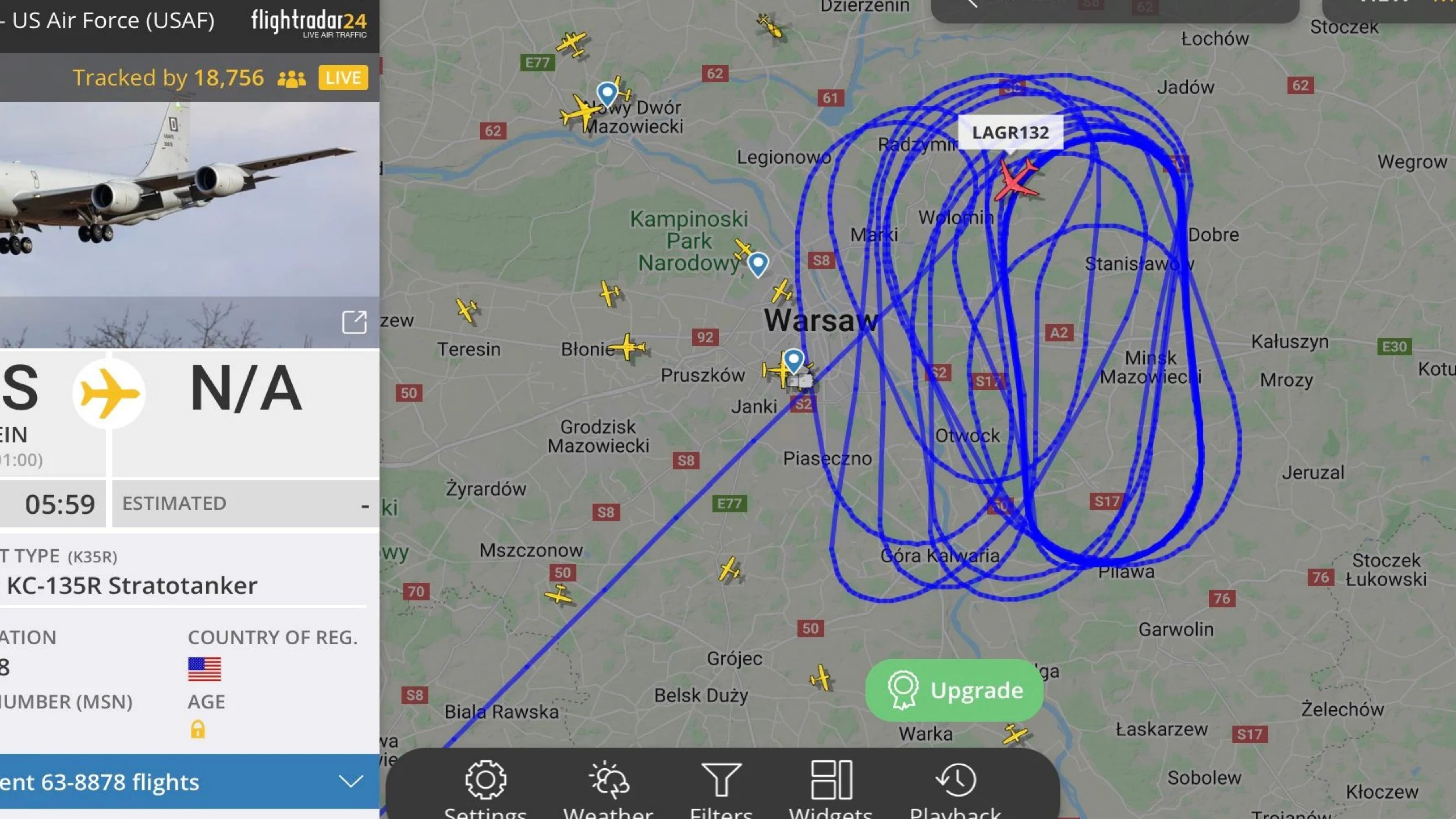
SESA2025 Mechanics of Flight
Cruise/Loiter Performance

