SESA2025 Mechanics of Flight
Lateral aerodynamic derivatives

Lecture 4.2

Lateral aerodynamic derivatives

Recap: Lateral dynamic mode approximations

Roll damping (subsidence)

$$\lambda = \frac{\mathring{L}_p}{I_{xx}}$$

Slow spiral mode

$$\lambda = -\frac{g}{U_\infty}\frac{\left(\mathring{L}_v\mathring{N}_r - \mathring{L}_r\mathring{N}_v\right)}{\left(\mathring{L}_v\mathring{N}_p - \mathring{L}_p\mathring{N}_v\right)}$$

Dutch roll

$$\omega_n^2 = \frac{\mathring{Y}_v\mathring{N}_r + mU_\infty\mathring{N}_v}{mI_{zz}} \qquad \zeta = \frac{-1}{2\omega_n}\left(\frac{\mathring{N}_r}{I_{zz}} + \frac{\mathring{Y}_v}{m}\right)$$

Objective:
understand the connection between aerodynamic derivatives and the geometric parameters of design

Dimensionless aerodynamic derivative definitions

$$\frac{\Delta Y_a}{\frac{1}{2}\rho U_\infty^2 S} = Y_v\left(\frac{v}{U_\infty}\right) + Y_p\left(\frac{pb}{U_\infty}\right) + Y_r\left(\frac{rb}{U_\infty}\right)$$ Side force

$$\frac{\Delta L_a}{\frac{1}{2}\rho U_\infty^2 Sb} = L_v\left(\frac{v}{U_\infty}\right) + L_p\left(\frac{pb}{U_\infty}\right) + L_r\left(\frac{rb}{U_\infty}\right)$$ Rolling moment

$$\frac{\Delta N_a}{\frac{1}{2}\rho U_\infty^2 Sb} = N_v\left(\frac{v}{U_\infty}\right) + N_p\left(\frac{pb}{U_\infty}\right) + N_r\left(\frac{rb}{U_\infty}\right)$$ Yawing moment

The wingspan b is the reference length for lateral derivatives

We expect significant contributions from the fin (to the yawing moment due to sideslip and yaw rate) and the wings (to the moments due to roll-rate in particular)

Fin derivatives

(a) yaw moment due to yaw rate

Fin incidence: $\alpha_{F,r} = \frac{r l_F}{U_\infty}$

Fin lift slope

Fin moment: $\Delta N = -\frac{1}{2}\rho U_\infty^2 S_F C_{L_{F,\alpha}} \alpha_{F,r} l_F$

Hence: $\frac{\Delta N}{\frac{1}{2}\rho U_\infty^2 S b} = -\frac{S_F}{S} C_{L_{F,\alpha}} \frac{l_F^2}{b^2} \left(\frac{rb}{U_\infty}\right)$

Using the dimensionless derivative definition:

$$\frac{\Delta N}{\frac{1}{2}\rho U_\infty^2 S b} = N_r \left(\frac{rb}{U_\infty}\right) + \cdots$$

