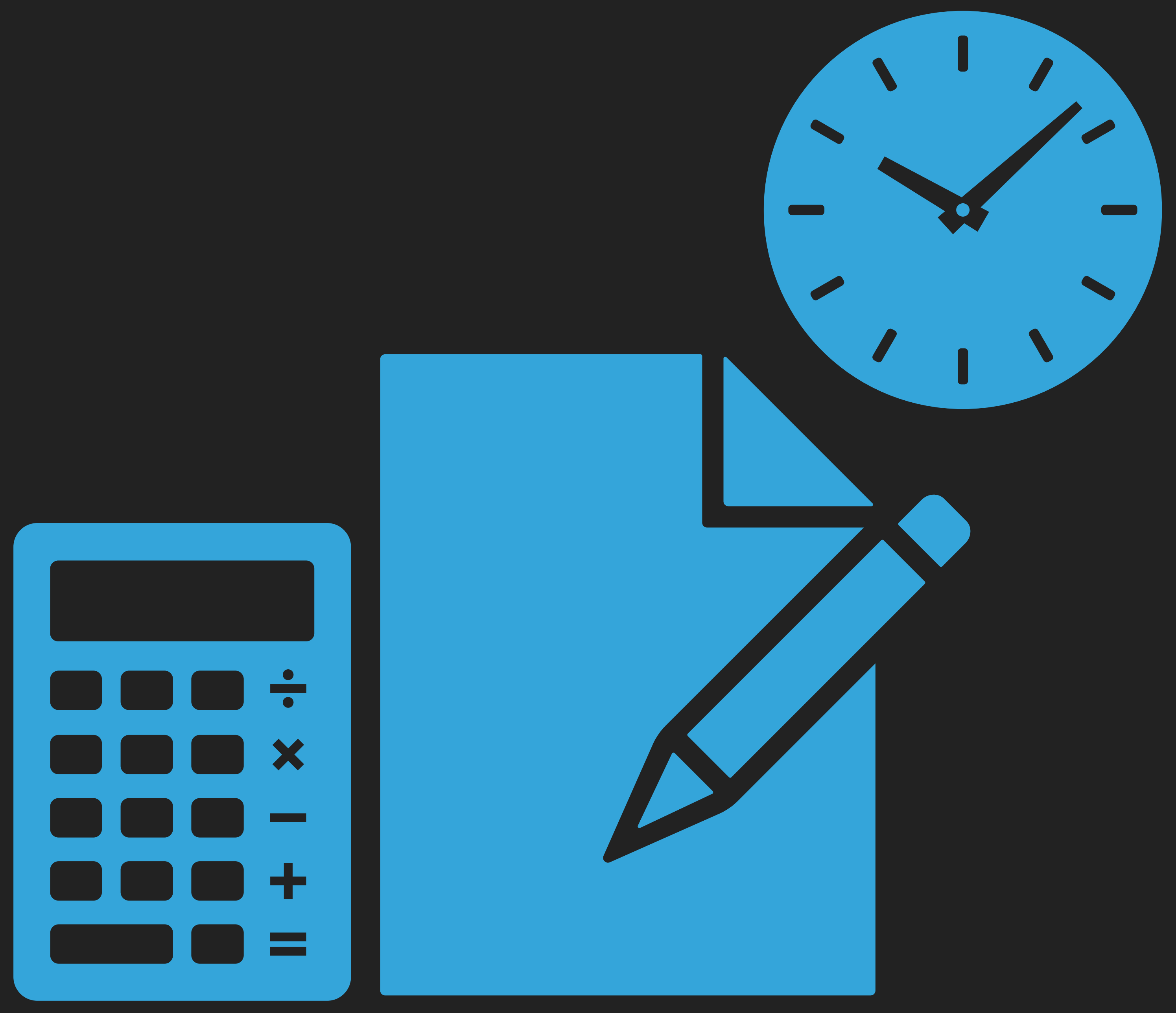
深宵教室 - DSE 必修模擬試題解答

2014 PAPER 1

此為參考2014試題之模擬試題，原版請另行購買

2014 PAPER 1

- Section A1
- Section A2
- Section B



2014 PAPER 1 - SECTION A1

Q1.) Simplified $\dfrac{(xy^{-2})^3}{y^4}$*, in positive indices*

* 參考課程 1.2

$$= x^{3} \cdot y^{-2\cdot 3-4}$$

$$= x^3 \cdot y^{-10}$$

$$= \frac{x^3}{y^{10}}$$

* 指數乘係加，除係減

* 指數負數，分母變分子，分子變分母

2014 PAPER 1 - SECTION A1

Q2.) Factorize $ab^2 + b^2 + a^2 - 2a - 3$

* 參考課程 2.5

$$ab^2 + b^2 + a^2 - 2a - 3$$

$$= b^2(a+1) + (a-3)(a+1)$$

$$= (a+1)(b^2 + a - 3)$$

* 抽 b^2
* 十字相乘 $(a-\alpha)(a-\beta) \rightarrow \alpha\beta = -3, \alpha + \beta = 2$

2014 PAPER 1 - SECTION A1

$Q3.)\ a.)\ Round\ up\ 123.45\ (to\ 1\ sig.fig.)$

$b.)\ Round\ off\ 123.45\ (to\ nearest\ integer)$

$c.)\ Round\ down\ 123.45\ (to\ 1\ d.p.)$

* 參考課程 1.1

$a.)\ 200$

$b.)\ 123$

$c.)\ 123.4$

* 進一至一位有效數

* 四捨五入之整數

* 捨去至小數後一個位

2014 PAPER 1 - SECTION A1

Q4.) The following shows the distribution of the numbers of children owned by family

Number of children	0	1	2	3
Number of family	7	14	15	4

Find the median, mode and the standard deviation of the distribution .

* 參考課程 4.2

$Median = 1$

$Mode = 2$

$Standard\ deviation = 0.889\ (to\ 3\ sig.fig.)$

* 中位數 = 中間的數值

* 眾數 = 頻數最多的數值

* 標準差 = (各數與平均的差)²平均值的開方

2014 PAPER 1 - SECTION A1

$Q5.)$ $Consider\ 2(3m+n)=m+7$

$Find\ the\ change\ in\ the\ value\ of\ n\ when\ m\ is\ increased\ by\ 2$

* 參考課程 2.1

$$6m+2n=m+7$$

$$\rightarrow 2n=7-5m$$

$$\rightarrow n=\frac{7-5m}{2}$$

$$\therefore The\ change\ of\ n=\frac{7-5(m+2)}{2}-\frac{7-5m}{2}$$

$$=-5$$

* 兩邊 -6m 並除 2

2014 PAPER 1 - SECTION A1

Q6.) The marked price of a good is \$255. A discount of 40 % is made . If the profit is 2 %
Find the cost of the good .

