深宵教室 - DSE 必修模擬試題解答

2012 PAPER 1

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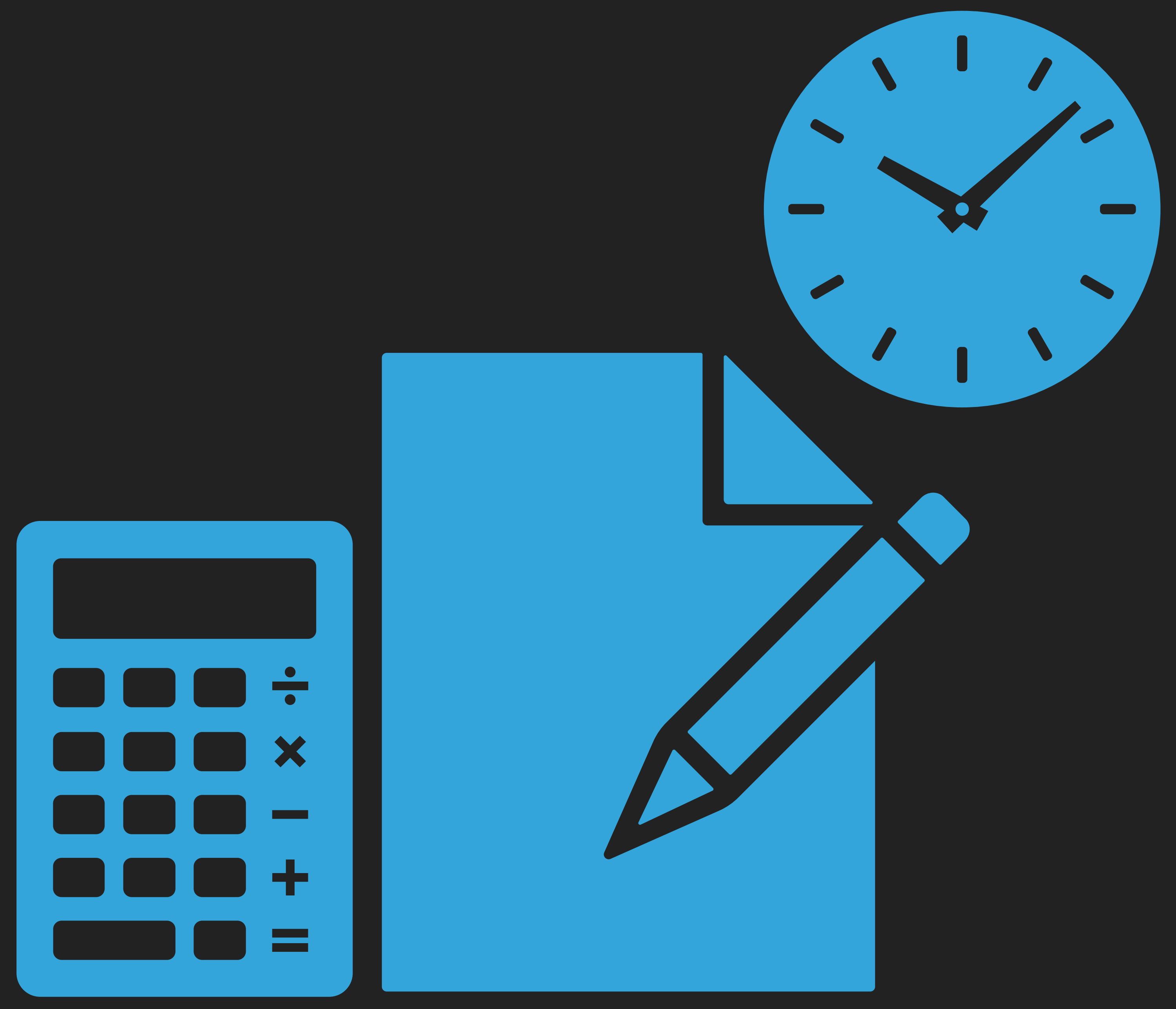
2012 PAPER 1

- Section A1
- Section A2
- Section B



2012 PAPER 1 - SECTION A1

Q1.) Simplified $\dfrac{m^{-12}n^{8}}{n^{3}}$, *in positive indices*

* 參考課程 1.2

$$= m^{\boxed{-12}} \cdot n^{\boxed{8-3}}$$

$$= \frac{n^5}{m^{12}}$$

* 指數乘係加，除係減
* 指數負數，分母變分子，分子變分母

2012 PAPER 1 - SECTION A1

$Q2.)\ \dfrac{3a+b}{8} = b-1,\ a = ?$

* 參考課程 2.1

$\rightarrow 3a + b = 8(b-1)$

$\rightarrow 3a + b = 8b - 8$

$\rightarrow a = \dfrac{7b-8}{3}$

* 兩邊乘 8

* 兩邊減 b 再除 3

2012 PAPER 1 - SECTION A1

Q3.) Factorize $x^2 - 6xy + 9y^2 + 7x - 21y$

* 參考課程 2.5

$$x^2 - 6xy + 9y^2 + 7x - 21y = (x - 3y)^2 - 7(x - 3y)$$

$$= (x - 3y)(x - 3y - 7)$$

* 恆等式 $(a - b)^2 \equiv a^2 - 2ab + b^2$
* 抽 -7

2012 PAPER 1 - SECTION A1

$Q4.)$ *The salary of Peter is 20% higher than that of Mary. The salary of Mary is 20% lower than that of Tom. The salary of Mary* $= \$480.$

a.) The salary of Peter = ?

b.) Who has the highest salary? Please explain.

