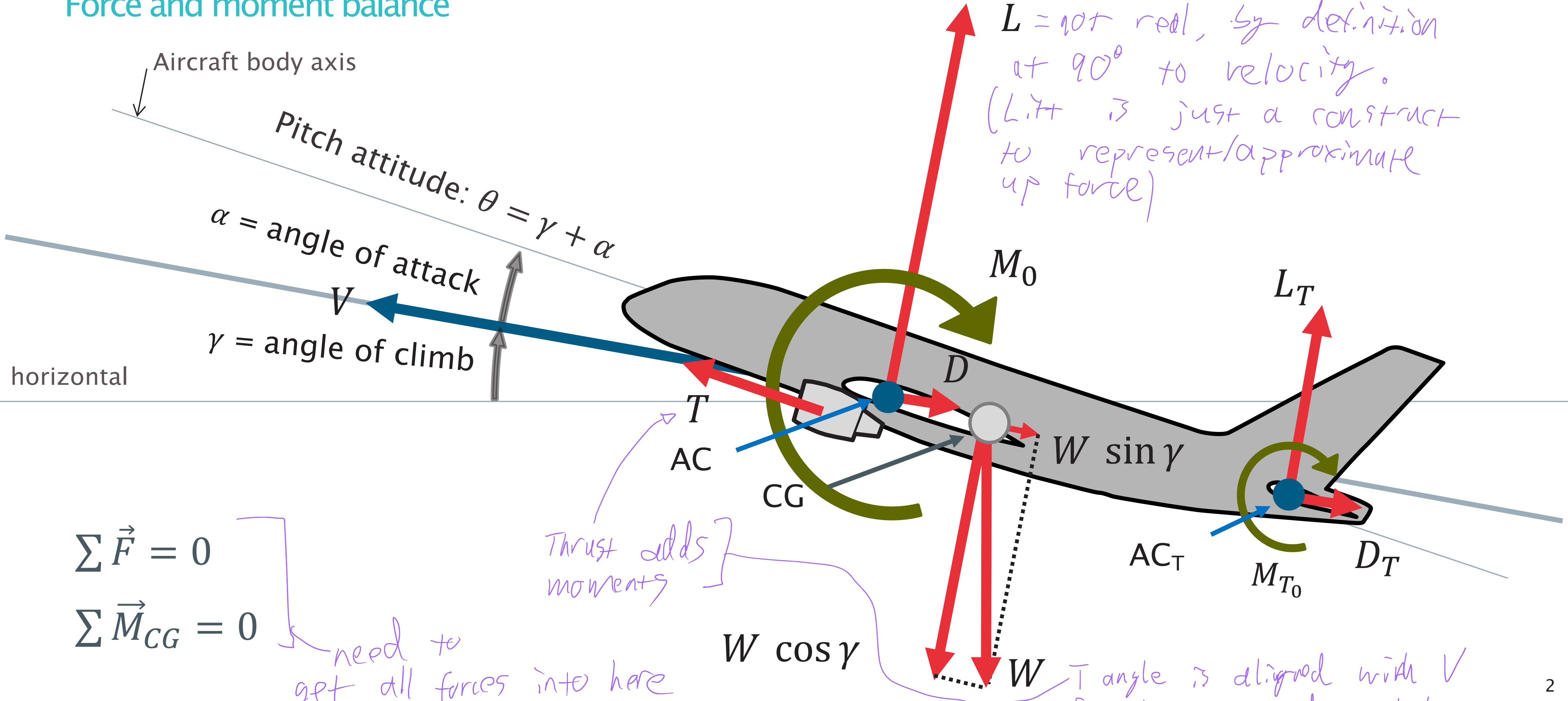
SESA2025 Mechanics of Flight
Equations of motion and tail plane equation

Lecture 1.1

Forces on an Aircraft (Steady, trimmed flight)

Force and moment balance



L = not real, by definition at 90° to velocity. (Lift is just a construct to represent/approximate up force)

Thrust adds moments

T angle is aligned with V in level steady flight

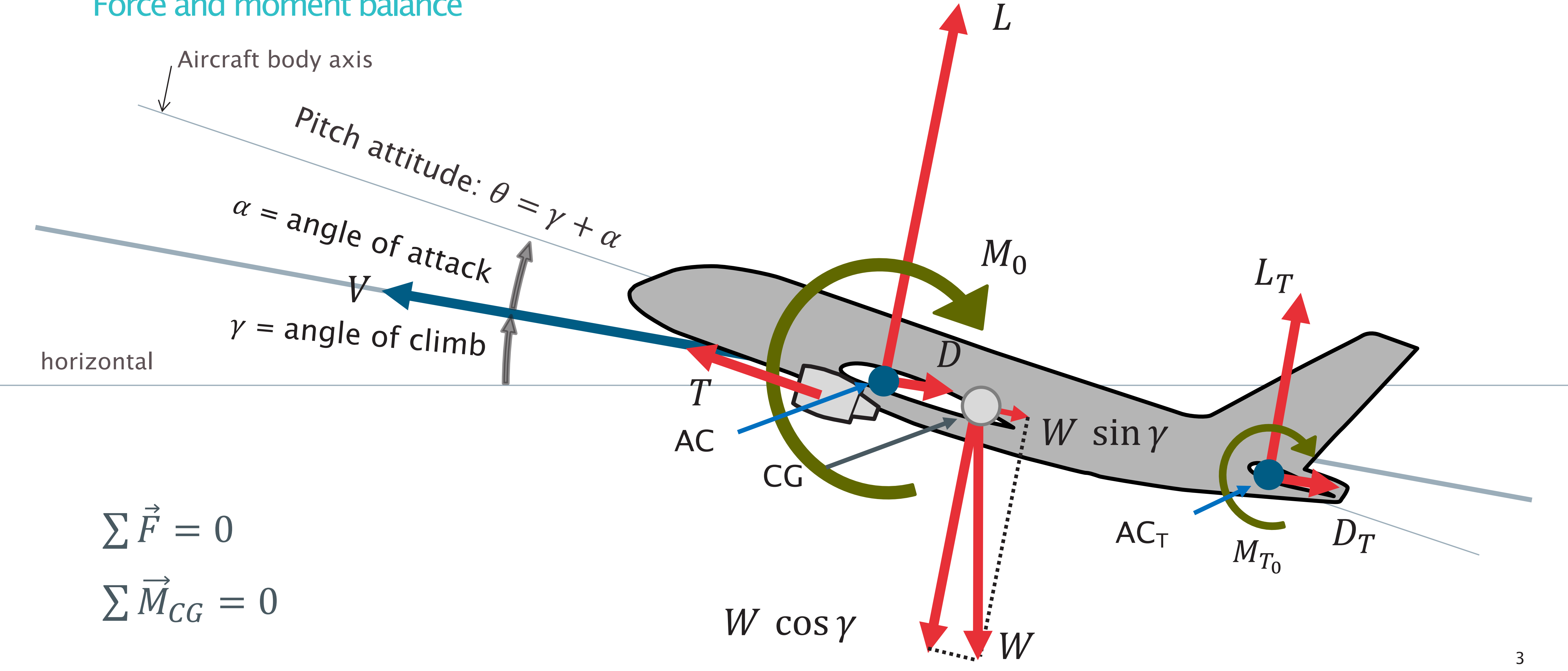
$$\sum \vec{F} = 0$$

$$\sum \vec{M}_{CG} = 0$$

need to get all forces into here

Forces on an Aircraft (Steady, trimmed flight)

Force and moment balance



$$\sum \vec{F} = 0$$

$$\sum \vec{M}_{CG} = 0$$

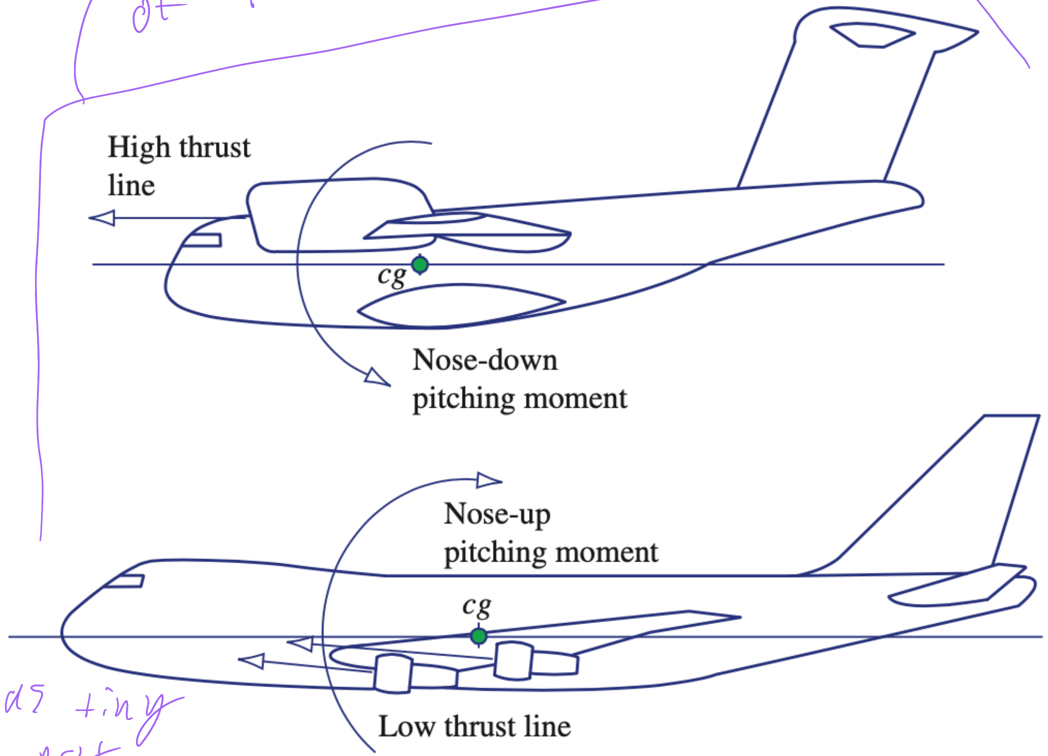
Forces on an Aircraft (Steady, trimmed flight)

