University of Southampton

Chapter 5: Mission Analysis
Workshop 1

Professor Hugh Lewis

Workshops - overview

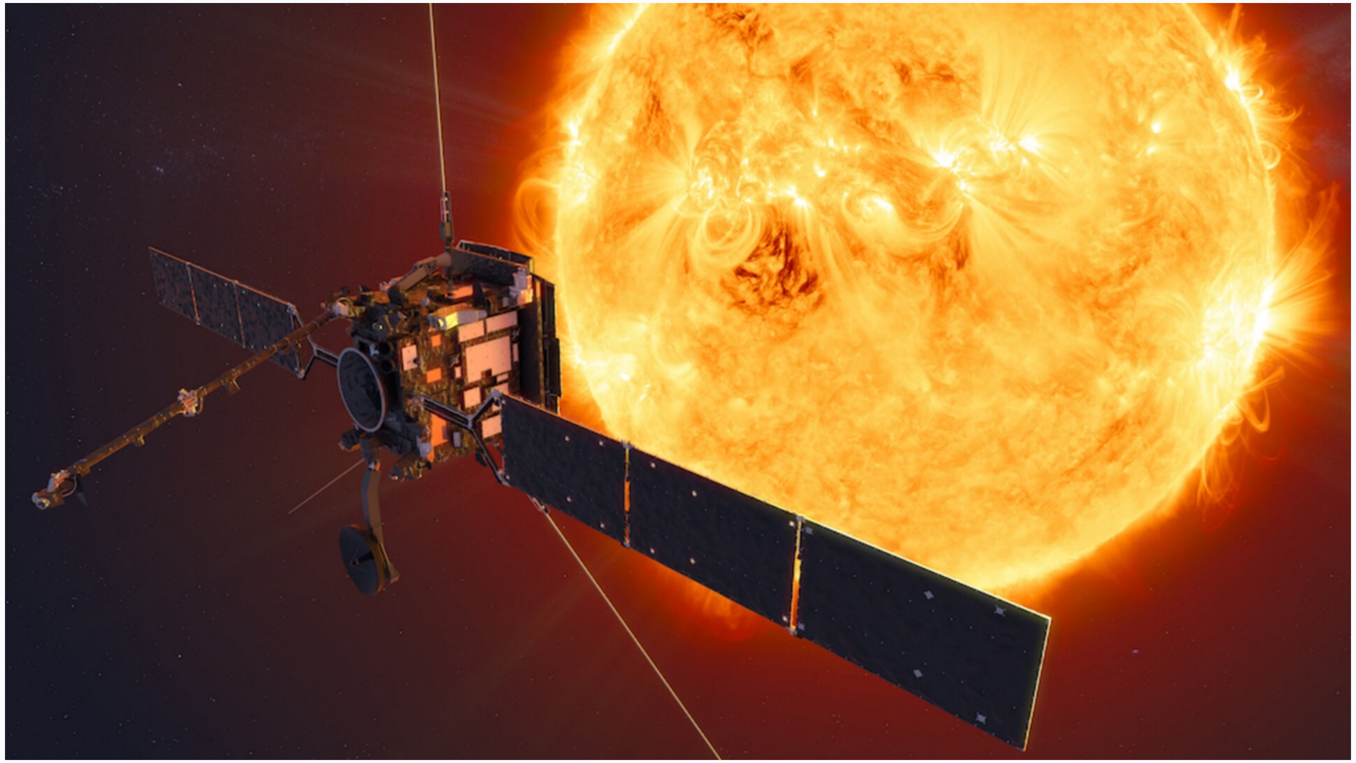
- Orbital elements demonstration
- Solar Orbiter worked example:
  - Ellipse equation
- Space tourism worked examples:
  - Sub-orbital flight, Blue Origin NS-18 (William Shatner)
  - SpaceX Crew-4 (Freedom) re-entry
  - SpaceX first Starship orbital flight
- DART worked example:
  - Characterising the effects of the DART impact on Dimorphos
- Space debris mitigation example:
  - FCC’s new 5-year de-orbit rule
- Quick quizzes
  - Check your understanding of orbits, ground tracks, and ACS

- If we are unable to complete everything in the workshop sessions, you can use the worked examples for self-study, revision, etc.

Orbital elements

Ellipse equation

- Solar Orbiter
  - Perihelion: 0.28 AU
  - Aphelion: 0.93 AU
- Calculate:
  - Semi-major axis
  - Eccentricity
  - Altitude at perihelion
  - (Speed at perihelion)



https://www.esa.int/Science_Exploration/Space_Science/Solar_Orbiter/Solar_Orbiter_factsheet

$\mu_S = 1.3271244 \times 10^{11}\ \mathrm{km^3 s^{-2}}$

$1\ \mathrm{AU} = 149{,}597{,}870.7\ \mathrm{km}$

$R_S = 696{,}342\ \mathrm{km}$

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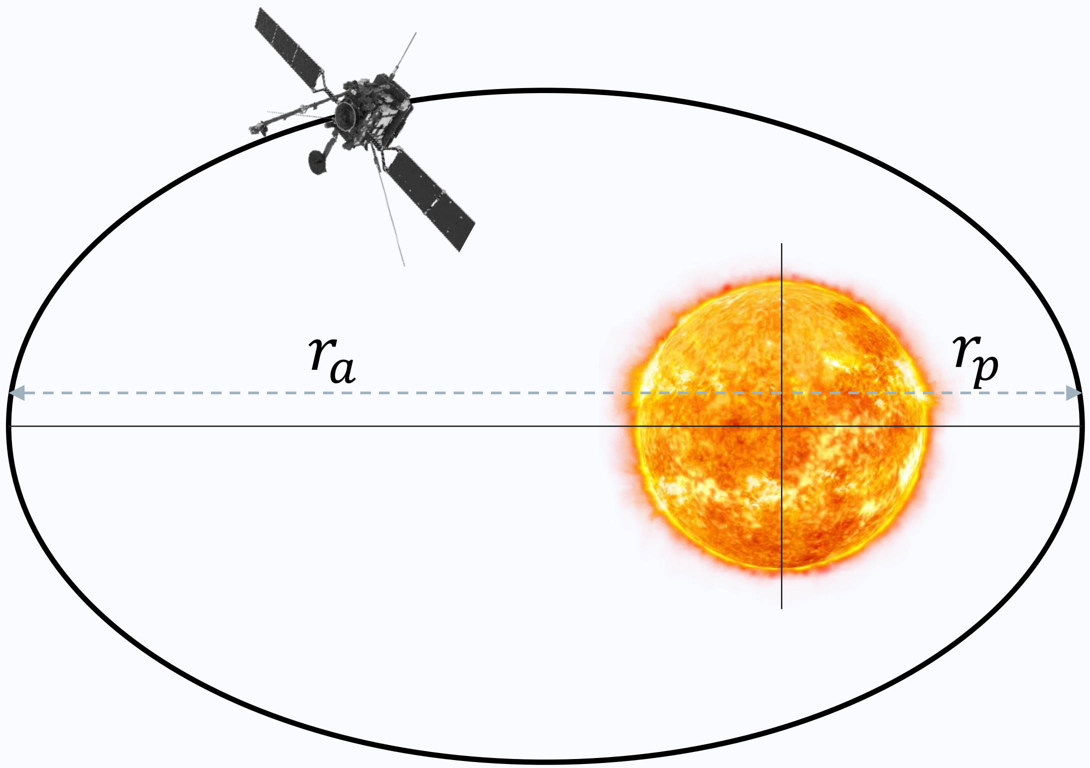
Ellipse equation

