Notes for GRE Physics

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2019-11

Preface

For test takers of GRE Physics, there are few materials to prepare. I took the test on Oct. 26, 2019 in Shanghai and got a full mark 990 (95%). Therefore, I plan to organize my notes based on Jeff's version...

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目录

1 Classic Mechanics

1. $\nabla \times \vec{F} = 0 \Rightarrow \vec{F} = -\nabla U.$

2. Projectile.

3. $$\frac{1}{k_t} = \frac{1}{k_1} + \frac{1}{k_2}; \qquad k_t = k_1 + k_2;$$

$$\lambda = \frac{R}{2}\sqrt{\frac{C}{L}} \quad \begin{cases} > 1 : \text{overdamped} \quad ; \\ = 1 : \text{critically damped} \quad ; \\ < 1 : \text{underdamped} \quad \Rightarrow \quad : \omega_d = \omega_0\sqrt{1 - \dfrac{b}{4mk}} : b \quad . \end{cases}$$

4. impulse.

5. the moment of inertia; fulcrum pendulum$\Rightarrow \omega = \sqrt{\dfrac{g}{l}}$.

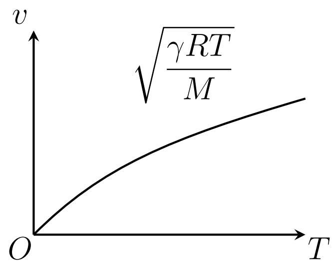
6. centripetal: ;
   centrifugal: .

7. $$\vec{F}_c = -2m\vec{\omega} \times \vec{v}$$

8. $$E = \frac{1}{2}mv^2 + V(r) = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + V(r) \qquad L = mr^2\dot{\theta}$$
$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r) = \frac{1}{2}m\dot{r}^2 + V_{eff}$$

9. $$\frac{1}{2}\rho v^2 + \rho g h + P = C;$$
$$v_i A_i = C;$$
$F_{drag} = -6\pi\mu R v_s$, μ v_s .

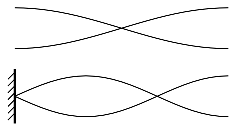
10. $v = \sqrt{\dfrac{T}{\mu}}$, T ,
μ



1: .

11. $$I = \frac{P_s}{4\pi r^2} \propto \frac{1}{r^2};$$
$$\beta = 10\log\frac{I}{I_0}(\text{dB}),\ I_0 \quad 10^{-12};$$
$$v = \sqrt{\frac{\gamma RT}{M}} \propto \sqrt{T},$$
1 .

12. =



13. $$L = n\frac{\lambda}{2};$$
$$L = \frac{2n+1}{4}\lambda$$
harmonics: $n = Round\left(\frac{f_2}{f_1}\right)$ beats= $n \times f_1 - f_2$.

14. $$m\frac{\mathrm{d}v}{\mathrm{d}t} + u\frac{\mathrm{d}m}{\mathrm{d}t} = 0,\ u$$

15.
$$L(q, \dot{q}, t) = T - U.$$
action $$S = \int L\mathrm{d}t.$$
actual path $\delta \int L\mathrm{d}t = 0,$ actual S

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0, \qquad : p = \frac{\partial L}{\partial \dot{q}},$$

$$ma = \frac{\mathrm{d}p}{\mathrm{d}t} = F.$$

$$H(q, p, t) = \sum_i p_i\dot{q}_i - L.$$

$$\dot{q} = \frac{\partial H}{\partial p},\ \dot{p} = -\frac{\partial H}{\partial q},\ \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t},$$

$$\frac{\partial H}{\partial q} = -\frac{\partial L}{\partial q} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = -\dot{p}.$$

2 Electrodynamics

1. ·

$$B = \frac{\mu_0}{4\pi}\int \frac{I \times \hat{r}}{r^2}\mathrm{d}l = \frac{\mu_0}{4\pi}\int \frac{J \times \hat{r}}{r^2}\mathrm{d}V.$$

2. Maxwell

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}; \\ \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}; \\ \nabla \cdot \vec{B} = 0; \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \Rightarrow \qquad : \vec{J}_D = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} = \dfrac{\partial \vec{D}}{\partial t}. \end{cases}$$

3. $\vec{P} = \dfrac{\sum \vec{p}_i}{\Delta V}$,

$$\left.\begin{array}{l} \vec{D} = \varepsilon_0 \vec{E} + \vec{P} \\ \vec{P} = \chi_e \varepsilon_0 \vec{E} \end{array}\right\} \Rightarrow \vec{D} = (1+\chi_e)\varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}.$$

4. $\vec{M} = \dfrac{\sum \vec{m}_i}{\Delta V}$,

$$\vec{B} = \mu \vec{H} = (1+\chi_M)\mu_0 \vec{H} = \mu_0 \vec{H} + \mu_0 \vec{M}.$$

5.

$$\begin{aligned} \rho_P = -\nabla \cdot \vec{P} &\Rightarrow \sigma_P = -(P_2 - P_1); \\ \rho_P + \rho_f = \varepsilon_0 \nabla \cdot \vec{E} &\Rightarrow \sigma_P + \sigma_f = \varepsilon_0 (E_2 - E_1); \\ \rho_f = \nabla \cdot \vec{D} &\Rightarrow \sigma_f = D_2 - D_1. \end{aligned}$$

6.

