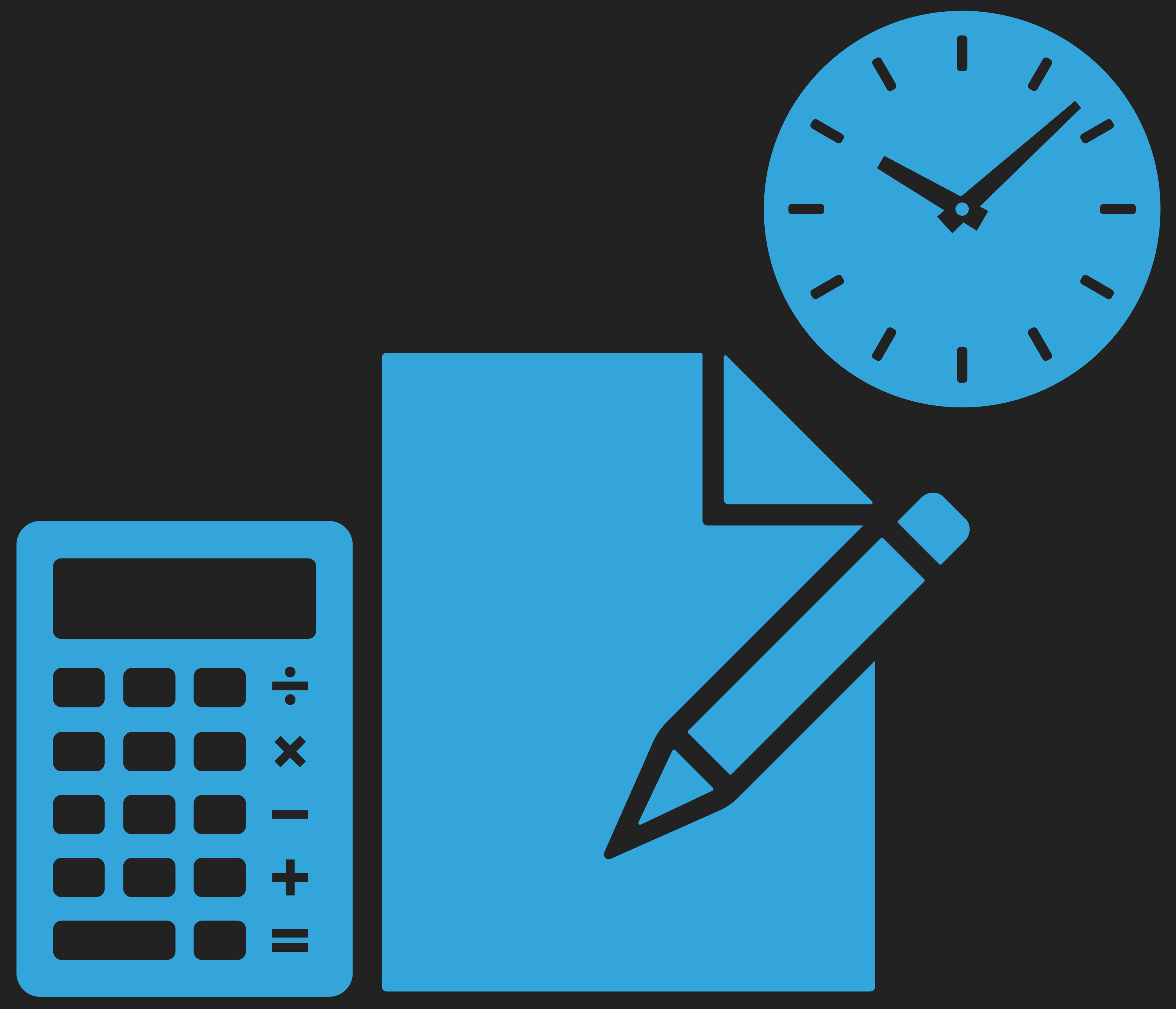
深宵教室 - DSE M1 模擬試題解答

2020

此為參考2020試題之模擬試題，原版請另行購買

2020

- Section A
- Section B



2020 - SECTION A

Q1.) Let X be discrete random vaiable, where a and p are constant, $p > 0$

k	0	1	2
$P(X=k)$	$1-4p$	ap	p

Given that $Var(2X+a^2) = 8E(aX-1)$, *find a and p.*

* 參考課程 4.1, 4.3 及 4.4

$1 - 4p + ap + p = 1 \rightarrow p(a-3) = 0 \rightarrow a = 3$

$E(X) = 0 \cdot (1-4p) + 1 \cdot ap + 2 \cdot p = p(a+2) = 5p$

$E(X^2) = 0^2 \cdot (1-4p) + 1^2 \cdot ap + 2^2 \cdot p = p(a+4) = 7p$

$Var(X) = E(X^2) - [E(X)]^2 = 7p - 25p^2$

$Given\ that\ Var(2X+9) = 8E(3X-1) \rightarrow 4Var(X) = 8[3E(X)-1]$

$\rightarrow 4[7p - 25p^2] = 8[3 \cdot 5p - 1] \rightarrow 25p^2 + 23p - 2 = 0$

$\rightarrow p = 0.08\ or\ -1\ (rejected)$

* 機率總加 = 1

* $E(X) = \sum_{i=1}^{n} k_i P(X = k_i)$

* $Var(X) = E(X^2) - [E(X)]^2$

* $E(aX+b) = aE(X) + b$

* $Var(aX+b) = a^2 Var(X)$

2020 - SECTION A

Q2.) Given that the probability of there is a dirt on a page of a photocopy = 0.2. Considering a document with 6 pages to be photocopied.

a.) Find the probability there is a dirt on the document.

b.) If there is fewer than 3 pages have dirt in the copy, the copy will be accepted.

i.) Find the probability the copy is acceptable.

ii.) Find the expected number of accepted copy between the first unaccepted copy to the next unaccepeted copy if the first copy is unacceptable.

