SESA3029 Aerothermodynamics

Lecture 5.3

Turbulent flow,

use of Reynolds analogy

Ground-based (near infra-red) visualisation of space shuttle transition phenomena (Horvath, 2012)

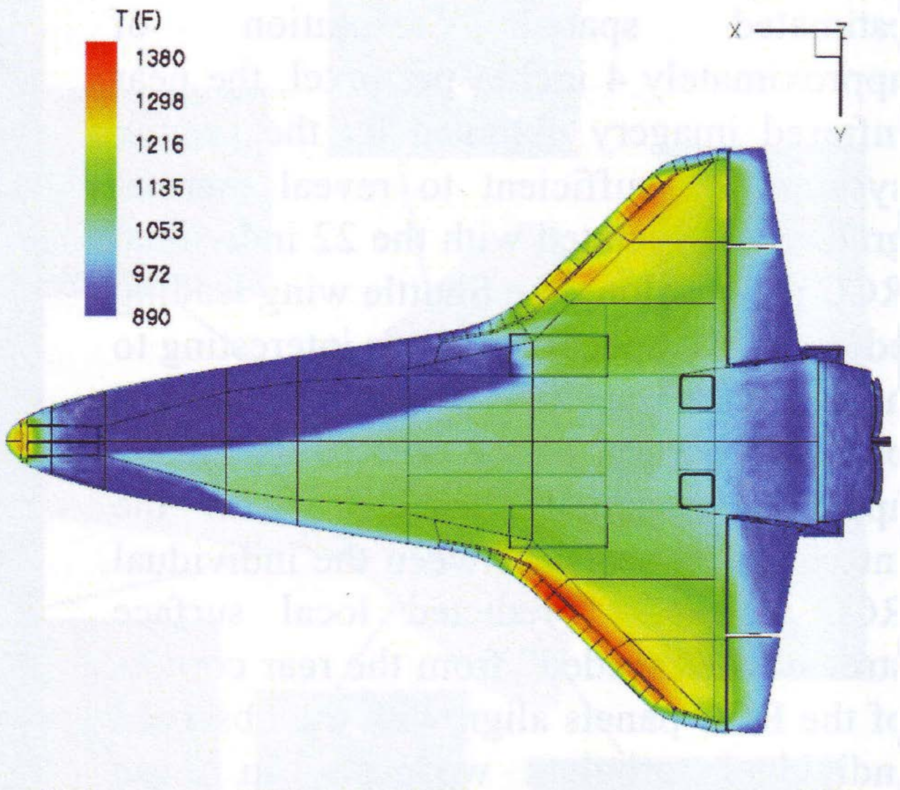


Figure 7b: STS-134 3-D temperature mapping.
Mach = 5.8, AOA = 28.8 deg

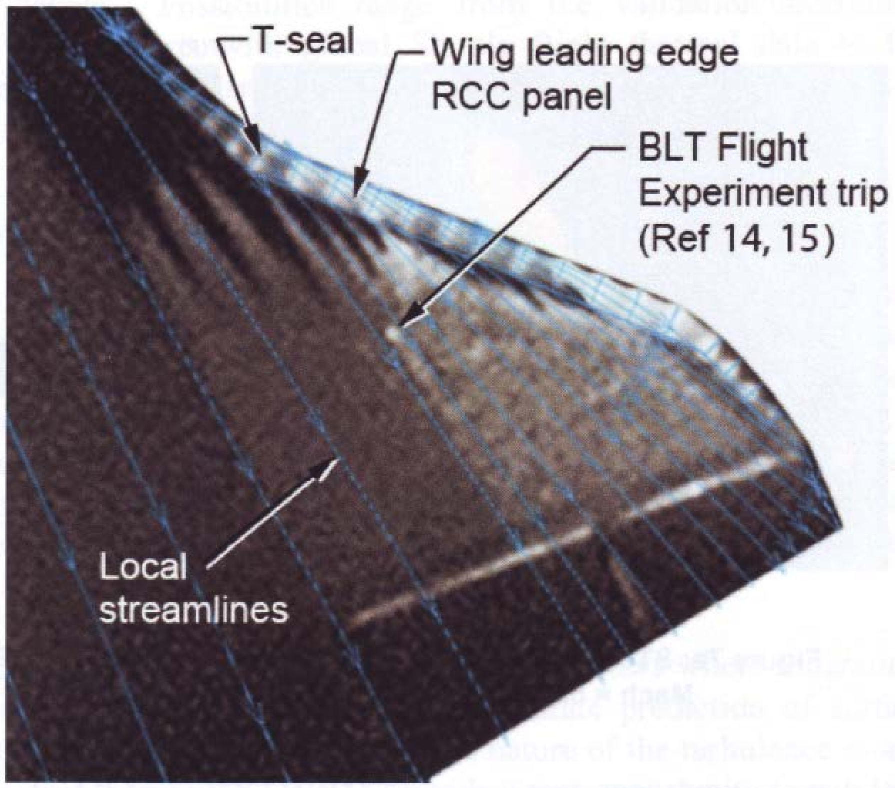


Figure 6: Transition patterns on Port wing. Turbulent wedges appear aligned with RCC panel T-seals. Mach = 5.8, AOA = 28.8 deg.

(STS=space transportation system; RCC=reinforced carbon-carbon; BLT=boundary-layer transition; AOA=angle of attack)

Some useful empirical correlations
(turbulent flow)

once again impirical formula

constants set to describe flow