* 參考課程 1.3

Let the cost of the good be \$C

$$2\% = \frac{\boxed{255(1-40\%)} - C}{C} x100\%$$

$$\rightarrow \boxed{153} = 1.02C \rightarrow C = 150$$

$$\therefore \text{The cost of the good} = \$150$$

* ▇ 打折後新價錢 = 價錢x(1-折扣)

* 利潤百份比 = (售價-成本) / 成本 x 100%

2014 PAPER 1 - SECTION A1

Q7.) Let $f(x) = 4x^3 - 5x^2 - 18x + c$, where c is constant. When $f(x)$ is divided by $(x-2)$, the remainder is -33.

a.) Factorize $f(x)$

b.) Is all roots of $f(x) = 0$ are rational number? Explain your answer.

* 參考課程 1.1, 2.4 及 2.6

a.) $f(2) = -33 \rightarrow c = -9$, $\because f(-1) = 0$, $\therefore$ $f(x) \equiv (x+1)(ax^2 + bx + c)$

By compare cofficient, $a = 4$, $b = -9$ and $c = -9$

i.e. $f(x) \equiv (x+1)(4x^2 - 9x - 9)$

b.) $f(x) = 0 \rightarrow x + 1 = 0$ or $4x^2 - 9x - 9 = 0$

$$\rightarrow x = -1 \text{ or } x = \frac{9 \pm \sqrt{(-9)^2 - 4(4)(-9)}}{2(4)} = 3 \text{ or } -0.75$$

i.e. All roots are rational number.

* 餘數定理

* 用二次方程根式解

2014 PAPER 1 - SECTION A1

Q8.) $P = (-3,5)$ and $Q = (2,-7)$. P is rotated anticlockwise about O through 270^0 while Q is translated leftwards by 21 units. Denoted P' and Q' be the new position of P and Q.

a.) Find P' and Q' coordination

b.) Is $PQ \perp P'Q'$? Explain your answer.

* 參考課程 3.3 及 3.8

a.) $P' = (5,3),\ Q = (-19,-7)$

b.) The slope of PQ, $m = \dfrac{5-(-7)}{-3-2} = -\dfrac{12}{5}$

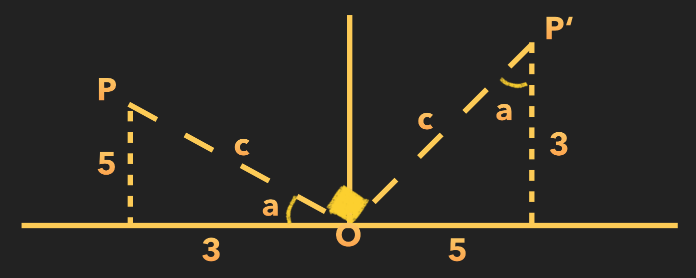
The slope of P'Q', $m' = \dfrac{3-(-7)}{5-(-19)} = \dfrac{5}{12}$

$\because \boxed{mm' = -1}$

$\therefore PQ \perp P'Q'$

* 逆時轉 270^0 = 順時轉 90^0

* 先畫圖理解 (全等三角形)

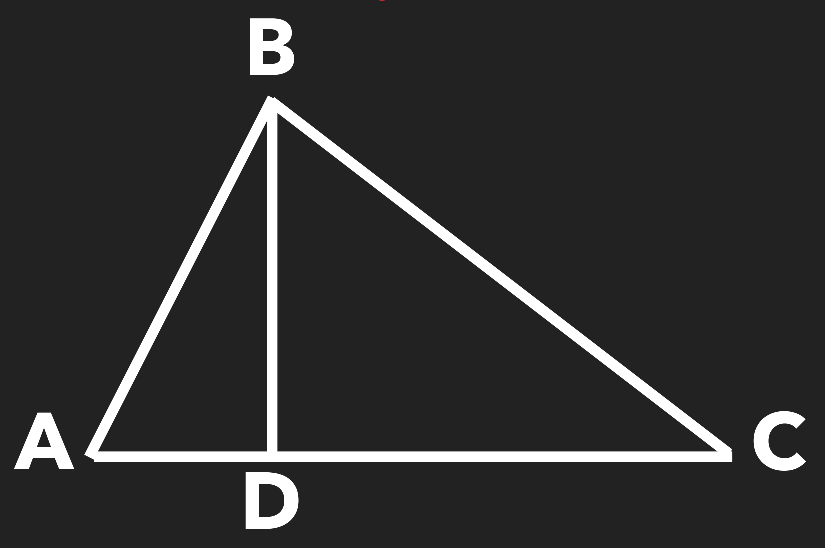


* 兩點斜率 $= \dfrac{y_2 - y_1}{x_2 - x_1}$

* 斜率相乘 = -1, 互相垂直

2014 PAPER 1 - SECTION A1

Q9.) In the following figure, $\angle BAC = \angle CBD$



a.) Prove $\Delta ABC \sim \Delta BDC$

b.) $AC = 25cm$, $BC = 20cm$ and $BD = 12cm$. Is $\angle BDC = 90^0$? Explain your answer.

* 參考課程 3.3

a.) $\angle BAC = \angle CBD$ (given)

$\angle ACB = \angle BCD$ (common)

$\angle ABC = \angle BDC$ ($\angle$s sum of Δ)

$\therefore \Delta ABC \sim \Delta BDC$ (AAA)

* 三角形內角和係180⁰

b.) $BD^2 + \boxed{DC}^2 = BD^2 + \left(\boxed{\frac{BC^2}{AC}}\right)^2$ (The property of $\sim \Delta s$, $\Delta ABC \sim \Delta BDC$)