- Semi-major axis:

$$a = \frac{1}{2}\left(r_p + r_a\right)$$

$$= \frac{1}{2}(0.28 + 0.93)$$

$$= 0.605\ \text{AU} = 90{,}506{,}711.75\ \text{km}$$



Ellipse equation

- Eccentricity:

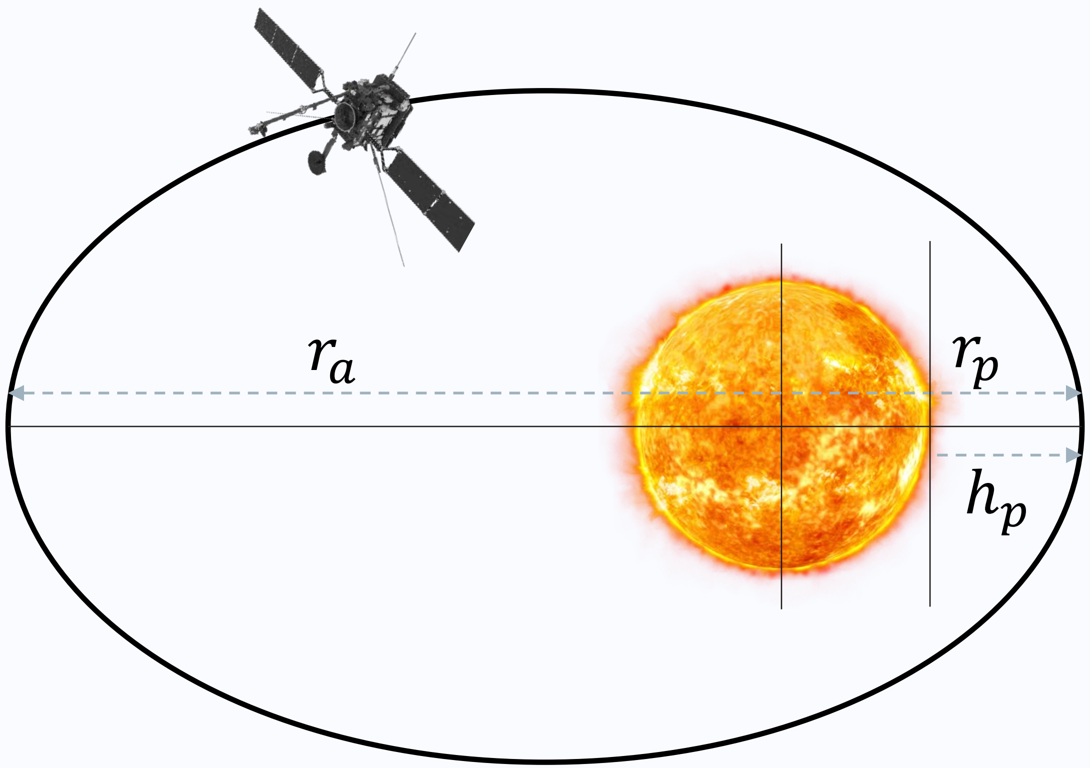
$$e = 1 - \frac{r_p}{a}$$

$$= 1 - \frac{0.28}{0.605} = 0.53719$$

- Altitude:

$$h_p = r_p - R_S$$

$$= 41{,}191{,}061.8 \text{ km}$$



Ellipse equation

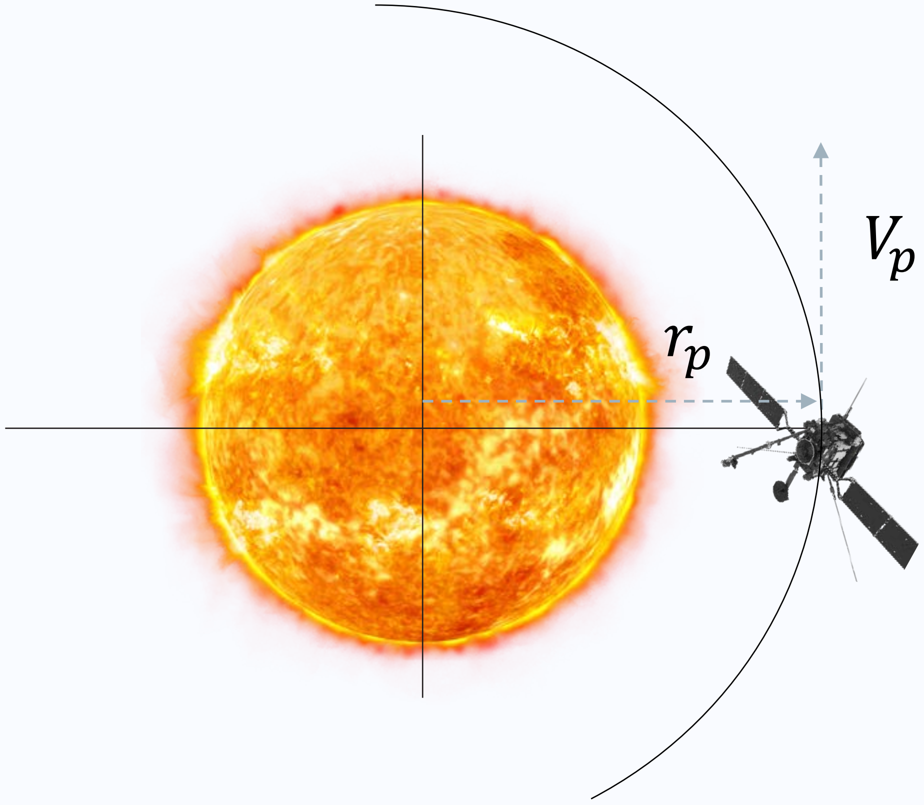
- Speed at perihelion:

$$\frac{V_p^2}{2} - \frac{\mu_S}{r_p} = -\frac{\mu_S}{2a}$$

$$V = \sqrt{\mu_S\left(\frac{2}{r_p} - \frac{1}{a}\right)}$$

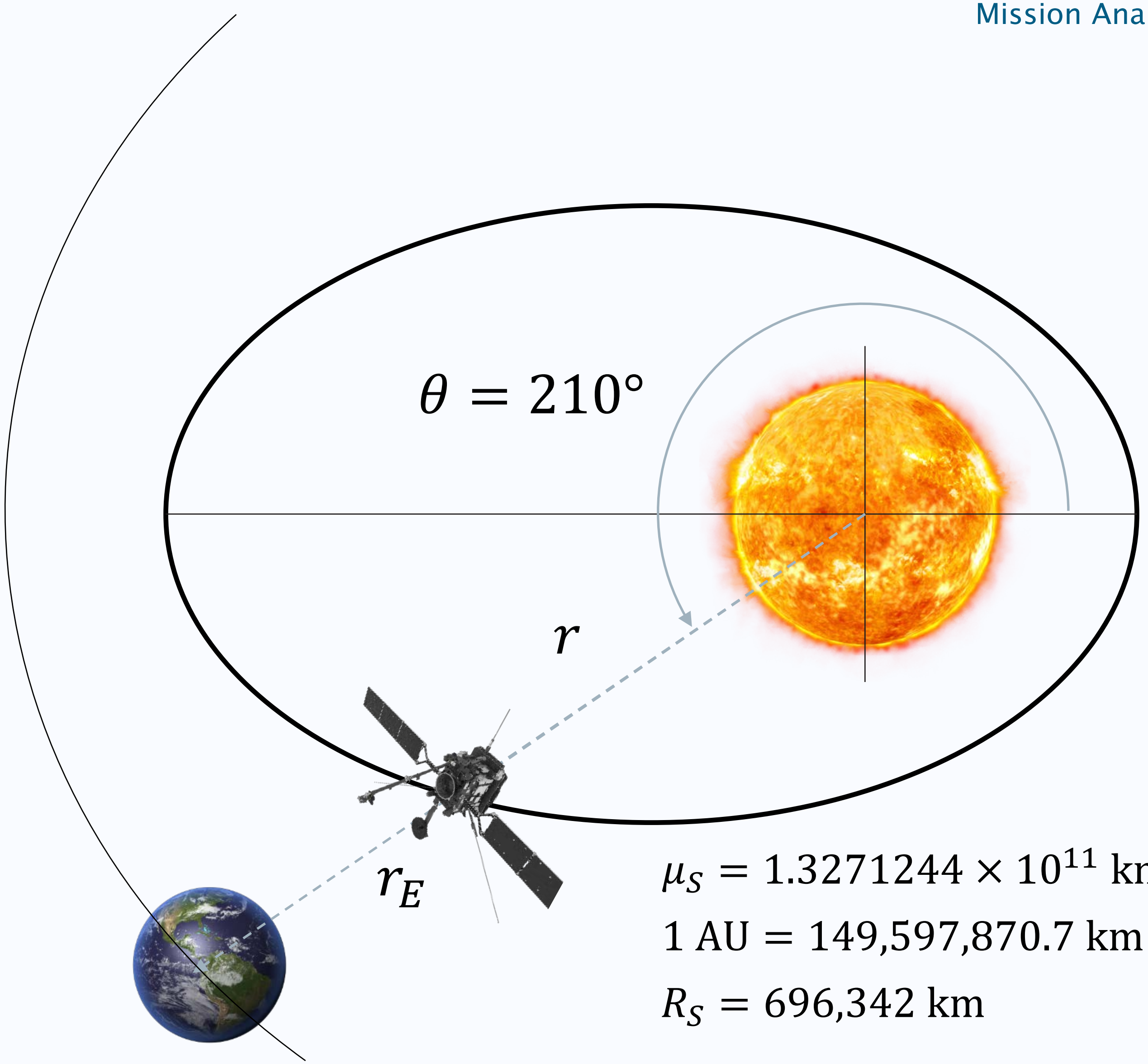
$$V = \sqrt{1.3 \times 10^{11}\left(\frac{2}{4.2 \times 10^7} - \frac{1}{9.1 \times 10^7}\right)}$$

$$= 69.79\ \text{km/s}$$



Ellipse equation

- If the Earth-Sun line and the spacecraft-Sun line are aligned when Solar Orbiter is at a true anomaly of $\theta = 210°$ calculate:
  - The radius of the Solar Orbiter orbit at this point
  - The Earth-Spacecraft distance (assuming Earth's orbit is circular)



$\mu_S = 1.3271244 \times 10^{11}\ \mathrm{km^3 s^{-2}}$

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$R_S = 696{,}342\ \mathrm{km}$

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Ellipse equation

- Use the ellipse equation:

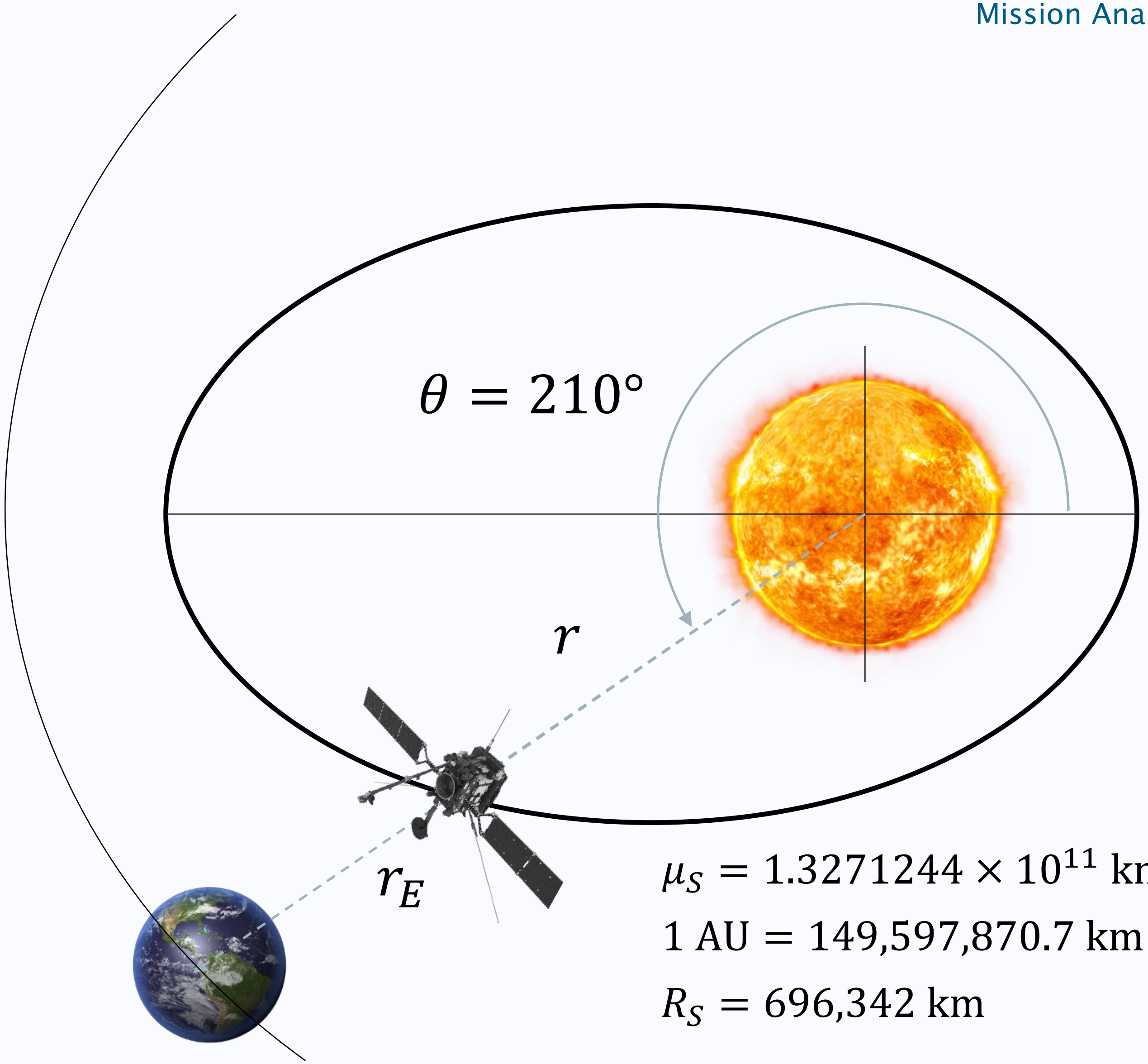
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