* 參考課程 4.4

a.) Let X be the random variable of the number of pages have dirt

$\rightarrow X \sim B(6, 0.2)$

$P(X \geq 1) = 1 - P(X = 0) = 1 - [1 - 0.2]^6 = 0.737856$

bi.) Let B be the event of $X < 3$

$P(B) = P(X = 0) + P(X = 1) + P(X = 2)$

$= (1 - 0.2)^6 + C_1^6(0.2)(1 - 0.2)^5 + C_2^6(0.2)^2(1 - 0.2)^4$

$= 0.90112$

bii.) The expected number $= \dfrac{1}{1 - 0.90112} - 1 = 9.1133$ *(to 4 d.p.)*

* P(Not A) = 1 - P(A)

* $P(X = k) = C_k^n p^k (1 - p)^{n-k}$

* $X \sim G(p), E(X) = \dfrac{1}{p}$

2020 - SECTION A

Q3.) Let A and B be the event such that $P(B|A) = \frac{1}{2}$ *and* $P(B) = \frac{1}{3} + P(A)$ *where* X' *is the complementary event of X, Suppose* $P(A' \cap B) = kP(A)$*, where k is constant.*

a.) Prove $k \neq 0.5$

b.) Is A and B are mutually exclusive? Explain your answer.

c.) Is A and B are independent? Explain your answer.

* 參考課程 4.1 及 4.2

a.) $P(A' \cap B) = P(A'|B)P(B) = [1 - P(A|B)]P(B)$

$$= [1 - \frac{P(B|A)P(A)}{P(B)}]P(B)$$

$$\rightarrow kP(A) = \frac{1}{3} + P(A) - \frac{1}{2}P(A) \rightarrow (k - \frac{1}{2})P(A) = \frac{1}{3}$$

$\therefore k \neq 0.5$

b.) $P(A) = \dfrac{2}{3(2k-1)}$ *and* $P(B) = \dfrac{1}{3} + P(A) = \dfrac{2k+1}{3(2k-1)}$

* P(Not A) = 1 - P(A)

* P(A & B)=P(A|B)P(B)=P(B|A)P(A)

