University of Southampton

Chapter 5: Mission Analysis
Workshop 2

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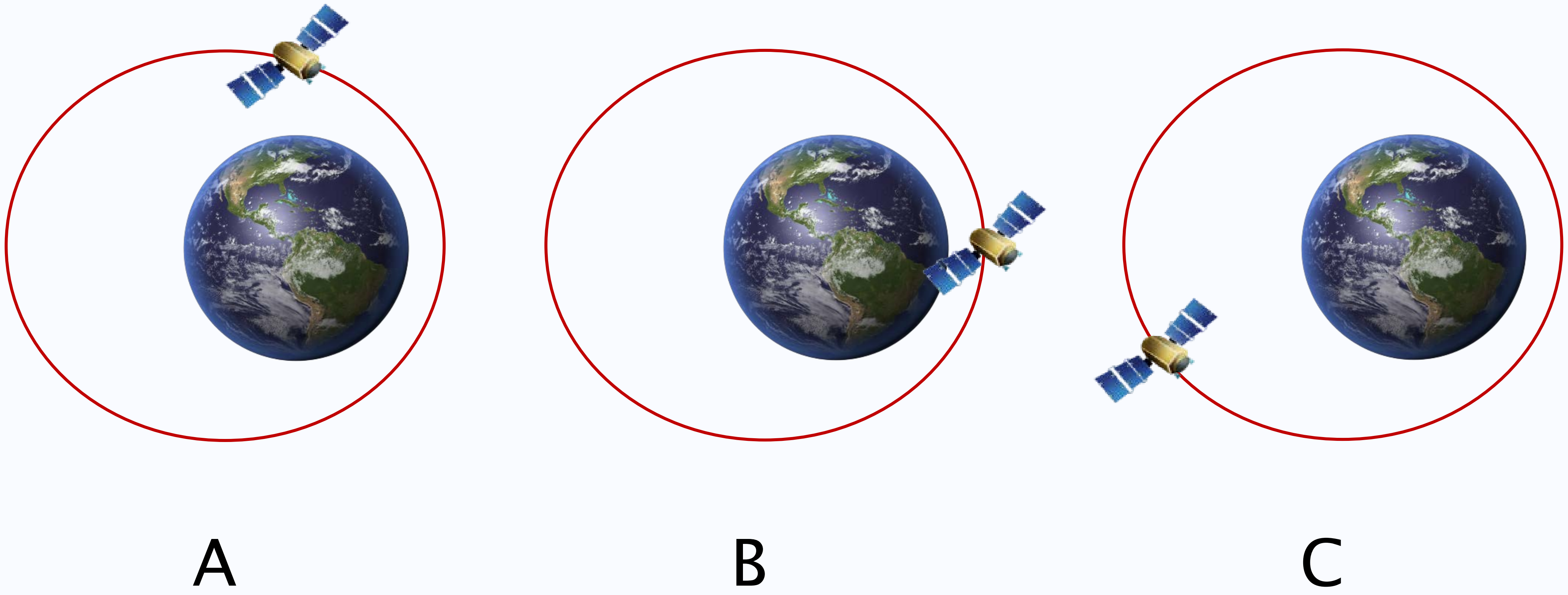
Workshops - overview

- Orbital elements demonstration
- Solar Orbiter worked example:
  - Ellipse equation
- Space tourism worked examples:
  - Sub-orbital flight, Blue Origin NS-18 (William Shatner)
  - SpaceX Crew-4 (Freedom) re-entry
  - SpaceX first Starship orbital flight
- DART worked example:
  - Characterising the effects of the DART impact on Dimorphos
- Space debris mitigation example:
  - FCC’s new 5-year de-orbit rule
- Quick quizzes
  - Check your understanding of orbits, ground tracks, and ACS

- If we are unable to complete everything in the workshop sessions, you can use the worked examples for self-study, revision, etc.

Orbital energy

- Quick quiz:
  - Which satellite has the **lowest** speed?



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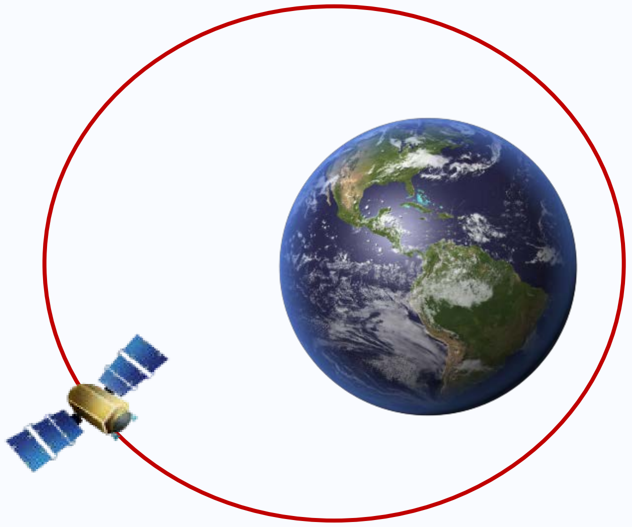
Orbital energy