$$\begin{cases} \qquad \vec{J}_M = \nabla \times \vec{M}; \\ \qquad \vec{J}_P = \dfrac{\partial \vec{P}}{\partial t}, \end{cases}$$

$$= \qquad + \qquad + \qquad \frac{1}{\mu_0}\nabla \times \vec{B} = \vec{J}_f + \vec{J}_M + \vec{J}_P + \vec{J}_D.$$

7. Maxwell

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}; \\ \nabla \cdot \vec{D} = \rho_f; \\ \nabla \cdot \vec{B} = 0; \\ \nabla \times \vec{H} = \vec{J}_f + \dfrac{\partial \vec{D}}{\partial t}. \end{cases}$$

1:

diamagnetic	$\mu < \mu_0$ $\chi_M < 0$	(Lenz's Law)
paramagnetic	$\mu > \mu_0$ $\chi_M > 0$	
ferromagnetic	$\mu \gg \mu_0$ $\chi_M \gg 0$	

8. $$\vec{S} = \vec{E} \times \vec{H}.$$
$$\vec{J} = \sigma \vec{E}.$$

9.
$$\begin{cases} \vec{e}_n \times (\vec{E}_2 - \vec{E}_1) = 0; \\ \vec{e}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}; \\ \vec{e}_n \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_f; \\ \vec{e}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0. \end{cases}$$

10. $$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t},$$
$$\omega = \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2),$$
$$\omega = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}), \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

11. $$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A},$$
$$\nabla \times \vec{E} = 0 \Rightarrow \vec{E} = -\nabla U - \frac{\partial A}{\partial t}.$$

12.
$$P_r = \frac{I}{c};$$
$$P_r = \frac{2I}{c}.$$

13. $$\vec{B} = \frac{1}{c} \vec{k} \times \vec{E} \Rightarrow B = \frac{E}{c}.$$

14. $$C = \frac{Q}{U}, \; E = \frac{1}{2} QU = \frac{1}{2} CU^2.$$

15. $$E = \frac{1}{2\pi\varepsilon_0} \frac{p}{z^3},$$

$$U = -\vec{p} \cdot \vec{E},\ \ U = -\vec{\mu} \cdot \vec{B}.$$

16. (solenoid) : $B = \mu_0 n I$

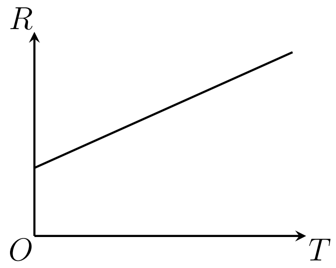
17. valence band ;
    conduction band .

18. BCS theory $\rightarrow$ bosonic state.
    Meissner effect: expel any magnetic field.

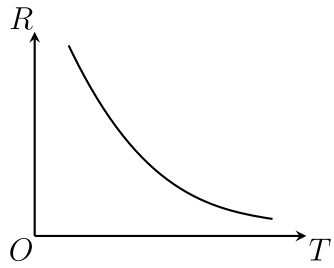
19. $$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}.$$

3 Electronics

1. Metal:



Semiconductor:



2. Capacitor: $q = c\varepsilon(1 - e^{-\frac{t}{\tau_c}})$, $\tau_C = RC$, $X_C = \dfrac{1}{\omega C} \propto \dfrac{1}{\omega}$.

3. Inductor: $\Phi = LI$, $\varepsilon = -\dfrac{\mathrm{d}\Phi}{\mathrm{d}t} = -L\dfrac{\mathrm{d}i}{\mathrm{d}t}$, $X_L = \omega L \propto \omega$,
$U = \dfrac{1}{2}LI^2$, $\tau_L = R/L$.

5. LC : $\omega = \dfrac{1}{\sqrt{LC}}$;

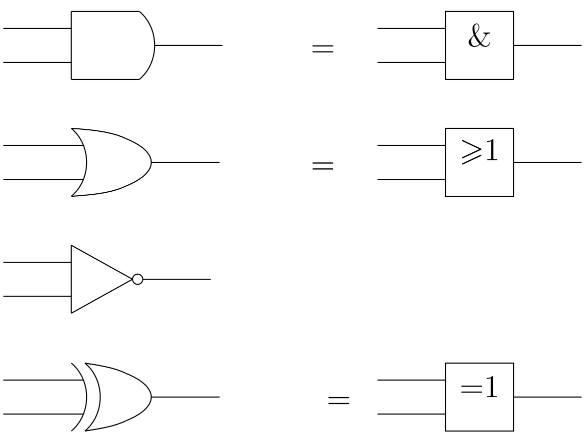
RLC : $\omega' = \sqrt{\omega^2 - \left(\dfrac{R}{2L}\right)^2}$;

: $Z = R + i\omega L + \dfrac{1}{i\omega C}$, $|Z| = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$;

: $\omega_{resonant} = \dfrac{1}{\sqrt{LC}}$.

6. Thevenin .

7. :



A B 0 A B 1.

4 Thermodynamics and Statistical Physics

1. $E = \frac{\nu}{2} N k_B T$, ν .

2. $pV = nRT = Nk_BT$. ($nR = Nk_B$)

$$n(v) = Av^2 e^{-mv^2/2k_BT},$$
$$v_{mode} = \sqrt{\frac{2RT}{M}};$$
$$v_{rms} = \sqrt{\frac{3RT}{M}};$$
$$v_{avg} = \sqrt{\frac{8RT}{\pi M}};$$
$$v_{mode} < v_{avg} < v_{rms}.$$

$$\lambda = \frac{1}{\sigma n} = \frac{1}{\sqrt{2}\pi d^2 n}, \ (\sigma \qquad)$$
$$\tau = \frac{\lambda}{v}, \qquad \frac{1}{2}mv^2 = \frac{\nu}{2}k_BT, \quad \tau = \frac{\sqrt{m}\lambda}{\sqrt{\nu k_B T}}.$$

3. $C_P - C_V = nR$;
Isothermal: ; Isobaric: ;
Isochoric: ; adiabatic: $\Rightarrow pV^\gamma = C$, ($\gamma = \frac{C_P}{C_V} > 1$,).