* 相似三角形, 邊比相等

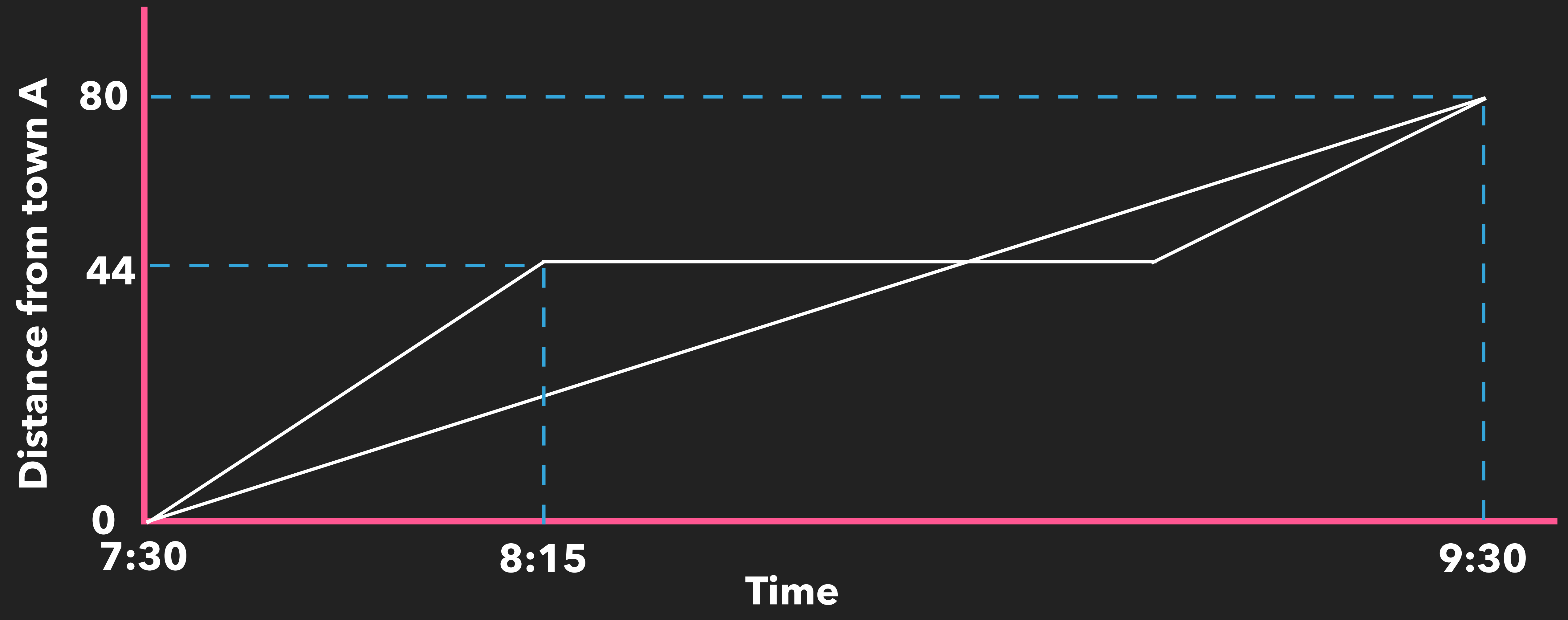
$= 12^2 + 16^2 = 400 = BC^2$

$\therefore \angle BDC = 90^0$ (converse of pyth. theorem)

* 畢氏定理逆定理

2014 PAPER 1 – SECTION A2

Q10.) The following graph show car A and car B travelling from town A and town B in between 7 : 30 and 9 : 30. Given that two towns are 80km apart .



a.) Find the distance for car A from town A at 8 : 15.

b.) When will car A and car B meet?

c.) Is the average speed of car B higher than that of car A between 8 : 15 to 9 : 30? Explain your answer .

* 參考課程 2.2

CONT'D

2014 PAPER 1 - SECTION A2

a.) $The\ distance = \frac{80}{\boxed{120}} \cdot \boxed{45} = 30\ km$

b.) $Let\ M\ min.\ be\ time\ for\ car\ A\ travel\ \boxed{44}\ km$

$$\frac{80}{120}M = 44 \rightarrow M = 66$$

$\therefore\ The\ first\ meet\ of\ two\ car\ at\ 8:36$

c.) $The\ average\ speed\ of\ car\ B\ after\ 8:15 = \frac{80-44}{\boxed{75}}$

$$= 0.48\ km/min$$

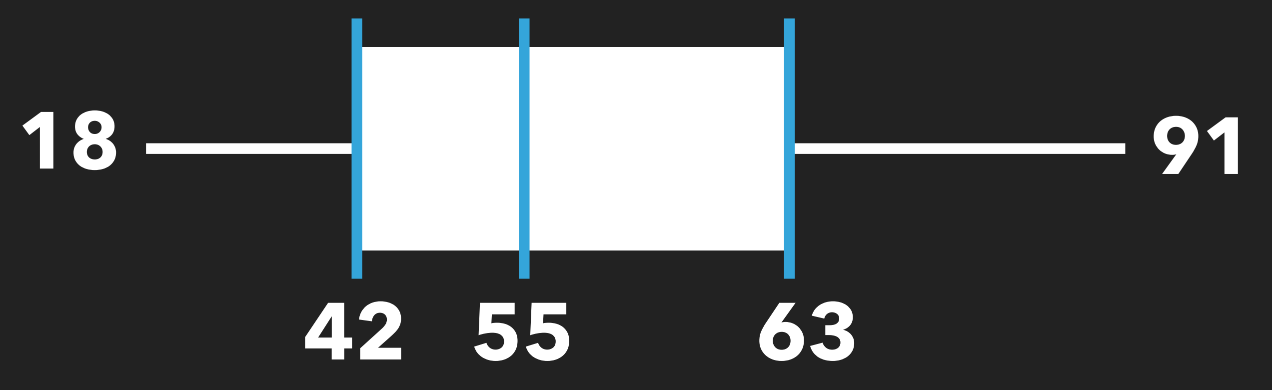
$$< \frac{80}{120}\ km/min$$

$\therefore\ The\ average\ speed\ of\ car\ B\ is\ lower\ that\ that\ of\ car\ A$
$after\ 8:15$

* 7:30-9:30 共用去 120 分鐘
* 7:30-8:15 共用去 45 分鐘
* 圖中 car A 同 car B 响 44 km 相交
* 8:15-9:30 共用去 75 分鐘

2014 PAPER 1 - SECTION A2

Q11.) The following box – and – whisker shows the distribution of the prices of 33 apples .
Given hat the mean is \$53



a.) Find the range and interquartile range .

b.) 4 apples are removed (\$32,\$34,\$58,\$59) . Find the the new mean and median .

* 參考課程 4.2

a.) $Range = \$73,\ Interquartile\ range = \$21$

b.) $The\ new\ mean = \dfrac{53 \cdot 33 - (32 + 34 + 58 + 59)}{33 - 4}$

$= \$54$

$The\ new\ median = \$55$

* Range = 最大最細值之差

* Interquatile Range = 第三及一四分位數之差

* 平均值 = 加總 / 總數量

* 中位數 = 由細到大排列好後，中間的數值

* 舊中位數(55)前後各拎走兩個值，中位數不變

2014 PAPER 1 - SECTION A2

Q12.) The circle, C passes through point A(6,11), with center G(0,3).
Given that there is moving point P, such that AP = GP. Denote L be the locus of P.
a.) Find the equation of C.
b.) Find the equation of L.
c.) Describe the geometric relation between L and AG.
d.) If L cuts C at Q and R, find the perimeter of AQGR.

