AIRCRAFT STRUCTURAL DESIGN
SESA3026

3. Aircraft Loads
3.1 Introduction, Symmetric Manoeuvre Loads

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Course Content

- ❑ Chapter 1 Aircraft structural design
- ❑ Chapter 2 Wing structural details
- ❑ Chapter 3 Aircraft loads

Part 1: Introduction, Symmetric Manoeuvre Loads

Part 2: Tail Loads, Gust Loads

Part 3: Total Loading on the Wing

Part 4: Comparison of Full Load and Light Load Cases

- ❑ Chapter 4 Fatigue of aircraft structures
- ❑ Chapter 5 Effects of dynamic response
- ❑ Chapter 6 Static aeroelasticity
- ❑ Chapter 7 Dynamic aeroelasticity

- ❑ **SN**
- ❑ **ADR**

Learning Outcomes

By the end of this section, you will be able to

- Appreciate the loading actions acting on an aircraft

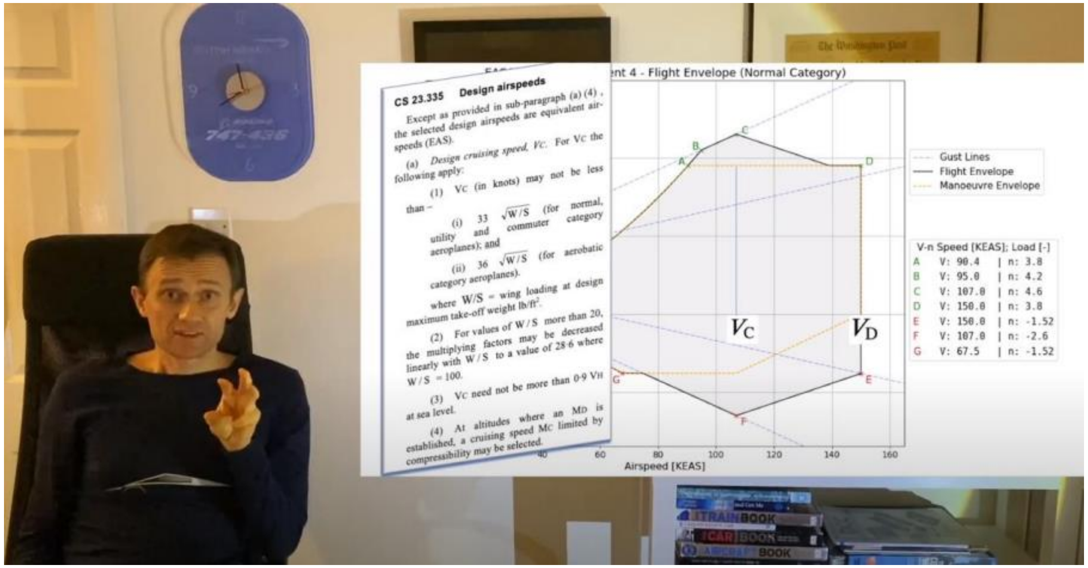
➢ https://www.southampton.ac.uk/courses/modules/sesa3026#aims_and_objectives

- Derive the load factor for a symmetric manoeuvre

- Illustrate the V-n diagram, explain the main features, and identify critical points of the flight envelope

