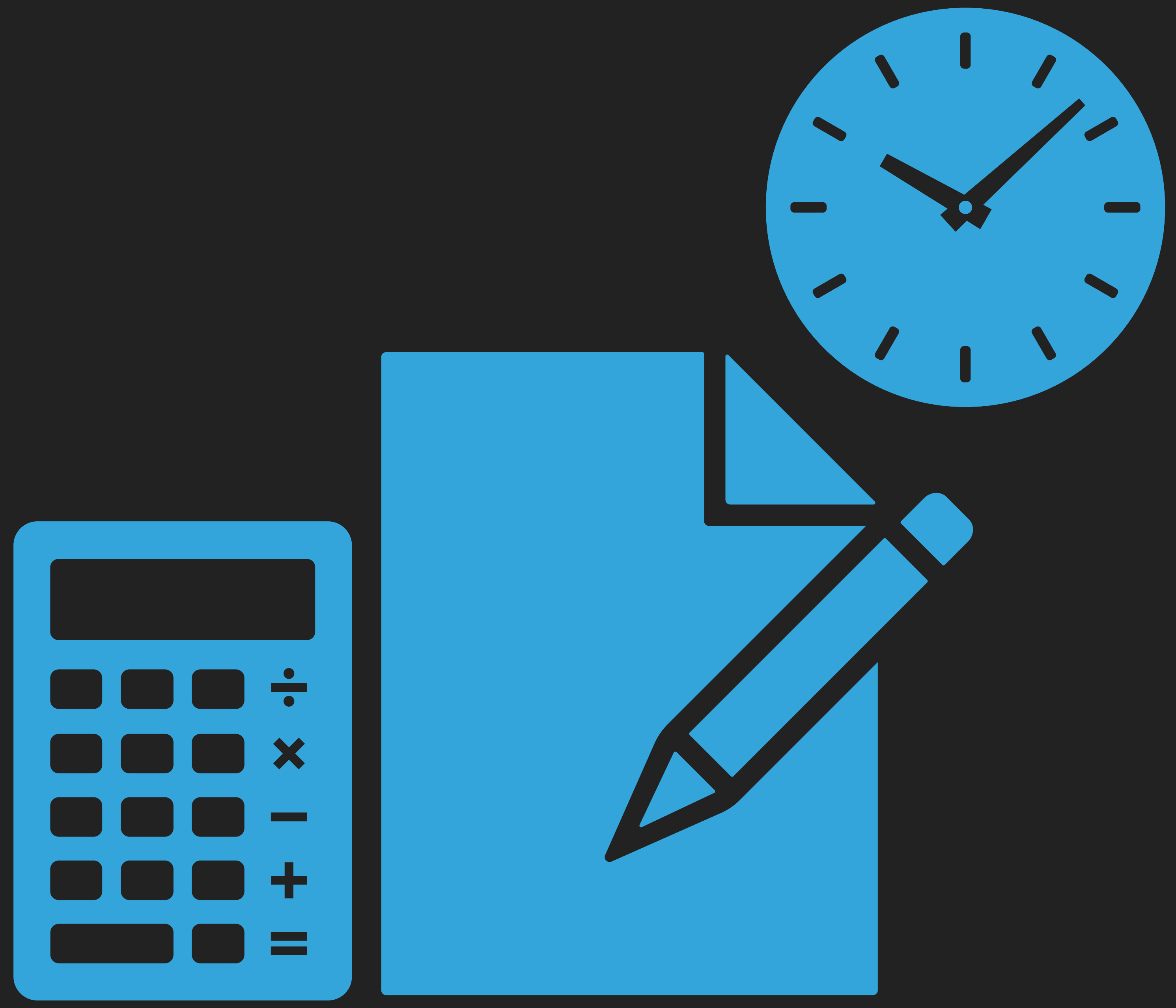
深宵教室 - DSE 必修模擬試題解答

2013 PAPER 1

此為參考2013試題之模擬試題，原版請另行購買

2013 PAPER 1

- Section A1
- Section A2
- Section B



2013 PAPER 1 - SECTION A1

Q1.) Simplified $\dfrac{x^{20}y^{13}}{(x^5y)^6}$, *in positive indices*

* 參考課程 1.2

$$= x^{20-5\cdot 6} \cdot y^{13-6}$$

$$= x^{-10} \cdot y^{7}$$

$$= \frac{y^7}{x^{10}}$$

* 指數乘係加，除係減

* 指數負數，分母變分子，分子變分母

2013 PAPER 1 - SECTION A1

$$Q2.)\ \frac{3}{h} - \frac{1}{k} = 2,\ k = ?$$

* 參考課程 2.1

$$\rightarrow \frac{3k - h}{hk} = 2$$

* 通分母

$$\rightarrow 3k - h = 2hk$$

* 兩邊乘 hk

$$\rightarrow k = \frac{h}{3 - 2h}$$

* 兩邊減 2hk 同加 h, 再除 3-2h

2013 PAPER 1 - SECTION A1

Q3.) Factorize $4m^2 - 25n^2 + 6m - 15n$

* 參考課程 2.5

$$4m^2 - 25n^2 + 6m - 15n$$

$$= (2m-5n)(2m+5n) + 3(2m-5n)$$

$$= (2m-5n)(2m+5n+3)$$

* 恆等式 $(a-b)(a+b) \equiv a^2 - b^2$
* 抽 3

2013 PAPER 1 - SECTION A1

Q4.) The price of 7 pens and 3 rulers is \$47, the price of 5 pens and 6 rulers is \$49

The price of pens = ?

* 參考課程 2.3

Let the price of pen be $\$x$

the price of ruler be $\$y$

$$\begin{cases} 7x + 3y = 47 & \text{———————} \quad (1) \\ 5x + 6y = 49 & \text{———————} \quad (2) \end{cases}$$

$$2x(1) - (2): \ 9x = 45 \rightarrow x = 5$$

$\therefore$ *The price of pens = \$5*

* 先 Let 未知數方便表達

* 消去法，目標整走個 y

2013 PAPER 1 - SECTION A1

Q5.) Solve $\dfrac{19-7x}{3} > 23-5x \text{ and } 18-2x \geq 0$

Hence, find out how many postive integers satisfy both inequalities.

