SESA2024 Astronautics

Chapter 5: Mission Analysis

Lecture 7 – Orbital elements

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Overview of lecture 7

- In this lecture we introduce the classical (Keplerian) orbital elements:
  - These describe the orbit:
    - Size
    - Shape
    - Orientation (with respect to a fixed frame of reference)

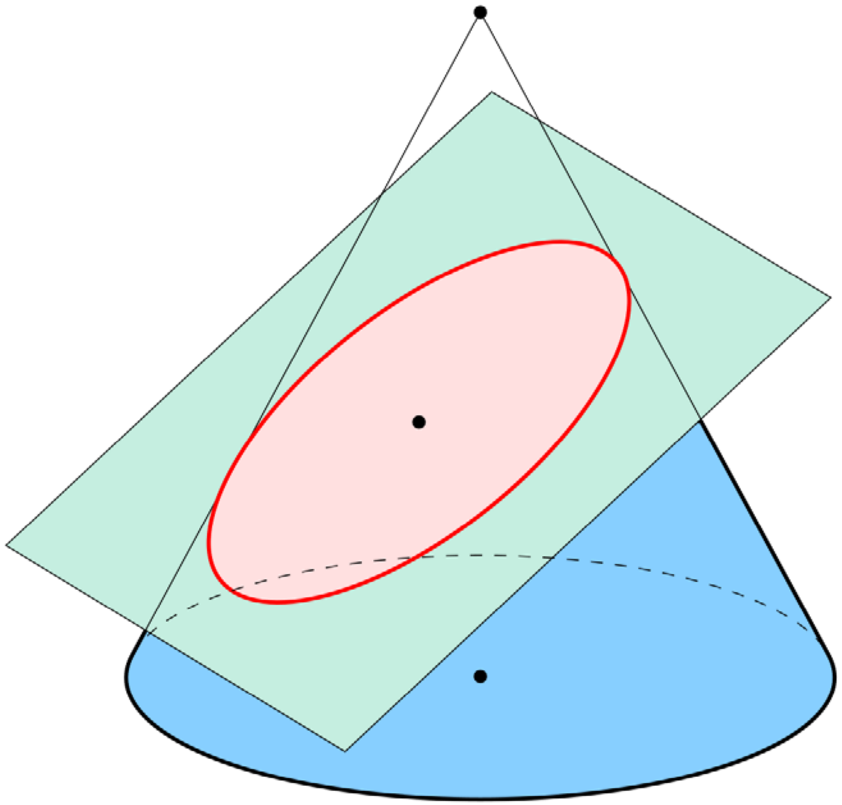
- In mission analysis, a key objective is to assign values to these orbital elements (i.e. to define the orbit for a particular mission) based on a set of payload requirements
  - And sometimes system requirements

Properties of an ellipse

- The ellipse equation is actually the equation of a conic section in polar form

$$r = \frac{a(1 - e^2)}{(1 + e\cos\theta)}$$

An ellipse (red) is obtained as the intersection of a cone (blue) with an inclined plane (green)



