SESA2025 Mechanics of Flight
Lateral dynamics

Lecture 3.8

Decoupled linearised equations

Longitudinal equations:

$$\Delta X = m\dot{u}$$

$$\Delta Z = m(\dot{w} - qU_\infty)$$

$$\Delta M = I_{yy}\dot{q}$$

Longitudinal equations
three equations
three unknowns: u, w, q

and lateral equations:

$$\Delta Y = m(\dot{v} + rU_\infty)$$

$$\Delta L = I_{xx}\dot{p} - I_{xz}\dot{r}$$

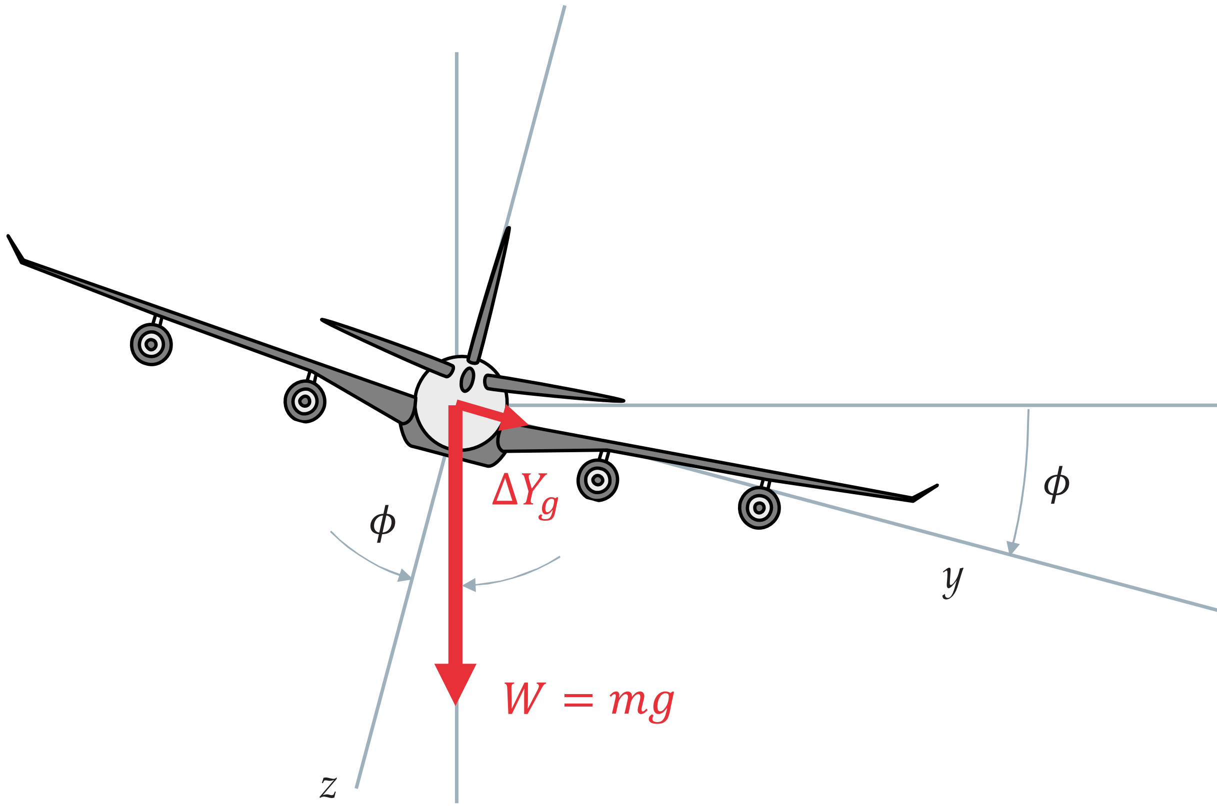
$$\Delta N = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

Lateral equations
three equations
three unknowns: v, p, r

Gravitational and aerodynamic out-of-balance contributions

assume zero climb angle

$$\Delta Y_g = mg\sin\phi$$
$$=_1 mg\phi \qquad \text{for small angles}$$



$$\Delta Y_a = \mathring{Y}_v v + \mathring{Y}_p p + \mathring{Y}_r r$$

$$\Delta L_a = \mathring{L}_v v + \mathring{L}_p p + \mathring{L}_r r$$

$$\Delta N_a = \mathring{N}_v v + \mathring{N}_p p + \mathring{N}_r r$$

Same method as before to convert dimensionless (Y_v, N_p etc) to dimensional derivatives, but now the reference length is the span, b

