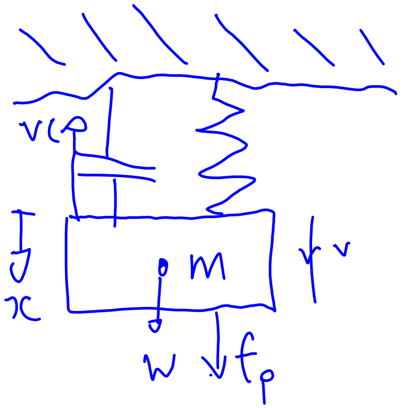
3. TRANSFER FUNCTIONS AND BLOCK DIAGRAMS

The theory of Laplace transforms is quite extensive, but only a small and isolated part is required for the initial study of dynamics and control. We will cover here:

- Laplace transforms
- Transfer function models of physical systems
  - Linear, time invariant models
- Block diagram form
  - manipulation
  - simplification



non linear problem

$F_g = mg$

$F_d = vc = \frac{dx}{dt} c$

$F_e = kx^3$ ← non linear spring

f_p = disturbance term

$ma = \sum F$

$$m\frac{d^2x}{dt^2} = -F_d - F_e + mg + f_p$$

$$= -\frac{dx}{dt}c - kx^3 + mg + f_p$$

$$f_p + mg = m\frac{d^2x}{dt^2} + \frac{dx}{dt}c + kx^3$$

linear approximation around x_0 (equilibrium $\frac{dx}{dt} = 0$)

$$f_p + mg = kx_0^3$$

use $x = x_0 + dx$

$$m\frac{d^2(x_0+x)}{dt^2} + c\frac{d(x_0+x)}{dt} + k(x_0+dx)^3 = f_e + mg$$

$\frac{d}{dt}x_0 = 0$

$$m\frac{d^2}{dt^2}dx + c\frac{d}{dt}dx + \underbrace{kx_0^3\left(1+\frac{dx}{x_0}\right)^3}_{} = f_c + mg$$

$$\rightarrow kx_0^3\left(1 + 3\frac{dx}{x_0} + 3\frac{dx^2}{x_0^2} + \frac{dx^3}{x_0}\right) \approx kx_0^3\left(1 + 3\frac{dx}{x_0}\right)$$

$$m\frac{d}{dt^2}dx + c\frac{d}{dt}dx + kx_0^3 + 3kx_0^2\,dx = f_0 + mg$$

this is now linear (remember x_0 is constant)

these cancel, assume $f_0 = 0$

$$m\frac{d^2dx}{dt} + c\frac{d\,dx}{dt} + 3kx_0^2\,dx = 0$$

linear ODE with constant coefficients! (easy)

call dx, x where x is small

$$m\frac{d^2x}{dt} + c\frac{dx}{dt} + 3kx_0x = 0$$

Laplace $\int \dots e^{-st}$

$$m\int_0^\infty \frac{d^2x}{dt}e^{-st}\,dt + c\int_0^\infty \frac{dx}{dt}e^{-st}\,dt + 3kx_0^2\int_0^\infty x e^{-st}\,dt = 0$$

$$c\int_0^\infty \frac{d}{dt}\left[xe^{-st}\right]dt$$

$$c\left[xe^{-st}\right]_0^\infty$$

In Topic 1 we saw that it is desirable to model a system as an input-output description:

$$y = Gu \tag{3.1}$$

In Topic 2 we found a generic description of the form:

$$\frac{d^n y(t)}{dt^n} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t) \tag{3.2}$$

using the Laplace transform we can express (3.2) in the form of (3.1).

If $U(s)$ and $Y(s)$ are the respective Laplace transforms of $u(t)$ and $y(t)$, and u and y are related by the DE (3.2) then:

$$Y(s) = G(s)U(s)$$

with

$$G(s) = \frac{b_m s^m + \cdots + b_0}{s_n + \cdots + a_0}$$

3.1 Laplace transforms

Using differential equations, it is difficult to model a system as a block diagram.

The Laplace transform is defined as

$$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt \tag{3.3}$$

where $s = \sigma + j\omega$ is a complex variable.

Knowing $f(t)$ and that the integral in (3.3) exists, it is possible to find the function $F(s)$, the Laplace transform of $f(t)$.

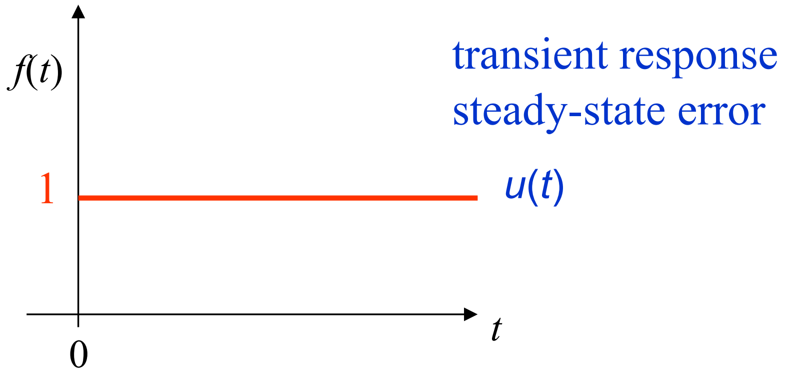
The Laplace transform changes a function of time, $f(t)$, into a new function of the complex variable s such that integration and differentiation are changed to algebraic operations.

Some important examples and theorems will now be considered.

3.2 Laplace transform examples

3.2.1 The unit step

$$u(t) = \begin{cases} 1 & if \quad t > 0 \\ 0 & if \quad t < 0 \end{cases}$$



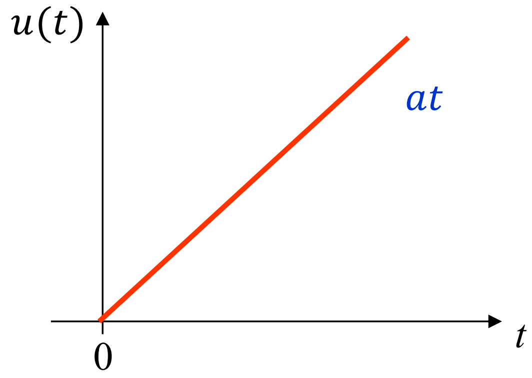
Its Laplace transform is obtained by substituting $u(t)$ in place of $f(t) = 1$ in (3.3), which gives:

$$\mathcal{L}[u(t)] = F(s) = \int_0^\infty 1e^{-st}dt = -\frac{e^{-st}}{s}\bigg|_0^\infty = \left(\frac{-0}{s} - \frac{-e^{-0}}{s}\right) = \frac{1}{s} \tag{3.4}$$

For a step of magnitude A, the transform of $Au(t)$ is A/s.

