

Data Analysis and Algorithm

Practical 2

Write a Program to implement Merge sort

&

Find the run time of the algorithm

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→ Write a program to implement Merge sort & find the running time of the algorithm

→ For algorithms such as selection sort or insertion sort when we increase the input size the time taken also increase by a big margin.

Thus we look at algorithm like merge sort which has runtime of $n \log n$ in all cases i.e. with large input size the run time is much better than insertion sort.

Merge sort uses the Divide & Conquer technique to first divide a large array into smaller section particularly 2 equal sections & sorting these section independently & recursively & at last combining.

Thus, the algorithm has 3 parts -

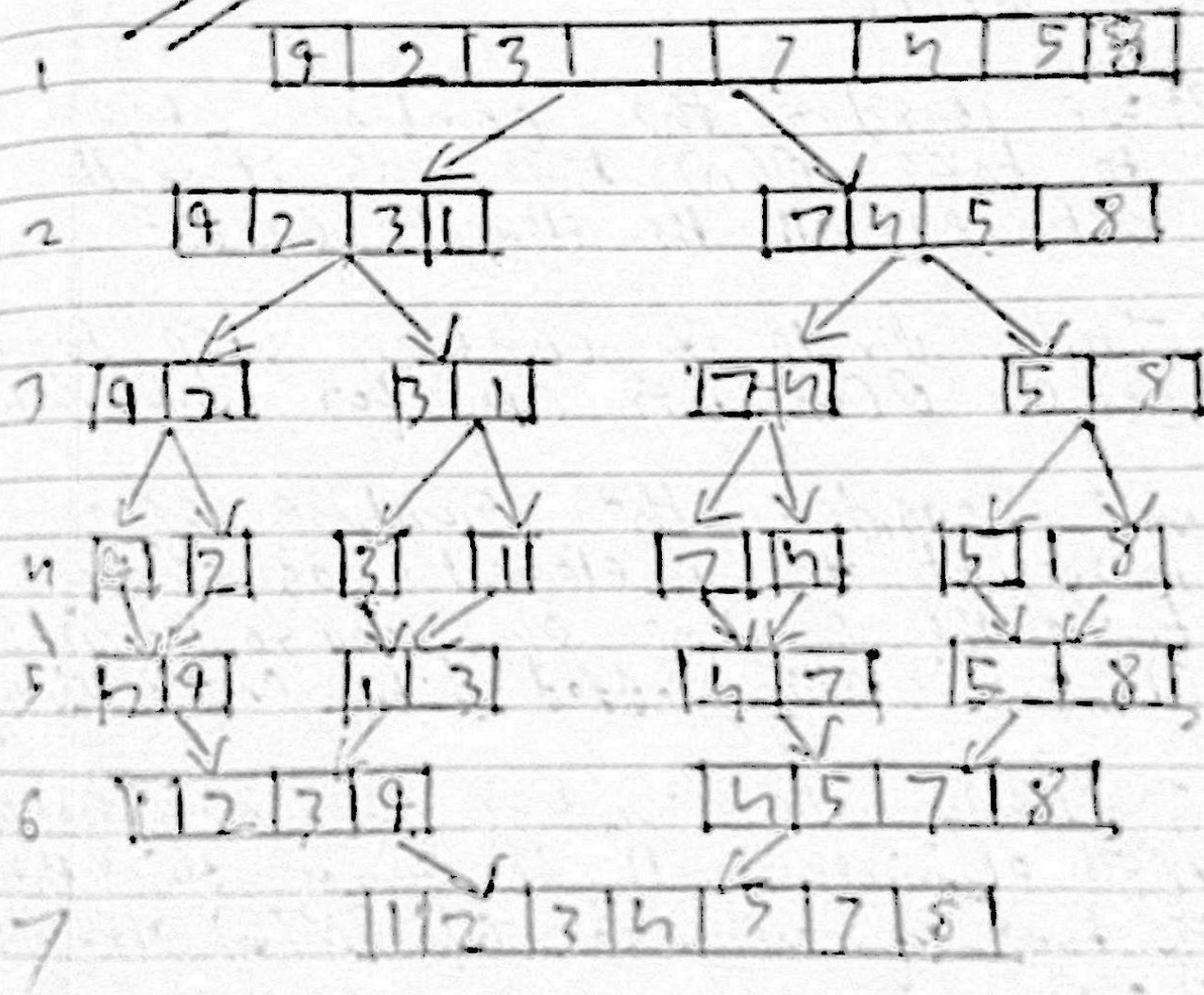
Divide, Conquer & combine

→ Divide → Divide the array into 2
— ie index $[p \dots q]$ ie $\frac{p+q}{2}$

& you get two arrays $\text{arr}[p, \text{part}]$ & $\text{arr}[\text{part}, q]$

Combine - merge the sorted subarray into one.

Example



where on steps 1-4 we divide & from 5-7 we conquer & merge

→ Runtime of merge sort algorithm

To calculate the runtime of merge sort we look at the 3 steps & the runtimes of the same.

→ we consider the divide step to have a constant time because in the step all we do is calculate the midpoint of array.

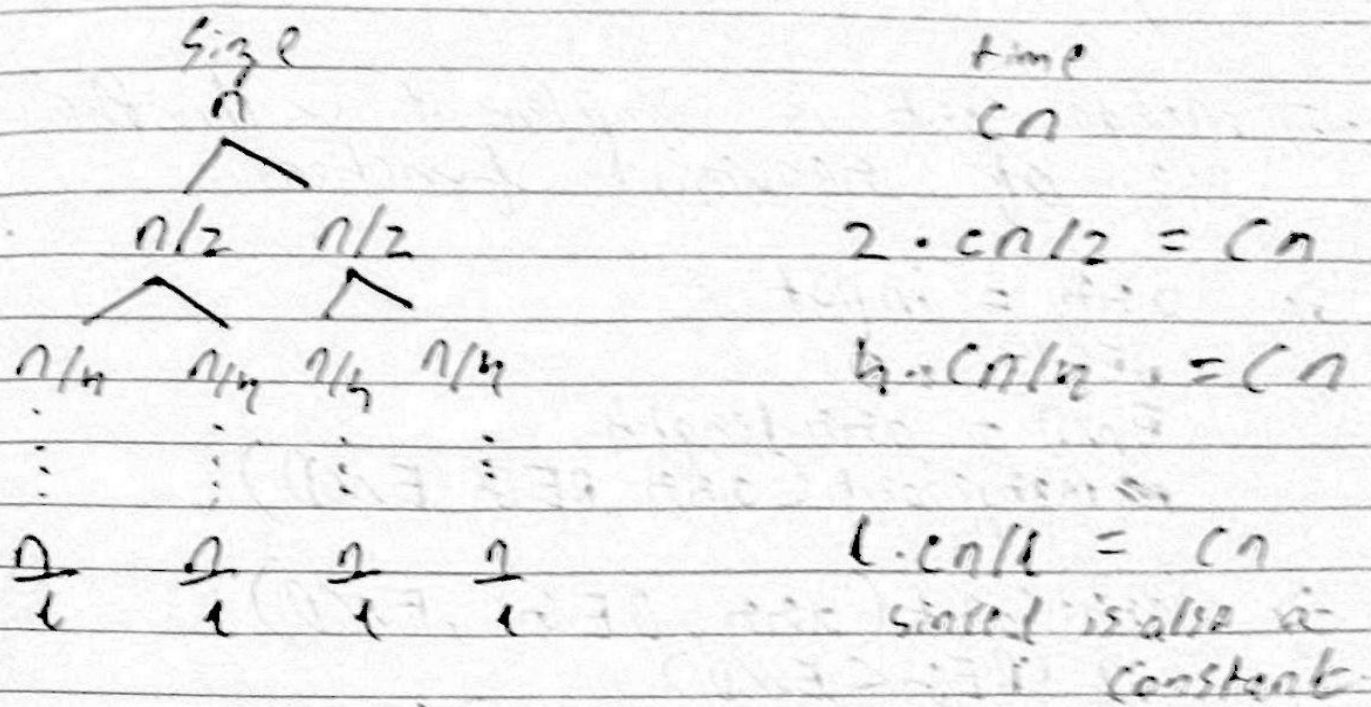
→ we consider the combine step to take $O(n)$ time as it will act on all the element once.

