SESA6085 – Advanced Aerospace Engineering Management

Lecture 9

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Importance Metrics

Importance of a Component

- Why is the importance of a component of interest?
- Once a model for a system's reliability is constructed it is necessary for design & reliability engineers to identify weaknesses in the system
  - Typically such systems can be composed of many subsystems with many hundreds of components in each
- Identified weaknesses can then be redesigned or upgraded
- Methods which can identify important components are therefore extremely useful

Importance of a Component

- In the majority of importance criteria the importance of each component is assessed based on observing the reliability of the system when
  - The component is working
  - The component is not working
- Before considering the methods lets first define the notation and any assumptions

Notation & Assumptions

- Let's assume we have n components in the system
- The components can have two states, working or failed with the state of the ith component defined as, $X_i = 0$ or $X_i = 1$
- The probability of the ith component not working is defined as q_i, i.e. $P(X_i = 0) = q_i$
- $\phi(\boldsymbol{X})$ denotes the state of the system, i.e. when $\phi(\boldsymbol{X}) = 0$ the system has failed and when $\phi(\boldsymbol{X}) = 1$ the system is working
- $\phi(\boldsymbol{X})$ – the system is dependent on the states $X_i \;\; i \in [1, n]$

Notation & Assumptions

- Define a system unreliability function $G(\boldsymbol{q})$ as

$$G(\boldsymbol{q}) = 1 - P[\phi(\boldsymbol{X}) = 1]$$

- Where $\boldsymbol{q}$ indicates that $G(\boldsymbol{q})$ is a function of the q_i for all $\boldsymbol{n}$ components
- Lets define two variants of $G(\boldsymbol{q})$, $G(0_i, \boldsymbol{q})$ and $G(1_i, \boldsymbol{q})$
  - $G(0_i, \boldsymbol{q})$ - system unavailability when component i is working i.e. $q_i = 0$
  - $G(1_i, \boldsymbol{q})$ - system unavailability when component i is not working i.e. $q_i = 1$
- This concept is used in all popular importance measures

Birnbaum's Importance Measure

- Defined as the probability that the i[th] component is critical to the system's functioning at time t

$$I_B^i(t) = \frac{\partial G(\boldsymbol{q}(t))}{\partial q_i(t)} = G(1_i, \boldsymbol{q}) - G(0_i, \boldsymbol{q})$$

System unavailability when *i* is not working

System unavailability when *i* is working

I_B Example

- A galvanometer (a device for measuring electrical resistance) contains four batteries with constant failure rates, $\lambda_1=0.005$, $\lambda_2=0.009$, $\lambda_3=0.003$ and $\lambda_4=0.05$

a) Calculate Birnbaum's importance measure for each battery when the batteries are all in series

b) Calculate Birnbaum's importance measure for each battery when the batteries are all in parallel

I_B Example

- Calculate the unreliability for each battery (t=40):

$$q_1 = 1 - R_1(t) = 1 - e^{-\lambda_1 t} = 0.181$$

$$q_2 = 0.302 \qquad q_3 = 0.113 \qquad q_4 = 0.864$$

- Define our expression for $P[\phi(\boldsymbol{X}) = 1]$ for a series system:

$$P[\phi(\boldsymbol{X}) = 1] = (1 - q_1)(1 - q_2)(1 - q_3)(1 - q_4)$$

- Our expression for $G(\boldsymbol{q})$ is therefore:

$$G(\boldsymbol{q}) = 1 - P[\phi(\boldsymbol{X}) = 1]$$

$$G(\boldsymbol{q}) = 1 - (1 - q_1)(1 - q_2)(1 - q_3)(1 - q_4)$$

I_B Example

- Calculate the importance measure for each battery

$$I_B^i(t) = G(1_i, \boldsymbol{q}) - G(0_i, \boldsymbol{q})$$

$$I_B^1(t) = [1 - (1-1)(1-q_2)(1-q_3)(1-q_4)\ ]$$
$$-[1 - (1-0)(1-q_2)(1-q_3)(1-q_4)\ ]$$

$$I_B^1(t) = (1-q_2)(1-q_3)(1-q_4) = 0.084$$

I_B Example

- Similarly:

$$I_B^2(t) = (1 - q_1)(1 - q_3)(1 - q_4) = 0.098$$

$$I_B^3(t) = (1 - q_1)(1 - q_2)(1 - q_4) = 0.077$$

$$I_B^4(t) = (1 - q_1)(1 - q_2)(1 - q_3) = 0.507$$

- Based on this measure battery number four has the highest importance
- Is this what you would expect?
- Yes, this battery has the highest failure rate and the reliability of a series system is driven by the least reliable component