* 參考課程 1.3

$a.)\ \text{The salary of Peter} = \$480(1+20\%)$

$= \$576$

* 新值 = 舊值 (1 + 百份比變化)

$b.)\ \text{Let the salary of Tom be } \$x$

$480 = x(1-20\%)$

$\rightarrow x = 600$

$\therefore$ *Tom has highest salary*

* 留意邊個係新值邊個係舊值

2012 PAPER 1 - SECTION A1

Q5.) The are 132 students in school consisting of 6 classes. Each class has same number of students. In each class, there are 4 more female than male. How many male students in school?

* 參考課程 1.3 及 2.3

Let x be the number of male students in school

Also, there are $132 \div 6 = 22$ *students in each class*

There are $\frac{x}{6}$ *male students in each class*

There are $\frac{x}{6} + 4$ *female students in each class*

$$\rightarrow \frac{x}{6} + \frac{x}{6} + 4 = 22 \rightarrow \frac{x}{3} = 18 \rightarrow x = 54$$

i.e. There are 54 male students in school.

* 先假設 x 為男生數量

* 將題目化做方程式

2012 PAPER 1 - SECTION A1

Q6.) Solve $\frac{4x+6}{7} > 2(x-3)$ *and* $2x - 10 \leq 0$

Hence, find out how many postive integers satisfy both inequalities .

* 參考課程 1.1 及 2.3

$$\frac{4x+6}{7} > 2(x-3) \text{ and } 2x - 10 \leq 0$$

$$\rightarrow 4x + 6 > 14x - 42 \text{ and } 2x \leq 10$$

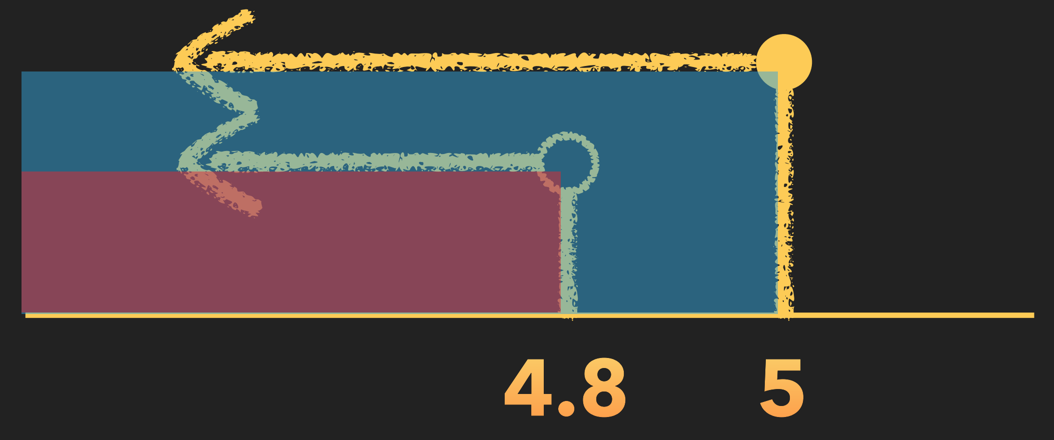
$$\rightarrow 4.8 > x \text{ and } x \leq 5$$

$$\rightarrow x < 4.8$$

i . e . The fulfilled postive integers are 1, 2, 3, 4

There are 4 *positive integers satisfy the inequalities .*

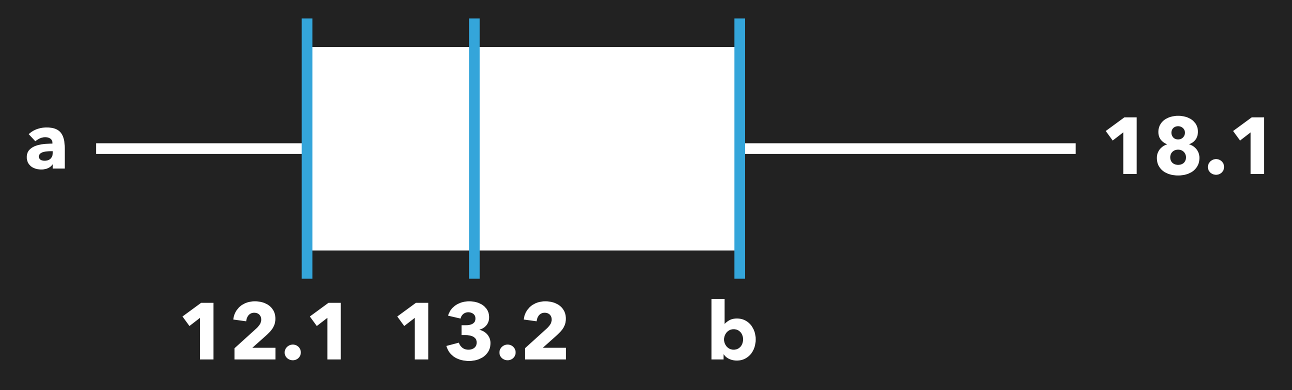
* And 指重疊地方



* 0 不是正整數

2012 PAPER 1 - SECTION A1

Q7.) The following box – and – whisker shows the distribution of times to finish 100m race by a group of students. Interquartile range = 3.2s, range = 6.8s



a.) Find a and b

b.) After training, the longest time taken by the student race less then 2.9s before the training. There is at least 25 % students show improvement after the training. Do you agree?

