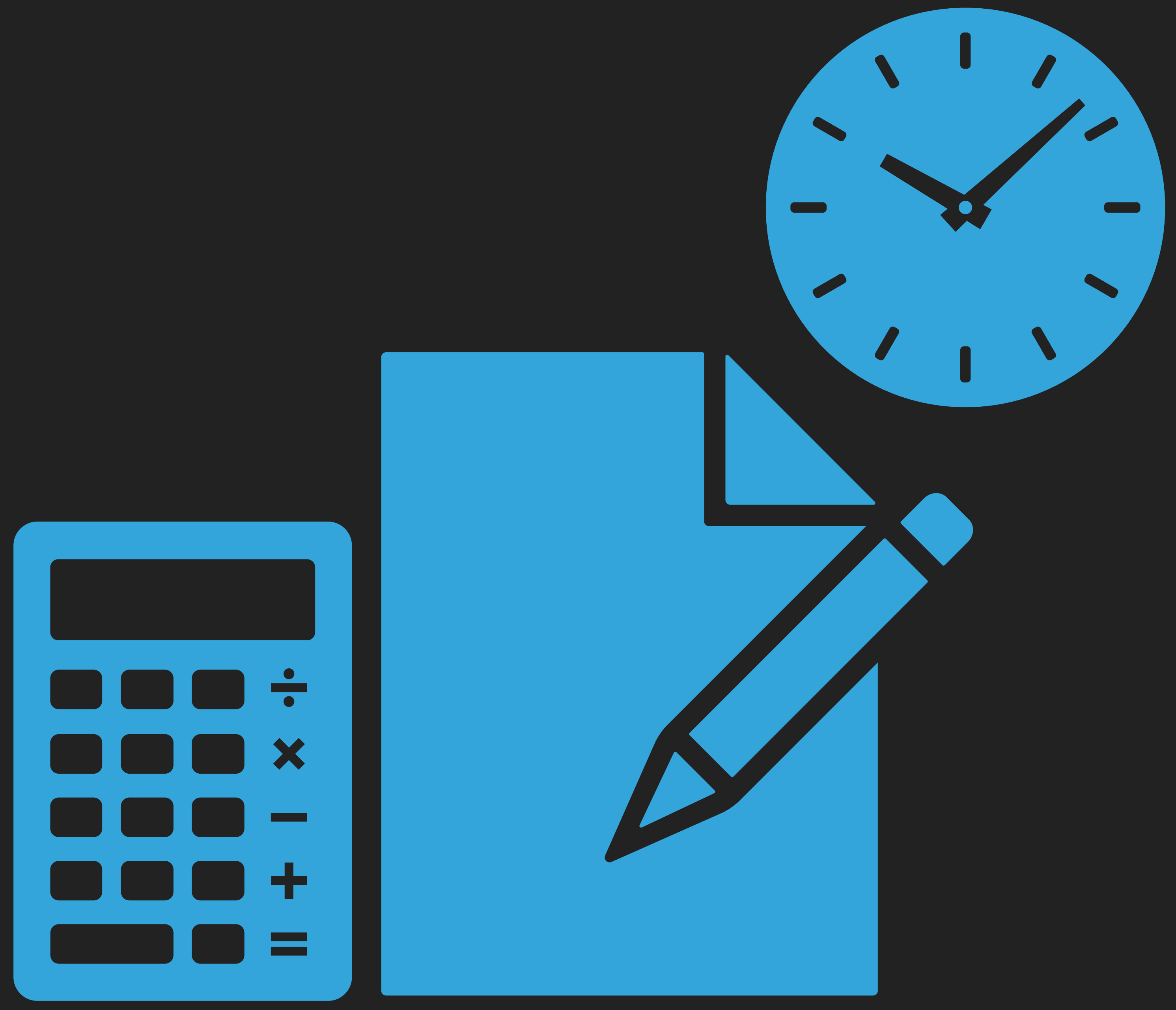
深宵教室 - DSE 必修模擬試題解答

2018 PAPER 1

此為參考2018試題之模擬試題，原版請另行購買

2018 PAPER 1

- Section A1
- Section A2
- Section B



2018 PAPER 1 - SECTION A1

$Q1.)\ \dfrac{a+4}{3} = \dfrac{b+1}{2},\ b = ?$

* 參考課程 2.1

$\rightarrow 2(a+4) = 3(b+1)$

* 兩邊交叉相乘分母

$\rightarrow 2a+8 = 3b+3$

$\rightarrow b = \dfrac{2a+5}{3}$

* 兩邊減 3 再除 3

2018 PAPER 1 - SECTION A1

Q2.) Simplified $\dfrac{xy^7}{(x^{-2}y^3)^4}$*, in positive indices*

* 參考課程 1.2

$$= x^{1-(-2\cdot 4)} \cdot y^{7-3\cdot 4}$$

$$= x^9 \cdot y^{-5}$$

$$= \frac{x^9}{y^5}$$

* 指數乘係加，除係減

* 指數負數，分母變分子，分子變分母

2018 PAPER 1 - SECTION A1

$Q3.)\ a.)\ Round\ up\ 265.473\ (to\ the\ nearest\ integer)$

$b.)\ Round\ down\ 265.473\ (to\ 1\ d.p.)$

$c.)\ Round\ off\ 265.473\ (to\ 2\ sig\ fig.)$

* 參考課程 1.1

$a.)\ 266$

$b.)\ 265.4$

$c.)\ 270$

* 進一至整數

* 捨去至小數後一個位

* 四捨五入之二位有效數字

2018 PAPER 1 - SECTION A1

Q4.) There are n white balls, 5 black balls and 8 red balls in the bag A ball is randomly selected from the bag . Given that the probability of the selected ball is red = 0.4. Find the value of n .

