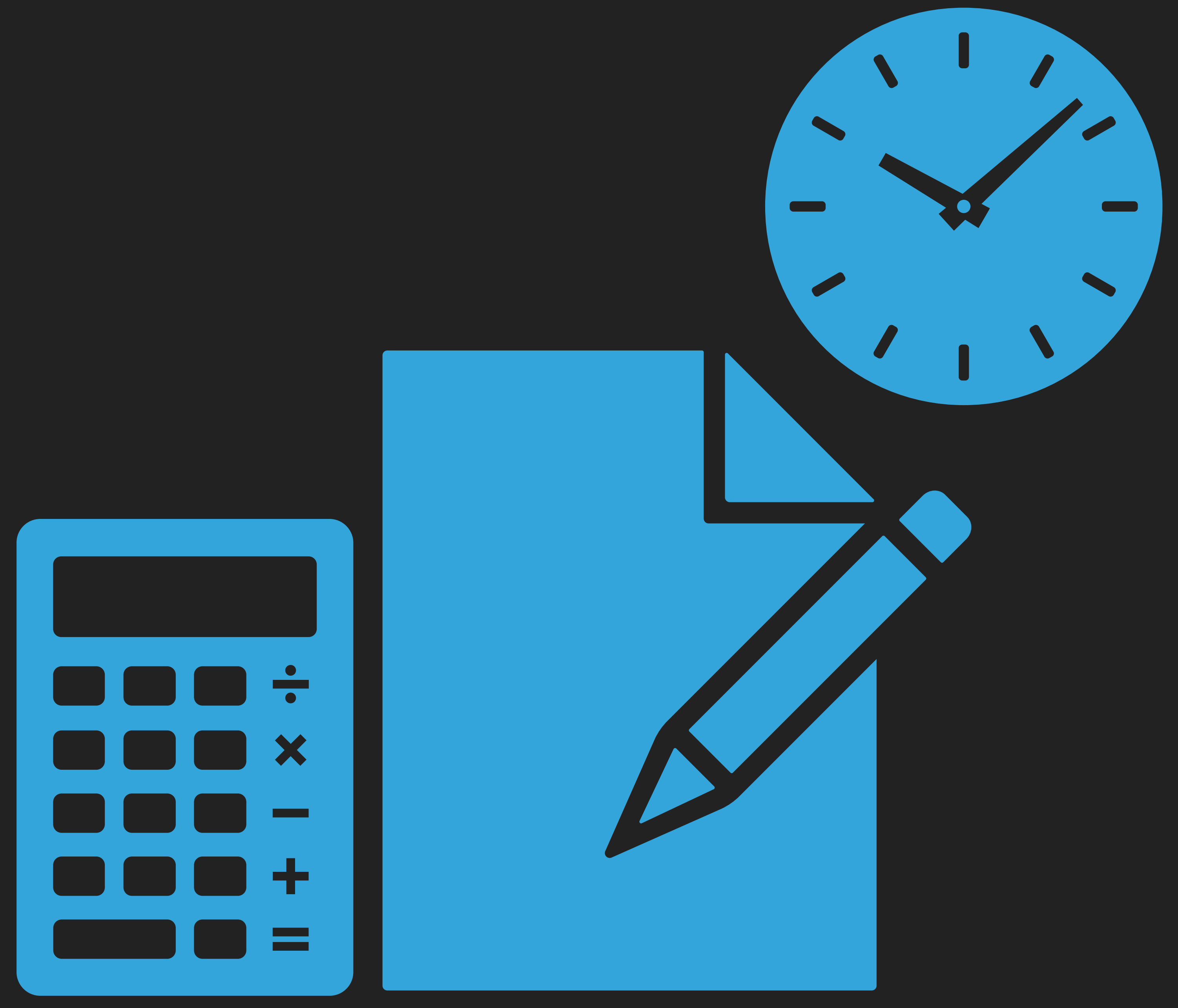
深宵教室 - DSE 必修模擬試題解答

2015 PAPER 1

此為參考2015試題之模擬試題，原版請另行購買

2015 PAPER 1

- Section A1
- Section A2
- Section B



2015 PAPER 1 - SECTION A1

Q1.) Simplified $\dfrac{x^9}{(x^3y^{-7})^5}$, *in positive indices*

* 參考課程 1.2

$$= x^{9-3\cdot 5} \cdot y^{-(-7)\cdot 5}$$

$$= x^{-6} \cdot y^{35}$$

$$= \frac{y^{35}}{x^6}$$

* 指數乘係加，除係減

* 指數負數，分母變分子，分子變分母

2015 PAPER 1 - SECTION A1

$$Q2.)\ \frac{4a+5b-7}{b}=8,\ b=?$$

* 參考課程 2.1

$$\rightarrow 4a+5b-7=8b$$

* 兩邊乘 b

$$\rightarrow 4a-7=3b$$

$$\rightarrow b=\frac{4a-7}{3}$$

* 兩邊減 5b 再除 3

2015 PAPER 1 – SECTION A1

Q3.) Box A contains 4 balls named 1, 3, 5, and 7 respectively while box B contains 5 balls named 2, 4, 6, 8 and 10 respectively . A ball is drawn from box A and box B . Find the probability of the sum of two balls less than 9

