UNIVERSITY OF SOUTHAMPTON SESA3029W1

SEMESTER 1 EXAMINATIONS 2018-19

TITLE: Aerothermodynamics

DURATION: 120 MINS

This paper contains **FIVE** Questions

Answer **ALL** questions on this paper. Question 1 is worth 36 marks and questions 2-5 are worth 16, 16, 12, and 20 marks, respectively.

An outline marking scheme is shown in brackets to the right of each question.

Isentropic flow **and** normal shock tables (11 sides) are provided. (In reading from tables, nearest values are acceptable unless explicitly stated otherwise.)

An oblique shock chart is provided.

Note that a formula sheet is provided at the end of this paper

Only University approved calculators may be used.

A foreign language direct 'Word to Word' translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

Unless otherwise stated, the working fluid should be taken as air with R=287 J/(kg K), c_p=1005 J/(kg K), γ=1.4, Pr=0.7, ρ=1.225 kg/m^3 and μ=1.79x10^{-5} Ns/m^2. 1bar=10^5 Nm^{-2}.

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Q.1

Figure Q.1(a) shows a planar converging-diverging nozzle exhausting into the atmosphere. The upstream stagnation pressure and temperature are p_0=600 kN/m^2 and T_0=1200 K respectively. The nozzle area ratio is 2.4 and the pressure of the surrounding atmosphere is p_{atm}=100 kN/m^2. At the exit a shock is formed with angle β and the jet edge is deflected inwards by an angle Δ relative to the nozzle wall, as shown.

(i) Find the static pressure and temperature in the throat (p_{throat}, T_{throat}) and the static pressure and temperature at the exit (p_{exit}, T_{exit}).

[6 marks]

(ii) Find the exit shock angle β and the jet edge angle Δ.

[6 marks]

(iii) Figure Q.1(a) only includes the first shock waves formed after the jet exit. State whether this is an over- or under-expanded configuration. Then, without doing any calculations, sketch two ways the pattern might continue downstream.

[6 marks]

Figure Q.1(b) shows a regular diamond-shaped aerofoil, with chord c=1 m and a leading-edge included semi-angle of 5°. It is placed at an incidence of α=5° in an M_∞=1.9 stream with upstream static pressure p_∞=70 kN/m^2.

(iv) Using the shock-expansion method, find the pressures on all the faces of the diamond and the resulting forces F_x and F_y as defined in the figure.

[15 marks]

(v) Sketch the behaviour of C_L vs M_∞ for a thin aerofoil, including comparisons with theoretical results in the subsonic and supersonic regimes.

[3 marks]

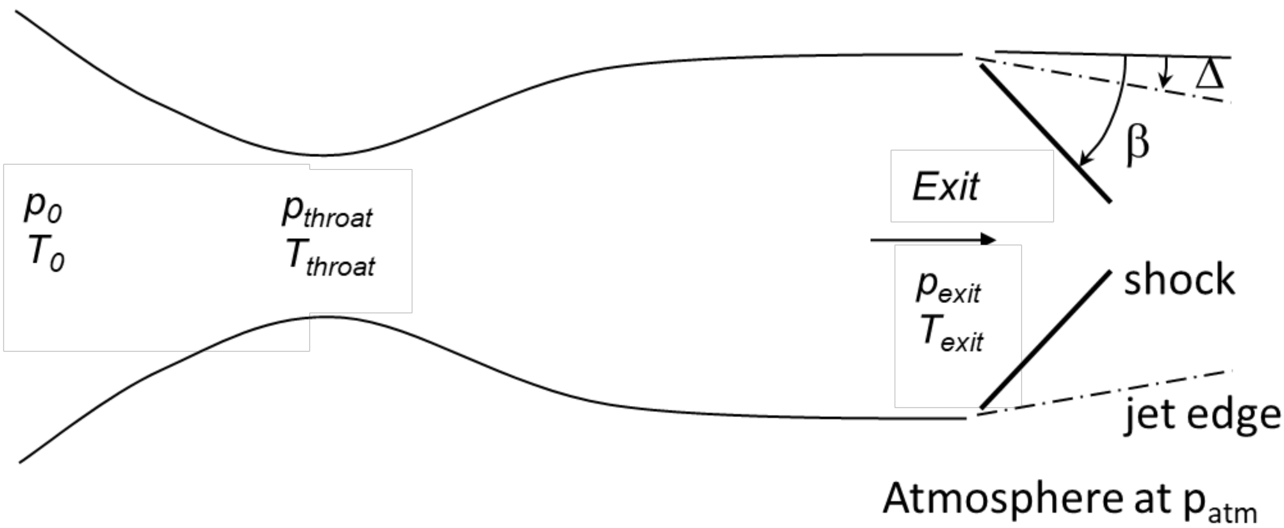

Figure Q.1(a)