* 參考課程 1.1 或 4.3

$$P(\text{the selected ball is red}) = \boxed{\frac{8}{n+5+8}} = 0.4$$

$$\rightarrow 8 = 0.4n + 5.2$$

$$\rightarrow n = 7$$

* 機率 = 紅波數量 / 波的總數

2018 PAPER 1 - SECTION A1

Q5.) Factorize $9x^3 - 18x^2y - xy^2 + 2y^3$

* 參考課程 2.5

$$= (9x^2)(x - 2y) + y^2(x - 2y)$$

$$= (x - 2y)(9x^2 - y^2)$$

$$= (x - 2y)(3x + y)(3x - y)$$

* 抽 $9x^2$
* 抽 y^2
* 恒等式 $a^2 - b^2 = (a - b)(a + b)$

2018 PAPER 1 - SECTION A1

$Q6.)\ Solve\ \dfrac{3-x}{2} > 2x+7\ and\ x+8 \geq 0$

Hence, find greatest integer satisfy the above inequalities .

* 參考課程 1.1 及 2.3

$\rightarrow 3 - x > 4x + 14\ and\ x \geq -8$

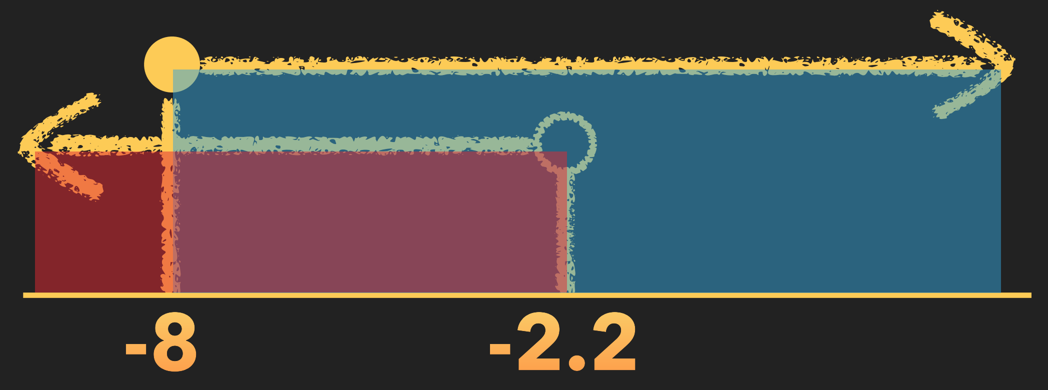
$\rightarrow 5x < -11\ and\ x \geq -8$

$\rightarrow x < -2.2\ and\ x \geq -8$

$\rightarrow -8 \leq x < -2.2$

$\therefore The\ greatest\ integer\ satisfy\ the\ inequalities = -3$

* and 指有重疊的地方



2018 PAPER 1 - SECTION A1

Q7.) The marked price of good A is 30 % above the cost. If a discount of 40 % at the marked price, there will be \$88 lost. Find the marked price of the good A.

* 參考課程 2.3

Let the marked price of good A be \$M

$$\text{Then, the cost of good } A,\ C = \boxed{\frac{M}{(1+30\%)}} = \frac{M}{1.3}$$

$$\therefore C - \boxed{M(1-40\%)} = 88 \rightarrow \frac{M}{13} - 0.6M = 88$$

$$\rightarrow M = 520$$

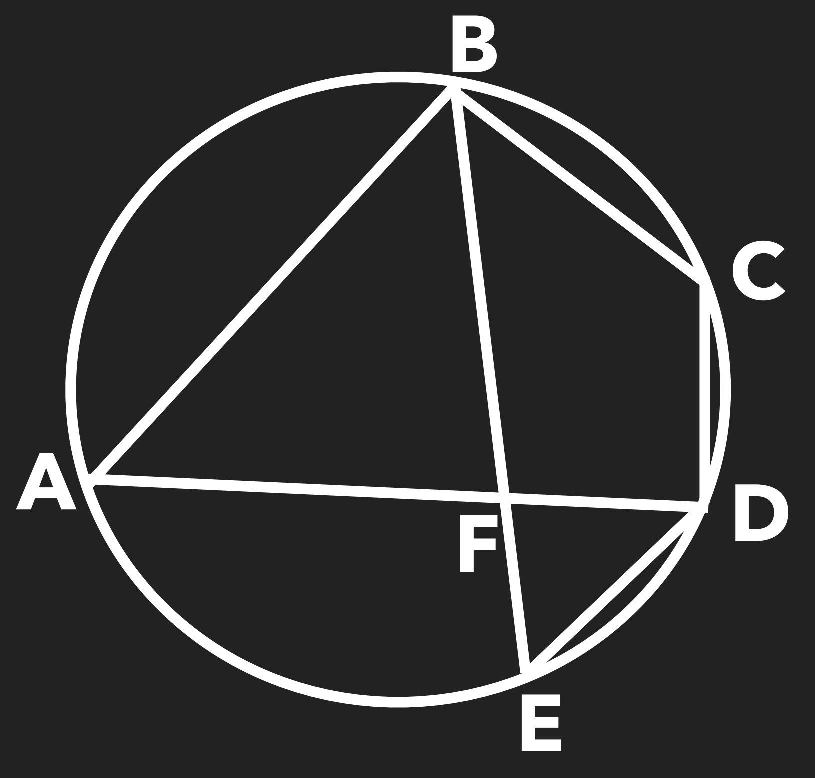
i.e. The marked price = \$520

* 百份比變化 = (新值 - 舊值) x 100% / 舊值

* 折扣後的售價

2018 PAPER 1 - SECTION A1

Q8.) In the following figure, $AB//ED$ and $\angle BCD = \theta$



Find $\angle BAD$ and $\angle EFD$ in term of θ.

* 參考課程 3.1, 3.2, 3.6 及 3.7

$\angle BAD = 180^0 - \theta$ (opp. $\angle$s, cyclic quad.)

