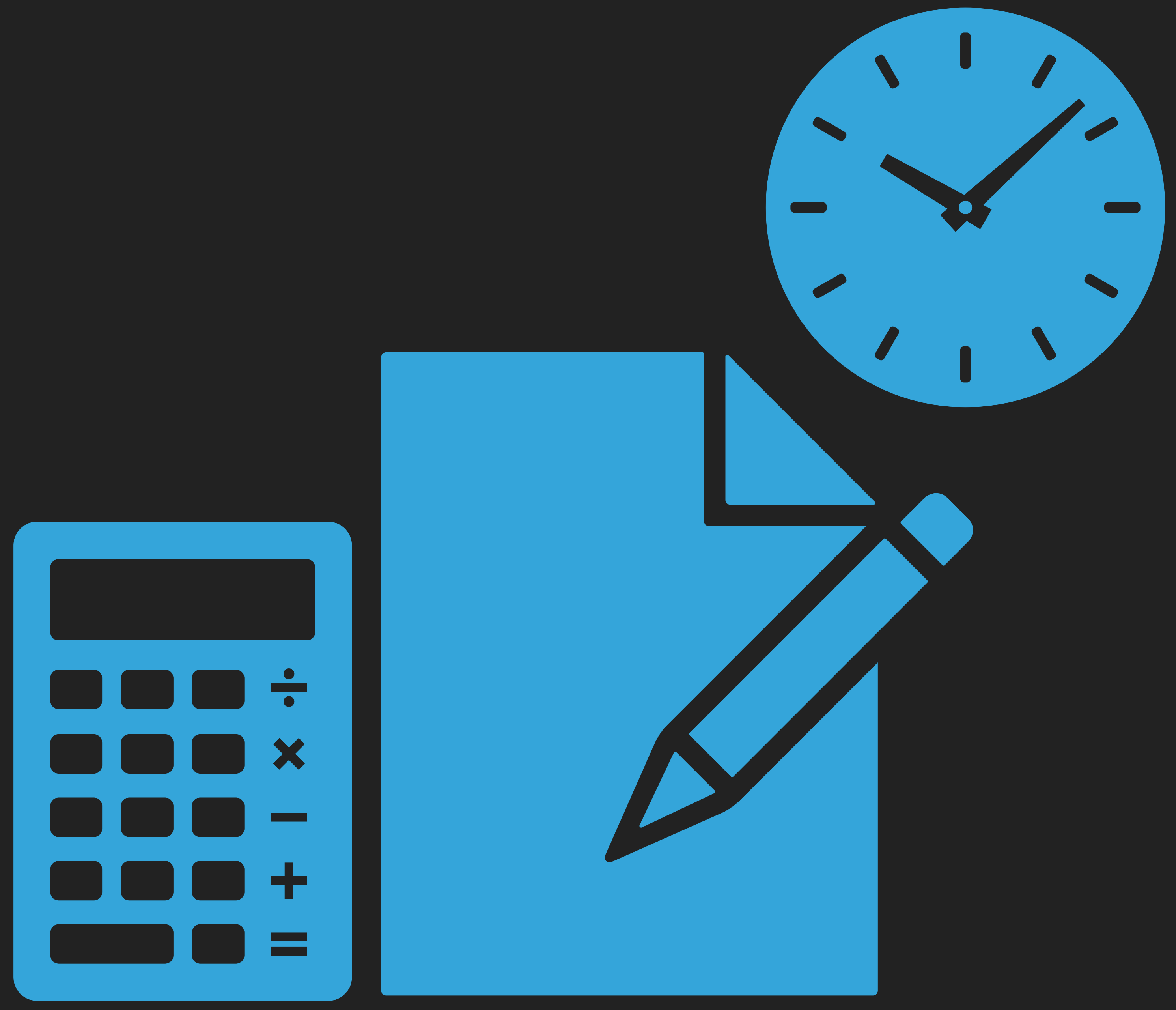
深宵教室 - DSE 必修模擬試題解答

2017 PAPER 1

2017 PAPER 1

- Section A1
- Section A2
- Section B



2017 PAPER 1 - SECTION A1

$Q1.)\ k = \dfrac{3x - y}{y},\ y = ?$

* 參考課程 2.1

$\rightarrow ky = 3x - y$

$\rightarrow y = \dfrac{3x}{k+1}$

* 兩邊乘y

* 兩邊加 y 再除 (k+1)