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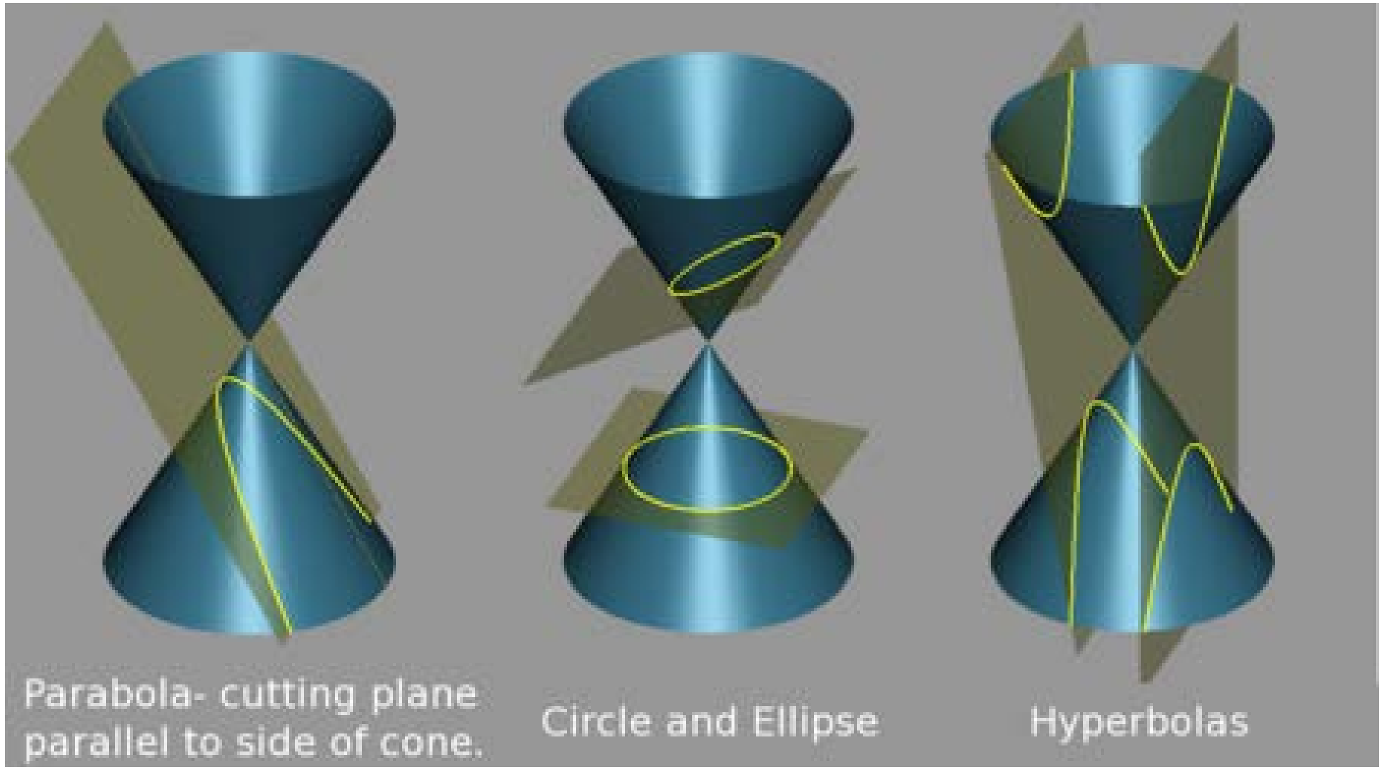


The trajectory is:

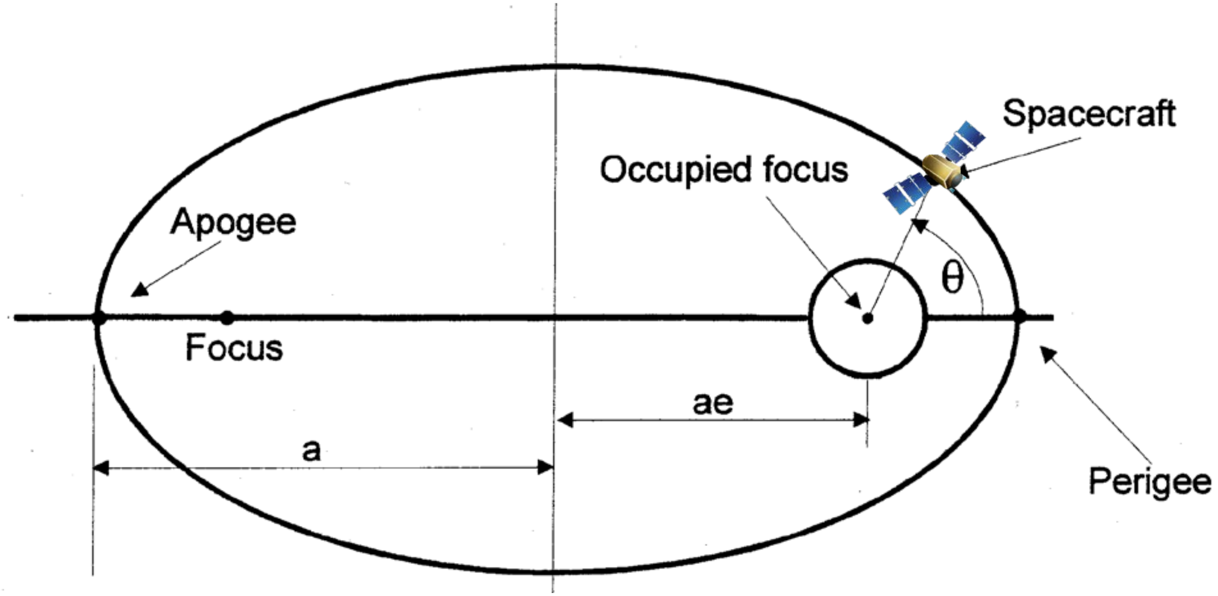
- circular for $e = 0$
- elliptic for $0 < e < 1$
- parabolic for $e = 1$
- hyperbolic for $e > 1$

