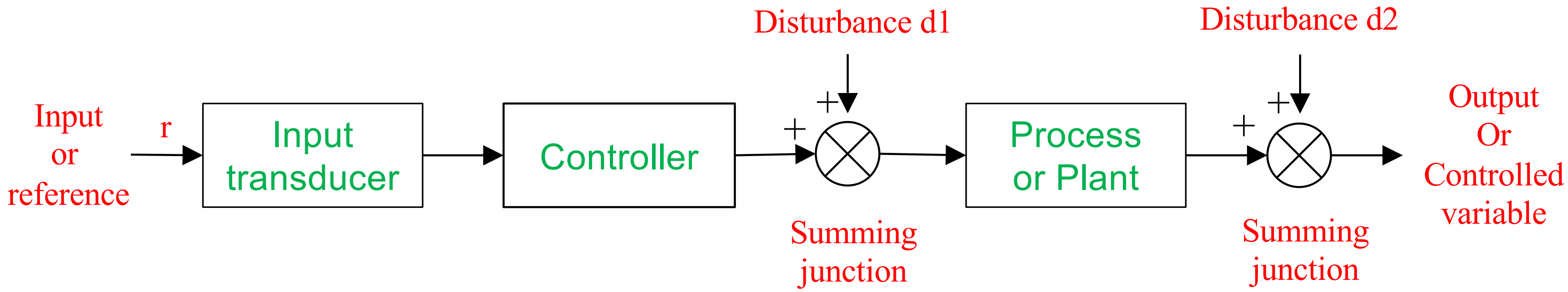
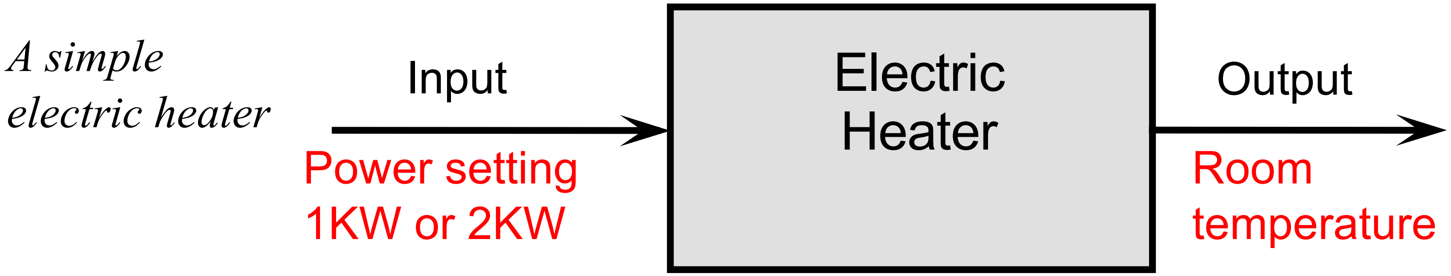
1.2(c) Feedback control theory: Open-loop systems



- Input transducer – converts the form of the input to that used by the controller
- Controller – drives a process or a plant
- Summing junctions – yield the algebraic sum of input signals and disturbances using associated signs
- Disturbances – noise or other forms of external excitations
- Plant can be for example, an air conditioning system. Output variable is temperature. Controller consists of fuel valves controlled by electronics.

Example:



This is a very simple control system.

The heater is turned on to warm up the room

The input is set to either off, 1kW or 2kW

The output is the room temperature.

This system can be improved by measuring room temperature and adjusting heater power depending on whether the room is hot or cold.