2017 PAPER 1 - SECTION A1

Q2.) Simplified $\dfrac{(x^4y^{-1})^3}{(x^{-2})^5}$, *in positive indices*

* 參考課程 1.2

$$= x^{12-(-2)\cdot 5} \cdot y^{-1\cdot 3}$$

$$= x^{22} \cdot y^{-3}$$

$$= \frac{x^{22}}{y^3}$$

* 指數乘係加，除係減

* 指數負數，分母變分子，分子變分母

2017 PAPER 1 - SECTION A1

Q3.) Factorize $x^2 - 4xy + 3y^2 + 11x - 33y$

* 參考課程 2.5

$= (x-3y)(x-y) + 11(x-3y)$

$= (x-3y)(x-y+11)$

* 十字相乘 $(a-\alpha)(a-\beta) \to \alpha\beta = -6, \alpha+\beta = -1$
* 抽 11

2017 PAPER 1 - SECTION A1

Q4.) There are good A and good B. The price of good A is \$126 while that of good B is \$78. Given that the number of good A sold is 5 times that of good B sold. The total money earned is \$50976. Find the total number of goods sold.

* 參考課程 2.3

Let the number of good A sold be a
the number of good B sold be b

$$\begin{cases} 50976 = 126a + 78b & \text{——— (1)} \\ a = 5b & \text{——————— (2)} \end{cases}$$

$$\text{Sub (2) into (1)} : b = 72 \rightarrow a = 360$$

$$\therefore \text{The total number of goods sold} = a + b$$
$$= 432$$

* 代入法代 (2) 入 (1) 搵 b ,再代 (2) 式搵 a

2017 PAPER 1 - SECTION A1

Q5.) Solve $7(x-2) \le \dfrac{11x+8}{3}$ *and* $6-x<5$

Hence, find the number of integer satisfy the above inequalities.

* 參考課程 1.1 及 2.3

$\rightarrow 21x - 42 \le 11x + 8 \text{ and } x > 1$

$\rightarrow 10x \le 50 \text{ and } x > 1$

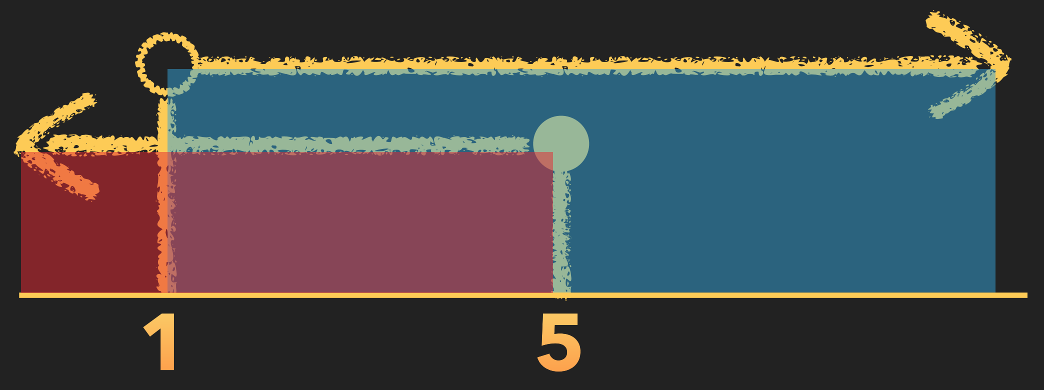
$\rightarrow x \le 5 \text{ and } x > 1$

$\rightarrow 1 < x \le 5$

The possible integers are 2, 3, 4, *and* 5

$\therefore$ *There are* 4 *number of integers satisfy the inequalities.*

* and 指有重疊的地方



2017 PAPER 1 - SECTION A1

Q6.) In a rectangle system, $O = (0, 0^0)$, $A = (-3, 4)$, $B = (9, \ -9)$. *A is rotated anticlockwise about O through* 90^0 *to* A'. B' *is the reflection image of B with respect to the x − axis.*

a.) Find A' *and* B'.

