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Robust Design and the Taguchi Method

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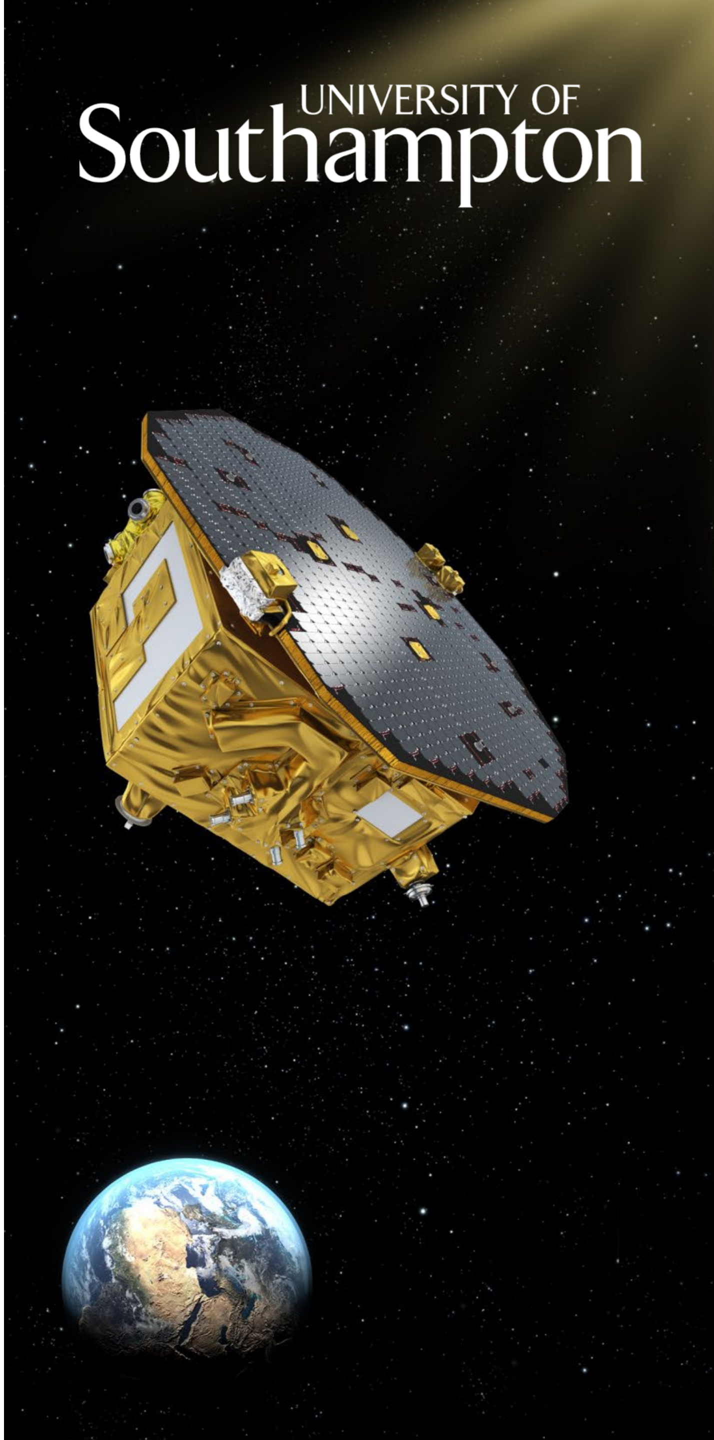
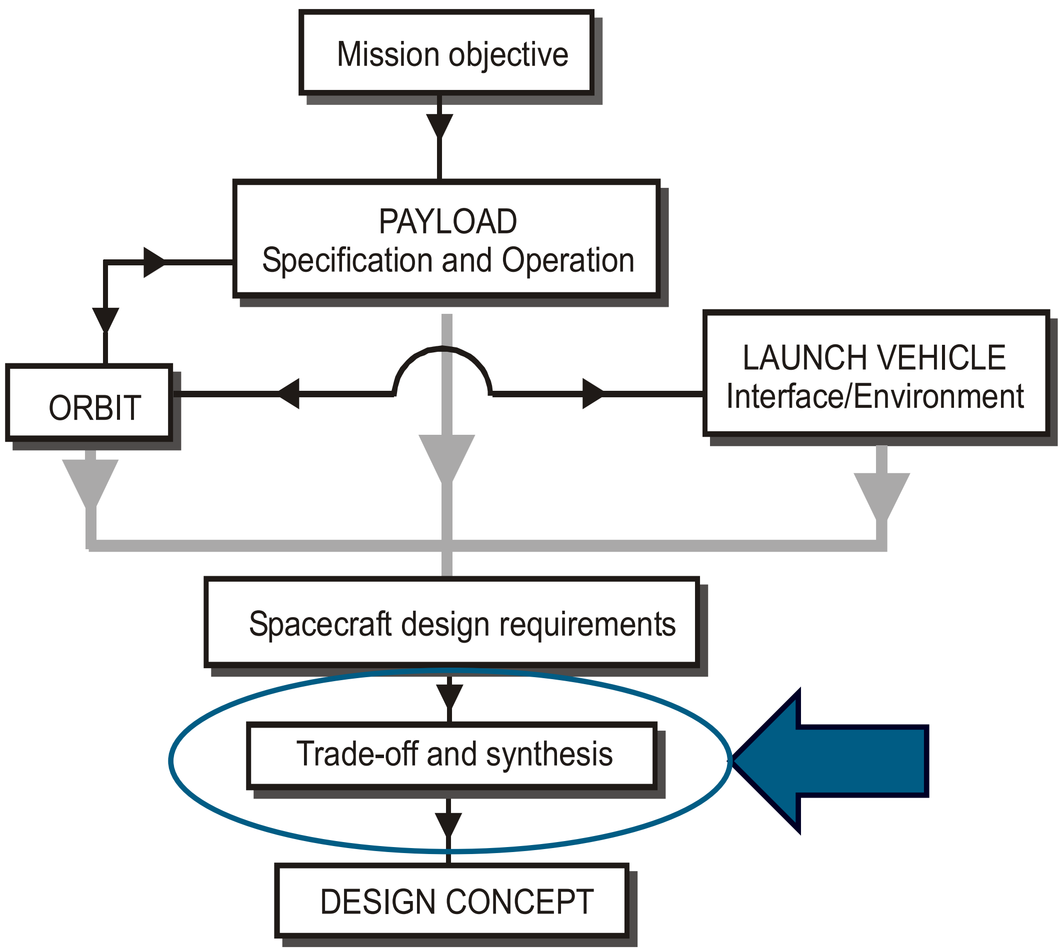
Robust Design

Robust design

- What is robust design?
- Taguchi method used for achieving
- Design parameters and noise factors
- Design of Experiments
- Signal-to-noise ratio
- Examples



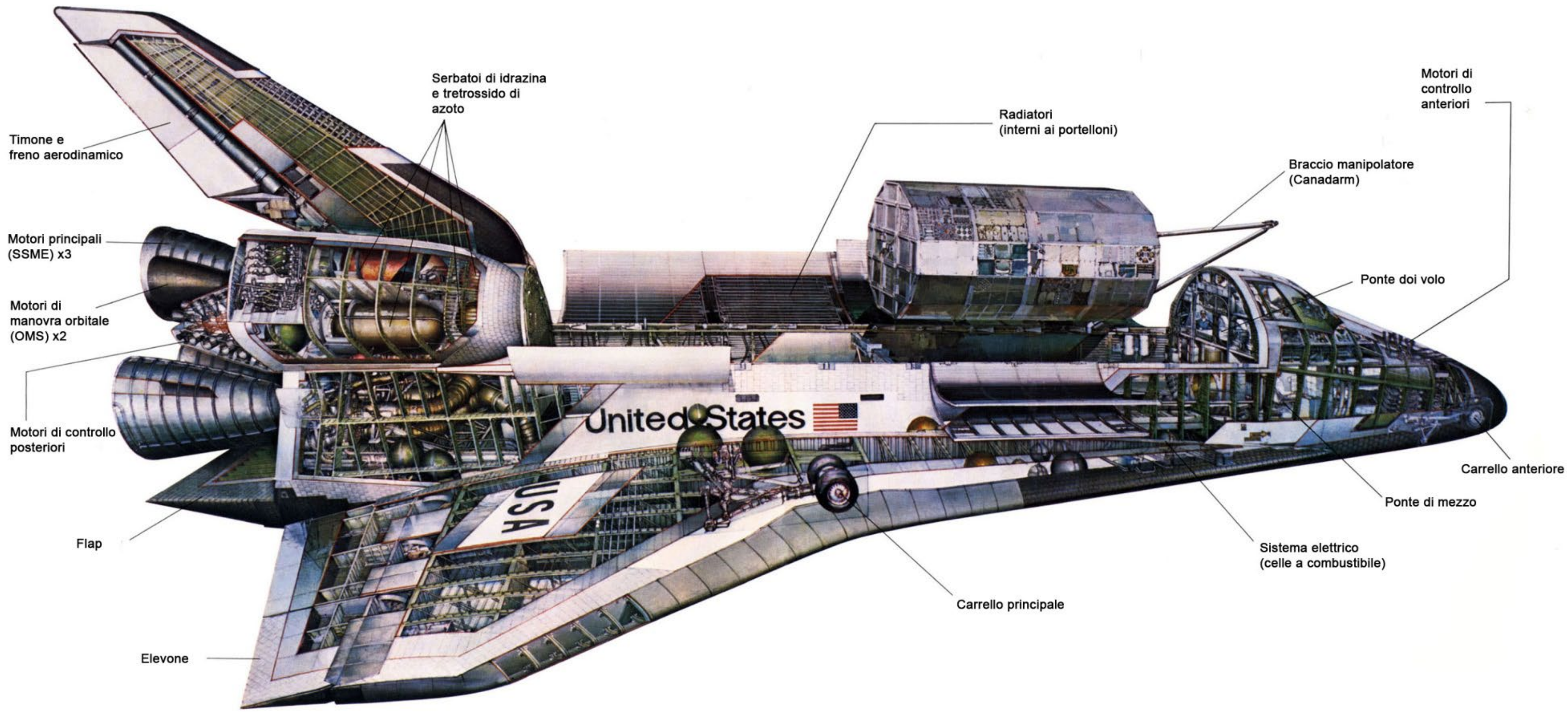
Robust design



What is robust design?