1.2(c) Feedback control theory: Open-loop systems

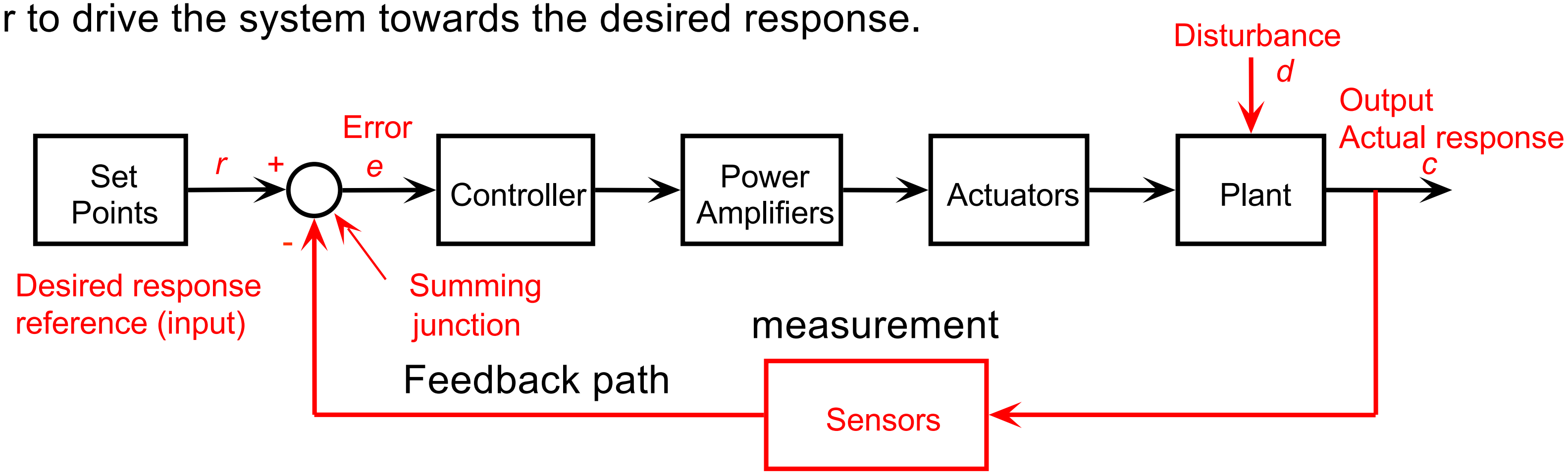
There is no direct connection between the parameter being “controlled” and the signal driving it – often the human operator.

Often the system behaviour is inherently unsatisfactory, e.g.:

- The system response is too slow
- The system is unstable
- External disturbance might unduly influence the system’s behaviour
- Components of the system might change their parameter values as they age and wear out affecting the response

1.2(c) Feedback control theory: Closed-loop systems

Control often involves measuring the system response (output) and comparing this with the desired behaviour to drive the system towards the desired response.



Generally, a controller (or compensator) is required to ensure that certain criteria or specifications are satisfied.

- disturbance rejection
- steady-state error
- transient response characteristics
- sensitivity to parameter changes in the plant

Open loop control systems:

- The input is chosen on the basis of experience of the system to give the desired output.
- Open loop control is simple, has low cost and is fairly reliable.
- They can be inaccurate as there is no correction for error.
- Can be very sensitive to changes in operation and/or calibration.

Closed loop control systems:

- A measurement signal is fed back to the controller that changes the system to ensure that pre-specified output requirements are met.
- An open loop system would be unsuitable when the operating conditions are not known *a priori*.
- Closed loop control systems can cope with disturbances.
- Closed loop control has greater complexity than an open loop and therefore costs more.
- They can cause instability (to be discussed in detail later).

1.3 Solution of practical control problems

1. choose sensors to measure the plant output,
2. choose actuators to drive the plant,
3. develop the plant, actuator and sensor equations (modelling),
4. design a controller based on the models and the control criteria,
5. evaluate the design analytically through simulation,
6. test the controller on the physical system, and
7. if the physical tests are unsatisfactory, iterate the design steps.

- **The system model could**:
    - be of very high order
    - suffer from numerical instabilities
    - or be described with nonlinear, time varying or partial differential equations.
- **Linear, time-invariant differential equation**:

$$\frac{d^n c(t)}{dt^n} + a_{n-1}\frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_1\frac{dc(t)}{dt} + a_0 c(t) = b_m\frac{d^m r(t)}{dt^m} + \cdots + b_1\frac{dr(t)}{dt} + b_0 r(t) \qquad (1.1)$$

    - Describes the relationship between the output, $c(t)$, to the input, $r(t)$, through the system parameters, a_i and b_j.
- Simplifying assumptions are used to reduce the number of terms in (1.1).
- Model needs to encompass the dynamics at the operating point and conditions of interest.
- A balance needs to be struck between complexity and accuracy that will achieve acceptable control.