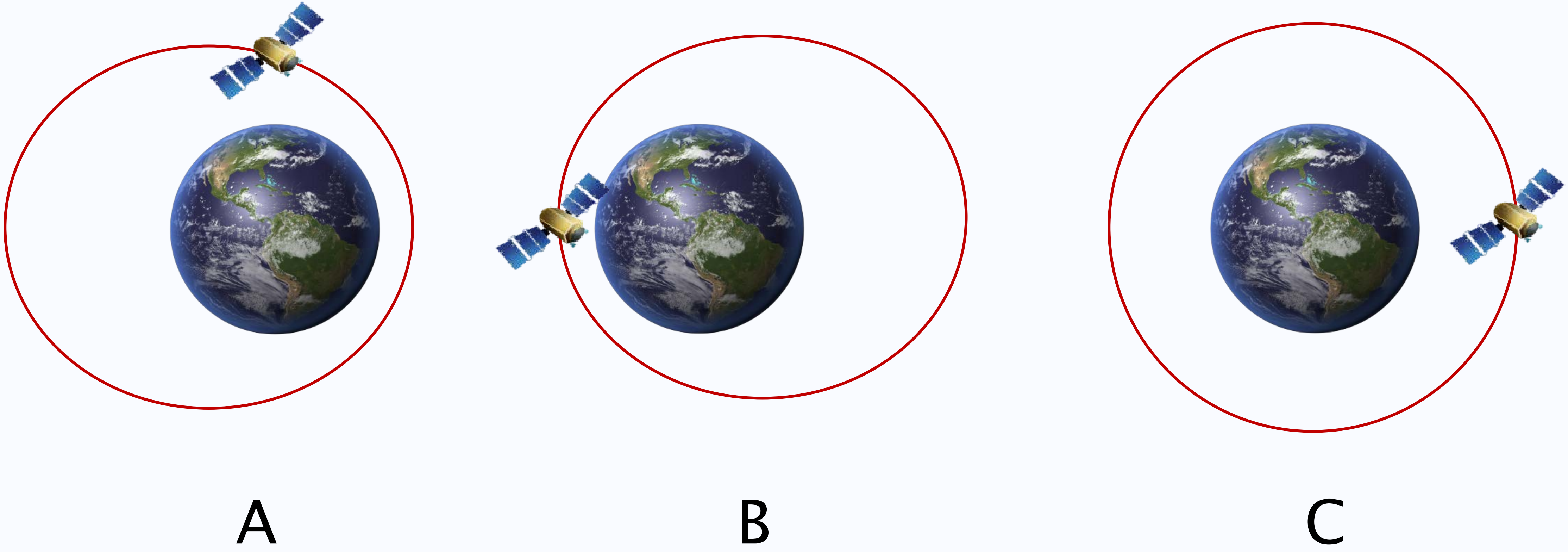
- Quick quiz answer: C
  - At this point in the orbit, the orbit radius (i.e. the line joining the spacecraft and planet) is the greatest of the three cases
  - Kepler's 2^{nd} Law tells us that the area swept out by this line in unit time is constant
  - As the line is longer, it must move more slowly to sweep out the same/constant area in each interval of time
  - Hence the satellite is travelling more slowly here than in A or B



C

Orbital energy

- Quick quiz:
  - Which satellite has the **shortest** orbital period?



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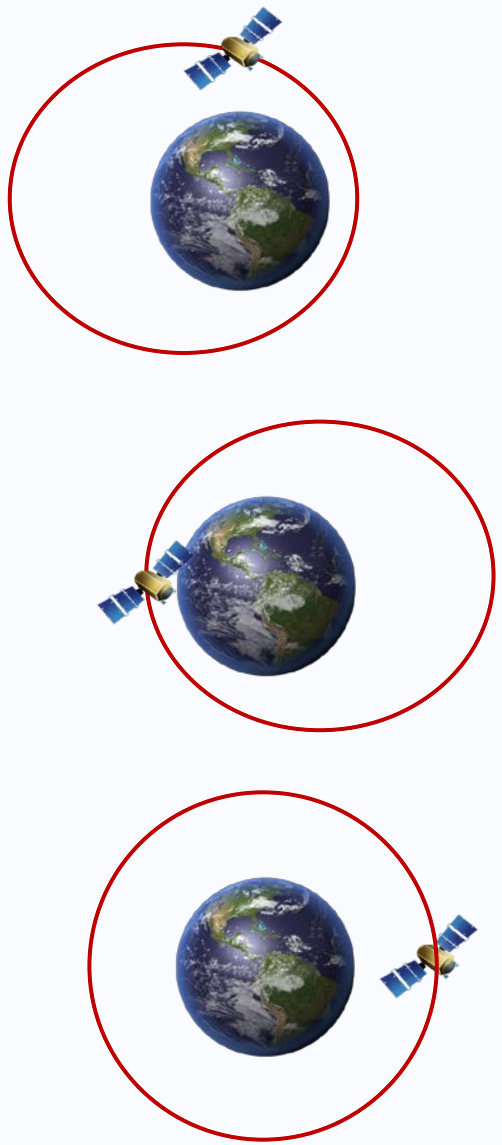
Orbital energy

