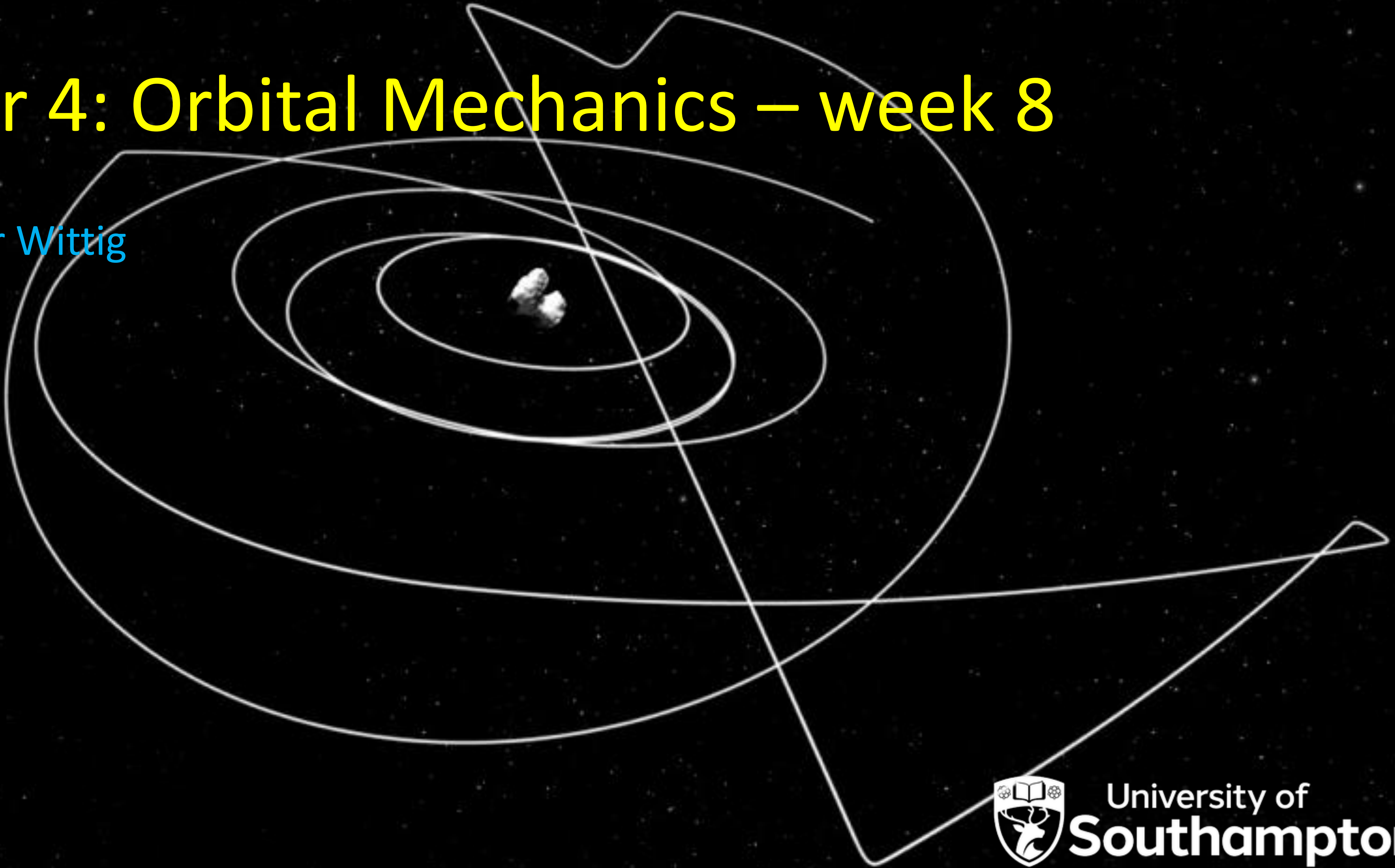
Advanced Astronautics (SESA3039)

# Chapter 4: Orbital Mechanics – week 8

Dr. Alexander Wittig



Rosetta lander deployment orbit (ESA)

Southampton AERO ASTRO

University of Southampton

# Introduction

- F2F lecture: Content of the week and "big picture"
  - Monday 13:00 in 35/1001
    Tuesday 15:00 in 35/1001
  - recorded, uploaded to Panopto
  - Slides available on BB
  - Questions welcome
    - You may want to look at the in-depth videos beforehand
- Pre-recorded Material
  - Filling in details from lectures
  - More in depth coverage of topics
  - **fully examinable material**
  - Videos and lecture notes on BB
- Tutorial sessions
  - Tuesdays 16:00 in 35/1001
  - Q&A on Problem Sheets
  - Tutors on hand for questions
- Quiz: 16/12
  - similar to tutorial sheet questions
  - numerical answers
  - same rules as other quizzes

# Problem sheet & Tutorials

## Problem Sheet

- Covers full 4 weeks of material
- Attempt questions on material of each week
- Additional questions available in textbooks

## Tutorial

- Tuesdays 16:00 in 35/1001
- Suggested in groups, self-organized
- **Active learning** session: prepare by attempting problem sheet
- Demonstrators available to help with questions
- 1 problem solved together (let me know preference 1 day before)

# Chapter 4 Roadmap

- **Week 8: Orbital Motion**
  - Math Basics
  - Spherical Trigonometry
  - Keplerian Motion from First Principles
- Week 9: Orbit Properties
  - Constants of Motion
  - Eccentricity Vector
  - Conic Sections
- Week 10: Orbit Representation
  - Coordinates
  - Dates & Times
  - Orbital Elements
- Week 11: Time Dependence
  - Eccentric Anomaly
  - Hyperbolic Anomaly
  - Kepler's Equations

# Textbooks

*Orbital Mechanics for Engineering Students*
**Curtis, H.**
Butterworth-Heinemann, 2013.
ISBN-13 978-0123747785

*Orbital Mechanics*
**Chobotov, V.**
AIAA Education Series, 2002
ISBN-13 978-1563475375

*Fundamentals of Astrodynamics and Applications*
**Vallado, D. A.**
Springer, 2007
ISBN-13 978-0387718316

*An Introduction to the Mathematics and Methods of Astrodynamics*
**Battin, R. H.**
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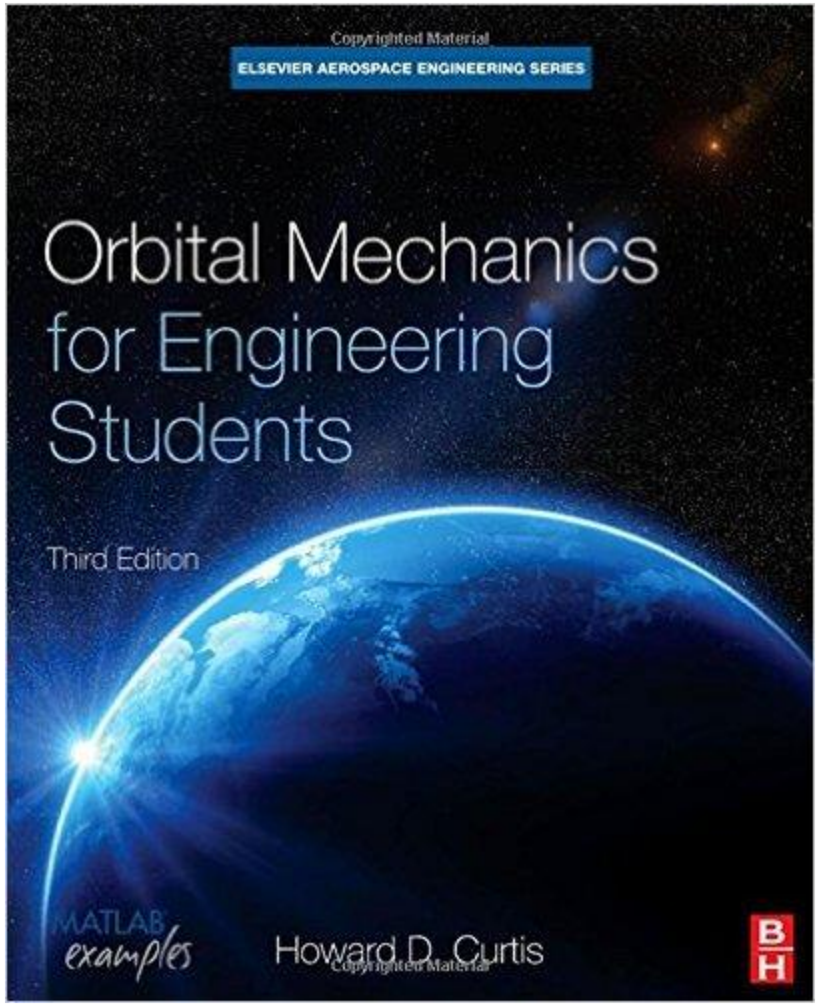
*Satellite Orbits: Models, Methods and Applications*
**Montenbruck, O. and Gill, E.**
Springer, 2011
ISBN-13 978-3540672807

Subjective observations:

- good introductory textbook
- very explicit mathematical derivations
- copious amount of worked examples
- not as complete as others
- not as useful as a reference

Chapter 4 will follow this textbook closely.

**Available as free full text with Soton Login from publisher!**




https://www.sciencedirect.com/book/9780080977478/orbital-mechanics-for-engineering-students

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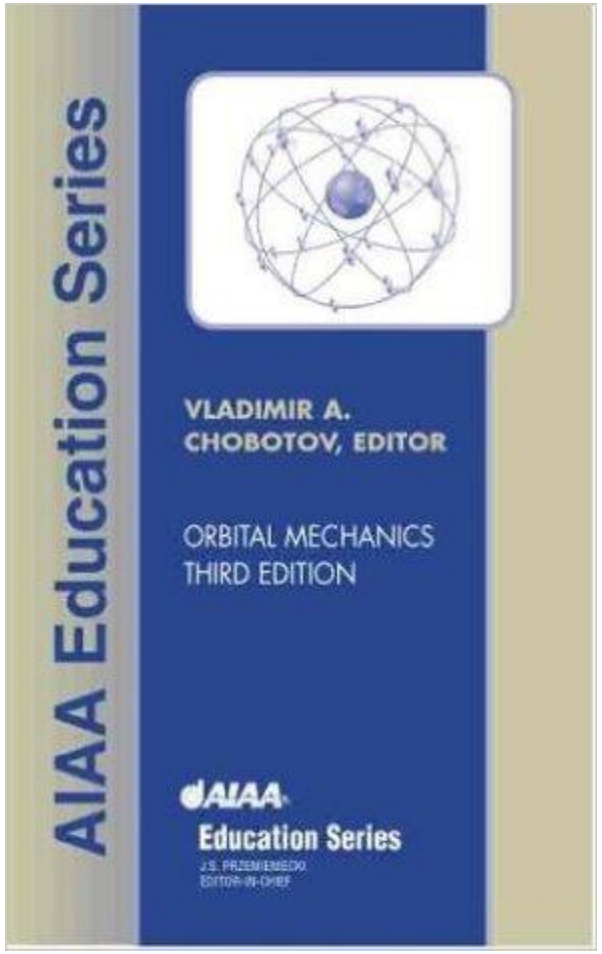
Subjective observations:

- wide range of topics covered
- mathematical derivations often challenging
- written as an expert text, not an introductory text
- includes references to research papers

eBook available at:
https://ebookcentral.proquest.com/lib/soton-ebooks/detail.action?docID=3111521



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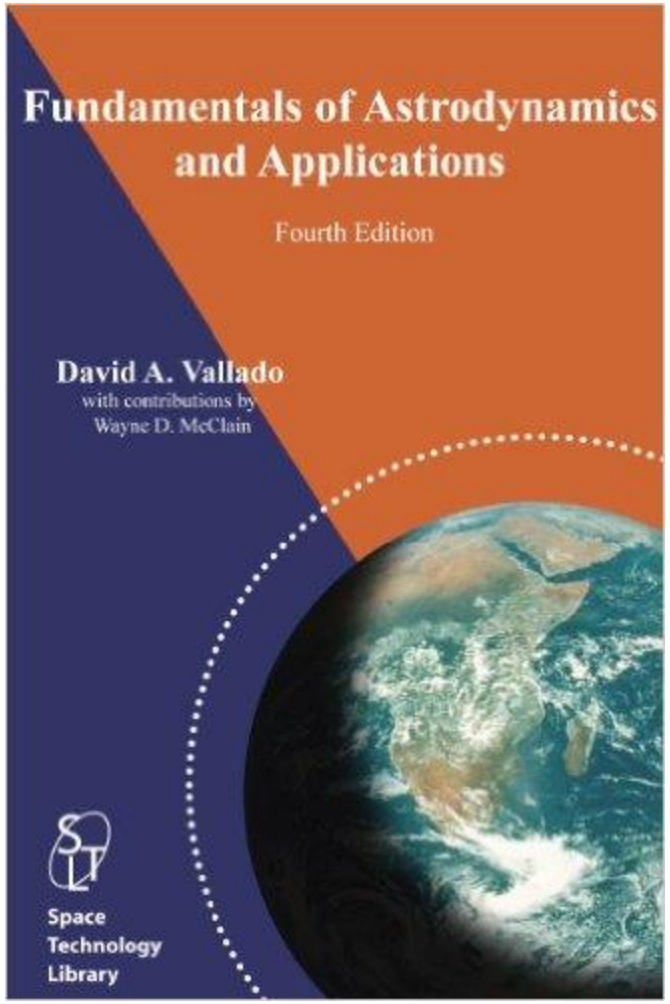
Springer, 2011
ISBN-13 978-3540672807

Subjective observations:

- good and quite readable, complete textbook
- understandable mathematical derivations
- focus on numerical implementation of algorithms (including code in various languages)
- also useful as a reference

Author worked on SGP4, the US airforce standard orbit propagator for NORAD TLEs.



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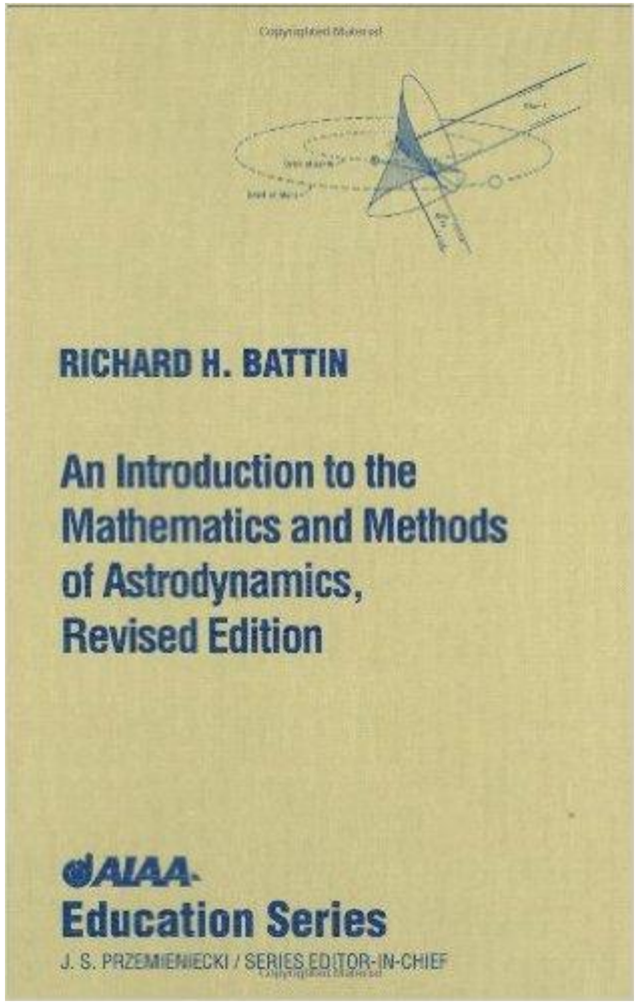
Subjective observations:

- most complete reference on the subject
- sometimes challenging mathematical derivations
- written by a mathematician!
- includes many exercises
- extremely useful as a reference

Author wrote the Apollo mission guidance computer code.

eBook available at:
[https://ebookcentral.proquest.com/lib/soton-ebooks/detail.action?docID=3111476](https://ebookcentral.proquest.com/lib/soton-ebooks/detail.action?docID=3111476)

Check out his MIT lecture
"*A Funny Thing Happened on the Way to the Moon*" for some history of the early NASA program on YT:
[http://y2u.be/ieiEoTo8-XY](http://y2u.be/ieiEoTo8-XY)



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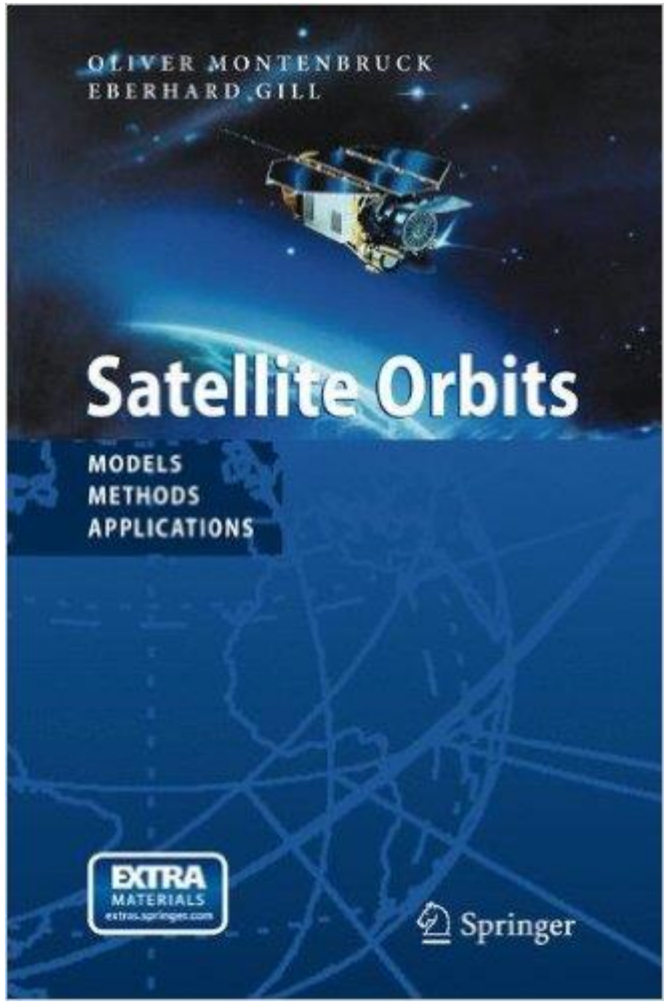
Subjective observations:

- focused specifically on satellites
- quite readable, few long derivations
- includes numerical and practical engineering considerations
- good textbook and reference on its limited subset of the subject



# Mathematical Concepts

# Learning outcomes

- Recall basic concepts of **vector algebra**
  - Vector identities
  - Coordinate representation

→ ensure you understand vector manipulation on your model of calculator

- Relate **Kinematics** and **Dynamics** in different frames
  - Accelerating reference frames
  - Equations of motion in accelerating reference frames

- Apply trigonometry to problems on the sphere
  - Trigonometric identities on the sphere

(watch recorded material)

# Vector Algebra & Mechanics

see Chapter 4 Video Lecture 1

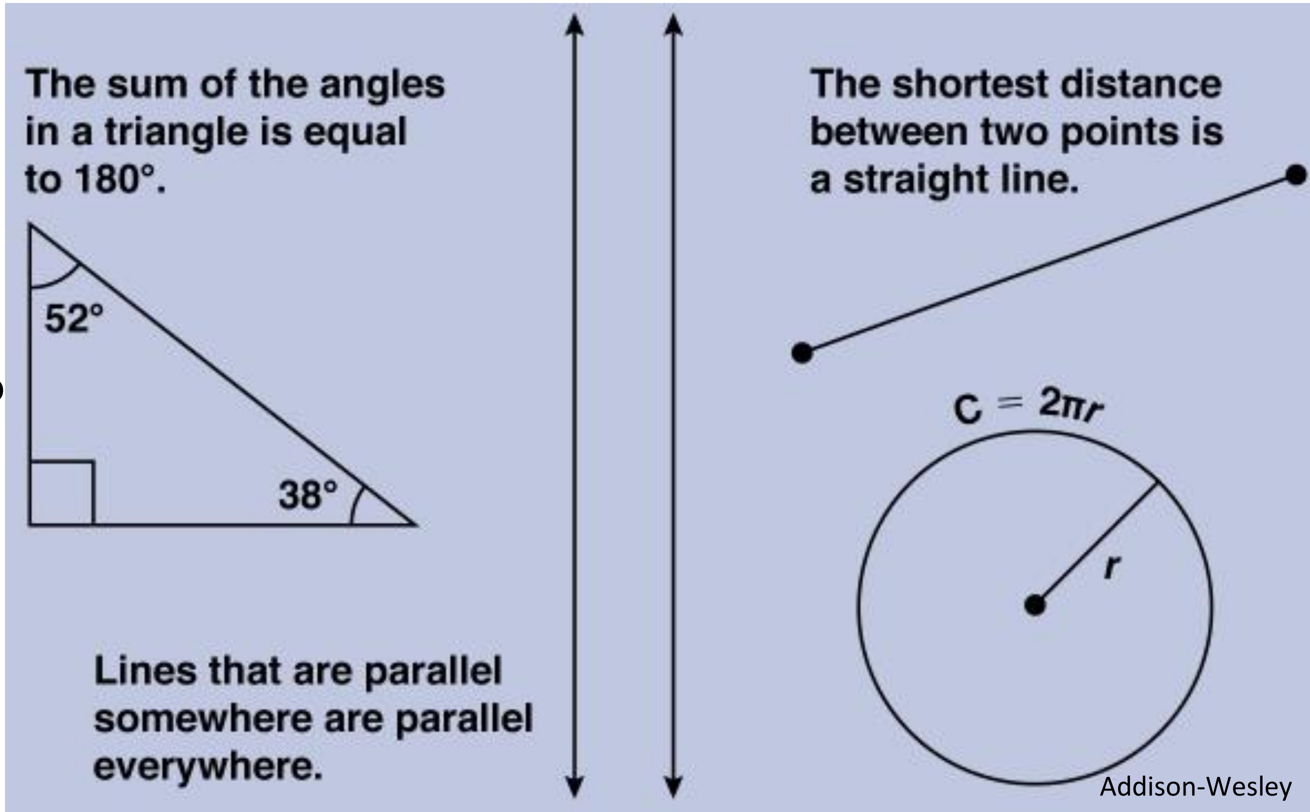
# Spherical Trigonometry

see also Chapter 4 Video Lecture 2

# Geometry in curved space

- Euclidean geometry is in flat space
- In curved space Euclidean properties generally **do not** hold
- Sphere is a simple curved space relevant in orbital mechanics and astronomy

- Line segment: shortest path between two points
  - Is it unique?
  - How often can different lines intersect?
  - How often do parallel lines intersect
  - How many lines are needed for a polygon?



