Notes for GRE Physics

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Preface

For test takers of GRE Physics, there are few materials to prepare. I took the test on Oct. 26, 2019 in Shanghai and got a full mark 990 (95%). Therefore, I plan to organize my notes based on Jeff's version...

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目录

1 Classic Mechanics

1. 保守力：$\nabla \times \vec{F} = 0 \Rightarrow \vec{F} = -\nabla U$.
2. 抛体：Projectile.
3. 弹簧串联：$\frac{1}{k_t} = \frac{1}{k_1} + \frac{1}{k_2}$; 并联：$k_t = k_1 + k_2$;

$$\lambda = \frac{R}{2}\sqrt{\frac{C}{L}} \quad \begin{cases} > 1 : \text{overdamped 过阻尼;} \\ = 1 : \text{critically damped 临界阻尼;} \\ < 1 : \text{underdamped 欠阻尼} \Rightarrow \text{频率}: \omega_d = \omega_0\sqrt{1 - \frac{b}{4mk}} : b\text{阻尼系数.} \end{cases}$$

4. 冲量：impulse.
5. 转动惯量：the moment of inertia; 支点：fulcrum；单摆：pendulum$\Rightarrow \omega = \sqrt{\frac{g}{l}}$.
6. centripetal: 向心的;
   centrifugal: 离心.
7. 科里奥利力：$\vec{F_c} = -2m\vec{\omega} \times \vec{v}$
8. 天体：$E = \frac{1}{2}mv^2 + V(r) = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + V(r)$，而 $L = mr^2\dot{\theta}$，
   故 $E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r) = \frac{1}{2}m\dot{r}^2 + V_{eff}$
9. 伯努利方程：$\frac{1}{2}\rho v^2 + \rho gh + P = C$;
   不可压缩流体：$v_i A_i = C$;
   斯托克斯定律（球体）：$F_{drag} = -6\pi\mu R v_s$, μ 为黏度系数，v_s 为下落最终速度.
10. 波在弦中传播速度：$v = \sqrt{\frac{T}{\mu}}$, T 为弦中张力，
    μ 为弦线密度;
11. 球面波强度：$I = \frac{P_s}{4\pi r^2} \propto \frac{1}{r^2}$;
    声强级：$\beta = 10\log\frac{I}{I_0}(\text{dB})$, I_0 为 10^{-12};
    理想气体中声波传播速度：$v = \sqrt{\frac{\gamma RT}{M}} \propto \sqrt{T}$,
    如图1所示.

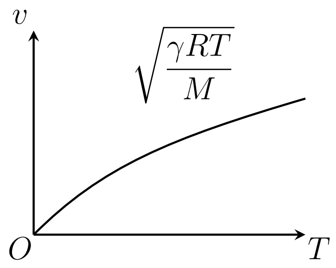


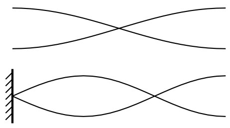
图 1: 理想气体中声波传播速度.

12. 允许的频率数 = 系统的自由度

13. 两端开口管：$L = n\dfrac{\lambda}{2}$;

    一端开口管：$L = \dfrac{2n+1}{4}\lambda$

    harmonics: $n = Round\left(\dfrac{f_2}{f_1}\right)$；beats= $n \times f_1 - f_2$.



14. 火箭运动：$m\dfrac{\mathrm{d}v}{\mathrm{d}t} + u\dfrac{\mathrm{d}m}{\mathrm{d}t} = 0$, u 为相对速度。

15. 分析力学:

    拉格朗日函数：$L(q, \dot{q}, t) = T - U$.

    action 作用函数：$S = \displaystyle\int L\mathrm{d}t$.

    哈密顿原理：actual path 满足 $\delta \displaystyle\int L\mathrm{d}t = 0$, 即 actual 作用函数 S 是极值。

    拉格朗日方程:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0,\ \text{广义动量: } p = \frac{\partial L}{\partial \dot{q}},$$

    类似于牛顿第二定律：$ma = \dfrac{\mathrm{d}p}{\mathrm{d}t} = F$.

    哈密顿量：$H(q, p, t) = \displaystyle\sum_i p_i \dot{q}_i - L$.

    正则方程:

$$\dot{q} = \frac{\partial H}{\partial p},\ \dot{p} = -\frac{\partial H}{\partial q},\ \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t},$$

    其中第二个式子应用了拉格朗日方程和广义动量的定义:

$$\frac{\partial H}{\partial q} = -\frac{\partial L}{\partial q} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = -\dot{p}.$$

2 Electrodynamics

1. 毕奥 · 萨伐尔定律：$B=\dfrac{\mu_0}{4\pi}\displaystyle\int\dfrac{I\times\hat{r}}{r^2}\mathrm{d}l=\dfrac{\mu_0}{4\pi}\displaystyle\int\dfrac{J\times\hat{r}}{r^2}\mathrm{d}V$.

2. Maxwell 方程组:

$$\begin{cases}\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t};\\ \nabla\cdot\vec{E}=\dfrac{\rho}{\varepsilon_0};\\ \nabla\cdot\vec{B}=0;\\ \nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}\Rightarrow\ \text{位移电流}:\ \vec{J}_D=\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}=\dfrac{\partial\vec{D}}{\partial t}.\end{cases}$$

3. 极化强度：$\vec{P}=\dfrac{\sum\vec{p_i}}{\Delta V}$,
   电位移矢量:

$$\left.\begin{aligned}\vec{D}&=\varepsilon_0\vec{E}+\vec{P}\\ \vec{P}&=\chi_e\varepsilon_0\vec{E}\end{aligned}\right\}\Rightarrow\vec{D}=(1+\chi_e)\varepsilon_0\vec{E}=\varepsilon_r\varepsilon_0\vec{E}.$$

4. 磁化强度：$\vec{M}=\dfrac{\sum\vec{m_i}}{\Delta V}$,
   磁场强度和磁感应强度的关系：$\vec{B}=\mu\vec{H}=(1+\chi_M)\mu_0\vec{H}=\mu_0\vec{H}+\mu_0\vec{M}$.

5. 在介质中:
   束缚电荷：$\rho_P=-\nabla\cdot\vec{P}\Rightarrow\sigma_P=-(P_2-P_1)$;
   净电荷：$\rho_P+\rho_f=\varepsilon_0\nabla\cdot\vec{E}\Rightarrow\sigma_P+\sigma_f=\varepsilon_0(E_2-E_1)$;
   自由电荷：$\rho_f=\nabla\cdot\vec{D}\Rightarrow\sigma_f=D_2-D_1$.

