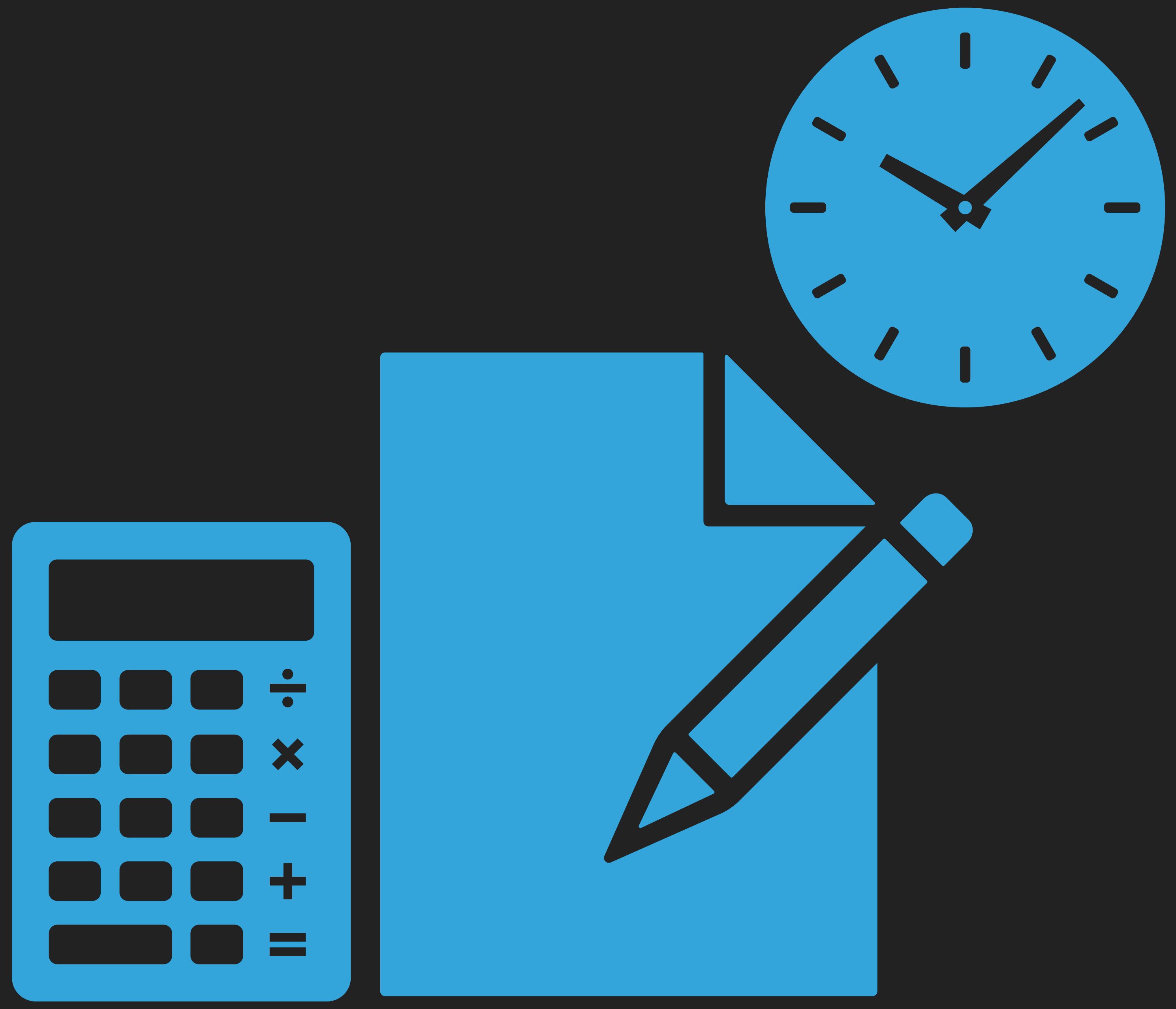
深宵教室 - DSE M1 模擬試題解答

2012

此為參考2012試題之模擬試題，原版請另行購買

2012

- Section A
- Section B



2012 - SECTION A

Q1.) The coefficient of x^2 of $e^{-2x}(1+3x)^n$ is 62. $n=$?

