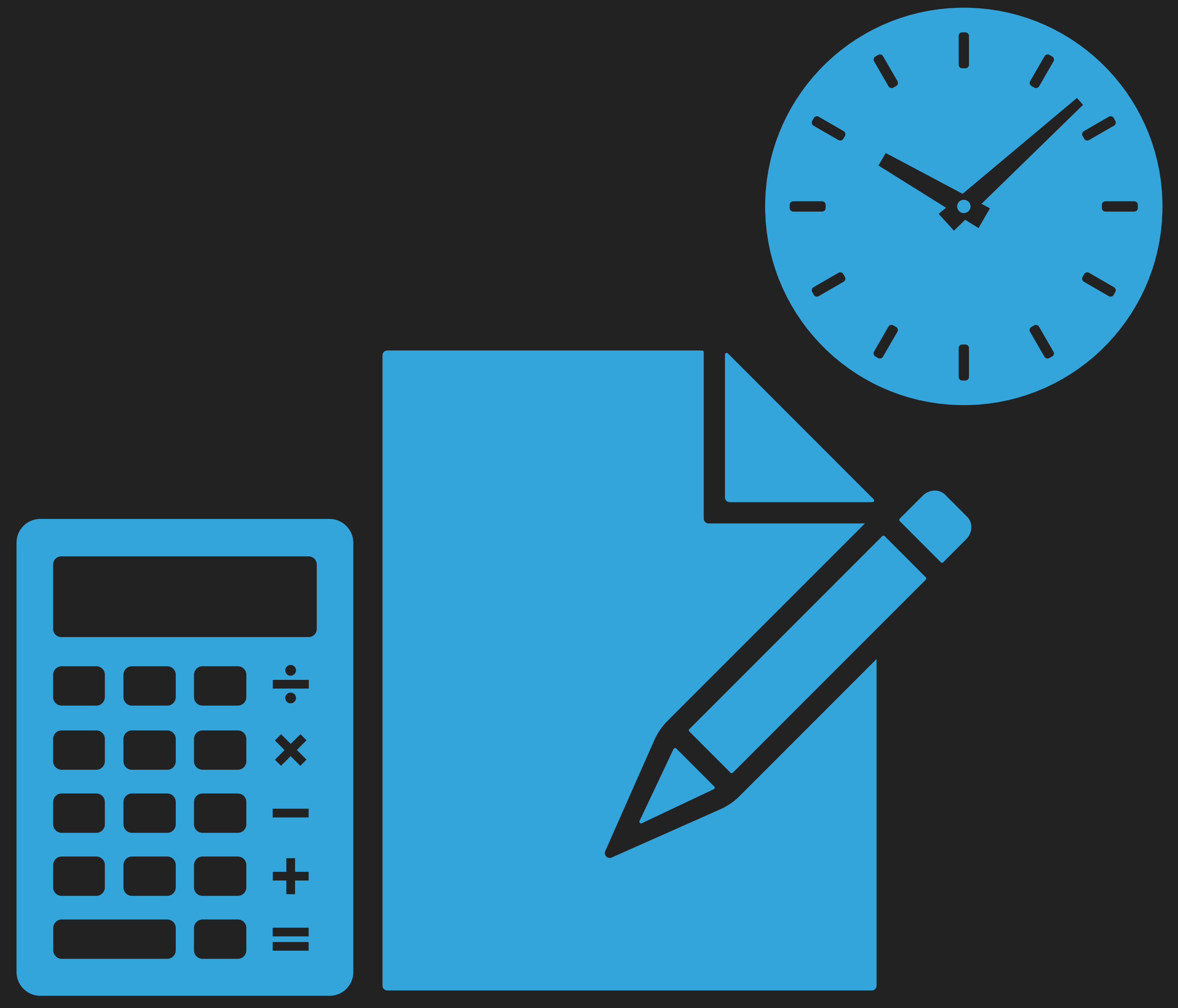
深宵教室 - DSE 必修模擬試題解答

2016 PAPER 1

此為參考2016試題之模擬試題，原版請另行購買

2016 PAPER 1

- Section A1
- Section A2
- Section B



2016 PAPER 1 - SECTION A1

Q1.) Simplified $\dfrac{(x^8y^7)^2}{x^5y^{-6}}$*, in positive indices*

* 參考課程 1.2

$$= x^{8\cdot 2-5}\cdot y^{7\cdot 2-(-6)}$$

$$= x^{11}\cdot y^{20}$$

* 指數乘係加，除係減

2016 PAPER 1 - SECTION A1

$Q2.)\ Ax = (4x + B)C,\ x = ?$

* 參考課程 2.1

$\rightarrow Ax = 4Cx + BC$

$\rightarrow (A - 4C)x = BC$

$\rightarrow x = \dfrac{BC}{A - 4C},\ where\ A \neq 4C$

$*\ If\ A = 4C,\ B = 0\ or\ (A = C = 0)$

* 兩邊減 4Cx

* 兩邊除 (A-4C)

* 分母為 0 是不容許

2016 PAPER 1 - SECTION A1

Q3.) Simplify $\dfrac{2}{4x-5}+\dfrac{3}{1-6x}$

* 參考課程 2.10

$$=\frac{2(1-6x)+3(4x-5)}{(4x-5)(1-6x)}$$

$$=\frac{2-12x+12x-15}{(4x-5)(1-6x)}=-\frac{13}{(4x-5)(1-6x)}$$

* 函數通分母

2016 PAPER 1 - SECTION A1

Q4.) Factorize $x^2 + xy - 6y^2 - 5x + 10y$

* 參考課程 2.5

$$= (x + 3y)(x - 2y) - 5(x - 2y)$$

$$= (x - 2y)(x + 3y - 5)$$

* 十字相乘 $(a - \alpha)(a - \beta) \rightarrow \alpha\beta = -6, \alpha + \beta = -1$
* 抽 5

2016 PAPER 1 – SECTION A1

Q5.) There is a 180 – staffs small company. The number of male staff is 40 % more than that of female staff. Find the number difference between male and female.

