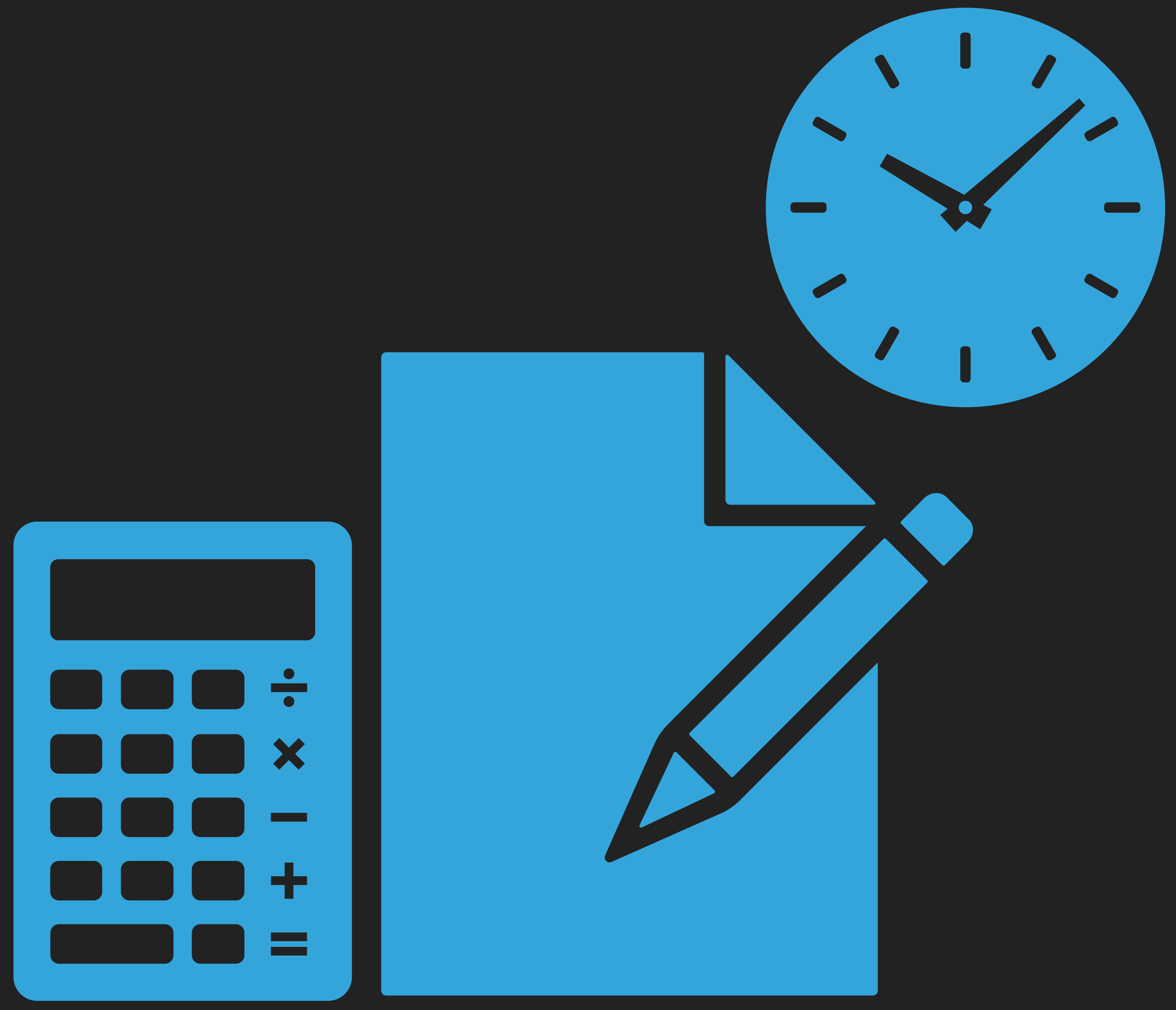
# 深宵教室 - DSE M1 模擬試題解答

# 2018

此為參考2018試題之模擬試題，原版請另行購買

# 2018

- Section A
- Section B



# 2018 - SECTION A

*Q1.) Let A and B be the event such that* $P(A) = 0.8$, $P(B|A) = 0.45$, *and* $P(B|A') = 0.6$ *where* $A'$ *is the complementary event of A, Find* $P(B)$, $P(A|B)$ *and* $P(A \cup B)$