* 參考課程 1.1 及 3.2

$$e^{-2x}(1+3x)^n = \left(1-2x+\frac{1}{2}(-2x)^2+\dots\right)\left(1+C_1^n(3x)+C_2^n(3x)^2+\dots\right)$$

The coefficient of $x^2 = 62$

$$\to C_2^n(9)-2C_1^n(3)+\frac{4}{2}=62$$

$$\to \frac{9n(n-1)}{2}-6n-60=0$$

$$\to 3n^2-7n-40=0$$

$$\to n=5 \text{ or } -\frac{8}{3} \text{ (rejected)}$$

$$* \quad e^x=\sum_{r=0}^{\infty}\frac{x^r}{r!}$$

$$* \quad (a+b)^n=\sum_{r=0}^{n}C_r^n a^r b^{n-r}$$

$$* \quad C_r^n=\frac{n!}{r!(n-r)!}$$

$$\to C_1^n=n \text{ and } C_2^n=\frac{n(n-1)}{2}$$

2012 - SECTION A

Q2.) The rate of change of V, $\frac{dV}{dt} = \frac{t}{\sqrt{4t+1}}$*, where* t = *the number of years since the beginning of 2012. At the beginning of 2012,* $V = 3$*. Find the percentage change of V from the beginning of 2012 to the beginning of 2014.*

* 參考課程 2.2, 2.3 及 2.4

$$V = \int \frac{tdt}{\sqrt{4t+1}} = \frac{1}{4}\int \frac{(4t+1)-1}{\sqrt{4t+1}}dt, \text{ let } u = 4t+1 \rightarrow du = 4dt$$

$$\rightarrow V = \frac{1}{16}\int \frac{u-1}{\sqrt{u}}du = \frac{1}{16}\int (\sqrt{u} - \frac{1}{\sqrt{u}})du$$

$$\rightarrow V = \frac{1}{16}[\frac{2}{3}(4t+1)^{\frac{3}{2}} - 2(4t+1)^{\frac{1}{2}}] + C, \text{ where } C \text{ is contant}$$

$$\therefore \text{The percentage change} = \frac{V(2) - V(0)}{V(0)} x100\,\%$$

$$= 27.7778\,\% \text{ (to 4 d.p.)}$$

* 積分類似微分的逆函數

* 積分代入法

* 2014年頭經歷左 2 年

* 唔須要搵 C, V(2)-V(0) 會 cancel

2012 - SECTION A

Q3.) Let $P = ae^{\frac{kt}{40}} - 5$. *The following table shows their relationship.*

t	2	4	6	8	10
P	2.36	2.81	3.23	3.55	4.01

Express $ln(P+5)$ *in term of* t. *Hence, find* a *and* k, *to nearest integer*

* 參考課程 1.1 及 3.1

$$ln(P+5) = ln(ae^{\frac{kt}{40}}) \rightarrow ln(P+5) = lna + \frac{k}{40}t$$

* ln(AB)=lnA+lnB

Then, consider the following table

t	2	4	6	8	10
$ln(P+5)$	2.00	2.06	2.11	2.15	2.20

$$\frac{k}{40} = \frac{5(64.1) - 30(10.52)}{5(220) - (30)^2} = 0.0245 \rightarrow k = 1 \text{ (to nearest integer)}$$

$$lna = \frac{(10.52)(220) - 30(64.1)}{5(220) - (30)^2} = 1.957 \rightarrow a = 7 \text{ (to nearest integer)}$$

* $m = \dfrac{n\sum x_iy_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$

* $c = \dfrac{\sum y_i \sum x_i^2 - \sum x_i \sum x_iy_i}{n\sum x_i^2 - (\sum x_i)^2}$

