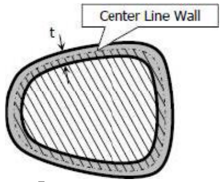
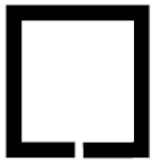
FEEG 2005
Structures: Lecture 5

Buckling 1: Instability in structures, Euler buckling theory

Summary of last lecture



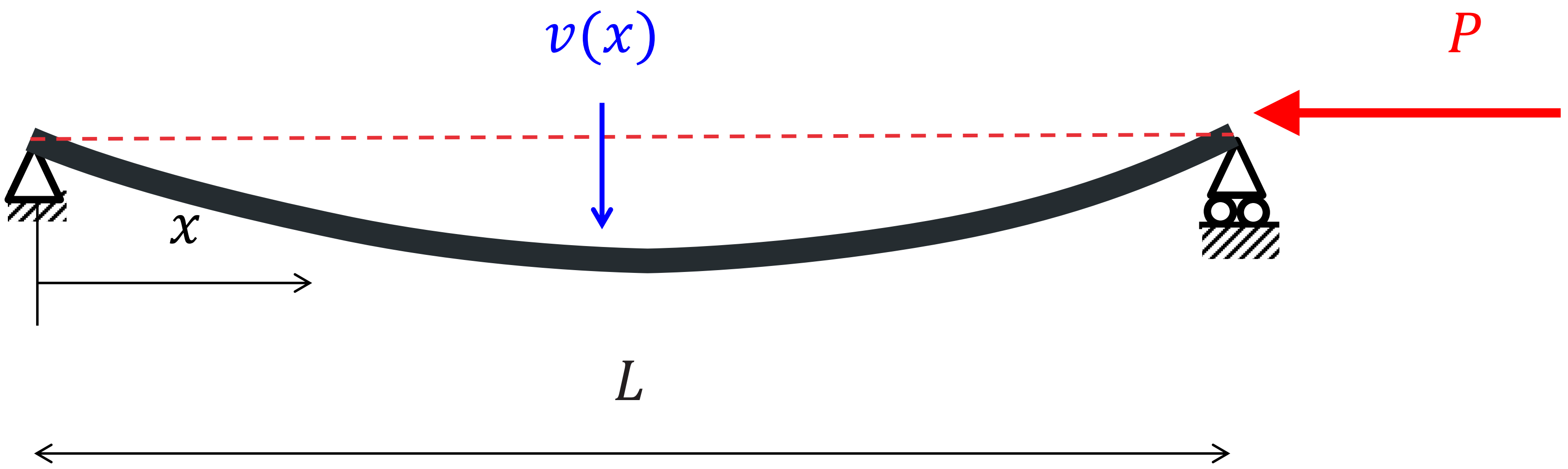


	Closed Sections:	Open Sections:
1) Stress:	$\tau_{max} = \dfrac{T_{max}}{2A_m t_{min}}$	$\tau_{i,max} = \pm \dfrac{Tt_i}{J}$
2) Deformation:	$\phi = \dfrac{TL}{GJ}$	$\phi = \dfrac{TL}{GJ}$
Torsional Constant:	$J = \dfrac{4A_m^2}{\oint \dfrac{ds}{t}}$	$J \approx \sum_i^N \dfrac{b_i t_i^3}{3}$

- Closed sections are much better at supporting torsion!

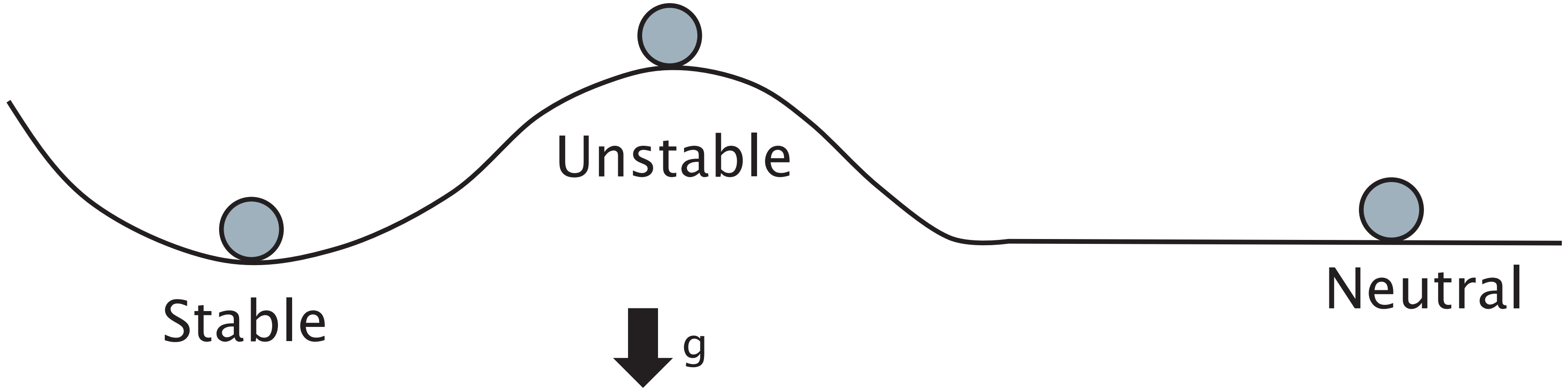
This lecture

- Instability and buckling of structures



Stability of equilibrium

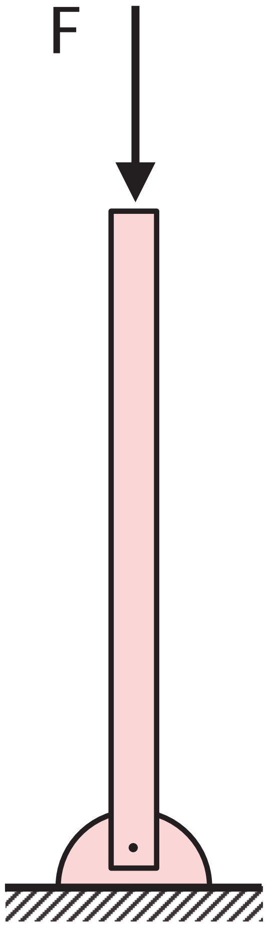
- So far, we discussed the equilibrium of internal and external forces.
- Applying a small disturbance to a system of forces in equilibrium can show if that system is (i) stable (ii) unstable, or (iii) Neutral. Note: all balls are in equilibrium.



Question

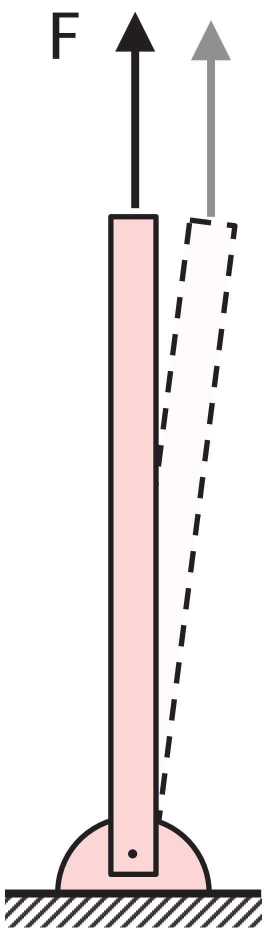
- Which statement is correct for the vertical end-pinned bar under compression?