* 參考課程 4.2

a.) $a = 18.1 - 6.8,\ b = 12.1 + 3.2$

$a = 11.3,\ b = 15.3$

b.) *The possible longest time taken after training*

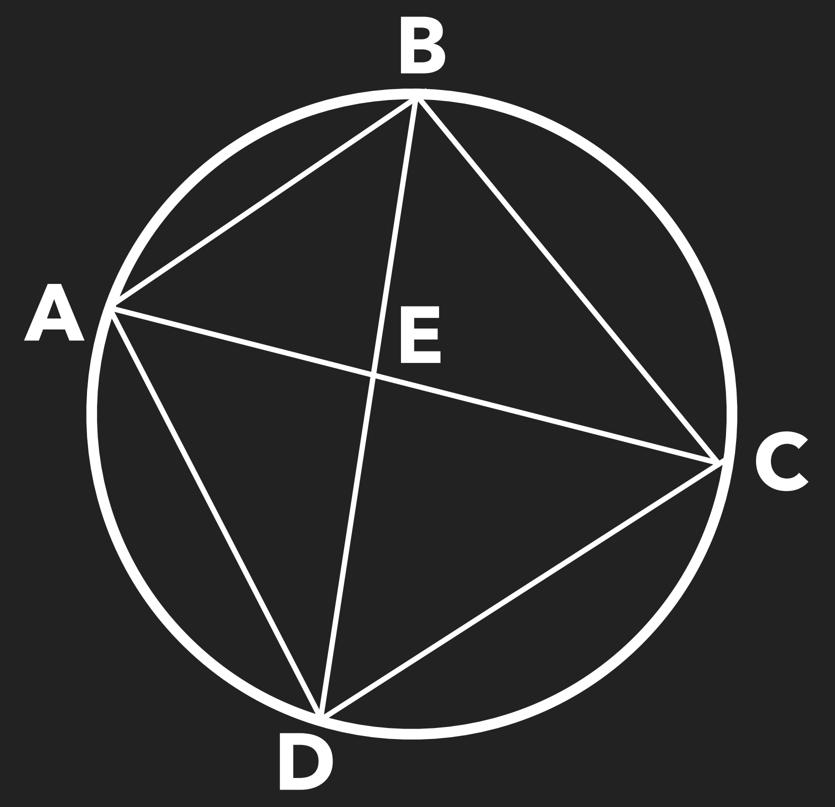
$= 18.1 - 2.9 = 15.2s < 15.3s$ (*The upper quartile*)

$\therefore$ *the statement is correct*

* Interquatile Range = 第三及一四分位數之差
* Range = 最大最細值之差

2012 PAPER 1 - SECTION A1

Q8.) In the figure, $BE = 8cm$, $CE = 20cm$, $DE = 15cm$ and $AB = 10cm$



a.) Find AE

b.) Are $AC \perp BD$? Explain your answer.

* 參考課程 3.1, 3.3 及 3.6

a.) $\angle AED = \angle BEC$ (vert. opp. $\angle$s)

* 對角相等

$\angle EAD = \angle EBC$ ($\angle$s in the same segment)

$\angle EDA = \angle ECB$ ($\angle$s in the same segment)

$\therefore \Delta AED \sim \Delta BEC$ (AAA)

* 三角相等，相似三角形

$\rightarrow \dfrac{AE}{BE} = \dfrac{DE}{CE} = \dfrac{15}{20} \rightarrow AE = 6cm$

* 相似三角形，邊比一樣

CONT'D

2012 PAPER 1 - SECTION A1

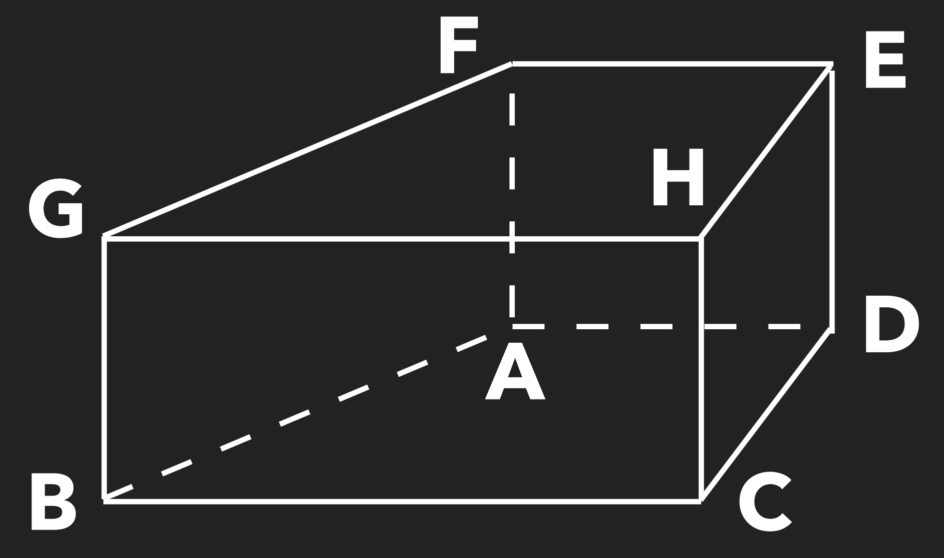
$$b.)\ In\ \Delta ABE,\ Consider\ BE^2 + AE^2 = 6^2 + 8^2$$
$$= 100$$
$$= AB^2$$

$$\therefore\ \angle AEB = 90^0 \rightarrow AC \perp BD\ (Coverse\ of\ pyth.\ theorem)$$