Lecture 1.6



- US Air Force (USAF)
flightradar24
LIVE AIR TRAFFIC
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Teresin
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Kałuszyn
Mrozy
Mińsk Mazowiecki
Grodzisk Mazowiecki
Piaseczno
Otwock
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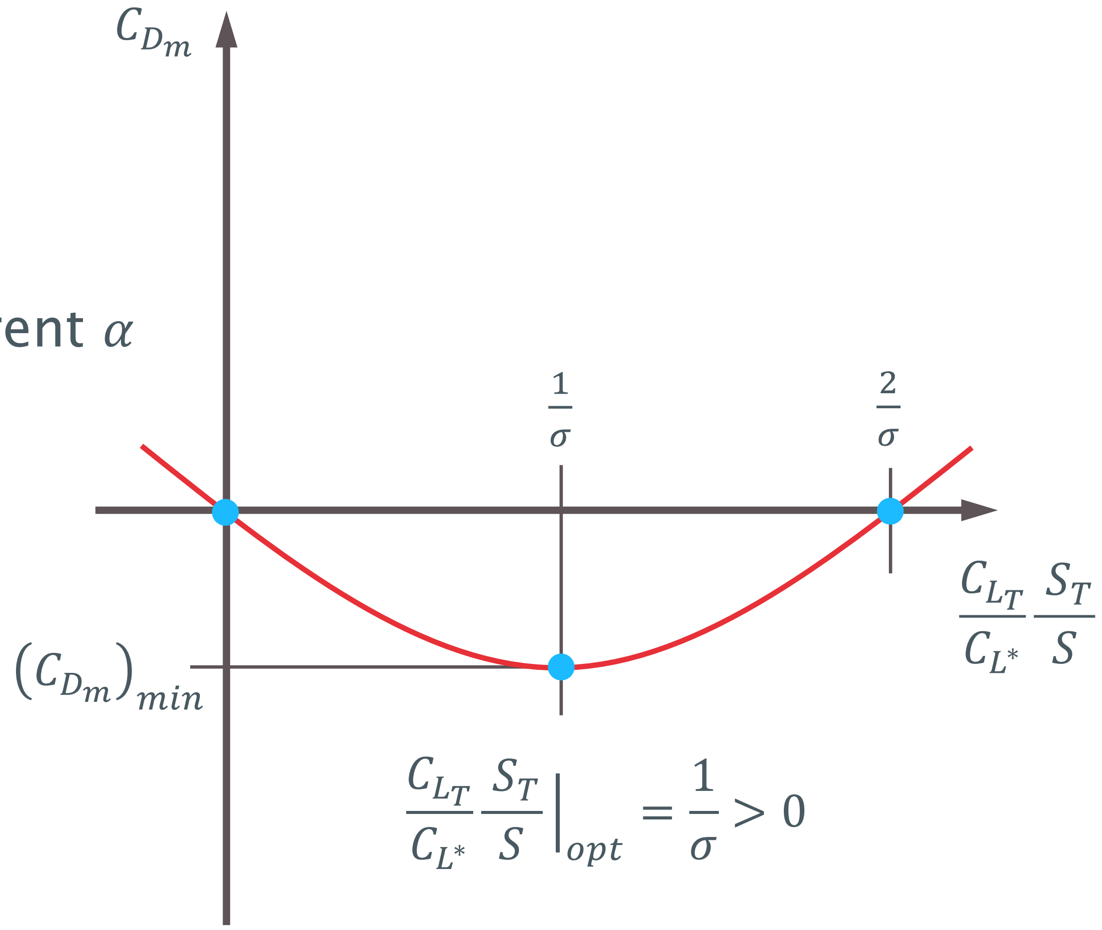
Żyrardów
Mszczonow
Góra Kalwaria
Pilawa
Stoczek Łukowski
Garwolin
Grójec
Belsk Duży
Biała Rawska
Warka
Żelechów
Łaskarzew
Sobolew
Kłoczew
Upgrade
Settings
Weather
Filters
Widgets
Playback

Notes on trim drag

To recap

- Purely aerodynamic analysis
  - Find $\frac{L_T}{L^*}$ then think about CG location
- For a fixed V, each $\frac{L_T}{L^*}$ corresponds to a different α



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Performance optimisation

Defining optimum flight conditions

- We have now quantified trim drag and found the optimal tailplane/total lift ratio

$$\left(C_{D_m}\right)_{min} = \frac{-C_{D_{L^*}}}{\sigma} \quad when \quad \underbrace{\frac{C_{L_T}}{C_{L^*}}\frac{S_T}{S} = \frac{1}{\sigma}}$$

use this requirement

- Use this information to find flight conditions for best cruise/loiter performance
  - Total lift and drag coefficients C_{L^*}, C_D
  - Flight velocity and angle of attack V, α
  - Elevator controls required η, β

find optimal values of these inputs

- Then find (optimal) c.g. position that trims the aircraft in these conditions (moment balance)

