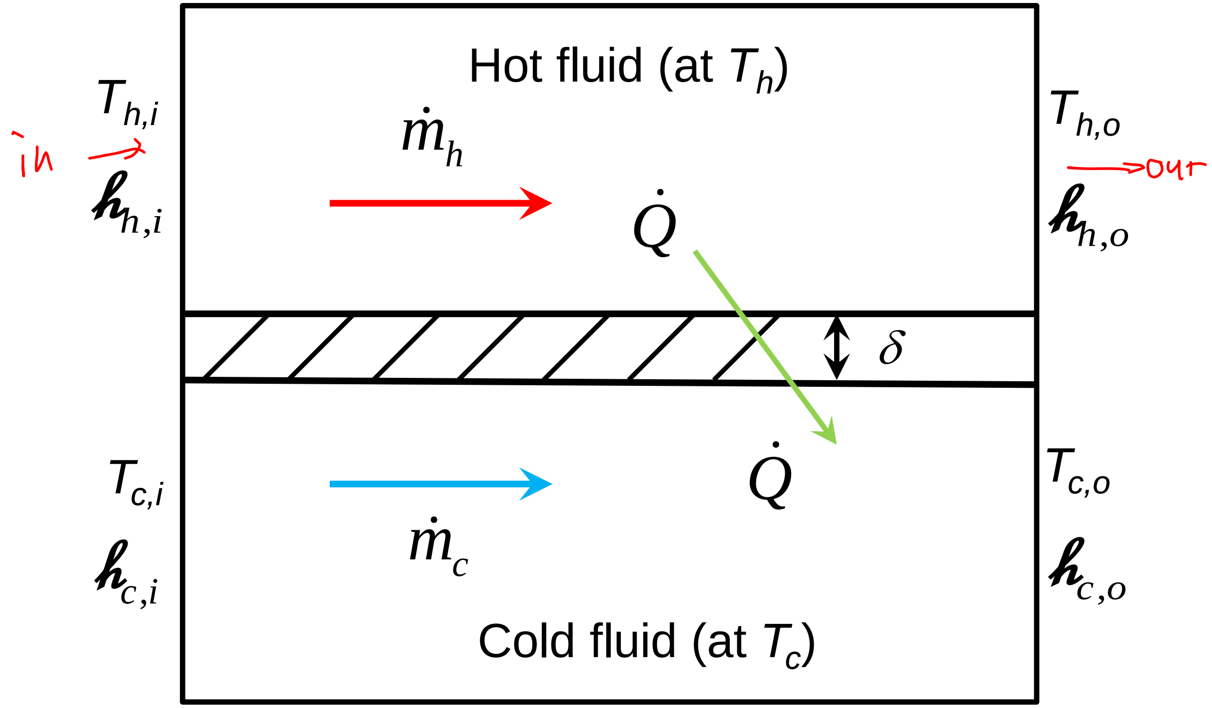
SESA3029
Aerothermodynamics

Lecture 5.8

Heat exchangers -

Log mean temperature difference method

Heat exchanger – General equations



h: hot
c: cold
i: inlet
o: outlet
$\dot{m}$ *: mass flow*
h *: enthalpy*
c_p*: fluid heat capacity*

Parallel-flow heat exchanger depicted but (1) and (2) are the same for the counter-flow heat exchanger!

obviously heat is conserved

Evaluation of heat transfer rate with inlet and outlet temperatures:

$$\dot{Q} = \dot{m}_h \left(h_{h,i} - h_{h,o} \right) = \dot{m}_h c_{p,h} \left(T_{h,i} - T_{h,o} \right) = C_h \left(T_{h,i} - T_{h,o} \right) \qquad (1)$$

$$\dot{Q} = \dot{m}_c \left(h_{c,o} - h_{c,i} \right) = \dot{m}_c c_{p,c} \left(T_{c,o} - T_{c,i} \right) = C_c \left(T_{c,o} - T_{c,i} \right) \qquad (2)$$

C_h and C_c are the ***capacities***. → non-specific constant pressure heat capacity

Heat exchanger – General equations

On a unit area basis:

series resistance?

$$\frac{1}{h'} = \frac{1}{h_h} + \frac{\delta}{k_w} + \frac{1}{h_c}$$

h_h and h_c are the heat transfer coefficients on **h**ot and **c**old side and k_w is the wall conductivity