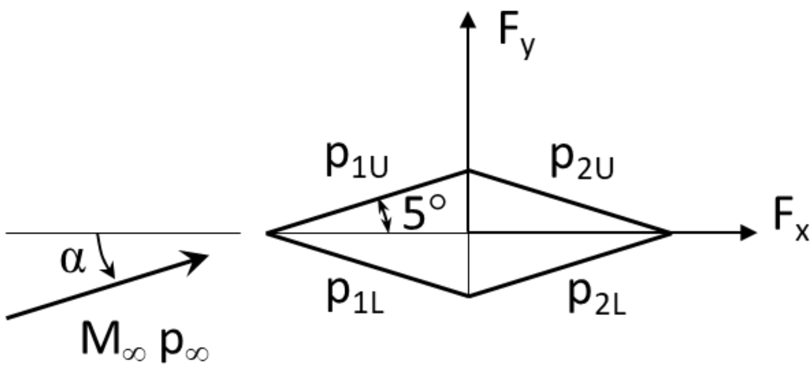

Figure Q.1(b)

TURN OVER

Q.2

(i) Starting from the compressible potential flow equation

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

derive Ackeret's formula

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

As part of the derivation, you should demonstrate that a constant velocity potential along lines with constant $\eta = x-\lambda y$ (where $\lambda=\sqrt{M_\infty^2-1}$) satisfies the governing equation. (Include all steps in the analysis. You can assume without proof that $C_p=-2u/U_\infty$ for small disturbances.)

[10 marks]

(ii) For the diamond-shaped aerofoil sketched in Figure Q.1(b) with M_∞=1.9 and p_∞=70 kN/m^2, use Ackeret's theory to determine the static pressure on each of the four faces (i.e. p_{1L},p_{1U},p_{2L} and p_{2U}).

[6 marks]

Q.3

(i) Figure Q.3 below shows four characteristic lines in a 2D convergent-divergent nozzle, designed to accelerate air to an exit Mach number M_e=1.8. Show that

$$\theta_{\max} = \frac{\nu(M_e)}{2}$$

where ν is the Prandtl-Meyer function.

[4 marks]

(ii) Complete the following table for a Method of Characteristics (MoC) calculation of a minimum length nozzle, based on Figure Q.3 using two characteristics. The design Mach number is M_e=1.8 at point 5, starting from characteristic lines 0-2 at a flow angle of θ_{max} and 0-1 at a flow angle of $0.5\theta_{max}$. All quantities have their usual notation.

[6 marks]

Point	R^+	R^-	θ	ν	M
1					
2					
3					
4					
5					1.8

(iii) Considering that point 0 is at the co-ordinates (x=0, y=1) and the centreline is at y=0, compute the (x, y) co-ordinates of points 1 and 2.

[6 marks]

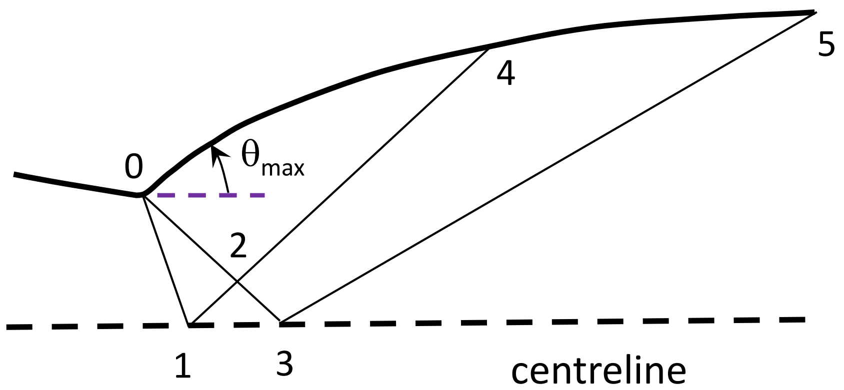

Figure Q.3

TURN OVER

Q.4

(i) Define Nusselt number, Reynolds number and Prandtl number and specify how they are related in Reynolds analogy

[4 marks]

(ii) Experimental tests using air as the working fluid are conducted on a portion of the turbine blade shown in the figure below. The heat flux to the blade at a particular point, given in non-dimensional coordinates *x**, on the surface is measured to be $\dot{q} = 95000\ \mathrm{W/m^2}$. To maintain a steady-state surface temperature of $T_s = 800^\circ\mathrm{C}$ heat transferred to the blade is removed by circulating a coolant inside the blade.