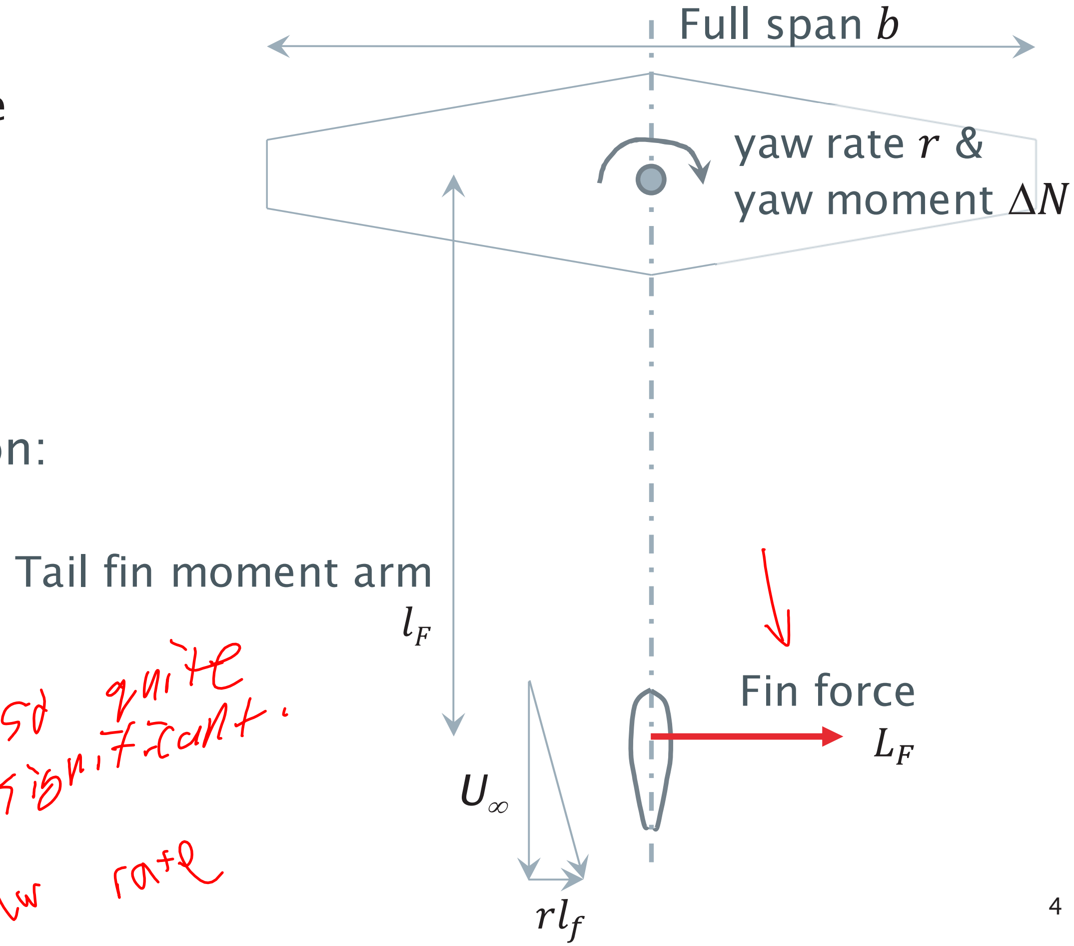
Final result (fin contribution to N_r):

$$N_r = -\frac{S_F}{S} C_{L_{F,\alpha}} \frac{l_F^2}{b^2}$$

damping

its squared, so quite significant.

yaw moment reaction from yaw rate



these proofs basically do the same thing, the isolated lift due to the input
note the opposite direction

Fin derivatives

(b) yaw moment due to sideslip

small angle approx

Fin incidence: $\alpha_{F,v} = \dfrac{v}{U_\infty}$

Fin lift slope

rearrange

Fin moment: $\Delta N = \frac{1}{2}\rho U_\infty^2 S_F \underbrace{C_{L_{F,\alpha}} \alpha_{F,v}}_{\doteq C_{L_F}} l_F$

Hence: $\dfrac{\Delta N}{\frac{1}{2}\rho U_\infty^2 S b} = \dfrac{S_F}{S} C_{L_{F,\alpha}} \dfrac{l_F}{b}\left(\dfrac{v}{U_\infty}\right)$

Using the dimensionless derivative definition:

$$\frac{\Delta N}{\dfrac{1}{2}\rho U_\infty^2 S b} = N_v\left(\frac{v}{U_\infty}\right) + \cdots$$

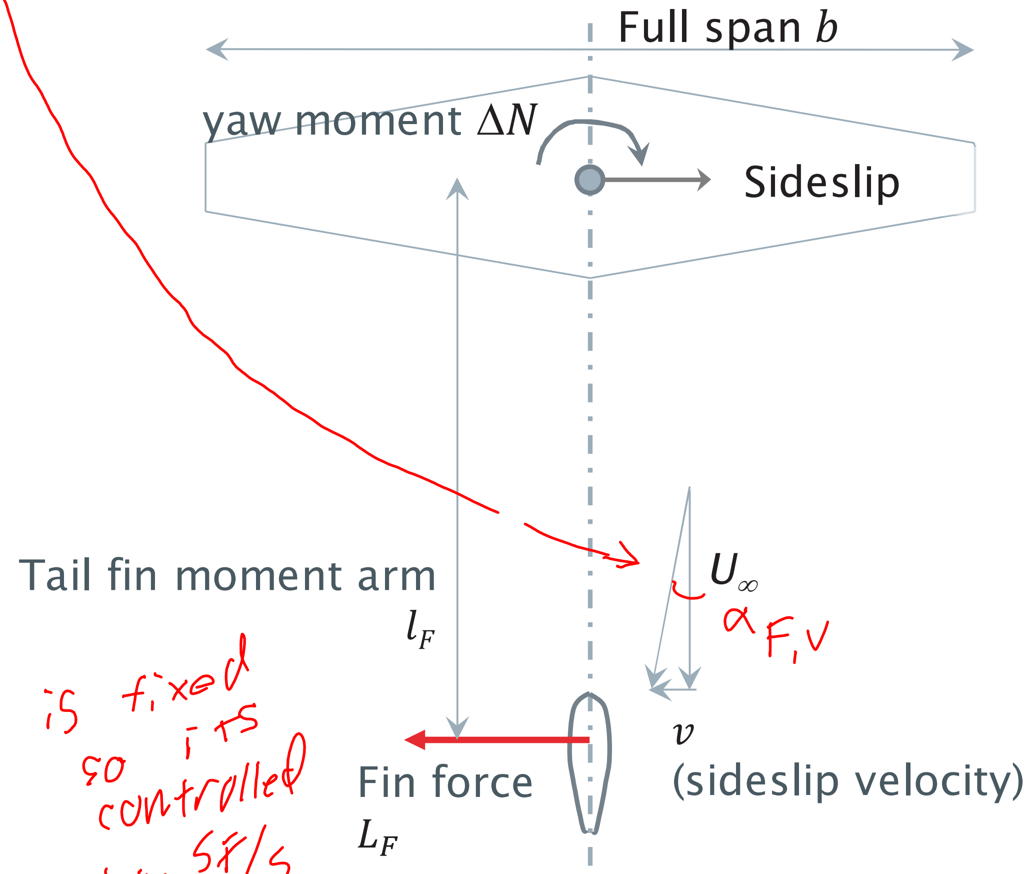
Final result (fin contribution to N_v):

