SESA2023 Week 1: Introduction

In this section we will give an introduction to the fundamental concepts and definitions for propulsion.

1.1 Learning outcomes

After completing this section you should be able to:

- Explain the physical principles for air- and spacecraft propulsion
- Explain the difference between rockets and air-breathing propulsion
- Explain the basic working principles of rockets, propellers, ramjets and gas turbines
- Use the concept of control volumes and momentum conservation to derive basic propulsion equations
- Understand the difference between the thermal, propulsive, and overall efficiency
- Derive efficiency equations and explain how they influence engine design considerations
- Use International Standard Atmosphere to obtain air properties at different altitudes

1.2 Thrust from rockets and jet engines

The primary goal of propulsion systems is to generate thrust. Here we will start from the momentum equation that you've learned in part 1 Thermofluids to derive the basic thrust equations for rockets and jet engines. In simple terms, thrust is generated whenever the momentum flux leaving an engine is larger than the momentum flux entering the engine. To make this more precise, and also take into account pressure differences between the inlet and outlet, we define a control volume (CV) and use the momentum conservation equation

$$\dot{M}_{x,out} - \dot{M}_{x,in} = \Sigma F_x, \tag{1.1}$$

where $\dot{M}_{x,out}$ is the momentum flux in the x-direction leaving the CV, $\dot{M}_{x,in}$ is the momentum flux in the x-direction entering the CV, and ΣF_x is the sum of all forces in the x-direction.

1.2.1 Momentum conservation applied to a rocket engine

We now apply the momentum conservation equation to a rocket, as shown in Figure 1.1. We do our analysis in the frame of reference of the rocket, so that the thrust is equal to the force required to hold the rocket (or control volume) in place. The control volume is drawn as a green dashed line, pressures are the blue arrows, and outlet velocity the orange arrow. Because a rocket has no inlet, we can immediately write

$$\dot{M}_{x,in} = 0. \tag{1.2}$$

Assuming a uniform exhaust (jet) velocity V_j relative to the rocket, then the momentum flux out is

$$\dot{M}_{x,out} = \dot{m} V_j, \tag{1.3}$$

where $\dot{m}$ is the mass flow rate of exhaust products. The exhaust products are simply the combusted fuel + oxidizer, so the mass flow rate of exhaust products is therefore equal to the sum of the mass flow rate of fuel and oxidizer. The forces in the x-direction that are acting on the CV are the forces due to the pressure and a resulting force F to hold the CV in place, so we get

$$\Sigma F_x = P_A A_j - P_j A_j + F, \tag{1.4}$$

where P_A is the atmospheric pressure, P_j is the local pressure at the exhaust plane, and A_j is the area of the exhaust plane. Combining all this in equation 1.1, we find the total thrust for a rocket to be

$$F = \dot{m} V_j + A_j (P_j - P_A). \tag{1.5}$$

Note that the thrust of a rocket will depend on the atmospheric pressure, and therefore on the altitude.

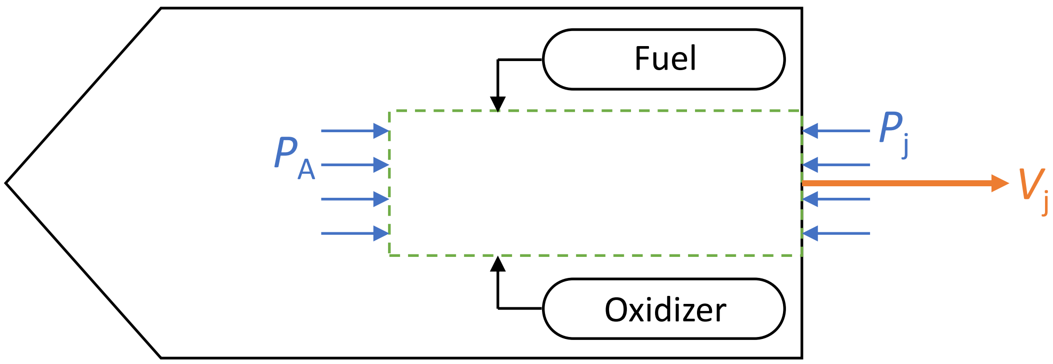

Figure 1.1: Control volume analysis of a rocket.

1.2.2 Momentum conservation applied to a jet engine

The momentum conservation analysis for a jet engine is very similar to that of a rocket engine, but with two differences:

1. We need to consider air entering the CV.
2. There is a difference between mass flow rate in and out of the CV due to the addition of fuel.