Performance optimisation

From the Breg

- For cruise :

- where d i ulsion type

- this mear
- and then

Performance optimisation

From the Breguet Range equations – SESA1015

- For cruise (or loiter) we want to fly in a (trimmed) condition where:

$$\left(\frac{C_{L^*}^{d}}{C_D}\right)_{\max}$$

- where d is a power depending on the required efficiency and propulsion type

set power

set thrust

Propulsion	Range	Endurance
Propeller/glider	$d = 1$	$d = 1.5$
Jet	$d = 0.5$	$d = 1$

- this means figuring out what is the optimal velocity V → +0
- and then the controls required for this flight condition

Performance optimisation

Drag polar/drag equation

- We need to use the drag polar (or drag equation):

$$C_D = C_{D_0} + C_{D_{L^*}} + C_{D_m}$$

- We need to express all components of drag as a function of the aircraft lift C_{L^*}
  - C_{D_0} is the (constant) zero-lift drag
  - $C_{D_{L^*}} \approx \frac{C_{L^*}^2}{\pi A e}$ is the lift dependent drag before trim (approx. as induced drag)
  - $C_{D_m} = \frac{-C_{D_{L^*}}}{\sigma}$ is the (minimum) trim drag

Performance optimisation

Rewrite further

- With simple algebra, we get:

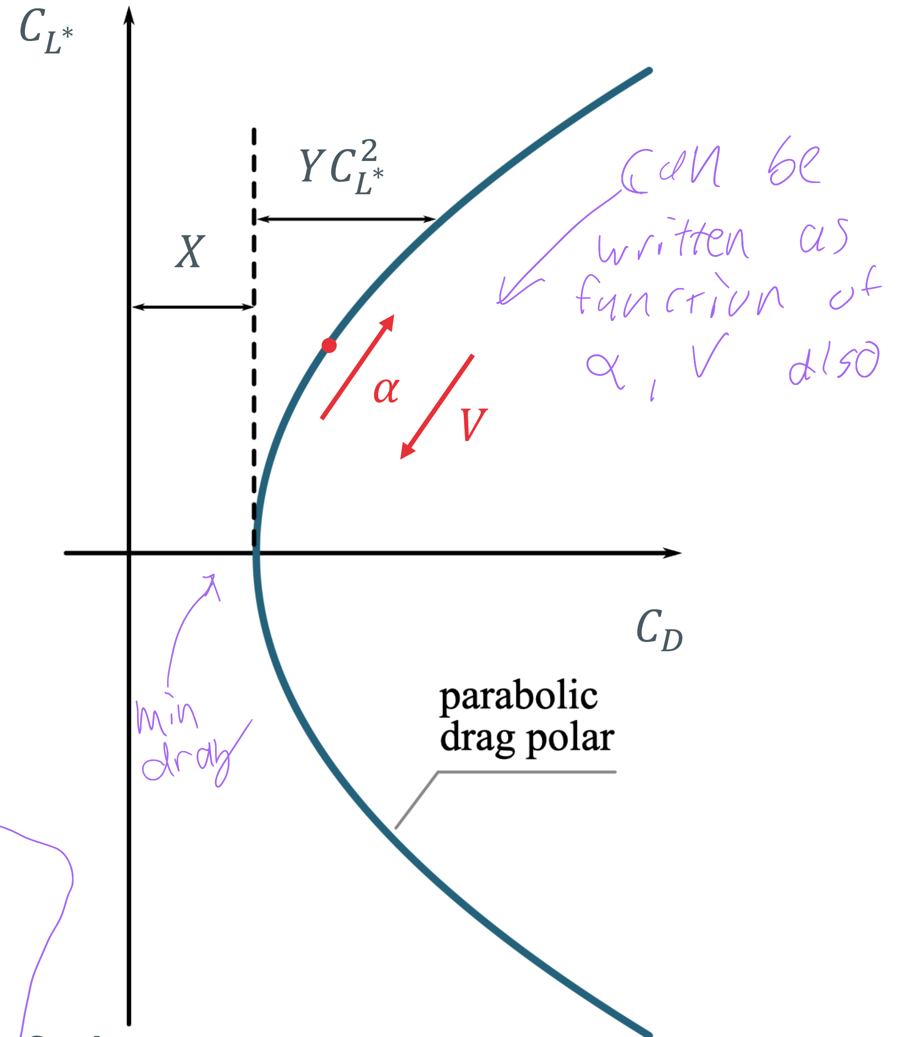
$$C_D = \underbrace{C_{D_0}}_{X} + \underbrace{\frac{1}{\pi A e}\left(1 - \frac{1}{\sigma}\right)}_{Y} C_{L^*}^2$$