sign, it yaws into the slip direction

$$N_v = \frac{S_F}{S} C_{L_{F,\alpha}} \frac{l_F}{b}$$

yaw moment reaction with sideslip

Practically this is fixed so its controlled by S_F/S and l_F/b

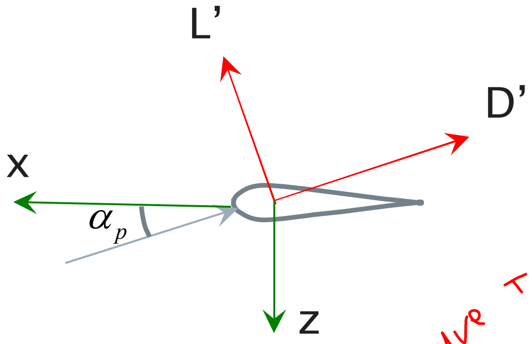


Wing derivatives

moments due to roll rate

Strip incidence
(due to roll rate p)

$$\alpha_p = \frac{py}{U_\infty}$$



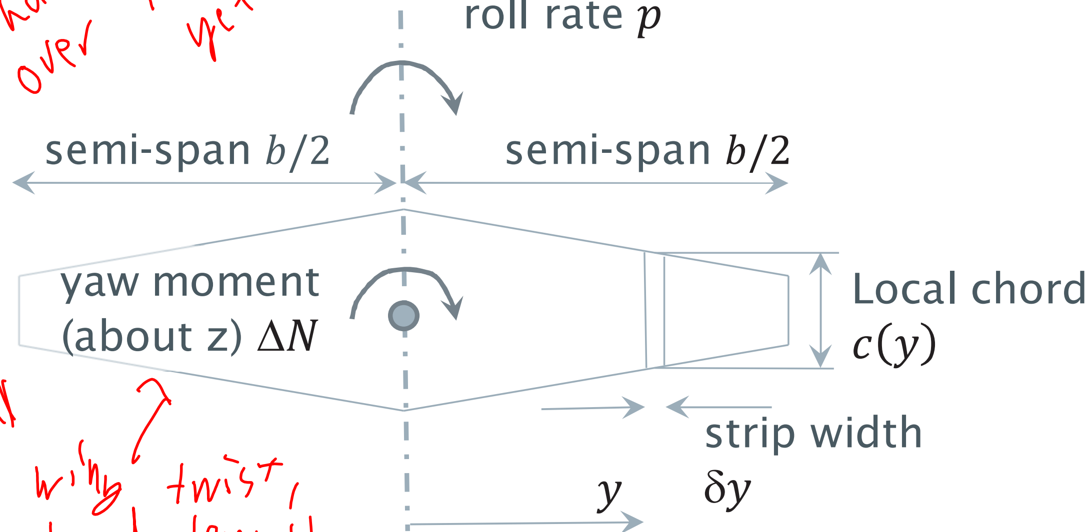
L' is the sectional lift
D' is the sectional drag

Out-of-balance forces on airfoil section per unit span (small angles)

$$\delta X = -\delta D' + L'\alpha_p$$

$$\delta Z = -\left(\delta L' + D'\alpha_p\right)$$

so have to calculate over small strips to get net forces.



wing twist, chord length change a lot with distance.