* 畢氏定理逆定理

2012 PAPER 1 - SECTION A1

Q9.) In the figure, $AD//BC$, $\angle BAD = 90^0$, $AB = 12cm$, $BC = 6cm$, $DE = 10cm$ *and the volume of* $ABCDEFGH = 1020cm^3$



a.) Find AD

b.) Find the total surface area of ABCDEFGH

* 參考課程 3.9

$$a.)\ \text{The volume} = \frac{(AD+BC)xAB}{2}xDE = 1020 \rightarrow AD = \frac{2(1020)}{12 \cdot 10} - 6$$

$$= 11cm$$

* 柱體體積 = 底面積 x 高

$$b.)\ CD = \sqrt{AB^2 + (AD - BC)^2} = \sqrt{12^2 + 5^2} = 13\ (pyth.\ theorem)$$

$$\text{The total surface area} = \frac{2(AD+BC)xAB}{2} + (AB + AD + BC + DC)xDE$$

$$= 624cm^2$$

* 表面面積 = 眼見面積

2012 PAPER 1 – SECTION A2

Q10.) The stem – and – leaf diagram below shows the number of hours on game in a week by students .

Stem (tens)	Leaf (units)
1	0 0 1 1 2 3 4 5 5 6 6 7 7
2	0 0 0 5 8
3	4 6

a.) Find the mean and median of the about record .

b.) 4 more results received and the mean of 4 results is 18. It found that two of them are 19 and 20

i.) What is the updated mean?

ii.) Is it possible for the updated median to be unchanged? Explain your answer .

* 參考課程 4.1 及 4.2

CONT'D

2012 PAPER 1 - SECTION A2

a.) *The mean* $= 18$, *The median* $= 16$

bi.) $$\text{The new mean} = \frac{20 \cdot 18 + 4 \cdot 18}{24} = 18$$

ii.) *Let the 4 records be a, b, 19 and 20*

Assume the updated median unchanges $= 16$

$$a, b \leq 16 \ (\because 19, 20 > 16)$$

$$\text{Also, } \frac{a + b + 19 + 20}{4} = 18 \rightarrow a + b = 33$$

There is no way $a + b = 33$, *while* $a, b < 16$

$\therefore$ *it is impossible for the median to be unchanged .*

* 平均值 = 加總 / 總數量
* 中位數 = 中間的數值

* 加總 = 平均值 x 總數量

* 要中位數一樣，兩個要細過等如 16
  兩個要大過等如 16

* 16+16 都係得 32

2012 PAPER 1 – SECTION A2

Q11.) Let \$C be the cost to paint Am^2 surface area of can, where C is sum of 2 parts. One is constant. Another is partly varied with A. Given that $A = 2$, $C = 62$ and $A = 6$, $C = 74$

a.) Find the cost to paint $13m^2$ surface area of can.

b.) There is a larger can similar to the above can. If the volume of the larger can is 8 times greater than that of above can. How much to paint the larger can?

* 參考課程 2.3, 2.4 及 3.9

a.) Let $C = k_1 + k_2 A$, where k_1, k_2 are real constant. Then,

$$\begin{cases} 62 = k_1 + 2k_2 & \text{———— (1)} \\ 74 = k_1 + 6k_2 & \text{———— (2)} \end{cases}$$

$(2) - (1): \quad 12 = 4k_2 \rightarrow k_2 = 3, k_1 = 56$

$\therefore C = 54 + 3A$

i.e. The cost to paint $13m^2$ surface area of can $= 54 + 3(13)$

$= \$95$

* 部分變量

* 消去法消去 k_1 搵 k_2，再代 (1) 式搵 k_1

CONT'D

2012 PAPER 1 - SECTION A2

b.) Let $A_2 m^2$ be the surface area of the larger can .

$$(\frac{A_2}{13})^{\frac{1}{2}} = 8^{\frac{1}{3}} \rightarrow A_2 = 4(13) = 52$$

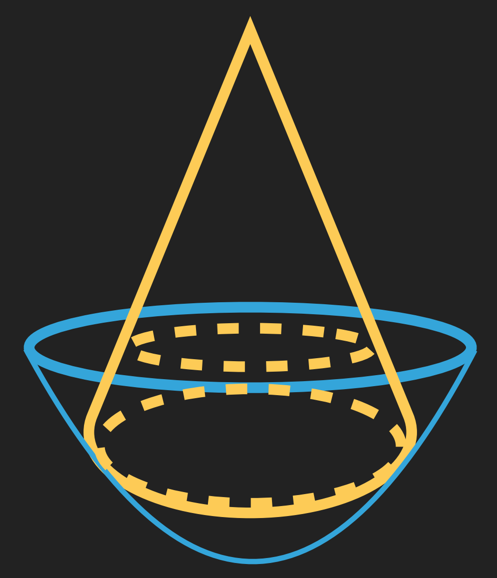
$$i.e.\ The\ cost\ to\ paint\ larger\ can = 54 + 3(A_2)$$

$$= \$212$$

* 相似圖形, 體積比 = (邊比)³
* 相似圖形, 面積比 = (邊比)²

2012 PAPER 1 - SECTION A2

Q12.) There is hemispherical vessel with radius = 60cm .
This vessel is fully filled with water, and a circular cone with radius = 48cm, and height = 96cm is held vertically as shown in below . Is the remaining water insider the vessel greater than $0.3m^3$? Explain your answer .