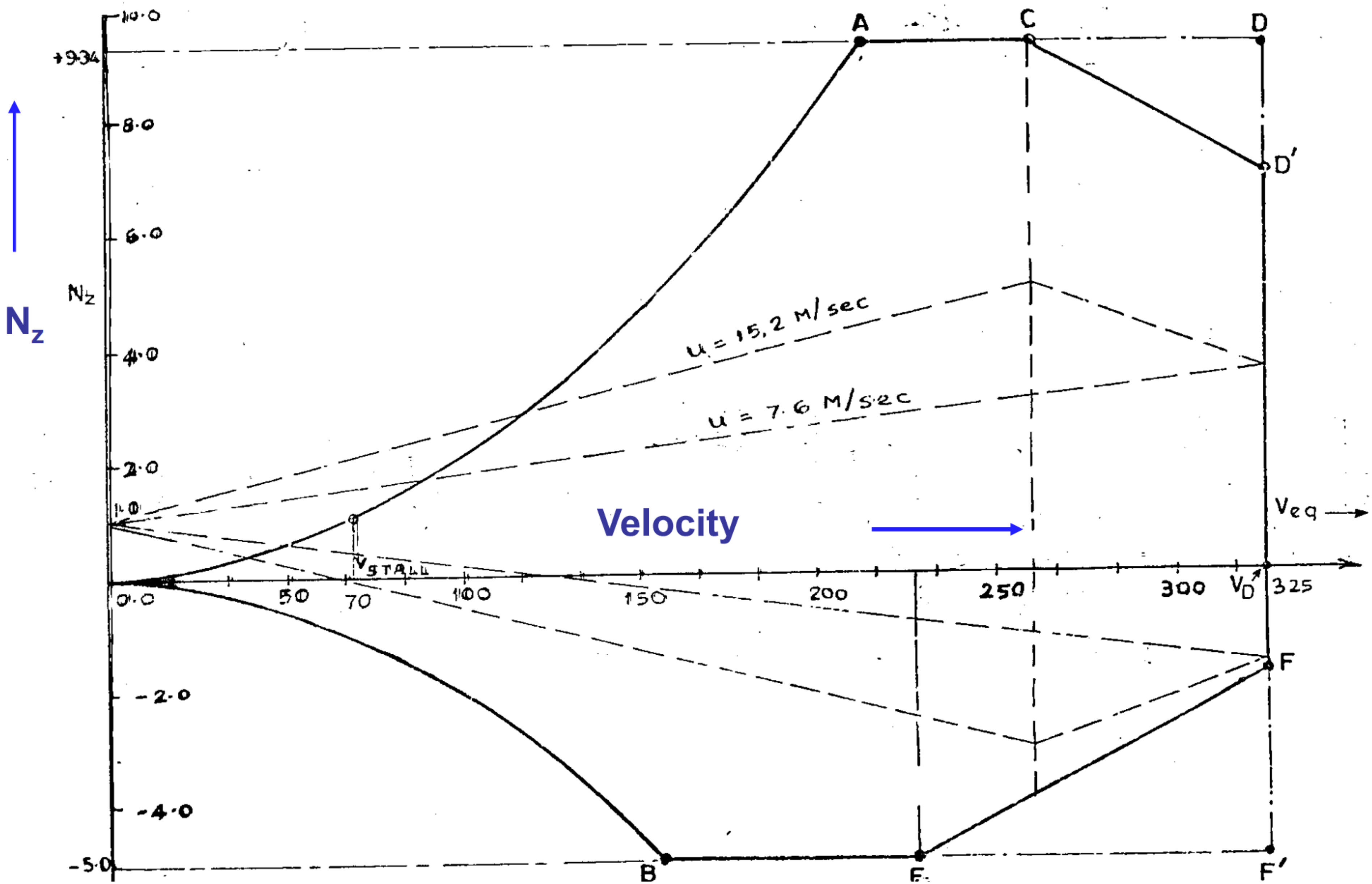
V-N Diagram

- V-N diagram definition
- a/c Load factors



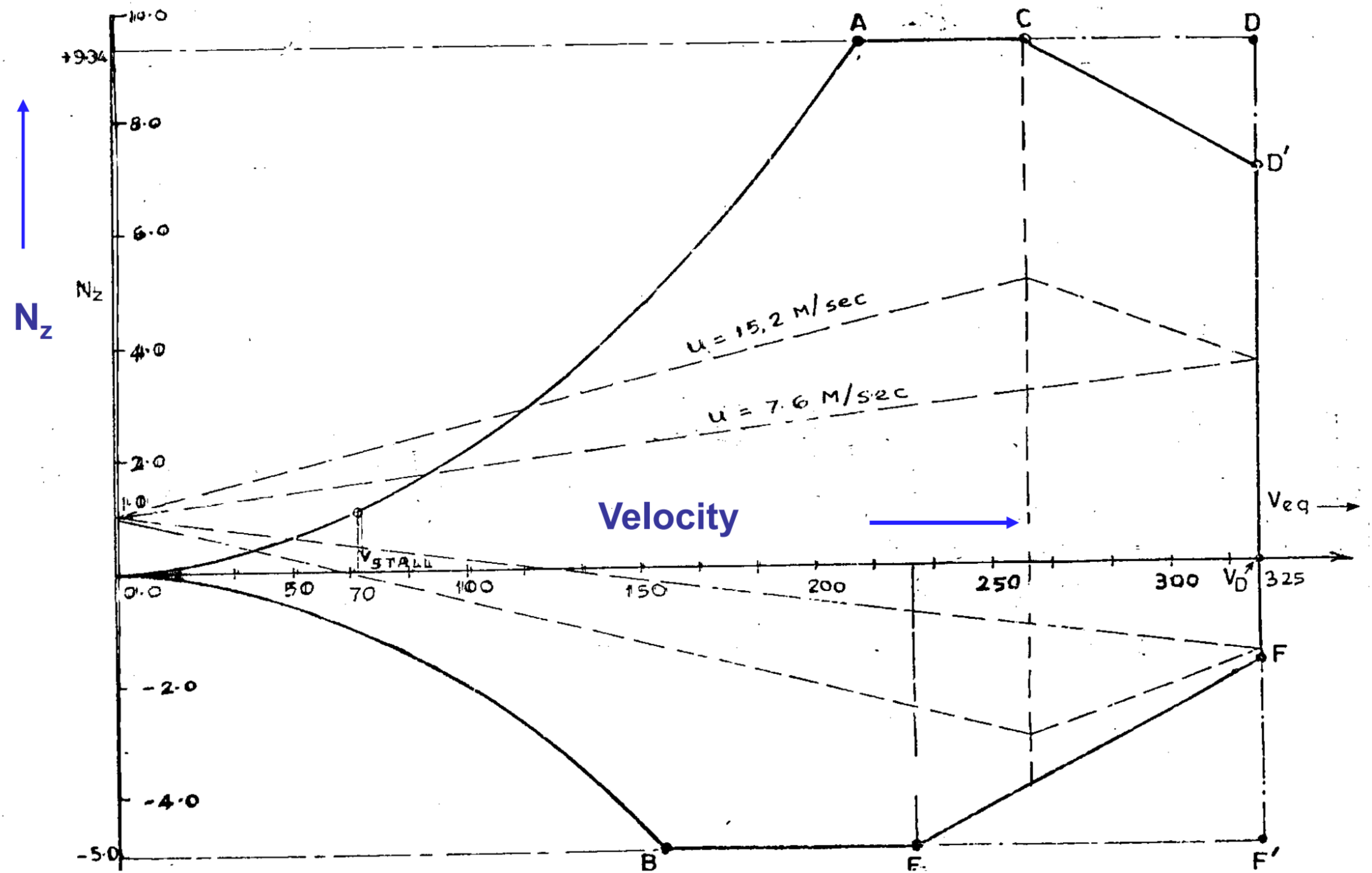
Must see: Prof. Sobester's video on V-n diagrams