We will focus only on trajectories for which:

$$0 \leq e < 1$$

Orbital elements

- Recall: the orbit of each planet is an ellipse with the Sun at one focus



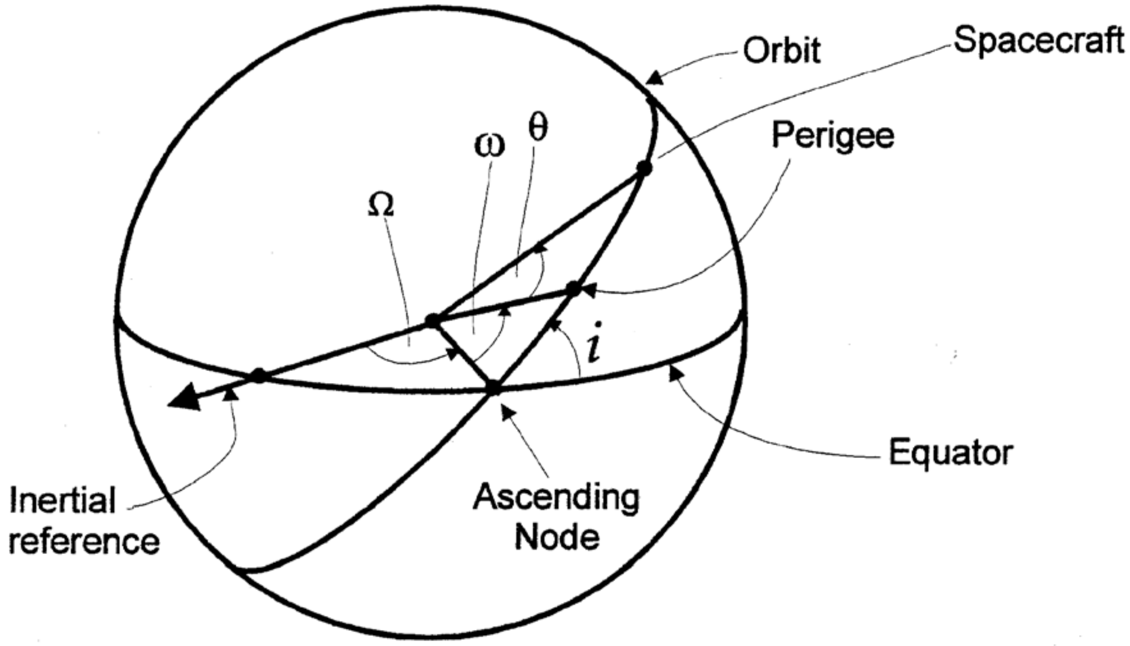
In-plane orbit elements

- size: semi-major axis, a
- shape: eccentricity, e
- orbital position: true anomaly, θ

* Note: the terms "apogee" and "perigee" are specific to orbit locations around the Earth, which is the focus of this course. The general terms for the equivalent locations of planets or bodies around the sun are: perihelion and aphelion.

