Part 4: Turbomachinery and Propellers

9 PRINCIPLES OF TURBOMACHINERY (WEEK 9)

9.1 LEARNING OUTCOMES

After studying this Section you will be able to:

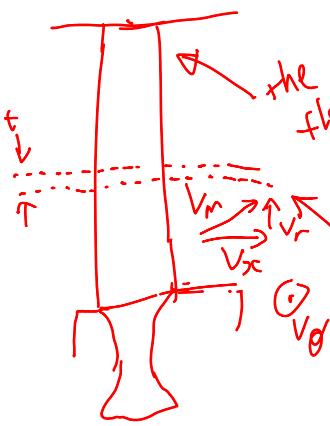
- Use the Euler work equation to relate turbomachinery blade geometry, changes in flow properties, and power transfer.
- Explain the working principles of turbomachinery
- Represent the change of thermodynamic state through turbomachinery stages on state diagrams.
- Draw and use velocity triangles to calculate flow angles and velocity components in absolute and rotating frames of reference.

9.2 INTRODUCTION

Turbomachinery transfers energy between a rotor and a fluid by the aerodynamic action of steadily-rotating rotor surfaces. Turbines transfer energy from the fluid to the rotor. Compressors transfer energy from the rotor to the fluid. Turbomachinery contrasts with positive-displacement expanders and pumps where the enclosed volume changes over time, for example by moving a piston.

The compressors and turbines in jet engines are obvious examples of turbomachines in propulsion systems, but other examples include aircraft propellers and fuel turbo-pumps in rocket engines. Two limiting cases of turbomachine can be distinguished: axial flow and radial flow machines. Axial flow turbomachines, such as the multis-stage compressor and high-pressure turbine stage shown in Figure 1 have minimal velocity in the radial direction. Radial flow machines, such as the rocket propellant pump in Figure 1, have practically no axial flow velocity at inlet to or outlet from the rotor.

In practice it is rare to have exclusively axial or radial flow in a turbo machine. For example, large civil turbofan engines are considered to be axial-flow machines, however there is usually a significant change in radius of the blading (requiring at least some flow in the radial direction) as the air is compressed by a factor of more than forty times through the compressor. Another example that is not purely 'axial' or 'radial' is the 'centrifugal' impeller in a turbocharger compressor which has axial flow at its inlet and radial flow at the outlet (see Figure 1)



the geometry and the flow at the top differs greatly to the root so we often work with stream tubes along the blade.

V_m = meridial velocity
V_r = radial velocity
V_x = axial
V_θ = circumferential/swirl

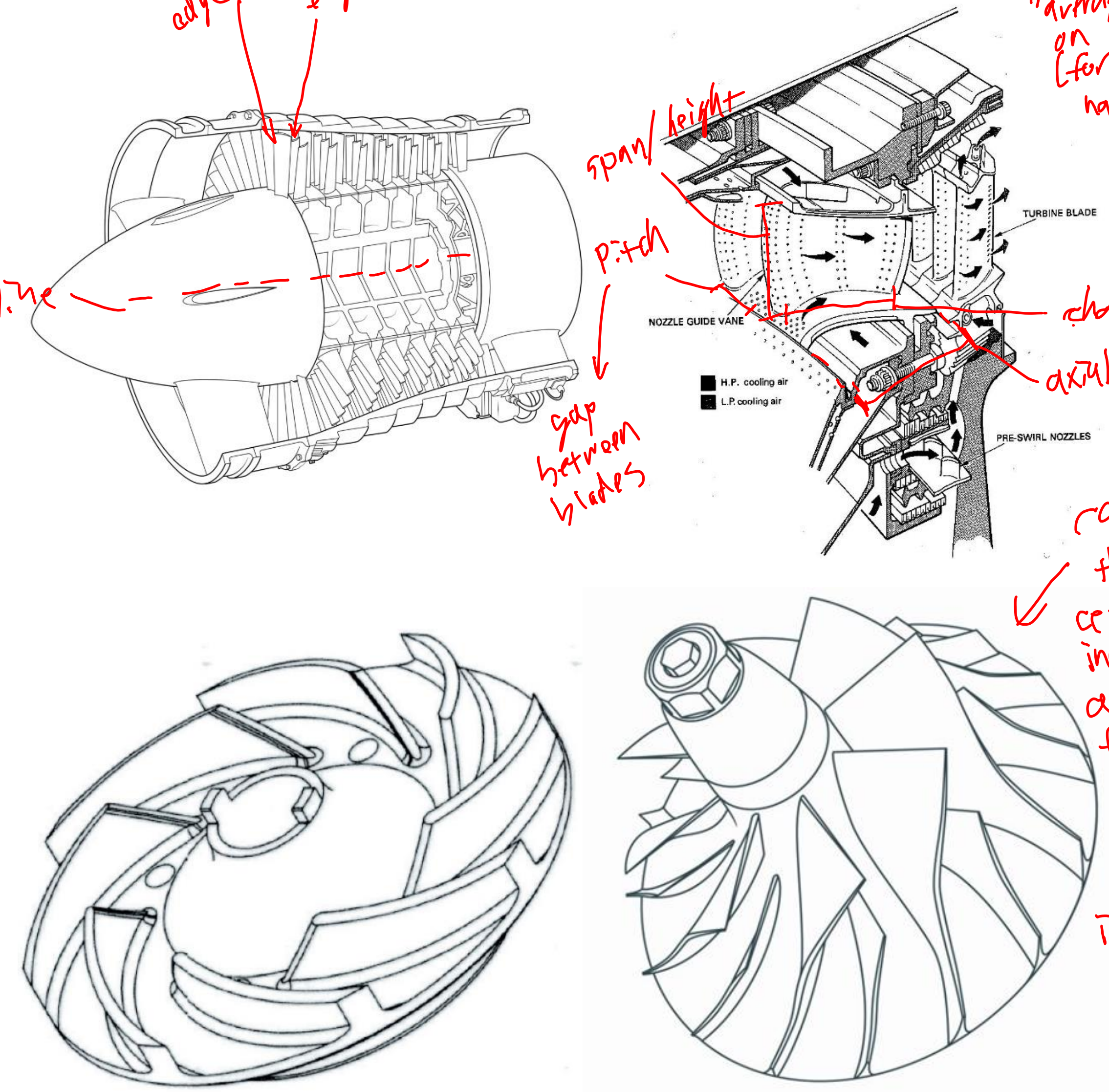


Figure 1. Multi-stage axial compressor; High pressure axial turbine stage (nozzle guide vane and first rotor); liquid-oxygen rocket pump impeller; turbocharger impeller.