* 參考課程 1.3

Let the number of female staff be x

Then, the number of male staff, $y = x(1+40\%)$

$= 1.4x$

Also, $x + 1.4x = 180 \rightarrow x = 75,\ y = 105$

$\therefore$, *The number difference* $= y - x$

$= 30$

* 新值 = 舊值 x (1+百份比變化)

2016 PAPER 1 - SECTION A1

Q6.) Solve $x + 6 < 6(x + 11)$ *or* $x \leq -5$

Hence, find out the greatest negative integer satisfy the above inequalities .

* 參考課程 1.1 及 2.3

$$x + 6 < 6(x + 11) \ or \ x \leq -5$$

$$\rightarrow x + 6 < 6x + 66 \ or \ x \leq -5$$

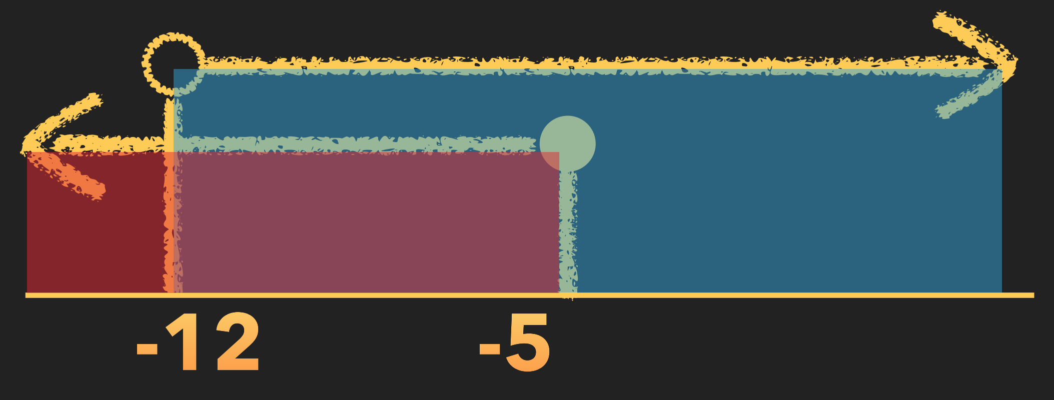
$$\rightarrow 5x > -60 \ or \ x \leq -5$$

$$\rightarrow x > -12 \ or \ x \leq -5$$

$$\rightarrow x = \text{ all real solution}$$

$\therefore -1$ *is the greatest integer satisfy the inequalities .*

* or 指有"顏色"的地方



2016 PAPER 1 - SECTION A1

Q7.) In a polar system, $O = (0, 0^0)$, $A = (12, 75^0)$, $B = (12, 135^0)$.

a.) Find $\angle AOB$

b.) Find the perimeter of ΔAOB.

c.) Find the number of folds of rotational symmetry of ΔAOB.

* 參考課程 3.2, 3.5 及 3.8

a.) $\angle AOB = 135^0 - 75^0 = 60^0$

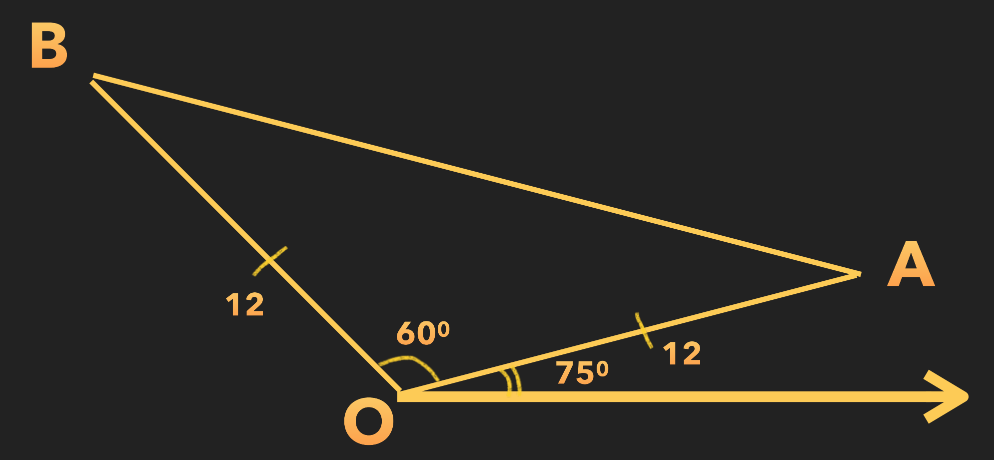
b.) ΔOAB *is an equitriangle*

$\therefore$ *The perimeter of* $\Delta AOB = 3 \cdot 12$

$= 36$ *unit*

c.) *The number of rotational symmetry* $= 3$

* 先畫圖理解



* 等腰三角形+60⁰=等邊三角形

* 等邊三角形有三條旋轉對稱

2016 PAPER 1 - SECTION A1

Q8.) f(x) is sum of two parts, one part varies as x^2 and other varies as x.