3.2.2 Ramp function

$$u(t) = \begin{cases} at & if \quad t > 0 \\ 0 & if \quad t < 0 \end{cases}$$



steady-state error
Tracking error

where $u(t)$ is from (3.4)

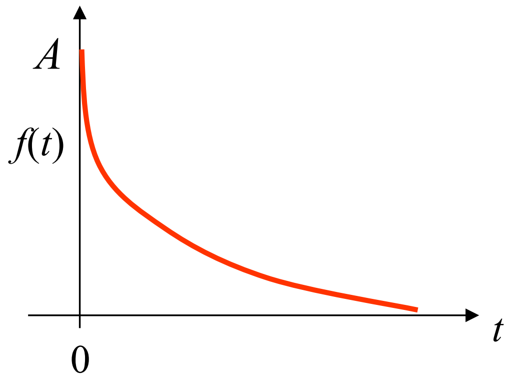
$$\mathcal{L}[u(t)] = F(s) = \int_0^{\infty} ate^{-st}dt = \frac{a}{s^2} \qquad (3.5)$$

Integration by parts yields

$$\int_0^{\infty} ate^{-st}dt = \left[-\frac{at}{s}e^{-st}\right]_0^{\infty} + \int_0^{\infty} \frac{a}{s}e^{-st}dt = \frac{a}{s^2}$$

3.2.3 Exponential function

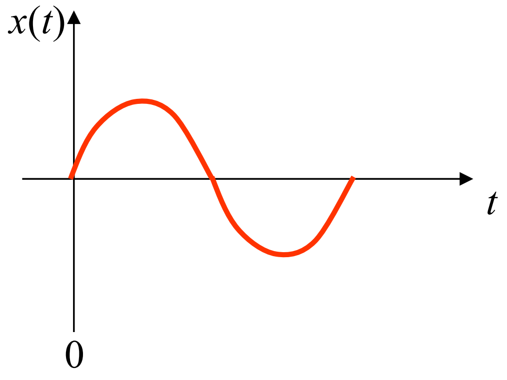
In many physical systems, the transient following a change of input appears as a decaying exponential $f(t) = Ae^{-\alpha t}$.



$$F(s) = \mathcal{L}[Ae^{-\alpha t}] = \int_{0-}^{\infty} Ae^{-\alpha t}e^{-st}dt = \frac{A}{s+\alpha} \qquad (3.6)$$

3.2.4 Sinusoid

$$x(t) = \sin \omega t$$



transient response
modelling
frequency response

We may use the sine formula to express $x(t)$ as:

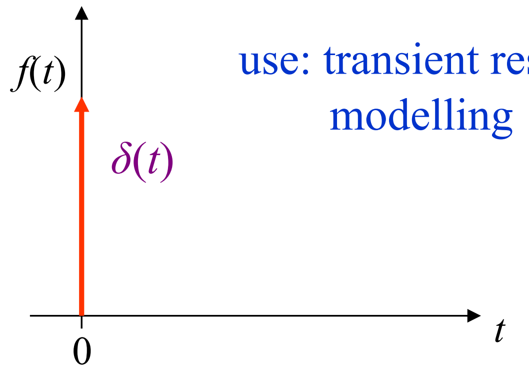
$$x(t) = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

It follows from the Laplace transform of an exponential that:

$$X(s) = \frac{1}{2j}\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) \quad = \frac{1}{2j} \times \frac{2j\omega}{s^2+\omega^2}$$

$$= \frac{\omega}{s^2+\omega^2}$$

3.2.5 Impulse (Dirac delta function)



use: transient response modelling

- Heuristic definition:

$$\delta(t) = \begin{cases} \infty & if \quad t = 0 \\ 0 & if \quad t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t)\, dt = f(0)$$

- Infinite magnitude, infinitesimal duration and unit area
- It's Laplace transform is:

$$F(s) = \mathcal{L}[\delta(t)] = \int_{0}^{\infty} \delta(t)e^{-st}\, dt = 1$$

- Some common Laplace transforms are given in Table 3.1 (also on Blackboard)

3.1 Table of Laplace transforms

Table 3.1: Common Laplace Transforms.

Name	$f(t)$	$F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	t	$\frac{1}{s^2}$
*n*th-order ramp	t^n	$\frac{n!}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
nth-order exponential	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
Damped sine	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped cosine	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
Diverging sine	$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
	$A \sin \omega t$	$\frac{A\omega}{s^2+\omega^2}$
Diverging cosine	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
	$e^{-\zeta\omega_n t} \sin\left[\omega_n\left(1-\zeta^2\right)^{1/2} t\right]; \zeta < 1$	$\frac{\omega_n\left(1-\zeta^2\right)^{1/2}}{s^2+2\zeta\omega_n s+\omega_n{}^2}$
	$1-e^{-\zeta\omega_n t}\left(1-\zeta^2\right)^{-1/2} \times \sin\left[\omega_n\left(1-\zeta^2\right)^{1/2} t+\phi\right]$	$\frac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n{}^2\right)}$

3.3 Properties of Laplace transforms

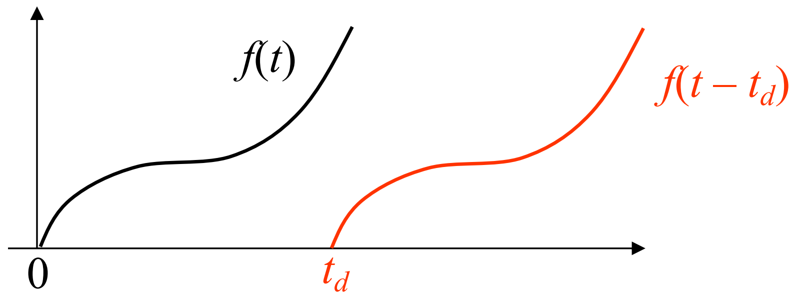
The following important rules are often very useful when using Laplace transforms:

1. Linearity: $f_1(t) + f_2(t) \rightarrow F_1(s) + F_2(s)$

2. Linearity: $f_1(t) - f_2(t) \rightarrow F_1(s) - F_2(s)$

3. Linearity: $af(t) \rightarrow aF(s)$

4. A function $f(t)$ which is delayed in time by t_d can be expressed as $f(t - t_d)$ and its Laplace transform is $e^{-t_d s}F(s)$



5. The Laplace transform of the derivative of a function $f(t)$ provided $f(0) = 0$ is:

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s)$$

6. The Laplace transform of the *n*-th integral of a function $f(t)$ is:

$$\mathcal{L}\left[\int \dots \int f(t) dt_1 \dots dt_n\right] = \frac{F(s)}{s^n}$$

7. The linearity theorem states that if $F_1(s) = \mathcal{L}[f_1(t)]$ and $F_2(s) = \mathcal{L}[f_2(t)]$, and if two coefficients c_1 and c_2 are independent of t then:

$$\mathcal{L}[c_1 f_1(t) + c_2 f_2(t)] = c_1 F_1(s) + c_2 F_2(s) \tag{3.7}$$

8. The final value theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) \tag{3.8}$$

This allows us to find the final, or *steady state*, value of $f(t)$ (i.e. its value as $t \to \infty$) from $F(s)$.

This theorem is valid only if the limit exists.

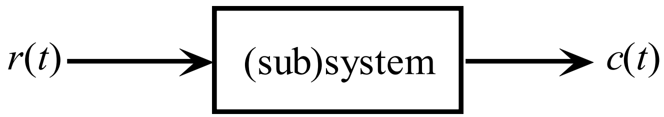
Table 3.2 (Wikipedia)

Properties of the unilateral Laplace transform

	Time domain	s domain	Comment
Linearity	$af(t)+bg(t)$	$aF(s)+bG(s)$	Can be proved using basic rules of integration.
Frequency-domain derivative	$tf(t)$	$-F'(s)$	F' is the first derivative of F with respect to s.
Frequency-domain general derivative	$t^n f(t)$	$(-1)^n F^{(n)}(s)$	More general form, nth derivative of $F(s)$.
Derivative	$f'(t)$	$sF(s)-f(0^-)$	f is assumed to be a differentiable function, and its derivative is assumed to be of exponential type. This can then be obtained by integration by parts
Second derivative	$f''(t)$	$s^2F(s)-sf(0^-)-f'(0^-)$	f is assumed twice differentiable and the second derivative to be of exponential type. Follows by applying the Differentiation property to $f'(t)$.
General derivative	$f^{(n)}(t)$	$s^nF(s)-\sum_{k=1}^{n}s^{n-k}f^{(k-1)}(0^-)$	f is assumed to be n-times differentiable, with nth derivative of exponential type. Follows by mathematical induction.
Frequency-domain integration	$\frac{1}{t}f(t)$	$\int_s^\infty F(\sigma)\,d\sigma$	This is deduced using the nature of frequency differentiation and conditional convergence.
Time-domain integration	$\int_0^t f(\tau)\,d\tau=(u*f)(t)$	$\frac{1}{s}F(s)$	$u(t)$ is the Heaviside step function and $(u*f)(t)$ is the convolution of $u(t)$ and $f(t)$.
Frequency shifting	$e^{at}f(t)$	$F(s-a)$	
Time shifting	$f(t-a)u(t-a)$	$e^{-as}F(s)$	$u(t)$ is the Heaviside step function
Time scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$	$a>0$
Multiplication	$f(t)g(t)$	$\frac{1}{2\pi i}\lim_{T\to\infty}\int_{c-iT}^{c+iT}F(\sigma)G(s-\sigma)\,d\sigma$	The integration is done along the vertical line $\mathrm{Re}(\sigma)=c$ that lies entirely within the region of convergence of F.[23]
Convolution	$(f*g)(t)=\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)\cdot G(s)$	
Complex conjugation	$f^*(t)$	$F^*(s^*)$	
Cross-correlation	$f(t)\star g(t)$	$F^*(-s^*)\cdot G(s)$	
Periodic function	$f(t)$	$\frac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)\,dt$	$f(t)$ is a periodic function of period T so that $f(t)=f(t+T)$, for all $t\geq 0$. This is the result of the time shifting property and the geometric series.

3.4 Laplace transforms of dynamic systems

Consider a simple single input, single output (SISO) system



The input, $r(t)$, and output, $c(t)$, variables are related by the standard linear differential equation:

$$\frac{d^n c(t)}{dt^n} + a_{n-1}\frac{d^{n-1}c(t)}{dt^{n-1}} + \cdots + a_1\frac{dc(t)}{dt} + a_0 c(t)$$
$$= b_m\frac{d^m r(t)}{dt^m} + b_{m-1}\frac{d^{m-1}r(t)}{dt^{m-1}} + \cdots + b_1\frac{dr(t)}{dt} + b_0 r(t)$$

From the linearity theorem (3.7) the Laplace transform can be written simply by transforming each term in turn.

If the initial conditions are assumed to be zero, the result is the common system representation of classical control theory:

$$\left(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right)C(s) = \left(b_m s^m + \cdots b_1 s + b_0\right)R(s)$$

where

$$C(s) = \mathcal{L}[c(t)] \qquad R(s) = \mathcal{L}[r(t)]$$

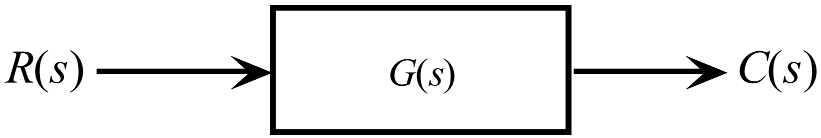
Definition: The **transfer function** of a (sub)system is the ratio of the Laplace transforms of its output and input, assuming zero initial conditions, i.e.

$$G(s) = \frac{C(s)}{R(s)} \tag{3.9}$$

The transfer function is given from (3.9) as

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \tag{3.10}$$

and the diagrammatic representation is replaced by



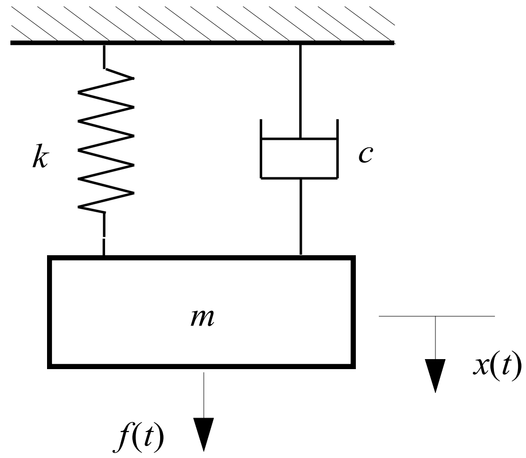
The following representation is often applied

$$C(s) = G(s)R(s) \tag{3.11}$$

In other words, the system output transform, $C(s)$, is equal to the transfer function, $G(s)$, times the system input transform, $R(s)$.