Simplifications and assumptions

depending on config the assumptions of the model will change

- small angle of attack
  - $T, D, D_T, W \sin\gamma$ on the same axis
- balance along path
  - $T = D + D_T + W \sin\gamma$
- symmetrical (and small) tailplane
  - $M_{T_0} = 0$ } neglecting
- neglect moments by T, D, D_T
  - see Cook sec 3.1.4

small and has tiny moment arc, not significant relative to lift



Forces on an Aircraft (Steady, trimmed flight)

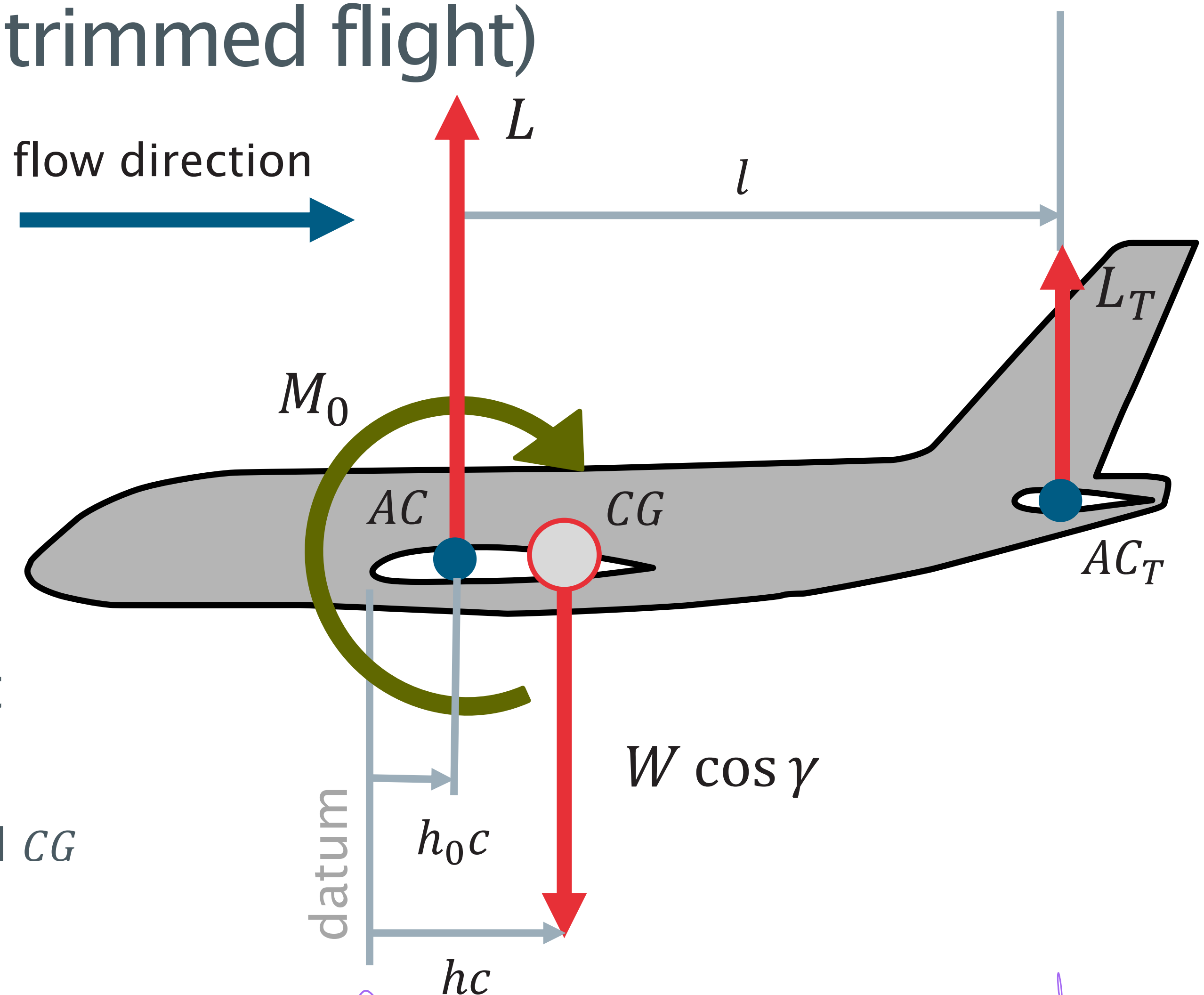
Final sketch

simplified from prev to show assumptions

Define positive directions

- $\boldsymbol{M_0}$: wing pitching moment about AC
  (+ve: nose up; constant with AoA)
- $\boldsymbol{l}$: distance from AC to AC_T
  (+ve: downstream)
- $\boldsymbol{h}$: distance from LE to CG divided by MAC
  (+ve: downstream)
- $\boldsymbol{h_0}$: distance from LE to AC divided by MAC
  (+ve: downstream)

Take force balance and moment balance around CG



somewhat arbitrary datum used as refrence for distance

Forces on an Aircraft (Steady, trimmed flight)

Trim conditions

focus on lift since drag is ez

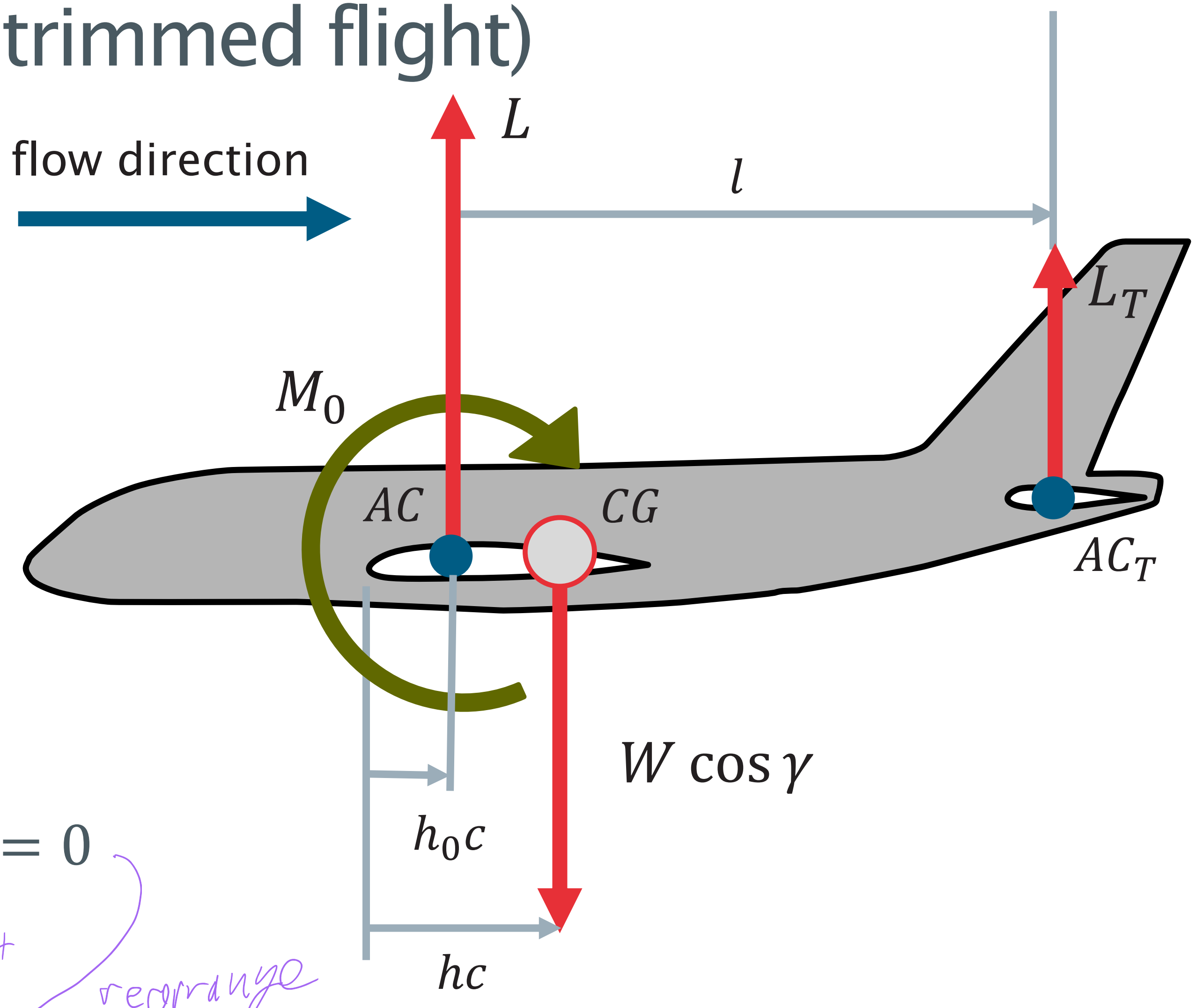
Aircraft in equilibrium $\Rightarrow \sum \vec{F} = 0$:

$$L + L_T - W\cos\gamma = 0$$

convenient "total lift"

$$\boxed{L^* \stackrel{\text{def}}{=} L + L_T} = W\cos\gamma$$

where L^* is the total aircraft lift



Moment balance about CG $\Rightarrow \sum \vec{M} = 0$:

$$\underbrace{M_0}_{\text{wing pitching m around AC}} + \underbrace{L(h - h_0)c}_{\text{lift moment main wing}} - \underbrace{L_T(l - (h - h_0)c)}_{\text{tail moment}} = 0$$

rearrange to clean up

Or

$$M_0 + \underbrace{(L + L_T)}_{=L^*}(h - h_0)c - L_T l = 0$$

Forces on an Aircraft (Steady, trimmed flight)

Nondimensional trim conditions

converting to coefficients

Now rewrite in coefficient form using:

note by using S_T C_{L_T} is a more intuitive number (by magnitude)

$$C_{L^*} = \frac{L^*}{\frac{1}{2}\rho V^2 S};\; C_L = \frac{L}{\frac{1}{2}\rho V^2 S};\; C_W = \frac{W}{\frac{1}{2}\rho V^2 S};$$

$$\boxed{C_{L_T}} = \frac{L_T}{\frac{1}{2}\rho V^2 \boxed{S_T}};\; C_{M_0} = \frac{M_0}{\frac{1}{2}\rho V^2 S c}$$

Resulting in:

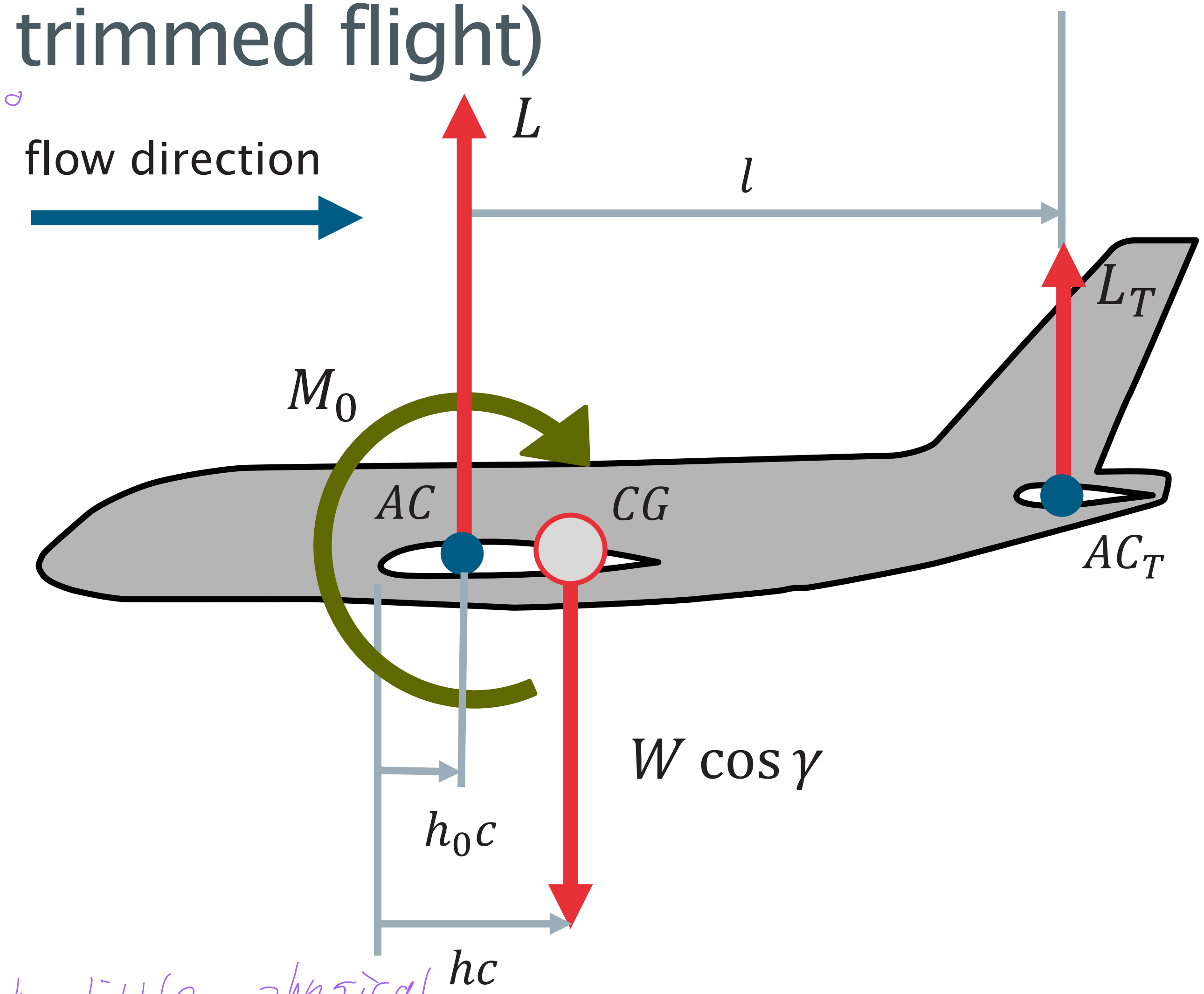
$$C_{L^*} = C_L + \boxed{C_{L_T}}\frac{S_T}{S} = C_W \cos\gamma$$

$$C_{M_0} + C_{L^*}(h - h_0) - C_{L_T} K = 0$$

with the tail volume fraction

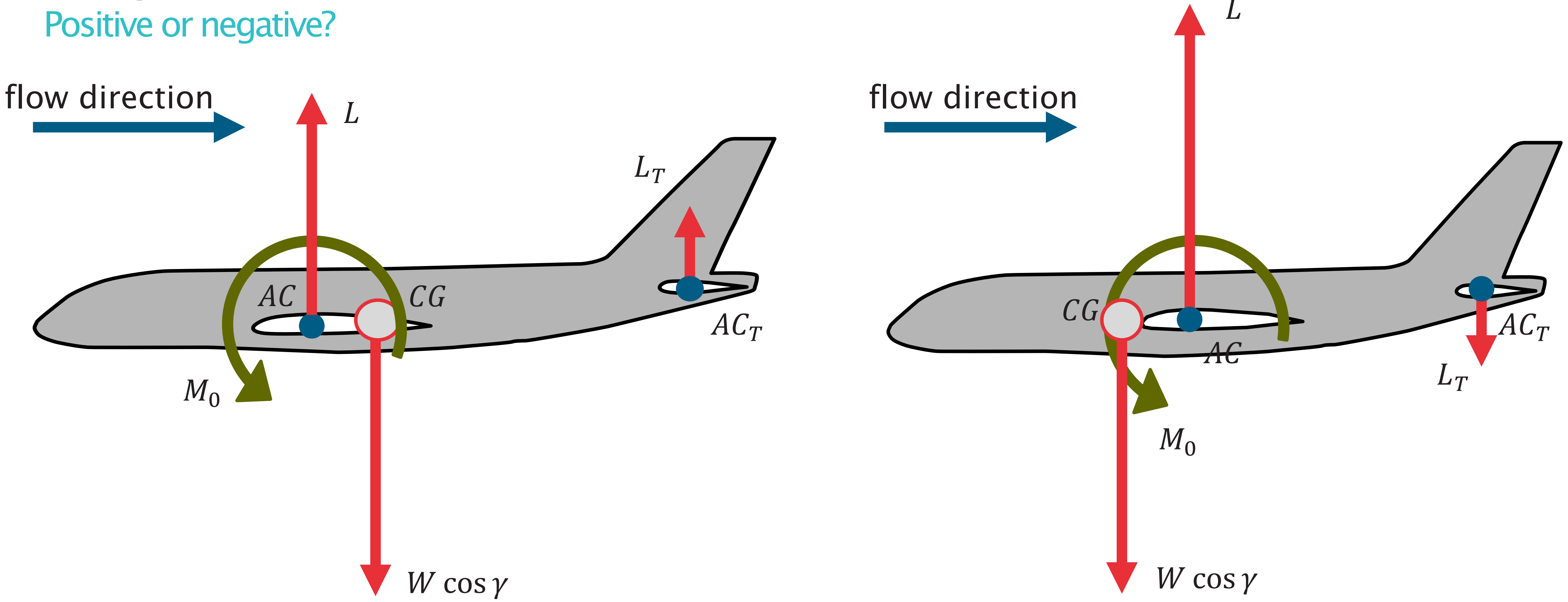
$$K \stackrel{\text{def}}{=} \frac{S_T l}{S c};$$

ratio of volumes with little physical meaning, just nice cleanup variable



Tailplane Lift

Positive or negative?