- Quick quiz answer: THEY ALL HAVE THE SAME ORBITAL PERIOD
  - The orbital period is only a function of the size of the orbit (i.e. the semi-major axis)
  - All cases have the same semi-major axis
  - The satellite will take the same time to make one revolution



Example 1: Blue Origin NS-18

13th October 2021
2nd crewed flight

Example 1: Blue Origin NS-18

- In the 10-minute flight, the crew capsule reached an apogee of 105,930 metres. If the Main Engine Cut-Off (MECO) occurred 131 seconds into the flight, when the booster was at its maximum speed of 999 m/s, calculate:
    - The time taken to reach this altitude after MECO
    - The altitude gained after MECO
- Sketch the altitude-time profile for this flight
- You can ignore the effects of aerodynamic drag

$$g = 9.81\ \mathrm{m/s^2}$$

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Example 1: solution

- Time from MECO to apogee:
  - Use constant acceleration formula $v = u + at$

$$t = \frac{v - u}{a} = \frac{0 - 999}{-9.81} = 101.835 \text{ sec}$$

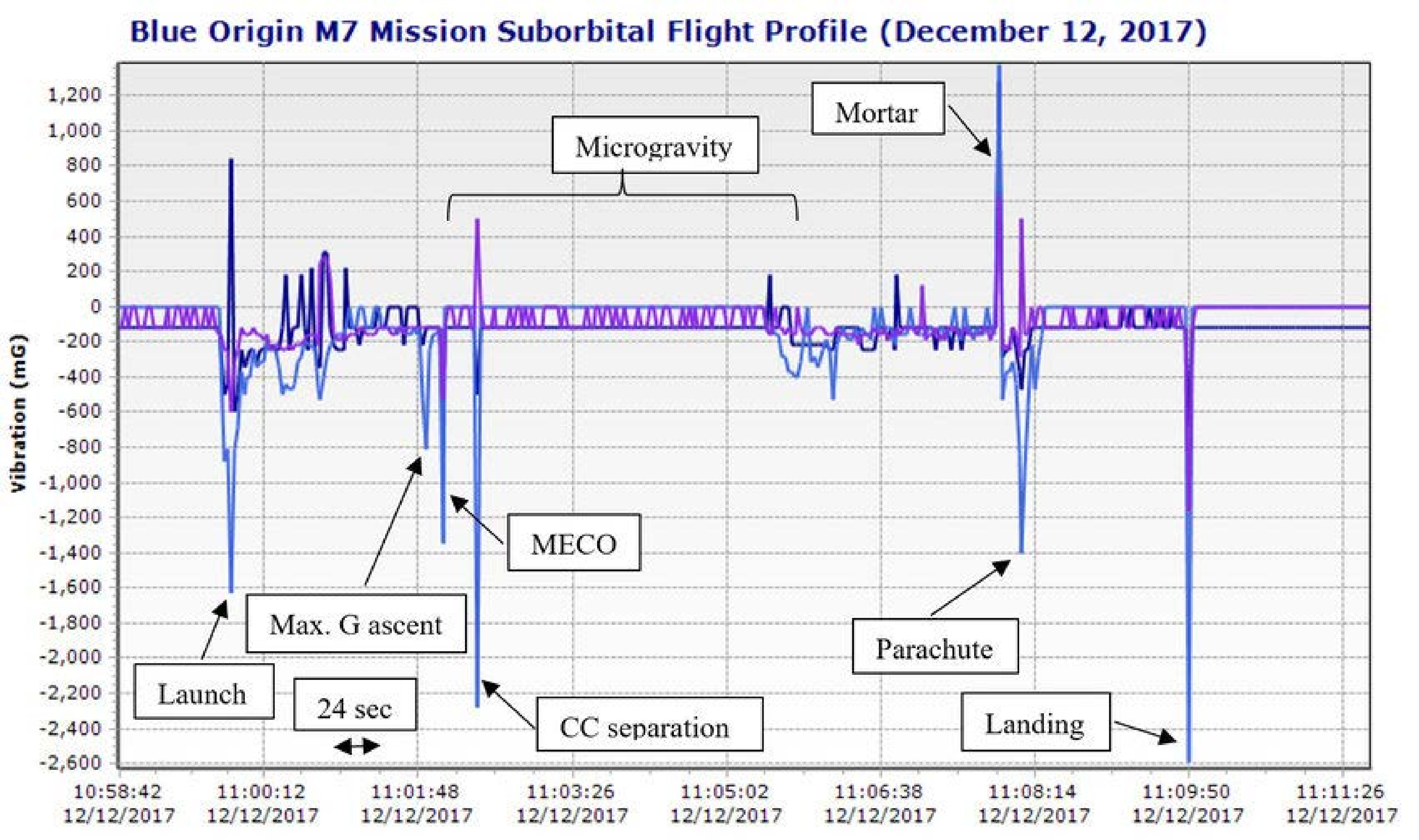
- Altitude gained after MECO:
  - Use constant acceleration formula $s = ut + \frac{1}{2}at^2$