In all cases, the device works by a change of angular momentum (swirl) in the flow producing/resulting from a torque on the rotor. The torque can be analysed with Newtons' Second Law of Motion (torque equals rate of change of angular momentum in the fluid), and the change in the thermodynamic properties of the fluid can be analysed using the First Law of Thermodynamics (stated as the Steady-Flow Energy Equation). Combining these two Laws leads to the Euler work equation.

9.3 THE EULER WORK EQUATION

The Euler equation describes the work input or output in a turbomachinery flow. Figure 2 shows flow through a hypothetical rotor with angular velocity Ω.

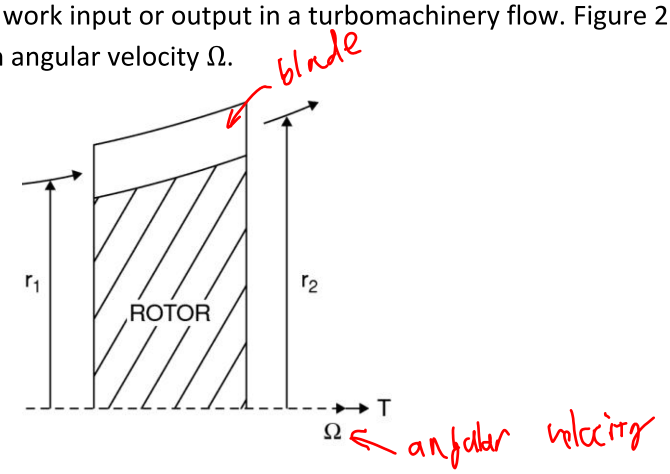


Figure 2. A hypothetical rotor for which flow enters at radius r_1 and leaves at radius r_2. The torque created is T and the rotor rotates at Ω radians/s.

Considering an infinitesimal packet of fluid with mass $\delta m = \dot{m}dt$ that enters the rotor. The moment of momentum of this packet is $\delta m r_1 V_{\theta 1}$ at inlet and $\delta m r_2 V_{\theta 2}$ at exit, where V_θ is the velocity component in the circumferential direction. The torque is equal to the rate of change of moment of momentum, given by

$$T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}).$$

and the power input is given by,

$$\dot{W}_x = T\Omega = \dot{m}\Omega(r_2 V_{\theta 2} - r_1 V_{\theta 1}),$$

$$\dot{W}_x = T\Omega = \dot{m}(U_2 V_{\theta 2} - U_1 V_{\theta 1}).$$

where U_1 and U_2 are the speed of the solid rotor at inlet and exit[1].

For an adiabatic flow with no change in gravitational potential energy, the steady flow energy equation gives $\dot{W}_x = \dot{m}\Delta h_0$. Substitution into the equation for $T\Omega$ gives the Euler equation:

$$\Delta h_0 = U_2 V_{\theta 2} - U_1 V_{\theta 1}.$$

In axial-flow turbomachines it is common to have $r_2 = r_1$, which allows simplification to

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 1})$$

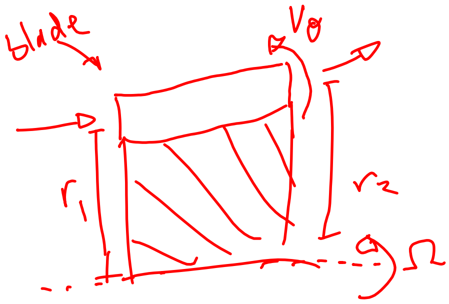
In many turbomachinery examples in this module we will work with an assumption that mean-line radius is constant, however this is not generally true, even in axial-flow turbomachines used in turbofan engines.

9.3.1 Centrifugal compressor example

A centrifugal compressor has an inflow at its centre and an outflow at its outer radius, as illustrated in Figure 3. The mass flow through the compressor is 10 kg.s^{-1}, the outer radius r_2 of the rotor is 0.1 m, the rotor turns at 20,000 rpm. The rotor has radial vanes – assume that the *relative* flow direction

[1]For a rotating solid object the speed of the surface of the object is radius multiplied by angular velocity: $U = r\Omega$.

Torque on rotor = Δ angular momentum flux (imparted on air)



$$T = (r_2 V_{\theta_2} - r_1 V_{\theta_1})\dot{m}$$

swirl

$$\text{Power} = T\Omega = \dot{m}\Omega(r_2 V_{\theta_2} - r_1 V_{\theta_1})$$

$u = r\Omega$ = blade speed

$$P = \dot{m}(u_2 V_{\theta_2} - u_1 V_{\theta_1})$$

SFEE: $\cancel{Q} - W = \dot{m}(h_{02} - h_{01})$

adiabatic

$$\boxed{h_{02} - h_{01} = u_2 V_{\theta_2} - u_1 V_{\theta_1}}$$

if $r_1 \approx r_2$ then $u_2 \approx u_1$

$$h_{02} - h_{01} = U(V_{\theta_2} - V_{\theta_1})$$

memorise derivation
equation not given!

applicability
- only applies over a rotor?
- no compressability assumptions
- no, but works for periodic unsteady flow
- assumes no heat transfer.

(V_2^{rel}) is aligned with the vanes as it leaves the rotor. Calculate the shaft torque and power required and the specific enthalpy change in the flow.

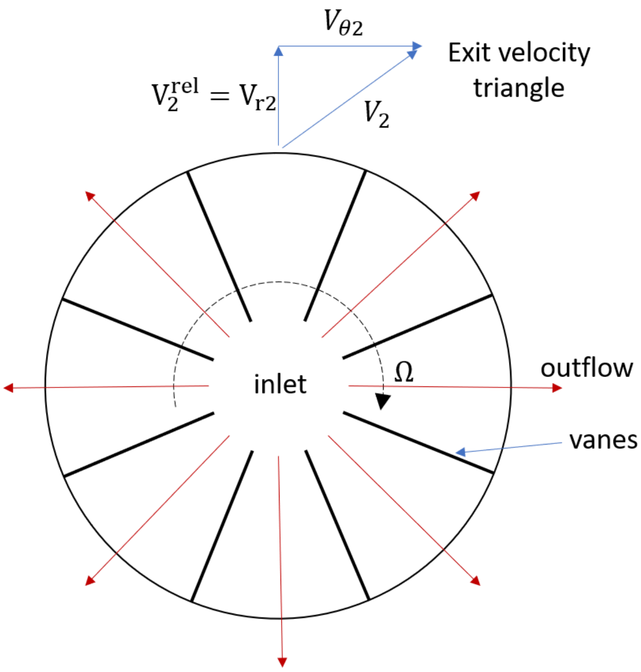


Figure 3. Impeller example.

The flow enters at the centreline without any angular velocity ($V_{\theta 1} = 0$).

Swirl velocity $V_{\theta 2}$ at exit is equal to blade velocity U.