$\angle ADE = \angle BAD$ (alt. $\angle$, $AB//ED$)

$\angle BED = \angle BAD$ ($\angle$s in the same segment)

In ΔEFD,

$\angle EFD + \angle FED + \angle FDE = 180^0$ ($\angle$s sum of Δ)

$\therefore \angle EFD = 180^0 - 2\angle BAD$

$= 2(\theta - 90^0)$

* 圓內四邊形, 對角相加 = 180⁰

* 平行線內錯角

* 弓內圓周角相等

* 三角形內角和 = 180⁰

2018 PAPER 1 - SECTION A1

Q9.) A moving ball travels from A to B with 72 km/h speed and then from B to C with 90 km/h speed. The ball travels from A to C (210 km) in 161 minutes. How long does the ball travel from A to B.

* 參考課程 1.3 及 2.3

Let the required time to travel from A to B be x hr
the required time to travel from B to C be y hr
The distance AB = 72x km, BC = 90y km, hence

$$\begin{cases} 72x + 90y = 210 & \text{——— (1)} \\ 60(x+y) = 161 & \text{——— (2)} \end{cases}$$

$$2x(1) - 3x(2) : 144x - 180x = 420 - 483 \rightarrow x = 1.75$$

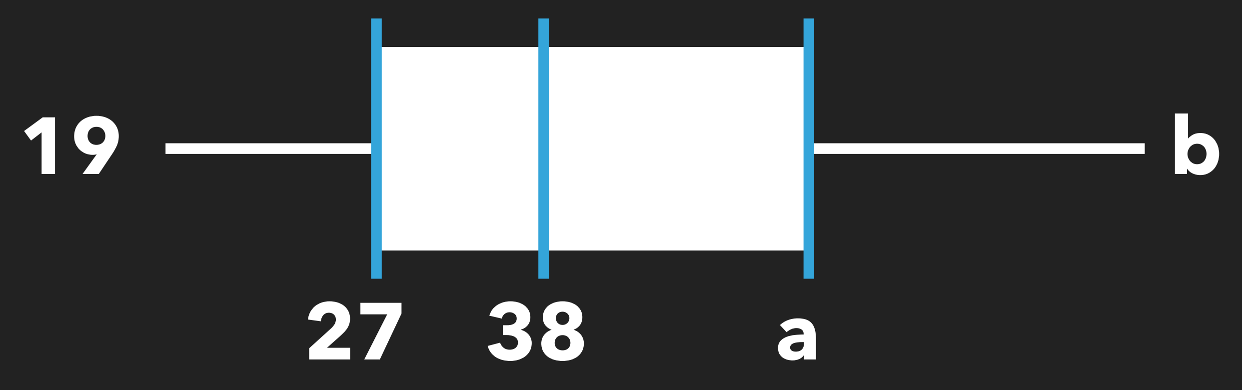
$\therefore$ *The ball takes 1.75 hrs from A to B*

* 距離 = 速度 x 時間

* 161 分鐘 = 161 / 60 小時

2018 PAPER 1 - SECTION A2

Q10.) The following box – and – whisker shows the distribution of the ages of a group of students in class A. Interquartile range = 21, range = 43



a.) Find a and b

b.) 5 more students are combined in the class A, in which 3 of them are of age 38 and the their range = 20. Will the range of the ages of class A unchange after the combination of these 5 students? Explain your answer.

* 參考課程 4.2

$a.)\ a - 27 = 21,\ b - 19 = 43$

$a = 48,\ b = 62$

b.) Assume the 5 students ages are :

$38, 38, 38, 38, 18,$ *which satisfy range* $= 20$

The combinated range $= 62 - 18 = 42 \neq 43$

$\therefore$ *The range will not always unchange*

* **Interquatile Range = 第三及一四分位數之差**
* **Range = 最大最細值之差**

* **舉反例証明錯**

2018 PAPER 1 - SECTION A2

Q11.) The following shows the result of the survey of the numbers of children in a typical family in Hong Kong . where k is a positive integer

Number of children	0	1	2	3	4
Number of families	k	2	9	6	7

a.) If mode = 2, the least and greatest value of k?

b.) If median = 2, the least and greatest value of k?

c.) If the mean = 2, find the value of k .

* 參考課程 4.1 及4.2

a.) The least value $= 1$
The greatest value $= 8$

b.) The least value $= 3$
The greatest value $= 19$

c.) $2 = \dfrac{2(1)+9(2)+6(3)+7(4)}{k+2+9+6+7} \rightarrow 2k+48=66 \rightarrow k=9$

* 眾數 = 出現最多

* 中位數 = 排序後中間的數值

* 平均值 = 數值總和 / 總數

2018 PAPER 1 - SECTION A2

$Q12.)$ *Let* $f(x) = 4x(x+1)^2 + ax + b$, *where a and b are constant. Given that* $x - 3$ *is a factor. and the remainder are* $2b + 165$ *for* $f(x)$ *is divided by* $x + 2$

a.) Find a and b

b.) Does $f(x) = 0$ *have at least one irrational root? Explain your answer.*

* 參考課程 1.1, 2.4 及 2.6

a.) Given that $f(3) = 0$ *and* $f(-2) = 2b + 165$

$$\begin{cases} 3a + b = -192 & \text{———— (1)} \\ 2a + b = -173 & \text{———— (2)} \end{cases}$$

