University of Southampton

Revision

Workshops 1 & 2 – 2020/21 Past Paper

Professor Hugh Lewis

Important context

- The final assessment for Astronautics in 2020/21 was a 24-hour online, open book exam. The format of this exam paper was substantially different from previous exam papers and from the 2022/23 paper.
- The questions were based on a single reference mission. So to tackle the questions we must first introduce the information related to this reference mission.
- Some numerical values were varied from student to student. Here, we will only focus on one set of values.
- Only the questions related to Chapter 5 and Chapter 11 are addressed in the revision workshops led by Hugh.
- The paper is not available from the past papers database.

Reference mission

You are considering the design of a European civilian remote-sensing satellite to monitor global land cover change for environmental and agricultural applications.

The satellite is expected to have a design lifetime of 12 years, to operate at an altitude between 750 km and 800 km (inclusive), and to provide complete (ideally, overlapping) coverage at the equator. It will carry a high resolution visible-waveband instrument with a field of view of 20.801°. This instrument should acquire images with a nominal spatial resolution of 10 metres (± 2 metres) per pixel at nadir, using a push-broom technique. The orbit will need to have a 10:30 am Local Solar Time (LST) descending node. In the initial spacecraft design, orbit control is expected to be achieved using thrusters.

Questions

2 (i)
Explain why an orbit that provides simultaneous Sun- and Earth-synchronism is important for this mission. Your answer should be around 50-100 words.

[2 marks]

2 (ii)
Using suitable calculations as evidence, propose ground track parameters (n, m) for the mission, where n is the number of orbits and m is the number of days before the ground track repeats ($m \leq 11$ days).

[10 marks]

2 (iii)
Using your ground track parameters, and assuming the satellite will be in a circular orbit, calculate the actual orbit altitude and inclination.

[2 marks]

Questions

2 (iv)

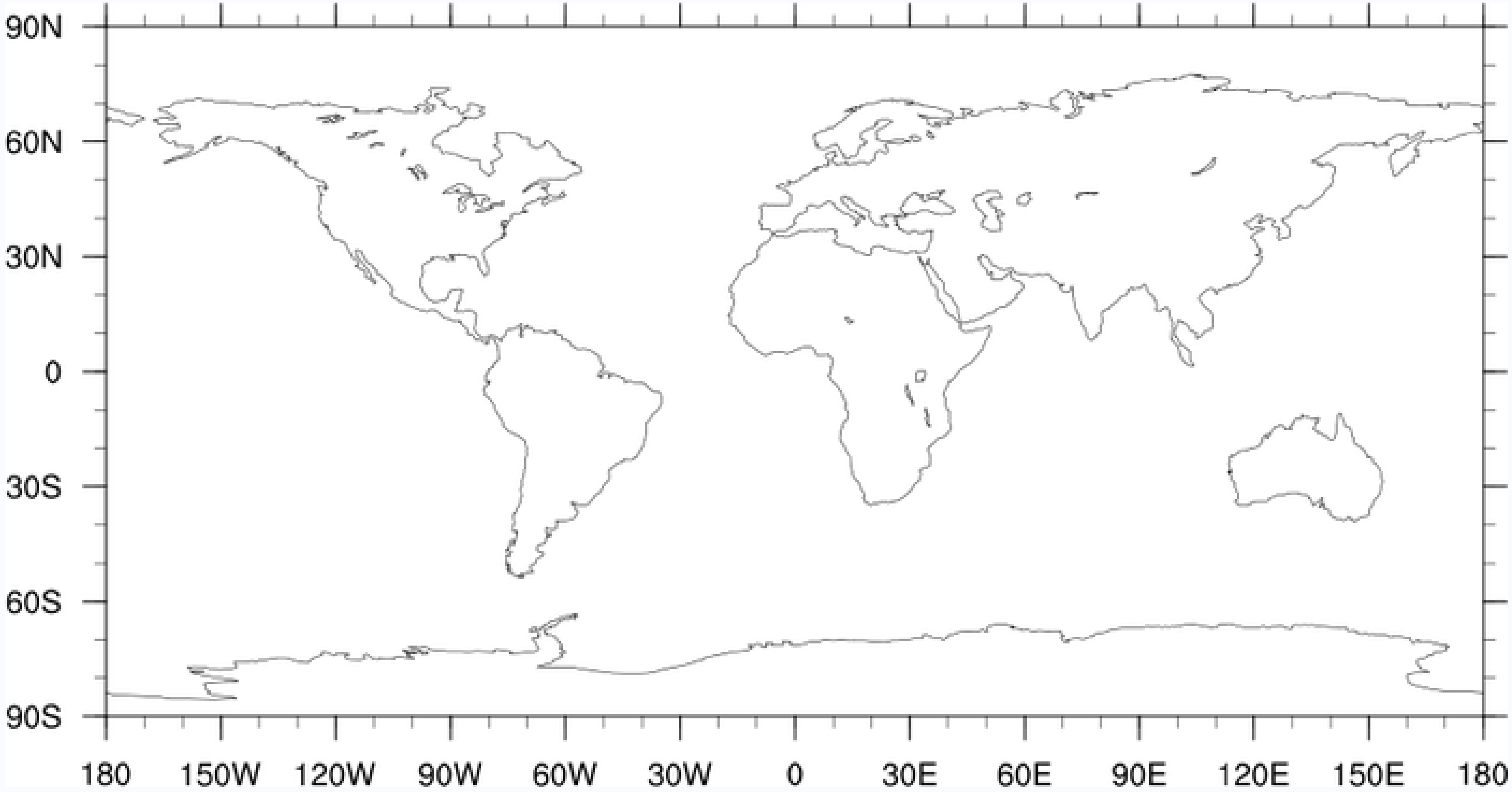
Taking into account that the orbit has a 10:30 am Local Solar Time (LST) descending node, use the map in Figure Q2 as a guide to sketch the ground track of the satellite over one orbit as it would appear at 12:00 (Noon) on the Spring Equinox. You should indicate the latitude and longitude of the locations of the ascending and descending nodes at the appropriate points on your sketch and mark the approximate locations on the ground track where the satellite enters and exits the eclipse. Note that you do not need to reproduce the continental outlines in your sketch, but you should provide appropriate, labelled axes and tick marks.

[6 marks]

Questions

2 (iv)

Figure Q2:



Questions

For the following calculations it can be assumed that the projected area of the spacecraft (in the direction of the velocity) is 20 m^2, the spacecraft wet mass is 1250 kg and the specific impulse of the propulsion system is 190 seconds.

3 (i)
If the required tolerance in the ground track, E_0, is 0.75 km at the equator, calculate the worst-case orbit control cycle time (i.e. the number of days before the orbit needs to be boosted) and the total height lost during one cycle. You may use values of atmospheric density, ρ, from Table Q3 (please take the value closest to the orbit altitude you identified in Q2).

[6 marks]

Questions

3 (ii)
If the orbit boost is achieved using a Hohmann Transfer, calculate the worst-case ΔV (“delta V”) needed to correct for atmospheric drag over the full mission lifetime and the corresponding propellant mass.

[10 marks]

3 (iii)
If the satellite suffered an anomaly that prevented it from boosting its orbit for a period of 1 year, explain qualitatively how its imaging mission would be affected. Your answer should be around 150-250 words.

[4 marks]

Questions

Table Q3:

Altitude (km)	ρ Low Solar Activity (kg/m^3)	ρ High Solar Activity (kg/m^3)
740	5.78×10^{-15}	4.35×10^{-14}
760	4.93×10^{-15}	3.44×10^{-14}
780	4.28×10^{-15}	2.75×10^{-14}
800	3.75×10^{-15}	2.22×10^{-14}

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Question 2 (i)

Explain why an orbit that provides simultaneous Sun- and Earth-synchronism is important for this mission. Your answer should be around 50-100 words.

[2 marks]

To meet the mission objectives (change detection) we need good illumination, invariant illumination (i.e. constant shadow length & orientation) and invariant viewing geometry.

Note this question does not ask “what is Sun- and Earth-synchronism?”

Question 2 (ii)

Using suitable calculations as evidence, propose ground track parameters (n, m) for the mission, where n is the number of orbits and m is the number of days before the ground track repeats ($m \leq 11$ days).