Dimensional vs dimensionless aerodynamic derivatives

Important for later

$$\mathring{Y}_v = Y_v \cdot \frac{1}{2}\rho U_\infty^2 S\left(\frac{1}{U_\infty}\right)$$
$$= Y_v \cdot \frac{1}{2}\rho U_\infty S$$

$$\mathring{Y}_p = Y_p \cdot \frac{1}{2}\rho U_\infty^2 S\left(\frac{b}{U_\infty}\right)$$
$$= Y_p \cdot \frac{1}{2}\rho U_\infty S b$$

etc

$$\mathring{L}_v = L_v \cdot \frac{1}{2}\rho U_\infty^2 S b\left(\frac{1}{U_\infty}\right)$$
$$= L_v \cdot \frac{1}{2}\rho U_\infty S b$$

$$\mathring{L}_p = L_p \cdot \frac{1}{2}\rho U_\infty^2 S b\left(\frac{b}{U_\infty}\right)$$
$$= L_p \cdot \frac{1}{2}\rho U_\infty S b^2$$

State equation (lateral motion)

Combine them all:

$$
\begin{array}{lcll}
\Delta Y = m\left(\dot{v} + rU_\infty\right) & & \Delta Y_a = \mathring{Y}_v v + \mathring{Y}_p p + \mathring{Y}_r r & & \Delta Y_g = mg\phi \\
\Delta L = I_{xx}\dot{p} - I_{xz}\dot{r} & = & \Delta L_a = \mathring{L}_v v + \mathring{L}_p p + \mathring{L}_r r & + & 0 \\
\Delta N = -I_{xz}\dot{p} + I_{zz}\dot{r} & & \Delta N_a = \mathring{N}_v v + \mathring{N}_p p + \mathring{N}_r r & & 0
\end{array}
$$

and rearrange them into matrix (state-space) form:

$$
\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 \\ 0 & -I_{xz} & I_{zz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix}
=
\begin{bmatrix} \mathring{Y}_v & \mathring{Y}_p & \mathring{Y}_r - mU_\infty & mg \\ \mathring{L}_v & \mathring{L}_p & \mathring{L}_r & 0 \\ \mathring{N}_v & \mathring{N}_p & \mathring{N}_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}
$$

State equation (Lateral motion)

Matrix (state space) form for decoupled linearised lateral motion:

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 \\ 0 & -I_{xz} & I_{zz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \mathring{Y}_v & \mathring{Y}_p & \mathring{Y}_r - mU_\infty & mg \\ \mathring{L}_v & \mathring{L}_p & \mathring{L}_r & 0 \\ \mathring{N}_v & \mathring{N}_p & \mathring{N}_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

Which can also be written as:

$$\mathbf{M}\dot{\mathbf{x}} = \mathbf{A}'\mathbf{x}$$

or

$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ with $\mathbf{A} = \mathbf{M}^{-1}\mathbf{A}'$

Example: Navion Rangemaster H

Bosscat65 on Wikimedia [accessed on 13 February 2019]

Navion Rangemaster H

Aircraft data sheet

Aircraft	
Mean aerodynamic chord (m)	1.679
Wing span (m)	10.18
Reference area (m^2)	17.09
Density (kg/m^3)	1.225
Flight Speed (m/s)	53.75
Aircraft mass (kg)	1247
I_{xx} (kg m^2)	1421
I_{yy} (kg m^2)	4068
I_{zz} (kg m^2)	4787
I_{xz} (kg m^2)	0

X_u	-0.1
X_w	0.073
Z_u	-0.806
Z_w	-4.57
$Z_{\dot{w}}$	-1.153
Z_q	-2.5
M_u	0.0
M_w	-1.159
$M_{\dot{w}}$	-3.102
M_q	-6.752

Y_v	-0.564
L_v	-0.074
L_p	-0.205
L_r	0.0535
N_v	0.0701
N_p	-0.02875
N_r	-0.0625

$X_{elevator}$	0.0
$Z_{elevator}$	-0.5
$M_{elevator}$	-1.35
$X_{throttle}$	1.0
Y_{rudder}	0.156
L_{rudder}	0.0118
N_{rudder}	-0.0717
$Y_{aileron}$	0.0
$L_{aileron}$	-0.1352
$N_{aileron}$	-0.00346

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Navion Rangemaster H

Numerical example

The state equation is a first order linear system of ODE:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

So we will look for solution of the following form: $\mathbf{x} = \mathbf{x}_0 e^{\lambda t}$; $\dot{\mathbf{x}} = \lambda \mathbf{x}_0 e^{\lambda t}$

which has nontrivial solutions is the determinant of $\mathbf{A}$ is zero.