* 參考課程 1.1 及 2.3

$$\frac{19-7x}{3} > 23-5x \text{ and } 18-2x \geq 0$$

$\rightarrow 19-7x > 69-15x \text{ and } 9 \geq x$

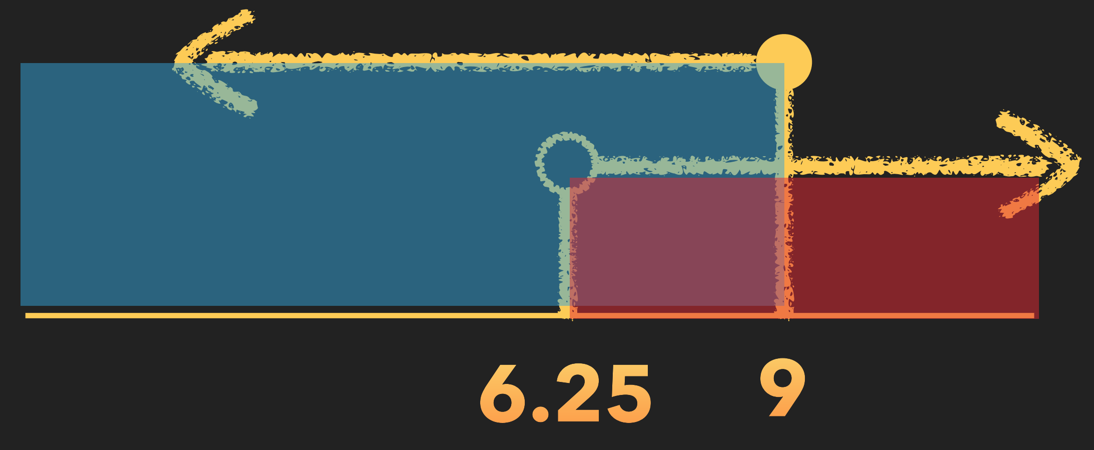
$\rightarrow x > 6.25 \text{ and } x \leq 9$

$\rightarrow 6.25 < x \leq 9$

i.e. The fulfilled postive integers are 7, 8, 9

There are 3 positive integers satisfy the inequalities.

* And 指重疊地方



2013 PAPER 1 - SECTION A1

Q6.) In a polar system, $O = (0, 0^0)$, $A = (26, 10^0)$, $B = (26, 130^0)$. Let L be the reflectional symmetry of ΔOAB.

a.) Describe the geometric relationship between L and $\angle AOB$

b.) Find the polar coordinates of the intersection of L and AB.

* 參考課程 3.1, 3.2 及 3.8

a.) *Angle Bisector*

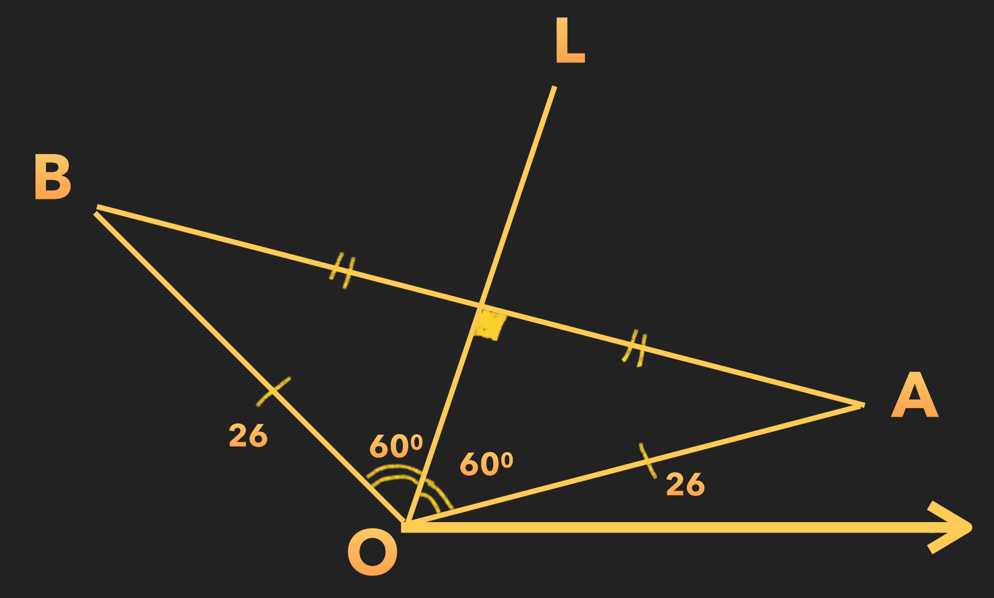
b.) ΔOAB is an isos. Δ

$\therefore$ The interception pt. = mid – pt. of AB (property of isos. Δ)