→ Thus, divide & combine step together take $O(2)$ or Cn for some constant

→ we consider the runtime of mergesort of n elements as sum of twice the runtime of mergesort of $(n/2)$ elements array added with Cn if divided continue

→ Similarly the runtime of mergesort of $(n/2)$ elements will be sum of twice the runtime of mergesort of $(n/4)$ elements + Cn

Thus we see,



i.e. total time of merge sort will be sum of merging time of each level.
i.e. for l levels we get $l \cdot cn$

Where l is given as

$$l = \log_2 n + 1$$

Eg:- $n=8$

$$l = \log_2 8 + 1$$

$$l = 4$$

i.e. for $n=8$ we have 4 levels.

thus we substitute $\log_2 n + 1$ in $l \cdot cn$ we get,

$$cn(\log_2 n + 1)$$

For Big O notation we discard constant c & low order term $(+1)$

thus get

$$O(n \log_2 n)$$

⇒ Algorithm for merge sort

⇒ Merge sort is implement with the use of recursive functions.

1) arr = input

BEG = 0

END = arr.length

mergeSort(arr, BEG, END)

2) mergeSort(arr, BEG, END)

if (BEG < END)

mid = (BEG + END) / 2

mergeSort(arr, BEG, MID)

mergeSort(arr, MID+1, END)

Merge(arr, BEG, MID, END)

3) Merge(arr, BEG, MID, END)

i = BEG

j = MID+1

x = 0

while

(i ≤ MID && j ≤ END)

if arr[i] < arr[j]

temp[x] = arr[i]

i = i + 1

else

temp[x] = arr[j]

j = j + 1

x = x + 1

```

4) if i > MID
    while j <= END
        temp[x] = arr[j]
        x = x + 1
        j = j + 1
else
    while i <= MID
        temp[x] = arr[i]
        x = x + 1
        i = i + 1

```

Step 1 \rightarrow calling code to initiate the function

Step 2 \rightarrow recursively calling merge sort with array split in half & calling merge to organize the array