* 參考課程 4.1 及 4.2

$$P(B) = P(B|A)P(A) + P(B|A')\boxed{P(A')}$$

$$= P(B|A)P(A) + P(B|A')[\boxed{1 - P(A)}] = 0.48$$

$$\boxed{P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)}$$

$$\rightarrow P(A|B) = \frac{P(B|A)P(A)}{P(B)} = 0.75$$

$$P(A \cup B) = P(A) + P(B) - \boxed{P(A \cap B)}$$

$$= P(A) + P(B) - P(A|B)P(B) = 0.92$$

* P(Not A) = 1 - P(A)

* P(A & B)=P(A|B)P(B)=P(B|A)P(A)

* 要減去重疊地方

# 2018 - SECTION A

*Q2.) Given that there is a proportion p of family keeping dogs . A random sample of 64 is drawn and find that β % confidence level of* $p = (0.0915, 0.3085)$

*a.) Find the sample proportion and β*

*b.) Using the above result, find the least number of family such that the probability of at least 1 of these family is greater than 0.999.*

* 參考課程 4.4 及 4.7

*Let* $p_s$ *be the sample proportion and α such that* $P(-\alpha < Z < \alpha) = \beta\,\%$

* 置信區間定義

$$Hence,\ p_s - \alpha\frac{\sqrt{p_s(1-p_s)}}{\sqrt{64}} = 0.0915 - (1)$$

$$p_s + \alpha\frac{\sqrt{p_s(1-p_s)}}{\sqrt{64}} = 0.3085 - (2)$$

$$(2) - (1) : \alpha\sqrt{p_s(1-p_s)} = 0.868 - (3)$$

$$Put\ (3)\ in\ (1) : p_s - \frac{0.868}{8} = 0.0915 \rightarrow p_s = 0.2$$

CONT'D

# 2018 - SECTION A

$In\ (3) : \alpha\sqrt{0.2(1-0.2)} = 0.868 \rightarrow \alpha = 2.17$

$Hence,\ P(-2.17 < Z < 2.17) = \beta\,\% = 2 \cdot 0.485 \rightarrow \beta = 97$

$b.)\ Let\ n\ be\ the\ number\ of\ family$

$X\ be\ the\ random\ vaiable\ follows\ B(n, p_s)$

$P(X \geq 1) = 1 - P(X = 0) > 0.999$

$\rightarrow P(X = 0) < 0.001 \rightarrow (1 - 0.2)^n < 0.001 \rightarrow n > log_{0.8}0.001$

$\rightarrow n > 30.95$

$\therefore\ The\ least\ number\ of\ family = 31$

* P(Not A) = 1 - P(A)

* $P(X = k) = C^n_k p^k (1-p)^{n-k}$

* log0.8 係負數, 互除要細轉大

# 2018 - SECTION A

*Q3.) The diameter of ball in market follows $N(9, 0.125^2)$ in cm. A ball is classified 'BIG', if the diameter of the ball is greater than 9.16cm. The diameter of balls in the market are measured one by one until a 'BIG' ball is found. Let $X$ be the random variable of the number of measurement taken. Find $P(X \le 3)$ and $E(X)$.*

* 參考課程 4.5

Let $B$ be event of a 'BIG' ball selected from market

$$\text{Given, } P(B) = P\left(Z > \frac{9.16 - 9}{0.125}\right) = P(Z > 1.28) = 0.1003$$

* 先計算 Z ~ N(0,1), 再對表

$$\because X \sim G(P(B))$$

$$\therefore P(X \le 3) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$= P(B) + [1 - P(B)]P(B) + [1 - P(B)]^2 P(B)$$

* $P(X = k) = (1 - p)^{k-1} p$

$$= P(B)\left[\frac{1 - [1 - P(B)]^3}{P(B)}\right] = 1 - [1 - P(B)]^3 = 0.2712 \text{ (to 4 d.p.)}$$