$$= (26\cos 60^0, 60^0 + 10^0)$$

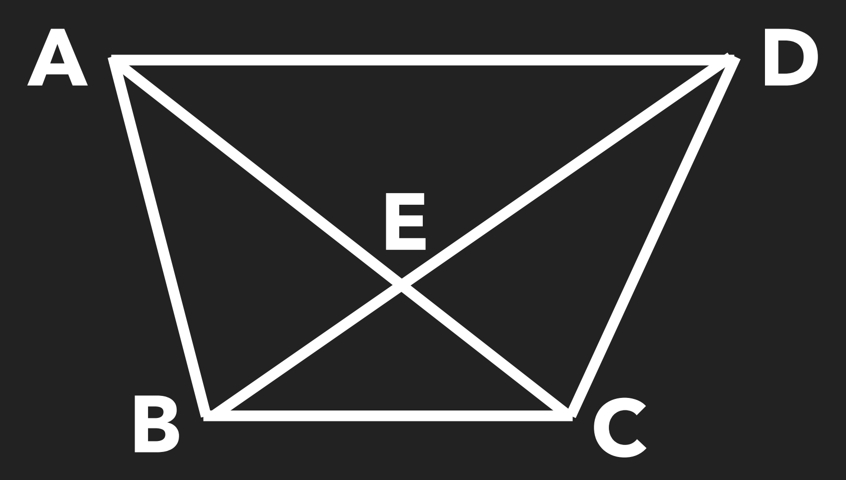
$$= (13, 70^0)$$

* 先畫圖理解



2013 PAPER 1 - SECTION A1

Q7.) In the following figure, $BE = CE$, $\angle BAC = \angle BDC$



a.) Prove $\Delta ABC \cong \Delta DCB$

b.) Find the numbers of pairs of congruent and similar Δ

* 參考課程 3.3

a.) $\angle BAC = \angle BDC$ (given)

$\angle ACB = \angle CBD$ (sides opp, eq. $\angle$s, $BE = CE$)

$BC = BC$ (common)

$\therefore \Delta ABC \cong \Delta DCB$ (AAS)

b.) There are 3 pairs of congruent Δ

There are 4 pairs of similar Δ

* 等腰三角形底角相等

* 共邊原因要寫

* $\Delta ABC \cong \Delta DCB$
$\Delta AEB \cong \Delta DEC$
$\Delta ADB \cong \Delta DAC$
$\Delta ADE \sim \Delta BEC$

* 全等三角形係相似三角形

2013 PAPER 1 - SECTION A1

Q8.) A pack of suger is standard if this measured as 100g correct to the nearest g

a.) Find the least weight of the standard pack of suger.

b.) Is it possible the total 32 standard pack of suger is measured as 3.1kg correct to 0.1kg? Explain your answer.

* 參考課程 1.1

$a.)\ The\ least\ weight = 99.5g$

* 四捨五入

$b.)\ Let\ be\ Xg\ the\ weight\ of\ a\ standard\ pack\ suger$

$$99.5 \leq X \leq 100.4$$

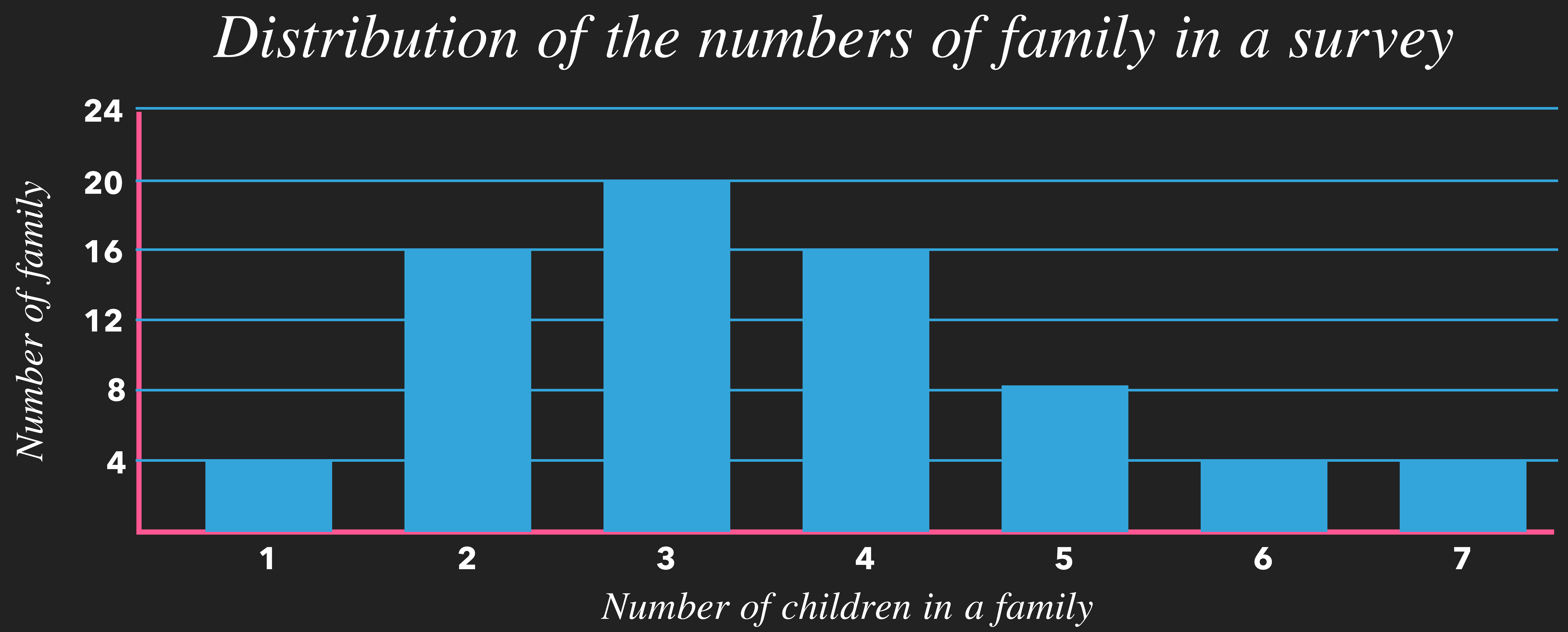
$$\rightarrow 3184 \leq 32X \leq 3216$$

$$\therefore 32X = 3.184kg = 3.2kg\ (to\ nearest\ 0.1kg)$$

It is impossible 32 standard pack of suger measured as 3.1kg

2013 PAPER 1 - SECTION A1

Q9.) The following bar chart shows the result of the survey of the numbers of children in a typical family in Hong Kong .



a.) Find the mean, inter – quartile range and standard deviation of the above result .

b.) If one family data is removed, which has 7 children . Find the change of the standard deviation .