Information provided:

- $750 \text{ km} \leq h \leq 800 \text{ km}$
- $\beta = 20.801°$ (Field of View)
- $m \leq 11$ days
- $R_E = 6378$ km
- $\mu_E = 398600 \text{ km}^3/\text{s}^2$

Ground track repeat parameters

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Astronautics – Revision

A reminder of our process:

*Changes to n result in smaller changes to the orbit period, compared with changes to m

1. Select any altitude, h, within the specified altitude range
2. Calculate the swath width: $d = 2h\tan\left(\frac{\beta}{2}\right)$
3. Calculate the orbital period: $\tau = 2\pi\sqrt{\frac{(R_E+h)^3}{\mu}}$
4. Calculate the repeat parameter: $n = \mathrm{int}\left(\frac{2\pi R_E}{d}\right)$
5. Calculate the repeat parameter: $m = \mathrm{int}\left(\frac{n\tau}{86400}\right)$
6. Re-calculate the orbital period: $\tau = \frac{m}{n}(86400)$
7. Is τ in the range?
8. No: Increase or decrease n (or m) by 1
9. Yes: Stop, keep values of n and m

*If the altitude range is NOT 700-900 km → Calculate the orbital periods for the altitude range: $\tau = 2\pi\sqrt{\frac{(R_E+h)^3}{\mu}}$

Calculating repeat parameters

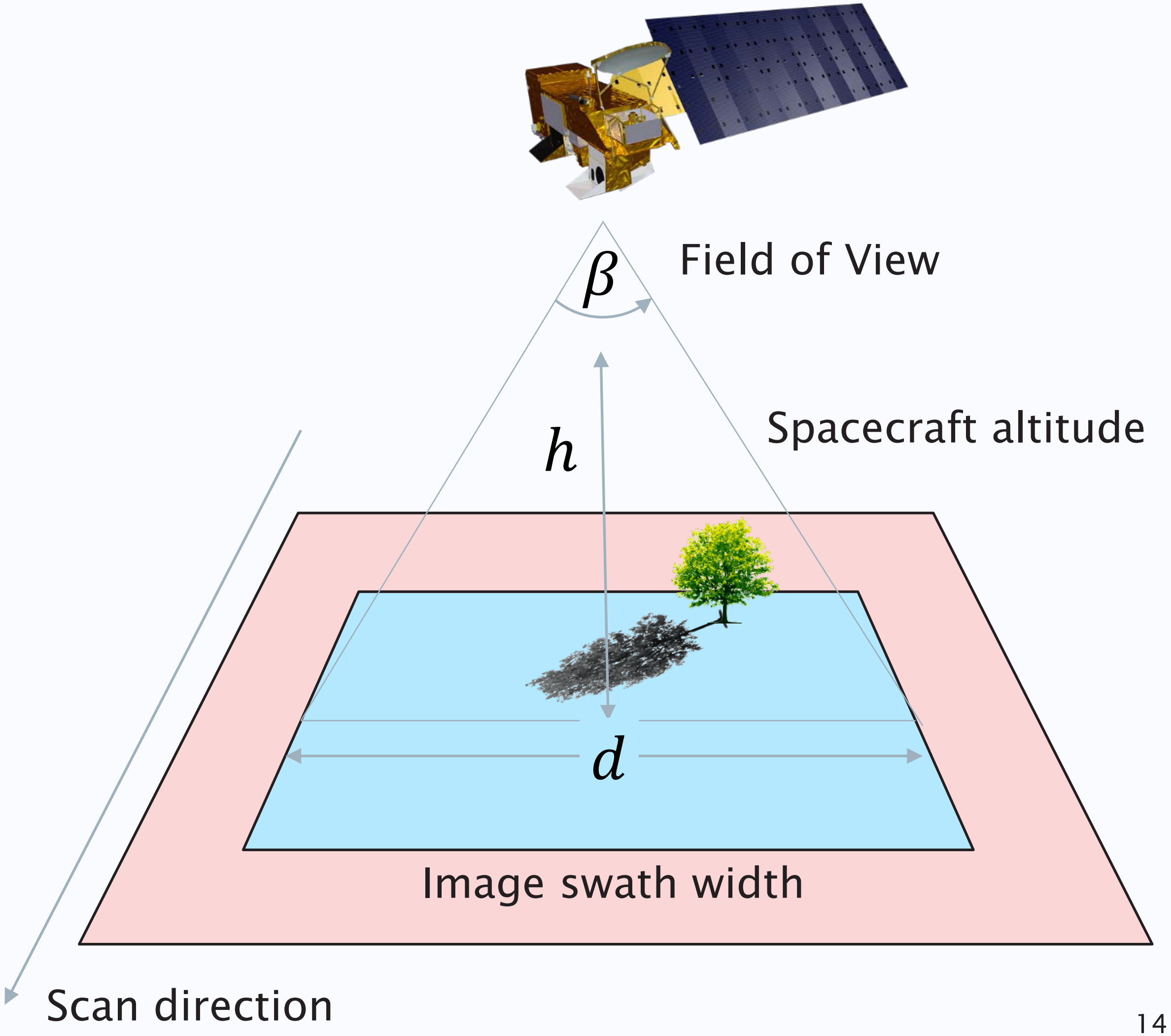
- Step 1: select an altitude within the specified range:
  - <u>Choose an altitude between 750 km and 800 km as our initial guess</u>

$$h = 775\ \text{km}$$

- Step 2: calculate the swath width:

$$d = 2h\tan\left(\frac{\beta}{2}\right)$$
$$= 2(775)\tan\left(\frac{20.801°}{2}\right)$$

$$\Rightarrow\quad d = 284.492\ \text{km}$$



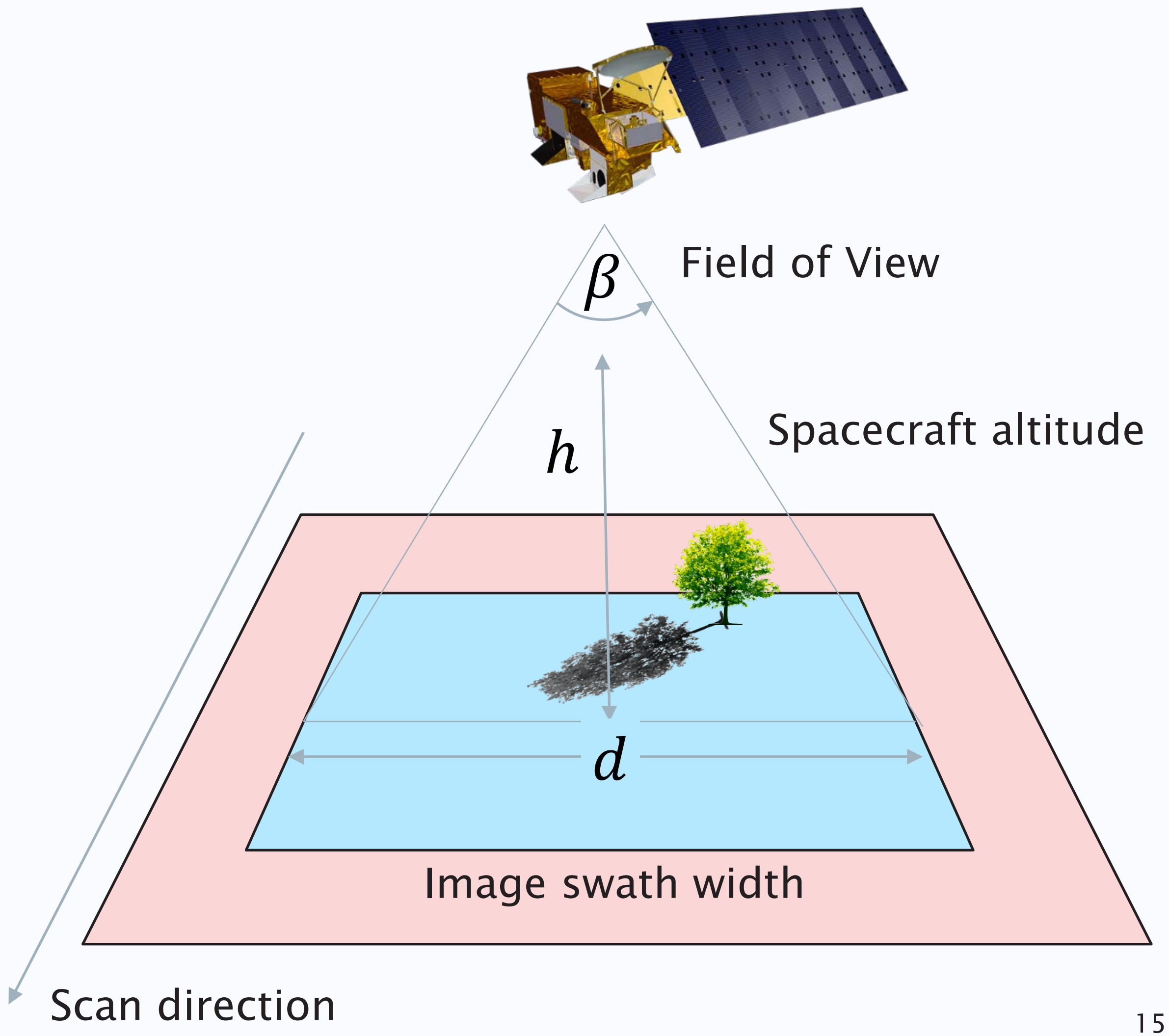
Calculating repeat parameters

- Step 3: calculate the orbit period:

$$\tau = 2\pi\sqrt{\frac{(R_E + h)^3}{\mu}}$$

$$= 2\pi\sqrt{\frac{(6378 + 775)^3}{398600}}$$

$$\Rightarrow \quad \tau = 6020.652\ \text{sec}$$



Calculating repeat parameters

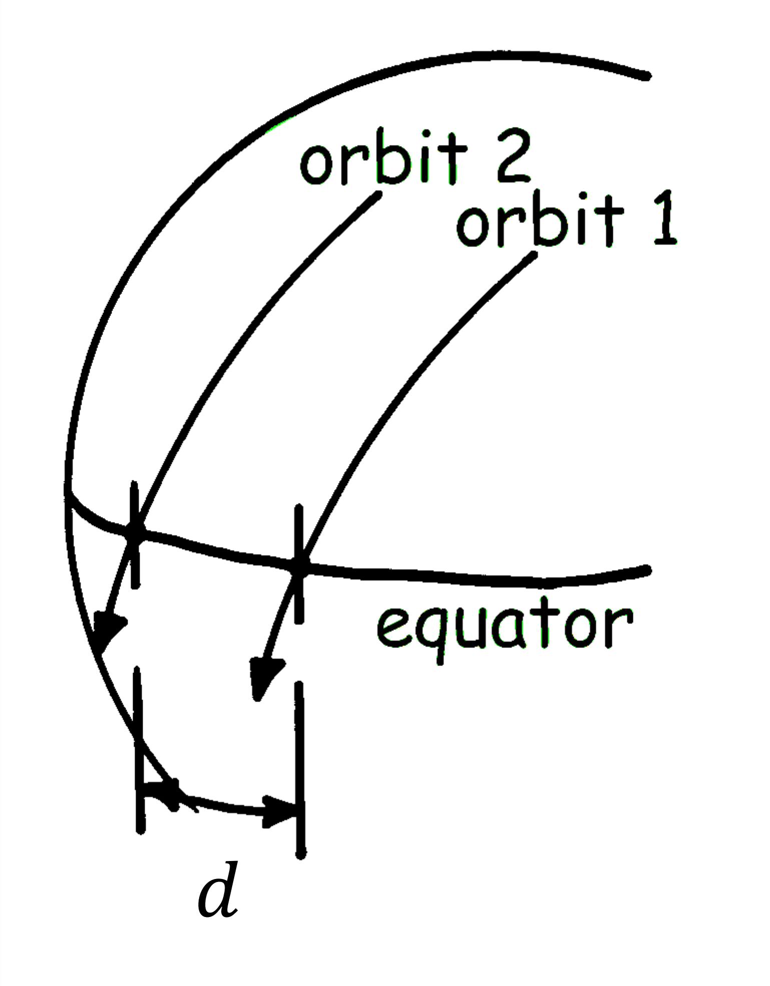
- Step 4: calculate the repeat parameter n:

$$n = \mathrm{int}\left(\frac{2\pi R_E}{d}\right)$$

$$= \mathrm{int}\left(\frac{2\pi(6378)}{284.492}\right)$$

$$\Rightarrow \quad n = \mathrm{int}(140.862)\ \text{orbits}$$

- Use $n = 140$ to maintain overlapping coverage



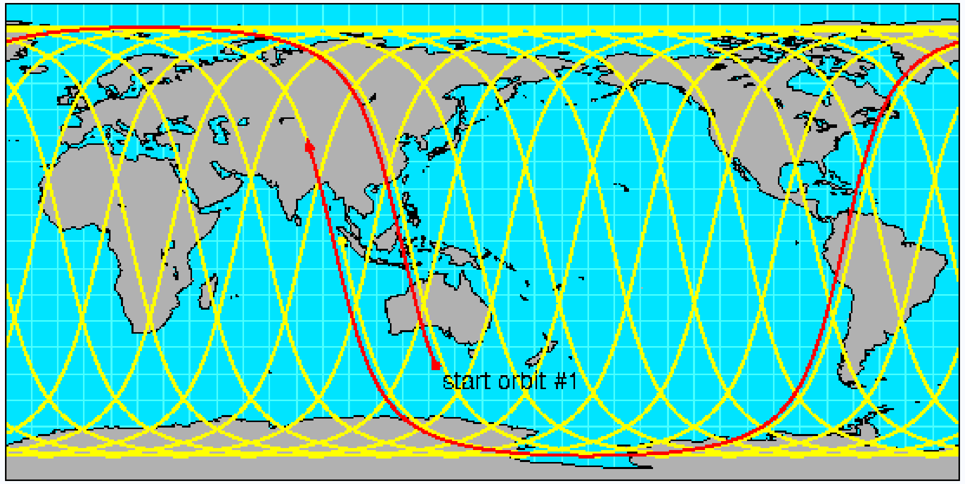
Calculating repeat parameters

- Step 5: calculate the repeat parameter m:

$$m = \mathrm{int}\left(\frac{n\tau}{86400}\right)$$

$$= \mathrm{int}\left(\frac{(140)(6020.652)}{86400}\right)$$

$$\Rightarrow m = \mathrm{int}(9.756)\ \text{days}$$



- Use $m = 10$ to maintain overlapping coverage

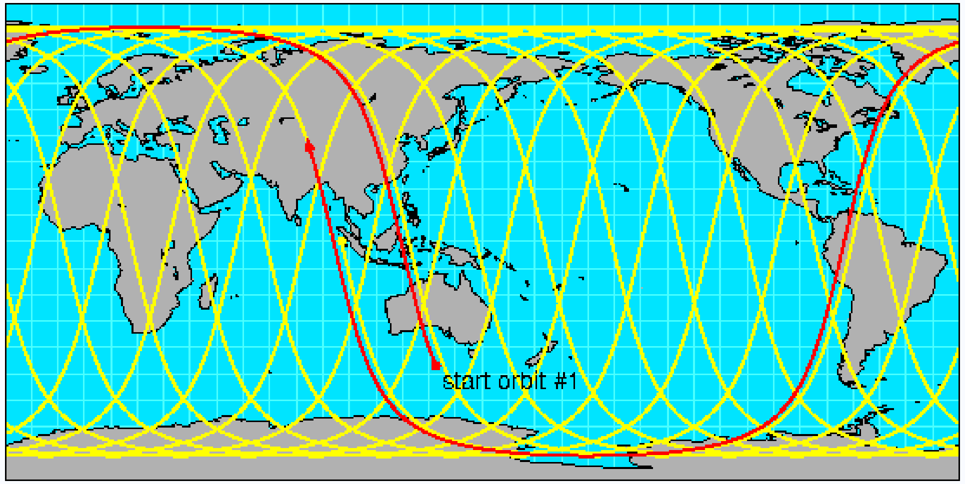
Calculating repeat parameters

- Step 6: calculate the actual orbit period for the ground track parameters (n, m):

$$\tau = \frac{m}{n}(86400)$$

$$= \frac{(10)}{(140)}(86400)$$

$$\Rightarrow \tau = 6171.429 \text{ sec}$$



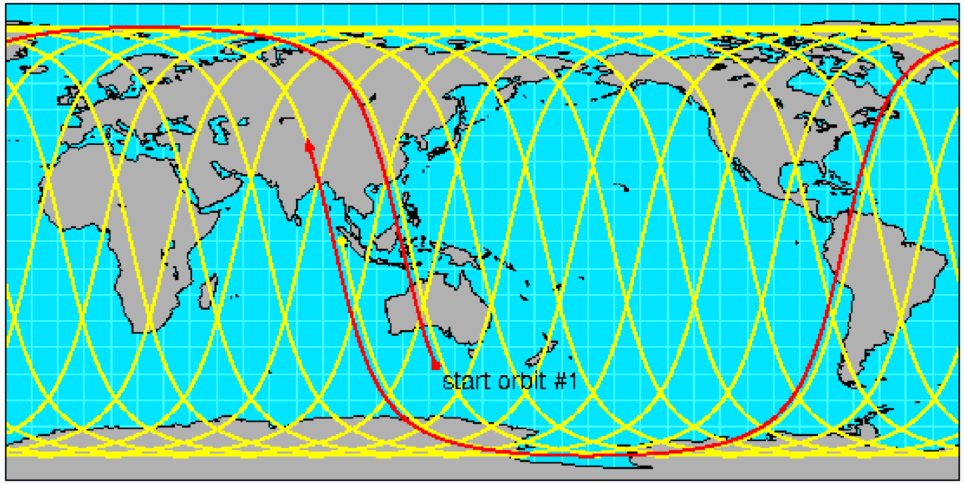
Calculating repeat parameters

- Step 7: is the orbital period within the range we need?