Addison-Wesley

# Geometry in curved space

here we define "lines" as great circle arcs

- Euclidean geometry is in flat space
- In curved space Euclidean properties generally **do not** hold
- Sphere is a simple curved space relevant in orbital mechanics and astronomy

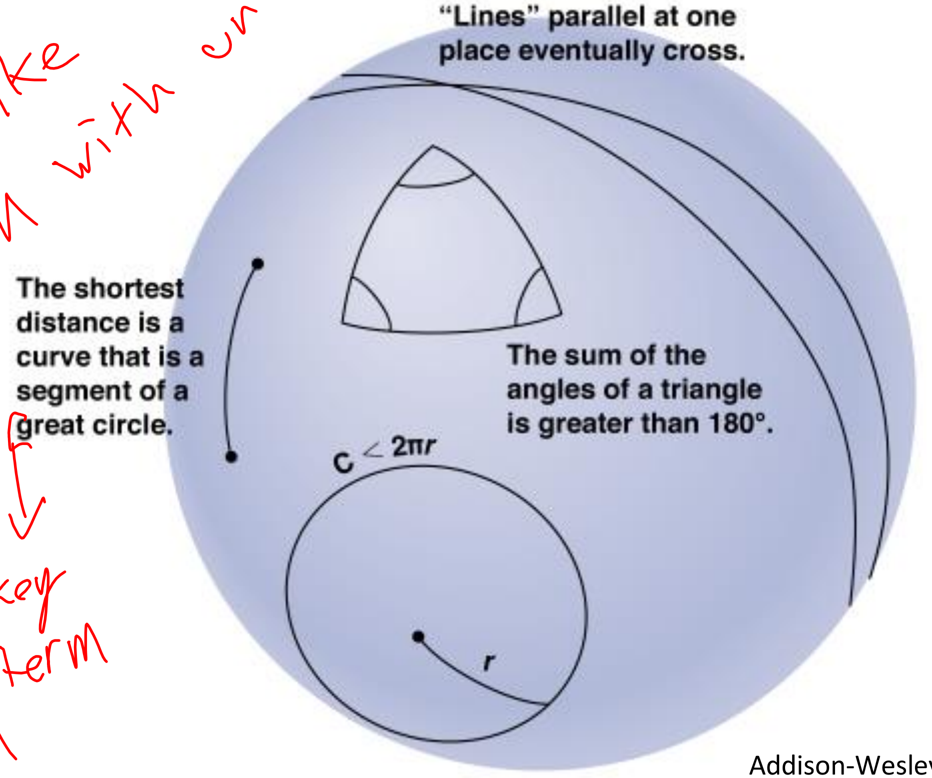
- Line segment: shortest path between two points
  - Is it unique?
  - How often can different lines intersect?
  - How often do parallel lines intersect
  - How many lines are needed for a polygon?

2

twice, unlike Euclidean with one

lines can be locally parallel

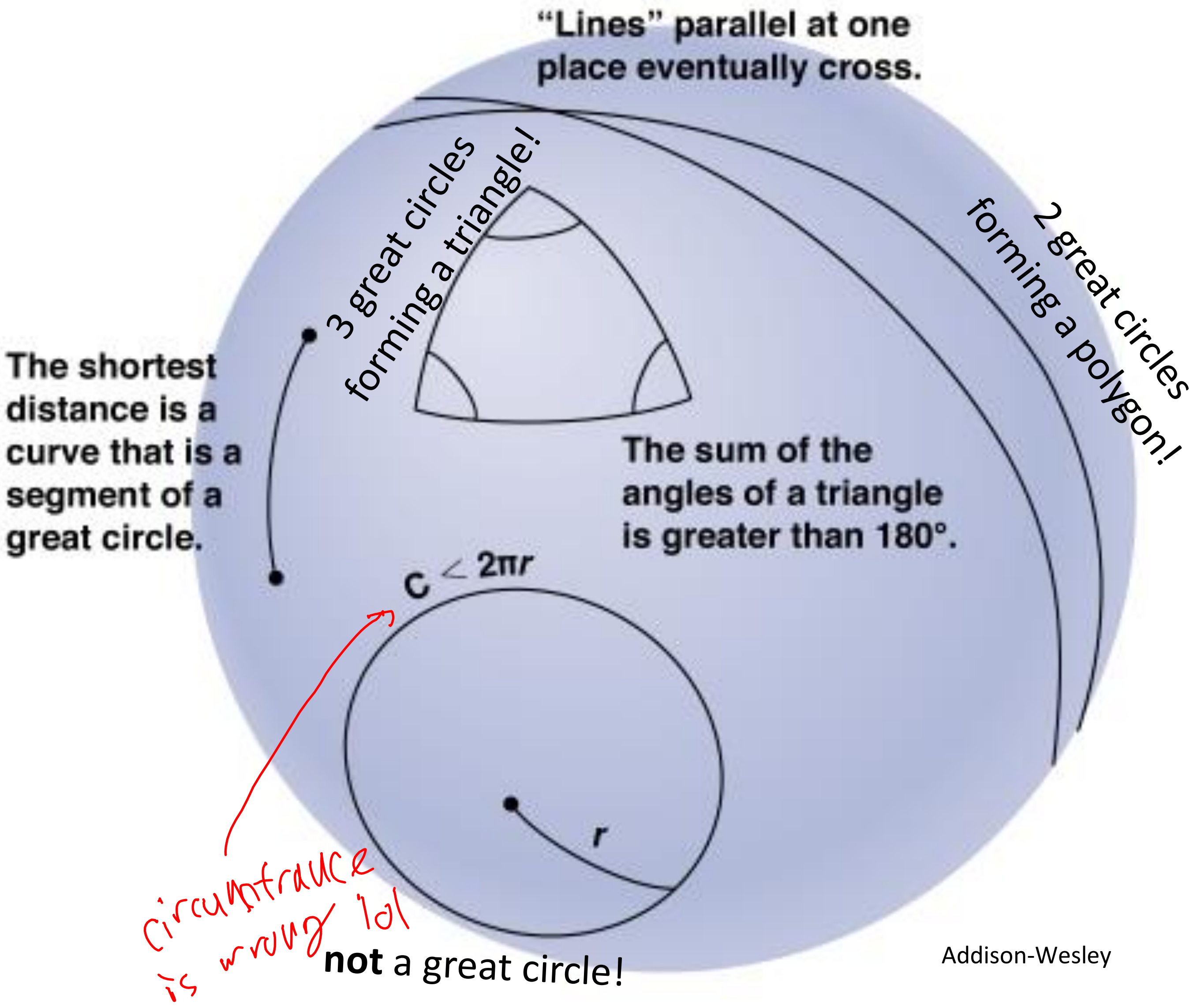
key term



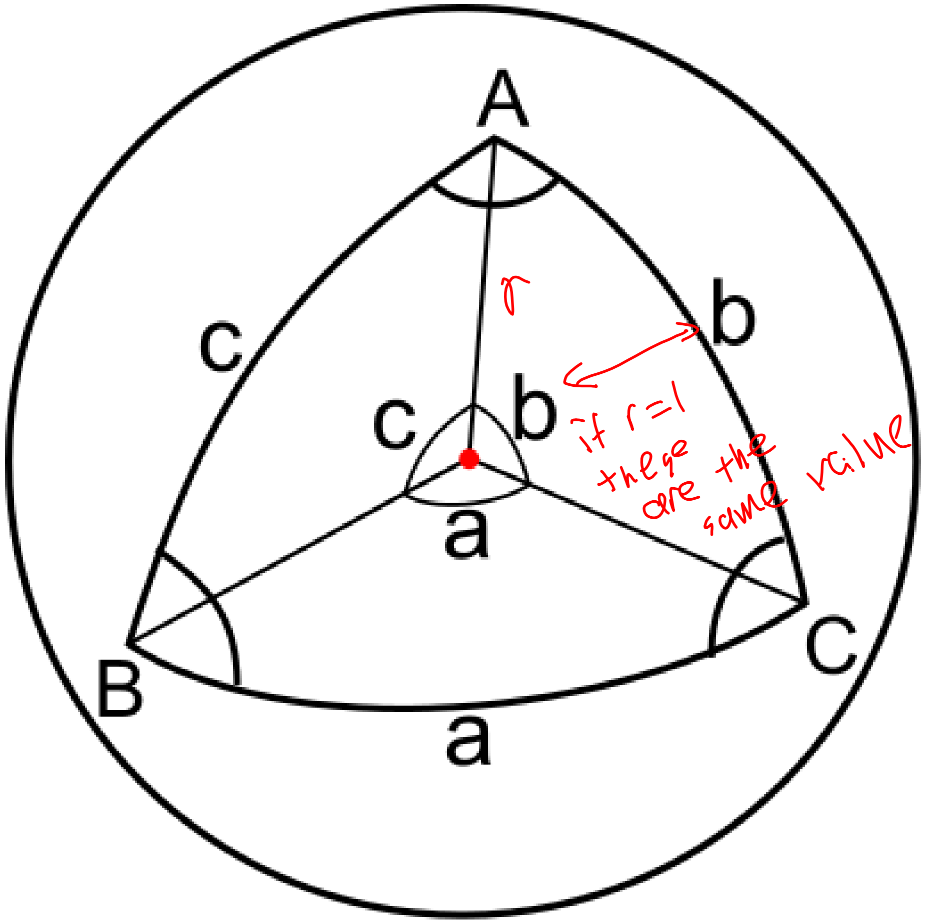

Addison-Wesley

# Spherical Geometry

- **Great circle**: circle on a plane including the center point of the sphere
- Line: segment of a **great circle**
- Polygon: closed loop of line segments on sphere surface