* 參考課程 4.1 及4.2

CONT'D

2013 PAPER 1 - SECTION A2

$a.)$ $The\ mean,\ \mu = 3.5,\ The\ inter-quartile\ range = 2.25$

$The\ standard\ deviation,\ sd = 1.5$

$b.)$ $The\ new\ standard\ deviation = 1.45229$

$\therefore\ the\ standard\ deviation\ drops\ 0.0477$

* 平均值 = 加總 / 總數量
* 四分位距 = 第一及三四分位數之差
* 標準差 = (各數與平均的差)²平均值的開方

2013 PAPER 1 – SECTION A2

Q10.) Consider the following ages distribution in class A and B

The ages of students in class A

17	18	21	21	22	22	23	23	23	31
31	34	35	36	47	47	58	68	69	69

The ages of students in class B

Stem (tens)	*Leaf (units)*
2	a 5 6 7
3	3 3 8
4	3
5	1 2 9
6	7 b

a.) Find the median and mode of the ages of class A

b.) It is given that the range of the ages distribution of class B is 47

(i) Find a and b

(ii) A students is randomly selected from class A and from class B. What is the probability of their ages difference exceeds 40?

* 參考課程 4.1 及 4.2

CONT'D

2013 PAPER 1 - SECTION A2

a.) $The\ median = 31,\ The\ mode = 23$

bi.) $The\ possible\ value\ of\ a\ are\ 0, 1, 2, 3, 4, 5$

$The\ possible\ value\ of\ b\ are\ 7, 8, 9$

$\because Range = 47 \rightarrow (60+b)-(20+a) = 47$

$\therefore\ (a, b) = (0,7)\ or\ (1,8)\ or\ (2,9)$

ii.) $P(the\ ages\ difference > 40)$

$$= P(Age\ from\ A = 17, 18)P(Ages\ from\ B = 59, 67, 6b) + P(Age\ from\ A = 21, 22, 23)P(Ages\ from\ B = 67, 6b) + P(Age\ from\ A = 68, 69)P(Ages\ from\ B = 2a, 25, 26, 27)$$

$$= \frac{2}{20}\frac{3}{13} + \frac{7}{20}\frac{2}{13} + \frac{3}{20}\frac{4}{13}$$

$$= \frac{8}{65}$$

* 中位數 = 中間的數值

* 眾數組 = 出現次數最多

* 全距 = 最大值 - 最細值

* 因為响 class A 抽人同响 class B 抽人係完全獨立事件, 所以 P(A and B)=P(A)P(B)

* 因為响 class A 抽人同响 class B 抽人係完全沒有重疊, 所以 P(A or B)=P(A)+P(B)

2013 PAPER 1 - SECTION A2

Q11.) Let l metres and W grams be the length of wire and the weight of the wire. It is known that W is sum of two parts. One part varies directly as l and another part varies directly as l^2. When $l = 1$, $W = 181$ and $l = 2$, $W = 402$

a.) Find the weight of 1.2m long wire.

b.) Find the length of the 594g wire.

* 參考課程 2.3, 2.4, 2.5 及 2.6

a.) Let $W = k_1 l + k_2 l^2$, where k_1, k_2 are real constant. Then,

$$\begin{cases} 181 = k_1 + k_2 & \text{———— (1)} \\ 402 = 2k_1 + 4k_2 & \text{———— (2)} \end{cases}$$

$(2) - 2(1):\ 40 = 2k_2 \to k_2 = 20,\ k_1 = 161$

$\therefore W = 161l + 20l^2$

i.e. The weight of 1.2m wire $= 161(1.2) + 20(1.2)^2 = 222g$

* 部分變量

* 消去法消去 k_1 搵 k_2，再代 (1) 式搵 k_1

CONT'D

2013 PAPER 1 - SECTION A2

b.) Let L metre be the length of 594g wire .

$$594 = 161L + 20L^2$$

$$\rightarrow 20L^2 + 161L - 594 = 0$$

$$\rightarrow \boxed{L = \frac{-161 \pm \sqrt{161^2 - 4(20)(-594)}}{2(20)}}$$

$$\rightarrow L = \frac{11}{4} \text{ or } \boxed{-\frac{54}{5} \ (rejected)}$$

$$\therefore \text{The length of 594g wire} = \frac{11}{4} m$$

* 二次方程根公式

* 長度無負數

2013 PAPER 1 - SECTION A2

$Q12.)\ Let\ f(x) = 3x^3 - 7x^2 + kx - 8,\ where\ k\ is\ constant.$

$a.)\ Factorize\ f(x)$

$b.)\ Is\ all\ roots\ of\ f(x) = 0\ are\ real\ number?\ Explain\ your\ answer.$

* 參考課程 1.1, 2.4 及 2.6

$a.)\ \because f(2) = 0\ \ \therefore f(x) \equiv (x-2)(ax^2 + bx + c)$

$By\ compare\ cofficient,\ a = 3,\ b = -1\ and\ c = 4$

$i.e.\ f(x) \equiv (x-2)(3x^2 - x + 4)$

$b.)\ f(x) = 0\ \rightarrow x - 2 = 0\ or\ 3x^2 - x + 4 = 0$ —— (*)

$In\ (*),\ \Delta = (-1)^2 - 4(3)(4) = -47 < 0$

$i.e.\ It\ is\ not\ all\ roots\ are\ real\ number.$

* 餘數定理
* 用判別式決定有幾多根
* 判別式為負數, 無實根根

2013 PAPER 1 - SECTION A2

Q13.) There is a larger circular cylinder with R cm base radius and a smaller one with r cm base radius, 10cm in height. It found that 2 larger cylinder can form 27 smaller one after melt down. The base area of these two cylinder is 1 : 9 (smaller : larger)

a.) $r : R = ?$

b.) Are they similar? Please explain your answer.