Given that $f(3) = 48$, $f(9) = 198$

a.) Find $f(x)$

b.) Solve $f(x) = 90$

* 參考課程 2.1, 2.3, 2.4, 2.5 及 2.6

a.) Let $f(x) = k_1x + k_2x^2$, where k_1, k_2 are real constant. Then,

$$\begin{cases} 48 = 3k_1 + 9k_2 & \text{———— (1)} \\ 198 = 9k_1 + 81k_2 & \text{———— (2)} \end{cases}$$

$(2) - 3x(1): \ 54k_2 = 54 \rightarrow k_2 = 1, k_1 = 13$

$\therefore f(x) = 13x + x^2$

b.) $f(x) = 90 \rightarrow x^2 + 13x - 90 = 0$

$$x = \frac{-13 \pm \sqrt{13^2 - 4(1)(-90)}}{2(1)} = -18 \text{ or } 5$$

* 部分變量

* 消去法消去 k_1 揾 k_2,再代 (1) 式揾 k_1

* 用二次方程根式解

2016 PAPER 1 - SECTION A1

Q9.) The following shows the distribution of the flowers' height

Frequency table

Height (m)	*Frequency*
0.1 – 0.3	a
0.4 – 0.6	4
0.7 – 0.9	b
1.0 – 1.2	c
1.3 – 1.5	15
1.6 – 1.8	3

Cumulative frequency table

Height less than (m)	*Cumulative Frequency*
0.35	2
0.65	x
0.95	13
1.25	y
1.55	37
1.85	z

a.) Find x, y, z.

b.) If one flower is randomly selected from the distribution, Find the probability of the selected flower's height is less than 1.25m but not less than 0.65m.

* 參考課程 4.1 及 4.3

CONT'D

2016 PAPER 1 - SECTION A1

$$a.)\ a = 2 \rightarrow \ x = 2 + 4 = 6$$

$$y = 37 - 15 = 22$$

$$z = 37 + 3 = 40$$

$$b.)\ The\ probability = \frac{22-6}{40} = \frac{2}{5}$$

* 累積頻數表將頻數表內特定範圍的數值相加

* 總共花朵的數量

* 總共花朵 (0.65m 至 1.25m) 的數量

2016 PAPER 1 - SECTION A2

Q10.) *Denote P is a moving point. It is equidistant from A(5,7) and B(13,1). Let L be the locus of P. L is intersects x – axis and y – axis at C and D respectively. Given that a circle, C, passes through O(0,0), C, and D.*

a.) Find the equation of L.

b.) Is the circumference of C > 30? Please explain your answer.

* 參考課程 3.1, 3.6 及 3.8

a.) The mid – pt of AB, $M = \left(\frac{5+13}{2}, \frac{7+1}{2}\right) = (9,4)$

The slope of $L = -1 \div \left(\frac{7-1}{5-13}\right) = \frac{4}{3}$

$\therefore L: y - 4 = \frac{4}{3}(x-9) \rightarrow 4x - 3y - 24 = 0$

b.) $C = (6,0)$ and $D = (0,-8)$

CD = The diameter of C (converse of ∠ in semi – circle)

* 垂直平分線, 搵中點及斜率相乘 = -1
* 中點公式 = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
* 兩點斜率 = $\frac{y_2-y_1}{x_2-x_1}$
* 點斜式

