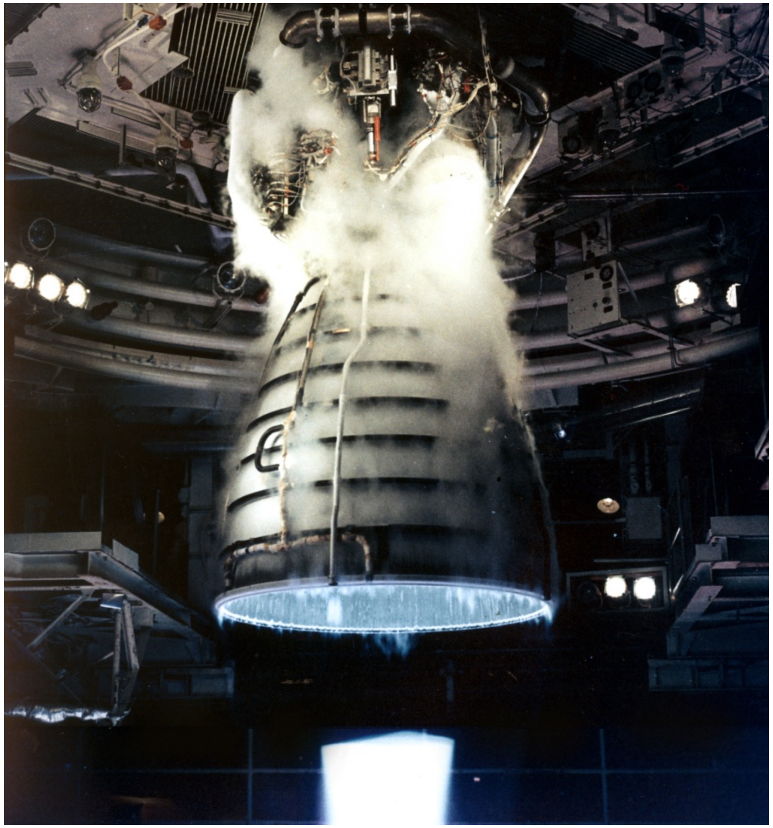
SESA3029 Aerothermodynamics

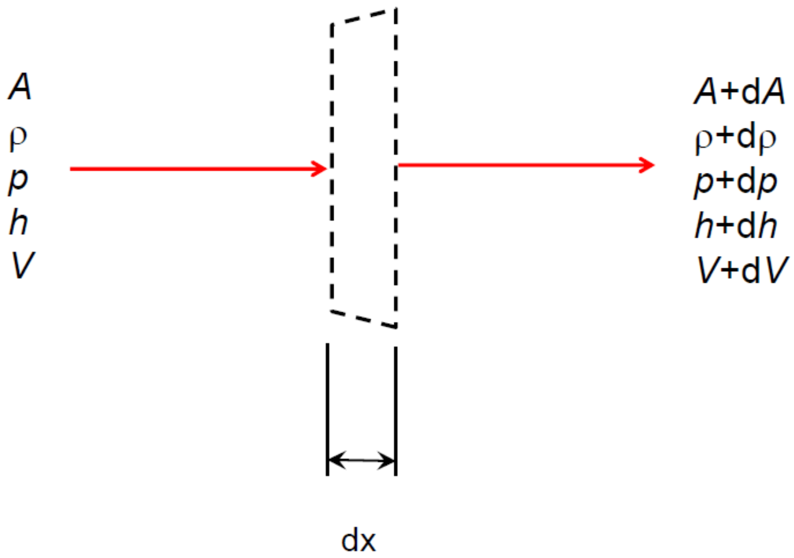


NASA Photograph

Lecture 2.6

Laval Nozzle

Area equation (differential form)