Example 3.1: Mass-spring-damper (from Section 2.3)

The differential equation describing the system is:



$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

Assuming that all initial conditions are zero and taking Laplace transforms:

$$ms^2X(s) + csX(s) + kX(s) = F(s)$$

Hence

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

Laplace tables can be used to solve these equations directly.

Example 3.2: System response with initial conditions

Having a definition for the transfer function provides a method for determining the system response, $c(t)$, to a given input, $r(t)$, with *specified initial conditions*.

For example, if:

$$\ddot{c} + a_1\dot{c} + a_0 c = r \tag{3.12}$$

with initial conditions $c(0) = \gamma_0$ and $\dot{c}(0) = \gamma_1$, then the transfer function is:

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s^2 + a_1 s + a_0}$$

and for a unit step input $r(t)$ the transform of the output due to $r(t)$ is $[R(s) = 1/s]$, and so we can write:

$$C_1(s) = \frac{1}{s(s^2 + a_1 s + a_0)}$$

$C_1(s)$ is the response of the system to a step input.

For the response to the initial conditions, $r = 0$, in (3.12), this equation is transformed using Rule 5 or Table 3.1 to

$$[s^2 C_2(s) - s\gamma_0 - \gamma_1] + a_1[sC_2(s) - \gamma_0] + a_0 C_2(s) = 0$$

rearranging yields

$$C_2(s) = \frac{(s + a_1)\gamma_0 + \gamma_1}{s^2 + a_1 s + a_0}$$

and hence the transform of the total output is $C(s) = C_1(s) + C_2(s)$

Finding an inverse of a Laplace transform gives us the response in the time domain.

For very simple cases we could use tables.

However, techniques are needed for more complex systems and these are introduced in Topic 4.

3.5 First and second order systems

The order of a system is equal to the highest power of derivative in the differential equation model. This corresponds to the highest power of s in the Laplace transform model.

For example, if the following differential equation describes a **first order** system

$$a_1 \frac{dy}{dt} + a_0 y = b_0 u$$

then the corresponding Laplace transform is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_1 s + a_0}$$

which can be arranged as

$$G(s) = \frac{b_0/a_0}{(a_1/a_0)\, s + 1} \equiv \frac{K_p}{\tau_p s + 1}$$

K_p is the system gain, which determines how much the output will change for a given input.
τ_p is the system time constant, and determines speed of response to a change of input.

A **second order** system could be described by

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_0 u$$

and the corresponding Laplace transform is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_2 s^2 + a_1 s + a_0}$$

which can be arranged as

$$G(s) = \frac{b_0/a_2}{s^2+(a_1/a_2)s+(a_0/a_2)} \equiv \frac{K\omega_n^2}{s^2+2\varsigma\omega_n+\omega_n^2} \qquad (3.12)$$

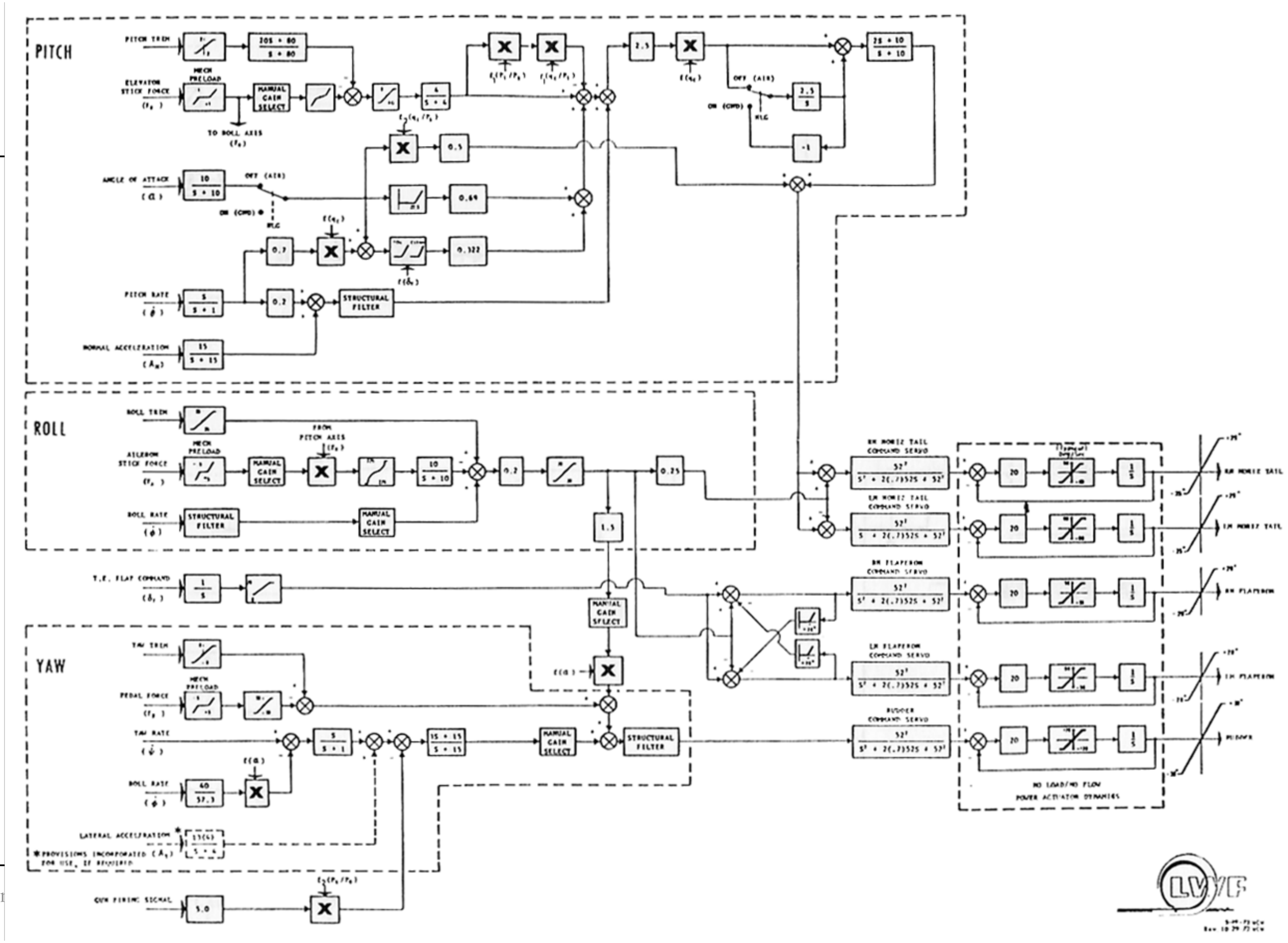
- K is the gain of the system
- ω_n is the natural frequency of oscillation
- ς is a measure of damping in the system