$$\tau = 6171.429 \text{ sec}$$

- For altitudes 750-800 km we want*:

$$\tau \approx 5989 \text{ to } 6052 \text{ sec}$$



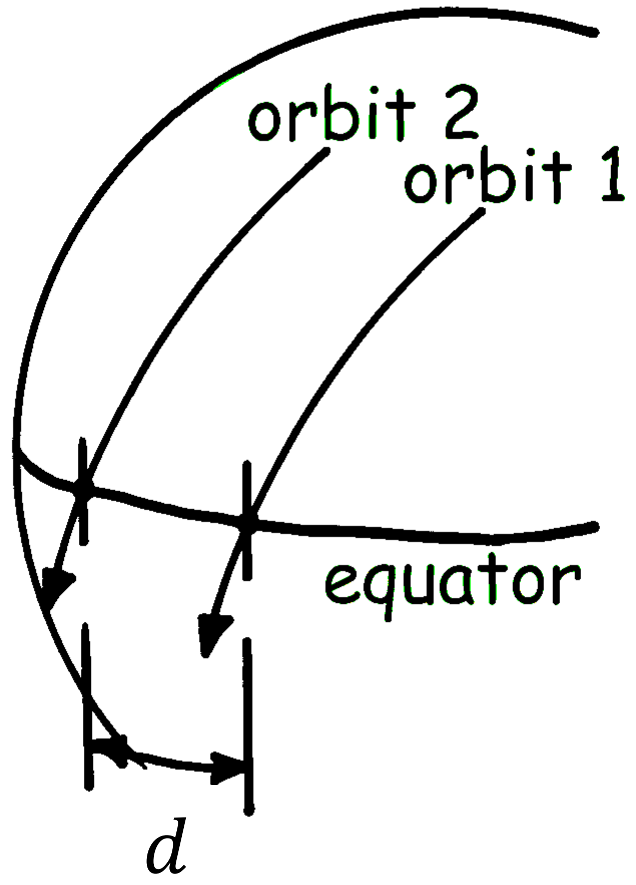
➡ Our answer is outside this range
- We need to adjust n or m

* Calculations are not shown

Calculating repeat parameters

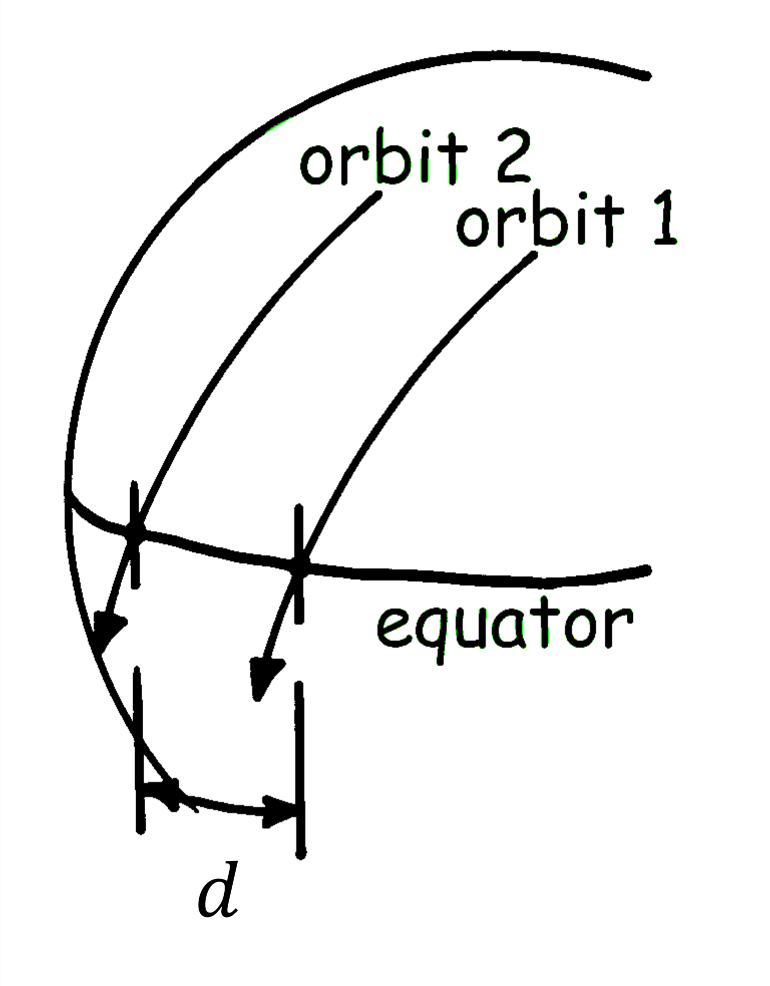
- Step 8: increase or decrease the repeat parameter n:
  - We have chosen to change n because
    - This gives us smaller changes in the orbital period

  - Should we increase or decrease the value of n?



Calculating repeat parameters

- Should we increase or decrease the value of n?
  - Look at the expression for the orbital period:
    - $\tau = \frac{m}{n}(86400)$
  - If we increase n the orbital period decreases (the altitude also decreases)
  - If we decrease n the orbital period increases (the altitude also increases)

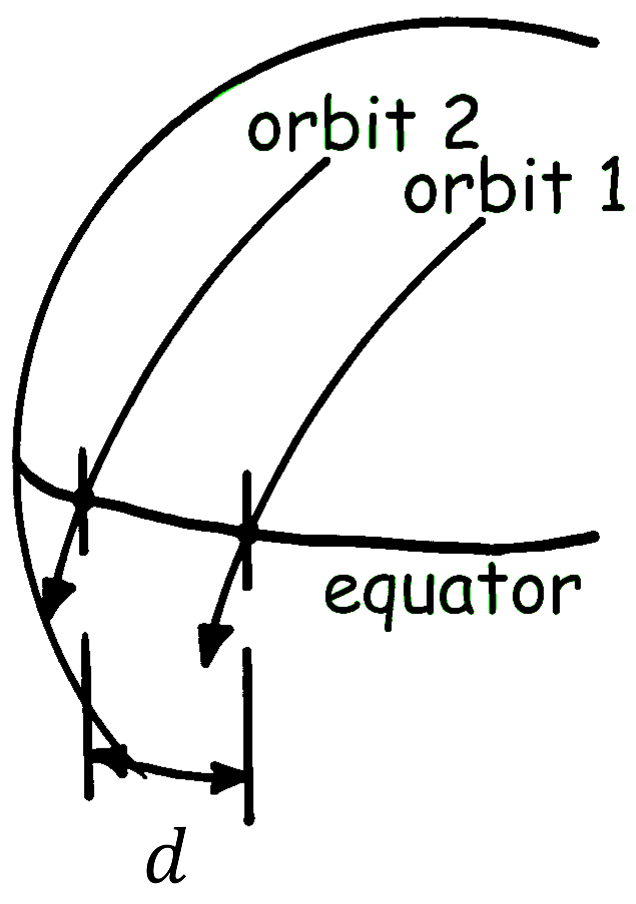


Calculating repeat parameters

- Should we increase or decrease the value of n?
  - Look at the orbital period:

$$\tau = 6171.429 \text{ sec}$$

  - We want $\tau \approx 5989 \text{ to } 6052 \text{ sec}$
    - The current value of τ is too high, so we need to increase n, as this will reduce the orbital period
  - Let's use: $n = 141 \text{ orbits}$