* 參考課程 3.2 及 3.9

a.) Let V_1 m^3 be the volume of the circular cone
V_2 m^3 be the volume of the hemispherical vessel

$$V_1 = \frac{1}{3}\pi(0.48)^2(0.96) = 0.0737\pi m^3$$

$$V_2 = \frac{1}{2}\frac{4}{3}\pi(0.6)^3 = 0.144\pi m^3$$

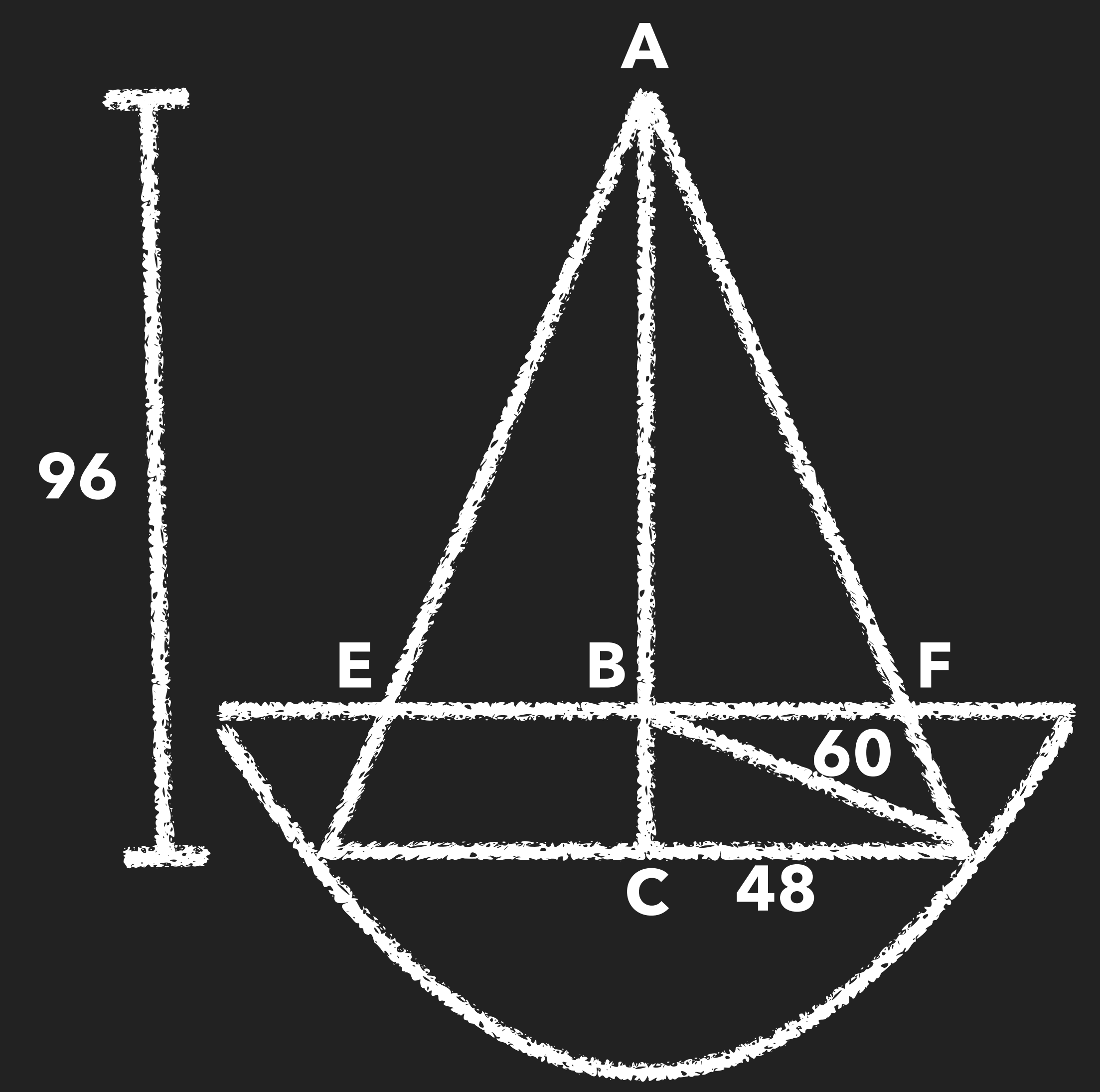
* 錐體體積 = 1\3 x 底面積 x 高

* 球體體積 = 4/3 x π x (半徑)³

CONT'D

2012 PAPER 1 – SECTION A2

a.) Consider the graph below :



$BC = \sqrt{60^2 - 48^2} = 36$

$AB = 96 - 36 = 60$

Let $V_3\ m^3$ *be the volume of the cone AEF*

$$\frac{V_3}{V_1} = \left(\frac{60}{96}\right)^3 \rightarrow V_3 = 0.01799\pi m^3$$

