University of Southampton

SESA6085 – Advanced Aerospace Engineering Management

Tutorial 1

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Q1.1

Q1.1

- A test set has a 98% probability of correctly classifying a faulty item as defective and a 4% probability of classifying a good item as defective.
- If in a batch of items tested 3% are actually defective, what is the probability that when an item is classified as defective, it is truly defective?
- How can we solve this?

$P(D|C) = 0.98$

$P(\bar{D}|C) = 0.04$

$P(D) = 0.03$

D = is defective

C = classified as defective

$$P(D|C) = \frac{P(C|D)P(D)}{P(C|D)P(D) + \cdots}$$

A1.1

- Let D represent the event that an item is defective and C represent the event that an item is classified defective.
- We need to find the probability of an item being truly defective, when it is classified as defective

$$P(D \mid C) = ?$$

A1.1

- Let's recap the question:
  - In a batch of items tested 3% are actually defective

$$P(D) = 0.03 \qquad \therefore P(\overline{D}) = 0.97$$

  - There is a 98% probability of correctly classifying a faulty item as defective

$$P(C \mid D) = 0.98$$

  - A 4% probability of classifying a good item as defective

$$P(C \mid \overline{D}) = 0.04$$

A1.1

- Using the binary partition form of Bayes' theorem, we find

$$P(D \mid C) = \frac{P(C \mid D)P(D)}{P(C \mid D)P(D) + P(C \mid \overline{D})P(\overline{D})}$$

$$P(D \mid C) = \frac{(0.98) \times (0.03)}{(0.98) \times (0.03) + (0.04) \times (0.97)}$$

$$P(D \mid C) = 0.43 = 43\%$$

Q1.2

Q1.2

In the test-firing of a missile, there are some events that are known to cause the missile to fail to reach its target. These events are listed below together with their approximate probabilities of occurrence during a flight and the probability of failure if each event occurs. Calculate the probability of each of these events being the cause in the event of a missile failing to reach its target.

Event	$P(A_i)$	$P(F\|A_i)$
Cloud reflection (A_1)	0.004	0.06
Precipitation (A_2)	0.011	0.03
Target evasion (A_3)	0.007	0.09
Electronic countermeasures (A_4)	0.05	0.07

How to proceed?

A1.2

- What are we trying to calculate?
- In our notation…

$$P(A_i|F)$$

- How can we calculate this?
- Recall the generalise form of Bayes theorem…

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$$

A1.2

- Therefore...

$$P(A_1|F) = \frac{P(B|A_1)P(A_1)}{\sum_{j=1}^{4} P(B|A_j)P(A_j)}$$

$$P(A_1|F) = \frac{0.00024}{0.0047} = 0.051$$

- Likewise...

$$P(A_2|F) = 0.070$$

$$P(A_3|F) = 0.134$$

$$P(A_4|F) = 0.745$$

A1.2

- A useful check in this case
- As all failures can only be attributed to the four events

$$\sum_{j=1}^{4} P\left(A_j|F\right) = 1.0$$

- Indeed…

$$0.051 + 0.070 + 0.134 + 0.745 = 1.0$$

Q2.1

Q2.1

- Using Excel, Matlab, Python or similar, create a plot of the PDF, CDF and reliability function of normal distribution with a $\mu = 5$ and $\sigma = 1$ between $t = 0$ and $t = 10$