b.) Is $AB \perp A'B'$? *Explain your answer.*

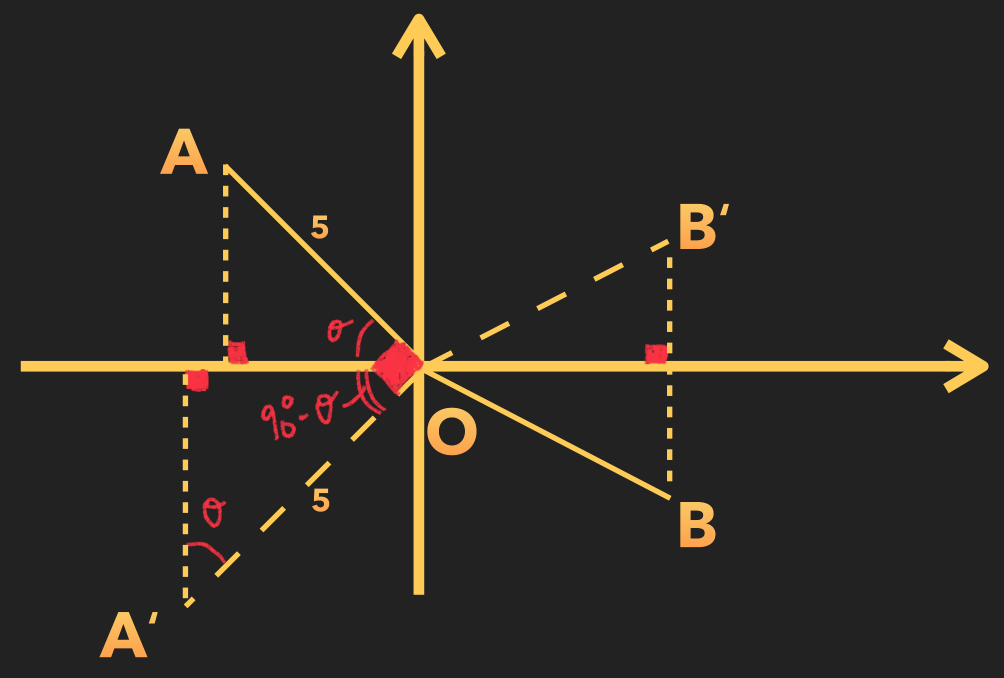
* 參考課程 2.8, 3.2 及 3.8

a.) $A' = (-4, \ -3)$, $B' = (9, 9)$

b.) The slope of AB x The slope of A'B' $= \boxed{\dfrac{-9-4}{9+3}}\boxed{\dfrac{9+3}{9+4}} = -1$

$\therefore AB \perp A'B'$

* 先畫圖理解

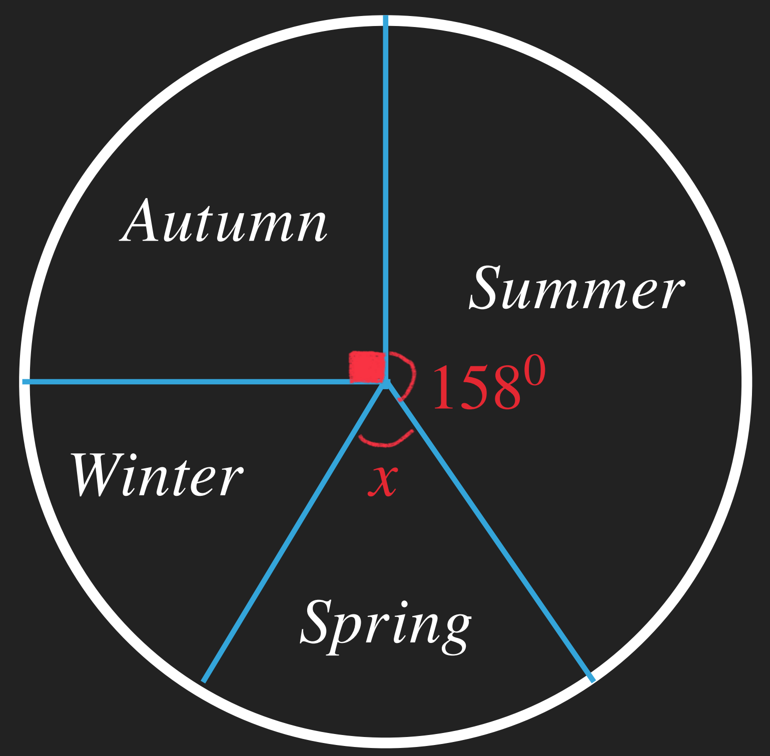


* $\boxed{}$ 兩點斜率 = $\dfrac{y_2 - y_1}{x_2 - x_1}$

* 兩條線斜率相乘 = -1, 互相垂直

2017 PAPER 1 - SECTION A1

Q7.) The following shows the season of birth of a group of student. If the probability of the randomly selected student was born in spring $= \frac{1}{9}$



a.) $x = ?$

b.) Given that there is 180 students born in Winter. Find the total number of students.

* 參考課程 4.1 及 4.3

a.) P(The selected student born in Spring) $= \frac{x}{360^0} = \frac{1}{9}$

$\rightarrow x = 40^0$

b.) The total number of student $= \frac{360^0}{360^0 - 90^0 - 158^0 - 40^0} x180$

$= 900$

* 圓形圖內角度比例 = 人數比例

2017 PAPER 1 - SECTION A1

Q8.) z is varies inversely as $\sqrt{x}$. *Given that* $x = 144$, $z = 81$.

a.) Find z in term of x

b.) Find the change of z if x increase from 144 to 324.

