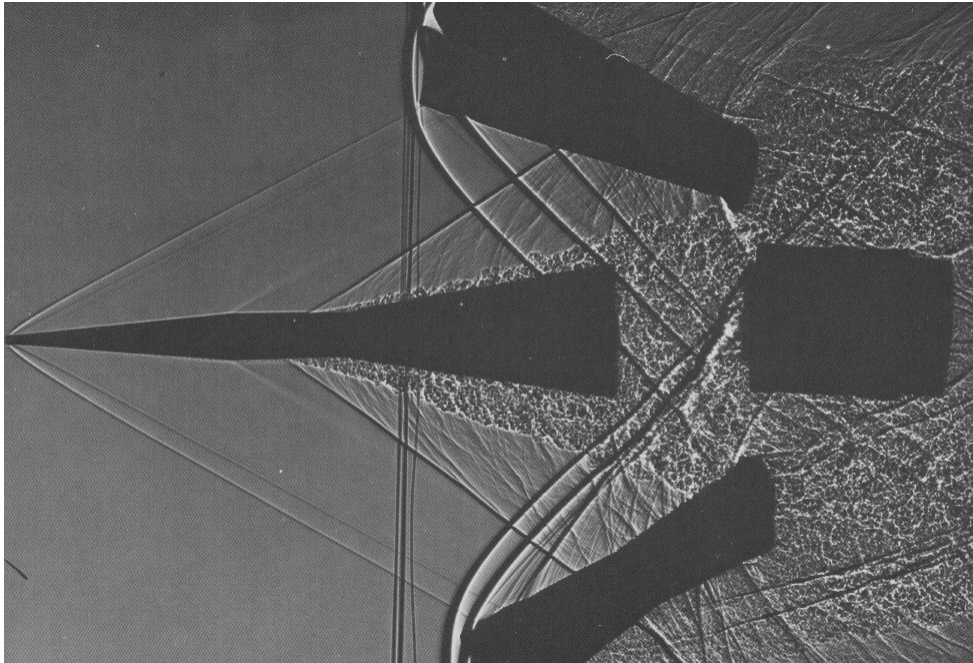


# SESA3029

## Aerothermodynamics

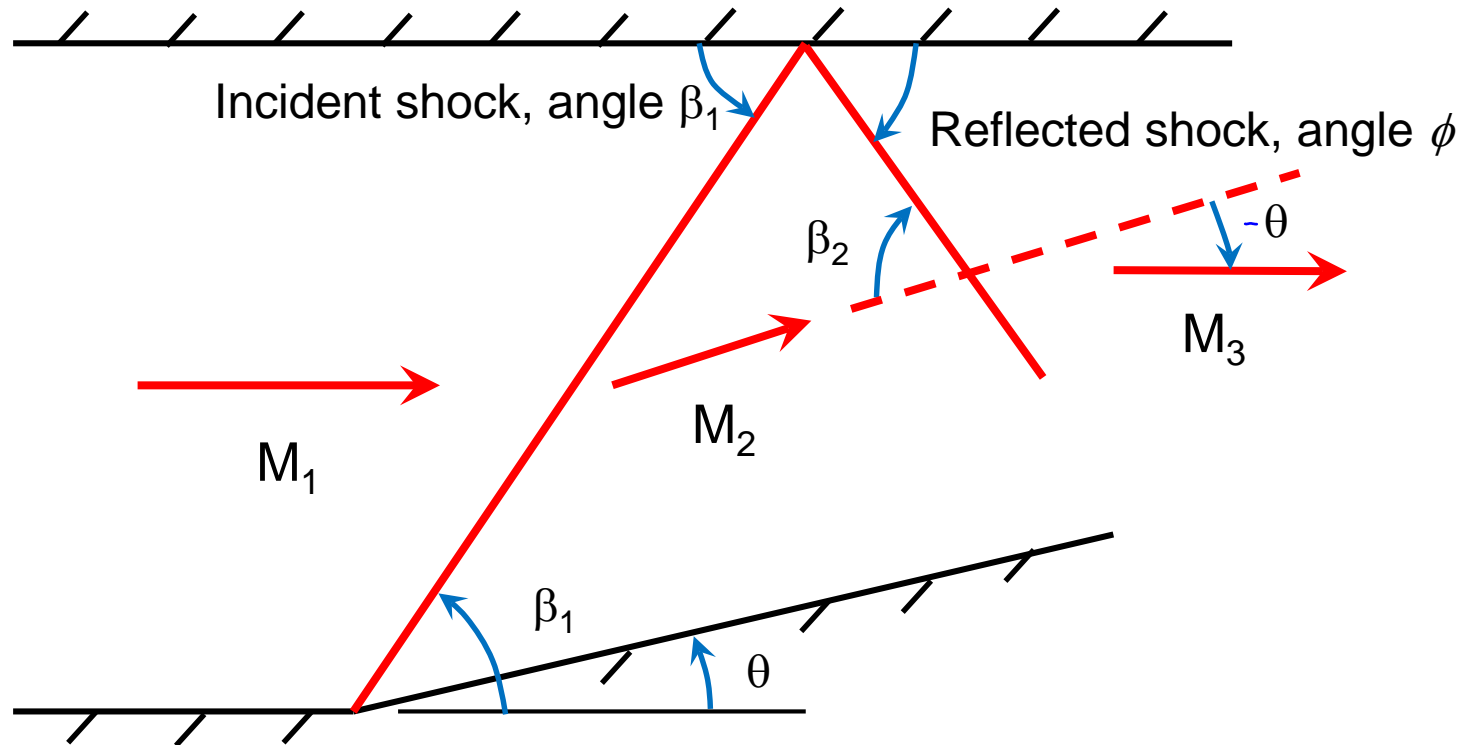


M=2 Separation of axisymmetric model from its support. Album of Fluid Motion.

### Lecture 2.3

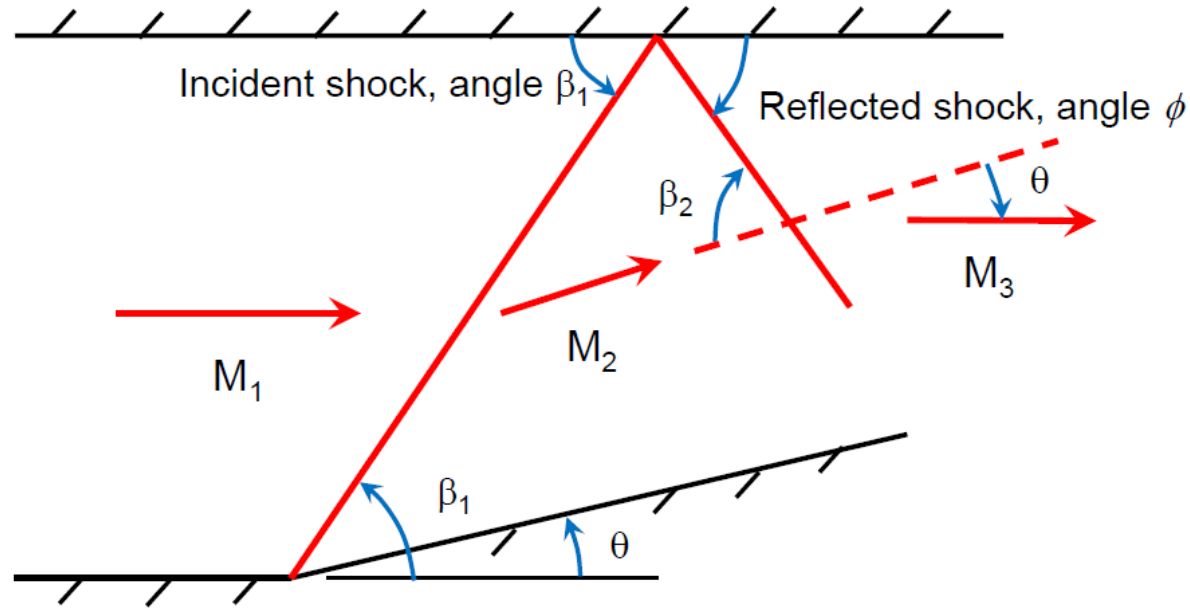
Shock interactions:  
regular reflections,  
Mach reflections and  
viscous effects