Reynolds' analogy states $$\mathrm{Nu} = \mathrm{C}\ \mathrm{Re}^{n}\ \mathrm{Pr}^{m}$$

Turbulent pipe flow (2000<Re<50000)
n=0.3 for cooling, n=0.4 heating

$$\mathrm{Nu} = 0.023\mathrm{Re}^{4/5}\ \mathrm{Pr}^{n}$$

fluid being heated

fluid being cooled

Turbulent boundary layer(Re>300,000):

$$\mathrm{Nu}_x = 0.0308\mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3}\ \text{(for uniform wall heat flux)}$$

$$\mathrm{Nu}_x = 0.0296\mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3}\ \text{(for uniform wall temperature)}$$

Ref: Bergman et al. 'Fundamentals of Heat and Mass Transfer'

Example: laminar-turbulent boundary layer

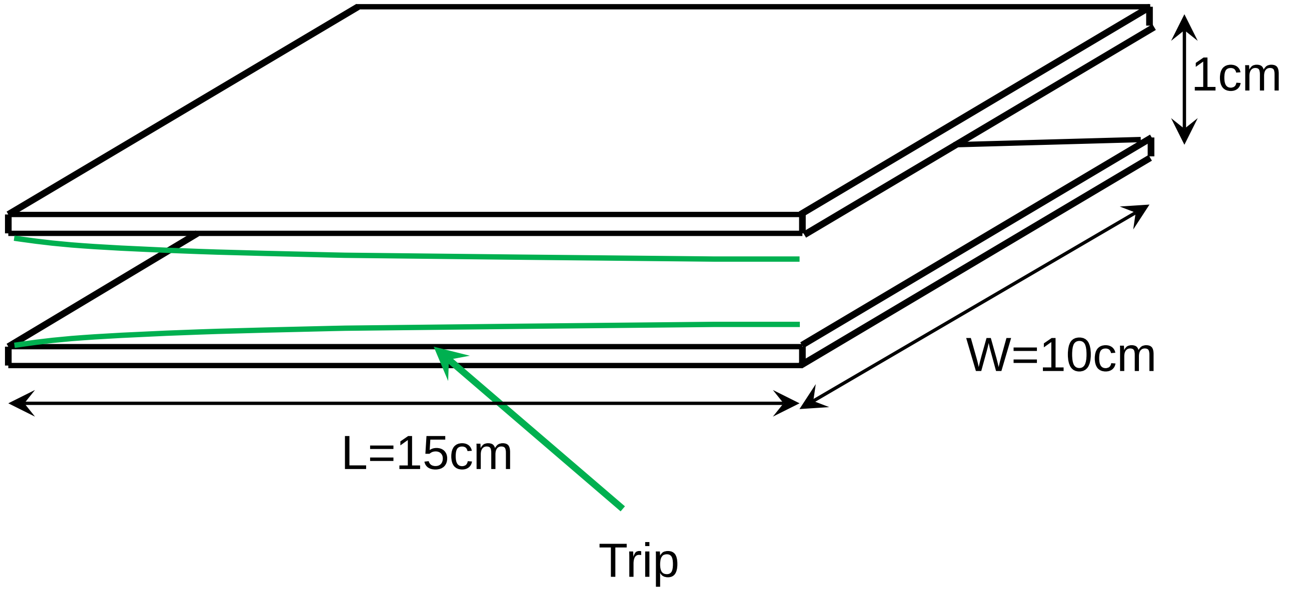
Two parallel plates at T=40°C

Air 8m/s, 2bar, 15 °C

μ=1.78x10^{-5} Ns/m^2

k=0.0248 W/(mK)