  - Use Astronomical Units (for simplicity)

$$r = \frac{0.605(1-0.53719^2)}{1+0.53719\cos 210°}$$

$$= 0.8048\ \text{AU}$$

$$= 120{,}402{,}656.8\ \text{km}$$



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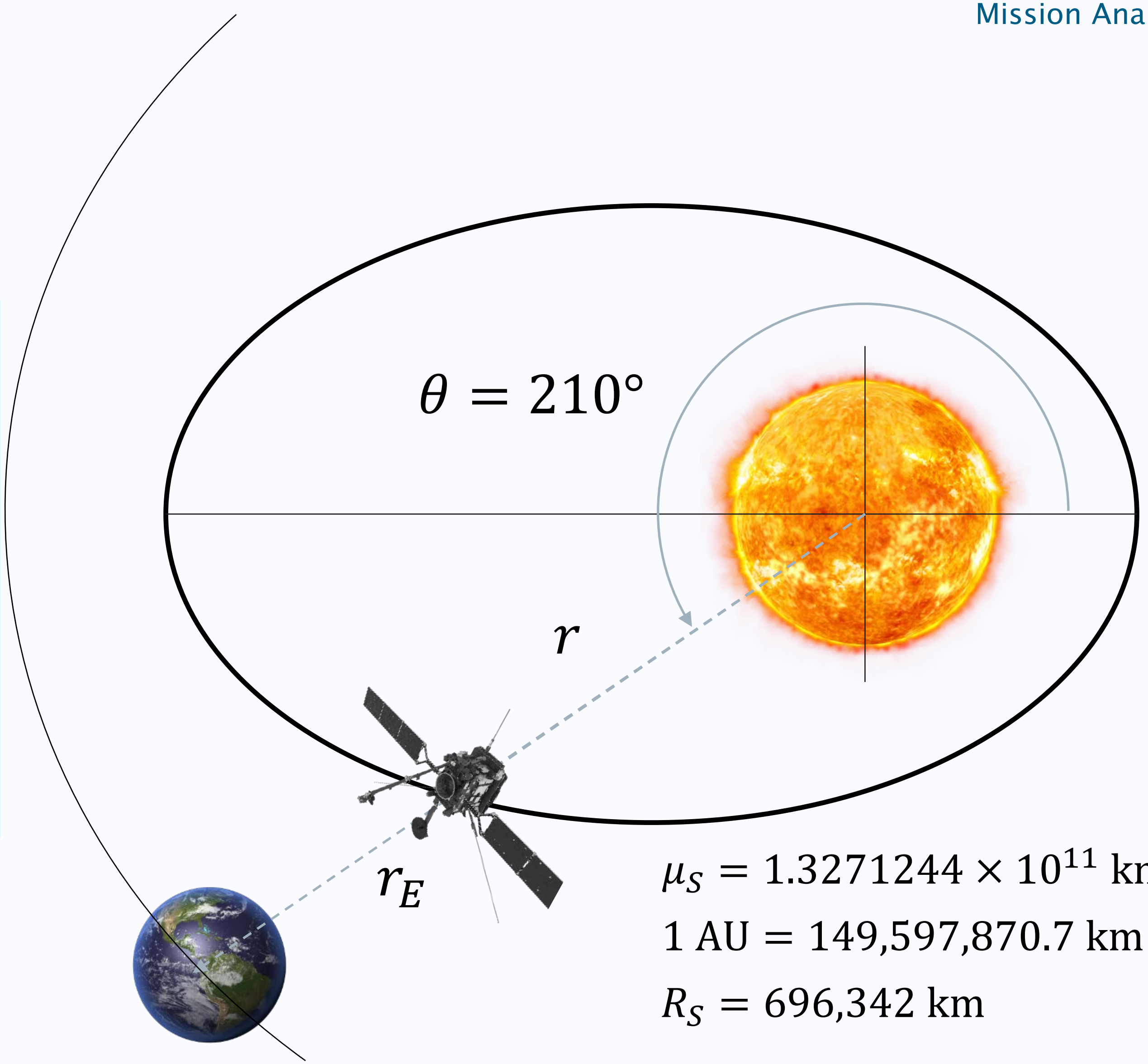
Ellipse equation

- Earth-spacecraft distance:

$$d = r_E - r$$
$$= 1 - 0.8048 \text{ AU}$$
$$= 0.1952 \text{ AU}$$
$$= 29{,}195{,}213.9 \text{ km}$$