* 參考課程 4.3

$$\text{The probabilty} = \frac{6}{20} = \frac{3}{10}$$

* 用列表列出所有可能

	1	3	5	7
2	3	5	7	*
4	5	7	*	*
6	7	*	*	*
8	*	*	*	*
10	*	*	*	*

2015 PAPER 1 - SECTION A1

Q4.) Factorize $x^3 + x^2y - 7x^2 - x - y + 7$

* 參考課程 2.5

$= x^2(x + y - 7) - (x + y - 7)$

$= (x + y - 7)(x^2 - 1)$

$= (x + y - 7)(x + 1)(x - 1)$

* 抽 x^2
* 恆等式 $a^2 - b^2 \equiv (a + b)(a - b)$

2015 PAPER 1 - SECTION A1

Q5.) Solve $\frac{7-3x}{5} \le 2(x+2)$ *and* $4x - 13 > 0$

Hence, find out the least integer satisfy both inequalities .

* 參考課程 1.1 及 2.3

$$\frac{7-3x}{5} \le 2(x+2) \text{ and } 4x - 13 > 0$$

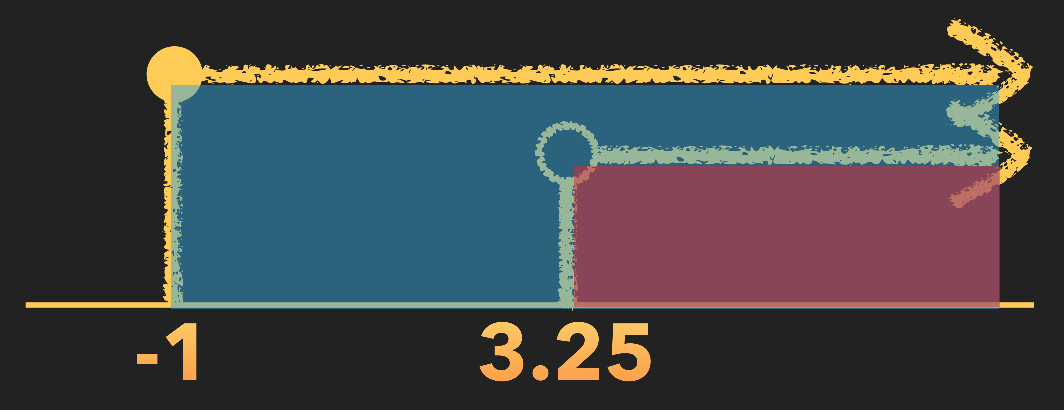
$$\rightarrow 7 - 3x \le 10x + 20 \text{ and } 4x > 13$$

$$\rightarrow x \ge -1 \text{ and } x > 3.25$$

$$\rightarrow x > 3.25$$

$\therefore$ *4 is the least integer satisfy the inequalities .*

* And 指重疊地方



2015 PAPER 1 - SECTION A1

Q6.) The cost of a good is \$250 and is sold at a 25 % discount with 20 % profit. Find the selling price and the marked price of the good.

* 參考課程 1.3

Let the selling price of the good be \$S
the marked price of the good be \$M

Then, $S = M(1 - 25\%)$ and $20\% = \dfrac{S - 250}{250} \times 100\%$

$\rightarrow S = 300 \text{ and } M = 400$

$\therefore$ *The selling price = \$300*
The marked price = \$400

* 打折後新價錢 = 價錢x(1-折扣)
* 利潤百份比 = (售價-成本) / 成本 x 100%

2015 PAPER 1 – SECTION A1

Q7.) The number of pens owned by Mary is 4 times that owned by Peter. They will have same number of pens if Mary gives 12 pens to Peter. Find the total number of pens owned by them.

* 參考課程 2.3

Let Mary has x pens

Peter has y pens

* 先 Let 未知數方便表達

$$\begin{cases} x = 4y \text{ ——————— } (1) \\ x - 12 = y + 12 \text{ ——— } (2) \end{cases}$$

