UNIVERSITY OF SOUTHAMPTON SESA3030W1

SEMESTER 1 ASSESSMENT PAPER 2021/22

TITLE: AEROSPACE CONTROL DESIGN

DURATION – 2 HOUR (2.5 hours if moved online)

This paper contains **FOUR** questions.

Answer **ALL** questions in **Section A** and **only TWO** questions in **Section B.**

Section A carries 40% of the total marks for the exam paper.

Section B carries 60% of the total marks for the exam paper.

An outline marking scheme is shown in brackets to the right of each question. Note that marks will only be awarded when appropriate working is given. All solutions should be hand-written and all steps should be shown to receive full credit. Provide explanations for every answer and indicate the unit(s) used in **ALL** calculations.

Additional generic rubric will apply in the event that the exam is moved online.

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SECTION A

A1.

(i) Reduce the block diagram shown in Figure A1-1 to a single block and write the overall transfer function $C(s)/R(s)$. Show all of the steps performed.

[5 marks]

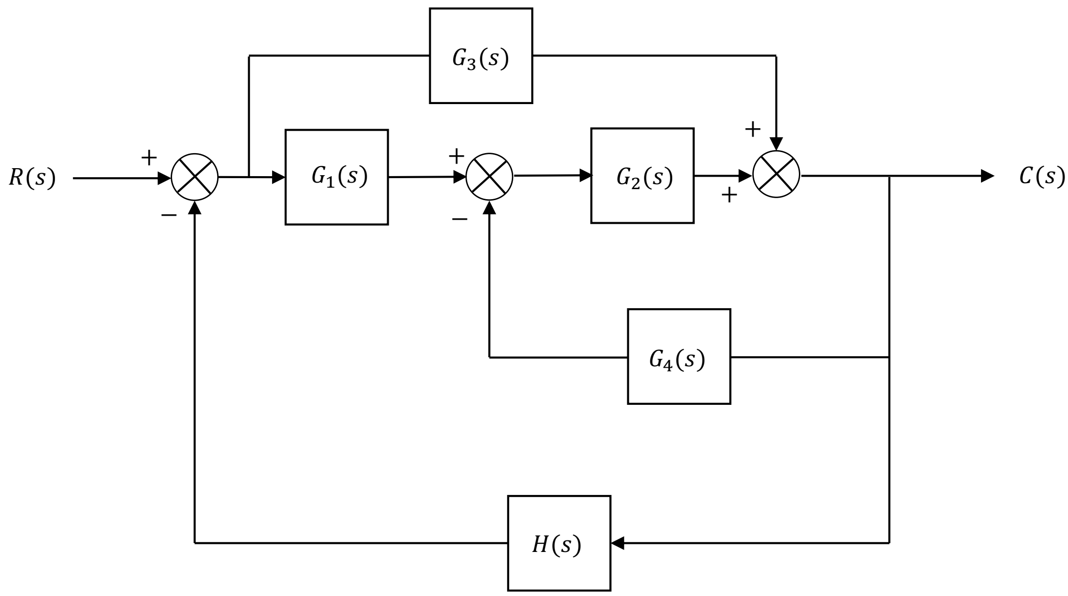

Figure A1-1

(ii) The output of a system given by transfer function $G(s)$ to a sinusoidal input $u(t) = \sin t$ is expressed in the time domain for $t \geq 0$ as:

$$y(t) = 0.5 \sin t - 1.5 \cos t + 2.5e^{-t} - e^{-2t}$$

Determine the transfer function $G(s)$. Hint: recall the Laplace transforms $\mathcal{L}[\sin t] = 1/(s^2 + 1)$ and $\mathcal{L}[\cos t] = s/(s^2 + 1)$.

[5 marks]

(iii) Give definitions for:

(a) the root locus,
(b) gain margin (GM),
(c) phase margin (PM).

[3 marks]

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(iv) Find the natural frequency (ω_n), damping factor (ζ), peak time (T_p), settling time (T_s), and percentage overshoot (PO) to a step input for the system given by the transfer function:

$$G(s) = \frac{4}{s^2 + s + 4}$$

[4 marks]

(v) Give examples of two types of nonlinearity and which parts they might represent in a system.

[3 marks]

(vi) Determine the gain and phase of the system at frequency $\omega = 1$ rad/sec:

$$G(s) = \frac{5s^2 + 3s + 2}{s^4 + 10s^2 + 3s + 3}$$

[4 marks]

(vii) Find a state space form for the system described by the transfer function:

$$G(s) = \frac{2s + 1}{s^2 + 7s + 9}$$

[5 marks]

(viii) Find a transfer function of a second order system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

That yields a 16.3% overshoot and a settling time 2 seconds.

[4 marks]

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(ix) Consider the differential equation:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = f(x)$$

where $f(x(t))$ is the input and is a function of the output variable $x(t)$. If $f(x) = \sin x$, linearize the differential equation around a) $x = 0$ and b) $x = \pi$.

[4 marks]

(x) (a) Give a brief justification for why it is convenient to use decibels (dB) in Bode magnitude plots.

(b) Assume a Bode plot shows a magnitude of 25dB. What is the gain of the corresponding transfer function?