Robust design:

- A design whose performance is insensitive to variations



What is robust design?

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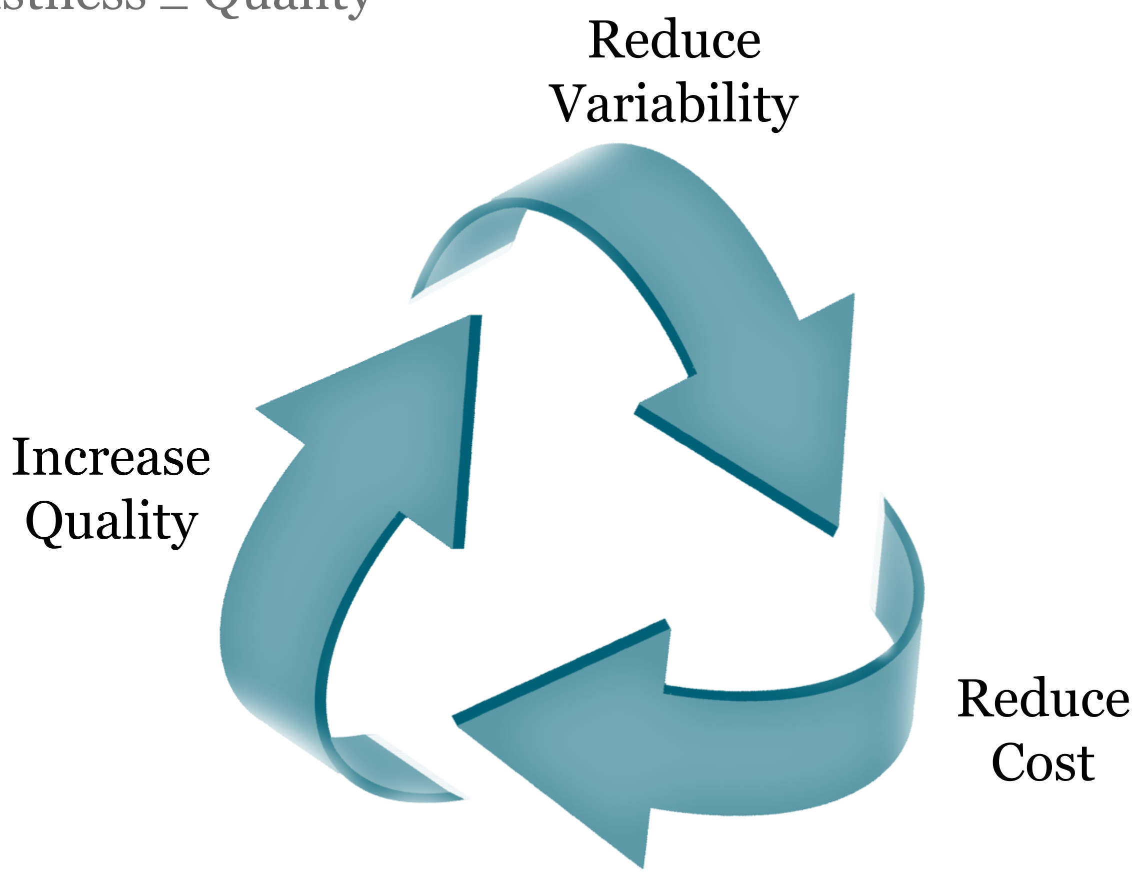
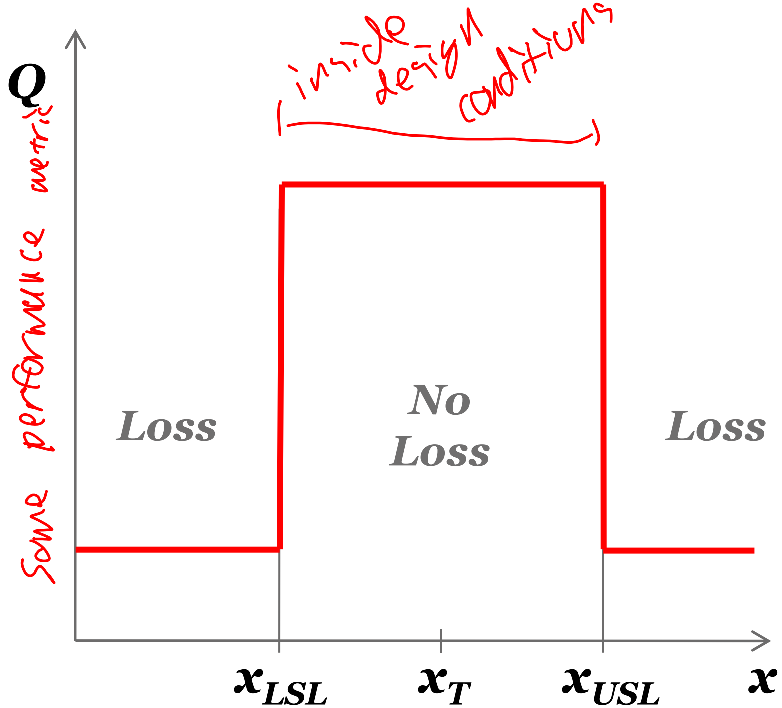
Robust design:

- Robustness ≅ Quality



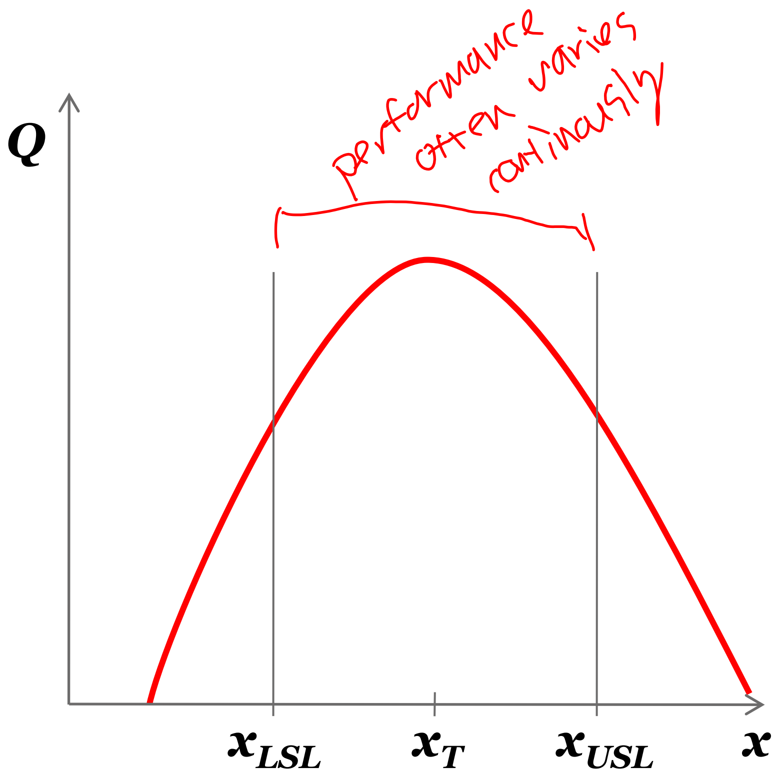
Traditional view

There is no loss in quality or value if the product performance is within some tolerance of the desired value



Actual view

Any deviation from the target value will result in a loss of quality



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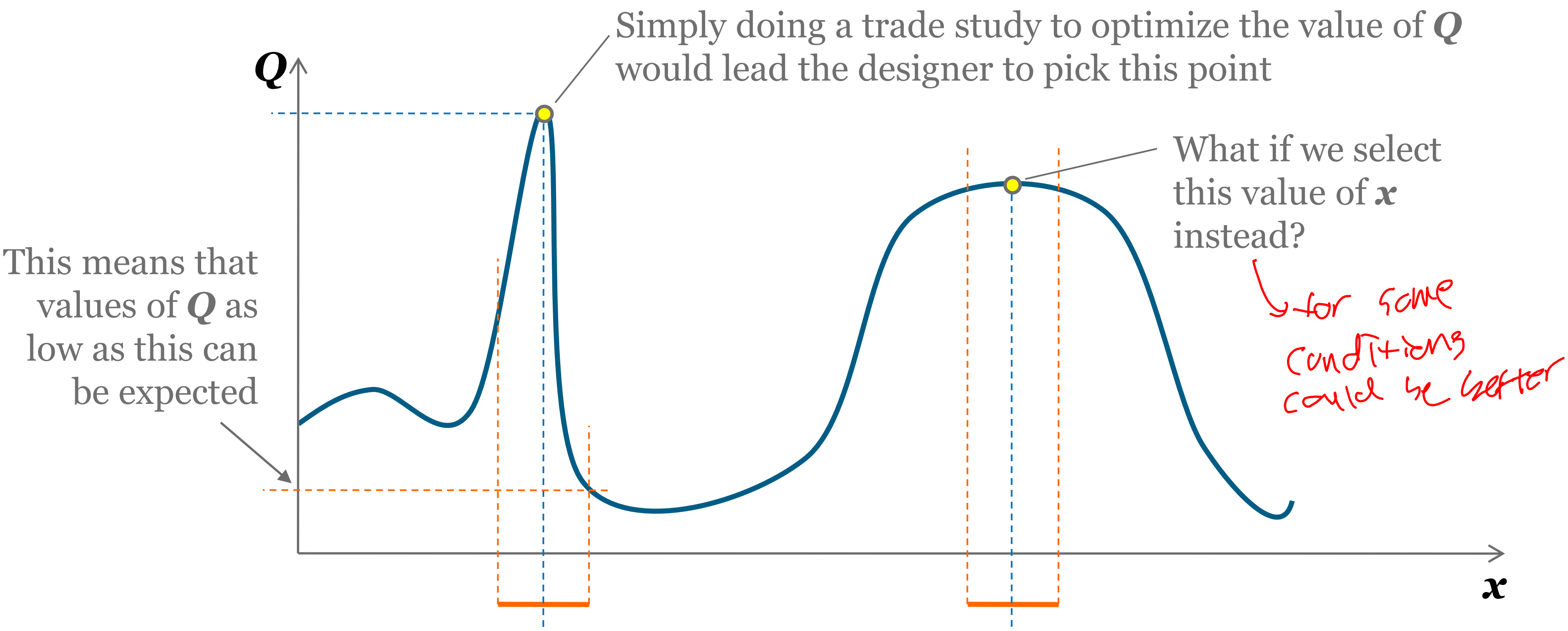
Robust design

