SESA3029 Aerothermodynamics

Lecture 1.3

1D gasdynamics

Intro : bernoulli can't be used anymore since we are working with high speeds (compressable)

Aerothermodynamics toolkit (1)

Perfect gas

$$\boxed{p = \rho R T}$$

$$R = \frac{\hat{R}}{\hat{M}} = \frac{\text{Universal gas constant}}{\text{Molar mass}}$$

Calorically perfect gas

$$\boxed{\begin{aligned} e &= c_v T \\ h &= c_p T \end{aligned}}$$

e=internal energy
h=enthalpy

$$\boxed{h = e + pv}$$

Entropy

$$\boxed{\mathrm{d}s = \left(\frac{\delta q}{T}\right)_{\text{rev}}}$$

Gibbs relations

$$\boxed{\begin{aligned} T\mathrm{d}s &= \mathrm{d}e + p\mathrm{d}\mathrm{v} \\ &= \mathrm{d}h - \mathrm{v}\mathrm{d}p \end{aligned}}$$

$$\boxed{s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)}$$

Isentropic flow

$$\boxed{\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

Aerothermodynamics toolkit (2)

(just more generalised bernoulli)

Steady flow energy equation

$$h + \frac{V^2}{2} = \text{constant} = \underbrace{h_0}_{\text{stagnation enthalpy}}$$

h_0 is called the stagnation enthalpy (the enthalpy that is recovered when we bring the fluid to rest adiabatically)

valid over shockwaves

no energy transfer

$$\boxed{\frac{a_0^2}{a^2} = 1 + \frac{\gamma - 1}{2} M^2}$$

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2}$$

For adiabatic flow brought to rest *isentropically* we also have:

no energy change

$$\boxed{\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}}$$

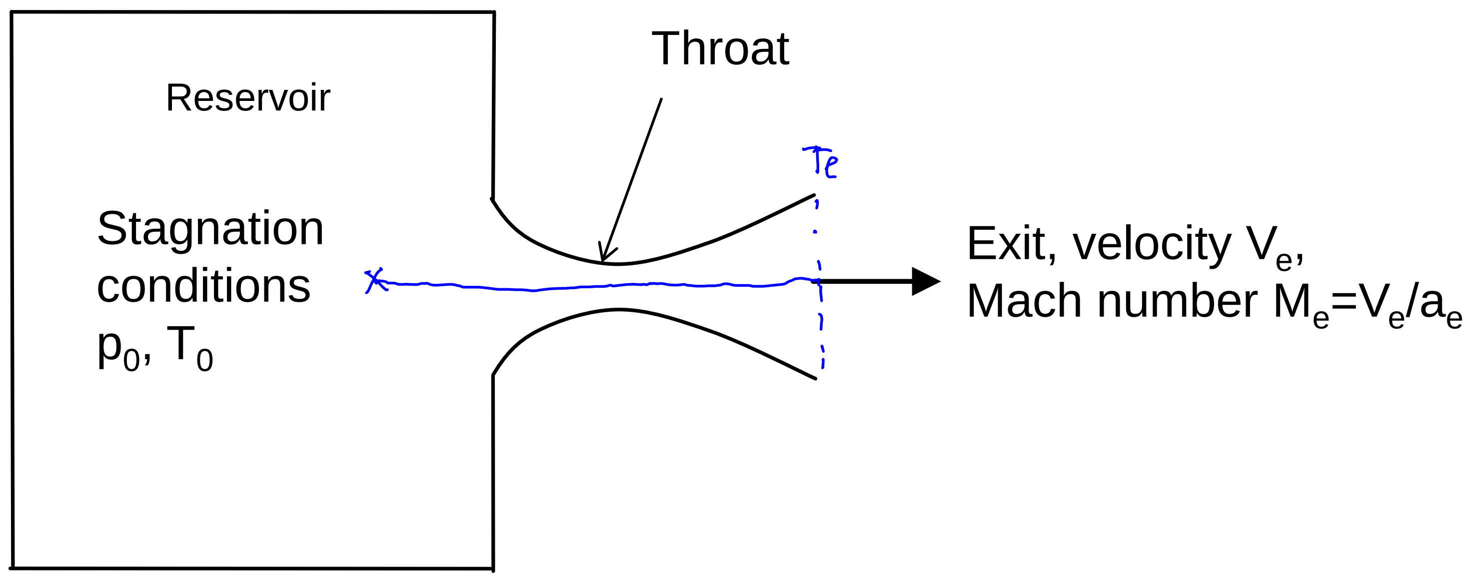
$$\boxed{\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}}$$