The swirl velocity at rotor exit is radius multiplied by angular velocity (in radians per second):

$$V_{\theta 2} = r_2\Omega = 0.1 \times 2\pi \times \frac{20{,}000 \text{ revs/minute}}{60 \text{ seconds/minute}} = 209.5\ m.s^{-1}$$

The shaft torque is:

$$T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = 10(0.1 \times 209.5 - 0) = 209.5\ Nm.$$

The shaft power input is:

$$\dot{W}_x = \dot{m}(U_2 V_{\theta 2} - U_1 V_{\theta 1}) = \dot{m} r_2 \Omega V_{\theta 2} = 10 \times \left(0.1 \times \frac{2\pi \times 20{,}000}{60}\right)^2 = 438.8\ kW.$$

9.3.2 Axial turbine example

The flow through an axial-flow turbine rotor has a constant 0.3 m radius and 1000 radians/s angular velocity, as illustrated in Figure 4. The inlet swirl velocity is 500 m/s and the stagnation enthalpy drop through the turbine stage is $h_{01} - h_{02} = 200$ kJ/kg. What is the swirl velocity at the rotor exit?

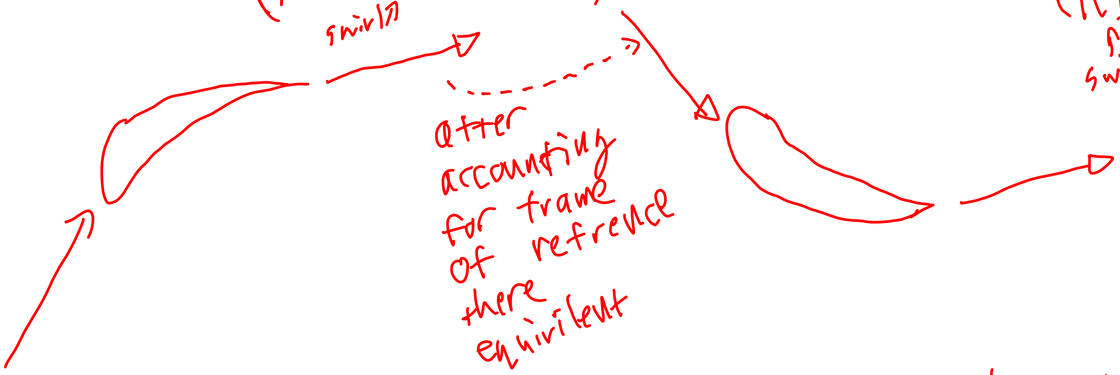
$$\dot{W}_x = \dot{m}\Omega(r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

$$\dot{w}_x = r\Omega(V_{\theta 2} - V_{\theta 1}) = h_{02} - h_{01}$$

$$V_{\theta 2} = V_{\theta 1} - \frac{h_{01} - h_{02}}{\mathrm{r}\Omega} = -167\ m.s^{-1}.$$

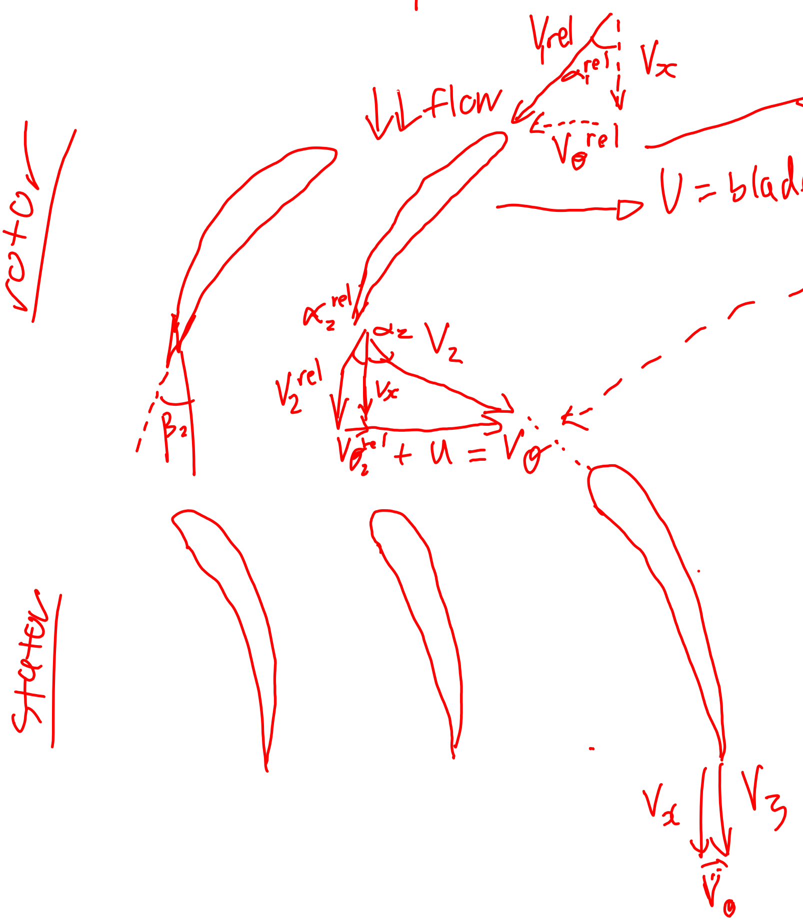
Rotor — adds swirl ($T \to KE + h_0 (+P)$)
swirl

Stator — removes swirl ($KE \to h_0 (+P)$)
swirl



after accounting for frame of refrence there equivilent

The KE added by the rotor $\approx$ KE $\to h_0$ in stator



$V_\theta^{rel} = V_\theta - U$ — converting V_θ relative to rotor into V_θ relative to stator (absolute ref frame)

U = blade speed.

Trig converting bits

$V_{\theta_2}^{rel} = V_{\theta_2} - U$

$V_2 = \sqrt{V_x^2 + V_{\theta_2}^2}$

$\alpha_2^{rel} \approx \beta_2$ — β_2 = camber angle out foil

$V_{\theta_2} = V_x \tan(\alpha_2^{rel}) = V_x \tan \beta$

$V_{\theta_2} = V_{\theta_2}^{rel} + U$

can be used to find Δh_0 → hence can find required β

Note that the angular velocity $V_{\theta 2}$ leaving the turbine is negative, implying that it is in the direction opposite to the blade motion.