https://www.youtube.com/watch?v=s-d5z-BQovY&ab_channel=AndrasSobester



V-N Diagram of HF- 24

V-N diagram is a graph of a/c velocity and the load factor



V-N Diagram of HF- 24

V-N diagram is a graph of a/c velocity and the load factor

Aircraft Load Factors

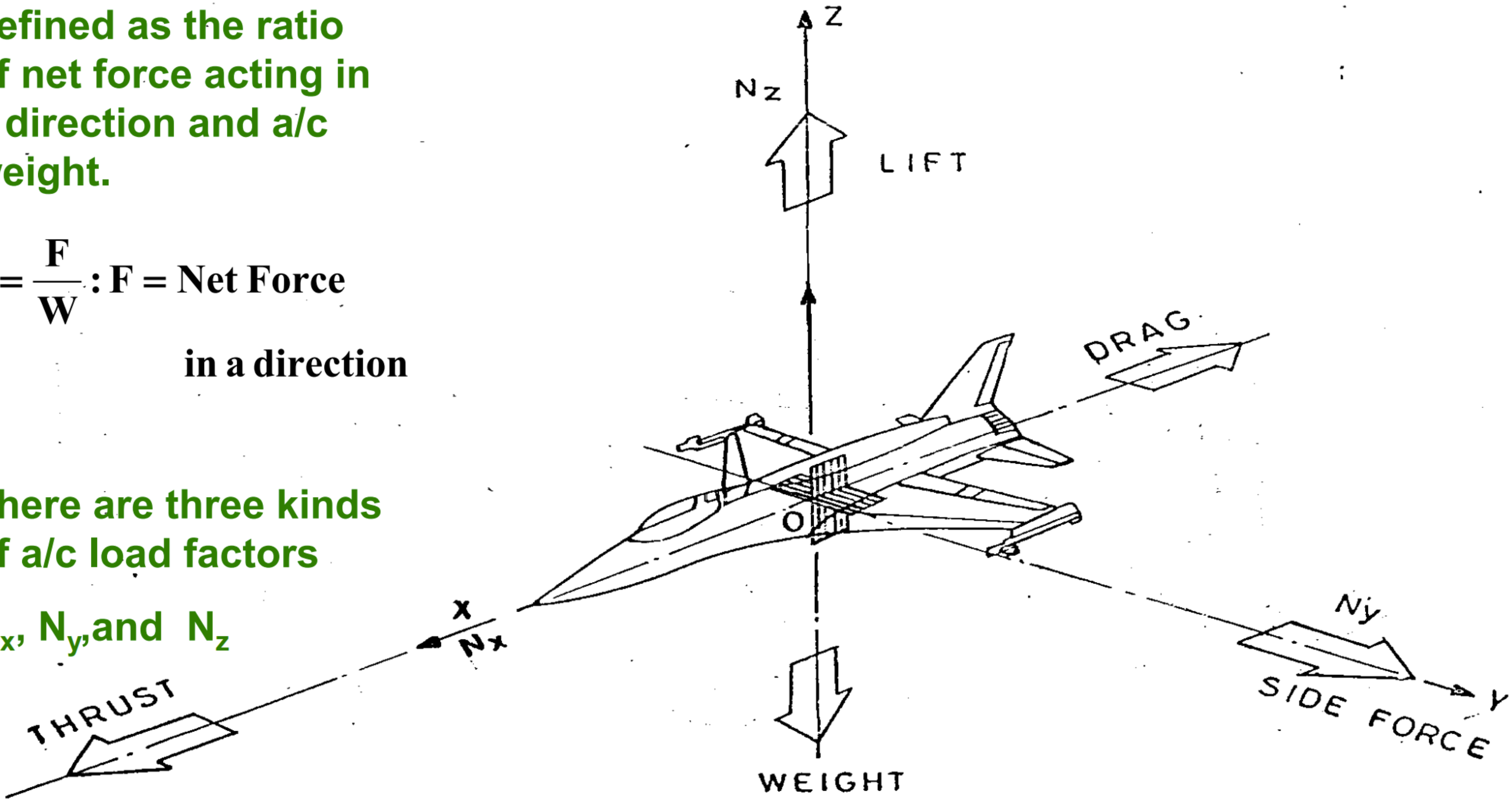
Load factor is defined as the ratio of net force acting in a direction and a/c weight.

$$N = \frac{F}{W} : F = \text{Net Force}$$

in a direction

There are three kinds of a/c load factors

N_x, N_y, and N_z



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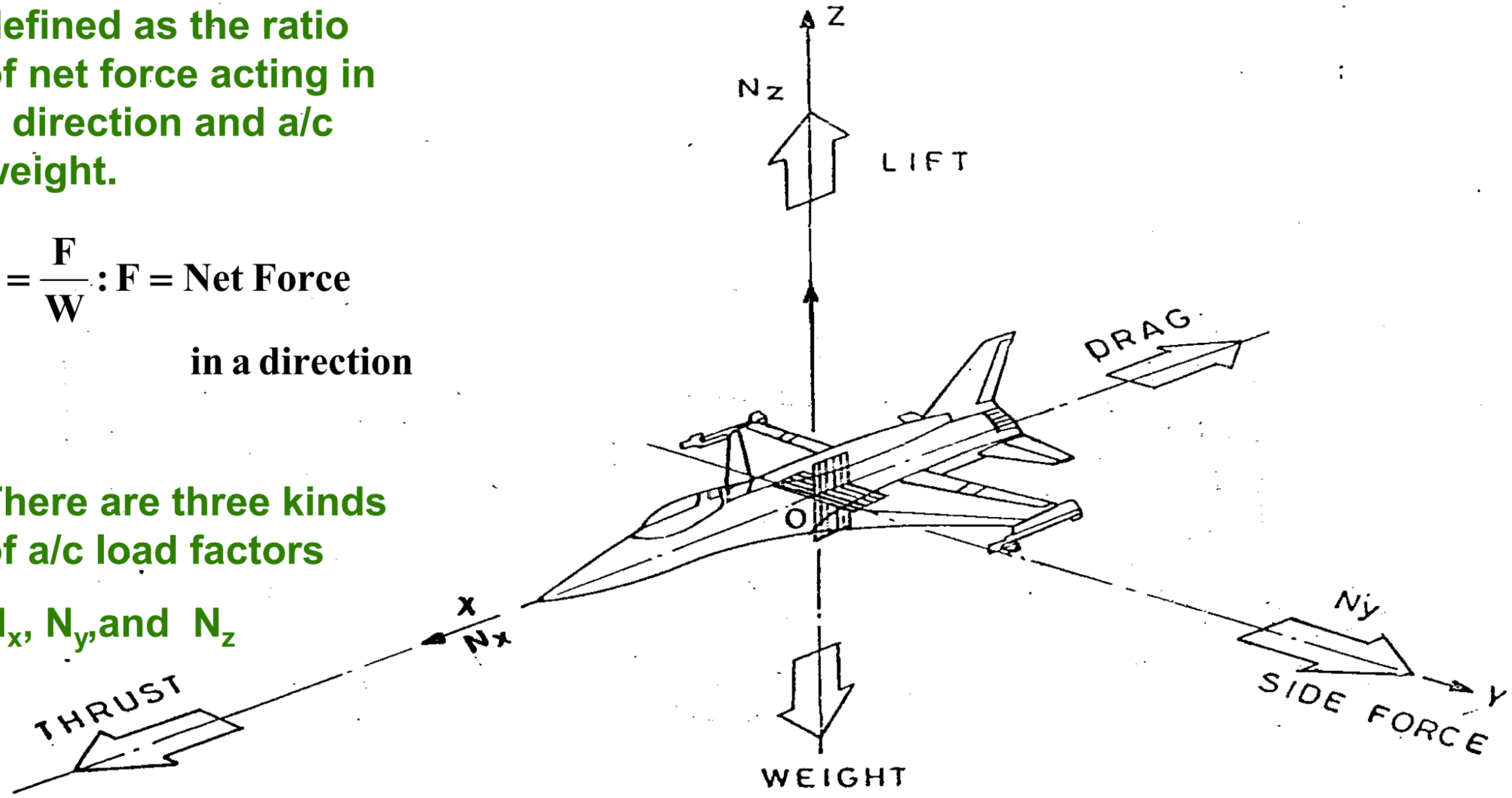

