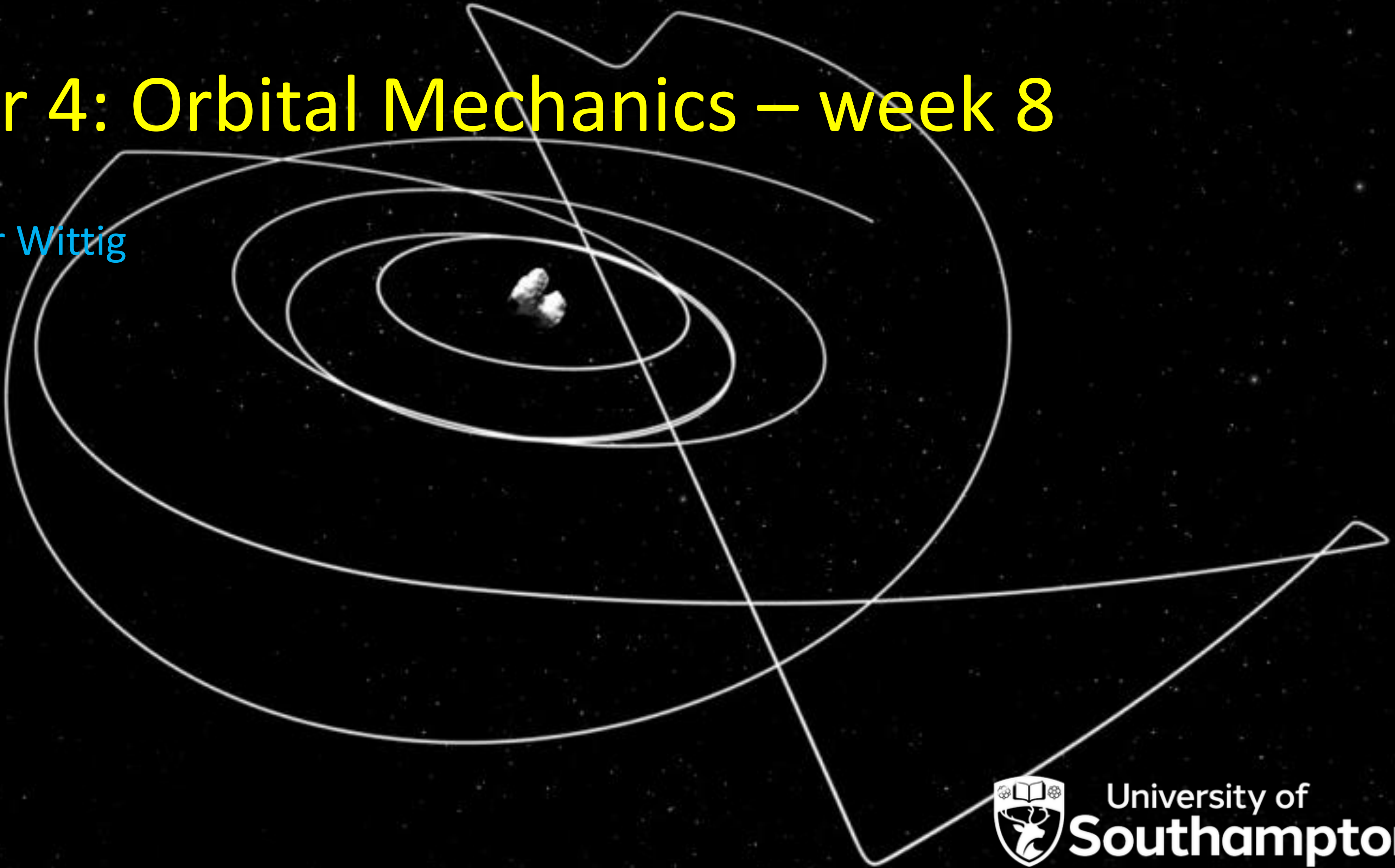
Chapter 4: Orbital Mechanics – week 8

Dr. Alexander Wittig



Rosetta lander deployment orbit (ESA)

Southampton AERO ASTRO

University of Southampton

Introduction

- F2F lecture: Content of the week and “big picture”
  - Monday 13:00 in 35/1001
    Tuesday 15:00 in 35/1001
  - recorded, uploaded to Panopto
  - Slides available on BB
  - Questions welcome
    - You may want to look at the in-depth videos beforehand
- Pre-recorded Material
  - Filling in details from lectures
  - More in depth coverage of topics
  - **fully examinable material**
  - Videos and lecture notes on BB
- Tutorial sessions
  - Tuesdays 16:00 in 35/1001
  - Q&A on Problem Sheets
  - Tutors on hand for questions
- Quiz: 16/12
  - similar to tutorial sheet questions
  - numerical answers
  - same rules as other quizzes

Problem sheet & Tutorials

Problem Sheet

- Covers full 4 weeks of material
- Attempt questions on material of each week
- Additional questions available in textbooks

Tutorial

- Tuesdays 16:00 in 35/1001
- Suggested in groups, self-organized
- **Active learning** session: prepare by attempting problem sheet
- Demonstrators available to help with questions
- 1 problem solved together (let me know preference 1 day before)

Chapter 4 Roadmap

- **Week 8: Orbital Motion**
  - Math Basics
  - Spherical Trigonometry
  - Keplerian Motion from First Principles
- Week 9: Orbit Properties
  - Constants of Motion
  - Eccentricity Vector
  - Conic Sections
- Week 10: Orbit Representation
  - Coordinates
  - Dates & Times
  - Orbital Elements
- Week 11: Time Dependence
  - Eccentric Anomaly
  - Hyperbolic Anomaly
  - Kepler's Equations

Textbooks

Orbital Mechanics for Engineering Students
Curtis, H.
Butterworth-Heinemann, 2013.
ISBN-13 978-0123747785

Orbital Mechanics
Chobotov, V.
AIAA Education Series, 2002
ISBN-13 978-1563475375

Fundamentals of Astrodynamics and Applications
Vallado, D. A.
Springer, 2007
ISBN-13 978-0387718316

An Introduction to the Mathematics and Methods of Astrodynamics
Battin, R. H.
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ISBN-13 978-1563473425

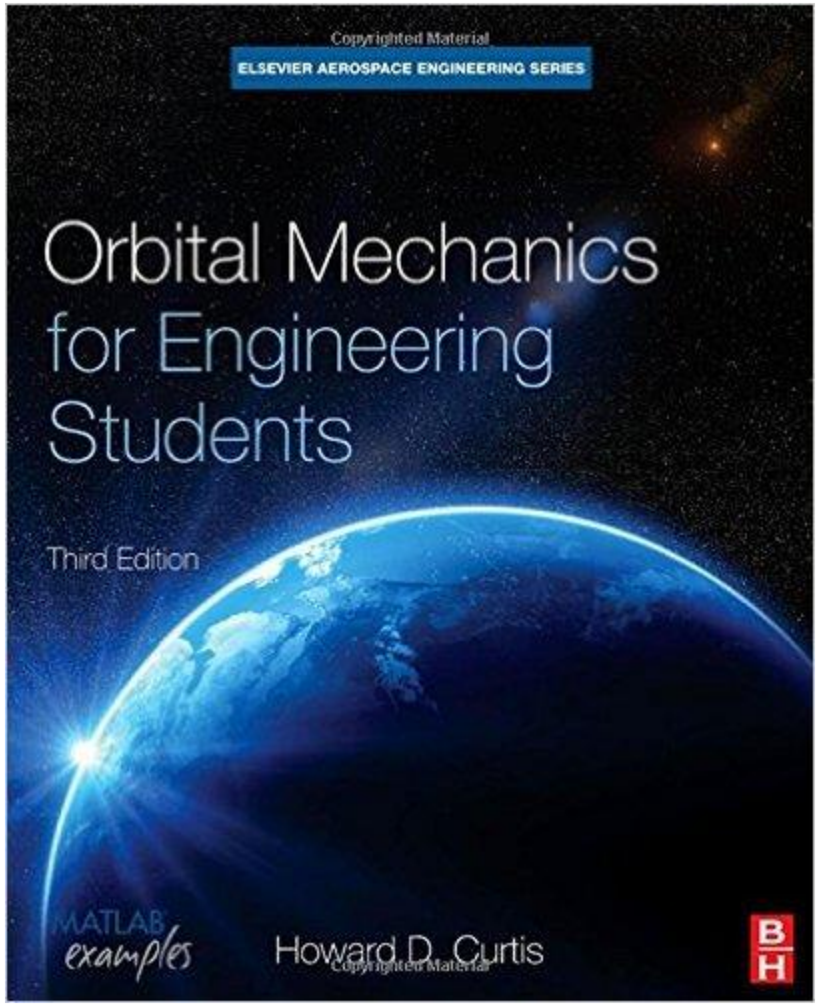
Satellite Orbits: Models, Methods and Applications
Montenbruck, O. and Gill, E.
Springer, 2011
ISBN-13 978-3540672807

Subjective observations:

- good introductory textbook
- very explicit mathematical derivations
- copious amount of worked examples
- not as complete as others
- not as useful as a reference

Chapter 4 will follow this textbook closely.

Available as free full text with Soton Login from publisher!



https://www.sciencedirect.com/book/9780080977478/orbital-mechanics-for-engineering-students

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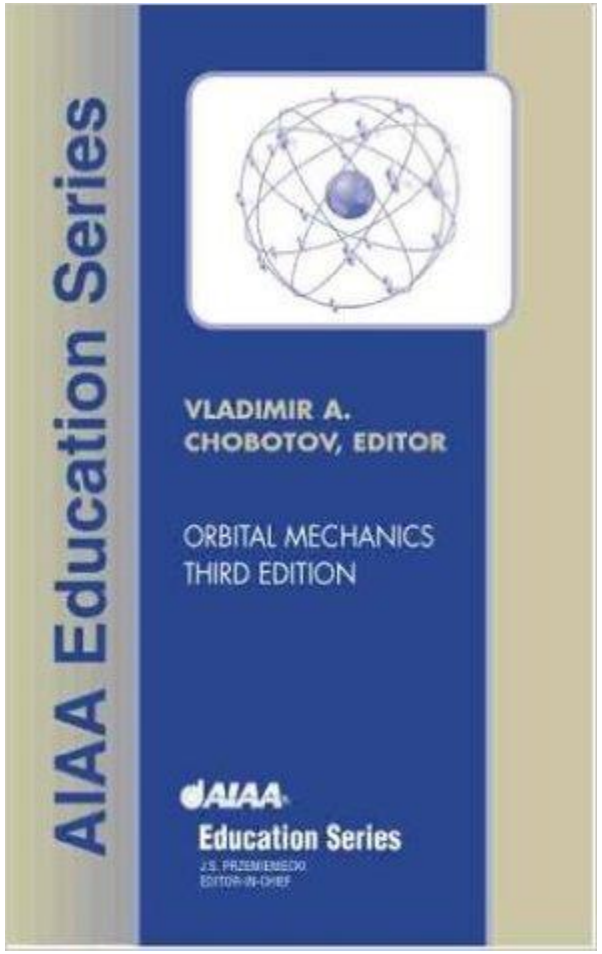
Subjective observations:

- wide range of topics covered
- mathematical derivations often challenging
- written as an expert text, not an introductory text
- includes references to research papers

eBook available at:
https://ebookcentral.proquest.com/lib/soton-ebooks/detail.action?docID=3111521



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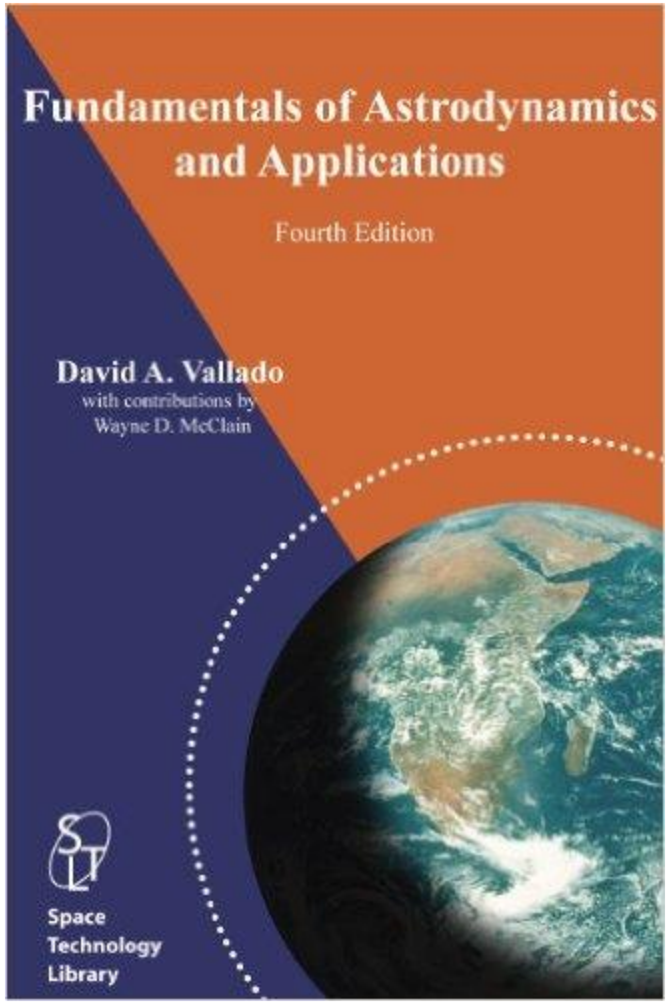
Springer, 2011
ISBN-13 978-3540672807

Subjective observations:

- good and quite readable, complete textbook
- understandable mathematical derivations
- focus on numerical implementation of algorithms (including code in various languages)
- also useful as a reference

Author worked on SGP4, the US airforce standard orbit propagator for NORAD TLEs.



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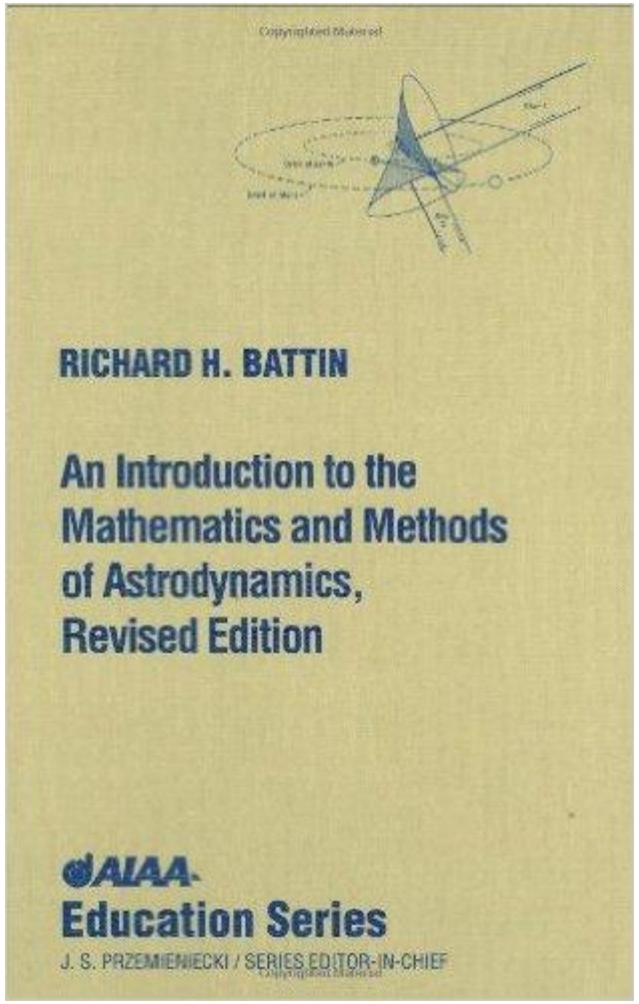
Subjective observations:

- most complete reference on the subject
- sometimes challenging mathematical derivations
- written by a mathematician!
- includes many exercises
- extremely useful as a reference

Author wrote the Apollo mission guidance computer code.

eBook available at:
https://ebookcentral.proquest.com/lib/soton-ebooks/detail.action?docID=3111476

Check out his MIT lecture "*A Funny Thing Happened on the Way to the Moon*" for some history of the early NASA program on YT:
http://y2u.be/ieiEoTo8-XY



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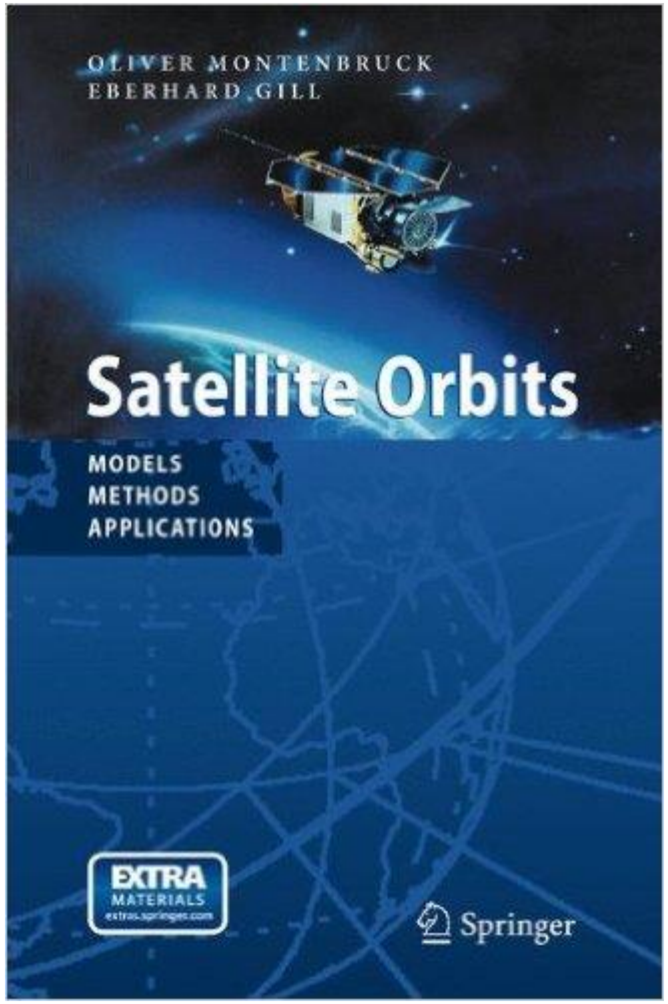
Subjective observations:

- focused specifically on satellites
- quite readable, few long derivations
- includes numerical and practical engineering considerations
- good textbook and reference on its limited subset of the subject



Mathematical Concepts

Learning outcomes

- Recall basic concepts of **vector algebra**
  - Vector identities
  - Coordinate representation

- Relate **Kinematics** and **Dynamics** in different frames
  - Accelerating reference frames
  - Equations of motion in accelerating reference frames

- Apply trigonometry to problems on the sphere
  - Trigonometric identities on the sphere

Vector Algebra & Mechanics

see Chapter 4 Video Lecture 1

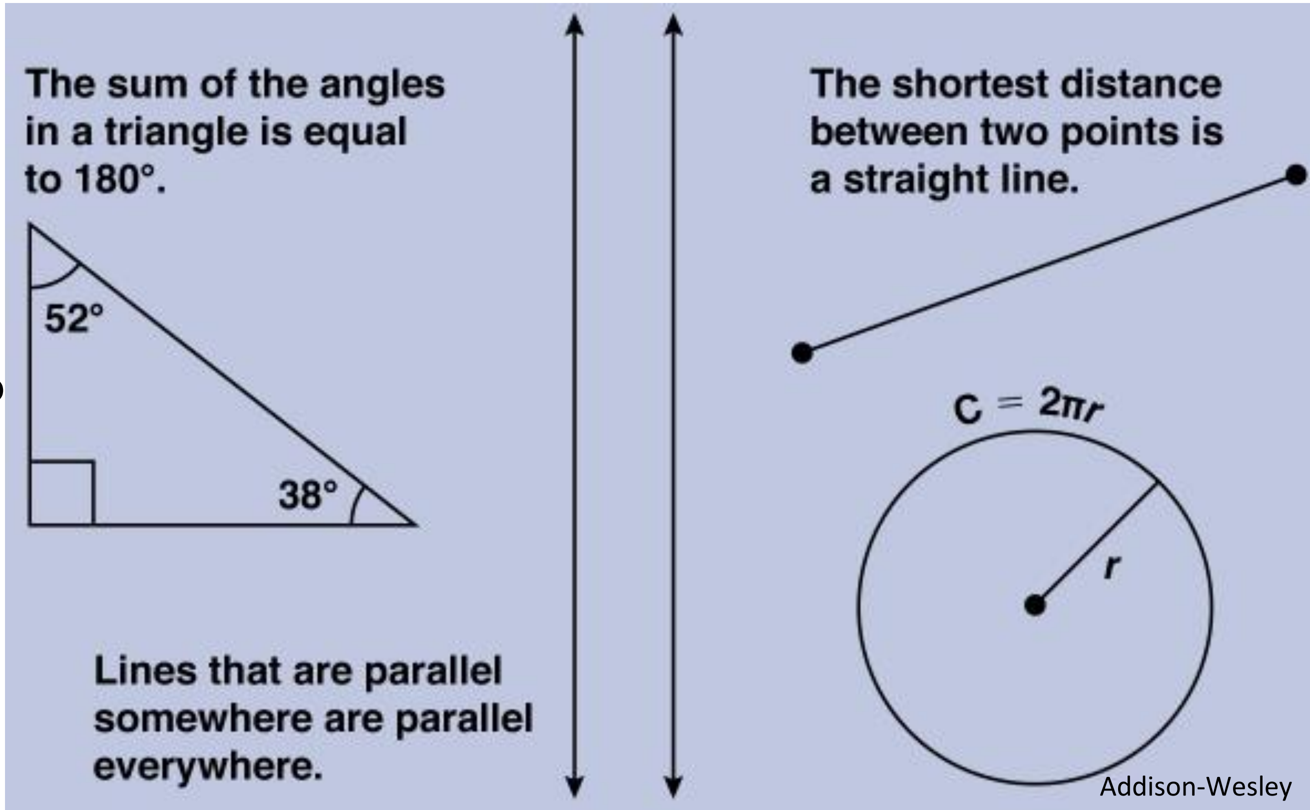
Spherical Trigonometry

see also Chapter 4 Video Lecture 2

Geometry in curved space

- Euclidean geometry is in flat space
- In curved space Euclidean properties generally **do not** hold
- Sphere is a simple curved space relevant in orbital mechanics and astronomy

- Line segment: shortest path between two points
  - Is it unique?
  - How often can different lines intersect?
  - How often do parallel lines intersect
  - How many lines are needed for a polygon?