* 參考課程 3.9

a.) $\pi r^2 : \pi R^2 = 1 : 9 \rightarrow r^2 : R^2 = 1 : 9 \rightarrow r : R = 1 : 3$

b.) *Let the height of the large cylinder be H cm*

$$\boxed{2\pi R^2 H} = \boxed{27\pi r^2(10)} \rightarrow H = \frac{27}{2}\left(\frac{r}{R}\right)^2(10) \rightarrow H = 15$$

$$\frac{\pi R^2(15)}{\pi r(10)} = \frac{27}{2} \neq \left(\frac{R}{r}\right)^3 \rightarrow \textit{They are not similar}$$

* 柱體體積 = 底面積 x 高

* 相似圖形, 體積比 = (邊比)³

2013 PAPER 1 - SECTION A2

Q14.) The circle, $C : x^2 + y^2 - 12x - 34y + 225 = 0$. *Denote the center of C by R*
Given that there is straight line, $L : 4x + 3y + 50 = 0$, *does not intercept with C*
Let point P lying on L be the point nearest to R
a.) Find the distance of PR
b.) Let Q be a moving point on C. When Q is nearest to P, what is the geometric relationship between P, Q, and R. Also, find Area of ΔOPQ : *Area of* ΔOQR.
where $O = (0,0)$.

* 參考課程 3.3 及 3.8

$$a.)\ R = \left(-\frac{-12}{2},\ -\frac{-34}{2}\right) = (6,\ 17)$$

$$\text{The radius of } C,\ r = \sqrt{6^2 + 17^2 - 225} = 10$$

$$\text{Let } P = (a,\ b)$$

* 圓形公式

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$
$$\rightarrow x^2 + y^2 - 2x_0x - 2y_0y^2 + (x_0^2 + y_0^2 - r^2) = 0$$

CONT'D

2013 PAPER 1 - SECTION A2

$$The\ slope\ of\ L = \frac{-4}{3}$$

$$\begin{cases} 4a + 3b = -50 & \text{———— (1)} \\ \dfrac{b-17}{a-6} \cdot \dfrac{-4}{3} = -1 & \text{———— (2)} \end{cases}$$

$$\begin{cases} 4a + 3b = -50 & \text{———— (1)} \\ 3a - 4b = -50 & \text{———— (2)} \end{cases}$$

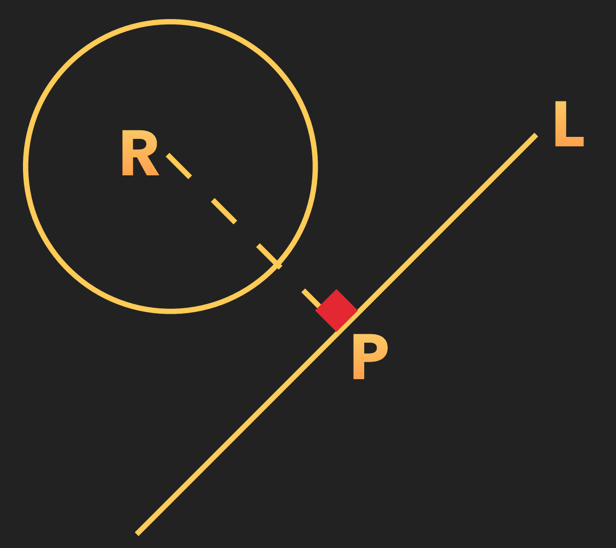
$$4(1) + 3(2) : 25a = (7)(-50) \rightarrow a = -14,\ b = 2$$

$$Hence\ P = (-14, 2),\ PR = \sqrt{(-14-6)^2 + (2-17)^2}$$

$$= 25\ unit$$

b.) P, Q and R are collinear

ΔOPQ *and* ΔOQR *have same height*



* 兩點斜率 $= \dfrac{y_2 - y_1}{x_2 - x_1}$

* 互相垂直, 斜率相乘 = -1

* 消去法整走 b, 再代 a 入 (1) 搵 b

* 兩點距離 $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

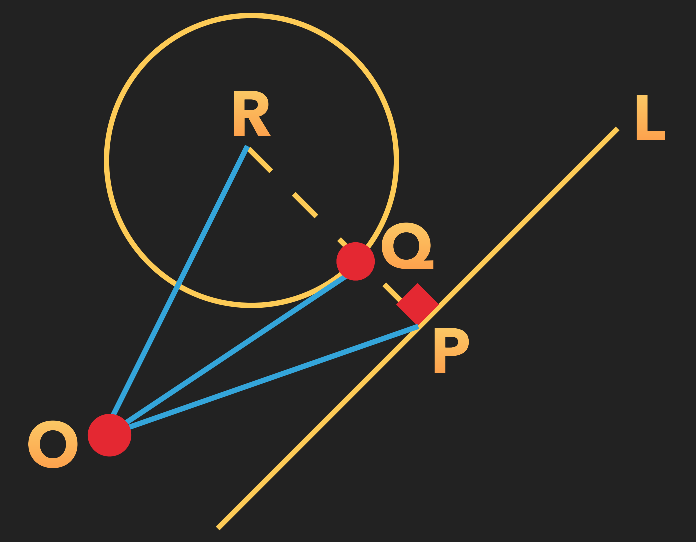
CONT'D

2013 PAPER 1 - SECTION A2

$$\therefore \boxed{Area\ of\ \Delta OPQ : Area\ of\ \Delta OQR = PQ : QR}$$

$$= PR - r : r$$

$$= 3 : 2$$



* 有共高三角形, 面積比 = 邊比

2013 PAPER 1 - SECTION B

Q15.) The box – and – whisker diagram below shows the score of the mathematic exam .
Studen A gets the highest score with standard score 3. Student B gets 65 marks with standard score 0.5.

a.) Find the mean

b.) Do half of the student have negative standard score? Explain your answer .