Addison-Wesley

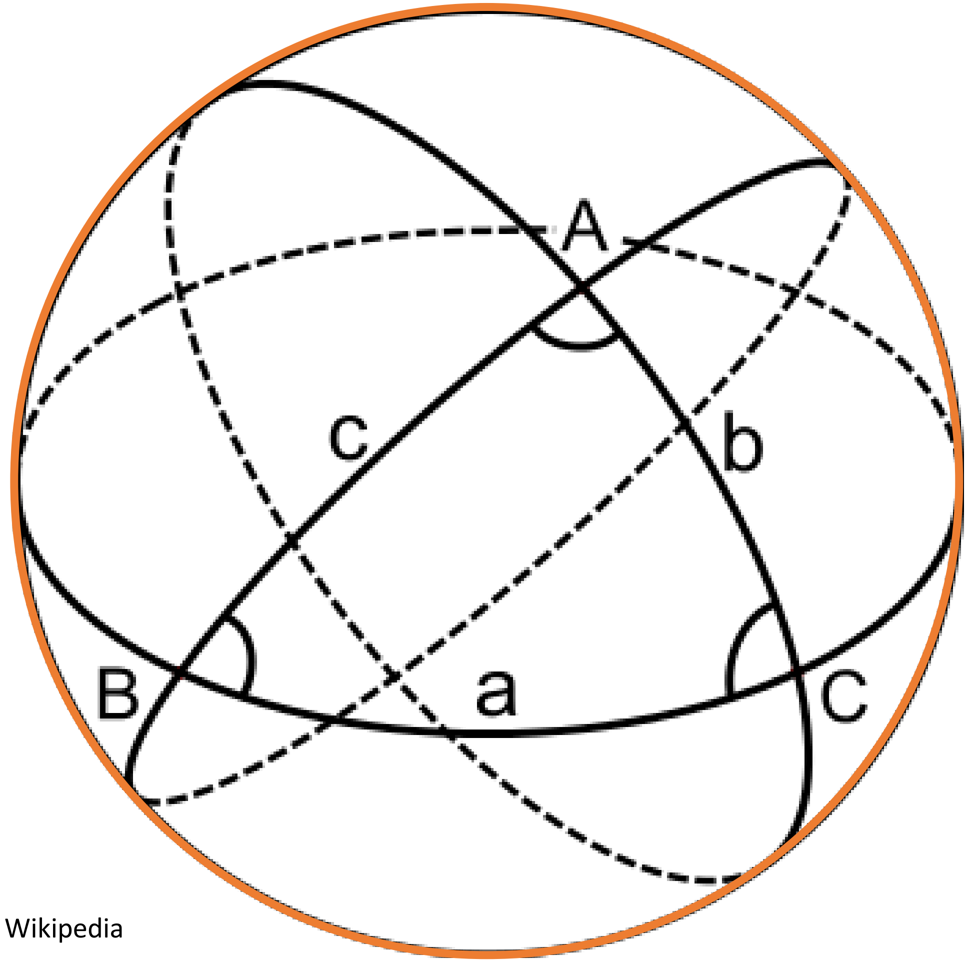



Wikipedia

Notation:

- Vertices and angles (in rad) at vertices denoted by capitals: A, B, C
- Angles are between tangent vectors to great circles, or equivalently between planes containing the great circles
- Sides and the angles (in rad) at sphere center denoted by lower case a, b, c
- Assuming a unit sphere (!) a, b, c are also lengths of sides (chords)

Three distinct great circles form a triangle




Wikipedia

Given angle A and side lengths b, c what is the missing length a?

- Planar case: law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

- But: planar trigonometry not true on sphere!

**Question**

How many spherical triangles are there in this picture?

## Cosine Rules

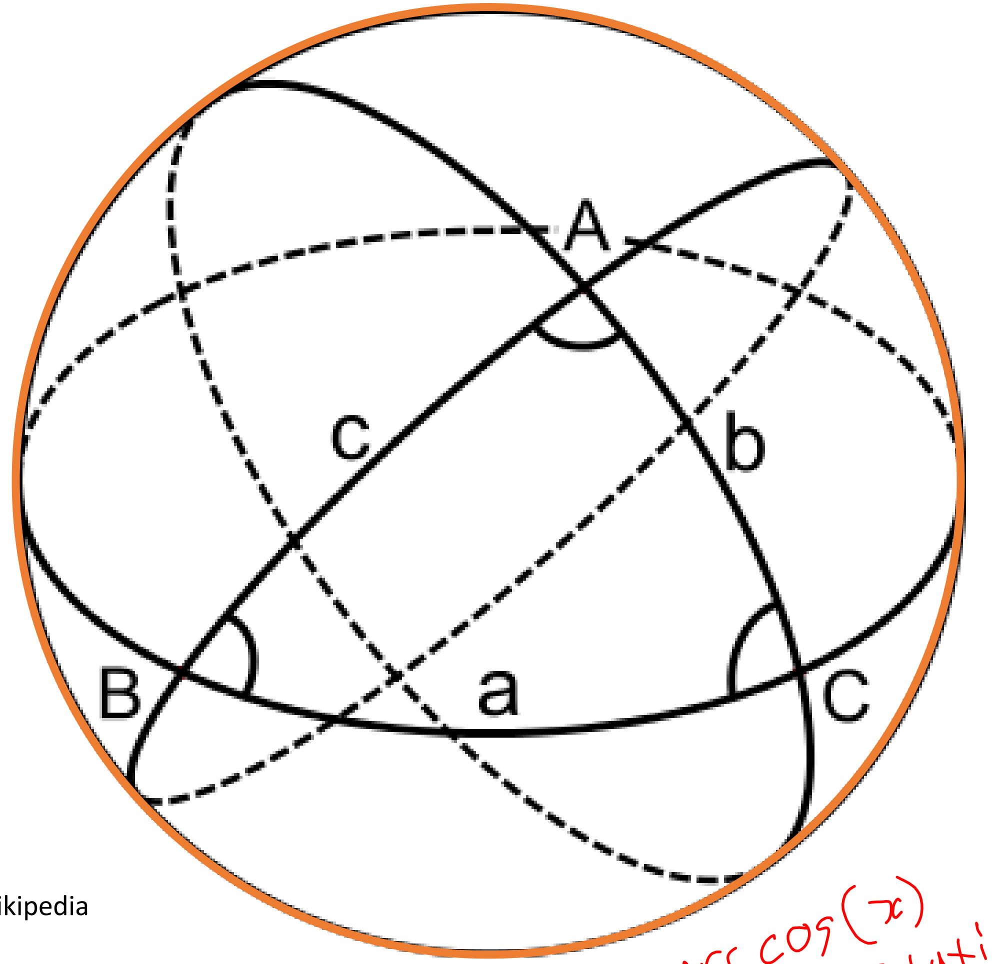
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

## Complementary Cosine Rules

$$\cos A = -\cos B \, \cos C + \sin B \, \sin C \, \cos a,$$
$$\cos B = -\cos C \, \cos A + \sin C \, \sin A \, \cos b,$$
$$\cos C = -\cos A \, \cos B + \sin A \, \sin B \, \cos c.$$

## Sine Rules

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$




Wikipedia

**CAREFUL WITH QUADRANT DISAMBIGUATION!** → arccos(x) has 2 solutions

One application: distances on a sphere (e.g. Earth):

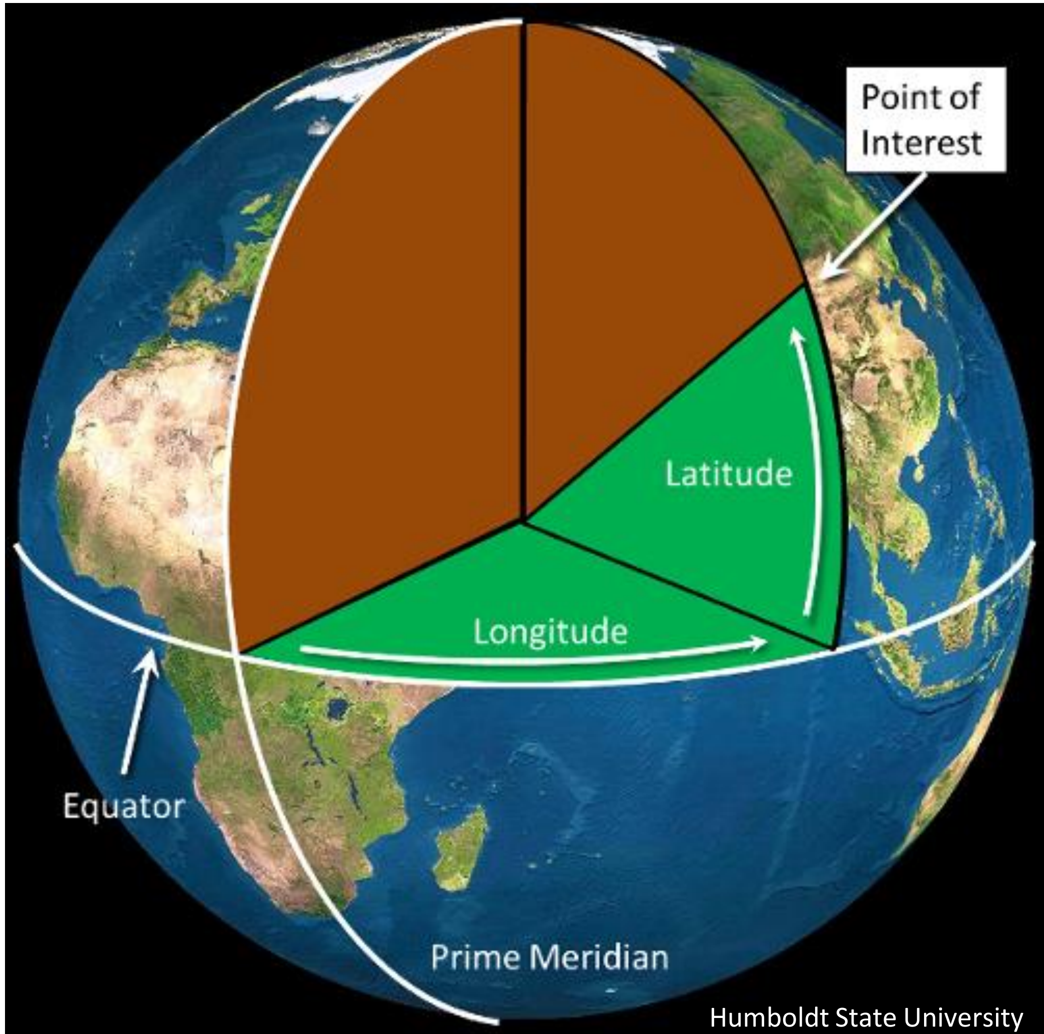
- Points located by latitude and longitude
- Prime meridian = 0 longitude
- Equator = 0 latitude

**Question**: given two points on Earth, what's the shortest distance between them?

$$\Phi_1, \lambda_1$$

$$\Phi_2, \lambda_2$$

$$d = ???$$




Humboldt State University

One application: distances on a sphere (e.g. Earth):