6. 在介质中:

$$\text{诱导电流}\begin{cases}\text{磁化电流}:\ \vec{J}_M=\nabla\times\vec{M};\\ \text{极化电流}:\ \vec{J}_P=\dfrac{\partial\vec{P}}{\partial t},\end{cases}$$

   故：总电流 = 自由电流 + 诱导电流 + 位移电流，即 $\dfrac{1}{\mu_0}\nabla\times\vec{B}=\vec{J}_f+\vec{J}_M+\vec{J}_P+\vec{J}_D$.

7. 介质中的 Maxwell 方程组:

$$\begin{cases}\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t};\\ \nabla\cdot\vec{D}=\rho_f;\\ \nabla\cdot\vec{B}=0;\\ \nabla\times\vec{H}=\vec{J}_f+\dfrac{\partial\vec{D}}{\partial t}.\end{cases}$$

表 1: 几种磁性之间的区别

diamagnetic 抗磁性	$\mu < \mu_0$ $\chi_M < 0$	没有不成对电子，磁场减弱 (Lenz's Law)
paramagnetic	$\mu > \mu_0$ $\chi_M > 0$	不成对电子有相同的自旋，增强磁场
ferromagnetic	$\mu \gg \mu_0$ $\chi_M \gg 0$	

8. 坡印廷矢量：$\vec{S} = \vec{E} \times \vec{H}$.
   欧姆定律微观形式：$\vec{J} = \sigma \vec{E}$.

9. 电磁场的变值关系：

$$\begin{cases} \vec{e}_n \times (\vec{E}_2 - \vec{E}_1) = 0; \\ \vec{e}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}; \\ \vec{e}_n \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_f; \\ \vec{e}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0. \end{cases}$$

10. 电磁场能量密度：$\dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \vec{E} \cdot \dfrac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \dfrac{\partial \vec{B}}{\partial t}$,
    故在真空中：$\omega = \dfrac{1}{2}(\varepsilon_0 E^2 + \dfrac{1}{\mu_0} B^2)$,
    在介质中：$\omega = \dfrac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})$, 注意光速 $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

11. 矢势：由麦克斯韦方程组中 $\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}$,
    则 $\nabla \times \vec{E} = 0 \Rightarrow \vec{E} = -\nabla U - \dfrac{\partial A}{\partial t}$.

12. 电磁波压力：
    全吸收：$P_r = \dfrac{I}{c}$;
    全反射：$P_r = \dfrac{2I}{c}$.

13. 电磁波中 $\vec{B} = \dfrac{1}{c} \vec{k} \times \vec{E} \Rightarrow B = \dfrac{E}{c}$.

14. 电容器：$C = \dfrac{Q}{U}$, $E = \dfrac{1}{2} QU = \dfrac{1}{2} CU^2$.

15. 偶极子电场：$E = \dfrac{1}{2\pi\varepsilon_0} \dfrac{p}{z^3}$,

电偶极子在电场中的能量和磁矩在磁场中的能量：$U = -\vec{p} \cdot \vec{E}$, $U = -\vec{\mu} \cdot \vec{B}$.

16. 螺线管 (solenoid) 内磁场: $B = \mu_0 n I$ （安培环路定理），理想螺线管外无磁场。

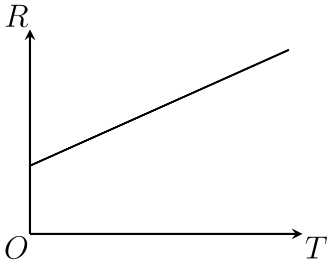
17. valence band 价带;
    conduction band 导带.

18. 超导：BCS theory：库珀对 $\to$ bosonic state.
    迈斯纳 Meissner effect: 超导态时 expel any magnetic field.

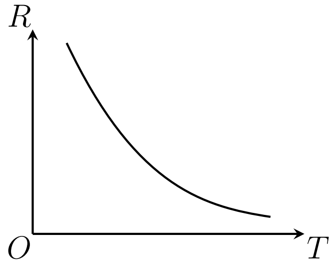
19. 电磁波的趋肤效应：$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$.

3 Electronics

1. Metal:



Semiconductor:



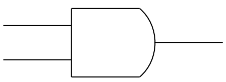
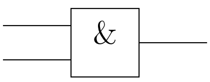
2. Capacitor: $q = c\varepsilon(1 - e^{-\frac{t}{\tau_C}})$, 其中 $\tau_C = RC$, 容抗 $X_C = \dfrac{1}{\omega C} \propto \dfrac{1}{\omega}$.

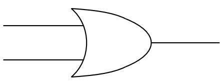
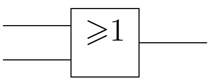
3. Inductor: 由 $\Phi = LI$ 定义, $\varepsilon = -\dfrac{\mathrm{d}\Phi}{\mathrm{d}t} = -L\dfrac{\mathrm{d}i}{\mathrm{d}t}$, 感抗 $X_L = \omega L \propto \omega$,
   能量 $U = \dfrac{1}{2}LI^2$, $\tau_L = R/L$.

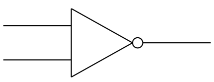
5. LC 回路频率: $\omega = \dfrac{1}{\sqrt{LC}}$;
   RLC 回路频率: $\omega' = \sqrt{\omega^2 - \left(\dfrac{R}{2L}\right)^2}$;
   总阻抗: $Z = R + i\omega L + \dfrac{1}{i\omega C}$, 故 $|Z| = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$;
   则共振频率: $\omega_{resonant} = \dfrac{1}{\sqrt{LC}}$.

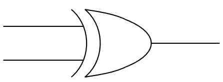
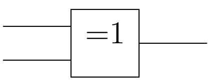
6. Thevenin 戴维南.

7. 门电路:

 与门 = 

 或门 = 

 非门

 异或门 = 

异或：输入端 A 和 B 相同为 0，A 和 B 不同则为 1.

4 Thermodynamics and Statistical Physics

1. 能均分定理：平均动能 $E = \frac{\nu}{2}Nk_BT$, ν 为自由度.

2. 理想气体：$pV = nRT = Nk_BT$. (注意 $nR = Nk_B$)
   麦克斯韦速率分布：$n(v) = Av^2e^{-mv^2/2k_BT}$, 推出：
   最可几速度：$v_{mode} = \sqrt{\frac{2RT}{M}}$;
   均方根速度：$v_{rms} = \sqrt{\frac{3RT}{M}}$;
   平均速度：$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$;
   其中：$v_{mode} < v_{avg} < v_{rms}$.

   平均自由程：$\lambda = \frac{1}{\sigma n} = \frac{1}{\sqrt{2}\pi d^2 n}$, ($\sigma$ 为碰撞截面积)
   则弛豫时间 $\tau = \frac{\lambda}{v}$, 利用能均分定理 $\frac{1}{2}mv^2 = \frac{\nu}{2}k_BT$, 故 $\tau = \frac{\sqrt{m}\lambda}{\sqrt{\nu k_BT}}$.