$$s = (999)(101.835) + \frac{1}{2}(-9.81)(101.835)^2$$

$$= 50866.5 \text{ m}$$

$$= 50.87 \text{ km}$$

Example 1: check



https://www.researchgate.net/publication/333810041_Challenges_of_ERAU's_First_Suborbital_Flight_Aboard_Blue_Origin's_New_Shepard_M7_for_the_Cell_Research_Experiment_In_Microgravity_CRExIM

Example 1: check

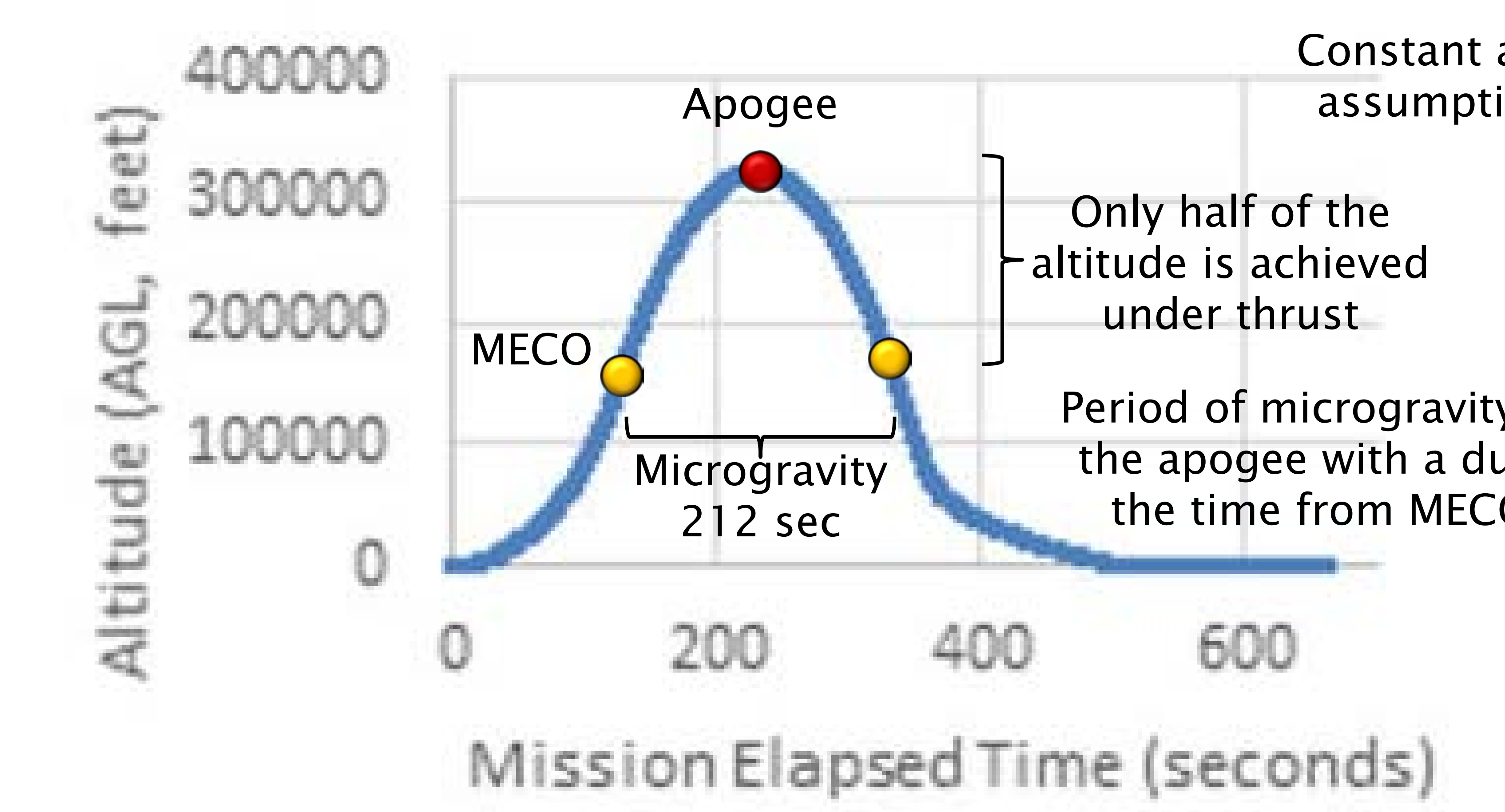
- Timeline derived from Blue Origin M7 mission vibration profile