1. The system is in equilibrium and stable.
2. The system is in equilibrium and unstable.
3. The system is not in equilibrium and is stable.
4. The system is not in equilibrium and is unstable.



https://vevox.app/#/m/185411783

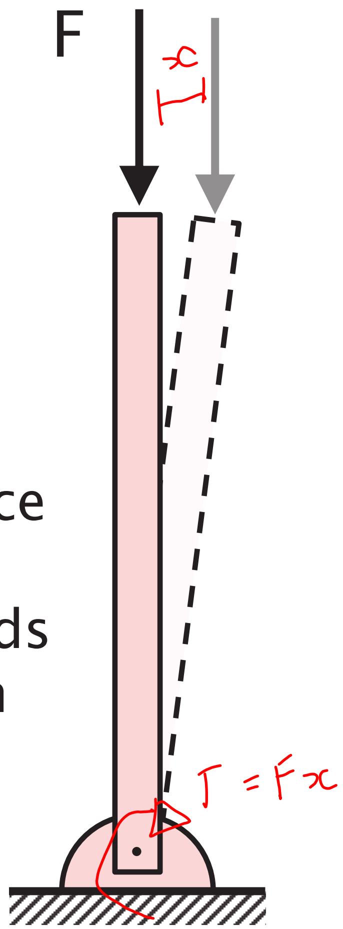
Vertical end-pinned bar, stable loading vs unstable loading



The force F restores the original equilibrium even after a small disturbance

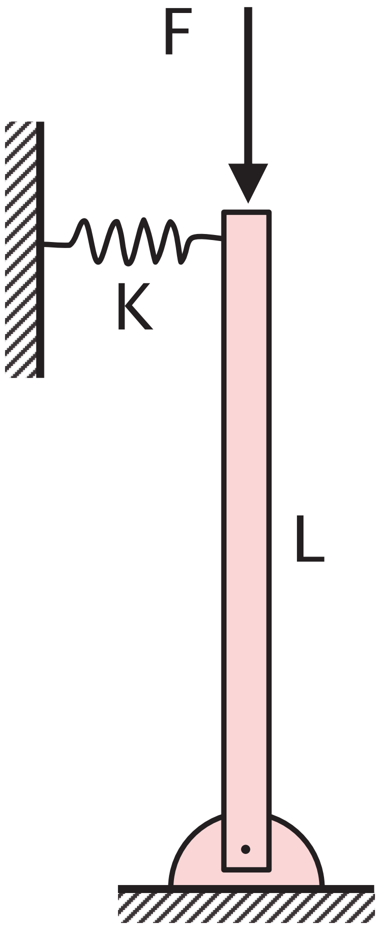
Stable

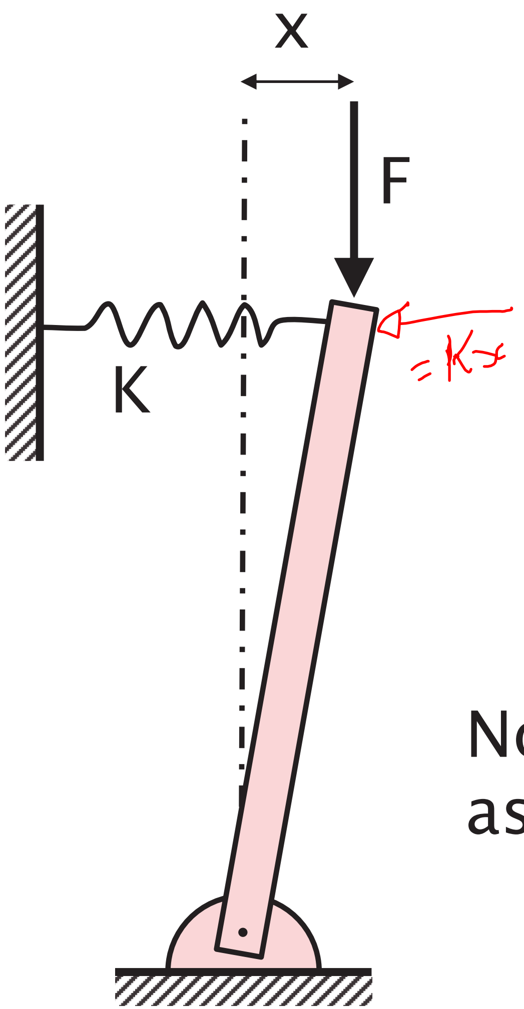
A small disturbance causes the bar accelerating towards horizontal position



Unstable

Vertical end-pinned bar with spring





1. Stable if Fx < kxL → F < KL

2. Unstable if Fx > kxL

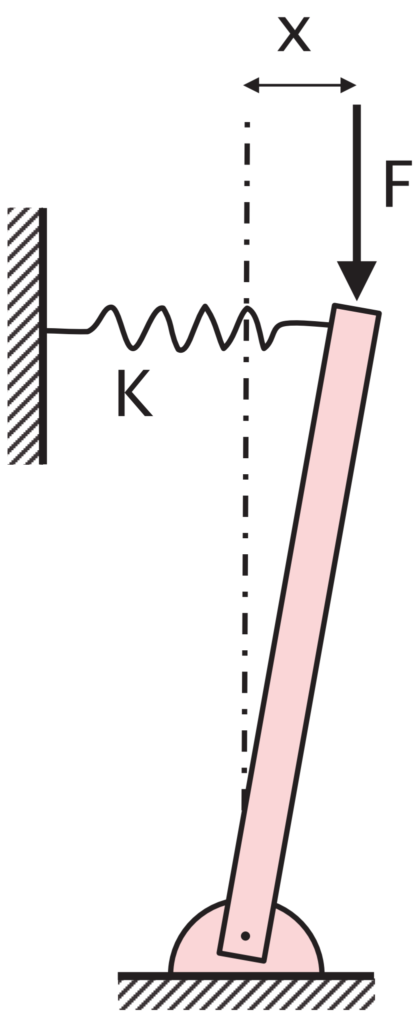
3. Neutral if Fx = kxL

Note: x and rotation angle are assumed to be small.

It depends …

Solving for the equilibrium

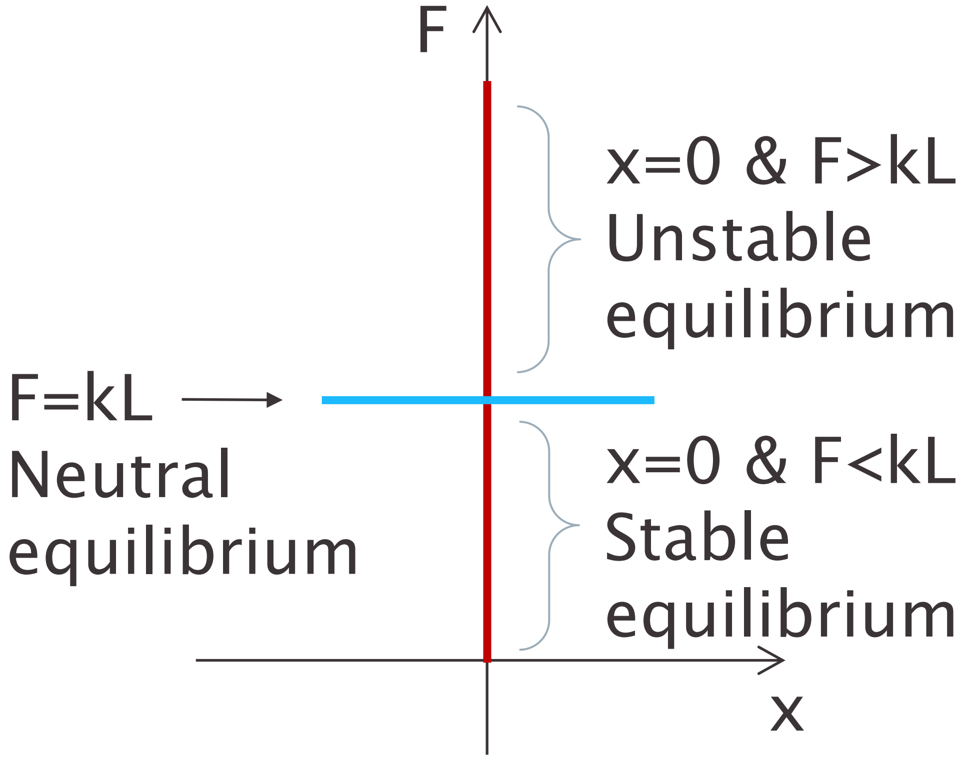
equilibrium if balanced spring n force



- $Fx = kxL \rightarrow \begin{cases} F = kL \\ \text{or} \\ x = 0 \end{cases}$

$x=0$ – equilibrium if centred regardless of force

Please contact me if interested to manufacture this system!