3. $C_P - C_V = nR$;
   Isothermal: 等温; Isobaric: 等压;
   Isochoric: 等容; adiabatic: 绝热 $\Rightarrow pV^\gamma = C$, ($\gamma = \frac{C_P}{C_V} > 1$, 故绝热线比等温线更陡).

4. Partition Function 配分函数：$Z = \sum_s g_s e^{-\beta\varepsilon_s}$ ($\beta = \frac{1}{k_BT}$), 其中 g_s 为简并度;
   多系统: $Z = \prod_{i=1}^{N} \zeta_i$; ($\zeta_i$ 为每个子系统的配分函数)
   故能量期望值为 $\langle E\rangle = \frac{1}{Z}\sum_s \varepsilon_s e^{-\beta\varepsilon_s} = -\frac{1}{Z}\frac{\partial Z}{\partial\beta} = -\frac{\partial \ln Z}{\partial\beta}$.
   每个态的概率 $P(s) = \frac{e^{-\beta\varepsilon_s}}{Z}$, 注意近似: 当 $X \gg 1$, $\ln X! \approx X(\ln X - 1)$.
   三种分布：
   玻尔兹曼分布：$n(\varepsilon_s) = \frac{g_s}{e^{(\varepsilon_s-\mu)\beta}} \Rightarrow a_l = \frac{\omega_l}{e^{(\varepsilon_l-\mu)\beta}}$;
   费米分布：$n(\varepsilon_s) = \frac{g_s}{e^{(\varepsilon_s-\mu)\beta}+1} \Rightarrow a_l = \frac{\omega_l}{e^{(\varepsilon_l-\mu)\beta}+1}$;
   波色分布：$n(\varepsilon_s) = \frac{g_s}{e^{(\varepsilon_s-\mu)\beta}-1} \Rightarrow a_l = \frac{\omega_l}{e^{(\varepsilon_l-\mu)\beta}-1}$;
   在经典极限条件 $\frac{a_l}{\omega_l} \ll 1$ 下，费米分布和波色分布都近似为玻尔兹曼分布.

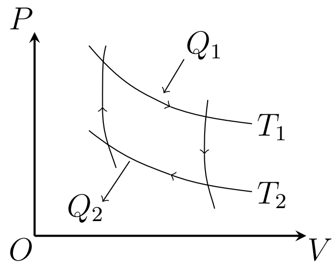
5. Entropy: $S = k_B \ln\Omega$, $\partial S = \frac{\partial Q}{T}$, 故 $\Delta S = \int_{T_1}^{T_2} \frac{\mathrm{d}Q}{T}$;
   $\mathrm{d}S \geqslant \frac{đQ}{T}$, 故 $\mathrm{d}U \leqslant T\mathrm{d}S - p\mathrm{d}V$, 且绝热过程 $\mathrm{d}s \geqslant 0$.

由热力学基本方程：$\mathrm{d}U = đQ - p\mathrm{d}V = T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N$, 故 $\dfrac{1}{T} = \left(\dfrac{\partial S}{\partial U}\right)_{V,N}$;

除此之外，熵和配分函数之间的关系: $S = \dfrac{\partial}{\partial T}(k_B T \ln Z)$.

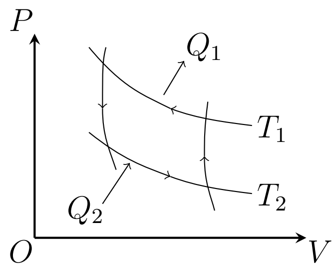
6. 活塞：piston.

7. 卡诺循环:



正卡诺循环:

对外做功 $\Rightarrow \eta = \dfrac{W}{Q_1} = \dfrac{Q_1 - Q_2}{Q_1} = 1 - \dfrac{T_2}{T_1}$;



逆卡诺循环:

制冷 $\Rightarrow \eta = \dfrac{Q_2}{W} = \dfrac{Q_2}{Q_1 - Q_2} = \dfrac{T_2}{T_1 - T_2}$;

8. 焓 Enthalpy: $H = E + pV$, $\mathrm{d}H = T\mathrm{d}S + V\mathrm{d}p$;

自由能 Helmholtz Free Energy: $F = E - TS$, $\mathrm{d}F = -S\mathrm{d}T - p\mathrm{d}V$;

吉布斯自由能 Gibbs Free Energy: $G = E - TS + pV$, $\mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}p$.

配分函数: $Z = e^{-\beta F}$;

由 $\mathrm{d}U \leqslant T\mathrm{d}S - p\mathrm{d}V$ 可知：等温等容：$\mathrm{d}F \leqslant 0$; 等温等压：$\mathrm{d}G \leqslant 0$.

9. 黑体辐射: $P = \sigma T^4$;

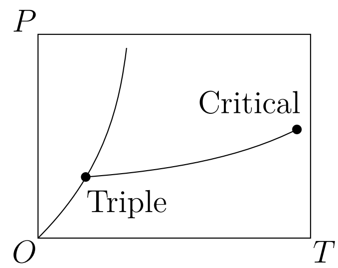
维恩位移定律: $\lambda_{max} \cdot T = Constant$;

普朗克分布: $I(\nu, T) = \dfrac{2h\nu^3}{c^2}\dfrac{1}{e^{\frac{h\nu}{k_B T}} - 1}$.

10. 相图:

Critical 临界点：气液之间无区别点;

Triple 三相电：气液固三相并存.



11. 相变：$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V$;

相变潜热：$L = T(S^{(2)} - S^{(1)})$; 体积突变：$\Delta V = V^{(2)} - V^{(1)}$.

一级相变：存在 L 和 ΔV; 二级相变（连续相变）：无相变潜热和体积突变.

12. 三维电子态密度

电子能量：$\varepsilon = \dfrac{p^2}{2m} = \dfrac{\hbar^2 k^2}{2m}$, 每个量子态在 k 空间占据体积为 $\dfrac{(2\pi)^3}{V}$,

故态密度 $N(E) = \dfrac{4\pi k^2 \cdot \mathrm{d}k}{(2\pi)^3/V \cdot \mathrm{d}\varepsilon}$;

若考虑电子自旋 $\times 2$, 则 $N(E) = \dfrac{2V}{(2\pi)^3} \dfrac{4\pi k m}{\hbar^2} \propto \sqrt{E}$.

5 Optics

1. 光速: $v = \frac{1}{\sqrt{\mu\varepsilon}}$.

2. 不确定度原理: $\Delta E \Delta \tau \approx \hbar \Rightarrow \Delta\tau\Delta\nu \approx 1$,
   相速度: $v_p = \frac{\omega}{k}$, 群速度: $v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}$,
   折射率: $n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}}$.

