SESA3029 Aerothermodynamics

Lecture 5.2

Convective heat transfer

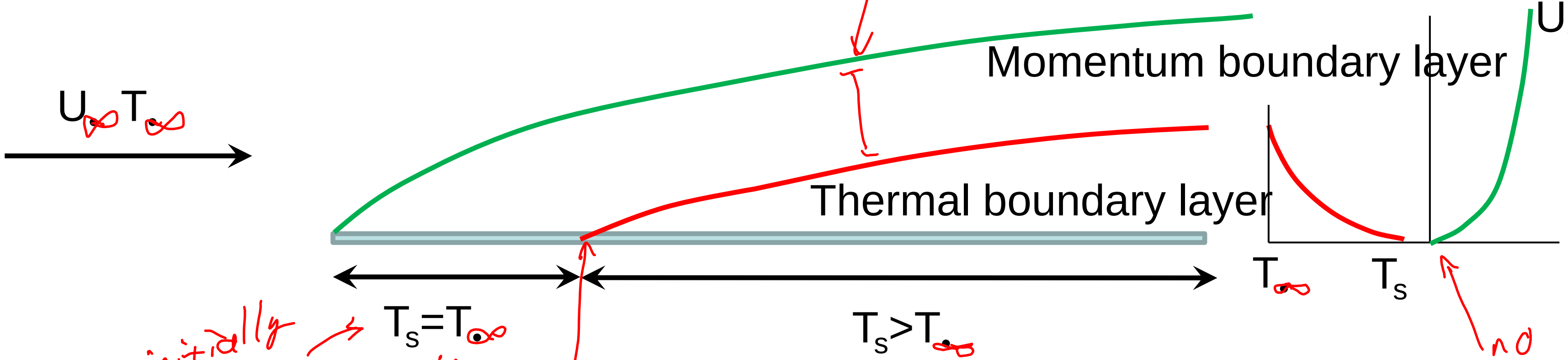
Convection heat transfer

importantly looking at flow — this is low speed

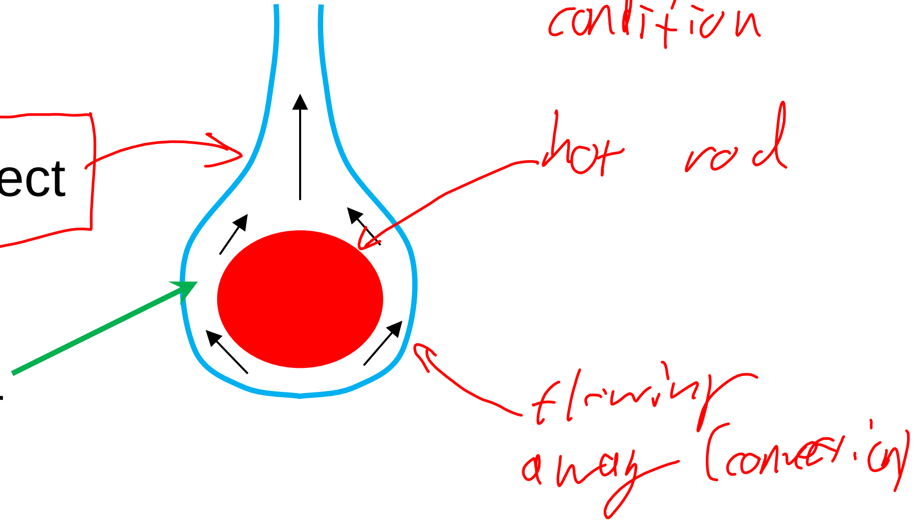
- Forced convection
  - In which a fluid passes over a hot surface, e.g. low-speed flow over a partially heated plate