CONT'D

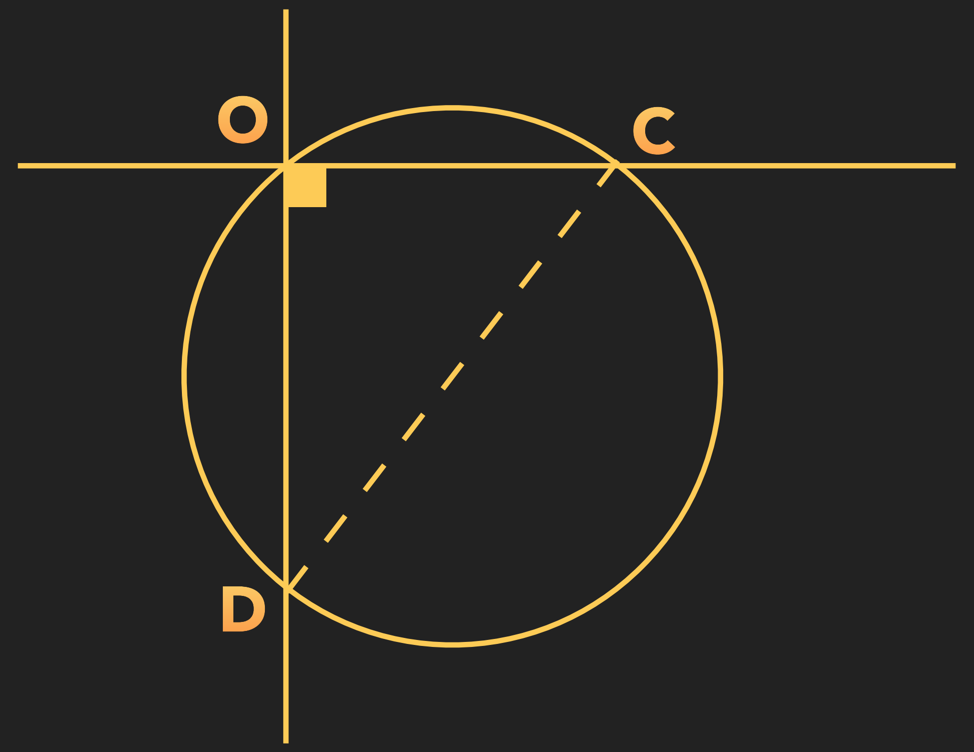
2016 PAPER 1 - SECTION A2

$$\therefore \textit{The circumference of } C = \sqrt{(0-6)^2+(-8-0)^2}\,\pi$$

$$\approx 31.4 > 30$$

$i.e.$ *The circumference of* $C > 30.$

* 距離公式 = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$



2016 PAPER 1 - SECTION A2

Q11.) There is inverted right circular cone vessel (20cm tall) with some milk that the depth of milk $= 12cm$. Then, $444\pi cm^3$ milk is poured into the vessel and the depth of milk become $= 16cm$.

a.) Find the final volume of milk in term of π.

b.) Is the final wet curved surface area at least $800cm^2$? Explain your answer.

* 參考課程 3.2 及 3.9

a.) Let $V_1\ cm^2$ be the volume of the final milk

$V_2\ cm^2$ be the volume of the initial milk

Then, $V_1 - V_2 = 444\pi$ and $V_2 = \boxed{(\frac{12}{16})^3 V_1}$

$$\rightarrow V_1(1 - \frac{9}{16}) = 444\pi \rightarrow V_1 = 768\pi$$

$\therefore$ The final volume of milk $= 768\pi\ cm^3$

* 相似圖形, 體積比 = (邊比)³

CONT'D

2016 PAPER 1 - SECTION A2

b.) *Let the radius of final milk cone be Rcm*

$$\boxed{V_1 = \frac{1}{3}\pi R^2(16)} \rightarrow R = 12cm$$

$$\therefore \text{The wet curved surface area} = \boxed{\pi(12)\sqrt{12^2+16^2}}$$