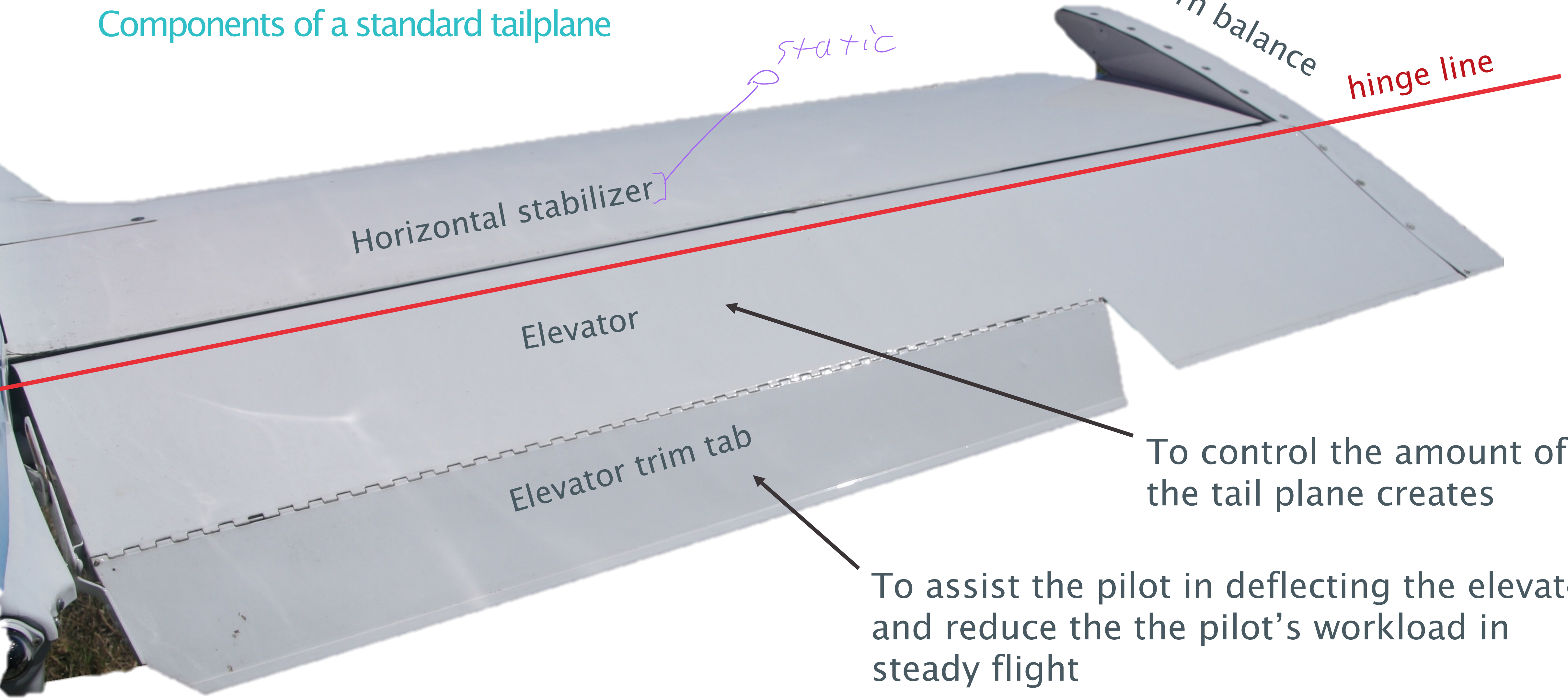
- Forward CG position benefits stability (but results in poorer handling quality at low speed)
- Tail plane design is affected: symmetric or negatively cambered airfoil sections