$(1) - (2) : a = -19 \rightarrow b = -135$

i.e. $(a, b) = (-19, \ -135)$

b.) Assume $f(x) = 4(x-3)(x^2 + Ax + B)$, *where A and B are constant.*

By comnpare cofficieent of x and constant, $A = 5$, $B = 11.25$

Hence, $f(x) = 0 \rightarrow x = 3$ *or* $x^2 + 5x + 11.25 = 0 - (*)$

In $(*)$, $\Delta = 5^2 - 4(11.25) = -20 < 0$, *There is no real root*

i.e. $f(x) = 0$ *has no irrational root.*

* 餘數定理

* 消去法, 用 (1) 式減 (2) 式約去 b 搵 a, 再代入 (1) 式搵 b

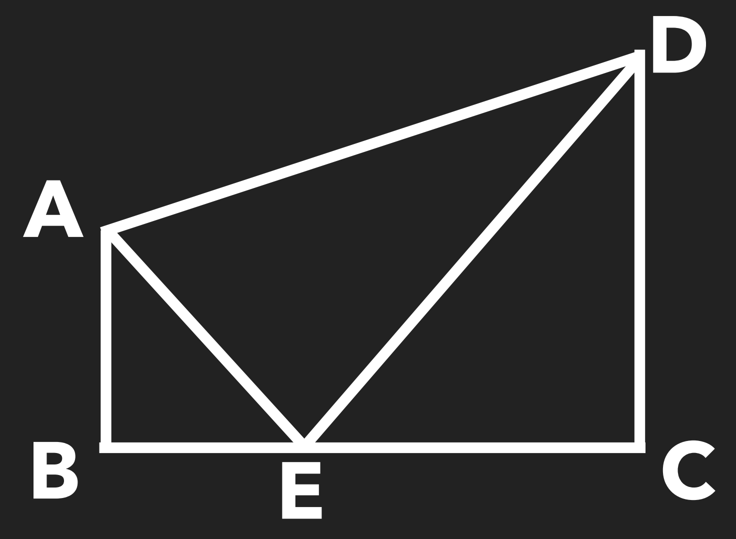
* 用二次方程判別式

2018 PAPER 1 - SECTION A2

Q13.) The following shows a trapezium with $\angle ABC = \angle AED = 90^0$ and $AB // CD$.

a.) Prove that $\Delta ABE \sim \Delta ECD$.

b.) Given that $AB = 15cm$, $AE = 25cm$ and $CE = 36cm$.
Is there a point F lying on AD such that $EF < 23cm$?
Explain your answer.



* 參考課程 3.1, 3.2 及 3.3

a.) $\angle ABE = \angle ECD = 90^0$ (given, $AB//DC$)

$\angle BAE = 90^0 - \angle AEB$ ($\angle s$ sum of Δ)

$\angle DEC = 90^0 - \angle AEB$ (alt. $\angle s$ on a st. line)

$\therefore \angle BAE = \angle DEC$

$\angle AEB = \angle CDE$ ($\angle s$ sum of Δ)

i.e. $\Delta ABE \sim \Delta CDE$ (AAA)

b.) $\because \Delta ABE \sim \Delta ECD \rightarrow \boxed{\dfrac{DE}{AE} = \dfrac{EC}{AB}} \rightarrow DE = 60cm$

* 三角形內角和 = 180⁰

* 直線上的角度加總 = 180⁰

* 三角形內角和 = 180⁰

* 相似三角形邊比相等

CONT'D

2018 PAPER 1 - SECTION A2

Let the shortest distance between point E and the line AD be H

where $AD = \sqrt{25^2 + 60^2} = 65cm\ (pyth.\ theorem)$

$$The\ area\ of\ \Delta AED = \frac{1}{2}(AE)(ED) = \frac{1}{2}(AD)(H)$$

$$\rightarrow H = \frac{AE \cdot ED}{AD} \approx 23.0769cm > 23cm$$

i.e. There is no point F lying on AD such that $EF < 23cm$

* 畢氏定理

2018 PAPER 1 - SECTION A2

Q14.) *There is a right circular cylinder with full of water, base radius = 8cm, height = 64cm.*
There is an empty circular cone, base radius = 20cm, height = 60cm are held inverted vertically. The water is now poured into the cone.
a.) *Find the volume of water in terms of* π
b.) *Find the depth of water inside the cone.*
c.) *Will water be overflowed if a metal sphere (radius = 14cm) immersed into the water? Explain your answer.*

* 參考課程 3.2 及 3.9

a.) Let $V_1\ cm^3$ be the volume of water

Then, $V_1 = 8^2(\pi)(64) = 4096\pi\ cm^3$

a.) Let $V_2\ cm^3$ be the volume of the cone

D cm be the depth of water

$$\frac{V_1}{V_2} = \left(\frac{D}{60}\right)^3 \rightarrow D^3 = (60)^3(4096\pi)\left(\frac{1}{3}60\cdot 20^2\pi\right) \rightarrow D = 48$$

* 柱體體積 = 底面積 x 高

* 錐體體積 = 1/3 x 底面積 x 高

* 相似圖形, 體積比 = (邊比)³

CONT'D

2018 PAPER 1 - SECTION A2

$\therefore$ *The depth of water is 48cm*

c.) *Let* $V_3\ cm^3$ *be the volume of the metal sphere*

Then, $V_3 = \frac{4}{3}14^3\pi = \frac{10976\pi}{3}cm^3$

$\because V_1 + V_3 \approx 7754.66\pi\ cm^3$ *and* $V_2 = \frac{1}{3}(20^2)\pi(60) = 8000\pi\ cm^3$

$\rightarrow V_1 + V_3 < V_2$

i.e. The water will not overflow.