Some General Points

V-N diagram is applicable only for symmetrical maneuvers in the vertical planes. Why?

Because **N_z has the highest numerical value** and in symmetrical maneuvers in vertical plane **N_x & N_y remain constant.**

V-N diagram is drawn only for N_z. Why?

Because the numerical values of N_x, N_y are small and can't lead to structural damage to a/c if they are too high.

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It can be seen that $N_z \alpha V^2$ and (AOA) How?

$$N = \frac{L}{W}$$

$L=1/2\rho_\infty v^2_\infty SC_L$: Lift

$L=1/2\rho_\infty v^2_\infty S(AOA)a_0$

where

ρ_∞ =density of air C_l = Lift Coefficient

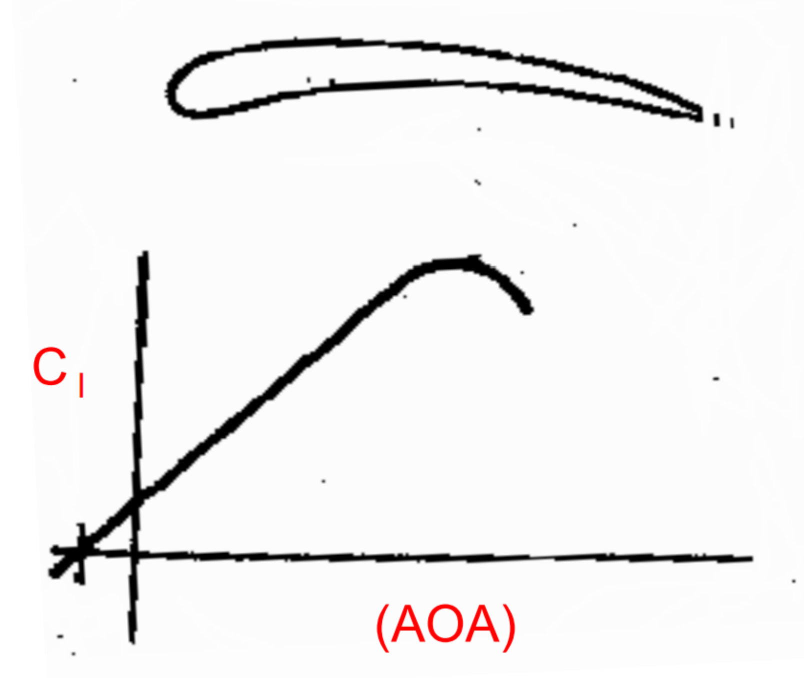
v = a/c speed S = wing area

a_0 = Lift curve slope

Thus $N_z \quad \alpha \quad \rho V^2$

and $N_z \quad \alpha \quad (AOA)$

Cambered Airfoil



$$N = \frac{L}{W}$$

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$L = 1/2 \rho_\infty v^2_\infty S(AOA) a_0$

where

ρ_∞ = density of air C_l = Lift Coefficient

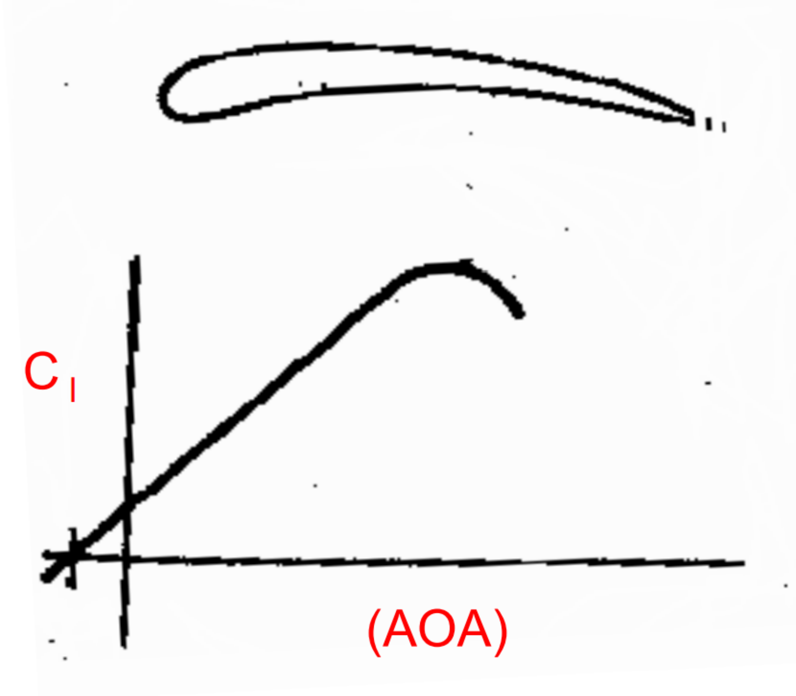
v = a/c speed S = wing area

a_0 = Lift curve slope

Thus $N_z \propto \rho V^2$

and $N_z \propto (AOA)$

Cambered Airfoil



But this would imply that we need to draw a different V-N diagram for every possible altitude.