- To define the PDF

```
=NORM.DIST(x,μ,σ,FALSE)
```

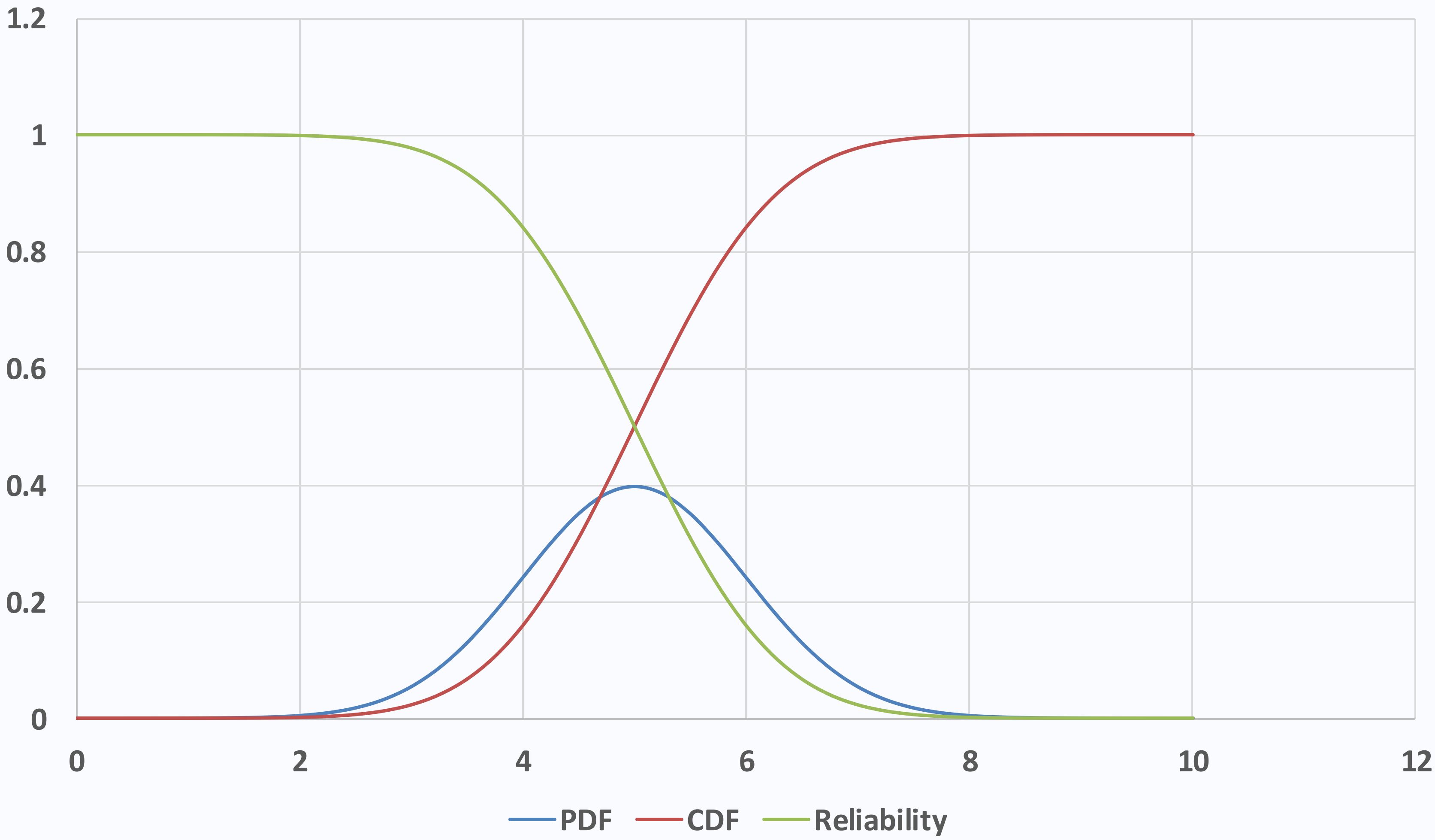
- To define the CDF

```
=NORM.DIST(x,μ,σ,TRUE)
```

- To define the reliability function

```
=1-NORM.DIST(x,μ,σ,TRUE)
```

A2.1



Q2.2

Q2.2

- The life of an incandescent lamp is s-normally distributed, with mean 1200hrs and standard deviation 200hrs. Calculate the probability that a lamp will last (a) at least 800hrs (b) at least 1600hrs.
- What are we trying to calculate here?

$$R(t)$$

- The reliability function gives the probability that the lamp will last to a time t
- How do we calculate this?