invalid over shockwaves

Tabulated in IFT

$$\sqrt{\frac{2}{\gamma-1}\left(\frac{T_0}{T}-1\right)} = M$$

Example: Nozzle Flow



For a nozzle flow with T_0=1000K and T_e=600K find M_e

From the problem statement $T_e/T_0=0.6$

Isentropic-flow table (γ = 1.4):

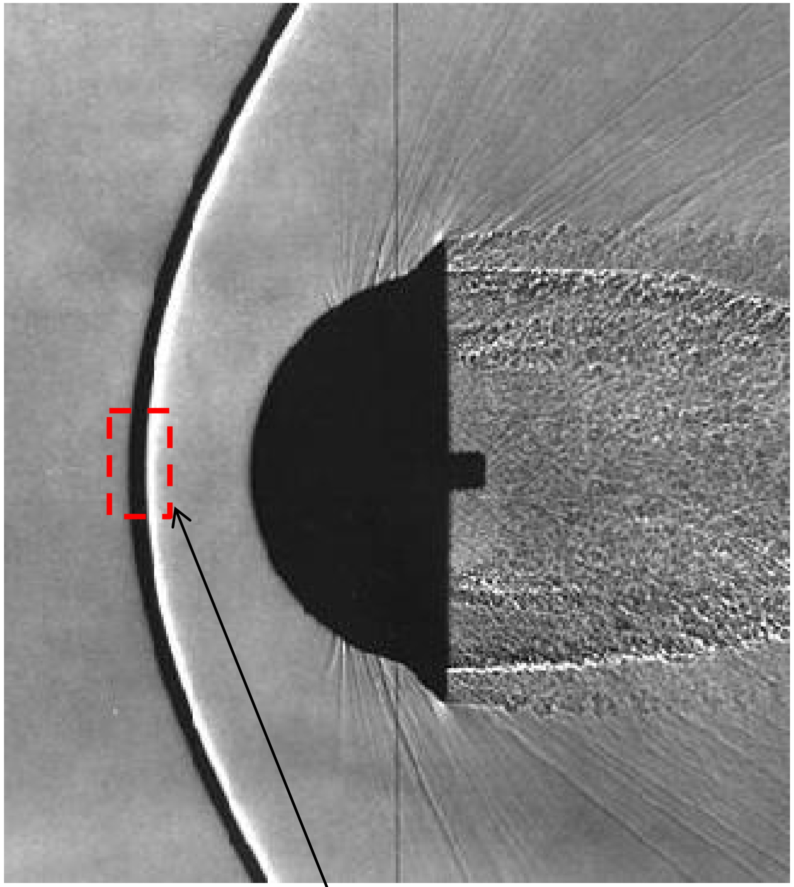
M	p/p_0	ρ/ρ_0	T/T_0	ν (deg.)	A/A^*
1.8000	0.1740	0.2868	0.6068	20.7251	1.4390
1.8200	0.1688	0.2806	0.6015	21.3021	1.4610
1.8400	0.1637	0.2745	0.5963	21.8768	1.4836
1.8600	0.1587	0.2686	0.5910	22.4492	1.5069
1.8800	0.1539	0.2627	0.5859	23.0190	1.5308