3. 球面反射: $\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} = \frac{2}{r}$, 放大率 magnification: $m = -\frac{s'}{s}$.
   球面折射: $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$.
   薄透镜有: $\frac{1}{f'} = (n_0 - 1)(\frac{1}{r_1} - \frac{1}{r_2})$, ($n_0$ 为透镜折射率),
   $\frac{n'}{s'} - \frac{n}{s} = \frac{n_0 - n}{r_1} + \frac{n' - n_0}{r_2}$,
   $\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$.

4. 瑞利 Rayleigh 判据（中央最大与第一极小重叠）像可分辨: $\theta_R = 1.22\frac{\lambda}{d} = 0.61\frac{\lambda}{r}$,
   故艾里斑半径: $r = \theta_R f = (1.22\frac{\lambda}{D})f$.

5. 多普勒效应: $\frac{1}{\lambda_s}(v \pm v_s) = \frac{1}{\lambda_D}(v \pm v_D)$.

6. 杨氏双缝干涉 (interference):
   最大: $d\sin\theta = m\lambda$, (d 为双缝间距);
   最小: $d\sin\theta = (m + \frac{1}{2})\lambda$.
   $y_m = \frac{m\lambda L}{d}$, ($L$ 为缝屏间距).

7. 单缝衍射 (diffraction):
   最大: $a\sin\theta = (m + \frac{1}{2})\lambda$, ($a$ 为缝宽).
   最小: $a\sin\theta = m\lambda$.

8. 求成像最 sharp 时照相机的小孔半径:
   即只有第一主极大在缝宽内 $y \approx a = D\cdot\theta$,
   而 $a\cdot\sin\theta \approx a\cdot\theta = \lambda$, 故 $a = \sqrt{\lambda D}$.

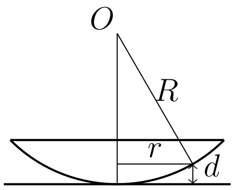
9. 光栅 (gratting) 衍射: 最大值 $d\sin\theta = k\lambda$, 光栅常数 $d = a + b$.

10. 布拉格衍射: $2d\sin\theta = k\lambda$.

11. 空气中薄膜 (第一次反射有半波损失): $2nd = (m + \frac{1}{2})\lambda$.

12. 迈克尔逊干涉仪:
插入折射率为 n 厚度为 L 的板: $N = \frac{2L}{\lambda}$, $N' = \frac{2Ln}{\lambda}$,
故: $N' - N = \frac{2L}{\lambda}(n-1)$;
单纯移动镜面: $\Delta N = \frac{2L}{\lambda}$.

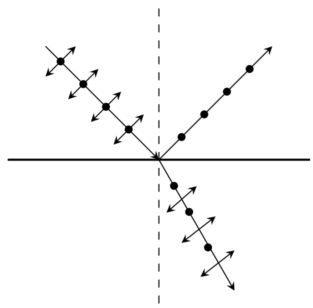
13. 牛顿环:



$$2d = (m + \frac{1}{2})\lambda,\ d = R - \sqrt{R^2 - r^2} \approx \frac{r^2}{2R},$$
故 $r = \sqrt{R(m + \frac{1}{2})\lambda}$.

14. 通过偏振片: $I = \frac{I_0}{2}$;
马吕斯定律: $I = I_0 \cos^2\theta$.

15. 斯涅耳定律: $n_1 \sin\theta_1 = n_2 \sin\theta_2$;
布鲁斯特角: $\theta = \arctan\left(\frac{n_2}{n_1}\right)$,
布鲁斯特角时反射光和折射光的偏振方向如右图所示.



16. 干涉光强 $I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$.

17. 光栅性能参数: <1> 角色散本领: $D_\theta = \frac{\delta\theta}{\delta\lambda} = \frac{K}{d\cos\theta} \Leftarrow (d\sin\theta = K\lambda$求导$)$.
<2> 线色散本领: $D_l = \frac{\delta l}{\delta\lambda} = D_\theta \cdot f = \frac{Kf}{d\cos\theta}$.
<3> 色分辨本领: $R = \frac{\lambda}{\Delta\lambda}$,
由 $d\sin\theta = K(\lambda + \Delta\lambda)$ 和第一极小 $d\sin\theta = (K + \frac{1}{N})\lambda$,
得到 $R = KN = \frac{Nd\sin\theta}{\lambda}$,
其中 Nd 为光栅宽度，所以使用光栅时最好全照亮.
<4> 色散范围 (自由光谱范围):

$$K(\lambda+\Delta\lambda)=(K+1)\lambda \Rightarrow \Delta\lambda=\frac{\lambda}{K}=\frac{\lambda^2}{d\sin\theta}.$$

18. 晶体光学:

    光轴: 不发生双折射.

    主平面: 光线和光轴确定的平面.

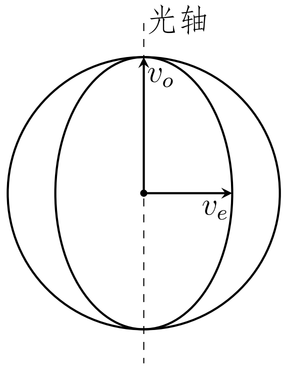
    o 光垂直于主平面, e 光平行于主平面.

    正晶体: $v_o > v_e,\ n_o < n_e$. (石英)

    负晶体: $v_o < v_e,\ n_o > n_e$. (方解石)

    较快光的光矢量振动的方向 $\to$ 快轴.

    较慢光的光矢量振动的方向 $\to$ 慢轴.



19. 正常色散柯西公式: $n=A+\dfrac{B}{\lambda^2}+\dfrac{C}{\lambda^4}$.

    色散率: $\nu=\dfrac{\mathrm{d}n}{\mathrm{d}\lambda}=-\dfrac{2B}{\lambda^3}<0$;

    而反常色散: $\nu>0$.

20. 散射强度: $I_\theta \propto \dfrac{1}{\lambda^4}$.

    分子小: 瑞利散射 (分子散射); 分子大: 米氏散射 (廷德尔散射).

6 Astrophysics and Relativity

18. 晶体光学: 1. Parsec: 秒差距 $= 3.26$ 光年.

$1^\circ = 60'$ arcminute 角分 $= 3600''$ arcsecond 角秒.

宇宙背景辐射: $\lambda T = C \Rightarrow T = \dfrac{C}{\lambda} \propto \dfrac{1}{r}(\lambda \propto r$, r 为宇宙半径)

2. 红移量: $z = \dfrac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}$, 当 $v \ll c$ 时, 有 $z \approx \dfrac{v}{c} = \beta$.