CONT'D

2020 - SECTION A

$$P(A \cap B) = \boxed{P(B \mid A)P(A)} = \frac{1}{3(2k-1)} \neq 0$$

$\therefore$ *A and B are not mutually exclusive*

c.) $$P(A)P(B) = \frac{2(2k+1)}{9(2k-1)^2}$$

If A and B are independent, then

$$\boxed{\frac{2(2k+1)}{9(2k-1)^2} = \frac{1}{3(2k-1)}} \rightarrow 2(2k+1) = 3(2k-1)$$

$$\rightarrow k = \frac{5}{2}$$

$\therefore$ *A and B are independent if* $k = \dfrac{5}{2}$

* **P(A & B)=P(A|B)P(B)=P(B|A)P(A)**
* **如果 mutually exclusive, P(A and B) = 0**
* **如果獨立事件, P(A & B)=P(A)P(B)**

2020 – SECTION A

$Q4.)$ *Peter wants to estimate the proportion p of the Youtuber in school. Given that there are 441 Youtubers in 841 randomly selected students in school.*

$a.)$ *Find the 95 % confidence interval for p.*

$b.)$ *The width of β % confidence interval for $p = 0.088$. Find β (to nearest integer)*

* 參考課程 4.6 及 4.7

$a.)$ *The sample proportion,* $p_s = \dfrac{441}{841}$

$$\text{The } 95\,\%\ C.I.\text{ for } p = \left(p_s - 1.96\sqrt{\frac{p_s(1-p_s)}{841}},\ p_s + 1.96\sqrt{\frac{p_s(1-p_s)}{841}}\right)$$

$$= (0.4906,\ 0.5581)\ (to\ 4\ d.p.)$$

$b.)$ *Let α be a value such that* $P(-\alpha < Z < \alpha) = \beta\,\%$, *then*

$$2\cdot\alpha\cdot\sqrt{\frac{p_s(1-p_s)}{841}} = 0.088 \rightarrow \alpha = 2.555 \rightarrow \beta = 99\ (to\ nearest\ integer)$$

* 95% 置信區間

* 對表搵答案

2020 – SECTION A

Q5.) Let $f(x) = (1+ke^x)^3$. If the constant term of $f(x) = 27$, find the coefficient of x^2.

* 參考課程 1.1 及 3.2

$$f(x) = 1 + 3ke^x + 3k^2e^{2x} + k^3e^{3x}$$

$$= (1 + 3k(1 + x + \frac{x^2}{2} + \ldots) + 3k^2(1 + 2x + \frac{(2x)^2}{2} + \ldots)$$

$$+k^3(1 + 3x + \frac{(3x)^2}{2} + \ldots))$$

Given that the constant term of $f(x) = 1 + 3k + 3k^2 + k^3 = 27$

$$\to (k+1)^3 = 3^3 \to k = 2$$

$\therefore$ The coefficient of $x^2 = 3k \cdot \frac{1}{2} + 3k^2 \cdot \frac{2^2}{2} + k^3 \cdot \frac{3^2}{2} = 63$

* $(a+b)^n = \sum_{r=0}^{n} C_r^n a^r b^{n-r}$

* $e^x = \sum_{r=0}^{\infty} \frac{x^r}{r!}$

* $C_r^n = \frac{n!}{r!(n-r)!}$

$\to C_1^n = n$ and $C_2^n = \frac{n(n-1)}{2}$

2020 – SECTION A

$Q6.)\ Let\ f(x) = x + 5x^{-1} + lnx^4, for\ x \neq 0.$

$a.)\ Is\ the\ min.\ value\ of\ f(x)\ >\ the\ max.\ value\ of\ f(x)?\ Explain\ your\ answer.$

$b.)\ Find\ 2\ horizontal\ tangents\ of\ the\ curve,\ C: y = f(x).$

* 參考課程 2.2, 2.4, 2.8 及 2.9

$a.)\ f'(x) = 1 - 5x^{-2} + 4x^{-1}$

$Let\ x_0\ be\ a\ non-zero\ real\ number\ such\ that\ f'(x_0) = 0$

$\rightarrow 1 - 5x_0^{-2} + 4x_0^{-1} = 0 \rightarrow x_0^2 + 4x_0 - 5 = 0$

$\rightarrow x_0 = -5\ or\ 1$

	x < -5	x = -5	-5 < x < 0	0 < x < 1	x = 1	x > 1
f′(x)	-	0	+	-	0	+
f(x)	Dec.		Inc.	Dec.		Inc.