Put (1) *into* (2) : $3y = 24 \rightarrow y = 8$ *and* $x = 32$

* 代入法, 在 (2) 式將 x 寫成 4y, 解 y 後再代 (1) 搵 x

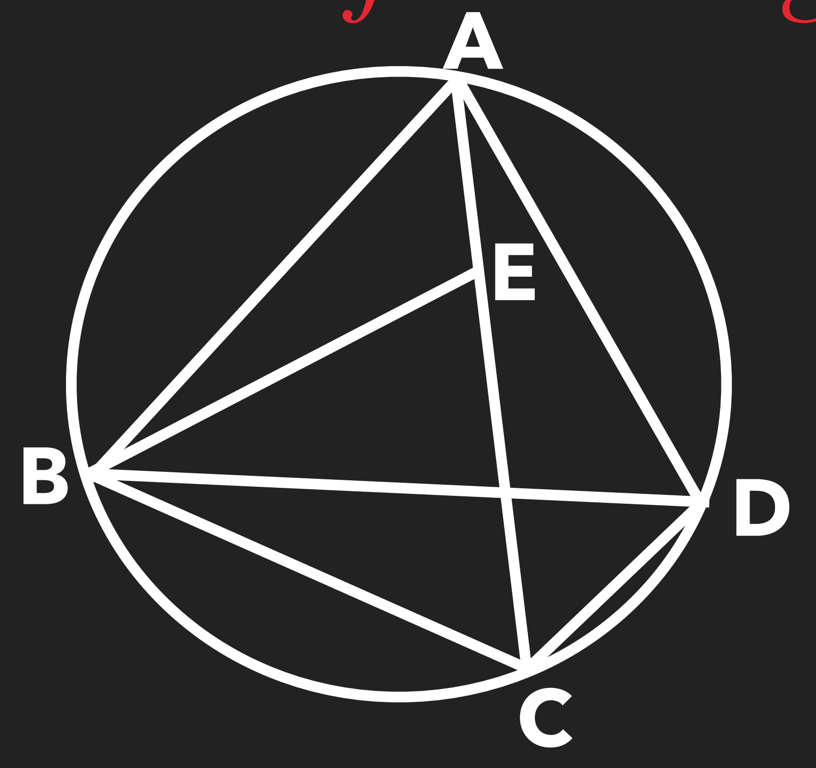
$\therefore$ *The total number of pens* $= x + y$

$= 40$

2015 PAPER 1 - SECTION A1

Q8.) In the following figure, $BC = CE$, $AB = AD$, $\angle ADB = 58^0$ and $\angle CBD = 25^0$

Find $\angle BDC$ and $\angle ABE$



* 參考課程 3.2 及 3.6

$In\ \Delta BAD,\ \angle DBA = \angle ADB = 58^0\ (base\ \angle s\ of\ isos\ \Delta)$ — * 等腰三角形底角相等

$2\angle ADB + \angle BAD = 180^0 \rightarrow \angle BAD = 64^0\ (\angle s\ sum\ of\ \Delta)$ — * 三角形內角和 = 180⁰

$\angle BDC = \angle BAC = \angle BAD - 25^0\ (\angle s\ in\ the\ same\ segment)$ — * 弓內圓周角相等

$= 39^0$

$Also,\ \angle ACB = \angle ADB = 58^0\ (\angle s\ in\ the\ same\ segment)$ — * 弓內圓周角相等

$In\ \Delta EBC,\ \angle CEB = \angle EBC\ (base\ \angle s\ of\ isos\ \Delta)$ — * 等腰三角形底角相等

$2\angle EBC + 58^0 = 180^0 \rightarrow \angle EBC = 61^0\ (\angle s\ sum\ of\ \Delta)$ — * 三角形內角和 = 180⁰

$\therefore \angle ABE = 58^0 + 25^0 - 61^0 = 22^0$

2015 PAPER 1 - SECTION A1

Q9.) The radius and the area of a sector are 12cm and 30π cm^2 respectively .

Find the angle of the sector and the perimeter of the sector in term of π

* 參考課程 3.6

Let θ be the angle of the sector

$$\frac{\theta}{360}\pi 12^2 = 30\pi \rightarrow \theta = 75$$

$\therefore$ The angle of the sector $= 75^0$

$$\text{The perimeter} = \frac{\theta}{360}(2\pi)(12) + 12 + 12$$

$$= (5\pi + 24)cm$$

* 面積 = 角度比例 x 圓面積

* 弧長 = 角度比例 x 圓周界

* 周界要包埋兩條半徑的長度

2015 PAPER 1 - SECTION A2

Q10.) It is given that S is sum of two parts, one part varies as a positive integer n and other is constant. When $n = 10$, $S = 10{,}600$, and when $n = 6$, $S = 9{,}000$.

a.) When $n = 20$, $S = ?$.

b.) Can $S = 18{,}000$?. Explain your answer.

* 參考課程 2.1, 2.3, 2.4, 2.5 及 2.6

a.) Let $S = k_1 + k_2 n$, where k_1, k_2 are real constant. Then,

$$\begin{cases} k_1 + 10k_2 = 10{,}600 \text{ ——— } (1) \\ k_1 + 6k_2 = 9{,}000 \text{ ——— } (2) \end{cases}$$

$(1) - (2):\ 4k_2 = 1600 \rightarrow k_2 = 400,\ k_1 = 6600$

$\therefore S = 6600 + 400n$ i.e. When $n = 20$, $S = 14{,}600$

b.) Consider, $18{,}000 = 6600 + 400n \rightarrow n = 28.5$

$\because$ n is not a positive integer

S cannot be $18{,}000$

* 部分變量

* 消去法消去 k_1 搵 k_2,再代 (1) 式搵 k_1

* n 要係正整數

2015 PAPER 1 - SECTION A2

Q11.) Let $f(x) = (x-2)^2(x+h)+k$, where h, k are constant. When $f(x)$ is divided by $(x-2)$, the remainder $= -5$. $f(x)$ is also divisible by $(x-3)$.

a.) Find h and k

b.) Is all roots of $f(x)=0$ are integer? Explain your answer.