* 參考課程 2.3, 4.1 及 4.2

a.) Let the mean be μ, the standard deviation be σ

Then, we have $\boxed{3 = \dfrac{90 - \mu}{\sigma}}$ and $\boxed{0.5 = \dfrac{65 - \mu}{\sigma}}$

By solving, the mean $= 60$

b.) The median $= 55 < 60\ (\mu)$

$\therefore$ Half of the student have negative standard score .

* ■ 標準分數 = 數據與平均值相差多少個標準差

* 用消去或代入法

* 中位數代表排序後中間的數據值

* 只要分數細個平均值, 標準分數為負數

2013 PAPER 1 - SECTION B

Q16.) A box contains 5 white balls and 11 blue balls. 6 balls are randomly drawn from box.

a.) Find the probability at least 4 white balls are drawn.

b.) Find the probability at least 3 blue balls are drawn.

* 參考課程 4.3 及 4.4

Denote nW be the n numbers of white balls drawn

nB be the n numbers of blue balls drawn

$$a.)\ P(at\ least\ 4W) = P(4W\ or\ 5W) = P(4W) + P(5W)$$

$$= \frac{C_4^5 C_2^{11}}{C_6^{16}} + \frac{C_5^5 C_1^{11}}{C_6^{16}} = \frac{1}{28}$$

$$b.)\ P(at\ least\ 3B) = P(at\ most\ 3W) = 1 - P(at\ least\ 4W)$$

$$= 1 - \frac{1}{28} = \frac{27}{28}$$

* 因為兩者並不存在同時發生的可能,
完全沒有重疊, 所以 P(A or B)=P(A)+P(B)

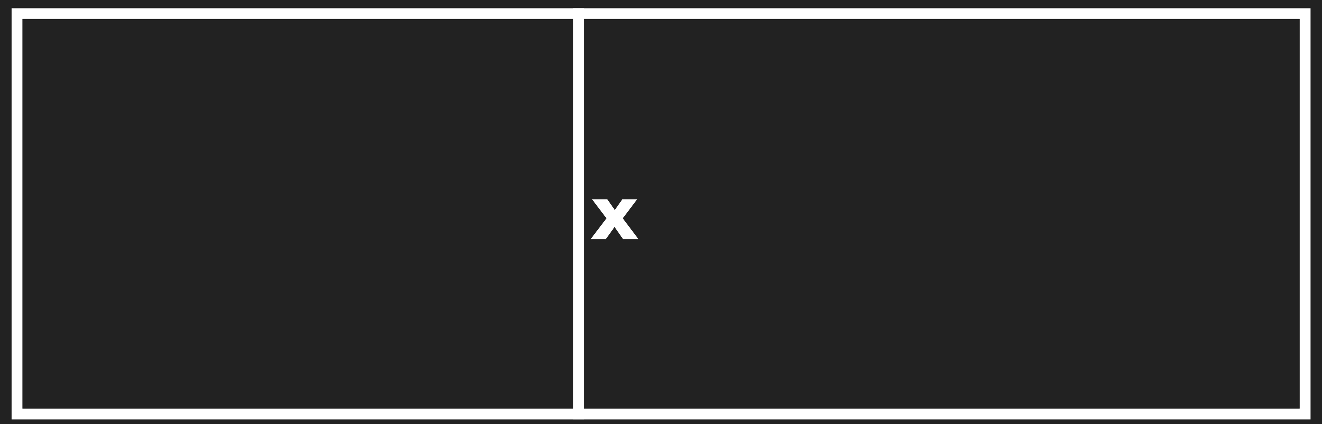
* (5 個白抽 4 個) x (11 個藍抽 2 個)

* (5 個白抽 5 個) x (11 個藍抽 1 個)

* P(最多3個白) = 1 - P(最小4個白)

2013 PAPER 1 - SECTION B

Q17.) A 108m long string is cut into two piece and form the following rectangle with a separation in the rectangle. Let the area of the rectangle be Am^2 and the length of the separation be xm.



a.) Express A in term of x

b.) Can A greater than $500m^2$? Explain your answer.

* 參考課程 2.5 及 2.6

$$a.)\ A = x\left(\frac{108-3x}{2}\right) = \frac{3}{2}(36x - x^2)$$

$$b.)\ A = \frac{3}{2}(36x - x^2) = -\frac{3}{2}[(x-18)^2 - 324]$$

$$\therefore \textit{The greaest value of } A = \frac{3}{2}(324) = 486 < 500$$

A cannot be greater than $500m^2$.