Addison-Wesley

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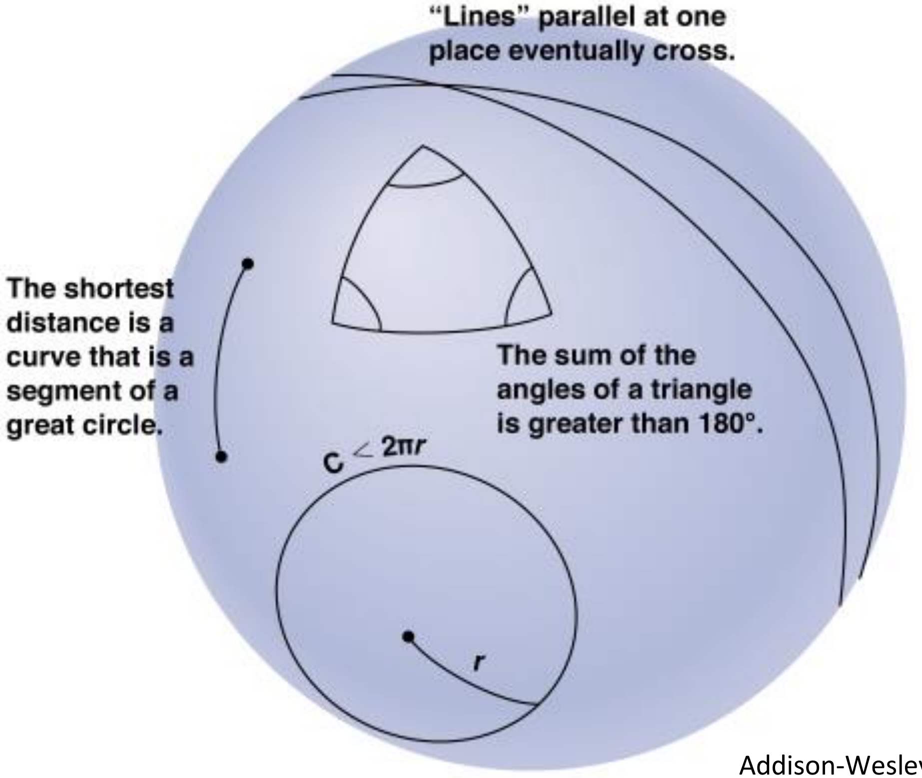
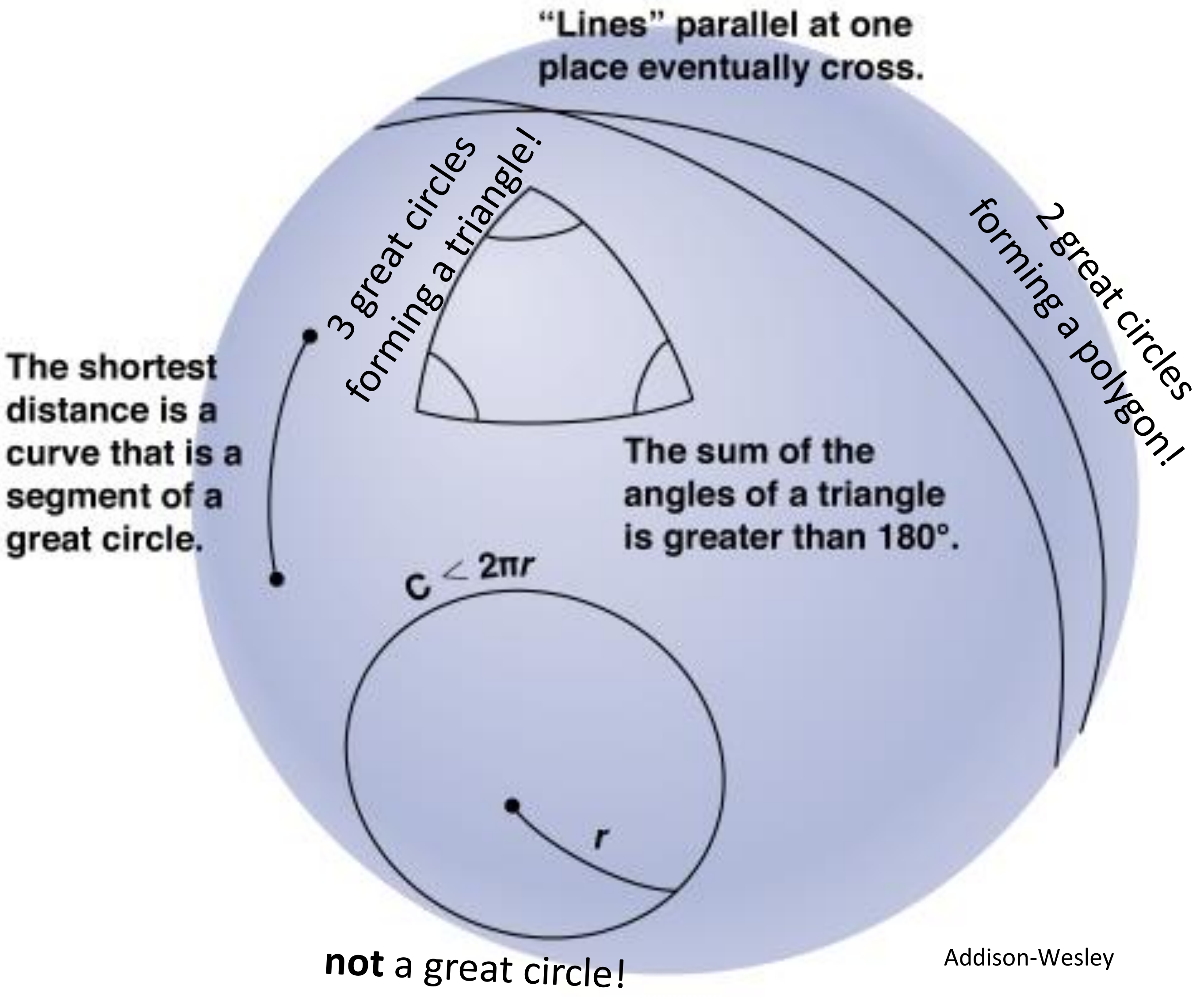


Addison-Wesley

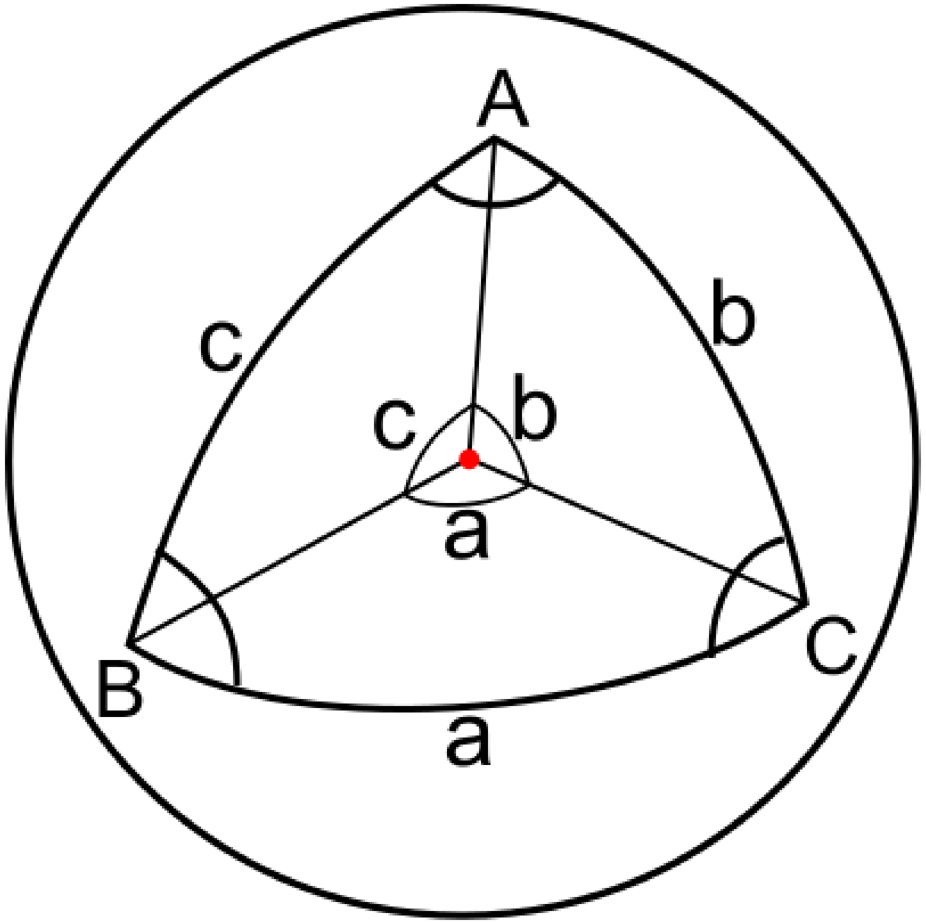
Spherical Geometry

- **Great circle**: circle on a plane including the center point of the sphere
- Line: segment of a **great circle**
- Polygon: closed loop of line segments on sphere surface