* 參考課程 1.3 及 2.4

a.) Let $z = \dfrac{k}{\sqrt{x}}$, where k is constant

$$\because x = 144,\ z = 81,\ 81 = \frac{k}{\sqrt{144}} \rightarrow k = 972$$

$$\therefore z = \frac{972}{\sqrt{x}}$$

b.) The change of $z = \dfrac{972}{\sqrt{324}} - \dfrac{972}{\sqrt{144}} = -27$

* 聯合變量

2017 PAPER 1 - SECTION A1

Q9.) A bottle is standard if its capacity measured as 200mL correct to the nearest 10mL

a.) Find the least capacity of the standard bottle .

b.) Is it possible the total 120 standard bottles is measured as 23.3L correct to the nearest 0.1L? Explain your answer .

* 參考課程 1.1

a.) *The least capacity* $= 195\ mL$

b.) *Let* X *be the capacity of a standard bottle*

$$195mL \le X \le 204mL$$

$$\rightarrow 23.4L \le 120X \le 24.48L$$

$\because$ *The least possible of* $120X = 23.4L$ (*to nearest* $0.1L$)

$\therefore$ *It is impossible* 120 *standard bottle measured as* 23.3L

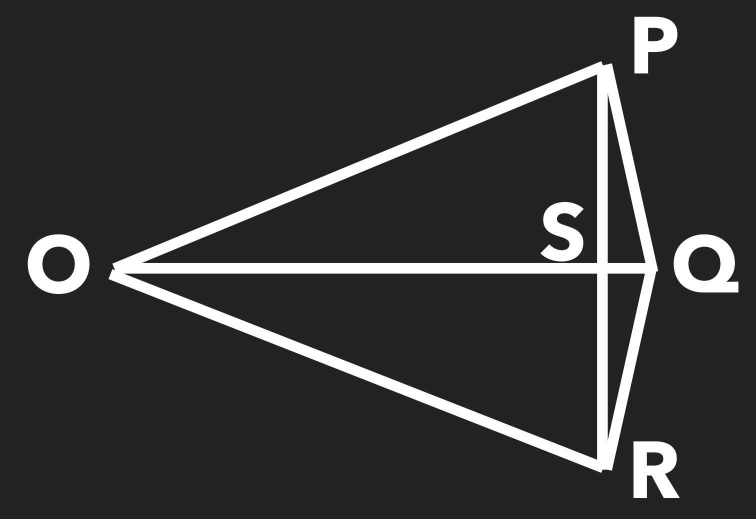
* 四捨五入

2017 PAPER 1 - SECTION A2

Q10.) The following shows a quadrilateral OPQR, with $OP = OQ = OR$.
S is a mid points of PR. Given that there is a circle with center O passing through P, Q and R. $OQ = 6cm$ *and* $\angle PRQ = 10^0$.

a.) Prove that $\Delta OPS \cong \Delta ORS$.

b.) Find the area of the sector OPQR in term of π.



* 參考課程 3.2, 3.3, 3.6 及 3.8

a.) $OP = OR, PS = RS$ (given) $OS = OS$ (common)

$\therefore \Delta OPS \cong \Delta ORS$ (SSS)

* 共邊要寫理由

b.) $\angle POQ = 2\angle PRQ = 20^0$ ($\angle$ at center twice $\angle$ at $\odot^{ce}$)

* 圓心角係圓周角兩倍

$\angle POR = \angle POQ + \angle QOR = 2x20^0 = 40^0$ ($\because \Delta OPS \cong \Delta ORS$)

The radius of the circle $= OQ = 6cm$

The are of the sector OPQR $= \boxed{\dfrac{40^0}{360^0}\pi(6)^2} = 4\pi\ cm^2$

* 扇形面積 = 角度比例 x 圓形面積

2017 PAPER 1 - SECTION A2

Q11.) The stem – and – leaf diagram below shows the hourly wages of a group of workers

Stem (tens)	Leaf (units)
6	1 1 1 3 4 6 8 9 9
7	a 7 7 8
8	1 b

Given that the mean and the range is \$70 and \$22 respectively.

a.) Find the median and the standard deviation.

b.) Find the probability of randomly selected worker in the group has more than \$70 hourly wage.

* 參考課程 4.1, 4.2 及 4.3

a.) $The\ range = (80+b) - 61 = 22 \rightarrow b = 3$

$$The\ mean = \frac{(70+a) + sum\ of\ the\ rest}{15} = 70$$

* 全距 = 最大值 - 最細值

* 平均值 = 加總 / 總數量

CONT'D

2017 PAPER 1 - SECTION A2

$$\rightarrow a = 2$$

$$\therefore \text{The median} = \$69$$

$$\text{The standard deviation} = \$7.33\ (\text{to } 3 \text{ sig . fig})$$

$$b.)\ \text{The probability} = \frac{6}{15} = \frac{2}{5}$$

* 中位數 = 中間的數值

* 標準分數 = 數據相差平均數幾多個標準差

2017 PAPER 1 - SECTION A2

Q12.) There are two similar solid right pyramids with square base. The ratio of their base area = 4 : 9. Their total volume is equal to a solid metal right prism with height = 20cm and the base area = $84cm^2$

a.) Find the volume of the larger pyramid

b.) Find the total surface area of the smaller pyramid if the height of the larger pyramid = 12cm.