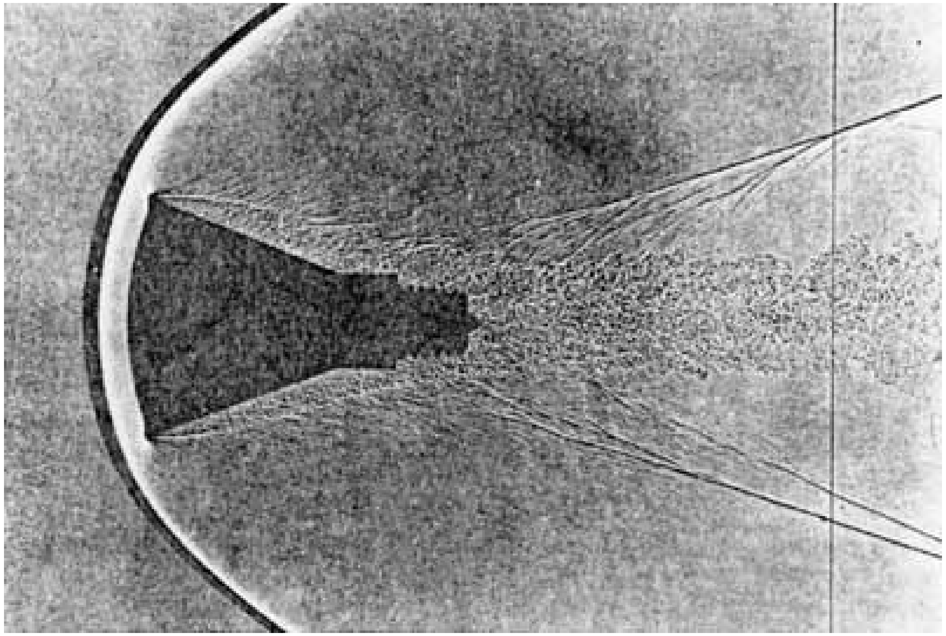
Nearest value M=1.82

Linear interpolation M=1.826 (1.82577 to 5 d.p.)

Exact (easy in this case) M=1.826 (1.82574 to 5 d.p.)

Bow shock wave



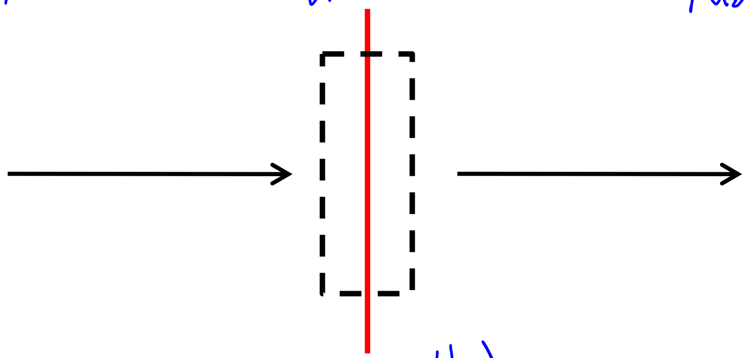


Control volume for analysis of a normal shock wave

Control volume for a normal shock wave (M_1>1)

supersonic

ρ_1
p_1
a_1
h_1
V_1



subsonic

ρ_2
p_2
a_2
h_2
V_2

laws across the shock (maintained)

$A_1 = A_2$ ∴ cancelled

$$\cancel{A_1}\,\boxed{\rho_1 V_1 = \rho_2 V_2}\,\cancel{A_2} \qquad (1)$$

Mass conservation

$$\boxed{\rho_1 V_1 \left(V_2 - V_1\right) = p_1 - p_2} \qquad (2)$$

Newton's second law

$$\boxed{a_0^2 = a_1^2 + \frac{\gamma - 1}{2} V_1^2 = a_2^2 + \frac{\gamma - 1}{2} V_2^2} \qquad (3)$$

Energy conservation

(total enthalpy is conserved)