Select a value of x to maximise Q



Taguchi Method

Taguchi method (1)

Taguchi method for robust design:

- Systemised statistical approach to product and process improvement developed by G. Taguchi
- Approach emphasises moving quality upstream to the design phase
- Based on the idea that minimising variation is the primary means of improving quality
- Special attention is given to designing systems such that their performance is insensitive to environmental changes

mass production

Taguchi method (2)

Overview:

Creative thinking

Identify Design Parameters

Identify Noise Factors

Construct Design of Experiments

Perform experiments

Analyse results

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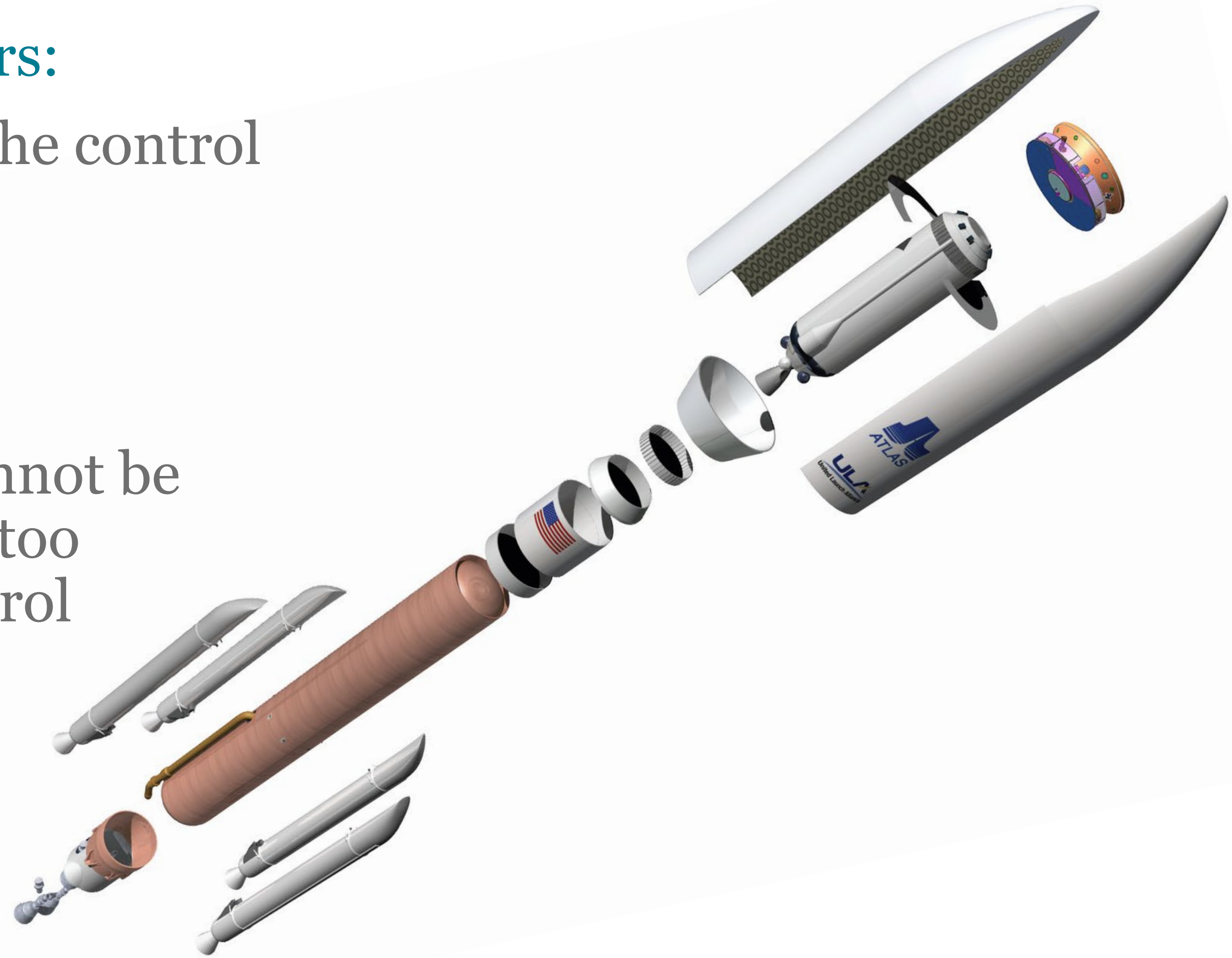
Taguchi method (3)

Design Parameters:

Variables under the control of the designer

Noise Factors:

Variables that cannot be controlled or are too expensive to control



DPs and NFs (1)

Example 1:

You are designing a spacecraft that will observe a comet from distances > 200 m using an imaging instrument

From the list of variables, which ones are Design Parameters and which ones are Noise Factors?

- Temperature in full Sun and in eclipse
- Material used to house instrument
- Number of cometary particles ejected from comet per cubic metre
- Size of cometary particles ejected
- Duration of imaging
- Size of instrument aperture

practically not controllable → noise factor

DPs and NFs (2)

Example 1:

You are designing a spacecraft that will observe a comet from distances > 200 m using an imaging instrument

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- Size of cometary particles ejected
- Duration of imaging
- Size of instrument aperture

DPs and NFs (3)

Example 2:

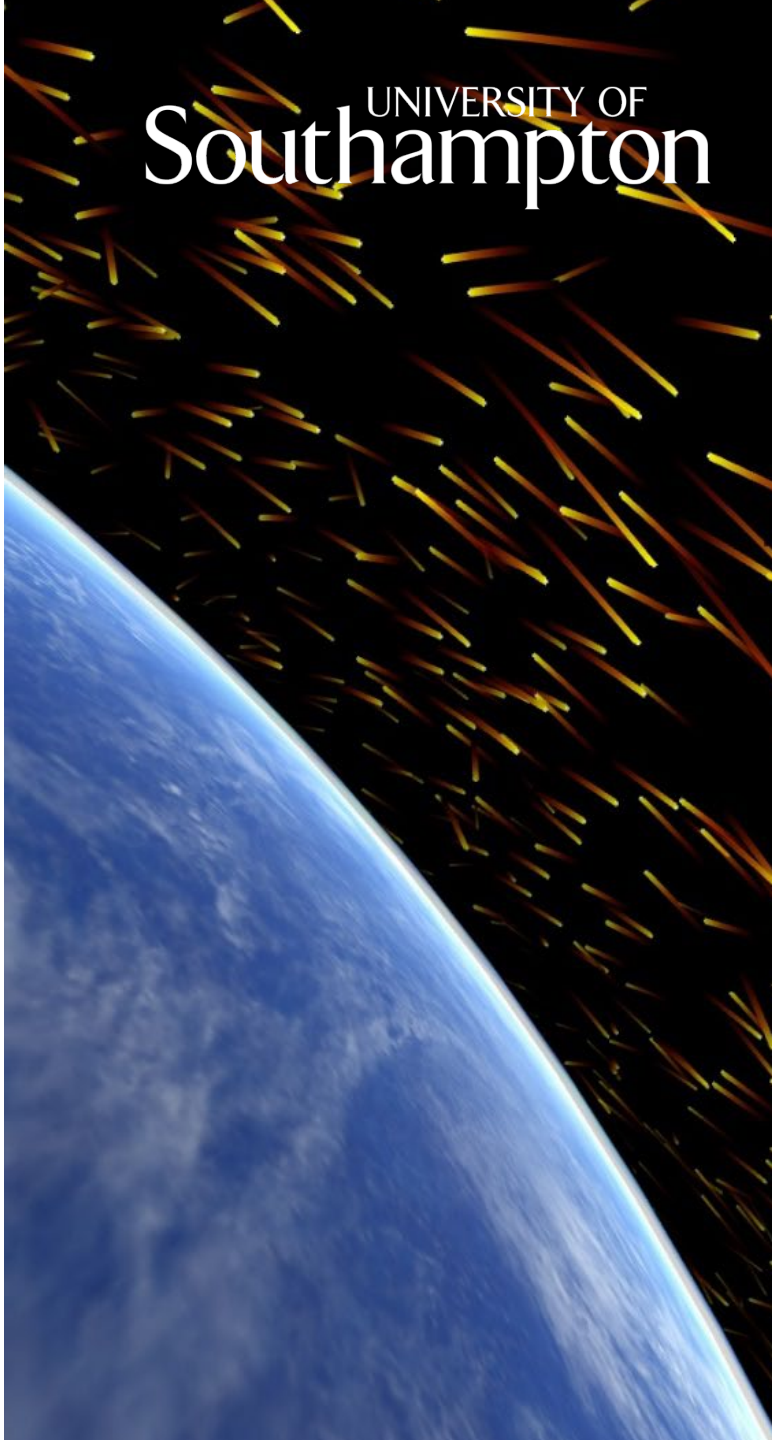
You are designing a deployable sail to provide end-of-life disposal for a spacecraft in LEO. The sail must survive impacts with small debris & burn up (demise) on re-entry.

From the list of variables, which ones are Design Parameters and which ones are Noise Factors?

- Local atmospheric density
- Altitude of demise

noise factors

- Sail area
- Orbital lifetime after sail deployment
- Number of small debris impacts before failure



DPs and NFs (4)

Example 2:

You are designing a deployable sail to provide end-of-life disposal for a spacecraft in LEO. The sail must survive impacts with small debris & burn up (demise) on re-entry.

From the list of variables, which ones are Design Parameters and which ones are Noise Factors?