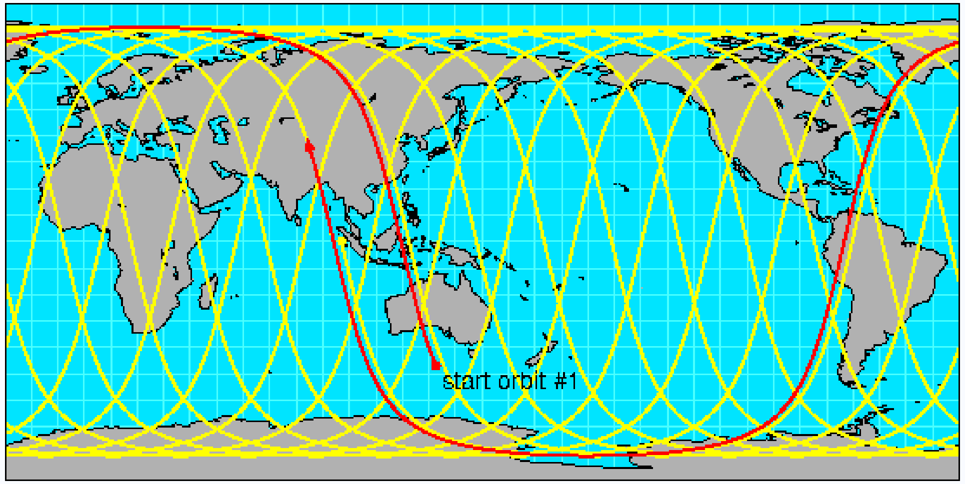
Calculating repeat parameters

- Back to Step 6 again: calculate the actual orbit period for the ground track parameters (n, m):

$$\tau = \frac{m}{n}(86400)$$

$$= \frac{(10)}{(141)}(86400)$$

$$\Rightarrow \tau = 6127.660 \text{ sec}$$



Calculating repeat parameters

- Back to Step 7 again: is the orbital period within the range we have specified?

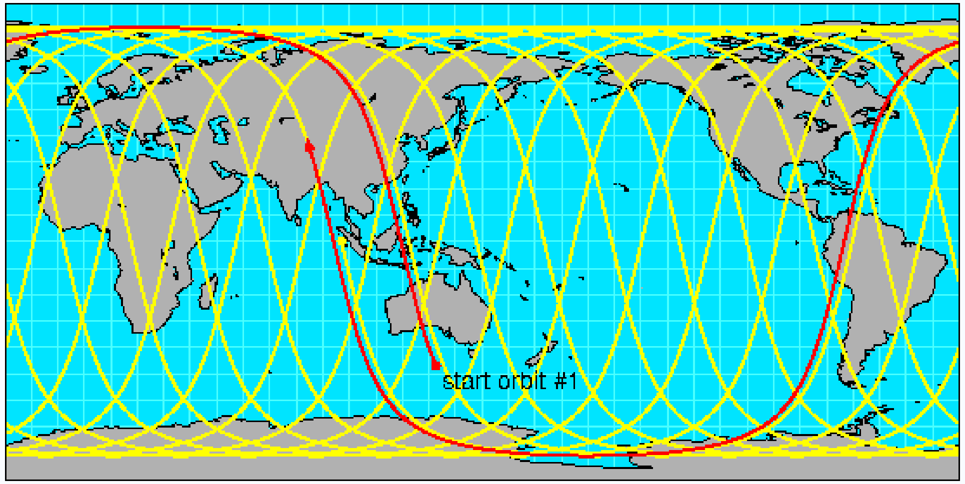
$$\tau = 6127.660 \text{ sec}$$

- For altitudes 750-800 km we want:

$$\tau \approx 5989 \text{ to } 6052 \text{ sec}$$

➡ Our answer is still outside this range

- Increase n again and repeat steps 6 and 7 until the orbital period is within the range



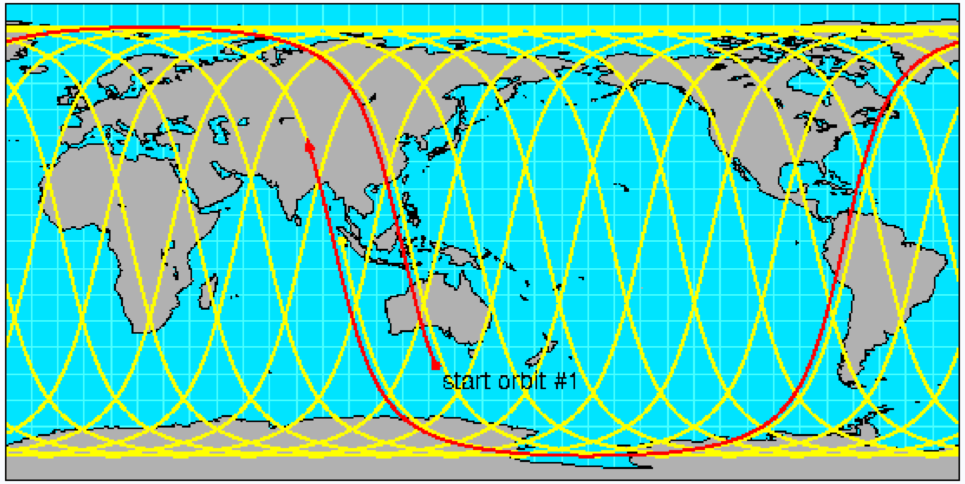
Calculating repeat parameters

- We arrive at $(n, m) = (143, 10)$:

$$\tau = \frac{m}{n}(86400)$$

$$= \frac{(10)}{(143)}(86400)$$

$$\Rightarrow \tau = 6041.958 \text{ sec}$$



- To gain full marks we must check the altitude is within the correct range (or that the period is within the correct range)

Question 2 (iii)

- Now we have the repeat parameters and the orbital period, we can calculate the precise values of the orbital elements (semi-major axis and inclination) that correspond with this Sun-synchronous and Earth-synchronous orbit:
  - Semi-major axis (from Kepler's 3rd Law):

$$a = \left\{\mu\left(\frac{\tau}{2\pi}\right)^2\right\}^{\frac{1}{3}} = \left\{398600\left(\frac{6041.958}{2\pi}\right)^2\right\}^{\frac{1}{3}} = 7169.865\ \mathrm{km}$$

  - And the altitude:

$$\Rightarrow\ h = a - R_E = 7169.865 - 6378 = 791.865\ \mathrm{km}$$

Question 2 (iii)

- Now we have the semi-major axis we can calculate the orbital inclination:
  - Orbital inclination (from nodal regression rate):

$$i = \cos^{-1}\left(\frac{0.986°}{(-2.0647 \times 10^{14})(a^{-3.5})}\right)$$

$$= \cos^{-1}\left(\frac{0.986°}{(-2.0647 \times 10^{14})(7169.865^{-3.5})}\right)$$