The inlet is pure air, and will enter the engine at a velocity V_{in} equal to the speed of the aircraft, which we will denote simply as V. Inside the engine, fuel will be added, so the mass flow rate at the outlet will be the mass flow rate of air + fuel, as shown in figure 1.2. Compared to the rocket, this changes our momentum flux terms to

$$\dot{M}_{x,in} = \dot{m}_a V, \tag{1.6}$$
$$\dot{M}_{x,out} = (\dot{m}_a + \dot{m}_f) V_j, \tag{1.7}$$

and our total thrust equation to

$$F = (\dot{m}_a + \dot{m}_f) V_j - \dot{m}_a V + A_j (P_j - P_A). \tag{1.8}$$

Introducing the fuel-air ratio

$$f = \frac{\dot{m}_f}{\dot{m}_a}, \tag{1.9}$$

we can rewrite the thrust equation as

$$F = \dot{m}_a \left[(1+f)V_j - V \right] + A_j(P_j - P_A). \tag{1.10}$$

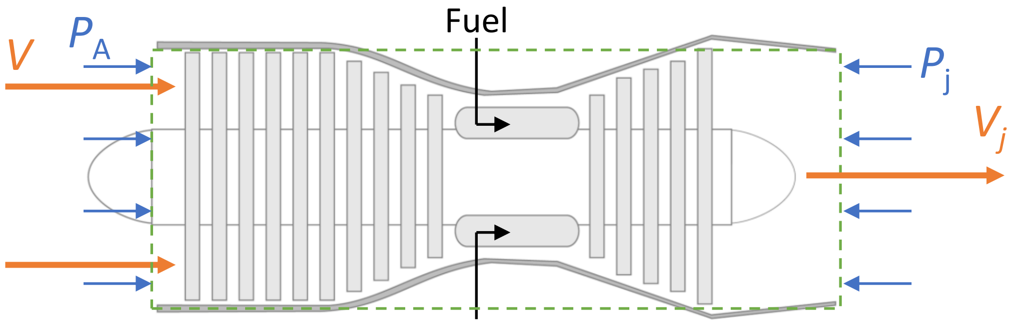

Figure 1.2: Control volume analysis of a jet engine.

1.3 Breguet range equation

One of the key properties of an aircraft is its range, i.e. how far an aircraft can fly without refuelling. We will here do this analysis based on a few key parameters to arrive at the Breguet range equation. Starting with assuming steady flight, so that the thrust F is equal to the drag D, and the lift L is equal to the weight W of the aircraft. Combining these equalities, we can relate the weight to the thrust and the lift-to-drag ratio L/D:

$$W = L = D\left(\frac{L}{D}\right) = F\left(\frac{L}{D}\right). \tag{1.11}$$

As the aircraft is consuming fuel, the weight decreases over time, which we can express as

$$\frac{dW}{dt} = -\dot{m}_f g_0, \tag{1.12}$$

with $\dot{m}_f$ the mass flow rate of fuel and g_0 the acceleration due to gravity. Introducing the *thrust specific fuel consumption*, which is defined as

$$\mathrm{TSFC} = \frac{\dot{m}_f}{F}, \tag{1.13}$$

we can rewrite equation (1.12) as

$$\frac{dW}{dt} = -\mathrm{TSFC} \times F \times g_0 = -\mathrm{TSFC} \times D \times g_0 = -g_0 \times \mathrm{TSFC}\frac{W}{L/D}, \tag{1.14}$$

which can be rearranged to

$$\frac{dW}{W} = -\frac{g_0 \times \mathrm{TSFC}}{L/D} dt. \tag{1.15}$$

This equation could be integrated to obtain the change in mass over a given time, but we are more interested in the distance travelled s. We can convert time to distance travelled easily using the speed of the aircraft V,

$$ds = V\,dt, \tag{1.16}$$

so that equation 1.15 becomes

$$\frac{dW}{W} = -\frac{g_0 \times \mathrm{TSFC}}{V \times L/D} ds, \tag{1.17}$$

which can be integrated from the initial distance (0) to the final distance (s), and the initial weight (W_1) to the final weight (W_2):

$$\int_{W_1}^{W_2} \frac{dW}{W} = -\frac{g_0 \times \text{TSFC}}{V \times L/D} \int_0^s ds, \tag{1.18}$$

which gives

$$\ln\left(\frac{W_2}{W_1}\right) = -\frac{g_0 \times \text{TSFC}}{V \times L/D} s, \tag{1.19}$$

or

$$s = -\frac{L}{D}\frac{V}{g_0 \times \text{TSFC}} \ln\left(\frac{W_2}{W_1}\right). \tag{1.20}$$

This last equation is the *Breguet range equation* and gives the range of an aircraft given its initial and final weight. You will find several versions of the same (or very similar) equations. For example, the lift coefficient C_L and drag coefficient C_D can be used to replace L and D:

$$s = -\frac{C_L}{C_D}\frac{V}{g_0 \times \text{TSFC}} \ln\left(\frac{W_2}{W_1}\right), \tag{1.21}$$

and the thrust specific fuel consumption can be replaced by other engine efficiency parameters as we will see in the following sections.

1.4 Efficiency

In this section we will introduce the different efficiency metrics that make up the overall efficiency of a propulsion system. We will first define the propulsive efficiency η_P, which together with the thermal efficiency η_{th} provides the overall efficiency

$$\eta_O = \eta_P \times \eta_{th}. \tag{1.22}$$

In words, the *thermal efficiency* indicates how well an engine can convert heat into kinetic energy, while the *propulsive efficiency* is a measure of how well an engine can convert the kinetic energy into displacement work. Multiplying these two efficiencies effectively removes the kinetic energy part, and thus directly indicates how well heat is converted into displacement work. After determining how to calculate these efficiencies, we will have a look at how the efficiencies can be increased and how this has influenced the design of propulsion systems.

1.4.1 Propulsive efficiency

The propulsive efficiency is defined as the ratio of aircraft power to the rate of kinetic energy of the working fluid produced by the engine:

$$\eta_P = \frac{\dot{W}_{aircraft}}{\dot{W}_{jet}} \tag{1.23}$$

The aircraft power is the rate useful work done by the engine. Useful work in this context means simply propelling the aircraft forward, so useful work is equal to the thrust multiplied by the displacement of the aircraft. The *rate* of useful work is therefore the thrust F (see section 1.2.2) multiplied by the speed of the aircraft V, which gives

$$\dot{W}_{aircraft} = F \times V = V\left[\dot{m}_a\left((1+f)V_j - V\right) + A_j(P_j - P_A)\right]. \tag{1.24}$$

The engine produces thrust by accelerating a working fluid, therefore increasing the kinetic energy of the working fluid. The rate at which the kinetic energy of the working fluid is increased is equal to the rate at which kinetic energy is leaving the engine minus the rate at which kinetic energy is entering the engine:

$$\dot{W}_{jet} = \tfrac{1}{2}(\dot{m}_a + \dot{m}_f)V_j^2 - \tfrac{1}{2}\dot{m}_a V^2, \tag{1.25}$$

or written making use of the air-fuel ratio f

$$\dot{W}_{jet} = \frac{1}{2}\dot{m}_a\left[(1+f)V_j^2 - V^2\right]. \tag{1.26}$$

Combining (1.24) and (1.26) gives us an expression for the propulsive efficiency

$$\eta_P = \frac{V\left[\dot{m}_a\left((1+f)V_j - V\right) + A_j(P_j - P_A)\right]}{\frac{1}{2}\dot{m}_a\left[(1+f)V_j^2 - V^2\right]}. \tag{1.27}$$

We can make two major simplifications to this equation. The first simplification comes from the design of most civil aircraft such that the jet pressure P_j is very close to the atmospheric pressure P_A. This is called a *fully expanded* jet, and sets the pressure term in the thrust equation to zero ($P_j - P_A = 0$)

$$\eta_P = \frac{V\left[(1+f)V_j - V\right]}{\frac{1}{2}\left[(1+f)V_j^2 - V^2\right]}. \tag{1.28}$$

The second simplification we can make is that the mass flow rate of air is typically much larger than the mass flow rate of fuel, such that $f \ll 1$, resulting in

$$\eta_P = \frac{2V(V_j - V)}{V_j^2 - V^2} = \frac{2}{\frac{V_j}{V} + 1}. \tag{1.29}$$

Note that we can use the same principles to simplify the thrust equation (1.10) to

$$F = \dot{m}_a(V_j - V). \tag{1.30}$$

Figure 1.3 shows a plot of equation (1.29), indicating how the propulsive efficiency changes with the ratio of the exhaust jet velocity to aircraft velocity. Considering a constant aircraft velocity, we can see that the propulsive efficiency drops with increasing exhaust speed, and that the efficiency increases as the exhaust speed approaches the aircraft speed. From a propulsive efficiency point of view, we therefore want to minimize the difference between V_j and V. However, equation (1.30) shows us that the thrust drops as V_j approaches V, and we need a finite amount of thrust – at a minimum to overcome the drag at steady flight.