In practice δ is usually taken as 0

Since the temperature difference $\Delta T = T_h - T_c$ is spatially varying, a suitable **mean** temperature difference needs to be found to evaluate the heat transfer rate of the overall system as

$$\dot{Q} = h' A \Delta T_m$$

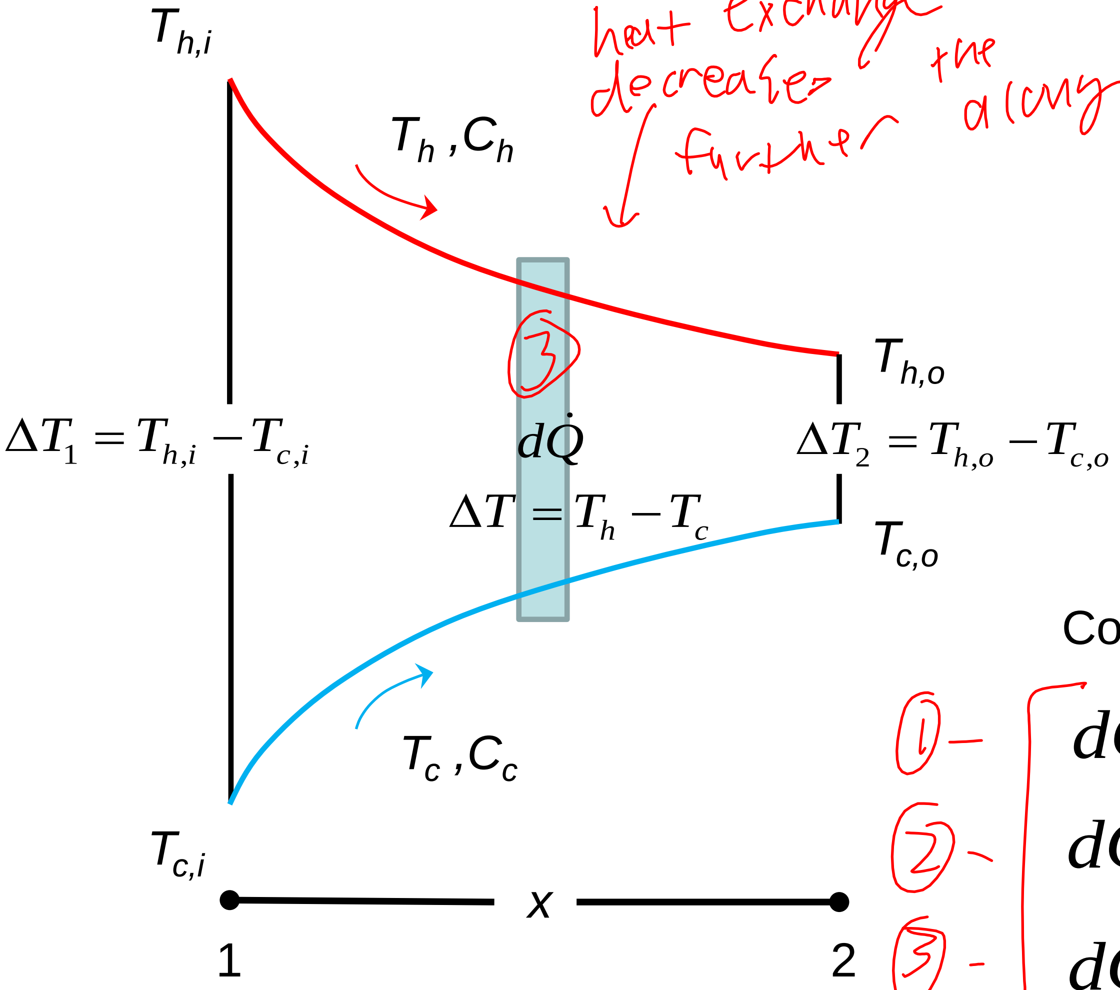
This leads to the *log mean temperature difference* method.