constants

- The simplified drag equation/drag polar is:

$$\boxed{C_D = X + Y C_{L^*}^2}$$

- The polar is parametrised by α (and implicitly by V)
- What happens if we change A? or use, e.g. laminar airfoils?



Performance optimisation

Optimise for cruise/loiter

- Using the drag polar, we need to find:

$\left(\frac{C_{L*}}{C_D}\right)_{max}$

$$\left(\frac{C_{L^*}^d}{X+YC_{L^*}^2}\right)_{\max}$$

- Which means we have to find:

note d variable not derivative symbol

$$\frac{\mathrm{d}}{\mathrm{d}C_{L^*}}\left(\frac{C_{L^*}^d}{X+YC_{L^*}^2}\right)=0 \;\rightarrow\; dC_{L^*}^{d-1}\left(X+YC_{L^*}^2\right)-C_{L^*}^d 2YC_{L^*}=0 \;\rightarrow\; \underbrace{dX-C_{L^*}^2Y(2-d)=0}$$

- Resulting in:

$\left(\frac{C_{L*}^d}{C_D}\right)_{max}$ when....

$$C_{L^*}=\sqrt{\frac{X}{Y}\frac{d}{(2-d)}}\;;\quad C_D=X\left(1+\frac{d}{2-d}\right)$$

so (subbing back to get C_D)

Performance optimisation

Optimise for cruise/loiter

- Replace

$$X = C_{D_0}\ ,\quad Y = \frac{1}{\pi A e}\left(1 - \frac{1}{\sigma}\right),\quad \text{and}\quad \sigma = 1 + \frac{S\pi A e}{S_T \pi A_T e_T}$$

- to get

$$C_{L^*} = \sqrt{C_{D_0}\left(\pi A e + \frac{S_T}{S}\pi A_T e_T\right)\frac{d}{(2-d)}}\ ;\quad C_D = C_{D_0}\left(1 + \frac{d}{2-d}\right)$$

- This C_D can be used to estimate fuel burn at cruise

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Performance optimisation

Optimise for cruise/loiter

- Best cruise/loiter performance

$$\left(\frac{C_{L^*}^{d}}{C_D}\right)_{\max} = \sqrt{\left(\pi A e + \frac{S_T}{S}\pi A_T e_T\right)\frac{d(2-d)}{4}\frac{1}{C_{D_0}}}$$

- To obtain the best cruise/loiter performance, we need:
  - Low zero-lift drag
  - High span efficiency
  - High aspect ratio

Performance\Propulsion	Range	Endurance
Propeller (or glider)	$d = 1$	$d = 1.5$
Jet	$d = 0.5$	$d = 1$

Performance optimisation

Solution process

$V_{opt} = \sqrt{\dfrac{W\cos\gamma}{\frac{1}{2}\rho S C_{L^*_{opt}}}}$

- Given air density, aircraft weight and wing area:
  - Determine velocity V from $L^* = \frac{1}{2}\rho V_{opt}^2 S C_{L^*_{opt}} = W\cos\gamma$
- Then determine the main wing's C_L

  using this eq (trim equation)

  - From equilibrium condition $\rightarrow$ $C_{L^*} = C_L + C_{L_T}\dfrac{S_T}{S}$

    will be best at $\frac{1}{\sigma}$ (prev lect)