The Zenith STOL

The Zenith STOL

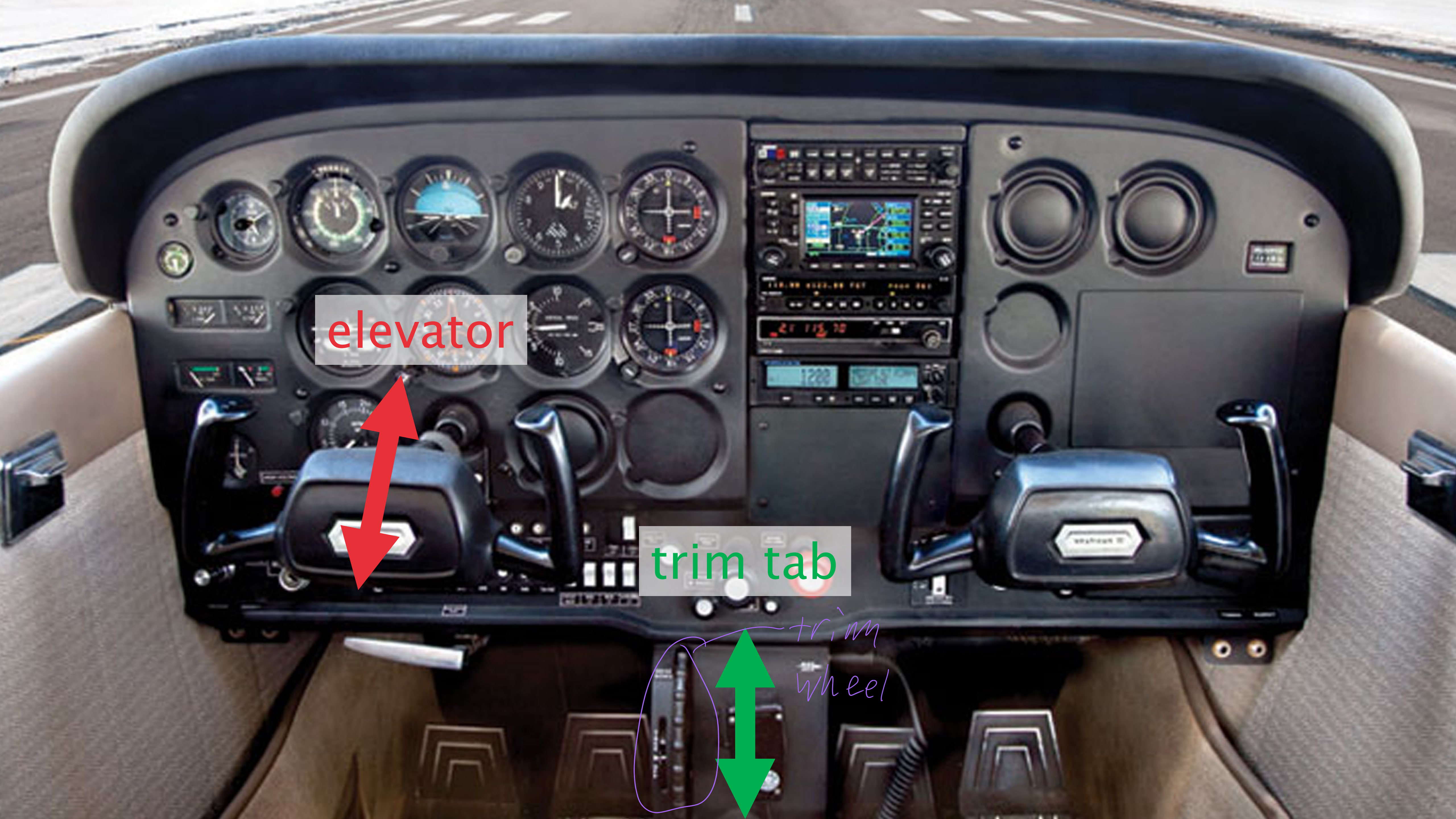
Tail plane

Components of a standard tailplane



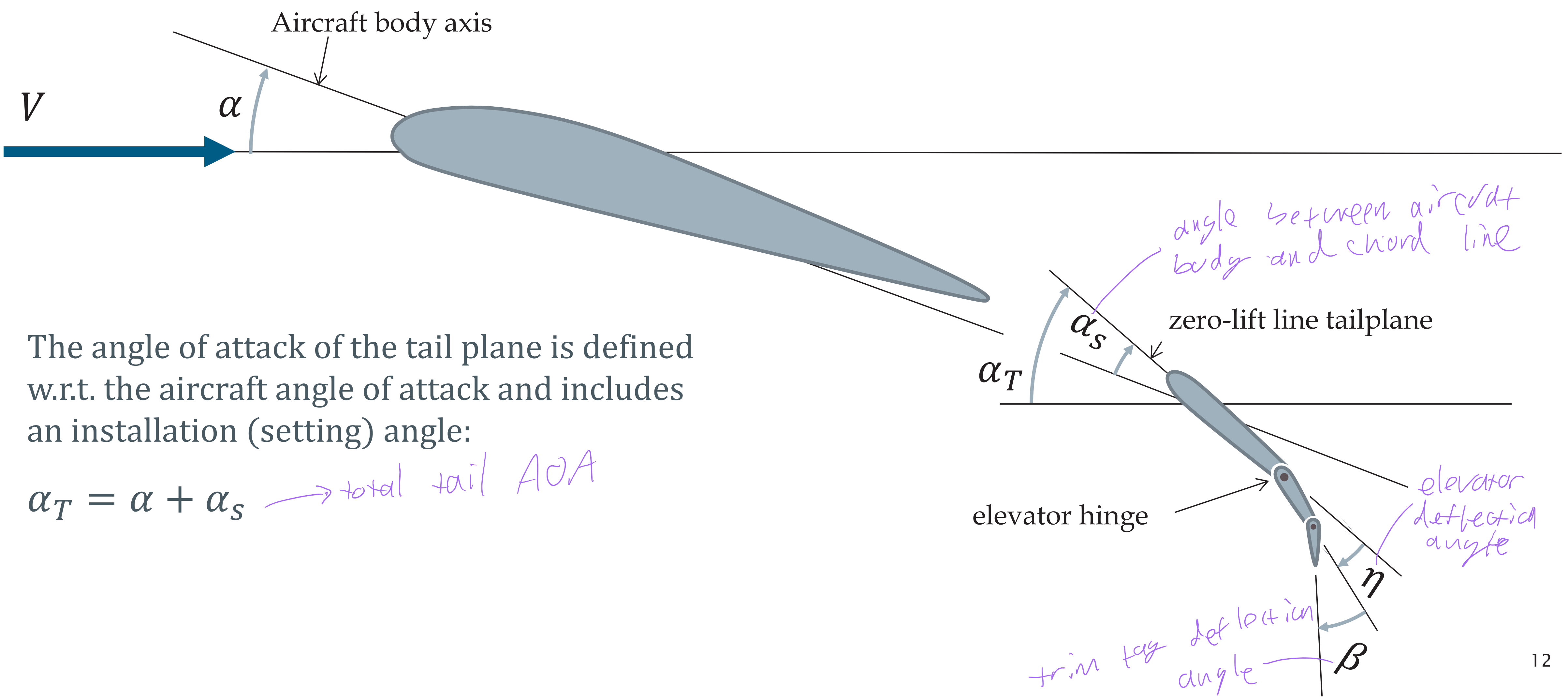
To control the amount of lift the tail plane creates

To assist the pilot in deflecting the elevator and reduce the the pilot’s workload in steady flight


elevator
trim tab
trim
wheel

Tail plane models

General definitions



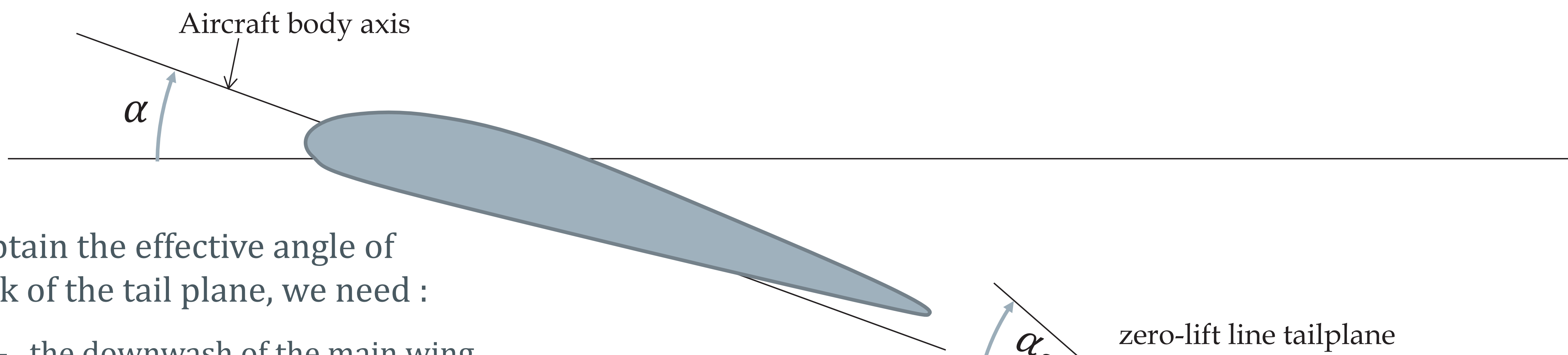
The angle of attack of the tail plane is defined w.r.t. the aircraft angle of attack and includes an installation (setting) angle:

$\alpha_T = \alpha + \alpha_s$ → total tail AOA

Tail plane angle of attack

Influence of downwash

→ cringe means $\vec{V}_\infty$ is not suitable for air hitting tail

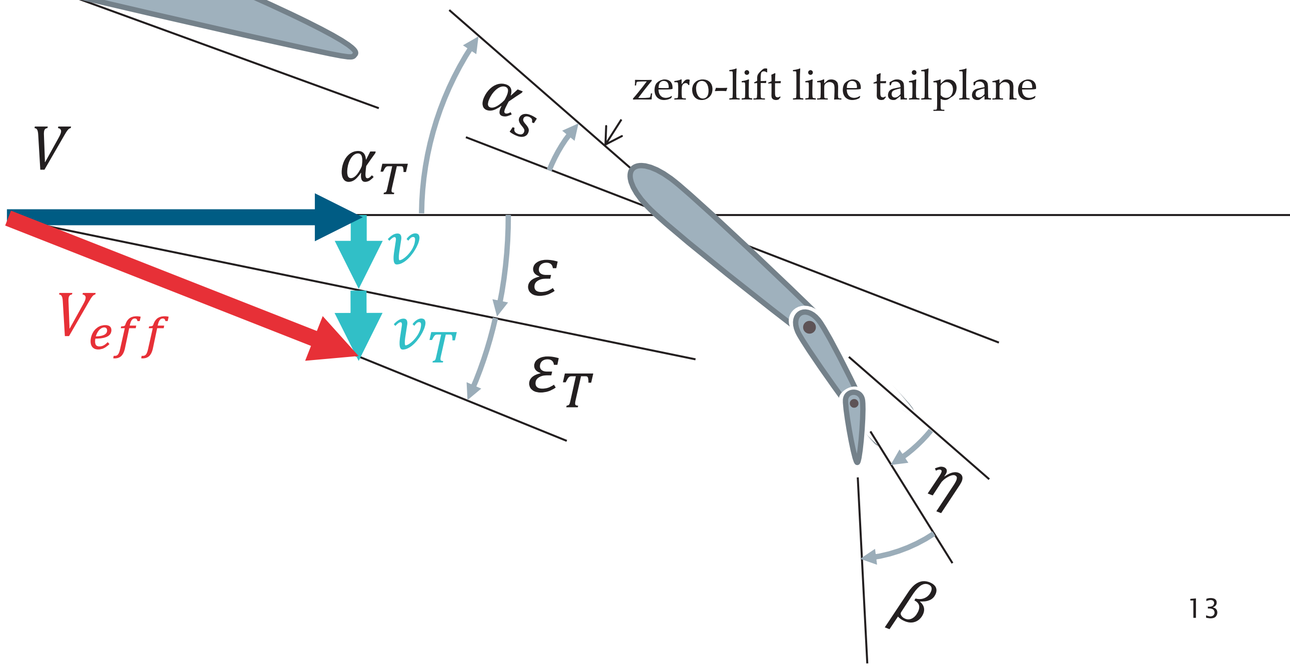


To obtain the effective angle of attack of the tail plane, we need :

- the downwash of the main wing
- the downwash of the tail itself

Resulting in:

$$\alpha_{T_{eff}} = \alpha_T \underbrace{- \varepsilon - \varepsilon_T}_{\text{downwash effect}}$$



Tail plane angle of attack

Influence of downwash II

Given:

$$\alpha_{T_{eff}} = \overbrace{\alpha + \alpha_s}^{\alpha_T = \alpha + \alpha_s} - \varepsilon - \varepsilon_T$$

Expand the downwash terms

$$\alpha_{T_{eff}} = \alpha + \alpha_s - \varepsilon_\alpha(\alpha - \alpha_0) - \frac{C_{L_T}}{\pi A_T e_T}$$