$\mu_S = 1.3271244 \times 10^{11}\ \text{km}^3\text{s}^{-2}$
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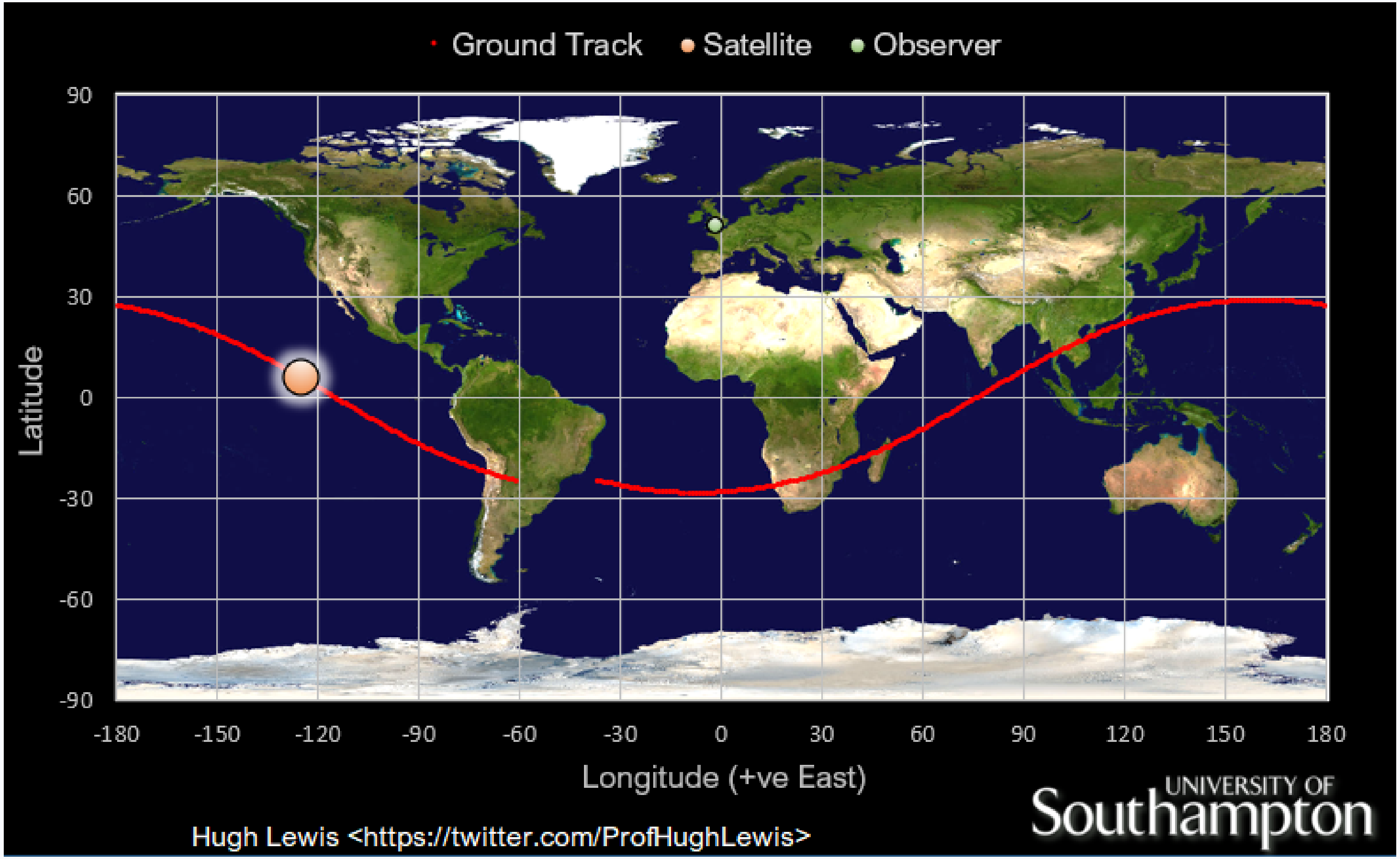
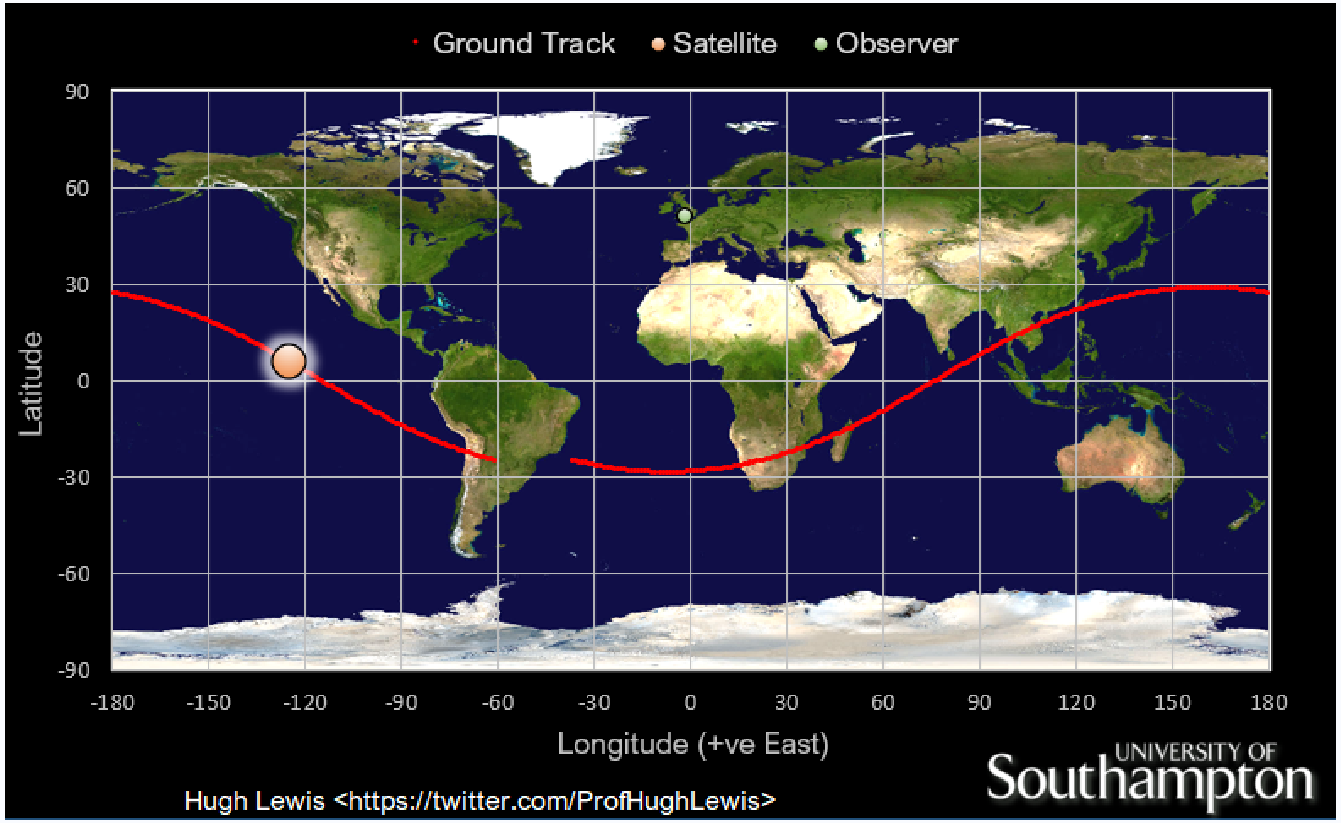
Ground track?



- NAVSTAR GPS
- HST
- SENTINEL 1B
- ISS

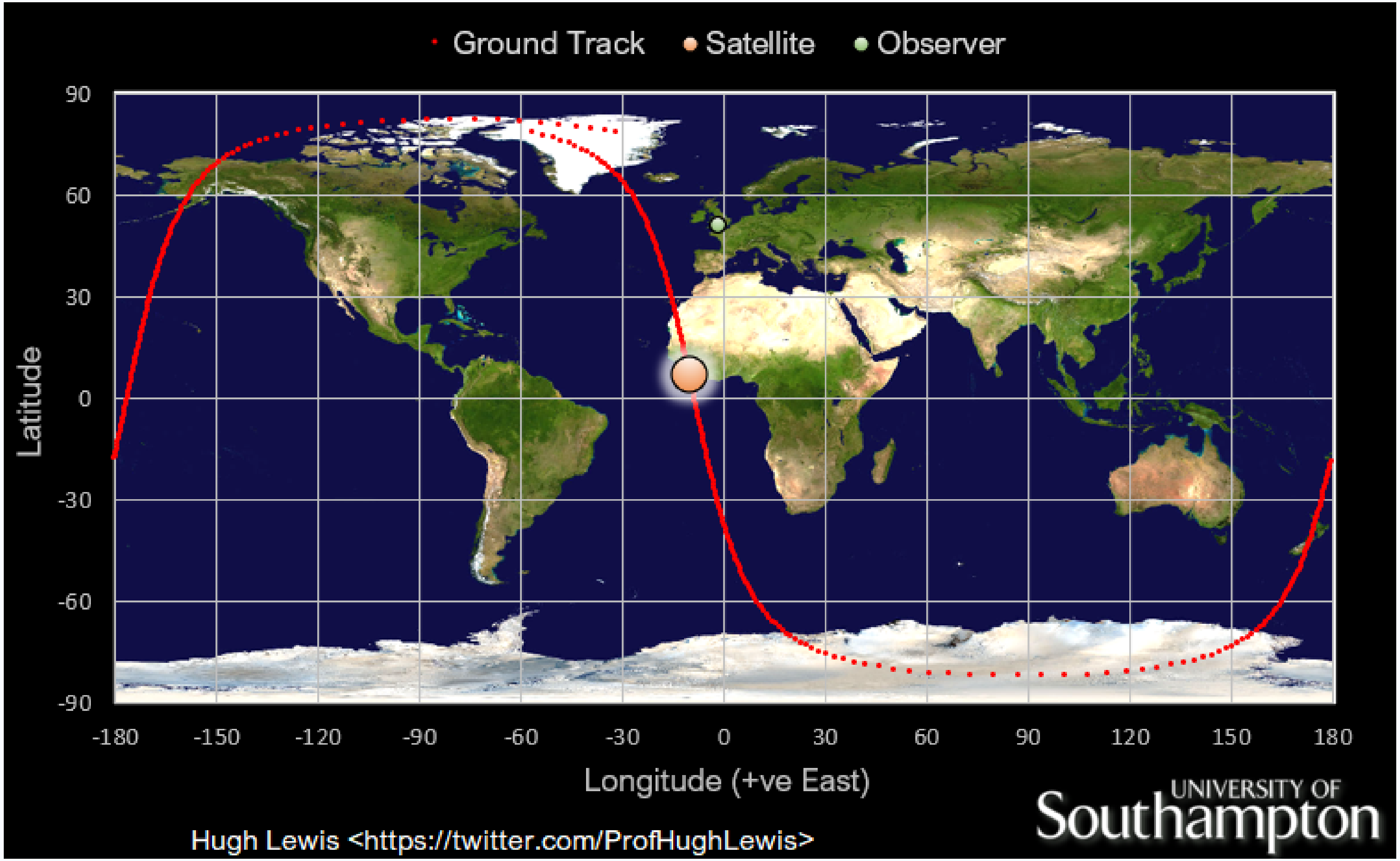
Ground track?



