SESA3029 Aerothermodynamics

Reaction Engines A2 (proposed Mach 5 vehicle)

Lecture 4.5

Ackeret theory for supersonic aerofoils

Recap: compressible potential flow (small disturbances assumed)

Velocity potential equation

$$\boxed{\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0}$$

$$\mathbf{u}'=\nabla\phi$$

$$C_p=-2\frac{u'}{U_\infty}$$

- Not valid near $M_\infty=1$ or for $M_\infty>5$
- We are considering solutions to this equation in two flow regimes
  - $M_\infty<0.8$ (elliptic equation: Prandtl-Glauert transformation – last time)

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

  - $1.2<M_\infty<5$ (hyperbolic equation)

Ackeret theory to solve $\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$

Supersonic flow, above the transonic regime

Define $\lambda=\sqrt{M_\infty^2-1}$ so that $\lambda^2\frac{\partial^2\phi}{\partial x^2}=\frac{\partial^2\phi}{\partial y^2}$

A wave equation: look for solutions where ϕ is constant along particular lines (characteristic lines)

$$\phi=\phi(\eta),\quad \eta=x-\lambda y$$

Check that solutions of the form $\phi(\eta), \quad \eta = x - \lambda y$

satisfy $\lambda^2 \dfrac{\partial^2 \phi}{\partial x^2} = \dfrac{\partial^2 \phi}{\partial y^2}$

$\dfrac{d\eta}{dx} = 1 \qquad \dfrac{d\eta}{dy} = -\lambda$

$$u' = \frac{\partial \phi}{\partial x} = \frac{d\phi}{d\eta}\boxed{\frac{\partial \eta}{\partial x}} = \frac{d\phi}{d\eta}$$

$$v' = \frac{\partial \phi}{\partial y} = \frac{d\phi}{d\eta}\boxed{\frac{\partial \eta}{\partial y}} = -\lambda\frac{d\phi}{d\eta}$$

$$u' = -\frac{v'}{\lambda}$$

Useful intermediate result

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{d}{d\eta}\left(\frac{d\phi}{d\eta}\right)\boxed{\frac{\partial \eta}{\partial x}} = \frac{d^2\phi}{d\eta^2}$$

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{d}{d\eta}\left(-\lambda\frac{d\phi}{d\eta}\right)\boxed{\frac{\partial \eta}{\partial y}} = \lambda^2\frac{d^2\phi}{d\eta^2}$$

