SESA3029 Aerothermodynamics

Lecture 5.9

Heat exchangers -

Number of transfer units method

Number of transfer units (NTU) concept

LMTD method leads to iteration if only the inlet temperatures are known. As an alternative, consider special case of infinitely long heat exchanger.

$$\rightarrow T_{h,o} = T_{c,o}$$

Maximal temp. difference in system: $T_{h,i} - T_{c,i}$

$$C_c < C_h : \quad \dot{Q}_{\max} = C_c \left(T_{h,i} - T_{c,i}\right)$$

and

$$C_h < C_c : \quad \dot{Q}_{\max} = C_h \left(T_{h,i} - T_{c,i}\right)$$

Altogether, we have $\dot{Q}_{\max} = C_{\min}\left(T_{h,i} - T_{c,i}\right)$ with $C_{\min/\max} = \min/\max\left\{C_h, C_c\right\}$

Effectiveness: $\varepsilon = \dfrac{\dot{Q}}{\dot{Q}_{\max}}$

Or using general heat exchanger relations (1) and (2)

$$\varepsilon = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} \qquad (10)$$

$$\varepsilon = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} \qquad (11)$$

If $T_{h,i}, T_{c,i}$ are known we have

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

where $\varepsilon = f\left(\mathrm{NTU}, \dfrac{C_{\min}}{C_{\max}}\right)$ with $\mathrm{NTU} = \dfrac{h'A}{C_{\min}}$

Parallel-flow heat exchanger

Assume $C_{\min} = C_h, C_{\max} = C_c$ (Same result for $C_{\min} = C_c$)

We know from (6) for the parallel-flow heat exchanger

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -h'A\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Using $\Delta T_1, \Delta T_2$ for the parallel-flow case and the assumptions from above this can be turned into

$$\ln\left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = -\frac{h'A}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)$$

or by using the definition of NTU and $C_r = \dfrac{C_{\min}}{C_{\max}}$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left(-\mathrm{NTU}\left(1 + C_r\right)\right) \qquad (12)$$

Under the present assumption we obtain for (10)

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} \qquad \rightarrow \qquad T_{h,o} = T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right) \tag{13}$$

and from the general relations (1), (2) under our assumptions

$$C_r = \frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}} \qquad \rightarrow \qquad T_{c,o} = T_{c,i} + C_r\left(T_{h,i} - T_{h,o}\right) \tag{14}$$

Using (13), (14) in (12) to replace $T_{h,o}$ and $T_{c,o}$

$$\frac{T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right) - T_{c,i} + C_r\left(T_{h,i} - T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right)\right)}{T_{h,i} - T_{c,i}} = \exp\left(-\mathrm{NTU}\left(1 + C_r\right)\right)$$

which can readily be turned into

$$1 - \varepsilon(1 + C_r) = \exp\left(-\mathrm{NTU}\left(1 + C_r\right)\right) \qquad \rightarrow \qquad \boxed{\varepsilon = \frac{1 - \exp\left(-\mathrm{NTU}\left(1 + C_r\right)\right)}{1 + C_r}} \tag{15}$$

Counter-flow heat exchanger

Again, we assume $C_{\min} = C_h, C_{\max} = C_c$ (Same result for $C_{\min} = C_c$)

Now start from counter-flow heat exchanger equation (9)

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -h'A\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

Using $\Delta T_1, \Delta T_2$ for the counter-flow case, one arrives similarly at

$$\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \exp\left(-\mathrm{NTU}\left(1 - C_r\right)\right) \qquad (16)$$