Mass conservation $$\frac{dA}{A}+\frac{dV}{V}+\frac{d\rho}{\rho}=0$$

Newton II $$\rho V\,\mathrm{d}V=-\mathrm{d}p$$

$$\boxed{\frac{\mathrm{d}A}{A}=\left(M^{2}-1\right)\frac{\mathrm{d}V}{V}}$$

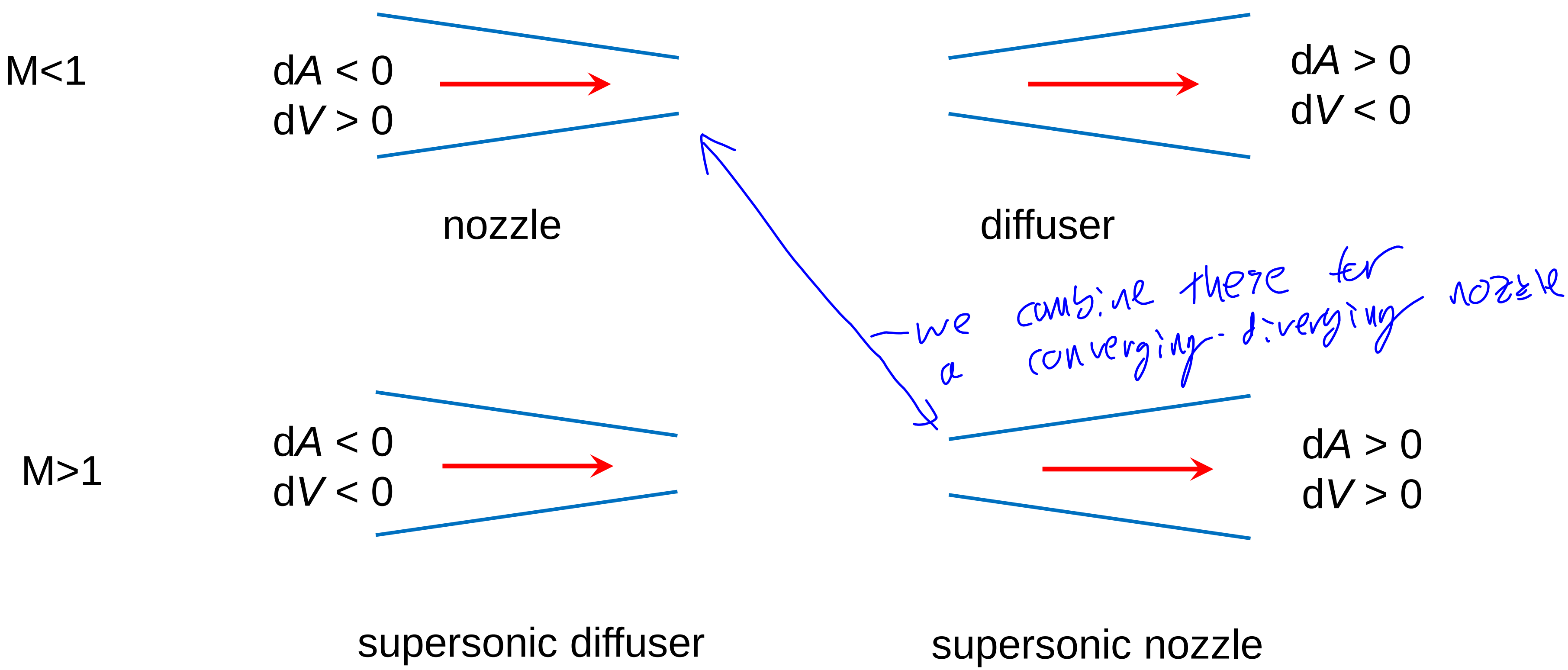
relationship between speed change and area change (larger gear)