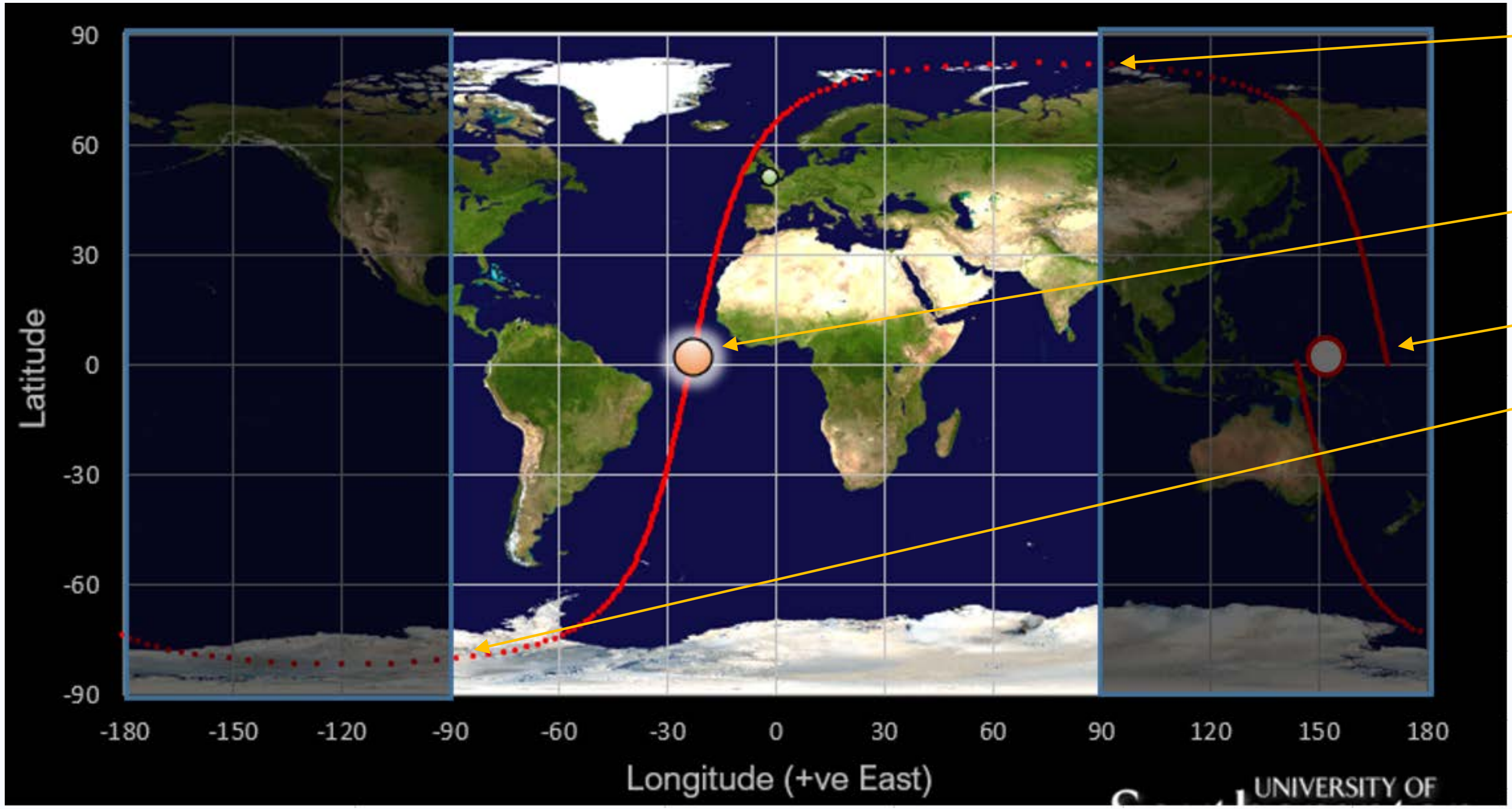
➡ $i = 98.571°$

Question 2 (iv) reminder

Taking into account that the orbit has a 10:30 am Local Solar Time (LST) descending node, use the map in Figure Q2 as a guide to sketch the ground track of the satellite over one orbit as it would appear at 12:00 (Noon) on the Spring Equinox. You should indicate the latitude and longitude of the locations of the ascending and descending nodes at the appropriate points on your sketch and mark the approximate locations on the ground track where the satellite enters and exits the eclipse. Note that you do not need to reproduce the continental outlines in your sketch, but you should provide appropriate, labelled axes and tick marks.

[6 marks]

Question 2 (iv)



Eclipse exit (approximate)

Descending node
(10:30 am LST)

Ascending node

Eclipse entry (approximate)

Northernmost and Southernmost latitudes of ground track must equal $(180 - i)°$

Reference mission (reminder)

You are considering the design of a European civilian remote-sensing satellite to monitor global land cover change for environmental and agricultural applications.

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[6 marks]

Questions (reminder)

3 (ii)
If the orbit boost is achieved using a Hohmann Transfer, calculate the worst-case ΔV (“delta V”) needed to correct for atmospheric drag over the full mission lifetime and the corresponding propellant mass.

[10 marks]

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If the satellite suffered an anomaly that prevented it from boosting its orbit for a period of 1 year, explain qualitatively how its imaging mission would be affected. Your answer should be around 150-250 words.

[4 marks]

Questions (reminder)

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Calculating orbit control parameters

- Question 3 (i)
  - Data:
    - $R_E = 6378$ km
    - $\mu_E = 398600\ \text{km}^3/\text{s}^2$.
    - $h = 791.865$ km
    - $S = 20\ \text{m}^2$
    - $m = 1250\ \text{kg}$
    - $C_D = 2.2$
    - $\rho = 2.22 \times 10^{-14}\ \text{kg/m}^3$ (this is the density for the closest altitude at high solar activity from Table Q3)
    - $E_0 = \pm 0.75$ km
    - $\omega_E = 0.004178°/s$

Calculating orbit control parameters

- First, we calculate the change in orbit height and the change in the orbit period per orbit:
  - The change in orbit height is:

$R_E + h$

$$\delta a = -2\pi\rho\frac{SC_D}{m}a^2 = -2\pi\left(2.22\times10^{-14}\right)\frac{(20)(2.2)}{1250}\left(7169.865\times10^3\right)^2$$

$$= -0.2524\ \mathrm{m/orbit}$$

  - The change in orbit period is:

$$\delta\tau = 3\pi\frac{a^{1/2}}{\mu^{1/2}}\delta a = 3\pi\frac{(7169.865)^{1/2}}{398600^{1/2}}\left(-0.2524\times10^{-3}\right)$$

$$= -3.1905\times10^{-4}\ \mathrm{s/orbit}$$

Calculating orbit control parameters

- Next, we calculate the longitude displacement at the limit of our ground track tolerance and the amount of time this represents (in terms of arriving early or late with respect to the nominal ground track):
  - The longitude displacement is:

$$\delta\lambda = \frac{2E_0}{R_E}\left(\frac{180°}{\pi}\right) = \frac{2(0.75)}{6378}\left(\frac{180°}{\pi}\right) = 0.01348°$$

  - This is equivalent to the time:

$$\Delta t_0 = \frac{\delta\lambda}{\omega_E} = \frac{0.01348°}{0.004178°/s} = 3.225\ \text{sec}$$

Calculating orbit control parameters

- Now we can calculate the number of orbit cycles for the drift from the Eastern threshold to the Western threshold of our ground track tolerance, and hence the number of orbits for the full orbit control cycle:
  - From the Eastern threshold to the Western threshold, the number of orbits is:

$$\Delta t_0 = \frac{\Delta\tau}{2}k^2 \Rightarrow k^2 = \frac{2\Delta t_0}{\Delta\tau} = \frac{2(3.225)}{-3.1905 \times 10^{-4}}$$

$$k = 142.19 \text{ orbits}$$

We need an integer value. Truncate this number to ensure we do not exceed ground track tolerance

  - So the full orbit control cycle will take: $2k = 284 \text{ orbits}$

Calculating orbit control parameters

- Finally, we calculate the total height lost over this full orbit control cycle and the time between orbit control manoeuvres:
  - The total height lost is:

$$\Delta a_{\text{total}} = 2k|\delta a| = 284(0.2524) = 71.683 \text{ m}$$

  - And the cycle time between manoeuvres:

$$\Delta T = 2k\tau_0 = 2k\left[2\pi\sqrt{\frac{{a_0}^3}{\mu}}\right] = 284\left[2\pi\sqrt{\frac{7169.865^3}{398600}}\right]$$

$$= 284[6041.958] = 1.72 \times 10^6 \text{ s} = 19.86 \text{ days}$$

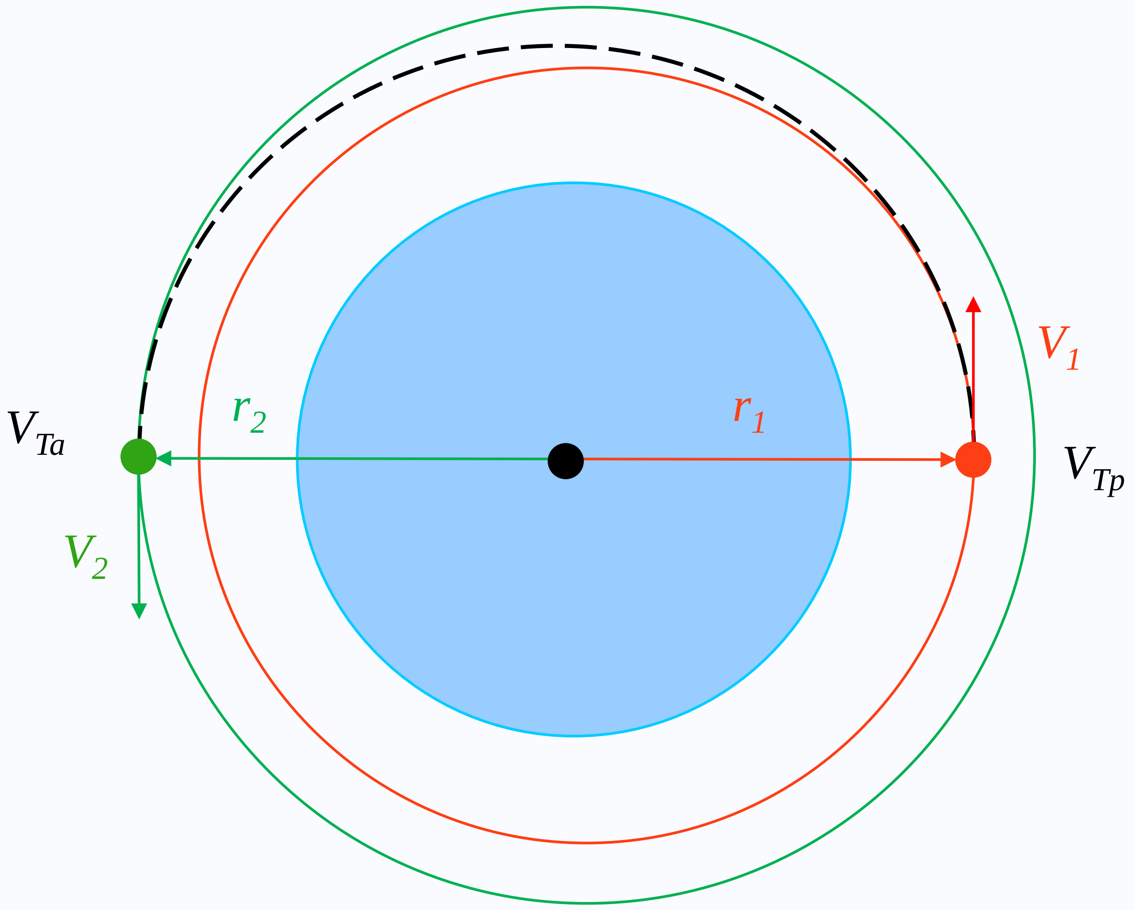
Question 3 (ii)