1.4 Summary

- The control problem has been introduced and the ideas of open and closed loop control considered.
- The range of criteria that a control system must meet is wide and often the choice of control is a compromise over the design criteria used.
- Mathematical models can describe the essential dynamics of the system we wish to control and these can be represented by block diagrams
- The response of system can vary widely depending on the form of controller used and its internal parameters, the control system can even make the plant unstable.

1.5 Problems

1. Describe two open loop control systems which could be found in the kitchen.
2. Describe two closed loop control systems that would be found in an aircraft.
3. Which of the following are open loop, closed loop and why?
    i. An electrical switch
    ii. A thermostatically controlled heater
    iii. The human perspiration system
    iv. A person driving a car.
4. Devise a closed loop automatic toaster and illustrate it in block diagram form.

Part II – A solved example

(solved in live lecture)

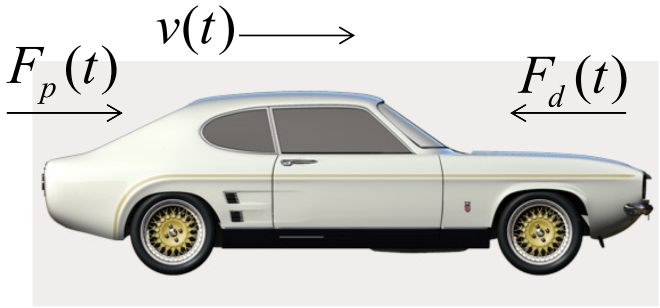
1.6 Example: car cruise control

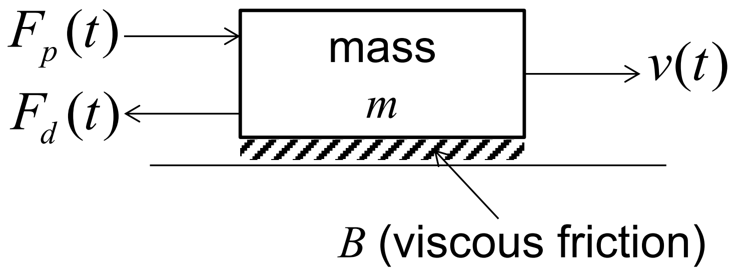
Objective: to maintain the speed of the vehicle to a pre-set value in the presence of external disturbances such as wind gusts and gravitational forces on an incline.

Additional objective: to improve the dynamic response of the vehicle when the driver “puts their foot down”.

What other objectives might we have?

(a) Formulate a simplified model of the car dynamics.





Assume two external forces:
$F_p(t)$ – propulsive force from engine
$F_d(t)$ – environment disturbance force

Car modelled as simple lumped mass, m, sliding on viscous friction element

Simplified to: $F_B = -Bv$

The simple force balance is then: $m\frac{dv}{dt} + Bv =$

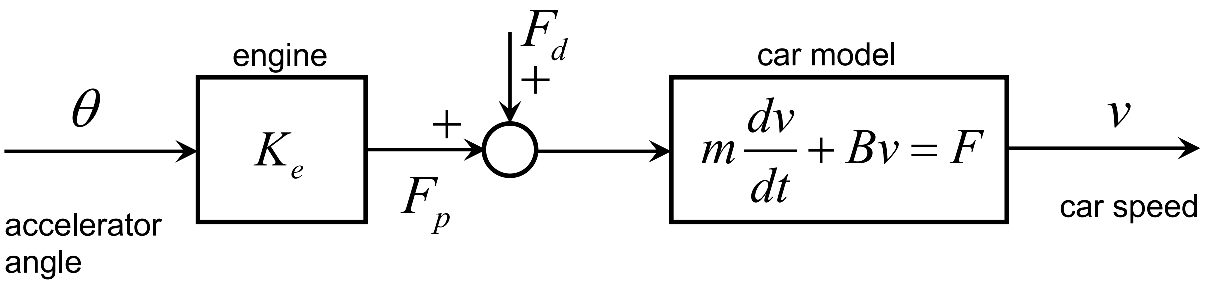
With the accelerator pedal angle, θ, we can write the propulsive force as:

$$F_p(t) = K_e\theta(t)$$

Where K_e is a constant and the simple force balance becomes:

$$m\frac{dv}{dt} + Bv =$$

We can now draw a block diagram of the system



(b) closed-loop control

Define error as the difference between desired speed, $v_d(t)$, and the measured speed, $v(t)$, such that