* ■ 球體體積 = 4/3 x 半徑³ x π

* 當球體及水的體積加總大過容器體積水便滿溢

2018 PAPER 1 - SECTION B

$Q15.)\ 8-digit\ number\ is\ formed\ by\ a\ permutation\ of\ 2,3,4,5,6,7,8\ and\ 9.$

$a.)\ Find\ the\ number\ of\ 8-digit\ number\ can\ form.$

$b.)\ Find\ the\ number\ of\ 8-digit\ number\ that\ the\ 1^{st}\ and\ the\ last\ digit\ are\ odd\ can\ form.$

* 參考課程 4.4

$a.)\ The\ number\ of\ 8-digit\ number\ can\ form = \boxed{P_8^8} = 40320$

$b.)\ The\ number\ of\ 8-digit\ number\ can\ form = \boxed{P_2^4} \cdot \boxed{P_6^6}$

$= 8640$

* 8 個數字個排序
* 頭同尾 3,5,7,9 兩個數字的排序
* 中間 6 個位 6 個數字(沒有頭尾)的排序

2018 PAPER 1 – SECTION B

$Q16.)$ *The* 3^{rd} *and the* 4^{th} *term of an geometric sequence are 720 and 864 respectively.*

a.) Find the 1^{st} *term of the sequence*

b.) Find the greatest value of n such that the sum of the $(n+1)^{th}$ *term and* $(2n+1)^{th}$ *term is less than* $5x10^{14}$.

* 參考課程 2.7

a.) Let $T(n) = ar^{n-1}$, where a and r are constant

$$T(3) = 720 \rightarrow ar^2 = 720 \quad - \quad (1)$$
$$T(4) = 864 \rightarrow ar^3 = 864 \quad - \quad (2)$$
$$(2)/(1) : r = 1.2 \rightarrow a = 500$$
$$\therefore The\ 1^{st}\ term = 500$$

b.) Consider, $T(n+1) + T(2n+1) < 5x10^{14}$ and $n \geq 1$

$$\rightarrow (1.2^n)^2 + (1.2^n) - 10^{12} < 0 \text{ and } n \geq 1$$
$$\rightarrow 1 \leq n < log_{1.2}999999.5 \approx 75.8$$
$$\therefore The\ greatest\ value\ of\ n = 75$$