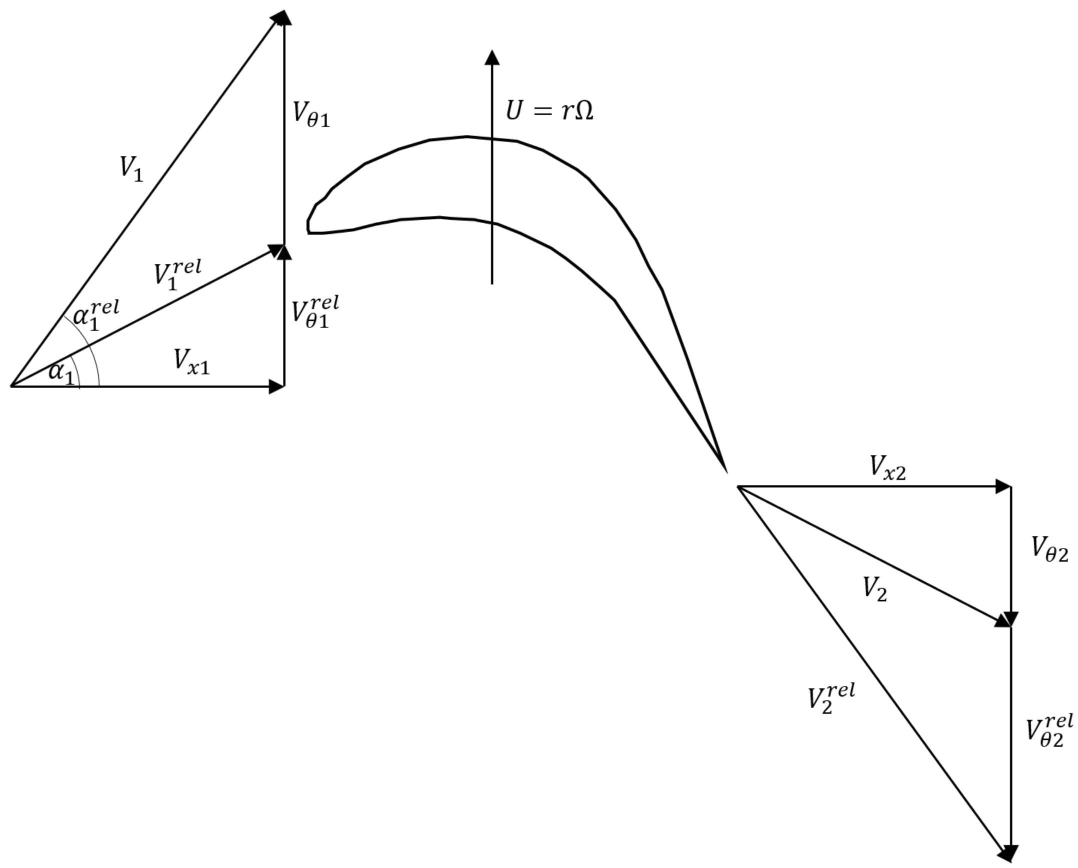


Figure 4. Turbine example.

Axial-flow turbomachinery is prevalent in aircraft engines as, compared to radial-flow or positive-displacement machines, it gives better efficiency at the high volume flow rates required for jet propulsion. The turbomachinery analysis in the remainder of the module focusses on axial-flow compressors and turbines, however the same principles apply to any turbomachine.

9.4 Description of axial-flow compressors and turbines

9.4.1 Axial-flow compressor

An axial flow compressor consists of rows of rotating and stationary blades called rotors and stators. Each rotor-stator pair is called a compressor stage. The pressure ratio for an individual axial-flow compressor stage is limited to less than 2 by the risk of flow separation on the aerofoils, so a multi-stage compressor is needed in order to achieve the overall pressure ratio needed for a gas turbine engine, as illustrated in Figure 5.

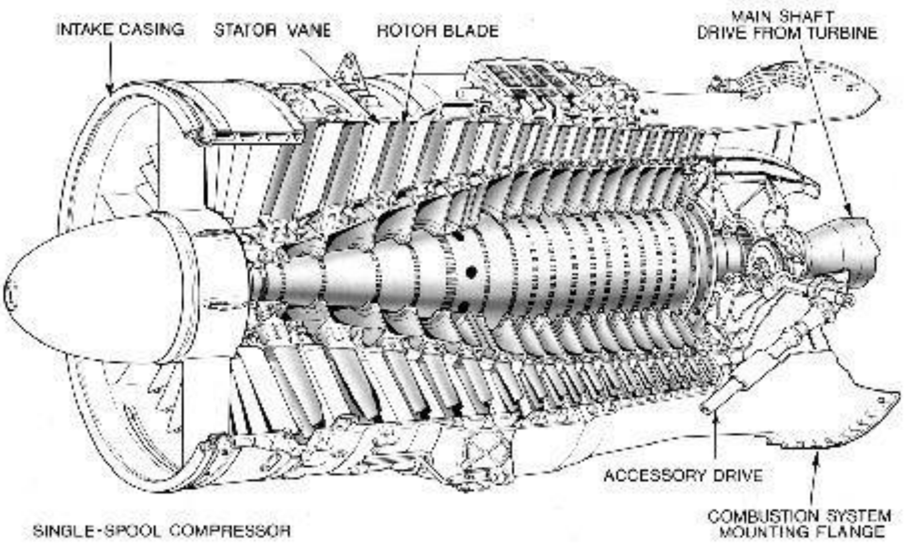


Figure 5. A typical multistage axial flow compressor (Rolls-Royce, 1992).

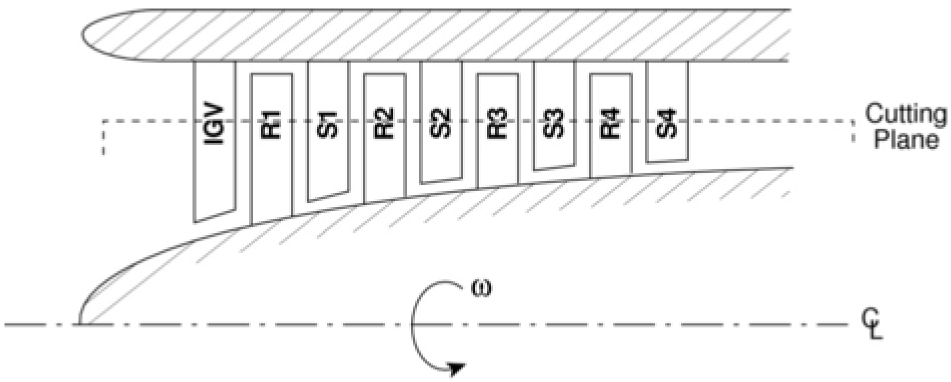


Figure 6. Schematic representation of an axial flow compressor.

