

SESA3029

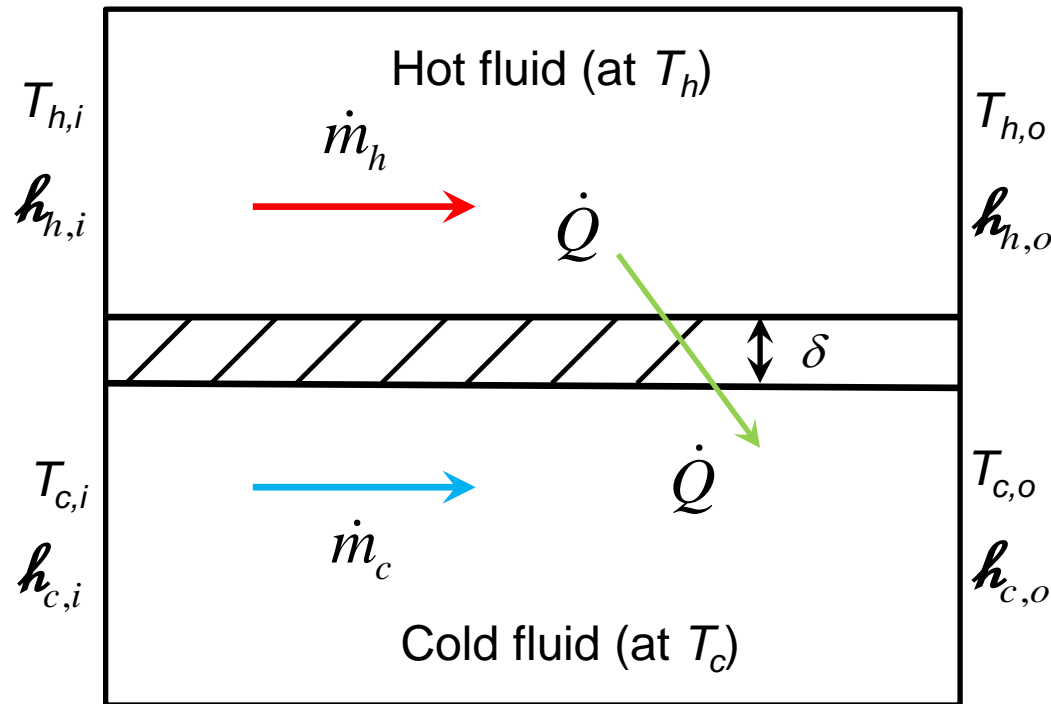
Aerothermodynamics

Lecture 5.8

Heat exchangers -

Log mean temperature difference method

Heat exchanger – General equations



*h: hot
c: cold
i: inlet
o: outlet*

\dot{m} : mass flow

h : enthalpy

C_p : fluid heat capacity

*Parallel-flow heat exchanger depicted
but (1) and (2) are the same for the
counter-flow heat exchanger!*

Evaluation of heat transfer rate with inlet and outlet temperatures:

$$\dot{Q} = \dot{m}_h (\mathbf{h}_{h,i} - \mathbf{h}_{h,o}) = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o}) = C_h (T_{h,i} - T_{h,o}) \quad (1)$$

$$\dot{Q} = \dot{m}_c (\mathbf{h}_{c,o} - \mathbf{h}_{c,i}) = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = C_c (T_{c,o} - T_{c,i}) \quad (2)$$

C_h and C_c are the **capacities**.

Heat exchanger – General equations

On a unit area basis:

$$\frac{1}{h'} = \frac{1}{h_h} + \frac{\delta}{k_w} + \frac{1}{h_c}$$

h_h and h_c are the heat transfer coefficients on **h**ot and **c**old side and k_w is the wall conductivity

