Useful formulae

Peak Time

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Percent Overshoot

$$PO = 100\exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

Settling Time

$$T = \frac{1}{\zeta\omega_n}, \qquad T_s = 4T$$

Second-Order Transfer Function

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Steady-State Error

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\,R(s)\left(1 - \frac{C(s)}{R(s)}\right) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

PID Controller

$$G_c = K_p + \frac{K_i}{s} + K_d s = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Nyquist Stability Criterion

$$N = P - Z$$

Eigenvalues

$$\det(A - \lambda I) = 0$$

Root Locus Rules:

Rule 4 – the number of asymptotes is the number of OL poles minus the number of OL zeros

Rule 5 – the angle of the asymptotes is given by $\alpha = \frac{\pm(2i+1)180}{n-m}$
Rule 6 – the intersection of the asymptotes with the real axis is given by $\rho_0 = \frac{(sum\ of\ OL\ poles)-(sum\ of\ OL\ zeros)}{n-m}$
Rule 7 – loci are symmetrical with respect to the real axis
Rule 8 – sections of the real axis that are to the left of an odd total number of OL poles and zeros on this axis form part of the loci
Rule 9 – points of breakaway or arrival at the real axis may exist.

Routh-Hurwitz:

For a generic characteristic equation of the form:

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

The Routh array is constructed as follows:

$$\begin{array}{cccccc}
s^n & a_n & a_{n-2} & a_{n-4} & \cdots & 0 \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots & 0 \\
s^{n-2} & b_1 & b_2 & b_3 & \cdots & 0 \\
s^{n-3} & c_1 & c_2 & c_3 & \cdots & 0 \\
\vdots & & & & & \\
s^1 & g_1 & 0 & & & \\
s^0 & h_1 & 0 & & &
\end{array}$$

$$b_1 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} = \frac{1}{a_{n-1}}(a_{n-1}a_{n-2} - a_n a_{n-3})$$

$$b_2 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix} = \frac{1}{a_{n-1}}(a_{n-1}a_{n-4} - a_n a_{n-5})$$

$$b_3 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix} = \frac{1}{a_{n-1}}(a_{n-1}a_{n-6} - a_n a_{n-7})$$

$$b_1 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix} = \frac{1}{b_1}(b_1 a_{n-3} - b_2 a_{n-1})$$

$$b_1 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix} = \frac{1}{b_1}(b_1 a_{n-5} - b_3 a_{n-1})$$

Observability – system is fully observable if following matrix is full-rank:

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

Controllability – system is fully controllable if following matrix is full-rank:

$$C = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

Determinant of a 3-by-3 matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + g\begin{vmatrix} b & c \\ e & f \end{vmatrix}$$

$$|A| = a(ei - fh) - d(bi - ch) + g(bf - ce)$$