$[x_i = t_i, y_i = ln(P_i + 5)]$

2012 - SECTION A

$Q4.)\ f(x) = (\frac{3x-1}{x-2})^{\frac{1}{3}},\ Find\ f'(x)\ and\ f''(3)$

* 參考課程 2.2 及 2.3

方法1

$$(x-2)[f(x)]^3 = 3x-1$$

$$\rightarrow \frac{d}{dx}[(x-2)[f(x)]^3] = \frac{d}{dx}(3x-1)$$

$$\rightarrow [f(x)]^3 + 3(x-2)[f(x)]^2 f'(x) = 3$$

$$\rightarrow f'(x) = \frac{3-[f(x)]^3}{3(x-2)[f(x)]^2} = \frac{3-\frac{3x-1}{x-2}}{3(x-2)(\frac{3x-1}{x-2})^{\frac{2}{3}}} = \frac{-5}{3(x-2)^{\frac{4}{3}}(3x-1)^{\frac{2}{3}}}$$

* Implicit 微分法

* Product rule 及 chain rule

方法2

$$ln[f(x)] = \frac{1}{3}ln(\frac{3x-1}{x-2})$$

* ln 微分法

CONT'D

2012 - SECTION A

$$\rightarrow ln[f(x)] = \frac{1}{3}[ln(3x-1) - ln(x-2)]$$

* ln(A/B) = lnA-lnB

$$\rightarrow \frac{f'(x)}{f(x)} = \frac{1}{3}[\frac{3}{3x-1} - \frac{1}{x-2}]$$

* Implicit 微分法

$$\rightarrow f'(x) = \frac{f(x)}{3}\frac{-5}{(3x-1)(x-2)} = \frac{-5}{3(x-2)^{\frac{4}{3}}(3x-1)^{\frac{2}{3}}}$$

$$Then,\ 3f'(x) = \boxed{-5(x-2)^{\frac{-4}{3}}(3x-1)^{\frac{-2}{3}}}$$

$$\rightarrow 3f''(x) = \boxed{-5\cdot\frac{-4}{3}(x-2)^{\frac{-7}{3}}(3x-1)^{\frac{-2}{3}} - 5\cdot(x-2)^{\frac{-4}{3}}\frac{-2}{3}(3x-1)^{\frac{-5}{3}}\cdot 3}$$

* Product rule

$$\rightarrow 3f''(3) = \frac{20}{3}(1)(8)^{\frac{-2}{3}} + \frac{30}{3}(1)(8)^{\frac{-5}{3}}$$

$$\rightarrow f''(3) = \frac{95}{144}$$

2012 - SECTION A

Q5.) Let the curve $C: y = f(x)$, where $f'(x) = e^{2x}$. Assume a tangent line L touch C at the point $A = (0,1)$. Find the equation of C and L. Hence, find the area of the region bounded by C, L and the line $x = 1$.

* 參考課程 2.2, 2.3, 2.4 及 2.9

$$Consider,\ f(x) = \int e^{2x}dx = \frac{e^{2x}}{2} + C,\ where\ C\ is\ constant$$