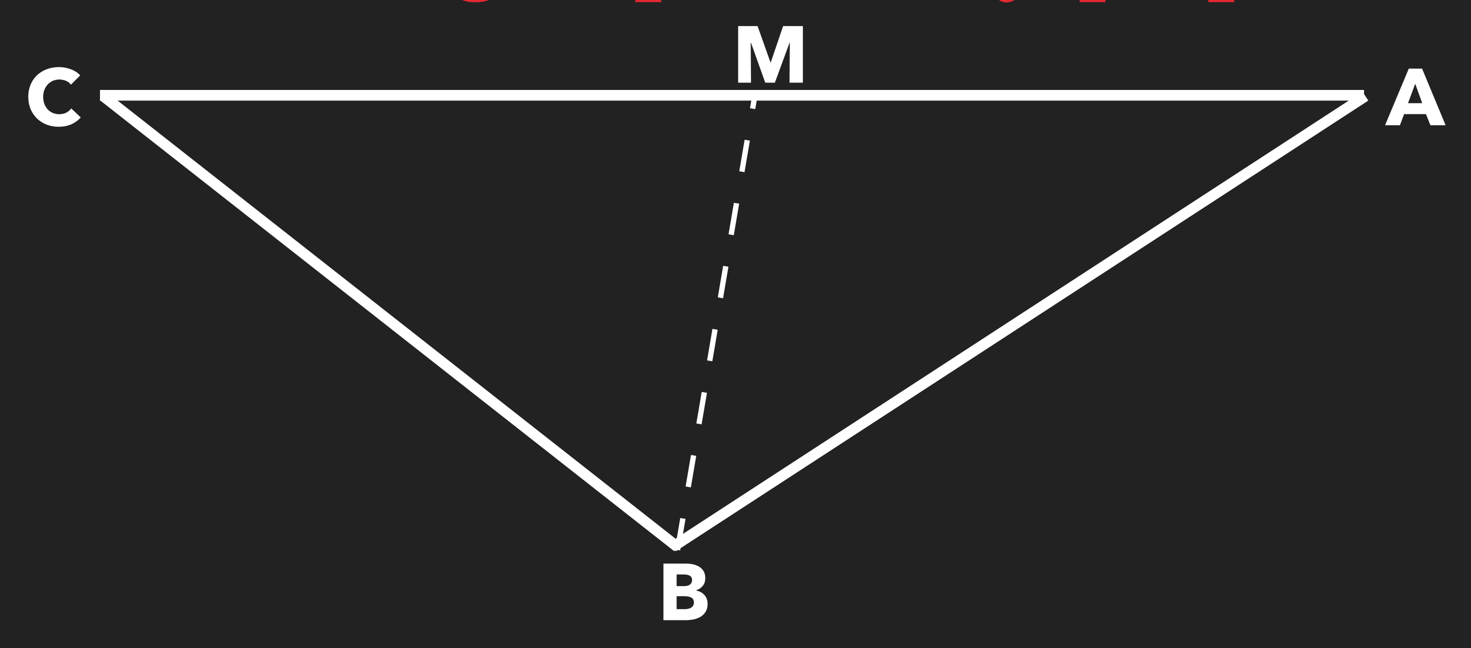
* 長方形周界 + x = 108 m

* 問最大最細值用頂點型

* 頂點 = (18, 486), 取 y 為最大值

2013 PAPER 1 - SECTION B

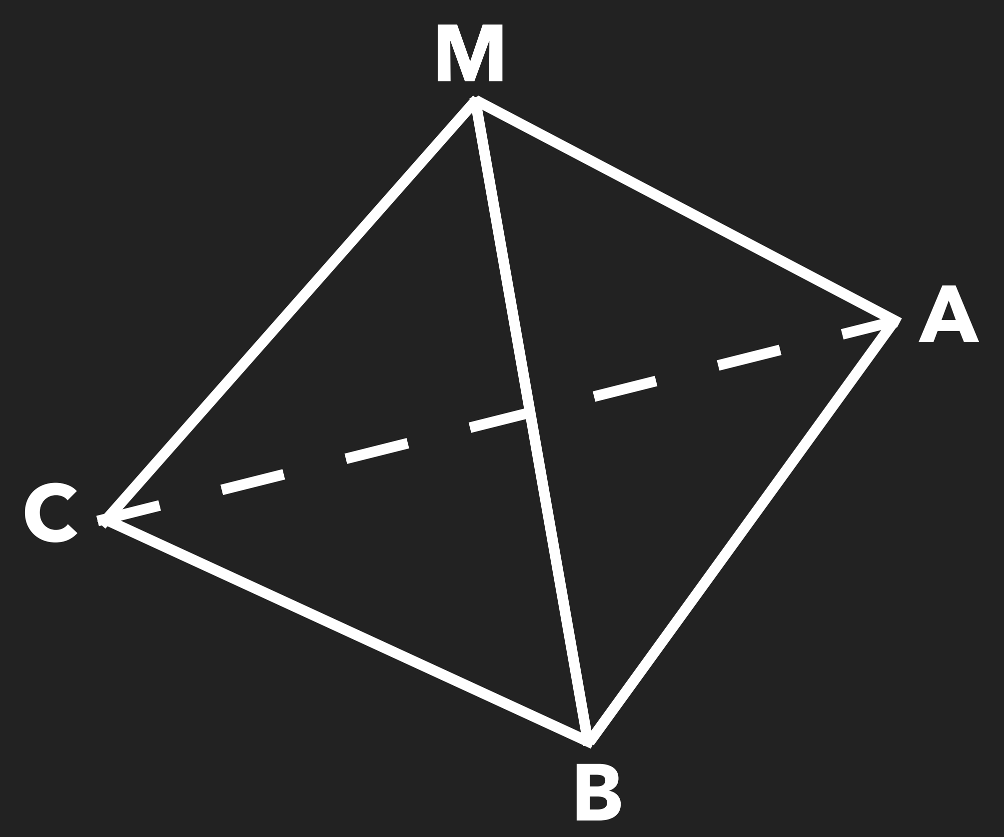
Q18.) The triangle piece of paper, ΔABC *with* $AB = 28cm$, $BC = 21cm$ *and* $AC = 35cm$



Let M lying on AC with $\angle BMC = 75^0$

a.) Find $\angle BCM$ *and CM*

Then, this paper is folded along BM and place the paper on a horizonal plane with AB BC lying on the plan. Given that $\angle AMC = 107^0$.



b.) (i) Find the distance between A and C on the plane

(ii) Let N lying on BC and $MN \perp BC$. *Is* $\angle MNA$ *the angle between plane ABC and the plane MBC? Explain your answer.*

* 參考課程 3.3, 及 3.10

CONT'D

2012 PAPER 1 - SECTION B

$a.)\ In\ \Delta ABC,\ using\ cosine\ law:$

$$\cos\angle BCM = \frac{21^2 + 35^2 - 28^2}{2(21)(35)}$$

$\rightarrow \angle BCM = 53.1^0\ (to\ 3\ sig.fig.)$

$In\ \Delta BMC,\ using\ sine\ law:$

$$\frac{CM}{\sin(180^0 - 75^0 - \angle BCM)} = \frac{21}{\sin 75^0}\ (\angle s\ sum\ of\ \Delta)$$