Orbital elements

- Orientation of orbit plane elements

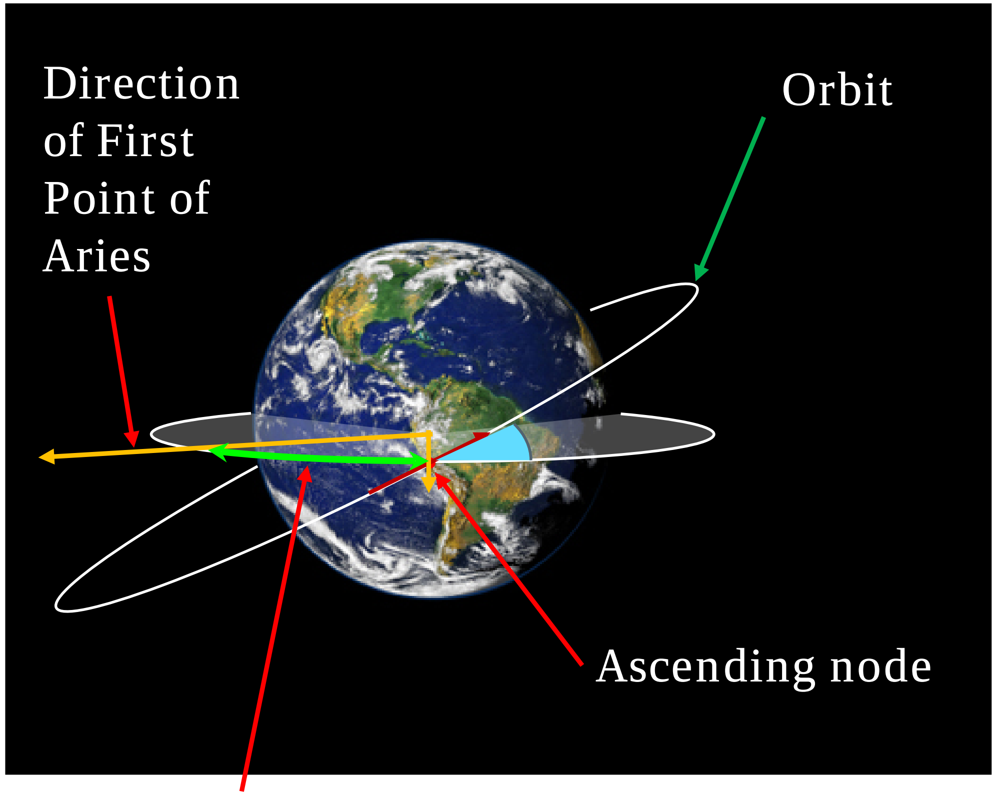


- orbital inclination, i
- right ascension of ascending node, Ω
- argument of perigee, ω

We need to define a datum point: the basis from which we can define the orientation of our orbit