- Points located by latitude and longitude
- Prime meridian = 0 longitude
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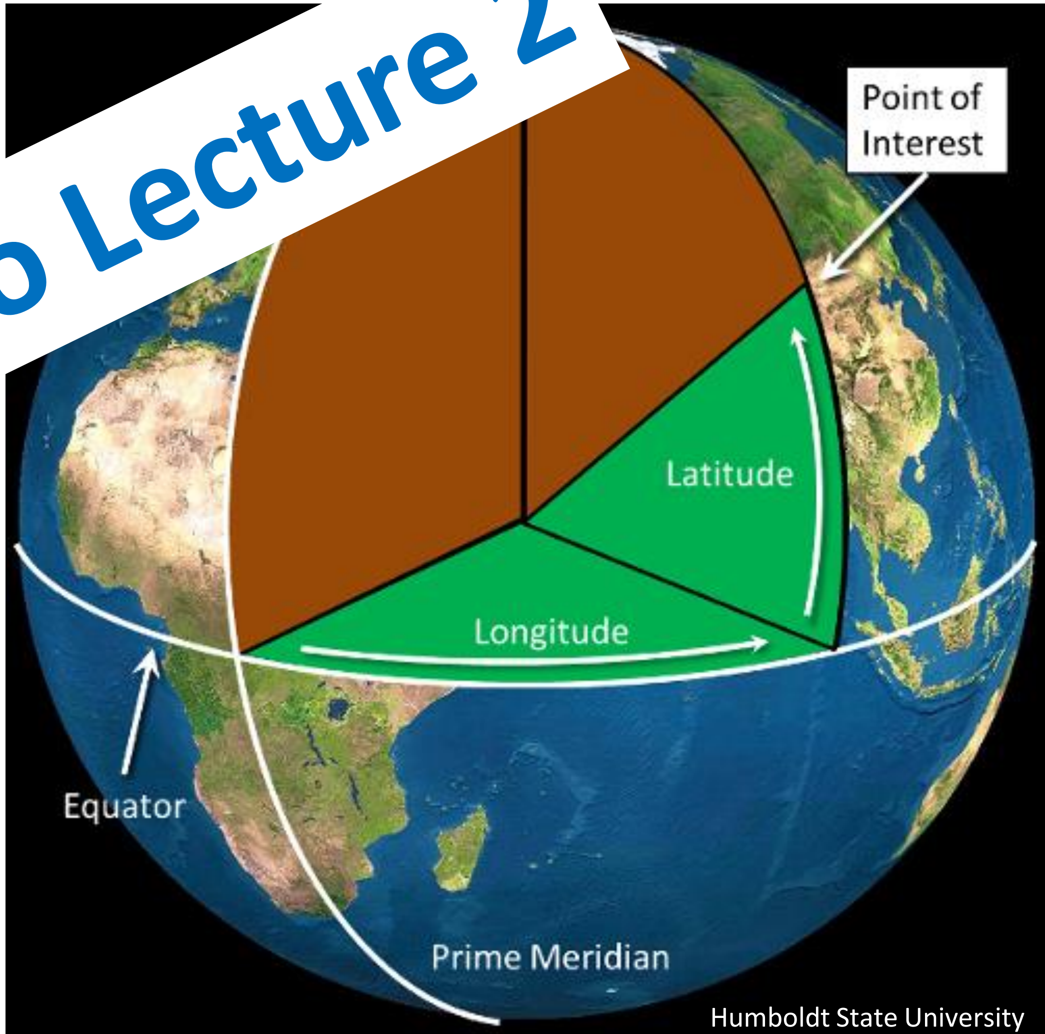
**see Chapter 4 Video Lecture 2**

**Question**: given two points … s
the shortest distance b…

$$\ldots_2, \lambda_2$$

$$d = ???$$




Humboldt State University

# Key words & Further Reading

- Cross products, vector identities, triple product
- Dynamics, Newton's law, inertial frame, accelerating frame, rotating frame, fictitious forces
- Ordinary Differential Equations, initial value problem, initial conditions
- trigonometry, law of sines, law of cosines, spherical trigonometry, distance on a sphere, latitude & longitude

**Further reading**

- Curtis, Chapter 1

Also:

- Chobotov, Chapter 1
- Batin, Chapter 2

# Orbits from First Principles

# Learning outcomes

- State solution of bound orbital motion of light satellite around a heavy primary
- Derive the correct solution for the unperturbed two body problem in inertial and non-inertial reference frames
- Quantify the accuracy of restricted 2BP
- Discuss the notion that the Earth orbits around the Sun

Previous lectures (Astronautics, Physics)

- Equation of motion:

$$\ddot{\vec{x}} = -\frac{GM}{|\vec{x}|^3}\vec{x} = -\frac{\mu}{|\vec{x}|^3}\vec{x}$$

$\vec{x}$ = vector to centre of body

*central force problem*

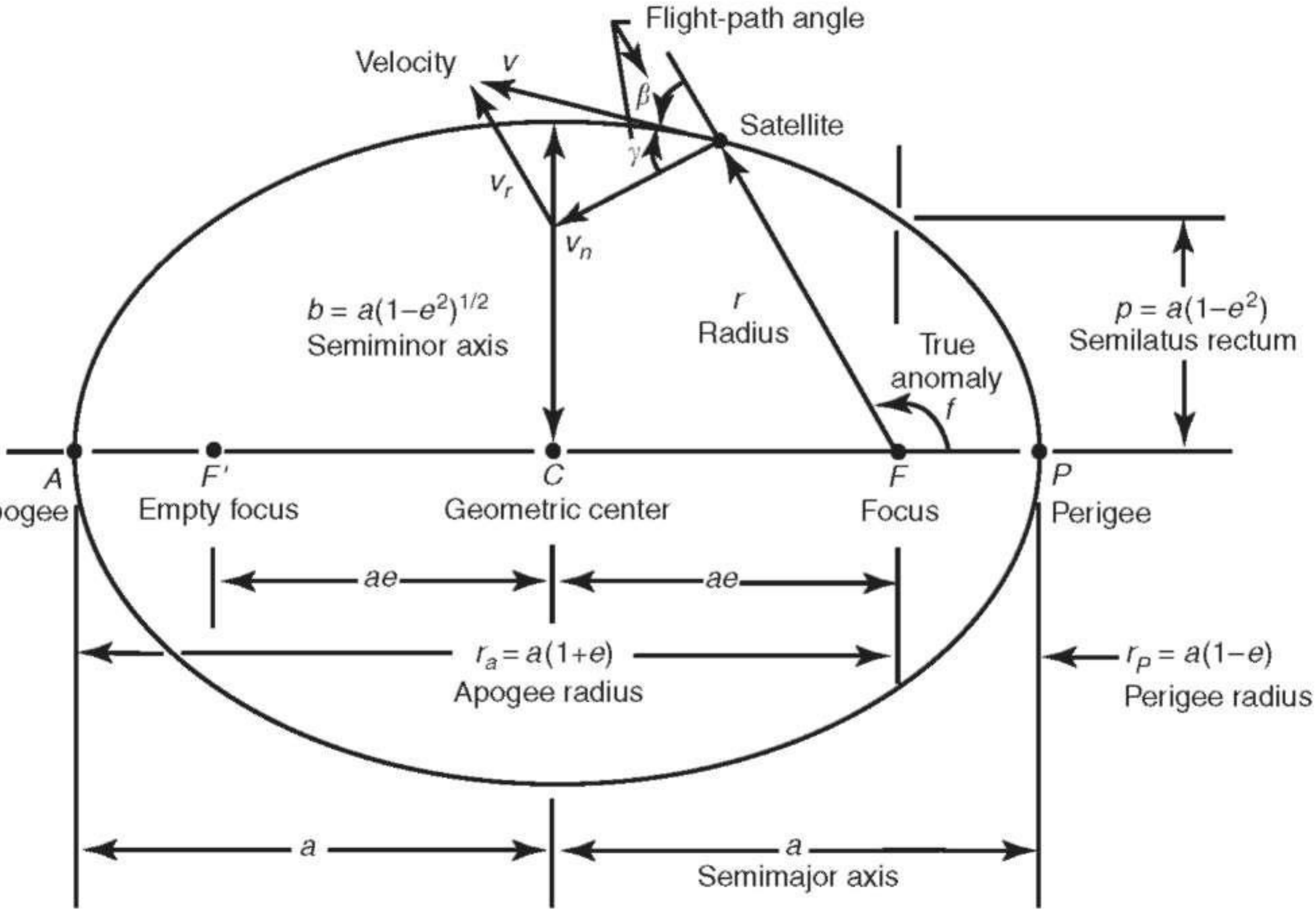
vector form of $y = \frac{\mu}{R^2}$

- Shape of solution:

$$r(\theta) = \frac{p}{1 + e\cos(\theta)}$$

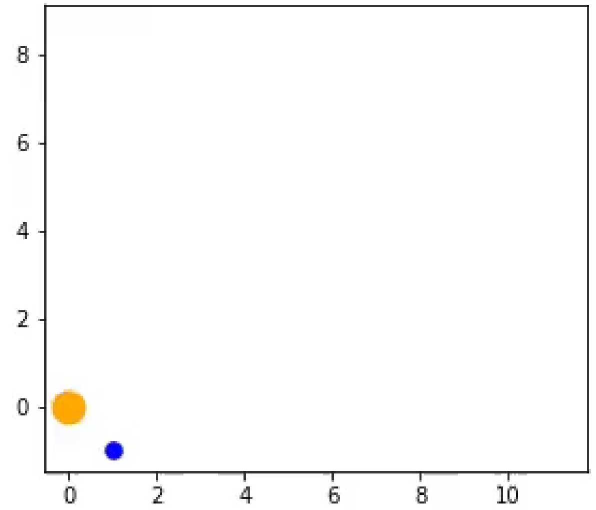
*conic section*

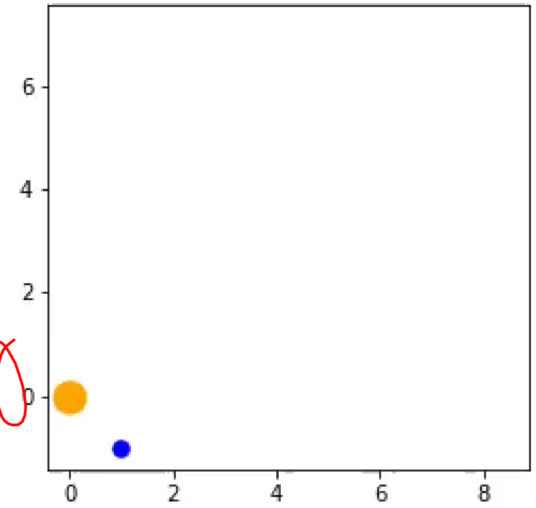
# Geometry of bounded 2BP motion



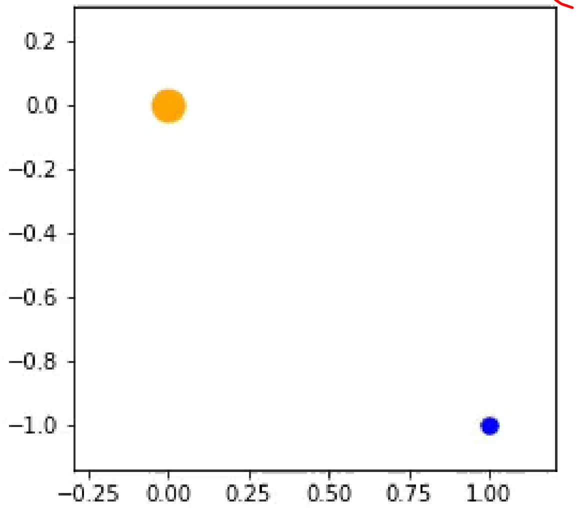
# Which one is 2BP motion?