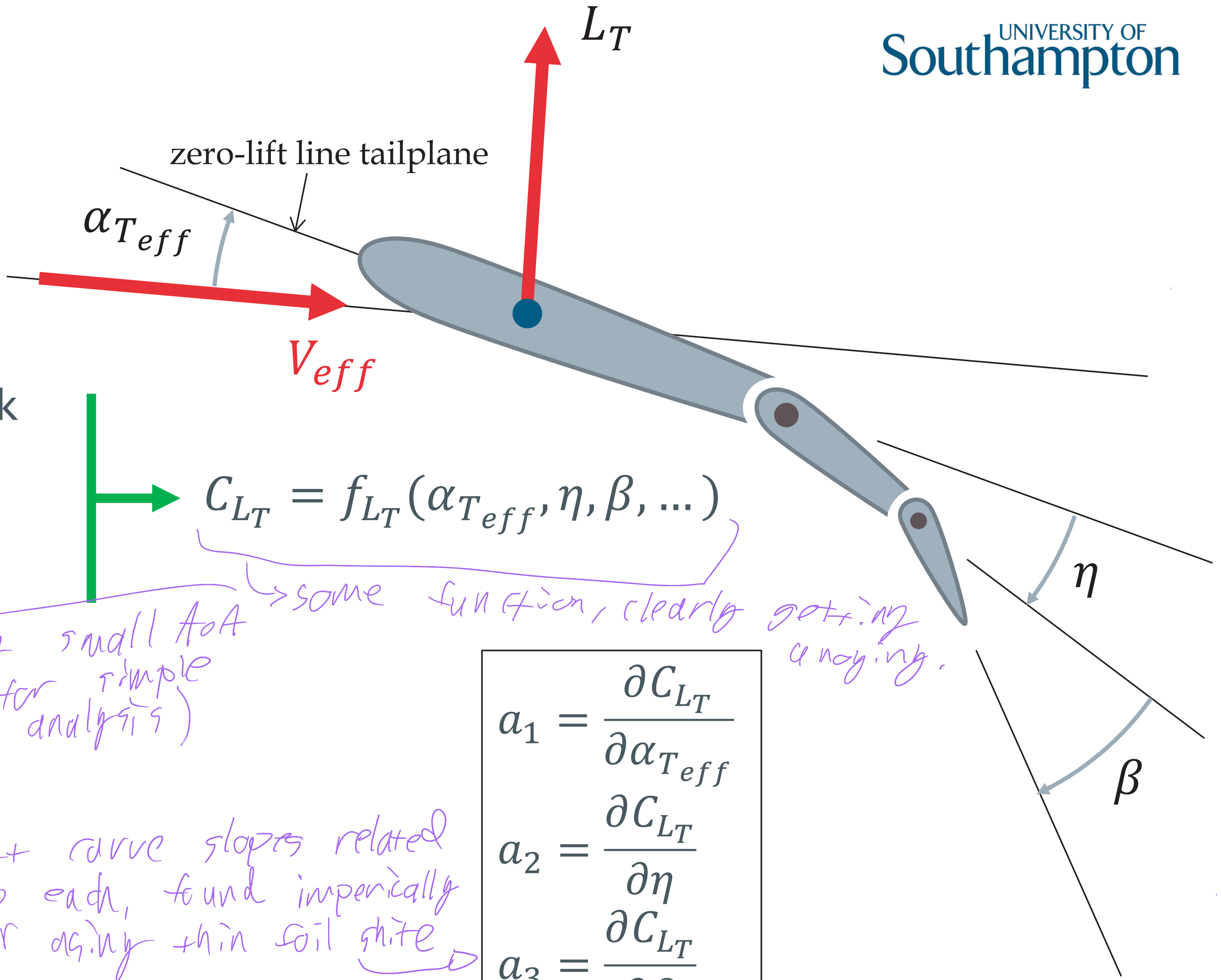
tail setting angle

zero lift angle of attack

tail downwash

Tail plane

Lift equation



Tail plane lift depends on:

$\alpha_{T_{eff}}$: tail plane angle of attack

η: elevator angle

β: elevator trim tab angle

$$C_{L_T} = f_{L_T}(\alpha_{T_{eff}}, \eta, \beta, \dots)$$

→ some function, clearly getting annoying.

A linear model is: assuming small AoA (suitable for simple analysis)

$$C_{L_T} = a_1 \alpha_{T_{eff}} + a_2 \eta + a_3 \beta$$

includes downwash

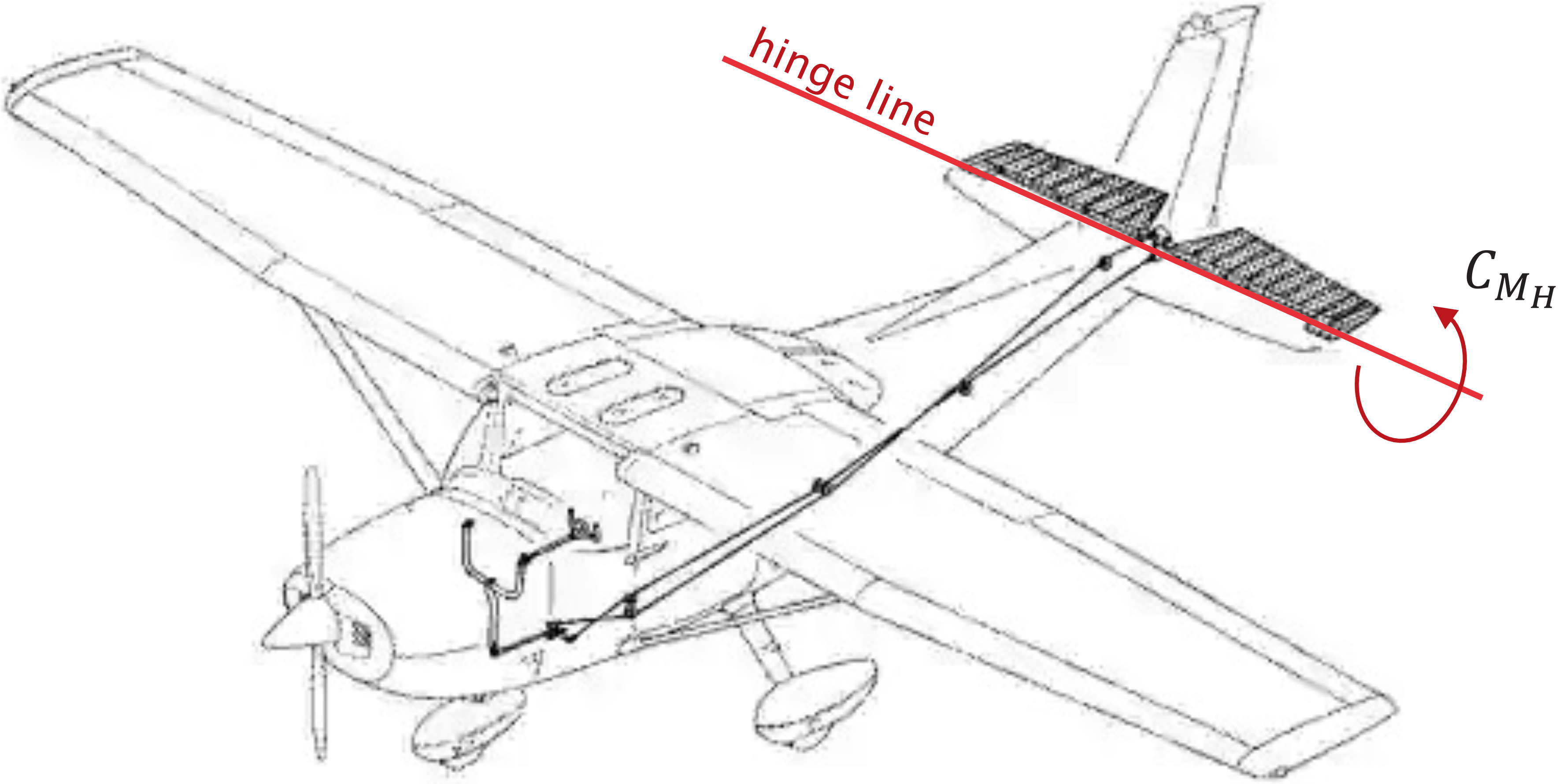
lift curve slopes related to each, found imperically or using thin foil shite →

$$\boxed{a_1 = \frac{\partial C_{L_T}}{\partial \alpha_{T_{eff}}} \quad a_2 = \frac{\partial C_{L_T}}{\partial \eta} \quad a_3 = \frac{\partial C_{L_T}}{\partial \beta}}$$

What would you do to estimate these coefficients?