Step 3 \rightarrow sorting the sub array

Step 4 \rightarrow merge the remaining array

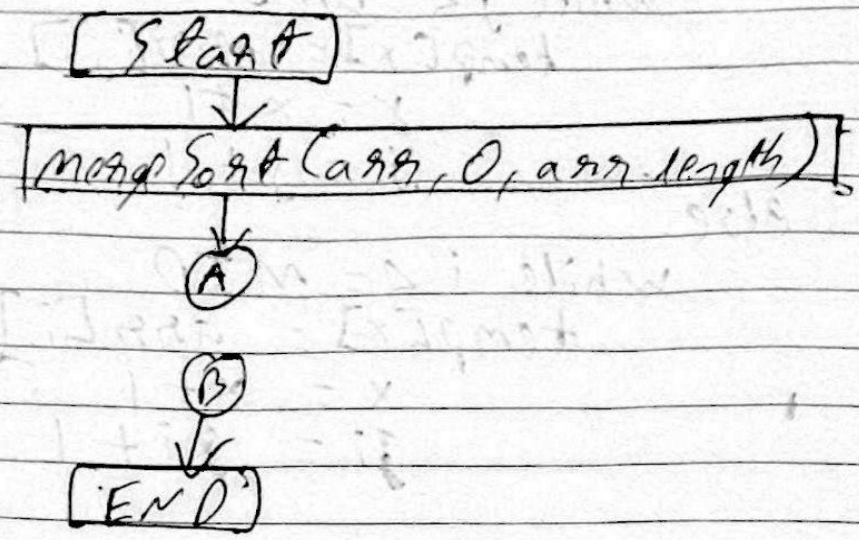
```

5) k = 0
    while k < x
        arr[k] = temp[k]
        k = k + 1

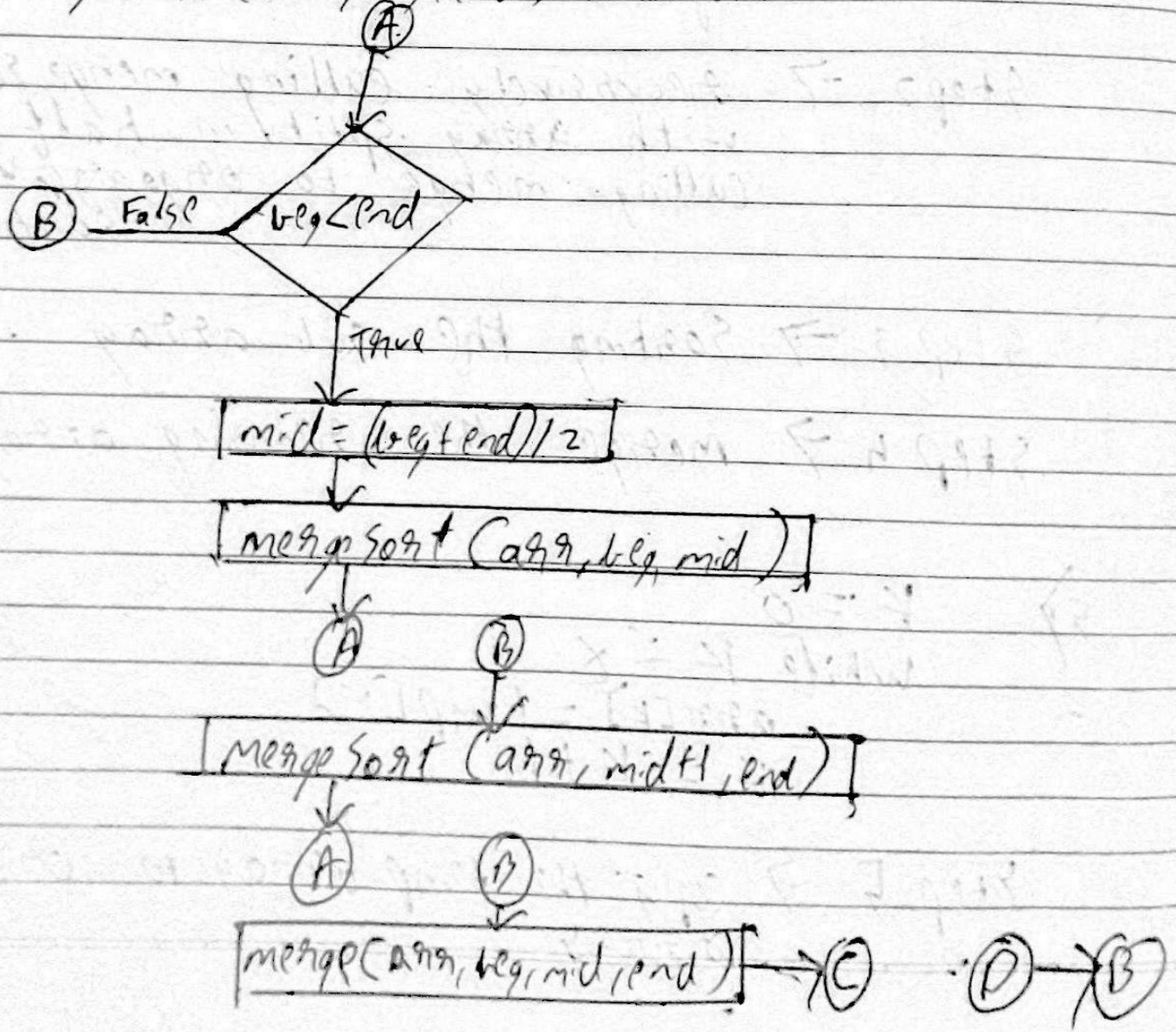