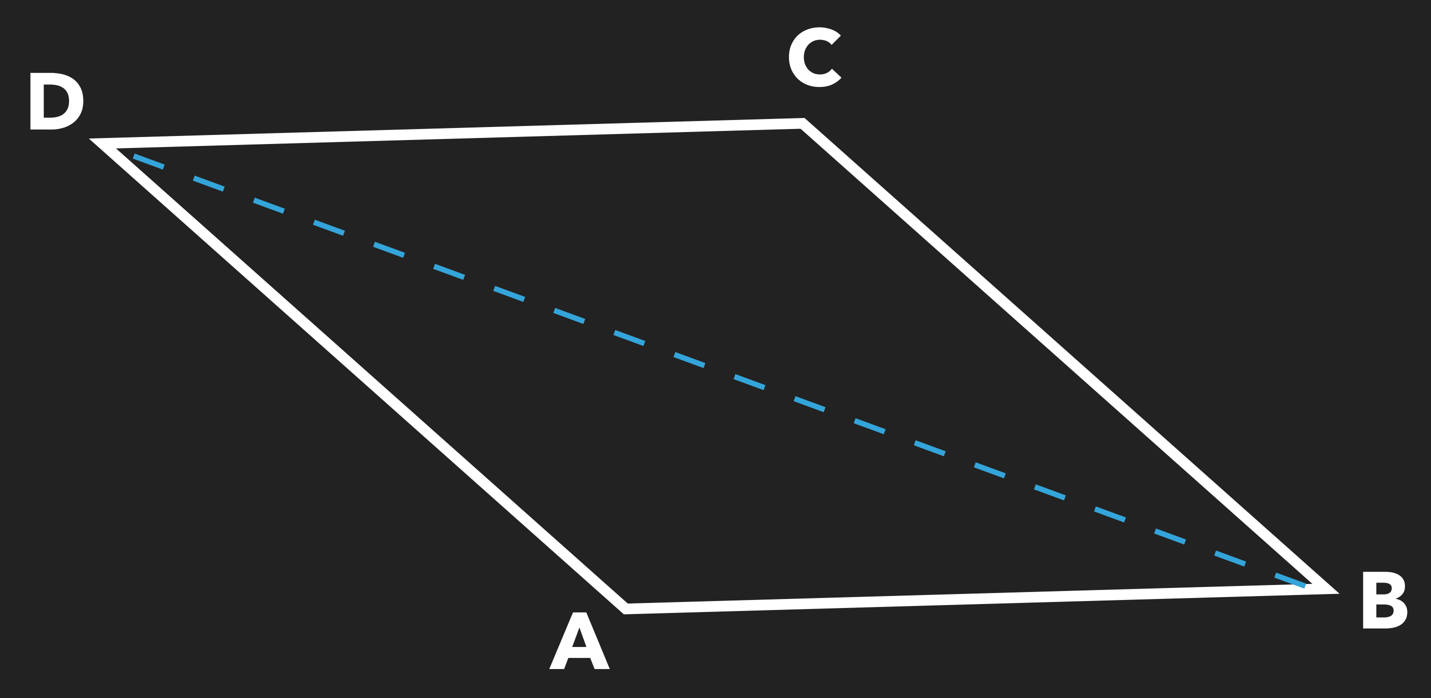
* 等比數列 = 首項 x (公比)$^{n-1}$

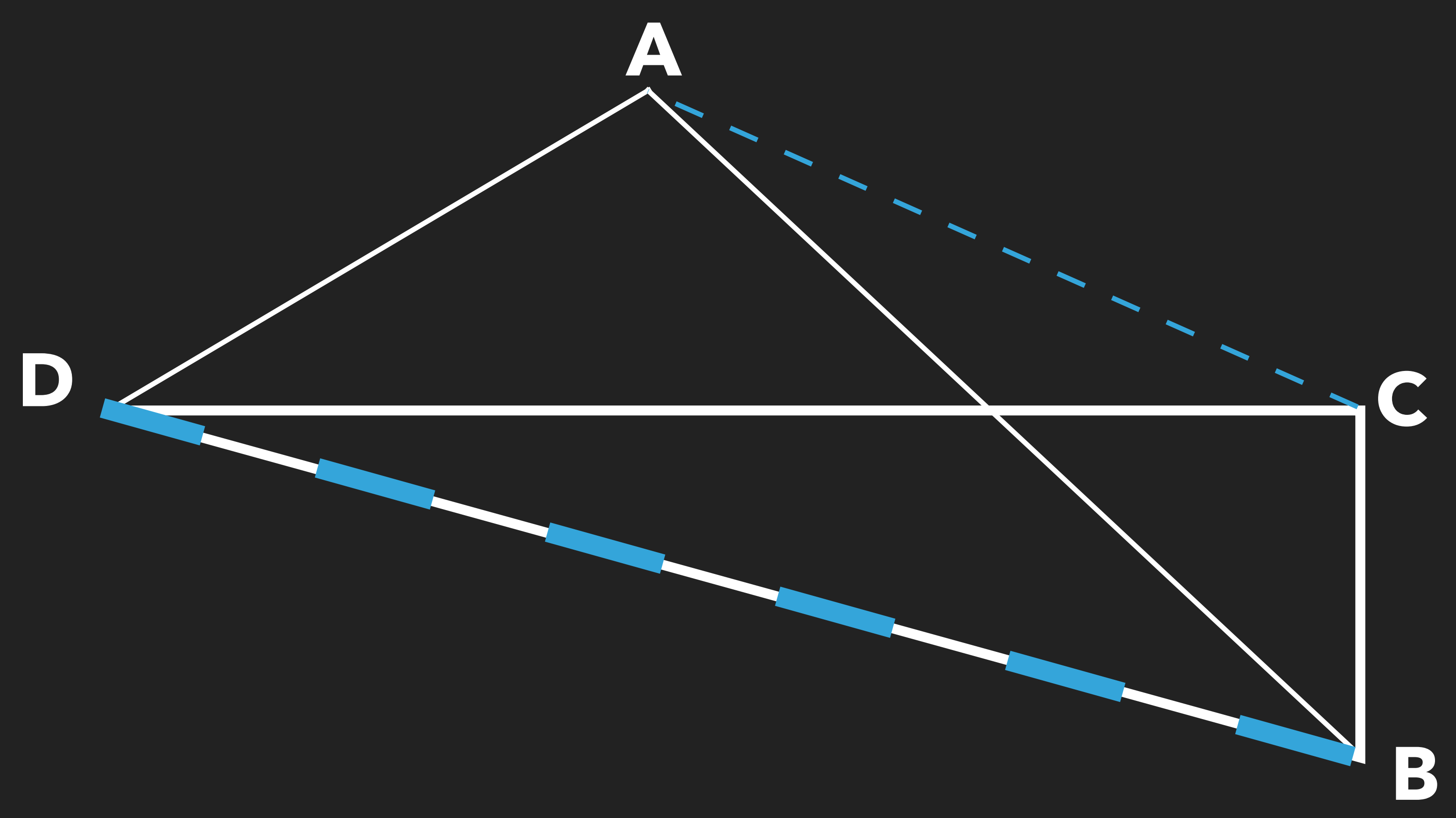
* (2) / (1) 搵 r 再代 (1) 搵 a

* 先解二次方程搵根, α, β
因為細過0, 所以答案喺最細最大根之間

2018 PAPER 1 – SECTION B

Q17.) In the following, ABCD is a parallelogram . Given that $AB = 60cm$, $\angle ABD = 20^0$, $\angle BAD = 120^0$ *. Then, the figure is folded along BD such that* $AC = 40cm$





a.) Find AD

b.) Find $\angle ABC$

c.) Find the angle between the plane ABD and the plane BCD

* 參考課程 3.3, 3.4 及 3.10

CONT'D

2018 PAPER 1 - SECTION B

$a.)$ *By sine law in* ΔADB,

$$AD = \frac{60\sin 20^0}{\sin(180^0 - 120^0 - 20^0)}, \ (\angle s \ sum \ of \ \Delta)$$

$$= 31.9cm \ (to \ 3 \ sig.fig.)$$

$b.)$ *By cosine law in* ΔABC *in the 3D figure,*

$$\cos\angle ABC = \frac{AB^2 + BC^2 - AC^2}{2AB \cdot BC} = \frac{AB^2 + AD^2 - AC^2}{2AB \cdot AD}$$

