University of Southampton

Chapter 5: Mission Analysis

Lecture 10 – Orbital energy worked example

Professor Hugh Lewis

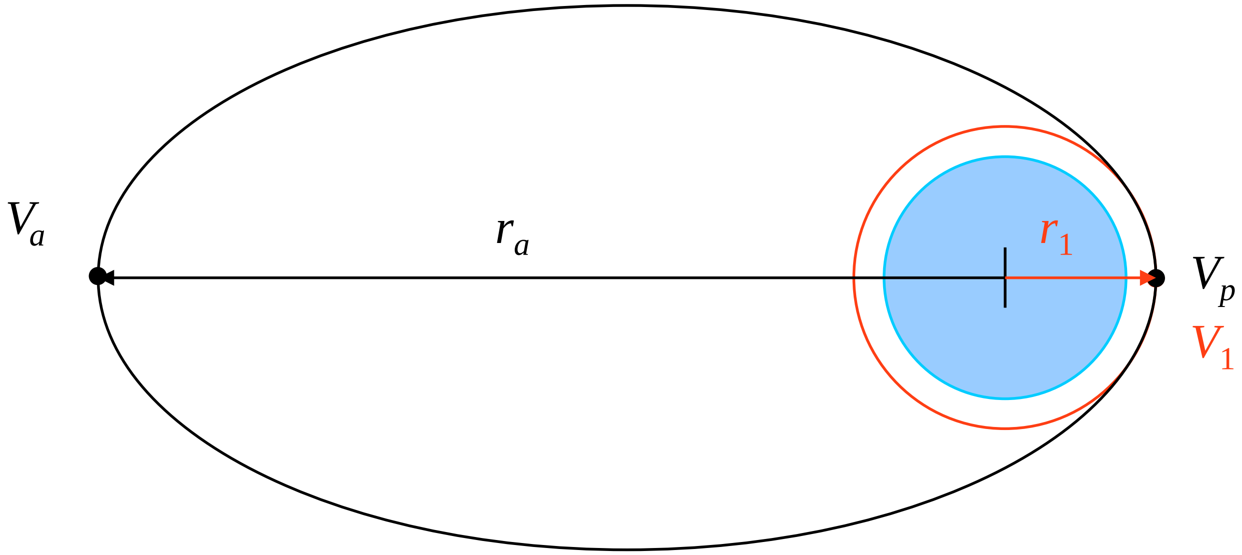
Overview of lecture 10

- This lecture presents a worked solution to a question about orbital motion
- The energy equation is needed for the answer (see lecture 9)
- The question appears on the next slide:
  - If you would like to attempt the question before continuing the lecture, simply pause or stop the recording
    - If you keep the Panopto player open you may still be able to see the solution even if you pause the recording on the question slide. If you prefer, you can minimise the Panopto window, open the lecture slides and advance to the question
  - Then resume the recording to see an explanation of the full worked solution

Orbital energy & motion

- Example calculation
  - Calculate the speed of a spacecraft: (i) In a circular orbit about the Earth at 200 km altitude, (ii) at the perigee and apogee of an elliptic orbit with

    $r_p = R_E + 200$ km, $r_a = R_E + 40{,}000$ km. Take $R_E = 6378$ km, $\mu_E = 398{,}600$ km^3/s^2.



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Orbital energy & motion

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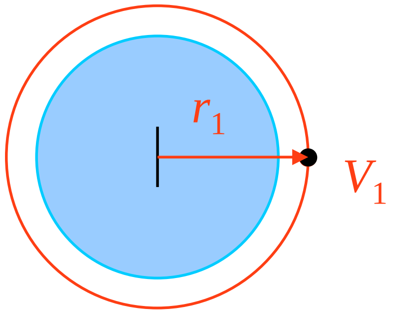

(i) Circular orbit: $V_1 = \sqrt{\dfrac{\mu_E}{r_1}} = \sqrt{\dfrac{398600}{6378 + 200}}$

[Remember that r is the orbit radius, i.e. the distance of the spacecraft from the Earth centre]

$$V_1 = 7.784 \text{ km/s}$$

Orbital energy & motion

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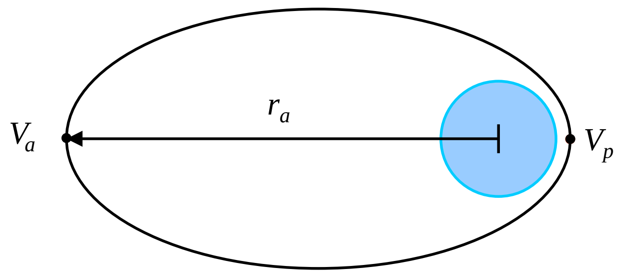

(ii) Elliptical orbit:

- First, compute the semi-major axis

$$a = \frac{1}{2}\left(r_p + r_a\right) = 26478\ \text{km}$$

[Remember that r is the orbit radius, i.e. the distance of the spacecraft from the Earth centre]

- Now use the energy equation…

Orbital energy & motion

- Example calculation
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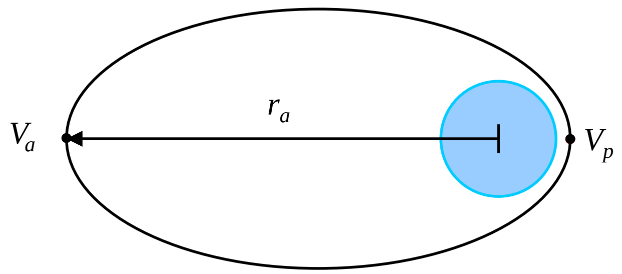

(ii) Elliptical orbit:

- Use the energy equation for speed at perigee & apogee:

$$V_p^2 = \mu\left(\frac{2}{r_p} - \frac{1}{a}\right), V_p = 10.302 \text{ km/s}$$

$$V_a^2 = \mu\left(\frac{2}{r_a} - \frac{1}{a}\right), V_a = 1.461 \text{ km/s}$$