# Problem sheet 1

04 October 2022 10:23

1)

a) $y'' + 3y' + 2y = 0$

$y = e^{\lambda x}$

$y' = \lambda e^{\lambda x}$

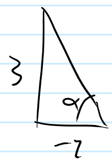
$y'' = \lambda^2 e^{\lambda x}$

$\lambda^2 e^{\lambda x} + 3\lambda e^{\lambda x} + 2e^{\lambda x} = 0$

$(\lambda^2 + 3\lambda + 2)e^{\lambda x} = 0$

$\lambda_1 = +1 \qquad \lambda_2 = +2$

$Ae^{+x} + Be^{+2x} = y$



b) $Ae^{(-2+3i)x} + Be^{(-2-3i)x}$

$Ae^{-2x}e^{3ix} + Be^{-2x}e^{-3ix}$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$e^{-2x}(Ae^{3ix} + Be^{-3ix})$

$(A(\cos 3x + i\sin 3x) + B(\cos 3x - i\sin 3x))$

$((A+B)\cos 3x + (A-B)i\sin 3x)$

$e^{-2x}(A\cos 3x + Bi\sin 3x)$ ✓

2)

a) $x^2\ddot{y} - 6y = 0$

$y = x^k$

$\dot{y} = kx^{k-1}$

$\ddot{y} = k(k-1)x^{k-2}$

$x^2 k(k-1)x^{k-2} - 6x^k = 0$

$x^k(k(k-1) - 6) = 0$

$k^2 - k - 6 = 0$

$k = 3 \qquad k = -2$

$Ax^3 + Bx^{-2} = y$ ✓

b) $x^2 \frac{d^2y}{dx^2} + cx\frac{dy}{dx} + dy = 0$

$t = \cancel{\log}\ \ln(x)$

**(b)** $x^2y'' + 3xy' + 10y = 0$

$$\frac{d^2y}{dt^2} + (c-1)\frac{dy}{dt} + dy = 0$$

$$\ddot{y} + 2\dot{y} + 10y = 0$$

$$e^{-t}(A\cos 3t + B\sin 3t) = y$$

$$e^{-\cancel{\log}\ln x}\left(A\cos(3\cancel{\log}\ln x) + B\sin(3\cancel{\log}\ln x)\right) = y$$

$$\frac{1}{x}\left(A\cos(3\ln x) + B\sin(3\ln x)\right) = y$$

3)

(a) $y'' - 2y' + y = 4e^x$

$$y = axe^x$$
$$\dot{y} = a(e^x + xe^x)$$
$$\ddot{y} = a(2e^x +$$

1)

c) $y'' - 4y' + y = 0$

$y_{1,2} = e^{\lambda x}$ $\quad \lambda = 2 \pm \sqrt{3}$

$$y = c_1 e^{x(2+\sqrt{3})} + c_2 e^{x(2-\sqrt{3})}$$ ✓

d) $y'' - 3y' = 0$ $\quad \lambda = 3, 0$

$$y = c_1 e^{3x} + c_2$$ ✓

2)

c) $x^2y'' + 5xy' + 4y = 0$

$k_1 = 2$ $\quad k_2 = 2$ ] dupe

$t = \ln x$

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 0$$

AE ...

$\lambda_1 = -2$ $\quad \lambda_2 = -2$

$$y = (c_1 t + c_2)e^{-2t}$$
$$= (c_1 \ln x + c_2)\frac{1}{x^2}$$

$$= (c_1 \ln x + c_2)\frac{1}{x^2}$$

$$x^2 \frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$$

$y = x^k$

$y' = k x^{k-1}$

$y'' = k(k-1)x^{k-2}$

$$\cancel{x^2} k(k-1)\cancel{x^{k-2}} + b\cancel{x} k \cancel{x^{k-1}} + c\cancel{x^k} = 0$$

$$k^2 - k + bk + c$$

$$k^2 - k(b-1) + c$$

3)

a)

$$y'' - 2y' + y = 4e^x$$

$k_{1,2} = 1$

$y_c = c_1 e^x + c_2 x e^x$

try: $y_p = A(x^2 e^x)$

$y_p' = A(2xe^x + x^2e^x)$

$y_p'' = A(2e^x + 2xe^x + 2xe^x + x^2e^x) = A(2e^x + 4xe^x + x^2e^x)$

$\frac{4e^x}{A} = (2e^x + \cancel{4xe^x} + \cancel{x^2e^x}) - 2(\cancel{2xe^x} + \cancel{x^2e^x}) + (\cancel{x^2e^x})$