$$= \frac{60^2 + 31.925^2 - 40^2}{2(60)(31.925)}$$

$$\rightarrow \angle ABC = 38.0^0 \ (to \ 3 \ sig.fig.)$$

$c.)$ *Let* M *be the point on* BD *such that* $AM \perp BD$

F be the point on CD such that $FM \perp BD$

* sine law 使用

* 三角形內角和 = 180⁰

* cosine law 使用

* BC=AD, 平行四邊形特性

CONT'D

2018 PAPER 1 - SECTION B

$$b.)\ In\ \Delta ADM,\ AM = ADsin\angle ADM = ADsin40^0$$

$$DM = ADcos\angle ADM = ADcos40^0$$

$$(where\ \angle ADM = 180^0 - 120^0 - 20^0 = 40^0)$$

$$In\ \Delta DMF,\ MF = DMtan\angle FDM = DMtan20^0$$

$$= ADcos40^0tan20^0$$

$$DF = \frac{DM}{cos\angle FDM} = \frac{ADcos40^0}{cos20^0}$$

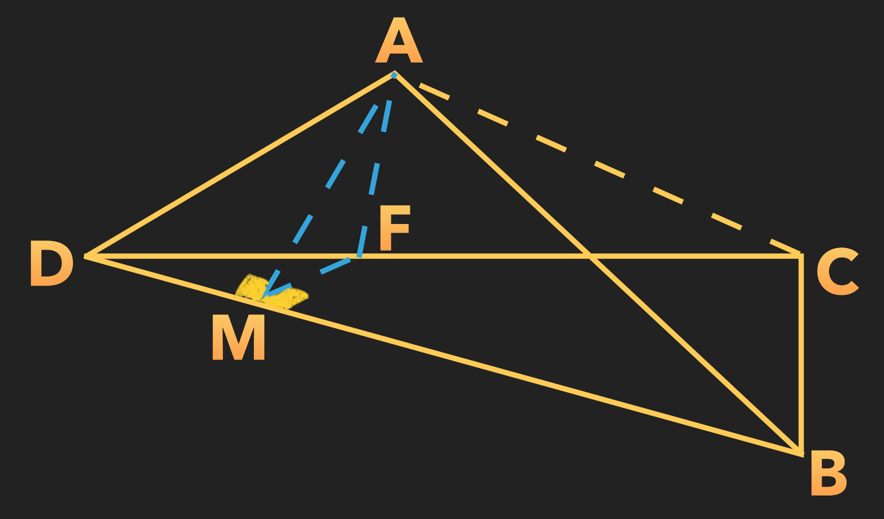
$$\because AD = CB\ (prop.\ of\ //gram)$$

$$DC = AB\ (prop.\ of\ //gram)$$

$$AC = CA\ (common)$$

$$\therefore \Delta ADC \cong \Delta CBA\ (SSS)$$

$$i.e.\ cos\angle ADC = cos\angle ABC$$



* 三角形內角和 = 180⁰

* 平行線內錯角相等

* 三邊相等, 全等三角形

CONT'D

2018 PAPER 1 - SECTION B

$In\ \Delta ADF,\ by\ cosine\ law,$

$$AF^2 = AD^2 + DF^2 - 2AD \cdot DF\cos\angle ADC$$

$$= AD^2[1 + (\frac{\cos 40^0}{\cos 20^0}) - 2(\frac{\cos 40^0 \cos\angle ABC}{\cos 20^0})]$$

$$= (0.37963549)AD^2$$

$In\ \Delta AMF,\ by\ cosine\ law,$

$$\cos\angle AMF = \frac{AM^2 + MF^2 - AF^2}{2AM \cdot MF}$$

$$= \frac{\cancel{AD^2}[\sin^2 40^0 + \cos^2 40^0 \tan^2 20^0 - 0.37963549]}{2\cancel{AD^2}\sin 40^0 \cos 40^0 \tan 20^0}$$

$$\rightarrow \angle AMF = 71.9^0\ (to\ 3\ sig.fig.)$$

$\therefore The\ angle\ between\ plane\ ABD\ and\ BCD = \angle AMF = 71.9^0$

* cosine law 使用

* cosine law 使用

2018 PAPER 1 - SECTION B

Q18.) It is given that $f(x)$ is sum of two parts, one part varies as a x^2 and one part varies as x. Given that $f(2) = 60$, $f(3) = 99$. Suppose Q is the vertex of $y = f(x)$ while R is the vertex of $y = 27 - f(x)$.

a.) Find $f(x)$.

b.) Find the coordinates of Q and R.

c.) Let $S = (56,0)$ and P be the circumcentre of ΔQRS. Describe the geometric relationship between P, Q and R.

* 參考課程 2.3, 2.4, 2.5, 3.2 及 3.8

a.) Let $f(x) = k_1x^2 + k_2x$, where k_1, k_2 are real constant. Then,

$$\begin{cases} 4k_1 + 2k_2 = 60 & \text{——} (1) \\ 9k_1 + 3k_2 = 99 & \text{——} (2) \end{cases} \rightarrow \begin{cases} 2k_1 + k_2 = 30 & \text{——} (1) \\ 3k_1 + k_2 = 33 & \text{——} (2) \end{cases}$$

$$(2) - (1) : k_1 = 3 \rightarrow k_2 = 24$$

$$\therefore f(x) = 3x^2 + 24x$$

* 部分變量

* 消去法消去 k₂ 搵 k₁ ,再代 (1) 式搵 k₂

CONT'D

2018 PAPER 1 – SECTION B

$b.)\ Let\ Q = (a, b)$

$f(x) = 3x^2 + 24x \equiv 3(x - a)^2 + b$

By compare cofficient of x and constant,

$$\begin{cases} -6a = 24 & \text{——— (1)} \\ 3a^2 + b = 0 & \text{——— (2)} \end{cases}$$

$a = -4,\ b = -48$

$\therefore Q = (-4, -48)$

$$\begin{aligned} Assume\ g(x) = 27 - f(x) &= 27 - 3(x + 4)^2 + 48 \\ &= -3(x + 4)^2 + 75 \end{aligned}$$