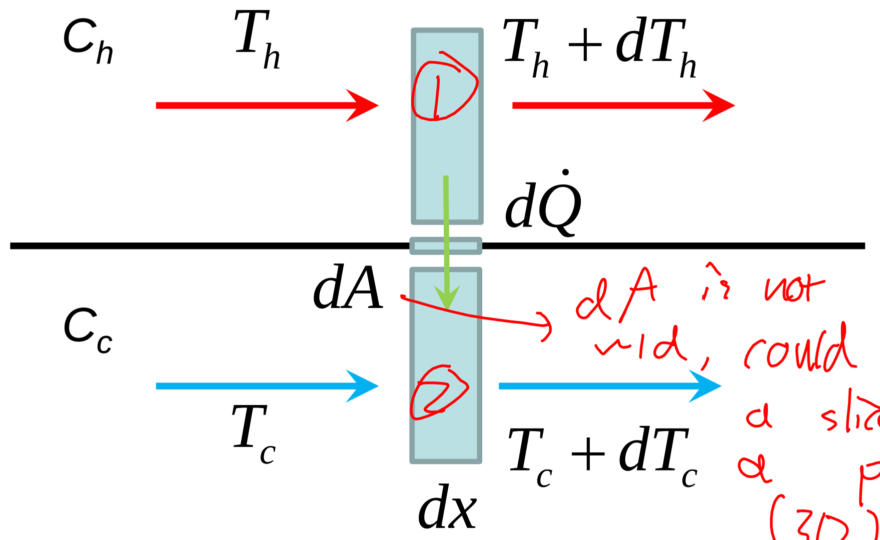
Derivation follows Bergman et al., pages 713-715 / Incropera et al., pages 659-662 closely.

no seperating solid boundry

Parallel-flow heat exchanger

heat exchange rate decreases the further along





dA is not nessisarily uniform, could be for a slice of a pipe! (3D)

Construct heat balance on elements *dx dA*:

1 – $$d\dot{Q} = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h \qquad (3)$$ heat loss side

2 – $$d\dot{Q} = \dot{m}_c c_{p,c} dT_c = C_c dT_c \qquad (4)$$ heat absorbed side

3 – $$d\dot{Q} = h' dA \Delta T \qquad (5)$$ total temp differ

assume loss-less system

Starting from $d(\Delta T) = dT_h - dT_c$ and using (3) and (4) together with (5), we obtain

subbed in and rearranged

$$d(\Delta T) = -h' dA \Delta T \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

Now integrate from location 1 to 2, i.e.,

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -h' \left(\frac{1}{C_h} + \frac{1}{C_c} \right) \int_{1 \,(\text{inlet})}^{2 \,(\text{outlet})} dA$$

and use the general heat exchanger relations (1) and (2) to replace the capacities to obtain

$$\ln(\Delta T_2) - \ln(\Delta T_1) = -h'A \left(\frac{T_{h,i} - T_{h,o}}{\cancel{Q}} \cancel{+} + \frac{T_{c,o} - T_{c,i}}{\cancel{Q}} \right) \quad \dot{Q} \qquad (6)$$

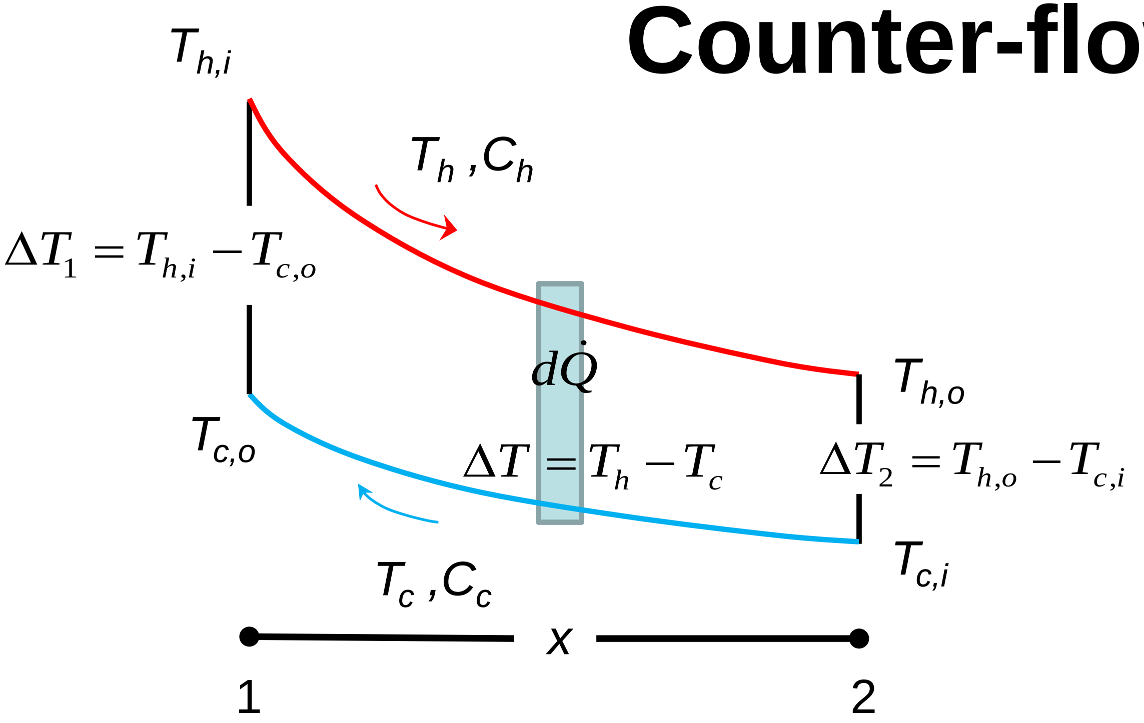
Inserting $\Delta T_1, \Delta T_2$ for the parallel-flow heat exchanger gives

