Statistics 305/605: Introduction to Biostatistical Methods for Health Sciences

Chapter 15, part 1: Contingency Tables

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Contingency Tables

- Contingency tables display the joint frequency distribution of two categorical variables.
- E.G.: Let's consider the data of Mungan et al. 2000 click on 21,737 bladder cancer patients
  - Two categorical variables: **gender**, which has 2 levels, and **cancer stage**, which has 4 levels.
  - The first few lines of the data file are as follows:

```
##   Gender Cancer.Stage
## 1   Male            I
## 2   Male            I
## 3   Male            I
## 4   Male            I
## 5   Male            I
## 6   Male            I
## 7   Male            I
## 8   Male            I
```

- The contingency table made by cross-tabulating the gender and cancer stage variables of the Mungan data is as follows:

```
##          Cancer.Stage
## Gender          I    II   III    IV
##   Female   3926   402   356   852
##   Male    12418   995   883  1905
```

Terminology: Cells of a Table

- The cells of the table are its entries.
  - In the table cross-tabulating the gender and cancer stage variables of the Mungan data, the first cell of the table is 3926

```
##          Cancer.Stage
## Gender         I    II   III    IV
##   Female  3926   402   356   852
##   Male   12418   995   883  1905
```

Terminology: Row and Column Variables

- The row variable in a table defines the rows, the column variable the columns.
  - In the table below, the row variable is `Gender` and the column variable is `Cancer.Stage`.

```
##          Cancer.Stage
## Gender        I    II   III    IV
##   Female  3926   402   356   852
##   Male   12418   995   883  1905
```

Terminology: Row and Column Margins

- The **row margin** is the tabulation of the row variable and the **column margin** is the tabulation of the column variable.
- For the Mungan data,
  - the row margin (tabulation of Gender) is 5536 and 16201 Females and Males, respectively
  - The column margin (tabulation of Cancer Stage) is 16344, 1397, 1239, 2757 for cancer stages I through IV, respectively.
- Exercise: verify these table margins yourself.

Adding Margins to a Table

- It is common practice to add margins to a contingency table.
- In the following, the row margins (first table) and column margins (second table) have been added:

```
##              I  II III   IV Total
## Female  3926 402 356  852  5536
## Male   12418 995 883 1905 16201
```

```
##              I   II  III   IV
## Female  3926  402  356  852
## Male   12418  995  883 1905
## Total  16344 1397 1239 2757
```

Conditional distribution of cancer stage given gender

```
##          Cancer.Stage
## Gender          I    II   III    IV
##   Female  3926   402   356   852
##   Male   12418   995   883  1905
```

- For each gender category, we can divide the counts in each row by the row total to get proportions.

```
##          Cancer.Stage
## Gender             I          II         III          IV
##   Female 0.70917630 0.07261561 0.06430636 0.15390173
##   Male   0.76649590 0.06141596 0.05450281 0.11758533
```

- This gives an estimate of the distributions of cancer stage within each gender.

Conditional distribution of gender given cancer stage

- Likewise, for each cancer stage category we can divide the counts in each column by the column total to get proportions.

```
##         Cancer.Stage
## Gender             I        II       III        IV
##   Female 0.2402105 0.2877595 0.2873285 0.3090316
##   Male   0.7597895 0.7122405 0.7126715 0.6909684
```

- This gives an estimate of the distributions of gender within each cancer stage.

Independence of Row and Column Variables.

- Suppose the conditional distribution of gender given cancer stage is 25% female and 75% male, regardless of cancer stage.

- What is the unconditional distribution of gender in this case (i.e., ignoring cancer stage)?

- If the conditional gender distribution is 25% female and 75% male in each cancer stage then, ignoring cancer stage and considering the unconditional distribution of gender, there will also be 25% females and 75% males.

- In this case, we say that gender and cancer stage are *independent*.

More generally

- If the conditional distributions of the row variable given the column variable are all the same, they will also be the same as the unconditional distribution of the row variable.
  - E.G., if the conditional gender distribution is 25% female and 75% male in each cancer stage, we will also have 25% females and 75% males unconditionally (i.e. ignoring cancer stage).
- We say that the column and row variables are *independent* because:
  - Knowing the value of the column variable tell us nothing about the row variable;
  - E.G. Knowing cancer stage tells us nothing about gender; so $P(\text{Gender} = \text{Female} \mid \text{Stage} = \text{I}) = P(\text{Gender} = \text{Female})$

- One can use the definition of conditional probability to show that independence of row and column variables is equivalent to the following two statements:
    1. The conditional distributions given the different levels of the row variable are all equal
    2. The conditional distributions given the different levels of the column variable are all equal.
- The opposite of independence is dependence, or an *association*.
- We next discuss how to test for association.