$\therefore$ *The volume of the remaining water* $= V_2 - (V_1 - V_3)$

$= 0.08829\pi\ m^3 < 0.3m^3$

i.e. The remaining water insider the vessel is less than $0.3m^3$

* 畢氏定理

* 相似圖形, 體積比 = (邊比)³

2012 PAPER 1 - SECTION A2

Q13.) a.) *Find the value of k if* $(x-2)$ *is the factor of* $kx^3-21x^2+24x-4$.

b.) *Assume, Q is a varible point moving along the curve* $C:\ y=15x^2-63x+72$ *in first quadrant. Let* $Q=(m,n)$. *Are there three different position of Q such that* $mn=12$? *Explain your answer.*

* 參考課程 2.1, 2.4 及 2.5

a.) Let $f(x)=kx^3-21x^2+24x-4$, such that $f(2)=0$

$\rightarrow k=5$

b.) $mn=12 \rightarrow m(15m^2-63m+72)=12$

$\rightarrow 5m^3-21m^2+24m-4=0$

$\rightarrow (x-2)(Ax^2+Bx+C)=0$

By compare coefficient, $A=5,\ B=-11,\ C=2$

$mn \equiv (m-2)(5m^2-11m+2) \equiv (m-2)^2(5m-1)$

i.e. $mn=12 \rightarrow (m-2)^2(5m-1)=0$ gives two solutions.

Hence, There are no three different position of Q

* 餘數定理

* Q 點响條 Curve 上，(m, n) 乎合 C 條公式

* 從 a.) 得知 f(m)=(m-2)(Am²+Bm+C)

* 用十字相乘方法

2012 PAPER 1 - SECTION A2

$Q14.)$ *The y − intercept of 2 parallel lines (l_1 and l_2) are −1 and −3. The x − intercept of l_1 is 3. P is a moving point such that the perpendicular distance to l_1 and to l_2 is equal. Denote the locus of P be Γ.*

a.) Find the equation of Γ

b.) Let $C: (x-6)^2 + y^2 = 4$ be a circle. Does Γ pass through the center of C? If l_1 cuts C at A and B, while Γ cuts C at H and K. Find Area of ΔAQH : Area of ΔBQK. where Q = the center of C.

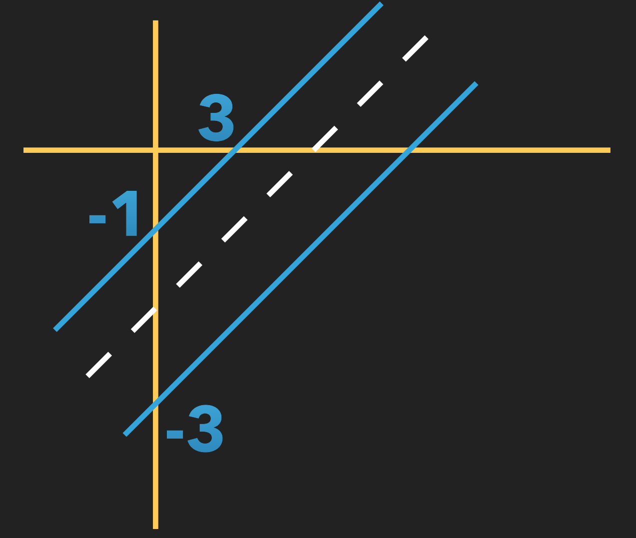
* 參考課程 3.6, 3.7 及 3.8

$$a.)\ \text{The slope } \Gamma = \text{The slope of } l_1 = \frac{0+1}{3-0} = \frac{1}{3}$$

$$\text{The } y-\text{intercept } \Gamma = \frac{-1-3}{2} = -2$$

$$\therefore\ \Gamma : y = \frac{x}{3} - 2$$

* 直線介乎响 l_1 l_2 中間



CONT'D

2012 PAPER 1 - SECTION A2

$b.)$ *The center of* C, $Q = (6, 0)$

$$0 = \frac{6}{3} - 2 \rightarrow \Gamma \textit{ passes through } Q$$

Hence $HQ = KQ =$ *radius of* C

Also, The height of $\Delta AQH =$ *The height of* ΔBQK $(l_1 // \Gamma)$

i.e. Area of ΔAQH : *Area of* $\Delta BQK = 1 : 1$

* 兩個三角形有共高，面積比 = 邊比

2012 PAPER 1 - SECTION B

Q15.) The standard deviation of a test result is 10 marks. Then all test scores are increased by 20 % and add extra 5 mark.

a.) Find the updated standard deviation.

b.) Is there any change in each standard score? Explain your answer.

* 參考課程 1.3 及 4.2

a.) $\text{The updated standard deviation} = 10(1+20\%)$

$$= 12$$

b.) Let x be the score, μ be the mean,

z be the standard score of x

$$\text{The updated standard score} = \boxed{\frac{(1.2x+5)-(1.2\mu+5)}{12}}$$

$$= \frac{x-\mu}{10} = z$$

i.e. There is no change in each standard score.

* 額外加減不會影響離散分佈

* 標準分數 = 數據與平均值相差多少個標準差

2012 PAPER 1 - SECTION B

Q16.) There is a 16 members task group formed by selected two representative form each each department (Total 8 departments). Then, 4 members are randomly selected.

a.) Find the probability that 4 selected members are from different departments.

b.) Find the probability that 4 selected members at most from 3 different departments.