  - From minimum trim drag condition $\rightarrow$ $\dfrac{C_{L_T}}{C_{L^*}}\dfrac{S_T}{S} = \dfrac{1}{\sigma}$

    find this

- And the main wing's angle of attack $\rightarrow$ α

  find

- Find $\alpha_{T_{eff}}$, knowing setting angle and downwash

  find using thingy eq

- Then find elevator controls η, β required to produce required C_{L_T}

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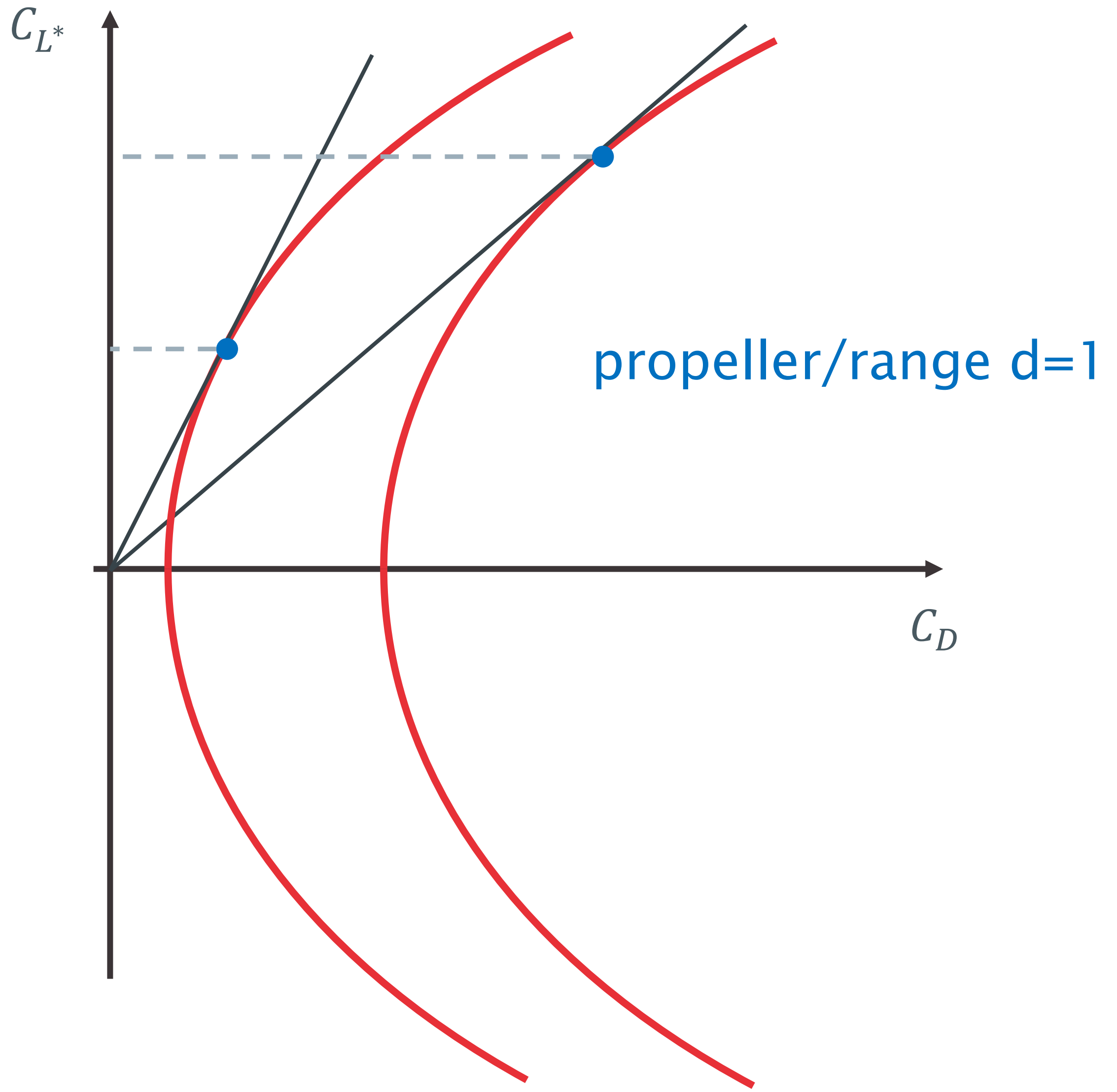
Performance optimisation

Example question

- Question:
  - What happens when C_{D_0} decreases?
- Answer:
  - C_{L^*} decreases, and so C_L and C_{L_T}
  - V increases
  - α decreases
- What if A increases?