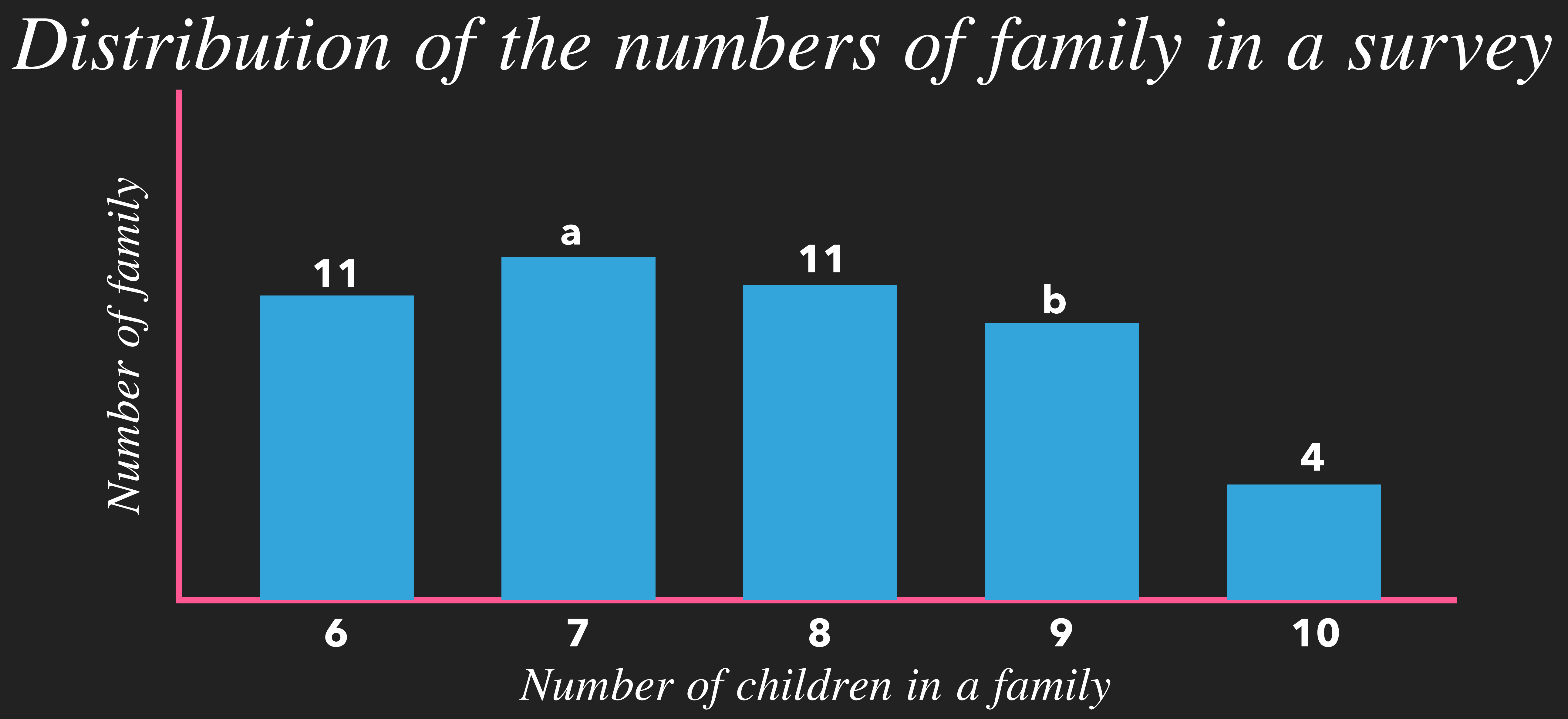
$$\approx 753.6\ cm^2 < 800\ cm^2$$

i.e. The final wet curved surface area is less than 800 cm^2

* 錐體體積 = 1/3 x 半徑² x π x 高
* 錐體曲面面積 = π x 半徑 x 斜邊

2016 PAPER 1 - SECTION A2

Q12.) The following bar chart shows the result of the survey of the numbers of children in a typical family in Hong Kong. where $a > 11$ *and* $4 < b < 10$*. The median* $= 7.5$



a.) Find a and b.

b.) 4 more family are counted. They all have different number of children in between the range of the above distribution. Find the greatest possible median and the least possible mean of the new distribution.

* 參考課程 4.1 及4.2

CONT'D

2016 PAPER 1 - SECTION A2

$a.)$ *The total number of family shall be even*

$\rightarrow 11 + a + 11 + b + 4 \text{ is even} \rightarrow a + b \text{ is even}$

$\text{Also, the median} = 7.5 \rightarrow 11 + a = 11 + b + 4 \rightarrow a = b + 4$

$a > 11 \rightarrow b + 4 > 11 \rightarrow b > 7 \text{ and given that } 4 < b < 10$

$\therefore 7 < b < 10, \text{ the possible } b = 8,9$

$\text{Hence, } (a, b) = (12,8) \text{ or } (13,9)$

$b.)$ $2 \text{ possible combination are } (6,7,8,9) \text{ and } (7,8,9,10)$

$\therefore \text{ The greatest possible median} = 8 \text{ for } (7,8,9,10)$

$\text{For } (6,7,8,9) \text{ with } (a, b) = (12,8) \rightarrow \text{Mean} = 7.6$

$\text{For } (6,7,8,9) \text{ with } (a, b) = (13,9) \rightarrow \text{Mean} = 7.68$

$\text{For } (7,8,9,10) \text{ with } (a, b) = (12,8) \rightarrow \text{Mean} = 7.6154$

$\text{For } (7,8,9,10) \text{ with } (a, b) = (13,9) \rightarrow \text{Mean} = 7.6923$

$\therefore \text{ The least possible mean} = 7.6$

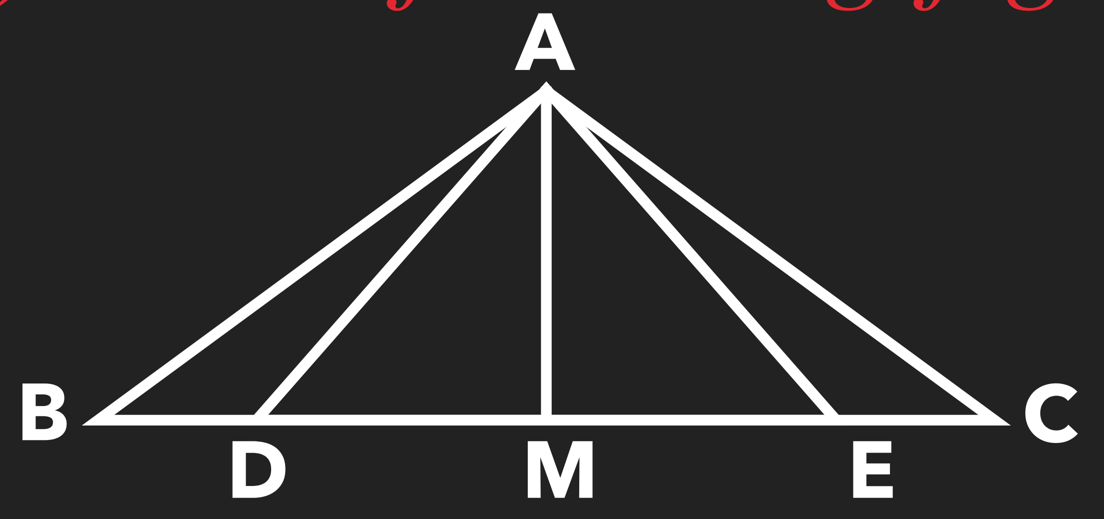
* 中位數是小數, 中間有兩個數, 總數是雙數
* 中位數是7.5, 中間果兩個數係 7 同 8

* 四個不同又要喺 6 至 10 之間

* 平均值 = 數值加總 / 總數

2016 PAPER 1 - SECTION A2

Q13.) In the following figure, $BD = CE$, $\angle ADC = \angle AEB$ *and* $DM = EM$



a.) Prove $\Delta ACD \cong \Delta ABE$

b.) Suppose $AD = 15cm$, $BD = 7cm$ *and* $DE = 18cm$

Is ΔABE *a right − angled triangle? Explain your answer.*

* 參考課程 3.2 及 3.3

a.) $\angle ADC = \angle AEB$ (given)

$AD = AE$ (sides opp, eq. ∠s) — * 底角相等, 等腰三角形

$CD = CE + ME + DM = BD + ME + DM = BE$ — * BD=CE (已知)