3.6 Block diagram reduction

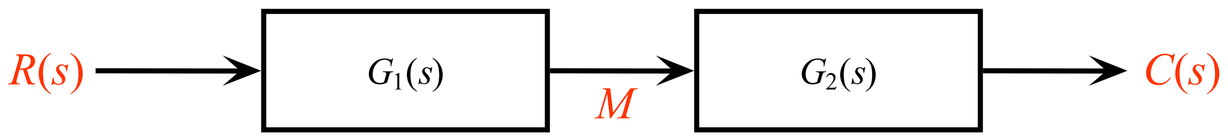
- The preceding discussion appears to imply that in order to obtain a transfer function relating input r to output c in a block diagram, the differential equations relating these variables need to be obtained first.

- Fortunately, this is not the case and this section will describe algebraic manipulations of subsystems (or blocks) that can be applied to obtain the transfer function(s) or reduce the complexity of the system.

- This is dealt with in great detail and depth in Chapter 5 of Nise and Chapter 3 of Franklin *et al*.


PITCH
ROLL
YAW
PITCH TRIM
ELEVATOR STICK FORCE
MECH PRELOAD
MANUAL GAIN SELECT
TO ROLL AXIS
ANGLE OF ATTACK
PITCH RATE
NORMAL ACCELERATION
STRUCTURAL FILTER
ROLL TRIM
AILERON STICK FORCE
ROLL RATE
T.E. FLAP COMMAND
YAW TRIM
PEDAL FORCE
YAW RATE
LATERAL ACCELERATION
GUN FIRING SIGNAL
RH HORIZ TAIL COMMAND SERVO
LH HORIZ TAIL COMMAND SERVO
RH FLAPERON COMMAND SERVO
LH FLAPERON COMMAND SERVO
RUDDER COMMAND SERVO
NO LOAD/NO FLOW POWER ACTUATOR DYNAMICS

Example 3.3: Reduce the *cascade,* or series connection, of the two blocks to a single block.



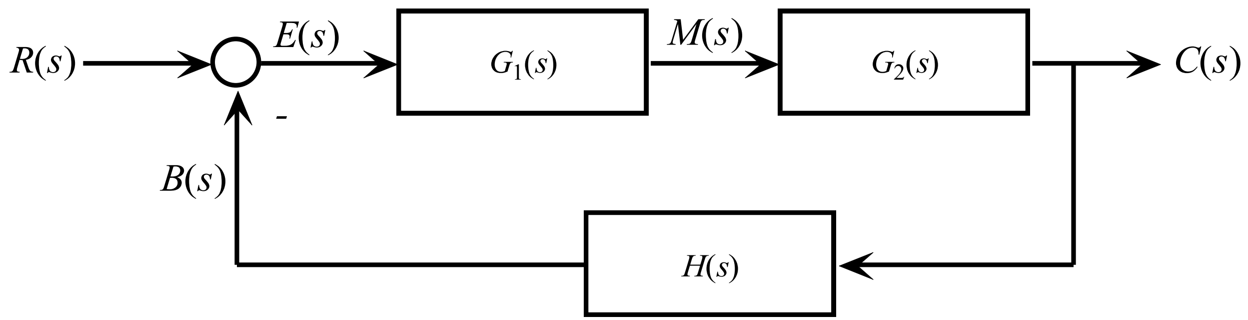
By definition $M = G_1R$ and $C = G_2M$.

Hence, substituting the later into the former yields

$$C = GR \qquad \text{where} \qquad G = G_1G_2 \tag{3.14}$$

The overall transfer function of a series of blocks equals the product of the individual transfer functions.

Example 3.4: Reduce the standard feedback loop to a single block.



By definition: $C = G_2 M$ $\quad M = G_1 E$ $\quad E = R - B$ $\quad B = HC$

Hence, substituting the later into the former yields: $C = G_1 G_2 E$ $\quad E = R - HC$

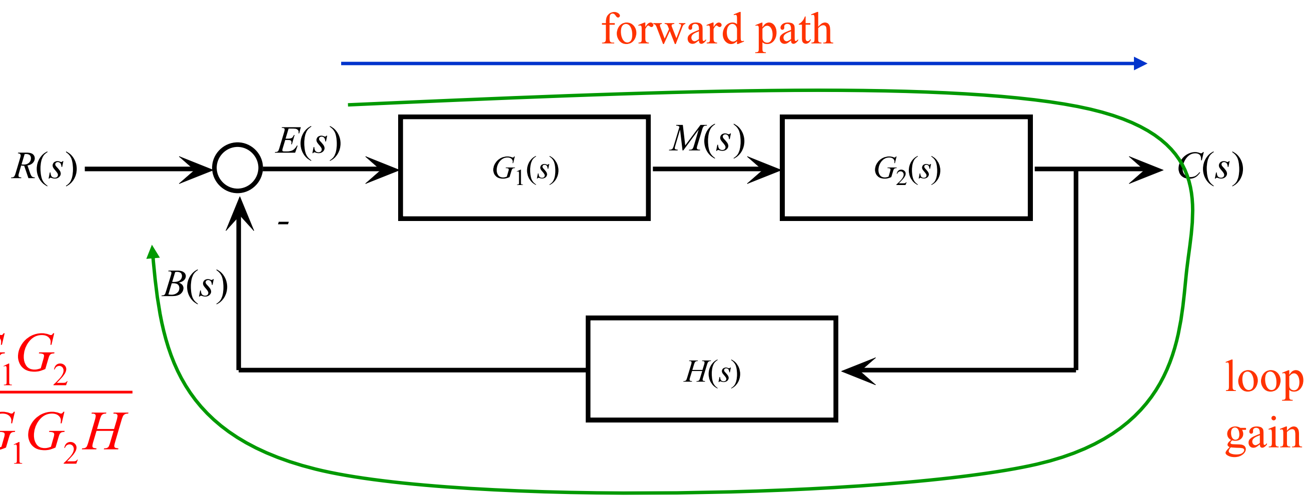
Eliminating E gives: $C = G_1 G_2 R - G_1 G_2 HC$