$\therefore R = (-4, 75)$

$c.)\ Let\ P = (m, n)$

$$where\ n = \frac{75 - 48}{2} = 13.5$$

* 頂點型態轉換, 可用 compare coefficient

* 外心 = 垂直中點線交點

CONT'D

2018 PAPER 1 - SECTION B

$Let\ the\ mid-pt.\ of\ QS\ be\ M$

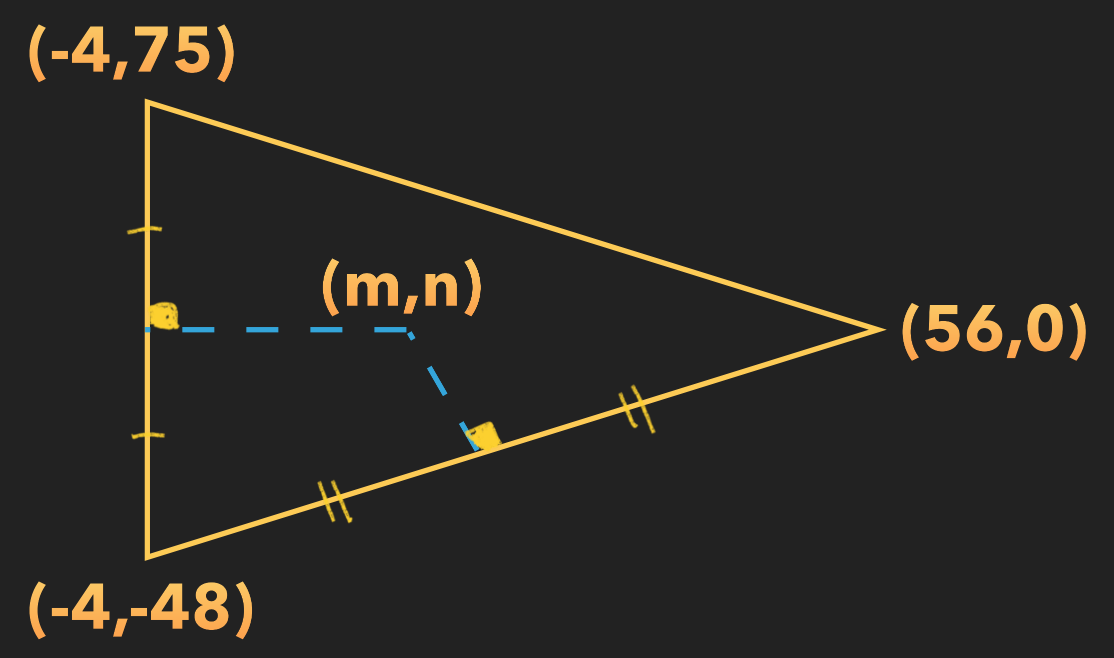
$$M=(\frac{56-4}{2},\frac{0-48}{2})=(26,-24)$$

$\because$ $the\ slope\ of\ PM\ x\ the\ slope\ of\ QS=-1$

$$\frac{13.5+24}{m-26}\cdot\frac{48}{60}=-1\rightarrow m=-4$$

$The\ x-coordinate\ of\ P,\ Q\ and\ R=-4$

$i.e.\ P,Q,R\ are\ collinear.$



* 兩線互相垂直, 斜率相乘 = -1

* 兩點斜率 = $\frac{y_2-y_1}{x_2-x_1}$

2018 PAPER 1 - SECTION B

Q19.) The circle, C has center = (8,2) and radius = r.

Denote a straight line, L : $kx - 5y - 21 = 0$, k is constant. L is a tangent to C

a.) Express r^2 in term of k.

b.) L passes through D(18, 39) :

i.) Find r.

ii.) Let E be the point y – intercept of L, F be a point that C is the inscribed circle of ΔDEF. Is ΔDEF an obtuse – angled Δ? Explain your answer.

* 參考課程 3.2, 3.7 及 3.8

a.) The equation of C : $(x-8)^2+(y-2)^2=r^2$

Consider,

$$\begin{cases}(x-8)^2+(y-2)^2=r^2 \quad \text{———} \quad (1)\\ kx-5y-21=0 \quad \text{————} \quad (2)\end{cases}$$

In (2) : $y=\frac{1}{5}(kx-21)$, sub into (1)

* 圓形公式: (x,y) 同圓心距離=半徑

* 用代入法建立二元方程

CONT'D

2018 PAPER 1 - SECTION B

$$\rightarrow (x-8)^2 + [\frac{1}{5}(kx-21)-2]^2 = r^2$$

$$\rightarrow (1+0.04k^2)x^2 - (16+2.48k)x + (102.44-r^2) = 0$$

$$\because L \text{ is the tangent of } C \rightarrow \boxed{\Delta = 0}$$

* L 是 C 的切線, 只有一個相交點, 判別式=0

$$\rightarrow (16+2.48k)^2 - 4(1+0.04k^2)(102.44-r^2) = 0$$

$$\rightarrow r^2 = 102.44 - \frac{(16+2.48k)^2}{4(1+0.04k^2)}$$

bi.) Since L passes through D,

$$\rightarrow k(18) - 5(39) - 21 = 0$$

$$\rightarrow k = 12$$

$$Hence,\ r^2 = 102.44 - \frac{(16+2.48(12))^2}{4(1+0.04(12)^2)} = 25 \rightarrow r = 5$$

CONT'D

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bii.) Let $O(8, 2)$ *be the center of* C.

$Then,\ E = (0,\ -4.2)$

$Assume,\ L\ touch\ C\ at\ R,\ then\ OR = r = 5$

Meanwhile,

$$OE = \sqrt{(8-0)^2 + (2+4.2)^2} = \sqrt{102.44}$$

$$OD = \sqrt{(8-18)^2 + (2-39)^2} = \sqrt{1469}$$

$$\sin\angle OED = \frac{OR}{OE}\ (\angle ORE = 90^0,\ tangent \perp radius)$$

$$\sin\angle ODE = \frac{OR}{OD}\ (\angle ORD = 90^0,\ tangent \perp radius)$$

* 代 x=0 入 L 搵 y 值

* 距離公式 = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

* 圓切線相交點與半經互相垂直

CONT'D

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$bii.)\ \angle ODF = \angle ODE,\ \angle OEF = \angle OED\ (tangent\ props)$

$\angle DFE = 180^0 - \angle FDE - \angle DEF$ $(\angle s\ sum\ of\ \Delta)$

$$\angle DFE = 180^0 - 2(\angle ODE - \angle OED)$$

$$= 180^0 - 2(sin^{-1}(\frac{OR}{OD}) - sin^{-1}(\frac{OR}{OE}))$$

$$\approx 105.8^0 > 90^0$$

$\therefore \Delta DEF\ is\ an\ obtuse-angled\ \Delta$

* 兩條切線構成一對全等三角形

* 三角形內角和 = 180⁰