Orbital elements

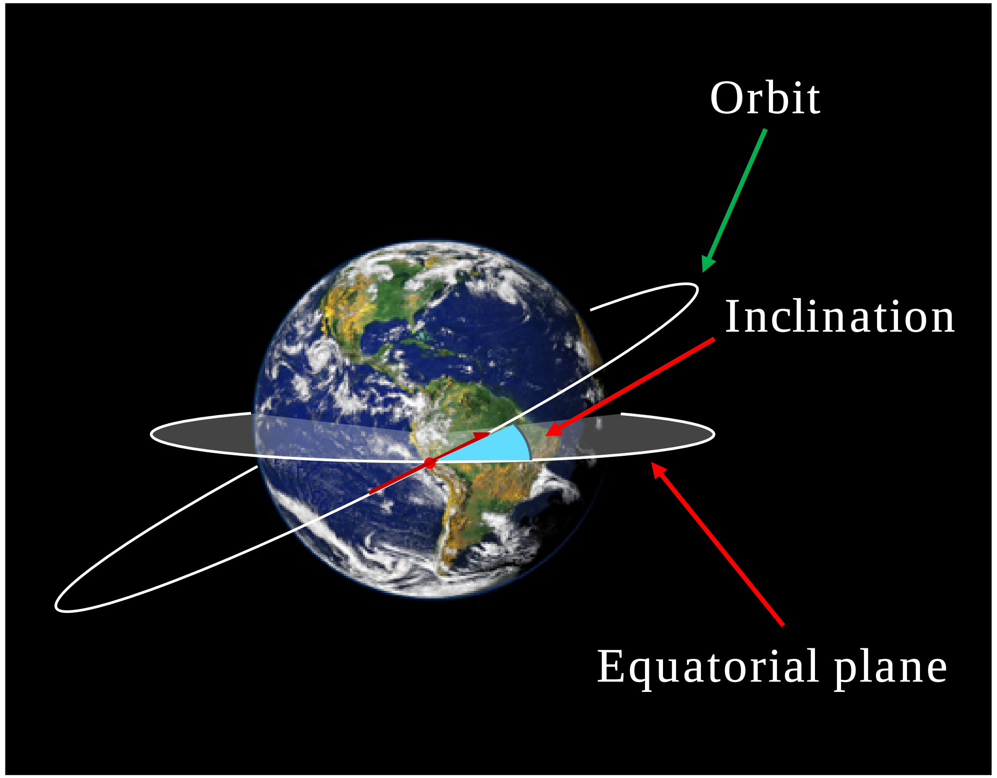
- Definition of our datum point
  - The position of the ascending node is defined from the 'First Point of Aries'.
  - This is in the direction of the Sun when the Earth is at the Spring Equinox position and lies in the Earth's equatorial plane.
  - The position of orbits around the Earth are defined by the angle between the line of the First Point of Aries to the position of the Ascending node.
  - This is called the 'Right Ascension of the Ascending node' – Ω.



Right Ascension of the Ascending node

Orbital elements

- Inclination
  - We define the orientation of the orbit plane with respect to the Earth's equatorial plane using the orbital inclination.
  - The inclination, i, is the angle between the orbit plane and the equatorial plane (as the satellite crosses the equator travelling from South to North, i.e. at the ascending node).
  - This angle lies in the range $[0, 180°]$.

