UNIVERSITY OF SOUTHAMPTON SESA3029W1

SEMESTER 1 EXAMINATIONS 2019-20

TITLE: Aerothermodynamics

DURATION: 120 MINS

This paper contains **FOUR** Questions

Answer **ALL** questions on this paper. Questions 1-4 are worth 32, 16, 20 and 32 marks respectively.

An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow **and** normal shock tables (11 sides) are provided. (In reading from tables, nearest values are acceptable unless explicitly stated otherwise.)

An oblique shock chart is provided.

Note that a formula sheet is provided at the end of this paper

Only University approved calculators may be used.

A foreign language direct 'Word to Word' translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), c_p=1004.5 J/(kg K), γ=1.4, Pr=0.7, ρ=1.225 kg/m^3 and μ=1.8x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

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Q.1

Figure Q.1 below shows a flat plate, set at zero degrees angle of attack to a uniform flow at M_∞=2.4, p_∞=60 kN/m^2 and T_∞=244 K. The plate is equipped with a trailing edge flap. The chord length of the plate is *c* and the flap has a length of *c*/4.

(i) Find the area ratio of the nozzle needed to produce such a flow and the required stagnation conditions p_0 and T_0

[6 marks]

(ii) Use the shock-expansion method to find the static pressure and Mach number on the upper and lower sides of the flap, for a flap deflection angle of η=9°.

[10 marks]

(iii) Find the lift coefficient C_L of the configuration, using *c* as the reference length.

[6 marks]

(iv) Use Ackeret's equation to show that the flap effectiveness for small angles of deflection can be predicted as

$$\frac{\mathrm{d}C_L}{\mathrm{d}\eta} = 0.458 \text{ rad}^{-1}$$

[6 marks]

(v) Find the relative error of Ackeret's method compared to the shock-expansion results for C_L at η=9° and discuss how the error depends on η.

[4 marks]

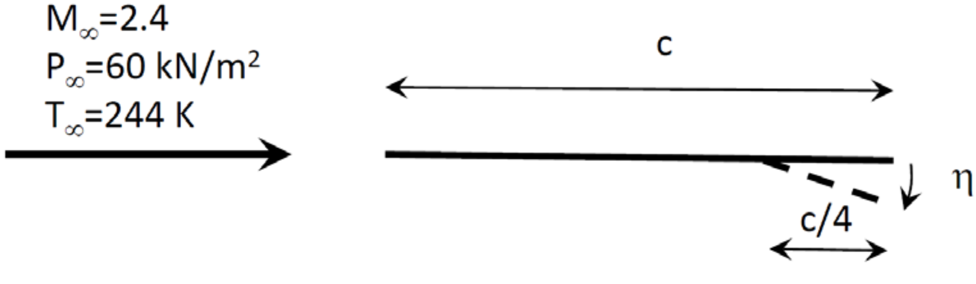

Figure Q.1

Q.2

(i) Starting from the compressible potential flow equation

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

derive the Prandtl-Glauert relation in the form

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

State all assumptions and explain how the final relation is useful.

[10 marks]

(ii) The effectiveness of the flap from **Figure Q.1** in incompressible flow (M_∞=0) can be derived using thin airfoil theory as

$$\left(\frac{\mathrm{d}C_L}{\mathrm{d}\eta}\right)_0=3.826\ \text{rad}^{-1}$$

Estimate the effectiveness at M_∞=0.75.

[4 marks]

(iii) Discuss why the effectiveness of trailing edge control devices is worse in supersonic flow compared to subsonic.

[2 marks]

TURN OVER

Q.3

(i) **Figure Q.3** below shows two characteristic lines 0-1-2-4 and 0-2-3-5 in a 2D convergent-divergent nozzle, symmetric about the centreline shown, designed to accelerate air to an exit Mach number M_e=2.2. Given that

$$\theta_{\max} = \frac{\nu(M_e)}{2},$$

where ν is the Prandtl-Meyer function, and that the starting characteristic lines at point 0 in **Figure Q.3** have flow angles of θ_{max} and $\theta_{max}/2$, find the Riemann invariants R^+ and R^- at point 2.

[4 marks]

(ii) Find the flow angle and Mach number at point 2.

[4 marks]

(iii) Given that point 0 is at (x=0, y=1) and the centreline is at y=0, find the co-ordinates of points 1 and 2.

[10 marks]

(iv) How would you extend this prototype calculation to obtain a full description of the nozzle geometry?

[2 marks]

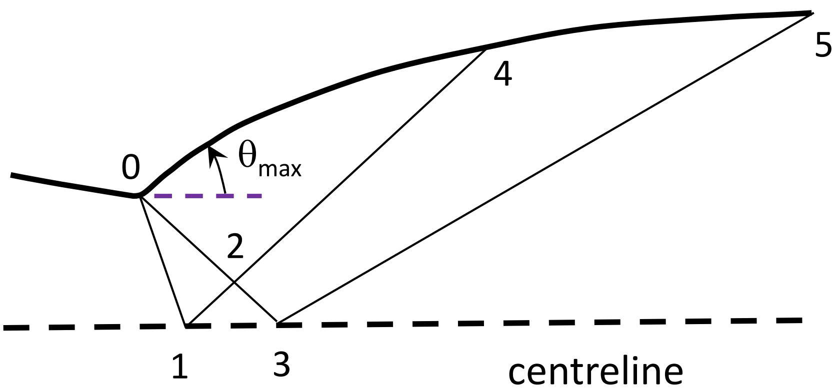

Figure Q.3

Q.4

(i) Define the Nusselt number (Nu), Stanton number (St), Prandtl number (Pr) and Reynolds number (Re) and show they are related by

$$\mathrm{Nu} = \mathrm{St}\,\mathrm{Re}\,\mathrm{Pr}\,.$$

[4 marks]

