Specimin test 16

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1. If

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, \quad \mathbf{B} = (4 \quad 0 \quad 1), \quad \mathbf{C} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix},$$

write down the order (in the form $m \times n$) of each of the above matrices.

$A: 2\times 2$ ✓ $B: 1\times 3$ ✓ $C: 2\times 3$ ✓ $D: 2\times 2$ ✓

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

State which of the following exist and express as a single matrix those which do:

(i) $\mathbf{A} + \mathbf{B}^T$

nope ✓

(ii) $2\mathbf{D} - \mathbf{A}$

$$2\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 3 & -1 \end{pmatrix}$$ ✓

(iii) $\mathbf{CB}$

nope ✓

(iv) $\mathbf{DC}$

$$\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix} = \begin{pmatrix} 2\times 2-1 & 1\times 2+0 & 2\times 0+3 \\ 2+1 & 1 & -3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 3 \\ 3 & 1 & -3 \end{pmatrix}$$ ✓

(v) $\mathbf{C}^T\mathbf{C}$

$$\begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 2 & -3 \\ 2 & 1 & 0 \\ -3 & 0 & 9 \end{pmatrix}$$ ✓

2. State the condition for a matrix $\mathbf{A}$ to be skew-symmetric.

$A^T = -A$ or $a_{ij} = a_{ji}$ for all elements of A ✓

3. For general matrices express each of the following in equivalent terms e.g. $(\mathbf{AB}^T)^T = (\mathbf{B}^T)^T\mathbf{A}^T = \mathbf{BA}^T$

(i) $(\mathbf{B}^T)^T = B$ ✓

$(XY)^T = Y^T X^T$ things flip

(ii) $(\mathbf{BA}^T)^T = B^T A$ ✗

$(BA^T)^T = (A^T)^T B^T = A B^T$

4. Assuming the required sums and products between the matrices **P** are **Q** exist, are the following relations always satisfied?

(i) $\mathbf{P}+\mathbf{Q}=\mathbf{Q}+\mathbf{P}$

yes ✓

(ii) $\mathbf{PQ}=\mathbf{QP}$

no ✓

5. For the matrix $\begin{pmatrix} 4 & 1 & 5 \\ -2 & 3 & -1 \\ -1 & 2 & 2 \end{pmatrix}$

evaluate

(i) the minor of the element in the second row and first column

(ii) the cofactor of the element in the third row and second column

(iii) the cofactor of the element in the first row and third column

i) $\det\begin{pmatrix} 1 & 5 \\ 2 & 2 \end{pmatrix} = 1\times 2 - 5\times 2 = -8$ ✓

ii) $\det\begin{pmatrix} 4 & 5 \\ -2 & -1 \end{pmatrix} = -4 - -10 = 6 \xrightarrow{\times -1} -6$ ✓

iii) $\det\begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix} = -4 - -3 = -1$ ✓

6. Evaluate directly the determinant in question 5 by expanding in terms of elements of the second row.

$$\begin{pmatrix} 4 & 1 & 5 \\ -2 & 3 & -1 \\ -1 & 2 & 2 \end{pmatrix}$$

$$--2\det\begin{pmatrix} 1 & 5 \\ 2 & 2 \end{pmatrix} + 3\det\begin{pmatrix} 4 & 5 \\ -1 & 2 \end{pmatrix} --1\det\begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$$

$$2(2-10) \quad +3(8+5) \quad +(8+1)$$

$$= 32$$ ✓

7. If $\mathbf{A}=\begin{pmatrix} 4 & 2 & 1 \\ 1 & -2 & 3 \\ 0 & 1 & -1 \end{pmatrix}$ determine $\operatorname{adj}\mathbf{A}$.

$$\begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 4 & 3 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 4 & +2 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix} \quad \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix} \quad \begin{pmatrix} 4 & 2 \\ 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & +1 & 1 \\ +3 & \cancel{-1}\ -4 & \cancel{-1}\ -4 \\ 8 & -11 & -10 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 3 & 8 \\ 1 & -4\ \cancel{1} & -11 \\ 1 & -4\ \cancel{1} & -10 \end{pmatrix}$$ ✗