$h_{01} = h_{02}$ ∴ $T_{01} = T_{02}$

Rearrange (2), using (1)

$$V_2 - V_1 = \frac{p_1}{\rho_1 V_1} - \frac{p_2}{\rho_2 V_2}$$

$$V_2 - V_1 = \frac{a_1^2}{\gamma V_1} - \frac{a_2^2}{\gamma V_2}$$

$$V_2 - V_1 = \frac{a_1^2}{\gamma V_1} - \frac{a_2^2}{\gamma V_2}$$

$$\boxed{a_0^2 = a_1^2 + \frac{\gamma - 1}{2} V_1^2 = a_2^2 + \frac{\gamma - 1}{2} V_2^2} \quad (3)$$

Combine with (3)

$$V_2 - V_1 = a_0^2 \left(\frac{1}{\gamma V_1} - \frac{1}{\gamma V_2} \right) - \frac{\gamma - 1}{2\gamma} (V_1 - V_2)$$

$$= a_0^2 \left(\frac{V_2 - V_1}{\gamma V_1 V_2} \right) + \frac{\gamma - 1}{2\gamma} (V_2 - V_1)$$

new

Cancel V_2-V_1

$$\frac{a_0^2}{\gamma V_1 V_2} = 1 - \frac{\gamma - 1}{2\gamma} = \frac{\gamma + 1}{2\gamma} \Rightarrow \boxed{V_1 V_2 = \frac{2 a_0^2}{\gamma + 1}} \quad (4)$$

Prandtl relation in terms of stagnation sound speed

and, hence, the jump relations by rearranging (1)-(4)

Summary of shock jump relations

$$\frac{\rho_2}{\rho_1} = \frac{M_1^2(\gamma+1)}{2+(\gamma-1)M_1^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma\left(M_1^2-1\right)}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1}$$

$$M_2^2 = \frac{2+(\gamma-1)M_1^2}{2\gamma M_1^2-(\gamma-1)}$$

All are functions of M_1 alone
Tabulated in NST

Quick examples

Example 1: find M_2 for a monatomic gas in the limit $M_1 \rightarrow \infty$

$$M_2^2 = \frac{2 + (\gamma - 1) M_1^2}{2\gamma M_1^2 - (\gamma - 1)} \qquad M_{2,\min} = \sqrt{\frac{\gamma - 1}{2\gamma}} = \frac{1}{\sqrt{5}}$$

What are the limiting relations for density, pressure and temperature?

Example 2: For air, given $p_2/p_1 = 1.25$ and $T_1 = 15\ ^\circ C$, find M_2 and V_2

Example using shock tables

Normal-shock table (γ = 1.4):

M_{n1}	M_{n2}	p_2/p_1	ρ_2/ρ_1	T_2/T_1
1.0000	1.0000	1.0000	1.0000	1.0000
1.0200	0.9805	1.0471	1.0334	1.0132
1.0400	0.9620	1.0952	1.0671	1.0263
1.0600	0.9444	1.1442	1.1009	1.0393
1.0800	0.9277	1.1941	1.1349	1.0522
1.1000	0.9118	1.2450	1.1691	1.0649
1.1200	0.8966	1.2968	1.2034	1.0776
1.1400	0.8820	1.3495	1.2378	1.0903
1.1600	0.8682	1.4032	1.2723	1.1029
1.1800	0.8549	1.4578	1.3069	1.1154

Given p_2/p_1=1.25 and T_1=15 °C, find M_2 and V_2

By linear interpolation

Interpolation factor (proportion of change from one row to the next)

$$f = \frac{1.25 - 1.2450}{1.2968 - 1.2450} = 0.09653$$

$$M_2 = 0.9118 + 0.09653 \times \left(0.8966 - 0.9118\right) = 0.9103$$

$$\frac{T_2}{T_1} = 1.0649 + 0.09653 \times \left(1.0776 - 1.0649\right) = 1.0661$$

$$T_2 = 1.0661 \times 288.15 = 307.2\ \mathrm{K}$$

$$V_2 = M_2 \sqrt{\gamma R T_2}$$

$$= 0.9103 \sqrt{1.4 \times 287 \times 307.2} = 319.8\ \mathrm{m/s}$$