SESA3029 Aerothermodynamics

2021-22 Examination Feedback

This should have been a straightforward exam to pass based on Q1 and Q2, but high marks also required a good attempt at Q3. Fairly generous partial credit was available in question 1 where the method was correct (as long as a method was shown) while questions 2 and 3 were similarly marked. Overall the top module mark was 87 (27 scores over 70). The module average was 54%.

Q1. This question included a selection of short numerical calculations to put into practice the various gas dynamics techniques learned on the module: (i) subsonic flow M=0.74 can be read from IFT giving V=Ma=231 m/s, (ii) $p_e/p_0$=0.0035 and $p_t/p_0$=0.5283 from IFT together with the given $p_e$ gives $p_t$=1811 kPa, (iii) standard expansion fan gives $M_2$=2.02 for which the Mach angle $\nu$=29.67°. This is 14.67° relative to the oncoming stream, taking account of the flow turning, (iv) note that since this is postulated as finding $C_p$ for a given $M_{CR}$ it doesn't require any iteration. You could have substituted into a derived (or memorised) $M_{CR}$ equation, or (quicker) the $C_p$ definition can be written in terms of $p/p_\infty$ and from IFT we have p/p0 and $p_\infty/p_0$ (at M=1 and M=$M_{CR}$ respectively), giving $C_p$=-1.061 and then PG correction gives $C_{p0}$=-0.815 (v) standard oblique shock calculation gives $p_{lower}$=107.7 kPA (a tolerance allowed for reading from chart) and standard expansion method gives $p_{upper}$=20.2 kPa, hence after multiplying by chord the normal force=10.5 kN/m, (vi) Ackeret gives $C_p$=±0.324 and normal force=9.8 kN/m, (vii) $M_2$=1.35 for which 15° is above $\theta_{max}$, so it must be a Mach reflection.

Q2. (i) The characteristic lines appear as two version of the standard method. Allowing for the fact that the upstream Mach number $M_t$=1 and $M_i$=1.6 not equal to one gives $\theta_i=(\nu(M_e)-\nu(M_i))/2$=14.38° and $\theta_t=\nu(M_i)/2$=7.43°, (ii) this required coverage (see lectures 2.6 and 2.7) of the different situations seen in design and off-design nozzles i.e. a design case, shock at exit, shock in different positions within nozzle, sonic throat, subsonic, over/under-expanded at exit. The only point of difference for the bell-shaped nozzle, compared to a standard Laval nozzle, is that the second design condition achieves the required shock-free flow at $M_i$. At that intermediate point the flow separates from the wall and continues to the exit with the same conditions as at point i.

Q3. The question corresponds to a small solution of Problem 5.132 from the textbook by Bergman et al.
(i) One discretizes the 1d heat diffusion equation $\frac{\partial q}{\partial x} + g = \rho c_p \frac{\partial T}{\partial t}$ for an interior cylindrical volume element around the node of value $T_i^n$ of side area $A_c = \pi \frac{d^2}{4}$ and circumferential length $P = \pi d$, considering that the convective heat flux $q_c = h(T_\infty - T_i^n)$ is as acting as source *g* on the outer surface area *P*Δ*x*. During the derivation the Fourier number $F = \frac{\Delta t}{\Delta x^2}\frac{k}{\rho c_p}$ and the Biot number $B = \frac{h \Delta x^2 P}{A_c k}$ are identified. The derivation for the free end node is canonical, considering that this volume element has length ½Δ*x* and convection is acting on the outer surface area *P*½Δ*x* as well as the free surface with area $A_c$. The additional parameter $D = \frac{h\Delta x}{k}$ is identified in the process. (ii) The non-dimensional parameter values are verified readily. Starting from $T_i^0 = 200$ and using $T_0^n = 200$, straightforward iteration on an equidistant 7-point mesh with 1-2*F*-*FB* = 0.3394, 1-2*F*-2*FD*-*FB* = 0.3010, 2*FD*+*FB* = 0.206 evaluates in 4 steps the minimally required temperature values

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 60.00 | 200.0 | 169.1 | 169.1 | 169.1 | 169.1 | 169.1 | 163.9 |
| 2 | 120.00 | | 151.1 | 143.6 | 143.6 | 143.6 | 142.4 | |
| 3 | 180.00 | | | 124.5 | 122.7 | 122.4 | | |
| 4 | 240.00 | | | | 105.8 | | | |

Interpolation gives 225.1s for the time when the centre node reaches 110°C. (iii) Interpolation in the centre evaluates the sought time for the 9-point mesh as 234.4s. First-order extrapolation in time for ratio 33.75s/60s yields 246.4s for Δt->0. Second-order extrapolation in space for ratio 2.25cm/3cm yields 246.4s for Δx->0. Both estimates are identical because the computations use the same Fourier number.

NDS/RD

14/2/2022