4. Partition Function $Z = \sum_s g_s e^{-\beta\varepsilon_s}$ ($\beta = \frac{1}{k_BT}$), g_s ;

: $Z = \prod_{i=1}^{N} \zeta_i$; ($\zeta_i$)

$$\langle E \rangle = \frac{1}{Z}\sum_s \varepsilon_s e^{-\beta\varepsilon_s} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta}.$$
$$P(s) = \frac{e^{-\beta\varepsilon_s}}{Z}, \qquad : \quad X \gg 1, \ln X! \approx X(\ln X - 1).$$

$$n(\varepsilon_s) = \frac{g_s}{e^{(\varepsilon_s-\mu)\beta}} \Rightarrow a_l = \frac{\omega_l}{e^{(\varepsilon_l-\mu)\beta}};$$
$$n(\varepsilon_s) = \frac{g_s}{e^{(\varepsilon_s-\mu)\beta}+1} \Rightarrow a_l = \frac{\omega_l}{e^{(\varepsilon_l-\mu)\beta}+1};$$
$$n(\varepsilon_s) = \frac{g_s}{e^{(\varepsilon_s-\mu)\beta}-1} \Rightarrow a_l = \frac{\omega_l}{e^{(\varepsilon_l-\mu)\beta}-1};$$
$$\frac{a_l}{\omega_l} \ll 1 \qquad .$$

5. Entropy: $S = k_B \ln \Omega$, $\partial S = \frac{\partial Q}{T}$, $\quad \Delta S = \int_{T_1}^{T_2} \frac{\mathrm{d}Q}{T}$;

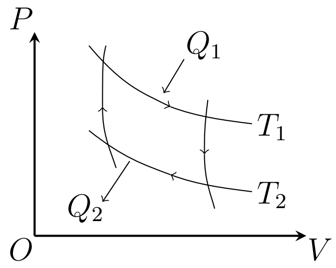
$\mathrm{d}S \geqslant \frac{đQ}{T}$, $\quad \mathrm{d}U \leqslant T\mathrm{d}S - p\mathrm{d}V$, $\qquad \mathrm{d}s \geqslant 0$.

$$\mathrm{d}U = \text{đ}Q - p\mathrm{d}V = T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N, \qquad \frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N};$$

$$:\ S = \frac{\partial}{\partial T}(k_B T \ln Z).$$

6. piston.

7.



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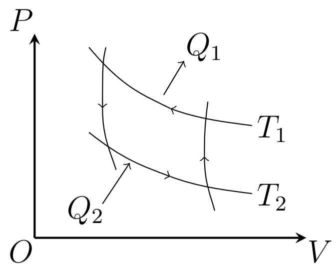
$$\Rightarrow \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_2}{T_1};$$



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$$\Rightarrow \eta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2};$$

8. Enthalpy: $H = E + pV$, $\mathrm{d}H = T\mathrm{d}S + V\mathrm{d}p$;

Helmholtz Free Energy: $F = E - TS$, $\mathrm{d}F = -S\mathrm{d}T - p\mathrm{d}V$;

Gibbs Free Energy: $G = E - TS + pV$, $\mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}p$.

: $Z = e^{-\beta F}$;

$\mathrm{d}U \leqslant T\mathrm{d}S - p\mathrm{d}V$ $\mathrm{d}F \leqslant 0$; $\mathrm{d}G \leqslant 0$.

9. : $P = \sigma T^4$;

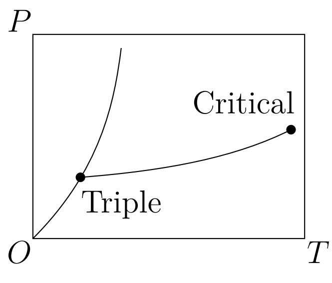
: $\lambda_{max} \cdot T = Constant$;

: $I(\nu, T) = \dfrac{2h\nu^3}{c^2}\dfrac{1}{e^{\frac{h\nu}{k_B T}} - 1}$.

10.

Critical ;

Triple .



11. $\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V$;

$$L = T(S^{(2)} - S^{(1)}); \qquad \Delta V = V^{(2)} - V^{(1)}.$$

L ΔV; .

12.

$$\varepsilon = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}, \qquad k \qquad \frac{(2\pi)^3}{V},$$

$$N(E) = \frac{4\pi k^2 \cdot \mathrm{d}k}{(2\pi)^3/V \cdot \mathrm{d}\varepsilon};$$

$$\times 2, \qquad N(E) = \frac{2V}{(2\pi)^3}\frac{4\pi k m}{\hbar^2} \propto \sqrt{E}.$$

5 Optics

1. : $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$.

2. : $\Delta E \Delta \tau \approx \hbar \Rightarrow \Delta\tau\Delta\nu \approx 1$,
: $v_p = \dfrac{\omega}{k}$, : $v_g = \dfrac{\mathrm{d}\omega}{\mathrm{d}k}$,
: $n = \dfrac{c}{v} = \sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}} \approx \sqrt{\dfrac{\varepsilon}{\varepsilon_0}}$.