- Local atmospheric density
- Altitude of demise
- Sail area
- Orbital lifetime after sail deployment
- Number of small debris impacts before failure

DPs and NFs (5)

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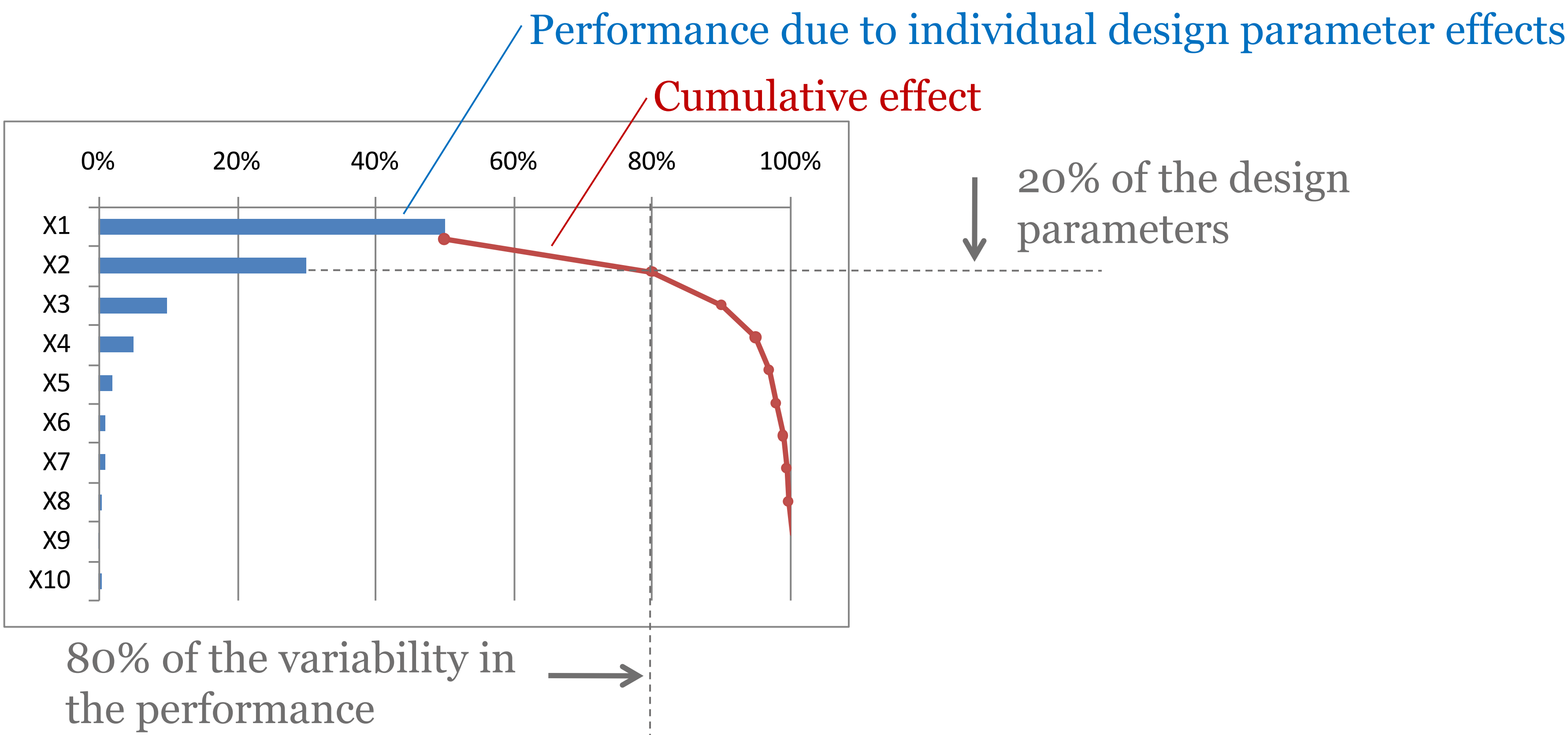
Design Parameters:

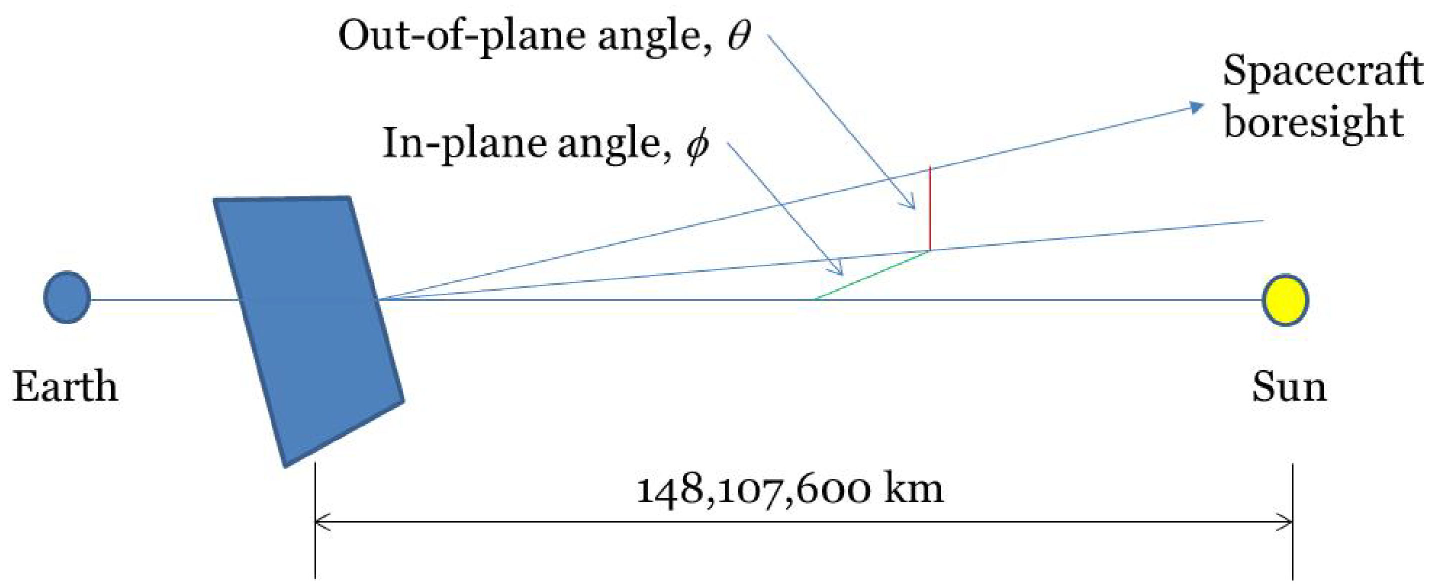
As a "rule of thumb": 20% of the possible set of design parameters for a system control 80% of the variability in the performance of the system



Performance characteristic (1)

Choose Design Parameters that maximise (or minimise) some performance characteristic Q whilst also minimising the variability in performance due to Noise Factors

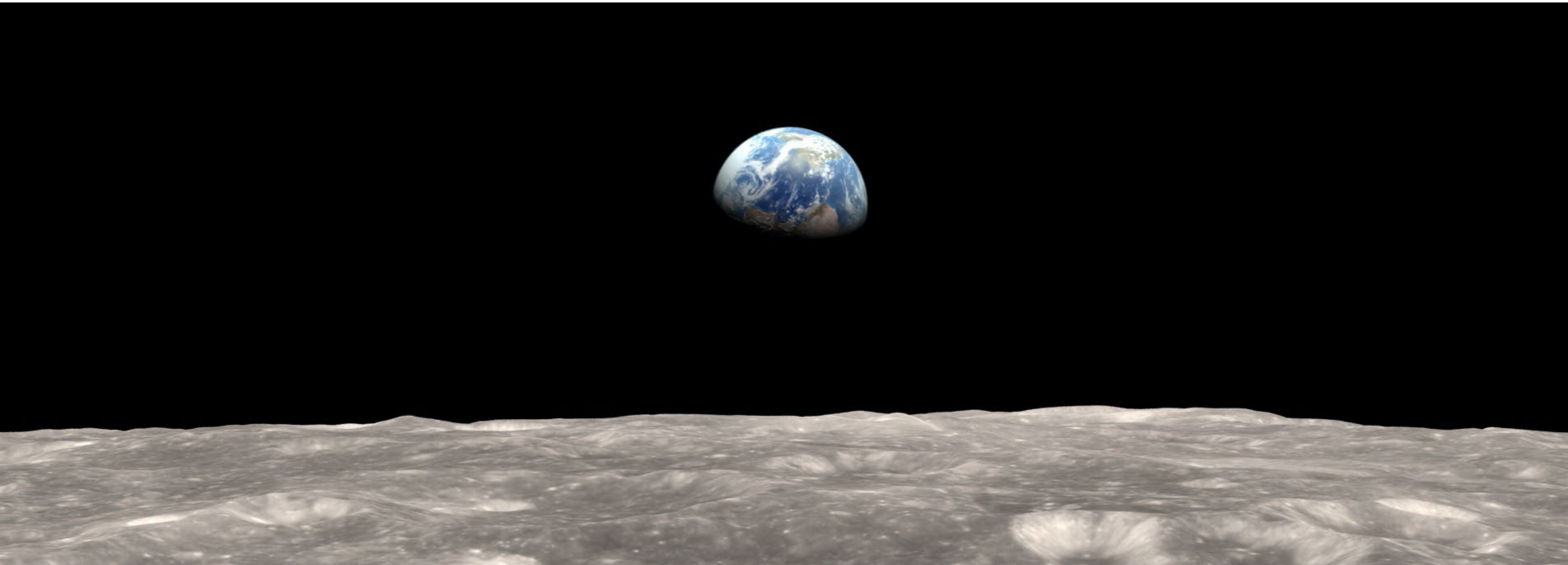
Example 1: maximise power available from body-mounted solar arrays



Performance characteristic (2)

Choose Design Parameters that maximise (or minimise) some performance characteristic Q whilst also minimising the variability in performance due to Noise Factors