* 參考課程 3.2 及 3.9

a.) Let V_1 cm^3 be the volume of the larger pyramid

V_2 cm^3 be the volume of the smaller pyramid

$$\text{Then, } \boxed{\frac{V_1}{V_1+V_2} = \frac{9^{\frac{3}{2}}}{4^{\frac{3}{2}}+9^{\frac{3}{2}}}} \rightarrow V_1 = \boxed{(84)(20)}\frac{27}{35} = 1296$$

$\therefore$ *The volume of larger pyramid = 1296 cm^3*

* 相似圖形, 體積比 = (邊比)³

面積比 = (邊比)²

* 柱體體積 = 底面積 x 高

CONT'D

2017 PAPER 1 - SECTION A2

$b.)\ Let\ the\ base\ area\ of\ larger\ pyramid\ be\ A_1$

$$\boxed{V_1 = \frac{1}{3}A_1(12)} \rightarrow A_1 = 324cm^2$$

$$\therefore The\ side\ length\ of\ the\ base,\ a_1 = \sqrt{324} = 18cm$$

$In\ the\ side\ \Delta\ of\ the\ larger\ pyramid,$

$$The\ height,\ \boxed{h_1 = \sqrt{12^2 + (\frac{18}{2})^2}} = 15cm$$

$$Hence\ the\ area\ of\ the\ side\ \Delta = \frac{1}{2}(18)(15) = 135cm^2$$

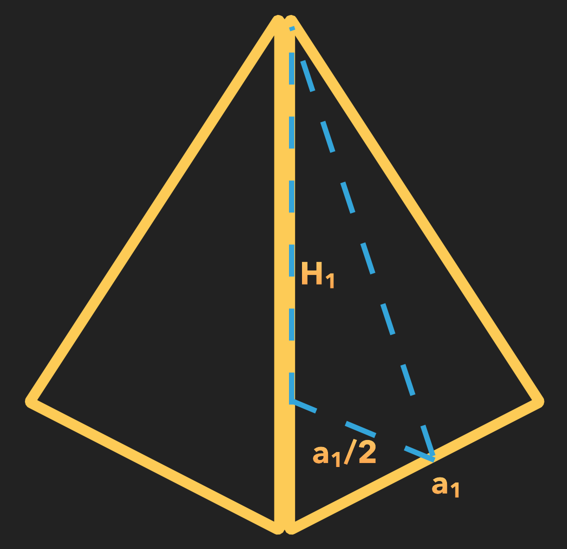
$$The\ total\ surface\ area\ of\ the\ larger\ pyramid = (4x135 + 324)$$

$$= 864\ cm^2$$

$$The\ total\ surface\ area\ of\ the\ smaller\ pyramid = \boxed{\frac{4}{9}(864)}$$

$$= 384\ cm^2$$

* 錐體體積 = 1/3 x 底面積 x 高



* 畢氏定理

* 相似圖形, 面積比全部一樣

2017 PAPER 1 - SECTION A2

Q13.) The circle, C passes through point $E(-6,5)$*, with center* $G(2,-1)$.

Given that $F=(-3,11)$

a.) Find the equation of C.

b.) Is F lie outside C? Explain your answer.

c.) Let H be the moving point on C. When H is farthest from F, Find the equation of the straight line passing through F and H.

* 參考課程 3.8

a.) *The radius of C* $=\sqrt{(2-(-6))^2+(-1-5)^2}=10$

The equation of C : $(x-2)^2+(y+1)2=10^2$

$(x-2)^2+(y+1)^2=100$

b.) $FG=\sqrt{(2-(-3))^2+(11-(-1))^2}=13>10$

$\because$ *FG* $>$ *the radius of C*

$\therefore$ *F lies outside C*

* 距離公式 = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

* 圓形公式 = $(x-h)^2+(y-k)^2=r^2$

* 某點至圓心距離 > 半徑, 某點在圓形之外

CONT'D

2017 PAPER 1 - SECTION A2

$c.) \because F, G \text{ and } H \text{ are collinear}$

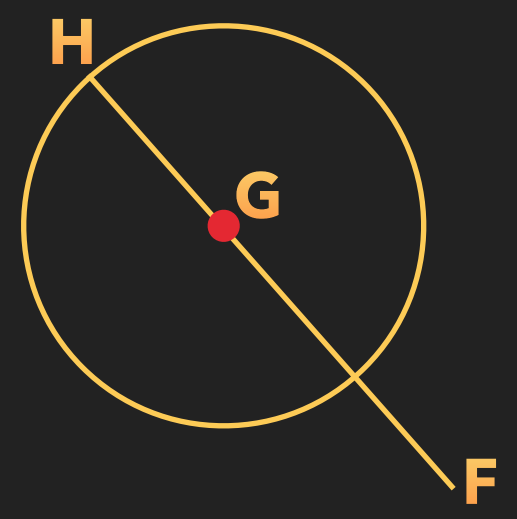
$\therefore \text{ the equation of the straight line :}$