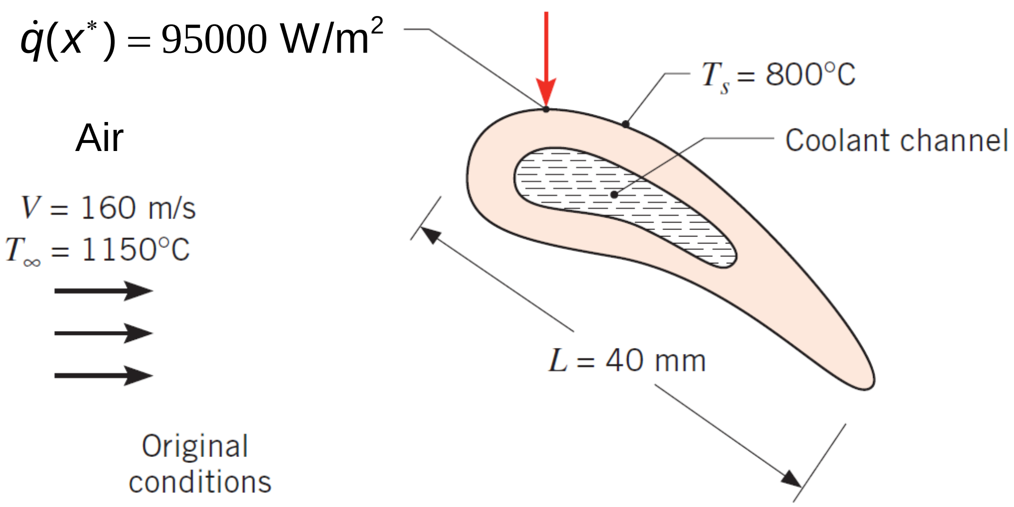

Figure Q.4

Determine the heat flux to the blade at *x** if its temperature is reduced to $T_s = 700^\circ\mathrm{C}$.

[3 marks]

(iii) Determine the heat flux at the non-dimensional location *x** for a similar turbine blade having a chord length of *L*=80mm, when the blade operates in an airflow at $T_\infty = 1150^\circ\mathrm{C}$ and $V = 80\ \mathrm{m/s}$, again with $T_s = 800^\circ\mathrm{C}$. Make use of Reynolds analogy to solve this problem.

[5 marks]

Q.5

The Python computer code given overleaf simulates the flow in an asymmetric convergent-divergent nozzle with MacCormack's finite difference method. The flow is treated as inviscid and quasi-one-dimensional. Quantities used in the program are normalised with conditions in the stagnation point (indexed by 0), the nozzle length L and the area of the throat A^* as

$$x^\dagger = \frac{x}{L},\quad V^\dagger = \frac{V}{a_0},\quad \rho^\dagger = \frac{\rho}{\rho_0},\quad p^\dagger = \frac{p}{\rho_0 a_0^2},$$
$$T^\dagger = \frac{T}{T_0},\quad E^\dagger = \frac{E}{a_0^2},\quad t^\dagger = \frac{t a_0}{L},\quad A^\dagger = \frac{A}{A^*}.$$

(i) Show that under this normalisation the relations

$$\frac{p}{p_0} = \gamma p^\dagger = \rho^\dagger T^\dagger$$

hold true.

[4 marks]

(ii) Determine the values of p_b/p_0, with p_b denoting the pressure at the outlet, for which one expects
(a) supersonic outflow,
(b) subsonic flow throughout the entire nozzle,
(c) a shock wave in the divergent section.

Employ the isentropic-flow and the normal shock tables and work as accurately as possible, using linear interpolation to obtain intermediate values.

[10 marks]

(iii) Relate your results from (ii) to the program output (given overleaf after the code). Which flow situation is presently approximated and why? Explain the need for altering the outflow boundary condition to simulate other flow regimes according to (ii) and how it is implemented in the program. Make reference to line numbers where appropriate.