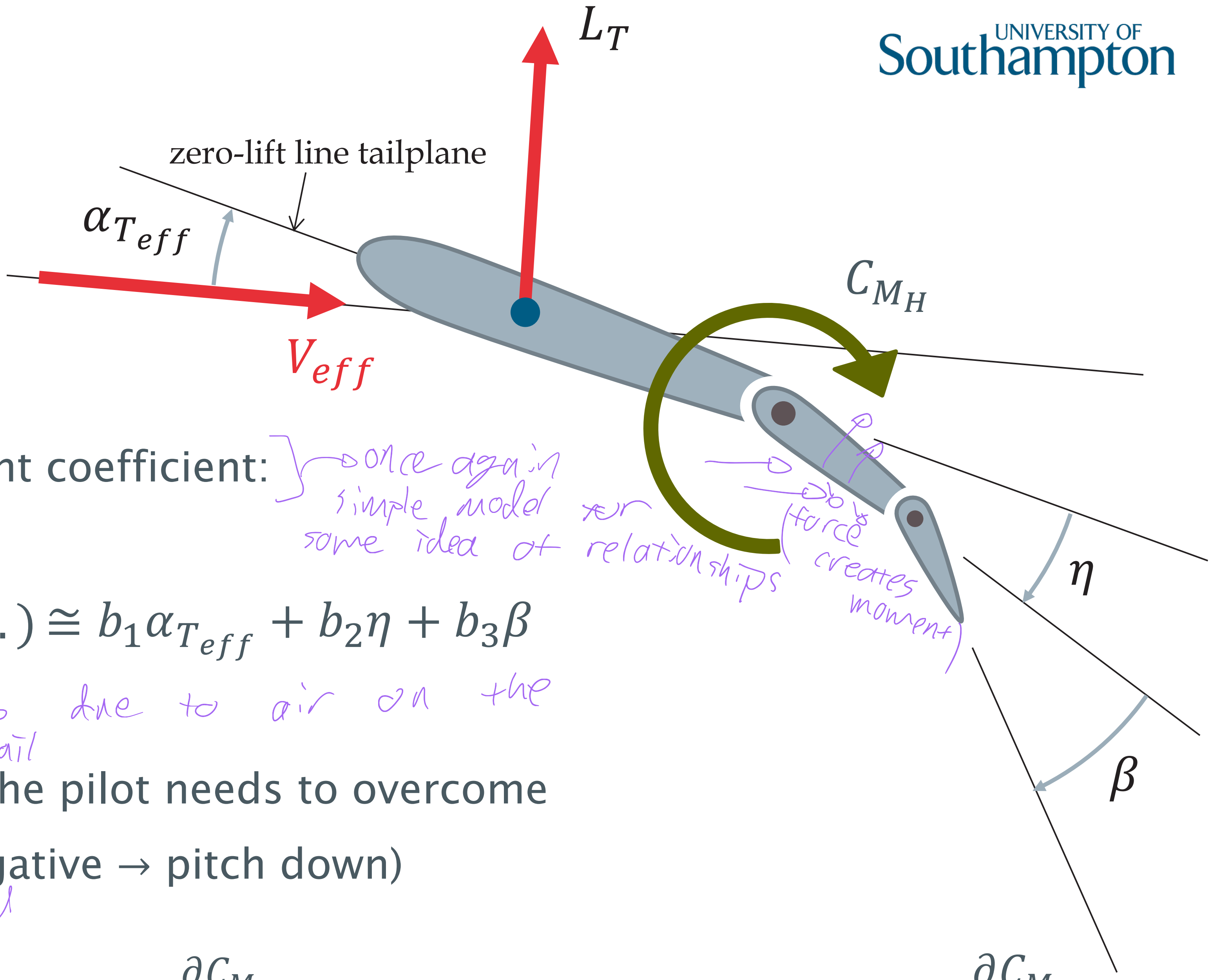
Tail plane

Aerodynamic forces produce a hinge moment



Tail plane

Hinge moment equation



Model for elevator hinge moment coefficient:

once again simple model for some idea of relationships

$$C_{M_H} = f_{M_H}(\alpha_{T_{eff}}, \eta, \beta, \dots) \cong b_1\alpha_{T_{eff}} + b_2\eta + b_3\beta$$

moment forces due to air on the deflected tail

This is the hinge moment that the pilot needs to overcome

(positive → pitch up, negative → pitch down)

describes force needed to deflect tail

$b_1 = \dfrac{\partial C_{M_H}}{\partial \alpha_{T_{eff}}}$ ⟶ floating tendency

$b_2 = \dfrac{\partial C_{M_H}}{\partial \eta}$ ⟶ restoring tendency

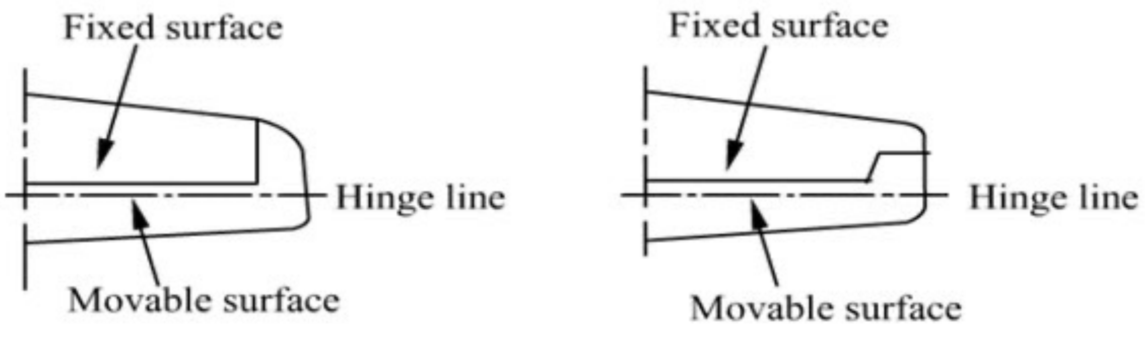
$b_3 = \dfrac{\partial C_{M_H}}{\partial \beta}$

Horn balance

Aerodynamic balancing: function and effect on modelling

The tail of the Spitfire, with unshielded horns elevator and rudder.

- Shielded/unshielded

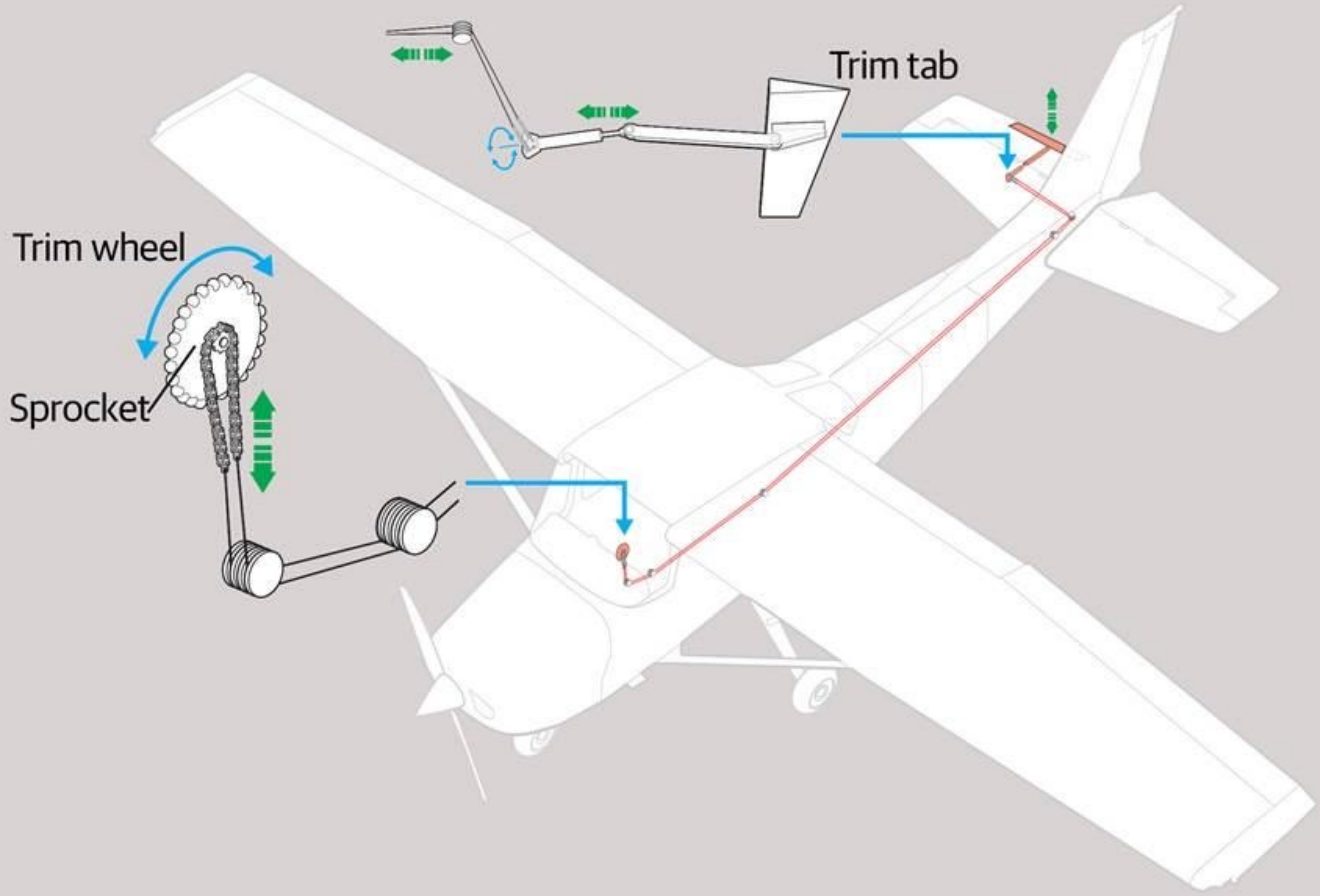


Unshielded Horn   Shielded Horn

- Reduces floating/restoring tendency

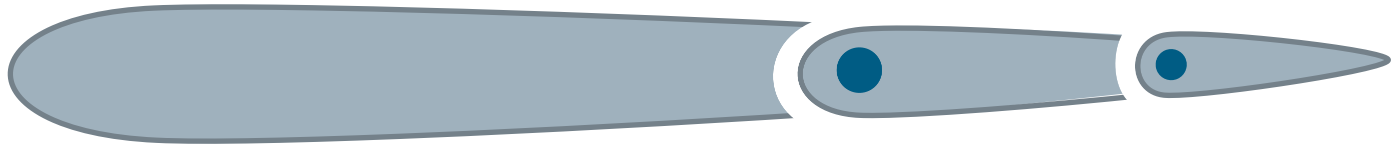
$$C_{M_H} = b_1 \alpha_{T_{eff}} + b_2 \eta + b_3 \beta$$

reduces moments acting on hinge so less force is needed to control it (biased)