And the ***closed loop transfer function*** is:

$$\frac{C}{R} = \frac{G_1 G_2}{1 + G_1 G_2 H} \tag{3.15}$$

This is an **extremely important result**, and stated in a somewhat generalized form:

- The *closed loop transfer function* of the standard loop equals the product of the transfer functions in the forward path divided by the sum of 1 and the loop gain function.
- The *loop gain func*tion is defined as the product of the transfer functions around the loop.



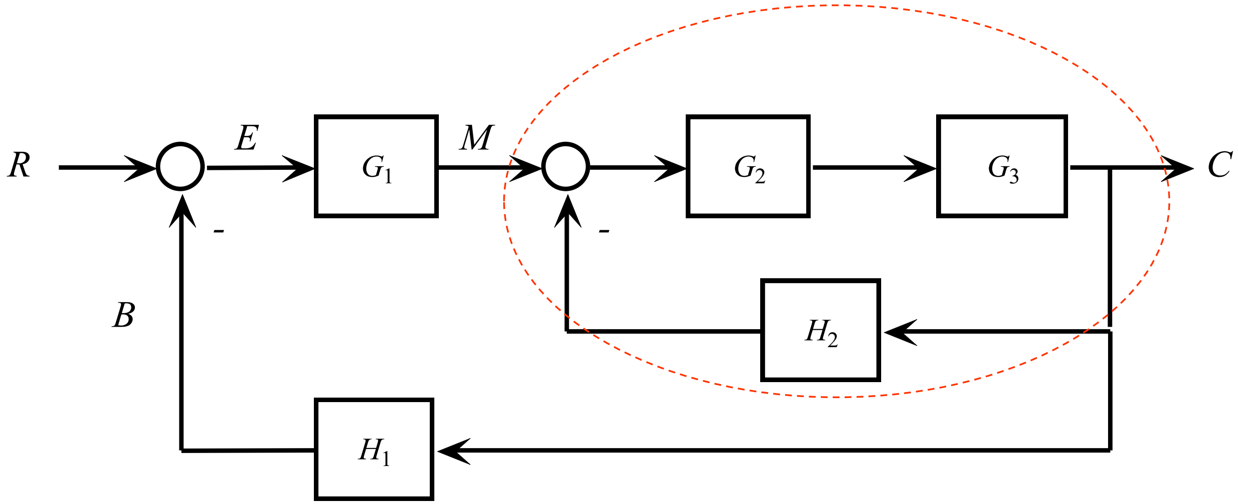
$$\frac{C}{R} = \frac{G_1 G_2}{1 + G_1 G_2 H}$$

Also, note that since $C = G_1 G_2 E$

$$\frac{E}{R} = \frac{1}{1+G_1 G_2 H} \qquad (3.16)$$

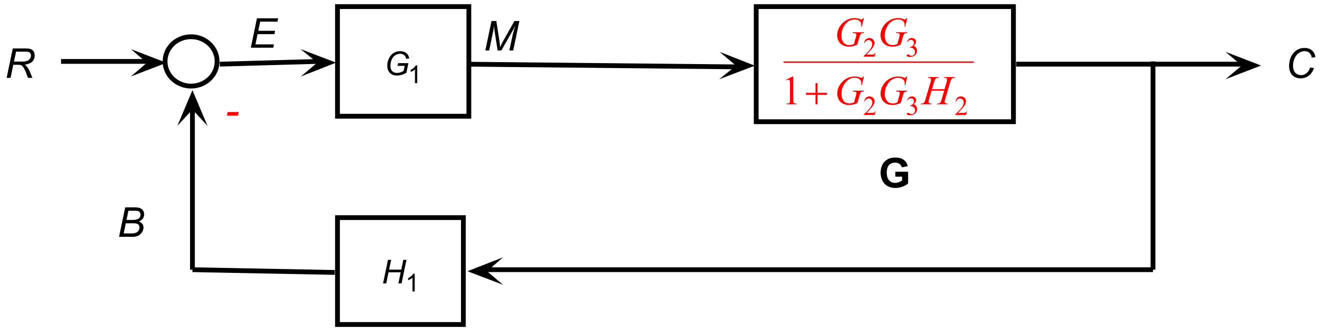
- If $H = 1$, then $E = R - C$ is the system error and E/R is the input-to-error transfer function.
- This permits the input to error response for a given input, $r(t)$, to be found directly.
- Transfer functions relating the input to any variable of interest can be found similarly.

Example 3.5: Minor feedback loops



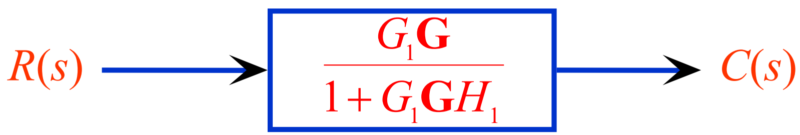
- This configuration is very common, e.g. in servomechanisms, and includes a minor feedback loop.
- Derivation by the C/R approach of Example 3.4 would be tedious.
- If the observations of (3.14) and (3.15) are used it becomes simple.