[6 marks]

TURN OVER

Python computer code for Q5 SESA3029W1

```
# Quasi-1D nozzle code
import numpy as np
import math as m

# Input data (or via function arguments)
gamma = 1.4
nstep = 10000
dt = 0.01
Cx = 0.2
p_p0 = 0.
pe = p_p0/gamma

# Define all arrays
Nx = 121
x, A, Afactor = np.zeros(Nx), np.zeros(Nx), np.zeros(Nx)
U, Up, F = np.zeros((Nx,3)), np.zeros((Nx,3)), np.zeros((Nx,3))
J, pA = np.zeros(Nx), np.zeros(Nx)

# Duct geometry
Lx = 3.0
dx = Lx/(Nx - 1.0)
x = np.arange(0, Lx+dx, dx)

# Initial condition
rho = 1.0
T = 1.0
V = 0.0
for i in range(0, Nx):
    if (x[i]<1.0):
        A[i] = 1.0 + 0.30*(x[i] - 1.0)**2
        Afactor[i] = 0.60*(x[i] - 1.0)/A[i]
    else:
        A[i] = 1.0 + 0.16*(x[i] - 1.0)**3
        Afactor[i] = 0.48*(x[i] - 1.0)**2/A[i]

    U[i][0] = rho*A[i]
    U[i][1] = U[i][0]*V
    U[i][2] = U[i][0]*(T/(gamma*(gamma - 1.0)) + 0.5*V**2)

# Main loop
for n in range(1,nstep):

    # Predictor
    for i in range(0, Nx):
        pA[i] = (gamma-1.0)*(U[i][2]-0.5*U[i][1]**2/U[i][0])
        F[i][0] = U[i][1]
        F[i][1] = U[i][1]**2/U[i][0] + pA[i]
        F[i][2] = (U[i][2]+pA[i])*U[i][1]/U[i][0]
        J[i] = pA[i]*Afactor[i]

    for i in range(0, Nx-1):
        Up[i] = U[i] + (-(F[i+1] - F[i])/dx)*dt  # Up=provisional update
        Up[i][1] = Up[i][1] + J[i]*dt

    Up[Nx-1] = U[Nx-1]  # use previous outflow solution to avoid nans

    # Corrector and full step, including shock capturing (S)
    for i in range(0, Nx):
        pA[i] = (gamma-1.0)*(Up[i][2]-0.5*Up[i][1]**2/Up[i][0])
        F[i][0] = Up[i][1]
        F[i][1] = Up[i][1]**2/Up[i][0] + pA[i]
        F[i][2] = (Up[i][2]+pA[i])*Up[i][1]/Up[i][0]
        J[i] = pA[i]*Afactor[i]

    for i in range(1, Nx-1):
        S = Cx*abs(pA[i+1]-2.0*pA[i]+pA[i-1])/(pA[i+1]+2.0*pA[i]+pA[i-1])
        U[i] = 0.5*(U[i] + Up[i] + (-(F[i] - F[i-1])/dx)*dt) + \
               S*(Up[i+1] - 2.0*Up[i] + Up[i-1])
        U[i][1] = U[i][1] + 0.5*J[i]*dt

```

Code continues on next page.