3. : $\dfrac{1}{s'} + \dfrac{1}{s} = \dfrac{1}{f} = \dfrac{2}{r}$, magnification: $m = -\dfrac{s'}{s}$.
: $\dfrac{n'}{s'} - \dfrac{n}{s} = \dfrac{n'-n}{r}$.
: $\dfrac{1}{f'} = (n_0 - 1)(\dfrac{1}{r_1} - \dfrac{1}{r_2})$, ($n_0$),
$\dfrac{n'}{s'} - \dfrac{n}{s} = \dfrac{n_0 - n}{r_1} + \dfrac{n' - n_0}{r_2}$,
$\dfrac{1}{s'} - \dfrac{1}{s} = \dfrac{1}{f'}$.

4. Rayleigh $\theta_R = 1.22\dfrac{\lambda}{d} = 0.61\dfrac{\lambda}{r}$,
: $r = \theta_R f = (1.22\dfrac{\lambda}{D})f$.

5. : $\dfrac{1}{\lambda_s}(v \pm v_s) = \dfrac{1}{\lambda_D}(v \pm v_D)$.

6. (interference):
: $d\sin\theta = m\lambda$, (d);
: $d\sin\theta = (m + \dfrac{1}{2})\lambda$.
$y_m = \dfrac{m\lambda L}{d}$, ($L$).

7. (diffraction):
: $a\sin\theta = (m + \dfrac{1}{2})\lambda$, ($a$).
: $a\sin\theta = m\lambda$.

8. sharp :
$$y \approx a = D\cdot\theta,$$
$a\cdot\sin\theta \approx a\cdot\theta = \lambda$, $a = \sqrt{\lambda D}$.

9. (gratting) : $d\sin\theta = k\lambda$, $d = a + b$.

10. : $2d\sin\theta = k\lambda$.

11. ( ): $2nd = (m+\frac{1}{2})\lambda$.

12. :

n L : $N = \dfrac{2L}{\lambda}$, $N' = \dfrac{2Ln}{\lambda}$,

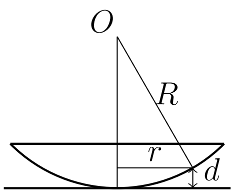
: $N' - N = \dfrac{2L}{\lambda}(n-1)$;

: $\Delta N = \dfrac{2L}{\lambda}$.

13. :

$$2d = (m+\frac{1}{2})\lambda,\ d = R - \sqrt{R^2 - r^2} \approx \frac{r^2}{2R},$$

$$r = \sqrt{R(m+\frac{1}{2})\lambda}.$$



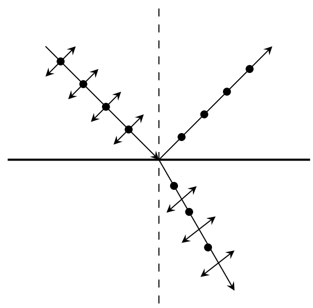
14. : $I = \dfrac{I_0}{2}$;

: $I = I_0\cos^2\theta$.

15. : $n_1\sin\theta_1 = n_2\sin\theta_2$;

: $\theta = \arctan\left(\dfrac{n_2}{n_1}\right)$,

.



16. $I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$.

17. : <1> : $D_\theta = \dfrac{\delta\theta}{\delta\lambda} = \dfrac{K}{d\cos\theta} \Leftarrow (d\sin\theta = K\lambda$).

<2> : $D_l = \dfrac{\delta l}{\delta\lambda} = D_\theta \cdot f = \dfrac{Kf}{d\cos\theta}$.

<3> : $R = \dfrac{\lambda}{\Delta\lambda}$,

$d\sin\theta = K(\lambda + \Delta\lambda)$ $d\sin\theta = (K + \frac{1}{N})\lambda$,

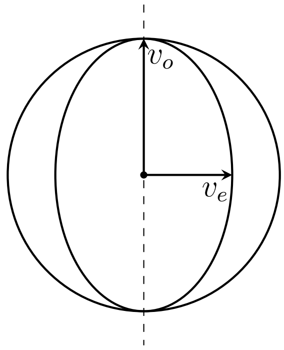
$R = KN = \dfrac{Nd\sin\theta}{\lambda}$,

Nd .

<4> ( ):

$$K(\lambda+\Delta\lambda)=(K+1)\lambda \Rightarrow \Delta\lambda=\frac{\lambda}{K}=\frac{\lambda^2}{d\sin\theta}.$$

18. :



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: .

o , e .

: $v_o > v_e,\ n_o < n_e$. ( )

: $v_o < v_e,\ n_o > n_e$. ( )

$\rightarrow$.

$\rightarrow$.

19. : $n = A + \dfrac{B}{\lambda^2} + \dfrac{C}{\lambda^4}$.

: $\nu = \dfrac{\mathrm{d}n}{\mathrm{d}\lambda} = -\dfrac{2B}{\lambda^3} < 0$;

: $\nu > 0$.

20. : $I_\theta \propto \dfrac{1}{\lambda^4}$.

: ( ); : ( ).

6 Astrophysics and Relativity

18. : 1. Parsec: $= 3.26$.

$1^\circ = 60'$ arcminute $= 3600''$ arcsecond .

: $\lambda T = C \Rightarrow T = \dfrac{C}{\lambda} \propto \dfrac{1}{r}(\lambda \propto r,\ r$)

2. : $z = \dfrac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}$, $v \ll c$, $z \approx \dfrac{v}{c} = \beta$.