Opposite dA for M<1 compared to M>1

M=1 at dA=0, but only a converging-diverging configuration can get there

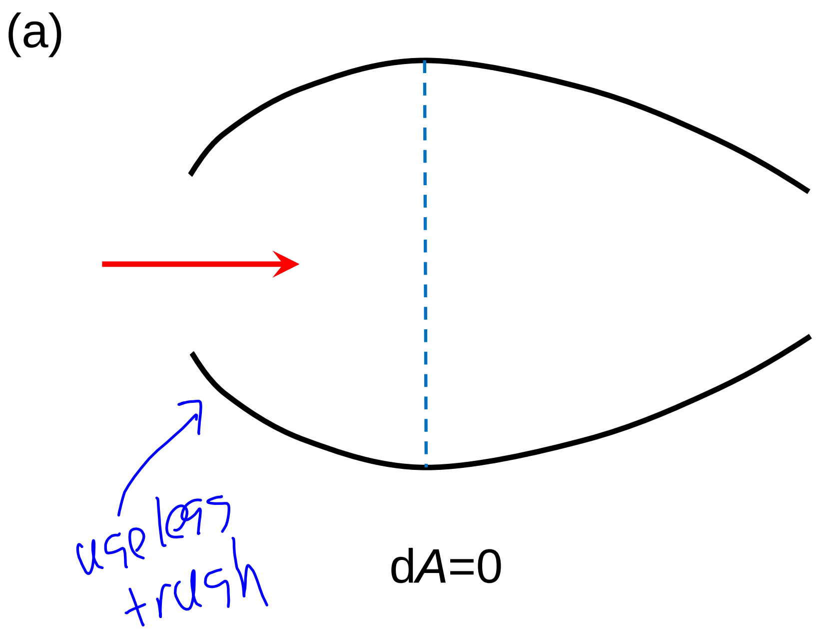
Application to nozzles and diffusers

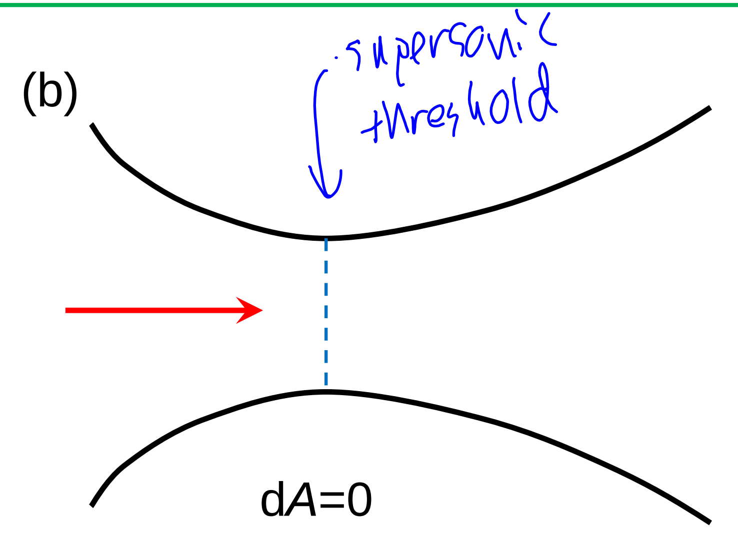
$$\frac{\mathrm{d}A}{A} = \left(M^2 - 1\right)\frac{\mathrm{d}V}{V}$$