Pr=0.72



a) Now assume that a trip wire is installed at x=7.5cm turning the flow abruptly turbulent

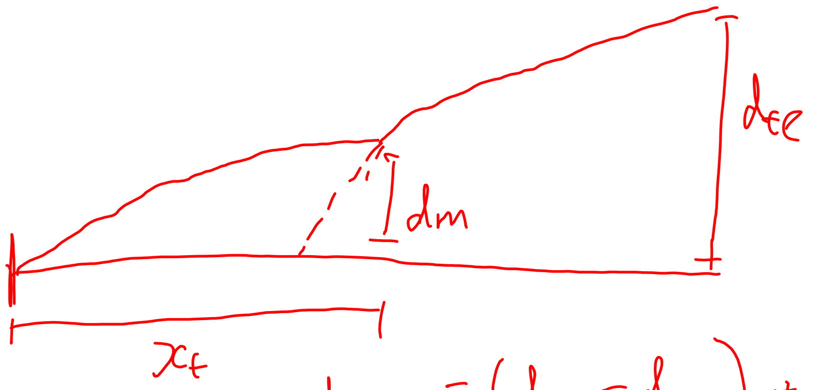
b) Find the heat transfer rate through each plate under the new conditions

BL thickness (laminar)

$$\delta = \frac{5x}{Re_x^{1/2}}$$

turbulant

$$\delta_t = \frac{0.37x}{Re_x^{1/5}}$$



$$\delta_{total} = (\delta_{te} - \delta_{tm}) + \delta_m =$$

$\delta_{te} = 5.03\,mm$

$\delta_m = 1.31\,mm$

$x_t = 0.075\,m$

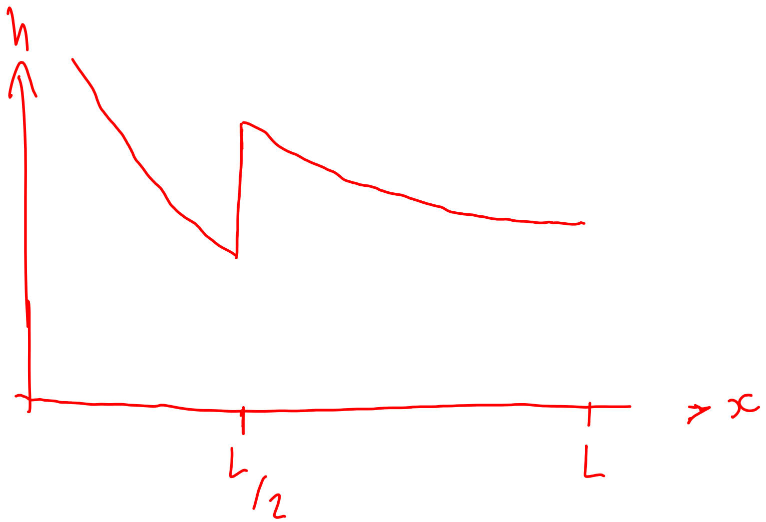
$$Nu_x = 0.0296\, Re_x^{4/5}\, Pr_x^{1/3}$$

turbulant case $\bar{h}$

av heat trans

$$\bar{h}_t = \frac{1}{L-x_t}\int_{x_t}^{L} \frac{Nu_x \cdot k}{x}\,dx = \frac{0.0296}{L-x_t}\left(\frac{\rho V}{\mu}\right)^{4/5} Pr^{1/3}\, k \underbrace{\int_{x_t}^{L} x^{-1/5}\,dx}_{=\frac{5}{4}x^{4/5}\Big|_{x_t}^{L}} = 69.09\ W/m^2K$$

$$\bar{h}_1 = \bar{h} \cdot \frac{\sqrt{0.15}}{\sqrt{0.075}} \doteq \bar{h}\sqrt{2} = 56.23 \ W/(m^2 K)$$



$$\dot{Q}_{total} = \left(\frac{\bar{h}_1 + \bar{h}_2}{2}\right) \times 0.15 \times 0.1 \times 25 = 23.5 \ W$$

