UNIVERSITY OF SOUTHAMPTON SESA3029W1

SEMESTER 1 EXAMINATIONS 2017-18

TITLE: Aerothermodynamics

DURATION: 120 MINS

This paper contains **FIVE** Questions

Answer **ALL** questions on this paper. Question 1 is worth 36 marks and questions 2-5 are worth 16 marks each.

An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow **and** normal shock tables (11 sides) are provided. (In reading from tables, nearest values are acceptable unless explicitly stated otherwise.)

An oblique shock chart is provided.

Note that a formula sheet is provided at the end of this paper

Only University approved calculators may be used.

A foreign language direct ‘Word to Word’ translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), c_p=1005 J/(kg K), γ=1.4, Pr=0.7, ρ=1.225 kg/m^3 and μ=1.79x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

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Q.1 Figure Q.1 opposite shows a converging-diverging nozzle that delivers air into a test section with uniform Mach 2.5 flow at temperature T=160 K and pressure p=20 kN/m^2. A flat plate is inserted into the test section as shown with angle of attack α=5°.

(i) Find the area ratio of the nozzle and the stagnation pressure, temperature and density (p_0, T_0 and ρ_0) of the upstream flow.

[6 marks]

(ii) Use shock-expansion theory to find the pressure and Mach number on the upper and lower sides of the flat plate.

[10 marks]

(iii) Find the pressure on the upper and lower sides of the plate according to Ackeret's theory.

[8 marks]

(iv) Find the maximum angle of attack α_{max} for there to be an attached oblique shock. Sketch the expected shock and expansion wave patterns for $\alpha<\alpha_{max}$ and $\alpha>\alpha_{max}$.

[6 marks]

(v) Discuss design requirements for efficient aerofoils in (a) the transonic and (b) the supersonic flight regimes.

[6 marks]

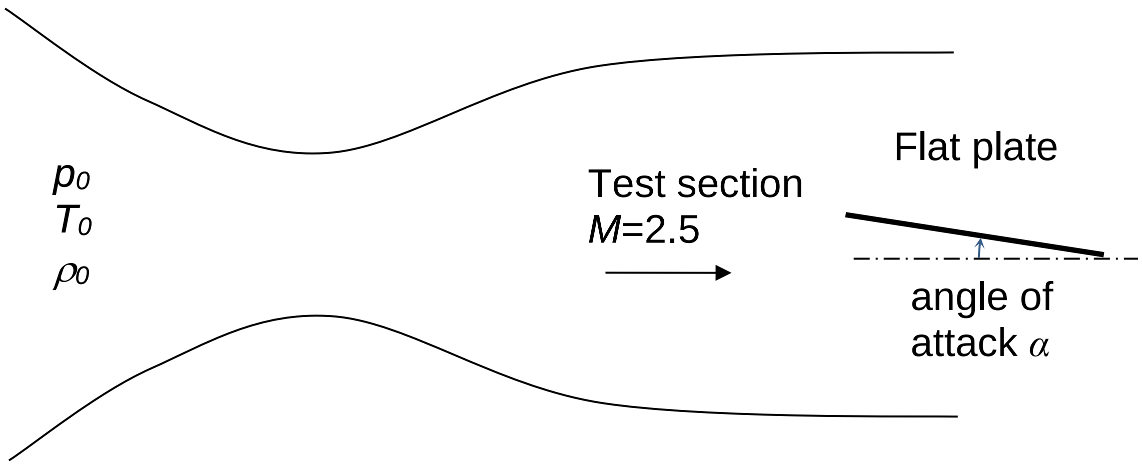

Figure Q.1

TURN OVER

Q.2

(i) For $M_\infty^2 < 1$ show how the compressible potential flow equation

$$\left(1 - M_\infty^2\right)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

can be reduced to Laplace's equation and hence derive the Prandtl-Glauert relation in the form

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2}}$$

[10 marks]

(ii) A multi-element aerofoil has a critical Mach number of 0.45. Find the minimum pressure coefficient in incompressible flow.

[6 marks]

Q.3

(i) Figure Q.3 below shows four characteristic lines in a 2D convergent-divergent nozzle, designed to accelerate air to Mach number M=3. Define the Riemann invariants at point 8 and state how they relate to points 4 and 7.

[4 marks]

(ii) At point 4: M=1.96, θ=24.5°, x=0.90, y=0.90, while at point 7: M=1.96, θ= 8.2°, x=1.74, y=0.33. Find M, θ, x and y at point 8.

[10 marks]

(iii) Define the simple region for the nozzle shown on figure Q3.