* 等比數列之和

$$E(X) = \frac{1}{P(B)} = 9.9701 \text{ (to 4 d.p.)}$$

* $X \sim G(p),\ E(X) = \frac{1}{p}$

# 2018 - SECTION A

*Q4.) Let X be discrete random vaiable, where m, p are a constant.*

| $k$ | $-2$ | $2$ | $m$ |
|---|---|---|---|
| $P(X=k)$ | $p$ | $0.25$ | $0.5$ |

*a.) Find Var(X) in term of m*

*b.) If $Var(2X-1) = 8E(2X-1)$, find m.*

* 參考課程 4.1, 4.3 及 4.4

$$a.)\ p + 0.25 + 0.5 = 1 \rightarrow p = 0.25$$

$$E(X) = -2\cdot 0.25 + 2\cdot 0.25 + m\cdot 0.5 = 0.5m$$

$$E(X^2) = (-2)^2\cdot 0.25 + (2)^2\cdot 0.25 + m^2\cdot 0.5 = 2 + 0.5m^2$$

$$Var(X) = E(X^2) - [E(X)]^2 = 2 + 0.5m^2 - 0.25m^2$$

$$= 0.25m^2 + 2$$

$$b.)\ Var(2x-1) = 8E(2X-1) \rightarrow 4Var(X) = 8[2E(X)-1]$$

$$\rightarrow m^2 + 8 = 8m - 8 \rightarrow m^2 - 8m + 16 = 0$$

$$\rightarrow (m-4)^2 = 0 \rightarrow m = 4\ (repeated)$$

* 機率總加 = 1

* $E(X) = \sum_{i=1}^{n} k_i P(X=k_i)$

* $Var(X) = E(X^2) - [E(X)]^2$

* $Var(aX+b) = a^2 Var(X)$

* $E(aX+b) = aE(X) + b$

* $a^2 - 2ab + b^2 \equiv (a-b)^2$

# 2018 - SECTION A

*Q5.) Let $f(x)$ be the continuous function and $f'(x) = (12x-48)(3x^2-24x+49)^{-2}$, for all real $x$. Given that the extreme value of $f(x) = 5$. Find $f(x)$ and $\lim_{x\to\infty} f(x)$.*

* 參考課程 2.1, 2.4, 2.6 及 2.7

$Obviously,\ f'(4) = 0$

| | x < 4 | x = 4 | x > 4 |
|---|---|---|---|
| f′(x) | - | 0 | + |
| f(x) | Dec. | | Inc. |

$\therefore (4, 5)$ *is the min. value.*

$$f(x) = 2\int \frac{6x-24}{(3x^2-24x+49)^2}dx = 2\int \frac{d(3x^2-24x+49)}{(3x^2-24x+49)^2}$$

$$= -\frac{2}{3x^2-24x+49} + C, \text{ where } C \text{ is constant.}$$

$$\because f(4) = 5 \to C = 7 \to f(x) = -\frac{2}{3x^2-24x+49} + 7$$

* 搵 turning point = 搵 $x_0$ 使度 $f'(x_0)=0$

* 利用表格計算 turning point 附近上升定下降

$f'(x) > 0 \to Increasing$

$f'(x) < 0 \to Decreasing$

* 積分三寶: 積分代入法

CONT'D

# 2018 - SECTION A

$$
\begin{aligned}
\text{Also, } \lim_{x\to\infty} f(x) &= -\lim_{x\to\infty} \frac{2}{3x^2-24x+49}+7 \\
&= -\lim_{x\to\infty} \cancel{\frac{2}{3(x-4)^2+1}}+7 \\
&= 7
\end{aligned}
$$

* 分母越來越大, f(x) 越細

* 印証 f(x) 係 continuous

# 2018 - SECTION A

*Q6.) Let $f(x) = (1-3x)^8(e^{kx} + e^{2x} - 1)$. If the coefficient of $x$ and $x^2$ are the same, find $k$.*