$$e(t) = v_d(t) - v(t)$$

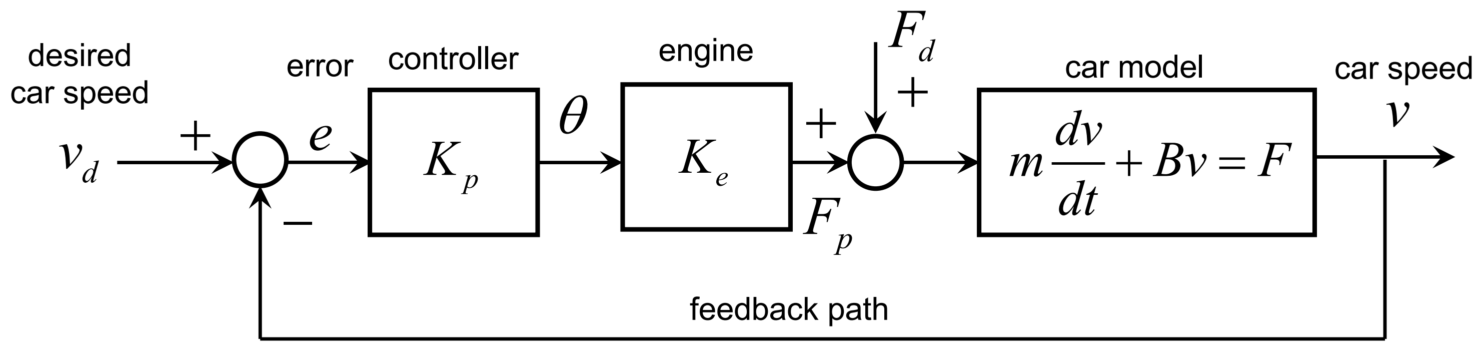
And we can then define a control law that tells us to increase the accelerator angle by an amount, K_p, proportional to the error:

$$\theta(t) = K_p e(t) =$$

The propulsive force acting on the car is now:

$$F_p(t) = K_e K_p e(t) = K_e K_p \left(v_d(t) - v(t) \right)$$

The closed-loop system block diagram is now:



This is known as *proportional control*

(c) Open-loop dynamic response

The force balance equation for the car dynamics are:

$$m\frac{dv}{dt} + Bv = K_e\theta + F_d(t)$$

How does the car respond to a change in accelerator position?

If the car is on level ground (no disturbance) $F_d(t)$ = 0 the above differential equation can be rewritten as

$$\frac{m}{B}\dot{v} + v = \frac{K_e}{B}\theta$$

And comparing with standard form of first-order ODE $\tau\dot{y} + y = f(t)$

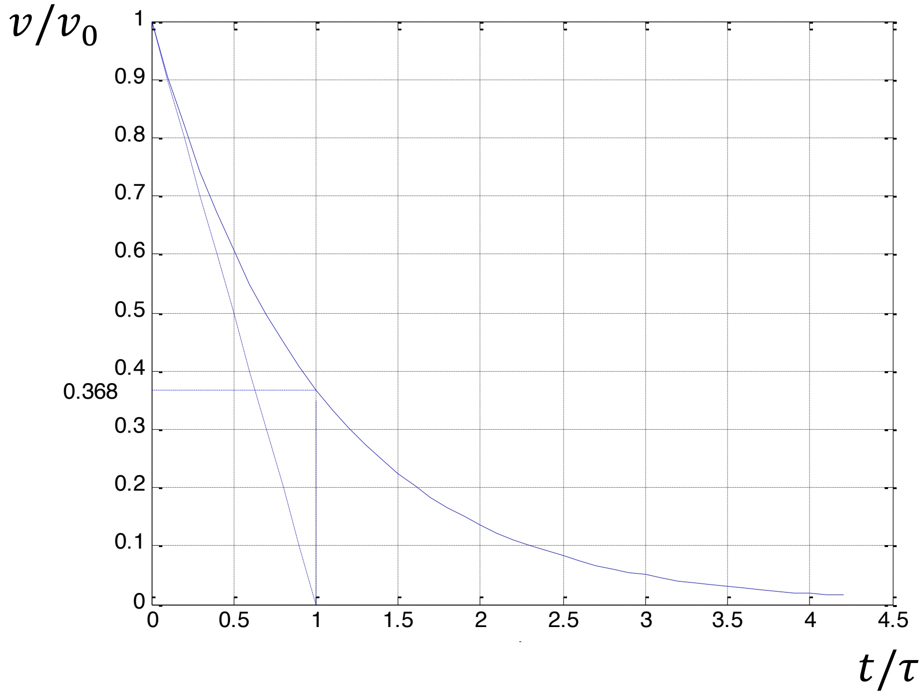
The time constant $\tau =$ and the forcing function $f(t) =$