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Step 5 \rightarrow copy the temp array to original array

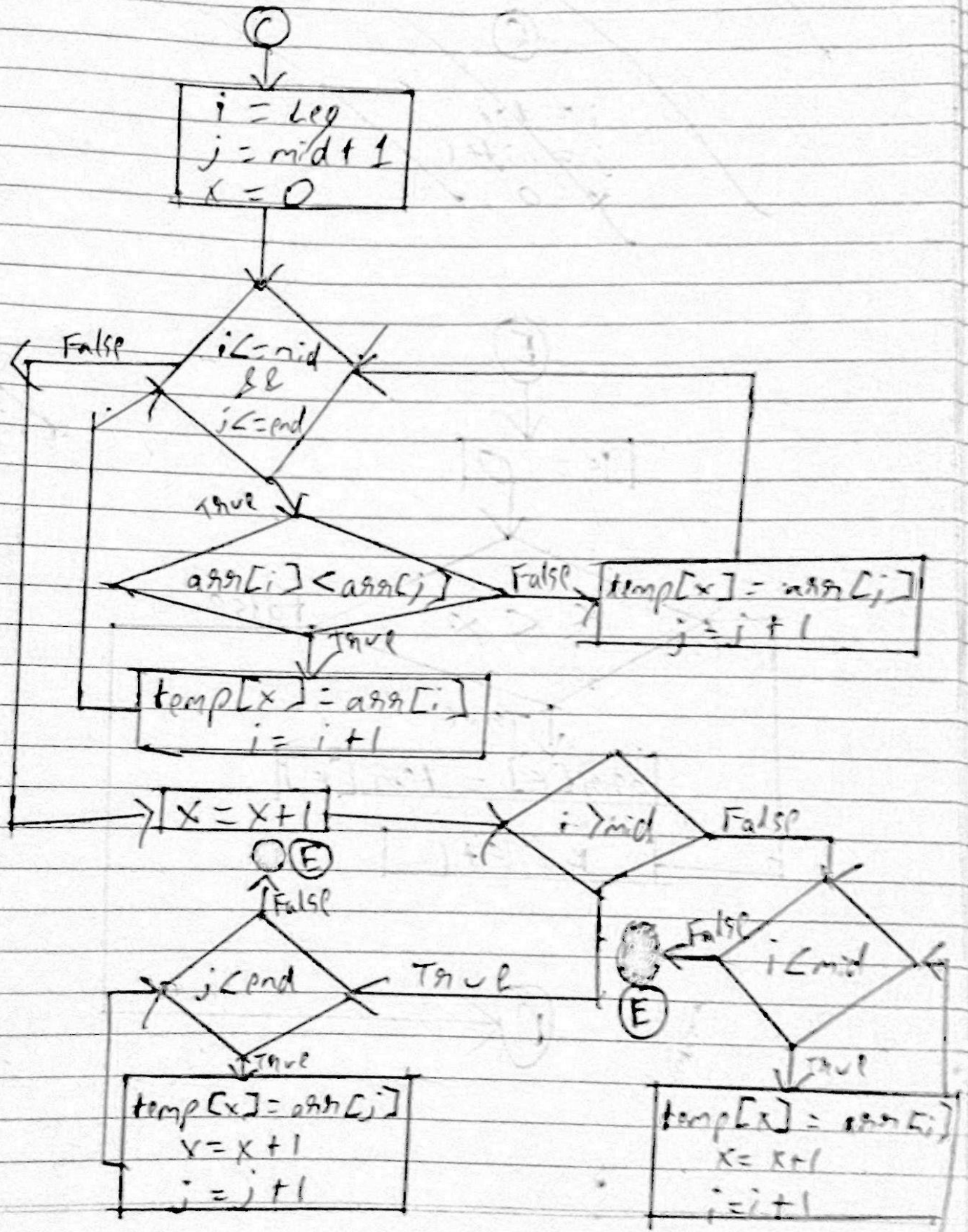
→ flowchart

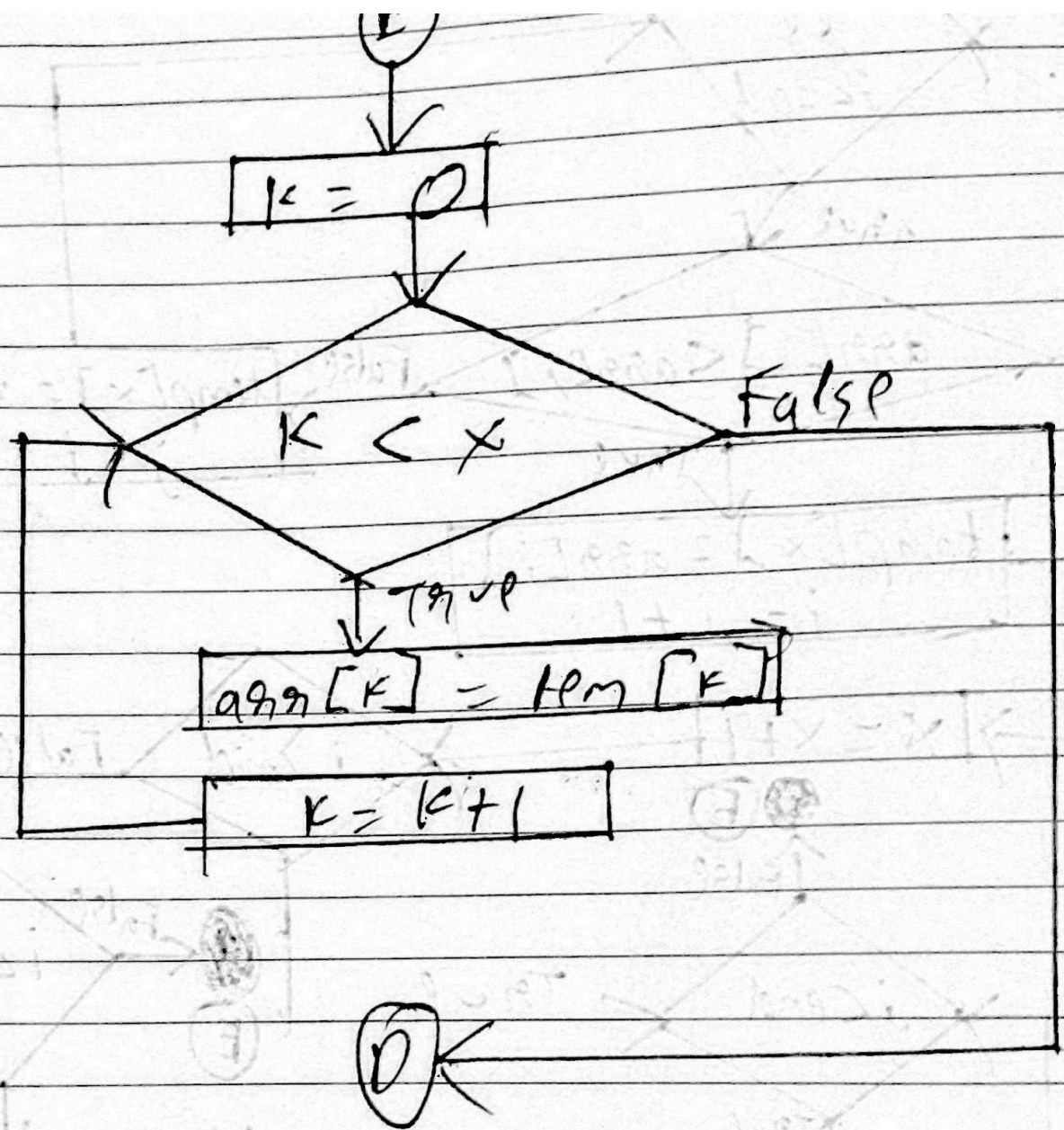


→ MergeSort(arr, beg, end)



→ Merge(arr, beg, mid, end)





Input

we use Python random library to create a list of n numbers ~~as~~ ~~as~~ to serve as an input.

⇒ Runtime

We use python timeit library
to run a timed test.

Program