I_B Example

- Define our expression for $P[\phi(\boldsymbol{X}) = 1]$ for a parallel system:

$$P[\phi(\boldsymbol{X}) = 1] = 1 - q_1 q_2 q_3 q_4$$

- Our expression for $G(\boldsymbol{q})$ is therefore:

$$G(\boldsymbol{q}) = q_1 q_2 q_3 q_4$$

- Making our importance measures:

$$I_B^1(40) = (1) q_2 q_3 q_4 - (0) q_2 q_3 q_4 = q_2 q_3 q_4 = 0.029$$

I_B Example

- Repeating the process for the remaining batteries:

$$I_B^2(40) = q_1 q_3 q_4 = 0.018$$

$$I_B^3(40) = q_1 q_2 q_4 = 0.047$$

$$I_B^4(40) = q_1 q_2 q_3 = 0.006$$

- This calculation suggests battery 3 is the most important
- Is this what you would expect?
- We know battery 4 is the least reliable battery so why is our measure stating that 3 is the most important?

Birnbaum's Importance Measure

- Birnbaum's importance measure relates to the probability that the system is in a state at t to which the functioning of the battery is critical
- As battery 3 fails last (it has the lowest failure rate) it is deemed to be the most critical

Criticality Importance

- Corresponds to the conditional probability that the system is in a state at time t such that the i^{th} component is critical and has failed by t
- This is based on the fact that it's easier to improve less reliable components than reliable ones

$$I_{CR}^{i}(t) = \frac{\partial G(\boldsymbol{q}(t))}{\partial q_i(t)} \times \frac{q_i(t)}{G(\boldsymbol{q}(t))}$$

$$I_{CR}^{i}(t) = \frac{[G(1_i, \boldsymbol{q}) - G(0_i, \boldsymbol{q})] \times q_i\ (t)}{G(\boldsymbol{q}(t))}$$

I_{CR} Example

- Let's calculate the criticality importance measure for our system of four batteries in series

$$G(\boldsymbol{q}) = 1 - (1 - q_1)(1 - q_2)(1 - q_3)(1 - q_4)$$

- I_{CR} is calculated as:

$$I_{CR}^{i}(t) = \frac{[G(1_i, \boldsymbol{q}) - G(0_i, \boldsymbol{q})] \times q_i\ (t)}{G(\boldsymbol{q}(t))}$$

- We know the first component of the top line, this is our expression for I_B

I_{CR} Example

- For the first battery the expression for I_{CR} is therefore:

$$I_{CR}^{1}(40) = \frac{(1-q_2)(1-q_3)(1-q_4)q_1}{1-(1-q_1)(1-q_2)(1-q_3)(1-q_4)} = 0.016$$

$$I_{CR}^{2}(40) = 0.032$$

$$I_{CR}^{3}(40) = 0.009$$

$$I_{CR}^{4}(40) = 0.471$$

- As with Birnbaum's measure the I_{CR} calculations indicate battery four as the most critical

I_{CR} Example

- Applying the same process to the parallel system

$$I_{CR}^{1}(40) = \frac{q_2 q_3 q_4 \times q_1}{q_1 q_2 q_3 q_4} = 1$$

$$I_{CR}^{2}(40) = 1.0$$

$$I_{CR}^{3}(40) = 1.0$$

$$I_{CR}^{4}(40) = 1.0$$

- With the parallel system the criticality importance measure places the same importance on all of the batteries a different result

Upgrading Function

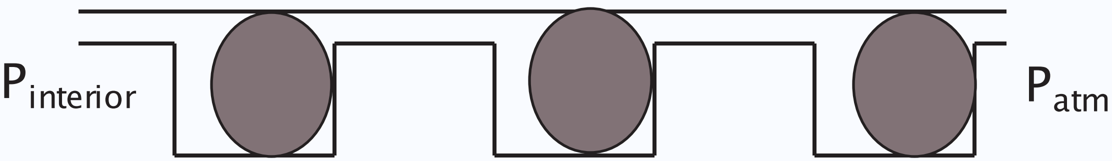
- This function is defined as the fractional reduction in the probability of the system failure when the failure rate of the i^{th} component is reduced

$$I_{UF}^{i}(t) = \frac{\lambda_i}{G(\boldsymbol{q}(t))} \times \frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_i}$$