Consider the “coast-down” response from an initial speed $v(0) = v_0$ with zero accelerator input $\theta = 0$.

The homogenous response is: $v(t) = v_0 e^{-\frac{t}{\tau}} = v_0 e^{-\frac{B}{m}t}$

$$e^{-1} = 0.3679$$
$$e^{-2} = 0.1353$$
$$e^{-3} = 0.0498$$
$$e^{-4} = 0.0183$$

After 4τ, $v < 0.02 v_0$



Accelerating the car from rest, i.e. $v(0) = 0$ and $\theta(0) = 0$, with a step change so that $\theta(t) = \theta$, the differential eqn is:

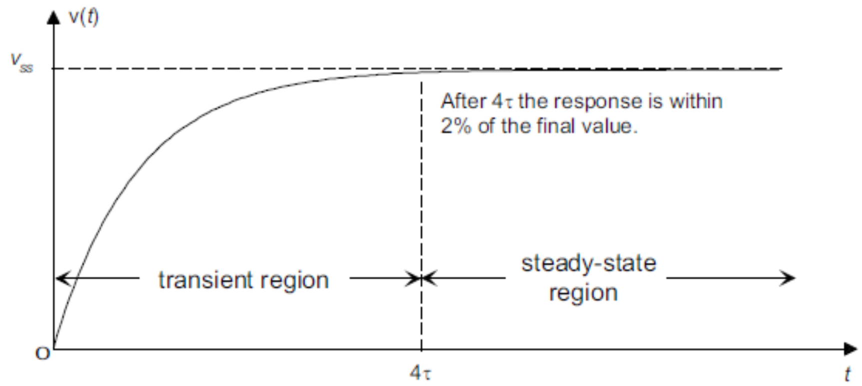
$$\frac{m}{B}\dot{v} + v = \frac{K_e}{B}\theta$$

i) By letting all derivatives become zero, the final (steady state) speed is:

$$v_{ss} =$$

ii) Solving the differential eqn gives the dynamic response

$$v(t) = v_{ss}(1 - e^{-\frac{t}{\tau}})$$
$$= \frac{K_e\theta}{B}(1 - e^{-\frac{B}{m}t})$$



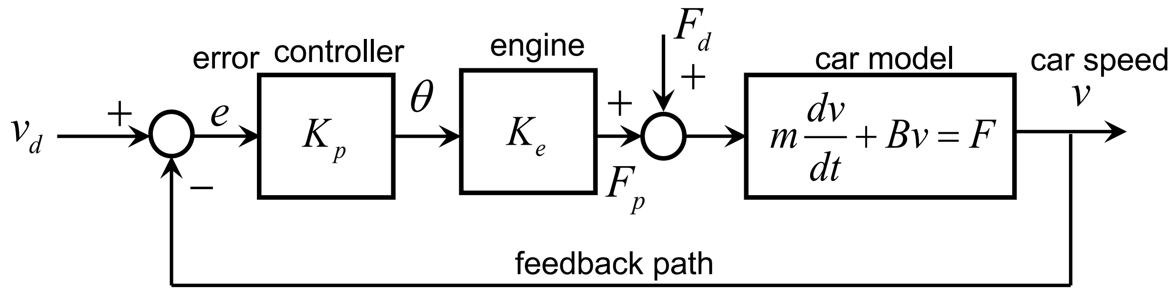
Two factors make the output different from the input:

1. States of physical systems cannot be changed instantaneously, this depends on how a physical device acquires or dissipates energy. Hence, there is a gradual change in position from 0 to v_{ss}. This part of the response is known as the *transient response*.