*** 參考課程 1.1 及 3.2**

$$(1-3x)^8(e^{kx} + e^{2x} - 1)^2 = (1 + C_1^8(-3x) + C_2^8(-3x)^2 + \ldots)(e^{kx} + e^{2x} - 1)$$

$$= (1 - 24x + 252x^2 + \ldots)(1 + kx + \frac{k^2x^2}{2} + \ldots + 1 + 2x + 2x^2 + \ldots - 1)$$

$$= (1 - 24x + 252x^2 + \ldots)[1 + (k+2)x + (\frac{k^2}{2} + 2)x^2 + \ldots]$$

*Given, The coefficient of $x$ = The coefficient of $x^2$*

$$\rightarrow (k+2) - 24 = (\frac{k^2}{2} + 2) - 24(k+2) + 252$$

$$\rightarrow k^2 - 50k + 456 = 0$$

$$\rightarrow k = 12 \text{ or } 38$$

* $(a+b)^n = \sum_{r=0}^{n} C_r^n a^r b^{n-r}$

* $e^x = \sum_{r=0}^{\infty} \frac{x^r}{r!}$

* $C_r^n = \frac{n!}{r!(n-r)!}$

$\rightarrow C_1^n = n$ and $C_2^n = \frac{n(n-1)}{2}$

# 2018 - SECTION A

*Q7.) Let the curve $C : y = f(x)$ and $f(x) = x^2(h-x)^{\frac{1}{2}}$, where, $0 < x < h$, for a constant $h$.*
*Given $f'(4) = 30$. Find the maximum point(s) of $C$ and all horizontal tangent to $C$*

* 參考課程 2.3 及 2.4

$$f(x) = \boxed{x^2(h-x)^{\frac{1}{2}}} \to f'(x) = \boxed{2x(h-x)^{\frac{1}{2}} - \frac{x^2}{2(h-x)^{\frac{1}{2}}}} = \frac{4hx - 5x^2}{2\sqrt{h-x}}$$

* 用 Product rule

$$f'(4) = 30 \to \frac{16(h-5)}{2\sqrt{h-4}} = 30 \to 4(h-5) = 15\sqrt{h-4}$$

$$\to 4(\sqrt{h-4})^2 - 15\sqrt{h-4} - 4 = 0$$

$$\to \sqrt{h-4} = 4 \text{ or } -0.25 \ (\text{rejected}, \sqrt{h-x} > 0)$$

$$\to h = 20$$

*Obviously, when* $x = 0$ (*rejected*, $x > 0$) *or* $x = 16$, $f'(x) = 0$

* 搵 turning point = 搵 $x_0$ 使度 $f'(x_0)=0$

CONT'D

# 2018 – SECTION A

| | $0 < x < 16$ | $x = 16$ | $x > 16$ |
|---|---|---|---|
| f′(x) | + | 0 | - |
| f(x) | Inc. | | Dec. |

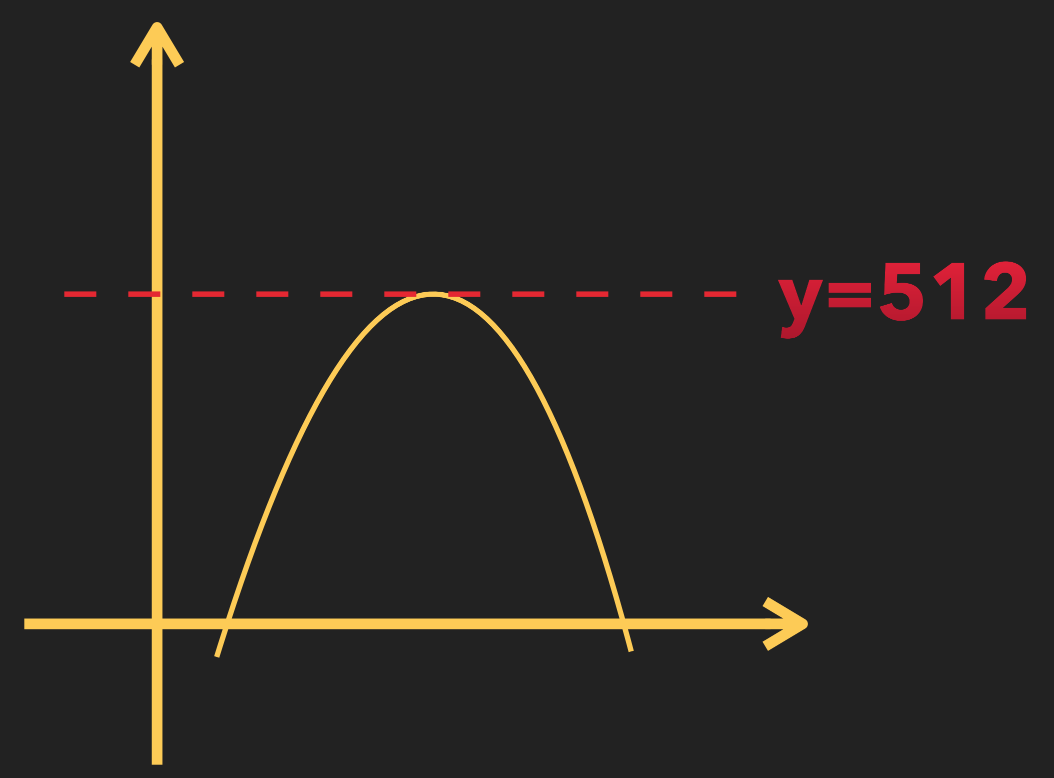
$$\therefore The\ max.\ pt.\ of\ f(x) = (16, f(16)) = (16, 512)$$