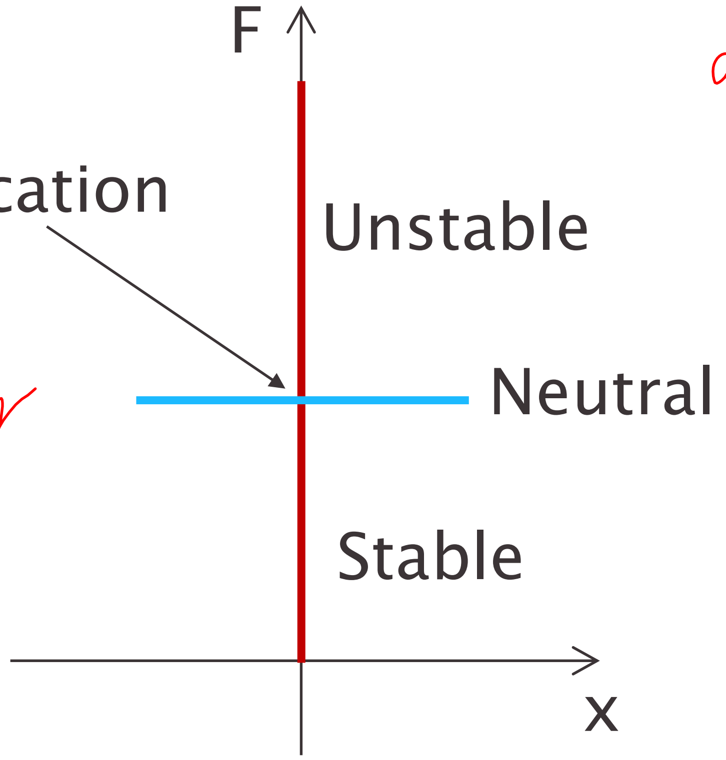


Plot shows equilibrium states at F and x

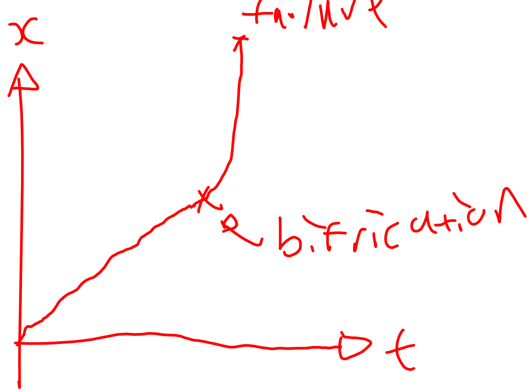
Bifurcation point

- Changes in stability (buckling), occur at bifurcation points.

This is where suddenly the beam fails as it goes from self correcting (stable) to accelerating deflection (unstable)



deflection time chart



assume $F \propto t$

Buckling categories

Classical or bifurcation buckling

- As the load passes through its critical stage, the structure passes from its unbuckled equilibrium configuration to an infinitesimally close buckled equilibrium configuration.

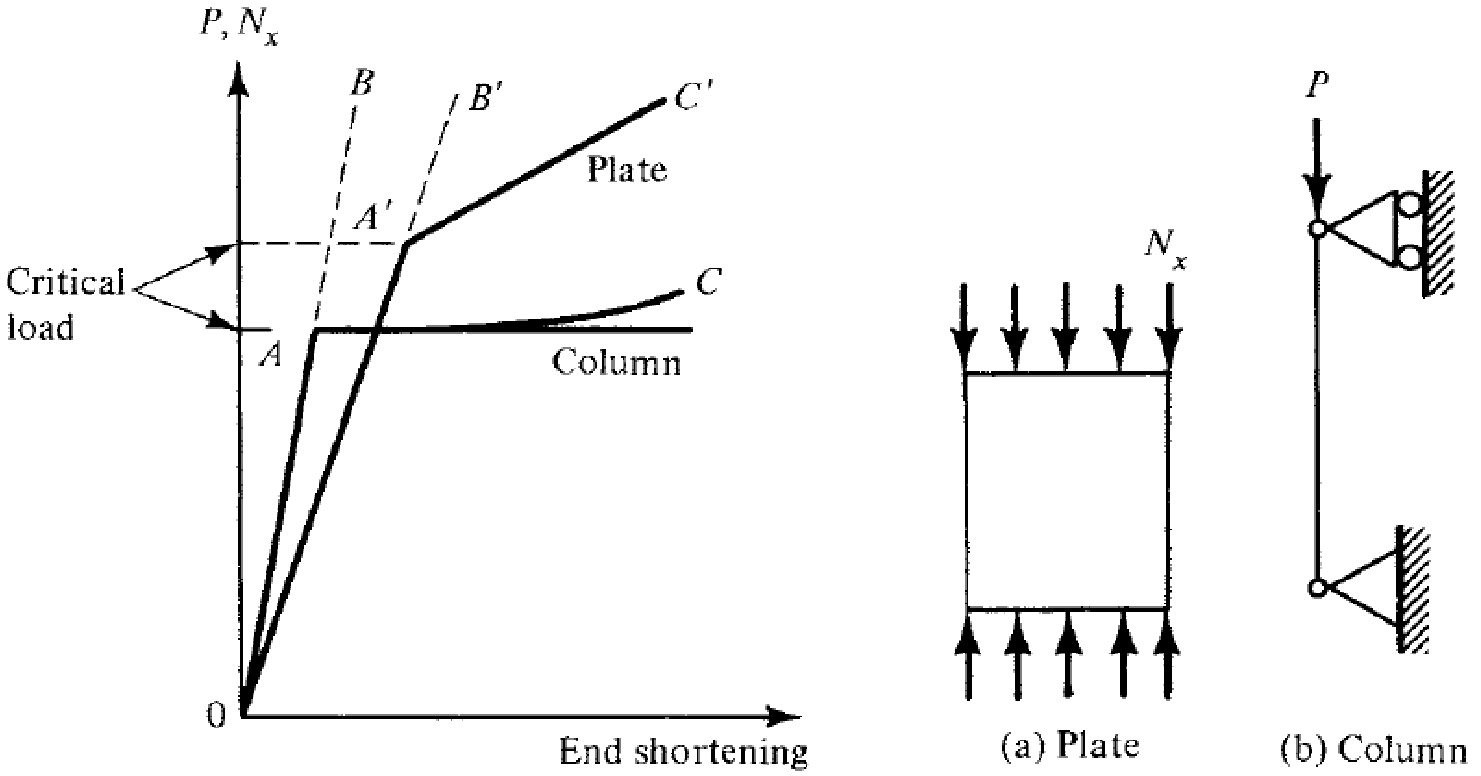


Figure from Fundamentals of Structural Stability by George J. Simitses and Dewey H. Hodges, available online

Finite-disturbance or snap-back buckling

- For some structures, the loss of stiffness after buckling is so great that the buckled equilibrium configuration can only be maintained by returning to an earlier level of loading.

