SCHOOL OF MATHEMATICS

MATHEMATICS FOR PART I ENGINEERING

Self-paced Course

MODULE 14 VECTORS I

Module Topics

1. Introductory remarks on scalars and vectors and the notation used for describing vectors

2. Addition and subtraction of vectors

3. Cartesian components of a vector

4. Scalar product of two vectors and applications

5. Vector product of two vectors and applications

A: Work Scheme based on JAMES (SIXTH EDITION)

1. This is the first of two modules on vectors. Turn to **J. p.230** and read the Introduction section **4.1**.

2. Study section **4.2.1** on Cartesian coordinates and direction cosines. The notations (x, y, z) and (x_1, x_2, x_3) are both commonly used. The axes are right-handed, so that if the y- and z- axes in Figure **4.1(a)** lie in the plane of the paper then the x- axis points **out** of the paper. Study Example **4.1**.

3. Study section **4.2.2** on scalars and vectors. In these modules, and in **J.**, vectors will be printed in bold face, e.g. **a**. In your written work, however, you should use the equivalent underlined notation $\underline{a}$. It is important that your own written work distinguishes between the vector $\underline{a}$ and its magnitude a.

Although **J.** introduces **antiparallel** to describe vectors which are parallel but point in opposite directions (i.e. have opposite senses) this terminology is not used by most authors.

4. Study section **4.2.3** on addition of vectors. The **parallelogram rule** and the **triangle law** are equivalent ways of defining addition. You can remember either but do look carefully at the directions of the vectors **a** and **b** in Figures **4.8** and **4.9**. The commutative law (a) and associative law (b) on the bottom of **p.236** and top of **p.237** show that vectors can be added in any order.

Figure **4.13** shows how to subtract vectors. Given **a** and **b** the vector $-\mathbf{b}$ is easily drawn. The quantity $\mathbf{a} - \mathbf{b}$ is then found from adding (by the parallelogram rule or the triangle law) the vectors **a** and $-\mathbf{b}$.

Study Example **4.4**. Then study Example **4.5**, drawing first the quadrilateral with the vertices in the stated order O, A, C and B. Study Example **4.6**, noting that the units of length are chosen so that 1 unit represents 1 N. Hence, the length OB equals the magnitude of $\mathbf{F}'$, $= 1$. Study Example **4.7**.

5. Study section **4.2.5** on Cartesian components - the manipulation of vectors in component form is very important. Study Example **4.10**, noting that in part (d) the unit vector $\hat{\mathbf{c}}$ is calculated by dividing **c** by its magnitude - this calculation is necessary in many situations. Study Example **4.11**.

Work through Example **4.12**. The theory involved in this example is relatively straightforward but you will need to do the algebra carefully to obtain the correct answers! Study Example **4.13**, noting that in calculating $\mathbf{F_2}$ you use the result that $\mathbf{F_2} = \text{magnitude} \times$ (unit vector in direction of $\mathbf{F_2}$).

*****Do Exercises 11, 12, 19 on p.250, and 9 on p.241*****

6. Turn to **p.251** and study section **4.2.8** on **scalar product**. On **p.252** the scalar product is defined **geometrically** and in **component form**. Both formulae are useful and must be known. The basic properties of scalar product are listed as (a) to (f) on **pp.252** to **254** and should be studied carefully. Property (f) is particularly important. Study Examples **4.17**, **4.18**, **4.19** and **4.20**, and then move on to study Example **4.22**.

7. Study the paragraph on the **component** of a vector in the direction of another vector, below the horizontal line on **p.257**, and work through Example **4.23**.

*****Do Exercises 27, 28(a),(c), 31, 32 on p.258*****

8. Study the introduction to **vector products** on **p.259**, stopping before the first application. This is the second commonly used product of vectors. Compare the similarities and differences between the definition **4.5** and the geometric definition of scalar product on **p.252**.

9. Study all the applications of vector products discussed on **pp.259** to **261**. The results **4.6** and **4.7** must be known - the order of the vectors on the right-hand sides of these equations is important. Note that the directions of the vector moment $\mathbf{M}$ and the angular velocity $\boldsymbol{\omega}$ are both along the axes of rotation of these quantities.