Hubble Space Telescope (HST)

Variable	Symbol	Value	Units
Semi-major axis	a	6914.5	km
Eccentricity	e	0.00024	
Inclination	i	28.4712	deg.
Right ascension of ascending node	Ω	10.3389	deg.
Argument of perigee	ω	241.637	deg.
Perigee altitude	h_p	534.8	km
Apogee altitude	h_a	538.2	km

Ground track?



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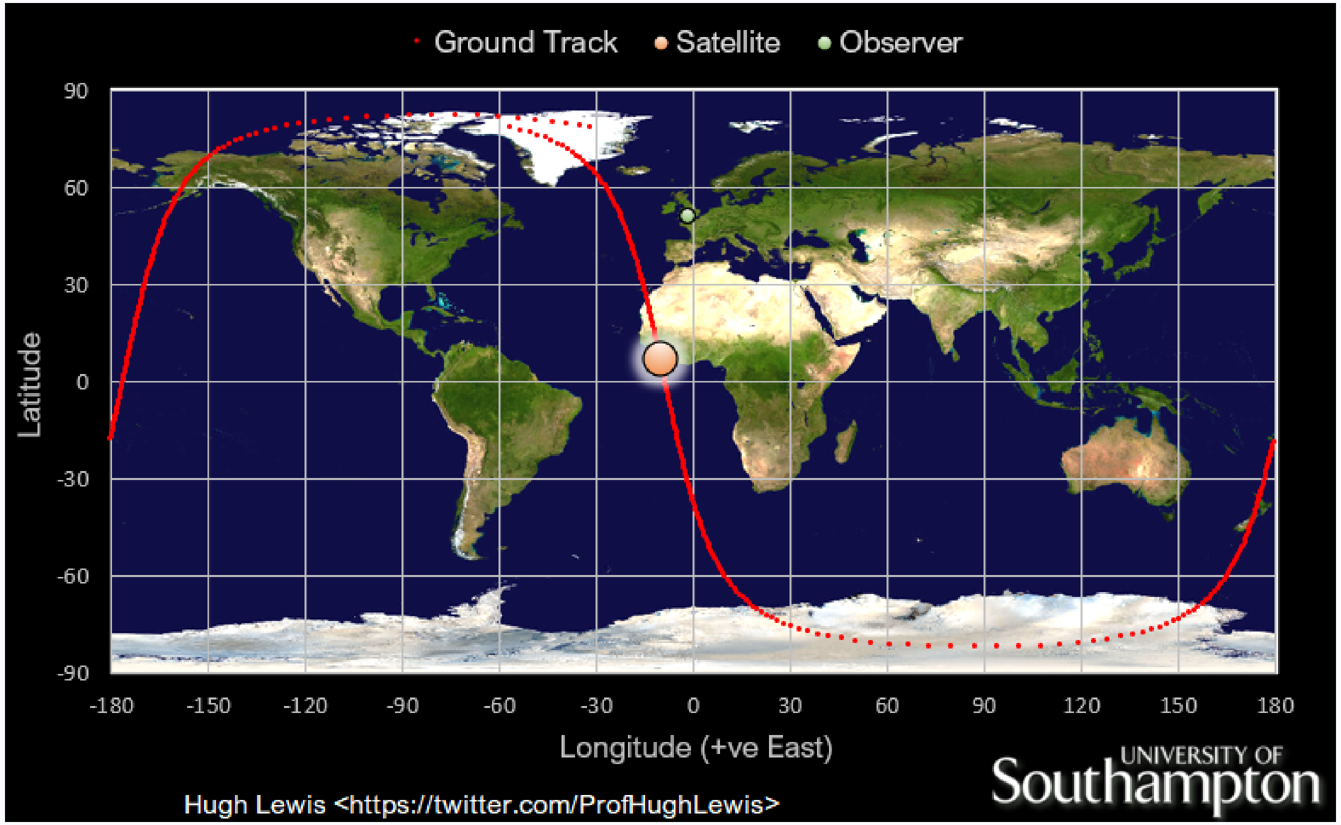
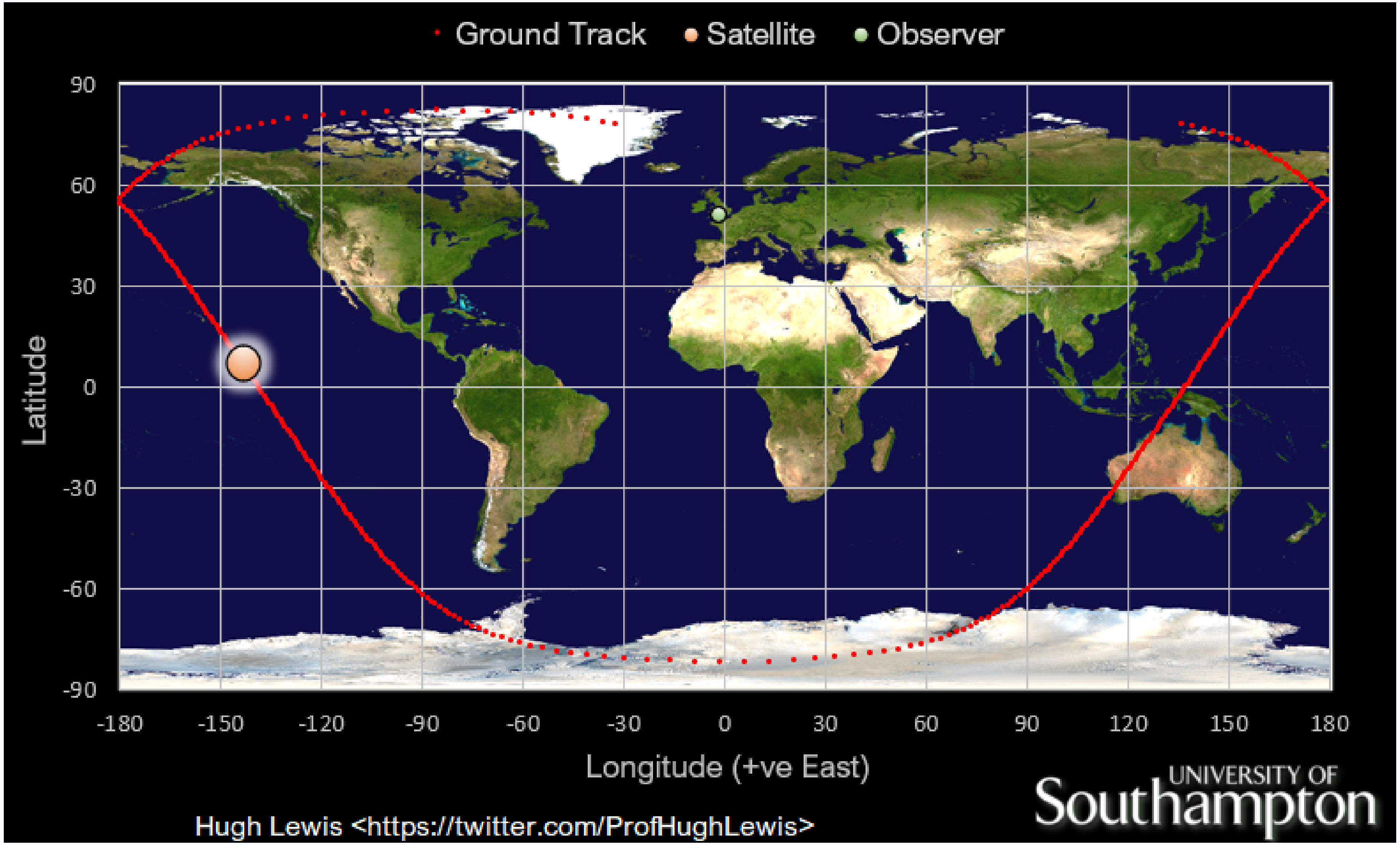
Ground track?