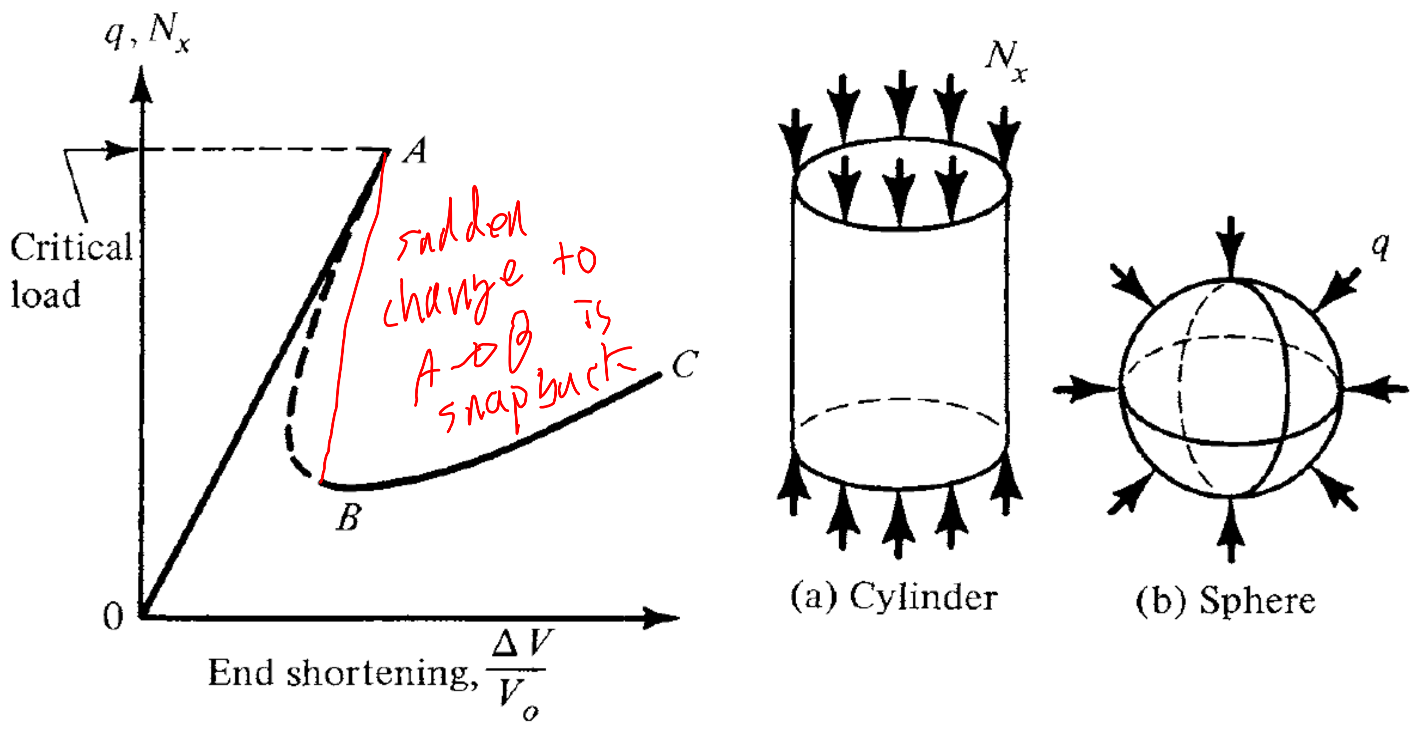


Figure from Fundamentals of Structural Stability by George J. Simitses and Dewey H. Hodges, available [online]

Oil-canning or snap-through buckling

Often occurs when controlling load/displacement.

By compressing it, it can support more load

- This phenomenon is characterized by a visible and sudden jump from one equilibrium configuration to another equilibrium configuration for which displacements are larger than in the first (nonadjacent equilibrium states).

but once it passes a flat state

it will rapidly fail

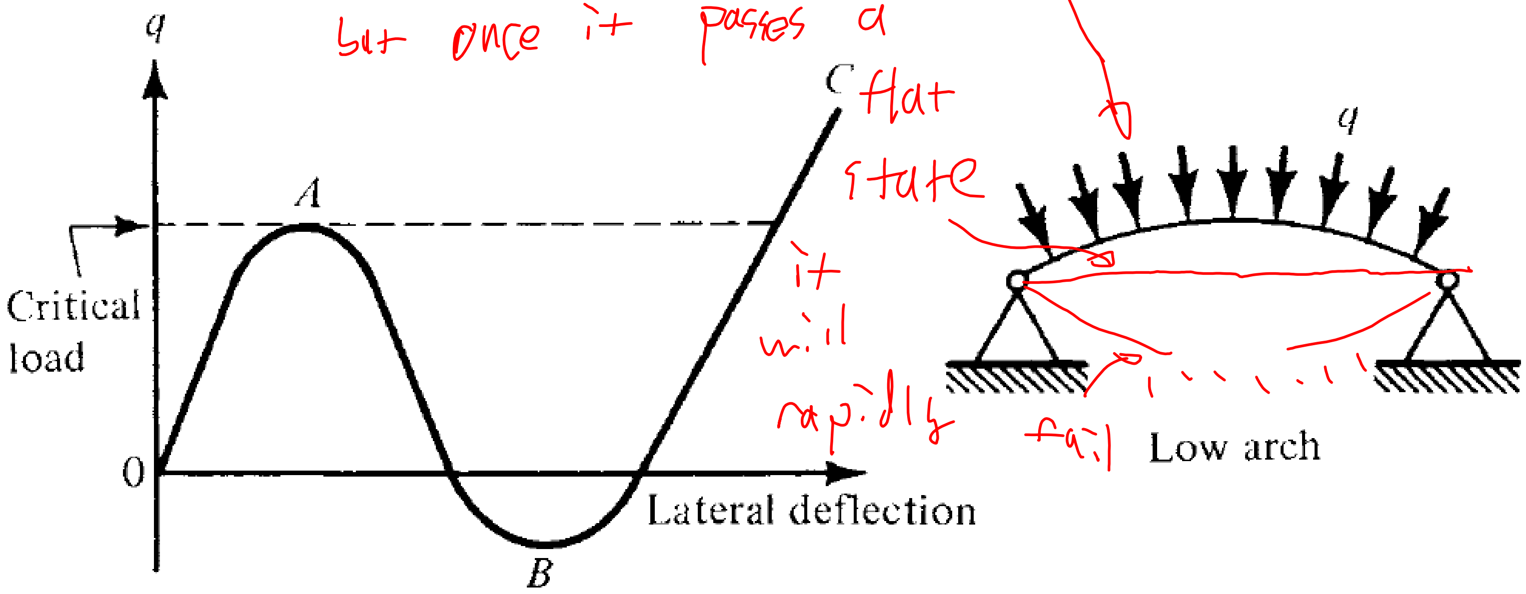


Figure from Fundamentals of Structural Stability by George J. Simitses and Dewey H. Hodges, available [online]