Event	Time	Elapsed Time	Elapsed time (sec)
Launch	10:59:53	00:00:00	0.0
MECO	11:02:04	00:02:11	131.0
CC Separation	11:02:25	00:02:32	152.0
Microgravity end	11:05:36	00:05:43	343.0
Parachute	11:08:04	00:08:11	491.0
Landing	11:09:50	00:09:57	597.0

https://www.researchgate.net/publication/333810041_Challenges_of_ERAU's_First_Suborbital_Flight_Aboard_Blue_Origin's_New_Shepard_M7_for_the_Cell_Research_Experiment_In_Microgravity_CRExIM

Example 1: sketch and insights

- Altitude profile from Blue Origin M7 mission and insights



Constant acceleration assumption is good

Only half of the altitude is achieved under thrust

Period of microgravity is centred on the apogee with a duration ~twice the time from MECO to apogee

https://www.researchgate.net/publication/333810041_Challenges_of_ERAU's_First_Suborbital_Flight_Aboard_Blue_Origin's_New_Shepard_M7_for_the_Cell_Research_Experiment_In_Microgravity_CRExIM

Example 1: Crew-4 re-entry

- After departing the ISS the *Freedom* moved to a circular orbit (assumed) at an altitude of 406 km (10 km below the ISS) and inclined at 51.6°. After jettisoning the trunk, the *Freedom* conducted a re-entry burn to descend in an elliptic trajectory to splashdown in the Atlantic Ocean at 20:55 UTC on 14th October 2022.
- If the landing was at the perigee of the elliptic descent trajectory and neglecting the effects of the atmosphere, calculate:
  - The semi-major axis and eccentricity of the elliptic re-entry trajectory;
  - The delta-v required for re-entry; and
  - The time (in UTC) when the re-entry burn takes place.
- Sketch the ground track of the elliptical re-entry trajectory

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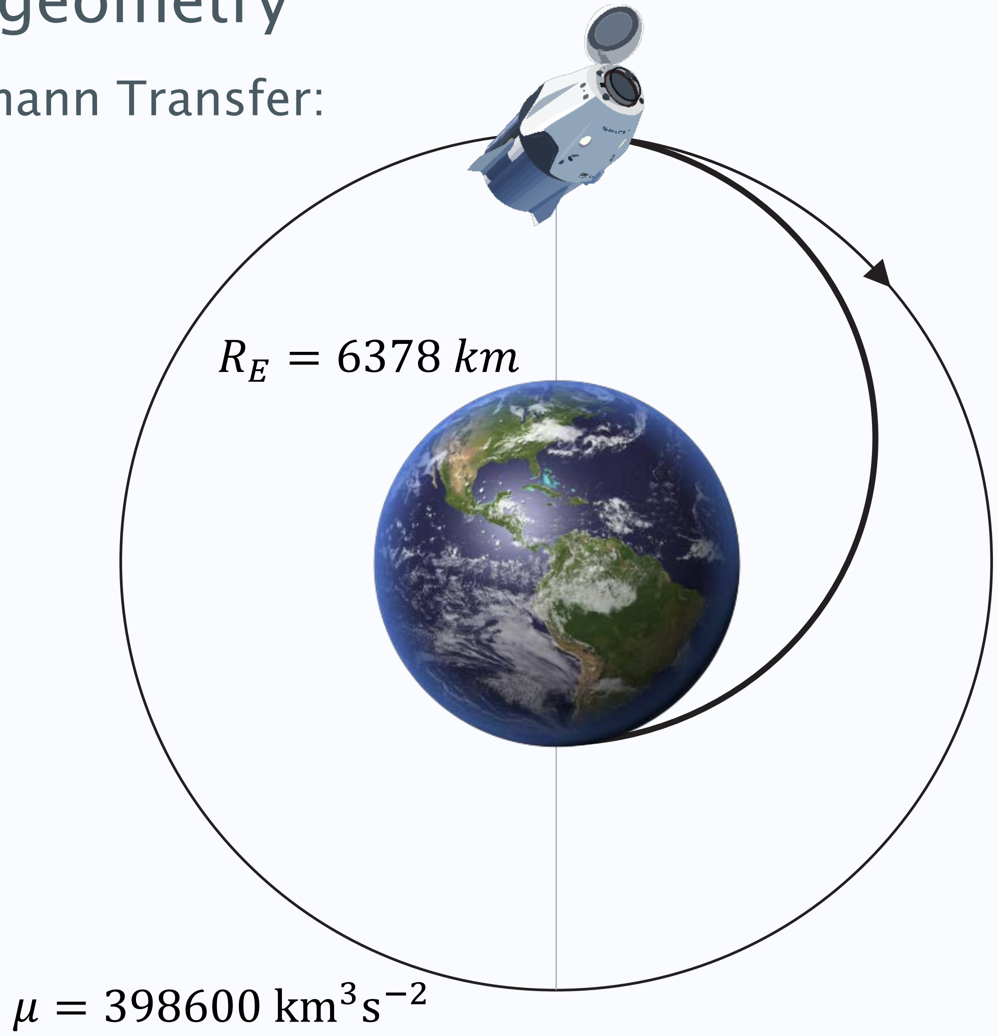
Example 2: *Freedom* Re-entry