* 參考課程 **1.1, 2.4 及 2.6**

a.) $f(2) = -5$ and $f(3) = 0 \rightarrow k = -5$ and $3 + h + k = 0$

By solving above, $h = 2$ and $k = -5$

b.) $f(x) = (x-2)^2(x+2) - 5 \equiv (x-3)(x^2+Ax+B)$

By comparison with constant and coefficient of x,

we have $f(x) = (x-3)(x^2+x-1)$

$\therefore f(x) = 0 \rightarrow (x-3) = 0$ or $(x^2+x-1) = 0$

$$\rightarrow x = 3 \text{ or } x = \frac{1}{2}(-1 \pm \sqrt{5})$$

i.e. It is not all roots are integer

* 餘數定理

* 因為 **f(x)** 可以俾 **(x-3)** 除得盡

* 用二次方程根公式

2015 PAPER 1 – SECTION A2

Q12.) The stem – and – leaf diagram below shows the distribution of student weight (in kg)

Stem (tens)	Leaf (units)
4	0 2 3 3 3 3 9
5	1 1 2 2 3 7 9
6	3 5 8 9
7	8 9

a.) Find the mean, median and range of the about record.

b.) 2 more students are recorded and it is found that the mean and the range is increased by 1. Find the weight of theses 2 students.

* 參考課程 4.1 及 4.2

a.) *The mean = 55kg, The median = 52kg, The range = 39kg*

b.) *Let the weight of 2 students be A kg and B kg*

$$A + B + 20(55) = 22(56) \rightarrow A + B = 132$$

There are 2 cases to be range = 40 with A + B = 132

* 平均值 = 加總 / 總數量
* 中位數 = 中間的數值
* 全距 = 最大值 - 最細值

* 平均值 = 加總 / 總數量

CONT'D

2015 PAPER 1 - SECTION A2

$Case\ 1:\quad When\ A = 80,\ then\ B = 132 - 80 = 52$

$Case\ 2:\quad When\ A = 39,\ then\ B = 132 - 39 = 93,$

$The\ range = 93 - 39 = 54 \neq 40$

$Case\ 2\ is\ impossible$

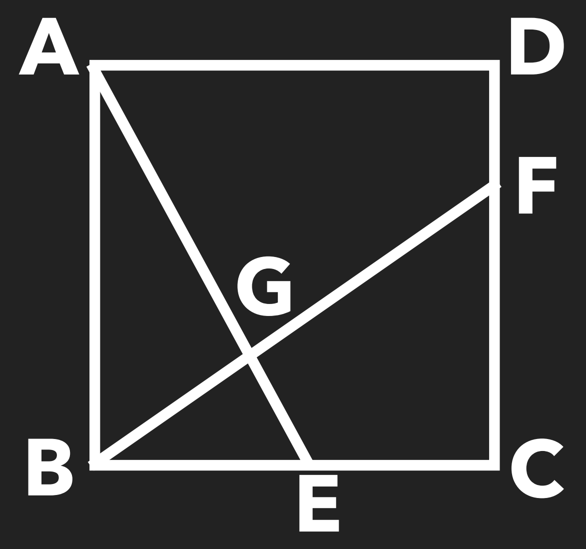
$\therefore The\ 2\ students\ weight\ 80kg\ and\ 52kg$

* 先考慮極端大+1, 搵 B 再檢查 range

* 先考慮極端細-1, 搵 B 再檢查 range

2015 PAPER 1 - SECTION A2

$Q13.)$ *In the following figure,* $ABCD$ *is square and* $AE = BF$



a.) Prove $\Delta ABE \cong \Delta BCF$

b.) Is ΔBGE a right − angled triangle? Explain your answer.

c.) $CF = 15cm$ and $EG = 9cm$, find BG.