core observation: temp boundary layer is same shape as momentum layer

This is subsonic



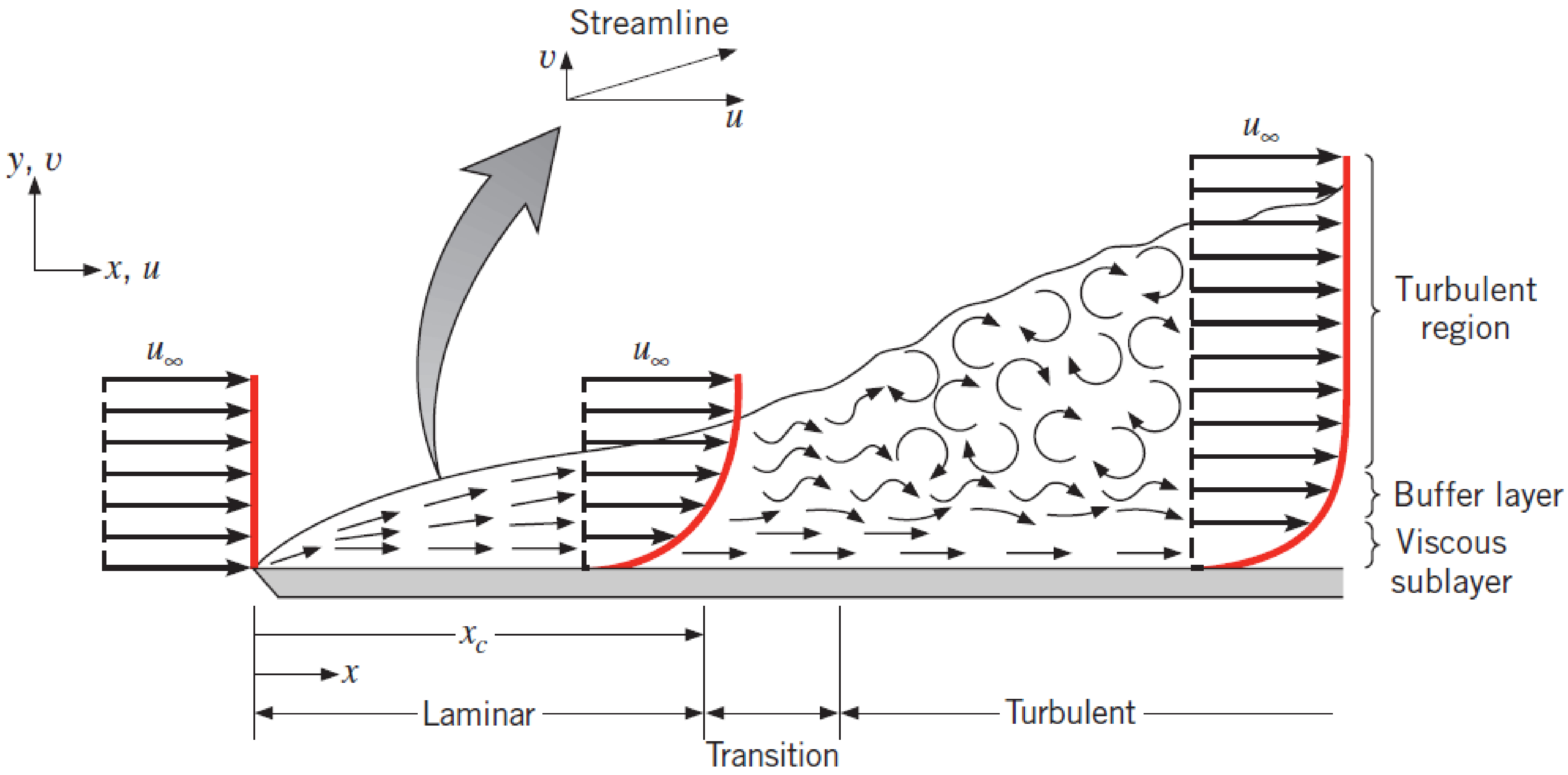
- Natural convection
  - In which the flow is generated by the hot object
  - e.g. stationary heated rod



Forced convection

- Laminar boundary layer
  - At lower Re and in quiet environments we have steady laminar flow, with no disturbances and zero relative velocity at the wall (no slip condition)
  - Internal energy conducted from the hot surface is swept downstream by convection leading to a thermal boundary layer
- Turbulent boundary layer
  - At higher Re and in noisy environments the boundary layer is likely to be turbulent with 3D irregular eddying motions
  - Large exchange of momentum and internal energy across the boundary layer.