10. Study carefully the properties (a) to (f) of vector products on **pp.261-263**. The Cartesian form **4.9** is again much used. You should be able to follow its derivation, but it is the result which is important. Don't worry if you do not know about determinants - this alternative method of calculating vector products will make sense after you have studied the modules on Matrices.

Study Example **4.24**, noting that in this and the following Examples you can evaluate the vector products using the procedure shown in Figure **4.39**. Study Examples **4.25, 4.26, 4.28, 4.29**.

*****Do Exercises 41, 51, 52 on p.269*****

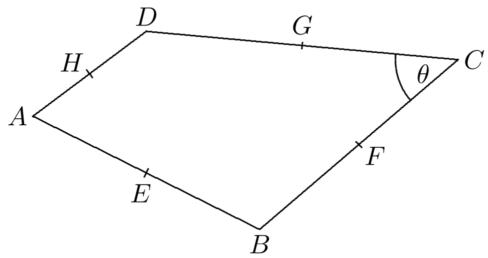
B: Work Scheme based on STROUD (SIXTH EDITION)

Work through Programme **6**, Vectors. The notation used by **S.** for vectors is non-standard. Instead of denoting a vector by $\overline{AB}$ or $\overline{a}$ as in **S.**, it is much more common to use $\overrightarrow{AB}$ or $\underline{a}$ or $\mathbf{a}$ (bold type).

The programme in **S.** provides a good introduction to vectors, their representation with respect to cartesian axes and to the definition and calculation of scalar and vector products. However, there is material in the syllabus for Module **14** that is missing (mainly on the applications to moments, angular velocity and areas) so you should go through a Work Scheme based on **J.** to discover the required extra information.

Specimen Test 14

1. $ABCD$ is a plane quadrilateral in which $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{CD} = \mathbf{c}$ and the midpoints of the sides are E, F, G, H, as shown



(i) Express the following vectors in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$: (a) $\overrightarrow{AC}$, (b) $\overrightarrow{AD}$, (c) $\overrightarrow{EG}$.

(ii) Write down $|\mathbf{b} + \mathbf{c}|$ in terms of b, c and $\cos\theta$

2. Suppose that, relative to a set of rectangular cartesian axes, P is a point with coordinates $(1, -1, 4)$, A is a point with coordinates $(0, 3, 1)$ and the origin O has coordinates $(0, 0, 0)$.
Write down in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$

(i) the position vector $\mathbf{r}$ of the point P, (ii) the vector $\overrightarrow{PA}$.

3. If $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$ find

(i) $|\mathbf{a}|$, (ii) $\hat{\mathbf{a}}$, (iii) $\mathbf{a} - 2\mathbf{b}$, (iv) the z-component of $\mathbf{b}$.

4. Given that $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ find

(i) $\mathbf{b} \,.\, \mathbf{c}$, (ii) the angle between $\mathbf{b}$ and $\mathbf{c}$.

5. A body moves from a point $A = (0, 1, 0)$ to a point $B = (1, 2, 2)$ (measured in metres) whilst being subjected to a force $\mathbf{F}$ of magnitude $6\,N$ in the direction of the vector $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

(i) Determine the vector force $\mathbf{F}$, (ii) calculate the work done by the force.

6. Given that $\mathbf{a} = 2\mathbf{i} + \mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ find

(i) $\mathbf{a} \times \mathbf{b}$, (ii) a unit vector perpendicular to $\mathbf{a}$ and $\mathbf{b}$.

7. A force defined by the vector $5\mathbf{i} + 2\mathbf{k}$ acts through the point $(2, 6, 1)$. Calculate the (vector) moment of this force about the point $(1, 1, 1)$ (the units being metres and newtons).

8. A body is rotating with angular velocity 4 rads/s in a positive sense about an axis from the point $(1, 0, 0)$ to the point $(2, 3, 2)$ (in metres). Calculate

(i) the angular velocity vector $\boldsymbol{\omega}$, (ii) the velocity of the point in the body at $(2, 1, 1)$.