The effect of the stator and rotor blades can be understood in terms of energy exchanges, with flow energy described by the total enthalpy:

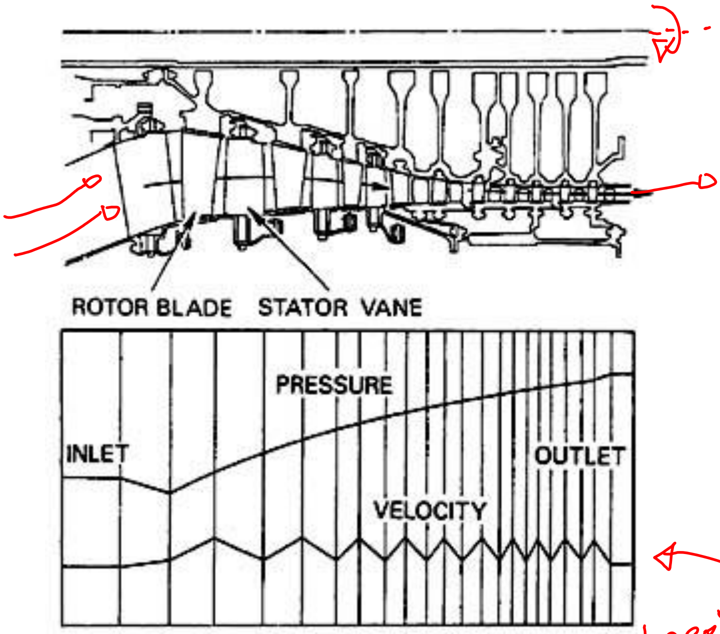
$$h_0 = h + \frac{1}{2}\left(V_x^2 + V_r^2 + V_\theta^2\right)$$

The torque on the moving rotor does work on the flow, increasing the swirl velocity V_θ and adding to the kinetic energy of the flow, $\left(V_x^2 + V_r^2 + V_\theta^2\right)/2$. The result is an overall increase in the total enthalpy through the rotor.

Since the stators do not move, they do no shaft work. Therefore the steady flow energy equation tells us that the total enthalpy is unchanged as the flow passes through the stators (assuming the flow is adiabatic). However the stators deflect the flow into the axial direction, reducing swirl and kinetic energy. Since the kinetic energy reduces but the total enthalpy is fixed, the result is that the static enthalpy (and static pressure) increase through the stator.

The combined effect of the rotor and stator is to increase the total enthalpy and pressure, without a substantial overall change in the flow velocity. The iterative pressure and velocity changes through a multi-stage compressor are illustrated in Figure 7.

Compressor :



KE added by rotor
& KE converted by stator

keeping KE low makes fluid management more practical

Figure 7. Pressure and velocity profiles through a multi-stage axial compressor (Rolls-Royce, 1992)

A compressor stage consist of a rotor followed by a stator. In some situations an additional stationary blade row of *inlet guide vanes* (IGV) may be added upstream of the first compressor rotor, swirling the flow in the direction of the blade rotation, in order to reduce the Mach number of the flow relative to the rotor blades and to improve their aerodynamic performance.

9.4.2 Axial-flow turbine

The axial-flow turbine serves to convert total enthalpy into shaft work. The axial-flow turbine is made up of a stator blade row followed by a rotor blade row, as illustrated in Figure 1. The stator consists of converging channels or nozzles that accelerate the flow, increasing its swirl. The swirling flow pushes the turbine rotor around allowing work to be extracted. There is a static pressure drop through both the stator and the rotor, and the favourable pressure gradient reduces the risk of boundary layer separation, allowing pressure ratios of around 4 in a single axial turbine stage. As a result, a single turbine stage may be used to power around six compressor stages in a gas turbine.

9.4.3 Anatomy of a blade row

A blade row may consist of tens or hundreds of individual blades arranged around the circumference of the annular flow passage. The annular flow passage is contained with an inner 'hub' and outer 'casing' as shown in Figure 8.

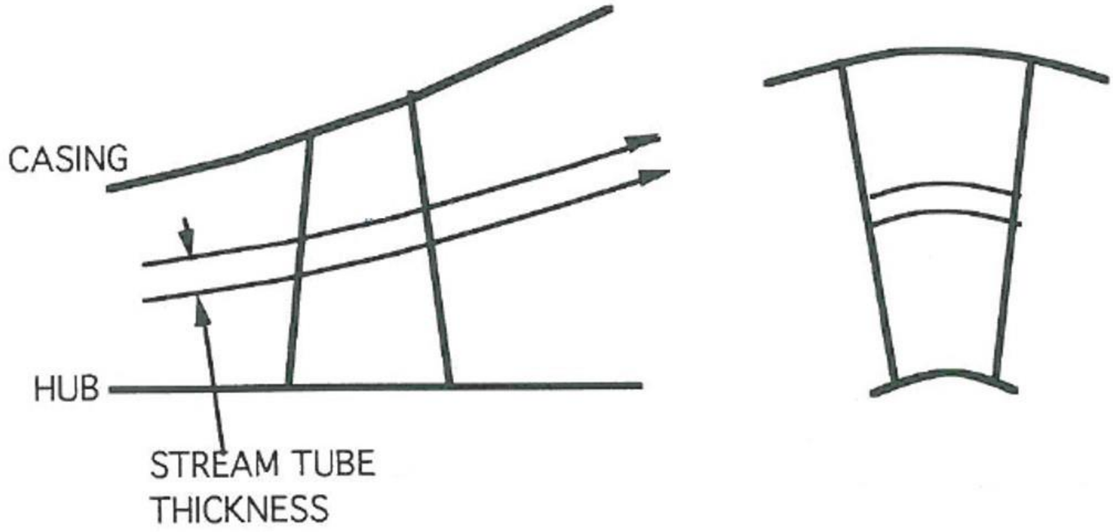


Figure 8. Flow model: the axi-symmetric meridional streamtube.

The individual blades have aerofoil cross-sections, causing the flow to turn and producing a pressure difference between the two surfaces of the blade. The pressure forces on the blade surfaces result in the torque on the rotor. The blades are described using much of the same vocabulary used to discuss wings: they have a root, tip, leading and trailing edge, camber, and span. In addition, the term ‘pitch’ is used to describe the distance between blades in the circumferential direction, as illustrated in Figure 9.

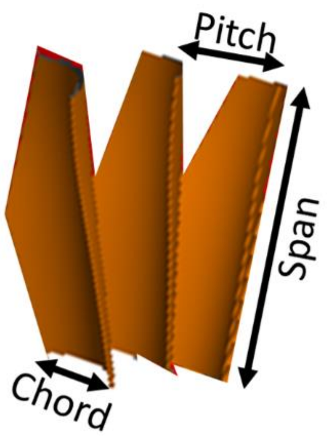


Figure 9. Illustration of the pitch-wise, span-wise and chord-wise directions.

As for an aircraft wing, the flow around the blades is three-dimensional, varying from root to tip. In addition, the flow into each blade passage is unsteady due to the wakes of the upstream rotor and stator blades. In order to introduce the principles of turbomachinery analysis in this module without undue complexity, we will neglect the effects of this unsteadiness (which happens to be a fairly good approximation) and neglect the variation in the flow between root and tip by considering flow on a representative ‘mid-line’ stream-surface through the blade row, as illustrated in Figure 8 and Figure 9. Neglect of three-dimensional effects is a more significant approximation, and one that we will remove if you go on to study Aircraft Propulsion in SESA6075.