So how do we eliminate this problem?

Equivalent Airspeed is used in calculations instead of True airspeed as found by Pitot-Static tube

- The velocity (True Airspeed [TAS]) indicated by the Airspeed Indicator is proportional to dynamic pressure
- Taking into account the errors in calibrated instruments we get the calibrated airspeed [CAS].
- And after taking into considerations the compressibility effects we get Equivalent airspeed [EAS] (so it is that speed at which the a/c would be flying at sea level under same conditions of pressure and temp.)
- By using this <u>equivalent speed</u> the variable 'ρ' can be eliminated
- So $N_z \quad \alpha \quad AOA$

  $\alpha \quad V_{eq}^2$ ONLY

Aircraft Loads

- Aerodynamic forces
- Self-weight
- Manoeuvres
- Taxiing
- Towing
- Crash landing

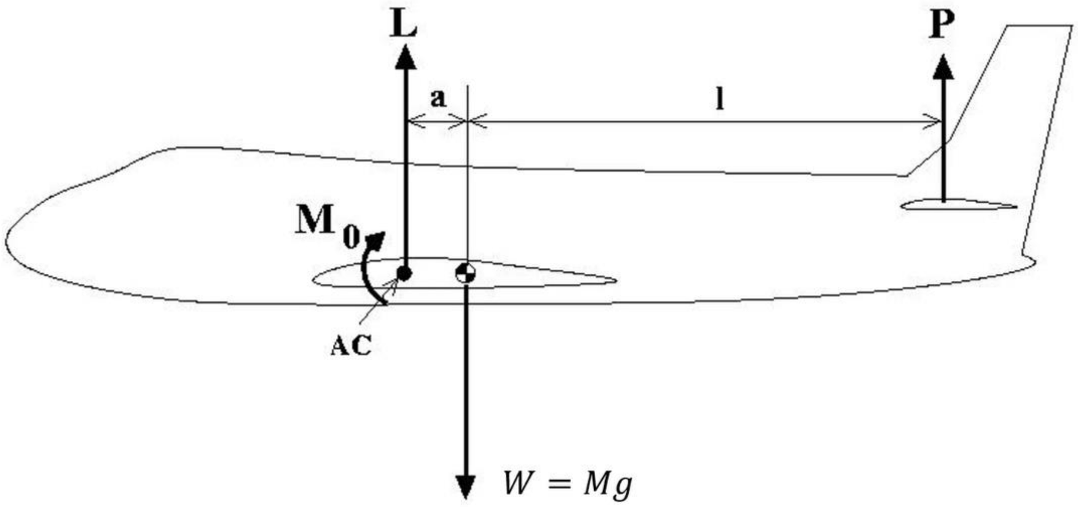
Each of these loading types are covered by various regulations

- This course: symmetric loads due to

✓ Manoeuvres

✓ gusts

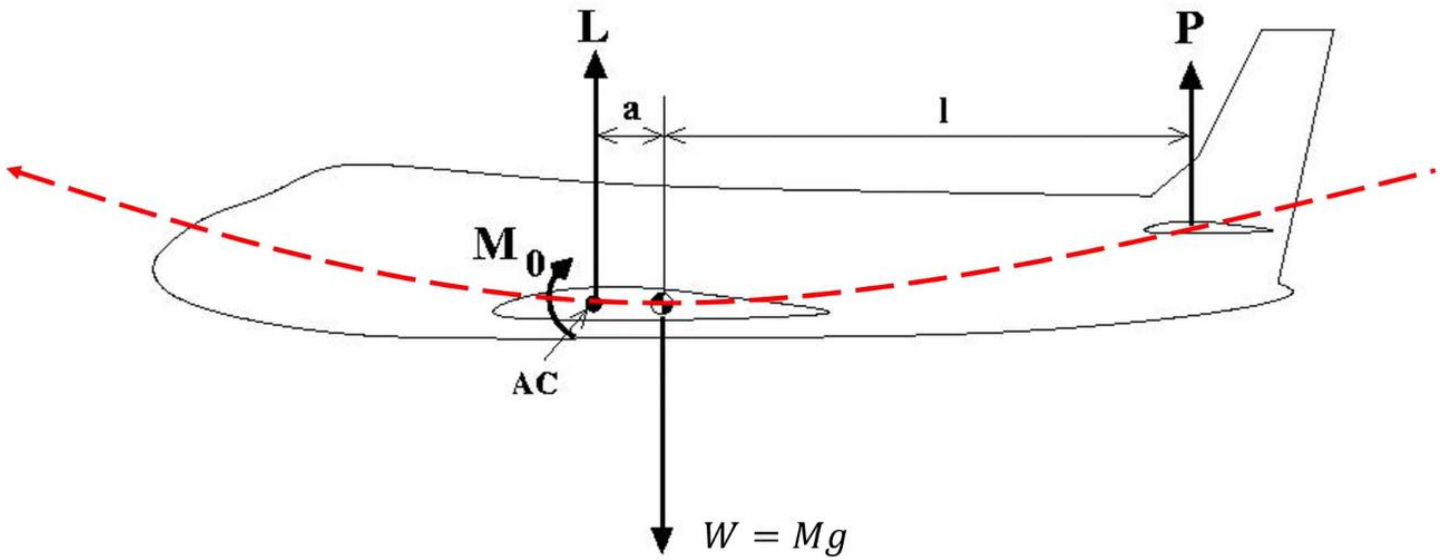
The Load Factor



Equilibrium vertical direction:

$$L + P = Mg \qquad\qquad = nW$$

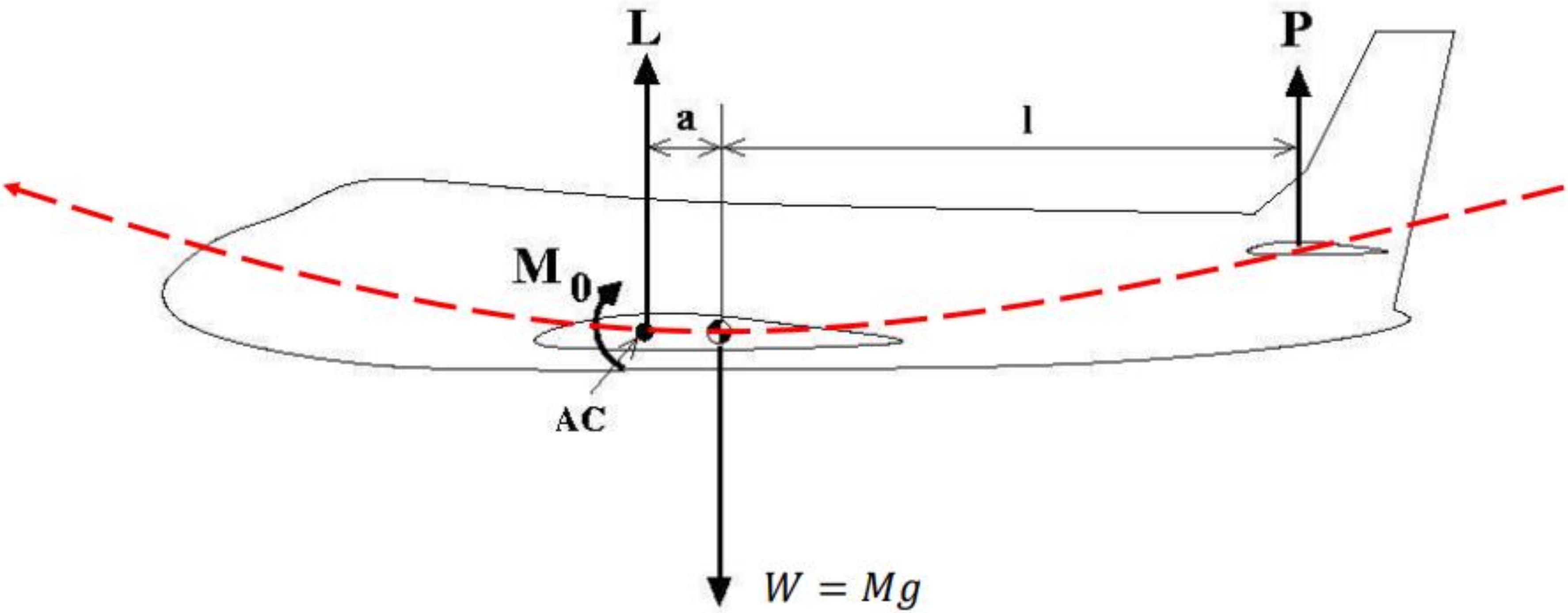
The Load Factor



Equilibrium vertical direction:

$$L + P = Mg + \underline{Ma_j} = Mg\left(1 + \frac{a_j}{g}\right) = nW$$

The Load Factor



Load factor: n

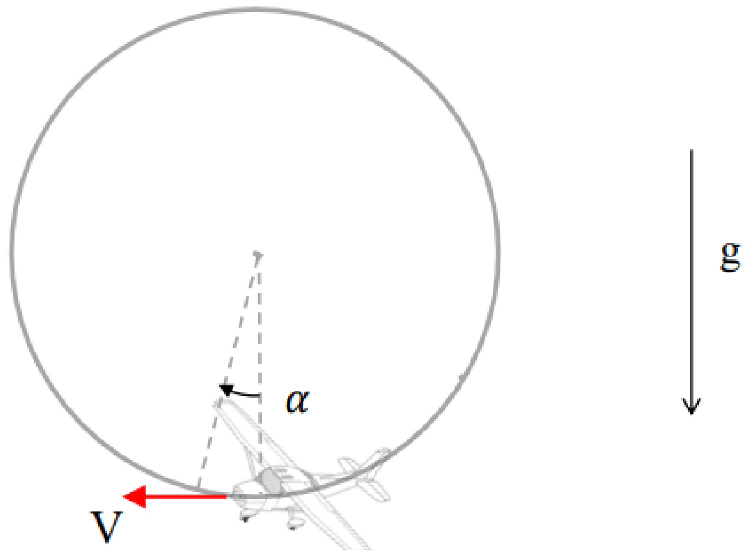
Equilibrium vertical direction:

$$L + P = Mg + \underline{Ma_j} = Mg\boxed{\left(1 + \frac{a_j}{g}\right)} = nW$$

Maximum Load Factor

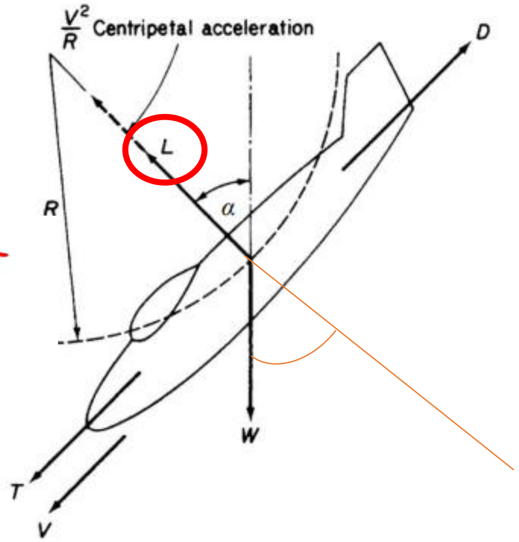
Example in a circular loop

Derive the analytical formulation that relates the load factor, n, to the aircraft angular position, α, when performing the loop sketched below. Then, indicate the point in which n is maximum and minimum. The gravity field acts in the downward direction.