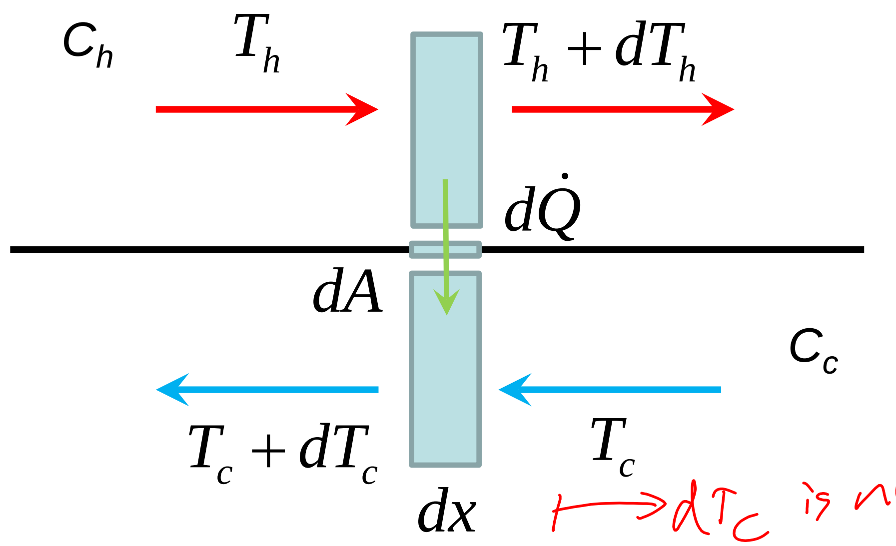
$$\ln\left(\frac{\Delta T_2}{\Delta T_1} \right) = -\frac{h'A}{\dot{Q}} (\Delta T_1 - \Delta T_2)$$

logarithmically weighted temp difference

from which we readily deduce $\dot{Q} = h'A\Delta T_{lm}$ with $\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

Counter-flow heat exchanger





$\mapsto dT_c$ is negative

Construct heat balance on elements *dx dA*:

$$d\dot{Q} = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h \qquad (7)$$

$$d\dot{Q} = -\dot{m}_c c_{p,c} dT_c = -C_c dT_c \qquad (8)$$

Starting from $d(\Delta T) = dT_h - dT_c$ and using (7) and (8) together with (5), we obtain after integration from position 1 to 2

$= \frac{h'A}{\dot{Q}} (\Delta T_2 - \Delta T_1)$

$$\ln(\Delta T_2) - \ln(\Delta T_1) = -h'A\left(\frac{T_{h,i} - T_{h,o}}{\dot{Q}} - \frac{T_{c,o} - T_{c,i}}{\dot{Q}}\right) \qquad (9)$$

i.e. $\dot{Q} = h'A\Delta T_{lm}$ with ΔT_{lm} as before.

Remember differences in ΔT_1 and ΔT_2!

Note that for this case the LMTD method is not applicable for $C_h = C_c$ and (5) needs to be integrated directly.