Note that in general both the radius and the thickness of the stream tube will vary through the blade row. However, for most axial machines we will assume that the radius stays constant. The component of velocity along the stream tube is called the meridional velocity V_m.

9.5 Blade-to-blade flow

When we view the flow through the blades on an axi-symmetric stream surface the blades appear as shown below.

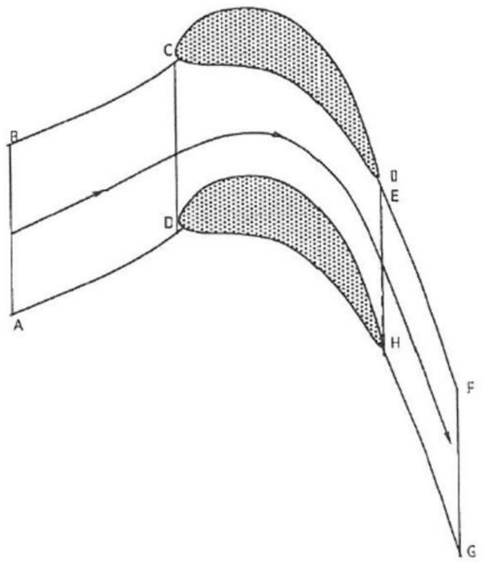


Figure 10. Illustration of the 'blade-to-blade' flow on the mid-line streamtube through an axial turbine blade row.

The flow through all blade-to-blade passages is the same and so conditions on BC are identical to those on AD and those on EF are identical to those on HG. The angle of incidence (sometimes called angle of attack) varies according to the flow direction leaving the preceding blades and the relative speed of the blade (if it is a rotor blade).

The situation is different at exit from the blade row. The Kutta-Joukowski condition at the blade trailing edge means that the flow must leave the trailing edge smoothly so that its direction cannot be greatly different from that of the metal at the trailing edge.

Once the flow has left the trailing edge, momentum conservation equation applied to a control volume such as EFGH shows that the angular momentum (rV_θ) is conserved until the flow meets another blade row.

9.6 Blade aerodynamics

The function of each blade row is to turn the flow direction and thereby change its swirl and its angular momentum. The flow turning is driven by a pressure gradient between the pressure and suction surfaces of the adjacent blades.

Compressor blading, like the example shown in Figure 11, turns the flow towards the axial direction, which tends to increase the cross-sectional area of the stream tube, thereby slowing the flow down and increasing its pressure (i.e. converting kinetic energy into static enthalpy, and converting dynamic

pressure into static pressure). Turbine blading, like the example shown in Figure 12, turns the flow away from the axial direction, which tends to decrease the cross-sectional area of the streamtube, thereby accelerating the flow, driven by a drop in pressure across the blade row.

The pressure rise / flow deceleration through compressor blading produces an adverse pressure gradient that promotes boundary layer growth and increases risk of boundary layer separation, increasing viscous losses and reducing the isentropic efficiency of the device. At off-design conditions, (especially low flow rates and/or high rotational speeds giving large angle of incidence) large-scale boundary layer separation may occur resulting in results in loss of compression, and this is described as ‘compressor stall’. In contrast, the overall pressure gradient in turbine blade rows is favourable, and the flow acceleration results in thin boundary layers that can support a greater amount of flow turning with lower risk of separation.

The pressure forces and the risk of flow separation on individual blades can be reduced either by increasing the number of stages (so that less pressure change and hence flow turning is needed in each stage) or by increasing the number of blades within each blade row (so that the pressure difference between pressure and suction surfaces of a blade are reduced). However the advantages of lower blade loading must be traded against the additional weight, cost and wetted surface area of increasing the number of stages and/or the number of blades in each blade row.

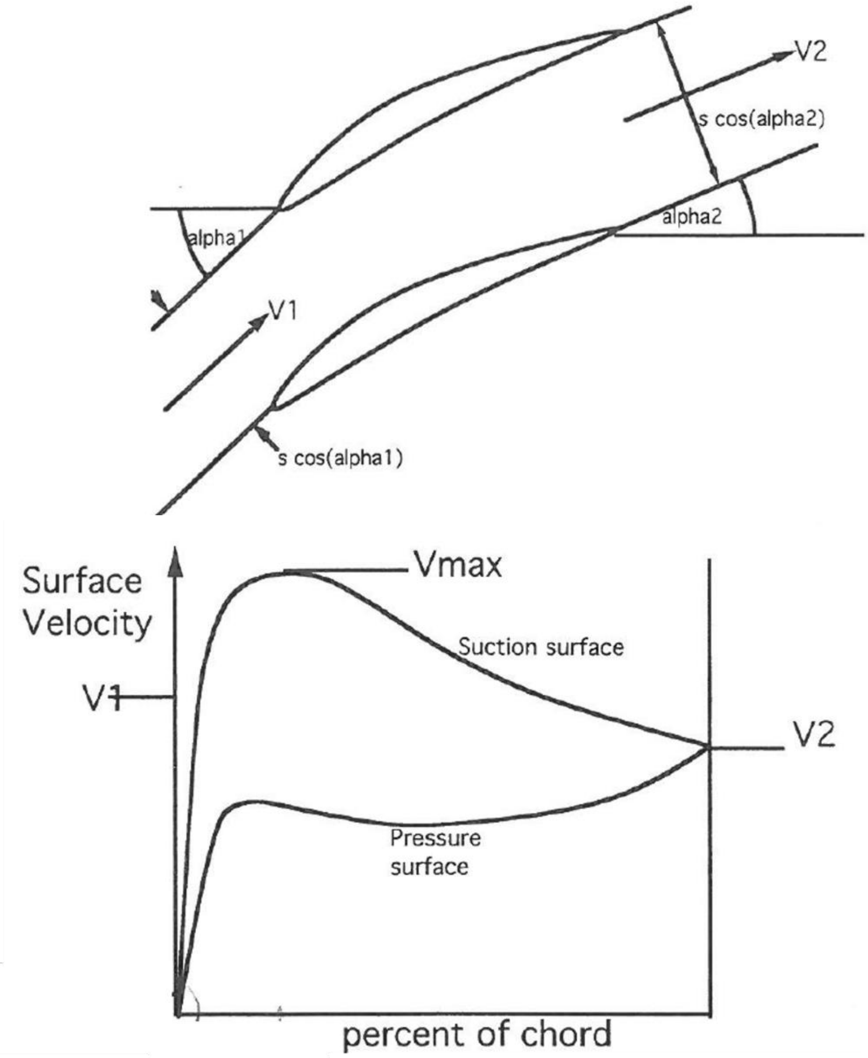


Figure 11. Typical compressor blade shape and surface velocity magnitude distribution (in frame of reference of the blades).