Using (13), (14) in (16) to replace $T_{h,o}$ and $T_{c,o}$

$$\frac{T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right) - T_{c,i}}{T_{h,i} - T_{c,i} - C_r\left(T_{h,i} - T_{h,i} + \varepsilon\left(T_{h,i} - T_{c,i}\right)\right)} = \exp\left(-\mathrm{NTU}\left(1 - C_r\right)\right)$$

which gives $$\frac{1-\varepsilon}{1-\varepsilon C_r} = \exp\left(-\mathrm{NTU}\left(1-C_r\right)\right)$$

and after some further simple rearrangements

$$\boxed{\varepsilon = \frac{1-\exp\left(-\mathrm{NTU}\left(1-C_r\right)\right)}{1-C_r \exp\left(-\mathrm{NTU}\left(1-C_r\right)\right)}} \qquad (17)$$

In contrast to (15), (17) is not defined for $\boxed{C_r=1.}$ This is the same as in the LMTD method.
For this specific case, we integrate (5) directly.

For $C_h=C_c$, we find from (7), (8) that $dT_h = dT_c \quad \rightarrow \quad d(\Delta T) = 0$

Hence $d\dot{Q} = h' dA \Delta T$ becomes $\dot{Q} = h' A \Delta T$ and with $\Delta T = \Delta T_2 = T_{h,o} - T_{c,i}$

and using the definition of effectiveness and (13), we obtain

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\mathrm{NTU}\, C_{\min}(T_{h,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})} = \mathrm{NTU}(1-\varepsilon) \qquad \rightarrow \qquad \boxed{\varepsilon = \frac{\mathrm{NTU}}{1+\mathrm{NTU}}}$$

Example: Counter-flow heat exchanger

Cold flow: capacity rate C_c=1500 W/K, h_c=275 W/(m^{2}K), $T_{c,i}$=15°C

Hot flow: capacity rate C_h=3000 W/K, h_h=400 W/(m^{2}K), $T_{h,i}$=150°C

1. Find NTU, taking surface area A=10.06m^2

$$\frac{1}{h'} = \frac{1}{h_c} + \frac{1}{h_h} = \frac{1}{275} + \frac{1}{400} \quad \rightarrow \quad h' = 162.963\,\mathrm{W/(m^2\,K)}$$

(Ignoring conduction)

$$C_r = \frac{1500}{3000} = \frac{1}{2} \qquad C_{\min} = \min\{1500, 3000\} \qquad \mathrm{NTU} = \frac{h'A}{C_{\min}} = \frac{162.963\mathrm{W/(m^2K)} \times 10.06\mathrm{m^2}}{1500\mathrm{W/K}} = 1.09294$$

2. Evaluate effectiveness with Eq. (17)

$$\varepsilon = \frac{1 - \exp\left(-1.09294(1-0.5)\right)}{1 - 0.5\exp\left(-1.09294(1-0.5)\right)} = 0.59255$$

Cold flow: capacity rate C_c=1500 W/K, h_c=275 W/(m^{2}K), $T_{c,i}$=15°C

Hot flow: capacity rate C_h=3000 W/K, h_h=400 W/(m^{2}K), $T_{h,i}$=150°C

3. Hence we can find the heat transfer rate

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}\left(T_{h,in} - T_{c,in}\right) = 0.59255 \times 1500\,\mathrm{W/K}(150 - 15)\mathrm{K} = 120\,\mathrm{kW}$$

4. Exit temperatures from Equations (1) and (2)

$$\dot{Q} = C_h\left(T_{h,i} - T_{h,o}\right) \quad \rightarrow \quad T_{h,o} = T_{h,i} - \frac{\dot{Q}}{C_h} = 150^{\mathrm{o}}\mathrm{C} - \frac{120\mathrm{kW}}{3000\mathrm{W/K}} = 110^{\mathrm{o}}\mathrm{C}$$

$$\dot{Q} = C_c\left(T_{c,o} - T_{c,i}\right) \quad \rightarrow \quad T_{c,o} = T_{c,i} + \frac{\dot{Q}}{C_c} = 15^{\mathrm{o}}\mathrm{C} + \frac{120\mathrm{kW}}{1500\mathrm{W/K}} = 95^{\mathrm{o}}\mathrm{C}$$