- Basic geometry
  - Hohmann Transfer:



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Example 2: solution (1)

- The semi-major axis and eccentricity of the elliptic re-entry trajectory
  - The re-entry burn takes place at the apogee of the elliptical re-entry trajectory → $\theta = 180°$:

$$r(\theta) = \frac{a_T(1 - {e_T}^2)}{1 + e_T \cos\theta} = \frac{a_T(1 - {e_T}^2)}{1 - e_T} = a_T(1 + e_T) = 6784 \text{ km}$$

  - The landing is at the perigee of the elliptical re-entry trajectory → $\theta = 0°$:

$$r(\theta) = \frac{a_T(1 - {e_T}^2)}{1 + e_T \cos\theta} = \frac{a_T(1 - {e_T}^2)}{1 + e_T} = a_T(1 - e_T) = 6378 \text{ km}$$

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Example 2: solution (2)

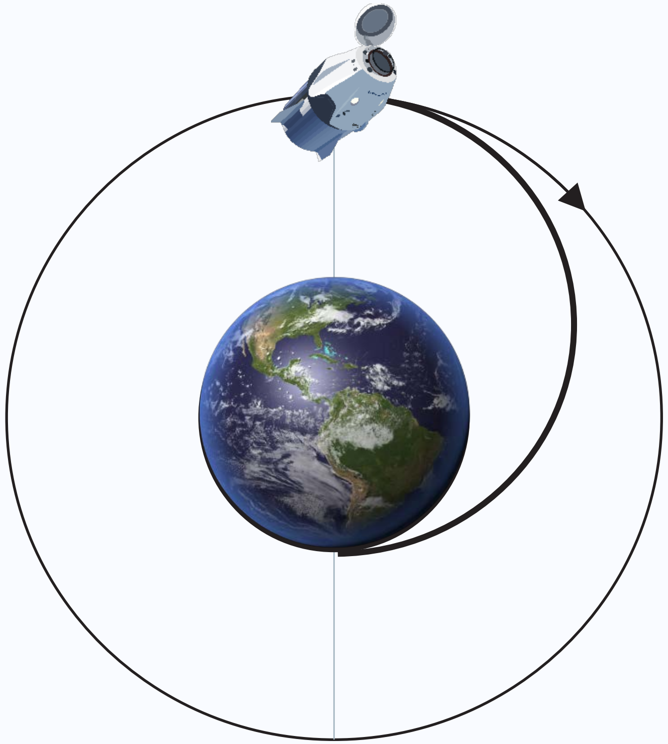
- Semi-major axis:

$$a_T = \frac{1}{2}(6784 + 6378)$$

$$= 6581 \text{ km}$$

- Substitute into either equation (perigee or apogee) to find eccentricity:

$$e_T = 0.030846$$



Example 2: solution (3)

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- *Freedom* is initially on a circular orbit:
  - Speed on initial circular orbit:

$$V_0 = \sqrt{\frac{\mu}{a_0}} = \sqrt{\frac{398600}{6784}} = 7.6652\ \mathrm{kms}^{-1}$$

- The speed of *Freedom* at the apogee of the re-entry orbit:
  - Using energy equation:

$$V_{Ta} = \sqrt{\mu\left(\frac{2}{r_{Ta}} - \frac{1}{a_T}\right)} = \sqrt{398600\left(\frac{2}{6784} - \frac{1}{6581}\right)}$$
$$= 7.5461\ \mathrm{kms}^{-1}$$