$$R(t) = 1 - F(t)$$

A2.2

Using Excel this is very simple…

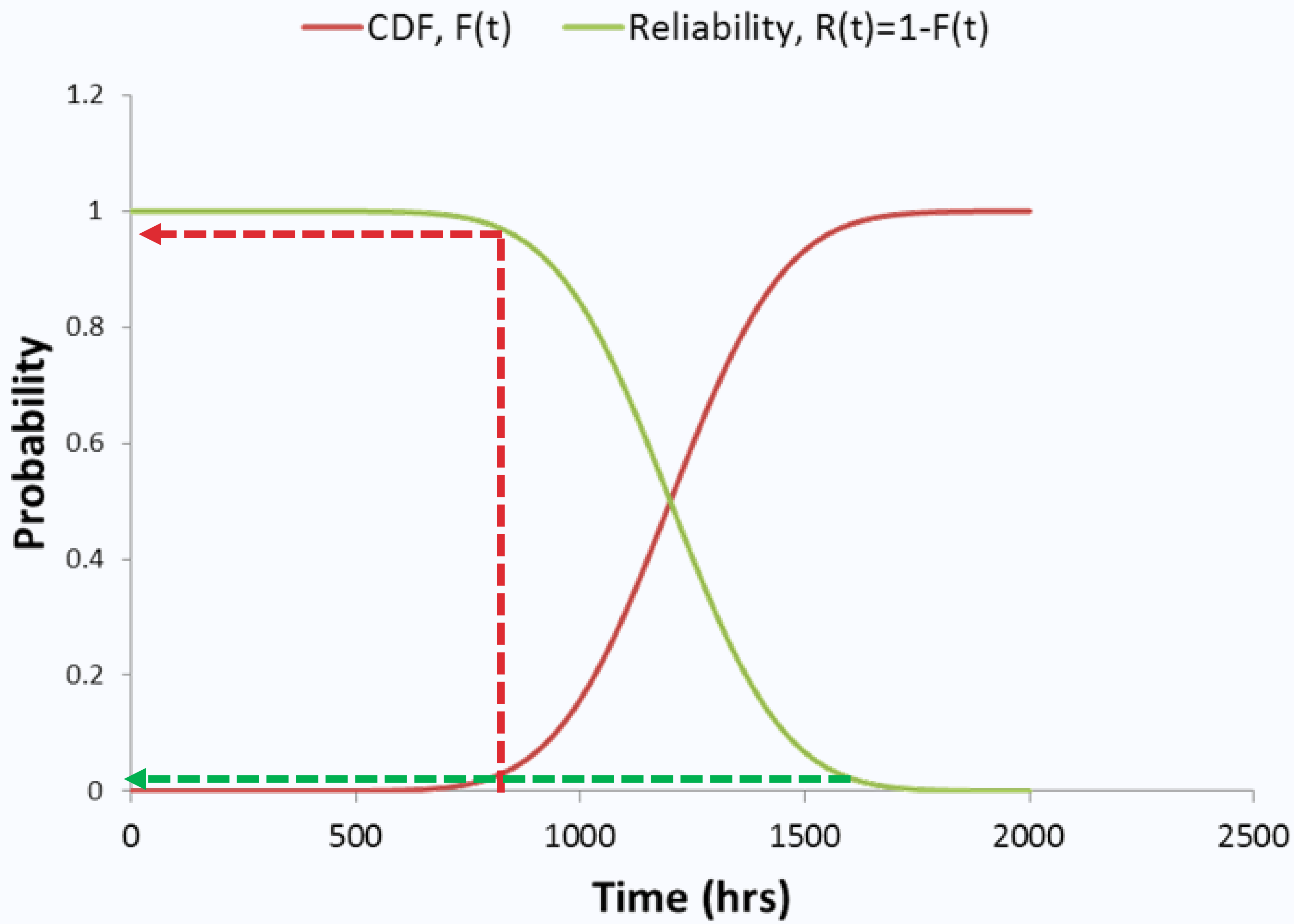
(a) R(800 hours) = `1-NORM.DIST(800,1200,200,TRUE)`