* 參考課程 3.3 及 3.8

$a.)\ C : (x-0)^2 + (y-3)^2 = AG^2$

$\rightarrow x^2 + (y-3)^2 = (6-0)^2 + (11-3)^2$

$\rightarrow x^2 + (y-3)^2 = 100$

$b.)\ Let\ P = (a,b),\ AP = GP \rightarrow AP^2 = GP^2$

$\rightarrow (a-6)^2 + (b-11)^2 = a^2 + (b-3)^2$

$\rightarrow 12a + 16b - 148 = 0 \ \rightarrow 3a + 4b - 37 = 0$

$\therefore$ *The equation of L :* $3x + 4y - 37 = 0$

* 圓形公式 $(x-x_0)^2 + (y-y_0)^2 = r^2$
* AG 係半徑, 距離公式 $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

* 距離公式 $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

CONT'D

2014 PAPER 1 – SECTION A2

c.) $\because \text{The slope of } L \cdot \text{The slope of } AG = -1$

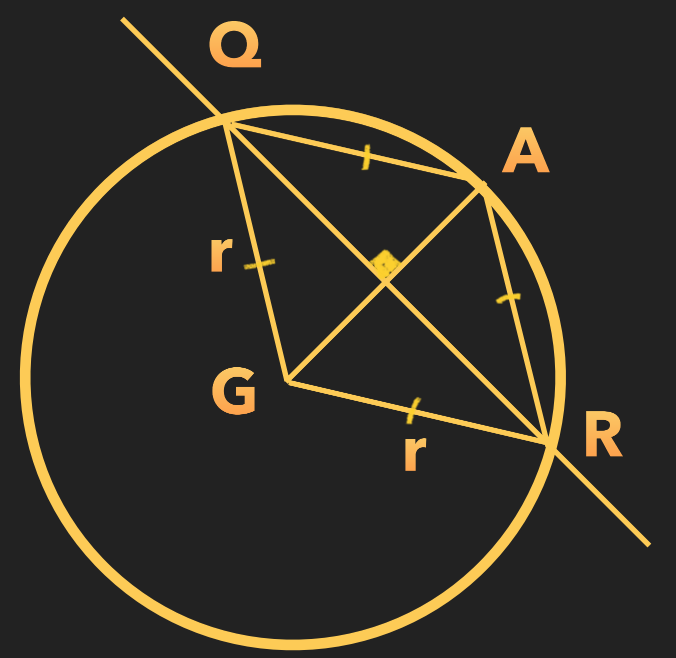
$\therefore L \perp AG$

d.) $\text{The perimeter} = 4 \cdot \text{radius of } C$

$= 40 \text{ unit}$

* 兩條線斜率相乘 = -1, 兩條線互相垂直

* AR=GR, AQ=GQ 因為 Locus



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Q13.) $f(x)$ is sum of two parts, one part varies as x^2 and other is constant.

Given that $f(2) = 59$, $f(7) = -121$

a.) Find $f(6)$

b.) Assume $A(6,a)$ and $B(-6,b)$ are points lying on $y = f(x)$.

Let $C = (c,0)$, find the area ΔABC.

* 參考課程 2.1, 2.3, 2.4, 2.5 及 2.6

a.) Let $f(x) = k_1x^2 + k_2$, where k_1, k_2 are real constant. Then,

$$\begin{cases} 59 = 4k_1 + k_2 & \text{——— (1)} \\ -121 = 49k_1 + k_2 & \text{——— (2)} \end{cases}$$

$(2) - (1)$: $45k_1 = -180 \rightarrow k_1 = -4$, $k_2 = 75$

$\therefore f(x) = -4x^2 + 75$ i.e. $f(6) = 69$

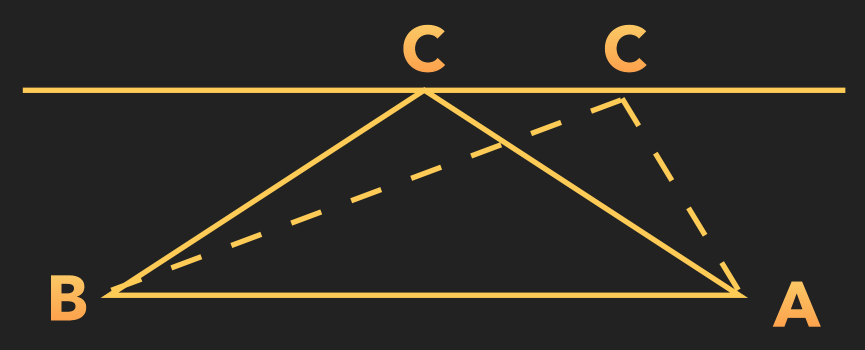
b.) $a = b = 69$

$$\therefore \text{The area of } \Delta ABC = \frac{69(6-(-6))}{2} = 414 \ unit^2$$

* 部分變量

* 消去法消去 k_2 搵 k_1 ,再代 (1) 式搵 k_2

* 三角形個高永遠係 69



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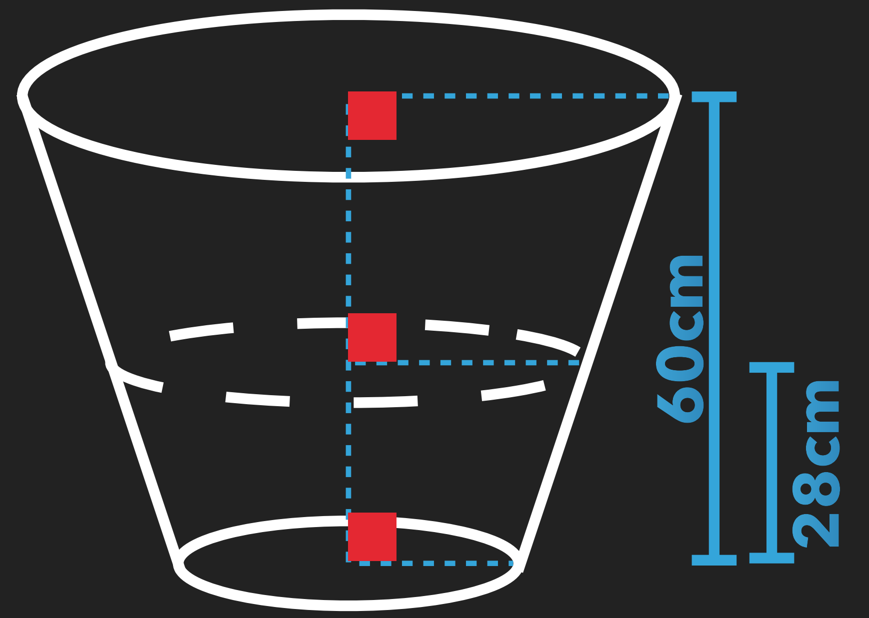
Q14.) There is vessel formed by cutting off the lower part of an inverted right circular cone with base radius $= 72cm$ and height $= 96cm$. The height of vessel $= 60cm$. Then, water is poured into vessel with the depth of water in the vessel $= 28cm$.



a.) Find the wet curved surface area in term of π.

b.) Is the volume of water in the vessel $> 0.1m^3$? Explain your answer.