In practice δ is usually taken as 0

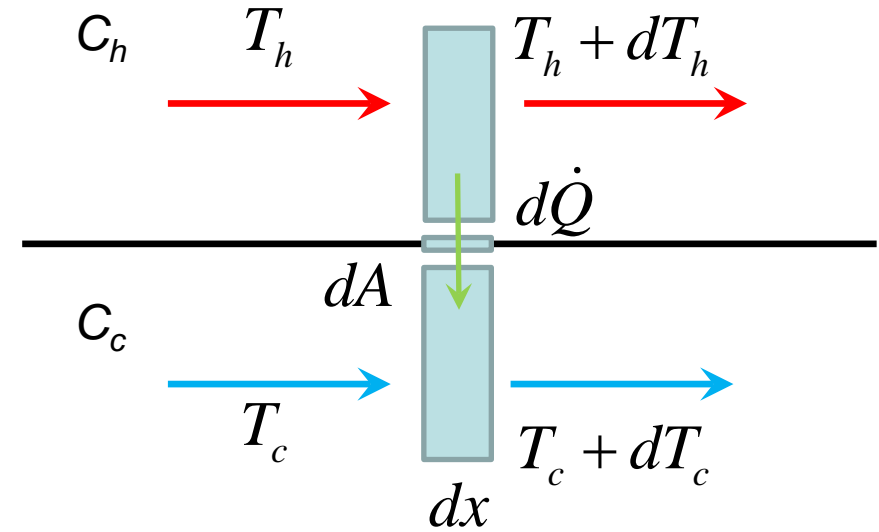
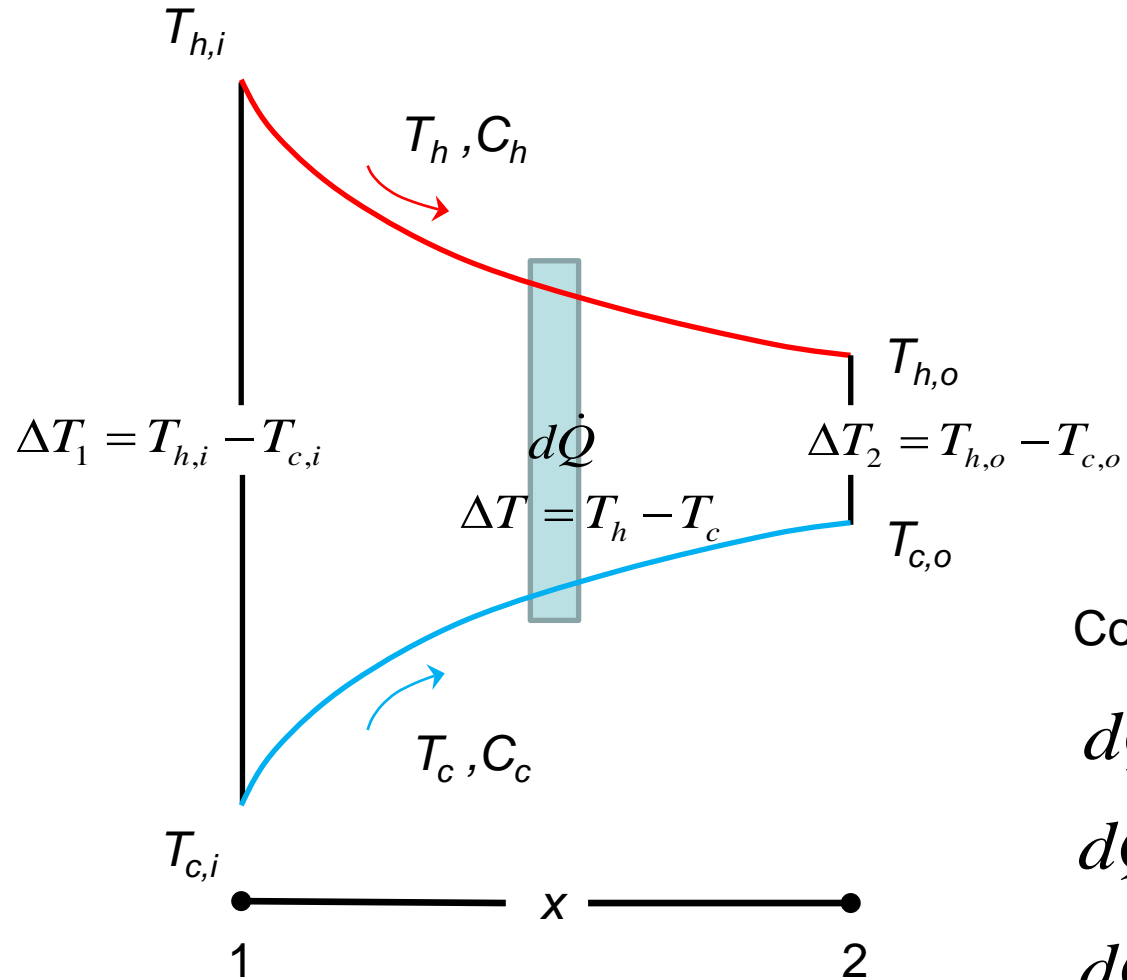
Since the temperature difference $\Delta T = T_h - T_c$ is spatially varying, a suitable **mean** temperature difference needs to be found to evaluate the heat transfer rate of the overall system as

$$\dot{Q} = h' A \Delta T_m$$

This leads to the *log mean temperature difference* method.