Example 2: minimise delta-v for lunar orbit insertion



Design of Experiment

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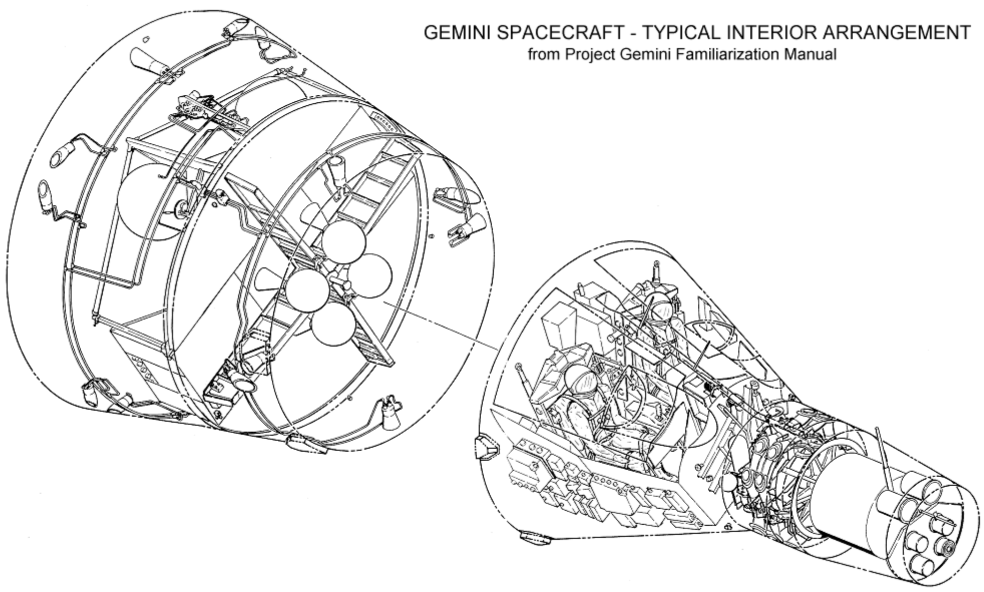
Design of Experiments (1)

We have m design parameters at s levels (i.e. the number of different values or options considered for the design parameters)

Ideal number of levels ≥ 3 to account for non-linearities

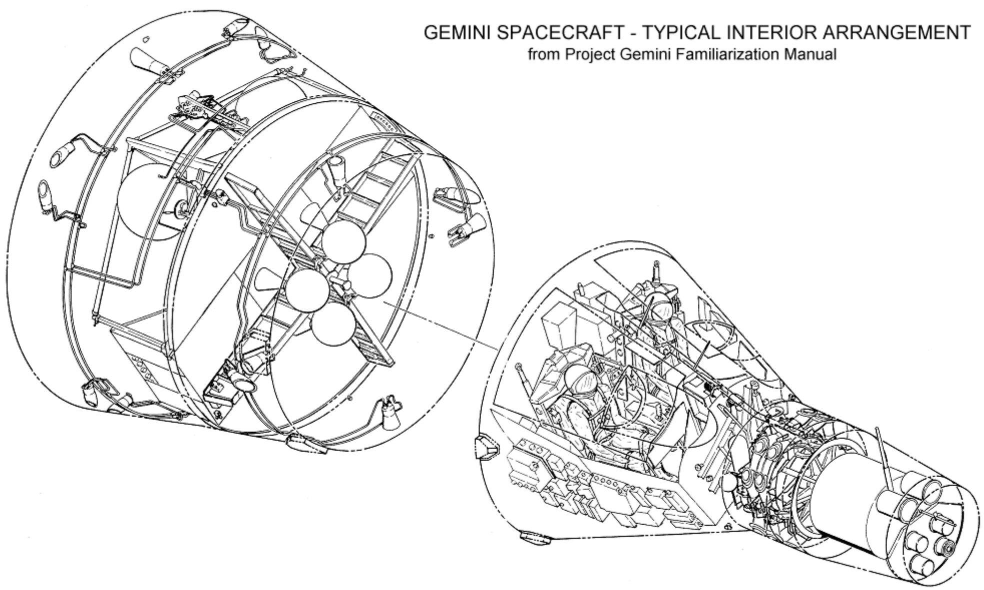
The number of possible test cases $= s^m$



Design of Experiments (2)

Ideally, all possible combinations of design parameters and noise factors would be investigated, allowing the selection of the best design parameters.

Cost and schedule constraints often prevent the analysis of all possible test cases → so we use methods to reduce count



Design of Experiments (3)

DoE is an information gathering exercise: a structured method for determining the relationship between process inputs and process outputs.

The aim is to choose what information to gather so that the relationship between inputs and outputs can be determined with the least amount of effort.

- One experiment/test = one set of design parameter and noise factor combinations

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Degrees of Freedom (1)

The system degrees of freedom (DOF):

$$\mathrm{DOF} = 1 + m(s - 1)$$

minimum number of experiment to relate design parameter to performance characteristic

Where:

- m = number of design parameters
- s = number of levels (i.e. the number of different values/options considered for the design parameters)

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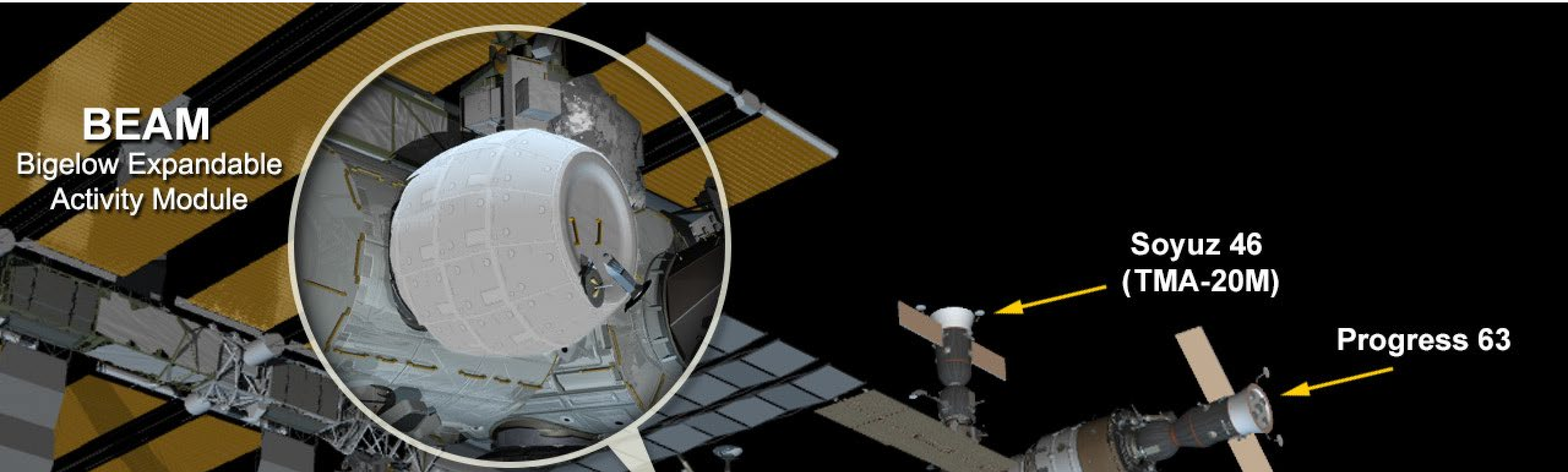
Degrees of Freedom (2)

Variable levels:

Example:

- Three design parameters for an inflatable module for the ISS:
  - Number of layers in the module wall: 1, 2 or 3
  - Density of material: low, moderate or high
  - Number of structural supports per 10 square metres: 1, 3 or 5



Degrees of Freedom (3)

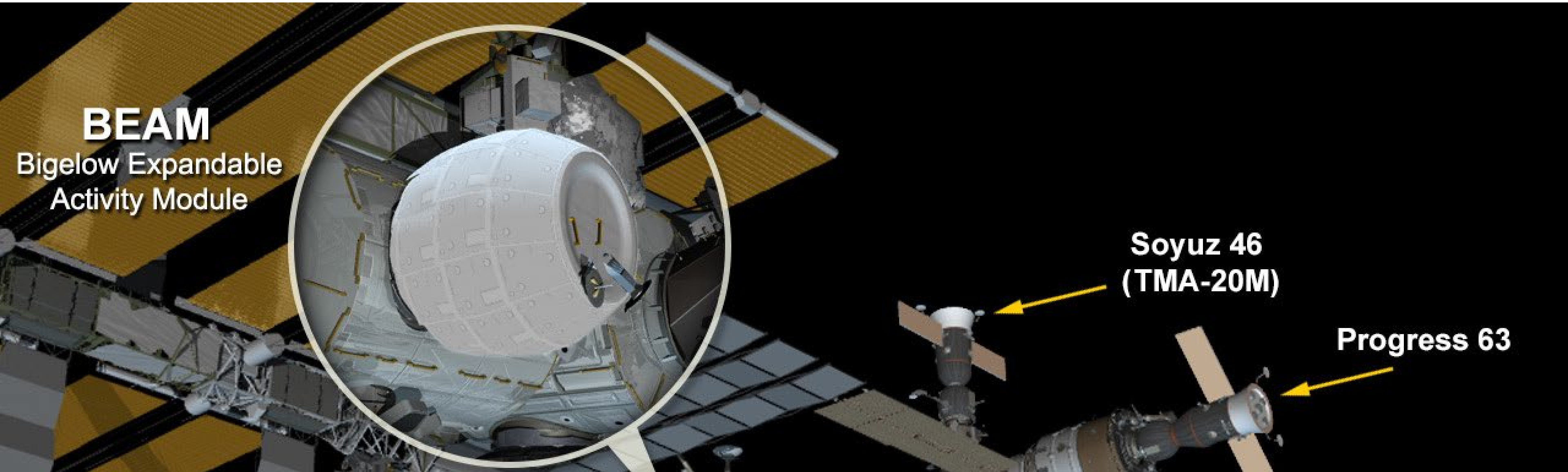
Variable levels:

Example:

- Three design parameters ($m = 3$) each with three levels ($s = 3$):

$$\text{DOF} = 1 + 3(3-1) = 7$$

instead of $3^3 \to 27$



Degrees of Freedom (4)

Variable levels:

Example:

- Three design parameters for the wheels of a Mars rover:
  - Wheel diameter: 20, 25, 30, or 35 cm
  - Wheel material: Titanium, steel, carbon fibre or aluminium
  - Rim thickness: 3 mm, 5 mm, 7 mm or 9 mm

Degrees of Freedom (5)

Variable levels:

Example:

- Three design parameters ($m = 3$) each with four levels ($s = 4$):

$$\mathrm{DOF} = 1 + 3(4 - 1) = 10$$

Degrees of Freedom (6)

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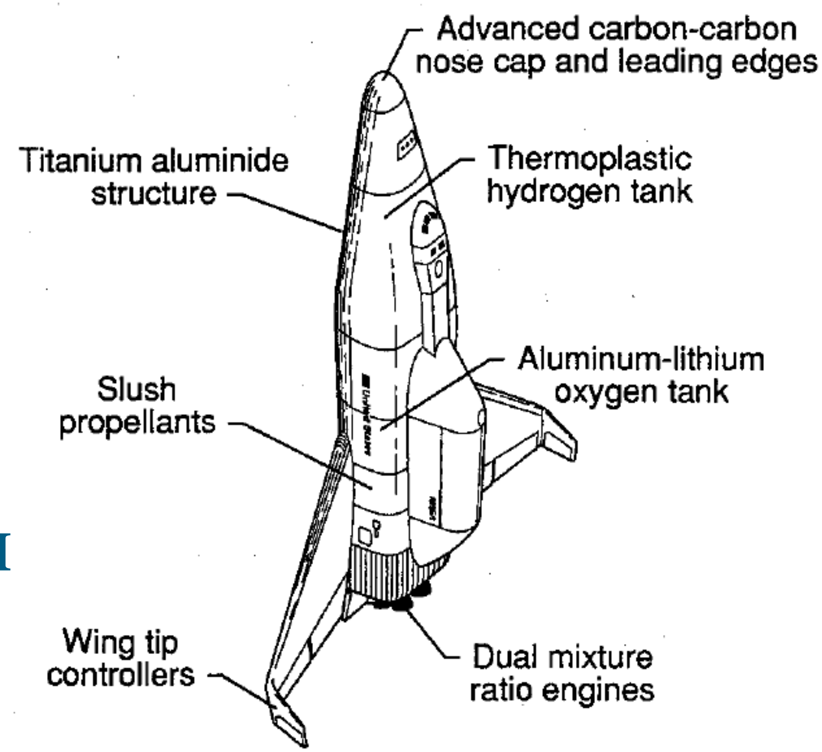
Variable levels:

Example:

- Propulsion system design for a SSTO vehicle:
  - O2/H2 mixture ratio: L, M, H
  - Area ratio: L, M, H
  - Chamber pressure: L, M, H
  - Mach number at transition: L, M, H



Stanley et al., 1992. Journal of Spacecraft and Rockets, 29 (4), 453-459

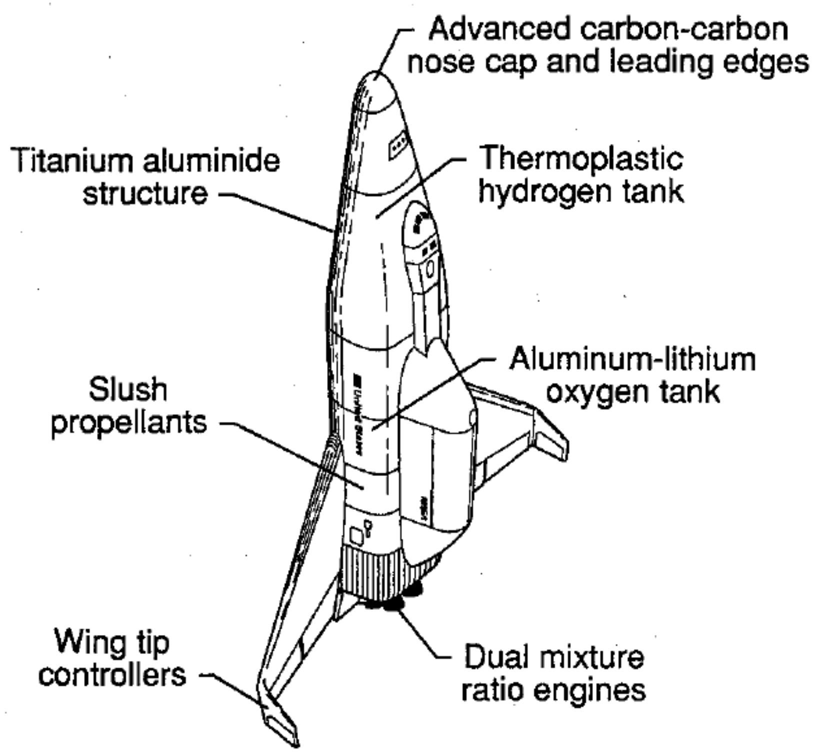
Degrees of Freedom (7)

Variable levels:

Example:

- Four design parameters ($m = 4$) each with three levels ($s = 3$):

$$\mathrm{DOF} = 1 + 4(3 - 1) = 9$$



Design of Experiments (4)

The Taguchi method uses **orthogonal matrices** from Design of Experiments theory to explore the parameter space with a significantly small number of experiments

- The orthogonal matrices provide a method for selecting a smaller subset of the parameter space
- The number of experiments (i.e. combinations of design parameters and noise factors)
  - **Must be ≥ DOF**

Orthogonal matrices (1)

- A fixed-element orthogonal array of s elements
- An $N \times m$ matrix:
  - Columns refer to the design parameters
  - Rows correspond to the test s of the design parameters
- In every pair of columns each of the possible ordered pairs of elements appears the same number of times
- Taguchi refers to the matrices as $L_N(s^m)$

Orthogonal matrices (2)

Notes

Here we have assumed that every design parameter has the same number of levels

- Not always the case
- Mixed-element orthogonal matrices
  - A matrix of N rows and $m + n$ columns
  - The first m columns have s elements each
  - The next n columns have t elements each
- Not considered here

Design of Experiments (5)

Example 1:

Three design parameters (A, B, C) at two different levels (L = Low, H = High)

$$m = 3 \quad s = 2$$

Design parameter names	Design parameter levels	
	L	H
A	A_L	A_H
B	B_L	B_H
C	C_L	C_H

Design of Experiments (6)

Example 1: L4 (2^3) orthogonal matrix

Experiment number	Design parameters		
	A	B	C
1	*L*	*L*	*L*
2	*L*	*H*	*H*
3	*H*	*L*	*H*
4	*H*	*H*	*L*

https://www.york.ac.uk/depts/maths/tables/orthogonal.htm

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Design of Experiments (7)

Example 2:

Four design parameters (A, B, C, D) at three different levels (L = Low, M = Medium, H = High)

$$m = 4 \quad s = 3$$

Design parameter names	Design parameter levels		
	L	M	H
A	A_L	A_M	A_H
B	B_L	B_M	B_H
C	C_L	C_M	C_H
D	D_L	D_M	D_H

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Design of Experiments (8)

Example 2: L9 (3^4) orthogonal matrix

Experiment number	Design parameters			
	A	B	C	D
1	*L*	*L*	*L*	*L*
2	*L*	*M*	*M*	*M*
3	*L*	*H*	*H*	*H*
4	*M*	*L*	*M*	*H*
5	*M*	*M*	*H*	*L*
6	*M*	*H*	*L*	*M*
7	*H*	*L*	*H*	*M*
8	*H*	*M*	*L*	*H*
9	*H*	*H*	*M*	*L*

https://www.york.ac.uk/depts/maths/tables/orthogonal.htm

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Noise Factors

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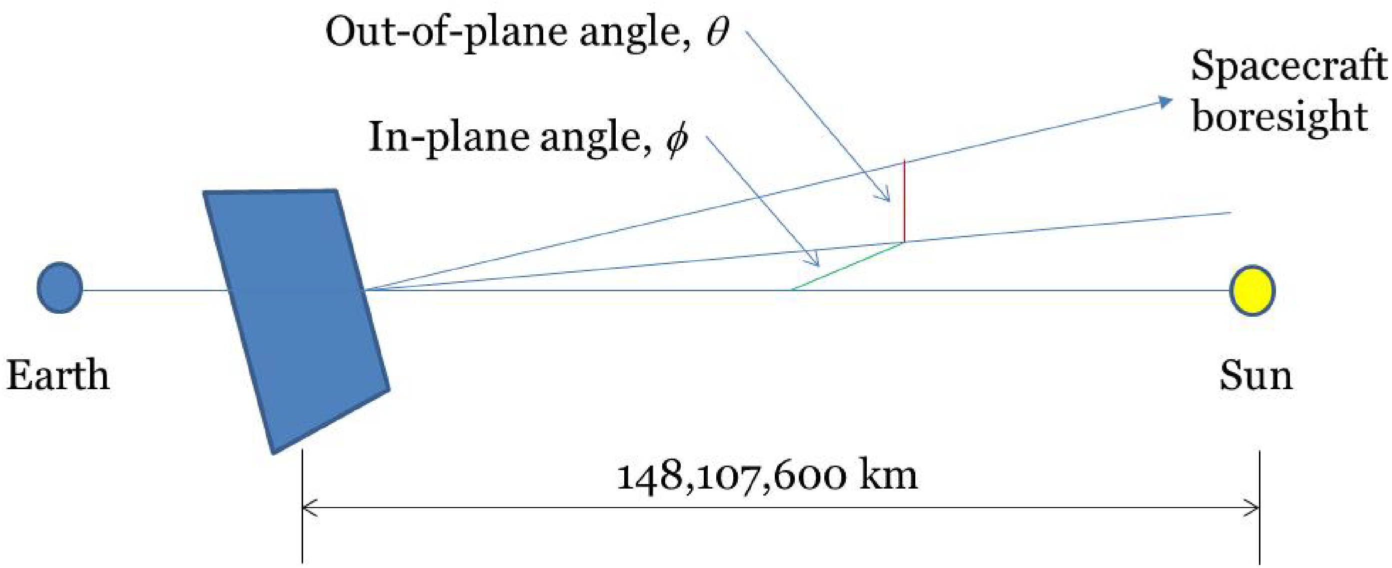
Inner & outer arrays (1)

Don't forget: there are noise factors too

experiment design can also consider these! To get useful data.