* 參考課程 3.2 及 3.9

a.) Let $A_1\ m^2$ be the curved area of circular cone

$A_2\ m^2$ be the curved area of the cut $-$ off circular cone

$$A_1 = \pi(0.72)(\sqrt{(0.72)^2 + (0.96)^2}) = 0.864\pi$$

$$A_2 = (\frac{0.96 - 0.6}{0.96})^2 A_1 = 0.1215\ \pi$$

$$\therefore \text{The wet curved surface area} = (\frac{0.96 - 0.6 + 0.28}{0.96})^2 A_1 - A_2$$

* 錐體曲面面積 = π x 半徑 x 斜邊

* 相似圖形, 面積比 = (邊比)²

CONT'D

2014 PAPER 1 - SECTION A2

$$= 0.2625\pi\ m^2$$

$b.)\ Let\ V_1\ m^3\ be\ the\ vol.\ of\ circular\ cone$

$V_2\ m^3\ be\ the\ vol.\ of\ the\ cut-off\ circular\ cone$

$$V_1 = \frac{1}{3}\pi(0.72)^2(0.96) \approx 0.166\pi$$

$$V_2 = \left(\frac{0.96-0.6}{0.96}\right)^3 V_1 \approx 0.0527\pi$$

$$\therefore The\ water\ vol. = \left(\frac{0.96-0.6+0.28}{0.96}\right)^2 V_1 - V_2 \approx 0.13\ m^3$$

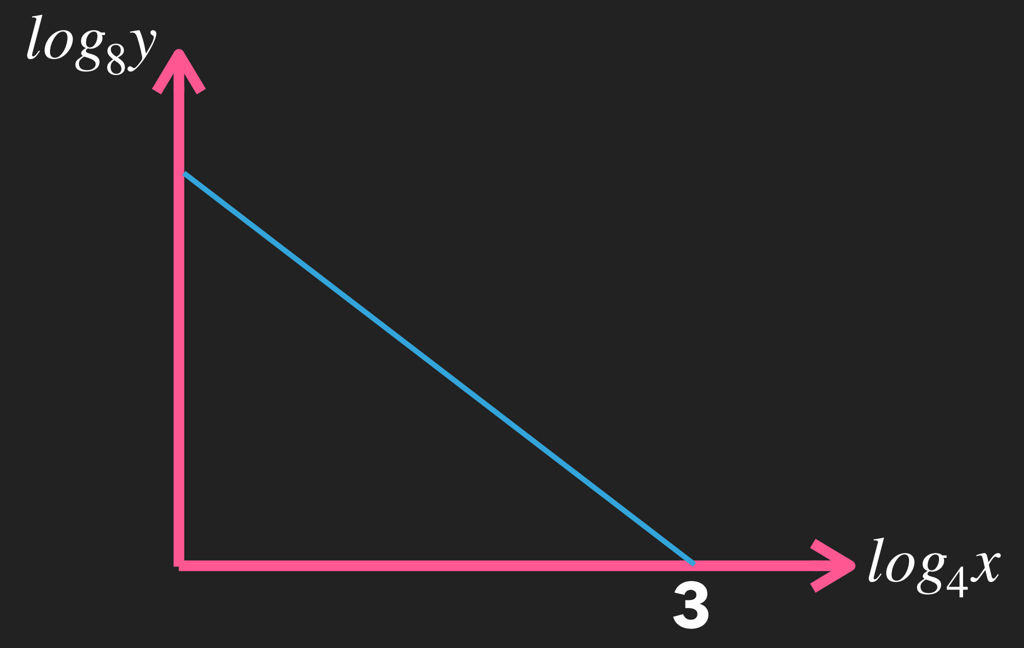
$\therefore The\ volume\ of\ water\ in\ the\ vessel > 0.1m^3.$

* 錐體體積 = 1/3 x 半徑² x π x 高
* 相似圖形, 體積比 = (邊比)³

2014 PAPER 1 - SECTION B

Q15.) The following figure shows the relationship between $\log_4 x$ and $\log_8 y$. Given that the slope of the straight line $= -\frac{1}{3}$

Express y in term of Ax^b where A and b are constant.



* 參考課程 2.2, 及 2.7

By slope formula :

$$\frac{\log_8 y - 0}{\log_4 x - 3} = -\frac{1}{3} \rightarrow 3\log_8 y = 3 - \log_4 x$$

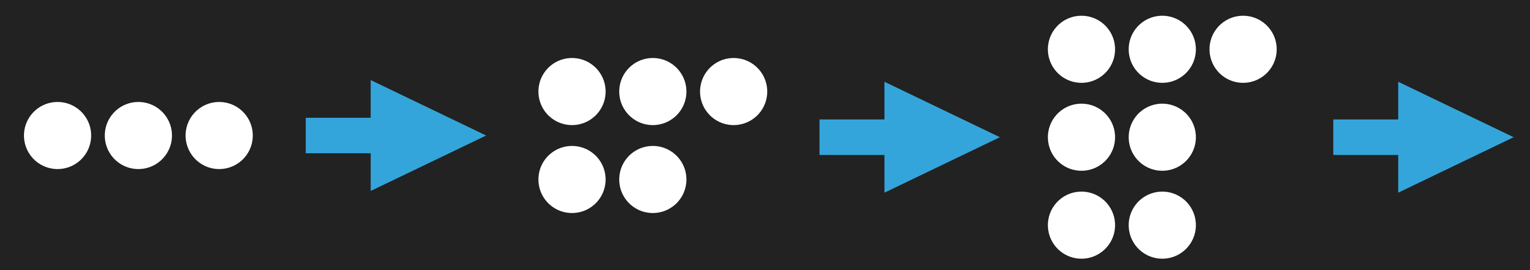
$$\rightarrow 3\frac{\log_2 y}{\log_2 8} = 3 - \frac{\log_2 x}{\log_2 4} \rightarrow \log_2 y = 3 - \frac{1}{2}\log_2 x$$

$$\rightarrow \log_2 y = \log_2 8 - \log_2\sqrt{x} = \log_2(\frac{8}{\sqrt{x}}) \rightarrow y = 8x^{-\frac{1}{2}}$$

* 斜率 = $\frac{y_2 - y_1}{x_2 - x_1}$
* log 轉 base 公式, $\log_a b = \frac{\log_c b}{\log_c a}$
* $n\log A = \log A^n$
* $\log A - \log B = \log\frac{A}{B}$

2014 PAPER 1 – SECTION B

Q16.) Consider the pattern below :