* 積分類似微分的逆函數

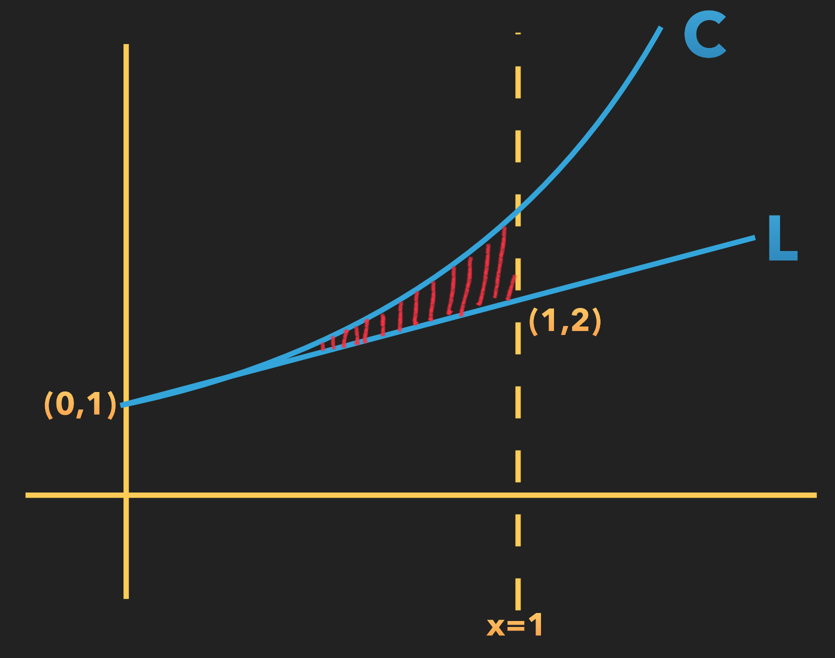
$$Given\ that\ f(0) = 1 \rightarrow C = \frac{1}{2} \quad \rightarrow C:\ y = \frac{1}{2}(e^{2x} + 1)$$

$$Then,\ L: y - 1 = f'(0)(x - 0) \quad \rightarrow L:\ y = x + 1$$

* 直線方程: 點斜式

$$The\ bounded\ area = \int_0^1 \frac{e^{2x} + 1}{2}dx - \frac{(2+1)(1)}{2}$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2} + x\right]_0^1 - \frac{3}{2} = \frac{e^2 - 5}{4}\ unit^2$$



2012 - SECTION A

Q6.) The weight (kg) of students in school follows normal distribution with the mean = 67kg and standard deviation = 15kg . The 36 random students area selected .

a.) Find the probability that the sample mean is over 70kg .

b.) Given that there are 9 students like basketball . Find the 95 % confidence interval for the proportion of the students like basketball .

* 參考課程 4.3, 4.5, 4.6 及 4.7

a.) Let X be the random variable of the weights of selected students .

$$\bar{X} = \frac{\sum_{i=1}^{36} X_i}{36}, \quad \text{Given that } X_i \sim (67, 15^2) \rightarrow \bar{X} \sim N(67, \frac{15^2}{36})$$

$$P(\bar{X} > 70) = P(\frac{\bar{X} - 67}{15/\sqrt{36}} > \frac{70 - 67}{15/\sqrt{36}}) = P(Z > 1.2) = 0.1151$$

b.) Let p_s be the proportion of the sample students like basketball .

p be the proportion of the students like basketball .

$* \quad E(\frac{\sum_{i=1}^{n} X_i}{n}) = \frac{\sum_{i=1}^{n} E(X_i)}{n} = E(X)$

$* \quad Var(\frac{\sum_{i=1}^{n} X_i}{n}) = \frac{\sum_{i=1}^{n} Var(X_i)}{n^2} = \frac{Var(X)}{n}$

* 先計算 Z ~ N(0,1), 再對表

CONT'D

2012 - SECTION A

$$Given\ that\ p_S = \frac{9}{36} = 0.25$$

$$For\ 95\,\%\ C.I.,\ P(-\alpha < \boxed{\frac{p_s - p}{\sqrt{p_s(1-p_s)/36}}} < \alpha) = 95\,\% \rightarrow \alpha = 1.96$$

$$\rightarrow p = (p_s - 1.96 \cdot \frac{\sqrt{p_s(1-p_s)}}{6},\ p_s + 1.96 \cdot \frac{\sqrt{p_s(1-p_s)}}{6})$$

$$\rightarrow p = (0.1085,\ 0.3915)\ (to\ 4\ d.p.)$$

* ■ $B(36, p) \rightarrow N(p, \frac{p(1-p)}{36})$

* 當樣本足夠大, 可用樣本標準差

2012 - SECTION A

Q7.) The number of goal of a randomly selected match of a team follow Possion Distribution,
$X \sim Po(\lambda)$. *Given that the probability of no goal by the team in a match* $= 0.1653$.

a.) $\lambda = ?$, *to 1 decimal place*

b.) Find the probability that the team score less than 3.

c.) Given that the number of goals by the team in any 2 matches is independent. Find the probability the total goals by the team of two matches is less than 3.

