SESA3029
Aerothermodynamics

Interim revision and test 3 Q&A
(Week 8)

Exam format

Answer ALL questions

Tables and chart

Formula sheet

Properties of air

UNIVERSITY OF SOUTHAMPTON SESA3029W1

SEMESTER 1 EXAMINATIONS 2023-24

TITLE: Aerothermodynamics

DURATION: 120 MINS

This paper contains **FOUR** Questions

Answer **ALL** questions on this paper. Questions 1, 2, 3 and 4 are worth 40, 20, 20 and 20 marks respectively (total 100 marks).

An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow **and** normal shock tables (11 sides) are provided. (In reading from tables, nearest values are acceptable unless explicitly stated otherwise.)

An oblique shock chart is provided.

Note that a formula sheet is provided at the end of this paper

Only University approved calculators may be used.

A foreign language direct 'Word to Word' translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), c_p=1005 J/(kg K), γ=1.4, Pr=0.7, ρ=1.225 kg/m^3 and μ=1.79x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

Calculators

Extract from regulations:

- All Casio Calculators are allowed but they must be Non-Programmable, Scientific models. Graphical calculators are not permitted. Casio Non-Programmable, Scientific models may be bought from any outlet and no longer require the University logo stamp. NO OTHER CALCULATOR MODELS OR BRANDS ARE ALLOWED. You can ONLY use Casio Models in your assessment. PLEASE DO NOT FORGET YOUR CALCULATOR AS SPARES ARE NOT AVAILABLE

Perfect gas equation of state:

$$p = \rho RT$$

Sound speed in a perfect gas:

$$a^2 = \gamma RT$$

Adiabatic flow:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

You do need to know what all the symbols mean

Mach angle:

$$\sin \mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta + \mu)_A + (\theta + \mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta - \mu)_B + (\theta - \mu)_P\right]$$

$$x_p = \frac{x_B \tan \alpha_{BP} - x_A \tan \alpha_{AP} + y_A - y_B}{\tan \alpha_{BP} - \tan \alpha_{AP}}$$

$$y_P = y_A + (x_P - x_A) \tan \alpha_{AP}$$

Velocity potential equation:

$$\left(1 - M_\infty^2\right)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Linearised pressure coefficient

$$C_p = -2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2}}$$

Ackeret formula:

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

Laminar pipe flow:

$$\mathrm{Nu} = 4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu} = 3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x = 0.453 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x = 0.332 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu} = 0.023 \mathrm{Re}^{4/5} \mathrm{Pr}^n$$

$$(n = 0.3 \text{ for cooling, } n = 0.4 \text{ for heating})$$

Turbulent boundary layer:

$$\mathrm{Nu}_x = 0.0308 \mathrm{Re}_x^{4/5} \mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x = 0.0296 \mathrm{Re}_x^{4/5} \mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Additional equations will be given in questions if needed

Exam scope

- Anything covered in the SESA3029 lectures is examinable
  - Expected to know the assumptions of all derivations, including any key definitions that emerge during derivations
- You won't be asked to reproduce long proofs such as:
  - Shock jump relations
  - Oblique shock equation
  - Rayleigh pitot formula
  - Prandtl-Meyer function
  - Euler equations and MoC
  - Velocity potential partial differential equation

Past exam papers

- The only slight change in the syllabus recently is a change of the finite difference example from nozzle flow (Euler CFD) to the heat equation
- See Blackboard, with numerical solutions and observations on typical errors
- Revisit the test questions for more practice (speed and accuracy); e.g. Blackboard Tests are good practice for exam question 1.

Test 3

QUESTION 1: The lift slope (dCL/dα) of a wing is measured in a wind tunnel as 6.64 at a Mach number 0.53. Find the equivalent lift slope in a low speed wind tunnel operating at 20 m/s under standard atmospheric conditions (15 deg C, 101.325 kPa). Give your answer to two decimal places.

QUESTION 2: A flat plate is placed at an angle of attack of 2.2 degrees in a Mach 3.0 stream with pressure 24.4 kPa. Use Ackeret's theory to find the pressure in kPa on the upper surface. Give your answer to one decimal place.