Velocity boundary layer on flat plate



Ref: Bergman et al. ‘Fundamentals of heat and mass transfer’

Convection heat transfer coefficient *h*

$$\dot{q} = h\,\Delta T$$

surface temp

freestream temp

In our boundary-layer example

$$\Delta T = T_s - T_\infty$$

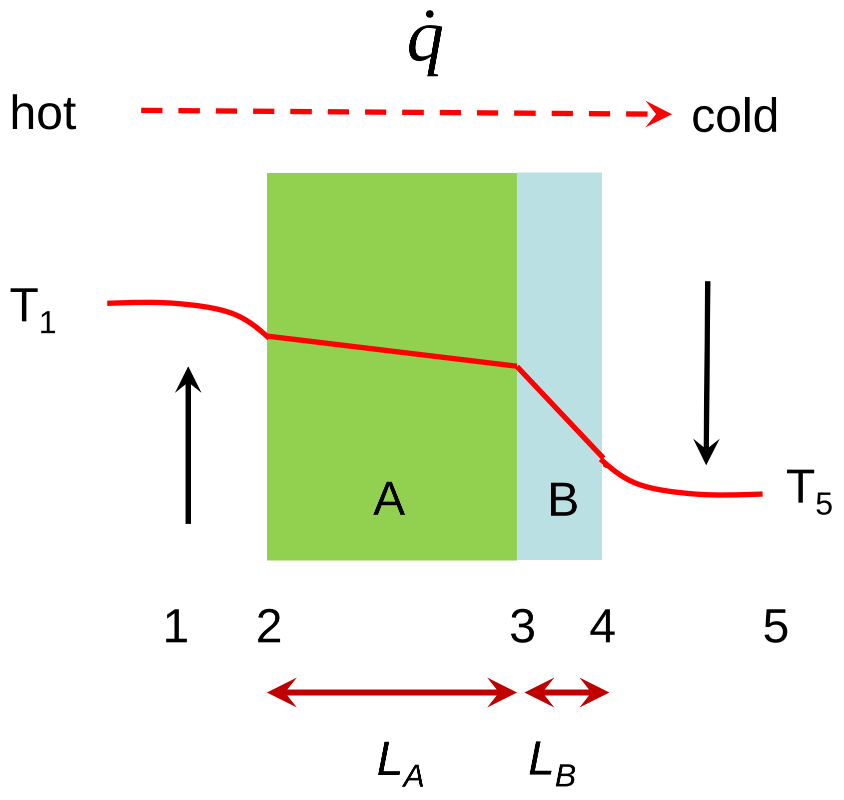
sign convention not rigid, so understand direction of heatflow.

The electrical analogy still holds, so we can define a thermal resistance as

$$R = \frac{1}{hA}$$

and easily combine with solid wall conduction

Example



$$RA = \overbrace{\frac{1}{h_1}}^{\text{hot side convection}} + \overbrace{\frac{L_A}{k_A} + \frac{L_B}{k_B}}^{\text{conduction}} + \overbrace{\frac{1}{h_5}}^{\text{cold side convection}}$$

$$\dot{q} = \frac{T_1 - T_5}{RA}$$

How to find *h*?

(very fucking difficult)

- Exact relations for some (laminar) flows
- Empirical relations involving dimensionless quantities
  - standard flow types only
- Solution of boundary-layer equations
  - thermal and momentum
- CFD

nice short cut

Dimensionless variables

Nusselt number $\mathrm{Nu} = \dfrac{hL}{k}$

Stanton number $\mathrm{St} = \dfrac{h}{\rho V c_p}$

don't need to know details, they are just useful for describing h

Expressed in terms of: Reynolds number $\mathrm{Re} = \dfrac{\rho V L}{\mu}$

Prandtl number $\mathrm{Pr} = \dfrac{c_p \mu}{k}$

Note that: $\boxed{\mathrm{Nu} = \mathrm{St}.\mathrm{Re}.\mathrm{Pr}}$

so we can interchange Nu and St

Some useful correlations (laminar flow)

through pipe → reached state ∴ steady conditions

Laminar pipe flow (Re<2000)

$$\mathrm{Nu} = 4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu} = 3.658 \text{ (for uniform wall temperature)}$$

over flat plate → not steady, varys along plate

Laminar boundary layer(Re<300,000)

$$\mathrm{Nu}_x = 0.453 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x = 0.332 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

General form (Reynolds' analogy)

$$\mathrm{Nu} = C\ \mathrm{Re}^n\ \mathrm{Pr}^m$$

Refs: Kays & Crawford 'Convective Heat and Mass Transfer',
Ref: Bergman et al. 'Fundamentals of Heat and Mass Transfer