Examples of buckling

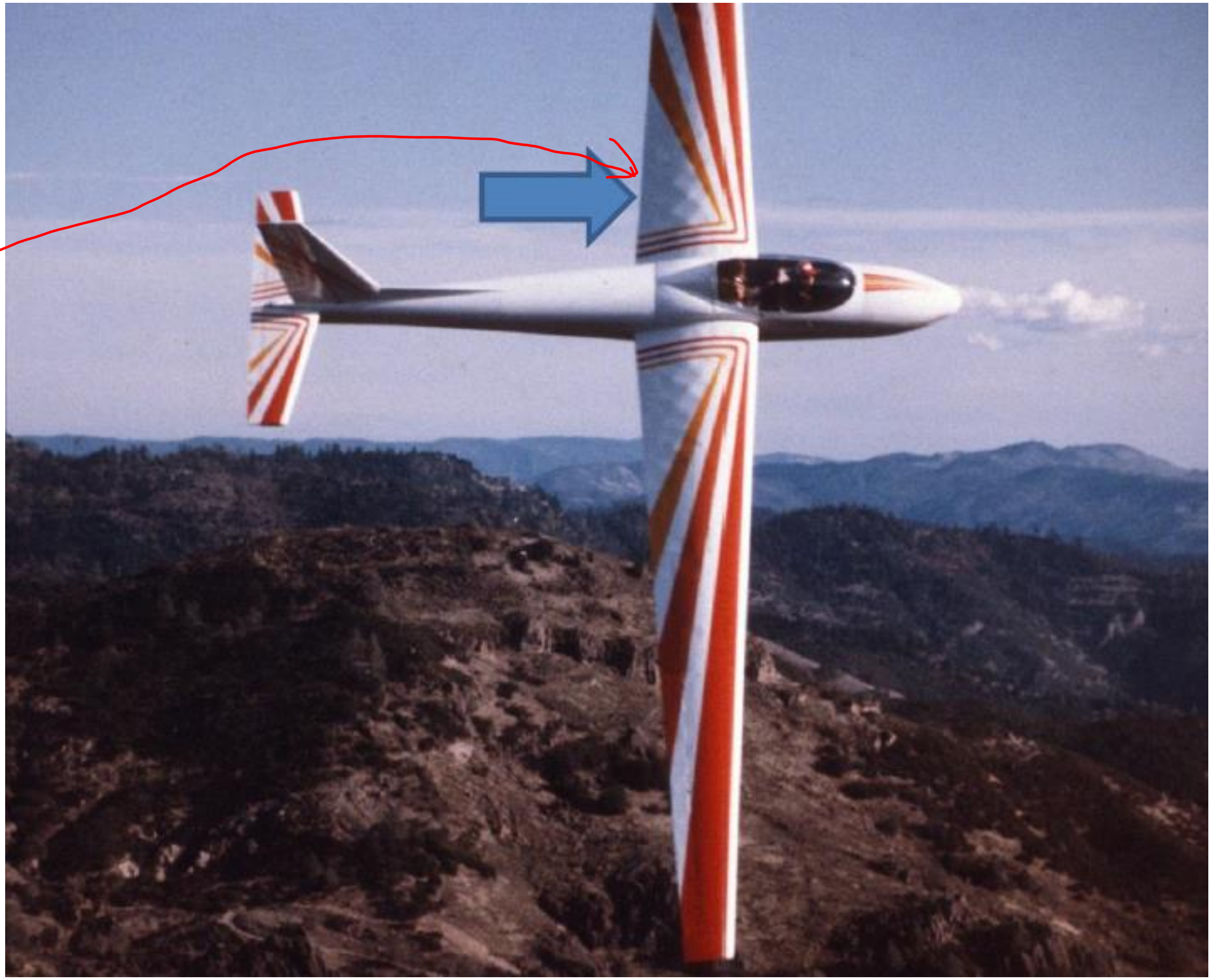
The containership "MOL Comfort" breaks in two

- On June 17 2008 Mitsui O.S.K. Lines' "MOL Comfort" experienced severe hogging and broke in two.

Local buckling of the top surfaces of the wings of a glider

Secondary structures can buckle without issues eg: skin

but primary eg: frame can't without issues



http://shellbuckling.com/index.php

Buckled wine tank following earthquake

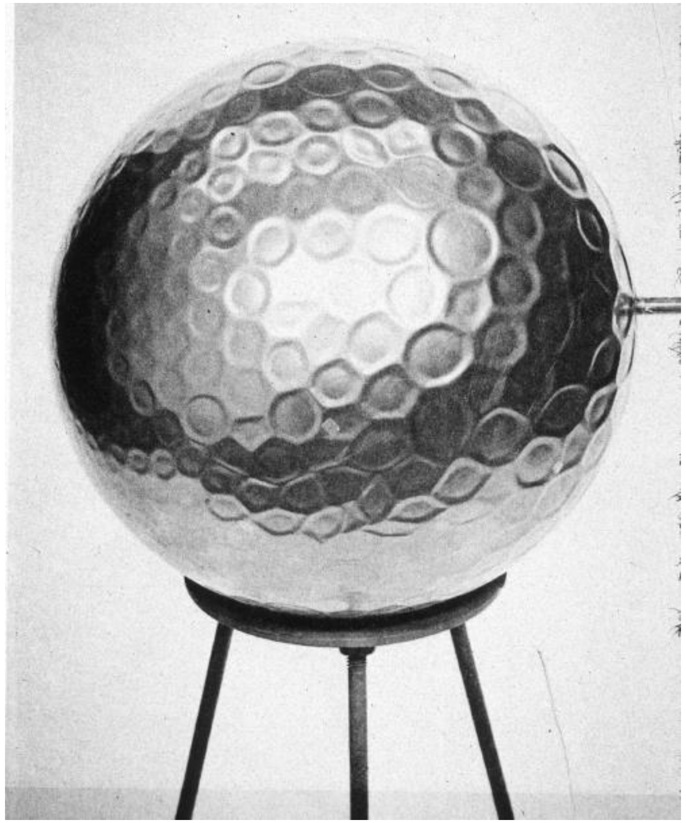
- The thin stainless steel wine tanks buckled in an earthquake near Livermore, California at the Wente Brothers winery in 1979.

buckled stainless steel wine tanks (shellbuckling.com)

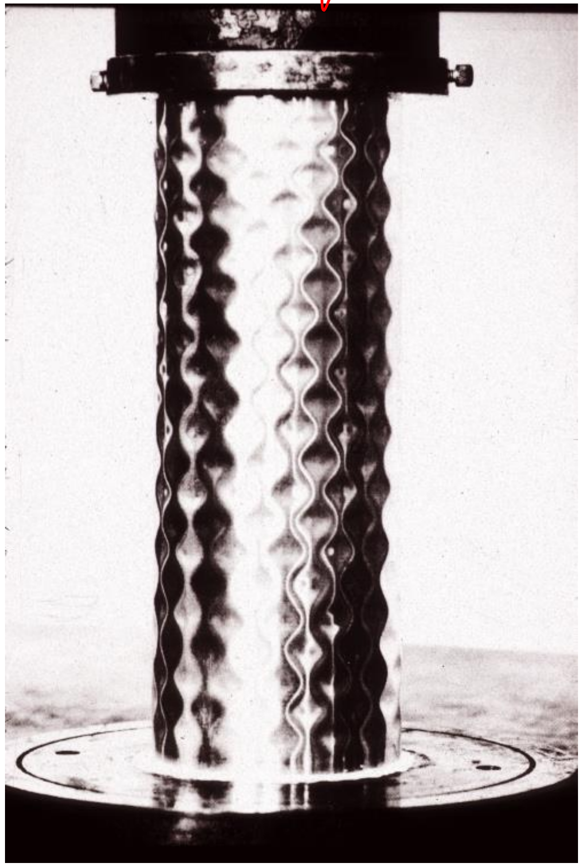
Thin shell cylinder and sphere

compression often has much more complex non linear relationships

- The post-buckling pattern (what you see in the pictures) is “artificially” stabilized because there is a solid mandrel inside the shell.