If the orbit boost is achieved using a Hohmann Transfer, calculate the worst-case ΔV ("delta V") needed to correct for atmospheric drag over the full mission lifetime and the corresponding propellant mass.

- Question 3 (ii)
  - Data:
    - $R_E = 6378$ km
    - $\mu_E = 398600\ \text{km}^3/\text{s}^2$
    - $h = 791.865$ km (from question 2)
    - $\Delta a_{\text{total}} = 71.683$ m (from question 3 (i))
    - $\Delta T = 19.86$ days (from question 3 (i))
    - $m = 1250\ \text{kg}$
    - $g_0 = 9.81\ \text{m/s}^2$
    - $I_{SP} = 190$ s

Delta-V & propellant calculation

- Draw a diagram:
  - A Hohmann transfer is used to "boost" the satellite from a low circular orbit to a high circular orbit.



Delta-V & propellant calculation

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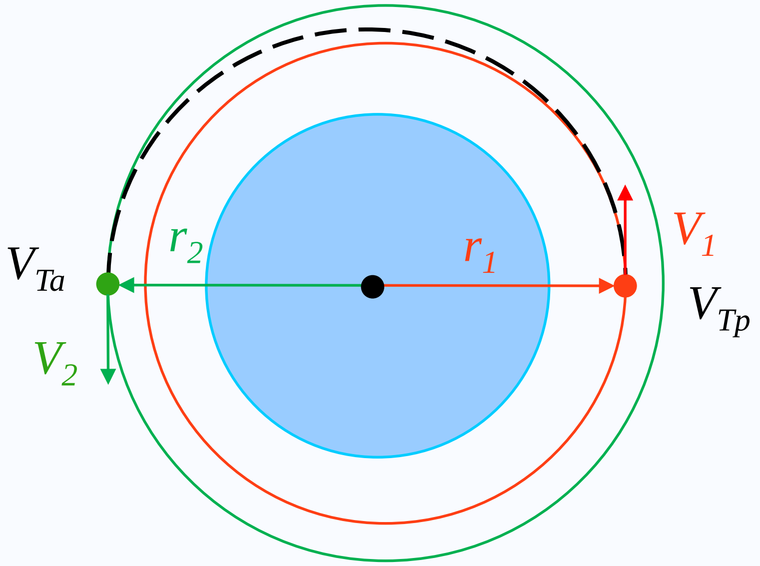
Astronautics – Revision



- The radius of the lower orbit is r_1

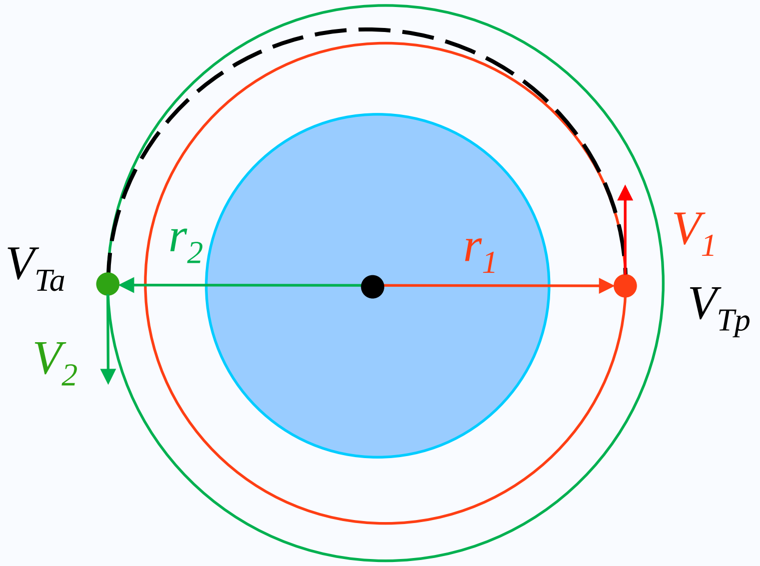
$$r_1 = 7169.865 - \frac{1}{2}(71.683 \times 10^{-3}) = 7169.8293 \text{ km}$$

- This is also the perigee of the elliptical transfer orbit.
- The radius of the higher orbit is r_2

$$r_2 = 7169.865 + \frac{1}{2}(71.683 \times 10^{-3}) = 7169.9010 \text{ km}$$

- This is also the apogee of the elliptical transfer orbit

Delta-V & propellant calculation

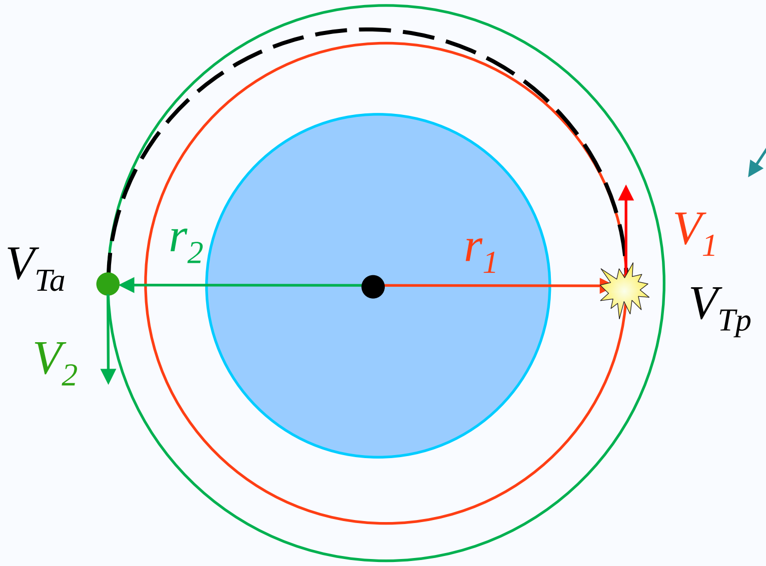


- The semi-major axis of the transfer orbit is a_T

$$a_T = \frac{1}{2}(r_1 + r_2) = 7169.865\ \text{km}$$

- This makes sense, given the assumptions we have made about the sizes of the low and high orbits.