$$\boxed{\frac{y-(-1)}{x-2} = \frac{11-(-1)}{-3-2}}$$

$$\rightarrow -5(y+1) = 12(x-2)$$

$$\rightarrow 12x + 5y - 19 = 0$$

* 直線方程, 兩點公式



2017 PAPER 1 - SECTION A2

Q14.) When $f(x) = 6x^3 - 13x^2 - 46x + 34$ is divided by $2x^2 + ax + 4$, the quotient $= 3x + 7$ the remainder $= bx + c$. Let $g(x)$ be a quadratic polynomial. When $g(x)$ is divided by $2x^2 + ax + 4$, the remainder $= bx + c$, where a, b and c are constant

a.) Find a

b.) Are all roots of $f(x) - g(x) = 0$ integers? Explain your answer.

* 參考課程 1.1, 2.4 及 2.6

a.) $f(x) \equiv (3x + 7)(2x^2 + ax + 4) + (bx + c)$

By comparison of coefficient of x^2,

$-13 = 14 + 3a \rightarrow a = -9$

b.) Let $g(x) \equiv A(2x^2 - 9x + 4) + (bx + c)$, where A is constant

Then, $f(x) - g(x) = 0 \rightarrow (2x^2 - 9x + 4)(3x + 7 + A) = 0$

$\rightarrow 2x^2 - 9x + 4 = 0 \ -(1)$ or $3x + 7 + A = 0 \ -(2)$

In (1), $x = 4$ or 0.5

i.e. There are not all integer roots for $f(x) - g(x) = 0$

* f(x) = 除數 x 被除數 + 餘數

* g(x) 係二次函數

* 用二次方程根公式

2017 PAPER 1 - SECTION B

Q15.) Let a and b be constant, such that a straight line $L : y = a + \log_b x$. *Given that* $(243,3)$ *lies on L and the* x *– inteception of* $L = 9$. *Express* x *in term of* y.

* 參考課程 1.2, 2.2, 及 2.7

From the information,

$$\begin{cases} 0 = a + \log_b 9 & \text{——— (1)} \\ 3 = a + \log_b 243 & \text{——— (2)} \end{cases}$$

$$(2) - (1) : 3 = \log_b 243 - \log_b 9 = \log_b\left(\frac{243}{9}\right)$$

$$\rightarrow b^3 = 27 \rightarrow b = 3$$

Hence, $a = -\log_3 9$

$$\text{And, } y = -\log_3 9 + \log_3 x = \log_3\left(\frac{x}{9}\right)$$

$$\rightarrow 3^y = \frac{x}{9} \rightarrow x = 3^y \cdot 3^2$$

$$\rightarrow x = 3^{y+2}$$

* $\log A - \log B = \log\dfrac{A}{B}$

* 消去法, 整走 a 搵 b 再代入 (1) 搵 a

* 指數乘係加，除係減

2017 PAPER 1 – SECTION B

$Q16.)$ *The spread of virus in town has been studied. It is given that the total infected area at the* 1^{st} *year is* $1.5x10^7 m^2$. *The spread decreases at constant rate* 10% *of the infected area of the previous year for the next year.*

a.) Find the total infected area in the first 20 years.

b.) Will the total infected area exceed $1.6x10^8\ m^2$*? Explain your answer.*

* 參考課程 1.2, 2.6 及 2.7

a.) Let $T(n)$ be the infected area $(10^7 m^2)$ in the n^{th} year

Then, $T(n) = 1.5(0.9)^{n-1}$

$$T(1) + T(2) + \ldots + T(20) = 1.5(1 + 0.9 + \ldots + 0.9^{19})$$

$$= \frac{1.5(1 - 0.9^{20})}{1 - 0.9} \approx 13.176$$

$\therefore$ The total infected area in the first 20 years

$= 1.32x10^8\ m^2$ (to 3 sig. fig)

* 將 10^7 納入 Let 範圍, 簡化往後表達

* 等比數列

* 等比數列之和

CONT'D

2017 PAPER 1 - SECTION B

$$b.)\ T(1) + T(2) + \ldots + T(\infty) = \boxed{\frac{1.5}{1-0.9}} = 15$$

$\therefore$ *The infected area will not exceed* $1.6x10^8 m^2$

* 等比數列之和 (至無限大)

2017 PAPER 1 - SECTION B

Q17.) There are 4 green balls, 7 blue balls and 8 red balls in the bag. If 5 balls are randomly selected from the bag at the same time. Find

a.) The probability of exactly 4 green balls are drawn.
b.) The probability of exactly 3 green balls are drawn.
c.) The probability of not more than 2 green balls are drawn.