```
1. import random
2. def merge(arr, l, m, r):
3.     n1 = m - l + 1
4.     n2 = r - m
5.
6.     # create temp arrays
7.     L = [0] * (n1)
8.     R = [0] * (n2)
9.
10.    # Copy data to temp arrays L[] and R[]
11.    for i in range(0, n1):
12.        L[i] = arr[l + i]
13.
14.    for j in range(0, n2):
15.        R[j] = arr[m + 1 + j]
16.
17.    # Merge the temp arrays back into arr[l..r]
18.    i = 0      # Initial index of first subarray
19.    j = 0      # Initial index of second subarray
20.    k = l      # Initial index of merged subarray
21.
22.    while i < n1 and j < n2:
23.        if L[i] <= R[j]:
24.            arr[k] = L[i]
25.            i += 1
26.        else:
27.            arr[k] = R[j]
28.            j += 1
29.        k += 1
30.
31.    # Copy the remaining elements of L[], if there
32.    # are any
33.    while i < n1:
34.        arr[k] = L[i]
35.        i += 1
36.        k += 1
37.
38.    # Copy the remaining elements of R[], if there
39.    # are any
40.    while j < n2:
41.        arr[k] = R[j]
42.        j += 1
43.        k += 1
44.
45.    # l is for left index and r is right index of the
46.    # sub-array of arr to be sorted
47.    def mergeSort(arr, l, r):
48.        if l < r:
49.            # Same as (l+r)//2, but avoids overflow for
50.            # large l and h
51.            m = l+(r-l)//2
52.            # Sort first and second halves
```