- First, the minor feedback loop C/M is reduced to a single block

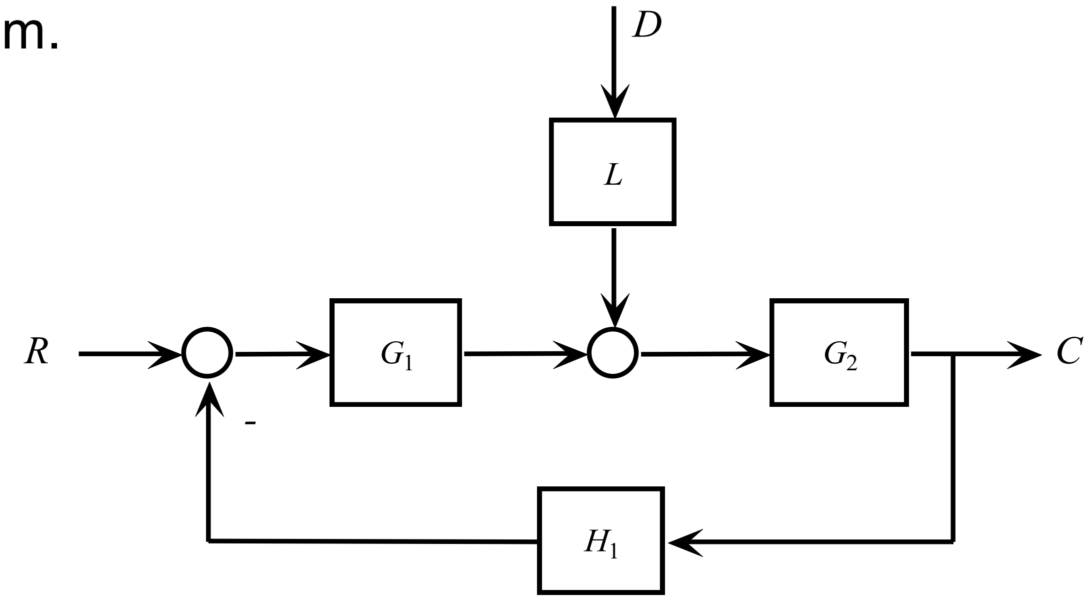


- and to C/R to achieve the closed loop feedback function

$$\frac{C}{R} = \frac{G_1\mathbf{G}}{1+G_1\mathbf{G}H_1} = \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2G_3H_1}$$



Example 3.6: A two-input system.



The additional input, D, often represents a disturbance.

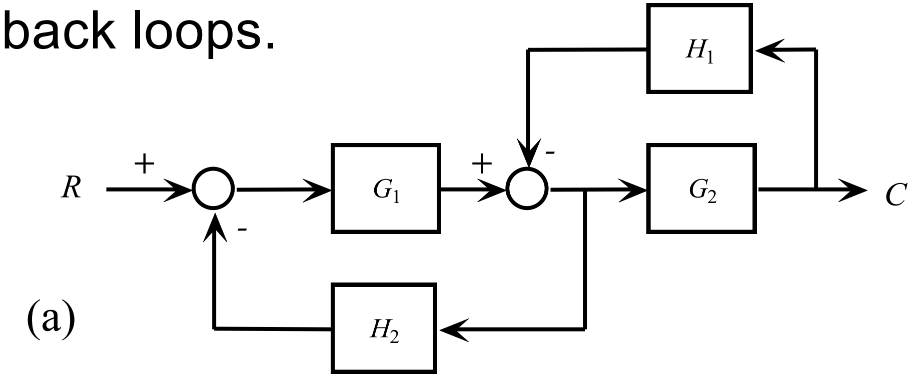
- The additional block L models such effects on the system.
- Principle of superposition applies (linear)
- Output is the sum of outputs due to each input separately.

- The output due to R is found as before (with $D = 0$)
- When finding output due to D, R is set to zero.
- Example 3.4 applies when finding the output due to D
- In this case the forward path only contains L and G_2.
- When $R = 0$, the minus sign for feedback at R can be moved to the summing junction after G_1 and L.

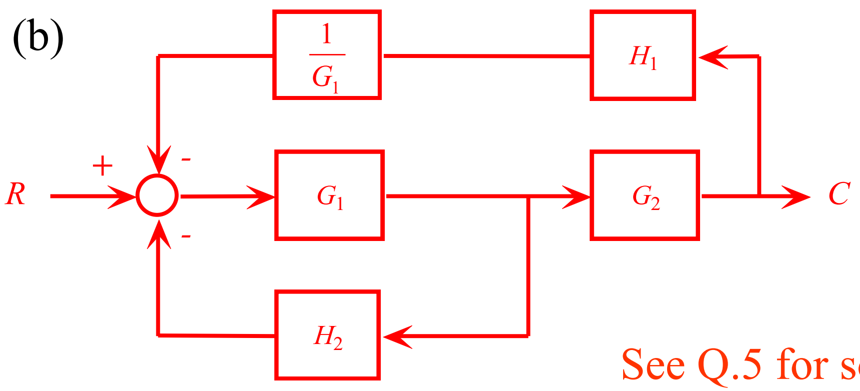
By inspection:

$$\frac{C}{D} = \frac{G_2 L}{1 + G_1 G_2 H_1} \qquad (3.17)$$

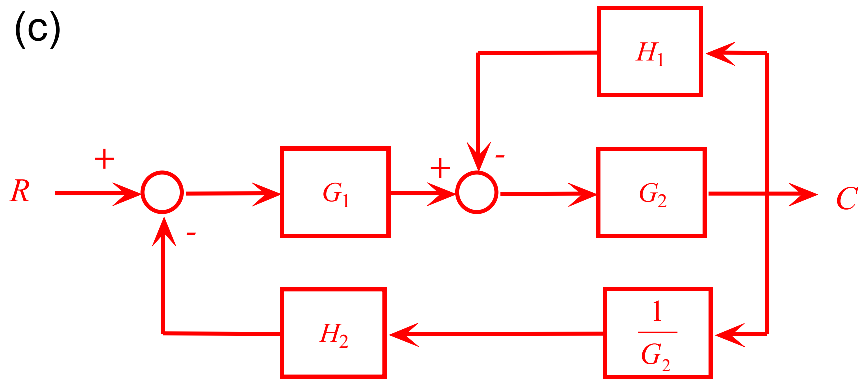
Example 3.7: Cross-coupled feedback loops.



The two feedback loops interfere with each other. The following diagrams (b) and (c) are equivalent to (a):



See Q.5 for solution.