- This metric can be applied to determine the optimal choice of upgrade

I_{UF} Example

- A booster rocket contains a set of three o-rings to prevent the leakage of gas



- The rings exhibit constant failure rates of λ_1=0.004, λ_2=0.009 and λ_3=0.025
- Calculate which of these o-rings is the most critical when in a series and 2-out-of-3 configuration

I_{UF} Example

- The formulae for the o-ring unreliabilities are:

$$q_1 = 1 - e^{-\lambda_1 t}$$

$$q_2 = 1 - e^{-\lambda_2 t}$$

$$q_3 = 1 - e^{-\lambda_3 t}$$

- Our expression for $G(\boldsymbol{q}(t))$ is:

$$G\big(\boldsymbol{q}(t)\big) = 1 - (1 - q_1)(1 - q_2)(1 - q_3)$$

- Which equates to:

$$G\big(\boldsymbol{q}(t)\big) = 1 - e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t}$$

I_{UF} Example

- Differentiating this with respect to each of the failure rates gives:

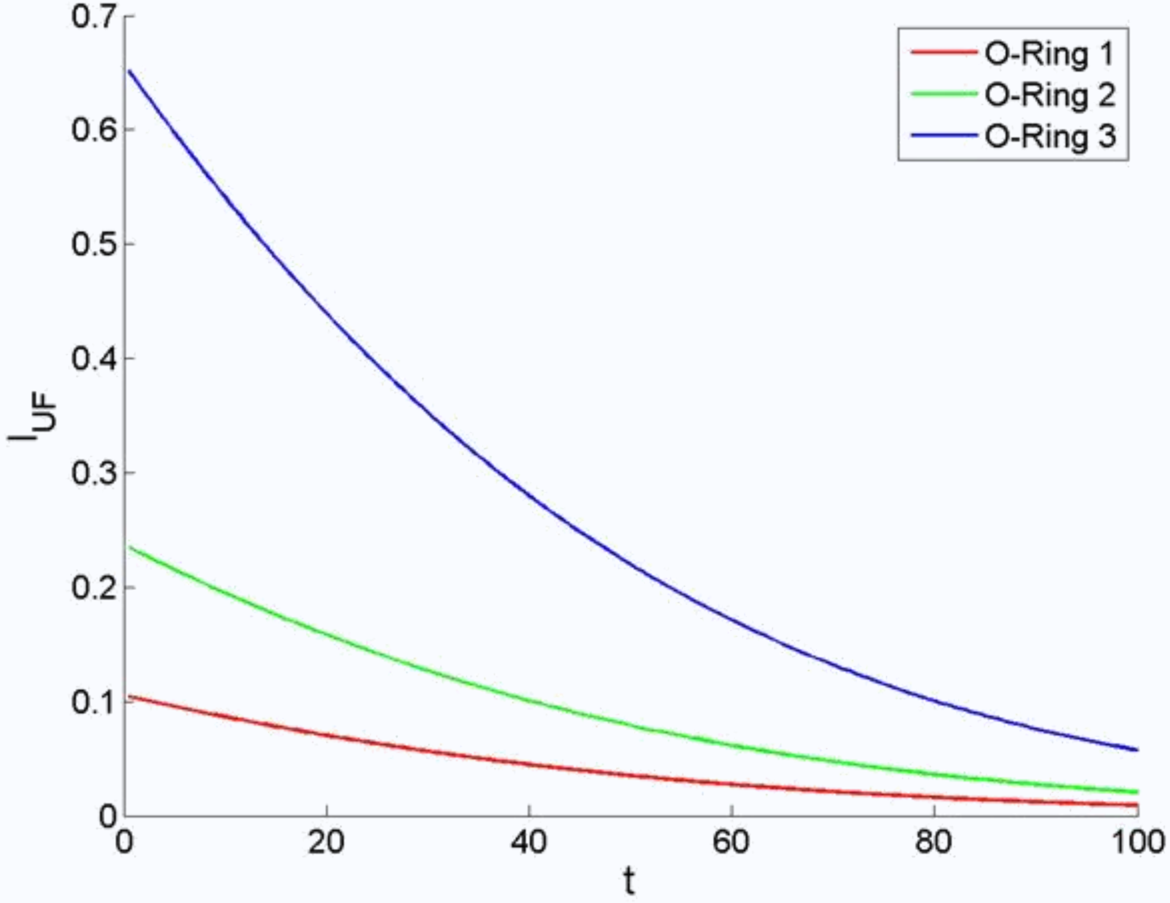
$$\frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_1} = \frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_2} = \frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_3} = te^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

- Which gives us the following expression for I_{UF}:

$$I_{UF}^{i}(t) = \frac{\lambda_i te^{-(\lambda_1+\lambda_2+\lambda_3)t}}{1 - e^{-(\lambda_1+\lambda_2+\lambda_3)t}}$$

I_{UF} Example

- For our series system of o-rings we obtain the following graph. Which is the most important?