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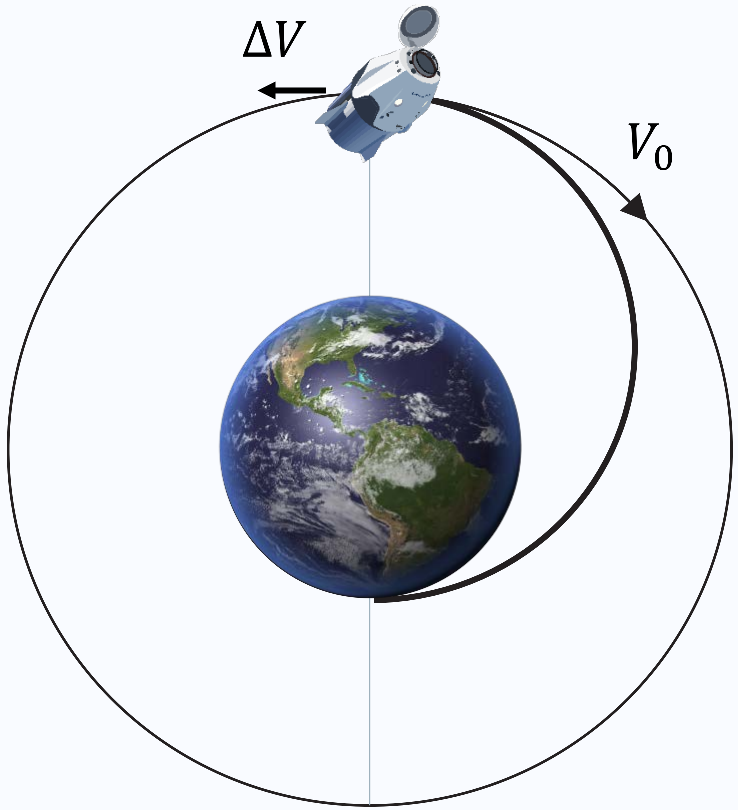
Example 2: solution (4)

- So, the delta-v is: $\Delta V = V_0 - V_{Ta}$
  $= 7.6652 - 7.5461\ \text{km}s^{-1}$
  $= 119.1\ \text{m}s^{-1}$



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Example 2: solution (5)

- The re-entry trajectory is one-half of a complete orbit:
  - Period (time needed for one orbit):

$$\tau = 2\pi\sqrt{\frac{a_T^3}{\mu}} = 2\pi\sqrt{\frac{6581^3}{398600}} = 5313.11\ \mathrm{sec}$$

- So time to complete one-half of the orbit:

$$t = 2656.56\ \mathrm{sec} = 44\ \mathrm{min}\ 16.56\ \mathrm{sec}$$

- So, time of re-entry burn was approx. 20:10:43 UTC

Example 1: actual timeline

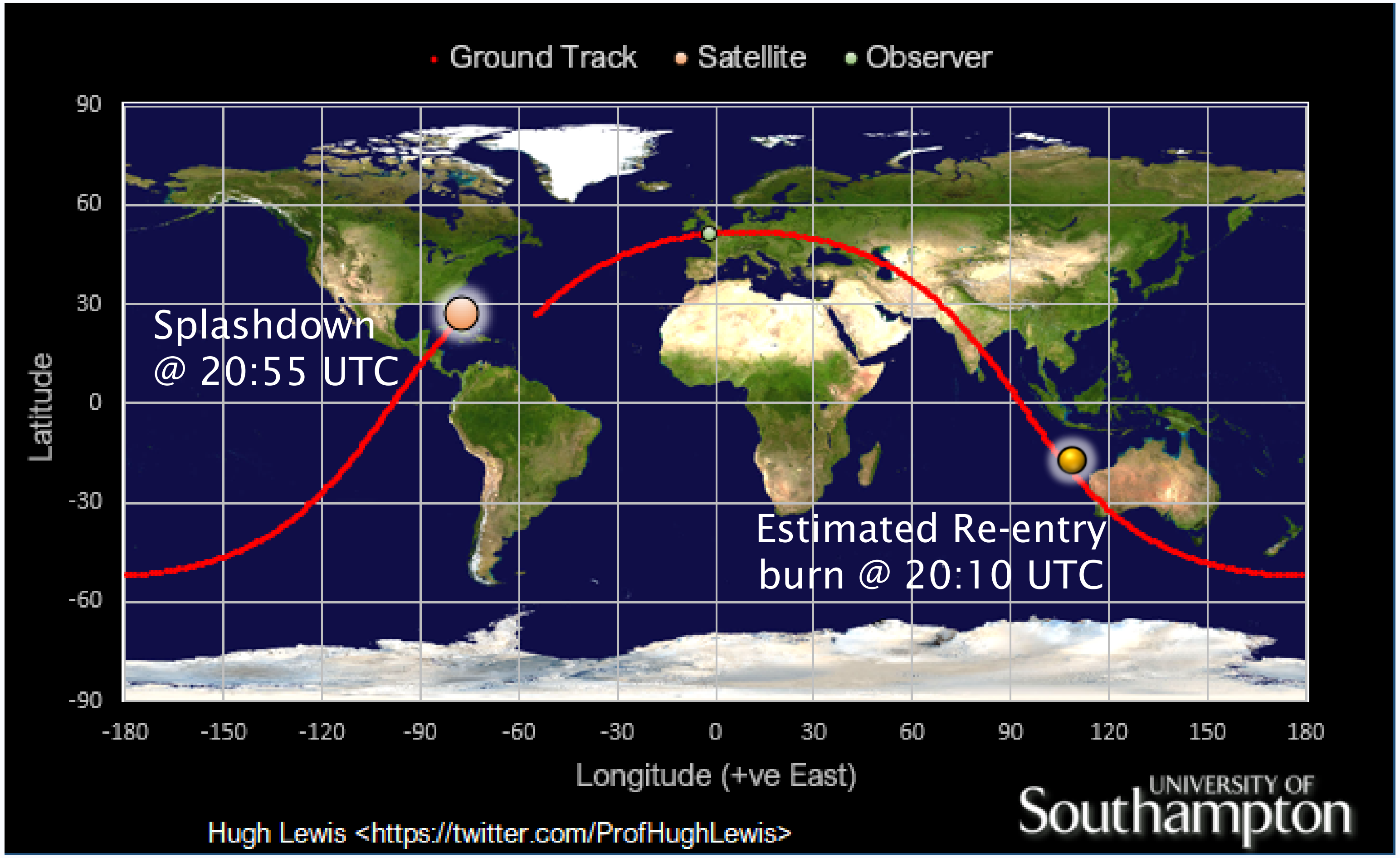
- Trunk jettison: 19:57 UTC
- Start of re-entry burn (approx.): 20:14 UTC
- End of re-entry burn: 20:26 UTC
- Burn duration 12 minutes
- End of burn to splashdown 29 minutes

