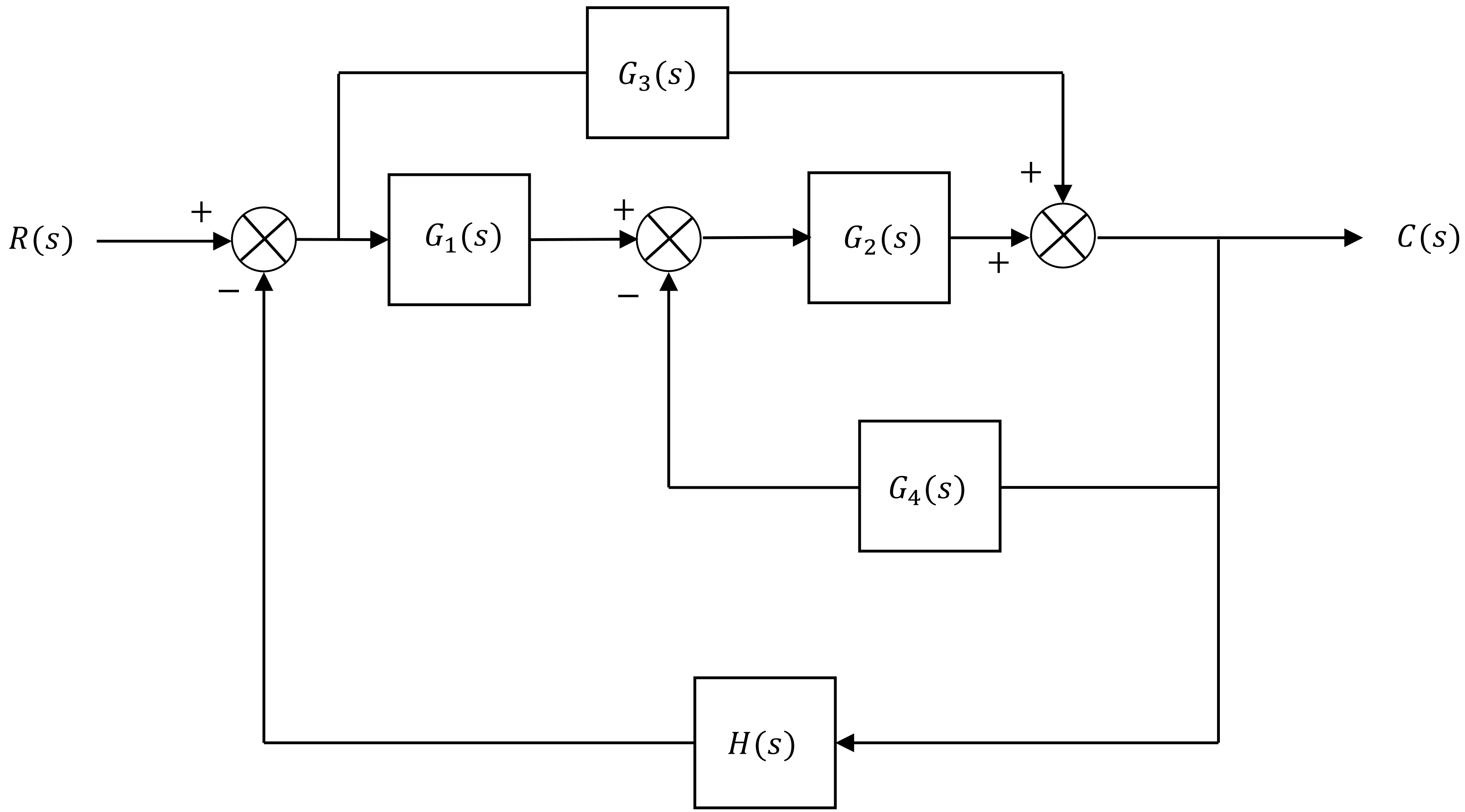
Q(A i): Simplify the block diagram