$Hence, the\ max.\ valve = f(-5) = 4ln5 - 6$

$the\ min.\ valve = f(1) = 6$

$\therefore The\ min.\ value > the\ max.\ value$

* $lnx^n = nlnx$

* 搵 turning point = 搵 x_0 使度 $f'(x_0)=0$

* 利用表格計算 turning point 附近上升定下降

$f'(x) > 0 \rightarrow Increasing$

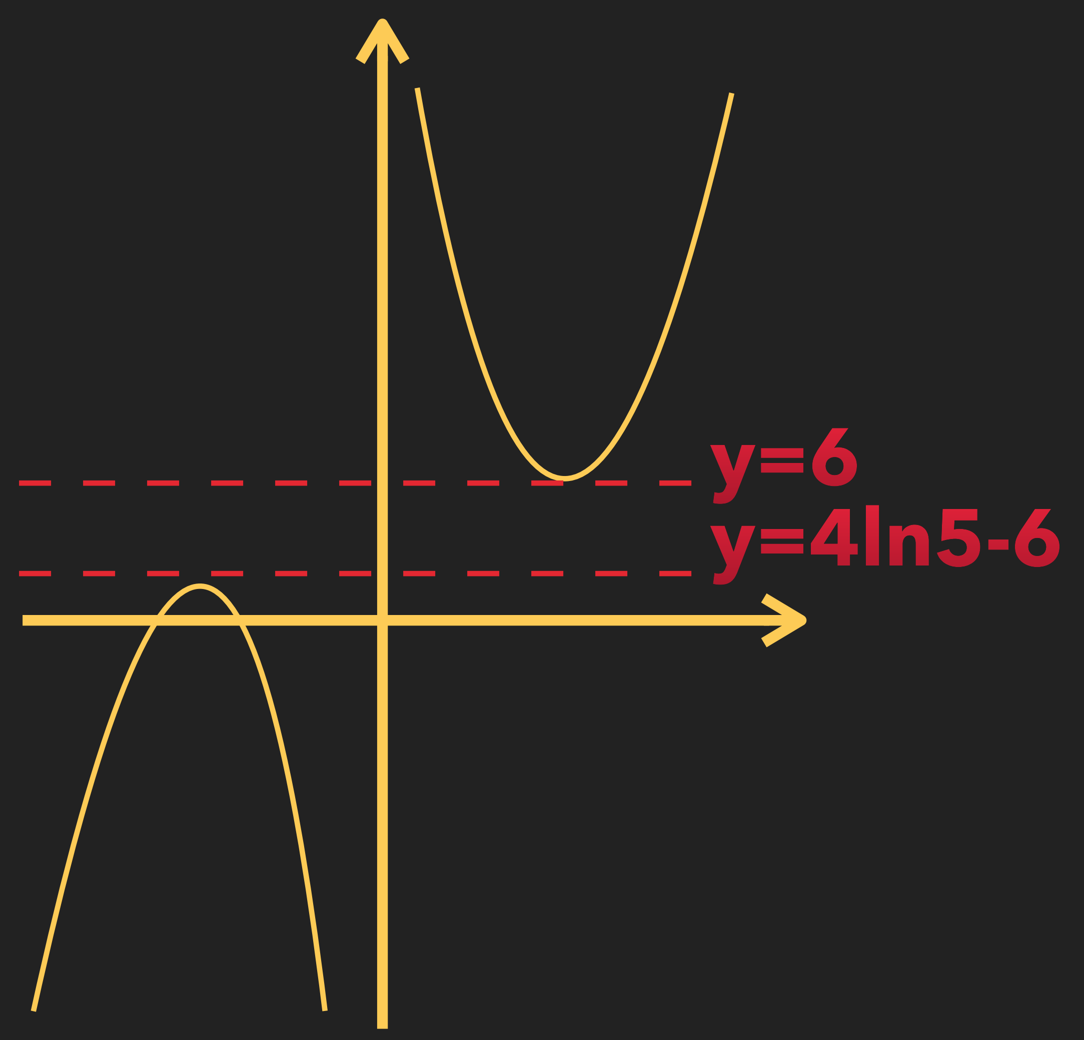
$f'(x) < 0 \rightarrow Decreasing$

CONT'D

2020 - SECTION A

$b.)$ *The horizontal tangents are,* $y = f(-5)$ *and* $y = f(1)$

$\rightarrow y = 4ln5 - 6$ *and* $y = 6$



2020 - SECTION A

Q7.) Let $V\text{cm}^3$ and $r\text{cm}$ be the volume and radius of a circular cylinder respectively. Given that the totla surface area of that cylinder $= 486\pi\text{cm}^2$. Find $\frac{dV}{dr}$ and justify if the volume exceed 5000cm^3.