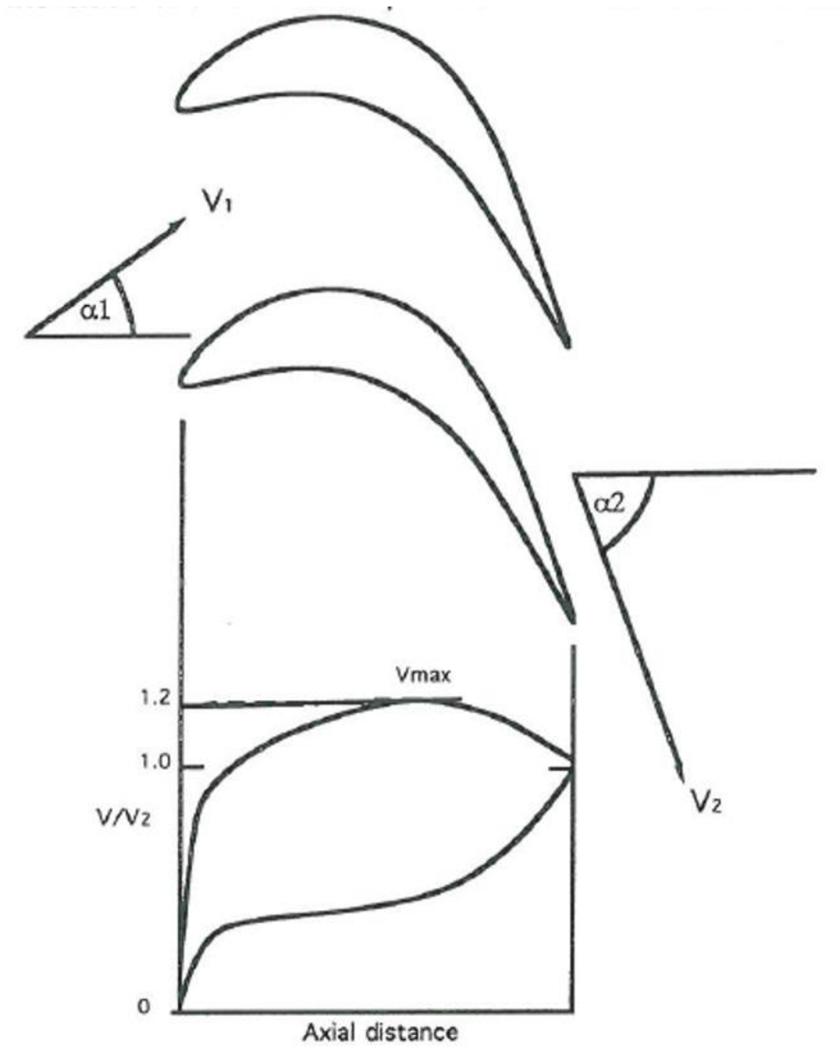


Figure 12. Typical turbine blade shape and surface velocity magnitude distribution (in frame of reference of the blades).

9.7 VELOCITY TRIANGLES

- Nomenclature
- Relationships
- Examples

Analysis of the velocities inside a turbomachine is made more complicated by the flow switching between rotating and stationary blade rows. Since we expect that the flow within any blade row will be approximately aligned with the surfaces of the blades, it is convenient to work in a frame of reference stuck to the blade row: when we analyse a stator blade row we use a stationary or *absolute* frame of reference; when we analyse a rotor blade row we use a *relative* frame of reference that translates with the blade speed $U = r\Omega$ (where r is the mid-line blade radius and Ω is the angular velocity of the rotor). When the flow moves between the stator and the rotor we can use velocity triangles (a.k.a. vector diagrams) to translate between the two frames of reference.

Figure 13 and Figure 14 show velocity triangles for a turbine and a compressor stage. Absolute flow angles and velocities are denoted by α and V. Relative flow angles and velocities are denoted α^{rel} and velocities V^{rel}. Flow angles are measured from the axial direction and considered positive when oriented towards the direction of the blade motion.

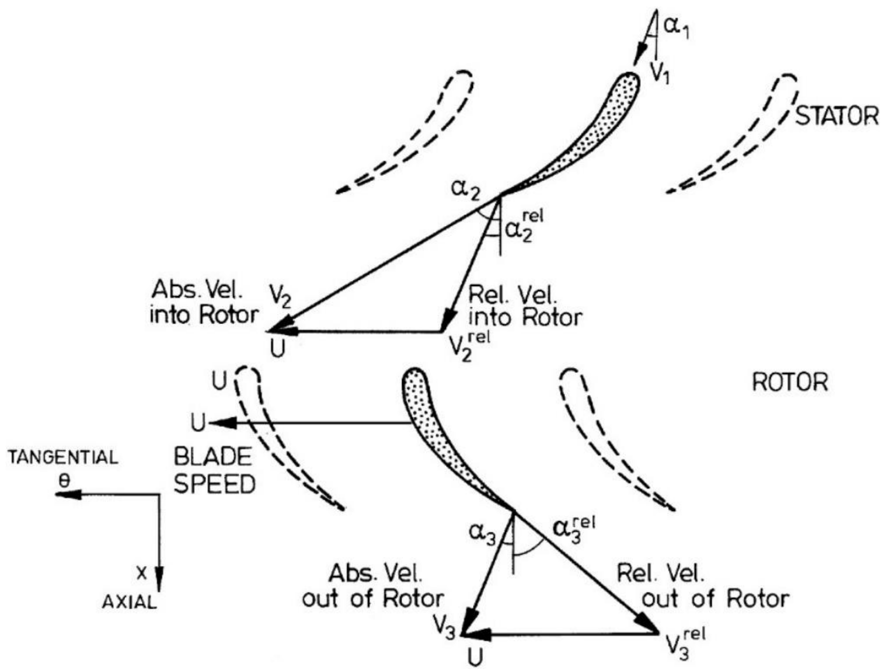


Figure 13. An axial turbine stage showing the velocity triangles into and out of the rotor.