```

53.         mergeSort(arr, l, m)
54.         mergeSort(arr, m+1, r)
55.         merge(arr, l, m, r)
56.         print(arr)
57.     return(arr)
58.
59. #driver code
60. if __name__ == "__main__":
61.     arr=[]
62.     for i in range (1,10):
63.         n = random.randint(0,1000)
64.         arr.append(n)
65.     mergeSort(arr, 0, len(arr)-1)
66.     print(mergeSort)

```

```

===== RESTART: C:\Users\ADMIN\Desktop\
[527, 771, 932, 266, 393, 59, 781, 553, 949]
[527, 771, 932, 266, 393, 59, 781, 553, 949]
[527, 771, 932, 266, 393, 59, 781, 553, 949]
[266, 393, 527, 771, 932, 59, 781, 553, 949]
[266, 393, 527, 771, 932, 59, 781, 553, 949]
[266, 393, 527, 771, 932, 59, 781, 553, 949]
[266, 393, 527, 771, 932, 59, 553, 781, 949]
[59, 266, 393, 527, 553, 771, 781, 932, 949]
[59, 266, 393, 527, 553, 771, 781, 932, 949]
>>>

```

Timing code

```

1. SETUP_CODE = '''
2. from __main__ import mergeSort,merge
3. import random
4. '''
5.
6. TEST_CODE = '''
7. arr=[]
8. input_size = 10
9. for i in range (1,input_size):
10.     n = random.randint(0,1000)
11.     arr.append(n)
12. mergeSort(arr, 0, len(arr)-1)'''
13.
14. times = timeit.timeit(setup = SETUP_CODE,
15.                        stmt = TEST_CODE,
16.                        number = 1)
17.
18. print(times)
19.

```

Output time with array of size 10

```

===== RESTART: C:/Users/ADMIN/Desktop/
[4, 279, 452, 588, 670, 775, 785, 820, 980]
0.020413899999999985

```

Output time with array of size 100000

```

0.8353626
>>>

```

7 Conclusion

We observe that merge sort takes very small amount of time when the input size is increased as compared to other linear algorithms.