Wing derivatives

Yaw moment due to roll rate

Section forces per unit span:

$$\delta X = -\delta D' + L' \alpha_p$$

$$\delta X = \tfrac{1}{2} \rho U_\infty^2 \, c(y) \left(-C_{d_\alpha}(y) \alpha_p + C_l(y) \alpha_p \right)$$

Multiply by strip width δy and moment arm y and convert into integral over span to get yaw moment:

$$\Delta N = -\int_{-b/2}^{b/2} \tfrac{1}{2} \rho U_\infty^2 c(y) \left(-C_{d\,\alpha}(y) + C_l(y) \right) \alpha_p \, y \, \mathrm{d}y$$

$$\frac{\Delta N}{\tfrac{1}{2} \rho U_\infty^2 S b} = -\left(\frac{pb}{U_\infty} \right) \frac{1}{S b^2} \int_{-b/2}^{b/2} \left(C_l(y) - C_{d_\alpha}(y) \right) c(y) y^2 \mathrm{d}y$$

Wing derivatives

Yaw moment due to roll rate

Using:

$$\frac{\Delta N}{\frac{1}{2}\rho U_\infty^2 S b} = N_p\left(\frac{pb}{U_\infty}\right) + \cdots$$

This results in:

$$N_p = -\frac{1}{Sb^2}\int_{-b/2}^{b/2}\left(C_l(y) - C_{d_\alpha}(y)\right)\, c(y)\, y^2 \mathrm{d}y$$

Includes taper effects with $c(y)$. Includes spanwise lift distribution and twist effects with $C_l(y)$.

Note that these are *sectional* lift and drag coefficients.

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Wing derivatives

Roll moment due to roll rate

sectional drag (ignoring tip vortices) is really small

$\boxed{C_{l_\alpha} \gg C_d}$

Section forces per unit span:

$$\delta Z = -\left(\delta L' + D'\alpha_p\right)$$

$$\delta Z = -\tfrac{1}{2}\rho U_\infty^2 c(y)\left(C_{l_\alpha}(y)\alpha_p + C_d(y)\alpha_p\right) \approx -\tfrac{1}{2}\rho U_\infty^2 c(y) C_{l_\alpha}(y)\alpha_p$$

Multiply by strip width δy and moment arm y and convert into integral over span to get roll moment (ΔL):

$$\Delta L = -\int_{-b/2}^{b/2} \tfrac{1}{2}\rho U_\infty^2 c(y)\left(C_{l_\alpha}(y)\alpha_p\right) y \, \mathrm{d}y \; ; \qquad \frac{\Delta L}{\tfrac{1}{2}\rho U_\infty^2 S b} = -\left(\frac{pb}{U_\infty}\right)\frac{1}{Sb^2}\int_{-b/2}^{b/2} C_{l_\alpha}(y) c(y) y^2 \mathrm{d}y$$

Resulting in:

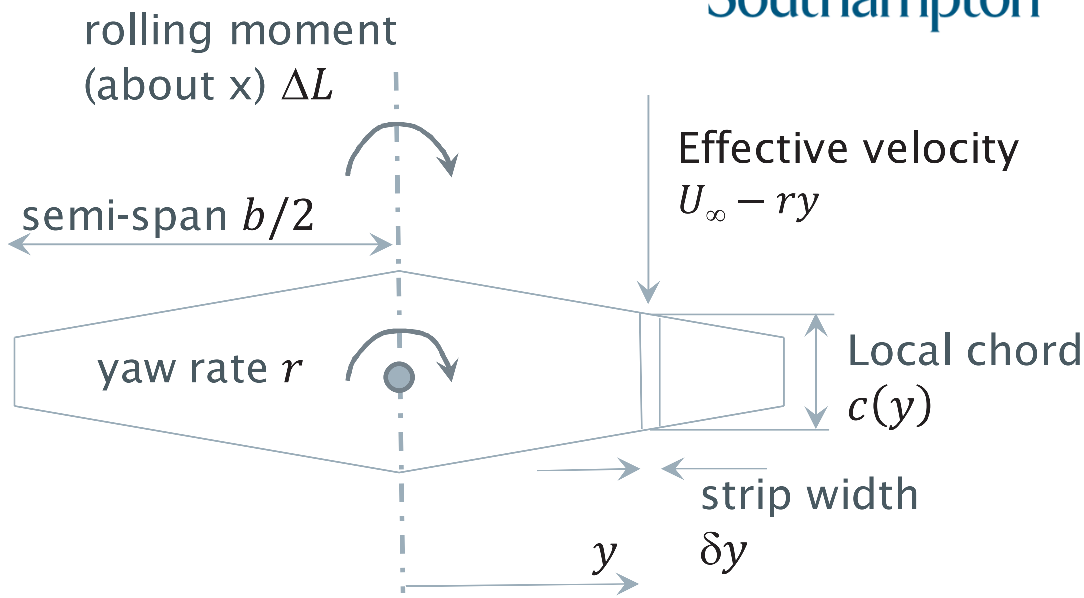
$$L_p = -\frac{1}{Sb^2}\int_{-b/2}^{b/2} C_{l_\alpha}(y) c(y) y^2 \mathrm{d}y$$