Addison-Wesley

Spherical Triangles



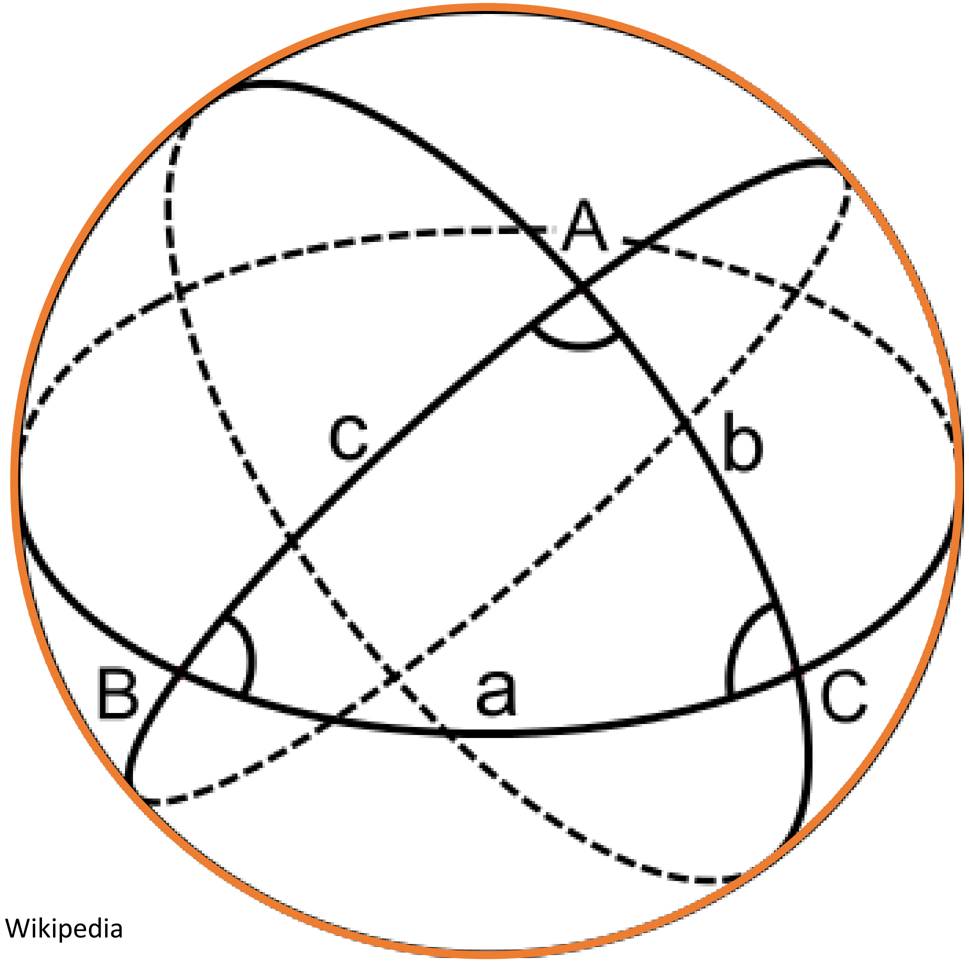
Wikipedia

Notation:

- Vertices and angles (in rad) at vertices denoted by capitals: A, B, C
- Angles are between tangent vectors to great circles, or equivalently between planes containing the great circles
- Sides and the angles (in rad) at sphere center denoted by lower case a, b, c
- Assuming a unit sphere (!) a, b, c are also lengths of sides (chords)

Spherical trigonometry

Three distinct great circles form a triangle



Wikipedia

Given angle A and side lengths b, c what is the missing length a?

- Planar case: law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

- But: planar trigonometry not true on sphere!

Question

How many spherical triangles are there in this picture?

Cosine Rules

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

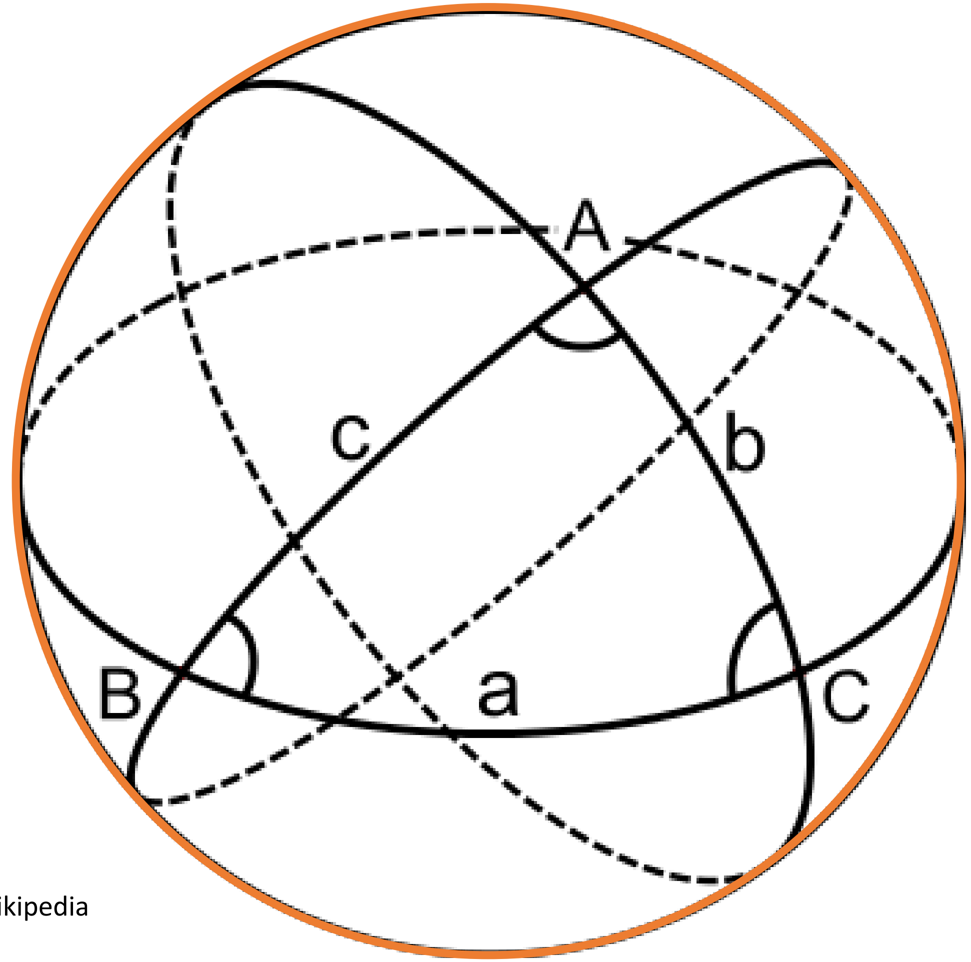
Complementary Cosine Rules

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a,$$
$$\cos B = -\cos C \cos A + \sin C \sin A \cos b,$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c.$$

Sine Rules

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

CAREFUL WITH QUADRANT DISAMBIGUATION!



Wikipedia

Distances on a sphere

One application: distances on a sphere (e.g. Earth):

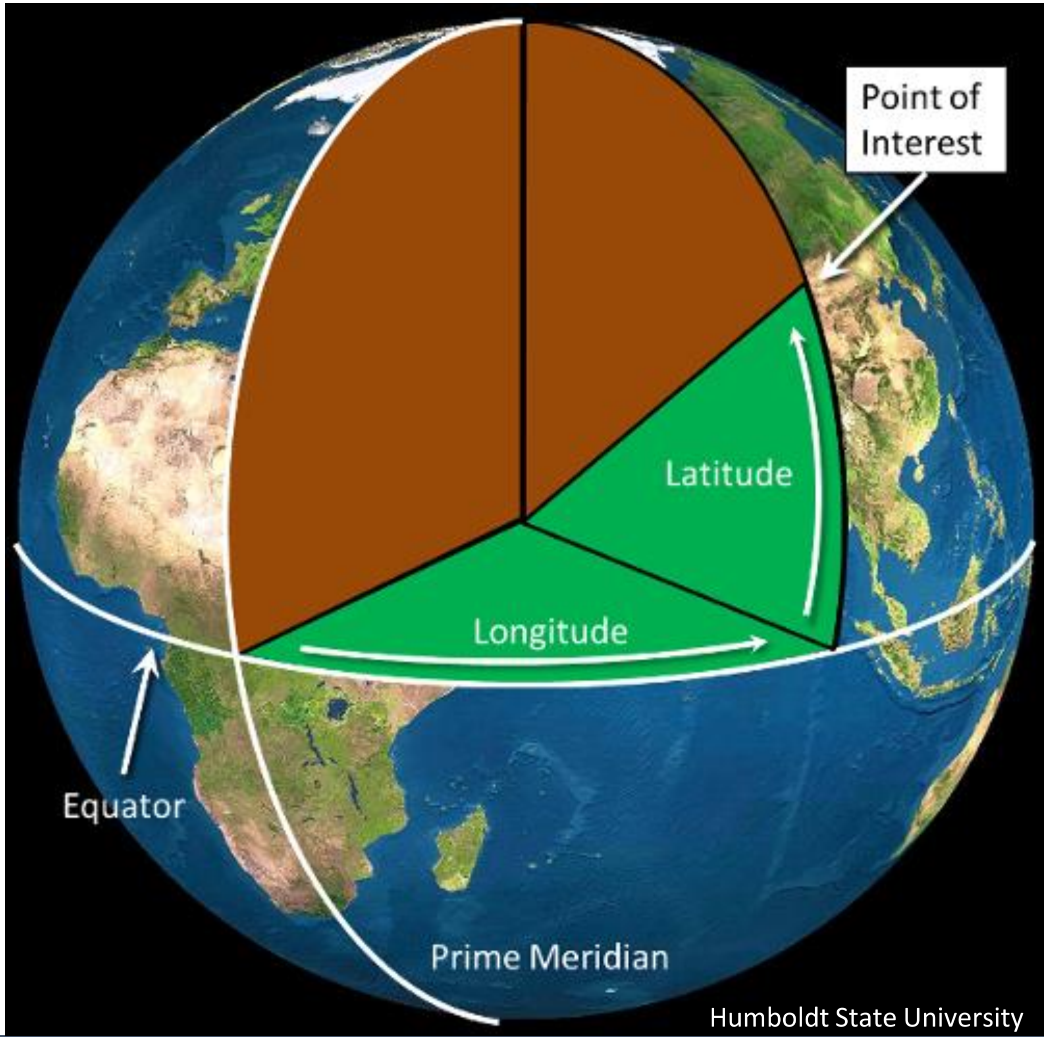
- Points located by latitude and longitude
- Prime meridian = 0 longitude
- Equator = 0 latitude

Question: given two points on Earth, what's the shortest distance between them?

$$\Phi_1, \lambda_1$$

$$\Phi_2, \lambda_2$$

$$d = ???$$



Humboldt State University

One application: distances on a sphere (e.g. Earth):