* 參考課程 4.3 及 4.4

$$a.)\ P(4\ members\ from\ 4\ different\ departments) = \frac{\boxed{C_4^8(C_1^2)^4}}{C_4^{16}}$$

$$= \frac{8}{13}$$

* (8 個部門抽 4 個) x 4(每個部門 2 個組合)

$$b.)\ P(4\ members\ at\ most\ from\ 3\ different\ departments)$$

$$= 1 - \frac{8}{13} = \frac{5}{13}$$

* P(4個最多3個唔同) = 1 - P(全部都唔同)

2012 PAPER 1 - SECTION B

Q17.) There is a circle, C with center = (6,10) and x − axis is the tangent of C.

a.) Find the equation of C

b.) There is a line with slope = − 1, y − intercept = k, cuts C at A and B.

Find mid − points of AB in terms of k.

* 參考課程 2.6, 3.7 及 3.8

$$a.)\ C : (x-6)^2 + (y-10)^2 = 10^2$$

$$\rightarrow (x-6)^2 + (y-10)^2 = 100$$

$$b.)\ \text{Let the line, } L : y = -x + k,\ A = (x_1, y_1),\ B = (x_2, y_2)$$

To find x_1, x_2, *sub L into C :*

$$(x-6)^2 + (x-11+k)^2 = 100$$

$$\rightarrow 2x^2 - 2(k-4)x + [(k-10)^2 - 64] = 0$$

$$\text{The mid-pt. of } AB = \left(\frac{x_1+x_2}{2},\ -\frac{x_1+x_2}{2} + k\right) = \left(\frac{k-4}{2},\ \frac{k+4}{2}\right)$$

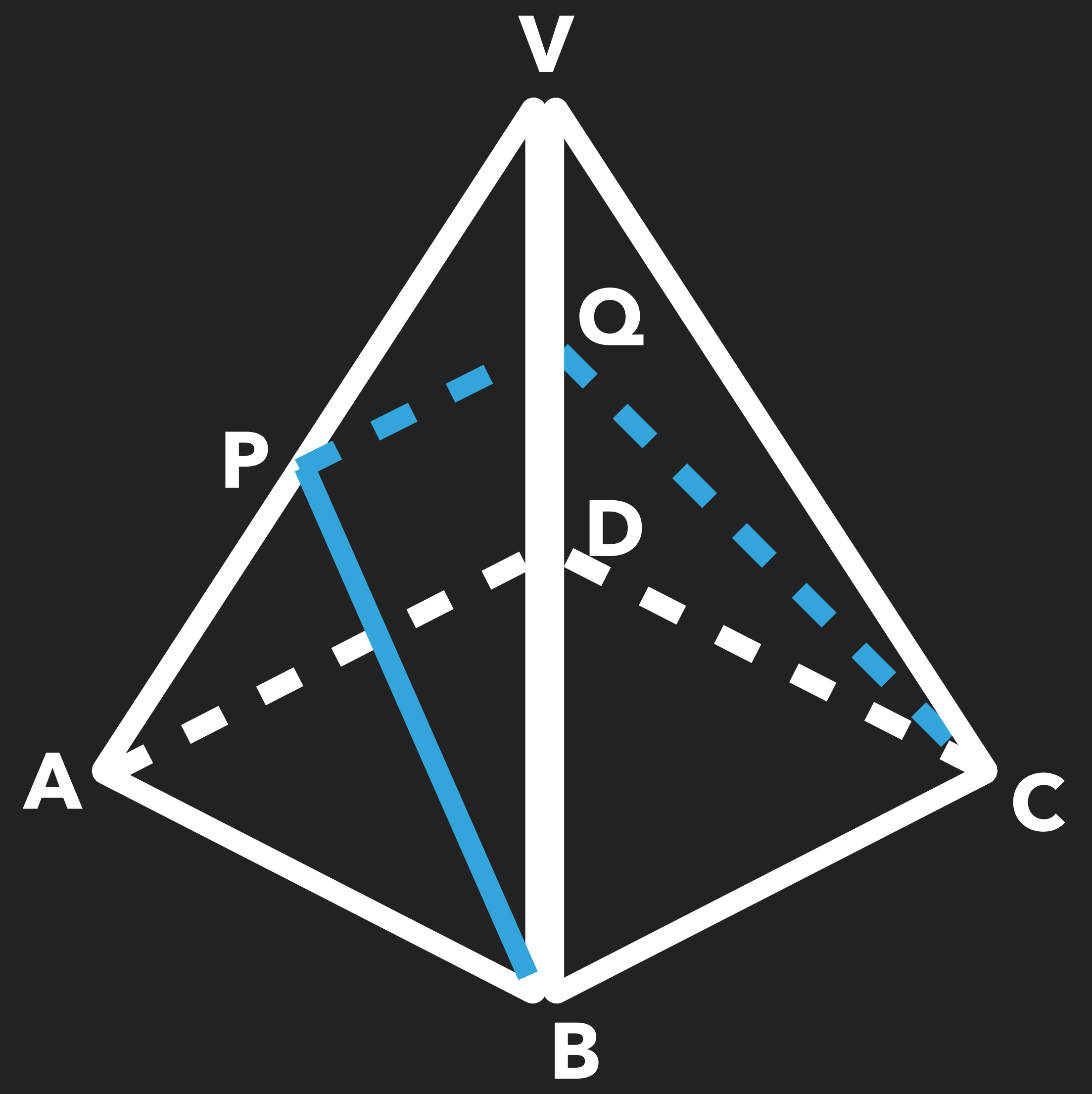
* x-axis 係切線，中心點個 y 就係半徑

* 直線方程: y = mx+c

* 根之和

2012 PAPER 1 - SECTION B

Q18.) *The following shows a square base (length = 20cm) right pyramid VABCD. Given that* $\angle VAB = 72^0$. *Let P and Q be point on VA and VD respectively such that PQ//BC and* $\angle PBA = 60^0$.