* 參考課程 4.3 及 4.4

$a.)\ P(X=0) = e^{-\lambda} \rightarrow e^{-\lambda} = 0.1653 \rightarrow \lambda = 1.8\ (to\ 1\ d.p.)$

$b.)\ P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$$= e^{-1.8} + \frac{e^{-1.8}(1.8)}{1!} + \frac{e^{-1.8}(1.8)^2}{2!} = 0.7306\ (to\ 4\ d.p.)$$

$c.)\ X_1 + X_2 \sim Po(2\lambda) \sim Po(3.6)$, *similarly in b.) with* $\lambda = 3.6$

$P(X_1 + X_2 < 3) = 0.3027\ (to\ 4\ d.p.)$

* *Possion Distribution,* $P(X=k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

* 因為獨立事件, E(X+Y)=E(X)+E(Y)

Var(X+Y)=Var(X)+Var(Y)

2012 - SECTION A

Q8.) Let X be the discrete random vaiablr with $E(X)=5.5$ *and*

k	1	3	4	6	9	13
$P(X=k)$	0.1	a	0.25	0.15	b	0.05

a.) Find a and b

b.) Assume F be event $X \geq 4$ *while G be event of* $X<8$. *Find* $P(F\cap G)$.
Are F and G independent events? Explain your answer.

* 參考課程 4.1, 4.2, 4.3 及 4.4

a.) $\sum_{i=1}^{6} P(X=k_i)=1$ and $\boxed{\sum_{i=1}^{6} k_iP(X=k_i)}=5.5$

$\rightarrow a+b=0.45$ and $a+3b=0.95 \rightarrow a=0.2$ and $b=0.25$

b.) $P(F\cap G)=P(4\leq X<8)=0.25+0.15=0.4$

Consider, $P(F)P(G)=P(X\geq 4)P(X<8)$

$=(0.25+0.15+0.25+0.05)(0.1+0.2+0.25+0.15)=0.49$

$\neq P(F\cap G)$

$\therefore$ F and G are not independent

* $\boxed{}$ $E(X)=\sum_{i=1}^{n} k_iP(X=k_i)$

* 用消去法整走 a 搵 b, 再代入搵 a

* 如果獨立事件, P(A & B)=P(A)P(B)

2012 - SECTION A

Q9.) Given that the test score of student have revised follow $N(59, 10^2)$ while that of student have not revised follow $N(35.2, 12^2)$. The passing score for the test is 43. There is 73 % have revised.

a.) Find the probability that randomly selected student pass the test.

b.) Find the probability student have not revised given that he passes the test.

c.) 10 randomly selected students pass the test. Find the probability there are exactly 4 of them had not revised.

* 參考課程 4.2 及 4.5

a.) Let X_1 be the ramdom variable of test score by not revised student

X_2 be the ramdom variable of test score by revised student

$$P(Pass) = 0.27P(X_1 \geq 43) + 0.73P(X_2 \geq 43)$$

$$= 0.27(0.2578) + 0.73(0.9452) = 0.759602$$

* 先計算 Z ~ N(0,1), 再對表

b.) $P(student\ not\ revised \mid he\ pass) = \dfrac{P(student\ not\ revised\ and\ pass)}{P(pass)}$

* P(A|B)=P(A & B)/P(B)