* 參考課程 3.2, 3.3 及 3.4

a.) $\angle ABE = \angle BCF = 90^0$ (prop. of square)

$AB = BC$ (prop. of square)

$AE = BF$ (given)

$\therefore \Delta ABE \cong \Delta BCF$ (RHS)

b.) In ΔBFC, $\angle BFC + \angle FBC = 90^0$ ($\angle$s sum of Δ)

$\angle AEB = \angle BFC$ ($\because \Delta ABE \cong \Delta BCF$)

In ΔBGE, $\angle BGE = 180^0 - (\angle AEB + \angle FBC)$ ($\angle$s sum of Δ)

$= 180^0 - (\angle BFC + \angle FBC) = 90^0$

$\therefore \Delta BGE$ is a right − angled triangle

* 正方形特性

* 共邊原因要寫

* 三角形內角和 = 180^0

* 三角形內角和 = 180^0

CONT'D

2015 PAPER 1 - SECTION A2

c.) $BE = CF = 15cm \ (\because \Delta ABE \cong \Delta BCF)$

$$BG = \sqrt{15^2 - 9^2} \ (pyth\,.\ theorem)$$

$$= 12cm$$

* 畢氏定理

2015 PAPER 1 – SECTION A2

$Q14.)$ *The circle, C passes through point* $P(4,-1)$ *and* $Q(-14,23)$, *with center* $G(h,k)$.
Let L be the perpendicular bisector of PQ

a.) Find the equation of L.

b.) Find the equation of C in term of h

c.) Assume $R(26,43)$ *also lies on C. Find the diameter of C.*

* 參考課程 3.1, 3.6 及 3.8

$$a.)\ \text{The mid}-\text{pt of } PQ,\ M=\left(\frac{4-14}{2},\frac{-1+23}{2}\right)=(-5,11)$$

$$\text{The slope of } L=-1\div\left(\frac{23+1}{-14-4}\right)=\frac{3}{4}$$

$$\therefore L: y-11=\frac{3}{4}(x+5)\rightarrow 3x-4y+59=0$$

$$b.)\ G \text{ lies on } L\rightarrow 3h-4k+59=0\rightarrow 4k=3h+59$$

$(\perp$ *bisector of chord passes through center)*

* 垂直平分線, 搵中點及斜率相乘 = -1

* 中點公式 = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

* 兩點斜率 = $\frac{y_2-y_1}{x_2-x_1}$

* 點斜式

* 弦垂直平分線通過圓心

CONT'D

2015 PAPER 1 - SECTION A2

$The\ radius\ of\ C,\ r = \sqrt{(h-4)^2 + (k+1)^2}$

$$\rightarrow r^2 = (h-4)^2 + (\frac{3h+59}{4} + 1)^2$$

$$\rightarrow 16r^2 = 16(h-4)^2 + (3h+63)^2$$

$The\ equation\ of\ C:\ (x-h)^2 + (y-k)^2 = r^2$

$$\rightarrow (x-h)^2 + (y - \frac{3h+59}{4})^2 = r^2$$

$$\rightarrow 16(x-h)^2 + (4y-3h-59)^2 = 16r^2$$

$$\rightarrow 16(x-h)^2 + (4y-3h-59)^2 = 16(h-4)^2 + (3h+63)^2$$

$$\rightarrow 2x^2 + 2y^2 - 4hx - (3h+59)y + 13h - 93 = 0$$

$c.)\ Sub\ (26{,}43)\ into\ C,\ \rightarrow h = 11,\ and\ k = \frac{3(11)+59}{4} = 23$

$$\therefore The\ diameter = 2\sqrt{(11-4)^2 + (23+1)^2} = 50\ unit$$

* 距離公式 = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

* 圓形公式: (x,y) 同圓心距離=半徑

2015 PAPER 1 - SECTION B

Q15.) The (mean, standard deviation) of the score of a large group of student in English and Chinese exam are (66, 12) and (52, 10) respectively.
The standard score of Peter in the English exam is -0.5.
a.) Find the score of Peter in the English exam.
b.) The score of Peter in the Chinese exam is 49. Does he perform better in English than in Chinese exam in related to the other students? Explain your answer.

* 參考課程 4.2

a.) The score of Peter in the English exam $= 66 - (0.5)(12)$
$= 60$

b.) The standard score of Peter in the Chinese exam

$$= \frac{49 - 52}{10} = -0.3 > -0.5 \text{ (Standard score in English)}$$

$\therefore$ *He performs better in Chinese exam.*

* 標準分數 = 數據相差平均數幾多個標準差

2015 PAPER 1 - SECTION B

Q16.) A box contains 5 red balls, 6 yellow balls and 3 white balls . 4 balls are randomly drawn .

a.) Find the probability that there is exactly 2 red balls .

b.) Find the probability that there is at least 2 red balls .

* 參考課程 4.3 及 4.4

Let P(NR) be the probability of N red balls are drawn .

$$a.)\ P(2R) = \frac{C_2^5 C_2^9}{C_4^{14}} = \frac{360}{1001}$$

$$b.)\ P(at\ least\ 2R) = 1 - P(0R) - P(1R)$$

$$= 1 - \frac{C_4^9}{C_4^{14}} - \frac{C_1^5 C_3^9}{C_4^{14}} = \frac{5}{11}$$

* 五個紅波內兩個組合

* 九個非紅波內兩個組合

* 九個非紅波內四個組合

* 五個紅波內一個組合

* 九個非紅波內三個組合

2015 PAPER 1 - SECTION B

$Q17.)$ *Let* $A(n) = 4n - 5$ *and* $B(n) = 10^{A(n)}$, *where n is positive integer*