= 0.9772

(b) R(1600 hours) = `1-NORM.DIST(1600,1200,200,TRUE)`

= 0.0228

A2.2

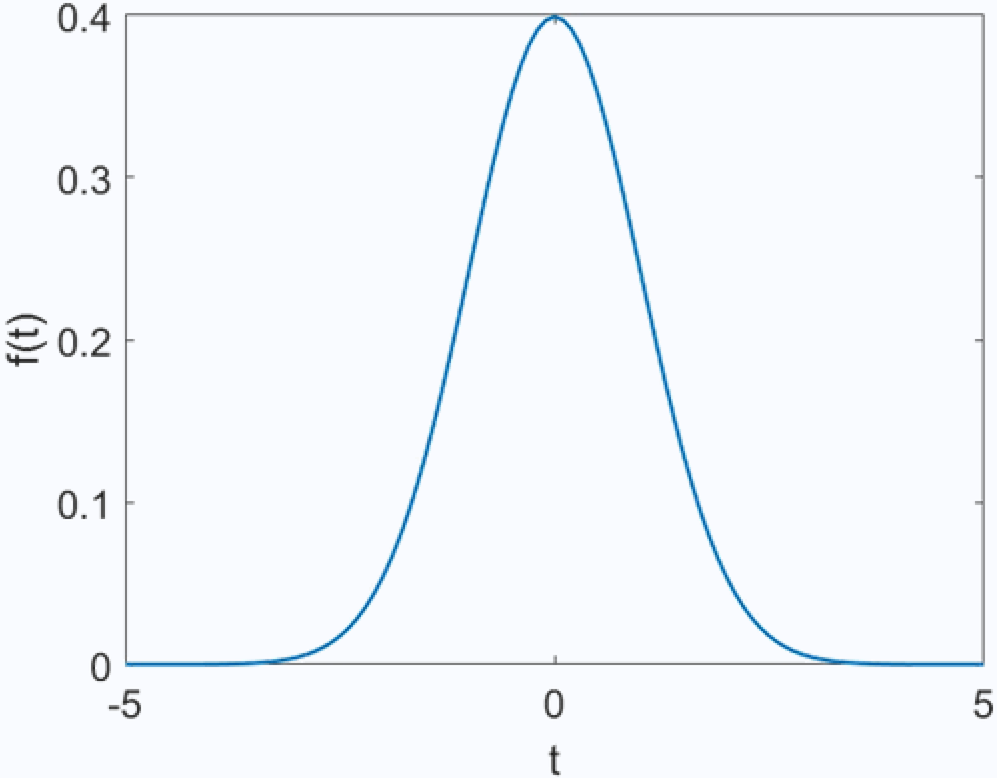


CDF, F(t)
Reliability, R(t)=1-F(t)
1.2
1
0.8
0.6
0.4
0.2
0
Probability
0
500
1000
1500
2000
2500
Time (hrs)

Standard Normal Distribution

What is the standard normal distribution?

- $\mu = 0$ and $\sigma = 1$



- Tables of the standard normal distribution enable calculations of f(t) and F(t) etc. without Excel, Matlab etc.

Standard Normal Distribution Tables

- From the workbook appendix…

Table 2: The PDF, $f(t)$, and CDF, $F(t)$, for the standard normal distribution with $\mu = 0$ and $\sigma = 1.0$