$\therefore \Delta ABE \cong \Delta BCF$ (SAS)

b.) $\because \angle AMD = 90^0$ (props of isos. ΔADE, $DM = ME$) — * 等腰三角形, 垂直平分線

$\therefore AM = \sqrt{AD^2 - DM^2}$ (pyth. theorem) $= 12\ cm$ — * 畢氏定理

Also, $AB^2 = AM^2 + BM^2$ (pyth. theorem) $= 400\ cm^2$

And, $AB^2 + AE^2 = 625 = 25^2 = BE^2$ ($BE = BD + DE$)

$\therefore \Delta ABE$ is a right − angled triangle (converse of pyth. theorem) — * 畢氏定理逆定理

2016 PAPER 1 - SECTION A2

Q14.) Let $f(x) = 6x^4 + 7x^3 + ax^2 + bx + c \equiv (lx^2 + 5x + 8)(2x^2 + mx + n)$*,*
where a, b, c, l, m, n *are constant. Given that the remainder are equal for* $f(x)$ *is divided by* $x + 2$ *and* $x - 2$*.*
a.) Find l*,* m*, and* n
b.) How many real roots of $f(x) = 0$*?*

* 參考課程 2.4 及 2.6

a.) $f(2) = f(-2) \to f(2) - f(-2) = 0 \to b = -28$

By comparison of coefficient of x^4*,* x^3*, and* x

$(For\ x^4): 2l = 6 \to l = 3$

$(For\ x^3): ml + 10 = 7 \to m = -1$

$(For\ x): 5n + 8m = b \to n = -4$

i.e. $(l, m, n) = (3, -1, -4)$

b.) $f(x) = 0 \to 3x^2 + 5x + 8 = 0 - (1)$ *or* $2x^2 - x - 4 = 0 - (2)$

In (1), $\Delta = 5^2 - 4(3)(8) = -71 < 0$*, There is no real root*

In (2), $\Delta = (-1)^2 - 4(-4)(2) = 33 > 0$*, There is 2 distinct real roots*

i.e. $f(x) = 0$ *has 2 distinct real roots.*

* 餘數定理

* 用二次方程判別式

2016 PAPER 1 – SECTION B

Q15.) There are 4 boys; and 5 girls made a queue. Find the probability of no boys are next to each other in the queue.

* 參考課程 4.4

$$\text{The probability} = \frac{P^5_5 P^6_4}{P^9_9} = \frac{5}{42}$$

○□○□○□○□○□○

○ 給4個男仔的排序

□ 給5個女仔的排序

* 9 個人的排序

* 5 個位置給 5 個女仔的排序

* 6 個位置給 4 個男仔排序

2016 PAPER 1 - SECTION B

Q16.) *The mean of the score of a large group of student in English exam = 61 marks*
The standard score of Peter and Mary are -2.6 *and* 1.4 *respectively. Peter gets 22 marks. Is the range of the exam at most 59 marks? Explain your answer.*

* 參考課程 4.1 及 4.2

a.) *Let the score of Mary in the English exam be x marks*
the standard deviation of the exam be σ

$$\sigma = \frac{22-61}{-2.6} = 15 \rightarrow x = 61 + 1.4\sigma = 82$$

$\because 82 - 22 = 60 > 59$

$\rightarrow$ *The possible score difference* > 59

i.e. The range of the exam is more than 59

* 標準分數 = 數據相差平均數幾多個標準差

* 全距 = 最大值 - 最細值

2016 PAPER 1 - SECTION B

Q17.) The 1^{st} and the 38^{th} term of an arithmetic sequence are 666 and 555 respectively.

a.) Find the common difference of the sequence.

b.) Find the greatest value of n such that the sum of the first n^{th} term of the sequence > 0

* 參考課程 2.2 及 2.6

a.) Let $A(n) = a + (n-1)d$, where a and d are constant

$A(1) = 666 \to a = 666,\ A(38) = a + 37d = 555 \to d = -3$

$\therefore$ The common difference $= -3$

b.) Consider, $A(1) + A(2) + \ldots + A(n) > 0$

$$\to \frac{n(666 + 666(n-1)(-3))}{2} > 0$$

$$\to n(3n - 1335) > 0 \to 0 < n < 445$$

$\therefore$ The greatest value of $n = 444$

* 等差數列 = 首項 + (項數 - 1) x 公差

* 等差數列之和=(首項+尾項)x項數/2

* 先解二次方程搵根, α, β

因為細過0, 所以答案晌最細最大根之間

2016 PAPER 1 - SECTION B

$Q18.)\ Let\ f(x) = -\frac{1}{3}x^2 + 12x - 121$

$a.)\ Find\ the\ vertex\ of\ y = f(x)$

$b.)\ The\ graph\ y = g(x)\ is\ obtained\ by\ tanslating\ y = f(x)\ vertically\ until\ it\ touches\ x - axis.$

$Find\ g(x).$

$c.)\ Describe\ the\ geometric\ meaning\ between\ y = f(x)\ and\ y = h(x) = -\frac{1}{3}x^2 - 12x - 121$

* 參考課程 2.5, 2.10 及 3.2

$a.)\ Let\ the\ vertex\ be\ (a, b),\ then$

$$f(x) = -\frac{1}{2}x^2 + 12x - 121 \equiv -\frac{1}{2}(x-a)^2 + b$$

* 二次函數轉換可用 compare coefficient

$By\ compare\ coefficient\ of\ x\ and\ constant,\ we\ have$

$a = 18,\ b = -13 \rightarrow The\ vertex = (18, -13)$

$b.)\ g(x) = -\frac{1}{3}(x-18)^2$

* 頂點由 (18,-13) 移到 (18,0)