2. When the system output has settled to a *steady-state* there may be some continuous error. This is the *steady-state error* and it arises due to manifold reasons. The level of acceptable steady-state error and its impact on system performance is of great interest to control engineers due to its effect on the operation of the system.

(d) Closed-loop controlled response

recall the closed-loop system



with the **control law** $\theta(t) = K_p e(t) = K_p\left(v_d(t) - v(t)\right)$

if $v < v_d$ then $e > 0$	set $\theta > 0$	depress accelerator
if $v = v_d$ then $e = 0$	set $\theta = 0$	do nothing
if $v > v_d$ then $e < 0$	set $\theta < 0$	reduce accelerator/apply brakes

The new differential equation is: $m\frac{dv}{dt} + Bv =$

rearranging ODE

$$m\frac{dv}{dt}+(B+K_pK_e)v=K_pK_ev_d(t)+F_d(t)$$

This is the **closed-loop differential equation** which can be written:

$$m\frac{dv}{dt}+(B+K_pK_e)v=K_pK_ev_d(t)+F_d(t)$$

and compared with the standard first-order form $\tau\dot{y}+y=f(t)$

$$\frac{m}{B+K_pK_e}\frac{dv}{dt}+v=\frac{K_pK_ev_d(t)}{B+K_pK_e}+\frac{1}{B+K_pK_e}F_d(t)$$

where $\tau =$ is the **closed-loop time constant** (recall that OL $\tau = m/B$)

The **feedback loop has modified the ODE** changing the time constant

(i) **Final or steady state response**

If we command the car to travel at a speed v_d, what speed will it actually reach?

With no external disturbance, $F_d(t) = 0$.

To find the steady state speed, v_{ss}, we need to set $\frac{dv}{dt} = 0$ and solve for v_{ss}:

$$v_{ss} = \frac{K_p K_e}{B + K_p K_e} v_d$$

Note that $v_{ss} < v_d$ for $B > 0$ **and that** increasing the controller gain K_p so that $K_p K_e \gg B$ leads to $v_{ss} \to v_d$.

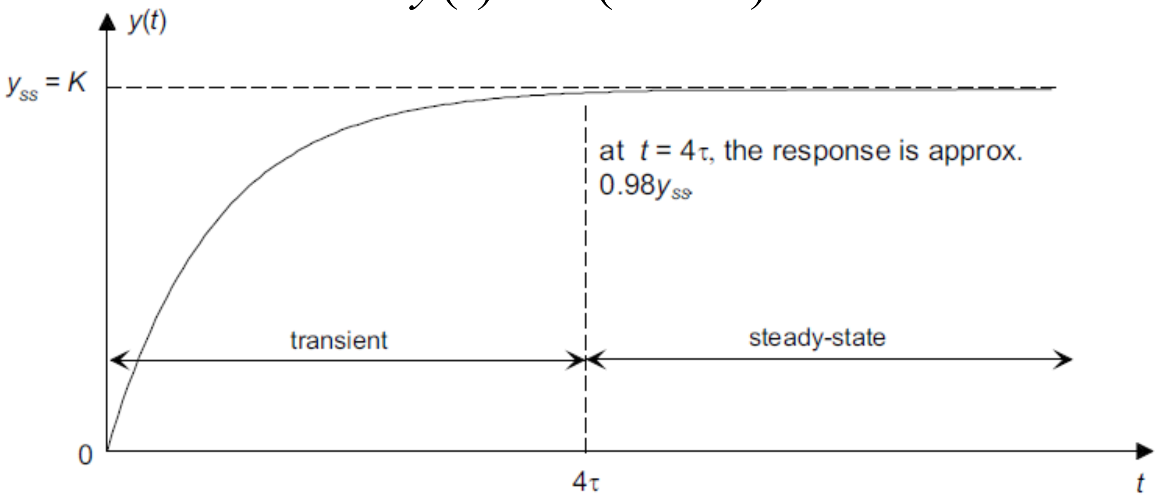
(ii) Effect of feedback on dynamic response