CONT'D

2012 - SECTION A

$$= \frac{0.27(0.2578)}{0.759602} = 0.0916\ (to\ 4\ d.p.)$$

$c.)$ $Consider,\ Y \sim B(10,\ 0.0916)$

$$The\ probability = P(Y = 4) = C_4^{10}(0.0916)^4(1 - 0.0916)^6$$

$$= 0.0083\ (to\ 4\ d.p.)$$

* 二項分佈

2012 - SECTION B

$$Q10.)\ Let\ I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt$$

a.) Estimate I by trapezoidal rule with 6 sub – interval. Is the result over – estimated? Explain your answer.

b.) Hence, by the a.) result, show that $\pi < 3.25$

* 參考課程 2.8, 3.3 及 4.5

$$a.)\ Let\ f(t) = \frac{1}{\sqrt{t}} e^{\frac{-t}{2}},\ the\ interval = \frac{4-1}{6} = 0.5$$

$$I \approx \frac{0.5}{2}[f(1) + 2f(1.5) + 2(2) + 2(2.5) + 2(3) + 2(3.5) + f(4)]$$

* 計算梯形面積的加總

$$= 0.6929\ (to\ 4\ d.p.)$$

$$Consider,\ t[f(t)]^2 = e^{-t} \rightarrow [f(t)]^2 + t2f(t)f'(t) = -e^{-t}$$

* Implicit 微分法

CONT'D

2012 - SECTION B

$$\rightarrow [f(t)]^2 + t2f(t)f'(t) = -t[f(t)]^2$$

$$\rightarrow 2f'(t) = -(1+t^{-1})f(t)$$

$$\rightarrow f''(t) = 0.5 \cdot [-(1+t^{-1})f'(t) + t^{-2}f(t)]$$

$$\because for\ 1 \le t \le 4, f(t) > 0\ and\ f'(t) < 0$$

$$\therefore f''(t) > 0,\ The\ result\ is\ over-estimated\ .$$

$$b.)\ I = \int_{1}^{2} \frac{1}{x} e^{\frac{-x^2}{2}} d(x^2),\ with\ t = x^2$$

$$= 2\int_1^2 e^{\frac{-x^2}{2}} dx = 2\sqrt{2\pi} \cdot \boxed{\frac{1}{\sqrt{2\pi}} \int_1^2 e^{\frac{-x^2}{2}} dx} = 2\sqrt{2\pi}(0.1359)$$

$$\because The\ result\ is\ over-estimated\ .$$

$$\therefore I < 0.6929 \rightarrow 8\pi(0.1359)^2 < (0.6929)^2$$

$$\rightarrow \pi < 3.25$$

* 個 f(t) 係 concave upward

* 用積分代入法, $t=x^2$

* 定積分代入要改範圍

* ▇ 用Normal Distribution 表搵面積

2012 - SECTION B

Q11.) Assume the rate of change of R,

$$\frac{dR}{dt} = \frac{a(30 - t) + 10}{(t - 35)^2 + b}, \text{ where } 0 \le t \le T \text{ is the number of days since the start, } a, b, T \in \mathbb{Z}^+$$

Given that R value increase at start and then the greatest value of 6 units at $t = 35$ and then decrease to the value at $t = 0$ when $t = T$. Also, the rate of decrease from $t = 40$ to $t = 41$ is $\ln \frac{61}{50}$ unit.

a.) Find a, b, T, and express R in term of t.

b.) For $0 \le t \le 35$, when will the rate of change of R attain the greatest?