Derivation follows Bergman et al., pages 713-715 / Incropera et al., pages 659-662 closely.

Parallel-flow heat exchanger



Construct heat balance on elements dx dA :

$$d\dot{Q} = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h \quad (3)$$

$$d\dot{Q} = \dot{m}_c c_{p,c} dT_c = C_c dT_c \quad (4)$$

$$d\dot{Q} = h' dA \Delta T \quad (5)$$

Starting from $d(\Delta T) = dT_h - dT_c$ and using (3) and (4) together with (5), we obtain

$$d(\Delta T) = -h' dA \Delta T \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

Now integrate from location 1 to 2, i.e.,

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -h' \left(\frac{1}{C_h} + \frac{1}{C_c} \right) \int_1^2 dA$$

and use the general heat exchanger relations (1) and (2) to replace the capacities to obtain

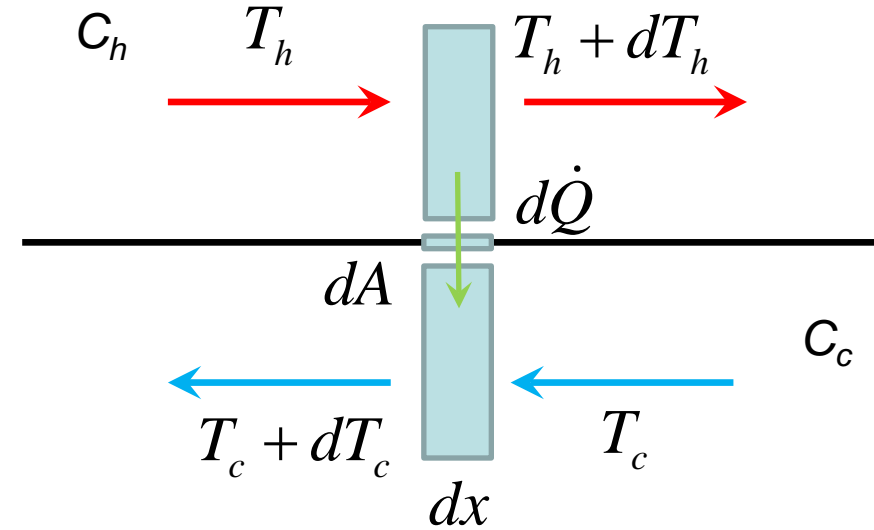
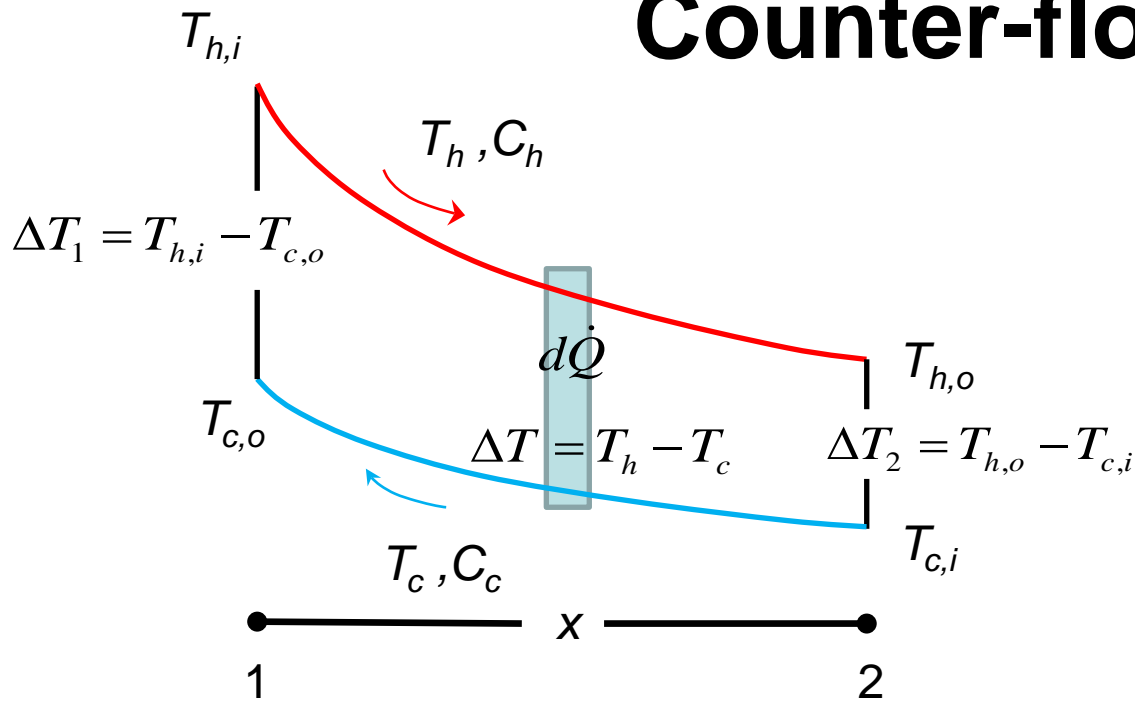
$$\ln(\Delta T_2) - \ln(\Delta T_1) = -h'A \left(\frac{T_{h,i} - T_{h,o}}{\dot{Q}} + \frac{T_{c,o} - T_{c,i}}{\dot{Q}} \right) \quad (6)$$