Lateral quartic with two real and two complex roots:

$$(\lambda + 0.0087)(\lambda + 8.4442)(\lambda^2 + 0.9744\lambda + 5.7040) = 0$$

$$\lambda_{3,4} = -0.4872 \pm i2.3381$$

A lightly damped mode, known as the slow spiral mode

A heavily damped mode, known as roll subsidence

An oscillatory mode, known as dutch roll

no oscillitary motion.

Spiral Mode (Slow Process)

In case of low wing with insufficient dihedral or sweep

- Slow process due to yaw damping & roll damping

Forces and moments in spiral divergence

- Sideslip causes side-force on fin in turn causing yaw, and aircraft enters a curved path. Extra velocity on outer wing causes roll leading to further sideslip and divergence. Dihedral or sweep will lead to opposite rolling moment tending to stabilise motion.

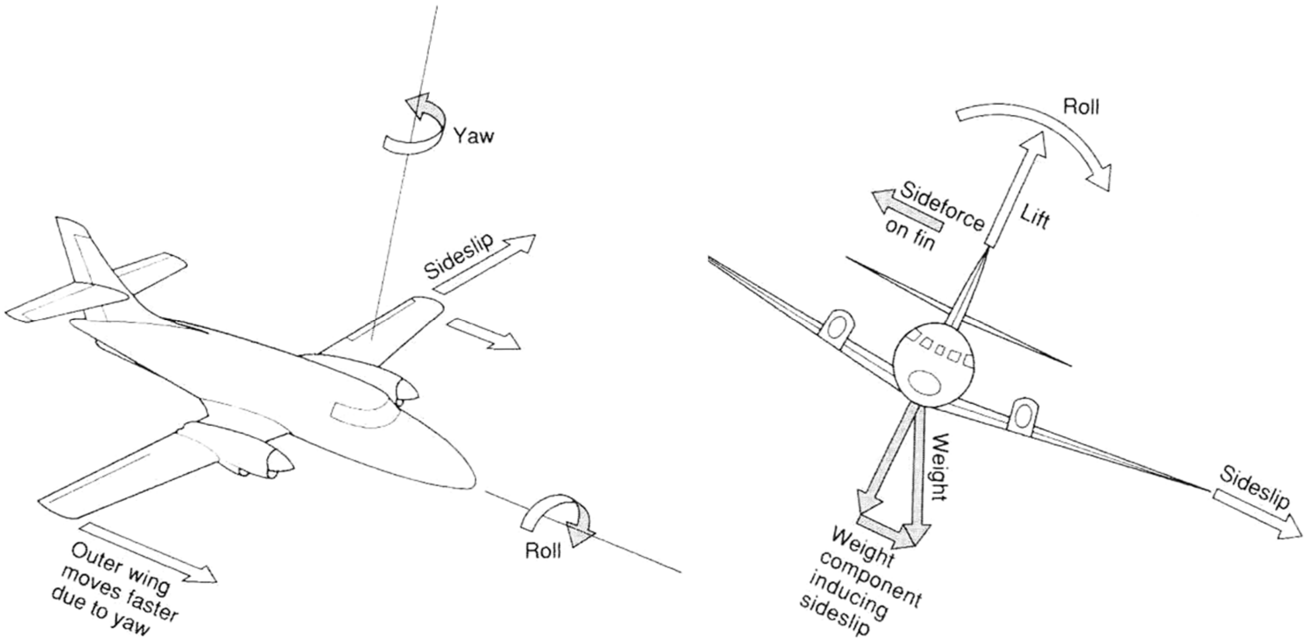

Figure from Barnard and Philpott (2010) *Aircraft Flight* 4th edition Prentice Hall