[2 marks]

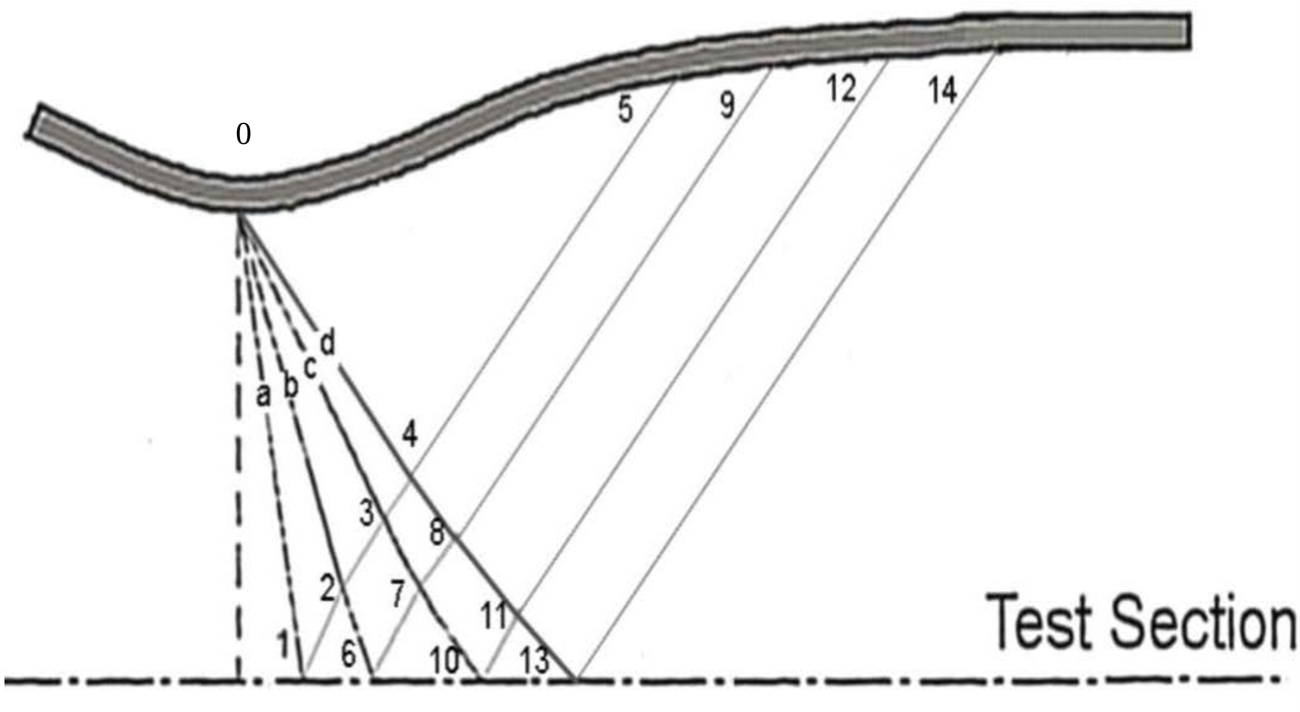

Figure Q.3

TURN OVER

Q.4

(i) An air flow (see rubric on front cover for properties) at 5 m/s with temperature 15°C develops a laminar boundary layer over a 12 cm long flat plate at a constant temperature of 40°C. For a total heat flux through the plate of 20 W, work out the required width of the plate.

[10 marks]

(ii) Sketch the shock layer near the leading edge of a spacecraft undergoing peak heating during re-entry after a lunar mission. Discuss the role of radiation from (a) the spacecraft and (b) the gas.

[6 marks]

Q.5

(i) Show that the one-dimensional isentropic Euler equations

$$\frac{\partial q}{\partial t}+\frac{\partial f(q)}{\partial x}=0, \text{ with } q=\begin{pmatrix}\rho \\ \rho u\end{pmatrix}, \quad f(q)=\begin{pmatrix}\rho u \\ \rho u^2+p\end{pmatrix}$$

and equation of state $p=C\rho^{\gamma}$, $C=\text{const.}$ can be written as

$$\frac{\partial q}{\partial t}+A\frac{\partial q}{\partial x}=0, \text{ with } A=\begin{pmatrix}0 & 1 \\ a^2-u^2 & 2u\end{pmatrix}$$

and hence show that the characteristic speeds are given by $u-a, u+a$. Note that the relation $a^2=\gamma\dfrac{p}{\rho}$ holds true as usual.

[10 marks]

(ii) A nozzle CFD calculation using a uniformly second order method gives a jet exit Mach number of 4.21 on an 80x80 grid and 4.35 on a 120x120 grid. Estimate the correct value of the jet Mach number.

[6 marks]

END OF PAPER (Formula sheet overleaf)

Useful Formulae

Perfect gas equation of state

$$p = \rho RT$$

Sound speed in a perfect gas

$$a^2 = \gamma RT$$

Adiabatic flow

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Mach angle:

$$\sin\mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta+\mu)_A + (\theta+\mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta-\mu)_B + (\theta-\mu)_P\right]$$

$$x_p = \frac{x_B \tan\alpha_{BP} - x_A \tan\alpha_{AP} + y_A - y_B}{\tan\alpha_{BP} - \tan\alpha_{AP}}$$

$$y_P = y_A + \left(x_P - x_A\right)\tan\alpha_{AP}$$

Velocity potential equation:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

Linearised pressure coefficient

$$C_p=-2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

Ackeret formula:

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

Laminar pipe flow:

$$\mathrm{Nu}=4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}=3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x=0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu}=0.022\,\mathrm{Pr}^{0.5}\,\mathrm{Re}^{0.8}$$

Turbulent boundary layer:

$$\mathrm{Nu}_x=0.029\mathrm{Re}_x^{0.8}\mathrm{Pr}^{0.6}$$