SENTINEL 1B

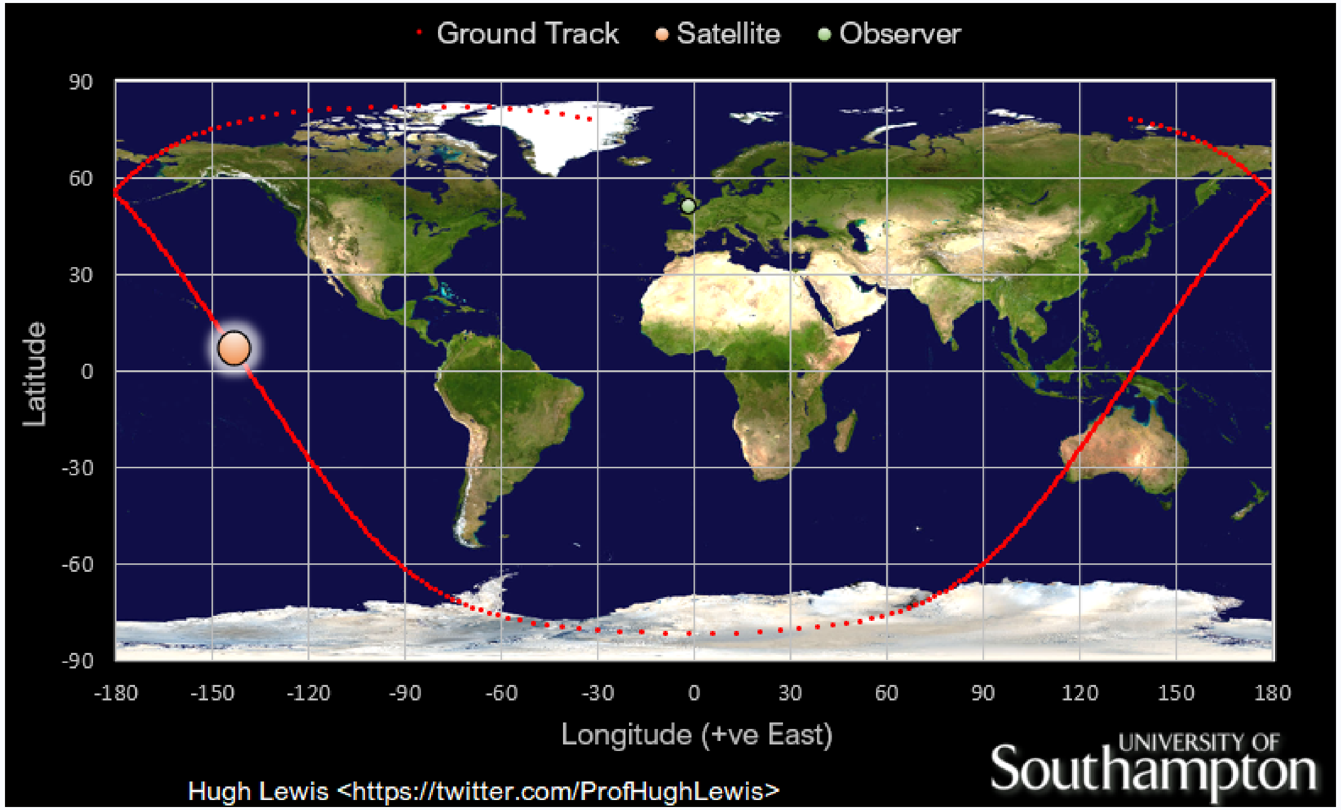
Variable	Symbol	Value	Units
Semi-major axis	a	7073.5	km
Eccentricity	e	0.00012	
Inclination	i	98.1814	deg.
Right ascension of ascending node	Ω	292.077	deg.
Argument of perigee	ω	81.0855	deg.
Perigee altitude	h_p	694.6	km
Apogee altitude	h_a	696.4	km

Ground track?