Trim tab
Trim wheel
Sprocket

Tail plane

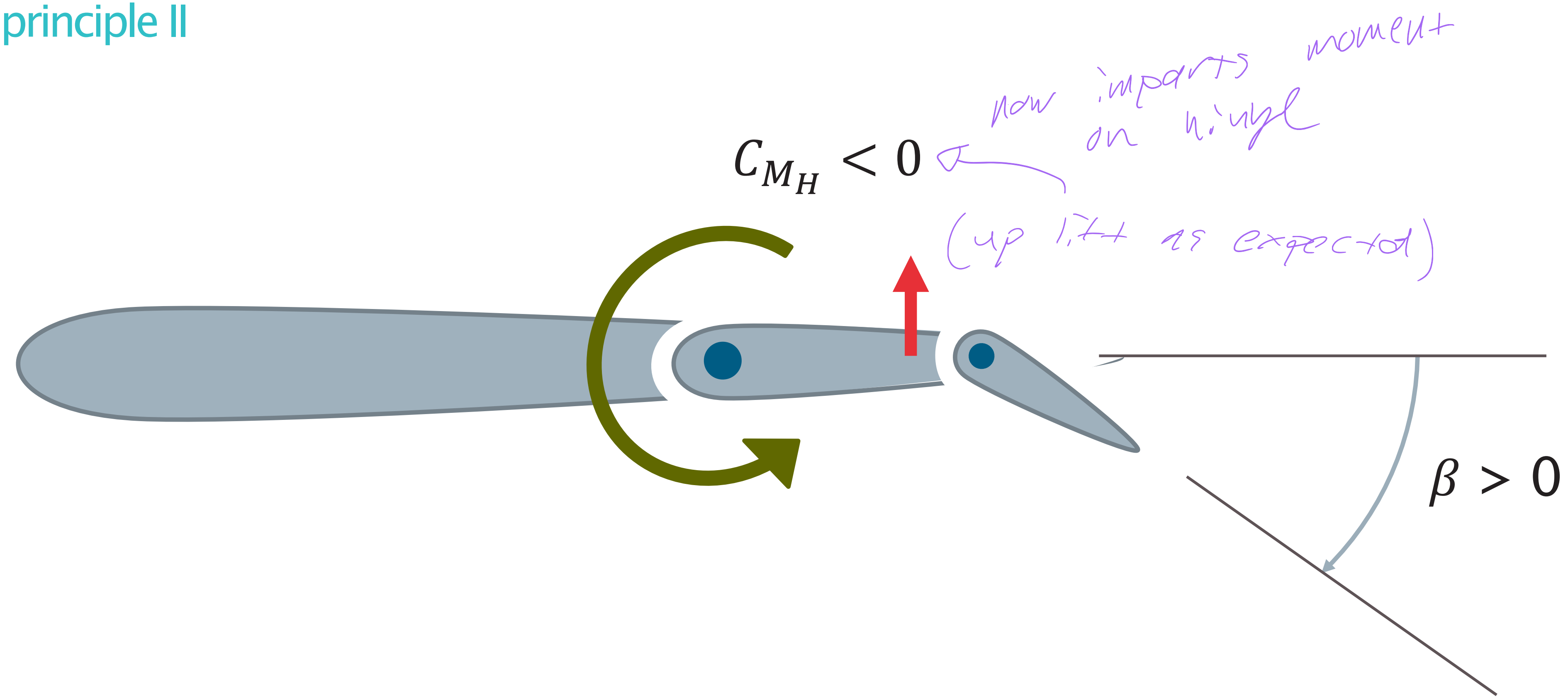
Trim tab working principle I



Assume we want to produce a negative tail lift, where should the elevator go?

Tail plane

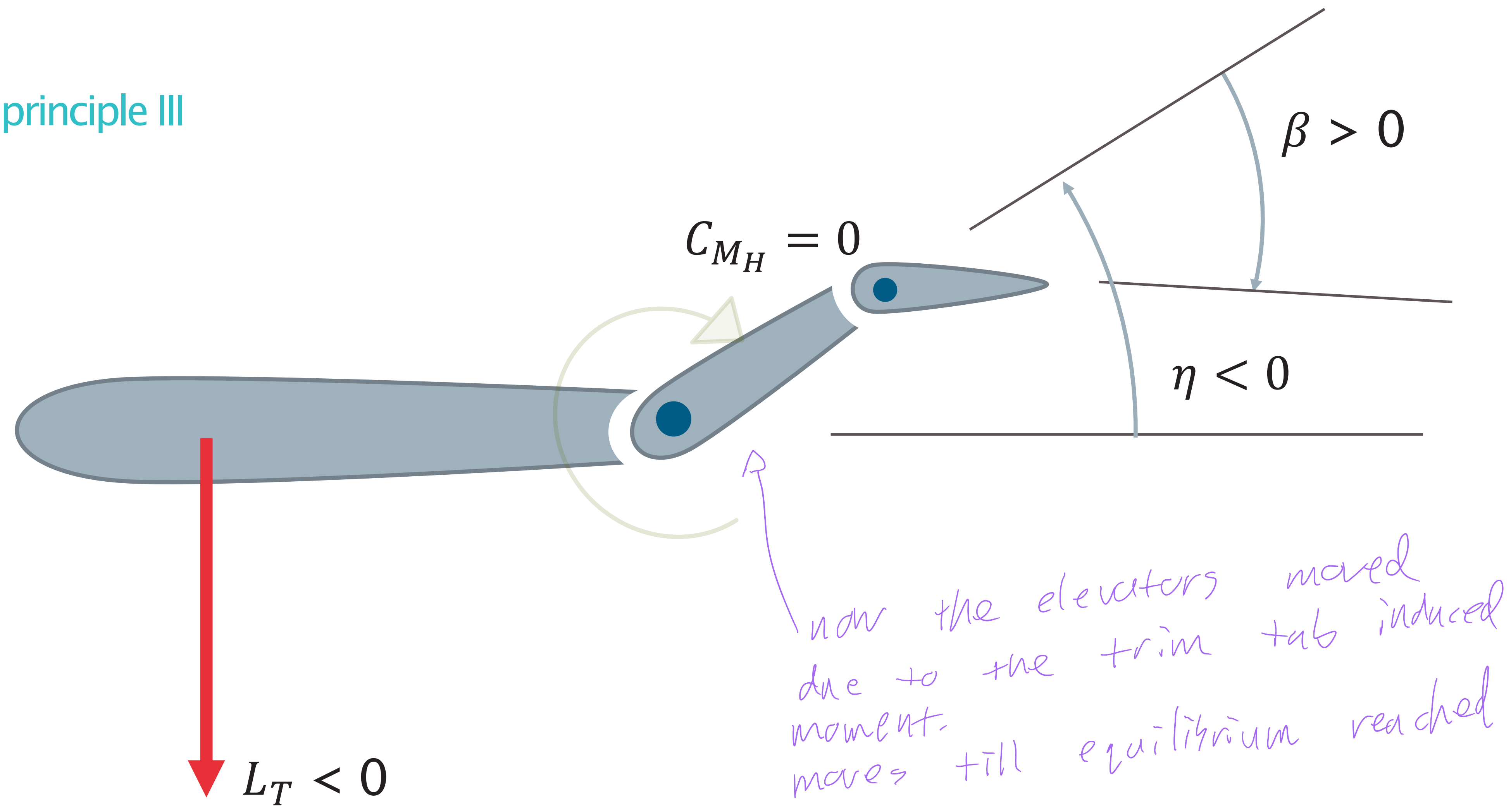
Trim tab working principle II



This configuration produces a pitch down moment on the elevator

Tail plane

Trim tab working principle III



The stick is free to move, but the aircraft is trimmed.