3. : $v = H_0 D \Rightarrow \dfrac{1}{H_0}$.

4. : Schwarzschid $\dfrac{1}{2}mc^2 - \dfrac{GMn}{r} = 0 \Rightarrow r = \dfrac{2GM}{c^2}$.

5. : $\beta = \dfrac{v}{c}$, $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} \approx 1 + \dfrac{1}{2}\dfrac{v^2}{c^2} \Rightarrow \dfrac{1}{\gamma} = 1 - \dfrac{1}{2}\dfrac{v^2}{c^2}$.

(x_1, x_2, x_3, ict).

:

$$\begin{cases} x' = \dfrac{x - vt}{\sqrt{1 - v^2/c^2}}, \\ y' = y,\ \ z' = z, \\ t' = \dfrac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}. \end{cases}$$

:

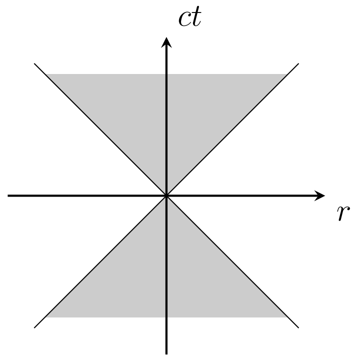
$$\begin{cases} u_x = \dfrac{u'_x + v}{1 + \dfrac{vu'_x}{c^2}},\ u_y = \dfrac{u'_y\sqrt{1 - \dfrac{v^2}{c^2}}}{1 + \dfrac{vu'_x}{c^2}},\ u_z = \dfrac{u'_z\sqrt{1 - \dfrac{v^2}{c^2}}}{1 + \dfrac{vu'_x}{c^2}}; \\ u'_x = \dfrac{u_x - v}{1 - \dfrac{vu_x}{c^2}},\ u'_y = \dfrac{u_y\sqrt{1 - \dfrac{v^2}{c^2}}}{1 - \dfrac{vu_x}{c^2}},\ u'_z = \dfrac{u_z\sqrt{1 - \dfrac{v^2}{c^2}}}{1 + \dfrac{vu_x}{c^2}}; \end{cases}$$

6. : $s^2 = c^2\Delta t^2 - (\Delta x)^2$.

<1> $s^2 = 0$, ;

<2> $s^2 > 0$, , ;

<3> $s^2 < 0$, , ;



$$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{m_0c^2}{\sqrt{1 - \dfrac{p^2c^2}{E^2}}}$$

$\Rightarrow E^2 = p^2c^2 + m_0^2c^4$.

$$p = \frac{h}{\lambda} = \frac{h\nu}{c} = \frac{E}{c}.$$

7. : $f' = \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} f($ $\beta,$ $\beta^2)$

: $f' = \sqrt{1 - \frac{v^2}{c^2}} f($ $).$

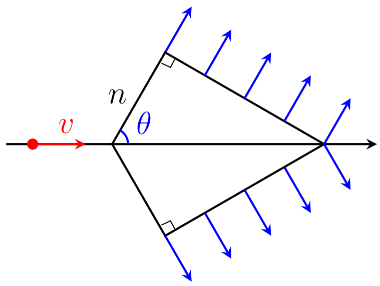
8. :

$$E'_x = E_x,\ E'_y = \gamma(E_y - vB_z),\ E'_z = \gamma(E_z + vB_y);$$
$$B'_x = B_x,\ B'_y = \gamma(B_y + \frac{v}{c^2}E_z),\ B'_z = \gamma(B_z - \frac{v}{c^2}B_y).$$

9. :

v $\frac{c}{n} < v,$,

$\cos\theta = \frac{c}{nv} = \frac{1}{n\beta},$ $\beta = \frac{v}{c}.$



2:

7 Quantum Mechanics

1. : $[AB, C] = A[B, C] + [A, C]B$; : $\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$.

:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{S} = \frac{\hbar}{2}\sigma$.

2. $$i\hbar\frac{\partial H}{\partial t} = \hat{H}\Psi,\ \Psi(t) = \Psi(0)e^{-\frac{iHt}{\hbar}}.$$

$$\hat{H}\Psi = E\Psi \Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi = E\Psi.$$

Q $\langle\hat{Q}\rangle = \langle\Psi|\hat{Q}\Psi\rangle = \int \Psi^*\hat{Q}\Psi \mathrm{d}x$.

3. $p \to \frac{\hbar}{i}\frac{\partial}{\partial x}$ $p \to -i\hbar\nabla$.

$[\hat{x}, \hat{p}] = i\hbar$.

$$Y_l^m = \Theta(\theta)e^{im\phi},\ Y_0^0 = \frac{1}{2}\sqrt{\frac{1}{\pi}}.$$

$$\cos m\phi = \frac{e^{im\phi} + e^{-im\phi}}{2} \Rightarrow$$ $m\hbar$ $-m\hbar$.

4. :

$$\begin{cases} \Psi(x) & ; \\ \dfrac{\mathrm{d}\Psi}{\mathrm{d}x} & ; \end{cases}$$

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-35}m.$$

: $\vec{J} = \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$.

5. : $|0\ 0\rangle = \frac{1}{\sqrt{2}}\uparrow\downarrow - \frac{1}{\sqrt{2}}\downarrow\uparrow$.