$\ldots\ldots$ *add 2 dots to the* $n-th$ *pattern*

Find the least value of m *such that the total number of dot* > 6888

* 參考課程 2.2, 2.3 及 2.6

Let $T(n)$ *be the number of dots at* $n-th$ *pattern*

$$T(n) = T(n-1) + 2 = T(n-2) + 2(2) = T(n-3) + 2(3)$$

$$= \ldots = T(1) + 2(n-1) = 3 + 2(n-1) = 2n + 1$$

Consider, $T(1) + T(2) + \ldots + T(m) > 6888$ *and* $m > 1$

$$\rightarrow \frac{m}{2}(T(1) + T(m)) > 6888 \text{ and } m > 1$$

$$\rightarrow m(m+2) > 6888 \text{ and } m > 1$$

$$\rightarrow m^2 + m - 6888 > 0 \text{ and } m > 1$$

$$\rightarrow m < -84 \text{ or } m > 82 \text{ and } m > 1$$

$\therefore$ *The least value of* $m = 83$

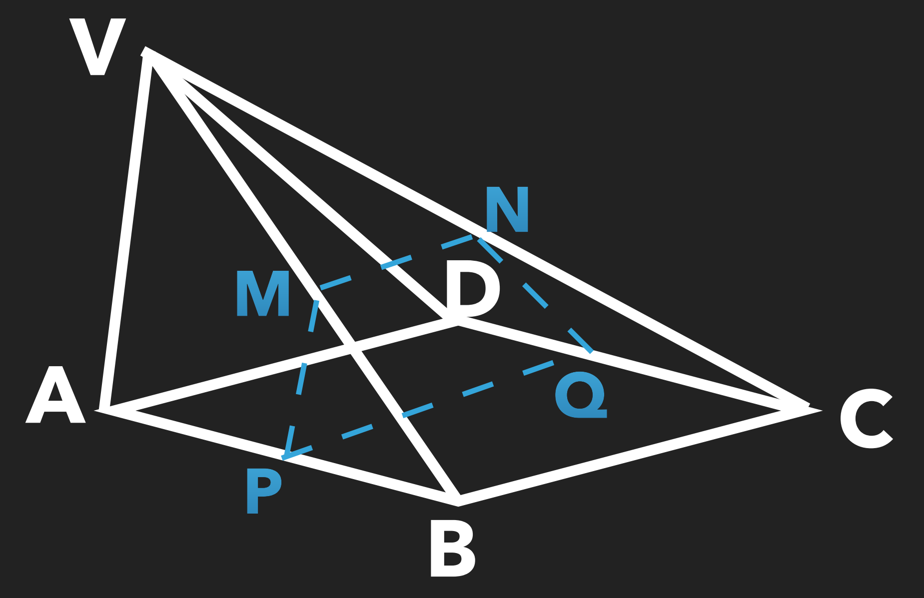
* 等差數列之和=(首項+尾項)x項數/2

* 先用二次方程根公式搵根, α, β

因為大過0, 所以答案係 x<最細根 or x>最大根

2014 PAPER 1 - SECTION B

Q17.) The following shows a solid pyramid VABCD a rectangle base. $AB = 18cm$, $BC = 10cm$, $VB = VC = 30cm$ and $\angle VAB = \angle VDC = 110^0$. P, Q, M and N are the mid-pt. of AB, CD, VB and VC.



a.) Find $\angle VBA$

b.) Is the area $PQMN < 70cm^2$? Explain your answer.

* 參考課程 3.3 及 3.10

$$a.)\ \sin\angle VAB = \frac{30}{18}\sin 110^0 \rightarrow \angle VAB = 34.32008^0$$

$$\therefore \angle VBA = 180^0 - 110^0 - \angle VAB \ (\angle sum\ of\ \Delta)$$

$$= 35.7^0\ (to\ 3\ sig.fig.)$$

$$b.)\ MB = \frac{30}{2} = 15,\ PB = \frac{18}{2} = 9$$

$$Also,\ \Delta VBC \sim \Delta VMN\ (AAA)$$

* sine law 使用

* 三角形內角和 = 180^0

* MN//BC 所以三隻角一樣

CONT'D

2014 PAPER 1 - SECTION B

$$\therefore MN = \frac{BC}{2} = \frac{10}{2} = 5$$

In ΔPMB, *by cosine law*

$$PM^2 = MB^2 + PB^2 - 2(MB)(PB)\cos\angle VBA$$
$$\approx 86.681$$

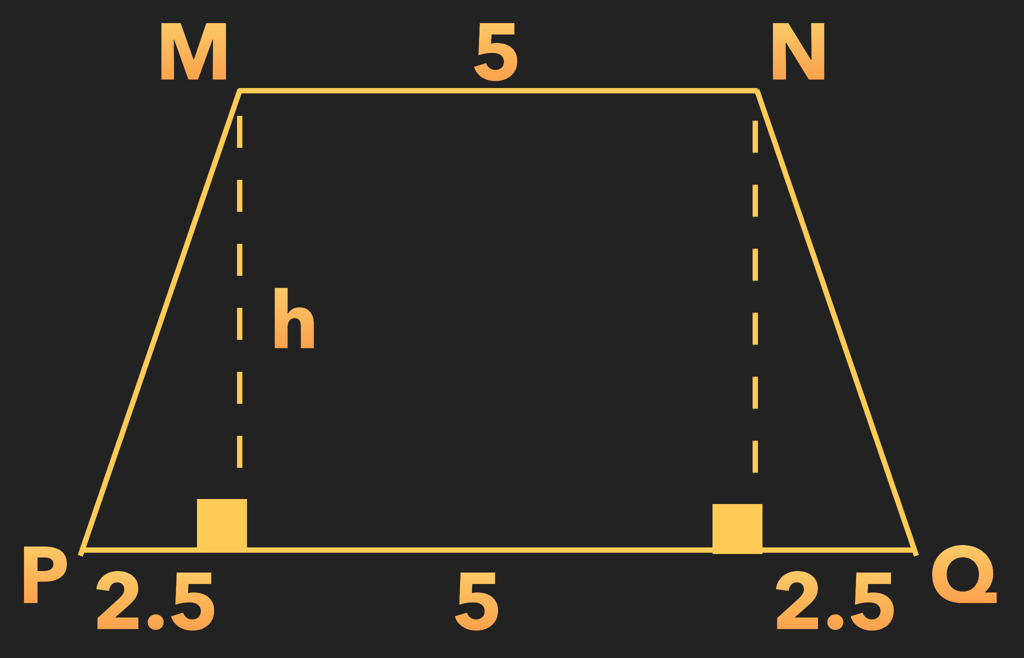
In MPNQ,

$$\text{the height of } MPNQ,\ h = \sqrt{PM^2 - (\frac{10-5}{2})^2}$$
$$\approx 8.9684$$