Coursework example: three noise factors at two levels

- In-plane angle: 0° and 15°
- Out-of-plane angle: 0° and 15°
- Solar array degradation factor: 0.1 and 0.3



Inner & outer arrays (2)

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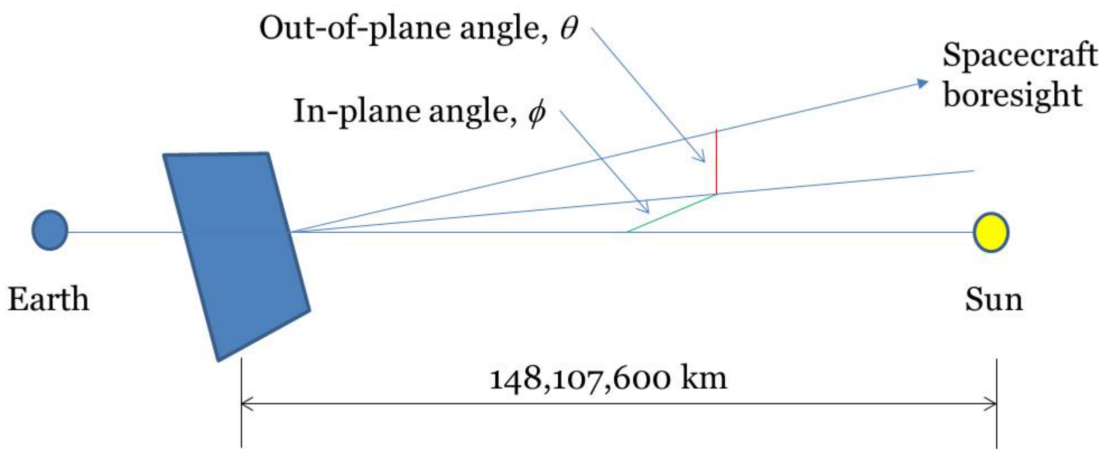
Coursework example: three noise factors at two levels

$$m = 3 \quad s = 2$$

Noise factor names	Noise factor levels	
	L	H
θ	0°	15°
ϕ	0°	15°
D	0.1	0.3



Inner & outer arrays (3)

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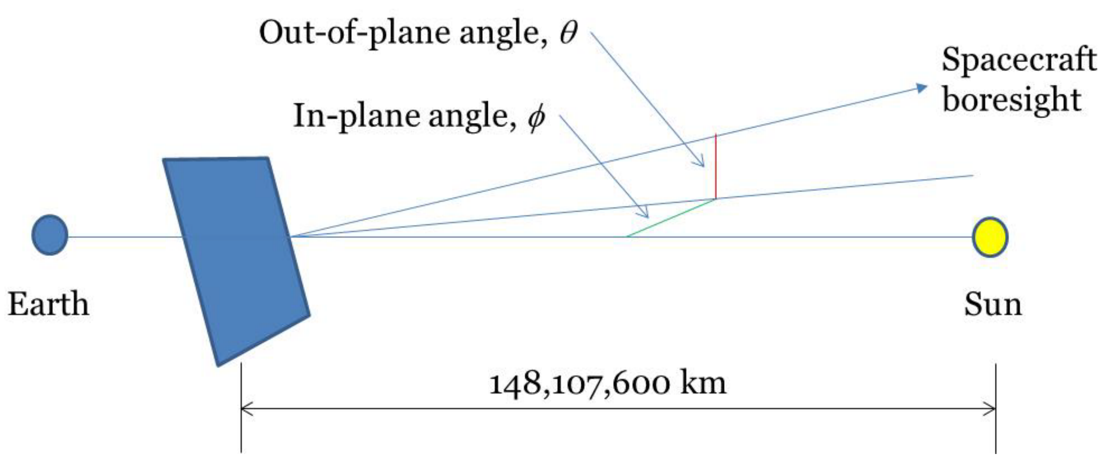
Coursework example: L4 (2^3) orthogonal matrix

Experiment number	Noise factors		
	θ	ϕ	D
1	0°	0°	0.1
2	0°	15°	0.3
3	15°	0°	0.3
4	15°	15°	0.1



Inner & outer arrays (4)

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table rotated compared to normal layout.

Outer array:

$L_4\,(2^3)$ orthogonal noise factor array

N_3	1	2	2	1
N_2	1	2	1	2
N_1	1	1	2	2
	1	2	3	4

Inner array:

$L_9\,(3^4)$ orthogonal design parameter array

	X_1	X_2	X_3	X_4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

$$Q_{11} = f\,\{X_1(1), X_2(1), X_3(1), X_4(1), N_1(1), N_2(1), N_3(1)\}$$

new experiment.

$$Q_{52} = f\,\{X_1(2), X_2(2), X_3(3), X_4(1), N_1(1), N_2(2), N_3(2)\}$$

Purpose of tagachi method is reduce test count with minimal loss in information.

Inner & outer arrays (5)

Number of experiments:

Example:

- L9 (3^4) orthogonal design parameter matrix:
  - $s^m = 3^4 = 81$ possible combinations of DPs
  - 9 tests from DoE
- L4 (2^3) orthogonal noise factor matrix:
  - $s^m = 2^3 = 8$ possible combinations of NFs
  - 4 tests from DoE
- $81 \times 8 = 648$ possible combinations of NFs and DPs
- $9 \times 4 = 36$ tests from DoE

	1	2	3	4
N_3	1	2	2	1
N_2	1	2	1	2
N_1	1	1	2	2

	X_1	X_2	X_3	X_4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

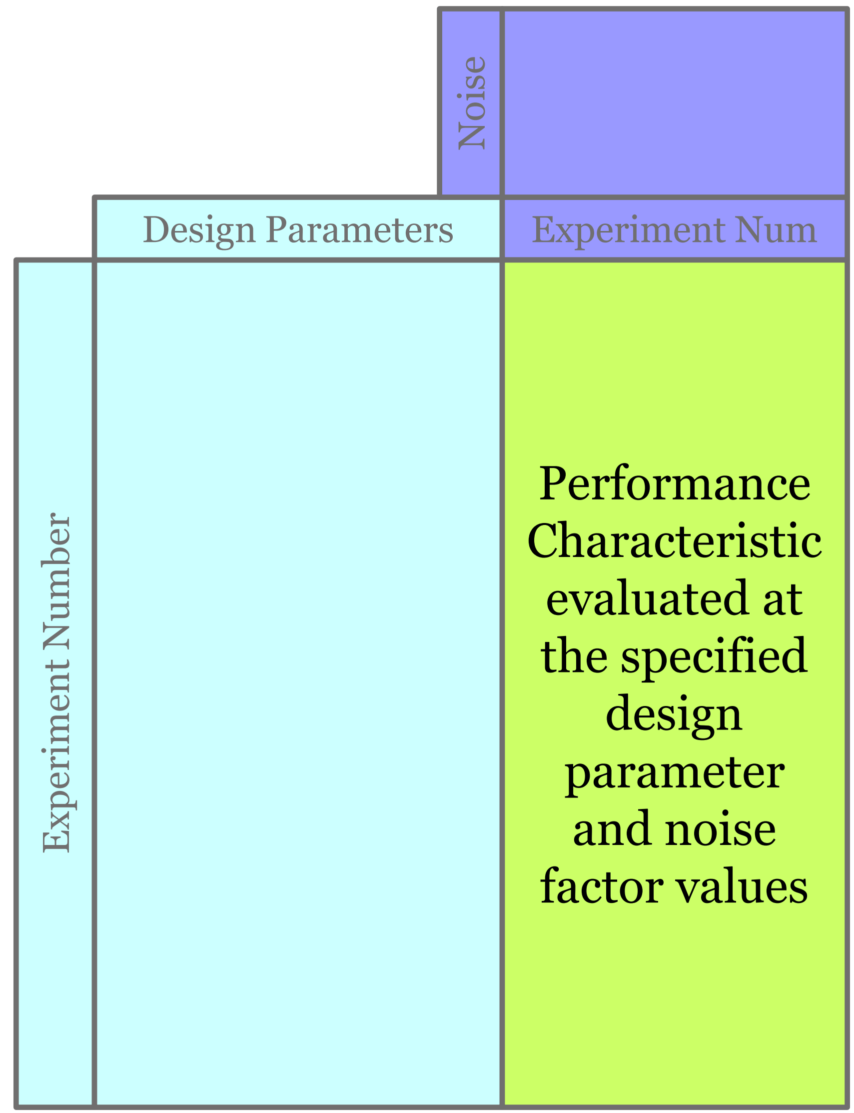
1,1 1,2 1,3 1,4
2,1 2,2 2,3 2,4
3,1 ... ...
... ...

28 tests

...

Inner & outer arrays (6)