:

$$\begin{cases} |1\ 1\rangle = \uparrow\uparrow \\ |1\ 0\rangle = \frac{1}{\sqrt{2}}\uparrow\downarrow + \frac{1}{\sqrt{2}}\downarrow\uparrow \\ |1\ -1\rangle = \downarrow\downarrow \end{cases}$$

6. :

$0 \leqslant x \leqslant a$, $\Psi_n(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi}{a}x),\ n = 1, 2, 3\ldots,$ $n-1$,

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}.$$

7. :

$$E_n = \hbar\omega(n + \frac{1}{2}),$$

$$\Psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\xi^2/2},\ \xi = \sqrt{\frac{m\omega}{\hbar}}x,\ \Psi_n \qquad n.$$

8. δ $V = -\alpha\delta(x)$.

$$E < 0 \Rightarrow x < 0, -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi = E\Psi, \qquad \frac{\partial^2}{\partial x^2}\Psi = \kappa^2\Psi,\ \kappa = \frac{\sqrt{-2mE}}{\hbar},$$

$\Psi(x) = Ae^{-\kappa x} + Be^{\kappa x}$. ($x \to -\infty$, $A = 0$)

$\Psi(x) = Fe^{-\kappa x}$ ($x > 0$).

$x = 0$ $B = F$,

$$-\frac{\hbar^2}{2m}\int_{-\varepsilon}^{\varepsilon}\frac{\partial^2\Psi}{\partial x^2}\mathrm{d}x + \int_{-\varepsilon}^{\varepsilon}V(x)\Psi\mathrm{d}x = \int_{-\varepsilon}^{\varepsilon}E\Psi(x)\mathrm{d}x,\ (\qquad)$$

$$\Delta\left(\frac{\partial\Psi}{\partial x}\right) = \frac{2m}{\hbar^2}\int_{-\varepsilon}^{\varepsilon}V(x)\Psi(x)\mathrm{d}x = -\frac{2m\alpha}{\hbar^2}\Psi(0).$$

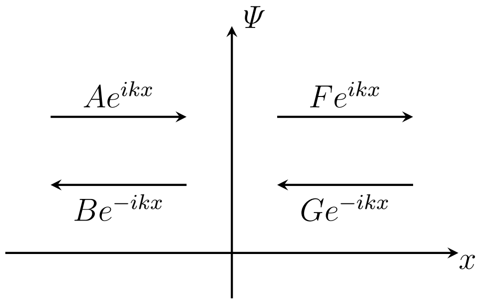
$$\Delta\left(\frac{\partial\Psi}{\partial x}\right) = 2B\kappa = -\frac{2m\alpha}{\hbar^2}B \Rightarrow \kappa = \frac{m\alpha}{\hbar^2},$$

$$\Psi(x) = \frac{\sqrt{m\alpha}}{\hbar}e^{-m\alpha|x|/\hbar^2},\ E = -\frac{m\alpha^2}{2\hbar^2},$$

.

1 .

$$T = 1 \Rightarrow k'b = n\pi.$$



3:

9. $E_n^{(1)} = \langle\Psi_n^{(0)}|\, H'\, |\Psi_n^{(0)}\rangle$.

$$\Psi_n^{(1)} = \sum_{m\neq n}\frac{\langle\Psi_m^{(0)}|\, H'\, |\Psi_m^{(0)}\rangle}{E_n^0 - E_m^0}\Psi_m^0.$$

$$E_n^{(2)} = \sum_{m\neq n}\frac{|\langle\Psi_m^{(0)}|\, H'\, |\Psi_m^{(0)}\rangle|^2}{E_n^0 - E_m^0} = \langle\Psi_n^{(1)}|\, H'\, |\Psi_n^{(1)}\rangle.$$

: $\langle\Psi|\, H\, |\Psi\rangle \leqslant E_g$,

$\Psi(x) = Ae^{-bx^2}$ $\qquad \Psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$.

10. :

$$\sigma_A^2\sigma_B^2 \leqslant (\frac{1}{2i}\langle[A, B]\rangle)^2.$$

$$\sigma_x\sigma_p \leqslant \frac{\hbar}{2},\ \sigma_E\sigma_t \leqslant \frac{\hbar}{2}.$$

$\rho\uparrow$, $\sigma_t\downarrow$, $\sigma_E\uparrow$, .

11. : $L^2\Psi = l(l+1)\hbar^2\Psi,\ L_z\Psi = m\hbar\Psi.$

: $[r_i, r_j] = [p_i, r_j] = 0,\ [r_i, p_j] = i\hbar\delta_{ij},$

$[L_x, L_y] = i\hbar L_z, [L^2, L_i] = 0 \Rightarrow [L^2, \vec{L}] = 0.$

$L_\pm = L_x \pm iL_y, \quad [L_z, L_\pm] = \hbar L_\pm,\ [L^2, L_\pm] = 0.$

$L_\pm f_l^m = \hbar\sqrt{l(l+1) - m(m\pm 1)} f_l^{m\pm 1}.$

12. : $E_n = -E_1(\frac{Z^2}{n^2}\frac{\mu}{\mu_e}),\ E_1 = \frac{q^4}{8\hbar^2\varepsilon_0^2} = 12.6eV.$

$\mu \approx \mu_e \approx m_e,$ positronium, $\mu = \frac{m_e^2}{m_e + m_e} = \frac{1}{2}m_e.$

$v_n = \frac{\alpha c}{n},$ α $\frac{1}{137}.$

$r_n \approx a_0(\frac{m_e}{\mu}\frac{n^2}{Z}),$ $a_0 \approx 0.53\mathring{A}.$

$\Psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a},$ $\propto (1 - \frac{r}{2a})e^{-r/2a}.$

13. $\mu = -\frac{e}{2m}L,$ $\mu_s = -\frac{ge}{2m}S,$ $g = 2,$ $\gamma = \frac{\mu_s}{S} = -\frac{ge}{2m}.$

14. Stern-Gerlach : Ag $5s^1$ $|\Psi\rangle = \frac{1}{\sqrt{2}}\uparrow\downarrow - \frac{1}{\sqrt{2}}\downarrow\uparrow$.