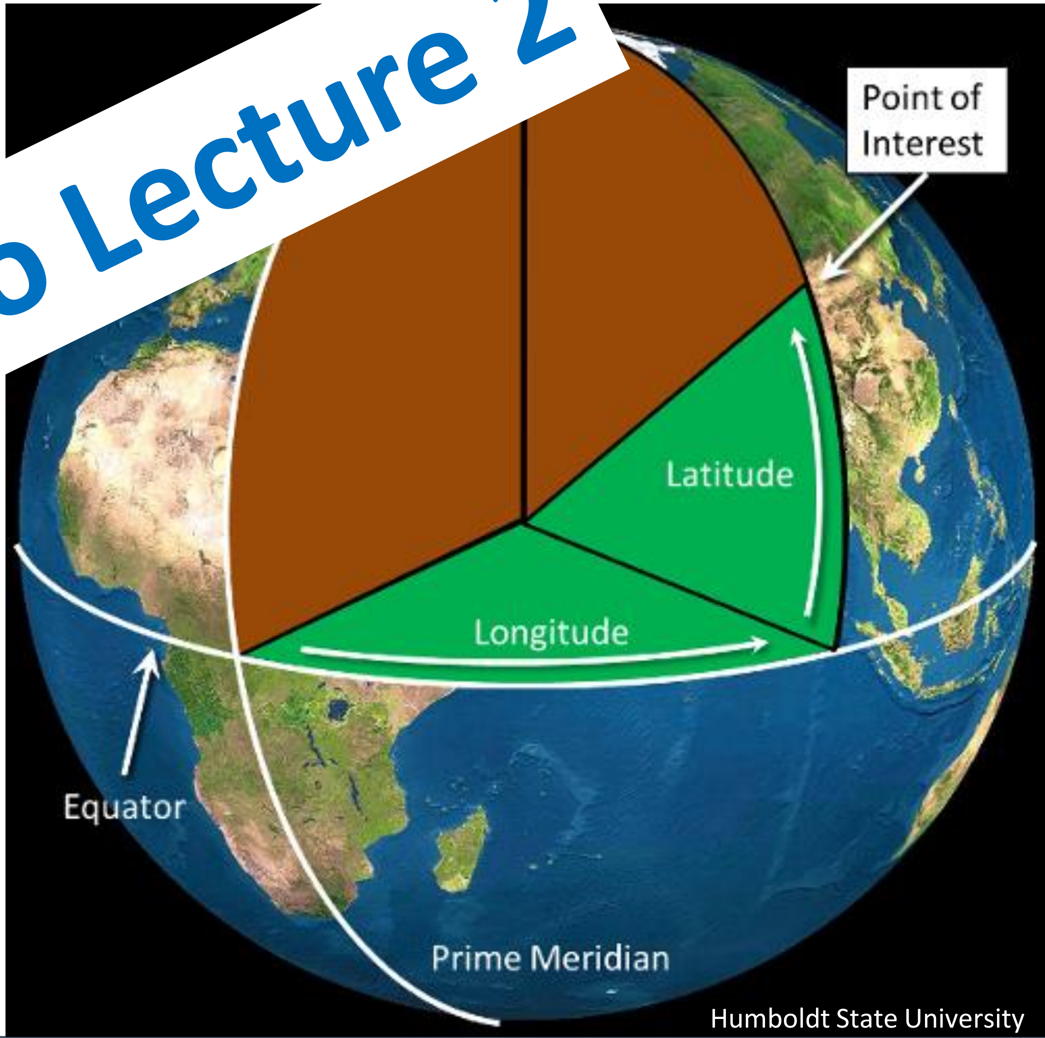
- Points located by latitude and longitude
- Prime meridian = 0 longitude
- Equator = 0 latitude

Question: given two points ... s
the shortest distance b...

$$\ldots, \lambda_2$$

$$d = ???$$

see Chapter 4 Video Lecture 2



Humboldt State University

Key words & Further Reading

- Cross products, vector identities, triple product
- Dynamics, Newton’s law, inertial frame, accelerating frame, rotating frame, fictitious forces
- Ordinary Differential Equations, initial value problem, initial conditions
- trigonometry, law of sines, law of cosines, spherical trigonometry, distance on a sphere, latitude & longitude

Further reading

- Curtis, Chapter 1

Also:

- Chobotov, Chapter 1
- Batin, Chapter 2

Orbits from First Principles

Learning outcomes

- State solution of bound orbital motion of light satellite around a heavy primary
- Derive the correct solution for the unperturbed two body problem in inertial and non-inertial reference frames
- Quantify the accuracy of restricted 2BP
- Discuss the notion that the Earth orbits around the Sun

Recap: Bound Orbits

Previous lectures (Astronautics, Physics)

- Equation of motion:

$$\ddot{\vec{x}} = -\frac{GM}{|\vec{x}|^3}\vec{x} = -\frac{\mu}{|\vec{x}|^3}\vec{x}$$

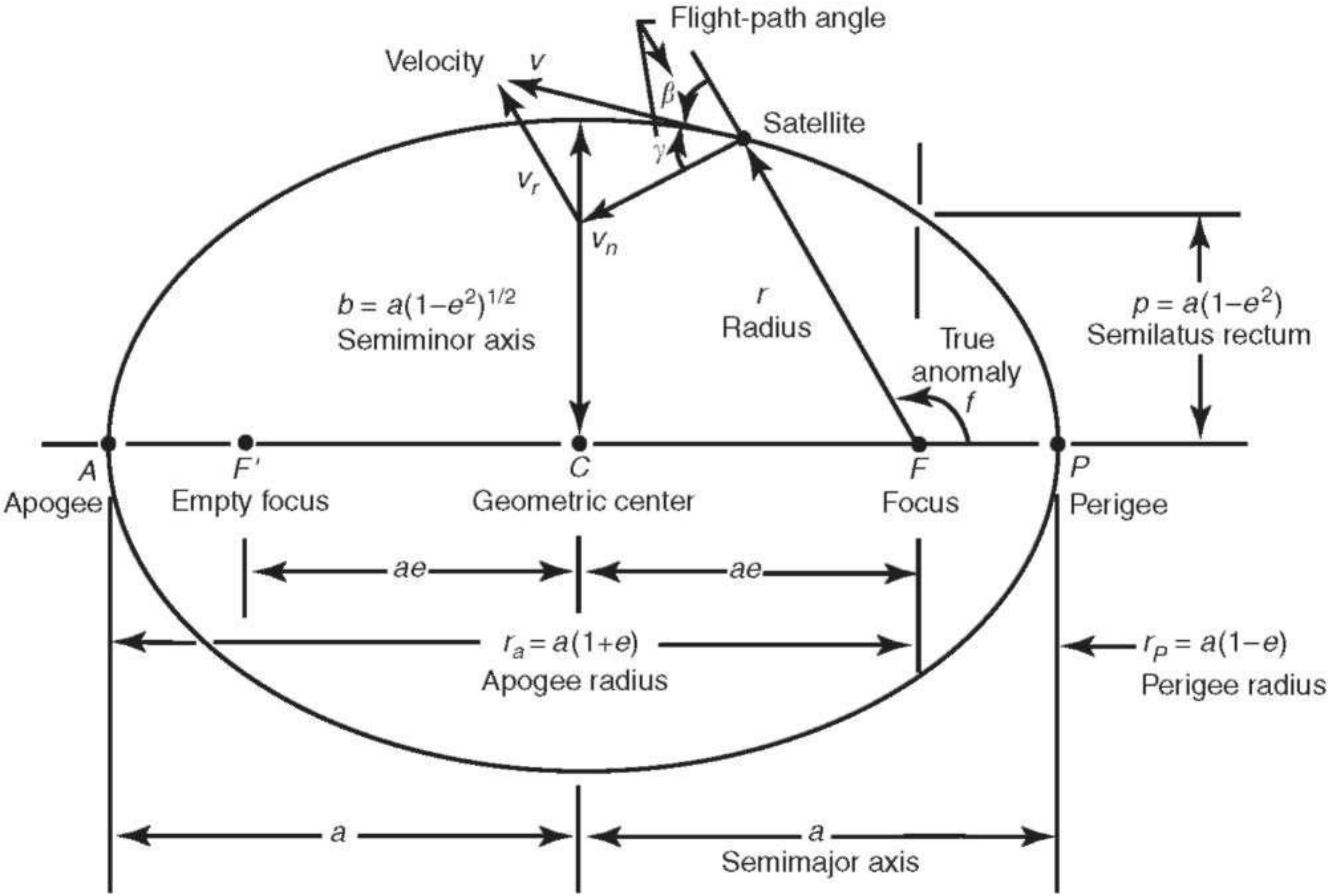
central force problem

- Shape of solution:

$$r(\theta) = \frac{p}{1 + e\cos(\theta)}$$

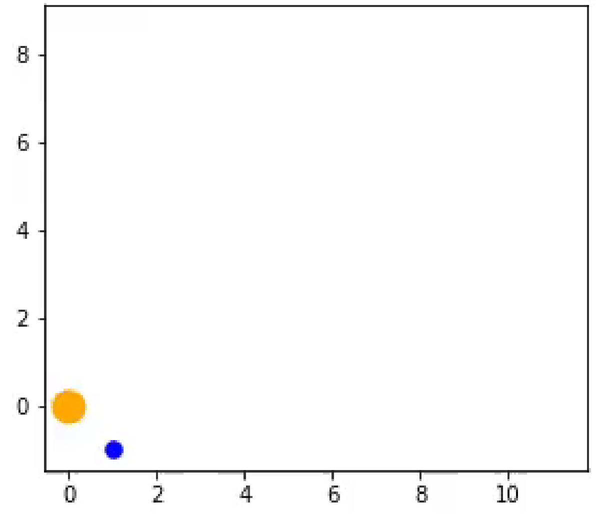
conic section

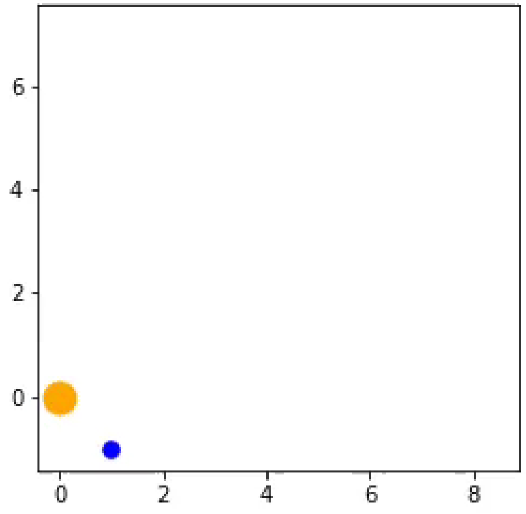
Geometry of bounded 2BP motion

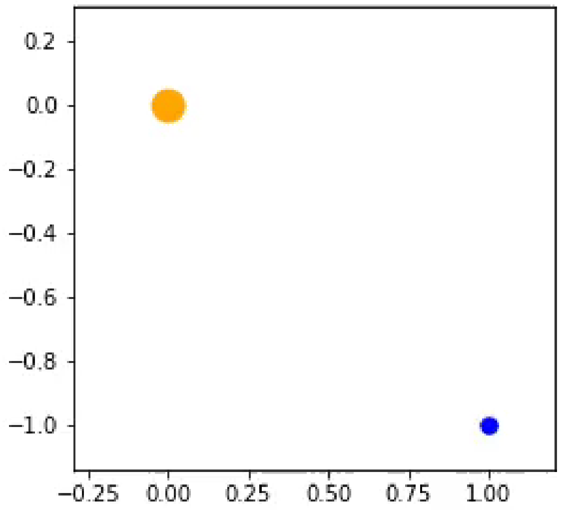


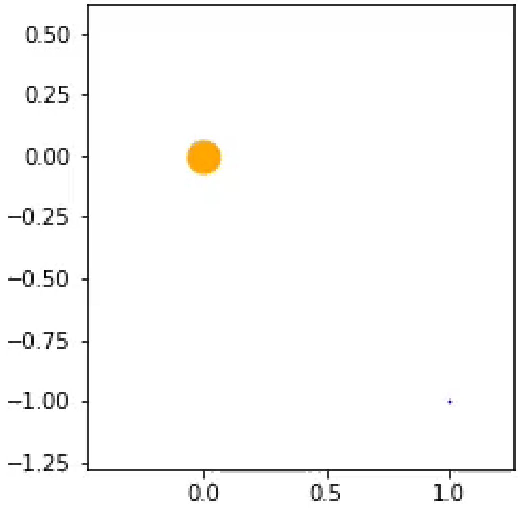
Which one is 2BP motion?