$$Hence\ the\ horizontal\ tangent:\ y = 512$$

* 利用表格計算 turning point 附近上升定下降

$$f'(x) > 0 \rightarrow Increasing$$

$$f'(x) < 0 \rightarrow Decreasing$$



# 2018 - SECTION A

$Q8.)\ Let\ the\ curve\ C : y = f(x), f(x) = \dfrac{(x-1)(lnx-1)}{x}, for\ x > 0$

$Find\ the\ area\ \ of\ the\ region\ bounded\ by\ C,\ and\ x-axis$

* 參考課程 2.8 及 2.9

$To\ find\ the\ x-interception\ of\ C,\ consider\ f(x) = 0 \rightarrow x = 1\ or\ e$

$\therefore\ The\ x-interception\ are\ 1\ and\ e$

$$The\ area,\ A = \left|\int_1^e \frac{(x-1)(lnx-1)}{x}dx\right|$$

$$= \left|\int_1^e lnxdx - \int_1^e dx - \int_1^e \frac{lnx}{x}dx + \int_1^e \frac{dx}{x}\right|$$

$$= \left|[xlnx]_1^e - \int_1^e dx - \int_1^e dx - \int_1^e lnx d[lnx] + \int_1^e \frac{dx}{x}\right|$$

$$= \left|[xlnx - 2x - 0.5(lnx)^2 + lnx]_1^e\right| = e - 2.5\ unit^2$$

* 積分三寶: 積分代入法

* 積分三寶: Integration by part

# 2018 – SECTION B

*Q9.) A batch of oranges are classified base on their weight, which follows $N(\mu, 16)$ in gram.*

| Weight ($x$ g) | $x \le a$ | $a < x < 260$ | $x \ge 260$ |
|---|---|---|---|
| Classification | Small | Medium | Large |

*Given that there are 10.56 % and 73.57 % of the oranges are Large and Medium respectively. Every 8 oranges are packed in a box. If there are at least 6 Medium oranges in a box, the box is named as regular.*

*a.) Find $\mu$ and $a$.*

*b.) Find the probability of a randomly selected box of oranges is regular.*

*c.) 3 boxes of oranges are randomly selected. Find the probability :*

*i.) 3 boxes of oranges are regular and there are totally 21 medium oranges and 3 small oranges.*

*ii.) There are totally 21 medium oranges and 3 small given that 3 boxes of oranges are regular.*

*iii.) 3 boxes of oranges are regular given there are totally 21 medium oranges and 3 small oranges in these 3 boxes.*



CONT'D

# 2018 - SECTION B

$a.)\ Given\ P(X \geq 260) = 0.1056\ and\ P(a < X < 260) = 0.7357$

$\rightarrow P(Z \geq \frac{260-\mu}{16}) = 0.1056\ and\ P(\frac{a-\mu}{16} < Z < \frac{260-\mu}{16}) = 0.7357$