- NAVSTAR GPS
- HST
- SENTINEL 1B
- ISS

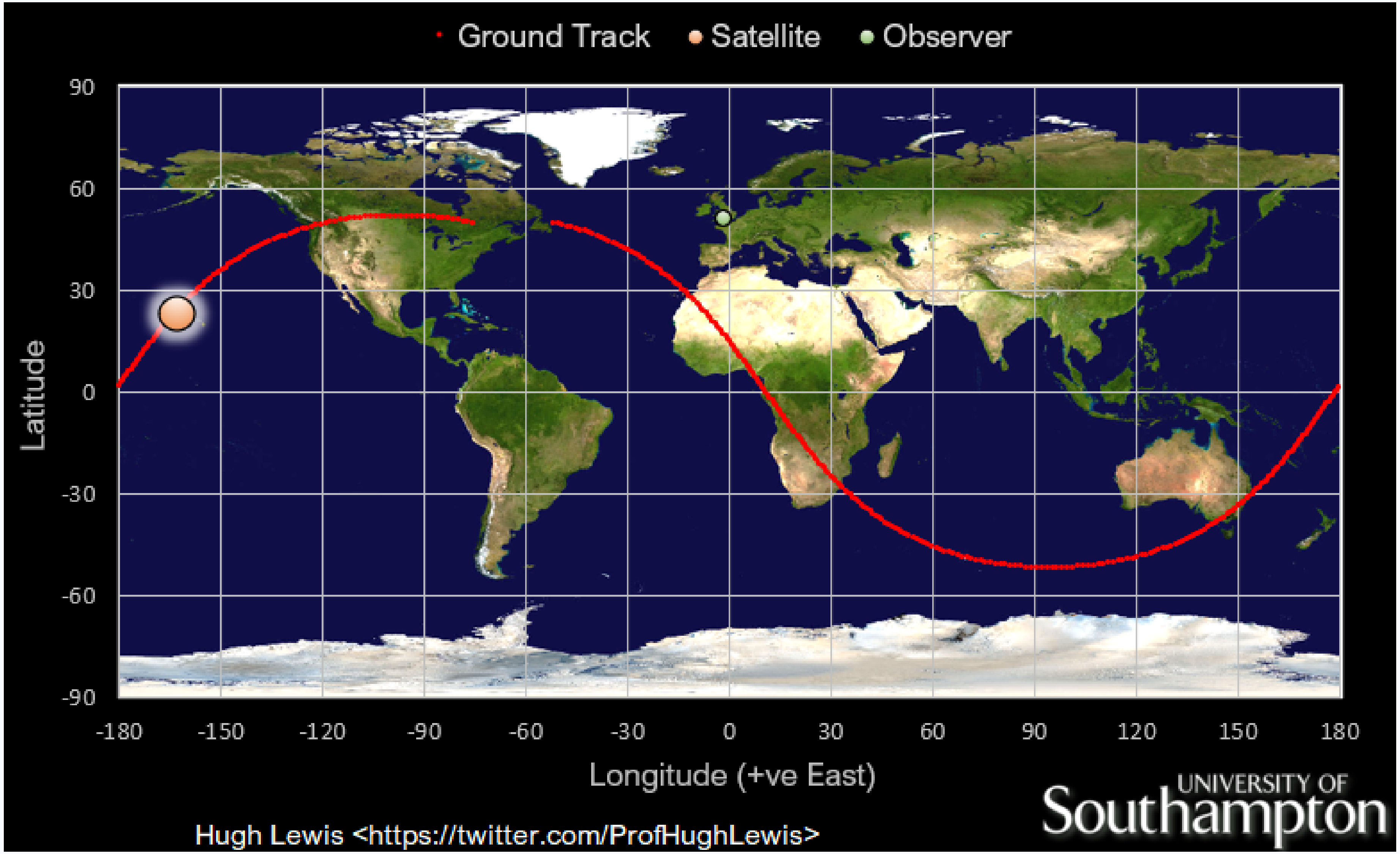
Ground track?



NAVSTAR GPS

Variable	Symbol	Value	Units
Semi-major axis	a	27717	km
Eccentricity	e	0.00187	
Inclination	i	98.1814	deg.
Right ascension of ascending node	Ω	292.077	deg.
Argument of perigee	ω	81.0855	deg.
Perigee altitude	h_p	21287.1	km
Apogee altitude	h_a	21390.9	km

Ground track?



- NAVSTAR GPS
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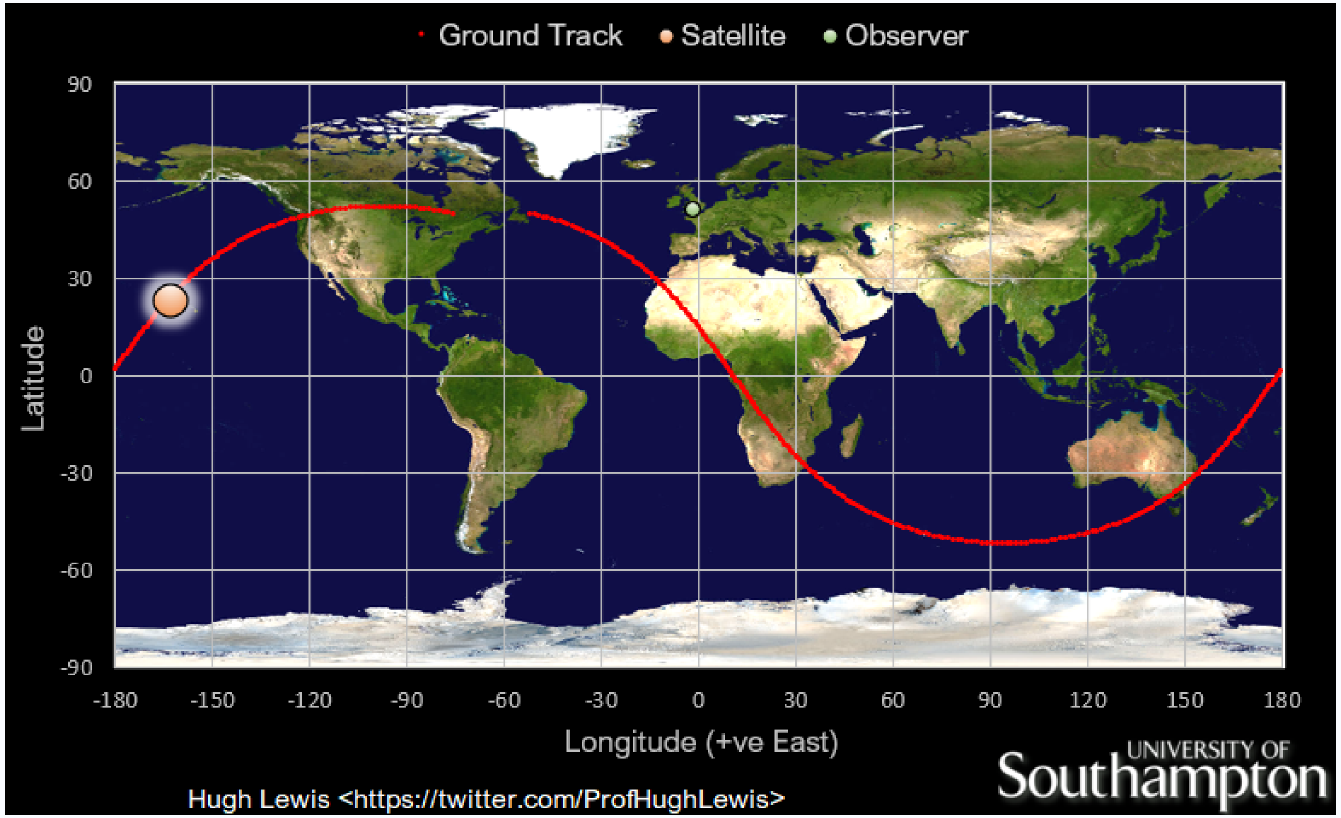
- SENTINEL 1B
- ISS

Ground track?



International Space Station (ISS)

Variable	Symbol	Value	Units
Semi-major axis	a	6798	km
Eccentricity	e	0.00042	
Inclination	i	51.6404	deg.
Right ascension of ascending node	Ω	117.399	deg.
Argument of perigee	ω	104.259	deg.
Perigee altitude	h_p	417.1	km
Apogee altitude	h_a	422.9	km

Activity

- The orbital motion (Celestial Mechanics) topic is covered in chapter 4 of Fortescue, Stark & Swinerd:
  - Read this chapter (up to an including the “Specifying the Orbit” section; there is no need to go further) in preparation for the next few lectures & to support your learning of this topic
  - Access to the e-book is available via the Library website: https://onlinelibrary.wiley.com/doi/book/10.1002/9781119971009