UNIVERSITY OF SOUTHAMPTON SESA3029W1

SEMESTER 1 EXAMINATIONS 2022-23

TITLE: Aerothermodynamics

DURATION: 120 MINS

This paper contains **FIVE** Questions

Answer **ALL** questions on this paper. Questions 1, 2, 3, 4 and 5 are worth 36, 16, 16, 14, and 18 marks respectively (total 100 marks).

An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow **and** normal shock tables (11 sides) are provided. (In reading from tables, nearest values are acceptable unless explicitly stated otherwise.)

An oblique shock chart is provided.

Note that a formula sheet is provided at the end of this paper

Only University approved calculators may be used.

A foreign language direct ‘Word to Word’ translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), c_p=1005 J/(kg K), γ=1.4, Pr=0.7, ρ=1.225 kg/m^3 and μ=1.79x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

Q.1 Figure Q.1 shows a triangular thin aerofoil at an incidence $\alpha = 10^{\circ}$ to an approaching stream at M_1=1.7 and p_1=20 kPa. The aerofoil chord length is c=15 cm. The lower surface is a straight line, while the maximum thickness is at the half chord location. The internal angle at the leading and trailing edges of the aerofoil is $\phi = \alpha = 10°$.

(i) Sketch the flow past the aerofoil, identifying clearly all the shock and expansion waves present.

[4 marks]

(ii) Find the pressure and Mach number on each surface using the shock-expansion method.

[14 marks]

(iii) Find the pitching moment about the leading edge according to the shock-expansion method.

[4 marks]

(iv) Find the surface pressures and the resulting pitching moment about the leading edge using Ackeret's theory. Comment on your results.

[10 marks]

(v) Without doing any calculations, explain how you could compute the location of the aerodynamic centre ($x_{AC}/c = -dC_{M,LE}/dC_L$) and where you expect it to be located

[4 marks]

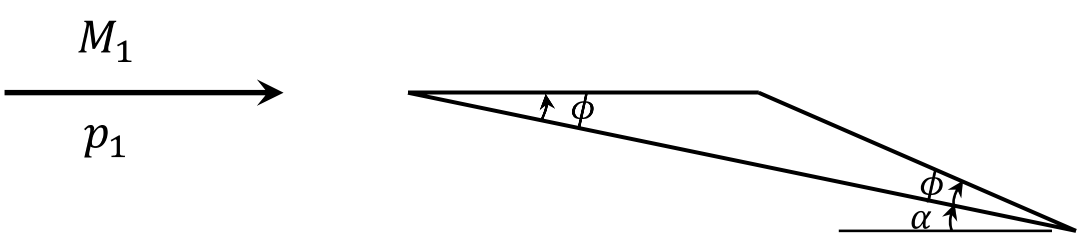

Figure Q.1

Q.2

(i) For $M_\infty^2 < 1$ show how the compressible potential flow equation

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

can be reduced to Laplace's equation and hence derive the Prandtl-Glauert relation in the form

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

[8 marks]

(ii) An aircraft wing with slat and flaps deployed has a minimum pressure coefficient, measured in a low-speed wind tunnel (effectively at Mach zero), of $C_{p0,min}$ = -5.4. The critical Mach number is estimated to be between 0.3 and 0.4. Find a better estimate.

[8 marks]

TURN OVER

Q.3

(i) Figure Q.3 below shows two characteristic lines 0-1-4 and 0-2-3-5 in a 2D convergent-divergent nozzle, symmetric about the centreline shown, designed to accelerate air to an exit Mach number M_e=1.9. Define the Riemann invariants R^+ and R^- and show that

$$\theta_{\max} = \frac{\nu\left(M_e\right)}{2}$$

where ν is the Prandtl-Meyer function.

[6 marks]

(ii) The starting characteristic lines at point 0 in figure Q.3 have flow angles of $\theta_{max}/2$ (the 0-1 line) and θ_{max} (the 0-2 line). Find the flow angle and Mach number at point 2.