3. 哈勃定律: $v = H_0 D \Rightarrow \dfrac{1}{H_0}$ 给出宇宙年龄.

4. 黑洞: Schwarzschid 半径 $\dfrac{1}{2}mc^2 - \dfrac{GMn}{r} = 0 \Rightarrow r = \dfrac{2GM}{c^2}$.

5. 洛伦兹变换: $\beta = \dfrac{v}{c}$, $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \approx 1 + \dfrac{1}{2}\dfrac{v^2}{c^2} \Rightarrow \dfrac{1}{\gamma} = 1 - \dfrac{1}{2}\dfrac{v^2}{c^2}$.

四维坐标 (x_1, x_2, x_3, ict).

坐标变换:

$$\begin{cases} x' = \dfrac{x - vt}{\sqrt{1 - v^2/c^2}}, \\ y' = y,\ z' = z, \\ t' = \dfrac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}. \end{cases}$$

速度变换:

$$\begin{cases} u_x = \dfrac{u'_x + v}{1 + \dfrac{vu'_x}{c^2}},\ u_y = \dfrac{u'_y\sqrt{1-\dfrac{v^2}{c^2}}}{1 + \dfrac{vu'_x}{c^2}},\ u_z = \dfrac{u'_z\sqrt{1-\dfrac{v^2}{c^2}}}{1 + \dfrac{vu'_x}{c^2}}; \\ u'_x = \dfrac{u_x - v}{1 - \dfrac{vu_x}{c^2}},\ u'_y = \dfrac{u_y\sqrt{1-\dfrac{v^2}{c^2}}}{1 - \dfrac{vu_x}{c^2}},\ u'_z = \dfrac{u_z\sqrt{1-\dfrac{v^2}{c^2}}}{1 + \dfrac{vu_x}{c^2}}; \end{cases}$$

6. 间隔: $s^2 = c^2\Delta t^2 - (\Delta x)^2$.

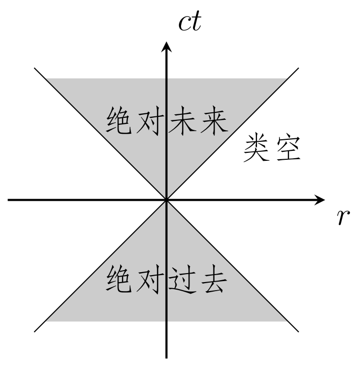
<1> $s^2 = 0$, 类光;

<2> $s^2 > 0$, 类时, 可用低于光速的作用来连接;

<3> $s^2 < 0$, 类空, 无因果关系，绝对异地;

$E = mc^2 = \dfrac{m_0c^2}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{m_0c^2}{\sqrt{1-\dfrac{p^2c^2}{E^2}}}$

$\Rightarrow E^2 = p^2c^2 + m_0^2c^4$.



对于光子有 $p = \dfrac{h}{\lambda} = \dfrac{h\nu}{c} = \dfrac{E}{c}$.

7. 相对论多普勒效应: $f' = \sqrt{\dfrac{1 \pm \beta}{1 \mp \beta}} f$(注意是 β, 不是 β^2)

横向多普勒效应: $f' = \sqrt{1 - \dfrac{v^2}{c^2}} f$(红移).

8. 电磁场:

$$E'_x = E_x,\ E'_y = \gamma(E_y - vB_z),\ E'_z = \gamma(E_z + vB_y);$$
$$B'_x = B_x,\ B'_y = \gamma(B_y + \frac{v}{c^2}E_z),\ B'_z = \gamma(B_z - \frac{v}{c^2}B_y).$$

9. 切连科夫辐射:

当介质中光速小于带电粒子速度 v 时即 $\dfrac{c}{n} < v$, 就会产生光子震波，如下图所示,

辐射角度满足 $\cos\theta = \dfrac{c}{nv} = \dfrac{1}{n\beta}$, 其中 $\beta = \dfrac{v}{c}$.

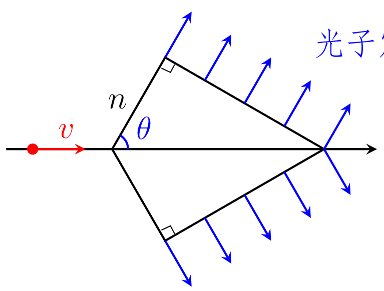


图 2: 切连科夫辐射

7 Quantum Mechanics

1. 对易: $[AB,C]=A[B,C]+[A,C]B$; 标准差: $\sigma=\sqrt{\langle j^2\rangle-\langle j\rangle^2}$.
   泡利矩阵:
$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$
   $\vec{S}=\frac{\hbar}{2}\sigma$.

2. 含时薛定谔方程 $i\hbar\frac{\partial H}{\partial t}=\hat{H}\Psi$, $\Psi(t)=\Psi(0)e^{-\frac{iHt}{\hbar}}$.
   定态薛定谔方程 $\hat{H}\Psi=E\Psi\Rightarrow-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi+V\Psi=E\Psi$.
   可观测量 Q 的期望值 $\langle\hat{Q}\rangle=\langle\Psi|\hat{Q}\Psi\rangle=\int\Psi^*\hat{Q}\Psi\mathrm{d}x$.

3. 动量对应算符 $p\rightarrow\frac{\hbar}{i}\frac{\partial}{\partial x}$ 或者 $p\rightarrow-i\hbar\nabla$.
   $[\hat{x},\hat{p}]=i\hbar$.
   球谐函数 $Y_l^m=\Theta(\theta)e^{im\phi}$, $Y_0^0=\frac{1}{2}\sqrt{\frac{1}{\pi}}$.
   若波函数为 $\cos m\phi=\frac{e^{im\phi}+e^{-im\phi}}{2}\Rightarrow$ 本征值为 $m\hbar$ 和 $-m\hbar$.

4. 标准边界条件:
$$\begin{cases}\Psi(x)\text{连续};\\ \frac{\mathrm{d}\Psi}{\mathrm{d}x}\text{在势能不为无穷大处连续};\end{cases}$$
   普朗克长度 $l_p=\sqrt{\frac{\hbar G}{c^3}}\approx10^{-35}m$.
   概率流: $\vec{J}=\frac{\hbar}{2mi}(\Psi^*\nabla\Psi-\Psi\nabla\Psi^*)$.

