

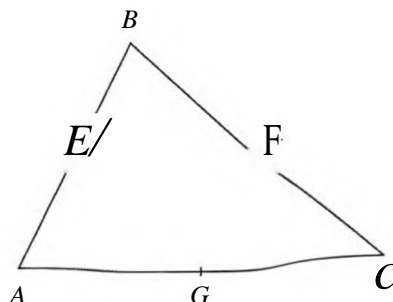
Name.....

Department.....

Whilst answering the test you may use the Formula Sheet or the Engineering Data Book, depending on your department, and a calculator but please do NOT consult the Module or any book.

### TEST 14.1

1.  $ABC$  is a plane triangle in which  $AB = a$ ,  $BC = b$  and the midpoints of the sides are  $E$ ,  $F$  and  $G$  as shown



Express the following vectors in terms of  $a$  and  $b$ :

$$\vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{EF} = \frac{1}{2} \vec{AC}$$

2. Suppose that the points  $A$  and  $B$  have coordinates  $(1, -1)$  and  $(2, 1, -2)$  respectively with respect to rectangular cartesian axes. Obtain  $\vec{AB}$  in terms of  $i$ ,  $j$  and  $k$ .

$$\vec{AB} = \vec{OB} - \vec{OA} = (2, 1, -2) - (1, -1, 0) = (1, 2, -2)$$

3. If  $\underline{a} = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$  and  $\underline{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$  find

(i)  $3\underline{b} - \underline{a}$   $3(\mathbf{i} + \mathbf{j} + \mathbf{k}) - (2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}) = \mathbf{i} + 9\mathbf{k}$

(ii)  $|\underline{a}|$   $|\underline{a}| = (2^2 + 3^2 + (-6)^2)^{\frac{1}{2}} = (4 + 9 + 36)^{\frac{1}{2}} = 7$

(iii) a unit vector parallel to  $\underline{a}$   
parallel to  $\underline{a} = \frac{\underline{a}}{|\underline{a}|} = \frac{1}{7}(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$

(iv) the component of  $\underline{b}$  in the direction of  $\underline{a}$ .  
cpt. of  $\underline{b}$  in direction of  $\underline{a} = \underline{b} \cdot \hat{\underline{a}} = (\mathbf{i}, \mathbf{j}, \mathbf{k}) \cdot \frac{1}{7}(2, 3, -6)$   
 $= \frac{1}{7}(2 + 3 - 6) = -\frac{1}{7}$

4. Given that  $\underline{a} = \mathbf{i} + \mathbf{k}$  and  $\underline{b} = \mathbf{j} + \mathbf{k}$  find

(i)  $\underline{a} \cdot \underline{b}$   $\underline{a} \cdot \underline{b} = (\mathbf{i}, 0, \mathbf{k}) \cdot (\mathbf{j}, \mathbf{k}, \mathbf{k}) = 0 + 0 + 1 = 1$

(ii) the angle between  $\underline{a}$  and  $\underline{b}$ .  
 $\cos \theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|} = \frac{1}{(1^2 + 1^2)^{\frac{1}{2}} (1^2 + 1^2)^{\frac{1}{2}}} = \frac{1}{\sqrt{2} \sqrt{2}} = \frac{1}{2}$   
 $\therefore \theta = \frac{\pi}{3} \quad (\text{or } 60^\circ)$

5. A force  $\underline{F}$  of magnitude 4 N acts on a body in the direction of the vector  $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ .

(i) Determine the vector force  $\underline{F}$ .

(ii) Calculate the work done by the force when the body is displaced by  $\mathbf{i} + \mathbf{j} + \mathbf{k}$ .

6. Given that  $\underline{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$  and  $\underline{b} = 2\mathbf{i} + \mathbf{k}$  find

$$\begin{aligned}\underline{a} \times \underline{b} &= (1, -1, 1) \times (2, 0, 1) \\ &= (-1(1) - 1(0), 1(2) - (1)(1), -1(0) - (1)(2)) \\ &= (-1 - 0, 2 - 1, 0 - 2) \\ &= \underline{(-1, 1, -2)}\end{aligned}$$

$\frac{1}{2} \begin{pmatrix} -1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$

(ii) a unit vector perpendicular to  $\underline{a}$  and  $\underline{b}$ .

$$\begin{aligned}\text{unit vector perp. to } \underline{a} \text{ and } \underline{b} &= \frac{\underline{a} \times \underline{b}}{|\underline{a} \times \underline{b}|} = \frac{(-1, 1, -2)}{((-1)^2 + 1^2 + 2^2)^{1/2}} \\ &= \underline{\underline{\frac{(-1, 1, -2)}{\sqrt{6}}}}\end{aligned}$$

7. A body is rotating with angular velocity 3 rad/s in a positive sense about an axis from the point (2, 1, -1) to the point (0, 3, -2) (distance in metres). Calculate

(i) the angular velocity vector  $\underline{\omega}$

$$\begin{aligned}\underline{\omega} &= 3 \frac{(-2, 2, -1)}{\sqrt{(-2)^2 + 2^2 + (-1)^2}} = \underline{\underline{\frac{(-2, 2, -1)}{\sqrt{9}}}} \\ &= \underline{\underline{\frac{(-2, 2, -1)}{3}}}\end{aligned}$$

(ii) the velocity of the body at the point (1, 2, 1).

$$\begin{aligned}\underline{v} &= \underline{\omega} \times \underline{r} = \frac{(-2, 2, -1)}{3} \times (1, 2, 1) \\ &= \frac{1}{3} \begin{vmatrix} -2 & 2 & -1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{vmatrix} = \frac{1}{3} (-2(2) - (-1)(2), -(-2)(1) - (-1)(1), -2(2) - (-1)(1)) \\ &= \frac{1}{3} (-4 + 2, 2 - 1, -4 + 1) = \underline{\underline{\frac{(-2, 1, -3)}{3}}}\end{aligned}$$