3.7 The bigger picture

time domain

$$u(t) \Rightarrow \frac{d^n y(t)}{dt^n} + \cdots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_0 u(t) \Rightarrow y(t)$$

$$U(s) = \mathcal{L}\left[u(t)\right]$$

The relationship between input-output signals in the time and Laplace domains is summarised by the transformations shown here

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right]$$

$$U(s) \Rightarrow Y(s) = G(s)U(s) \Rightarrow Y(s)$$

Laplace domain

3.8 Summary

- Laplace transforms can be used to represent both differential equations and system inputs and outputs.
- The notion of a transfer function, describing the relationship between an input and an output, has been introduced.
- Block diagram algebra can be used to reduce systems of differential equations to a simpler form.
- In subsequent Topics, these transfer functions will be used to determine the response of a dynamic system to inputs, the stability of such a system and the use of feedback to control systems closed loop.

3.9 Problems

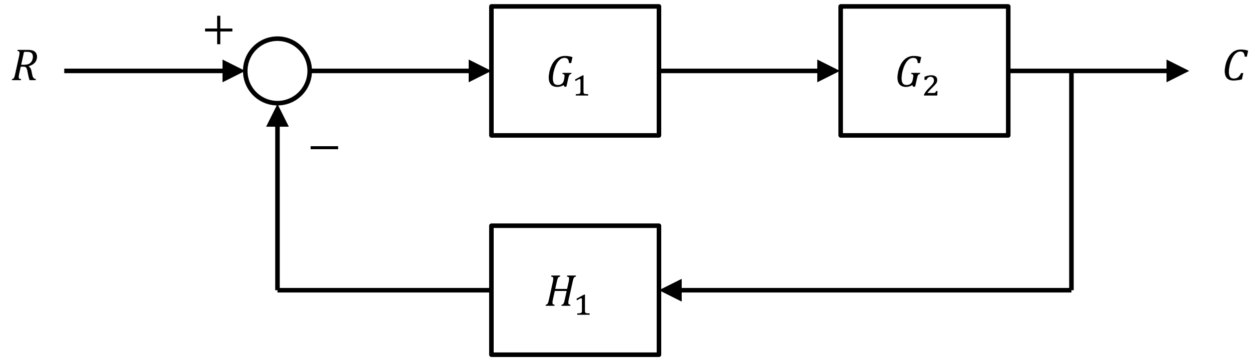
1. Determine Laplace transforms of (a) t^2 and (b) $t^2 \exp(-2t)$.
2. Determine inverse transforms of (a) $2/s$ and (b) $3/(2s+1)$
3. Determine the Laplace transform of a step change in voltage from $0V$ to $4V$ at time $t = 0$.
4. What is the transfer function of the system described by the following input-output relationship?

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x + \frac{dx}{dt}$$

5. Verify that neither of the transformations in Example 3.7 changes the system, and that applying (3.14) twice to (b) or (c) yields the closed loop transfer function:

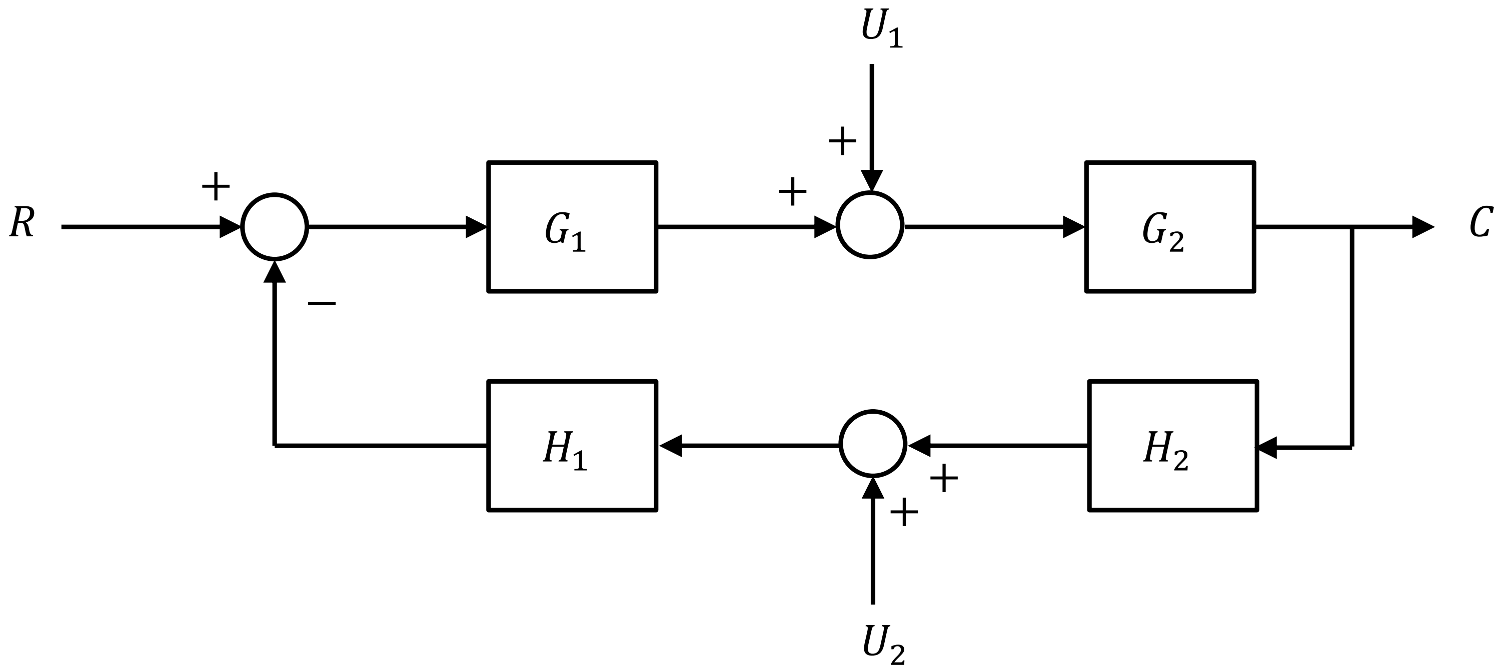
$$\frac{C}{R} = \frac{G_1 G_2}{1 + G_1 H_2 + G_2 H_1}$$

6. Determine the closed loop transfer function for the following system:



What is the closed loop transfer function if $G_1 = 1/(3s + 1)$, $G_2 = 5/(s + 1)$ and $H_1 = 1$?

7. Determine the closed loop transfer function for the following system:



8. Reduce the following diagram to canonical form $C = GR$ and find the closed loop transfer function G: