SESA3029 Aerothermodynamics

Bell X-1

Lecture 4.6

Optimum shape of aerofoils for supersonic flight?

F-22

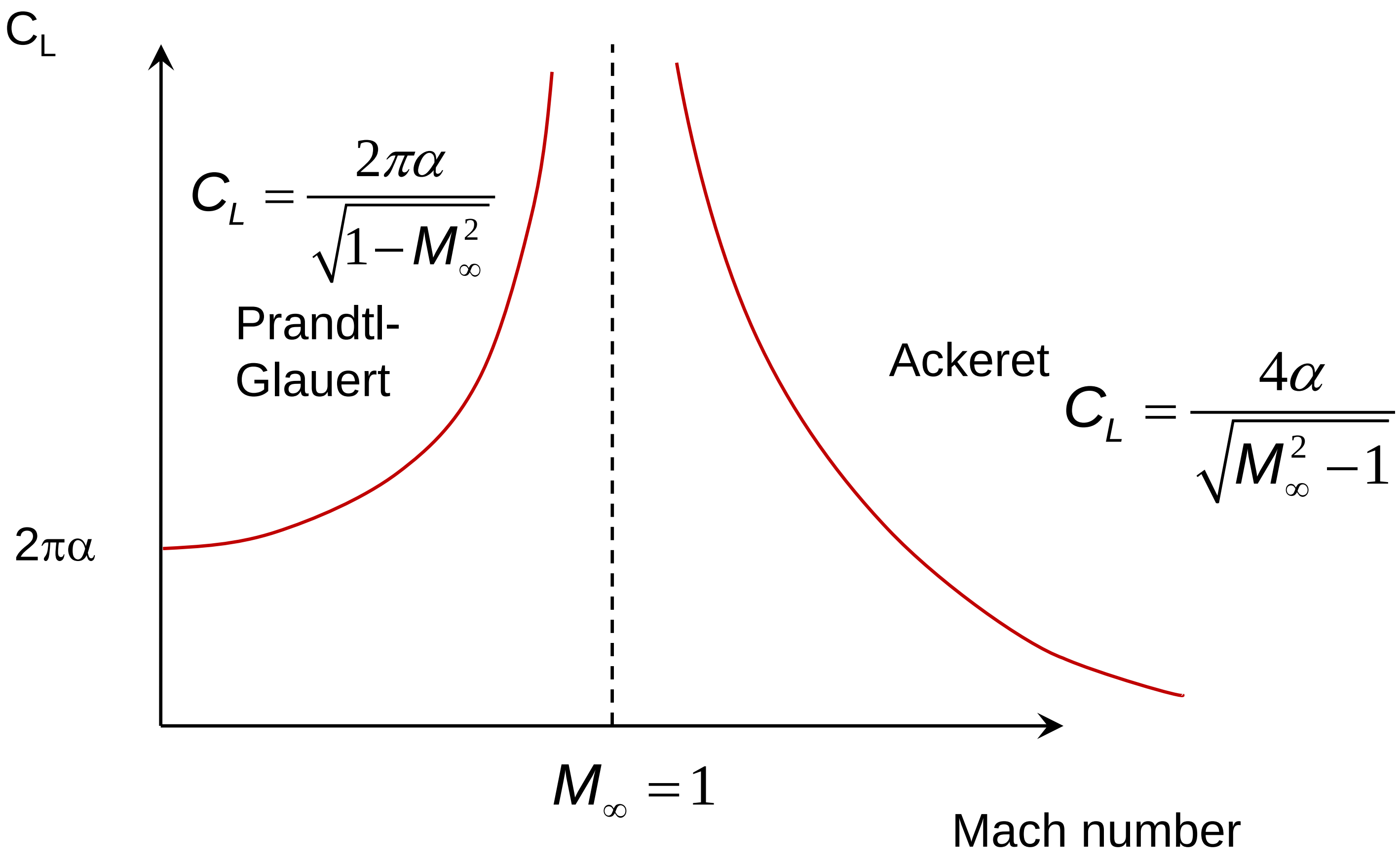
Recap: inviscid theory for a flat plate

both suck at $M \approx 1$

CFD used here

$$\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}}$$

Ackeret formula



General airfoil shape

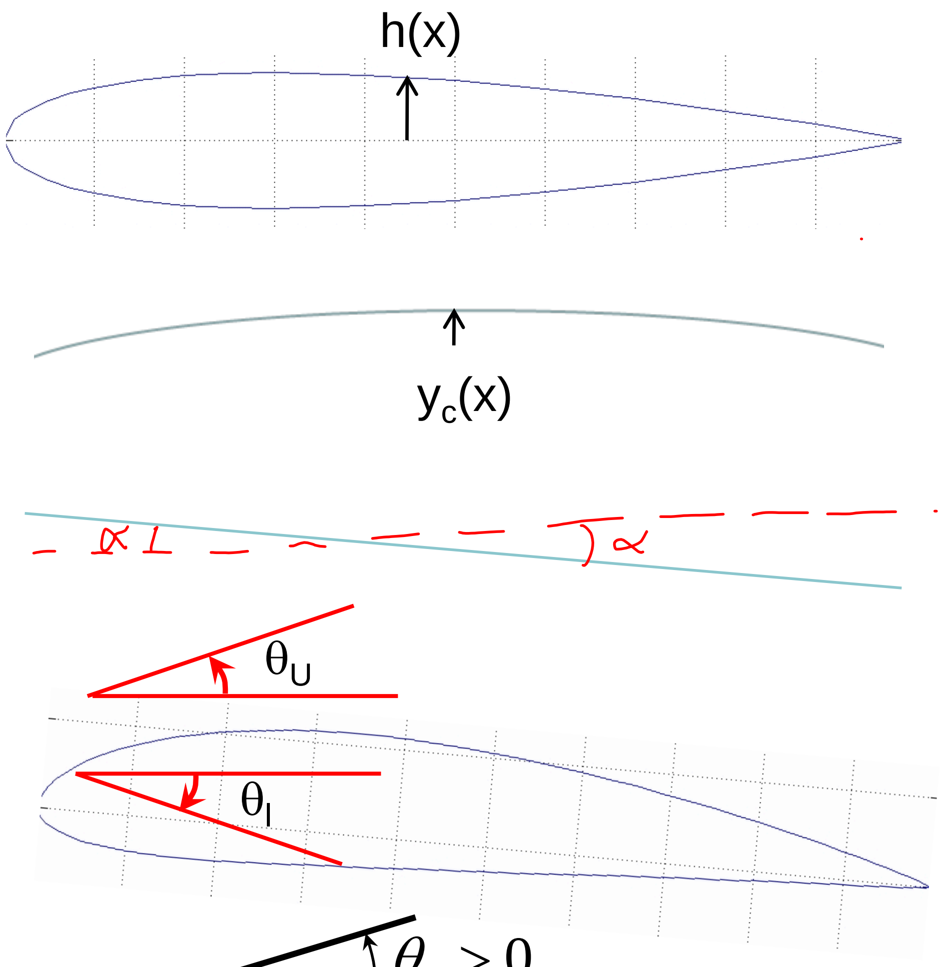
needed so we can parametrise shape for optimization process!

Thickness

+camber

+incidence α

= airfoil

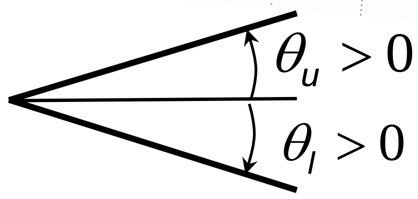


Camber line

Chord line

u=upper
l=lower

Ackeret $$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$



Sign convention

$$\theta_u = \frac{dh}{dx} + \frac{dy_c}{dx} - \alpha$$

local theta (upper)

local theta (lower)

$$\theta_l = \frac{dh}{dx} - \frac{dy_c}{dx} + \alpha$$

Lift coefficient

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

$$\theta_l = \frac{dh}{dx} - \frac{dy_c}{dx} + \alpha$$

$$\theta_u = \frac{dh}{dx} + \frac{dy_c}{dx} - \alpha$$

already using small angle approx!

$$C_L = \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right) dx$$

sub in and expand

$$= \frac{4}{c\sqrt{M_\infty^2 - 1}}\int_0^c \left(\alpha - \frac{dy_c}{dx}\right) dx$$