$$\text{The area of } MPNQ \approx \frac{1}{2}(MN + PQ)h = 67.3\ cm^2$$

$$\therefore \text{The area of } PQMN < 70cm^2$$

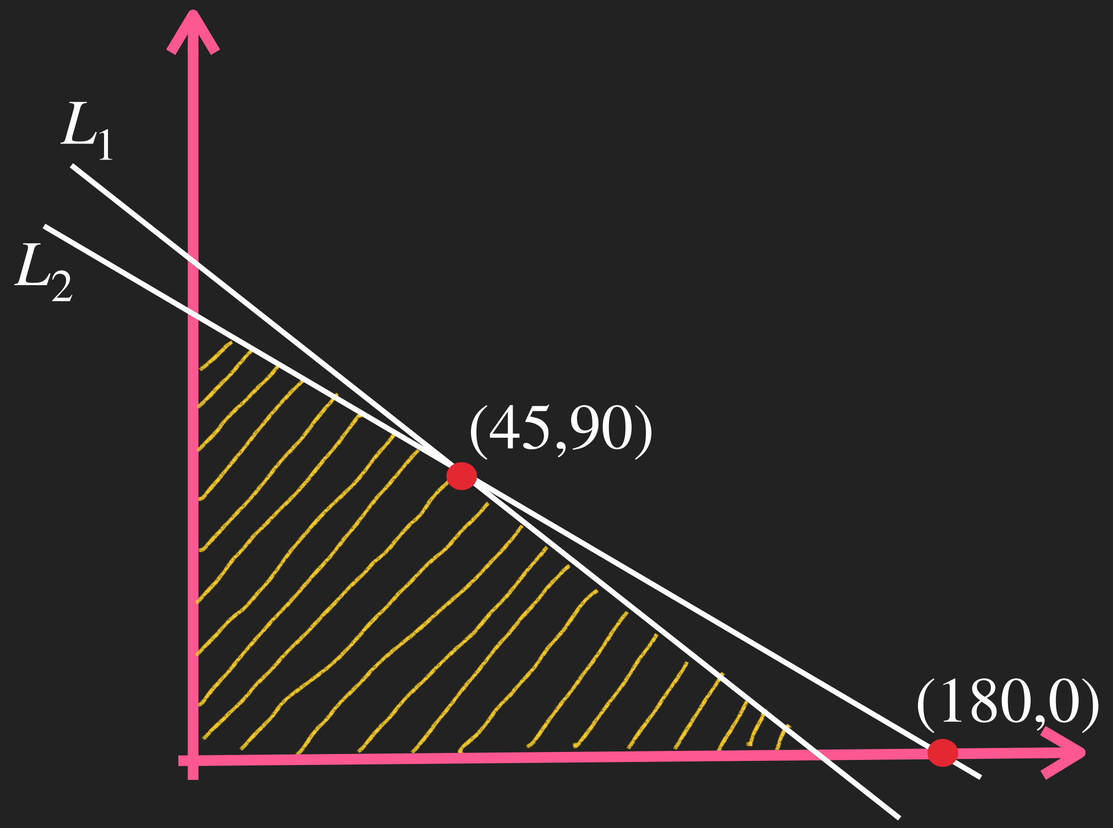
* 相似三角形三邊比例相等



* 畢氏定理

2014 PAPER 1 – SECTION B

Q18.) The following shaded area in the diagram shows the systems of inequalities.



$L_1 : 6x + 7y = 900$

$L_2 : y = mx + c$, *where* m, c *are constant*

a.) Find the systems of inequalities.

b.) There are two factory, A and B. Each factory A requires 6 man – hours for cutting and 2 man – hours for packing, while factory B requires 7 man – hours for cutting and 3 man – hours for packing. A business man owns 900 man – hours for cutting and 360 man – hours for packing. The business man can earned \$440 from A while can earned \$665 from B. Suppose the business man has serveral number of A and B. Can he earn exceed \$80,000? Please explain your answer.

* 參考課程 2.2, 及 2.3

CONT'D

2014 PAPER 1 - SECTION B

$$a.)\ m = \frac{90-0}{45-180} = -\frac{2}{3}$$

$Put\ (180{,}0)\ into\ L_2\ to\ find\ c,\ \rightarrow c = 120$

$\therefore L_2:\ 2x+3y=360,\ Hence,\ the\ systems\ of\ inequality =$

$$\begin{cases} x \geq 0,\ y \geq 0 \\ 6x+7y \leq 900 \\ 2x+3y \leq 360 \end{cases}$$

$b.)\ Let\ x\ be\ the\ number\ of\ factory\ A$

$y\ be\ the\ number\ of\ factory\ B$

$P\ be\ the\ total\ profit$

$$Then,\ \begin{cases} x \geq 0,\ y \geq 0 \\ 6x+7y \leq 900 \\ 2x+3y \leq 360 \end{cases}$$

* 斜率 = $\dfrac{y_2 - y_1}{x_2 - x_1}$

CONT'D

2014 PAPER 1 - SECTION B

$$P = 440x + 665y, \textit{ from the graph}$$

$$At\ (0,0):\ \ P = 0 < 80{,}000$$

$$At\ (0,120):\ \ P = 79{,}800 < 80{,}000$$

$$At\ (150,0):\ \ P = 66{,}000 < 80{,}000$$

$$At\ (45,90):\ \ P = 79{,}650 < 80{,}000$$

$$\therefore \textit{ He cannot earn more than } \$80{,}000$$

* 極端數值發生在所有線相交點

2013 PAPER 1 – SECTION B

Q19.) *Mary and Peter play a game consisting of 2 rounds. In 1^{st} round, they take turns to throw a fair die. The player who first get number '6' wins the 1^{st} round. They play the game until one of them win. Mary throws the die first.*

a.) Find the probabilty that Mary wins the 1^{st} round.

b.) In 2^{nd} round, balls drop into the following device that contains 8 tubes.

Winner of 1^{st} round can choose 2 balls or 3 balls to play the game

Option 1 – 2 balls are dropped one by one. Player get 10 points if 2 balls in 1 tube. Player get 5 points if 2 balls fall into adjacent tubes. Otherwisw, no points.

