SESA2024 Astronautics

Chapter 7: Propulsion – Solutions

1. Refer to Chapter 7 of notes.
Total impulse of rocket is
$I = \int_0^t T dt,$ where the thrust T is given by $T = \sigma V_{ex}$.
$\therefore\ I = \int_0^t \sigma V_{ex} dt = \int_0^t -\frac{dM}{dt} V_{ex} dt,$ since $\sigma = -\frac{dM}{dt},$ where M is the vehicle mass. Hence,
$I = -V_{ex} \int_0^t \frac{dM}{dt} dt = -V_{ex} \int_{M_0}^{M_b} dM = V_{ex}(M_0 - M_b) = V_{ex} M_e,$ where M_e is the fuel mass.
Hence the specific impulse is defined as the impulse per weight of fuel, so that
$I_{sp} = \frac{I}{M_e g_0} = \frac{V_{ex} M_e}{g_0 M_e} = \frac{V_{ex}}{g_0} \Rightarrow V_{ex} = g_0 I_{sp}.$
One way to see the importance of the specific impulse is to consider the field-free rocket equation,
$\Delta V = V_{ex} \log_e (M_0 / M_b) = g_0 I_{sp} \log_e (M_0 / M_b).$
For a fixed initial mass and a fixed amount of propellant, then the mass ratio M_0 / M_b is a constant, and the rocket equation reduces to
$\Delta V = I_{sp} K$, where K is a constant. Hence the ΔV of the vehicle is directly proportional to the specific impulse.

2. The liquid hydrazine (N_2H_4) is fed to the thruster under pressure, and when it is required to fire, a valve opens (electrical power required) to allow the propellant to flow via an injector onto a catalytic bed. This bed usually comprises platinum and iridium spread over a 'large' surface area of aluminium oxide. The catalytic reaction is exothermic (heat producing) and decomposes the liquid hydrazine into hot nitrogen, ammonia and hydrogen gases. These hot products are expanded through a nozzle to produce a thrust, with a specific impulse in the region of 230 seconds.

3. ***Impulse thrust*** – due to the momentum flow of the exhaust products, and ***pressure thrust*** – due to a difference between the pressure of the exhaust products and the pressure of the ambient atmosphere across the exit plane of the rocket nozzle.

4. ***Hypergolic*** fuel is a combination of propellant and oxidiser, such that combustion occurs immediately the two substances are mixed – i.e. there is no

requirement for initiating combustion. An example of this is the combination of monomethylhydrazine (MMH) and nitrogen tetroxide (N_2O_4) – the latter being the oxidiser.

5. Three functions:
   - Primary propulsion: orbit transfer manoeuvres
   - Secondary propulsion: orbit control manoeuvres
   - Secondary propulsion: attitude control

6. Possible answers include the following:

System	✓	✕
Liquid monoprop	Low complexity Small minimum impulse	Fuel handling hazardous Relatively low I_{sp} (~ 230 sec)
Liquid biprop	More than one engine burn (restart capability) Relatively high I_{sp} (~ 310 sec for MMH/N_2O_4, ~ 450 sec for LOX/LH_2)	Relatively complex feed system for hypergolic combinations (to prevent premature mixing of propellant and oxidiser) Fuel Handling hazardous
Solid prop	Low complexity Ease of storage	One engine firing only Relatively low I_{sp} (~ 260 sec)
Cold gas	Low complexity Small minimum impulse Ease of handling fuel (e.g. Nitrogen)	Very low I_{sp} (~ 50 sec) Low total impulse
Ion prop	High I_{sp} (~ 4000 sec) High ΔV capability	High electrical power requirement (and its system impacts) Potential contamination of S/C by propellant
Nuclear prop	High I_{sp} (~ 1000 sec) Relatively high thrust for long durations	Not 'green' Radiation hazard

7. Ion propulsion has the highest specific impulse

8. Liquid bipropellant; Solid propellant; possibly Nuclear (not demonstrated)

9. Generally, solid propellant systems are not used for attitude control because of their 'one-shot' characteristic – i.e. one engine firing only. An exception to this is the attitude control of launch vehicles using solid propellant. The engine nozzle(s) can be gimballed to stabilise and control the vehicle's attitude, by modulating the effective engine thrust vector around the vehicle's centre of mass to produce 'control torques'. For example, the Space Shuttle.

10. ***Resistojet:*** $I_{sp} = 700\,\text{sec},\ \eta = 0.9.$

Effective exhaust velocity, $V_{ex} = g_0 I_{sp} = 6867$ m/sec. Then the thrust is given by

$T = \sigma V_{ex} \;\; \Rightarrow \;\; \sigma = T/V_{ex} = \dfrac{50 \times 10^{-3}}{6867} = 7.2812 \times 10^{-6}$ kg/sec.

Then from the notes, we have

W = $(1/2\eta)\,\sigma V_{ex}^2 = (1/(2 \times 0.9))\,(7.2812 \times 10^{-6})\,(6867)^2 = 190.75$ W

Arcjet: $I_{sp} = 1500$ sec, $\eta = 0.3$. Similar calculations give

$V_{ex} = 14715$ m/sec, $\sigma = 3.3979 \times 10^{-6}$ kg/sec, W = 1226 W

Ion: $I_{sp} = 5000$ sec, $\eta = 0.75$. In this case we have,

$V_{ex} = 49050$ m/sec, $\sigma = 1.0194 \times 10^{-6}$ kg/sec, W = 1635 W

Clearly the power overhead when flying the arcjet and ion thrusters is significant, and this will have a major impact on the mass of the power subsystem, and on the size of the solar array needed (if using photovoltaics) – which will have an influence on the configuration.

11. Produce real curves computationally.
The curves have a maximum value, since as V_{ex} increases so the power increases. This in turn increases the mass of the power plant required, and this will compromise the dynamical performance – i.e. it will decrease the acceleration for a given thrust level. Consequently, the achievable ΔV is constrained.
For this type of system, therefore, we should operate near the maximum value on each curve. That is, we should optimise in terms of choosing an appropriate V_{ex} for a given mission ΔV, for a particular payload fraction.

12. Why is the use of a chemical propulsion system not feasible? – estimate the fuel mass required for a ΔV of 17 km/sec using a typical chemical propulsion V_{ex}.

Burn time $t_b = 2 \text{ years} = 6.31152 \times 10^7$ sec.
For $M_p/M_0 = 0.1$, then peak ΔV occurs around $V_{ex}/V_c \approx 0.65$, from plot produced in previous question. Then from the notes, this corresponds to $\Delta V/V_c = 0.651$ (the near equality of these two values is coincidental!). These values give the 'characteristic velocity' of the system as
$V_c = \Delta V/0.651 = 26114$ m/sec,
and the exhaust velocity as
$V_{ex} = 0.65\, V_c = 16974$ m/sec,
$I_{sp} = V_{ex}/g_0 = 1730$ sec
This suggests an ion thruster is appropriate.
Equation (7.16) of notes gives
$\Rightarrow \;\; \beta \approx 5.4$ W/kg.

This suggests the use of RTGs is appropriate as a power source.
If $M_0 = 3000\,\text{kg}$ then
payload mass is $M_p = 0.1(3000) = 300\,\text{kg}$,
propellant mass is $M_e = 1898$ kg,
power plant mass is $M_w = 802$ kg,
power requirement is $W = \beta M_W \approx 4.3\,\text{kW}$,
propellant flow rate is $\sigma = M_e / t_b \approx 3.01 \times 10^{-5}$ kg/sec,
and the thrust level is $T = \sigma V_{ex} = 0.51$ N