$a.)$ *Find* $A(1) + A(2) + \ldots + A(n)$ *in term of* n

$b.)$ *Find the greatest value of* n *such that* $\log(B(1)B(2)\ldots B(n)) \leq 8000.$

* 參考課程 2.2, 2.3, 2.6 及 2.7

$a.)$ *Let* $S(n) = A(1) + A(2) + \ldots + A(n)$

$$= 4(1 + 2 + \ldots + n) - 5n$$

$$= 4\frac{n(n+1)}{2} - 5n = 2n^2 - 3n$$

$b.)$ *Consider,* $\log(B(1)B(2)\ldots B(n)) \leq 8000$ *and* $n \geq 1$

$\rightarrow \log(B(1)) + \log(B(2)) + \ldots + \log(B(n)) \leq 8000$ *and* $n \geq 1$

$\rightarrow S(n) \leq 8000$ *and* $n \geq 1$

$\rightarrow 2n^2 - 3n - 8000 \leq 0$ *and* $n \geq 1$

$\rightarrow 1 \leq n \leq 64$

$\therefore$ *The greatest value of* $n = 64$

* 等差數列之和=(首項+尾項)x項數/2

* log(AB) = logA+logB

* 先用二次方程根公式搵根, α, β

因為細過0, 所以答案喺最細最大根之間

2015 PAPER 1 - SECTION B

Q18.) Let $f(x) = 2x^2 - 4kx + 3k^2 + 5$ and $g(x) = 2 - f(x)$, where k is constant.

a.) Does $y = f(x)$ cut the $x-axis$? Explain your answer.

b.) Find the vertex of $y = f(x)$ in term of k

c.) Denote S and T are the moving points of $y = f(x)$ and $y = g(x)$ respectively. Does the circumcenter of ΔOST lies on $x-axis$ when S ans T are nearest to each other? Explain your answer.

* 參考課程 2.5, 2.10 及 3.2

a.) Consider $f(x) = 0$, $\Delta = (4k)^2 - 4(2)(3k^2 + 5)$

$$= 16k^2 - 24k^2 - 40$$

$$= -8(k^2 + 5) < 0$$

$\therefore y = f(x)$ cuts the $x - axis$.

b.) Let the vertex be (a, b), then

$$f(x) = 2x^2 - 4kx + 3k^2 + 5 \equiv 2(x-a)^2 + b$$

By compare coefficient of x and constant, we have

$$-4k = -4a \text{ and } 3k^2 + 5 = 2a^2 + b$$

* 如果不切 x 軸, f(x)=0 沒有實根, 判別式<0

* 二次函數轉換可用 compare coefficient

CONT'D

2015 PAPER 1 - SECTION B

$\rightarrow (a, b) = (k, k^2 + 5)$

c.) *When S and T are nearest to each other,*

$S = \text{the vertex of } y = f(x) = (k, k^2 + 5)$

$T = \text{the vertex of } y = g(x) = (k, 2 - (k^2 + 5))$

$= (k, -k^2 - 3)$

$\therefore \text{The mid-pt. of } ST = (k, 1)$

$i.e\ x-\text{axis is not the } \perp \text{ bisector of } ST$

$\rightarrow \text{The circumcenter of } \Delta OST \text{ does not lie on } x-\text{axis}.$