I_{UF} Example

- For our 2-out-of-3 system, $P[\phi(\boldsymbol{X}) = 1]$ equals the probability of all 3 o-rings working plus the probability of any combination of 2 o-rings working
- From this the expression for $G(\boldsymbol{q})$ can be found:

$$G(\boldsymbol{q}) = q_1q_2 + q_1q_3 + q_2q_3 - 2q_1q_2q_3$$

- Confirm this for yourself

I_{UF} Example

- Expanding out $G(\boldsymbol{q})$ to include our expressions for q_i:

$$G(\boldsymbol{q}) = 1 - e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_1+\lambda_3)t} - e^{-(\lambda_2+\lambda_3)t} + 2e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

- From which we can calculate derivatives:

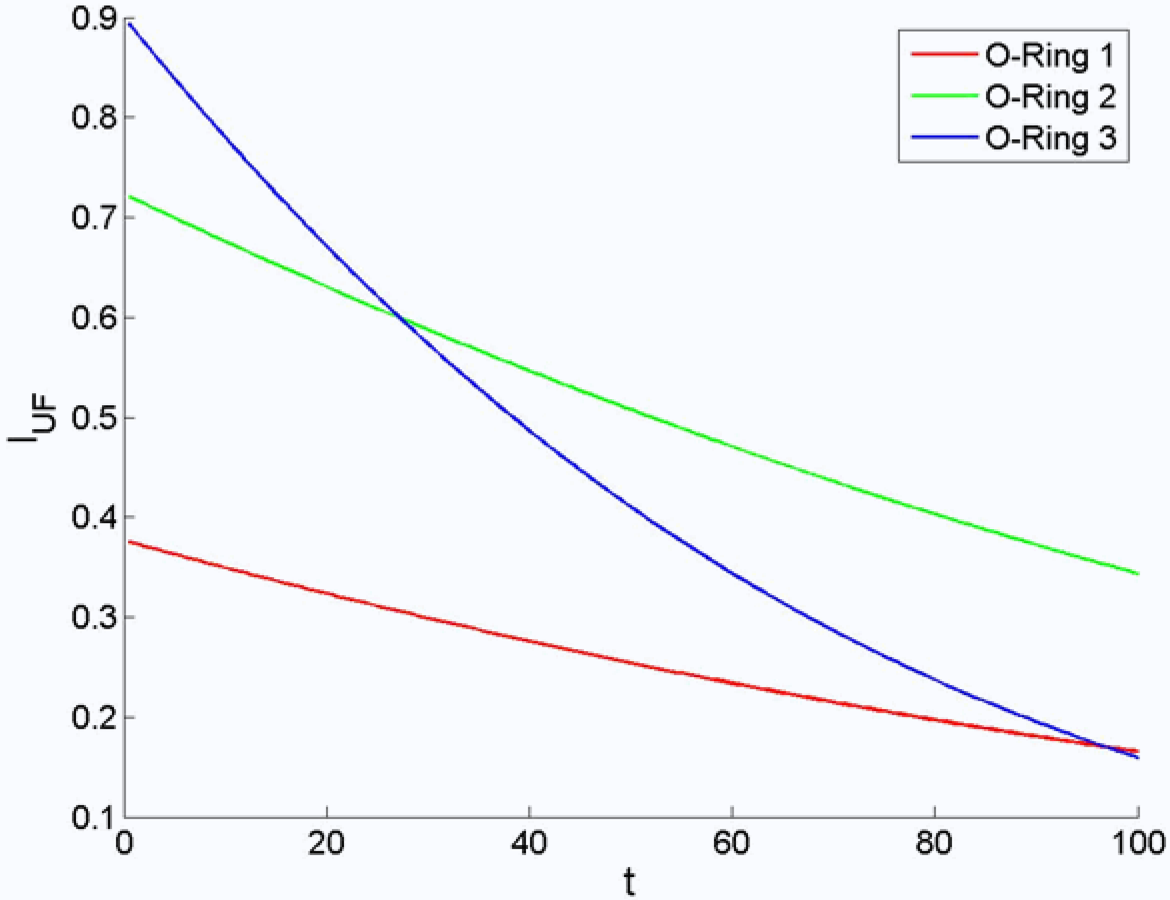
$$\frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_1} = te^{-(\lambda_1+\lambda_2)t} + te^{-(\lambda_1+\lambda_3)t} - 2te^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