15. - : .

16. : $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi).$ ( )

17. split with j: <1> ; <2> - .

18. $D(\theta) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega},$ $\mathrm{d}\Omega = \sin\theta \mathrm{d}\theta \mathrm{d}\phi.$

19. ...

8 Solid State Physics

1. $V = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)$.

   : $\vec{b}_1 = 2\pi \dfrac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$, $\vec{b}_1 = 2\pi \dfrac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$, $\vec{b}_1 = 2\pi \dfrac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$.

2. : $E_F = \dfrac{\hbar^2 k_F^2}{2m}$, $k_F = (3\rho\pi^2)^{\frac{1}{3}}$, ρ .

   : $T = 0$, $f_0(E) = \begin{cases} 1, E \leqslant E_F \\ 0, E > E_F \end{cases}$,

$$N(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$$

$$N = \int_0^\infty f_0(E) \cdot N(E)\mathrm{d}E = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \times \int_0^{E_F} E^{\frac{1}{2}}\mathrm{d}E.$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}, \qquad k_F = \left(\frac{3\pi^2 N}{V}\right)^{\frac{1}{3}}.$$

3. : $\dfrac{\mathrm{d}^2 E}{\mathrm{d}k^2} = \dfrac{\hbar^2}{m}$, $\quad m^* = \dfrac{\hbar^2}{\left(\dfrac{\mathrm{d}^2 E}{\mathrm{d}k^2}\right)}$.

9 Particle Physics

1. J/Ψ meson : charm quark anti charm quark .
   Deutron: .
   : photon= 1, $=\frac{1}{2}$, proton= $\frac{1}{2}$.

2.

$$\text{hadron} \begin{cases} \text{meson (boson): a quark and an antiquark.} \\ \text{baryon (fermion): 3 quarks}(\frac{1}{3}). \end{cases}$$

baryon number $B(\pm 1)$.

$$\text{lepton} \begin{cases} \text{electron}(e^-) \quad \text{electron neutrino}(\nu_e), \\ \mu \quad \text{muon}(\mu^-) \quad \mu \quad \text{muon neutrino}(\nu_\mu), \\ \tau \quad \text{tau}(\tau) \quad \tau \quad \text{tau neutrino}(\nu_\tau). \end{cases}$$

: lepton number L conservation.

3. π , μ .

4. μ : $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$.
   α : ${}^AN \to {}^{A-4}L + {}^4He$.
   β : β^- : $N \to O + e^- + \bar{\nu}_e$ $(n \to p + e^- + \bar{\nu}_e)$,
   β^+ : $N \to O + e^+ + \nu_e$ $(p \to n + e^+ + \nu_e)$.
   β^- .

10 Atomic Physics and Nuclear Physics

1. ${}^{2S+1}L_J$, Lithium, Sodium.

2. : $\Delta m_l = \pm 1, 0$; $\Delta l = \pm 1$; $\Delta j = 0, \pm 1$; $\Delta m_s = 0$.
   : <1> $S \uparrow$, $E \downarrow$;
   <2> $L \uparrow$, $E \downarrow$;
   <3> , $J = |L - S|$, $E \downarrow$,
   , $J = L + S$, $E \downarrow$,

3. : $U = -\vec{\mu} \cdot \vec{B} = -\frac{ge}{2m} \vec{J} \cdot \vec{B}$.
   : $\mu_B = \gamma \hbar = \frac{e\hbar}{2m}$.

$h\nu' = h\nu + (m_2 g_2 - m_1 g_1)\mu_B B$, $S = 0$, $g_1 = g_2 = 1$, $\Delta m = 0, \pm 1$,

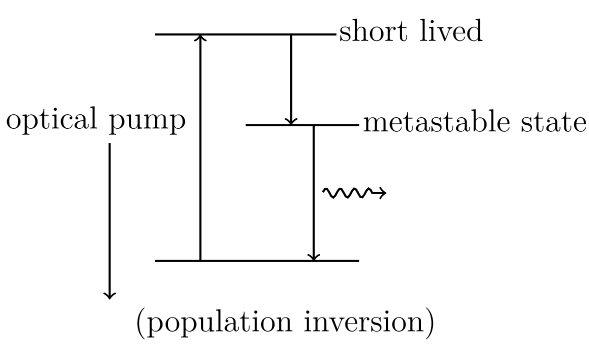
$$:h\nu' = h\nu + \begin{pmatrix} \mu_B B \\ 0 \\ -\mu_B B \end{pmatrix} \begin{matrix} \sigma^+ \\ \pi \\ \sigma^- \end{matrix}$$

$$\begin{cases} B & \sigma^{\pm}\ 2 \\ B & 3 \end{cases}$$

3. (E): $U = \vec{p} \cdot \vec{E} = -qEr\cos\theta$.

4. X (exicite inner electrons of element near the nuclear)→Auger transition .
   shielding : $E = 13.6eV(\frac{3}{4})(Z-1)^2$.

5. ( ): ω ϕ ...
   : 1. coherent.
   2. monochromatic.
   3. minimal divergence.
   4. high intensity.