* 參考課程 2.2 及 2.3

Let S be the total surface area of the cylinder in cm^2

h be the height of the cylinder in cm

$$S = 2\pi rh + 2\pi r^2 \text{ and } V = \pi r^2 h \rightarrow S = \frac{2V}{r} + 2\pi r^2$$

$$\rightarrow 243\pi = \frac{V}{r} + \pi r^2$$

$$\text{Hence, } 0 = \boxed{\frac{dV}{dr} \cdot \frac{1}{r} - \frac{V}{r^2}} + 2\pi r \rightarrow \frac{dV}{dr} = 243\pi - 3\pi r^2$$

$$= 3\pi(81 - r^2)$$

* 圓柱體表面面積=曲面面積+兩個圓面積

* 圓柱體體積=圓面積 x 高

* 用 Product rule

CONT'D

2020 – SECTION A

$$Obviously,\ when\ r = 9,\ \frac{dV}{dr} = 0,\ where\ r > 0$$

	0 < r < 9	r = 9	r > 9
V′(r)	+	0	-
V(r)	Inc.		Dec.

$$Hence,\ the\ max.\ value\ of\ V = 9[243\pi - \pi(9)^2]$$
$$\approx 4581 < 5000$$

$$\therefore\ The\ volume\ does\ not\ exceed\ 5000cm^3$$

* 搵 turning point = 搵 x_0 使度 $f'(x_0)=0$
* 利用表格計算 turning point 附近上升定下降

$f'(x) > 0 \rightarrow Increasing$

$f'(x) < 0 \rightarrow Decreasing$

2020 - SECTION A

$Q8.)\ Let\ the\ curve\ C: y = f(x), f(x) = xe^{mx},\ where\ m\ is\ non-zero\ constant$

$Find\ m\ if\ the\ area\ \ of\ the\ region\ bounded\ by\ C, x-axis,\ and\ x = 1\ is\ \frac{1}{m}$

* 參考課程 2.8 及 2.9

$To\ find\ the\ x-interception\ of\ C,\ consider\ f(x) = 0 \rightarrow x = 0$

$\therefore\ \ The\ x-interception\ is\ 0$

$Let\ h(x) = e^{mx} \rightarrow lnh(x) = mx \rightarrow h'(x) = mh(x)$

$Hence, \frac{1}{m} = \int_0^1 xh(x)dx = \int_0^1 xd[\frac{h(x)}{m}] \rightarrow 1 = [xh(x)]_0^1 - \int_0^1 h(x)dx$

$\rightarrow 1 = [xh(x) - \frac{h(x)}{m}]_0^1 = e^m - \frac{e^m}{m} + \frac{1}{m} \rightarrow 1 - \frac{1}{m} = e^m(1 - \frac{1}{m})$

$\rightarrow (1 - \frac{1}{m})(1 - e^m) = 0 \rightarrow m = 1\ or\ 0\ (rejected)$

* ln 微分法

* 積分三寶: Integration by part

* 面積分母不能是0

2020 – SECTION B

Q9.) *Peter leaves home at* $7:10am$ *and walks to train station to catch train. The time taken for Peter's walk follows* $N(15, 2^2)$ *in minutes. There are two trains departing at* $7:23am$ *and* $7:30$ *respectively. Given Peter catches the eariest departing train when he arrives.*

a.) *Find the probability Peter catches the train departing at* $7:23am$ *on a certain day.*

b.) *Find the probability Peter catches the train departing at* $7:30am$ *on a certain day.*

c.) *Everyday, John walks to the same train station to catch train. He catches the eariest departing train when he arrives. The probability of he catches the train at* $7:23am$ *and at* $7:30am$ *are* 0.3015 *and* 0.6328 *respectively.*

i.) *Find the probability the* 4^{th} *day in a week is* 2^{nd} *time they catch the same train.*

ii.) *Given they catch the same train on 2 certain day. Find the probability they catch the train departing at* $7:30am$ *on these 2 days.*

iii.) *Given they catch the same train on 4 certain day. Find the probability they catch the train departing at* $7:23am$ *on at least 1 of these 4 days.*

iv.) *What is the latest time for Peter to leave home if he have a higher chance to catch the train departing at* $7:23am$ *than that of John? Give your answer to nearest minute.*