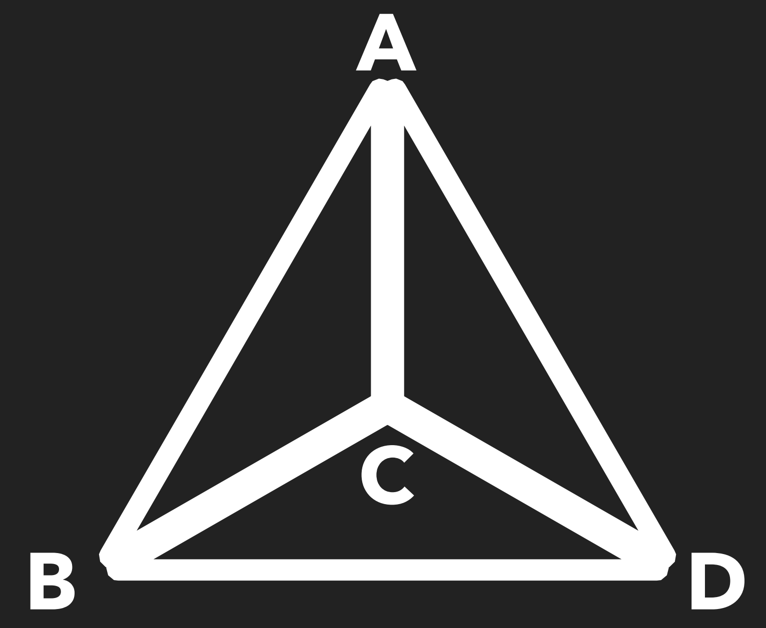
$c.)\ \because h(x) = f(-x),$

$\therefore y = h(x)\ is\ the\ reflection\ of\ y = f(x)\ along\ y - axis$

* 負變正, 正變負

2015 PAPER 1 - SECTION B

Q19.) The following shows a tetrahedron with $\angle BAD = 86^0$, $\angle CBD = 43^0$, $AB = 10cm$, $AC = 6cm$, $BC = 8cm$, *and* $BD = 15cm$



a.) Find CD and $\angle ABD$

b.) Is the angle between AB and the plane BCD $= \angle ABC$? *Explain your answer.*

* 參考課程 3.2, 3.3 及 3.10

a.) By cosine law in ΔBCD,

$$CD = \sqrt{8^2 + 15^2 - 2(8)(15)cos43^0} = 10.7\ cm\ (to\ 3\ sig.fig.)$$

* cosine law 使用

By sine law in ΔABD,

$$sin(180^0 - 86^0 - \angle ABD) = \frac{10sin86^0}{15},\ (\angle s\ sum\ of\ \Delta)$$

* sine law 使用

* 三角形內角和 = 180⁰

$$\rightarrow \angle ABD = 52.3^0\ (to\ 3\ sig.fig.)$$

CONT'D

2016 PAPER 1 – SECTION B

$b.)$ *By sine law in* ΔABD,

$$AD^2 = \left(\frac{15\sin(\angle ABD)}{\sin 86^0}\right)^2 \approx 141.5465$$

$$AC^2 + CD^2 \approx 150.49 \neq AD^2$$

$$\therefore \angle ACD \neq 90^0 \ (converse\ of\ pyth.\ theorem)$$

$$i.e.\ Angle\ between\ AB\ and\ the\ plane\ BCD \neq \angle ABC$$

* sine law 使用

* 畢氏定理逆定理

* C 點不是 A 點在 BCD 的投影

2016 PAPER 1 - SECTION B

Q20.) ΔOPQ is an obtuse – triangle, where its in – center, I, circumcenter, J and P are collinear. Given that, $O=(0,0)$, $P=(x_1,19$, $Q=(40,30)$. C is circle passing through O, P, and Q.

a.) Prove $OP=PQ$, hence find the equation of C

b.) Let L_1 and L_2 be two tangent to C. Both of them have slope $\frac{3}{4}$. L_1 has greater y – intercept than L_2. Denote S and T be the x – interception and y – interception of L_1 respectively while U and V are the x – interception and y – interception of L_2 respectively. Is the area of the trapezium $STUV > 17{,}000$? Explain your answer.

* 參考課程 2.5, 2.10 及 3.2

a.) PI is an angle bisector of $\angle OPQ$

PJ is $\perp$ bisector of OQ

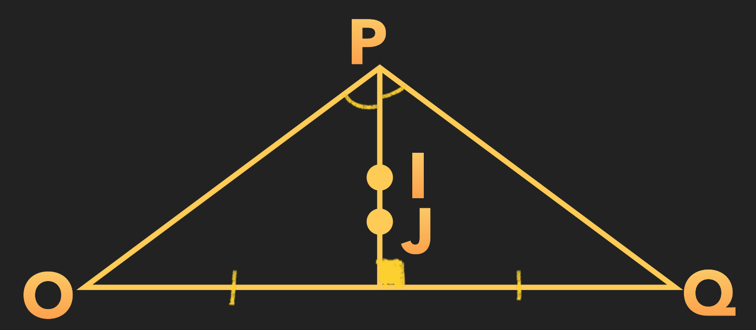
$\because$ P, I, J are collinear.