Radius of objects proportional to mass



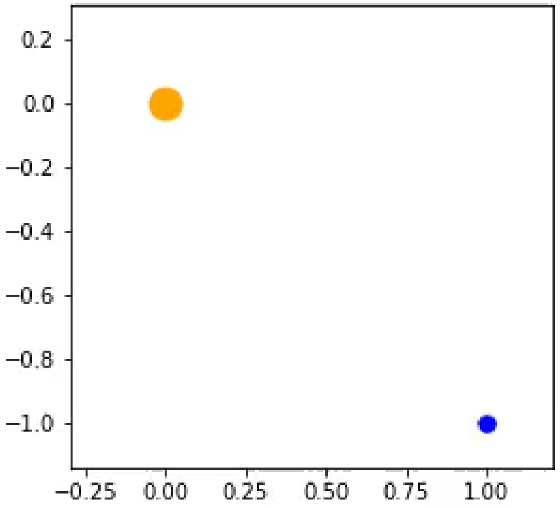






Reference frames

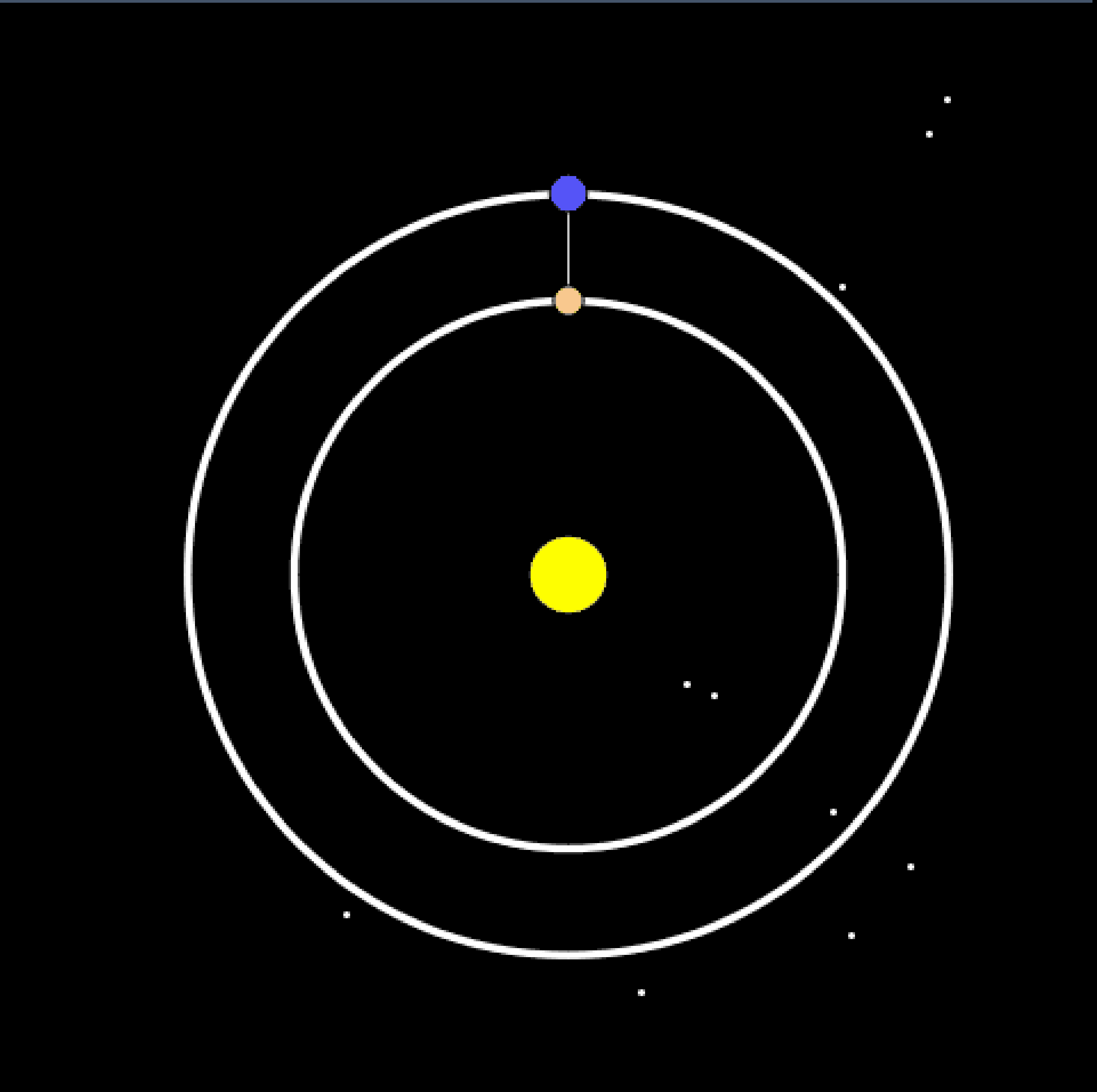
- **All** of these were 2BP motion!
- Motion can be viewed from different reference frames
- **Bonus question**: which ones were **inertial** frames?



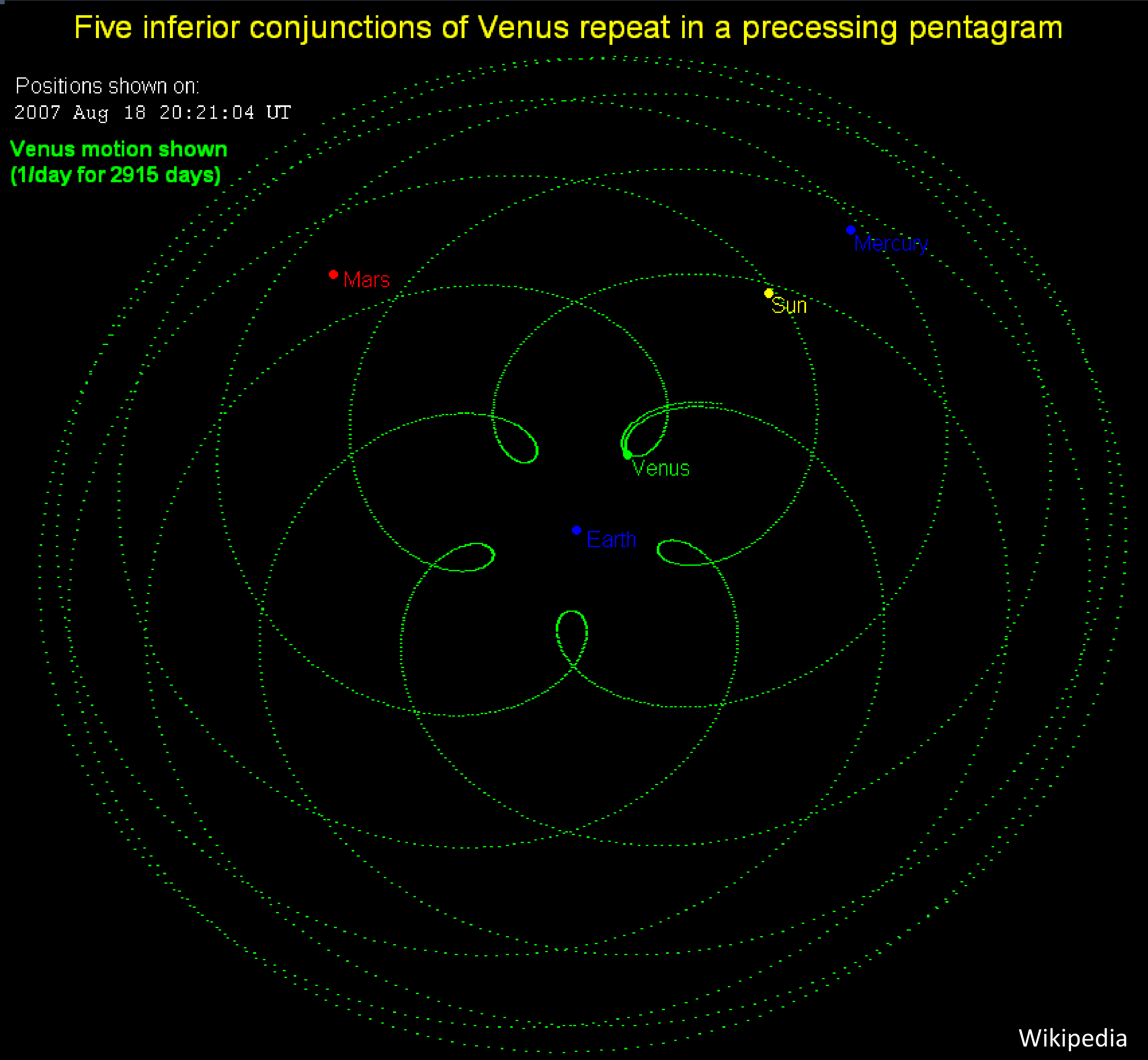
The only non-inertial frame!