$\rightarrow \frac{260-\mu}{16} = 1.25\ and\ \frac{a-\mu}{16} = -1$

$\rightarrow \mu = 240\ and\ a = 224$

$b.)\ Let\ nR\ be\ the\ event\ of\ n\ numbers\ of\ regular\ box\ is\ selected.$

$p_m = P(224 < X < 260),\ p_l = P(X \geq 260),\ p_s = 1 - p_m - p_l$

$P(1R) = C_6^8 p_m^6 (1-p_m)^2 + C_7^8 p_m^7 (1-p_m) + C_8^8 p_m^8$

$= 0.6426\ (to\ 4\ d.p.)$

$ci.)\ Let\ (x, y, z)\ be\ the\ event\ of\ a\ selected\ box\ contains\ x\ small$
$y\ medium\ and\ z\ large\ oranges.$

$The\ probability = P_3^3 P(0,8,0)P(1,7,0)P(2,6,0) + P(1,7,0)^3$

* 先計算 Z ~ N(0,1), 再對表

* $P(X=k) = C_k^n p^k (1-p)^{n-k}$

* 1箱有8中, 1箱有7中1細, 1箱有6中2細

* 3箱有7中1細,

CONT'D

# 2018 - SECTION B

$$= 6(p_m^8)(C_1^8 p_s p_m^7)(C_2^8 p_s^2 p_m^8) + (C_1^8 p_s p_m^7)^3$$

$$= 0.0118 \ (to\ 4\ d.p.)$$

$$ii.)\ The\ probability = \frac{P(3R\ and\ total\ 21\ medium\ and\ 3\ small)}{P(3R)}$$

$$= \frac{0.0118}{0.6426^3} = 0.0444 \ (to\ 4\ d.p.)$$

$$iii.)\ The\ probability = \frac{P(3R\ and\ total\ 21\ medium\ and\ 3\ small)}{P(total\ 21\ medium\ and\ 3\ small)}$$

$$= \frac{0.0118}{C_{21}^{24} p_m^{21} p_s^3} = 0.9188 \ (to\ 4\ d.p.)$$

* $P(X = k) = C_k^n p^k (1-p)^{n-k}$

* 條件概率

* 24個橙有21中3細

# 2018 – SECTION B

Q10.) *In a school, A student is defined as 'Good' if he is late for fewer than 2 times in that month. The lateness of Peter in a month follows* $Po(1.8)$. *In the coming 4 months, the school propose 2 bouns scheme.*

*Proposal 1*

| *Number of month with 'Good'* | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|
| *Bouns* | \$5000 | \$2500 | \$1500 | \$600 | \$0 |

*Proposal 2*

| *Number of lateness in these 4 months* | $< 5$ | *otherwise* |
|---|---|---|
| *Bouns* | \$8000 | \$0 |

a.) *Which proposal favours to Peter? Explain your answer.*

b.) *Given that the number of Peter's early leave in a month follows* $Po(\lambda)$. *Assume whether Peter is late and whether he leaves early are independent event. Given that the sum of his lateness and the number of early leaves in a certain month is 2, the probability he is late for 2 times and does not early leave in that month* $= 0.36$. *Find* $\lambda$.

* 參考課程 4.2 及 4.4

CONT'D

# 2018 – SECTION B

a.) *Let X be random variable of Peter's number of lateness in certain month.*

*Let G be the event of Peter is defined as 'Good' in certain month.*

$$P(G), p_g = P(X < 2) = P(X = 0) + P(X = 1) = e^{-1.8} + e^{-1.8}(1.8)$$
$$= 0.462837$$

*The expected bouns for proposal 1*

$$= 5000p_g^4 + 2500C_3^4 p_g^3(1 - p_g) + 1500C_2^4 p_g^2(1 - p_g)^2 + 600C_1^4 p_g(1 - p_g)^3$$
$$\approx \$1491$$

*Also,* $4X \sim Po(4 \cdot 1.8 = 7.2)$

*The expected bouns for proposal 2* $= 8000P(4X < 5)$

$$= 8000e^{-7.2}[1 + 7.2 + \frac{7.2^2}{2!} + \frac{7.2^3}{3!} + \frac{7.2^4}{4!}] \approx \$1244$$

$\because$ *The expected bouns for proposal 1 > that of proposal 2*

$\therefore$ *Proposal 1 favours to Peter.*

* $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

* 二項分佈: B(4, $p_g$)