Two possibilities for dA=0 and M=1 at throat

$$\frac{\mathrm{d}A}{A} = \left(M^2 - 1\right)\frac{\mathrm{d}V}{V}$$

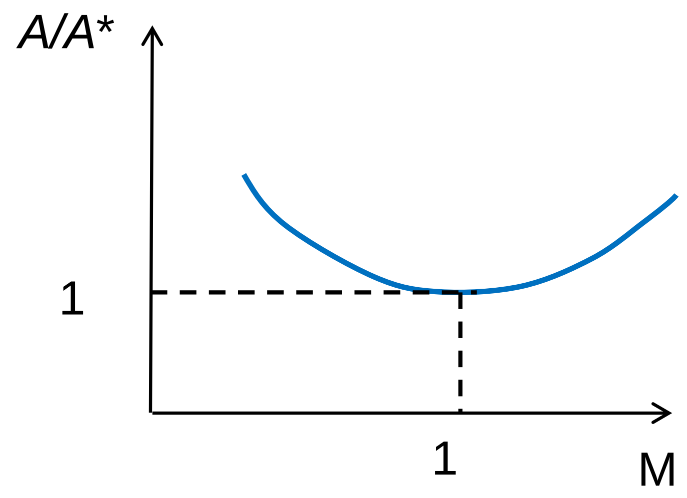




Only the converging-diverging configuration (b) can get to M=1 at throat

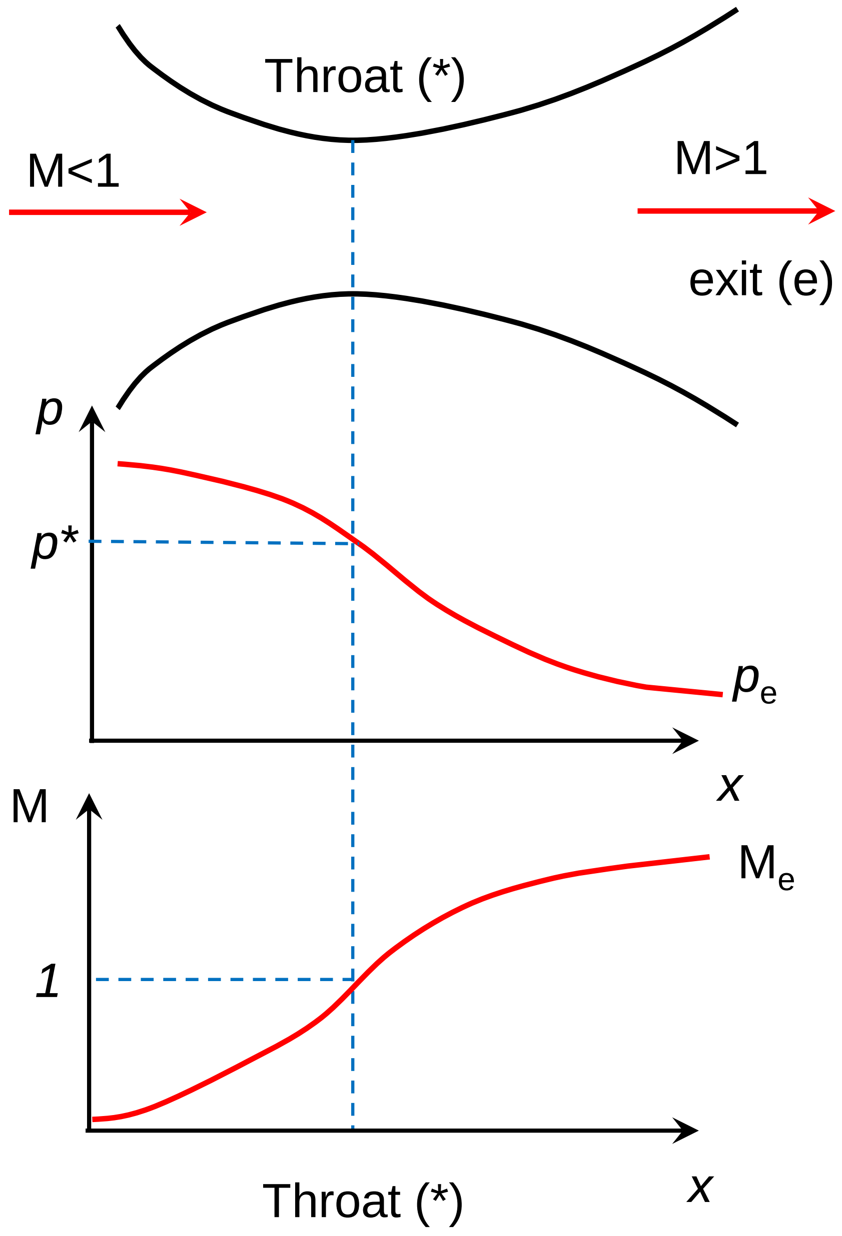
Area-Mach number relation

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$



- Tabulated in IFT
  - For any *M* we can find *A/A**
  - For any *A/A** (>1) there are two solutions for *M*, one subsonic, one supersonic
  - ‘Design condition’ of a supersonic (Laval) nozzle has M<1 upstream, M>1 downstream of throat with no shocks

Supersonic design condition of a Laval nozzle



Mass flowrate

$$\dot{m} = \rho A V$$
$$= \rho^* A^* a^*$$
$$= \rho^* A^* \sqrt{\gamma R T^*}$$

and for sonic conditions

$$\frac{T_0}{T^*} = \frac{\gamma + 1}{2}$$

$$\frac{\rho_0}{\rho^*} = \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}}$$

hence $$\dot{m} = \rho_0 A^* \sqrt{\gamma R T_0} \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1} + \frac{1}{2}}$$

$$\boxed{\dot{m} = \frac{p_0}{\sqrt{R T_0}} A^* \sqrt{\gamma} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}}$$

Stagnation conditions

Throat area

Gas properties

N.B. – no influence of back pressure or other downstream conditions as long as flow is choked