[6 marks]

(iii) Find the angles of the characteristic lines 2-4 and 3-5 (hint: a full calculation of all the points is not required)

[4 marks]

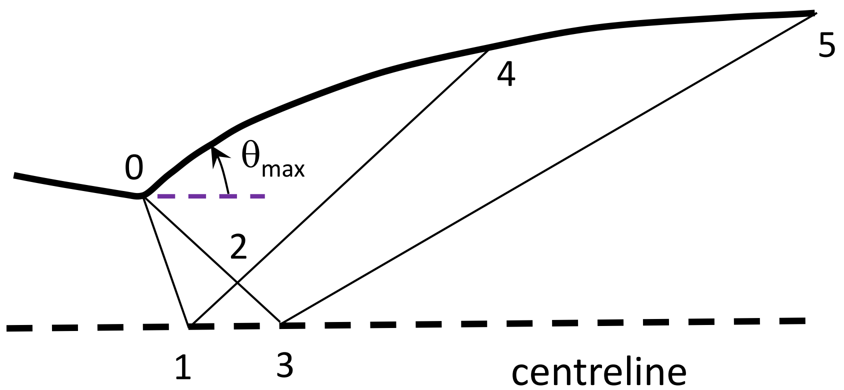

Figure Q.3

Q.4 A monolithic block of quadratic cross-section 20 cm x 20 cm is exposed to constant temperature boundary conditions of 300 K on the left and right side and 500 K on the top and bottom side. Apply the finite difference method for the steady heat diffusion equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

to estimate the temperatures in the interior on a grid of step sizes $\Delta x = \Delta y = 5$ cm.

(i) Take advantage of symmetries to derive the required four algebraic stencil relations.

[6 marks]

(ii) Solve the resulting linear 4x4 system to evaluate the discrete temperature values.

[8 marks]

TURN OVER

Q.5 A shell-and-tube counterflow heat exchanger must be designed to heat 2.5 kg/s of water from 15 to 85°C. The heating is to be accomplished by passing hot engine oil, which is available at 160°C, through the shell side of the exchanger. The oil is known to provide an average heat transfer coefficient of h = 400 W/m^2·K on the outside of the tubes. Ten tubes pass the water through the shell. Each tube is thin walled, of diameter 25 mm and makes eight passes through the shell.

Use c_p = 2350 J/kg·K for oil and c_p = 4181 J/kg·K, $\mu = 5.48\times10^{-4}$ Ns/m^2, k = 0.643 W/m·K, Pr = 3.56 for water.

(i) If the oil leaves the exchanger at 100°C, what is its flow rate?

[6 marks]

(ii) Compute the Reynolds number of the flow in the tubes and use the result to estimate the combined heat transfer coefficient h'.

[6 marks]

(iii) Apply the log mean temperature difference method to compute the length of the tubes and the pass length.

[6 marks]

END OF PAPER (Formula sheet overleaf)

Useful Formulae

Perfect gas equation of state

$$p = \rho R T$$

Sound speed in a perfect gas

$$a^2 = \gamma R T$$

Adiabatic flow

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Mach angle:

$$\sin\mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta+\mu)_A + (\theta+\mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta-\mu)_B + (\theta-\mu)_P\right]$$

$$x_p = \frac{x_B \tan\alpha_{BP} - x_A \tan\alpha_{AP} + y_A - y_B}{\tan\alpha_{BP} - \tan\alpha_{AP}}$$

$$y_P = y_A + (x_P - x_A)\tan\alpha_{AP}$$

TURN OVER

Velocity potential equation:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

Linearised pressure coefficient

$$C_p=-2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

Ackeret formula:

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

Laminar pipe flow:

$$\mathrm{Nu}=4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}=3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x=0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu}=0.023\mathrm{Re}^{4/5}\,\mathrm{Pr}^n$$

$$(n=0.3 \text{ for cooling, } n=0.4 \text{ for heating})$$

Turbulent boundary layer:

$$\mathrm{Nu}_x=0.0308\mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.0296\mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