5. 单态: $|0\ 0\rangle=\frac{1}{\sqrt{2}}\uparrow\downarrow-\frac{1}{\sqrt{2}}\downarrow\uparrow$.
   三重态:
$$\begin{cases}|1\ 1\rangle=\uparrow\uparrow\\ |1\ 0\rangle=\frac{1}{\sqrt{2}}\uparrow\downarrow+\frac{1}{\sqrt{2}}\downarrow\uparrow\\ |1\ -1\rangle=\downarrow\downarrow\end{cases}$$

6. 无穷深势阱:
   在 $0\leqslant x\leqslant a$, 波函数 $\Psi_n(x)=\sqrt{\frac{2}{a}}\sin(\frac{n\pi}{a}x)$, $n=1,2,3\ldots$, 有 $n-1$ 个节点,

能量 $E_n = \dfrac{n^2\pi^2\hbar^2}{2ma^2}$.

7. 谐振子:

能量 $E_n = \hbar\omega(n+\dfrac{1}{2})$,

波函数 $\Psi_0 = \left(\dfrac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\xi^2/2}$, $\xi = \sqrt{\dfrac{m\omega}{\hbar}}x$, Ψ_n 的奇偶性同 n.

8. δ 函数势 $V = -\alpha\delta(x)$.

考虑束缚态 $E < 0 \Rightarrow x < 0, -\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}\Psi = E\Psi$, 即 $\dfrac{\partial^2}{\partial x^2}\Psi = \kappa^2\Psi,\ \kappa = \dfrac{\sqrt{-2mE}}{\hbar}$,

故 $\Psi(x) = Ae^{-\kappa x} + Be^{\kappa x}$. ($x \to -\infty$ 需收敛, 故 $A = 0$)

同理 $\Psi(x) = Fe^{-\kappa x}$ $(x > 0)$.

边界条件 $x = 0$ 时有 $B = F$,

且 $-\dfrac{\hbar^2}{2m}\displaystyle\int_{-\varepsilon}^{\varepsilon}\dfrac{\partial^2\Psi}{\partial x^2}\mathrm{d}x + \int_{-\varepsilon}^{\varepsilon}V(x)\Psi\mathrm{d}x = \int_{-\varepsilon}^{\varepsilon}E\Psi(x)\mathrm{d}x$, (积分薛定谔方程)

$\Delta\left(\dfrac{\partial\Psi}{\partial x}\right) = \dfrac{2m}{\hbar^2}\displaystyle\int_{-\varepsilon}^{\varepsilon}V(x)\Psi(x)\mathrm{d}x = -\dfrac{2m\alpha}{\hbar^2}\Psi(0)$.

故 $\Delta\left(\dfrac{\partial\Psi}{\partial x}\right) = 2B\kappa = -\dfrac{2m\alpha}{\hbar^2}B \Rightarrow \kappa = \dfrac{m\alpha}{\hbar^2}$,

故 $\Psi(x) = \dfrac{\sqrt{m\alpha}}{\hbar}e^{-m\alpha|x|/\hbar^2},\ E = -\dfrac{m\alpha^2}{2\hbar^2}$,

可见只有一个束缚态解.

有限深势阱至少有 1 个束缚态解.

共振透射 $T = 1 \Rightarrow k'b = n\pi$.

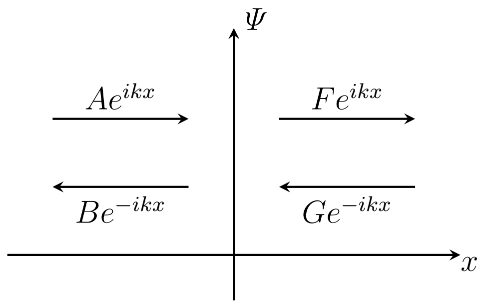


图 3: 边界反射与透射

9. 一阶微扰 $E_n^{(1)} = \langle\Psi_n^{(0)}|\,H'\,|\Psi_n^{(0)}\rangle$.

一阶波函数 $\Psi_n^{(1)} = \displaystyle\sum_{m\neq n}\dfrac{\langle\Psi_m^{(0)}|\,H'\,|\Psi_m^{(0)}\rangle}{E_n^0 - E_m^0}\Psi_m^0$.

二阶微扰 $E_n^{(2)} = \displaystyle\sum_{m\neq n}\dfrac{|\langle\Psi_m^{(0)}|\,H'\,|\Psi_m^{(0)}\rangle|^2}{E_n^0 - E_m^0} = \langle\Psi_n^{(1)}|\,H'\,|\Psi_n^{(1)}\rangle$.

变分法: $\langle\Psi|\,H\,|\Psi\rangle \leqslant E_g$,

试探函数一般可取 $\Psi(x) = Ae^{-bx^2}$ 或氢原子基态波函数 $\Psi_{100} = \dfrac{1}{\sqrt{\pi a^3}}e^{-r/a}$.

10. 不确定原理:

$\sigma_A^2\sigma_B^2 \leqslant (\dfrac{1}{2i}\langle[A,B]\rangle)^2$.

如 $\sigma_x\sigma_p \leqslant \dfrac{\hbar}{2}$, $\sigma_E\sigma_t \leqslant \dfrac{\hbar}{2}$.

气体密度 $\rho\uparrow$, $\sigma_t\downarrow$, $\sigma_E\uparrow$, 故谱线越宽.

11. 角动量: $L^2\Psi = l(l+1)\hbar^2\Psi$, $L_z\Psi = m\hbar\Psi$.

   对易: $[r_i, r_j] = [p_i, r_j] = 0,\ [r_i, p_j] = i\hbar\delta_{ij}$,

   $[L_x, L_y] = i\hbar L_z, [L^2, L_i] = 0 \Rightarrow [L^2, \vec{L}] = 0$.

   令 $L_\pm = L_x \pm iL_y$, 则 $[L_z, L_\pm] = \hbar L_\pm,\ [L^2, L_\pm] = 0$.

   且 $L_\pm f_l^m = \hbar\sqrt{l(l+1) - m(m\pm1)} f_l^{m\pm1}$.

12. 氢原子: $E_n = -E_1(\frac{Z^2}{n^2}\frac{\mu}{\mu_e}),\ E_1 = \frac{q^4}{8\hbar^2\varepsilon_0^2} = 12.6eV$.

   一般情况 $\mu \approx \mu_e \approx m_e$, 对于 positronium, $\mu = \frac{m_e^2}{m_e + m_e} = \frac{1}{2}m_e$.

   轨道电子速度 $v_n = \frac{\alpha c}{n}$, 其中 α 为精细结构常数 $\frac{1}{137}$.

   轨道半径 $r_n \approx a_0(\frac{m_e}{\mu}\frac{n^2}{Z})$, 其中 $a_0 \approx 0.53\mathring{A}$.

   基态波函数 $\Psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$, 第一激发态 $\propto (1 - \frac{r}{2a})e^{-r/2a}$.

13. 磁矩 $\mu = -\frac{e}{2m}L$, 自旋 $\mu_s = -\frac{ge}{2m}S$, 朗德因子 $g = 2$, 旋磁比 $\gamma = \frac{\mu_s}{S} = -\frac{ge}{2m}$.