$$\frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_2} = te^{-(\lambda_1+\lambda_2)t} + te^{-(\lambda_2+\lambda_3)t} - 2te^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

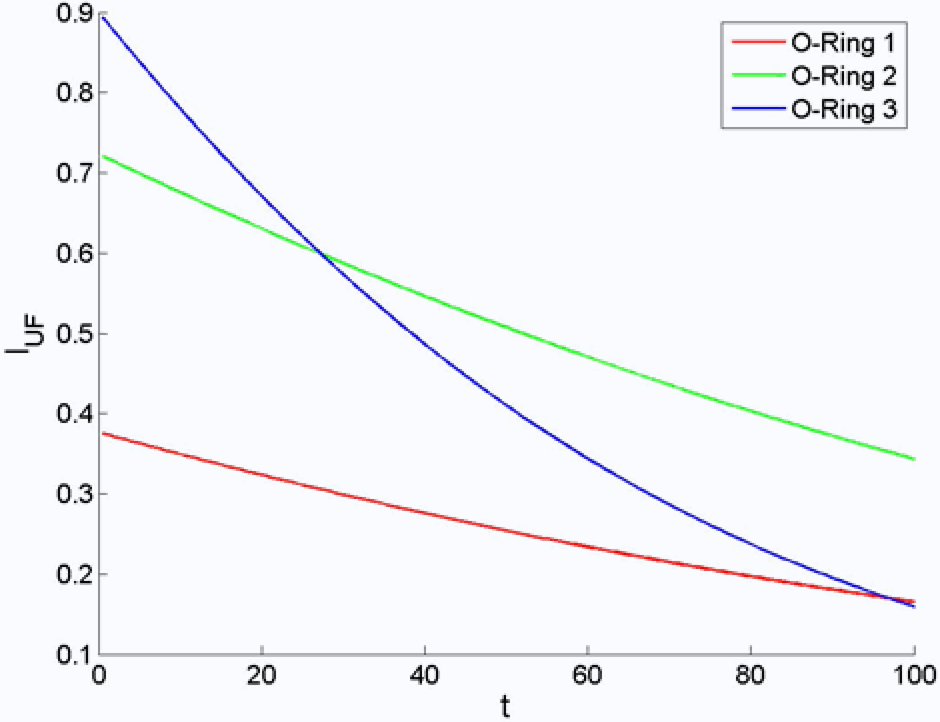
$$\frac{\partial G(\boldsymbol{q}(t))}{\partial \lambda_3} = te^{-(\lambda_1+\lambda_3)t} + te^{-(\lambda_2+\lambda_3)t} - 2te^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

I_{UF} Example

- An expression for I_{UF} can be formed producing the following graph:



Importance Vs. Time



- Note that the most important component changes depending on the time
- It's an important observation that all such metrics are a function of time

Other Functions

- Fussell-Vesely importance:
  - Probability that the system's life coincides with the cut-set containing the *i*th component

$$I_{FV}^{i}(t) = \frac{G_i(\boldsymbol{q}(t))}{G(\boldsymbol{q}(t))}$$

- Barlow-Proschan importance:
  - Conditional probability that component *i* causes the system to fail in the time interval t_0, t_f

$$I_{BP}^{i}(t) = \frac{\int_{t_0}^{t_f} \frac{\partial G(\boldsymbol{q}(t))}{\partial q_i} \frac{dq_i}{dt} dt}{\sum_{k=1}^{n} \int_{t_0}^{t_f} \frac{\partial G(\boldsymbol{q}(t))}{\partial q_k} \frac{dq_k}{dt} dt}$$

Importance Calculation Issues

- We’ve seen a number of different metrics for determining the importance of a component in a system
- It is important to remember that no single importance metric is valid for all system configurations
  - We’ve seen very different results with our parallel batteries
- A combination of metrics may be a better approach, for example, a weighted sum of different metrics
- The conclusion is dependent on time

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