For the first-order ODE

$$\tau \frac{dy}{dt} + y = Ku(t)$$

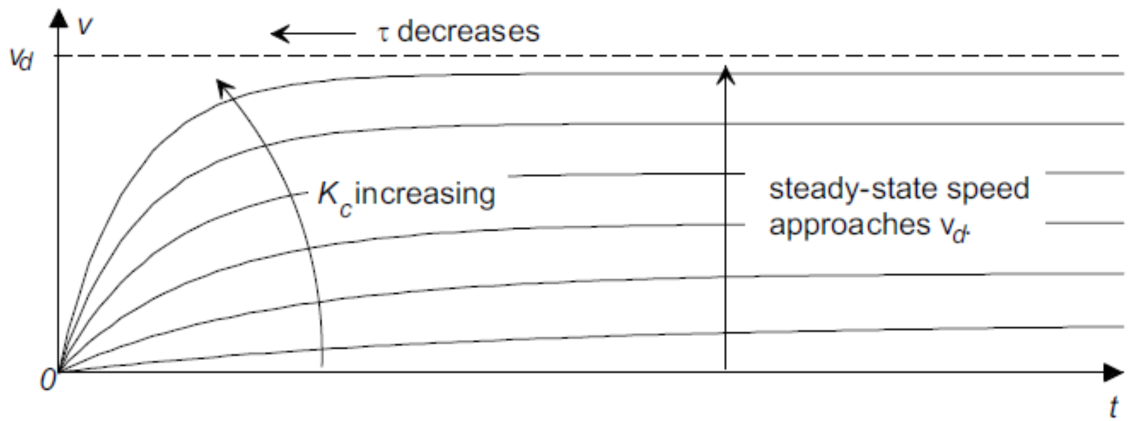
with a constant input $u(t) = 1$ and initial conditions $y(0) = 0$ the dynamic response is:

$$y(t) = K(1 - e^{-\frac{t}{\tau}})$$



With proportional feedback control the steady state speed, $v_{ss} = \frac{K_p K_e}{B + K_p K_e} v_d$, is a function of controller gain, K_p.

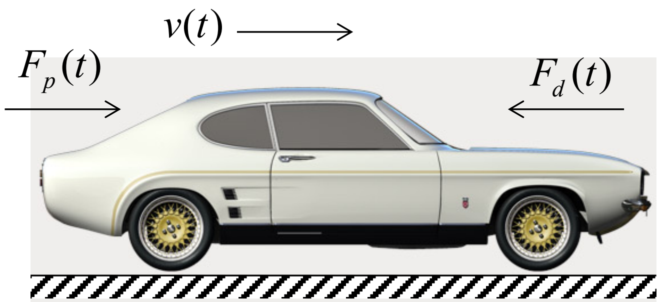
The closed loop time-constant τ is: $\tau = \frac{m}{B + K_p K_e}$, also a function of K_p.

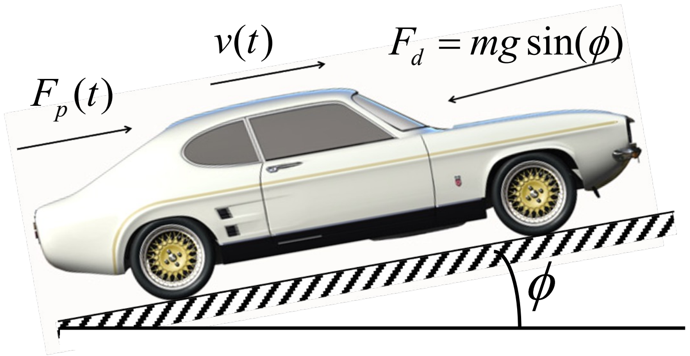


As K_p increases, τ decreases so the car responds more quickly to changes in accelerator pedal angle.

(e) Effect of disturbances

What happens when the car goes up a hill under closed-loop control?





On level ground, $F_d = 0$ and $v_{ss} = \frac{K_p K_e}{B+K_p K_e} v_d$. On an incline $F_d = -mg\sin\phi$

closed-loop diff. eq.: $\frac{m}{B+K_p K_e}\frac{dv}{dt} + v = \frac{K_p K_e v_d(t)}{B+K_p K_e} + \frac{1}{B+K_p K_e} F_d(t)$ (introduced earlier)

The steady state speed is $v_{ss} = \frac{K_p K_e}{B+K_p K_e} v_d - \frac{mg\sin\phi}{B+K_p K_e}$

As controller gain, K_p, is increased the impact of the disturbance is reduced