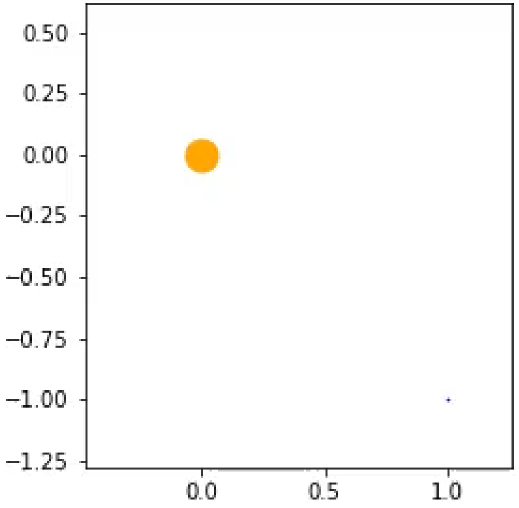
**Radius of objects proportional to mass**




(these are gifs showing 2 body problems)

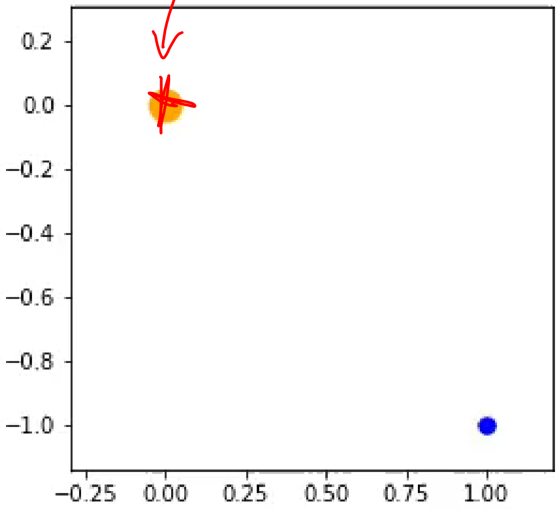


# Reference frames

- **All** of these were 2BP motion!
- Motion can be viewed from different reference frames
- **Bonus question**: which ones were **inertial** frames?

being used as the reference

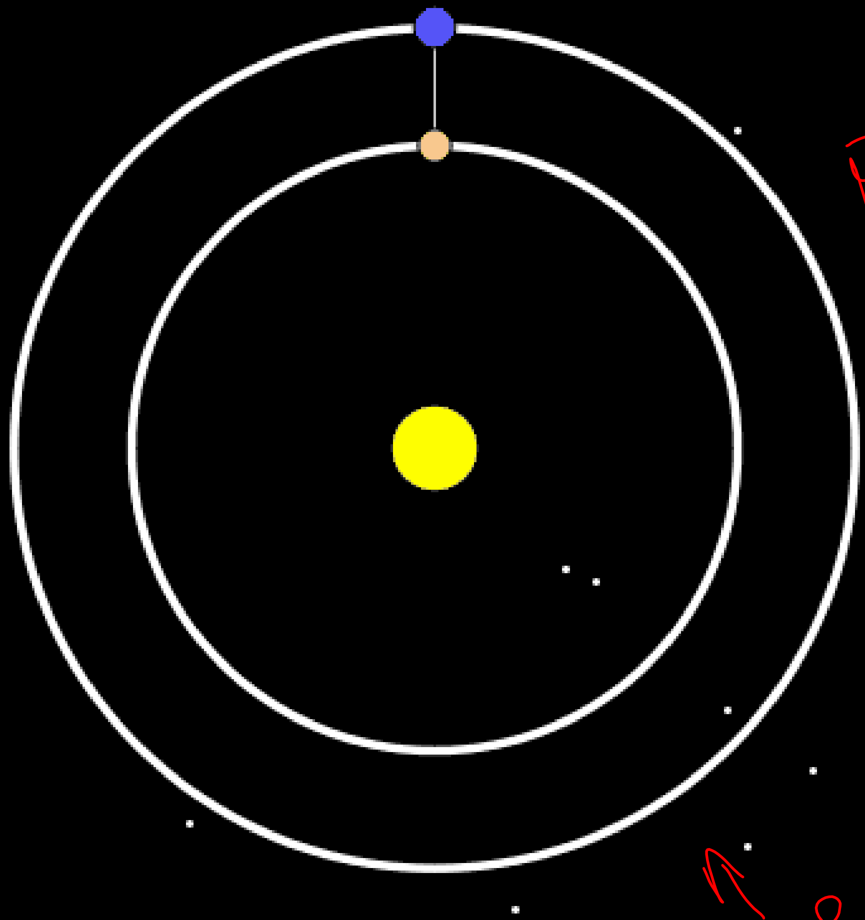



The only
non-inertial frame!

NASA

# Reference frames

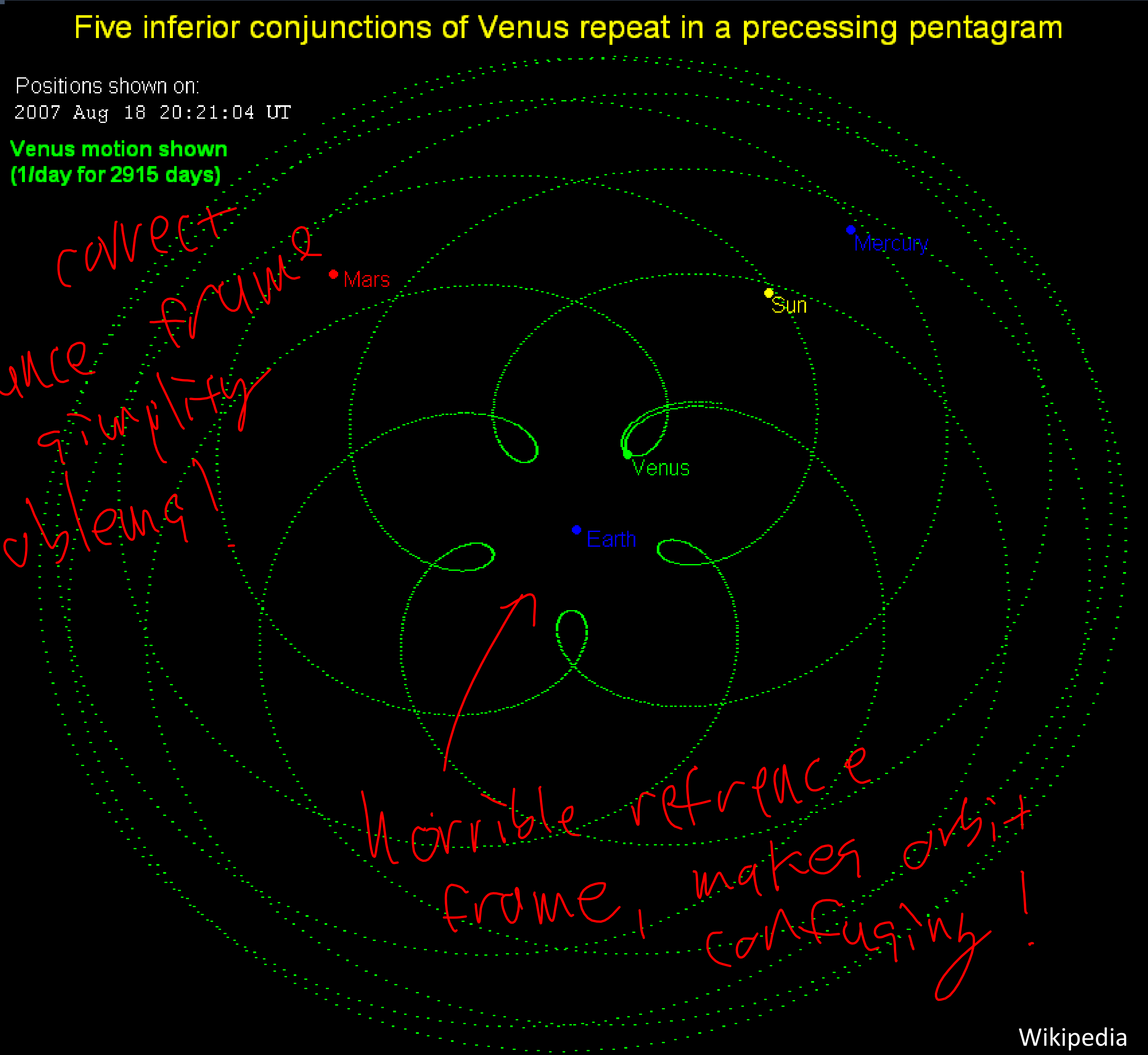
Gif!



Pick correct refrance frame to simplify problem!