* $E(X) = \sum_{i=1}^{n} k_i P(X = k_i)$

* 比較邊個期望值最大

CONT'D

# 2018 - SECTION B

$b.)$ *Let* $Y$ *be random variable of Peter's number of early leaves in certain month.*

*Given that* $Y \sim Po(\lambda)$ *and* $P(X=2 \text{ and } Y=0 \mid X+Y=2)=0.36$

$$\rightarrow \frac{P(X=2 \text{ and } Y=0 \cap X+Y=2)}{P(X+Y=2)}=0.36$$

$$\rightarrow \frac{P(X=2)P(Y=0)}{P(X=0)P(Y=2)+P(X=1)P(Y=1)+P(X=2)P(Y=0)}=0.36$$

$$\rightarrow \frac{\frac{e^{-1.8}1.8^2}{2!}e^{-\lambda}}{e^{-1.8}\frac{e^{-\lambda}\lambda^2}{2!}+e^{-1.8}(1.8)\cdot e^{-\lambda}\lambda+\frac{e^{-1.8}1.8^2}{2!}e^{-\lambda}}=0.36$$

$$\rightarrow 1.62=0.36(0.5\lambda^2+1.8\lambda+1.62) \rightarrow 0.18\lambda^2+0.648\lambda-1.0368=0$$

$$\rightarrow \lambda=1.2 \text{ or } -0.864 \text{ (rejected)}$$

* 條件概率

* 沒有遲到, 2 次早走

* 1 次遲到, 1 次早走

* 2 次遲到, 沒有早走

* $P(X=k)=\frac{e^{-\lambda}\lambda^k}{k!}$

# 2018 - SECTION B

$Q11.)$ Let $f(t) = 60(1+10t)e^{-2t}$ and $I = \int_{0.1}^{0.5} f(t)dx$. There are two method to estimate $I$.

$M1:$ By the trapezoidal rule with 4 sub – interval
$M2:$ By replacing $f(t) = 50(1+10t)(1+2t)^{-1}$

a.) Find the estimation based on $M1$ and $M2$.
b.) Determine if over – estimation or under – estimation for $M1$
c.) Which method is more accurate? Explain your answer.

* 參考課程 2.2, 2.3, 3.2 及 3.3

a.) Let $I_1$ be the estimation by $M1$
$I_2$ be the estimation by $M2$

$$I_1 = \frac{0.5-0.1}{4 \cdot 2}[f(0.1)+2f(0.2)+2f(0.3)+2f(0.4)+f(0.5)]$$
$$= 50.2513 \text{ (to 4 d.p.)}$$

* 計算梯形面積的加總

CONT'D

# 2018 - SECTION B

$$I_2 = 50\int_{0.1}^{0.5}\frac{1+10t}{1+2t}dx = 50\int_{0.1}^{0.5}\boxed{5-\frac{4}{1+2t}}dt = 50[5t-2ln(1+2t)]_{0.1}^{0.5}$$

$$= 48.9174\ (to\ 4\ d.p.)$$

$$b.)\ Consider\ f'(t) = \boxed{60[10e^{-2t} - 2(1+10t)e^{-2t}]}$$

$$= 600e^{-2t} - 2f(t)$$

$$\rightarrow f''(x) = -1200e^{-2t} - 2f'(t) = -2400e^{-2t} + 4f(t)$$

$$= -2400e^{-2t} + 240(1+10t)e^{-2t}$$

$$= 240(1+10(t-1))e^{-2t}$$

$$\because 0.1 \le t \le 0.5 \rightarrow -8 \le 1+10(t-1) \le -4$$

$$\therefore f''(t) < 0,\ where\ 0.1 \le t \le 0.5$$

$$i.e.I_1\ is\ under-estimated$$

$$c.)\ Since\ I > I_1 > I_2,\ (I_1\ is\ under-estimated)$$

$$i.e.I_1\ is\ more\ accurate\ than\ I_2$$

* 積分三寶: Partial fraction

* Product rule

* 個 f(t) 係凹口向下

# 2018 - SECTION B

Q12.) The number of bacteria, $N(t)$ (in millions) under controlled condition is recorded. $Q(t) = lnr + (sln3)t$, where $Q(t)$ is the room temperture ($^oC$) against time $t$ (in hours) $s$ and $r$ are constant. The slope and the $y-$interception of the linear function are $-0.1ln9$ and $ln9$ respectively. The experiment start at $t = 0$ and end at $t = 20$

Given that $Q(t) = ln(\frac{120 - 3N(t)}{N(t)})$

a.) Will there are 4 million bacteria in that condition? Explain your answer.

b.) Describe how $N'(t)$ varies during the experiment.