(ii) The upper surface of a monolithic block of homogenous material (see **Figure Q.4**) is exposed to a tangential high-temperature flow of air (Pr=0.71) of velocity 40m/s, temperature 1200K and pressure 150kPa. The block has length 80cm, width 20cm and height 10cm. The block's underside is fixed at a temperature of 300K

Use the viscosity–temperature relationship for air given by

$$\frac{\mu}{\mu_{ref}} = \left(\frac{T}{T_{ref}}\right)^{0.75}$$

with $\mu_{ref} = 1.8 \times 10^{-5}$ Ns/m^2 and T_{ref} = 288K to estimate the Reynolds number (based on the high temperature flow properties) at the end of the upper surface and confirm that the boundary layer satisfies $\mathrm{Re}_x < 300{,}000$, thereby remaining laminar.

[4 marks]

(iii) A finite difference (FD) program approximating the steady heat diffusion equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

has been written to study the heat transfer within the block. The program is to be verified for the equidistant 5x3 grid depicted in **Figure Q.4**

TURN OVER

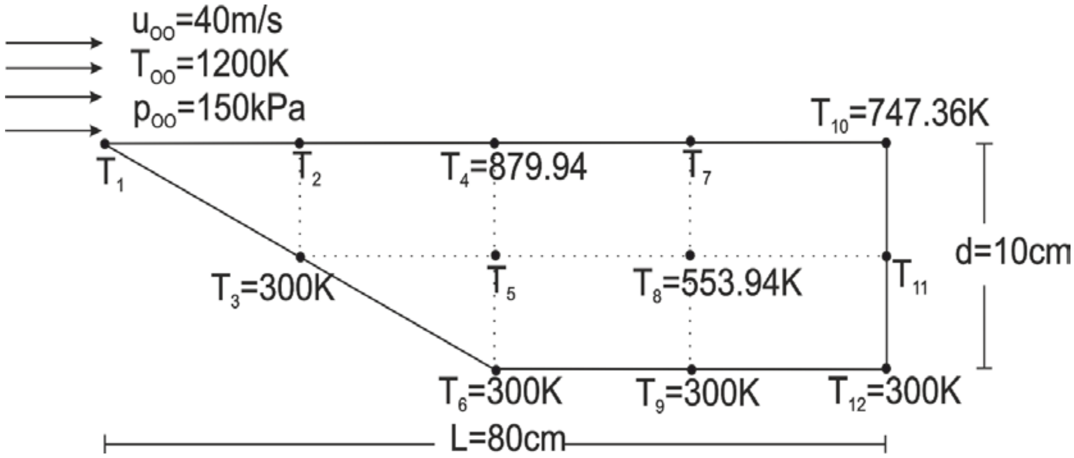

Figure Q.4

A uniformly constant convective heat flux through the upper surface is assumed. The total convective heat transfer rate through that surface is 1400W. The boundary on the right side of the block is adiabatic. At all other boundaries a constant temperature of 300K is assumed (see **Figure Q.4**).

(a) Evaluate the temperature in the surface boundary points 1,2 and 7.

[8 marks]

(b) Construct the FD stencils in points 5 and 11 and use those to compute the values for T_5 and T_{11} when the following values are returned by the program: T_4=879.94K, T_8=553.94K, T_{10}=747.36K. Finally, insert previously obtained results in the FD stencil in point 8 to verify the program's prediction for T_8.

[10 marks]

(iv) The FD program is run for equidistant grids of different resolutions. For the same location the following values are obtained: 430.1716K (33x17 grid), 429.2628K (65x33 grid), 429.0301K (129x65 grid). Based on these results confirm that the order of the numerical method is approximately 2 and – assuming uniformly second-order convergence – estimate the correct temperature at the location of interest.

[6 marks]

END OF PAPER (Formula sheet on next page)

Useful Formulae

Perfect gas equation of state

$$p = \rho R T$$

Sound speed in a perfect gas

$$a^2 = \gamma R T$$

Adiabatic flow

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Mach angle:

$$\sin \mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta + \mu)_A + (\theta + \mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta - \mu)_B + (\theta - \mu)_P\right]$$

$$x_p = \frac{x_B \tan \alpha_{BP} - x_A \tan \alpha_{AP} + y_A - y_B}{\tan \alpha_{BP} - \tan \alpha_{AP}}$$

$$y_P = y_A + \left(x_P - x_A\right) \tan \alpha_{AP}$$

TURN OVER

Velocity potential equation:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

Linearised pressure coefficient

$$C_p=-2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

Ackeret formula:

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

Laminar pipe flow:

$$\mathrm{Nu}=4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}=3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x=0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu}=0.022\,\mathrm{Pr}^{0.5}\,\mathrm{Re}^{0.8}$$

Turbulent boundary layer:

$$\mathrm{Nu}_x=0.029\mathrm{Re}_x^{0.8}\mathrm{Pr}^{0.6}$$