Example: Counter-flow heat exchanger

Cold flow: capacity rate C_c=1500 W/K, h_c=275 W/(m^{2}K), $T_{c,i}$=15°C

Hot flow: capacity rate C_h=3000 W/K, h_h=400 W/(m^{2}K), $T_{h,i}$=150°C

Find area A for which hot outlet temperature is $T_{h,o}$=110°C.

1. Compute heat transfer rate from Eq. (1)

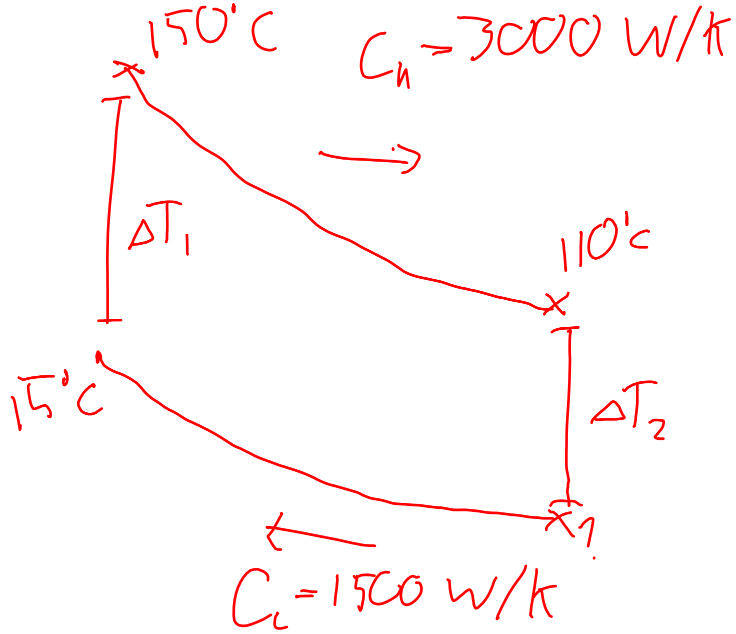
$$\dot{Q} = C_h\left(T_{h,i} - T_{h,o}\right) \quad \rightarrow \quad \dot{Q} = 3000\,\mathrm{W/\,K}\left(150 - 110\right)\mathrm{K} = 120\mathrm{kW}$$

2. Compute $T_{c,o}$ from Eq. (2)

$$\dot{Q} = C_c\left(T_{c,o} - T_{c,i}\right) \quad \rightarrow \quad T_{c,o} = T_{c,i} + \frac{\dot{Q}}{C_c} = 15^{\mathrm{o}}\mathrm{C} + \frac{120\mathrm{kW}}{1500\mathrm{W/K}} = 95^{\mathrm{o}}\mathrm{C}$$

3. Compute overall convective heat transfer rate (ignoring conduction)

$$\frac{1}{h'} = \frac{1}{h_c} + \frac{1}{h_h} = \frac{1}{275} + \frac{1}{400} \quad \rightarrow \quad h' = 162.963\,\mathrm{W/(m^2\,K)}$$



$$\dot{Q} = C_h (T_{h_i} - T_{h_o}) = 120000\,W$$

$$\dot{Q} = C_c (T_{co} - T_{ci}) \longrightarrow T_{co} = \frac{\dot{Q}}{C_c} + T_{ci} = T_{co} = 95°C$$

Cold flow: capacity rate C_c=1500 W/K, h_c=275 W/(m^{2}K), $T_{c,i}$=15°C

Hot flow: capacity rate C_h=3000 W/K, h_h=400 W/(m^{2}K), $T_{h,i}$=150°C

4. Compute log mean temperature difference for counter-flow heat exchanger

$$\Delta T_1 = T_{h,i} - T_{c,o} = 150^{\circ}\mathrm{C} - 95^{\circ}\mathrm{C} = 55\mathrm{K}$$

$$\Delta T_2 = T_{h,o} - T_{c,i} = 110^{\circ}\mathrm{C} - 15^{\circ}\mathrm{C} = 95\mathrm{K}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = \frac{95\mathrm{K} - 55\mathrm{K}}{\ln(95\mathrm{K} / 55\mathrm{K})} = 73.187\mathrm{K}$$

5. Compute area from $\dot{Q} = h' A \Delta T_{lm}$

$$A = \frac{\dot{Q}}{h' \Delta T_{lm}} = \frac{120\mathrm{kW}}{162.963\mathrm{W/(m^2K)} \times 73.187\mathrm{K}} = 10.06\mathrm{m}^2$$