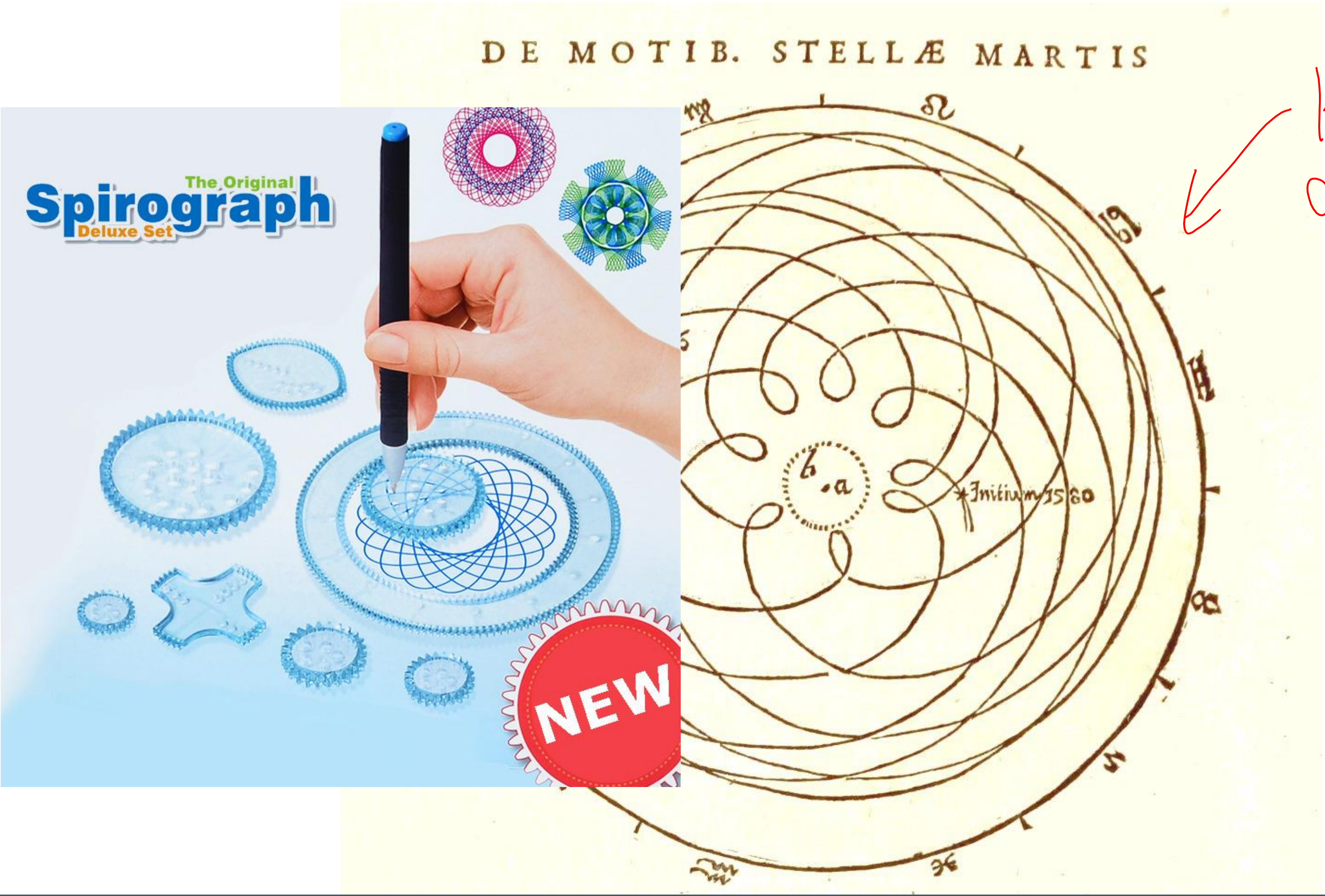
nice frame

Matthew Henderson




horrible refrence frame, makes orbit confusing!

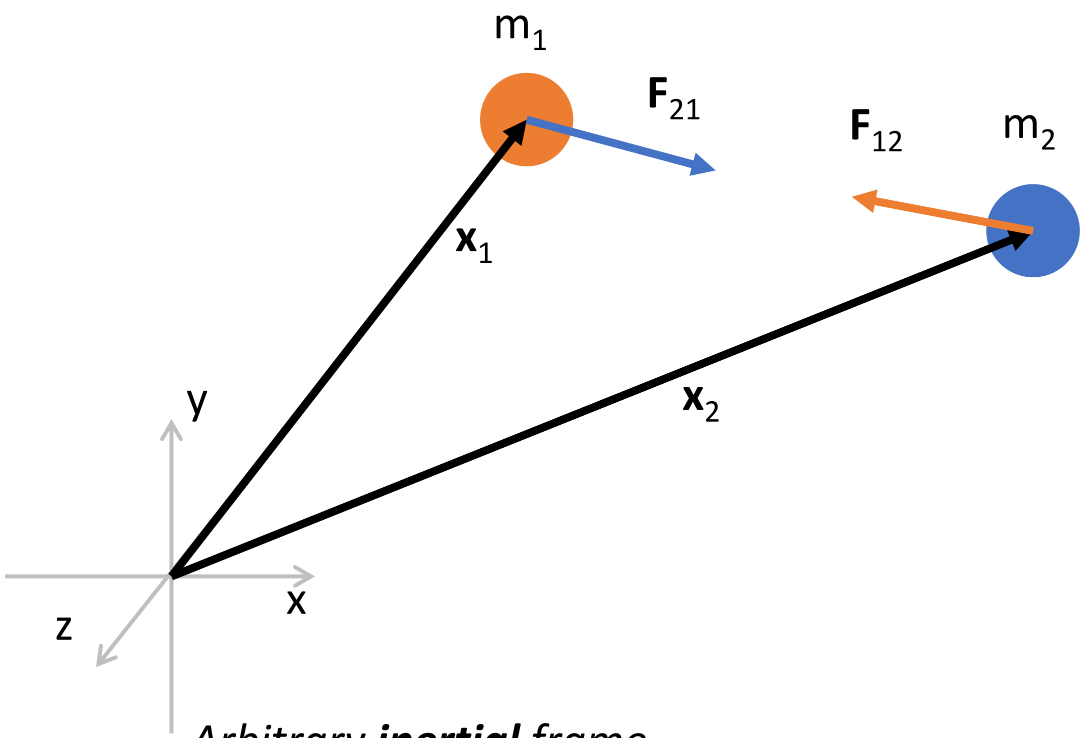
Wikipedia



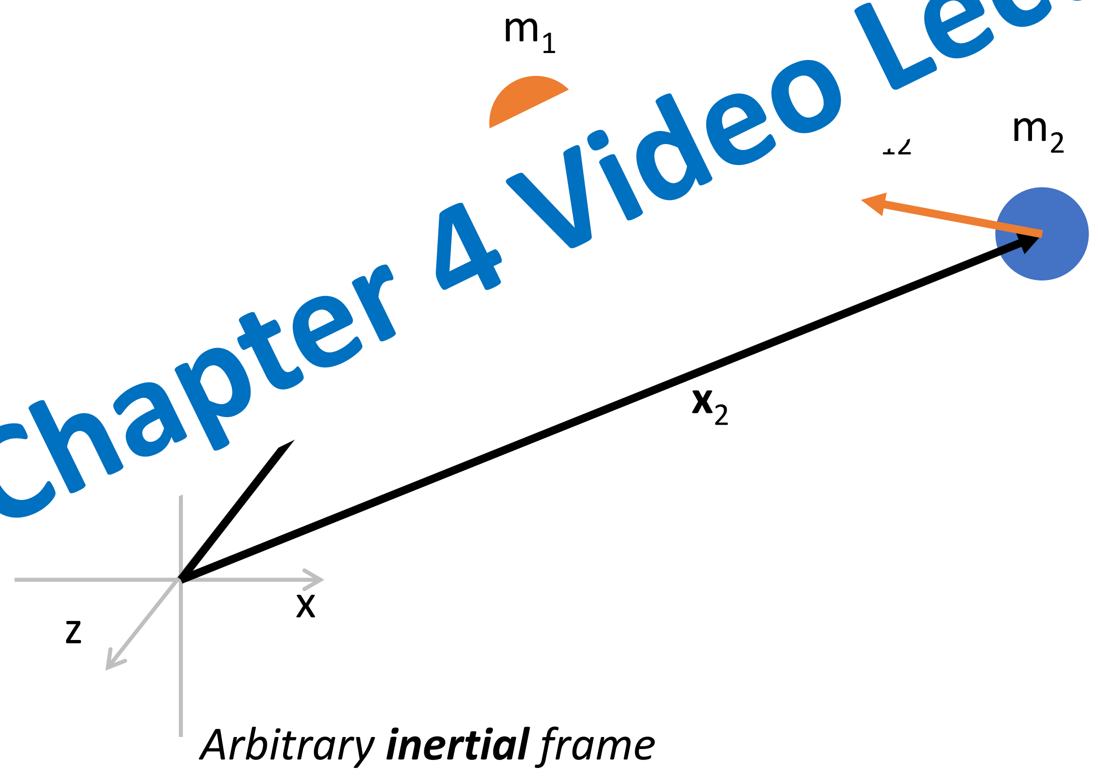

keplers observation of mars' position

Johannes Kepler/Astronomia Nova

# Back to Basics: First Principles




*Arbitrary **inertial** frame*




see Chapter 4 Video Lecture 3

## After some math: Motion splits in two

- Center of mass: moves uniformly

of "combined" thing

$$\vec{x}_{cm} = \frac{m_1 \vec{x}_1 + m_2 \vec{x}_2}{m_1 + m_2}$$

$$\ddot{\vec{x}}_{cm} = \vec{0}$$

$\dot{\vec{x}}_{cm} = \text{constant}$ uniform motion! (no acceleration)

- Relative distance: moves on conic section

$$\vec{d} = \vec{x}_2 - \vec{x}_1$$

$$\ddot{\vec{d}} = -\frac{\mu}{|\vec{d}|^3} \vec{d}$$

# Restricted 2BP

Observations:

- Both bodies on ellipses around barycenter
- Heavy mass moves closer to barycenter than light mass

Key word!

$$\vec{x}_1 = \vec{x}_{cm} - \frac{m_2}{m_1 + m_2}\vec{d}$$

$$\vec{x}_2 = \vec{x}_{cm} + \frac{m_1}{m_1 + m_2}\vec{d}$$

If $m_1 \ggg m_2$ :

- $\mu \approx Gm_1$
- $\vec{x}_{cm} \approx \vec{x}_1$

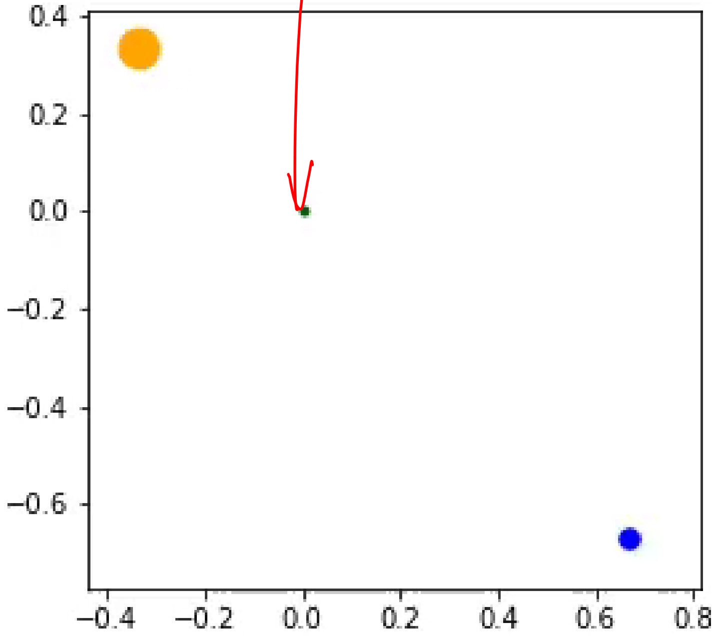
This is an inertial frame

since it's not accelerating!