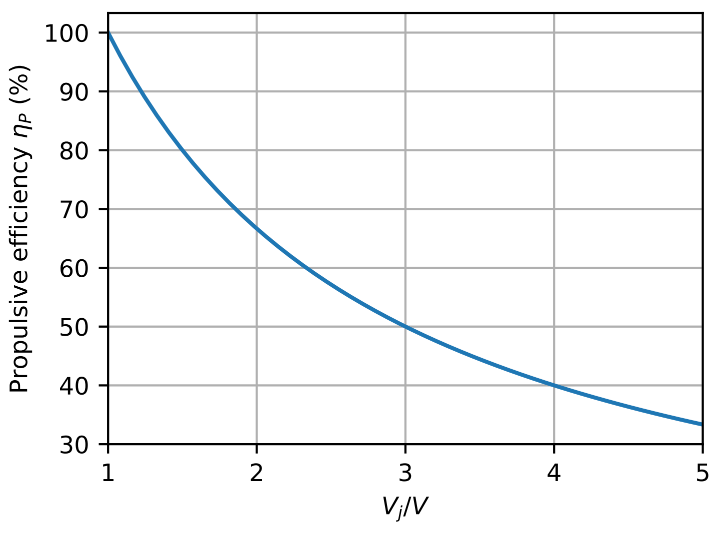

Figure 1.3: Propulsive efficiency as a function of the exhaust jet-aircraft velocity ratio.

So how can we increase the propulsive efficiency without reducing the thrust? Looking at equations (1.29) and (1.30), we see that there is one parameter that changes the thrust, but does not influence the propulsive efficiency: the mass flow rate of air. By increasing $\dot{m}_a$, we can try to keep the thrust constant as we decrease the jet velocity. This fundamental principle is why the industry is moving towards larger engines: to maximize the air intake. This does come with its own design challenges, such as an increase in drag, ground clearance, and engine weight.

1.4.2 Thermal and overall efficiency

Before deriving the thermal efficiency, we will first find an expression for the overall efficiency, because this is a relatively straightforward definition. What we want to engine to provide is thrust power $F \times V$, as we have determined earlier, and what we have to provide is basically heat by burning fuel. The amount of heat that we supply is the mass flow rate of fuel $\dot{m}_f$, multiplied by the *lower calorific value* (LCV) of the fuel, which is the amount of heat that the fuel releases per unit mass. So we can write the overall efficiency as

$$\eta_O = \frac{\dot{W}_{aircraft}}{\dot{Q}_{in}} = \frac{F \times V}{\dot{m}_f\, LCV}. \tag{1.31}$$

Using the approximations of a fully expanded exhaust and $f \ll 1$, this simplifies to

$$\eta_O = \frac{\dot{m}_a (V_j - V) V}{\dot{m}_f\, LCV} = \frac{(V_j - V)V}{f \times LCV}. \tag{1.32}$$

Or, using the definition of thrust specific fuel consumption, defined in equation (1.13), we can also write

$$\eta_O = \frac{F}{\dot{m}_f} \frac{V}{LCV} = \frac{1}{TSFC} \frac{V}{LCV}. \tag{1.33}$$

The thermal efficiency of a jet engine, as explained above, is the measure of how well an engine can convert heat into kinetic energy. The input is the heat released by the fuel per unit time $\dot{Q}_{in}$, and the output is the rate of increase in kinetic energy of the working fluid $\dot{W}_{jet}$. Again assuming $f \ll 1$, this becomes

$$\eta_{th} = \frac{\dot{W}_{jet}}{\dot{Q}_{in}} = \frac{\frac{1}{2}\dot{m}_a (V_j^2 - V^2)}{\dot{m}_f\, LCV} = \frac{1}{2} \frac{V_j^2 - V^2}{f \times LCV} \tag{1.34}$$

You can verify that multiplying the thermal efficiency (1.34) with the propulsive efficiency (1.29) indeed results in the overall efficiency (1.32).

The reason for splitting up the overall efficiency in the thermal and propulsive part is so that we can more easily see how to optimize each component. We already observed that the propulsive efficiency can be improved by reducing the difference between the jet speed and aircraft speed, and consequently the need for large air mass flow rates. For the thermal efficiency, we can use our knowledge of Brayton cycles from part 1 thermofluids. We will go into more detail later in the module, but for an ideal Brayton cycle the thermal efficiency will increase with the overall pressure ratio, which is directly related to the temperature ratio. We see the result of this in the trend of production engines shown in Figure 1.4 The pressure ratio has increased from about 15 to well over 40 in about 40 years. Similarly the turbine inlet temperature has increased from just over 1000 K to nearly 1800 K. Note that this is well above the material limit for the turbine blades, which is made possible largely due to blade cooling, which we will also look at in more depth later in the module.