# Regular reflection



## Analysis of regular reflection

Example  $\theta=10^\circ$ ,  $M_1=3.6$ ,  $p_1=40$  kPa



1. From  $M_1$  and  $\theta$ , find  $\beta_1$  from shock calculator

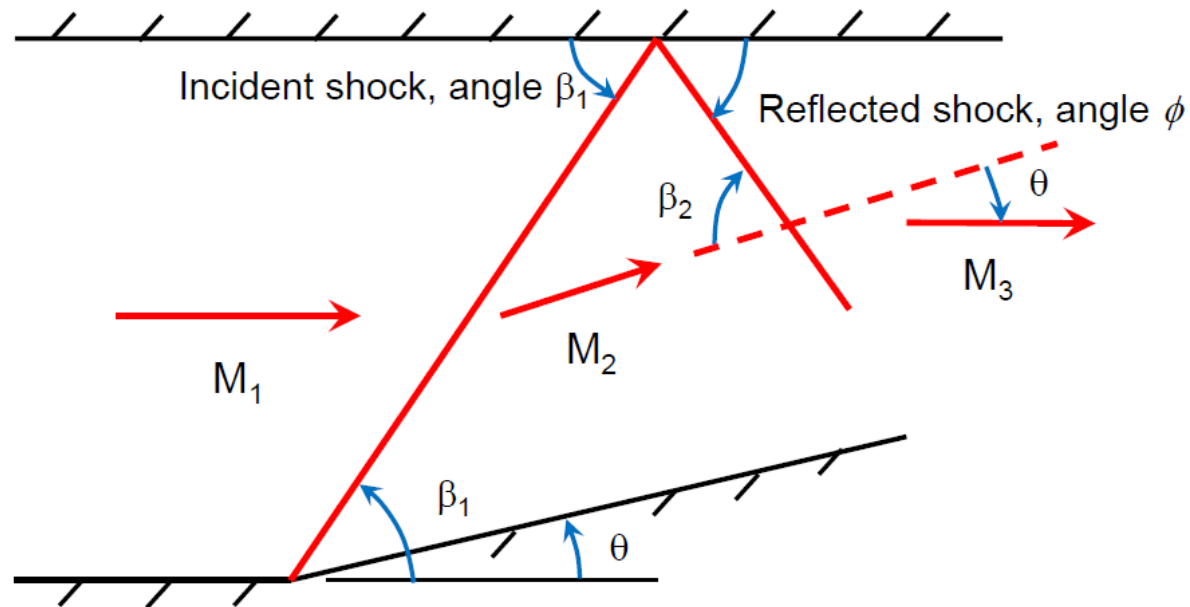
$$\beta_1=23.9^\circ$$

2. Set  $M_{n1}=M_1\sin\beta_1$  and use NST to get  $M_{n2}$ ,  $p_2/p_1$ , etc.

$$M_{n1}=1.459, M_{n2}=0.716$$
$$p_2/p_1=2.317$$

3. Find  $M_2=M_{n2}/\sin(\beta_1-\theta)$

$$M_2=2.98$$



4. Flow turning angle on upper boundary is also  $\theta$  (to bring streams parallel again). Hence, from  $M_2$  and  $\theta$ , find  $\beta_2$  from OSC

$$\beta_2 = 27.5^\circ$$

5.  $M_{n2}' = M_2 \sin \beta_2$

$$M_{n2}' = 1.376$$

6.  $M_{n3}'$  and  $p_3/p_2$  from NST

$$M_{n3}' = 0.750 \quad p_3/p_2 = 2.042$$

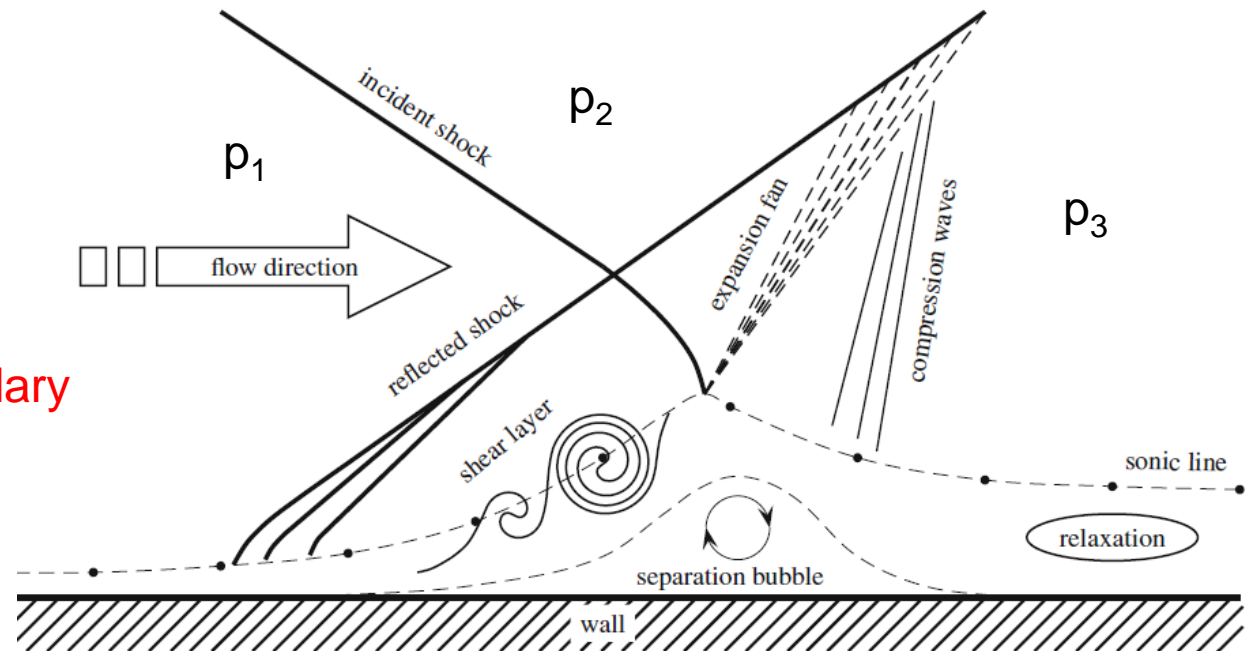
7.  $M_3 = M_{n3}' / \sin(\beta_2 - \theta)$

$$M_3 = 2.49, \quad p_3 = 189 \text{ kPa}$$

Note that  $\phi = \beta_2 - \theta = 17.5^\circ$  is not equal to  $\beta_1$  ( $24^\circ$ ), hence the reflection is not specular

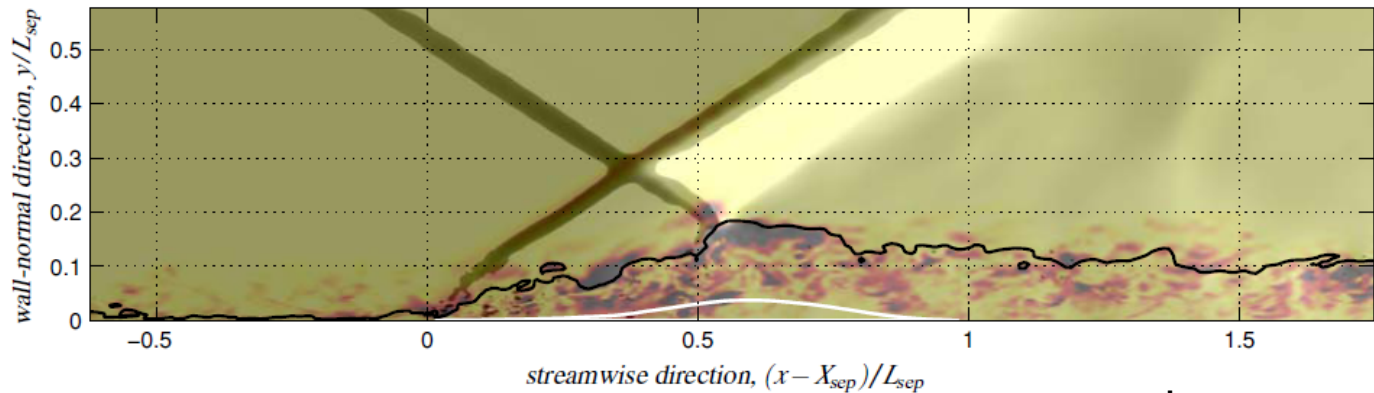
# Shock/boundary-layer interaction

Schematic:



Petrov formula: boundary  
layer separation if  
 $p_3/p_1 > 0.287 + 0.713M_1$

CFD  
(large eddy  
simulation)



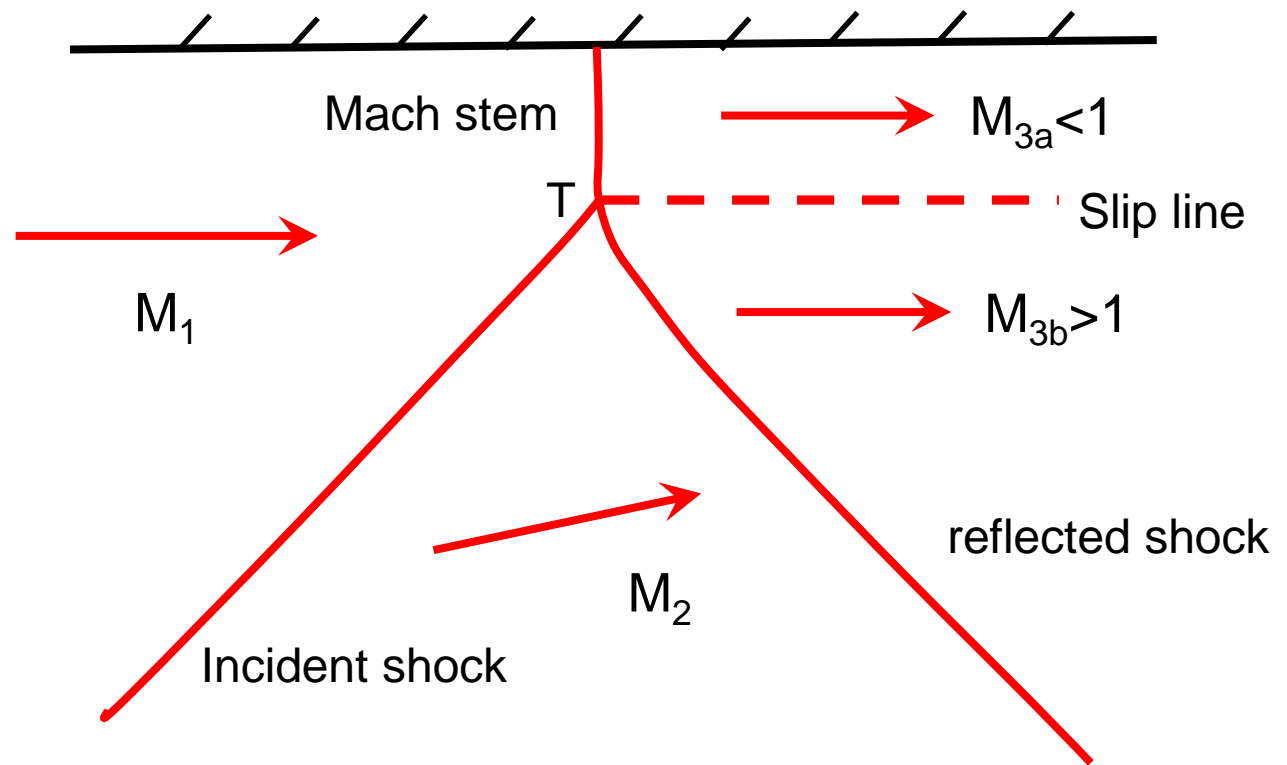
+movie,  $p_w$ ,  $c_f$  sketch

Returning to inviscid flow, if  $\theta > \theta_{\max}$  for  $M_1$  we know we get a detached shock system (last lecture)

Another possibility is if  $\theta < \theta_{\max}$  for  $M_1$  but  $\theta > \theta_{\max}$  for  $M_2$  i.e. if the reflected shock can't exist as an oblique shock solution

In this case we get what is known as a Mach reflection

# Mach reflection



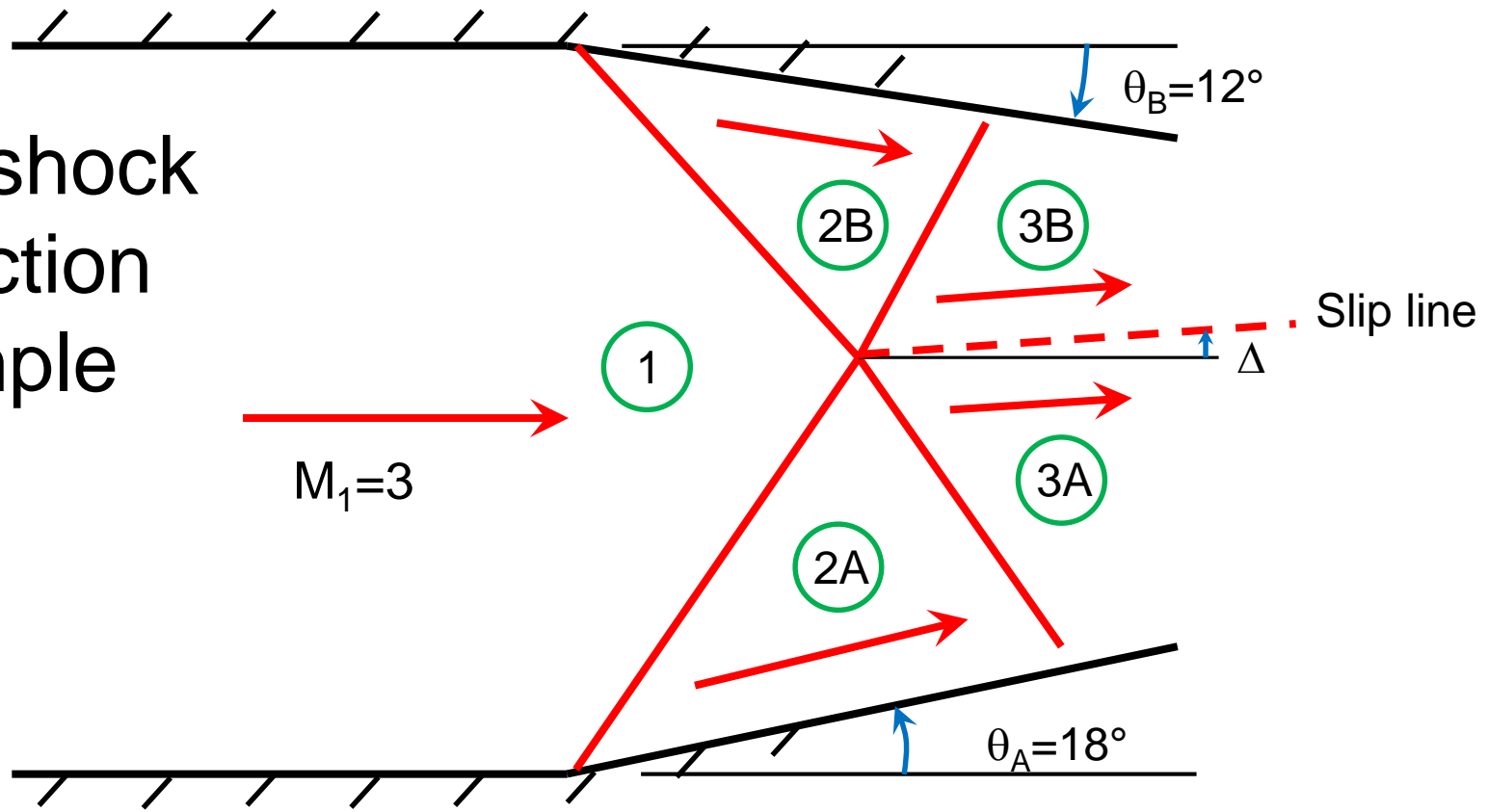
Curved shocks near the triple point T

Normal shock near wall, called Mach stem

Across a slip line (contact discontinuity) the pressure and flow direction must match, but the velocity, Mach number, temperature and density can be different

Solution is dependent on downstream conditions

# Shock-shock interaction example



Find flow conditions in 2A and 2B by usual oblique shock method

Conditions in 3A are not equal to 3B - in general we will need a slip line

Turning angle from 2A to 3A is  $\theta_A - \Delta$

Turning angle from 2B to 3B is  $\theta_B + \Delta$

Adjust  $\Delta$  until  $p_{3A} = p_{3B}$

**Solution  $\Delta = 5.85^\circ$**



# Aside: How does entropy vary across a shock wave?

Gibbs relationship:

$$Tds = dh - vdp$$

$$dh = c_p dT \quad pv = RT$$

Rearrange. Divide by T,  
perform substitutions

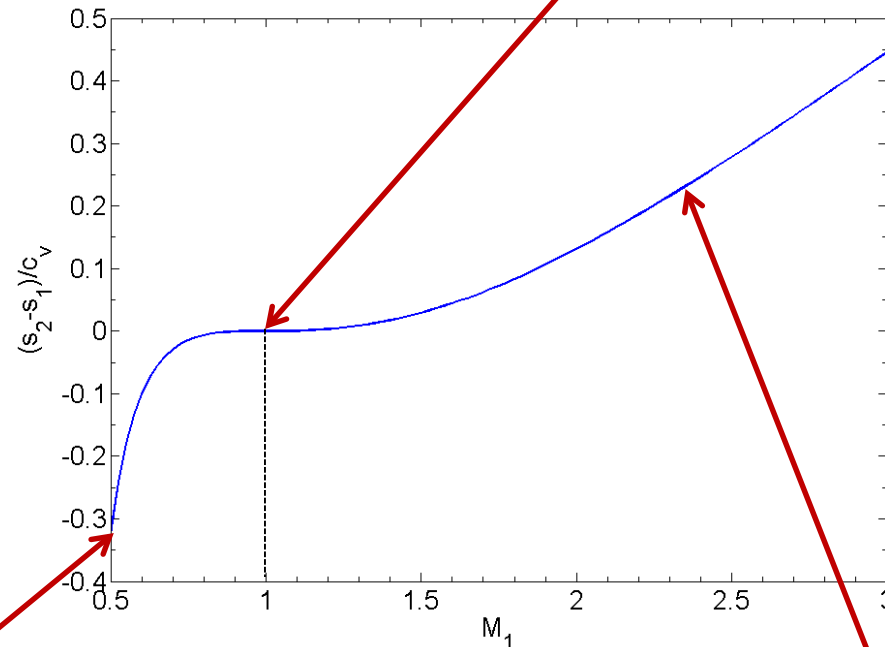
$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

Integrate across shock

$$s_2 - s_1 = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right)$$

# Plot entropy change vs. normal Mach number

Can show that for  $M_1 = 1$ ,  $\frac{ds}{dM_1} = 0$



For  $M_1 < 1$ ,  $s_2 - s_1 < 0$

unphysical

For  $M_1 > 1$ ,  $s_2 - s_1 > 0$

physical

# Deductions from the 2<sup>nd</sup> Law of Thermodynamics for shock waves

1)  $M_1$  cannot be less than 1

- Entropy cannot decrease in adiabatic flow
- Shocks cannot exist in subsonic flow

2) For  $M_1 > 1$ ,  $p_2/p_1 > 1$

- Shock waves are compressive
- No such thing as an expansion shock

3) Isentropic flow only valid for  $M_1$  very close to 1

- i.e. weak expansions and compressions (sound waves)

4) Entropy increase across shock waves is due to internal dissipation, which is irreversible