[3 marks]

SECTION B

B1. You are tasked with designing a controller that keeps a wing's angle of attack fixed at the reference value r. The actuation u is provided by a trailing edge flap which is allowed to deflect when the wing encounters external disturbances d such as gusts. The output y is the lift coefficient, which can be written as the sum of the effects due to actuation and external disturbances, i.e. $y = G_{yu}u + G_{yd}d$.

[30 marks total]

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(i) Consider a controller where the actuation u is a sum of a feedforward component $G_{ff} = G_{yu}^{-1} G_{yd}$ and a unity gain feedback component G_{fb}. Draw a block diagram of the system.

[6 marks]

(ii) The plant dynamics are given by the following transfer function.

$$G_{yu} = \frac{a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0}$$

Sketch the approximate Bode magnitude and phase plots of the system.

[8 marks]

(iii) Although the coefficients of the transfer function G_{yu} are not given, how might you be able to find them experimentally?

[4 marks]

(iv) Ignoring the feedforward component of the controller, design the feedback component of the controller such that the closed-loop system attenuates noise at high frequencies and provides reference tracking for lower frequencies. Explain why your choice of controller works by sketching the modified Bode magnitude and phase plots of the open-loop transfer function.

[8 marks]

(v) Let's now consider the feedforward component of the control. Explain how its role differs from that of the feedback component.

[4 marks]

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B2. The dynamics of a spacecraft are given by the following ordinary differential equations

$$\dot{x} = -3x + 3$$
$$\dot{y} = (x + y)^2 - x$$
$$\dot{z} = \sin z$$

[30 marks total]

(i) Find all of the equilibrium points for this system.

[4 marks]

(ii) Linearise the system around the equilibrium point $(x_0, y_0, z_0) = (1,0,0)$.

[12 marks]

(iii) Assuming no control, express the linearised system in state-space form.

[3 marks]

(iv) Is the system stable and why?

[3 marks]

(v) We would like to design a controller for the altitude (z-coordinate) such that its eigenvalue is shifted to $\lambda_3 = -1$. Find the gain of a proportional controller that can correctly shift this eigenvalue from the value you found in part (iv).

[5 marks]

(vi) What are the limitations of the linearised model? How can it be made more realistic?

[3 marks]

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B3. You have been tasked with designing an adaptive helicopter seat that reduces vibrations during flight. The plant model has been simplified to the following transfer function

$$G(s) = \frac{1}{s^2 - 2s + 101}$$

[30 marks total]

(i) The Bode plots for this system are given in Figure B3-1. Explain why this is the correct Bode plot for the system.

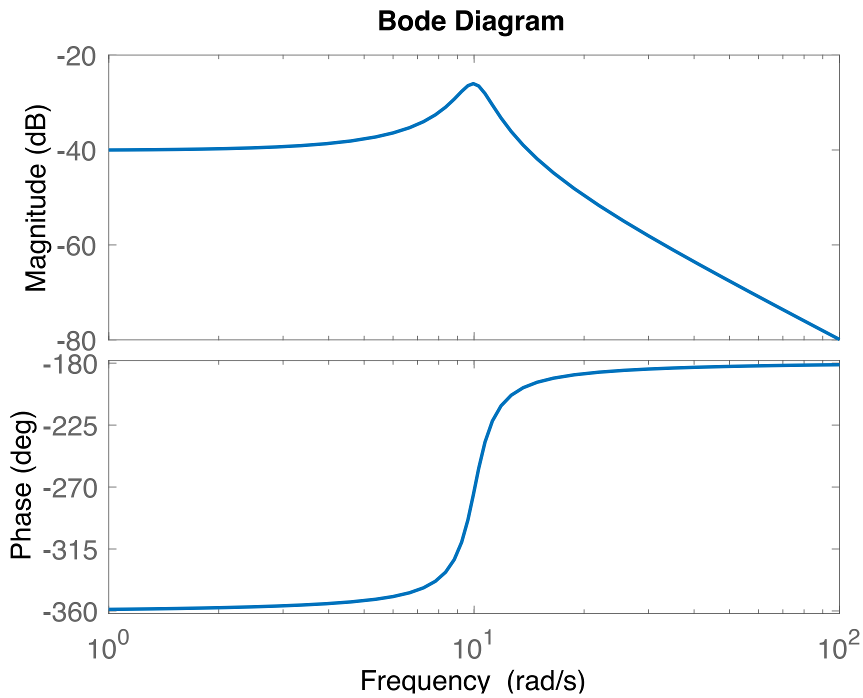

Figure B3-1

[6 marks]

(ii) Sketch the Nyquist plot for this system. Using the Nyquist stability criterion, is the system stable or unstable?

[6 marks]

(iii) Using the Nyquist plot, explain why a proportional controller cannot stabilise this system even for large gains.

[2 marks]

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(iv) Using a PD controller and setting $T_d = 1$, what is the minimum K_p needed to stabilise the system?

[4 marks]

(v) Setting K_p to double the minimum value you found in question B3-(iv), sketch the new Bode plots. (Hint: You will need to estimate the maximum gain that occurs near the resonant frequency.)

[6 marks]

(vi) Using your modified Bode plots, sketch the new Nyquist plot. Is the system stabilised according to the Nyquist criterion?

[6 marks]

END OF PAPER

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