Figure 1.5 shows the trends in efficiencies of different engines. Note that current engines have a propulsive efficiency between 0.6 and 0.7, and a thermal efficiency between 0.5 and 0.6. This puts their overall efficiency between 0.3 and 0.4, and shows a need for improvement.

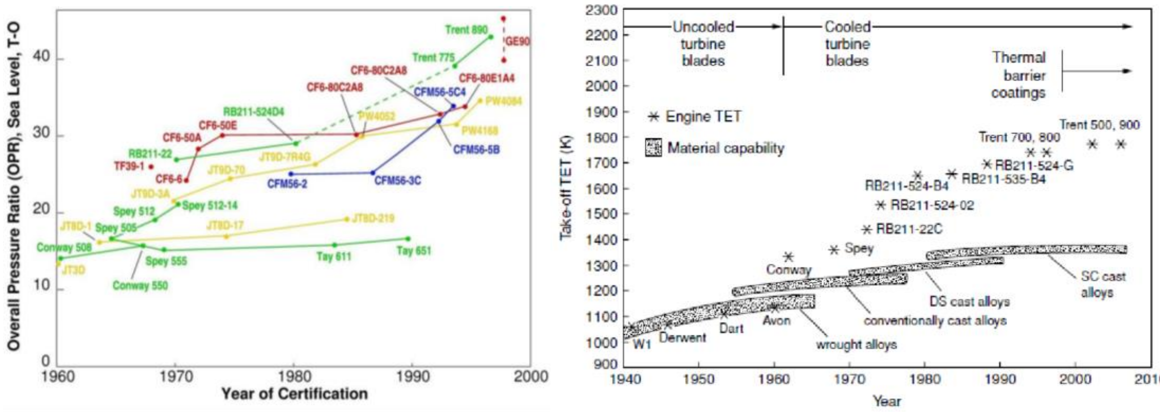

Figure 1.4: Pressure ratio trends for commercial transport engines (Epstein 1998) and trends in turbine inlet temperature (Jet Propulsion, Cumpsty).

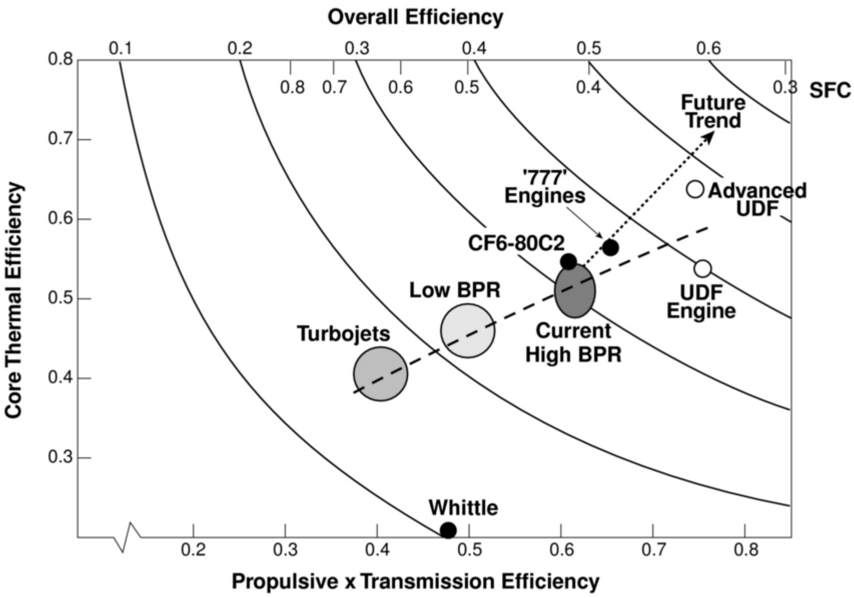

Figure 1.5: Trends in aircraft efficiency (Pratt & Whitney)

1.5 Altitude Tables

Because we design engines to operate at altitude, we need to know how the properties of air change. To take a consistent approach, we will make use of the International Standard Atmosphere. Tables with properties (pressure, temperature, and density) can be found in Appendix A of Mattingly and Boyer, or on Blackboard under Data Books. The tables give ratios to reference values defined as

$$\delta = \frac{P}{P_{\text{ref}}}, \tag{1.35}$$

$$\theta = \frac{T}{T_{\text{ref}}}, \tag{1.36}$$

$$\sigma = \frac{\rho}{\rho_{\text{ref}}}, \tag{1.37}$$

with the reference values given below the tables.

Instead of using the tables, you can also use the Jupyter notebook provided in the additional material for week 1.