Option 2 – 3 balls are dropped one by one. Player get 50 points if 3 balls in 1 tube. Player get 10 points if 3 balls fall into adjacent tubes. Player gets 5 Otherwisw, no points.

i.) Find the expected points if player plays Option 1.

ii.) Find the expected points if player plays Option 2.

iii.) Find the probability Mary gets no points if Mary choose Options 1 in 2^{nd} round

* 參考課程 2.7, 4.3 及 4.4

CONT'D

2014 PAPER 1 - SECTION B

$a.)\ Let\ P(M)\ be\ the\ probabilty\ of\ Mary\ gets\ '6'$

$P(P)\ be\ the\ probabilty\ of\ Peter\ gets\ '6'$

$P(M')\ be\ the\ probabilty\ of\ Mary\ gets\ not\ '6'$

$P(P')\ be\ the\ probabilty\ of\ Peter\ gets\ not\ '6'$

$The\ probability\ Mary\ win\ 1^{st}\ round,\ P(M\ win)$

$$= P(M) + P(M')P(P')P(M) + [P(M')P(P')]^2P(M) + \dots$$

$$= \frac{1}{6}[1 + (\frac{5}{6})^2 + (\frac{5}{6})^4 + \dots] = \frac{1}{6}[\frac{1}{1-\frac{25}{36}}] = \frac{6}{11}$$

$bi.)\ Let\ P(2\ in\ 1)\ be\ the\ probability\ 2\ balls\ fall\ into\ same\ tube$

$P(2\ in\ 2)\ be\ the\ probability\ 2\ balls\ fall\ into\ adjacent\ tube$

$The\ expected\ points = 10P(2\ in\ 1) + 5P(2\ in\ 2)$

$$= 10(\frac{1}{8}) + 5[(\frac{2}{8})(\frac{1}{8}) + (\frac{6}{8})(\frac{2}{8})] = 2.34\ (to\ 3\ sig\ fig)$$

* 每一次擲骰子都是獨立事件
所以 P(A and B)=P(A)P(B)
每一個可能性都沒有交集
所以 P(A or B)=P(A)+P(B)

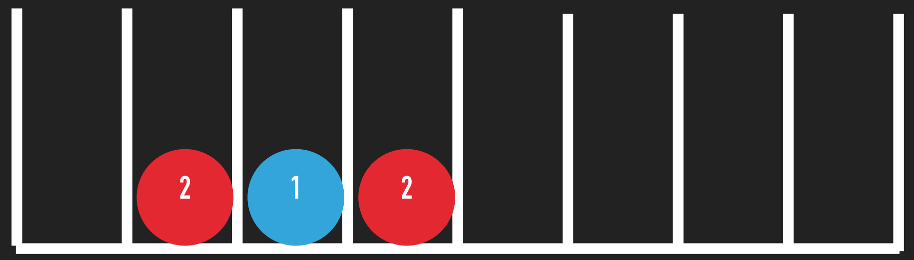
* 等比數列之和 (無限)

* 期望值 = 所有(所得 x 機率)

* 8 個可能性 (兩波一筒) / 8^2 可能

* 如果第1個波在兩側 (共 2 個位)

* 如果第1個波在中間 (共 6 個位)



CONT'D

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bii.) Let $P(3\ in\ 1)$ be the probability 3 balls fall into same tube
$P(3\ in\ 3)$ be the probability 3 balls fall into adjacent tube
$P(3\ in\ 2)$ be the probability 2 balls fall into adjacent tube

$$\text{The expected points} = 50P(3\ in\ 1) + 10P(3\ in\ 3) + 5P(3\ in\ 2)$$

$$= 50\left(\frac{1}{64}\right) + 10\left(\frac{6P_3^3}{8^3}\right) + 5\left(\frac{7P_2^3}{8^3}\right) = 1.89\ (to\ 3\ sig\ fig)$$

biii.) $P(\text{Mary get no points})$

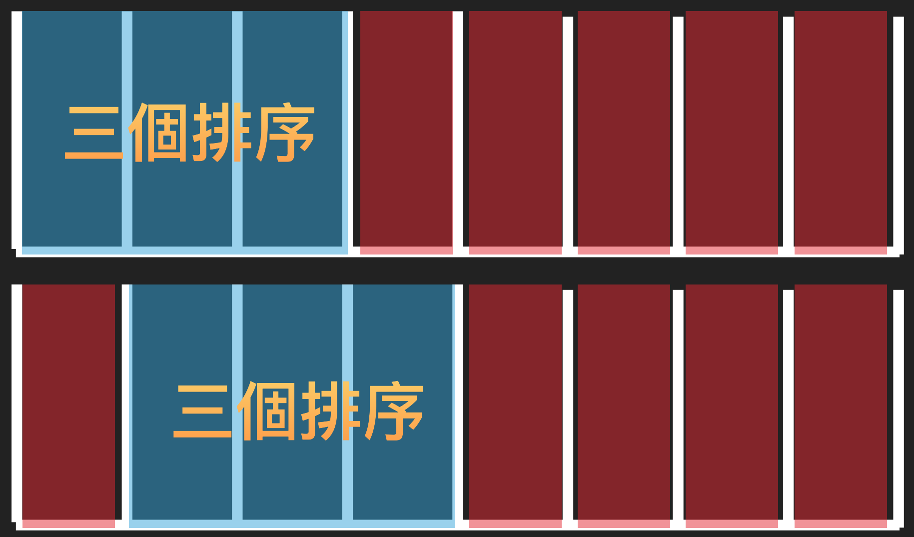
$$= (1 - P(M\ win) + P(M\ win)[1 - P(2\ in\ 1) - P(2\ in\ 2)]$$

$$= (1 - \frac{6}{11}) + \frac{6}{11}[1 - \frac{1}{8} - \frac{7}{32}] = 0.8125$$

* 期望值 = 所有(所得 x 機率)

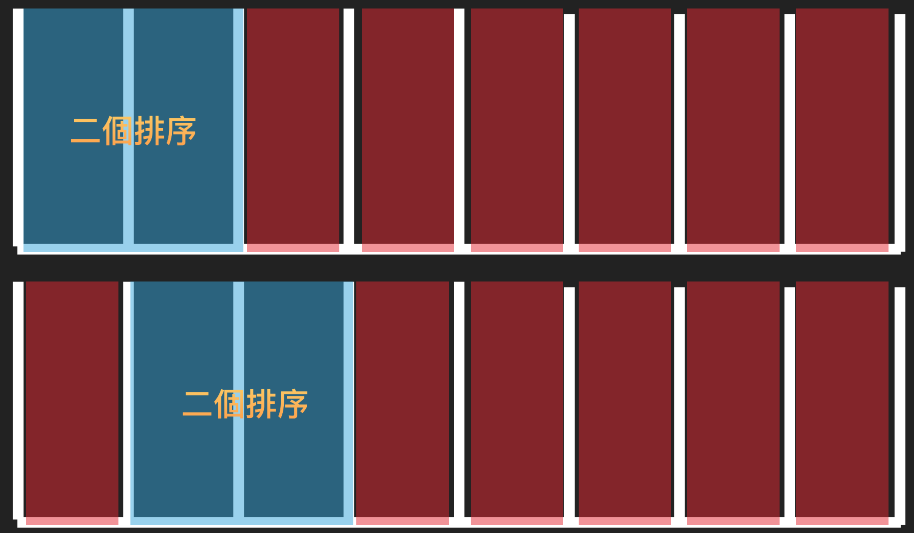
* 8 個可能性 (三波一筒) / 8^3 可能

* 假設 3 個波在一起為一個空間



.... 共 6 種可能

* 假設 2 個波在一起為一個空間



.... 共 7 種可能