Note that these are *sectional* lift and drag coefficients.

restoring moment in response to roll

Wing derivatives

Roll moment due to yaw rate



Effective velocity:

$$U_{eff} = U_\infty - ry$$

Similar to previous derivative, ignore contribution of drag to changes in Z force

Change in down-force, final – initial

$$\delta Z = -\left(\tfrac{1}{2}\rho(U_\infty - ry)^2 c(y) C_l(y) - \tfrac{1}{2}\rho U_\infty^2 c(y) C_l(y)\right)$$

$$\delta Z =_1 \rho r y\, U_\infty c(y) C_l(y)$$

Multiply by strip width δy and moment arm y and convert into integral over span to get roll moment:

$$\Delta L = \int_{-b/2}^{b/2} \left(\rho r y\, U_\infty c(y) C_l(y)\right) y\, \mathrm{d}y$$

Wing derivatives

Roll moment due to yaw rate

Multiply by strip width δy and moment arm y and convert into integral over span to get roll moment:

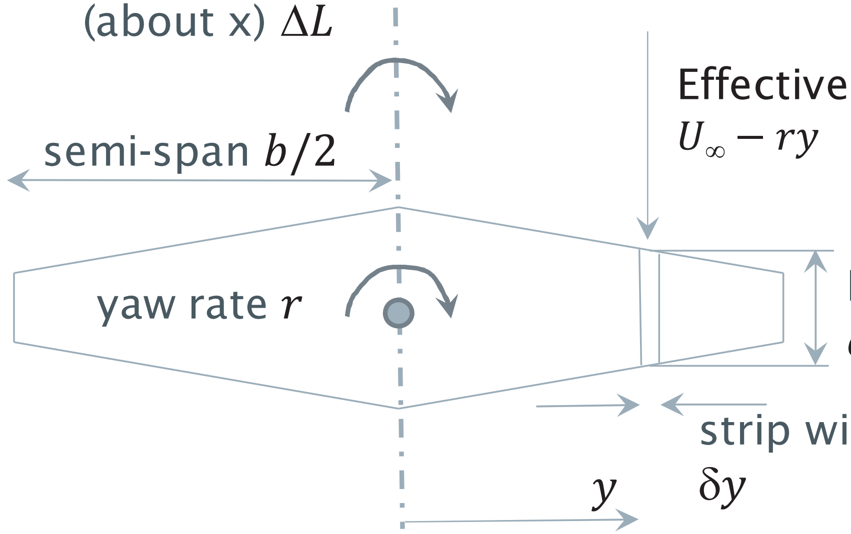
$$\Delta L = \int_{-b/2}^{b/2} \left(\rho r y\, U_\infty c(y) C_l(y)\right) y\, \mathrm{d}y$$

$$\frac{\Delta L}{\frac{1}{2}\rho U_\infty^2 S b} = \left(\frac{rb}{U_\infty}\right) \frac{2}{Sb^2} \int_{-b/2}^{b/2} c(y) C_l(y)\, y^2\, \mathrm{d}y$$

Resulting in:

$$L_r = \frac{2}{Sb^2} \int_{-b/2}^{b/2} c(y) C_l(y)\, y^2\, \mathrm{d}y$$

sign, rolls in direction of yaw



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Lateral derivative summary

We now have good estimates for L_p, L_r and N_p (mainly from wings) and N_v and N_r (mainly from fin)

Other derivatives:

- Y_v (<0) difficult to estimate, with contributions from fuselage, fin and wing (dihedral/anhedral)

sideforce drag

- L_v also difficult to estimate, contributions from fuselage, fin and wing (dihedral/anhedral, mounting position high/low and sweep)

- Y_p and Y_r are negligible (conventional aircraft)

restoring roll from sideslip

drag from forward and back of CG cancel-ish to be neglected.