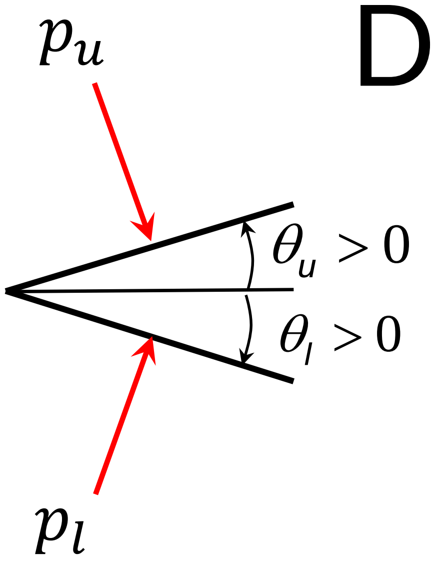
Thickness term cancels out

$= \int_0^c \frac{dy_c}{dx} dx = \int_0^0 dy_c = 0$ · y_c is zero at both ends

$$= \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

Since y_c=0 at lower and upper limits

Same result as for flat plate. No influence of camber or thickness on lift in this inviscid thin airfoil limit.



Drag coefficient

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

$$\theta_l = \frac{dh}{dx} - \frac{dy_c}{dx} + \alpha$$

$$\theta_u = \frac{dh}{dx} + \frac{dy_c}{dx} - \alpha$$

$$C_D = \frac{1}{c}\int_0^c \left(C_{p,l}\theta_l + C_{p,u}\theta_u\right)dx \;=\frac{2}{c\sqrt{M_\infty^2-1}}\int_0^c \left(\theta_l^2+\theta_u^2\right)dx$$

expand and cancel

$$=\frac{2}{c\sqrt{M_\infty^2-1}}\int_0^c \left[\alpha^2 + 2\alpha\cancelto{0}{\left(\frac{dh}{dx}-\frac{dy_c}{dx}\right)} + \left(\frac{dh}{dx}-\frac{dy_c}{dx}\right)^2 + \alpha^2 - 2\alpha\cancelto{0}{\left(\frac{dh}{dx}+\frac{dy_c}{dx}\right)} + \left(\frac{dh}{dx}+\frac{dy_c}{dx}\right)^2\right]dx$$

$$\left(\frac{dh}{dx}-\frac{dy}{dx}\right)^2 + \left(\frac{dh}{dx}+\frac{dy}{dx}\right)^2 = \underbrace{2\frac{dh}{dx}^2 + 2\frac{dy}{dx}^2}$$

cancel terms

$$C_D = \frac{4}{c\sqrt{M_\infty^2-1}}\int_0^c \left[\alpha^2 + \underbrace{\left(\frac{dh}{dx}\right)^2 + \left(\frac{dy_c}{dx}\right)^2}\right]dx$$

$$C_D = \frac{4}{c\sqrt{M_\infty^2 - 1}} \int_0^c \left[\alpha^2 + \left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)^2 + \underbrace{\left(\frac{\mathrm{d}y_c}{\mathrm{d}x}\right)^2}_{\text{can bring this to zero easily}} \right] dx$$

Incidence, thickness and camber all contribute to drag, so we prefer thin uncambered sections for supersonic flight

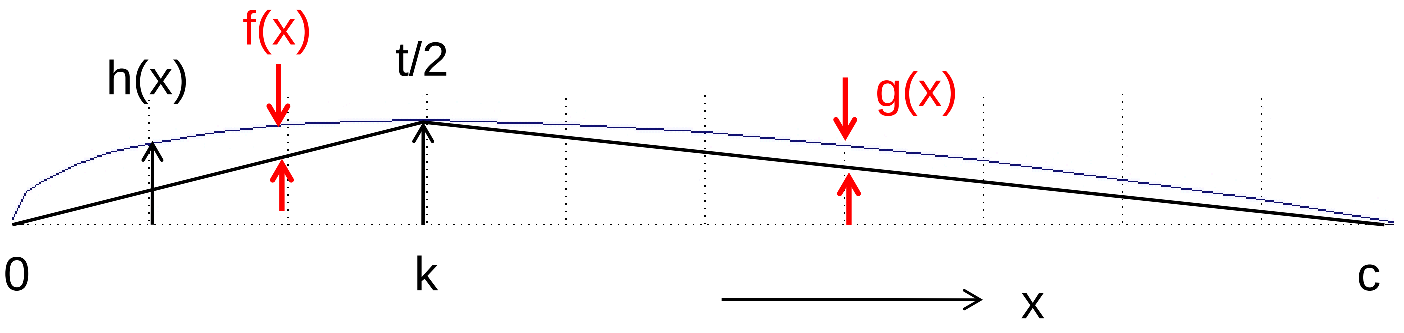
$\frac{dy_c}{dx} = 0$

For minimum wave drag we need to minimise the thickness term

$$\int_0^c \left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)^2 \mathrm{d}x$$

Thickness distribution for minimum wave drag

f(x) and g(x) are functions describing foil geometry change from straight lines!