Python computer code for Q5 SESA3029W1

```
    # Inflow condition (isentropic flow from stagnation conditions)
    Vin = 2.0*U[1][1]/U[1][0] - U[2][1]/U[2][0]  # extrapolate velocity
    Tin = 1.0 - 0.5*(gamma - 1.0)*Vin**2
    rhoin = (1.0 + 0.5*(gamma - 1.0)*Vin**2/Tin)**(-1.0/(gamma - 1.0))

    U[0][0] = rhoin*A[0]
    U[0][1] = U[0][0]*Vin
    U[0][2] = U[0][0]*(Tin/(gamma*(gamma - 1.0)) + 0.5*Vin**2)

    # Outflow condition
    U[Nx-1] = 2*U[Nx-2] - U[Nx-3]
    if pe>0.0:
        U[Nx-1][2] = pe*A[Nx-1]/(gamma-1.0)+0.5*U[Nx-1][1]**2/U[Nx-1][0]

# Table output
for i in range(0, Nx):
    rhop = U[i][0] / A[i]
    Vp = U[i][1]/U[i][0]
    Tp = gamma * (gamma - 1.0) * (U[i][2] / U[i][0] - 0.5 * Vp*Vp);
    Mp = Vp/m.sqrt(Tp)
    pp = Tp*rhop
    print("%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f" % \
            (x[i],A[i],pp,rhop,Vp,Tp,Mp))
```

```
Output:
  0.0000   1.3000   0.8305   0.8758   0.5083   0.9483   0.5220
  0.0250   1.2852   0.8253   0.8719   0.5165   0.9466   0.5309
  0.0500   1.2708   0.8202   0.8680   0.5247   0.9449   0.5398
  0.0750   1.2567   0.8148   0.8639   0.5331   0.9432   0.5489
  0.1000   1.2430   0.8093   0.8597   0.5415   0.9413   0.5582
  0.1250   1.2297   0.8036   0.8554   0.5502   0.9395   0.5676
  0.1500   1.2167   0.7979   0.8511   0.5589   0.9375   0.5772
  0.1750   1.2042   0.7920   0.8465   0.5677   0.9355   0.5870
  ...
  0.9250   1.0017   0.5559   0.6575   0.8787   0.8456   0.9556
  0.9500   1.0008   0.5467   0.6497   0.8901   0.8415   0.9703
  0.9750   1.0002   0.5376   0.6419   0.9014   0.8375   0.9850
  1.0000   1.0000   0.5262   0.6322   0.9155   0.8324   1.0034
  1.0250   1.0000   0.5261   0.6321   0.9156   0.8323   1.0036
  1.0500   1.0000   0.5248   0.6310   0.9172   0.8317   1.0057
  1.0750   1.0001   0.5226   0.6291   0.9199   0.8307   1.0093
  1.1000   1.0002   0.5197   0.6266   0.9235   0.8294   1.0140
  ...
  2.6000   1.6554   0.1325   0.2361   1.4810   0.5613   1.9769
  2.6250   1.6866   0.1279   0.2302   1.4904   0.5557   1.9994
  2.6500   1.7187   0.1235   0.2245   1.4997   0.5501   2.0220
  2.6750   1.7519   0.1192   0.2189   1.5089   0.5446   2.0446
  2.7000   1.7861   0.1151   0.2135   1.5180   0.5391   2.0673
  2.7250   1.8213   0.1111   0.2081   1.5269   0.5337   2.0901
  2.7500   1.8575   0.1072   0.2029   1.5357   0.5283   2.1129
  2.7750   1.8948   0.1034   0.1978   1.5444   0.5229   2.1358
  2.8000   1.9331   0.0998   0.1928   1.5530   0.5176   2.1587
  2.8250   1.9725   0.0963   0.1879   1.5615   0.5123   2.1816
  2.8500   2.0131   0.0929   0.1831   1.5699   0.5071   2.2046
  2.8750   2.0547   0.0896   0.1785   1.5781   0.5019   2.2276
  2.9000   2.0974   0.0864   0.1739   1.5862   0.4968   2.2506
  2.9250   2.1413   0.0834   0.1695   1.5942   0.4917   2.2736
  2.9500   2.1864   0.0804   0.1652   1.6021   0.4866   2.2967
  2.9750   2.2326   0.0776   0.1610   1.6098   0.4817   2.3196
  3.0000   2.2800   0.0748   0.1569   1.6176   0.4766   2.3431
```

END OF PAPER (Formula sheet overleaf)

Perfect gas equation of state

$$p = \rho RT$$

Sound speed in a perfect gas

$$a^2 = \gamma RT$$

Adiabatic flow

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

Mach angle:

$$\sin \mu = \frac{1}{M}$$

Trigonometric relations for method of characteristics:

$$\alpha_{AP} = \frac{1}{2}\left[(\theta + \mu)_A + (\theta + \mu)_P\right]$$

$$\alpha_{BP} = \frac{1}{2}\left[(\theta - \mu)_B + (\theta - \mu)_P\right]$$

$$x_p = \frac{x_B \tan \alpha_{BP} - x_A \tan \alpha_{AP} + y_A - y_B}{\tan \alpha_{BP} - \tan \alpha_{AP}}$$

$$y_P = y_A + (x_P - x_A) \tan \alpha_{AP}$$

Velocity potential equation:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

Linearised pressure coefficient

$$C_p=-2\frac{u'}{U_\infty}$$

Prandtl-Glauert transformation

$$C_p=\frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

Ackeret formula:

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}}$$

Laminar pipe flow:

$$\mathrm{Nu}=4.364 \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}=3.658 \text{ (for uniform wall temperature)}$$

Laminar boundary layer:

$$\mathrm{Nu}_x=0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall heat flux)}$$

$$\mathrm{Nu}_x=0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \text{ (for uniform wall temperature)}$$

Turbulent pipe flow:

$$\mathrm{Nu}=0.022\,\mathrm{Pr}^{0.5}\,\mathrm{Re}^{0.8}$$

Turbulent boundary layer:

$$\mathrm{Nu}_x=0.029\mathrm{Re}_x^{0.8}\mathrm{Pr}^{0.6}$$