Critical conditions for choked flow

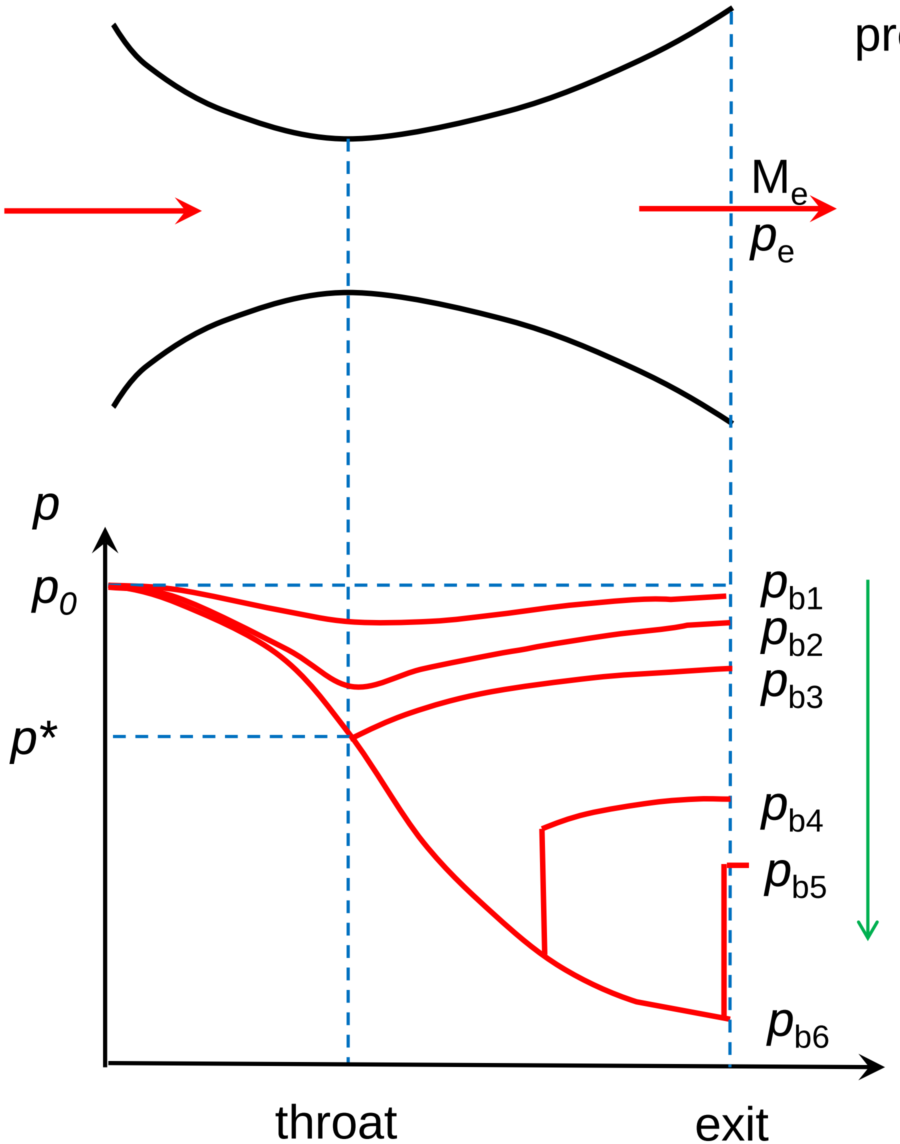
$$\frac{p_0}{p^*} = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma}{\gamma - 1}} \qquad \text{or} \qquad \frac{p^*}{p_0} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} = 0.528 \quad \text{for } \gamma = 1.4$$

- If $p_{\text{throat}} > p^*$ flow is subsonic throughout
- If $p_{\text{throat}} = p^*$ flow is choked
  - (can’t have $p_{\text{throat}} < p^*$)

Effect of back pressure

Environment is at 'back' pressure p_b

Reservoir conditions p_0, T_0



Reducing back pressure

p_{b1} – low subsonic throughout (effectively incompressible)

p_{b2} –subsonic throughout (compressible)