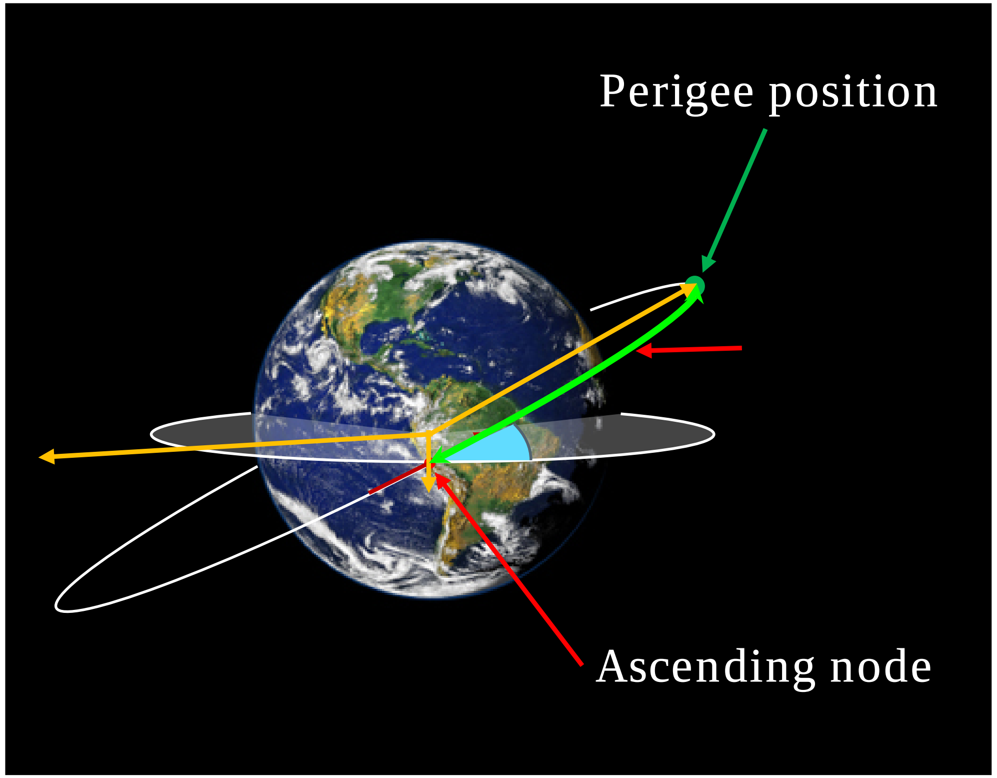


Orbital elements

- Argument of Perigee

  - The next important piece of information that needs to be defined is where the perigee of the orbit is.

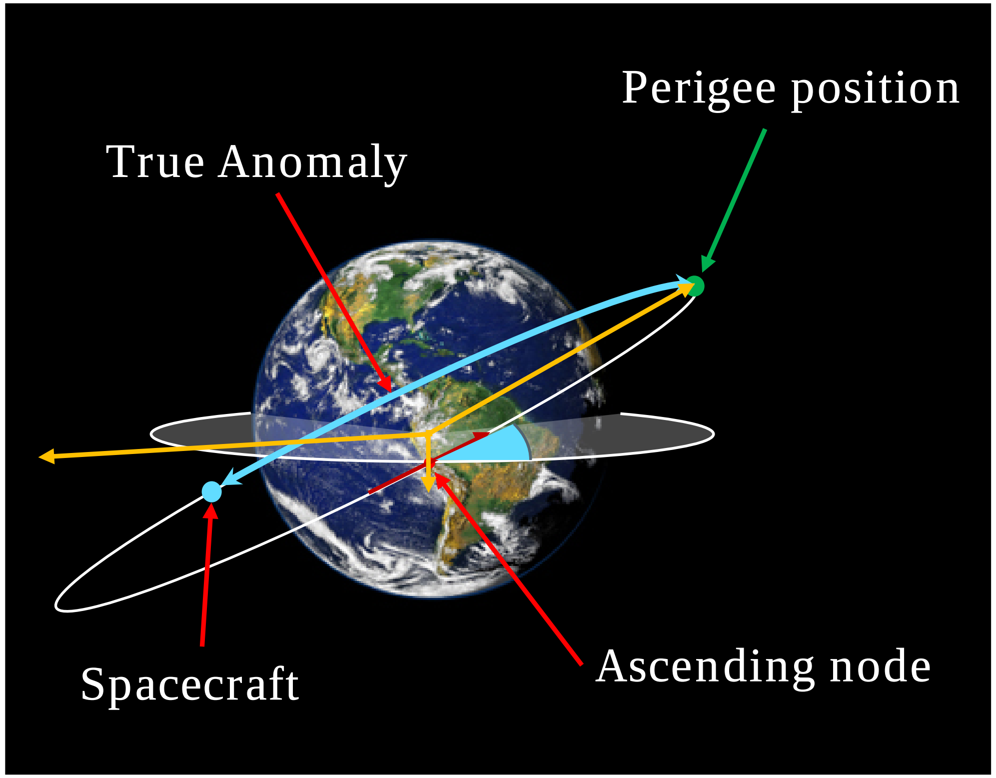
  - This angle is measured round from the Ascending node, in the orbital plane, to the Perigee position. The angle is called the 'Argument of Perigee' – ω.