QUESTION 3: A diamond shaped airfoil with a thickness to chord ratio of 0.06 is placed at zero degrees angle of attack in a wind tunnel at Mach number 2.5 and pressure 11.9 kPa. Find the pressure in kPa on the leeward surface according to Ackeret's theory. Give your answer to one decimal place.

QUESTION 4: Estimate the critical Mach number for an airfoil which has a minimum pressure coefficient of -1.92 in incompressible flow. Give your answer to two decimal places, accurate to plus/minus 0.02

Q.3

(i) Figure Q.3 below shows two characteristic lines 0-1-4 and 0-2-3-5 in a 2D convergent-divergent nozzle, symmetric about the centreline shown, designed to accelerate air to an exit Mach number M_e=1.9. Define the Riemann invariants R^+ and R^- and show that

$$\theta_{\max} = \frac{\nu(M_e)}{2}$$

where ν is the Prandtl-Meyer function.

[6 marks]

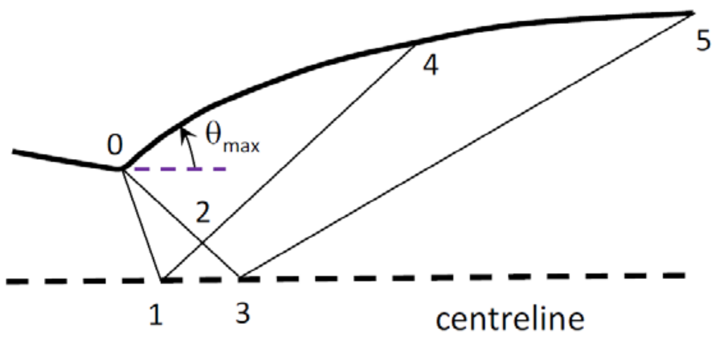


Figure Q.3

(ii) The starting characteristic lines at point 0 in figure Q.3 have flow angles of $\theta_{max}/2$ (the 0-1 line) and θ_{max} (the 0-2 line). Find the flow angle and Mach number at point 2.

[6 marks]

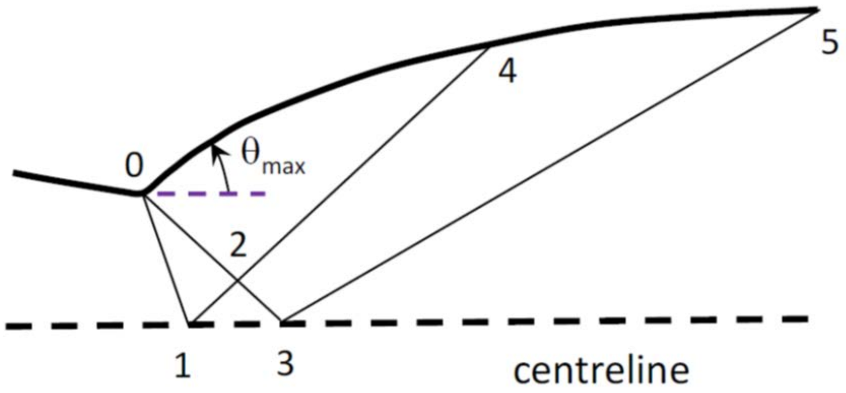
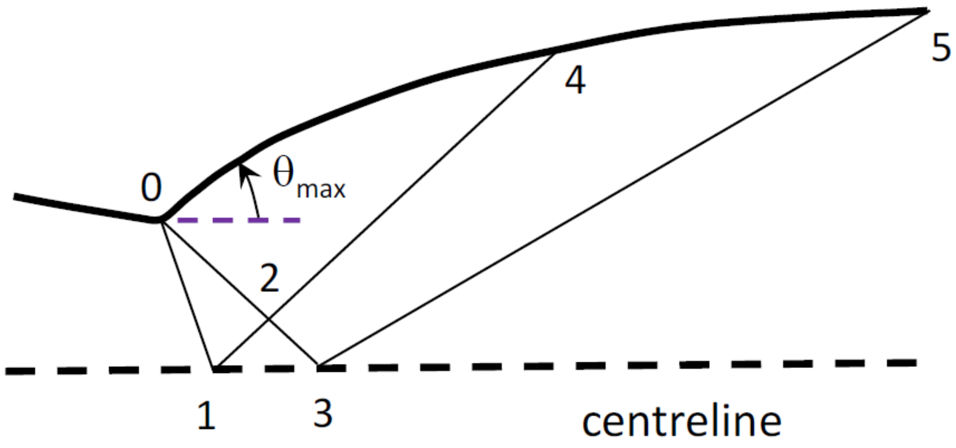