t	$f(t)$	$F(t)$	t	$f(t)$	$F(t)$	t	$f(t)$	$F(t)$	t	$f(t)$	$F(t)$
0	3.989e-01	5.000e-01	1.05	2.299e-01	8.531e-01	2.05	4.879e-02	9.798e-01	3.05	3.810e-03	9.989e-01
0.05	3.984e-01	5.199e-01	1.1	2.179e-01	8.643e-01	2.1	4.398e-02	9.821e-01	3.1	3.267e-03	9.990e-01
0.1	3.970e-01	5.398e-01	1.15	2.059e-01	8.749e-01	2.15	3.955e-02	9.842e-01	3.15	2.794e-03	9.992e-01
0.15	3.945e-01	5.596e-01	1.2	1.942e-01	8.849e-01	2.2	3.547e-02	9.861e-01	3.2	2.384e-03	9.993e-01
0.2	3.910e-01	5.793e-01	1.25	1.826e-01	8.944e-01	2.25	3.174e-02	9.878e-01	3.25	2.029e-03	9.994e-01
0.25	3.867e-01	5.987e-01	1.3	1.714e-01	9.032e-01	2.3	2.833e-02	9.893e-01	3.3	1.723e-03	9.995e-01
0.3	3.814e-01	6.179e-01	1.35	1.604e-01	9.115e-01	2.35	2.522e-02	9.906e-01	3.35	1.459e-03	9.996e-01
0.35	3.752e-01	6.368e-01	1.4	1.497e-01	9.192e-01	2.4	2.239e-02	9.918e-01	3.4	1.232e-03	9.997e-01
0.4	3.683e-01	6.554e-01	1.45	1.394e-01	9.265e-01	2.45	1.984e-02	9.929e-01	3.45	1.038e-03	9.997e-01
0.45	3.605e-01	6.736e-01	1.5	1.295e-01	9.332e-01	2.5	1.753e-02	9.938e-01	3.5	8.727e-04	9.998e-01
0.5	3.521e-01	6.915e-01	1.55	1.200e-01	9.394e-01	2.55	1.545e-02	9.946e-01	3.55	7.317e-04	9.998e-01
0.55	3.429e-01	7.088e-01	1.6	1.109e-01	9.452e-01	2.6	1.358e-02	9.953e-01	3.6	6.119e-04	9.998e-01
0.6	3.332e-01	7.257e-01	1.65	1.023e-01	9.505e-01	2.65	1.191e-02	9.960e-01	3.65	5.105e-04	9.999e-01
0.65	3.230e-01	7.422e-01	1.7	9.405e-02	9.554e-01	2.7	1.042e-02	9.965e-01	3.7	4.248e-04	9.999e-01
0.7	3.123e-01	7.580e-01	1.75	8.628e-02	9.599e-01	2.75	9.094e-03	9.970e-01	3.75	3.526e-04	9.999e-01
0.75	3.011e-01	7.734e-01	1.8	7.895e-02	9.641e-01	2.8	7.915e-03	9.974e-01	3.8	2.919e-04	9.999e-01
0.8	2.897e-01	7.881e-01	1.85	7.206e-02	9.678e-01	2.85	6.873e-03	9.978e-01	3.85	2.411e-04	9.999e-01
0.85	2.780e-01	8.023e-01	1.9	6.562e-02	9.713e-01	2.9	5.953e-03	9.981e-01	3.9	1.987e-04	1.000e+00
0.9	2.661e-01	8.159e-01	1.95	5.959e-02	9.744e-01	2.95	5.143e-03	9.984e-01	3.95	1.633e-04	1.000e+00
0.95	2.541e-01	8.289e-01	2	5.399e-02	9.772e-01	3	4.432e-03	9.987e-01	4	1.338e-04	1.000e+00
1	2.420e-01	8.413e-01									

How to use these tables

- Any normal distribution is a scaling and translation of the standard distribution
- To use the standard normal distribution first shift the t by μ and then divide by σ
- For our previous example we wish to find $R(1600)$ when μ =1200 and $\sigma = 200$
- Converting this to our standard distribution

$$t_{std} = \frac{(t - \mu)}{\sigma} = 2.0$$

How to use these tables

- Which we can look up on our table…

t	$f(t)$	$F(t)$	t	$f(t)$	$F(t)$
0	3.989e-01	5.000e-01	1.05	2.299e-01	8.531e-01
0.05	3.984e-01	5.199e-01	1.1	2.179e-01	8.643e-01
0.1	3.970e-01	5.398e-01	1.15	2.059e-01	8.749e-01
0.15	3.945e-01	5.596e-01	1.2	1.942e-01	8.849e-01
0.2	3.910e-01	5.793e-01	1.25	1.826e-01	8.944e-01
0.25	3.867e-01	5.987e-01	1.3	1.714e-01	9.032e-01
0.3	3.814e-01	6.179e-01	1.35	1.604e-01	9.115e-01
0.35	3.752e-01	6.368e-01	1.4	1.497e-01	9.192e-01
0.4	3.683e-01	6.554e-01	1.45	1.394e-01	9.265e-01
0.45	3.605e-01	6.736e-01	1.5	1.295e-01	9.332e-01
0.5	3.521e-01	6.915e-01	1.55	1.200e-01	9.394e-01
0.55	3.429e-01	7.088e-01	1.6	1.109e-01	9.452e-01
0.6	3.332e-01	7.257e-01	1.65	1.023e-01	9.505e-01
0.65	3.230e-01	7.422e-01	1.7	9.405e-02	9.554e-01
0.7	3.123e-01	7.580e-01	1.75	8.628e-02	9.599e-01
0.75	3.011e-01	7.734e-01	1.8	7.895e-02	9.641e-01
0.8	2.897e-01	7.881e-01	1.85	7.206e-02	9.678e-01
0.85	2.780e-01	8.023e-01	1.9	6.562e-02	9.713e-01
0.9	2.661e-01	8.159e-01	1.95	5.959e-02	9.744e-01
0.95	2.541e-01	8.289e-01	2	5.399e-02	9.772e-01
1	2.420e-01	8.413e-01			