Still need to ensure aircraft is trimmed (balance of moments)

Performance optimisation

Moment balance to find the best CG position

- Recall for moment balance we have:

$$0 = C_{M_0} + C_{L^*}(h - h_0) - C_{L_T}K$$

- Which we can rewrite:

to convenient form

$$h = h_0 - \frac{C_{M_0}}{C_{L^*}} + \frac{C_{L_T}}{C_{L^*}}\frac{S_T l}{Sc}$$

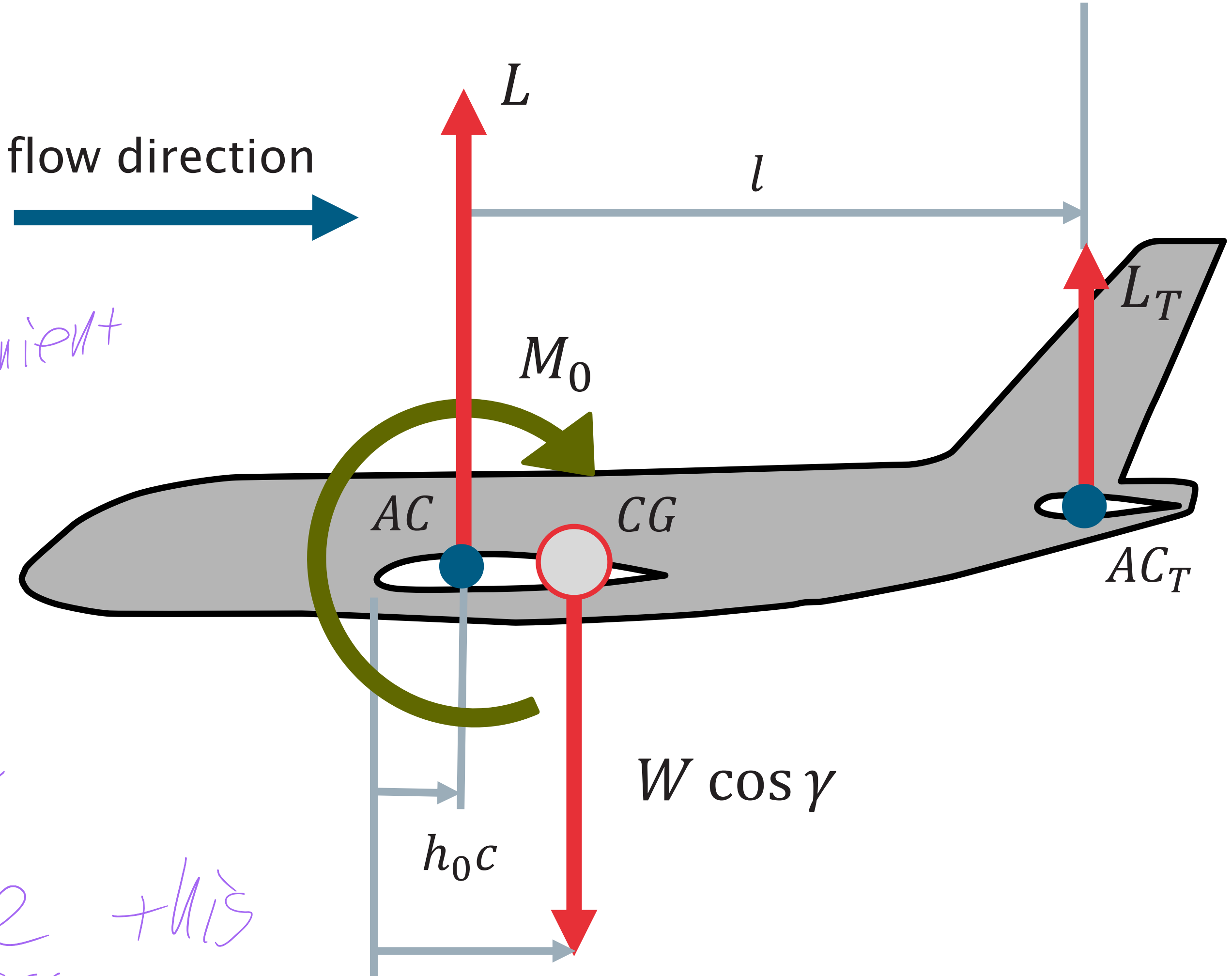
- Now $(C_{D_m})_{min} \rightarrow \frac{C_{L_T}}{C_{L^*}}\frac{S_T}{S} = \frac{1}{\sigma}$

combine and rearrange

- So we have:

$$h_{opt} = h_0 - \frac{C_{M_0}}{C_{L^*}} + \frac{l}{c\sigma}$$

note this is different for endurance and range



This optimal CG position may be too aft to ensure adequate stability