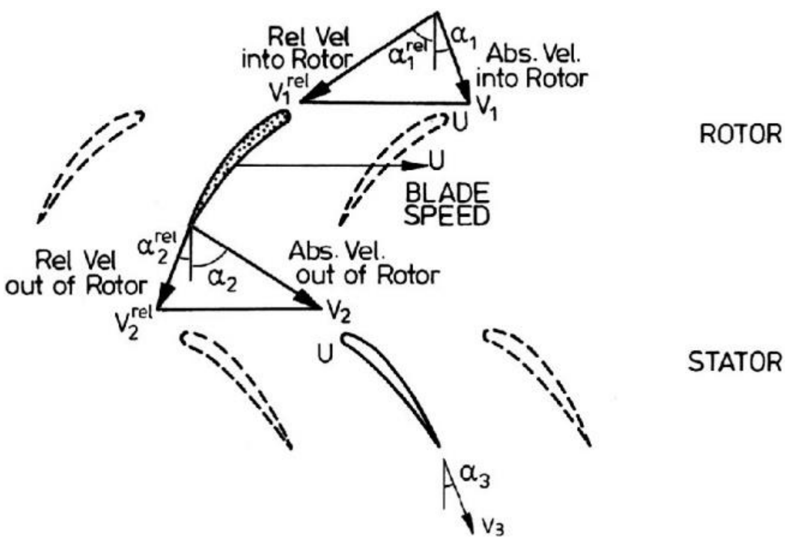


Figure 14. An axial compressor stage showing the velocity triangles into and out of the rotor.

Neglecting radial flow (which is reasonable for most axial-flow turbomachines), the velocity vector V can be decomposed into axial and circumferential components, V_x and V_θ. The circumferential velocity also gets referred to as the swirl velocity and as the tangential velocity.

The axial velocity is identical in the absolute and relative frames of reference. The relative swirl velocity is related to the absolute swirl velocity by

$$V_\theta^{rel} = V_\theta - U.$$

The various velocity components can be evaluated using trigonometry, for example:

$$V_{x1} = V_1 \cos\alpha_1, \quad V_{x2} = V_2 \cos\alpha_2, \quad V_{x3} = V_3 \cos\alpha_3,$$
$$V_{\theta 1} = V_{x1} \tan\alpha_1, \quad V_{\theta 2} = V_{x2} \tan\alpha_2, \quad V_{\theta 3} = V_{x3} \tan\alpha_3$$

It is common practice to keep the axial velocity V_x approximately constant through a blade row, or indeed through most of an axial-flow compressor or turbine. It is also usually safe to neglect the radial flow velocity unless dealing with a radial (centrifugal) compressor or turbine

9.7.1 Work transfer

The Euler work equation shows that the drop in *absolute* stagnation enthalpy through the stage in Figure 13 is given by

$$\Delta h_0 = U_3 V_{\theta 3} - U_2 V_{\theta 2} = U(V_{\theta 3} - V_{\theta 2})$$

with the restriction to streamlines at constant radius such that $U = U_2 = U_3$. Note that Δh_0 is negative for a turbine. This can be rewritten in terms of relative tangential velocity as,

$$\Delta h_0 = U\left(V_{\theta 3}^{rel} - V_{\theta 2}^{rel}\right)$$

where V_θ and V_θ^{rel} are positive if in the same sense as the blade speed. If the axial velocity is chosen to be constant, then,

$$\Delta h_0 = U V_x (\tan\alpha_3 - \tan\alpha_2)$$

which it can be shown is also equal to

$$\Delta h_0 = U V_x \left(\tan\alpha_3^{rel} - \tan\alpha_2^{rel}\right).$$

This work transfer takes place even if there are losses present. The effect of the losses however is to make the pressure drop greater than it would otherwise be in an isentropic turbine.

9.7.2 Velocity triangle example

Consider a constant radius streamtube flowing through an axial compressor stage. The compressor blade speed is 300 m.s^{-1}. The compressor is designed to operate with +26° flow angle at entry to and exit from the stage. The axial velocity is constant through the stage and equal to 0.55 times the blade speed. The enthalpy rise through the stage is required to be 0.45 times the blade speed squared.

Find all absolute and relative velocity components at rotor inlet and exit.

$$\mathrm{V_x = 0.55U = 165\,\frac{m}{s}}$$

$$\Delta h_0 = 0.45\mathrm{U}^2 = 40.5 \text{ kJ/kg}$$

The velocity triangles are the same shape as in Figure 14.

Absolute frame of reference at rotor inlet:

$$\mathrm{V}_1 = \frac{V_x}{\cos \alpha_1} = 183.6\ m/s$$

$$V_{\theta 1} = V_x \tan \alpha_1 = 80.5\ m/s$$

Relative frame of reference at rotor inlet:

$$V_{\theta 1}^{rel} = V_{\theta 1} - U = -219.5\ m/s$$

$$V_1^{rel} = \sqrt{{V_{\theta 1}^{rel}}^2 + V_x^2} = 274.6\ m/s$$

$$\alpha_1^{rel} = \tan^{-1}\left(\frac{V_{\theta 1}^{rel}}{V_x}\right) = -53.1°$$

Absolute frame of reference at rotor exit:

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 1})$$

$$\mathrm{V}_{\theta 2} = V_{\theta 1} + \frac{\Delta h_0}{U} = 215.5\ m/s$$

$$\mathrm{V}_2 = \sqrt{V_{\theta 2}^2 + V_x^2} = 271.4\ m/s$$

$$\alpha_2 = \tan^{-1}\left(\frac{V_{\theta 2}}{V_x}\right) = 52.6°$$

Relative frame of reference at rotor exit:

$$V_{\theta 2}^{rel} = V_{\theta 2} - U = -84.5\ m/s$$

$$V_2^{rel} = \sqrt{{V_{\theta 2}^{rel}}^2 + V_x^2} = 185.4\ m/s$$

$$\alpha_2^{rel} = \tan^{-1}\left(\frac{V_{\theta 2}^{rel}}{V_x}\right) = -27.1°$$

Note that although the absolute velocity increases through the rotor, the velocity decreases in the frame of reference of the moving blades, corresponding to increasing flow area and increasing static pressure.

Note that all of the change in stagnation enthalpy occurs in the rotor (there is no shaft work done in the stator)

Note that this example is a ‘repeating stage’, meaning that the flow direction and velocity are equal at stage inlet and outlet. Having several repeating stages in series is a reasonable approximation to the design of a multistage compressor.

9.8 SUMMARY

- Compressors and turbines consist of rows of stationary blades (stators) and rows of rotating blades (rotors).
- The pressure rises through a compressor because the *relative* velocity decreases through each blade row. The compressor rotor requires an input of shaft work.
- The flow accelerates through the turbine blade rows due to the pressure drop, and the change of angular momentum flux through the rotor produce a shaft work output.
- Compressor flows are prone to flow separation and stall because of the adverse pressure gradient, and only work efficiently for a narrow range of inflow angles close to zero incidence.
- It is convenient to calculate flow velocities in the *relative* frame of reference -- use velocity triangles to convert between this and the *absolute* frame of reference.
- The work exchange in a turbine or compressor blade row is given by the Euler work equation

$$\Delta h_0 = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$

and for two-dimensional flows at constant radius, $U_2 = U_1$.