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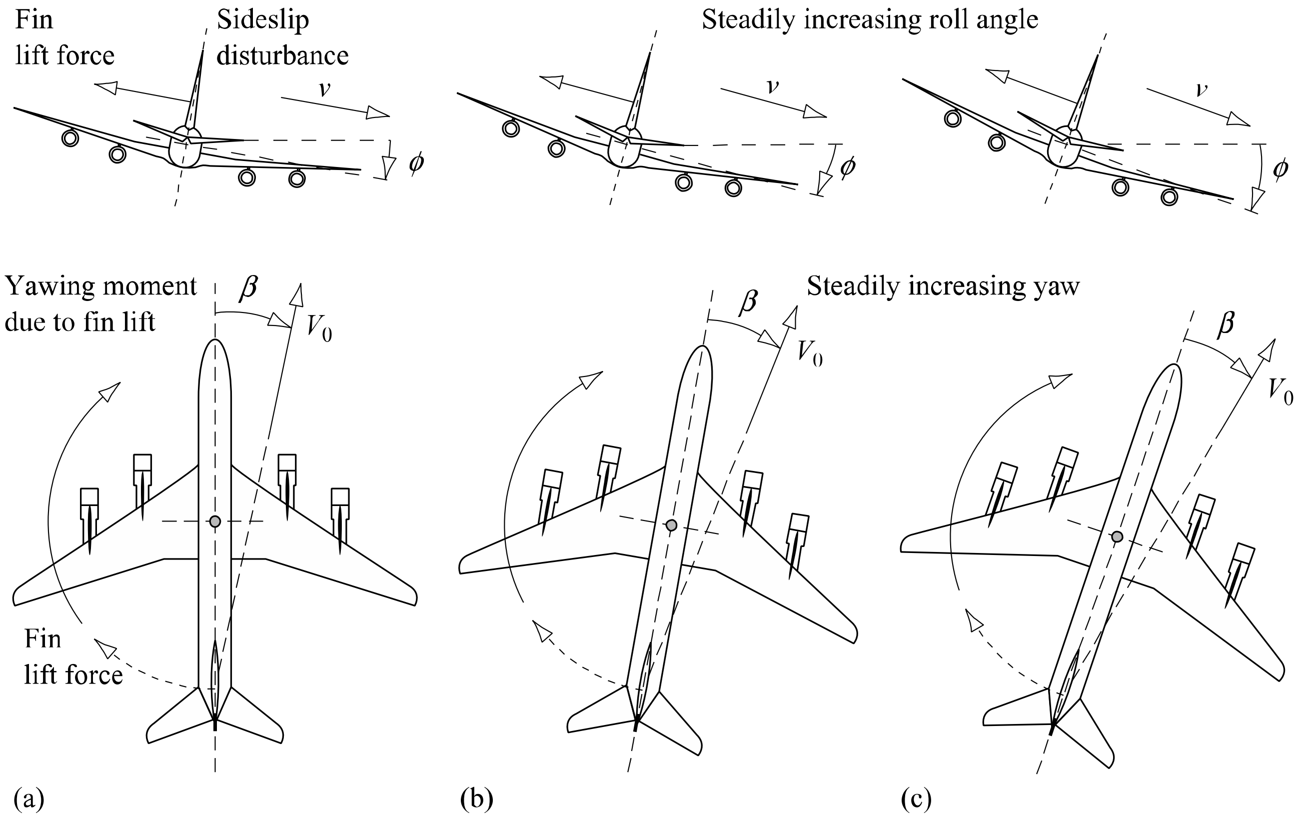


Cook 2007 *Flight Dynamics* 2nd edition, Elsevier

it's often so slow that the pilot barely has to try to stop it, it is unstable

Roll damping/roll subsidence

→ caused by the drag of the large flat wings trying to move up/down. very strong damping

Pure roll of the aircraft is a good model

ie drop everything apart from the roll rate and its derivative:

$$I_{xx}\dot{p} = \mathring{L}_p p$$

Let $p = p_0 e^{\lambda t}$

$$I_{xx}\lambda p_0 e^{\lambda t} = \mathring{L}_p p_0 e^{\lambda t}$$

Resulting in:

$$\lambda = \frac{\mathring{L}_p}{I_{xx}}$$

Navion $\lambda = -8.4117$
(exact $\lambda = -8.4442$)