Orbital elements

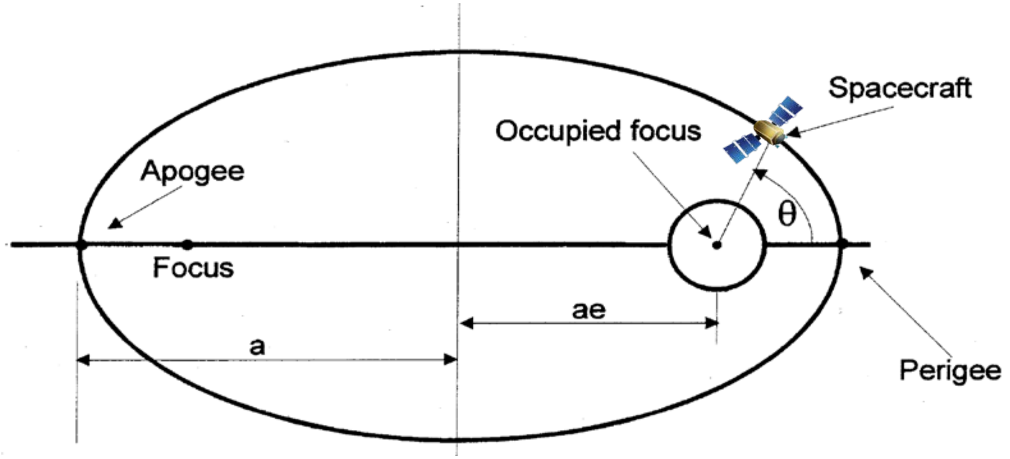
- Spacecraft position

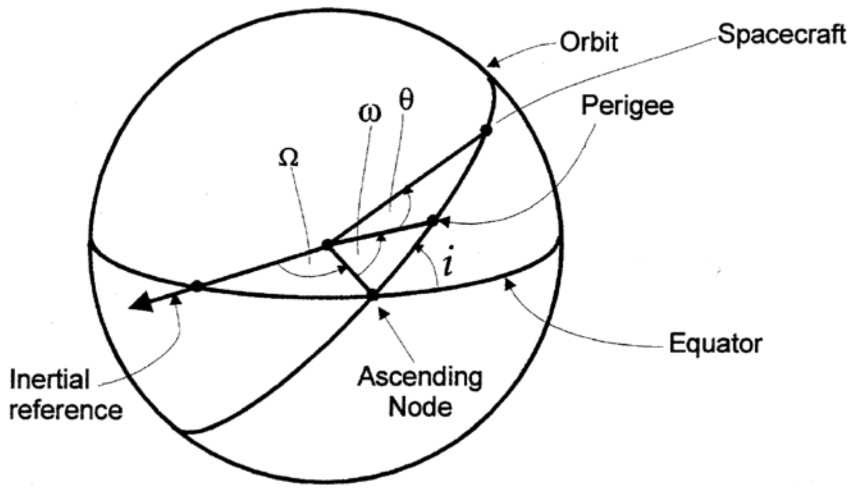
  - The position of the spacecraft in the orbit can then be specified as the angle around the orbit from the Perigee position.
  - This quantity featured in our derivation of the equation of motion and the ellipse equation.
  - This is called the 'True Anomaly' - θ.



Orbital elements

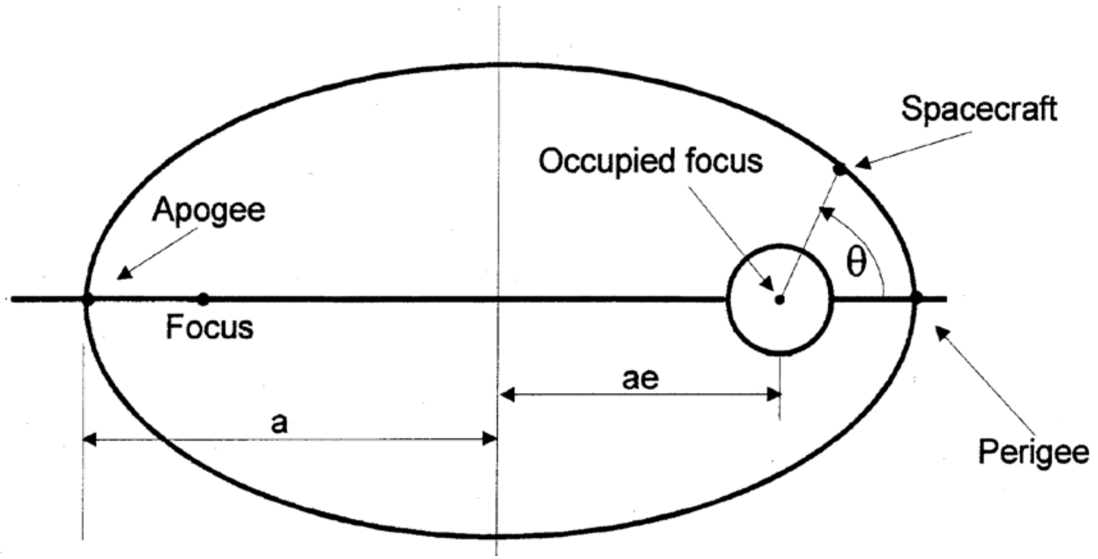
- Six (Keplerian) elements
  - Right Ascension of the Ascending node: Ω
  - Inclination: i
  - Argument of Perigee: ω
  - Semi-major axis: a
  - Eccentricity: e
  - True Anomaly: θ