* 參考課程 4.2, 4.4 及 4.5

CONT'D

2020 – SECTION B

$a.)\ Let\ T\ be\ the\ random\ variable\ of\ the\ time\ taken\ for\ Peter's\ walk.$

$$The\ probability = P(T < 13) = P\left(Z < \frac{13-15}{2}\right) = P(Z < -1)$$

$$= 0.1587$$

$$b.)\ The\ probability = P(13 < T < 20) = P\left(\frac{13-15}{2} < Z < \frac{20-15}{2}\right)$$

$$= P(-1 < Z < 2.5) = 0.8351$$

$ci.)\ Let\ p\ be\ the\ probability\ they\ catch\ the\ same\ train\ on\ a\ certain\ day$

$$p = P(T < 13) \cdot 0.3015 + P(13 < T < 20) \cdot 0.6328 = 0.576299$$

$$The\ required\ probability = C_1^3 p(1-p)^2 \cdot p = 0.1789\ (to\ 4\ d.p.)$$

$$ii.)\ The\ required\ probability = \frac{[P(13 < T < 20) \cdot 0.6328]^2}{p^2}$$

$$= 0.8408\ (to\ 4\ d.p.)$$

* 先計算 Z ~ N(0,1), 再對表
* Peter 同 John 搭 7:23
* Peter 同 John 搭 7:30
* $P(X = k) = C_k^n p^k (1-p)^{n-k}$ 3日有1日同車
* 第四日第2次同車
* 條件概率

CONT'D

2020 - SECTION B

iii.) *The required probability*

$$= \frac{p^4 - [P(13 < T < 20) \cdot 0.6328]^4}{p^4}$$

$$= 0.2927 \ (to\ 4\ d.p.)$$

iv.) *Let the minutes for Peter's walk,* $T' \sim N(M, 2^2)$

$$P(T' < 13) > 0.3015 \rightarrow P(Z < \frac{13 - M}{2}) > 0.3015$$

$$\rightarrow \frac{13 - M}{2} > -0.52$$

$$\rightarrow M < 14.04$$

$\therefore$ *The latest time for Peter leave home* $= 7:09am$

* $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 = (a+b)^4 - b^4$
  當中 a+b = p

* Peter 須加快平均步速去追火車

* 先計算 Z ~ N(0,1), 再對表

* 7:23am - 14 分鐘

2020 – SECTION B

Q10.) A man throw a fair die 4 times and he gets points as follow :

Die result	1	2	3	4	5	6
Points recieved	10	10	10	25	50	25

a.) Find the probability he gets total 200 points

b.) Find the probability he gets total not less than 150 points

c.) If he gets total less than 150 points, he can join the game . He need to press the button 3 times . A number of balls will show up on the screen for each button pressed . If there are 1 to 4 balls, the result is 'Good', 5 balls is 'Excellence', otherwise, 'Fair' . He can have a cup if he gets 1 'Excellence' and 2 'Good', a backpack if for 2 'Excellence' and 1 'Good', and a car for 3 'Excellence' . If he cannot join the next game, he can have a lucky draw with 0.01 chance to win a cup . Find the probability that;

i.) He win a backpack .

ii.) He win a car given that he join the game and win a prize .

iii.) He cannot join the game given that he win a cup .