- F(1600) = 0.9772
- R(1600) = 0.0228 as before

How to use these tables

- How about R(800)?

$$t_{std} = \frac{(t-\mu)}{\sigma} = -2.0$$

- But recall that our table only starts at 0! How do we solve this problem
- Remember that our standard distribution is symmetrical

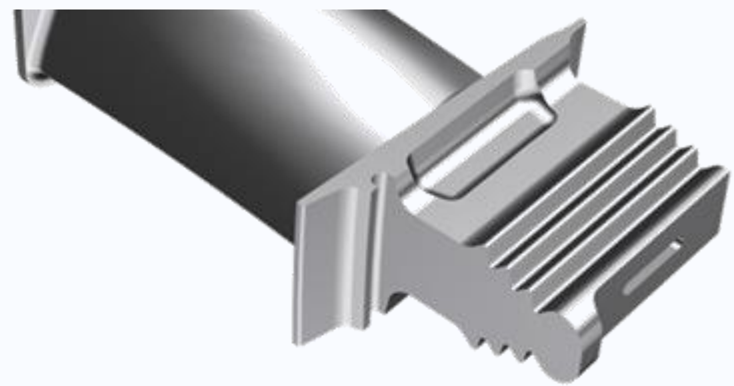
$$F(t_{std}) = R(-t_{std})$$

- Therefore for $t_{std} = -2.0$ we use $t_{std} = 2.0$
- R(800) is therefore $F(t_{std} = 2.0)$ from our standard table
- R(800) = 0.9772 as before