since $x_1 = \frac{L}{2}$

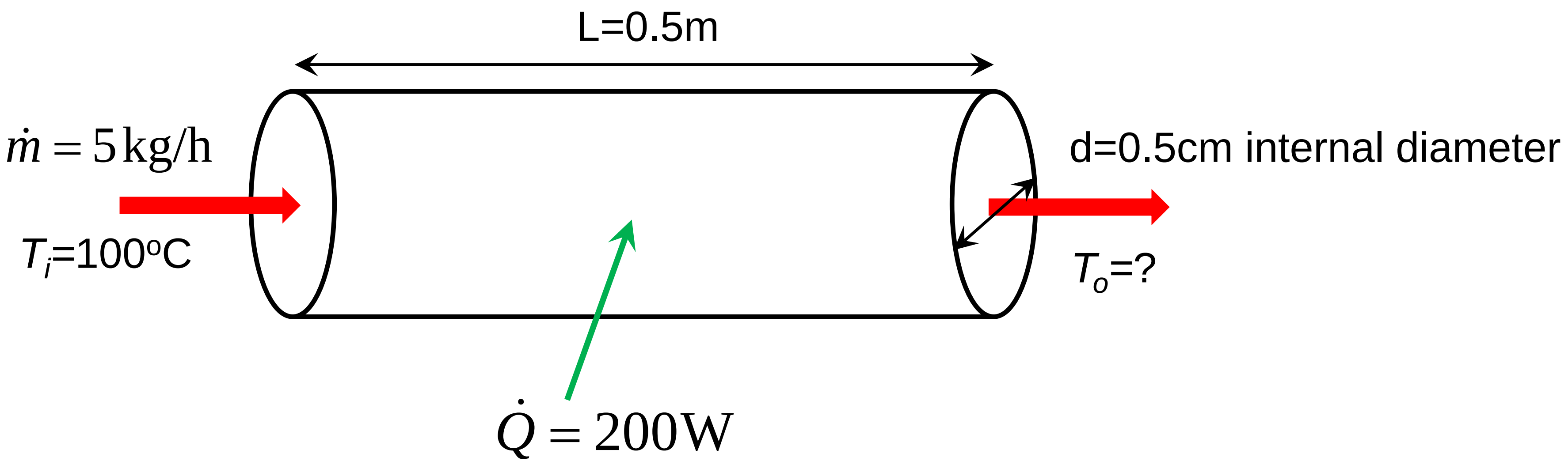
Example: Heat transfer in turbulent pipe flow

Air at 100°C

μ=2.48x10^{-5} Ns/m^2

k=0.037 W/(mK)

Pr=0.68



Air flows at 5kg/h through an electrically heated pipe of length 0.5m and internal diameter 0.5cm. The air enters at 100°C and the electrical power dissipation is 200W.
Find the

a) mixed mean outlet temperature
b) Reynolds number
c) maximum wall temperature

$c_p T = h$ ← enthalpy

$h \dot{m} = \text{watts}$

$$\dot{m} C_p T_{in} + \dot{Q} = m C_p T_{out}$$

$$T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m} C_p}$$

$$= T_{in} + \frac{\dot{Q}\mu}{\dot{m} Pr k}$$

$$= 100 + \frac{200 \times 2.48 \times 10^{-5}}{\frac{5}{3600} \times 0.68 \times 0.037} = 242°C$$

$$Pr = \frac{C_p \mu}{k}$$

if prandtl is not 0.71 we want to calc C_p

$$Re = \frac{\rho V d}{\mu}$$

$$\dot{m} = \rho V A_p \rightarrow \rho V = \frac{\dot{m}}{A_p}$$

$$= \frac{\dot{m} d}{A_p \mu}$$

$$= \frac{\frac{5}{3600} \times 0.005}{\pi \frac{0.005^2}{4} \times}$$

watch recording

$$Nu = 0.023\, Re^{4/5}\, Pr^{0.4} = \frac{hd}{k}$$

$$h = \frac{0.023\, Re^{4/5}\, Pr^{0.4}\, k}{d}$$

$$= \frac{0.023\,(14\ldots)^{0.8}\, 0.68^{0.4}\, 0.037}{0.005} = 307 \text{ W/m}^2\text{K}$$

$$\dot{q} = h(T_s - T) = \frac{\dot{Q}}{A_s} \qquad A_s = \pi d L$$

$$T_{s\,max} = T_{out} + \frac{\dot{Q}}{h A_s} = 242 + \underbrace{\frac{200}{307\pi\, 0.005 \times 0.5}}_{=82.9\text{ K}} = 324.9$$

max temp in fluid