14. Stern-Gerlach 实验: Ag 原子 $5s^1$ 电子 $|\Psi\rangle = \frac{1}{\sqrt{2}}\uparrow\downarrow - \frac{1}{\sqrt{2}}\downarrow\uparrow$ 经过磁场分裂为两束.

15. 弗兰克-赫兹实验: 非弹性碰撞才会吸收电子能量.

16. 康普顿散射: $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$. (务必记住此公式)

17. 精细结构 split with j: <1> 相对论修正; <2> 自旋-轨道耦合.

18. 横截面微分 $D(\theta) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$, 其中 $\mathrm{d}\Omega = \sin\theta\mathrm{d}\theta\mathrm{d}\phi$.

19. 更多请参看量子力学笔记部分...

8 Solid State Physics

1. $V = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)$.

   倒格子: $\vec{b}_1 = 2\pi \dfrac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$, $\vec{b}_1 = 2\pi \dfrac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$, $\vec{b}_1 = 2\pi \dfrac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$.

2. 自由电子气: $E_F = \dfrac{\hbar^2 k_F^2}{2m}$, $k_F = (3\rho\pi^2)^{\frac{1}{3}}$, ρ 为电子数密度.

   推导: 在 $T = 0$ 时, 费米分布 $f_0(E) = \begin{cases} 1, E \leqslant E_F \\ 0, E > E_F \end{cases}$,

   态密度 $N(E) = \dfrac{V}{2\pi^2} \left(\dfrac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$

   $$N = \int_0^\infty f_0(E) \cdot N(E) \mathrm{d}E = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \times \int_0^{E_F} E^{\frac{1}{2}} \mathrm{d}E.$$

   可得 $E_F = \dfrac{\hbar^2}{2m} \left(\dfrac{3\pi^2 N}{V}\right)^{\frac{2}{3}}$, 故 $k_F = \left(\dfrac{3\pi^2 N}{V}\right)^{\frac{1}{3}}$.

3. 有效质量: $\dfrac{\mathrm{d}^2 E}{\mathrm{d}k^2} = \dfrac{\hbar^2}{m}$, 故 $m^* = \dfrac{\hbar^2}{\left(\dfrac{\mathrm{d}^2 E}{\mathrm{d}k^2}\right)}$.

9 Particle Physics

1. J/Ψ meson 介子: 由一个 charm quark 和一个 anti charm quark 组成.
   Deutron: 氘核.
   自旋: 光子 photon= 1, 电子 $=\frac{1}{2}$, 质子 proton= $\frac{1}{2}$.

2.
$$\text{hadron 强子}\begin{cases}\text{meson 介子 (boson): a quark and an antiquark.}\\ \text{baryon 重子 (fermion): 3 quarks}(\frac{1}{3}).\end{cases}$$

baryon number $B(\pm1)$.

$$\text{lepton 轻子}\begin{cases}\text{电子 electron}(e^-)\text{ 和电子中微子 electron neutrino}(\nu_e),\\ \mu\text{子 muon}(\mu^-)\text{ 和}\mu\text{子中微子 muon neutrino}(\nu_\mu),\\ \tau\text{子 tau}(\tau)\text{ 和}\tau\text{子中微子 tau neutrino}(\nu_\tau).\end{cases}$$

轻子数守恒: lepton number L conservation.

3. 汤川秀树预言的介子是 π 介子, 不是 μ 子.

4. μ 子衰变: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$.
   α 衰变: ${}^{A}N \to {}^{A-4}L + {}^{4}He$.
   β 衰变: $\beta^-: N \to O + e^- + \bar{\nu}_e$ $(n \to p + e^- + \bar{\nu}_e)$,
   $\beta^+: N \to O + e^+ + \nu_e$ $(p \to n + e^+ + \nu_e)$.
   β^- 衰变更普遍，因为中子数量更多.

10 Atomic Physics and Nuclear Physics

1. $^{2S+1}L_J$, 锂 Lithium, 钠 Sodium.

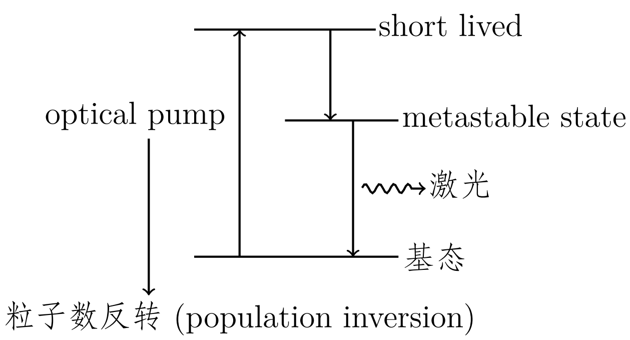
2. 选择定则: $\Delta m_l = \pm 1, 0$; $\Delta l = \pm 1$; $\Delta j = 0, \pm 1$; $\Delta m_s = 0$.
   洪特法则: <1> $S\uparrow$, $E\downarrow$;
   <2> $L\uparrow$, $E\downarrow$;
   <3> 如果壳层少于半满, $J = |L - S|$, $E\downarrow$,
   如果壳层多于半满, $J = L + S$, $E\downarrow$,

3. 塞曼效应: $U = -\vec{\mu}\cdot\vec{B} = -\dfrac{ge}{2m}\vec{J}\cdot\vec{B}$.
   玻尔磁子: $\mu_B = \gamma\hbar = \dfrac{e\hbar}{2m}$.
   $h\nu' = h\nu + (m_2g_2 - m_1g_1)\mu_B B$, 正常塞曼效应 $S = 0$, $g_1 = g_2 = 1$, $\Delta m = 0, \pm 1$,
   $$故:h\nu' = h\nu + \begin{pmatrix} \mu_B B \\ 0 \\ -\mu_B B \end{pmatrix} \begin{matrix} \sigma^+左旋 \\ \pi线偏振 \\ \sigma^-右旋 \end{matrix}$$
   $$\begin{cases} 平行于B方向只能看到\sigma^{\pm}\ 2条 \\ 垂直B方向可以看到3条 \end{cases}$$

3. 斯塔克效应 (E): $U = \vec{p}\cdot\vec{E} = -qEr\cos\theta$.

4. X 射线 (exicite inner electrons of element near the nuclear)→Auger transition 俄歇跃迁.
   考虑 shielding 作用: $E = 13.6eV(\dfrac{3}{4})(Z-1)^2$.

5. 激光 (受激发射): 发射光子与入射光子有相同的 ω、ϕ 和偏振方向...
   条件: 1. 相干 coherent.
   2. 单色 monochromatic.
   3. minimal divergence.
   4. high intensity.