Sphere under external pressure



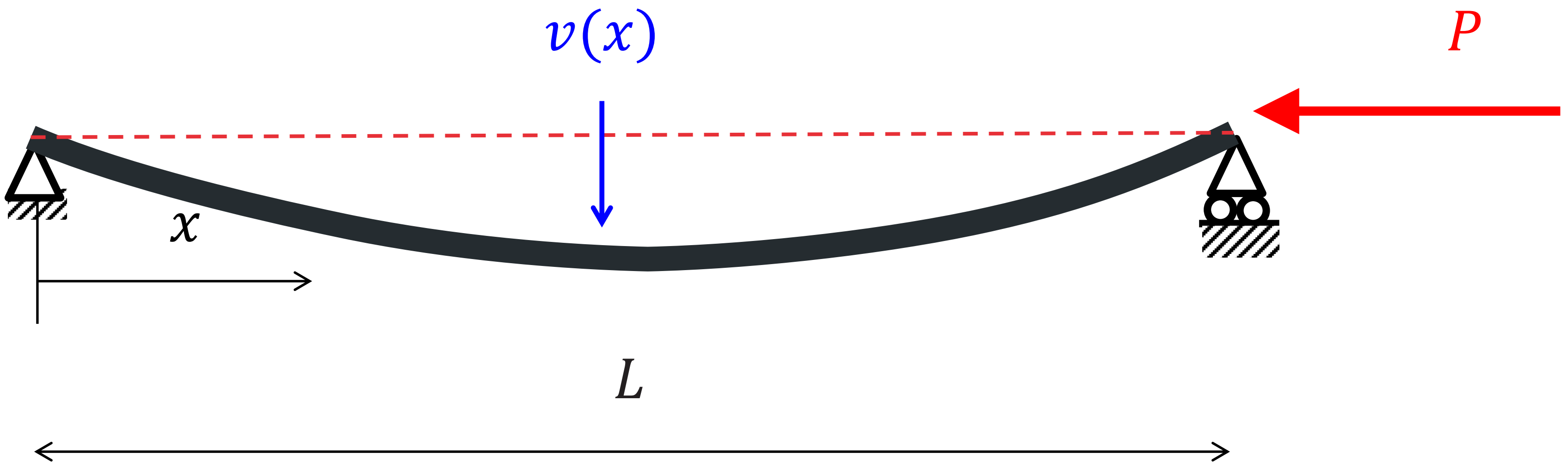
Cylinder under axial compression

Pictures from [very thin buckled externally pressurized spherical shell (shellbuckling.com)]

Euler buckling theory for beams

Euler buckling theory

- Motivation: Finding the critical load beyond which the beam becomes unstable.



Question

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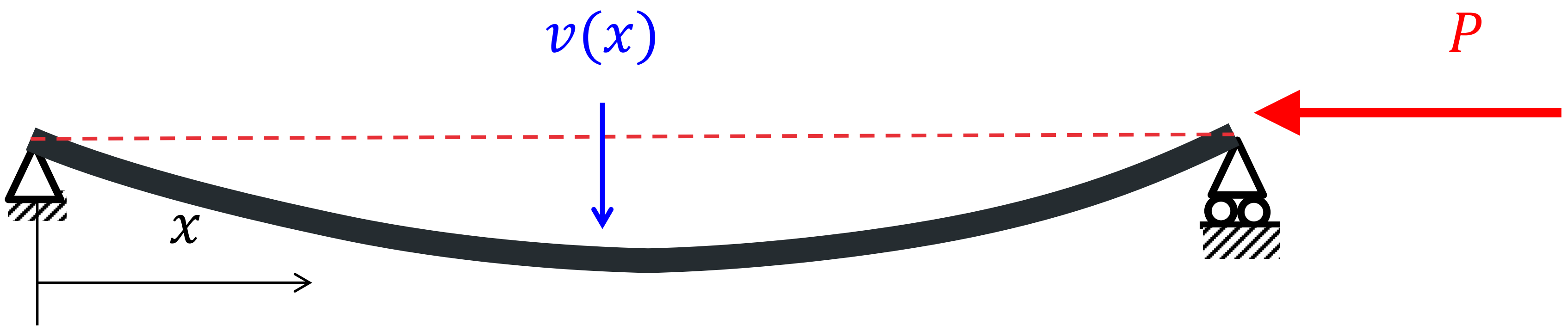
- Which statement is incorrect for an Euler beam under compression?

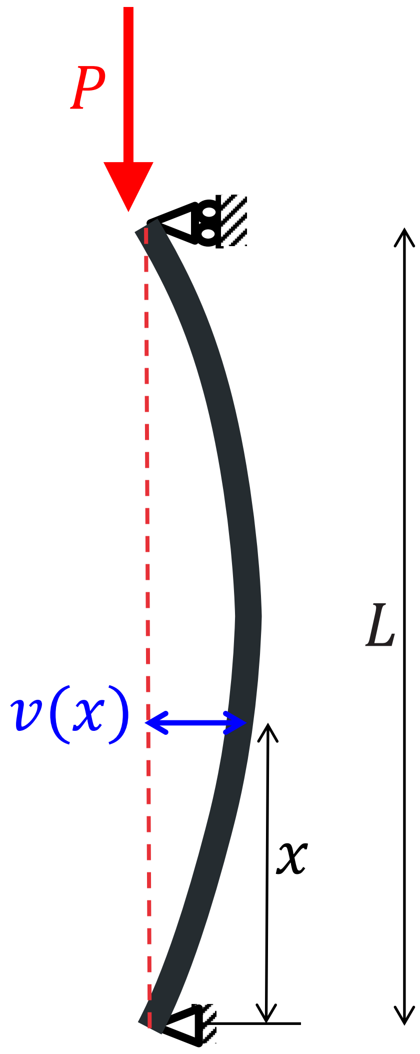
1. The stability of the beam is not a function of the disturbance magnitude.
2. There are infinite number of solutions that can satisfy the BCs.
3. Materials with higher Yield stress buckle at higher loads.
4. Sections with lower I buckle at lower loads.

often won't reach yield



Assumptions

- The column is initially perfectly straight with a uniform section
- Homogenous, isotropic, linear elastic material behaviour
- The compressive load is applied through the centroid



Free body diagram of part of the beam

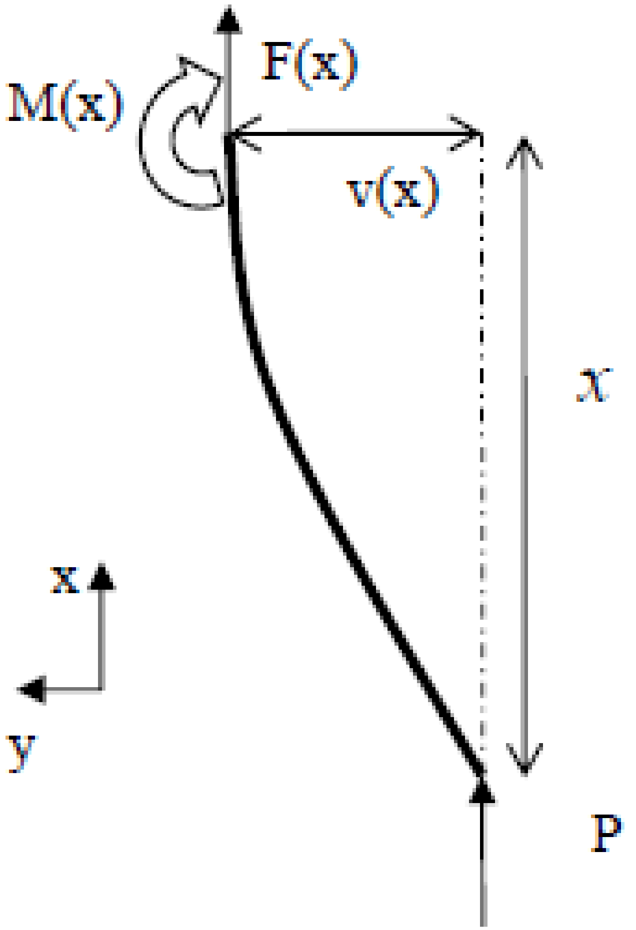
- Draw an FBD of the buckled column, take a cut and apply equilibrium: $\not$ moment equilibrium

$$M(x) - Pv(x) = 0 \Rightarrow M(x) = Pv(x)$$

Whereas $M(x)$ is the internal moment at x and $v(x)$ is the deflection in y direction.