* 參考課程 2.3, 2.4, 2.6 及 2.7

CONT'D

2012 - SECTION B

$$a.)\ \frac{dR}{dt}\Big|_{t=35} = 0 \rightarrow -5a + 10 = 0 \rightarrow a = 2$$

$$Consider,\ R(t) = \int \frac{2(30-t)+10}{(t-35)^2+b}dt$$

$$= \int -\frac{2(t-35)}{(t-35)^2+b}dt$$

$$= \int -\frac{d[(t-35)^2+b]}{(t-35)^2+b}$$

$$= -\ln|(t-35)^2+b| + C,\ where\ C\ is\ constant.$$

$$Given\ that\ R(T) = R(0),\ R(40) - R(41) = \ln\frac{61}{50}\ and\ R(35) = 6$$

$$R(T) = R(0) \rightarrow \ln|(T-35)^2+b| = \ln|(35)^2+b|$$

* f′(t_0) = 0 當 t = t_0 係 turning point

* 積分類似微分的逆函數

* 用積分代入法

2012 - SECTION B

$$\rightarrow (T-35)^2 = 35^2 \rightarrow T = 70 \ or \ 0 \ (rejected)$$

$$i.e.\ T = 70$$

$$R(40) - R(41) = ln\frac{61}{50} \rightarrow ln\left|\frac{36+b}{25+b}\right| = ln\frac{61}{50}$$

$$\rightarrow \frac{36+b}{25+b} = \frac{61}{50} \quad \rightarrow \ b = 25$$

$$R(35) = 6 \rightarrow C = ln25 + 6$$

$$i.e.\ R(t) = -ln\left|(t-35)^2 + 25\right| + ln25 + 6$$

$$b.)\ R'(t) = -\frac{2(t-35)}{(t-35)^2+25} \rightarrow [(t-35)^2+25]R'(t) = -2(t-35)$$

$$\rightarrow 2(t-35)R'(t) + [(t-35)^2+25]R''(t) = -2$$

$$Let\ t_0 \in \mathbb{R}\ such\ that\ R''(t_0) = 0$$

* lnA - lnB = lnA/B

* 要搵 Rate of change 的最大值

* Implicit 微分法

* 搵 turning point = 搵 t_0 使度 $R''(t_0)=0$

2012 - SECTION B

$$Then\ 2(t_0-35)R'(t_0)=-2 \rightarrow 2(t_0-35)\frac{2(t_0-35)}{(t_0-35)^2+25}=2$$

$$Let\ u=t-35, \rightarrow 4u^2=2(u^2+25) \rightarrow u^2=25$$

$$i.e.\ t-35=\pm 5 \rightarrow t=30\ or\ 40\ (rejected,\ 0 \le t \le 35)$$

	0 < t <30	t =30	30 < t < 35
R''(t)	+	0	-
R'(t)	Increasing		Decreasing

$$\therefore The\ rate\ of\ change\ of\ R\ attain\ the\ greatest\ when\ t=30$$

* 利用表格計算 turning pt. 附近情況

$f'(x)>0 \rightarrow increasing$

$f'(x)<0 \rightarrow decreasing$

2012 – SECTION B

Q12.) Given that the waiting time (in minutes) of a randomly selected person in a queue follow a normal distribution with mean $= \mu$ and standard deviation $= 9$.
A size 16 random sample is conducted to study the μ with the following waiting record

56	36	48	63	57	41	50	43
56	55	62	46	55	69	38	50

a.) Find the 90% confidence interval of μ

b.) Find the min. sample size for the width of 90% confidence interval of μ less than 6 minutes.

c.) Suppose $\mu = 51.5$. Person waits more than 65 minutes in the queue will be angry.

i.) Find the probability that there is less than 2 people are angry within 10 randomly selected people in the queue.

ii.) Find the probability that the 5^{th} person is angry for the 20^{th} person in the queue.