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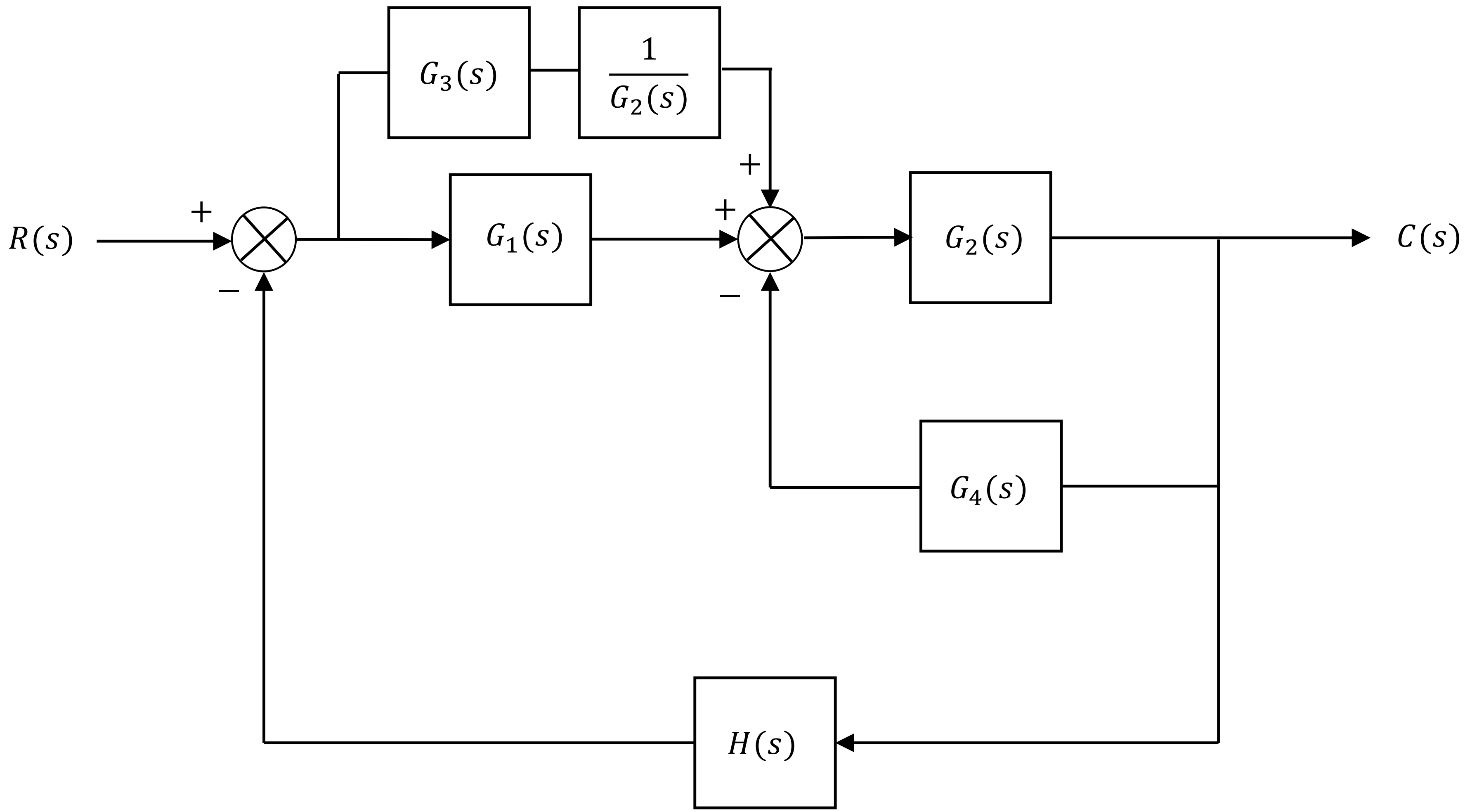