* g(x) 頂點 y 的數值=2-f(x) 頂點 y 的數值

$g(x) = 2 - f(x) = 2 - (2(x-a)^2 + b)$

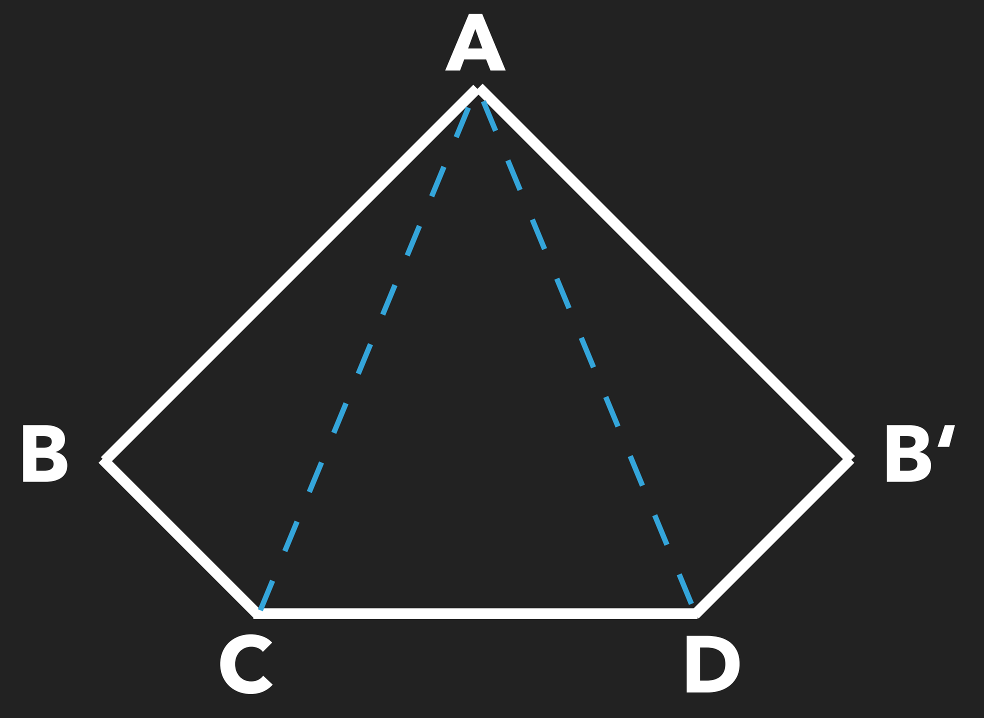
$= -2(x-a)^2 + (2-b)$

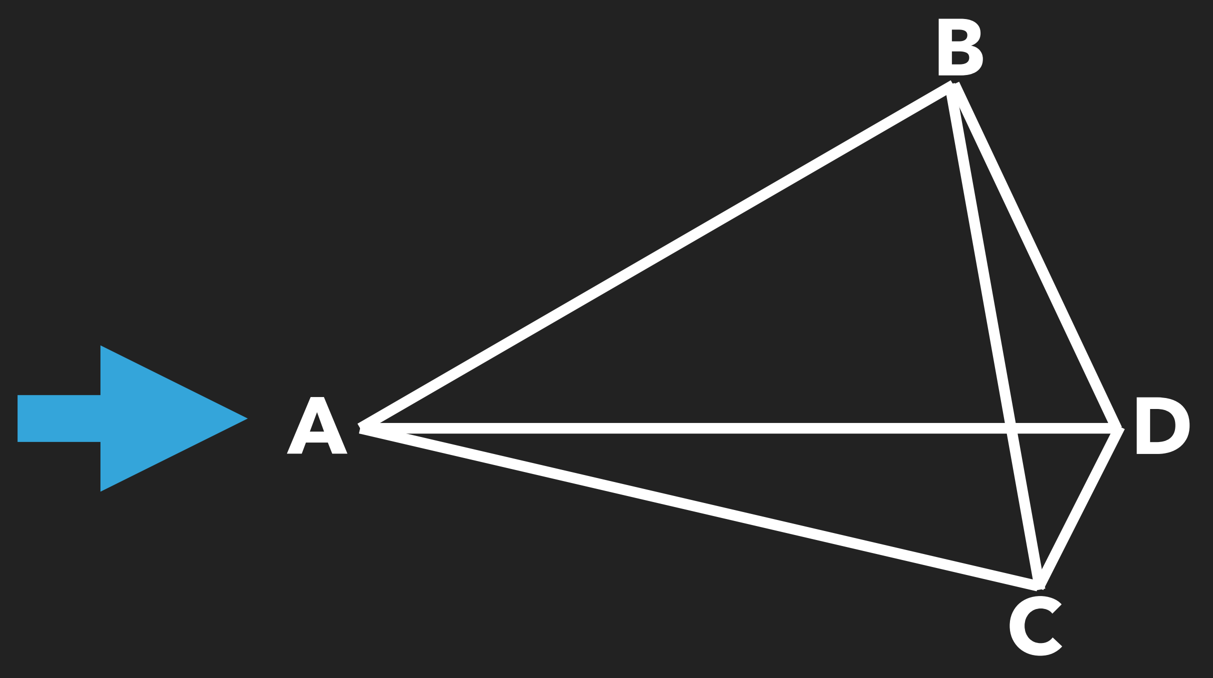
* 中點公式 = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

* Circumcenter 係各邊的垂直平分線的相交

2015 PAPER 1 - SECTION B

Q19.) *The following shows a pentagonal paper card with* $AB = AB' = 40cm$, $BC = B'D = 24cm$ *and* $\angle ABC = \angle AB'D = 80^0$, *Suppose that* $105^0 \leq \angle BCD \leq 145^0$.





a.) *Find* AC *and* $\angle ACB$

b.) *Describe the area of the paper varies when* $\angle BCD$ *rises from* 105^0 *to* 145^0. *Explain your answer.*

c.) *Assume* $\angle BCD = 132^0$, *the paper card is folded along* AC *and* AD *such that* AB *join to* AB' *and a pyramid* $ABCD$ *is formed as above figure. Find the volume of the pyramid* $ABCD$.