- From Statics, we know that:

$$M(x) = -EI\frac{d^2v(x)}{dx^2}$$



Finding the differential equation

- Combine the equations to obtain the second order differential eq. for the buckled column:

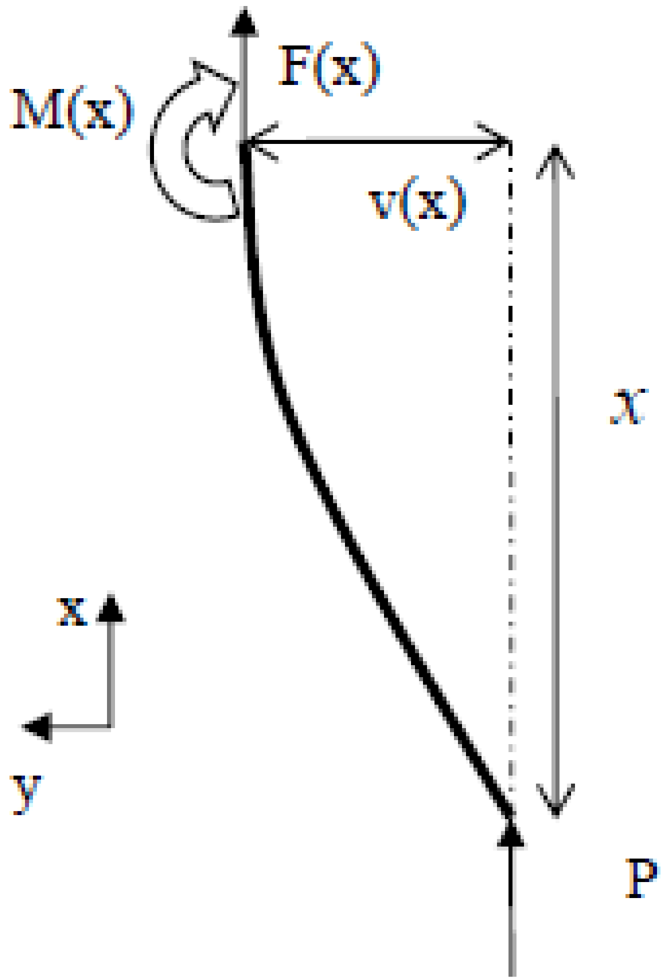
$$\frac{d^2v(x)}{dx^2} + \frac{P}{EI}v(x) = 0$$

$$\frac{d^2v(x)}{dx^2} + \mu^2 v(x) = 0 \ \text{ where } \ \mu^2 = \frac{P}{EI}$$

- The general solution for this equation is:

$$v(x) = Asin(\mu x) + Bcos(\mu x)$$

- Check this by differentiating and subbing into the DE!



Applying boundary conditions and finding P

- Applying the boundary conditions of the column (assume pinned-pinned):
  - $v(0) = 0$, so $B = 0$

    top and bottom have no displacement

    Two possible solution cases

  - $v(L) = 0$, so $Asin(\mu L) = 0 \Rightarrow \begin{cases} (1) \quad A = 0 \Rightarrow v(x) = 0 \\ \text{or} \\ (2)\ \mu = \frac{n\pi}{L} \Rightarrow v(x) = Asin(\frac{n\pi}{L}x) \end{cases}$ where n = 1,2,3…

    trivial solution

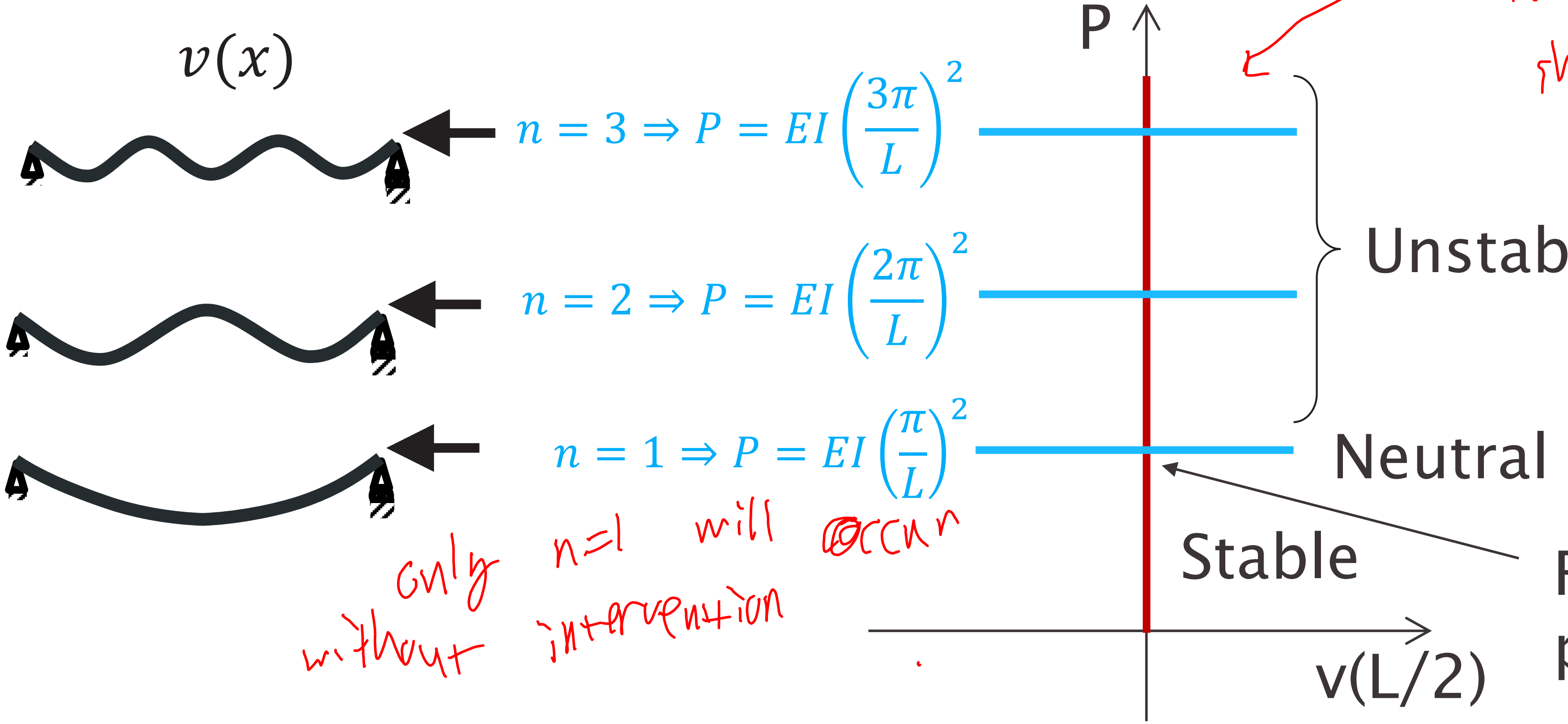
    n number of solutions

- Substituting v(x) into $\frac{d^2v(x)}{dx^2} + \frac{P}{EI}v(x) = 0$ to find P:
  - If $v(x) = 0 \Rightarrow 0 + \frac{P}{EI}0 = 0 \Rightarrow$ P can be any number! trash
  - If $v(x) = Asin(\frac{n\pi}{L}x) \Rightarrow -A\left(\frac{n\pi}{L}\right)^2 sin\left(\frac{n\pi}{L}x\right) + \frac{P}{EI}Asin\left(\frac{n\pi}{L}x\right) = 0 \Rightarrow P = EI\left(\frac{n\pi}{L}\right)^2$