$= 2e^x$

$2/A = 1$

$2 = A$ $\therefore$ valid sol

$$y = e^x(c_1 + x c_2) + 2x^2e^x \checkmark \quad = e^x(c_1 + xc_2 + 2x^2)$$

cleaner

b)

$$y'' - 2y' + 2y = 4e^x \sin x$$

$k = 1 \pm i$

$y_c = e^x(c_1\cos(x) + c_2\sin(x))$ $\checkmark$

$y_p = Ae^x\sin x + Be^x\cos x = e^x(As + Bc)$

$y_p' = A(e^x\sin x - e^x\cos x) + B(e^x\cos x + e^x\sin x)$
$= (A+B)(e^x\sin x) + (B-A)(e^x\cos x) = e^x(As + Bs + Bc - Ac)$

$y_p'' = (A+B)(e^x\sin x - e^x\cos x) + (B-A)(e^x\cos x + e^x\sin x)$
$= e^x(\cancel{As} - Ac + Bs - \cancel{Bc} + \cancel{Bc} + Bs - Ac - \cancel{As})$
$= e^x(2Bs - 2Ac)$

$4e^x s = e^x(2Bs - 2Ac) - 2e^x(As + Bs + Bc - Ac) + 2e^x(As + Bc)$

(it all cancels down to $2\sin x = 0$, is wrong)

stuck

stupid boy, $Ae^x\sin x$ in invalid since its a part of the complementary function!

$y_p = Ae^x x\sin x + Be^x x\cos x$
$= Aexs + Bexc$

$y_p' = Aexs + Aes - Aexc + Bexc + Bec + Bexs$
$= (A+B)exs + (B-A)exc + Aes + Bec$

$y_p'' = (A+B)(exs + es - exc) + (B-A)(exc + ec + exs) + A(es - ec) + B(ec + es)$

$= Aexs + Aes - Aexc + Bexs + Bes - Bexc + Bexc + Bec + Bexs - Aexc - Aec - Aexs + Aes - Aec + Bec + Bes$

$= 2Aes - 2Aexc + 2Bexs + 2Bes - 2Aec + 2Bec$

$4e^x\sin x = (2Aes - 2Aexc + 2Bexs + 2Bes - 2Aec + 2Bec) - 2((A+B)exs + (B-A)exc + Aes + Bec)$

$+2(Aexs + Bexc)$

$4es = 2Aes - 2Aexc + 2Bexs + 2Bes - 2Aec + 2Bec - 2Bexs + 2Aexc - 2Aes - 2Bec$

$4es = \quad +2Bes - 2Aec$

$2\sin x = B\sin x - A\cos x$

$B = 2 \qquad A = 0$

$\therefore y = e^x(c_1\cos(x) + c_2\sin(x)) + 2e^x x\cos x$

should be negative (sad)

$= e^x(c_1\cos(x) + c_2\sin(x) + 2x\cos x)$

c) $y'' - 4y' + 4y = xe^{2x}$

$\lambda_{1,2} = 2$

$y_c = e^{2x}(c_1 + c_2 x)$

$y_p = (x^2e^{2x})A$

$\dot{y}_p = (2xe^{2x} + 2x^2e^{2x})A$

$\ddot{y}_p = (4xe^{2x} + 2e^{2x} + 4xe^{2x} + 4x^2e^{2x})A$

$= (8xe^{2x} + 4x^2e^{2x} + 2e^{2x})A$

$\frac{1}{A}xe^{2x} = (8xe^{2x} + 4x^2e^{2x} + 2e^{2x}) - 4(2xe^{2x} + 2x^2e^{2x}) + 4(x^2e^{2x})$

$\frac{1}{A}x = 8x + 4x^2 + 2 - 8x - 8x^2 + 4x^2$

$\frac{1}{A}x = 8x + 2 - 8x$ Still no good, you need to use y = A x^3 e^(2x)