$$\mu = G(m_1 + m_2)$$

*Restricted Two-Body Problem*

eg: satellite and Earth

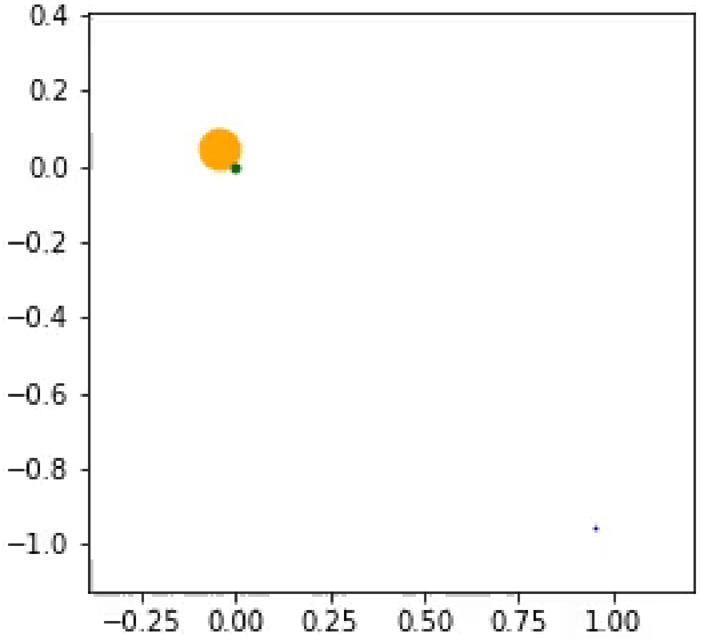



centre of mass

(include gifs in notes)

mass ratio: 1/2

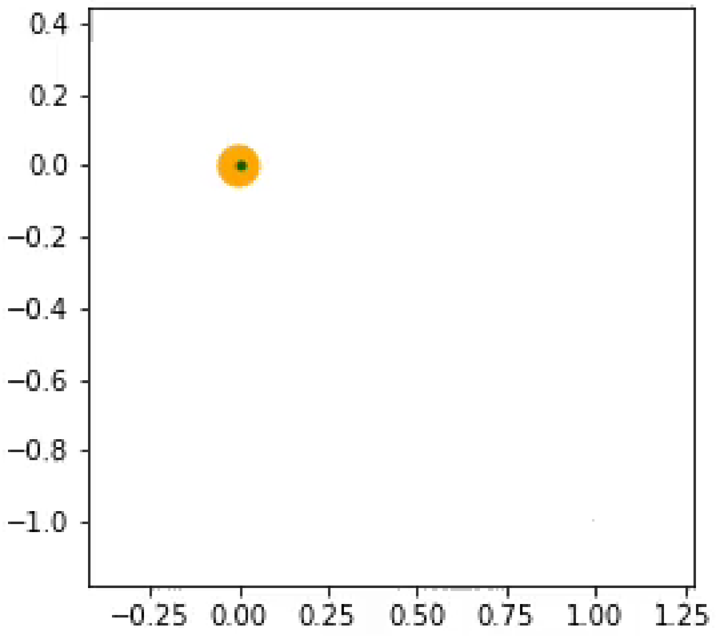
mass ratio 1:20




mass ratio: 1/20

# Restricted 2BP

mass ratio 1:200




mass ratio: 1/200

# Summary

- Full 2BP separates
  - Center of mass (barycenter) moves along line at constant velocity
  - Relative motion: moves on conic section

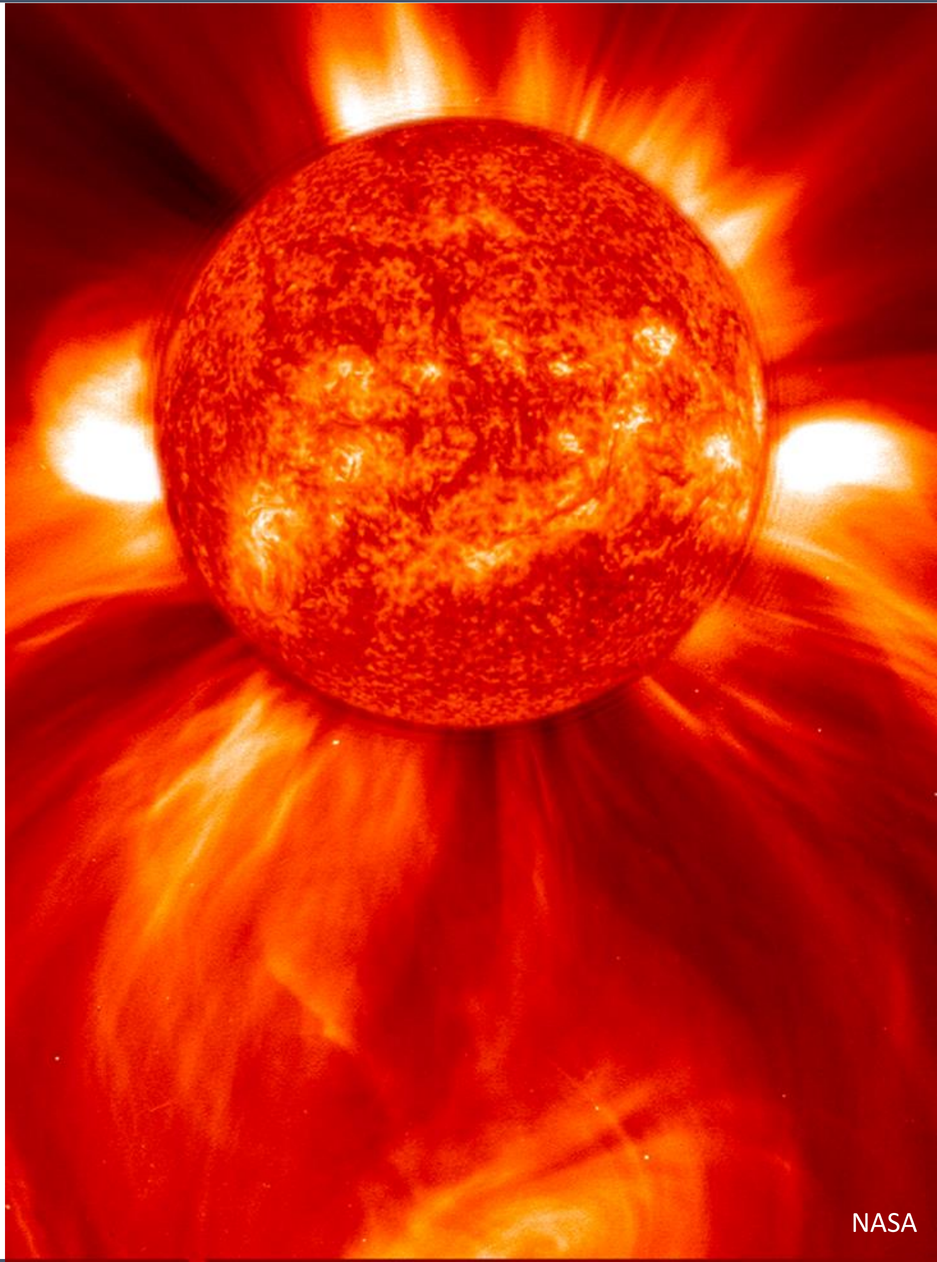
This holds for 2, 3, 4, 5.... body's

- Barycentric frame is inertial

poggers

- Both bodies orbit barycenter

- Large mass ratio: heavy body ≈ barycenter (restricted 2BP)



NASA

Advanced Astronautics (SESA3039)

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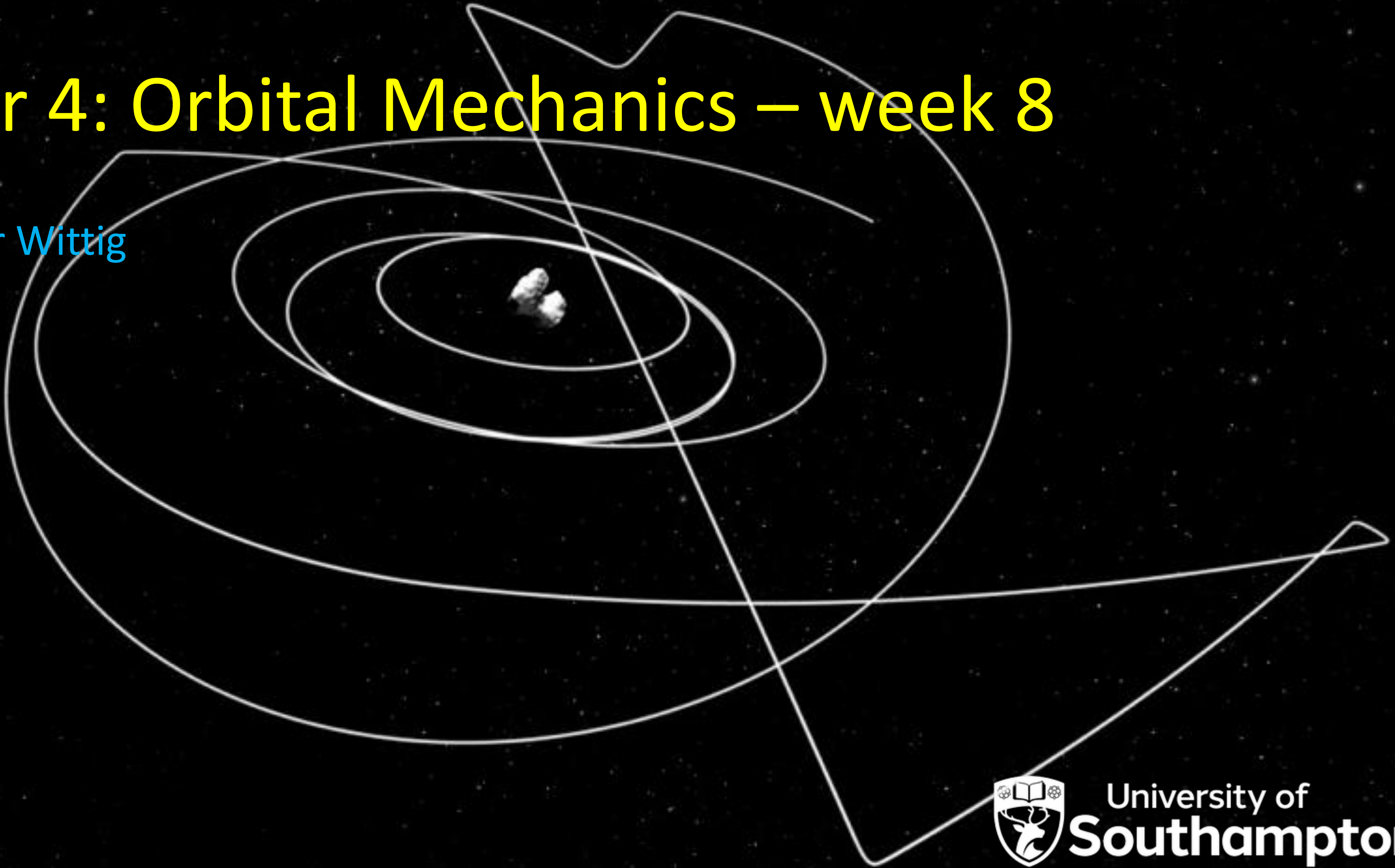
Dr. Alexander Wittig



Rosetta lander deployment orbit (ESA)

Southampton AERO ASTRO

University of Southampton