Orbital elements

- Perigee and apogee



When the true anomaly $\theta = 0°$

$$r = r_p = \frac{a(1-e^2)}{(1+e\cos 0°)} = \frac{a(1-e^2)}{(1+e)}$$

So the perigee radius is: $r_p = a(1-e)$

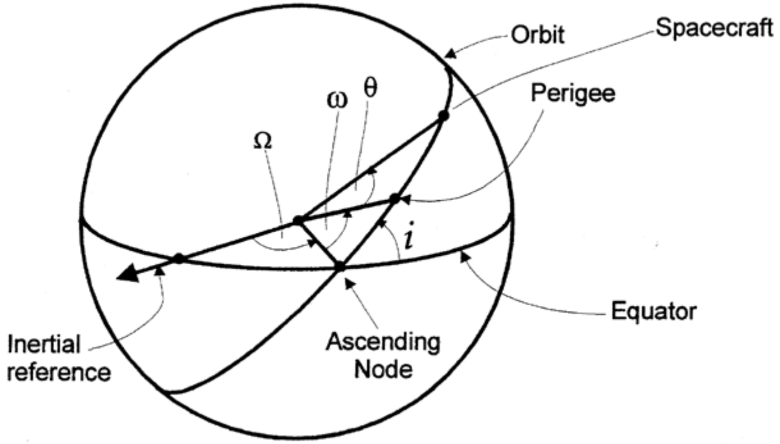
Similarly, we can find the apogee radius when the true anomaly $\theta = 180°$

$$r_a = a(1+e)$$

Orbital elements

- Keplerian elements are undefined in some situations
  - Right Ascension of the Ascending node:
    - Ω is not defined as $i \rightarrow 0$
    - ω is not defined as $e \rightarrow 0$ (there is no defined perigee)

  We can use a different element set for these situations



- We will assume that the motion is unperturbed
  - All Keplerian elements, except θ, remain constant

Activity

- Orbit visualisation in Microsoft Excel (available on Blackboard)

Orbit and Visualisation Control Panel

Change the orbital elements and use the view settings to visualise the orbit

Variable	Symbol	Value	Units
Semi-major axis	a	6778	km
Eccentricity	e	0.001	
Inclination	i	51.5	deg.
Right ascension of ascending node	Ω	60	deg.
Argument of perigee	ω	30	deg.
Perigee altitude	h_p	393.2	km
Apogee altitude	h_a	406.8	km

Observer	
Latitude:	50.9 deg. N
Longitude:	1.4 deg. W
Date:	16/09/2020
Time (UTC	12:00:00

Satellite position:	True Anomaly:	30 deg.
View settings:	Zoom:	2
	Azimuth:	-18 deg.
	Elevation:	18 deg.

Acknowledgements:	
Based on Perspective1.xls by:	George Lungu <excelunusual.com>
Keplerian to Cartesian conversion from:	Richard Bate <Fundamentals of astrodynamics>
This visualisation:	Hugh Lewis <https://twitter.com/ProfHughLewis>