Q2.4

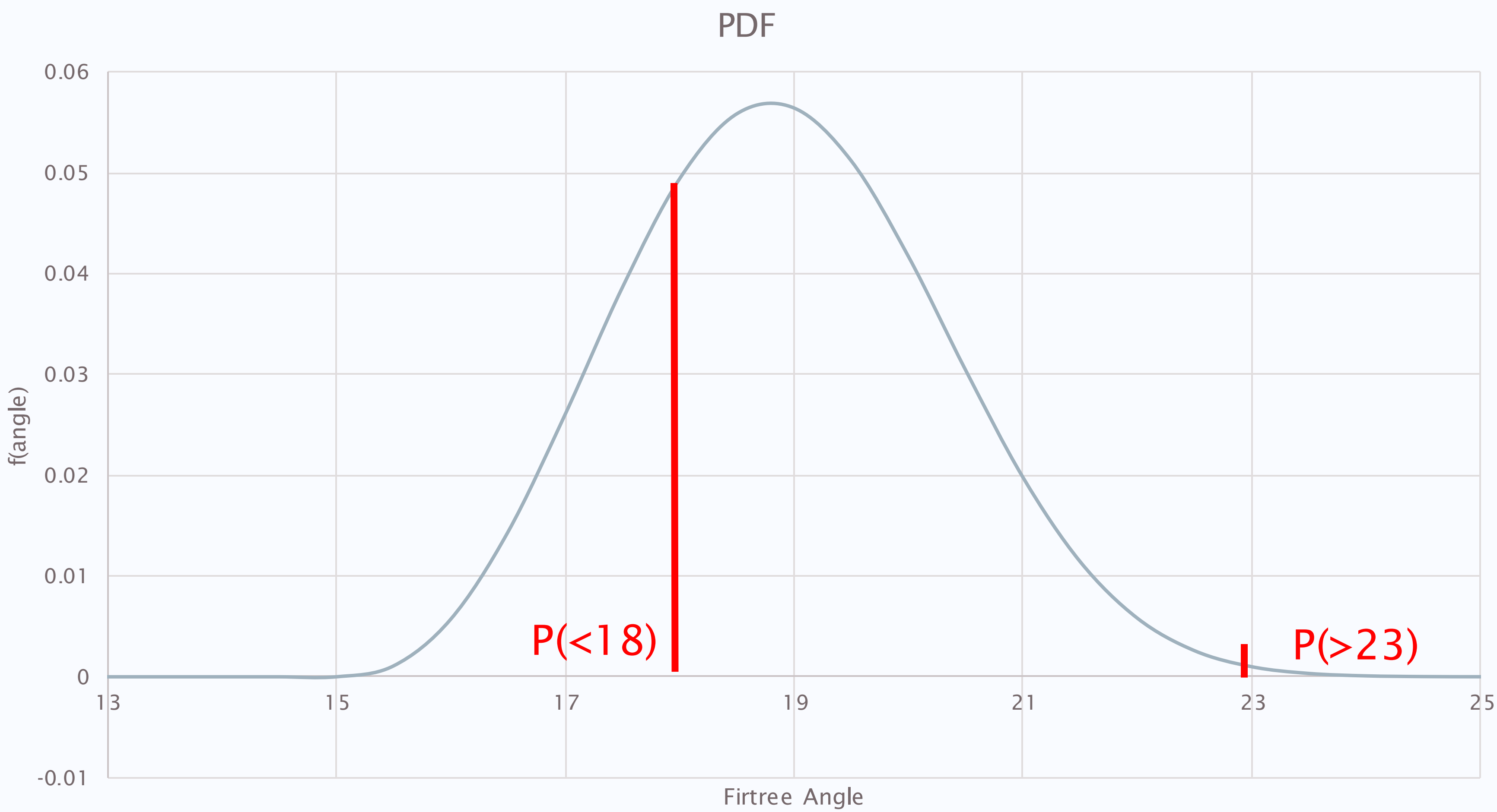
Q2.4

- After a turbine blade has been manufactured the angle of its firtree face is measured. Previous data suggests that the angle of the face varies according to a 3 parameter Weibull distribution with $\beta = 3.5$, $\eta = 6.0$ and $\gamma = 15^\circ$



- A turbine blade is rejected if the angle of its firtree is less than 18° and greater than 23°. What is the probability that a turbine blade is rejected?

A2.4



PDF
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
f(angle)
13
15
17
19
21
23
25
Firtree Angle
P(<18)
P(>23)

A2.4

- The probability of rejection is given by

$$P(\text{Rejected}) = \int_{-\infty}^{18} f(x)\,\mathrm{d}x + \int_{23}^{\infty} f(x)\,\mathrm{d}x$$

should be 15 (still works)

- How do we calculate this?

$$P(\text{Rejected}) = F(18) + R(23)$$

A2.4

- The Weibull distribution has nicely defined equations for the PDF and CDF

$$f(t) = \begin{cases} \dfrac{\beta}{\eta^{\beta}}(t-\gamma)^{\beta-1}\exp\left[-\left(\dfrac{t-\gamma}{\eta}\right)^{\beta}\right] & (\text{for } t \geq 0) \\ 0 & (\text{for } t < 0) \end{cases}$$