Solution

Consider the sketch shown on the right



1. Write equilibrium of forces in radial direction

$$L - mg\cos\alpha \quad L - mg\cos\alpha = \frac{mV^2}{R}$$

2. Load factor

$$\frac{L}{W} - \cos\alpha = \frac{V^2}{gR}$$

$$n = \cos\alpha + \frac{V^2}{gR}$$

Solution

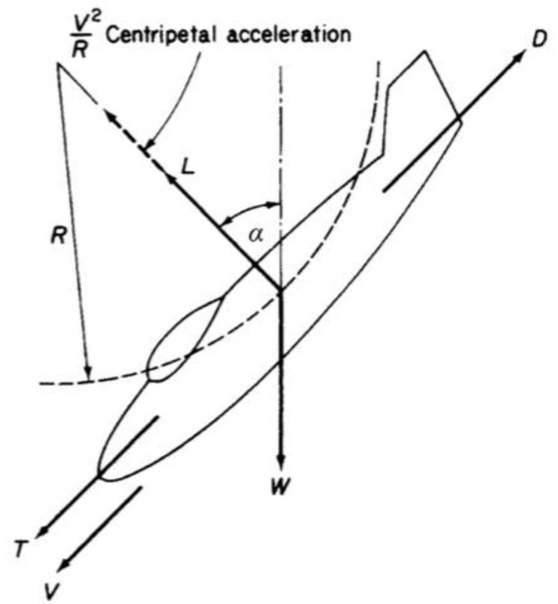
Consider the sketch shown on the right

1. Write equilibrium of forces in radial direction

$$L = W\cos\alpha + M\frac{V^2}{R}$$

2. Load factor

$$n = \frac{L}{W} = \frac{W}{W}\cos\alpha + \frac{M}{W}\frac{V^2}{R} = \boxed{\cos\alpha + \frac{V^2}{gR}}$$

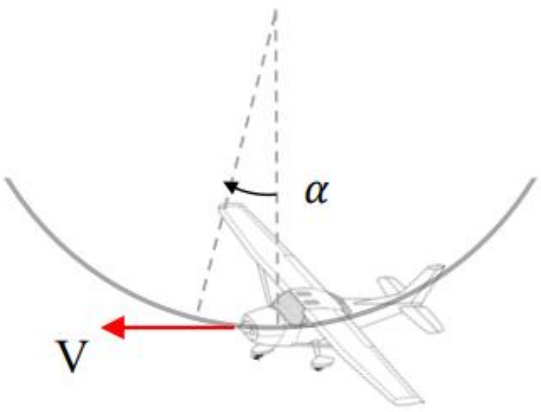


- Note n varies during the manoeuvre.
- The **max value** occurs when the aircraft is at the lowest point of the loop, $\alpha = 0^\circ$.
- The **min value** occurs when the aircraft is at the highest point, $\alpha = 180^\circ$; in this case, the pilot is upside down and the centrifugal acceleration directed upward acts in the opposite direction to the gravity field.

Symmetric Manoeuvre Loads

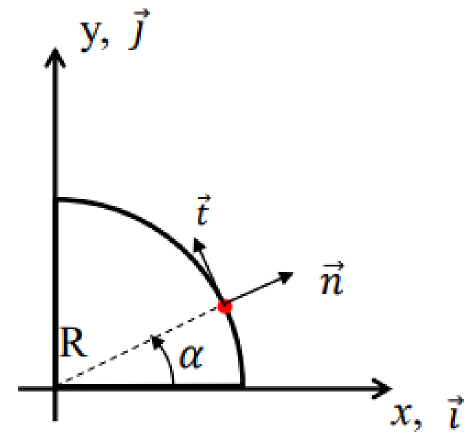
A Circular Manoeuvre at Constant Rate of Pitch

An aircraft travelling at a constant EAS, V = 435 km/h, performs the symmetric manoeuvre sketched below. If the curved part of the path is an arc of a circle, covered with a constant rate of pitch, $\dot{\alpha} = 7$ deg/s, calculate the load factor, n. Assume the aircraft is at its lowest position.



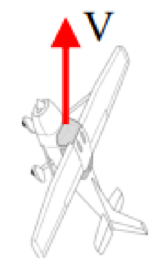
Solution

1. Kinematic analysis



- position: $\vec{p} = R(\cos\alpha \cdot \vec{\imath} + \sin\alpha \cdot \vec{\jmath})$ $\alpha = \alpha(t)$
- velocity: $\bar{v} = R\left(-\dot{\alpha}\sin\alpha\,\underline{i} + \dot{\alpha}\cos\alpha\,\underline{j}\right)$
- acceleration: $\bar{a} = R\left[\left(-\ddot{\alpha}\sin\alpha - \dot{\alpha}^2\cos\alpha\right)\underline{i} + \left(\ddot{\alpha}\cos\alpha - \dot{\alpha}^2\sin\alpha\right)\underline{j}\right]$

2. Use problem data



- velocity:
- acceleration:

$$\bar{v} = R\left(-\dot{\alpha}\sin\alpha\,\underline{i} + \dot{\alpha}\cos\alpha\,\underline{j}\right) \quad (1)$$

$$\bar{a} = R\left[\left(-\cancelto{0}{\ddot{\alpha}}\sin\alpha - \dot{\alpha}^2\cos\alpha\right)\underline{i} + \left(\cancelto{0}{\ddot{\alpha}}\cos\alpha - \dot{\alpha}^2\sin\alpha\right)\underline{j}\right]$$

$$= R\left(-\dot{\alpha}^2\cos\alpha\,\underline{i} - \dot{\alpha}^2\sin\alpha\,\underline{j}\right) \quad (2)$$

3. Calculate acceleration

$(1): |\bar{v}| = R\dot{\alpha}$ $(2): |\bar{a}| = R\dot{\alpha}^2 \Rightarrow |\bar{a}| = |\bar{v}|\dot{\alpha}$

4. Load factor

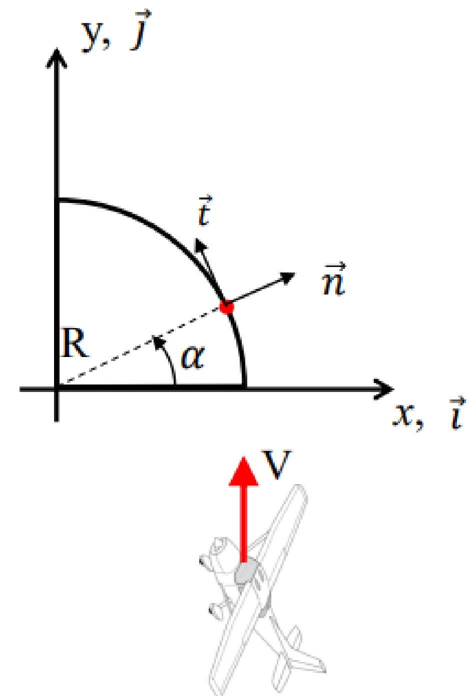
$n = 1 + \dfrac{a}{g}$ $n = \cos\alpha + \dfrac{V^2}{gR}$