Inserting $\Delta T_1, \Delta T_2$ for the parallel-flow heat exchanger gives

$$\ln \left(\frac{\Delta T_2}{\Delta T_1} \right) = -\frac{h'A}{\dot{Q}} (\Delta T_1 - \Delta T_2)$$

from which we readily deduce $\dot{Q} = h'A\Delta T_{lm}$ with $\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

Counter-flow heat exchanger



Construct heat balance on elements dx dA :

$$d\dot{Q} = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h \quad (7)$$

$$d\dot{Q} = -\dot{m}_c c_{p,c} dT_c = -C_c dT_c \quad (8)$$

Starting from $d(\Delta T) = dT_h - dT_c$ and using (7) and (8) together with (5), we obtain after integration from position 1 to 2

$$\ln(\Delta T_2) - \ln(\Delta T_1) = -h'A \left(\frac{T_{h,i} - T_{h,o}}{\dot{Q}} - \frac{T_{c,o} - T_{c,i}}{\dot{Q}} \right) \quad (9) \quad \text{i.e. } \dot{Q} = h'A \Delta T_{lm} \text{ with } \Delta T_{lm} \text{ as before.}$$

Remember differences in ΔT_1 and ΔT_2 !

Note that for this case the LMTD method is not applicable for $C_h = C_c$ and (5) needs to be integrated directly.

Example: Counter-flow heat exchanger

Cold flow: capacity rate $C_c=1500$ W/K, $h_c=275$ W/(m²K), $T_{c,i}=15^\circ\text{C}$

Hot flow: capacity rate $C_h=3000$ W/K, $h_h=400$ W/(m²K), $T_{h,i}=150^\circ\text{C}$

Find area A for which hot outlet temperature is $T_{h,o}=110^\circ\text{C}$.

1. Compute heat transfer rate from Eq. (1)

$$\dot{Q} = C_h (T_{h,i} - T_{h,o}) \rightarrow \dot{Q} = 3000 \text{ W/K} (150 - 110) \text{ K} = 120 \text{ kW}$$

2. Compute $T_{c,o}$ from Eq. (2)

$$\dot{Q} = C_c (T_{c,o} - T_{c,i}) \rightarrow T_{c,o} = T_{c,i} + \frac{\dot{Q}}{C_c} = 15^\circ\text{C} + \frac{120 \text{ kW}}{1500 \text{ W/K}} = 95^\circ\text{C}$$

3. Compute overall convective heat transfer rate (ignoring conduction)

$$\frac{1}{h'} = \frac{1}{h_c} + \frac{1}{h_h} = \frac{1}{275} + \frac{1}{400} \rightarrow h' = 162.963 \text{ W/(m}^2 \text{ K)}$$

Cold flow: capacity rate $C_c=1500 \text{ W/K}$, $h_c=275 \text{ W/(m}^2\text{K)}$, $T_{c,i}=15^\circ\text{C}$

Hot flow: capacity rate $C_h=3000 \text{ W/K}$, $h_h=400 \text{ W/(m}^2\text{K)}$, $T_{h,i}=150^\circ\text{C}$

4. Compute log mean temperature difference for counter-flow heat exchanger

$$\Delta T_1 = T_{h,i} - T_{c,o} = 150^\circ\text{C} - 95^\circ\text{C} = 55\text{K}$$

$$\Delta T_2 = T_{h,o} - T_{c,i} = 110^\circ\text{C} - 15^\circ\text{C} = 95\text{K}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = \frac{95\text{K} - 55\text{K}}{\ln(95\text{K} / 55\text{K})} = 73.187\text{K}$$

5. Compute area from $\dot{Q} = h'A\Delta T_{lm}$

$$A = \frac{\dot{Q}}{h'\Delta T_{lm}} = \frac{120\text{kW}}{162.963\text{W/(m}^2\text{K)} \times 73.187\text{K}} = 10.06\text{m}^2$$