6. :
   Diod Laser: active medium is semiconducting. (p-n)
   Gas Laser: active medium is free gas. (⟶atoms)

7. Nucleus is most likely to emit electrons in the direction opposite to the magnetic field.

8. $r \approx r_0 A^{\frac{1}{3}}$, $r_0 \approx 1.2fm$, : $E_B = \sum_i m_i c^2 - Mc^2$, nucleon.

9. : $N = N_0 e^{-\lambda t}$, $R = -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N$, $t_{1/2} = \frac{\ln 2}{\lambda}$,

mean life: $\tau = \frac{1}{\lambda}$.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -(\lambda_A + \lambda_B)N \Rightarrow N = N_0 e^{-(\lambda_A+\lambda_B)N}, \qquad \frac{1}{t_{1/2}} = \frac{1}{t^A_{1/2}} + \frac{1}{t^B_{1/2}}.$$

10. $Gray = \frac{Energy}{Unit\ mass}$, $1Gy = \frac{1J}{1kg}$.

Bremsstrahlung: $P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} \propto q^2 \propto a^2$; $P = \frac{\omega^4}{12\pi\varepsilon_0 c^3} p^2$.

11. : $\frac{1}{2}mv^2 = k\frac{2e^2}{r}$.

12. $\frac{1}{\lambda} = R(\frac{1}{n^2} - \frac{1}{n'^2})$:

$n = 1$, ;

$n = 2$, $\begin{cases} H-\alpha & , 3 \to 2 \\ H-\beta & , 4 \to 2 \\ H-\gamma & , 5 \to 2 \end{cases}$

$n = 3$, .

11 Mathematics and Ohters

1. $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$.

2. Fourier Series: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$,

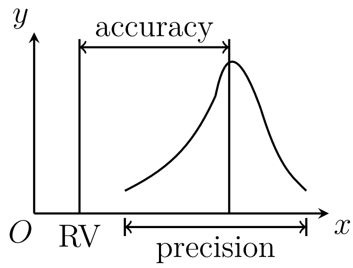
$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos(\frac{n\pi t}{L})\mathrm{d}t,\ b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin(\frac{n\pi t}{L})\mathrm{d}t.$$

$$: f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\mathrm{d}t.$$

3. :

$$\left.\begin{array}{l}\text{Accuracy}\\ \text{Precision}\end{array}\right\}$$

RV(Real Value) .



4. : $P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$,

$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$, .

$\sigma = \sqrt{\mu}$, N , $\sigma_N = \frac{\sigma}{\sqrt{N}}$.

$3-\sigma$: 0.6826, 0.9544, 0.9974.

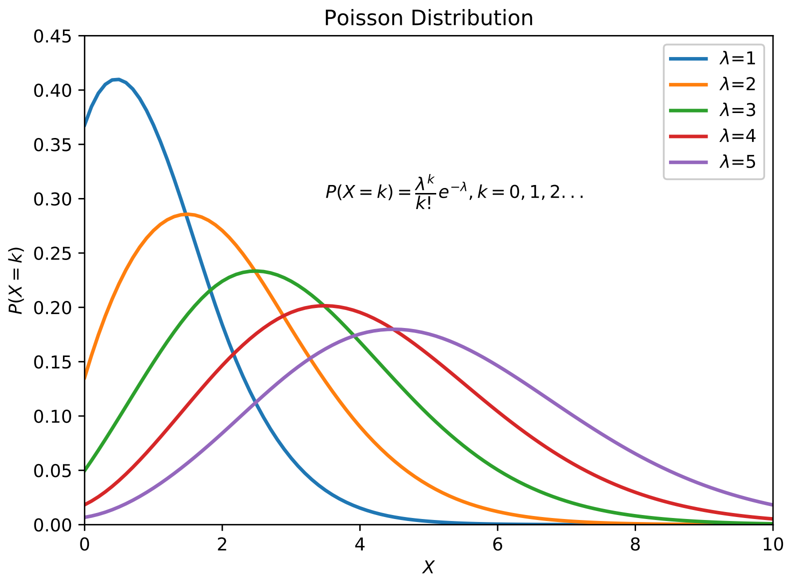


图 4: Poisson Distribution.

5. :

<1> $x = a + b + \cdots \Rightarrow \mathrm{d}x = \mathrm{d}a + \mathrm{d}b + \cdots \Rightarrow (\delta x)^2 = (\delta a)^2 + (\delta b)^2 + \cdots;$

<2> $x = a \cdot b \cdots \Rightarrow \dfrac{\mathrm{d}x}{x} = \dfrac{\mathrm{d}a}{a} + \dfrac{\mathrm{d}b}{b} + \cdots \Rightarrow (\dfrac{\delta x}{x})^2 = (\dfrac{\delta a}{a})^2 + (\dfrac{\delta b}{b})^2 + \cdots.$

N x_i σ_i, $\omega_i = \dfrac{1}{\sigma_i^2}$,

$$x_{avg} = \frac{\sum \omega_i x_i}{\sum \omega_i},\ \sigma_{avg} = \frac{1}{\sqrt{\sum \omega_i}}.$$

$$\omega_i = \omega, \qquad x_{avg} = \frac{\sum x_i}{N},\ \sigma_{avg} = \frac{\sigma}{\sqrt{N}}.$$

6.

$$\pi(10^{-14})^2 m^2 = 10^{-28} m^2;$$

$$\sigma = 10^{-18} m^2;$$

$$n = 10^{25} m^{-3};$$

$$l_p = \frac{1}{\sigma n} = 10^{-7} m.$$