Let h(x) be the addition of a straight line segment and an arbitrary function (zero at start and end points)

For x<k $\quad h(x) = \dfrac{tx}{2k} + f(x)$ $\quad \dfrac{d}{dx}$

$$\frac{dh}{dx} = \frac{t}{2k} + \frac{df}{dx}$$

For x>k $\quad h(x) = \dfrac{t(c-x)}{2(c-k)} + g(x)$ $\quad \dfrac{d}{dx}$

$$\frac{dh}{dx} = \frac{-t}{2(c-\cancel{x})} + \frac{dg}{dx}$$

$$\text{Minimise} \quad \int_0^c \left(\frac{dh}{dx}\right)^2 dx = \int_0^k \left[\left(\frac{t}{2k}\right)^2 + \overset{0}{\cancel{\frac{t}{k}\frac{df}{dx}}} + \left(\frac{df}{dx}\right)^2\right] dx + \int_k^c \left[\left(\frac{t}{2(c-k)}\right)^2 - \overset{0}{\cancel{\frac{t}{(c-k)}\frac{dg}{dx}}} + \left(\frac{dg}{dx}\right)^2\right] dx$$

same for t

g is zero at c and k

since terms are squared

f and *g* only add drag, so minimum drag for *f=g=0* i.e. faceted airfoils

just straight lines!

$\int \frac{dg}{dx} dx = 0$

What is the optimum k?

$$\int_0^c \left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)^2 \mathrm{d}x = \int_0^k \left(\frac{t}{2k}\right)^2 \mathrm{d}x + \int_k^c \left(\frac{t}{2(c-k)}\right)^2 \mathrm{d}x$$

$$= \frac{t^2}{4}\left[\frac{1}{k} + \frac{1}{c-k}\right]$$

$$\frac{1}{c}\int_0^c \left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)^2 \mathrm{d}x = \left(\frac{t}{c}\right)^2$$

Best k when we have a minimum drag contribution

$$\frac{\mathrm{d}}{\mathrm{d}k}\left[\frac{1}{k} + \frac{1}{c-k}\right] = 0$$

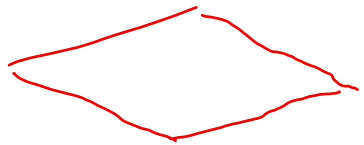
$$\frac{-1}{k^2} + \frac{1}{(c-k)^2} = 0$$

$$k^2 = c^2 - 2kc + k^2$$

$$k = \frac{c}{2}$$

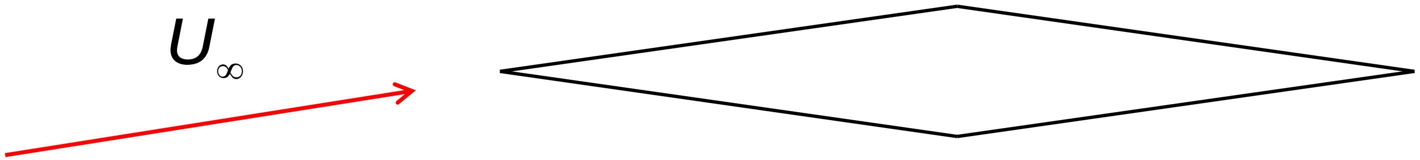
Maximum thickness at half chord

i.e. a diamond shaped airfoil has the lowest wave drag



Optimum (minimum wave drag) airfoil for supersonic flight

design will clearly suck for subsonic flow!



$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \qquad C_D = \frac{4}{\sqrt{M_\infty^2 - 1}}\left[\alpha^2 + \left(\frac{t}{c}\right)^2\right]$$

- Lowest drag when as thin as possible (within structural constraints)
- Ignores effects of viscosity and off-design issues (take-off, landing and low-speed performance)

(X-1-1) NACA 65-110
(X-1-2) NACA 65-108
(X-1E) NACA 64A004

F-22: also NACA sections

64A005.92 (root) and 64A004.29 (tip)