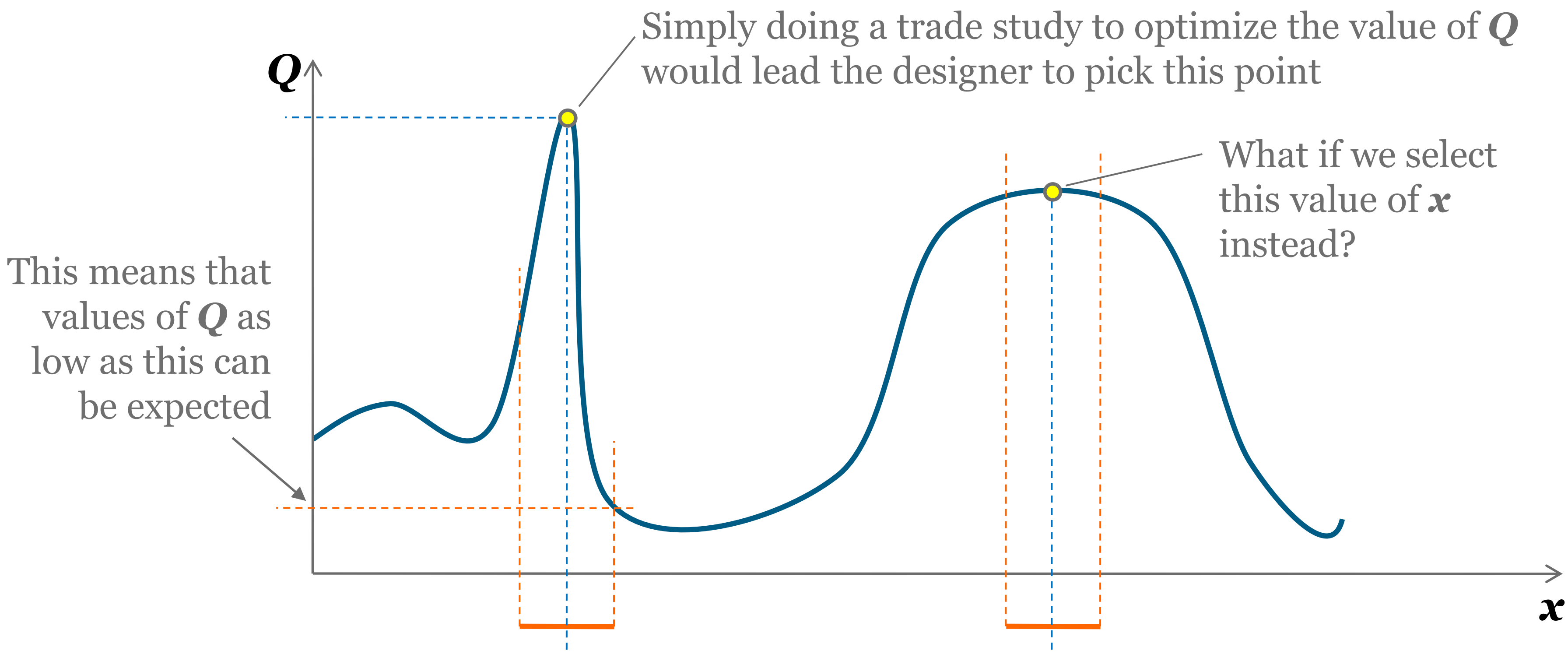
				N_3	1	2	2	1
				N_2	1	2	1	2
				N_1	1	1	2	2
	X_1	X_2	X_3	X_4	1	2	3	4
1	1	1	1	1				
2	1	2	2	2				
3	1	3	3	3				
4	2	1	2	3				
5	2	2	3	1				
6	2	3	1	2				
7	3	1	3	2				
8	3	2	1	3				
9	3	3	2	1				



- Inner Array – design parameter matrix
- Outer Array – noise factor matrix

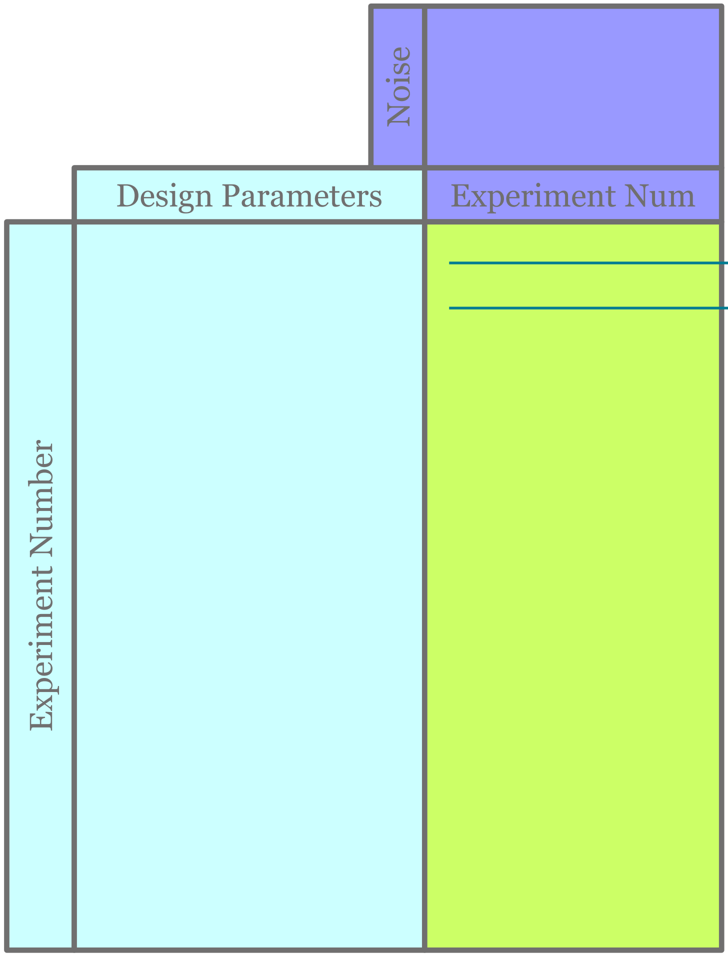
Recall: robust design

Select a value of $\boldsymbol{x}$ to maximise $\boldsymbol{Q}$ whilst also minimising variation



Robust design

Compute signal-to-noise ratio:



Compute signal-to-noise for each row i

$$S/N_i$$

Minimising performance characteristic:

$$S/N_i = -10\log_{10}\left(\frac{1}{NT}\sum_{j=1}^{NT} Q_{ij}{}^{2}\right)$$

Maximising performance characteristic:

$$S/N_i = -10\log_{10}\left(\frac{1}{NT}\sum_{j=1}^{NT} \frac{1}{Q_{ij}{}^{2}}\right)$$

where NT = number of trials

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Analysis of S/N (1)

Identify the instances of each design parameter at each level and compute the average of the corresponding S/N values:

Example 1: L4 (2^3) orthogonal matrix each experiment has a signal to noise ratio

Experiment number	Design parameters		
	A	B	C
1	*L*	*L*	*L*
2	*L*	*H*	*H*
3	*H*	*L*	*H*
4	*H*	*H*	*L*

Design parameter A is at level 1 (L) in experiments 1 and 2

$$\text{Avg.}\, S/N_{A(1)} = \frac{S/N_1 + S/N_2}{2}$$

Design parameter A is at level 2 (H) in experiments 3 and 4

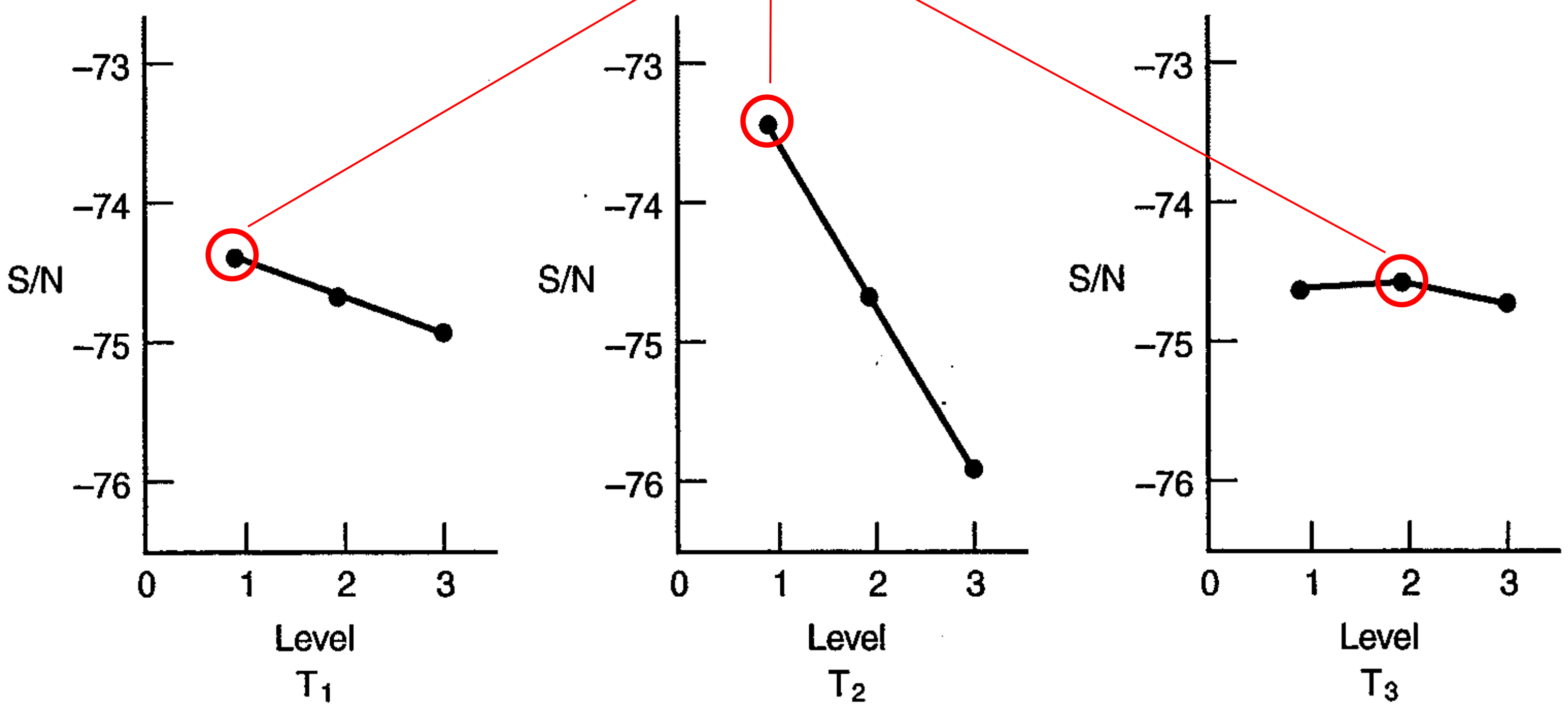
$$\text{Avg.}\, S/N_{A(2)} = \frac{S/N_3 + S/N_4}{2}$$

Analysis of S/N (2)

Plot the average S/N values for each design parameter:

ALWAYS aim to maximise S/N

In this example, these are the best cases.



Limitations of Taguchi method

- Inner and outer array structure assumes no interaction between design parameters and noise factors
- Only accounts for one performance characteristic
- Assumes continuous functions (in computation of performance characteristic)

Better Design of Experiments and analysis methods can address these limitations

Robust design & Taguchi method

- Some questions to reflect on:
  - What would happen if a the design of a spacecraft sub-system was based only on maximising the performance?
  - In the context of spacecraft systems design why is it important to minimise the variation in performance?
  - What is the difference between a design parameter and a noise factor?
  - Why is Design of Experiments needed?

Homework: reading

- On Blackboard:
  - Example: Stanley et al., 1992. Journal of Spacecraft and Rockets, 29 (4), 453-459