NASA

Reference frames

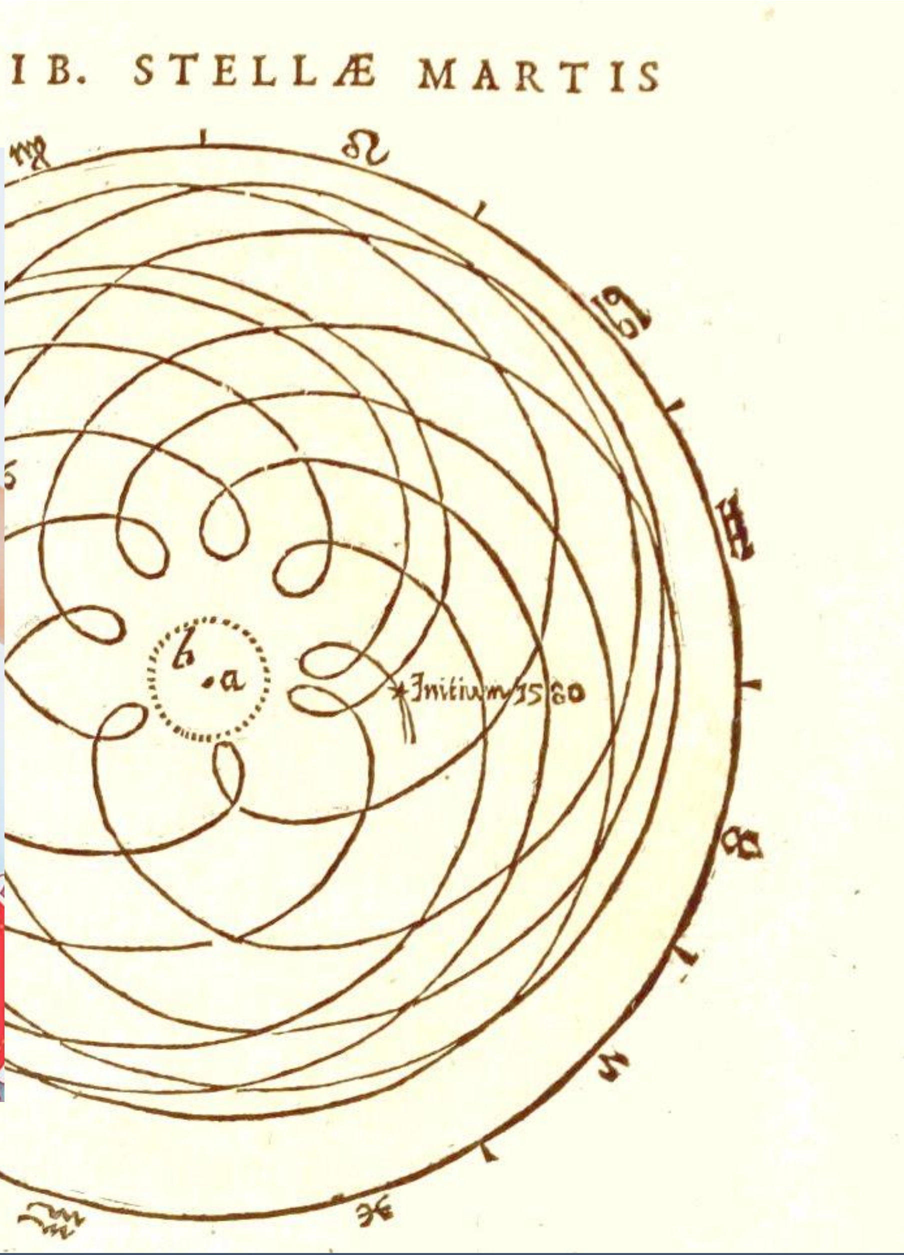


Matthew Henderson



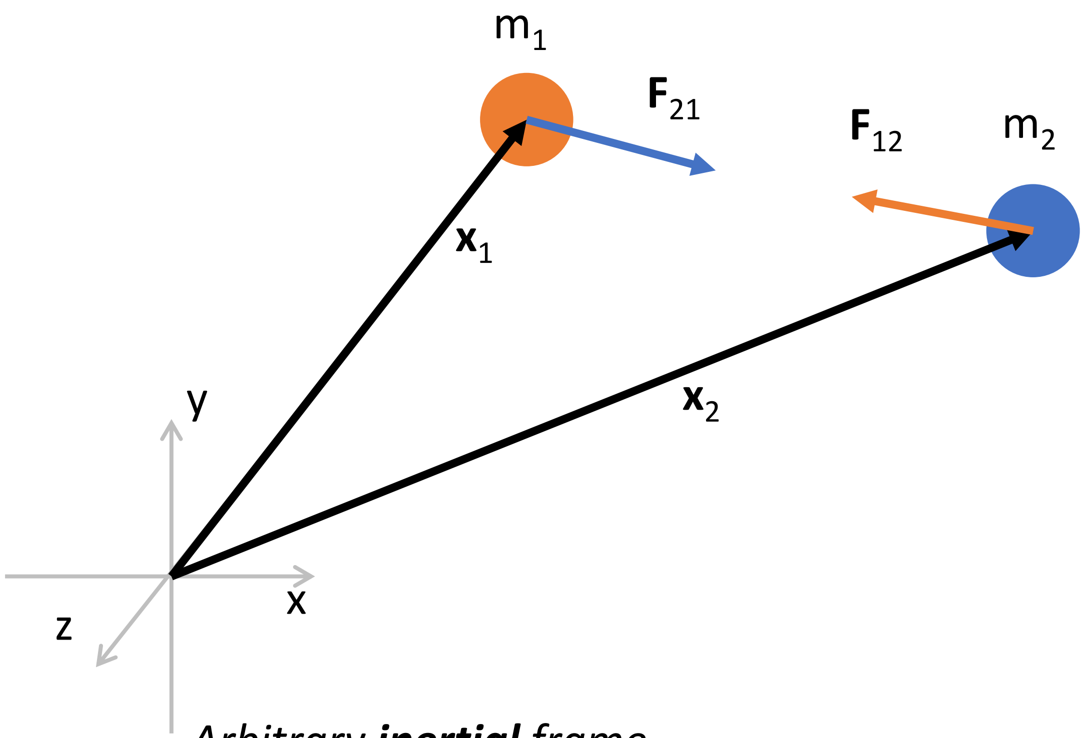
Wikipedia

Reference frames

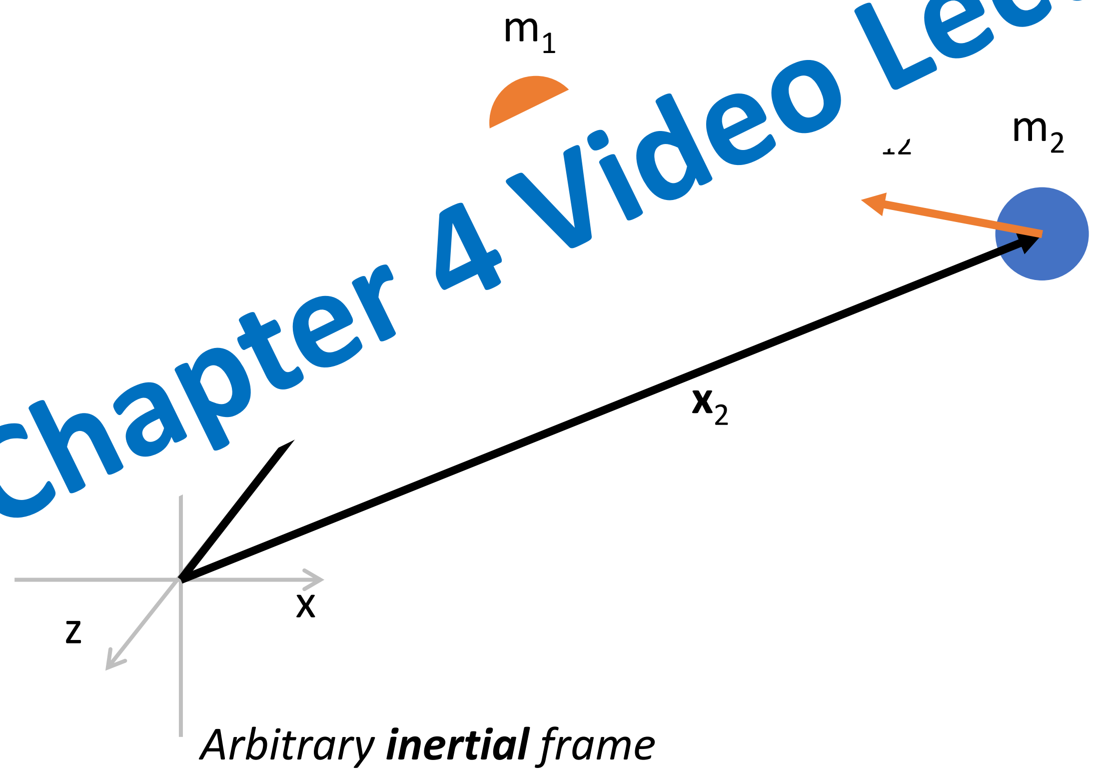


Johannes Kepler/Astronomia Nova

Back to Basics: First Principles



*Arbitrary **inertial** frame*



see Chapter 4 Video Lecture 3

After some math: Motion splits in two

- Center of mass: moves uniformly

$$\vec{x}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

$$\ddot{\vec{x}}_{cm} = \vec{0}$$

- Relative distance: moves on conic section

$$\vec{d} = \vec{x}_2 - \vec{x}_1$$

$$\ddot{\vec{d}} = -\frac{\mu}{|\vec{d}|^3}\vec{d}$$

Observations:

- Both bodies on ellipses around barycenter
- Heavy mass moves closer to barycenter than light mass

$$\vec{x}_1 = \vec{x}_{cm} - \frac{m_2}{m_1 + m_2}\vec{d}$$

$$\vec{x}_2 = \vec{x}_{cm} + \frac{m_1}{m_1 + m_2}\vec{d}$$

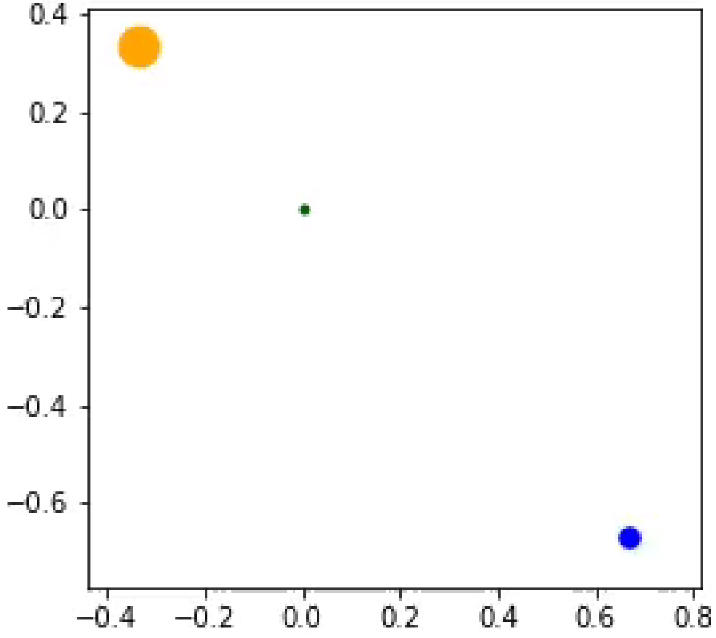
If $m_1 \ggg m_2$:

- $\mu \approx Gm_1$
- $\vec{x}_{cm} \approx \vec{x}_1$

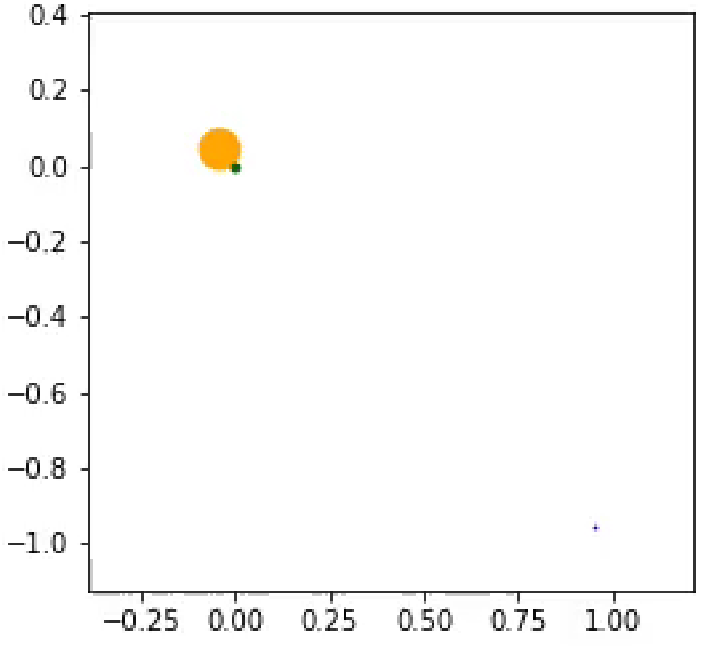
$$\mu = G(m_1 + m_2)$$

Restricted Two-Body Problem

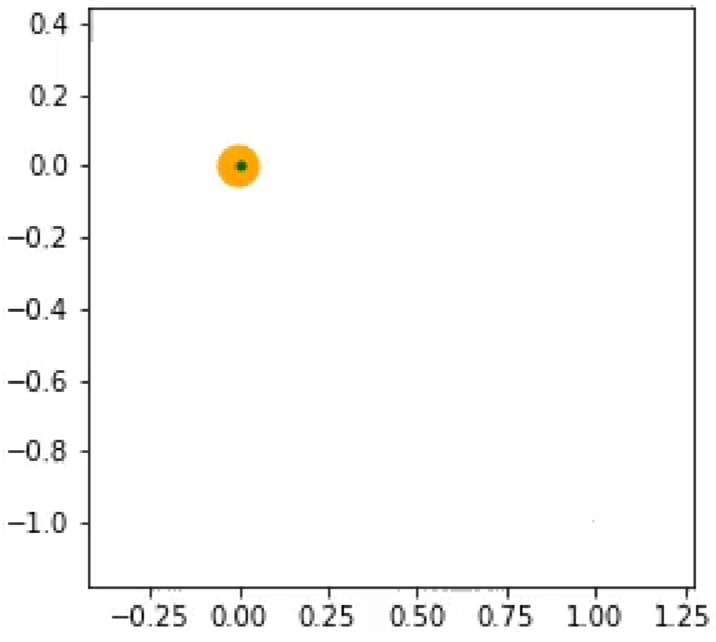
Restricted 2BP



mass ratio: 1/2



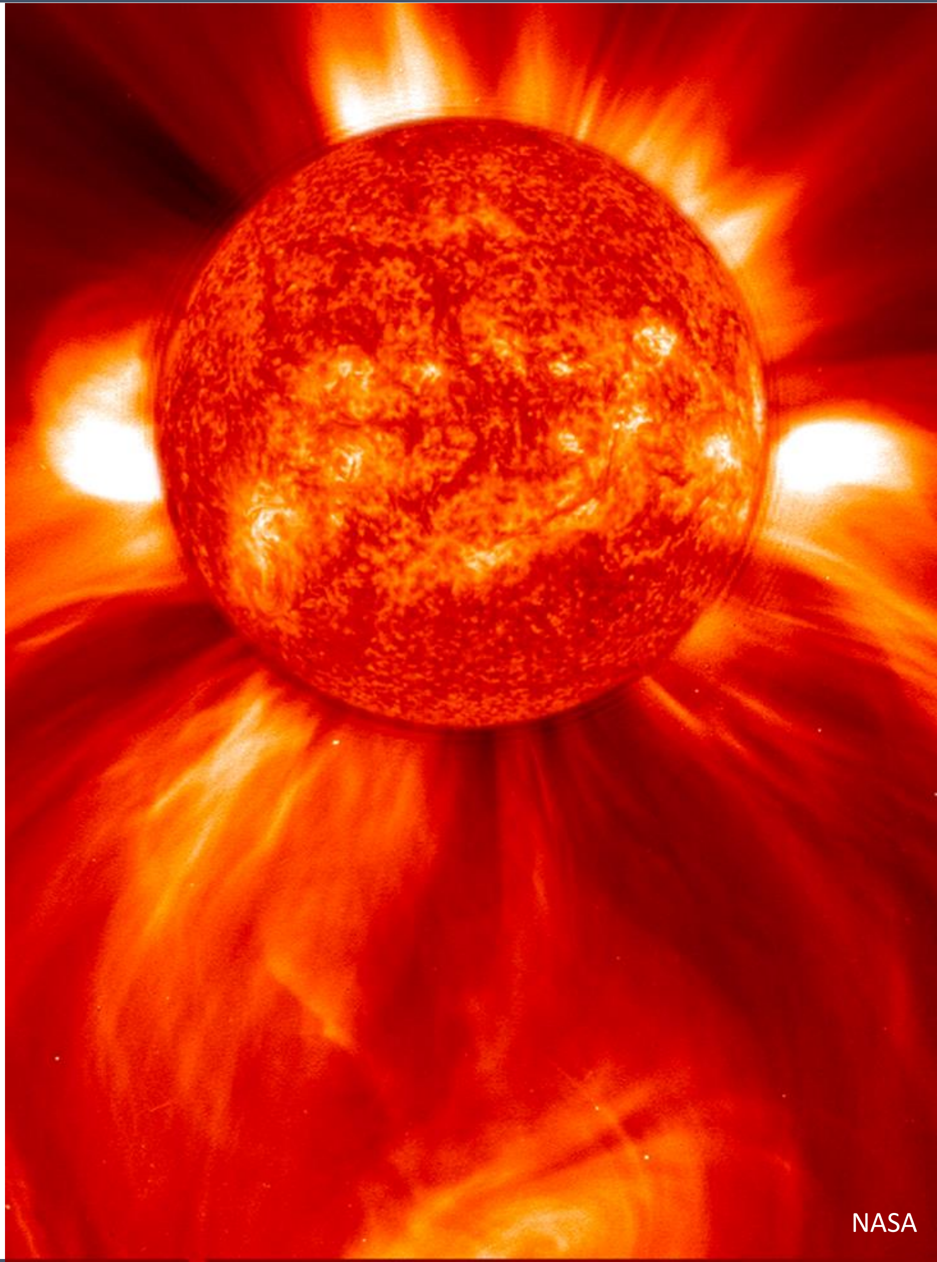
mass ratio: 1/20



mass ratio: 1/200

Summary

- Full 2BP separates
  - Center of mass (barycenter) moves along line at constant velocity
  - Relative motion: moves on conic section
- Barycentric frame is inertial
- Both bodies orbit barycenter
- Large mass ratio: heavy body ≈ barycenter (restricted 2BP)



NASA

Advanced Astronautics (SESA3039)

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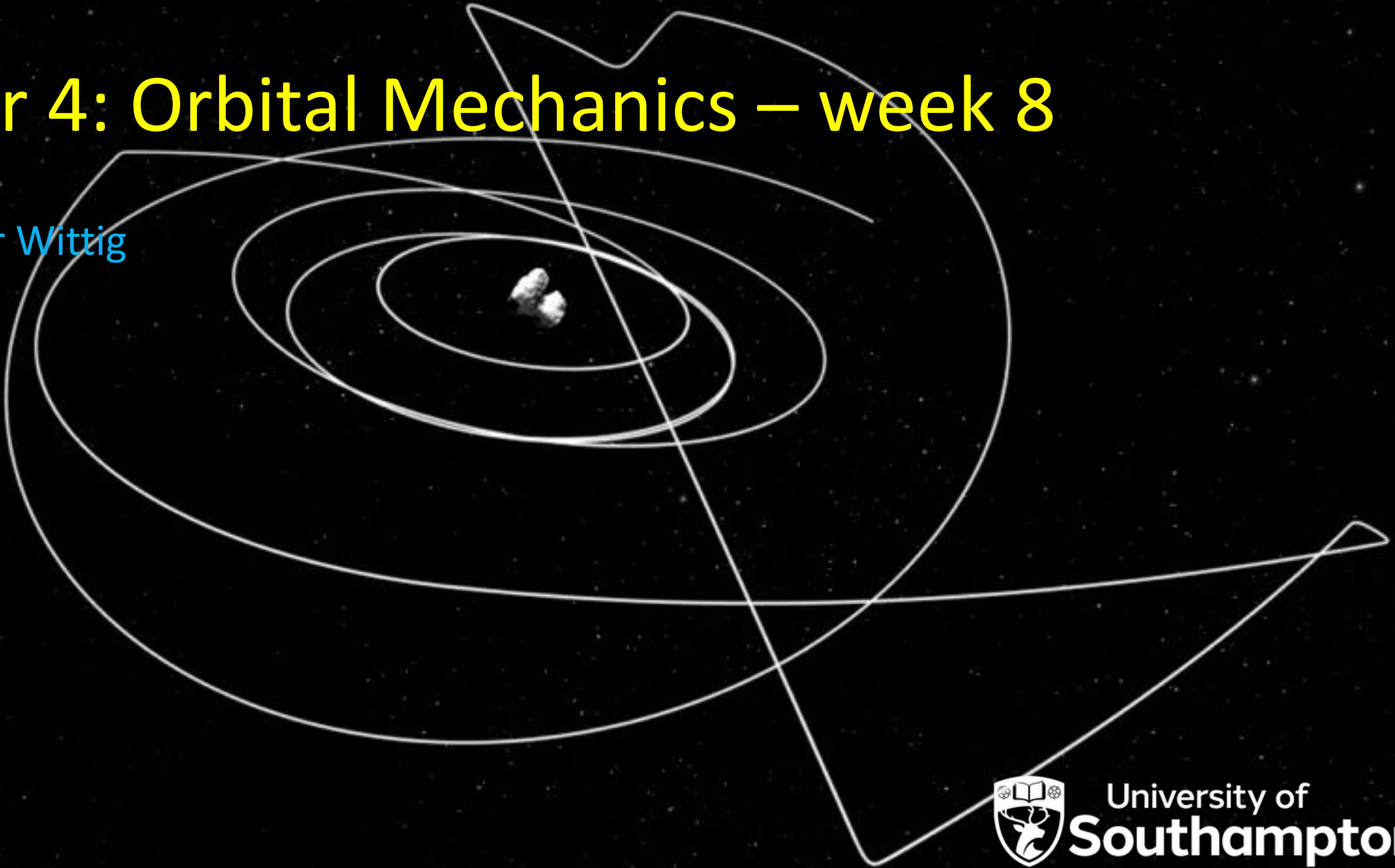

Rosetta lander deployment orbit (ESA)

Southampton AERO ASTRO

University of Southampton