* 參考課程 4.4

Let NG be the event of the N numbers of green balls are drawn.

$$a.)\ P(4G) = \frac{C_4^4 C_1^{15}}{C_5^{19}} = \frac{5}{3876}$$

$$b.)\ P(3G) = \frac{C_3^4 C_2^{15}}{C_5^{19}} = \frac{35}{969}$$

$$c.)\ P(\leq 2G) = 1 - P(3G) - P(4G) = \frac{3731}{3876}$$

* 19 個波 5 個組合
* 4 個波 4 個綠波組合
* 15 個波 1 個非綠波組合
* 4 個波 3 個綠波組合
* 15 個波 2 個非綠波組合

2017 PAPER 1 - SECTION B

$Q18.)$ *Let the curve* $\Gamma : y = 2x^2 - 2kx + 2x - 3k + 8$, *where k is real constant.*
Denote a straight line $L : y = 19$.

$a.)$ *Do* L *and* Γ *intersect at 2 distinct points? Explain your answer.*

$b.)$ *Denote A and B be the intersect point of* L *and* Γ. *Is* $AB < 4$? *Explain your answer.*

* 參考課程 2.6

$a.)$ *Consider,*

$$\begin{cases} y = 2x^2 - 2kx + 2x - 3k + 8 & \text{——— (1)} \\ y = 19 & \text{——— (2)} \end{cases}$$

Sub (2) *into* (1) : $2x^2 - 2(k-1)x - (3k+11) = 0 \quad -(*)$

In $(*)$: $\Delta = 4(k-1)^2 + 4(2)(3k+11)$

$= 4(k^2 + 4k + 23) = 4[(k+2)^2 + 19] > 0$

$\therefore$ *L and* Γ *intersect at 2 distinct points.*

* 用代入法建立二次方程
* 用判別式決定有幾多根
* 頂點式決定大過 0 定細過 0

CONT'D

2017 PAPER 1 - SECTION B

$b.)\ Let\ A = (\alpha,0),\ B = (\beta,0)$

$$|AB|^2 = (\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$
$$= (\alpha + \beta)^2 - 4\alpha\beta$$
$$= (\frac{2k-2}{2})^2 - 4(\frac{3k+11}{2})$$
$$= k^2 + 4k + 23$$
$$= (k+2)^2 + 19 \geq 19$$

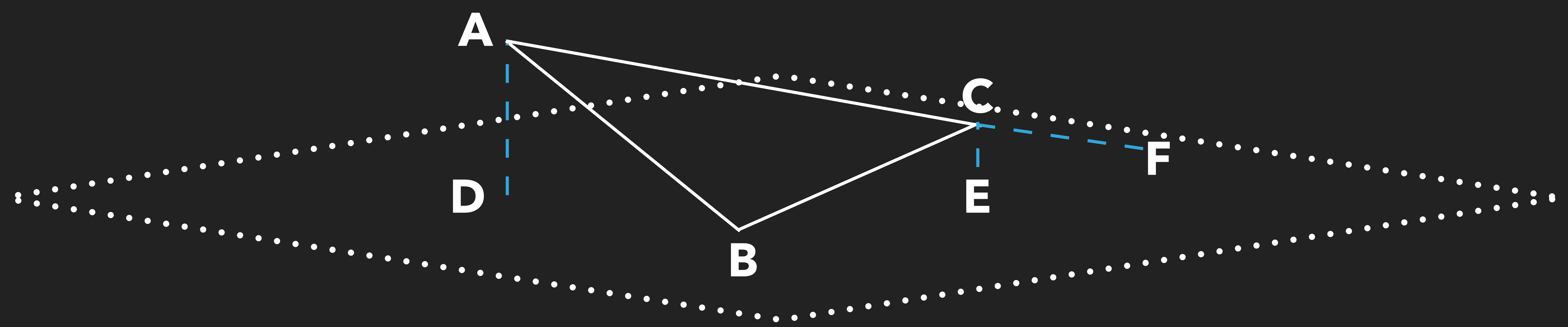
$Hence,\ |AB| \geq \sqrt{19} > 4$

$\therefore\ AB\ is\ not < 4$

* 兩根之和
* 兩根之積
* 頂點式決定最細值

2017 PAPER 1 – SECTION B

Q19.) In the following ΔABC, $BC = 24cm$, $\angle BAC = 30^0$ and $\angle ACB = 42^0$. Then ΔABC lies on the horizontal ground such that only the vertex B touches the ground. D and E are the projection of A and C on the ground respectively, with $AD = 10cm$ and $CE = 2cm$. F is produced when AC meets the ground.



a.) Find AC and CF

b.) Find the area of ΔABF

c.) Find the inclination of ΔABC to the ground.

d.) Is the area of $\Delta BDF > 460cm^2$? Explain your answer.