$$F(t) = 1 - \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]$$

- We can just use these directly
  - P(Rejected) = 0.0846 + 0.0647 = 0.149

Q2.5

Q2.5

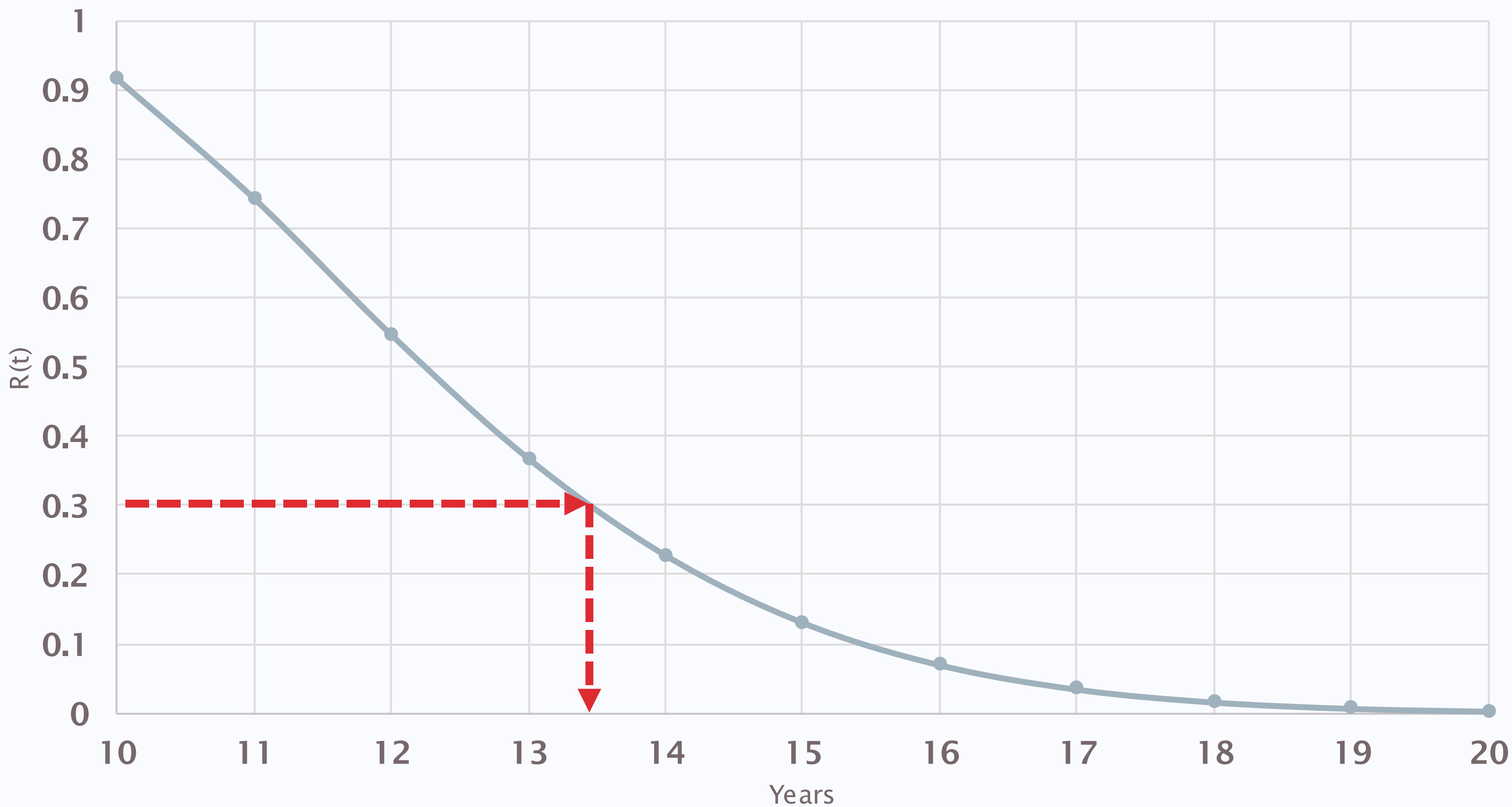
- The reliability of a communications satellite is described by a Weibull distribution with parameters, $\beta = 1.75$, $\eta = 4.0$ and $\gamma = 9$, with units of years. The satellite operator intends to replace the system when its reliability drops to 30%. How many years after the system is first launched should it be replaced?
- How to approach this problem?

$$t = ?$$

$$R(t) = 0.3$$

A2.5

- We can interpret this graphically…



A2.5

- Both Matlab and Python have inverse CDF functions that can be used to give the exact solution
  - Matlab - `icdf('Weibull’,0.7,4.0,1.75)+9  = 13.448`
  - Why 0.7? → RF not CDF
  - What is the +9 for?
- Remember these are often for two parameter Weibull distributions and don’t therefore contain the failure-free time which is effectively a shift in the location of the distribution
- What are these functions actually doing?

A2.5

- Recall that for a 2 parameter Weibull distribution the CDF is defined as

(solve for t)

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

- Essentially the inverse CDF rearranges this equation to make t the subject

$$t = \eta[-\ln(1 - F(t))]^{1/\beta}$$

- Some CDFs have no closed form inverse and so a goal seek optimisation may have to be performed
- For the 3 parameter inverse CDF don't forget to add γ to the solution

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