$\therefore \Delta OPQ$ is an isos. Δ, (prop. of isos. Δ)

i.e. $OP=PQ$ (prop of isos. Δ)

* 內心定義

* 外心定義



CONT'D

2016 PAPER 1 – SECTION B

$Let\ M\ be\ the\ mid-pt.\ of\ OQ = (20,15)$

$The\ slope\ of\ PM\ x\ The\ slope\ of\ OQ = -1$

$$\rightarrow \frac{19-15}{x_1-20}\ x\ \frac{3}{4} = -1$$

$$\rightarrow x_1 = 17$$

$Let\ C:\ x^2+y^2+Dx+Ey+F=0$

$Sub,\ O=(0,0),\ P(17,19),\ and\ Q=(40,30)\ into\ C,\ we\ have$

$F=0$

$$\begin{cases} 40D+30E=-2500 & -(1) \\ 17D+19E=-650 & -(2) \end{cases}$$

$By\ solving\ the\ above,\ \rightarrow D=-112\ and\ E=66$

$i.e.\ C: x^2+y^2-112x+66y=0$

* 中點公式 = $(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$
* 兩線互相垂直, 斜率相乘 = -1
* 兩點斜率 = $\frac{y_2-y_1}{x_2-x_1}$
* 三點定圓

CONT'D

2016 PAPER 1 – SECTION B

$b.)$ *The center of* $C = (56, \ -33)$

$\therefore$ *The diameter,* $D = 2\sqrt{56^2 + 33^2} = 130$

$\because$ *The slope of* L_1 *and* $L_2 = \dfrac{3}{4} \rightarrow tan\angle SUV = \dfrac{3}{4}$

$$\therefore sin\angle SUV = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5},\ cos\angle SUV = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$$

$$\sin\angle SUV = \frac{D}{SU} \rightarrow SU = \frac{D}{sin\angle SUV} = \frac{5D}{3}$$

$$\sin(90^0 - \angle SUV) = \frac{D}{TV} \rightarrow TV = \frac{D}{cos\angle SUV} = \frac{5D}{4}$$

$$The\ area\ of\ STUV = \frac{SUxTV}{2} = \frac{25D^2}{24} \approx 17{,}604 > 17{,}000$$

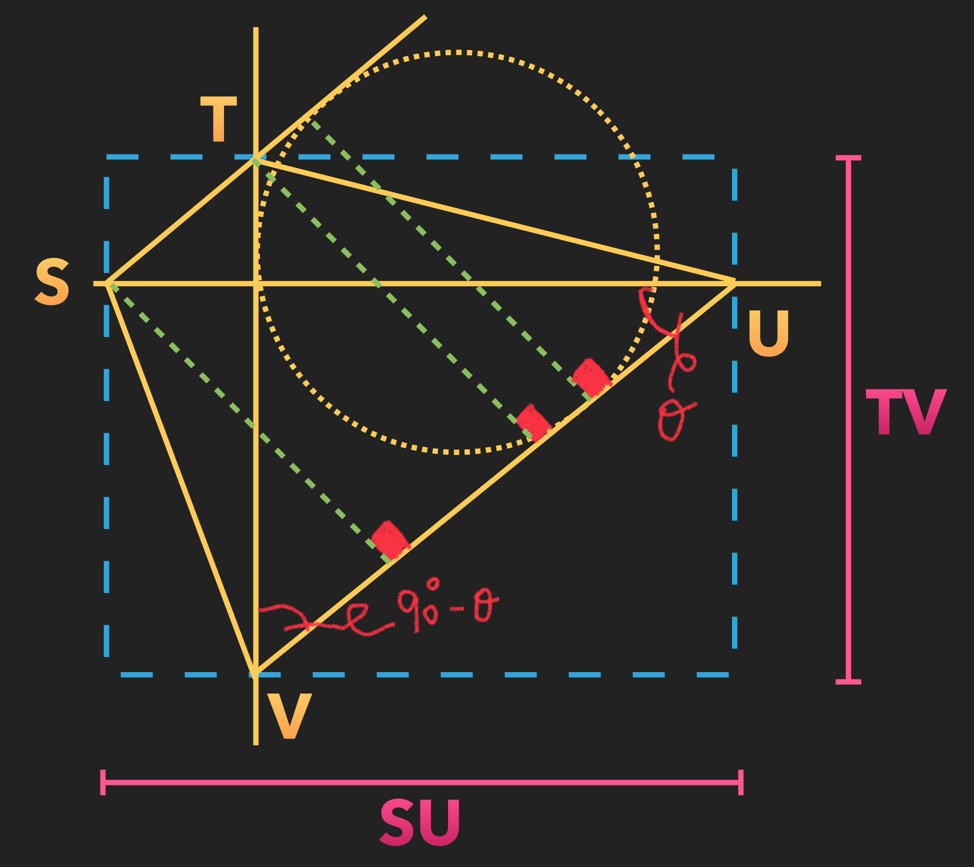
$\therefore$ *The area of the trapezium STUV* $> 17{,}000$

* $x^2 + y^2 + Dx + Ey + F = 0$

$Center = (-\dfrac{D}{2}, -\dfrac{E}{2})$

* 距離公式 = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

* 斜率 = tan (線同 X軸的夾角)



* 梯形STUV面積 = 藍色長方形面積 / 2

= SUxTV / 2