* 參考課程 4.2 及 4.4

CONT'D

2020 – SECTION B

$a.)\ The\ required\ probability = (\frac{1}{6})^4 = \frac{1}{1296}$

$b.)\ The\ required\ probability = \frac{1}{1296} + C^4_3(\frac{1}{6})^3(\frac{5}{6}) + C^4_2(\frac{1}{6})^2(\frac{2}{6})^2$

$= \frac{5}{144}$

$ci.)\ Let\ N\ be\ the\ random\ variable\ of\ the\ numbers\ of\ balls\ shown\ up$
$for\ each\ button \sim Po(5),\ BP\ be\ the\ event\ he\ win\ a\ backpack$

$Let\ p_e = P(N=5) = \frac{e^{-5}5^5}{5!}$

$p_g = P(1 \le N \le 4) = e^{-5}(5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!})$

$P(BP) = \frac{5}{144} \cdot C^3_2 p_e^2 p_g = 0.0014\ (to\ 4\ d.p.)$

* 4 次每次要 50 分
* 4次有 3次 50分
* 4次有 2次 50分 及 2次 25分
* $P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$
* 有最少 150 分
* 2 個 Excellence 及 1 個 Good

CONT'D

2020 – SECTION B

$ii.)$ *Let CUP be the event he win a cup and join the game*

CAR be the event he win a car

$$P(CUP) = \frac{5}{144} \cdot \boxed{C_1^3 p_e p_g^2} \text{ and } P(CAR) = \frac{5}{144} \cdot \boxed{p_e^3}$$

$$\text{The required probability} = \frac{P(CAR)}{P(CAR \cap CUP \cap BP)}$$

$$= \frac{p_e^3}{\boxed{p_e^3 + C_1^3 p_e p_g^2 + C_2^3 p_e^2 p_g}} = \frac{p_e^3}{\boxed{(p_e + p_g)^3 - p_g^3}} = 0.0374 \ (to\ 4\ d.p.)$$

$$iii.) \text{ The required probability} = \frac{(1 - \frac{5}{144}) \cdot 0.01}{(1 - \frac{5}{144}) \cdot 0.01 + P(CUP)}$$

$$= 0.7373 \ (to\ 4\ d.p.)$$

* 2 個 Good 及 1 個 Excellence

* 3 個 Excellence

* 條件概率

* $(a+b)^n = \sum_{r=0}^{n} C_r^n a^r b^{n-r}$

* 條件概率

2020 - SECTION B

Q11.) The rate of change of the number of adults ($x10^3$ per month) visiting the mall is

$A(t) = 5\ln(t^2 - 8t + 20)$, where t is the number of months elapsed since the mall opening.

a.) Find the estimation of the total visiting number of the adults by trapezoidal rule with 5 sub-intervals for first 2 months. Determine if the the estimation is over-estimated.

b.) The rate of change of the number of children ($x10^3$ per month) visiting mall is

$B(t) = 3^{2t+2}(1+3^{2t})^{-1}$, where t is the number of months elapsed since the mall opening.

i.) Find the total visiting number of the children for first 2 months.

ii.) Does the difference of the total number of adults and that of children exceed 40% of the total number of adults for the first 2 months? Explain your answer.

* 參考課程 2.2, 2.3, 3.2 及 3.3

a.) Let I_1' be the estimation of $I_1 = \int_0^2 A(t)dt$ (in 10^3)

$$I_1' = \frac{2-0}{5\cdot 2}[A(0) + 2A(0.4) + 2A(0.8) + 2A(1.2) + 2A(1.6) + A(2)]$$

* 計算梯形面積的加總

$$= 25.5486x10^3 \text{ (to 4 d.p.)}$$

CONT'D

2020 - SECTION B

$$Consider\ A'(t) = \frac{10(t-4)}{t^2-8t+20} = \frac{10(t-4)}{(t-4)^2+4}$$

$$\to A''(t) = \frac{10}{(t-4)^2+4} - \frac{20(t-4)^2}{[(t-4)^2+4)]^2}$$

$$= \frac{10[4-(t-4)^2]}{[(t-4)^2+4)]^2} = \frac{10(6-t)(t-2)}{[(t-4)^2+4)]^2} < 0, for\ 0 \le t \le 2$$

$i.e. I_1'\ is\ under-estimated$

$$b.)\ Let\ I_2 = \int_0^2 B(t)dt = \int_0^2 \frac{3^{2t+2}}{1+3^{2t}}dt = 9\int_0^2 \frac{9^t}{1+9^t}dt$$

$$Let\ y = 9^t \to lny = tln9 \to dy = y \cdot ln9 \cdot dt$$

$$\to I_2 = \frac{9}{ln9}\int_1^{81} \frac{1}{1+y}dy = \frac{9}{ln9}[ln(1+y)]_1^{81} = \frac{9ln41}{ln9}$$