* 參考課程 2.1, 2.2, 2.3 及 2.4

a.) Obviously, $sln3 = -0.1ln9$ and $lnr = ln9 \rightarrow s = -0.2$ and $r = 9$

$$Hence, \boxed{ln9 - (0.2ln3)t} = ln(\frac{120 - 3N(t)}{N(t)})$$

$$\rightarrow \boxed{ln(9 \cdot 3^{-0.2t})} = ln(\frac{120 - 3N(t)}{N(t)})$$

* 直線方程: y=(斜率)x+(y截距)

* lnA+lnB=lnAB

CONT'D

# 2018 – SECTION B

$$\rightarrow (3 \cdot 3^{-0.2t} + 1)N(t) = 40 \rightarrow N(t) = \frac{40}{3^{1-0.2t} + 1}$$

$Assume\ N(t_0) = 4,\ where\ 0 \leq t_0 \leq 20,$

$$N(t_0) = \frac{40}{3^{1-0.2t_0} + 1} = 4 \rightarrow 10 = 3^{1-0.2t_0} + 1$$

$$\rightarrow 3^2 = 3^{1-0.2t_0} \rightarrow 1 - 0.2t_0 = 2 \rightarrow t_0 = -5\ (rejected)$$

$\therefore There\ will\ not\ be\ 4\ million\ bacteria\ in\ that\ condition$

$$b.)\ Let\ y(t) = 3^{1-0.2t} \rightarrow ln(y(t)) = (1 - 0.2t)ln3 \rightarrow \frac{y'(t)}{y(t)} = -0.2ln3$$

*** Implicit 微分法**

$$\rightarrow y'(t) = (-0.2ln3)y(t)$$

$$Hence,\ N(t) = \frac{40}{y(t) + 1} \rightarrow N'(t) = -\frac{40y'(t)}{(y(t) + 1)^2} = \frac{8ln3y(t)}{(y(t) + 1)^2}$$

CONT'D

# 2018 – SECTION B

$$\rightarrow (y(t)+1)^2N'(t) = 8ln3y(t)$$

$$\rightarrow 2(y(t)+1)y'(t)N'(t) + (y(t)+1)^2N''(t) = 8ln3y'(t)$$

$$\rightarrow \frac{16ln3y(t)y'(t)}{y(t)+1} + (y(t)+1)^2N''(t) = 8ln3y'(t)$$

$$\rightarrow \frac{-3.2(ln3)^2[y(t)]^2}{y(t)+1} + (y(t)+1)^2N''(t) = -1.6(ln3)^2y(t)$$

$$\rightarrow (y(t)+1)^3N''(t) = -1.6(ln3)^2y(t)(y(t)+1) + 3.2(ln3)^2[y(t)]^2$$

$$\rightarrow N''(t) = \frac{1.6(ln3)^2y(t)(y(t)-1)}{(y(t)+1)^3}, \ where\ y(t) > 0$$

$$When\ N''(t) = 0 \rightarrow y(t) = 1 \rightarrow 3^{1-0.2t} = 3^0 \rightarrow t = 5$$

* **Product rule**

* $N'(t) = \dfrac{8ln3y(t)}{(y(t)+1)^2}$

* $y'(t) = -0.2ln3y(t)$

* **搵 turning point = 搵 $t_0$ 使度 $N''(t_0)=0$**

CONT'D

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| | 0 < t < 5 | t = 5 | 5 < t < 20 |
|---|---|---|---|
| N''(t) | + | 0 | - |
| N'(t) | Inc. | | Dec. |

$\therefore N'(t)$ *increase from* $0 \leq t < 5$, *at max . when* $t = 5$
*and then decrease from* $5 < t \leq 20$

* 利用表格計算 turning point 附近上升定下降

$f'(x) > 0 \rightarrow Increasing$

$f'(x) < 0 \rightarrow Decreasing$