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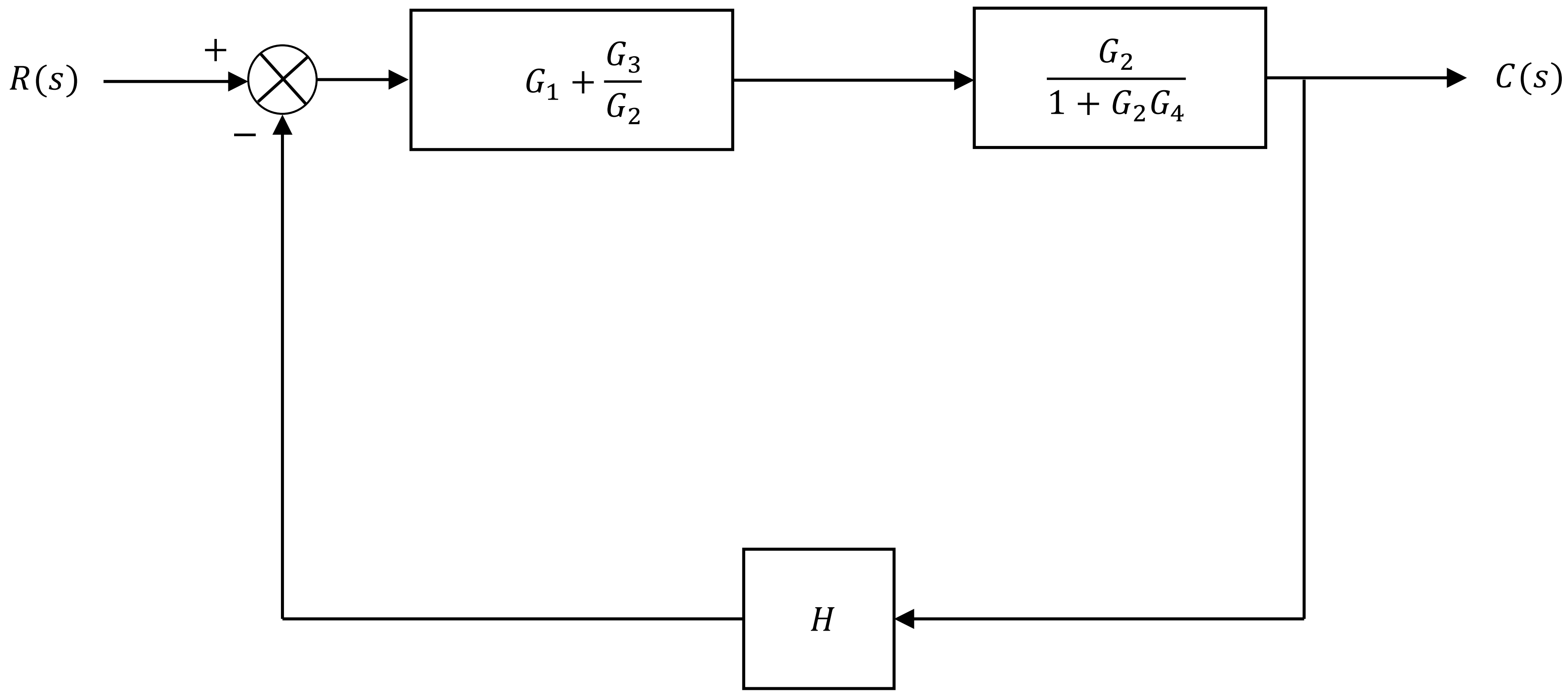

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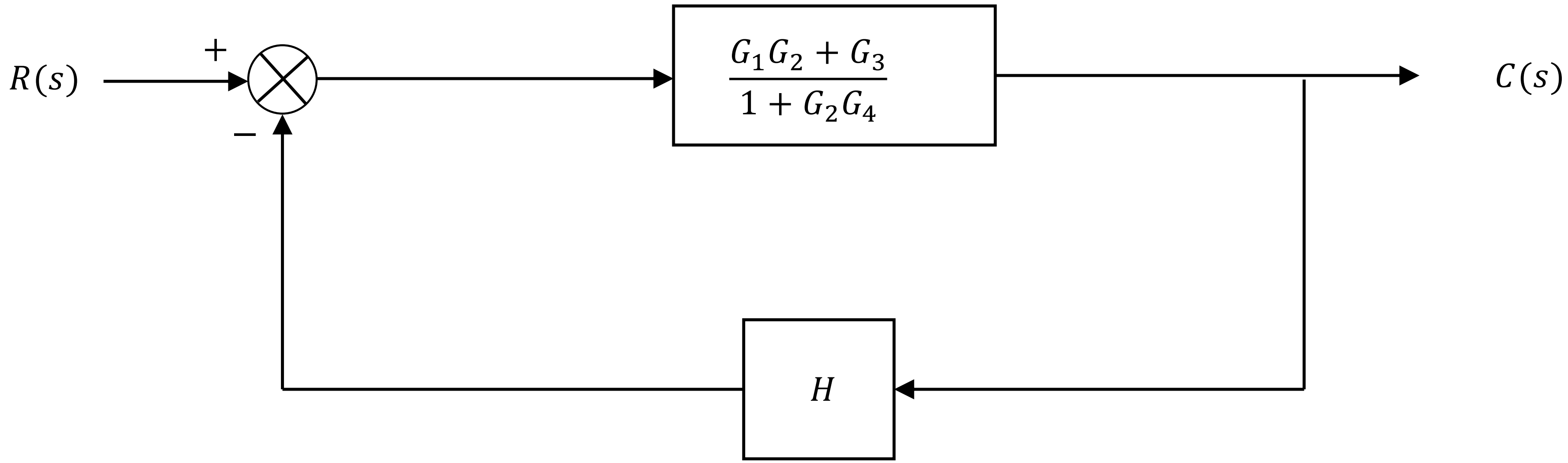


