Specimin test 6

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2. State how many arbitrary constants you expect to find in the general solution to the following differential equation, and then find the general solution:

3 ✓

$$\frac{d^3x}{dt^3} = t + \sin(2t).$$

$\frac{d}{dx}\sin = \cos$

$\frac{d}{dx}\cos = -\sin$

$$\frac{d^2x}{dt^2} = \frac{t^2}{2} - \frac{\cos 2t}{2} + A$$

$$\frac{dx}{dt} = \frac{t^3}{6} - \frac{\sin 2t}{4} + At + B$$

$$x(t) = \frac{t^4}{24} + \frac{\cos 2t}{8} + \frac{At^2}{2} + Bt + C$$ ✓

3. (i) Determine the general solution of the equation $xt^3\frac{dx}{dt} = 1$.

$$\int x\ dx = \int \frac{1}{t^3}\ dt$$

$$\frac{x^2}{2} = -\frac{1}{2}t^{-2} + A$$

$$x = \pm\sqrt{A - \frac{1}{t^2}}$$

(ii) Find the solution of the differential equation $\frac{dx}{dt} = (x+1)t^2$ given that $x(0) = 1$.

$$\int \frac{1}{x+1}\ dx = \int t^2\ dt$$

$$\ln(x+1) = \frac{t^3}{3} + A$$

$$x = Ae^{t^3/3} - 1$$

$$1 = A - 1$$

$$A = 2$$

$$x = 2e^{t^3/3} - 1$$ ✓

4. Find the general solution to the following equations:

(i) $2\frac{d^2x}{dt^2} + 3\frac{dx}{dt} - 2x = 0$

$m_1 = \frac{1}{2}$ $m_2 = -2$

$x = Ae^{t/2} + Be^{-2t}$ ✓

(ii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

$m_1 = m_2 = 3$

$x = (At + B)e^{3t}$ ✓

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5. Find the solution to the differential equation

$$2\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0 \qquad \text{given that} \quad x(0) = 0, \quad \frac{dx}{dt}(0) = 1.$$

$$m = \frac{-1 \pm i}{2}$$

$$x = \left(A\cos\tfrac{t}{2} + B\sin\tfrac{t}{2}\right)e^{-\frac{t}{2}}$$

$$\frac{dx}{dt} = \ldots -\frac{1}{2}A\cos\tfrac{t}{2} \ldots + \frac{B}{2}\cos\tfrac{t}{2} + \ldots$$

$$1 = -\frac{A}{2} + \frac{B}{2}$$

$$0 = A$$

$$B = 2$$

$$x = e^{-\frac{t}{2}}\left(2\sin\tfrac{t}{2}\right)$$ ✓