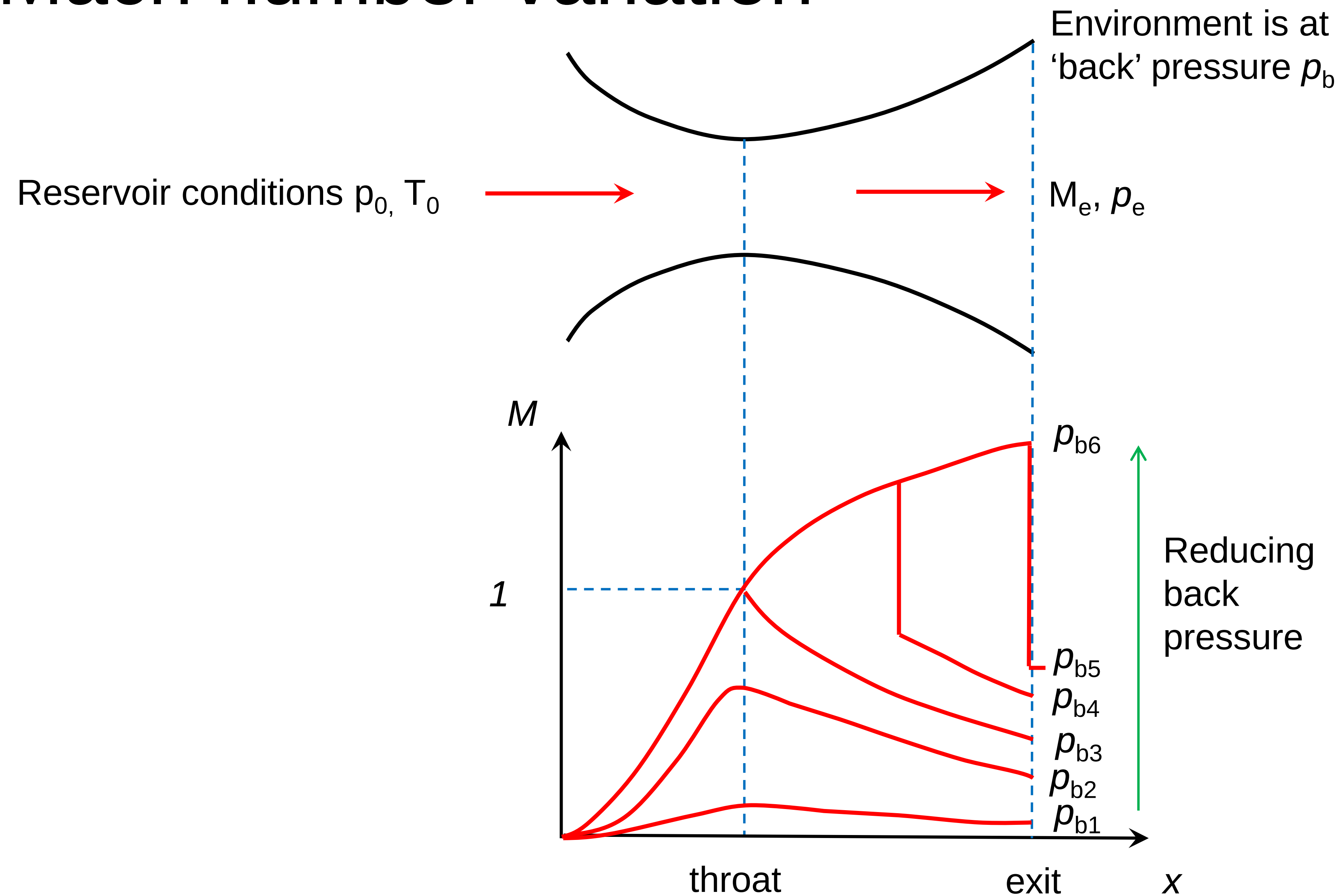
p_{b3} –sonic at throat, subsonic in diverging section

p_{b4} –shock in diverging section

p_{b5} –shock at exit

p_{b6} –design condition (supersonic exit)

Mach number variation



Example

For a Laval nozzle with a design exit Mach number of M_e=2.2 find p_{b6}, p_{b3} and p_{b5} in terms of p_0

Isentropic-flow table (γ = 1.4):

M	p/p_0	ρ/ρ_0	T/T_0	ν (deg.)	A/A^*
2.2000	0.0935	0.1841	0.5081	31.7325	2.0050
2.2200	0.0906	0.1800	0.5036	32.2494	2.0409
2.2400	0.0878	0.1760	0.4991	32.7629	2.0777
2.2600	0.0851	0.1721	0.4947	33.2730	2.1153
2.2800	0.0825	0.1683	0.4903	33.7796	2.1538

Design condition M_e=2.2, A_e/A^*=2.005 and p_{b6}/p_0=0.0935

For same area ratio there is a subsonic solution

Isentropic-flow table (γ = 1.4):

M	p/p_0	ρ/ρ_0	T/T_0	ν (deg.)	A/A^*
0.3000	0.9395	0.9564	0.9823	n/a	2.0351
0.3200	0.9315	0.9506	0.9799	n/a	1.9219
0.3400	0.9231	0.9445	0.9774	n/a	1.8229
0.3600	0.9143	0.9380	0.9747	n/a	1.7358
0.3800	0.9052	0.9313	0.9719	n/a	1.6587

Linear interpolation for $A_e/A^*=2.005$ gives $M_e=0.305$ and $p_{b3}/p_0=0.937$

To get p_{b5} we add a normal shock at the exit i.e. from M=2.2

NST for M=2.2 gives $p_{b5}/p_{b6}=5.480$

$p_{b5}=p_{b6} \times p_{b5}/p_{b6} = 0.0935p_0 \times 5.480 = 0.512p_0$