Q(A i): Simplify the block diagram

$$R(s) \longrightarrow \boxed{\dfrac{G_1G_2 + G_3}{1 + G_2G_4 + HG_3 + HG_1G_2}} \longrightarrow C(s)$$

Q(A ii): To find the transfer function corresponding to the time response:

$$\frac{1}{2}\sin t - \frac{3}{2}\cos t + \frac{5}{2}e^{-t} - e^{-2t}$$

First find the Laplace transform of the response and arrange:

$$\frac{1}{2}(\sin t - 3\cos t + 5e^{-t} - 2e^{-2t})\,\theta(t) \rightarrow$$

$$\frac{1}{2}\frac{1}{s^2+1} - \frac{3}{2}\frac{s}{s^2+1} + \frac{5}{2}\frac{1}{s+1} - \frac{1}{s+2} =$$

$$\frac{5}{(s^2+1)(s+1)(s+2)}$$

Then since the Laplace transform of $\sin t$ is $1/(s^2+1)$ we obtain the transfer function:

$$G(s) = \frac{5}{(s+1)(s+2)}$$

Q(A iii): root locus, gain margin, and phase margin

The root locus is a graphical technique used to examine how the roots and poles of a system move in the *s*-plane with variation of a certain system parameter. For example, it can be used to determine if a system will become unstable for certain values of gain, *K*, or the behaviour of oscillatory pole-pairs.

Define the terms gain and phase margin.

Gain margin: GM_{dB} is the distance of the magnitude below 0 dB where the phase angle is –180°

e.g. for second order system $\rightarrow\infty$

gain in dB that is permissible before system becomes unstable

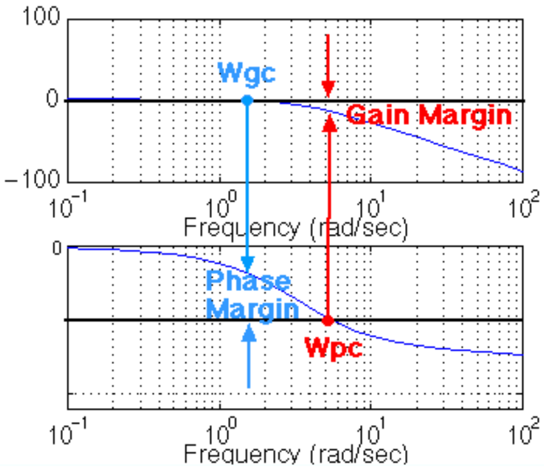
or extra gain required for AR to equal 1

Phase margin: ϕ_m is the sum of 180° and the phase angle where $|G_cGH| = 1$ (i.e. 0 dB).

I.e extra phase lag tolerable before instability

e.g. 90° for second order system (approx)

+ve gain and phase margins imply stability.



Q(A iv): Consider transfer function:

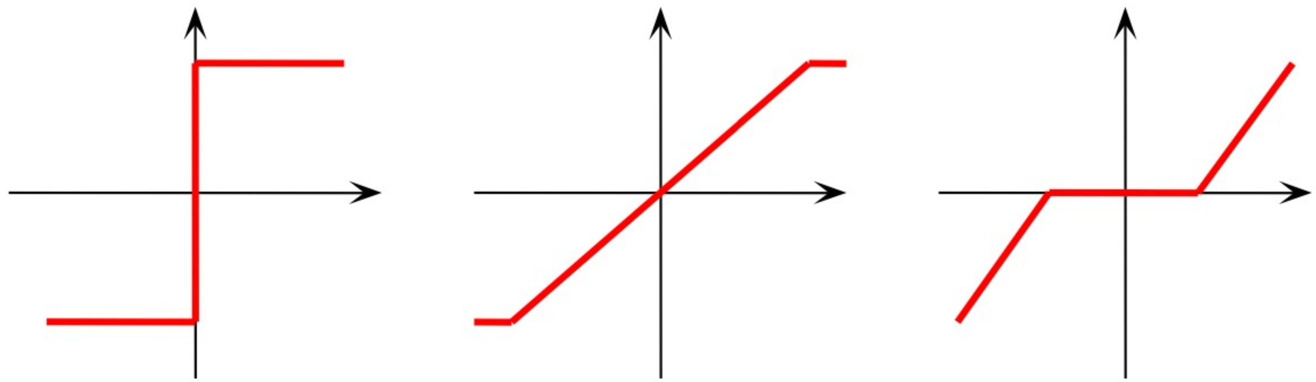
$$G(s) = \frac{4}{s^2 + s + 4}$$

- Natural frequency is: $\omega_n = \sqrt{4}s^{-1} = 2\ s^{-1}$
- Damping factor: $2\zeta\omega_n = 1 \rightarrow \zeta = \frac{1}{2\omega_n} = 0.25$
- Peak time: $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{2\pi}{\sqrt{15}}$ sec ≈ 1.62 sec
- Settling time: $T_s = 4T = \frac{4}{\zeta\omega_n} = 8$ sec
- Percentage overshoot: $PO = 100\exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 100\exp\left(-\frac{\pi}{\sqrt{15}}\right) = 44.43\%$

Q(A v): Two types of non-linearities:

Examples of system nonlinearities

(a) Relay | (b) Saturation | (c) Dead-zone



(a) Relay	(b) Saturation	(c) Dead-zone
switch solenoid	limit max/min flow, speed, acceleration	gear backlash neutral zones tolerances

nonlinearities frequently occur in actuating components of systems

Q(A vi): Gain and Phase at $\omega = 1$ rad/s

$$G(s) = \frac{5s^2 + 3s + 2}{s^4 + 10s^2 + 3s + 3}$$

$$G(j\omega)_{\omega\to 1} = \frac{-5\omega^2 + 3j\omega + 2}{\omega^4 - 10\omega^2 + 3j\omega + 3} \xrightarrow{\omega=1} \frac{-5 + 3j + 2}{1 - 10 + 3j + 3} = \frac{-3 + 3j}{-6 + 3j} = \frac{1 - j}{2 - j} = \frac{3}{5} - \frac{j}{5}$$

Gain:

$$|G(j\omega)_{\omega\to 1}| = \frac{\sqrt{10}}{5} = 0.63$$

Phase:

$$\arg G(j\omega)_{\omega\to 1} = \arg\left(\frac{3}{5} - \frac{j}{5}\right) = -18.43^{\circ}$$

Q(A vii): State space representation:

Handle nominator and denominator of the transfer function separately.

Denominator:

- Convert to ode: $\ddot{y} + 7\dot{y} + 9y = r$
- Identify state variables: $x_1 = y, x_2 = \dot{y}$ and so $\dot{x}_1 = \dot{y}, \dot{x}_2 = \ddot{y}$
- So: $\dot{x}$_2 $+ 7x_2 + 9x_1 = r$ and:

$$\begin{bmatrix} \dot{x_1} \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

Nominator:

- $x_1 = x_1;\ \dot{x}_1 = x_2$
- $y = 2x_2 + x_1$
- $y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Q(A viii): find transfer function

The PO is given by expression: $PO = 100 \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$ which allows to evaluate damping factor $\zeta = 0.5$.

The settling time reads: $T_s = \frac{4}{\zeta\omega_n}$ which after evaluating ζ from PO allows to evaluate $\omega_n = 4$ rad/s.

Q(A xi): Linearization:

Expand the function $f(x) = f(x_0 + \chi) = f(x_0) + f'(x_0)\chi + \cdots$

So, in our case: $\sin x = \sin x_0 + \cos x_0\ \chi + \cdots$

a) $x_0 = 0 \rightarrow \sin x \approx x$ and (we now call χ as x)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = x$$

b) $x_0 = \pi \rightarrow \sin x \approx -x$ and (we now call χ as x)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = -x$$

Q(A x): Justification why dB as useful:

Using dB allows to express the magnitude of a transfer function formed as a product of transfer functions of several subsystems as a sum of magnitudes of transfer functions of these subsystems. Phase is also a sum of phases of subsystems. Convenient for interpreting Bode plots.

$M_{dB} = 25\ dB$. Then since $M_{dB} = 20 \log M$, we can express $M = 10^{M_{dB}/20} = 10^{1.25} = 17.78$