Solution

$$n = 1 + \frac{V^2}{gR}$$

$$n = 1 + \frac{R\dot{\alpha}^2}{g}$$

$$n = 1 + \frac{a}{g}$$



$$V = 435\ km/h = 120.83\ m/s$$

$$\dot{\alpha} = 7\ deg/s = 0.12217\ m/s$$

$$n = 1 + \frac{120.83 \cdot 0.12217}{9.81} \Rightarrow n = 2.5$$

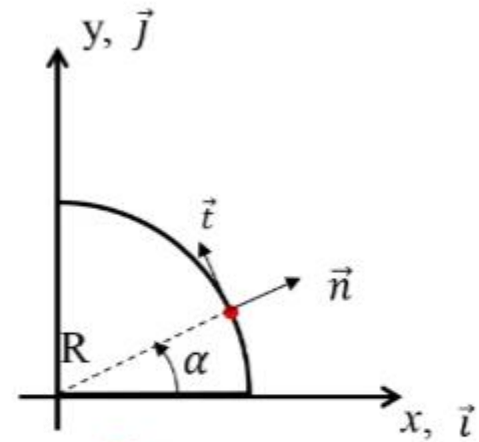
Solution

1. Kinematic analysis

$\alpha = \alpha(t)$

- position: $\vec{p} = R(\cos\alpha \cdot \vec{i} + \sin\alpha \cdot \vec{j})$
- velocity: $\bar{v} = R\left(-\dot{\alpha}\sin\alpha\,\underline{i} + \dot{\alpha}\cos\alpha\,\underline{j}\right)$ ①
- acceleration: $\bar{a} = R\left[\left(-\cancel{\ddot{\alpha}}^{\,0}\sin\alpha - \dot{\alpha}^2\cos\alpha\right)\underline{i} + \left(\cancel{\ddot{\alpha}}^{\,0}\cos\alpha - \dot{\alpha}^2\sin\alpha\right)\underline{j}\right]$

$= R\left(-\dot{\alpha}^2\cos\alpha\,\underline{i} - \dot{\alpha}^2\sin\alpha\,\underline{j}\right)$ ②



2. Use problem data

①: $|\bar{v}| = R\dot{\alpha}$

②: $|\bar{a}| = R\dot{\alpha}^2 \Rightarrow |\bar{a}| = |\bar{v}|\dot{\alpha}$

- velocity: $n = 1 + \dfrac{a}{g}$
- acceleration: $n = \cos\alpha + \dfrac{V^2}{gR}$

$\alpha = 0$

$n = 1 + \dfrac{V^2}{gR} = 1 + \dfrac{R\dot{\alpha}^2}{g} = 1 + \dfrac{a}{g}$

3. Calculate acceleration

$V = 435\,\text{km/h} = 120.83\,\text{m/s}$

$\dot{\alpha} = 7\,\text{deg/s} = 0.12217\,\text{rad/s}$

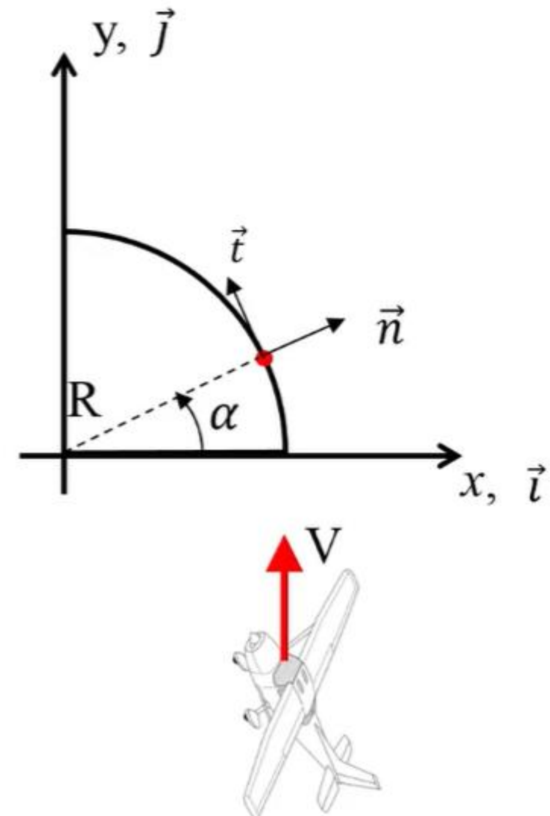
4. Load factor

$n = 1 + \dfrac{V\dot{\alpha}}{g}$

$n = 1 + \dfrac{120.83 \cdot 0.12217}{9.81} \Rightarrow n = 2.5$

Solution

1. Kinematic analysis
   - position: $\vec{p} = R(\cos\alpha \cdot \vec{\imath} + \sin\alpha \cdot \vec{\jmath})$
   - velocity: $\dots$
   - acceleration: $\vec{a} = \cancel{R\ddot{\alpha}} \cdot \vec{t} - R\dot{\alpha}^2 \cdot \vec{n}$



2. Use problem data
   - $\dot{\alpha}$ = const. $\Rightarrow a = |\vec{a}| = |R\dot{\alpha}^2|$
   - velocity: $V = R\dot{\alpha}$

3. Calculate acceleration $a = R\dot{\alpha}^2 = \ldots = V\dot{\alpha} = 14.8 m/s^2$

4. Load factor $n = \left(1 + \dfrac{a}{g}\right) = 2.5$

Maximum Load Factor

Load factor

$$L + P = nW$$

$n = 1$: level and straight flight

$n = 0$: zero-g (parabolic flight)

$$n = \frac{L + P}{W}$$

https://www.youtube.com/watch?v=mQ1ZozXp2tw&ab_channel=BloombergQuicktake

Maximum value of n just before stall

$$\boxed{n_{MAX} = \frac{0.5\rho V^2 S C_L^{MAX}}{W}}$$

determined by **total aircraft lift coefficient, C_L^{MAX}**