SESA2023 Week 6: Rockets

This week we look at rockets. We will start with some definitions for rocket performance and rocket nozzles, followed by staging, liquid and solid propellant rockets, and rocket power cycles.

6.1 Learning outcomes

After completing this section you should be able to:

- Explain how rockets fundamentally produce thrust and how to maximize thrust.
- Explain how rockets can use fuel in the most economical way.
- Understand and use the performance parameters for rocket propulsion systems.
- Understand and use the rocket equation to relate mass ratios to velocity changes.
- Calculate the ratio of payload to initial mass for a multi-stage launch vehicle.
- Explain the advantages and disadvantages of solid and liquid propellant rockets.
- Explain how the grain of a solid rocket propellant determines the thrust profile.
- Explain the working of four common rocket engine cycles for liquid fuel cycles, indicating the advantages and disadvantages.

6.2 Introduction

As we have seen in week 1, in the absence of a pressure contribution, the thrust of a rocket is the product of the mass flow rate of exhaust gases $\dot{m}$ with the velocity V_j out of the nozzle.

$$F = V_j \dot{m} \tag{6.1}$$

The main aim of a rocket engine is therefore to maximize the exhaust velocity. This way, the propellant is used in its most economical way.

Including the pressure contribution, the thrust of a rocket engine is given by

$$F = V_j \dot{m} + A_e(P_e - P_A) \tag{6.2}$$

with A_e the exit area of the nozzle, P_e the pressure of at the nozzle exit, and P_A the ambient pressure or back pressure. A schematic of a rocket nozzle is shown in figure 6.1. As we've seen in week 3, we can increase the exit velocity by increasing the stagnation pressure and temperature. In the schematic shown, this means maximizing the stagnation pressure P_{0c} and stagnation temperature T_{0c} in the combustion chamber.

Assuming the nozzle is adiabatic (although this is not generally true, as we will see later in section 6.5), then we can say that there is no change in stagnation temperature between the combustion chamber and the nozzle exit ($T_{0c} = T_{0e}$). If we further assume the exhaust to be fully expanded, then

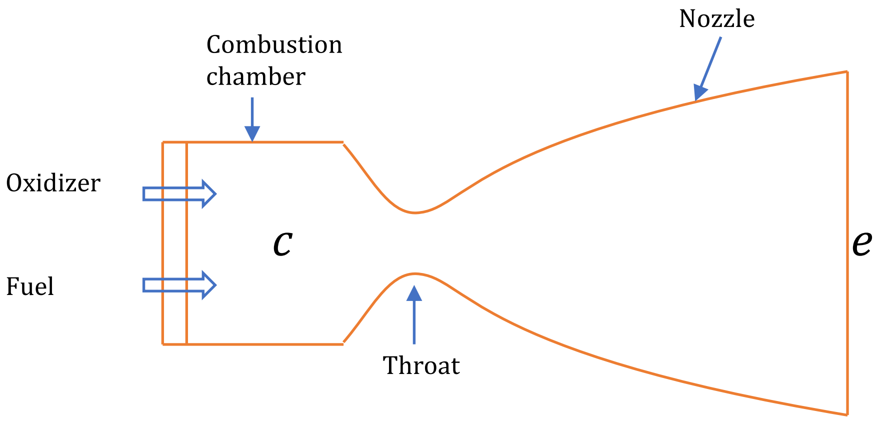

Figure 6.1: Rocket nozzle schematic.

the exhaust pressure is equal to the atmospheric pressure ($P_e = P_A$). Finally, assuming an isentropic nozzle, then we can write for the exit velocity

$$V_e = V_j = \sqrt{2c_p(T_{0e} - T_e)} \tag{6.3}$$

with

$$T_e = T_{0c}\left(\frac{P_e}{P_{0c}}\right)^{\frac{\gamma-1}{\gamma}}, \tag{6.4}$$

resulting in

$$\boxed{V_j = \sqrt{2c_p T_{0c}\left[1 - \left(\frac{P_e}{P_{0c}}\right)^{\frac{\gamma-1}{\gamma}}\right]}} \tag{6.5}$$

which is the most ideal jet velocity for a given stagnation temperature and pressure in the combustion chamber.

6.2.1 Performance parameters

We have a few parameters that are used to quantify the performance of rockets, with the first one the specific impulse I_{sp}, which is defined as the total impulse

$$I_T = \int_0^t F dt \tag{6.6}$$

divided by the weight of propellant (fuel + oxidizer) consumed $g_0 m_p$, with g_0 the standard acceleration of gravity at sea level on Earth, giving

$$I_{sp} = \frac{I_T}{g_0 m_p} \tag{6.7}$$

In steady state (constant F and constant $\dot{m}$, this becomes

$$\boxed{I_{sp} = \frac{F}{g_0 \dot{m}}} \tag{6.8}$$

Because in reality the flow our of a nozzle will not be uniform, we define an effective exhaust velocity c_e as

$$\boxed{c_e = \frac{F}{\dot{m}}} \tag{6.9}$$

which would be the exhaust velocity that gives the actual thrust if it was uniform.

The characteristic velocity C^* is a measure of the velocity through the throat of a nozzle, which is a measure of the combustion performance and is independent of the nozzle downstream of the throat because the nozzle will be choked.

C* is effectively an indication of combustion performance.

$$\boxed{C^* = \frac{P_c A_t}{\dot{m}}} \tag{6.10}$$

with A_t the throat area of the nozzle.

The performance of the nozzle is provided by the nozzle thrust coefficient

$$\boxed{C_F = \frac{F}{P_c A_t},} \tag{6.11}$$

Note that the effective exhaust velocity is simply the product of the characteristic velocity with the nozzle thrust coefficient

$$c_e = C^* C_F \tag{6.12}$$

and we can recover the specific impulse as

$$I_{sp} = \frac{C^* C_F}{g_0}. \tag{6.13}$$

6.3 Staging

Launch vehicles come in a broad range of sizes and configurations. The size and configuration depends on the specific mission of a launch vehicle. An important aspect of rocket design is staging, which greatly reduces the ratio of the initial total mass of a rocket to the payload mass. In this section we will look at what determines the design of a launch vehicle.

6.3.1 Mass ratios

The main masses that determine the performance of a rocket are the payload mass m_{pl}, the propellant mass m_p, and the dead weight (or structure) mass m_{dw}. Combining these masses, we can obtain the initial mass

$$m_0 = m_{pl} + m_p + m_{dw} \tag{6.14}$$

and the burnout mass

$$m_{bo} = m_{pl} + m_{dw}, \tag{6.15}$$

which is the mass of a vehicle after all the fuel has been consumed. The mass ratio MR is defined as the initial mass m_0 divided by the burnout mass m_{bo}

$$\boxed{\mathrm{MR} = \frac{\mathrm{m}_0}{\mathrm{m}_{\mathrm{bo}}}} \tag{6.16}$$

We will further make use of the ratio of payload mass to initial mass

$$\lambda = \frac{m_{pl}}{m_0} \tag{6.17}$$

and the the dead weight mass to initial mass

$$\delta = \frac{m_{dw}}{m_0} \tag{6.18}$$

Using these two ratios, we can now rewrite MR as

$$\mathrm{MR} = \frac{1}{\lambda + \delta}, \tag{6.19}$$

which is a useful set of parameters when calculating the effectiveness of staging, as we will see later in this section.

6.3.2 Rocket equation

We will now look at a fundamental equation for rocket propulsion, known as the Tsiolkovsky rocket equation. We can derive by starting from our standard equation for thrust without a pressure contribution

$$F = \dot{m} c_e \tag{6.20}$$

If we write the mass flow rate as a differential, and the force as mass times acceleration ($\mathrm{d}V/\mathrm{d}t$), we obtain

$$m\frac{\mathrm{d}V}{\mathrm{d}t} = -c_e\frac{\mathrm{d}m}{\mathrm{d}t} \tag{6.21}$$

with V the velocity of the rocket. Note that a positive mass flow rate $\dot{m}$ corresponds to a negative change in mass (a decrease in mass), which is why the negative sign appears: $\dot{m} = -\frac{\mathrm{d}m}{\mathrm{d}t}$. We now re-arrange the equation so we can integrate to obtain a change in velocity and a change in mass:

$$\int_0^{\Delta V} \mathrm{d}V = -c_e \int_{m_0}^{m_{bo}} \frac{1}{m}\mathrm{d}m \tag{6.22}$$

results in

$$\boxed{\Delta V_{\text{ideal}} = c_e \ln\left(\frac{m_0}{m_{bo}}\right) = c_e \ln\left(\frac{1}{\lambda+\delta}\right)} \tag{6.23}$$

This equation shows that a change in velocity of the rocket can be increased by either increasing the effective exhaust velocity, or by increasing the ratio of initial mass to burnout mass.

The ΔV provided by this equation is the ideal value, which does not take into account losses due to drag and gravity. Including these losses, we can write

$$\Delta V = \Delta V_{\text{ideal}} - \Delta V_g - \Delta V_D \tag{6.24}$$

or written out fully as

$$\Delta V = c_e \ln\left(\frac{m_0}{m_{bo}}\right) - \int_0^t g\sin\psi \,\mathrm{d}t - \int_0^t \frac{D}{m} dt \tag{6.25}$$

with ψ the angle with respect to the horizontal ($\psi = 90°$ equals a vertical take-off), and D the drag force.

The drag is determined by the local air density, the velocity, the frontal area S and the drag coefficient C_D:

$$D = \tfrac{1}{2}\rho V^2 S C_D \tag{6.26}$$

To minimize the drag experienced, a slow vertical take-off is preferred. A lower speed will decrease the drag due to the V^2 term, and a vertical take-off provides the quickest decrease in air density. As the air density decreases, the speed can be increased without a significant drag penalty. The gravitational loss on the other hand, is minimized by maximizing the acceleration, so that the time that gravity counteracts the acceleration is minimized. Gravity loss is further reduced by minimizing the angle ψ. Note that both requirements to minimize gravitational losses are in direct conflict with the requirements to minimize drag losses. In practice, launches start vertically with relatively low speeds and accelerations, which occurs quite naturally because the mass is highest at lift-off. After that, the angle ψ is reduced quite rapidly to reduce gravity losses in a manoeuvre called a *gravity turn*.

6.3.3 Multi-stage rockets

We will now apply the above principles to a multi-stage rocket. To simplify our analysis, we will neglect gravity and drag losses. If we consider a two-stage launch vehicle, then the payload of the first stage, is simply the initial weight of the second stage:

$$(m_{pl})_1 = (m_0)_2, \tag{6.27}$$

which we can write more generally as

$$(m_{pl})_i = (m_0)_{i+1}, \tag{6.28}$$

The initial mass of any stage if given by

$$(m_0)_i = \frac{(m_{pl})_i}{\lambda_i} \tag{6.29}$$

We can determine the value for λ using the rocket equation, from which we get

$$\frac{1}{\lambda + \delta} = \exp\left\{\frac{\Delta V}{c_e}\right\} \tag{6.30}$$

or

$$\lambda_i = \exp\left\{-\left(\frac{\Delta V}{c_e}\right)_i\right\} - \delta_i \tag{6.31}$$

Starting with the final payload to be delivered, the initial mass of the final stage can be determined given a specific ΔV for the final stage, and rocket characteristics that will provide δ and c_e for the stage. Once the initial mass of the final stage has been determined, the same procedure can then be applied to the stage that becomes before that.

The parameter λ provides the ratio of the payload mass to the initial mass of a rocket, which is what we seek to increase. Adding a stage will increase λ, where the overall ratio can be expressed as the product of the ratio of each stage (e.g., $\lambda = \lambda_1 \times \lambda_2$). The gain of adding stages is the largest when going from one to two stages. Adding further stages will result in smaller gains, while each stage will add complexity to the overall launch vehicle. The number of stages therefore needs to be carefully considered depending on the payload mass and required ΔV.

6.4 Solid and liquid propulsion systems

The main distinction that can be made between rocket propellants is solid and liquid fuel. In this section we will look at the main characteristics of both types. Both solid and liquid are chemical propulsion systems, effectively meaning that fuel is burned. Another type of propulsion to be aware of is electric propulsion. Electric propulsion is mainly used for spacecraft attitude and orbit control, but will not be discussed here.

The main components of a chemical propulsion system are the propellant storage, a combustion chamber, and an exhaust nozzle. If the propellant is liquid, then additionally a propellant feed and mixing system is required.

6.4.1 Solid propellant rockets

Figure 6.2 shows an example of a solid propellant rocket. The solid propellant typically contains both the fuel and the oxidizer. There is a perforation through the center of the rocket, which acts as the combustion chamber, and will increase in size as the propellant is consumed.

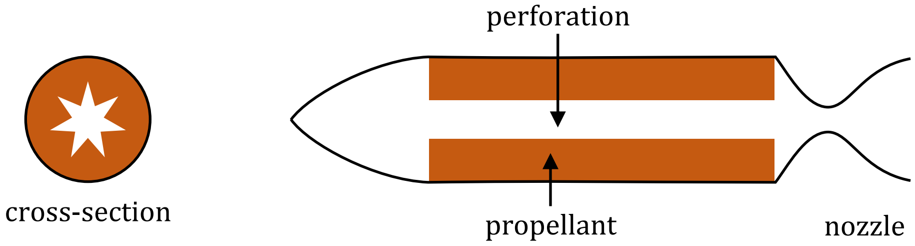

Figure 6.2: Schematic view of a solid propellant rocket with a cross section of the perforation.

The main advantage of a solid propellant rocket is that there is no need for a propellant feed system. The combustion chamber is also embedded in the fuel storage, requiring only a nozzle to accelerate the exhaust gases.

The main disadvantage of a solid propellant rocket is that once the rocket is ignited, there is no further control possible, and the rocket cannot be stopped and started again. Because control over the rocket is not possible during operation, the thrust profile (thrust as a function of time) needs to be embedded in the rocket design. This is done by designing a specific shape of the perforation, called the *grain*. After ignition, the propellant will be consumed at the exposed surface as shown in figure 6.3. If the perforation was circular, then the area of exposed propellant would increase as the propellant is consumed. This would result in an increased rate of propellant consumption, and consequently an increase of thrust with time. Figure 6.3 shows how different grain designs influence the thrust profile.

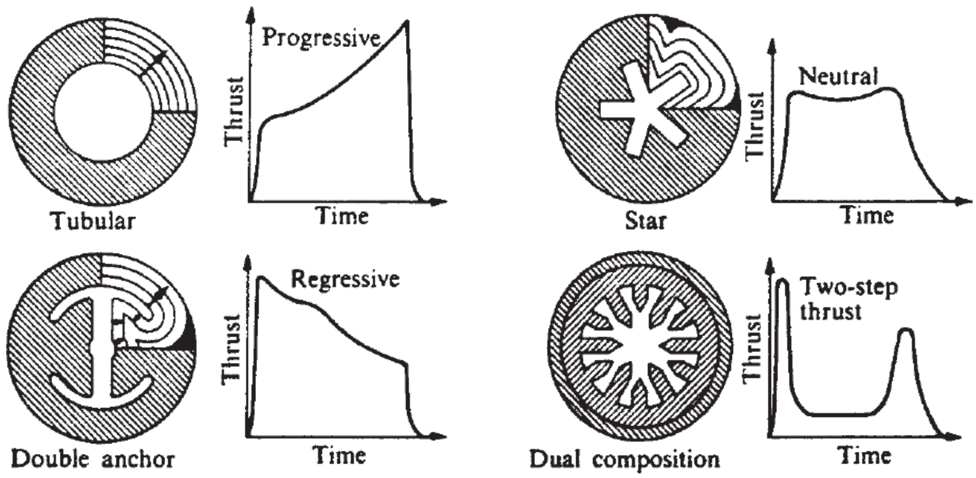

Figure 6.3: Grain designs with thrust profiles as a function of time (Hill and Peterson, 1992).

6.4.2 Liquid propellant rockets

Figure 6.4 shows an example of a liquid propellant rocket. It is clear that it is a more complex system than a solid propellant rocket: fuel and oxidizer pumps are required to feed the propellant into the combustion chamber. Because the pumps require power, a gas turbine is typically used to drive the pumps. Different configurations are possible, which we will look at in section 6.5.

The complexity of the propellant feed system is a main disadvantage, together with the additional weight and cost of the system. Further, the storage of the propellants is a more challenging task compared to solid rockets. The main advantage however, is that the thrust can be controlled. The engines can also be stopped and started again, making it a much more flexible propulsion system. Finally, the specific impulse of liquid propellant rockets is typically larger compared to solid propellant rockets.

6.4.3 Types of main engines

We will now list some groups of rocket engines with their typical requirements. Solid propellant is typically only used for boosters.

1. Boosters
   The function of boosters is to provide a high thrust (3000 - 8000 kN) over a relatively short period of time (< 150 s). They can be either solid or liquid.

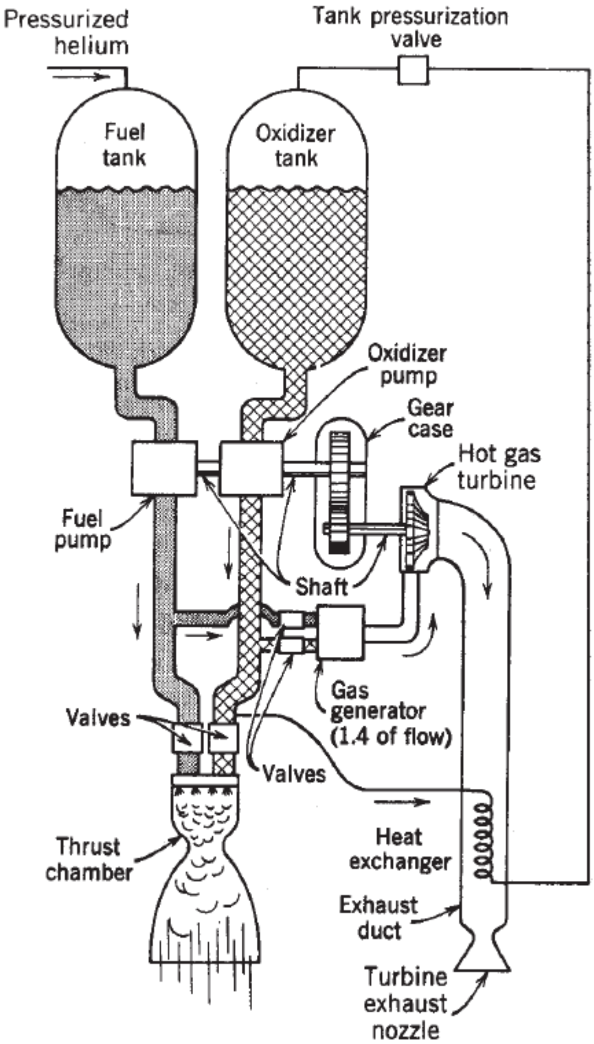

Figure 6.4: Schematic view of a liquid propellant rocket (Sutton, 1992).

2. Core engines
   Core engines provide a lower thrust than boosters (1000 - 2000 kN), but have a higher specific impulse and longer operating time of around 600 s. They operate at boosters at lift-off.

3. Upper stages
   The thrust of upper stages varies from 30 - 150 kN with operating times varying between 600 - 1100 s. A key feature of upper stage engines is vacuum ignition capability.

6.5 Rocket engine power cycles

In this last section we will have a closer look at power cycles for liquid propellant rocket engines. As explained in the introduction, the aim is to increase the exit velocity of exhaust gases out of the nozzle, because this will provide thrust in the most economical way. This requires us to maximize both the pressure and the temperature in the combustion chamber. The main purpose of the engine power cycle is to increase the pressure in the combustion chamber. We will look at four common types of power cycles: pressure driven, the expander cycle, the gas generator (GG) cycle, and the staged combustion (SC) cycle.

As we will see below, in most applications the nozzle will be used as a heat exchanger to preheat the fuel. This means that there is heat being removed from the exhaust gases, demonstrating that our assumption of an adiabatic nozzle is not entirely valid. For simplicity we will still assume an adiabatic nozzle as an approximation for nozzle calculations.

6.5.1 Pressure driven

The pressure driven power cycle is the simplest method to deliver liquid propellant to the combustion chamber. The fuel and oxidizer tanks are pressurized, so that only valves are required to control the flow rate of propellant. Figure 6.5 shows a schematic view of a pressure driven cycle.

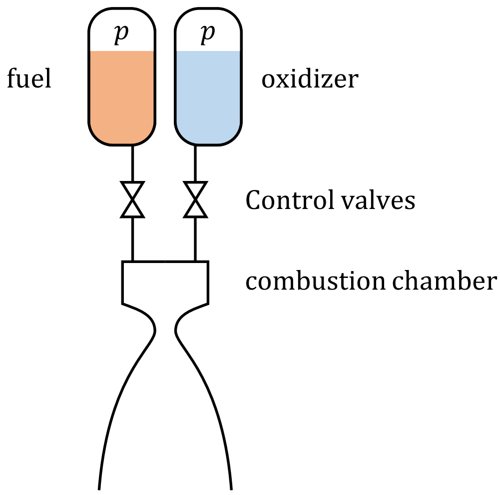

Figure 6.5: Schematic view of a pressure driven propellant delivery system.

The pressure can be from a secondary gas, typically helium, but can also be achieved by self-pressurization, for example by vaporization of part of the liquid propellant. The main advantage of pressure driven cycles is the simplicity, which reduces cost and weight. It also does not require a start-up to start operating. A major limitation is the pressure that can be achieved, mainly due to limitations to the pressure that propellant tanks can withstand. This limits the thrust that can be achieved, and is therefore not suitable for first stage launch vehicles. It is, however, commonly used in low thrust applications.

6.5.2 Expander cycle

In the expander cycle, the fuel is heated in a heat-exchanger on the nozzle (which is also providing the necessary cooling of the nozzle), and is then passed through a turbine. This turbine then drives the fuel and oxidizer pumps, as shown in figure 6.6.

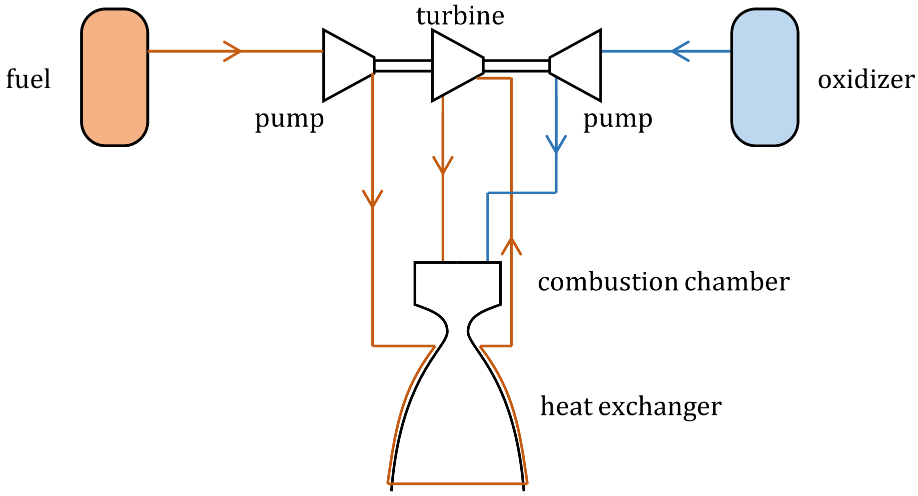

Figure 6.6: Schematic view of an expander cycle.

A variation of this cycle is possible where only a part of the fuel is entering the turbine, with the fuel going through the turbine being discarded. This means that some fuel is lost, but enables the turbine to operate on a larger pressure ratio, because the fuel leaving the turbine does not need to be pressurized.

The expander cycle is more complex than the pressure driven system, but it is still a relatively simple system. Because the turbine is driven just by fuel, and not exhaust products, this reduces the wear on the turbine. The main disadvantage is that the power that the turbine can deliver is limited by the amount of heat that can be extracted from the heat exchanger at the nozzle. This results in a size limitation, because an increase in engine size results in a volume that increases with the size cubed, while the surface area (and therefore the amount of heat that can be extracted) increases with the size squared. Further, the expander cycle cannot start by itself: the heat exchanger will only work once the engine is running.

6.5.3 Gas generator cycle

The gas generator cycle, shown in figure 6.7, works similar to the expansion cycle, except that the turbine is driven by a separate combustion process. This removes the disadvantage of the expansion cycle because the heat required to drive the turbine is not limited anymore by the heat exchanger. The turbine can therefore provide much more power to the pumps, resulting in higher combustion chamber pressures. Throttling is possible by controlling the valves that regulate the amount of fuel and oxidizer are entering the pre-combustion chamber. The disadvantages are that it further increases complexity and cost, and there is an increased turbine wear because exhaust gases are going through the turbine.

6.5.4 Staged combustion cycle

The staged combustion cycle, shown in figure 6.8 is a variation on the gas generator cycle, where *all* the fuel passes through the pre-combustion chamber and the turbine. Only a limited amount of oxidizer will enter the pre-combustion chamber, so most of the fuel will not be combusted.

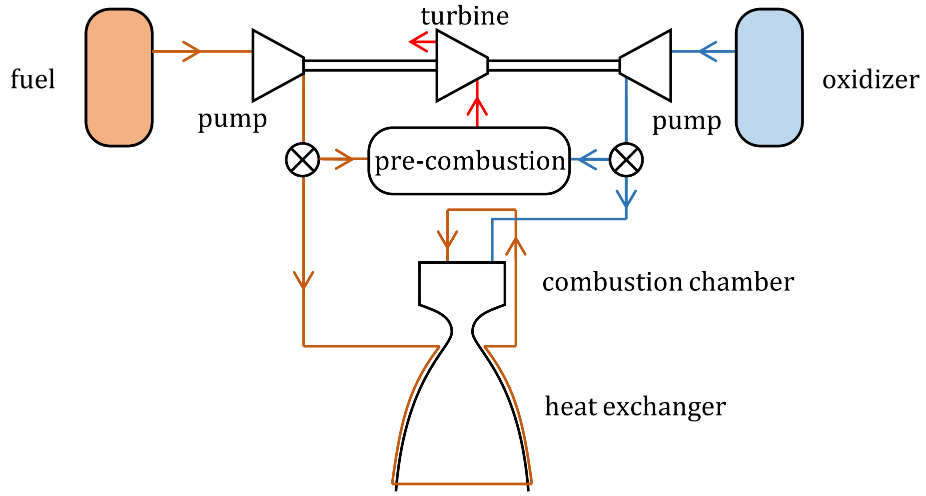

Figure 6.7: Schematic view of a gas generator cycle.

The staged combustion cycle will produce the highest thrust of the four cycles that we discussed. Notable engines that used the SC cycle are the Space Shuttle Main Engine, generating chamber pressures over 200 bar, and the Ariane Vulcain. Disadvantages are that the conditions are harsh for the turbine, and the complexity of the system.

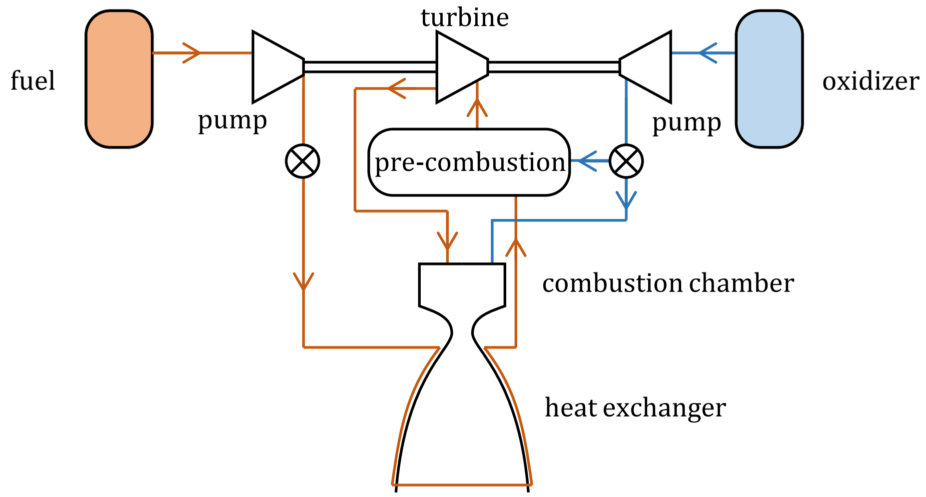

Figure 6.8: Schematic view of a staged combustion cycle.