* 參考課程 3.3 及 3.10

a.) *By cosine law in* ΔABC,

$$AC = \sqrt{40^2 + 24^2 - 2(40)(24)cos80^0}$$
$$= 42.9\ cm\ (to\ 3\ sig.fig.)$$

* cosine law 使用

CONT'D

2015 PAPER 1 – SECTION B

$By\ sine\ law\ in\ \Delta ABC,$

$$\sin\angle ACB = \frac{40\sin 80^0}{AC} \rightarrow \angle ACB = 66.6^0\ (to\ 3\ sig.\ fig.)$$

* sine law 使用

$b.)\ The\ area\ of\ \Delta ABC\ and\ \Delta AB'D,\ A_1 = \frac{1}{2}(40)(24)\sin 80^0$

$= 480\sin 80^0$

* 三角形面積 = 1/2 absinC

$$AD = \sqrt{40^2 + 24^2 - 2(40)(24)\cos 80^0} = AC$$

$The\ area\ of\ \Delta ACD,\ A_2 = \frac{1}{2}(AC)(AD)\sin\angle CAD$

$= \frac{1}{2}AC^2\sin(180^0 - 2\angle ACD)$

$= \frac{1}{2}AC^2\sin(2\angle ACD)$

* 三角形內角和 = 180⁰

* sin(180⁰ - x) = sin x

CONT'D

2015 PAPER 1 - SECTION B

$$= \frac{1}{2} AC^2 sin(2(\angle BCD - 66.6^0))$$

$$105^0 \leq \angle BCD \leq 145^0$$

$$\rightarrow 76.8^0 \leq 2(\angle BCD - 66.6^0) \leq 156.8^0$$

Since, sinθ increase from 76.8^0 *to* 90^0 *and then decrease*

and the area of paper card $= 2A_1 + A_2$

$$= 2(480)sin80^0 + \frac{1}{2} AC^2 sin(2(\angle BCD - 66.6^0))$$

The area of paper card increase from 105^0 *to* $\frac{90^0}{2} + 66.6^0$

$= 111.6^0$ *and then decrease .*

* sin x 不斷上升當係 0 到 90^0

* sin x 不斷下降當係 90^0 到 180^0

CONT'D

2015 PAPER 1 - SECTION B

$c.)\ Let\ M\ be\ the\ mid-pt\ of\ CD,$

$H\ be\ the\ projection\ of\ B\ on\ \Delta ACD$

$AM = AC\sin\angle ACD \qquad CM = AC\cos\angle ACD$

$BM^2 = BC^2 - CM^2$

$$\cos\angle BMA = \frac{BM^2 + AM^2 - BA^2}{2(BM)(AM)}$$

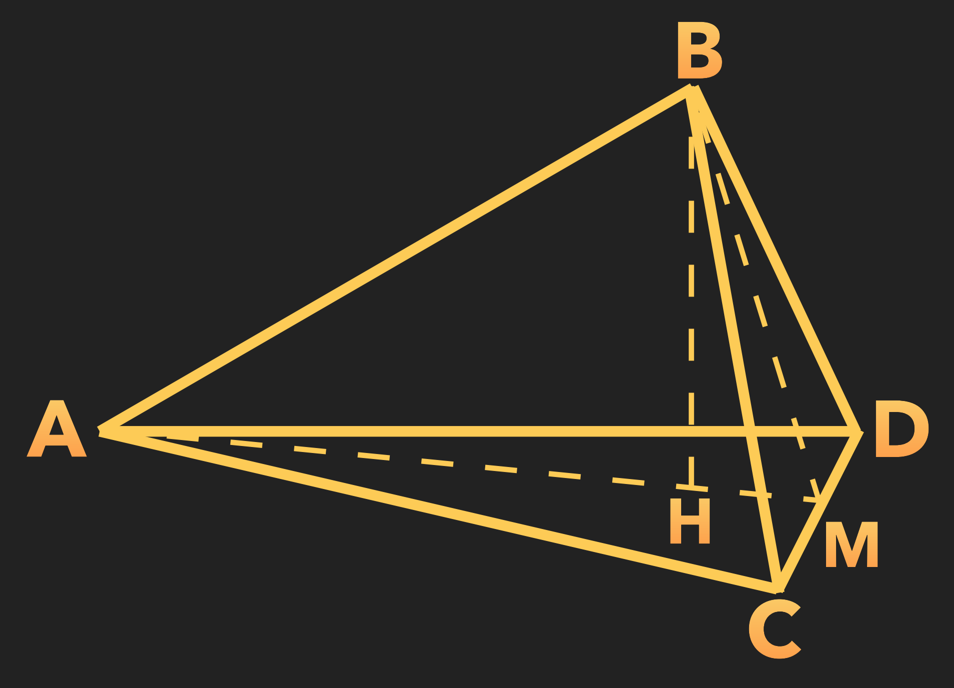
$$BH = BM\sin\angle BMA = BM\sqrt{1-\cos\angle BMA}$$

$$A_2 = \frac{1}{2}AC^2\sin(2(132^0 - 66.6^0))$$

$$\therefore The\ volume = \frac{1}{3}A_2 \cdot BH$$

$$= 3690\ cm^3\ (to\ 3\ sig.fig.)$$

* 錐體體積 = 1/3 x 底面積 x 高



* 畢氏定理
* cosine law 使用