Dutch Roll Mode

Lateral equivalent of short period oscillation mode

Due to weaker directional stability:

fin less effective than tailplane at damping

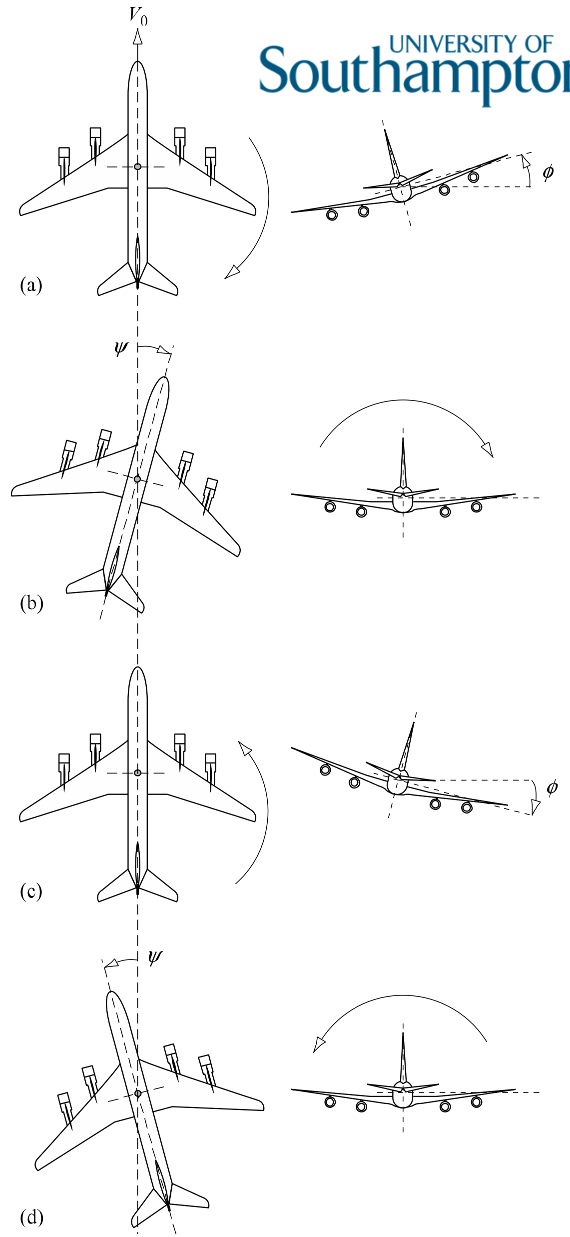
Associated with flying quality:

provoking nausea

Consider a disturbance from straight-level flight

Primary effect: Oscillation in yaw
(yaw rate r vs sideslip v)

Secondary effect: Typical yaw-roll motion



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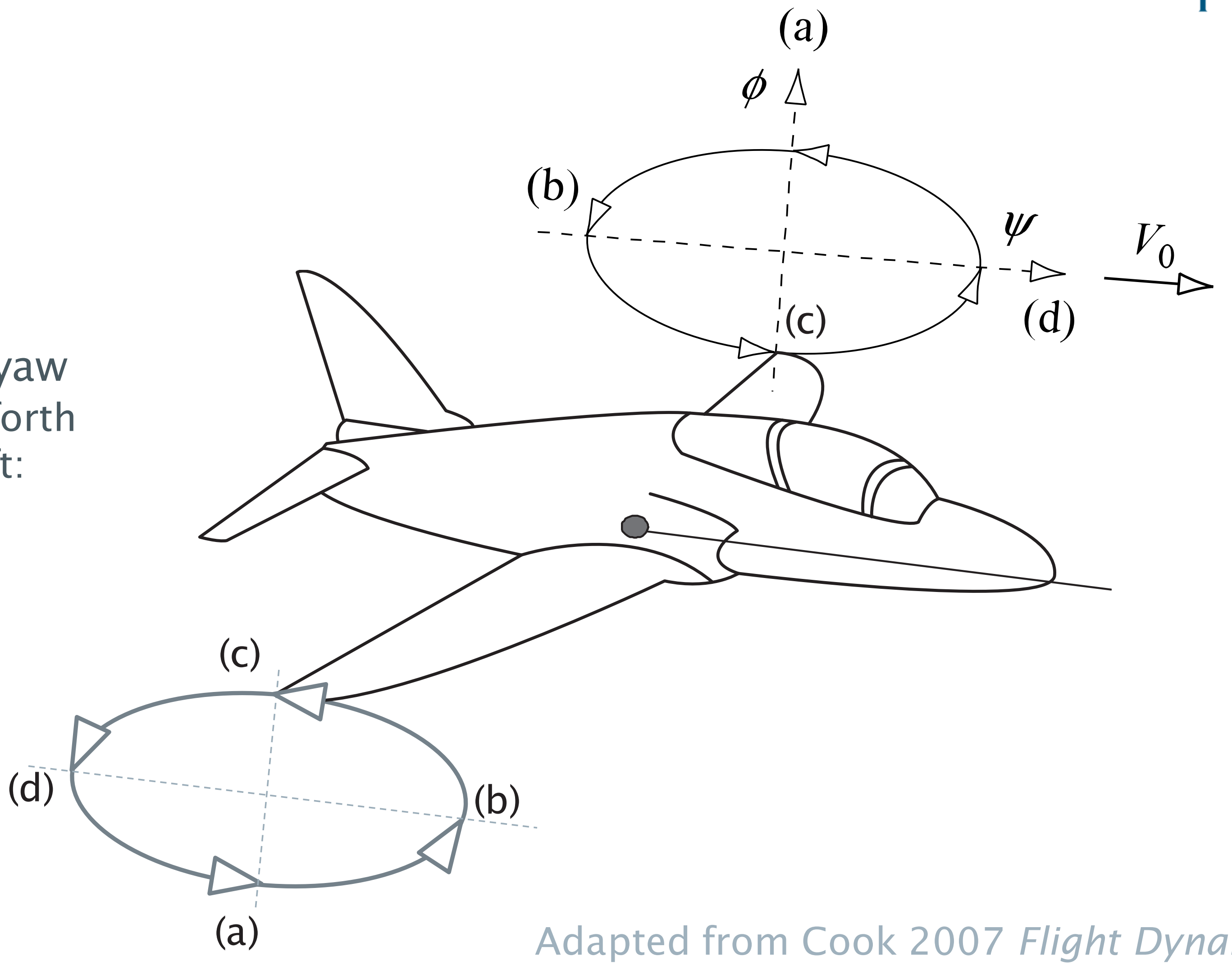
Dutch Roll Mode