$\rightarrow CM = 17.1cm\ (to\ 3\ sig.fig.)$

$bi.)\ In\ \Delta AMC,\ using\ cosine\ law:$

$$AC^2 = CM^2 + AM^2 - 2(CM)(AM)\cos\angle AMC$$
$$= CM^2 + (35 - CM)^2 - 2(CM)(35 - CM)\cos 107^0$$

$\rightarrow AC = 28.1cm\ (to\ 3\ sig.fig.)$

* 三條邊搵角用 cosine law

* 三角形內角和 = 180⁰

* 三條邊搵角用 cosine law

CONT'D

2013 PAPER 1 - SECTION B

$bii.)\ In\ \Delta MNC,\ CN = CM\cos\angle BCM$

$Consider,\ AC^2 + CN^2\ and\ AB^2 + BN^2$

$AC^2 + CN^2 = (28.1)^2 + (17.1\cos(53.1^0))^2 = 895.025$

$AB^2 + BN^2 = 28^2 + (21 - 17.1\cos(53.1^0))^2 = 899.193$

$\because AC^2 + CN^2 \neq AB^2 + BN^2$

$\therefore \angle ANC \neq 90^0\ (converse\ of\ pyth.\ theorem)$

$i.e.\ \angle MNA \neq the\ angle\ between\ plane\ ABC\ and\ MBC$

* 利用畢氏逆定理証明直角三角形

* 如果係平面夾角, AN 同 BC 須互相垂直

2013 PAPER 1 - SECTION B

Q19.) The spread of virus in town has been studied. It is given that the total infected area at the end of 1^{st} year is $9x10^6 m^2$. The spread increases at constant rate $r\%$ of the infected area at the end of the year for the next year. For each end year, there is $3x10^5 m^2$ infected area drop. Given that the infected area is $1.026x10^7 m^2$ at the end of 3^{rd} year.

a.) Express, in term of n, the infected area at the end of n^{th} year

b.) When will the infected area exceed $4x10^7 m^2$?

c.) There is vaccine that the total cover area at the end of n^{th} year, $A(n) = (a(1.21)^n + b)m^2$. Given that $A(1) = 1x10^7 m^2$, and $A(2) = 1.063x10^7 m^2$. Can the virus infected area be greater than the vaccine cover area in a certain end of year? Explain your answer.

* 參考課程 1.2, 2.6 及 2.7

CONT'D

2012 PAPER 1 - SECTION B

$a.)$ *Let* $T(n)$ *be the infected area* $(10^6 m^2)$ *in the end of* n^{th} *year*

$$\begin{cases} T(1) = 9 & \text{———— (1)} \\ T(3) = 10.26 & \text{———— (2)} \end{cases}$$

$$where\ T(n) = T(n-1)R - 0.3,\ (R = 1 + r\%)$$

$$\therefore T(3) = T(2)R - 0.3 = (T(1)R - 0.3)R - 0.3$$

$$\rightarrow 10.26 = (9R - 0.3)R - 0.3 = 9R^2 - 0.3R - 0.3$$

$$\rightarrow 9R^2 - 0.3R - 10.56 = 0$$

$$\rightarrow R = \frac{0.3 \pm \sqrt{0.3^2 + 4(9)(10.56)}}{2(9)}$$

$$\rightarrow R = 1.1\ or\ -1.07\ (rejected)$$

$$\therefore T(n) = 1.1T(n-1) - 0.3$$

* 將 10⁶ 納入 Let 範圍, 簡化往後表達

* 二次方程尋根公式

* 增長須為正數

CONT'D

2013 PAPER 1 - SECTION B

$= 1.1(1.1T(n-2)-0.3)-0.3 = 1.1^2T(n-2)-0.3(1+1.1)$

$= 1.1^2(1.1T(n-3)-0.3)-0.3(1+1.1)$

$= 1.1^3T(n-3)-0.3(1+1.1+1.1^2)$

$= 1.1^{n-1}T(1)-0.3(1+1.1+1.1^2+...+1.1^{n-2})$

$= 9(1.1)^{n-1}-0.3(\frac{1.1^{n-1}-1}{0.1}) = 6(1.1)^{n-1}+3$

The infected area at the end of n^{th} *year* $= [6(1.1)^{n-1}+3]x10^6m^2$

b.) Solve $T(n) > 40 \rightarrow 1.1^{n-1} > \frac{37}{6} \rightarrow n > log_{1.1}(\frac{37}{6})+1$

$\rightarrow n > 20.086$

$\therefore$ *At the end of* 21^{th} *year, the infected area* $> 4x10^7m^2$

* 搵 Pattern, 唔好計數字

* 3 對應的數字

* 等比數列之和

* log 函數為指數逆函數

CONT'D

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$c.)$ $\begin{cases} A(1) = 1x10^7 \text{ ——————— } (1) \\ A(2) = 1.063x10^7 \text{ ———— } (2) \end{cases}$

$(2) - (1): \ a(1.21^2 - 1.21) = 0.063x10^7$

$$a = \frac{30}{121}x10^7, \ b = 0.7x10^7$$

* 用消去法將 b 整走再代入 (1) 搵 b

$Consider, \ T(n) - A(n) > 0$

$$\rightarrow [6(1.1)^{n-1} + 3] - [\frac{300}{121}\boxed{(1.21)^n} + 7] > 0$$

$$\rightarrow [6(1.1)^{n-1} + 3] - [\frac{300\boxed{(1.21)}}{121}\boxed{(1.1^{n-1})^2} + 7] > 0$$

$$\rightarrow -3x^2 + 6x - 4 > 0, \ where \ x = (1.1)^{n-1}$$

* 將 10^6 抽走方便計算

* 指數乘係加, 另 $1.21=1.1^2$

CONT'D

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$c.)\quad \rightarrow -3(x-1)^2 - 1 > 0$

$\rightarrow$ *There is no solution for* x

$\therefore$ *There is no way infected area > vaccine cover area .*

* 由於沒有實根, 所以用頂點式睇吓最大值符唔符合不等式要求