* 參考課程 4.4, 4.5, 4.6 及 4.7

CONT'D

2012 – SECTION B

$a.)\ Let\ \bar{X}\ be\ the\ sample\ mean\ of\ the\ study.$

$$\bar{X} = \frac{\sum_{i=1}^{16} X_i}{16},\ where\ X_i\ is\ the\ sample\ data$$

$$\rightarrow \bar{X} = 51.5625\ minutes$$

$$\therefore 90\%\ C.I.\ for\ \mu = \left(\bar{X} - 1.645 \cdot \frac{9}{\sqrt{16}},\ \bar{X} + 1.645 \cdot \frac{9}{\sqrt{16}}\right)$$

$$= (47.86125,\ 55.26375)$$

$b.)\ Let\ the\ sample\ size\ be\ n$

$$The\ width\ of\ the\ interval = 2 \cdot 1.645 \cdot \frac{9}{\sqrt{n}} < 6 \rightarrow n > 24.35$$

$$\therefore The\ min.\ sample\ size = 25$$

* 90% 置信區間

CONT'D

2012 - SECTION B

$ci.)$ *Let X be the random variable of the waiting time of a randomly selected person from the queue*

$$P(Person\ Angry) = P(X > 65) = P\left(Z > \frac{65 - 51.5}{9}\right)$$

$$= P(Z > 1.5) = 0.0668$$

Consider the random variable, $Y \sim B(10, 0.0668)$

$$The\ probability = P(Y=0) + P(Y=1)$$

$$= (1 - 0.0668)^{10} + C_1^{10}(0.0668)(1 - 0.0668)^9$$

$$= 0.8594\ (to\ 4\ d.p.)$$

$$ii.)\ The\ probability = (0.0668) \cdot C_4^{19}(0.0668)^4(1 - 0.0668)^{15}$$

$$= 0.0018\ (to\ 4\ d.p.)$$

* 先計算 Z ~ N(0,1), 再對表

* $P(Y = k) = C_k^n p^k (1 - p)^{n-k}$

* 頭 19 個有 4 個 angry, 第 20 個 angry

2012 - SECTION B

Q13.) Given that number of drunk driver found in a test follows Possion Distribution with the mean = 2.3 person

a.) Find the probability that at least 2 drunk driver found in a test.

b.) Find the probability more than not more than 4 drunk drivers are found in the test, given that there are at least 2 drunk drivers are found in the test.

c.) 3 numbers of test are conducted. Given that those testes are independent.

i.) Find the porobability that the 3^{rd} test is the first test have at least 2 drunk drivers found.

ii.) Find the probability that there is at least 2 drunk drivers found in each test with total 10 drunk drivers found in 3 test.

* 參考課程 4.3 及 4.4

a.) Let $X \sim Po(2.3)$ be the random variable of the drunk drivers found

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-2.3} - e^{-2.3}(2.3)$$

$$= 0.6691 \ (to\ 4\ d.p.)$$

* P(Not A) = 1 - P(A)

* $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

CONT'D

2012 – SECTION B

$$b.)\ P(X \le 4 \mid X \ge 2) = \frac{P(2 \le X \le 4)}{P(X \ge 2)} = \frac{P(X=2)+P(X=3)+P(X=4)}{P(X \ge 2)}$$

$$= \frac{\frac{e^{-2.3}(2.3)^2}{2!} + \frac{e^{-2.3}(2.3)^3}{3!} + \frac{e^{-2.3}(2.3)^4}{4!}}{0.6691}$$

$$= 0.8748\ (to\ 4\ d.p.)$$

* P(A|B)=P(A & B)/P(B)

$c.)$ *Let* X_1, X_2, X_3 *be the random variable of drunk drivers found on each 3 tests.*

* A 同 B 獨立, P(A and B)=P(A)P(B)

$$i.)\ The\ probability = P(X_1 < 2)P(X_2 < 2)P(X_3 \ge 2)$$

$$= (1-0.6691)^2(0.6691) = 0.0732\ (to\ 4\ d.p.)$$

$$ii.)\ The\ probability = C^3_2 P(X=2)^2 P(X=6) + P^3_1 P(X=3)P(X=4)P(X=5)$$
$$+ C^3_2 P(X=4)^2 P(X=2) + C^3_2 P(X=3)^2 P(X=4)$$

$$= 0.0471\ (to\ 4\ d.p.)$$

* 列舉所有可能性