Example: boundary layer cooling

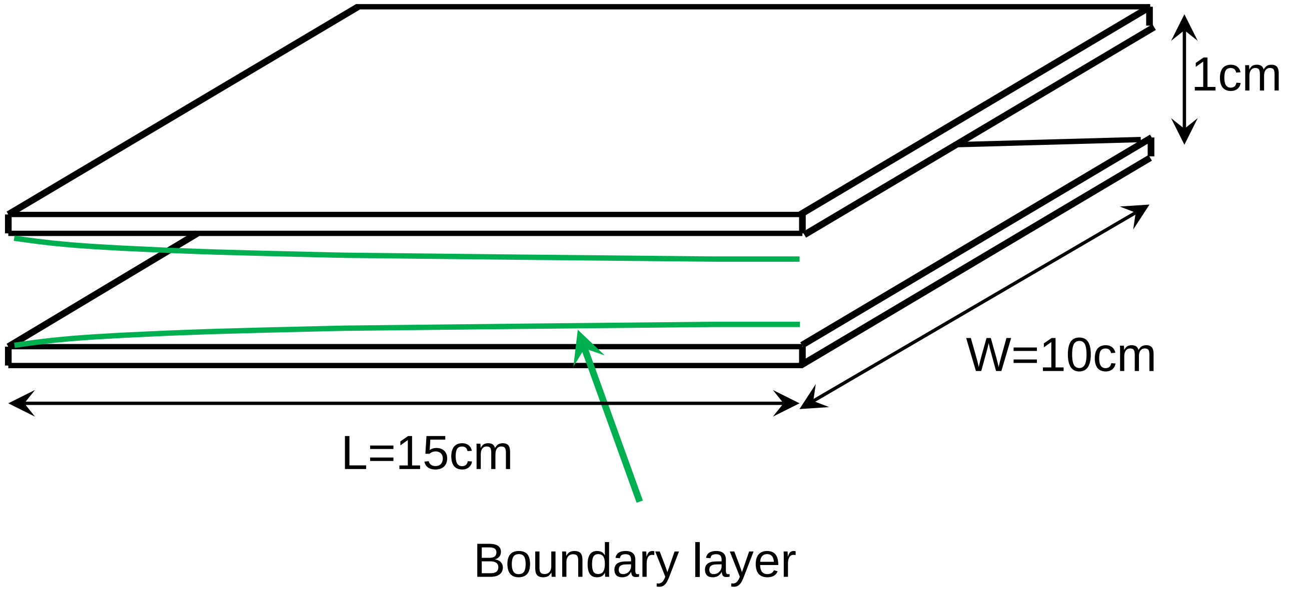
Two parallel plates at T=40°C

Air 8m/s, 2bar, 15 °C

μ=1.78x10^{-5} Ns/m^2

k=0.0248 W/(mK)

Pr=0.72



a) Check whether the flow is laminar or turbulent
b) Find the heat transfer rate through each plate
c) Check the boundary layer thickness at the outflow

$Nu = St \cdot Re \cdot Pr$

average heat transfer coefficient

$$h(x) = \frac{Nu_x\, u}{x} \qquad \bar{h} = \frac{1}{L}\int_0^L h(x)\,dx$$

use in calc

$$\bar{h} = \frac{1}{L}\int_0^L 0.332\left(\frac{\rho V x}{\mu}\right.$$

(unfinished)

$$\dot{Q} = \bar{h} A (T - T_\infty)$$

$$\bar{h} = \frac{0.332\left(\frac{\rho V}{\mu}\right)^{N_2} P_r^{N_3} k}{L}\int_0^L x^{-N_2}\,dx$$

$\bar{h} = 39.76\ W/(m^2 K)$

$\dot{Q} = 39.76 \times 0.1 \times 0.15 \times 25 = 14.91\ W$

$P = 2\times10^5$ $R = 287$ $T = 288.15$

from data

$\rho = \frac{P}{RT} = 2.42\ kg/m^3$

$Re = \frac{\rho V L}{\mu} = 163261$ (at end of plate)

laminar!

$\delta_L = \frac{5L}{Re_L^{\frac{1}{2}}} = 1.86\times10^{-3}\ m$

layer thickness

hence boundary layers don't meet