Load-deflection graph

1. $v(x) = 0$ and P can be any number.
2. $v(x) = A\sin(\frac{n\pi}{L}x)$ and $P = EI\left(\frac{n\pi}{L}\right)^2$ with n=1,2,3…

n solutions, depending on loaded shape you can change loading capability

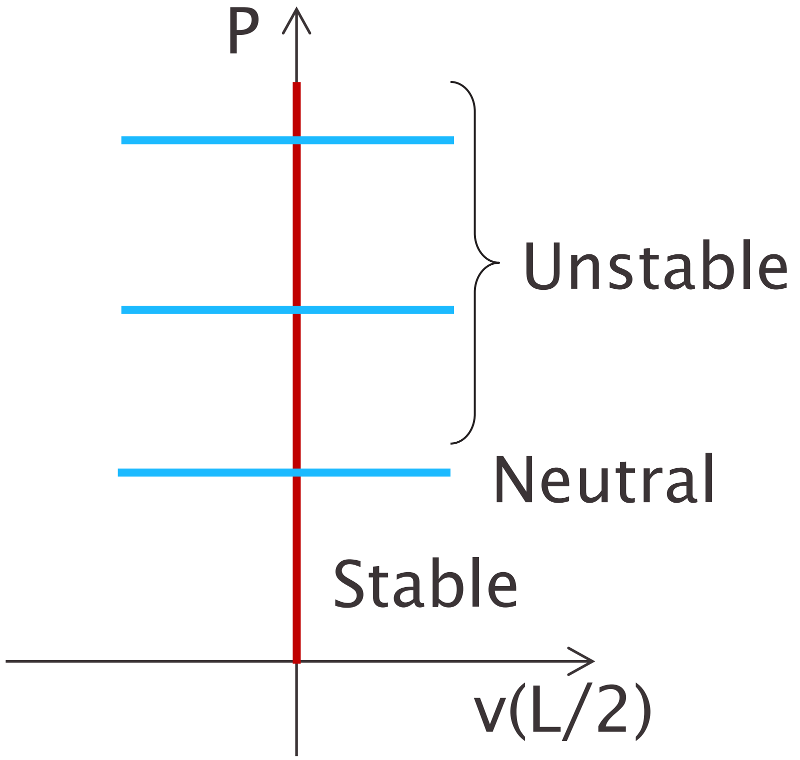


Only n=1 will occur without intervention

To get a solution at n>1 you need to artificially change conditions

Buckling of an Axially Loaded Column

- The first buckling mode requires the lowest force so this one is 'critical': $\boxed{P_{crit} = \frac{\pi^2 EI}{L^2}}$ For pinned-pinned BCs
- If P equals or exceeds 'P_{crit}' then the column is unstable and will fail by buckling.
- This equation tells us:
  - Stiffer materials (higher E) are more resistant to buckling
  - The column will buckle about the axis with lowest I
  - The critical load decreases with L^2

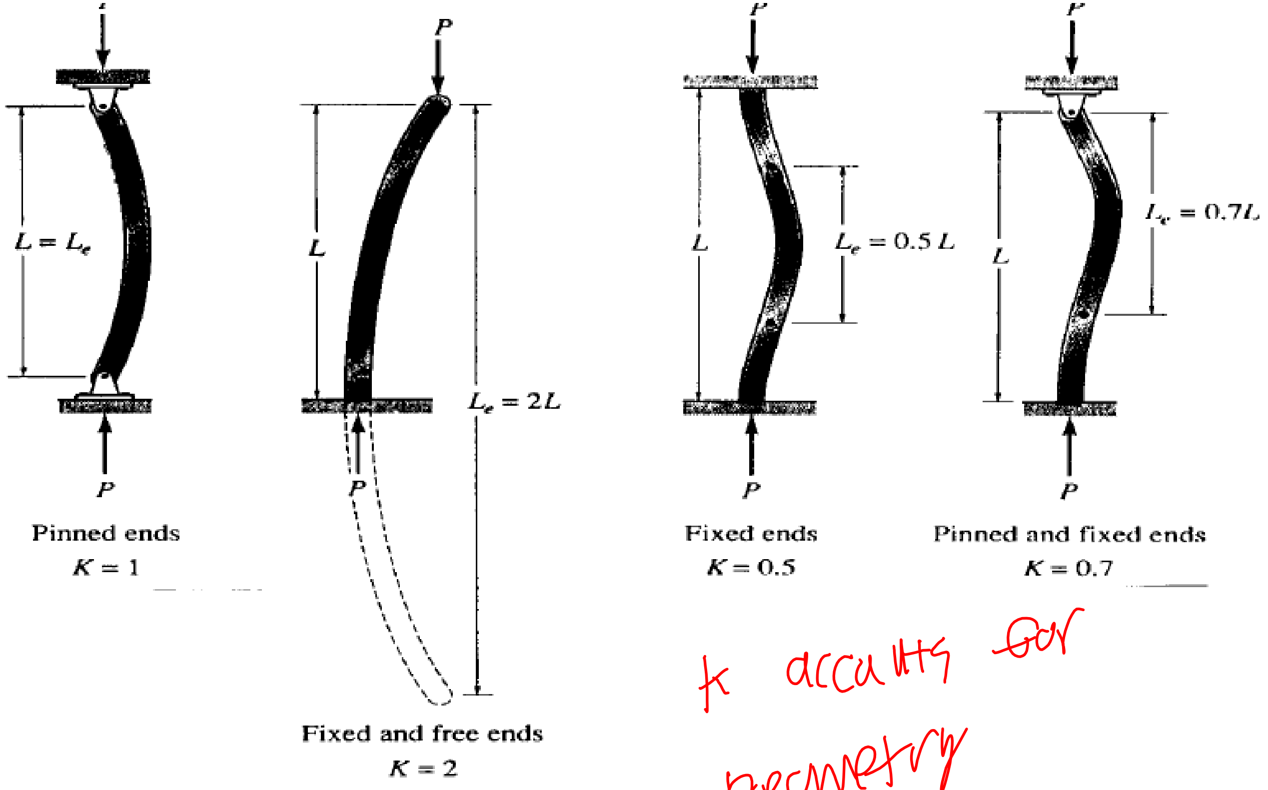


Effect of Boundary Conditions

- Previously we assumed pinned-pinned boundary conditions, the general form of the critical buckling load equation is:

$$\boxed{P_{crit} = \frac{\pi^2 EI}{(KL)^2}}$$

- Where 'K' is dependent on the
  - Pinned-pinned: $K = 1$
  - Fixed-free: $K = 2$
  - Fixed-fixed: $K = 0.5$
  - Fixed-pinned: $K = 0.7$



K accounts for geometry

Next lecture: response of imperfect columns

- We will consider imperfect/realistic columns:
  - Eccentric loading
  - Initially curved columns