Delta-V & propellant calculation



- We can now use the energy equation twice, where the low circular orbit and the transfer orbit intersect:

On the circular low orbit:

$$V_1 = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{398600}{7169.8293}} = 7.456143 \text{ km/s}$$

At the perigee of the elliptical transfer orbit:

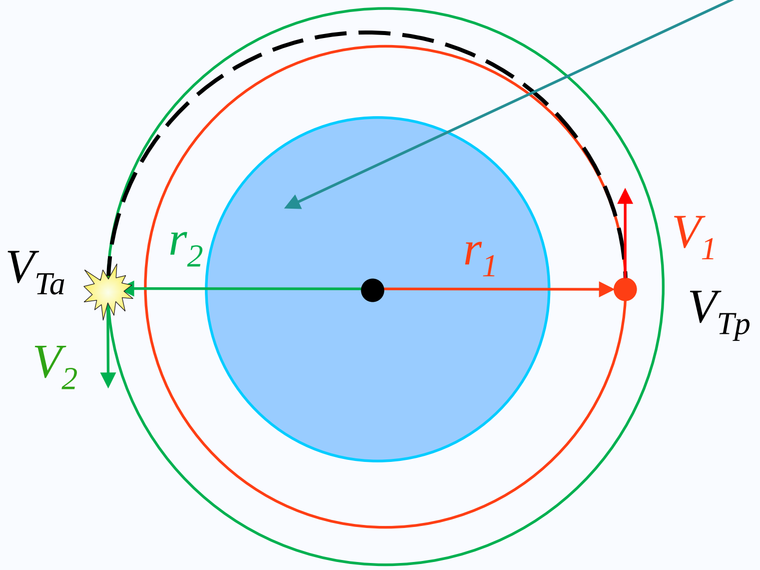
$$V_{Tp} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_T}\right)} = \sqrt{398600\left(\frac{2}{7169.8293} - \frac{1}{7169.865}\right)}$$

$$= 7.456162 \text{ km/s}$$

- The difference is the ΔV_{Tp} for the first burn

Be aware that we need to maintain a high level of precision in the values we calculate because the difference in altitude is so small!

Delta-V & propellant calculation



- We can now use the energy equation twice, where the transfer orbit and the high circular orbit intersect:
  - At the apogee of the elliptical transfer orbit:

$$V_{Ta} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_T}\right)} = \sqrt{398600\left(\frac{2}{7169.9010} - \frac{1}{7169.865}\right)}$$

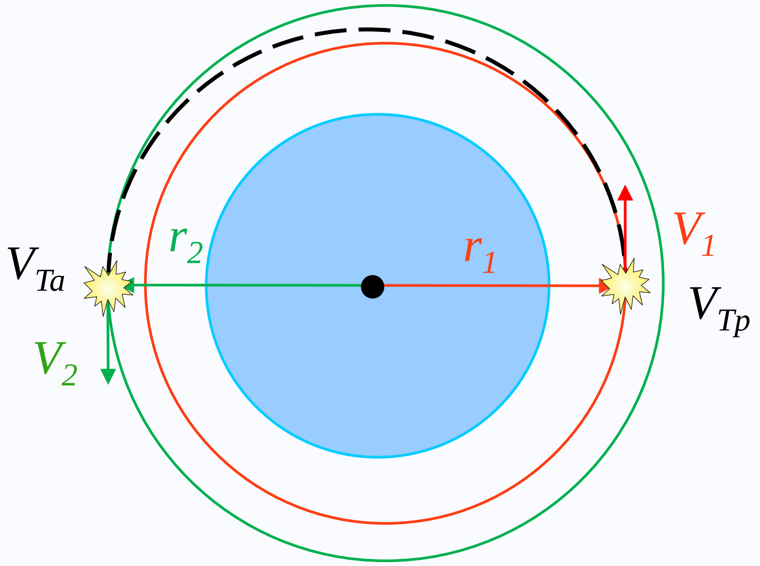
$$= 7.456087 \text{ km/s}$$

  - On the high circular orbit:

$$V_2 = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{398600}{7169.9010}} = 7.456106 \text{ km/s}$$

- The difference is the ΔV_{Ta} for the second burn

Delta-V & propellant calculation



- For the first burn:

$$\Delta V_{Tp} = V_{Tp} - V_1 = 7.456162 - 7.456143\text{km/s}$$
$$= 1.8636 \times 10^{-5}\ \text{km/s}$$

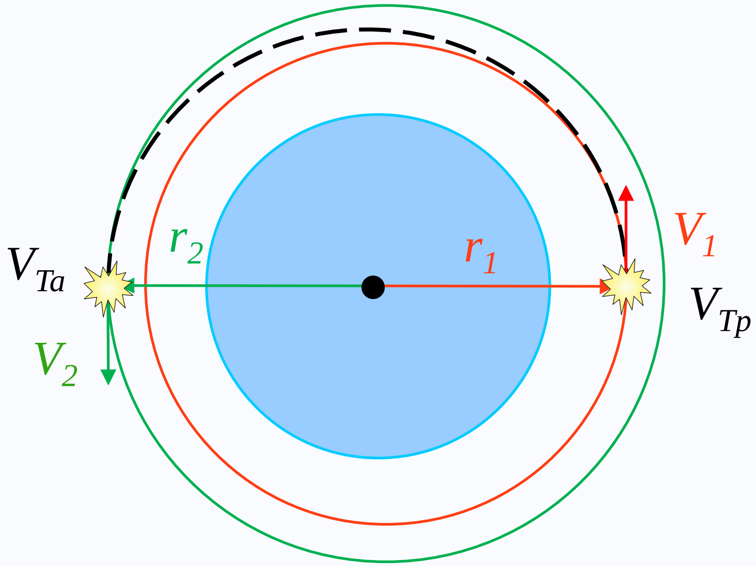
- For the second burn:

$$\Delta V_{Ta} = V_2 - V_{Ta} = 7.456106 - 7.456087\text{km/s}$$
$$= 1.8636 \times 10^{-5}\text{km/s}$$

- Hence the total ΔV is:

$$\Delta V = \Delta V_{Tp} + \Delta V_{Ta}$$
$$= 3.7273 \times 10^{-5}\ \text{km/s}$$

Delta-V & propellant calculation



- Over the 12 year lifetime the number of orbit cycles is:

$$N = \mathrm{int}\left(\frac{12(365.25)}{19.86}\right) = 220$$

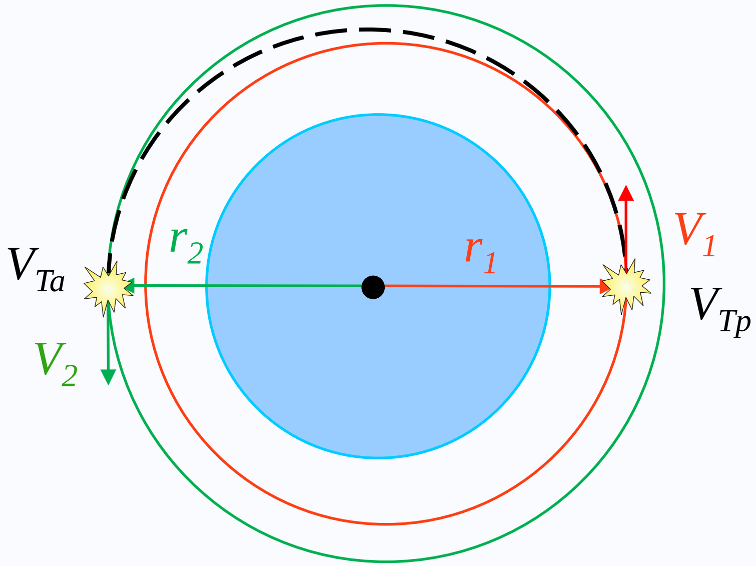
- Hence the total ΔV is:

$$\Delta V_{tot} = N \times \Delta V = 220 \times 3.7273 \times 10^{-5}$$

$$= 0.0082\ \mathrm{km/s}$$

$$= 8.2\ \mathrm{m/s}$$

Delta-V & propellant calculation



- The exhaust velocity is:

$$V_{ex} = g_0 I_{sp} = 9.81 \times 190 = 1863.9 \text{ m/s}$$

- Finally, the propellant mass is:

$$M_p = M_0\left(1 - e^{-\frac{\Delta V_{tot}}{V_{ex}}}\right)$$

$$= 1250\left(1 - e^{-\frac{8.2}{1863.9}}\right)$$

$$= 5.487 \text{ kg}$$

Question 3 (iii)

If the satellite suffered an anomaly that prevented it from boosting its orbit for a period of 1 year, explain qualitatively how its imaging mission would be affected. Your answer should be around 150-250 words.

[4 marks]

Answer (note not all words are needed. Highlighted text is important):

Altitude would decay: this would result in the spacecraft arriving early at the equator and the ground track shifting to the East. Orbit period would decrease, so more orbits in one day (might still be able to cover the equator). Inclination needed for sun-synchronous orbit depends on the size of the orbit, so the spacecraft would stop arriving at the same local solar time. Rate of change of right ascension is proportional to semi-major axis to the power 3.5, so as altitude decreases, rate of change of right ascension would also increase to values above 0.986 degrees/per day needed for sun-synchronous behaviour: LST at descending node will drift from 10:30 am. In summary: the invariant illumination and invariant viewing geometry conditions would not be possible, although imaging could still be done. We would have variable shadow lengths and orientations, and different view angles of target, making change detection very difficult.