6. 几种常用激光:
   Diod Laser: active medium is semiconducting. (p-n 结)
   Gas Laser: active medium is free gas. (电流 $\xrightarrow{激发}$atoms)

7. Nucleus is most likely to emit electrons in the direction opposite to the magnetic field.

8. $r \approx r_0 A^{\frac{1}{3}}$, $r_0 \approx 1.2fm$, 结合能: $E_B = \sum_i m_i c^2 - Mc^2$, 核子 nucleon.

9. 半衰期: $N = N_0 e^{-\lambda t}$, $R = -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N$, $t_{1/2} = \frac{\ln 2}{\lambda}$,
   平均寿命 mean life: $\tau = \frac{1}{\lambda}$.
   若 $\frac{\mathrm{d}N}{\mathrm{d}t} = -(\lambda_A + \lambda_B)N \Rightarrow N = N_0 e^{-(\lambda_A+\lambda_B)N}$, 则 $\frac{1}{t_{1/2}} = \frac{1}{t_{1/2}^A} + \frac{1}{t_{1/2}^B}$.

10. 单位 $Gray = \frac{Energy}{Unit\ mass}$, 即 $1Gy = \frac{1J}{1kg}$.
    韧致辐射 Bremsstrahlung: 对电荷 $P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} \propto q^2 \propto a^2$; 对电偶极子 $P = \frac{\omega^4}{12\pi\varepsilon_0 c^3} p^2$.

11. 卢瑟福散射: $\frac{1}{2}mv^2 = k\frac{2e^2}{r}$.

12. $\frac{1}{\lambda} = R(\frac{1}{n^2} - \frac{1}{n'^2})$:
    $n = 1$, 莱曼系;
    $n = 2$, 巴耳末系 $\begin{cases} H-\alpha\text{线},\ 3 \to 2 \\ H-\beta\text{线},\ 4 \to 2 \\ H-\gamma\text{线},\ 5 \to 2 \end{cases}$
    $n = 3$, 帕刑系.

11 Mathematics and Ohters

1. $\sum n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

2. Fourier Series: $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\dfrac{n\pi x}{L}) + b_n \sin(\dfrac{n\pi x}{L})$,

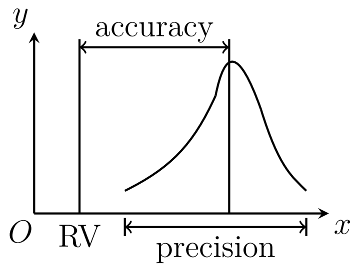
   其中 $a_n = \dfrac{1}{L}\int_{-L}^{L} f(t)\cos(\dfrac{n\pi t}{L})\mathrm{d}t$, $b_n = \dfrac{1}{L}\int_{-L}^{L} f(t)\sin(\dfrac{n\pi t}{L})\mathrm{d}t$.

   傅里叶变换: $f(\omega) = \dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\mathrm{d}t$.

3. 误差分析:

   $\left.\begin{array}{l}\text{Accuracy}\\ \text{Precision}\end{array}\right\}$ 区别

   RV(Real Value) 代表实际值.



4. 泊松分布: $P_\mu(\nu) = e^{-\mu}\dfrac{\mu^\nu}{\nu!}$,

   即 $P(X=k) = e^{-\lambda}\dfrac{\lambda^k}{k!}$, 如下图所示.

   标准差 $\sigma = \sqrt{\mu}$, 若重复 N 次, 则 $\sigma_N = \dfrac{\sigma}{\sqrt{N}}$.

   $3-\sigma$ 原则对应概率: 0.6826, 0.9544, 0.9974.

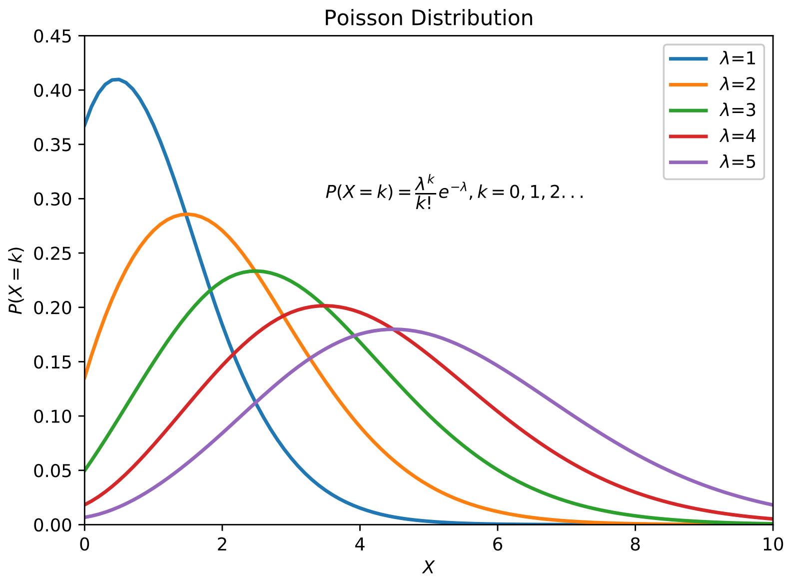


图 4: Poisson Distribution.

5. 不确定度:

   <1> $x = a + b + \cdots \Rightarrow \mathrm{d}x = \mathrm{d}a + \mathrm{d}b + \cdots \Rightarrow (\delta x)^2 = (\delta a)^2 + (\delta b)^2 + \cdots$;

   <2> $x = a \cdot b \cdots \Rightarrow \frac{\mathrm{d}x}{x} = \frac{\mathrm{d}a}{a} + \frac{\mathrm{d}b}{b} + \cdots \Rightarrow (\frac{\delta x}{x})^2 = (\frac{\delta a}{a})^2 + (\frac{\delta b}{b})^2 + \cdots$.

   N 次测量得到的值和不确定度分别为 x_i 和 σ_i, 则每次的权重 $\omega_i = \frac{1}{\sigma_i^2}$,

   则 $x_{avg} = \frac{\sum \omega_i x_i}{\sum \omega_i}$, $\sigma_{avg} = \frac{1}{\sqrt{\sum \omega_i}}$.

   若多次测量均有 $\omega_i = \omega$, 则 $x_{avg} = \frac{\sum x_i}{N}$, $\sigma_{avg} = \frac{\sigma}{\sqrt{N}}$.

6. 原子核横截面积 $\pi(10^{-14})^2 m^2 = 10^{-28} m^2$;

   空气分子碰撞横截面积 $\sigma = 10^{-18} m^2$;

   空气粒子数密度 $n = 10^{25} m^{-3}$;

   空气分子平均自由程 $l_p = \frac{1}{\sigma n} = 10^{-7} m$.