Statistics 305/605: Introduction to Biostatistical Methods for Health Sciences

Chapter 15, part 2: Chi-Square Tests

Jinko Graham

2018-10-06

Chi-square tests of association

- In chapter 14, we tested for association between two categorical variables by testing for differences between proportions
  - Call the test from Chapter 14 the Z test.
- Applied the Z test to data from the WHI.
  - Recall that the WHI randomized 16,608 post-menopausal women aged 50-79 years to receive either hormone replacement therapy (estrogen plus progestin EP$+$;$n_1 = 8506$), or a placebo (EP$-$; $n_2 = 8102$).
- We tested for a difference in the proportions of women with invasive breast cancer (BC$+$) in the hormone replacement therapy (EP$+$) and placebo (EP$-$) groups.

The WHI Data click

- The first few rows of the dataset are as follows:

```
##    EP  BC
## 1 EP+ BC+
## 2 EP+ BC+
## 3 EP+ BC+
## 4 EP+ BC+
## 5 EP+ BC+
## 6 EP+ BC+
## 7 EP+ BC+
## 8 EP+ BC+
```

- A cross-tabulation of the BC and EP variables in the dataset is:

```
##      BC
## EP     BC-  BC+
##   EP- 7980  122
##   EP+ 8340  166
```

Association between HRT and breast cancer

- The table of proportions below gives the conditional distributions of BC status given EP status.
  - The proportions in each row add to 1.
- BC and EP are associated if their conditional distributions are different.

```
##      BC
## EP           BC-        BC+
##   EP- 0.98494199 0.01505801
##   EP+ 0.98048436 0.01951564
```

- Previously, we used the Z test for different conditional distributions
- Looks for differences in the proportion of BC$+$ in the EP$-$ and EP$+$ groups.
- Can be applied to data in 2×2 tables

Chi-square test of association

- When applied to 2×2 tables, the Z test for a difference in proportions is equivalent to the chi-square (χ^2) test.
- But the chi-square test has the advantage of generalizing from 2×2 tables to $r \times c$ tables, for $r \geq 2$ rows and $c \geq 2$ columns.
- Compares *observed* cell counts to *expected* counts
  - The expected count is the count we would expect if the null hypothesis of no association were true (details deferred).
- The form of the statistic is

$$X^2 = \sum_{\text{cells}} \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Sampling distribution of X^2

- Under the null hypothesis of no association between row and column variables, the test statistic X^2 is approximately distributed as a chi-square distribution with $(r-1) \times (c-1)$ degrees of freedom (df).
- Computer software gives upper tail probabilities of different chi-square distributions.

Chi-square test for WHI example

- We can perform the chi-square test in R (see demo).

```
##
##  Pearson's Chi-squared test
##
## data:  wtab
## X-squared = 4.8387, df = 1, p-value = 0.02783
```

- At the 5% level, there is statistical evidence of an association between hormone-replacement therapy and invasive breast cancer.

Continuity correction

- The continuity correction to the X^2 test improves the χ^2 approximation for 2×2 tables.
- The corrected version of the statistic is:

$$X^2 = \sum_{\text{cells}} \frac{(|\text{observed} - \text{expected}| - 0.5)^2}{\text{expected}}$$

Chi-square test with continuity correction for WHI example

```
##
##  Pearson's Chi-squared test with Yates' continuity correction
##
## data:  wtab
## X-squared = 4.5807, df = 1, p-value = 0.03233
```

- At the 5% level, there is still evidence of an association between hormone replacement therapy and invasive breast cancer.

Expected counts

- As mentioned earlier, these are calculated under the null hypothesis of no association between the 2 variables in the table.
- Let's first discuss expected counts for the WHI example. Later, we'll generalize to arbitrary $r \times c$ tables.

```
##        BC
## EP      BC-   BC+
##    EP- 7980   122
##    EP+ 8340   166
```

- If H_0 holds and HRT has no effect on breast cancer, the proportion of BC$+$ in each EP group should be the same and can be estimated by pooling:

$$\hat{p} = \frac{122 + 166}{7980 + 122 + 8340 + 166} = \frac{288}{16608} = 0.01734.$$

- Here is the table of counts with column and row margins added:

```
##           BC- BC+ rowTot
## EP-      7980 122   8102
## EP+      8340 166   8506
## colTot 16320 288  16608
```

- Focusing on the 1st row of the table, under H_0 of no association, we expect that, of the 8102 women who are EP$-$,
  - $8102 \times \hat{p} = 140.5$ would be BC$+$, and
  - $8102 \times (1 - \hat{p}) = 7961.5$ would be BC$-$.
- Similarly, focusing on the 2nd row, we expect that, under H_0 of no association, of the 8506 women who are EP$+$,
  - $8506 \times \hat{p} = 8506 \times \frac{288}{16608} = 147.5$ would be BC$+$, and
  - $8506 \times (1 - \hat{p}) = 8506 \times \frac{16320}{16608} = 8358.5$ would be BC$-$.

Expected counts, notation

- Notation from the text:

		BC		
		BC+	BC−	
EP	EP+	a	b	$a+b$
	EP−	c	d	$c+d$
		$a+c$	$b+d$	n

  where $n = a + b + c + d$
- The pooled estimate of the proportion of BC+ women is $\hat{p} = (a + c)/n$.
- Expected count for the EP+ and BC+ cell:
  - Of the $(a + b)$ women who are EP+, we expect that $(a + b) \times \hat{p} = (a + b)(a + c)/n$ would be BC+
- Notice that the expected count is of the form row total $(a + b)$ times column total $(a + c)$ divided by table total (n). This is a generalizable pattern ...

Expected counts: general formula

- For $r \times c$ tables, the expected count for the cell in the ith row and jth column is the ith row total times the jth column total divided by table total.

Accuracy of the χ^2 approximation ($r \times c$ tables)

- The χ^2 approximation for the null distribution of the test statistic is considered accurate when
  1. No more than 20% of cells have expected counts < 5, and
  2. All expected cell counts are ≥ 1.
- Note: These rules-of-thumb are intended regardless of whether or not we use the continuity correction for 2×2 tables.

Accuracy of the χ^2 approximation in WHI example

- The expected cell counts are as follows:

```
##       BC
## EP          BC-      BC+
##   EP- 7961.503 140.4971
##   EP+ 8358.497 147.5029
```

- *All* expected cell counts are greater than 5, and so the χ^2 approximation is considered accurate.

Sampling

- The chi-square test is appropriate under different sampling schemes such as:
  1. Take simple-random samples (SRSs) from each of c populations and classify individuals in each SRS according to one categorical variable with r levels
  2. Take one SRS from a single population and classify individuals according to two categorical variables, one with c levels and the other with r levels
- The first scheme includes case-control sampling ($c = 2$)
  - e.g. an SRS of size $n_1 = 500$ from the case population for non-Hodgkin lymphoma and an SRS of size $n_2 = 500$ from the control population and then classify them according to whether or not they are exposed to some pesticide ingredient.
- The second scheme pertains to the WHI study
  - A sample of size $n = 16608$ was drawn from the population of post-menopausal women and then cross-classified according to whether or not they were randomized to receive HRT and whether or not they developed invasive breast cancer.