* Chain rule
* Product rule + Chain rule
* $a^2-b^2=(a+b)(a-b)$
* 個 A(t) 係凹口向下
* 積分三寶: 積分代入
* 定積分代入要改範圍

CONT'D

2020 - SECTION B

$\therefore$ *Total visiting number of the children for first 2 months* $= 9\log_9 41x10^3$

$$* \ \frac{\log_c a}{\log_c b} = \log_b a$$

$$c.)\ Consider\ \left|\frac{I_1 - I_2}{I_1}\right| = \left|1 - \frac{I_2}{I_1}\right| > \left|1 - \frac{I_2}{I_1'}\right| \approx 59\,\%,\ where\ I_1' < I_1$$

$\therefore$ *The difference exceed* $40\,\%$ *of total number of adults*

2020 – SECTION B

$Q12.)$ $Given\ that\ P = \dfrac{32}{a^{5+bt}+8},\ where\ a\ and\ b \in \mathbb{R},\ t \geq 0$

$a.)\ Express\ ln(\dfrac{32}{P} - 8)\ as\ a\ linear\ function\ of\ t$

$b.)\ Given\ that\ the\ linear\ function\ in\ a.)\ passing\ through\ (1, ln2)\ and\ (0, ln32).\ Find\ a,\ b,\ P'(t)\ and\ P''(t)$

$c.)\ Find\ P\ for\ t\ tends\ to\ infinity\ and\ the\ value\ of\ P\ when\ P'(t)\ attain\ to\ its\ greatest\ value.$

* 參考課程 2.1, 2.2, 2.3, 2.4 及 3.1

$$a.)\ ln(\frac{32}{P} - 8) = \boxed{ln(a^{5+bt})} = \boxed{5lna + blna \cdot t}$$

$b.)\ Given\ that\ the\ function\ passing\ through\ (1, ln2)\ and\ (0, ln32)$

$\rightarrow 5lna = ln32\ and\ (5+b)lna = ln2$

$\rightarrow a = 2\ and\ b = -4$

* ln(AB) = lnA + lnB

* 直線方程, y = (斜率)x + (y-intercept)

CONT'D

2020 - SECTION B

$$Hence,\ P(t) = \frac{32}{y(t)+8},$$

$$where\ y(t) = 2^{5-4t}$$

$$\to \ln y(t) = (5-4t)\ln 2 \to y'(t) = -4\ln 2 \cdot y(t)$$

$$Then,\ P'(t) = -\frac{32y'(t)}{[y(t)+8]^2} = \frac{128\ln 2 \cdot y(t)}{[y(t)+8]^2}$$

$$P''(t) = \frac{128\ln 2 \cdot y'(t)}{[y(t)+8]^2} - \frac{2 \cdot 128\ln 2 \cdot y(t) \cdot y'(t)}{[y(t)+8]^3}$$

$$= \frac{128\ln 2 \cdot y'(t)}{[y(t)+8]^3}(y(t)+8-2y(t))$$

$$= -\frac{512(\ln 2)^2 \cdot y(t) \cdot (8-y(t))}{[y(t)+8]^3}$$

* ln 微分法
* Chain rule
* Product rule + Chain rule

CONT'D

2020 - SECTION B

$c.)\ Assume\ P''(t_0) = 0 \rightarrow y(t) = 8 \rightarrow t = 0.5$

	0 < t < 0.5	t = 0.5	t > 0.5
V''(t)	+	0	-
V'(t)	Inc.		Dec.

$\therefore P'(t)\ attains\ greatest\ value\ when\ t = 0.5$

$P(0.5) = 2$

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{32}{y(t)+8},\ where\ \lim_{t\to\infty} y(t) = \lim_{t\to\infty} \frac{2^5}{2^{4t}} = 0$$

$$\rightarrow \lim_{t\to\infty} P(t) = \frac{32}{8} = 4$$

* 搵 turning point = 搵 t_0 使度 $P''(t_0)=0$

* 利用表格計算 turning point 附近上升定下降

$f'(x) > 0 \rightarrow Increasing$

$f'(x) < 0 \rightarrow Decreasing$