Typical yaw-roll motion
(secondary effect)

Due to the oscillation in yaw
⇒ wingtip moving back & forth
⇒ oscillatory differential lift: wingtip moving forward generates more lift.
⇒ oscillatory roll motion



Adapted from Cook 2007 *Flight Dynamics* 2nd edition, Elsevier

Dutch Roll Mode

Typical yaw-roll motion (secondary effect)

Path traced by starboard wing tip in one Dutch roll cycle

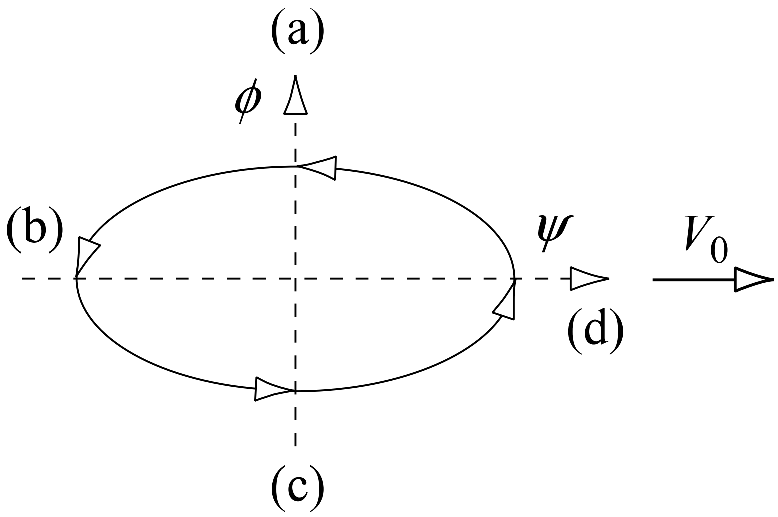
(a) Port (left) wing yaws aft with wing tip high

(b) Port (left) wing reaches maximum aft yaw angle as aircraft rolls through wings level in positive sense

(c) Port (left) wing yaws forward with wing tip low

(d) Port (left) wing reaches maximum forward yaw angle as aircraft rolls through wings level in negative sense

Oscillatory cycle then repeats decaying to zero with positive damping



Cook 2007 *Flight Dynamics* 2nd edition, Elsevier

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Summary of Lateral Modes

Lateral eigenvalues typically consist of a stable real eigenvalue (roll damping), a stable complex pair (Dutch roll) and a marginally stable or unstable real eigenvalue (spiral).

The roll damping mode affects p and ϕ

The Dutch roll is a coupled oscillatory yaw-rate–sideslip mode (r and v) that exhibits a typical **roll–yaw motion** (p and r, or ϕ and ψ)

The spiral mode affects mainly ϕ and hence r.