$$\lambda^2 \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial y^2}$$

∴ √ Satisfies the equation

same equation

CheckBox1

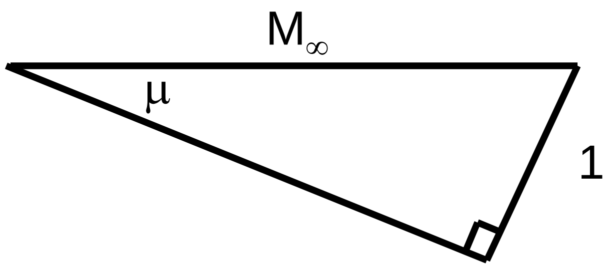
Ackeret theory

Wave form $\phi = f(\eta), \quad \eta = x - \lambda y$

Solutions are constant along lines of $y = \frac{x}{\lambda} + c$

$$\frac{dy}{dx} = \frac{1}{\lambda} = \frac{1}{\sqrt{M_\infty^2 - 1}}$$

Recall the Mach triangle

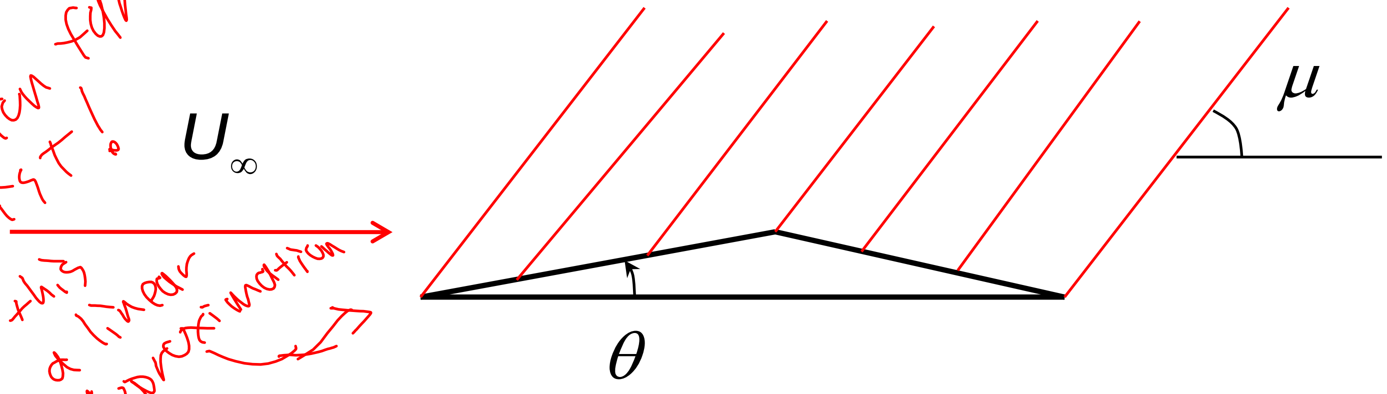


i.e. solutions are constant along Mach lines, since

$$\tan\mu = \frac{1}{\sqrt{M_\infty^2 - 1}} = \frac{dy}{dx}$$

Match Mach lines to surface boundary condition

note no shocks or expansion fan exist! since this is a linear approximation



A 'simple' flow in MoC terminology

Flow is tangent to the surface, so for small surface angles $\theta \approx \tan\theta = \dfrac{v'}{U_\infty}$

Recall $u' = -\dfrac{v'}{\lambda}$

so $C_p = -2\dfrac{u'}{U_\infty} = \dfrac{2v'}{\lambda U_\infty} = \dfrac{2\theta}{\lambda}$

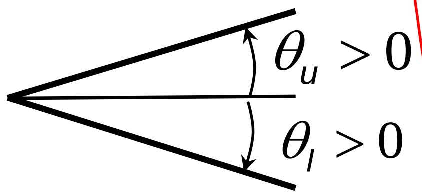
$$\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}}$$

Ackeret formula

Application: Flat plate at incidence in supersonic flow

Surface slope
(upper, lower)

$$\theta_u = -\alpha \qquad \theta_l = +\alpha$$



Sign convention

Ackeret

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,u} = \frac{-2\alpha}{\sqrt{M_\infty^2 - 1}} \qquad C_{p,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$

Normal force coefficient

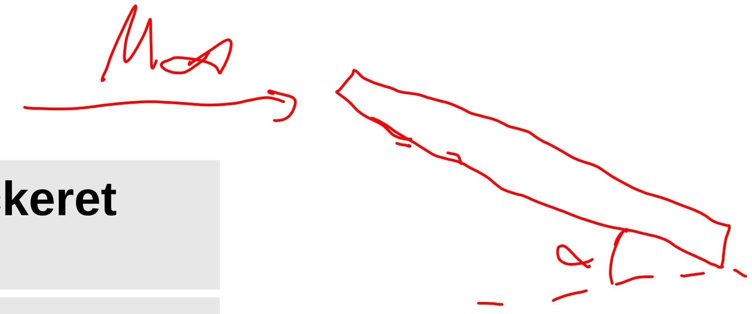
$$C_N = \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right)dx = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$C_L = C_N \cos\alpha \approx \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \qquad C_D = C_N \sin\alpha \approx \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

Wave drag is predicted!

Numerical example

- For a flat plate at 10 degrees incidence in a stream at Mach 2, compare the Ackeret predictions of lift and drag coefficients with the shock-expansion method



	Shock-expansion (computer solution)	**Ackeret**
C_L	0.4075	0.4031
C_D	0.0719	0.0703

$\alpha = 10^\circ \rightarrow \alpha = 10 \frac{\pi}{180}$ rad

$M_\infty = 2$

then just sub into formulas from prev slide

Summary (inviscid thin airfoil theory)