* 參考課程 3.2, 3.3 及 3.10

CONT'D

2017 PAPER 1 – SECTION B

a.) By sine law in ΔABC,

$$AC = \frac{24\sin(180^0 - 30^0 - 42^0)}{\sin 30^0}, \ (\angle s \ sum \ of \ \Delta)$$

$$= 48\sin 72^0 = 45.7cm \ (to \ 3 \ sig.fig.)$$

Besides,

$\angle ADF = \angle CEF = 90^0$ (given)

$\angle AFD = \angle CFE$ (common)

$\angle DAF = \angle ECF$ ($\angle s$ sum of Δ)

$\therefore \Delta ADF \sim \Delta CEF$ (AAA)

$$i.e.\ \frac{CF}{AC + CF} = \frac{CE}{AD} = \frac{1}{5}$$

$\rightarrow 4CF = AC$

$\rightarrow CF = 11.4cm$ (to 3 sig. fig.)

* sine law 使用

* 三角形內角和 = 180^0

* DEF 條線係 ACF 條線的投影線

* 三對角相等 = 相似三角形

* 相似三角形 = 邊比一樣

CONT'D

2017 PAPER 1 - SECTION B

$$b.)\ The\ area\ of\ \Delta ABC,\ A_1 = \frac{1}{2} AC \cdot BC \sin\angle ACB$$

$$= 576 \sin 72^0 \sin 42^0$$

$$\because The\ area\ of\ \Delta ABF : A_1 = AF : AC$$

$$\rightarrow The\ area\ of\ \Delta ABF,\ A_2 = 576 \sin 72^0 \sin 42^0 \frac{5}{4}$$

$$= 458\ cm^2\ (to\ 3\ sig.fig.)$$

$$c.)\ Let\ M\ be\ the\ point\ on\ BF\ such\ that\ AM \perp BF$$

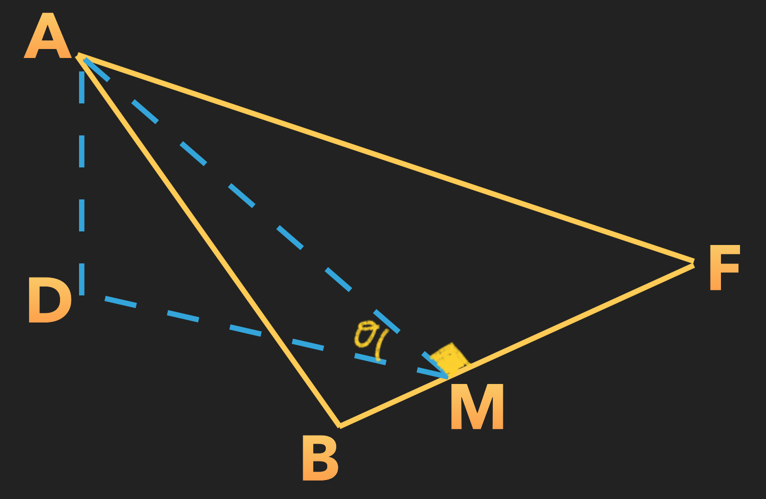
$$A_2 = \frac{1}{2} AM \cdot BF \rightarrow AM = \frac{2A_2}{BF}$$

$$\rightarrow AM = \frac{2A_2}{\sqrt{BC^2 + CF^2 - 2BC \cdot CF \cos\angle BCF}}$$

* 三角形面積 = 1/2 ab sin C

* 兩個三角形有共高, 面積比 = 邊比

* 4CF = AC, 所以 AF:AC = 5:4



* cosine law 使用

CONT'D

2017 PAPER 1 - SECTION B

$$\rightarrow AM \approx 27.4640$$

$$\therefore \text{The inclination}, \theta = \sin^{-1}\frac{AD}{AM} = \sin^{-1}\frac{10}{27.640}$$

$$= 21.4^0 \text{ (to 3 sig . fig.)}$$

$$d.) \text{ The area of } \Delta BDF = \frac{1}{2}BF \cdot DM = \frac{1}{2}BF \cdot AM\cos\theta$$

$$= A_2\cos\theta$$

$$\leq A_2 = 458cm^2$$

$$\therefore \text{The area of } \Delta BDF < 460cm^2$$

* 三角形 ABF 面積

* cos 角度介乎 -1 同 1 之間