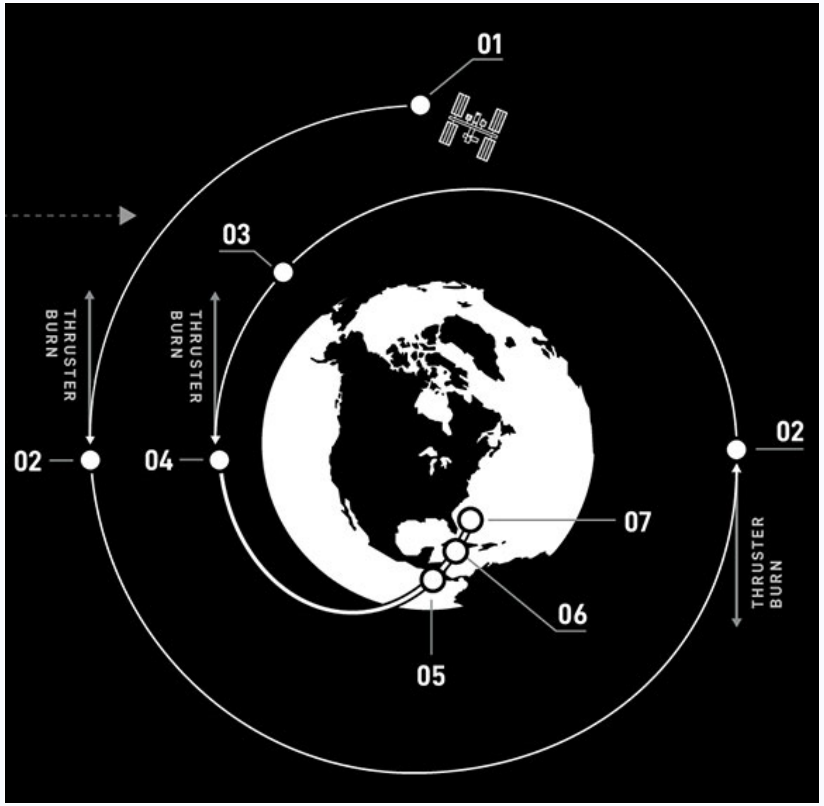
https://twitter.com/SpaceX

- Our assumptions:
  - No atmosphere
  - Impulsive manoeuvre
    - What will these do to the flight time?

Example 1: Crew-4 re-entry

- Sketch of the ground-track: *Freedom* towards splashdown in the Atlantic Ocean off the Florida Coast





Source: SpaceX

Activity

- The orbital motion (Celestial Mechanics) topic is covered in chapter 4 of Fortescue, Stark & Swinerd:
  - Read this chapter (up to an including the “Specifying the Orbit” section; there is no need to go further) in preparation for the next few lectures & to support your learning of this topic
  - Access to the e-book is available via the Library website: https://onlinelibrary.wiley.com/doi/book/10.1002/9781119971009