Figure Q.3

(iii) Find the angles of the characteristic lines 2-4 and 3-5 (hint: a full calculation of all the points is not required)

[4 marks]



(iii) Find the drag coefficient on the aerofoil sketched in figure Q.2 below, when it is placed at an angle of attack of 0° in a Mach 3.0 flow stream. The aerofoil has a chord c=1.0 m, a flat lower surface and an upper surface shape given by y=0.2x(1-x) m, with x measured from the leading edge along the chord line.

[10 marks]

(iv) Discuss whether this would be a suitable aerofoil for a supersonic aircraft?

[4 marks]

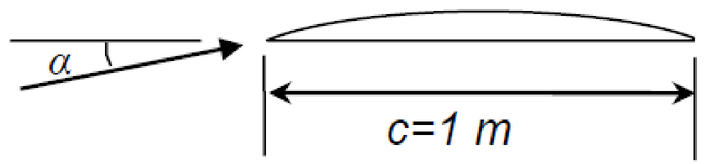
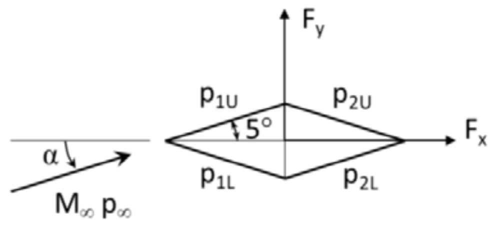


Figure Q.2

(ii) For the diamond-shaped aerofoil sketched in Figure Q.1(b) with M_∞=1.9 and p_∞=70 kN/m^2, use Ackeret's theory to determine the static pressure on each of the four faces (i.e. p_{1L},p_{1U},p_{2L} and p_{2U}).

[6 marks]

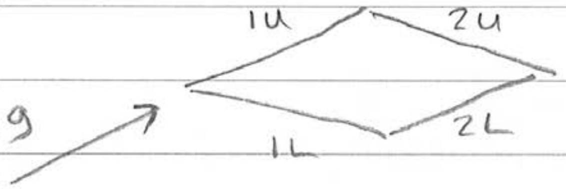


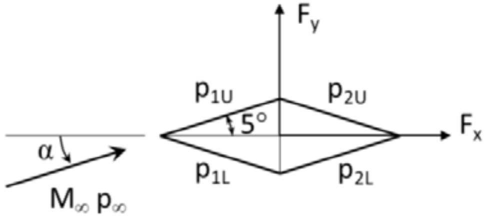
α=5 deg

(ii) For the diamond-shaped aerofoil sketched in Figure Q.1(b) with M_∞=1.9 and p_∞=70 kN/m^2, use Ackeret's theory to determine the static pressure on each of the four faces (i.e. p_{1L},p_{1U},p_{2L} and p_{2U}).

[6 marks]

Diamond aerofoil





$M_\infty = 1.9$

$P_\infty = 70$ kN/m^2

α=5 deg

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2-1}} = 1.238\,\theta$$

$$C_p = \frac{P-P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = \frac{2(P-P_\infty)}{P_\infty \gamma M_\infty^2} = \frac{2}{\gamma M_\infty^2}\left(\frac{P}{P_\infty}-1\right)$$

$$P = P_\infty\left(1+\frac{\gamma M_\infty^2}{2}C_p\right) \qquad P = 70\,(1+2.527\,C_p)$$

1U	:	$\theta = 0$	$C_p = 0$	$p = 70$ kN/m^2
1L	:	$\theta = 10\pi/180$	$C_p = 0.216$	$p = 108$ kN/m^2
2U		$\theta = -10\pi/180$	$C_p = -0.216$	$p = 31.8$ kN/m^2
2L		$\theta = 0$	$C_p = 0$	$p = 70$ kN/m^2