a.) *Find AP*

b.) *Let* α *be the angle between the plane PBCQ and the plane ABCD*

β *be the angle between the line PB and the plane ABCD.*

Is $\alpha > \beta$? *Explain your answer.*

* 參考課程 3.3, 及 3.10

CONT'D

2012 PAPER 1 – SECTION B

$a.)$ $In\ \Delta ABP,\ using\ sine\ law:$

$$\frac{AP}{\sin 60^0} = \frac{20}{\sin(180^0 - 60^0 - 72^0)} \rightarrow AP = \frac{20\sin 60^0}{\sin 48^0}$$

$$\rightarrow AP = 23.3cm\ (to\ 3\ sig\ fig)$$

$b.)$ $Let\ M\ be\ the\ pt.\ on\ AD\ such\ that\ PM \perp AD$

$N\ be\ the\ pt.\ on\ BC\ such\ that\ PN \perp BC$

$$AM = BN = AP\cos 72^0,\quad PM = AP\sin 72^0,\quad MN = 20$$

$$MB^2 = AM^2 + 20^2\ (pyth.\ theorem)$$

$$PB = \frac{20\sin 72^0}{\sin 48^0}\ (By\ sine\ law)$$

$$PN^2 = PB^2 - BN^2\ (pyth.\ theorem)$$

* 三角形內角和 = 180⁰

* 畢氏定理

CONT'D

2012 PAPER 1 - SECTION B

$$\boxed{\cos\alpha = \frac{PN^2 + MN^2 - PM^2}{2PN \cdot MN}} \quad (By\ cosine\ law)$$

$$\rightarrow \quad \cos\alpha = 0.521 \rightarrow \alpha = 58.6^0$$

$$\boxed{\cos\beta = \frac{PB^2 + MB^2 - PM^2}{2PB \cdot MB}} \quad (By\ cosine\ law)$$

$$\rightarrow \quad \cos\beta = 0.566 \rightarrow \beta = 55.5^0$$

$Hence,\ \beta < \alpha$

* 三邊用 cosine law 搵角

* 代入所有然後先一次用計算機

* 代入所有然後先一次用計算機

2012 PAPER 1 – SECTION B

$Q19.)$ Let $A(n)$ be the tonnes of goods in the nth year handled by factory X. $A(n) = ab^{2n}$ Given that X handle $254{,}100$ tonnes of good in 1st year, and $307{,}461$ tonnes of good in the 2nd year. a and b are positive real constant.

a.) Find a, b and the sum of all tonnes of goods after nth year

b.) After 4th years operation of X, factory Y starts to operate. Let $B(m) = 2ab^{m}$ be the tonnes of goods handled by Y in mth year.

i.) Is the tonnes of goods handled by Y is less than that handled by X in each year? Explain your answer.

ii.) When will the total tonnes of goods handled by X and Y exceed $20{,}000{,}000$ since the operation of X?

* 參考課程 2.6 及 2.7

CONT'D

2012 PAPER 1 - SECTION B

$a.)$ *Given that :*

$$\begin{cases} ab^2 = 254{,}100 \text{ ———————— } (1) \\ ab^4 = 307{,}461 \text{ ———————— } (2) \end{cases}$$

$(2) \div (1) : b^2 = 1.21 \rightarrow b = 1.1$

$Put\ b = 1.1\ into\ (1) : a = 210{,}000$

$\therefore (a = 210{,}000,\ b = 1.1)$

$$Let\ S_1(n) = A(1) + A(2) + \ldots + A(n)$$

$$= ab^2(1 + b^2 + \ldots + b^{2n-2}) = \frac{ab^2(1 - b^{2n})}{1 - b^2}$$

$$= 1{,}210{,}000(1.21^n - 1)$$

$bi.)$ *Solve* $B(m) < A(m + 4)$

* 用兩式相除整走 a

* 等比數列之和

CONT'D

2012 PAPER 1 - SECTION B

$$\rightarrow 2ab^m < ab^{2(m+4)} \rightarrow ab^m(2 - b^{m+8}) < 0$$

$$\rightarrow 2 < (1.1)^{m+8} \rightarrow m > \boxed{\frac{log2}{log1.1}} - 8 \approx -0.73$$

$\therefore$ *The tonnes of goods by Y is less than that handled by X in each year*

$$bii.)\ Let\ S_2(m) = B(1) + B(2) + \ldots + B(m)$$

$$= 2ab\boxed{(1 + b + \ldots + b^{m-1})} = \frac{2ab\boxed{(1 - b^m)}}{\boxed{1 - b}}$$

$$Solve\ S_1(n) + S_2(n-4) > 20{,}000{,}000\ and\ n > 0$$

$$\rightarrow 1{,}210{,}000(1.1^n)^2 + 3{,}155{,}522.16(1.1^n) - 25{,}830{,}000 > 0\ and\ n > 0$$

$$\rightarrow 1.1^n > 3.4968 \rightarrow n > \frac{log3.4968}{log1.1} \approx 13.13$$

$\therefore$ *The total tonnes of goods > 20,000,000 in 14th year*

* 指數用 log
* 等比數列之和